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EFFICIENCY AND EQUITY: IS THERE A TRADE-OFF?

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ABSTRACT

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Efficient measures are often not implemented because of their potentially damaging effects on distribution, yet these distributional effects are scarcely studied in economics because of the idea that they are case specific. In this paper we show that when we can separate the effect on efficiency from the effect on distribution, that is when Gorman aggregation applies, the well-known result that aggregate effects can be computed independently of the distribution can be accompanied by a similar result on distribution; the effect on distribution is also distribution independent and can be computed using only aggregate effects. In a world with no lump-sum transfers and agent heterogeneity, we thus have an expeditious rule for defining a class of measures, which for a significative group of agent characteristics, are social welfare improving without requiring the knowledge of the characteristic distribution or the specific form of the social welfare function.

JEL Classification: D31, D61, D63
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NON-TECHNICAL SUMMARY

Welfare economics has for a long period used the notion of efficiency as its main indicator. This avoids not only individual comparisons, but also the construction of a social welfare function which could represent the preferences of a community. The classification of a certain policy measure as efficient is thus a judgement-free concept. This notion implies that given the resources and the technology every household could be better off after the policy measure. But to be sure that each household is really better off the government must be able to make transfers between agents that do not make those agents change their decisions (lump-sum transfers). Yet most of the literature on optimal government decisions has been based on the assumption that governments do not have this type of instrument available (a differentiated or even a common lump-sum tax). This implies that the potential positive individual welfare effect can not be realized. Given this fact, efficient measures are often not implemented because of their potentially damaging effects on distribution. On the other side, the distributional effects are scarcely studied in economics because of the idea that they are case specific.

In this paper we show that when we can obtain the effect on efficiency independently from the consideration of agent heterogeneity, we can get the result that the effect on utility distribution is also independent of the distribution of characteristics between agents, and can be computed using only aggregate effects. To know whether a policy is efficient or not, independently of agent heterogeneity, means that we can compute the change in the equilibrium of the economy using only the situation of the average (representative) agent of the economy. If we have a different individual configuration, but the same average, the change in the equilibrium is not affected. In this case the classification of the policy as efficient implies that this representative agent increases utility. As we are interested in knowing what happens to every agent in the economy we must study the effect of the policy change on the utility distribution of the whole economy. The importance of our result is that we can obtain the effect on the utility distribution without calculating the effect on each individual's utility. We show that we just need to know the change in the equilibrium prices of the economy to find the effect on the utility distribution. This result means that in a world without lump-sum transfers and agent heterogeneity, we have at our disposal an expeditious rule to separate the class of measures which are social welfare improving, without requiring the knowledge of the characteristic distribution or the specific form of the social welfare function, from the class of measures that, being efficient, increase the welfare of some agents and reduce the welfare of others. In this case the defence of the measure must involve the calculation of the exact impact on each agent's utility, and must use a specific social welfare function that allows the comparison of the gains and losses of individual utilities. In the first class there exists no trade-off between efficiency and equity, but this trade-off is present in the second.

We show that these results apply when agents in one economy can be differentiated by different non-human wealth and/or by different labour efficiency levels. We also show that the type of policies for which our result is relevant is a significant group of those studied. Examples are the abolition of capital income taxation, the reduction of inflation and international capital liberalization.

The distribution of utilities that we refer to in this paper is the normative concept of equity or of inequality. We explain in the paper why such a concept is relevant in dynamic general equilibrium models as opposed to the notion of income or wealth distribution, which should be regarded as positive or descriptive instruments. Hence, we should not say that a country improved its income distribution since the change in distribution can be accompanied by an identical, better or worse utility distribution. As the evolution of individual income and wealth is endogenous to each agent, it can not characterize their well-being. This is dependent on utilities over time and can only be changed when some exogeneous variable for the agent is altered.

I- INTRODUCTION

The classification of a policy measure as efficient, in the sense of originating a potential Pareto movement, has dominated the so called normative economics. The success of this concept is that it freed the economist from interpersonal utility comparisons, necessary to make welfare judgments. Then the concept of efficiency appears as a weaker concept of welfare economics but at the same time as a judgment free concept. Recently we find some discomfort with that measure. The validity of the concept is highly dependent on the existence of differentiated lumpsum transfers. Just in this case to implement an efficient policy measure is equivalent to a Pareto movement in the economy. At the same time the main body of studies on efficiency preclude by assumption the existence of lumpsum taxation in the characterization of the optimal outcomes. This asymmetry of positions led to the fact that the defense and implementation of such measures has always been accepted with some reservations. These reservations being increased in this decade with the renewed interest in the study of models with heterogeneous agents. Once we recognize the non existence of lumpsum transfers we can say that the notion of efficiency lost interest since the proper redistribution policy after the efficient policy could entail big losses that could neutralize the desirability of the measure. one reason This is the so called trade-off between efficiency and distribution.

