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JEL Classification: E52, F31, F41

Keywords: monetary policy rules, Exchange rate management, External habit, Risk premium

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The Exchange Rate as an Instrument of Monetary Policy

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Most of the theoretical research in small open economies has typically focused on corner solutions regarding the exchange rate: either the currency rate is fixed by the central bank or it is left to be freely determined by market forces. We build an open-economy model with external habits in consumption to study the properties of a new class of monetary policy rules, in which the exchange rate serves as the instrument for stabilizing business cycle fluctuations. Instead of using a short-term interest rate, the monetary authority announces a path for currency appreciation or depreciation as a reaction to fluctuations in inflation and the output gap. We find that, under a wide range of modeling assumptions, the exchange rate rule outperforms a standard Taylor rule in terms of stabilizing both output and inflation. The reduction in volatility is more pronounced for more open economies and for economies with lower sensitivity to movements in the interest rate. We show that differences between the two rules are driven by two key factors: (i) paths of the nominal exchange rate and the interest rate under each rule, and (ii) the time variation in the risk premium, which leads to deviations from uncovered interest parity.

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1 Introduction

Understanding the properties of alternative monetary policy rules and designing rules that maximize social welfare are important objectives both from a policy point of view and from an analytical perspective. The seminal paper by Taylor (1993) has generated an entire sub-field of monetary economics that studies the properties of alternative interest rate rules within empirical or theoretical frameworks. The majority of these papers focus on models of closed economies. The Taylor rule prescribes that the monetary authority adjusts interest rates in response to deviations in inflation from a prespecified implicit or explicit inflation target and to fluctuations in the output gap. The research based on Taylor's work has extended the policy rule by including additional variables, by implementing different estimation techniques to determine the reactions of central banks to key macroeconomic variables, and by building theoretical models to study the properties of these rules. Gali (2008) provides an insightful overview of this literature.

In small open economies, however, the exchange rate is an important element of the transmission of monetary policy (Svensson (2000)). In these economies, central banks generally prefer to keep the exchange rate under tight control. The economic literature incorporates exchange rate policies of various central banks in two ways. First, a large number of papers evaluate the costs and benefits of fixed exchange rates (including Friedman (1953) and Flood & Rose (1995)). A second approach to incorporating the exchange rate into discussions of monetary policy is by augmenting a closed-economy Taylor rule with the rate of currency depreciation. Under this approach the monetary policy instrument, the interest rate, reacts not only to inflation and the output gap but also to movements in the exchange rate. For example, De Paoli (2009) derives an optimal monetary policy rule within a DSGE model and shows that by putting some weight on real exchange rate fluctuations, a central bank can achieve improvements in social welfare.

In discussing how monetary authorities deal with exchange rates, monetary policy research focuses on corner solutions: either the currency rate is fixed by the central bank or the government, or it is left to be determined by market forces.[1] Of course, there is a large body of empirical research on managed floating. Especially after the seminal paper of Reinhart & Rogoff (2004) there has been growing interest in how the intermediate cases fare in terms of economic performance. However, in both theoretical and empirical papers, the exchange rate is treated as a resultant variable. In this paper, we evaluate the properties of an alternative class of policy rules where the exchange rate is used as an instrument of monetary policy. The exchange rate is

[1] A notable exception is the target zone literature started by Krugman (1991).

adjusted by the central bank in a manner similar to the use of the interest rate as an operating instrument. We build a model in which the rate of currency appreciation is determined as a reaction to the rate of inflation and the output gap. Our goal is to establish whether there are economic structures for which the use of this exchange rate-based rule delivers lower economic fluctuations compared to policies based on interest rate rules.

The motivation for writing the model comes from the Monetary Authority of Singapore (MAS). Unlike other central banks, MAS does not rely on the overnight interest rate or any monetary aggregate to implement its monetary policy. Since 1981, the operating instrument has been the exchange rate. Khor, Lee, Robinson & Supaat (2007), McCallum (2007), and MAS (2012) offer detailed descriptions of the policy regime in Singapore. Many authors view Williamson (1998) and Williamson (2001) as providing the analytical foundations of this system, since he proposed an intermediate exchange rate system in the form of an adjustable crawling peg with a band. The system is also referred to as BBC (basket, band, crawl) because the currency can be pegged against a basket of currencies in order to minimize misalignment with major trading partners. The crux of Williamson's argument is that the crawl or the level of the exchange rate must be adjusted to reflect differences in inflation and productivity trends between the domestic and foreign economies. In other words, the exchange rate must move over time towards its equilibrium value.

There can be little doubt that having the exchange rate aligned with the fundamentals is a valuable goal in the long run. In this paper, we argue that MAS goes beyond a simple adjustment towards equilibrium and uses the exchange rate to stabilize the business cycle. At business cycle frequencies, the central bank reacts to changes in inflation and the output gap. According to some casual metrics - macroeconomic stability, inflation, currency volatility, etc. - this policy has been quite successful over the past twenty years. And yet, it is not clear whether the success of the central bank is due to the novel policy rule or to some other factors such as prudent fiscal policy. Therefore, an important question to ask is whether other countries would benefit from adopting a rule where the exchange rate is moved on a continuous basis as a reaction to the level of inflation and the output gap. To understand the benefits of this rule relative to a standard interest rate rule, we need the discipline of a model with optimizing agents, market clearing, and other features that are characteristic of recent advances in monetary policy analysis. Our paper aims to analyze the properties of exchange rate rules by building a DSGE model where the monetary authority adjusts the exchange rate in response to deviations of inflation from a target and fluctuations in the output gap.