This paper proposes a definition and a very simple way of identification of a class of policy measures in which such problems can be avoided. We use quite standard conditions, namely that the classification of the policy as efficient do not goes through the determination of the effect on utility of each heterogeneous agent, that is that we have conditions that guaranty Gorman aggregation. We prove that we can order the utility distributions before and after the policy as Lorenz dominated what implies that we can classify univocally the efficient moves in terms of the effects on utility distribution. But more relevant is the fact that we can identify the class of heterogeneous characteristics where the classification of the policy in terms of the effect on distribution is not dependent on those heterogeneity: it depends just on the aggregate effects of the policy. At the same time that aggregation implies that efficiency characterization is independent of the distribution of characteristics, for a certain class of heterogeneity the characterization in terms of the effect on the distribution of utilities is also distribution independent. It is relevant that the identified class covers the more important heterogeneity characterization used in general equilibrium aggregate models, namely the heterogeneity over an aggregate of private wealth and the heterogeneity over labor efficiency. The importance of the characterization in terms of utility distribution is that we can recognize, using just aggregate variables, the efficient policies which are not only Pareto potential but effective Pareto improving movements. That is we can reduce in a very significant way the class of policies for which the classification as efficient is a weak argument.

We begin by considering that heterogeneity among households is given just by a different stock of initial wealth. We will extend this type of heterogeneity but we will work in a deterministic context, so that ex ante and ex post differences are identical, and we are not interested in explaining the existing heterogeneity but just to understand its effects. In section II we describe a particular set up and we define the conditions under which an endogenous labor, infinite horizon problem where heterogeneity is due to different initial wealth stocks, is aggregable. Section III studies the effects on the utilities distribution of a policy change, in the set up described in II. In section IV we generalize the results and identify the necessary conditions to guaranty that in a multivariable heterogeneity set up the result is maintained. The classification of a policy as efficient and the class of changes of environment that can be classified in terms of welfare without recourse to the initial distribution and the functional form of the social welfare function is stated in section V. In section VI we exemplify with well known policy changes which are potential Pareto improving and section VII discusses the relation between our results and the generally utilized positive measures of wealth and income distribution. Section VIII provides the conclusions.

II- AGGREGATION

The possibility to evaluate the efficiency property of a policy without having to know, in a first stage, the individual effective effects of the policy, plus the possibility to do such classification with no recourse to the characteristics distribution of the economy lead us to choose the set up of this paper as one where we have aggregation in the sense of Gorman (1953).

This choice is also derived from the way the literature on efficiency was developed. This literature usually classifies the measure as efficient or not, that is a qualitative analysis. Recently there were some attempts to measure efficiency benefits or alternatively distortion costs. But even the qualitative studies were developed in representative agent models. Very few studies were done in general equilibrium macro models that imposes heterogeneity and were not aggregable, e.g. Garcia- Milá, Marcet and Ventura (1995)). We think that the reason for that choice in the development of the efficiency literature is related to the computable difficulties of solving non aggregable models and the related calibration procedure, but essentially to the fact that this literature wants to have general rules that can be checked afterwards for a particular situation, e.g. Ramsey rules. This can only be done in models with the properties of aggregation.

In this section we consider that heterogeneity among households originates exclusively from different levels of non-human wealth, i.e physical capital and private and government bonds.

As stressed and demonstrated by Gorman (1953) the necessary and sufficient condition for aggregation is that Engel curves are parallel straight lines for different individuals, for the same prices. So one important restriction is that prices are identical for every agent. Engle curves are defined by Gorman not in the usual way, that is goods demands as functions of income, but as goods demand as function of an utility index. He shows that the two concepts are equivalent.

In the dynamic infinite horizon problems that we are going to discuss the exogenous counterpart of income is the stock of non human wealth. So aggregation implies in this set up that the good demands and labor supplies are linear on wealth for each agent and with a common slope.

Pollack (1971) shows which functional forms of additive utility functions have the property of linear Engel curves. As we are going to allow for time separability we use an extension of his results for the introduction of leisure, when consumption and leisure are not necessarily additive. Given the functional forms used in the literature with a representative agent, and leisure as a good, we choose the general form of the utility function for agent i[1]:

$$U_i = \sum_{t=0}^{\infty} \beta^t (u(C_{it}, N_{it}))^{1-\sigma} / (1-\sigma) \qquad \sigma>0 \qquad 0<\beta<1 \tag{1}$$

where C_i and N_i represent respectively the choices of consumption and work hours of agent i.

The restrictions on u(C,N) to guaranty aggregation are that either u is homothetic or u is additive and linear in at least one of the arguments.

Each agent maximizes that function subject to

$$\sum_{t=0}^{\infty} d_t^1 \; C_{it} = \sum_{t=0}^{\infty} d_t^2 \; W_t \; N_{it} + A_{i0} \tag{2}$$

where d_t^1, d_t^2, W_t and A_{i0} represent, respectively, the net price of consumption, the discount factor, the net wage at period t and the initial level of non-human wealth of agent i (stock of physical capital and private and government bonds). We have markets for consumption goods, labor and the three assets, physical capital, private and government bonds, and the markets are competitive. Then prices are exogenous to each agent.

Then we have

$$C_{it} = \alpha_t + \chi_t \; A_{it} \qquad t\geq 0 \tag{3}$$

$$N_{it} = \varepsilon_t + \phi_t \; A_{it}$$

where α, χ, ε and ϕ are functions of the whole sequence of d^1, d^2 and W , from t to infinite.

[1]The results are robust to a quadratic or exponential function of u().

There are two main classes of momentary utility functions that are interesting most commonly. The first class is the isoelastic utility function (CRRA) and perhaps the most utilized in studies with a representative agent:

$$u(C_{1t},N_{1t}) = C_{1t}(1-N_{1t})^{\theta} \qquad \theta>0 \qquad (4)$$

For this class it is straightforward to see that

$$\varepsilon_t = 1-[\theta(1-\tau_{ct})/W_t]\ \alpha_t \qquad \text{and } \phi_t = -\theta(1-\tau_{ct})/W_t\ \chi_t. \qquad (5)$$

This implies, as noticed by Garcia- Milá, Marcet and Ventura (1995) for $\sigma=0$, that agents that have higher consumption have also higher leisure,that is work less. This is counter factual because from panel data studies we observe that richer agents consume more but work more or less the same that poor agents. Therefore we are also interested in another class of momentary utility functions that is able to reproduce more accurately cross section data. That is the one proposed by Greenwood, Hercowitz and Huffman (1988), (GHH preferences):

$$u(C_{1t},N_{1t}) = C_{1t}- \mu\ N_{1t}^{\nu} \qquad \mu>0\ ,\ \nu>1 \qquad (6)$$

and where

$$\varepsilon_t = (W_t/(1-\tau_{ct})\mu\nu)^{\frac{1}{\nu-1}} \qquad \text{and } \phi_t = 0. \qquad (7)$$

This reflects the well known property of this class of functions that there are no wealth effect on labor and which results in this problem in the fact that rich and poor will work exactly the same, in each period.
Therefore the representative agent decisions can be described by

$$C_{rt} = \alpha_t + \chi_t\ A_{rt} \qquad t\geq 0 \qquad (8)$$
$$N_{rt} = \varepsilon_t + \phi_t\ A_{rt}$$

where A_{rt} is the average non human wealth of the economy.

To characterize the equilibrium we need some more information on the set up: Technology is described by a production function $Y(K_t, N_t)$, where K represents the average stock of physical capital. This function satisfy the usual properties to guaranty the existence of a maximum for firms. Additionally I will suppose that this function has constant returns to scale. Capital accumulation is done without adjustment costs and subject to a constant proportional depreciation rate, δ. Government has per capita expenditures, G_t and taxes consumption expenditures at a rate τ_{ct}, labor income at the rate τ_{Nt} and capital income net of depreciation at the rate τ_{Kt}. Public debt is represented by B_t^g.

Then aggregate equilibrium is given by (8) and by[2,3]

$$Y_t = F(K_t, N_{rt}) = K_{t+1}-(1-\delta)K_t + C_{rt}+G_t \tag{9}$$

$$A_{rt} = K_t + B_t^g$$

$$d_t^1 = \frac{(1+\tau_{ct})}{\prod_{s=0}^{t}(1+R_s)}, \quad d_t^2 = \frac{1}{\prod_{s=0}^{t}(1+R_s)}, \quad R_t=(1-\tau_{Kt})(F_{1t}-\delta) \quad \text{and} \quad W_t=(1-\tau_{Nt})F_{2t}$$

So technology, preferences and government policy determines the equilibrium, without information on the distribution of A_{io}. Changes of policy will change this equilibrium.

Equilibrium price sequence, d_t^1, d_t^2 and W_t, which result from the solution of the aggregate equilibrium just defined, plus (3) allow for the calculation of realizations for each agent.

[2]I define partial derivative of F() in relation to the ith argument as $F_i()$.

[3]In same examples we can modify this equilibrium condition to accommodate foreign trade.

III- EFFECTS ON THE DISTRIBUTION OF UTILITIES

In this section we show that the effect on the distribution of utilities can be determined just by looking at the effects of the policy change on the equilibrium price sequence. So not only the aggregate effects of the move are independent of the distribution but the proper effect on the utility distribution is independent of the distribution.

Households are indexed by i, where i=1,.. N. They are ordered according to increasing non human wealth.
We define utility distribution as the relative position of these agents in terms of utility. We begin by showing how can we can get a utility distribution given the distribution of initial non human wealth. Secondly we will see how this utility distribution changes by a policy change.

Definition: Let us define as individual transformed wealth

$$A_i^T = \sum_{t=0}^{\infty} d_t^2 \, F(W_t) + A_{i0}. \qquad (10)$$

For CRRA preferences we have $F(W_t)=W_t$ and for GHH preferences

$$F(W_t) = \frac{(W_t/(1-\tau_{ct}))^{\nu/(\nu-1)}}{(\mu\nu)^{1/\nu-1}} \left[1 - \frac{\mu}{(\mu\nu)^{\nu}}\right].$$

Notice that transformed wealth is equivalent to the present valued expenditures on the goods on which utility is homothetic. By definition CRRA is homothetic in C and (1-N) and for GHH preferences the optimum of each agent can be solved into two stages: at the first one we compute the level of hours of work of each agent that for each period it is a function on the net wage of that period and in the second stage we substitute this optimum values at the momentary utility functions and we can compute the optimal sequence of consumption; at this stage the problem is equivalent to an isoelastic in consumption utility problem with a subsistence level each period.

So utility is homogeneous in $C_{it} - \mu \, ((W_t(1-\tau_{ct}))/\mu\nu)^{\frac{\nu}{\nu-1}}$.

LEMMA 1: The Lorenz curve of the distribution of the individual utilities coincide with the Lorenz curve of the distribution of the transformed wealth to the degree of homogeneity of the lifetime utility. That is given the sequence of prices $p_t=p(d_t^1,d_t^2,w_t)$, the logarithm of the ratio U_i/U_j is proportional to the logarithm of the ratio of the transformed wealth of agent i and j.

Proof:

We will begin by prooving that, for a given sequence of prices $p_t=p(d_t^1,d_t^2,w_t)$, the logarithm of the ratio U_i/U_j is proportional to the logarithm of the ratio of the transformed wealth of agent i and j.

As we transform the problem in a homothetic utility problem and it has linear decision functions we have that:

$$\gamma(p)X_{i0} = A_i^T \tag{11}$$

where X is the vector of goods on which utility is homothetic.

This implies that

$$X_{i0} = \lambda_{ij} X_{j0} \tag{12}$$

where $\lambda_{ij} = \frac{A_i^T}{A_j^T}$.

As every agent faces the same sequence of prices we have that

$$X_{it} = \lambda_{ij} X_{jt} \quad t \geq 0 \tag{13}$$

Substituting into the utility function we obtain that

$$U_i/U_j = \lambda_{ij}^{(1-\sigma)m} = \left[\frac{A_j^T}{A_i^T}\right]^{(1-\sigma)m} \tag{14},$$

where m is the degree of homogeneity of u(X).

Then it is straightforward to see that

$\sum_{i=1}^{k} U_i / \sum_{i=1}^{N} U_i = \sum_{i=1}^{k} (A_i^T)^{(1-\sigma)m} / \sum_{i=1}^{N} (A_i^T)^{(1-\sigma)m}$, $k\geq 1$, what proves the coincidence of the Lorenz curves.■

For CRRA preferences we have

$C_{it} = \lambda_{ij}\, C_{jt}$ and $1-N_{it} = \lambda_{ij}\, (1-N_{jt})$ $\quad t\geq 0$ and $m = 1+\theta$,

and for GHH preferences $C_{it} - \mu\, N_t^{\nu} = \lambda_{ij}(C_{jt} - \mu\, N_t^{\nu})$ $\quad t\geq 0$ and $m = 1$.

Now we show how does this utility distribution change when given a policy change we change from an equilibrium characterized by equilibrium prices p^B to another characterized by equilibrium prices p^A.

Definition[4]: The policy change is utility distribution improving, neutral or worsening when

$$[U_i^A/U_k^A]/[U_i^B/U_k^B] \qquad i=1,\ldots,N. \quad k=1,\ldots,i-1.$$

is respectively lower, equal or higher than one.

This definition is equivalent to the so called Lorenz dominance, corresponds, with identical means, to second degree stochastic dominance, and graphically corresponds to non intersecting Lorenz curves.

[4]This definition is not dependent on individual cardinality and is maintained if we apply to every individual utility function the same monotonic transformation In this case the λ_{ij} and the change in λ_{ij} is, respectively, the compensation (in consumption in the CRRA and in X in the GHH function) and the change in the compensation between agent i and j.

PROPOSITION 1: *Given a change of policy the initial and final utility distributions can always be ranked through the concept of Lorenz dominance. The initial utility distribution dominates, is identical or is dominated by the final one depending on* $\sum_{t=0}^{\infty} d_t^A F(W_t^A) / \sum_{t=0}^{\infty} d_t^B F(W_t^B)$ *being respectively higher, equal or lower than zero.*

Proof:

By lemma 1 we saw that $U_j^A/U_k^A = \lambda_{jk}^{A\,(1-\sigma)m}$.

Therefore $[U_i^A/U_k^A] / [U_i^B/U_k^B]$ is higher, equal or lower than one iff $\lambda_{ij}^A / \lambda_{ij}^B$ is higher, equal or lower than zero. As $\lambda_{ij} = \frac{A_j^T}{A_i^T}$, it is immediate to show that the sign of $\lambda_{ij}^A / \lambda_{ij}^B - 1$, for i>j, it is always identical to the sign of $[\sum_{t=0}^{\infty} d_t^A F(W_t^A) / \sum_{t=0}^{\infty} d_t^B F(W_t^B)] - 1$. That is, as by lemma 1 we show that there is a coincidence between utility distribution and transformed wealth distribution the change on transformed wealth distribution is the key to understand the change on utility distribution. As the sign of the change on λ_{ij} is independent of i and j and depends just on the sign of $\sum_{t=0}^{\infty} d_t^A F(W_t^A) - \sum_{t=0}^{\infty} d_t^B F(W_t^B)$, we can rank the two transformed wealth distribution using the Lorenz dominance concept and so we also can rank the utility distribution using the same criteria. ■

In summary we show that the effect on the utility distribution depends only on the effect of the policy change on the factor $\sum_{t=0}^{\infty} d_t F(W_t)$, and as that effect does not depend on the wealth distribution, the effect on utility distribution is characteristic distribution independent. So the computation of the effect of the policy using the representative agent not only determines whether the policy is efficient or not but also whether the policy is utility distribution improving or not.

IV- GENERALIZATION

We show above that when agents are heterogeneous in their nonhuman wealth holdings, when the set up is the dynamic general equilibrium one usually utilized for fiscal policy exercises, and when we have Gorman aggregation we can classify the policy not only in terms of its efficiency results but also in terms of the effects on the utility distribution, without any knowledge on the specific wealth distribution that exists at the economy.

Now the relevant question is what determines that result, or how can this result be generalized. The intuition of the result for the presented set up is the following: Agents face exogenous transformed wealth and this is composed of two parts: one is non human wealth, heterogeneous among agents and exogenous to the policy change; the other is identical for each agent and endogenous to the change of environment. When the change of environment increases this homogeneous parcel and by definition has no impact on the heterogeneous parcel it is obvious that distribution improves.

PROPOSITION 2: *Proposition 1 can be generalized for every environment where:*
i) a definition of transformed wealth is possible and
ii) transformed wealth can be decomposed into the heterogeneous component, that is exogeneous, and the rest. This decomposition is still possible when transformed wealth is the sum of two heterogeneous parcels.
The generalization it is not possible for more than two heterogeneous characteristics.

Proof: The way the proof of proposition 1 was developed implies that the effect on utility distribution is given by the impact of the policy measure on λ_{1j}. That is that we need to identify a measure of *wealth* whose distribution is related to the distribution of utilities. And this is always possible in the representative agents problems that we are treating. Generalizing the conditions stated at section II, maintaining the intertemporal separability, implies that necessary conditions for aggregation are that either u is homogeneous in the arguments or is linear in one of the arguments, that is it can be transformed into a Stone Geary function, and U is homogeneous in u. In both cases it is possible the definition of A^T, even if we change the set up to an endowment economy, to an exogeneous labor economy, to a monetary economy, to an open economy etc.

Additionally we need to know the effect on λ_{ij} independently on the characteristics distribution. Then it is immediate to see that the only way to have this result is that the impact on λ_{ij} goes through the homogeneous part, that is the heterogeneous component has to be exogeneous to the policy change. This means that the result can be applied for every situation where agents are discriminated by one characteristic but more important it can be extended to some circumstances where there are simultaneous two characteristics which define each individual. In this case the condition to get the result of proposition 1 is that transformed wealth is the sum of two heterogeneous components. In this case it is always possible to transform the problem into a standard one where transformed wealth has just one heterogeneous exogeneous parcel. We are going to prove through an example of a specific two individual characteristics, since it is easy to see that there are no particular aspects in this case. Moreover we think that it is an interesting case to discuss.

Suppose that agent i is characterized by A_{io} and by E_i, where E_i represents his labor efficiency level. If we suppose that these individual efficiency levels (E_i) are constant over time we have a natural extension of our results. Households decisions are derived from the maximization of equation 1 subject to

$$\sum_{t=0}^{\infty} d_t^1 \, C_{it} = \sum_{t=0}^{\infty} d_t^2 \, E_i \, W_t \, N_{it} + A_{i0} \qquad (2')$$

Aggregation still holds since we have now

$$C_{it} = \alpha_{it} + \chi_t \, A_{it} \qquad t \geq 0 \qquad (3')$$
$$N_{it} = \varepsilon_{it} + \phi_t \, A_{it}$$

Transformed wealth would be defined as

$$A_i^T = E_i \sum_{t=0}^{\infty} d_t^2 \, F(W_t) + A_{i0}. \qquad (10')$$

Notice that transformed wealth is the sum of two heterogeneous components.

If we redefine that transformed wealth as

$$\tilde{A}^T_{10} = \sum_{t=0}^{\infty} d_t^2 \; F(W_t) + \tilde{A}_{10} \qquad (10'')$$

where $\tilde{A}_{lo} = A_{lo} / E_l$. Our variables are redefined as $\tilde{C}_{lt}=C_{lt}/E_l$ for CRRA preferences and for the second class, GHH preferences, if we define $X_{lt}= C_{lt}- \mu \; (E_l W_t(1-\tau_{ct})/\mu\nu)^{\frac{\nu}{\nu-1}}$, $\tilde{X}_{lt}= X_{lt}/E_l$. Given this we have that $\tilde{U}_l=U_l/(E_l)^{1-\sigma}$. Given these transformations the problem is identical to the described for the original variables. Now we have $\tilde{\lambda}_{lj}= \lambda_{lj}[E_l/E_j]^{1-\sigma}$. Therefore it is easy to check that Proposition 1 is applies for this case of additional heterogeneity. The effect of the policy on $\tilde{\lambda}_{lj}$ depends exactly on the same term as in proposition 1 and as $[E_l/E_j]^{1-\sigma}$is exogeneous it determines the effect on λ_{lj}.

So we show in this example how we can transform the problem, in this case transforming the variables into efficiency units, so that with two types of heterogeneity the result still holds.

The interest of this result comes from the fact that the class of heterogeneity for which it applies include the case that we think is more interesting and, naturally, the more utilized in general equilibrium macro models. This is a set up similar to the one defined in section II but where agents are differentiated not only by initial non human wealth but also by labor efficiency.■

V- WELFARE

As stated in the introduction the judgment free characteristic of the efficient classification loses meaning once we are in a world where it is not possible to apply discriminated lumpsum transfers. When this is the case there is no sense in constructing Pareto frontiers, and inquiring what is the effect of a policy on that frontier but we have to compare just the effective situations which characterize the economy before and after the policy change.

A policy is said to be efficient when it leads to a potential Pareto movement. In models with aggregation, that is, with a representative agent, the notion of efficiency is associated with an increase of utility of the representative agent.

Just by the work of Dow and Werlang (1988) we get a formal proof of that statement. They show that it exists always a function $H(U_1,\ldots, U_N)$, increasing in each argument, which takes at each situation an identical value of the value of the representative agent utility function. Therefore an increase of the utility of the representative agent can be associated with a higher Pareto frontier and consequently to a potential Pareto movement. This is very clear when we have identical and homogeneous of degree one utility functions. In that case both the Pareto frontier and the function H coincide and is the sum of agents utilities. More general cases, even the one we treated here of identical preferences but not homogeneous of degree one are not so straightforward. So we will associate an efficient policy with the one that increases the utility of the representative agent, r.

The non existence of lumpsum transfers implies that, in general, we have to abandon the general statement of welfare economics of the impossibility of utility measurability and comparability, even when we want to compare measures which are already classified by efficiency.

Ethical requirements to compare utilities can be traced to Dalton (1920). He discusses the social welfare evaluation of income (here utility) distribution and Atkinson (1970) recently follow's Dalton approach. They propose individualistic social welfare functions, that is a sum of individual utilities. Dagum (1990) introduces two basic ethical principles to deal with the distribution and social welfare. He proposes that we need i) The Aversion to Inequality Principle, that states that a society prefers less inequality and ii) The Aversion to Poverty Principle, that says that a society prefers larger average income (here utility). The first principle implies that the social welfare function is quasi concave and the second that is increasing.

PROPOSITION 3: *Welfarism and the weak Pareto principle, together with full comparability and ordinal scale measurability, are sufficient to define a class of policies as welfare increasing (decreasing). Belong to this class every policy that is efficient (not efficient) and that verifies the improvement (worsening) in utility distribution stated in proposition 1. Moreover this class is identified without recurs to the characteristics distribution.*

Efficient (not efficient) policies which are Lorenz dominated by (dominate) the *anterior situation imply generalized Lorenz curves that intersect and so to be compared we need to know the functional form of the social welfare function, that is we need to go much further in terms of ethical judgments on interpersonal utilities comparisons.*

Proof: Using the results of last sections we can easily see that there exist a class of measures that implies an improvement on the utility distribution in the sense of Lorenz dominance. When additionally these policies are efficient the increase of the utility of the representative agent implies a generalized Lorenz dominance and more than this it implies necessarily that every agent has a higher level of utility than before. We just need the weak Pareto principle, additionally to the strict dependence of social welfare on individual utilities, to state that welfare increased. This classification of welfare increasing measures do not depend on the existing characteristic distribution and, as we saw, we need just full comparability and ordinal measurability to get proposition 1. The complement to this class of efficient measures is the one in which the change of policy implies a distribution that is Lorenz dominated by the anterior situation. In this case we need to know the specific form of the social welfare function to take a decision.■

It is important to notice that the class of efficient measures which are distribution improving are an important result given its parsimony in information: we just need to know the aggregate effects and to impose full measurability. When we take the other type of efficient measures, while for their identification the same type of information is needed, to classify the policies as welfare improving or not we need a incredible additional amount of information. Not only the form of the social welfare function implies, as we said, more ethical judgments on interpersonal comparability, but more important, to apply that function we need to know the utility individual levels before and after the distribution, that is we need to take into account the specific distribution on characteristics.

In summary, in a world with no differentiated lumpsum transfers, the simple and complete coverage classification in terms of distribution improving (worsening) developed in sections III and IV is a expeditious way to distinguish policies which should (or should not) be implemented once they are qualified as efficient (non-efficient) from others policies that can not be welfare classified without taking into account distribution considerations.

VI- DISCUSSION OF SOME EFFICIENT MOVES

Last section results would not be very interesting if the class of policies that could be easily classified in terms of welfare were not significative. We show in this section that this is not the case by presenting some important efficient measures which can be classified according to proposition 3 as welfare improving.

Efficiency measured by a potential Pareto movement or an increase in the utility of the representative agent (U_r), can be related to a price effect, and to a wealth effect, a change in A_r^T, (the transformed wealth of the representative household). This fact can be easily seen if we construct the representative agent problem as the

$$\text{MAX } U_r = \sum_{t=0}^{\infty} \beta^t (u(X_{rt}))^{1-\sigma} / (1-\sigma) \qquad \sigma>0$$

$$\sum_{t=0}^{\infty} d_t^1 \, m \, X_{rt} = A_{r0}^T$$

where X and m are respectively (C, 1-N) and $1+\theta$ for isoelastic preferences and

$C - \mu \, (W(1-\tau_c)/\mu\nu)^{\frac{\nu}{\nu-1}}$ and 1, for GHH preferences.

The move has a positive impact on U_r if at least one d_t^1 or if A_{r0}^T increases. This means that, as the effect on distribution depends on the effect on A_{r0}^T, this effect is in general indeterminate. We summarize this fact in the following lemma:

LEMMA 2: To guaranty that a policy measure is efficiency increasing it is necessary that: i) That change is reflected into a positive price effect, that is for some non zero elements of the price sequence $d_t^A < d_t^B$,[5] and /or ii) That change is reflected

[5]A decline in W_t is always efficiency decreasing. In class 1 , the is the price effect is more then compensated by the negative wealth effect, and class 2 has no price effect on the second level resolution.

into a positive wealth effect, that is $\sum_{t=0}^{\infty} d_t^A F(W_t^A) > \sum_{t=0}^{\infty} d_t^B F(W_t^B)$. Then we can have efficient moves which are either distribution improving, neutral or worsening.

In this section we will discuss some well studied efficient moves and try to classify them into distribution improving, neutral or worsening.

CASE 1: Decreasing capital taxation in a small open economy with the territorial regime. (see Correia (1993))

As the marginal intertemporal rate of substitution is given in this case by the international real rate of interest and if we assume that our small open economy is symmetrical to the rest of the world, the change from a positive rate of capital taxation to a lower one (until zero) will, maintain the economy at the steady state maintaining every d_t, for $t>0$. As capital labor ratio increases in the new situation and what happens to N depends on preferences d_0 depend on preferences. Anyway for the two types of preferences we have that in equilibrium the net wage rate is higher. This implies always a positive effect on the utility of the representative agent and an improvement in distribution. So this measure has always a positive effect on welfare.

CASE 2: Reduction of the inflation tax in a model that introduces money as intermediate good, and without capital. (see Correia and Teles (1995))

When the alternative tax is an income tax as we have no capital and consequently the marginal productivity of labor is constant, net wage declines always. This implies a worsening of the distribution: [The decline in inflation benefits relatively more creditors then debtors]. Nevertheless when the alternative tax is a consumption tax as we can translate the inflation tax as an equivalent consumption tax the decline on inflation has as only effect the decline of a composite tax on consumption. Then the wage rate is maintained and we have no distributional effects: the measure now is neutral which implies that every agent increases utility proportionally to the representative agent and so the measure has always a positive effect on welfare.

CASE 3: Liberalization of capital movements in a small open economy.

If we suppose that the small economy is symmetrical to the rest of the world and that the rest of the world is at the steady state the opening of capital markets will be efficient if the small economy is not at the steady state.

When capital labor ratio is lower then the steady state the opening of the capital market will avoid the transitional period, allowing the existence of external debt. Again in this case the effect is to increase the level of d_tW_t every period. Consequently we have a positive effect on distribution and a positive effect on welfare.

Alternatively when the capital labor ratio is higher then the steady state, when the economy with access to the international market will became a net creditor, the effect is a decline in d_tW_t at every period. The opening is distribution worsening and at this level of generality the effect on welfare is uncertain.

CASE 4: Elimination of capital taxation into a closed economy

This case has no uniform general answer in terms of the effect on the relative transformed wealth. We know that the simple move to a zero capital taxation can even not be efficient. So we will compare, through a numerical example a change from a level of 50% capital income taxation to zero capital taxation and later a more efficient move that additionally imposes a positive tax on capital income at period zero.[6] So what we want to compare is two moves, where one is nearer the Ramsey, or Chamley solution (Chamley (1986)), so is more efficient then the other.

Our results are that the less efficient move is associated with a worsening in distribution and that the more efficient move is distribution improving.

This example is important because it stresses the fact that an efficient move can change from distribution worsening to improving once we approach the second best solution. That is it is not always possible to classify a move in one or other category as happens in the given examples but its classification can depend on the *size* of the efficient move. In this example the increase of efficiency modified the

[6]Chari, Christiano and Kehoe (1994) show that the positive efficiency effect of Ramsey solution is due mainly to the initial period high taxation of capital income. They show that the constant zero taxation has lower, and potentially negative, efficiency effects.

distribution classification.

VII- INCOME AND WEALTH DISTRIBUTIONS

In this paper we are interested in the notion of utility distribution. As we saw this distribution is associated with the distribution of initial financial wealth. That is, it depends on the heterogeneity that is exogeneous to each agent. We find usually another notion of distribution, a purely positive notion, that associates distribution with the relative position, at a point in time, of income or wealth of households.

Recently we can find some works which determine the evolution of wealth distribution in dynamic models. We have a class of studies that study dynamic distributional effects of idiosyncratic uninsurable shocks, e.g. Atkenson and Lucas (1992)[7], Ayagari (1994) and Bertola (1994) and another class that studies the evolution of distribution under certainty, e.g. Chatterjee (1994).

Chatterjee shows that the evolution of distribution depends on the functional form of the utility function and on the aggregate path to be above or under the steady state. This author works in a dynamic neoclassical model with exogenous labor and in a particular environment where wages are distributed as proportion to shares. Given this hypothesis his conclusions can be read for wealth or income distribution.

We can extend this result to our environment. Even when we suppose that heterogeneity is given by different initial financial wealth the conditions that gave the evolution of wealth over time are dependent on the type of preferences being the CRRA or the GHH type.

Our results are that in CRRA preferences wealth distribution improves over time when the rate of growth of net wages is superior to the rate of growth of consumption, that is when labor supply declines over time. With GHH preferences wealth distribution is always worse off as time goes by.

We do not think that we can have an equivalent general result in terms of the income

[7]This work studies the efficient evolution of distribution over time, that is the time distribution when welfare is maximized.It is normative in the sense that it is the best solution given idiosyncratic shocks and private information and is contrasted with other inefficient environments. It is positive in the sense that for each case the authors describe the wealth evolution over time.

distribution over time. It seems that this evolution is case specific.

We think that it is important to stress that this evolution of wealth (or income) distribution is just a positive description. It can have no normative or value judgment content. So the observation that a country *improved* its wealth (or income) distribution is an abuse of language. Nothing can be said in normative terms of an indicator that is a mix of the effects of endogenous decisions and a different initial, exogenous, situation of new households. It is important this distinction since this is a characteristic of the dynamic thinking of the economy.

In static and exogenous labor models it is a fact that the existence of a situation where agent i has a higher wealth, or income, that agent j means that this agent will have at that period of time a higher level of utility. And, more important if we compare two situations where one has a narrow distribution of income or wealth differences of utility at this situation will also narrow. Therefore the positive measure of the change of distribution over time could say something in terms of welfare distribution.

When we introduce endogenous labor choices income distribution, as income is no more an exogenous variable, is not necessarily correlated with wealth and the association of changes in wealth or income distribution over time with changes in relative utility position has to be done cautiously. When we change to dynamic models we can not take as exogenous, to each agent, the evolution over time of wealth or of income. Therefore the normative distribution notion has always to be related, in these dynamic models, to the notion of utility distribution that we used in this paper. And that utility distribution can not be read over time but across policies, or environments, in the infinite horizon agent paradigm. The dynastic interpretation of this type of models allows us to compute some utility distribution effects over time for the same environment: if, for example wealth distribution improves monotonically over time, and life duration of each generation is identical across heterogeneous "agents", wealth distribution improvement has to be compared with the new endogeneous term of transformed wealth (which is identical to the one of their parents except that do not have the terms of the parents life) to compute the effect on utility distribution over time. There is no reason why the change of wealth distribution will dominate the endogenous term of transformed wealth, in the change of the utility distribution. There is no hope that this utility generation distribution have the special characteristic discussed in this paper of being distribution independent.

VIII- CONCLUSIONS

The study of the social welfare in a world without differentiated lumpsum taxes is the object of the modern welfare economics (Stiglitz 1987) This implies that distribution measures through incentive restrictions, are efficiency damaging and that the efficiency-equity trade-off is always present. The main results of this paper are:

1) We identified a class of measures where the trade-off between efficiency and equity do not exist. The reason being that the policies are per se welfare improving and therefore do not need to be accompanied by distortionary redistributive measures.

2) In the universe of aggregable models, and of heterogeneity given in proposition 2, there a clear cut and very simple division between the efficient measures that belong to this class and the others.

3) To determine the effect of the policy on utility distribution it is sufficient to know the aggregate effects of policies, that is distribution effects are distribution independents.

4) This class is a significative one in the modern second best literature.

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