Understanding the costs and benefits of an exchange rate policy rule within a fully specified model is not a trivial task. The immediate revelation is that if the model features an uncovered interest parity condition (UIP), then interest rate and exchange rate rules might generate similar, if not identical, outcomes. In our model, there are two reasons why the outcomes for the two rules differ. First, the actual implementation of the exchange rate rule is important. While the central bank technically can replicate any interest rate rule by moving the exchange rate today and announcing depreciation consistent with UIP, it is not the way that our rule operates. In our model, the exchange rate today is predetermined and the central bank announces the depreciation rate from time t to $t+1$. This implies for example, that the model may not feature the standard overshooting result as the currency rate both today and at $t+1$ are determined by the monetary authority. The simulations of our model suggest that this feature does generate differences between the two rules.

Furthermore, these differences are amplified when UIP fails. Indeed, Alvarez, Atkeson & Kehoe (2007) argue forcefully that a key part of the impact of monetary policy on the economy goes through conditional variances of macroeconomic variables rather than conditional means. In terms of the UIP condition, their paper implies that the interest parity condition has a time-varying risk premium. Interest in time-varying risk premium has been growing in recent years. In the context of the interest parity condition, Verdelhan (2010) shows how consumption models with external habit formation can generate counter-cyclical risk premium that matches key stylized facts quite successfully. In our model, we adopt a similar approach by allowing external habit formation. To show the importance of the counter-cyclical risk premium, we report results for the first-order approximation, which wipes out the risk premium from UIP, and for the third-order approximation, which preserves time variation in the risk premium.[2]

We start by writing down a relatively standard New-Keynesian small open economy model as in Gali & Monacelli (2005) that we extend to include external habit in consumption. We then analyze the performance of the model under two different policy rules: a standard Taylor rule in which the monetary authority sets interest rates, and an alternative monetary rule in which the monetary authority sets the depreciation rate of the nominal exchange rate. We show that if UIP holds (i.e. we use first-order approximation), these rules generate different responses to shocks.

[2]An alternative route for introducing risk premium in the UIP condition is by building in incomplete financial markets, as in Schmitt-Grohé & Uribe (2003), Turnovsky (1985), Benigno (2009) and De Paoli (2009). Under incomplete markets, deviations from UIP come from costs of adjusting holdings of foreign bonds. This requires the introduction of the country's net foreign asset (NFA) position in the model dynamics. The cost of holding foreign bonds introduces a time-varying risk premium and deviations from UIP.

The Taylor rule implies overshooting of the exchange rate following a shock, generating a higher volatility of the exchange rate and other economic variables. We then introduce deviations from UIP. The goal is to analyze the performance of the two competing rules when the one-to-one relationship between exchange rates and interest rates breaks down. In this case, the differences between the two rules, in terms of the response of the economy to shocks, is amplified. The main reason is that the implementation of the monetary rule has an effect on the volatility of the risk premium through a precautionary saving motive. The Taylor rule features overshooting of the exchange rate, and it generates larger fluctuations of inflation and output gap, as the larger volatility of exchange rates increases the risk premium. The opposite is true for the exchange rate rule, as the monetary authority adjusts its path of appreciation to smooth economic fluctuations by generating a a less-volatile exchange rate. In this regard, the exchange rate rule is also different from a peg, in which the monetary authority fixes the exchange rate to a specified value. [3]

To shed more light on the mechanism, we follow the methodology in Backus, Gavazzoni, Telmer & Zin (2010) and derive an analytical solution for the risk premium under the two monetary rules. We do it in the context of an endowment economy in which all variables are jointly log-normal. We find that the risk premium exhibits different behavior depending on the rule that the monetary authority follows. For the same parameter values, the exchange rate rule implies a lower risk premium than the Taylor rule. To get a better understanding of these differences, we decompose the risk premium into the conditional volatility of the exchange rate depreciation and the conditional covariance between the stochastic discount factor and the exchange rate depreciation. The exchange rate rule generates both less fluctuations of the exchange rate depreciation and lower covariance. Therefore, the the risk premium is lower under this rule.

Finally, we find that the differences between the two rules are amplified when the economy is exposed to foreign shocks, suggesting that exchange rate rules may be more successful at smoothing economic fluctuations in small open economies more exposed to these shocks. Indeed, we find that the volatility of inflation under an exchange rate rule is significantly lower when the degree of openness of the economy is larger. Furthermore, we find that the degree of interest-rate sensitivity matters as well – as interest rate sensitivity increases, the interest-rate rule improves significantly

[3] Our results are consistent with those in Chow, Lim & McNelis (2014), who estimate a DSGE model for the Singapore economy under the two rules and find that the exchange rate rule outperforms the Taylor rule in reducing fluctuations of inflation. Different from their paper, we have a more general framework that can be applied to any small open economy. More importantly, our goal is to analyze the mechanisms behind the different performance of the two rules, especially those driven by the existence of a countercyclical risk premium that introduces deviation from UIP.

in terms of lowering both output and inflation volatility. Thus it is conceivable that for large, relatively closed economies with a large number of sectors being sensitive to interest rate movements, a Taylor-like rule will generate superior outcomes.

The rest of the paper proceeds as follows. Section 2 lays out the details of the model. Section 3 presents the quantitative analysis. Section 4 analyzes the role of time-varying risk premium in generating different outcomes for the two instrument rules. Section 5 provides a summary of our key findings, some ideas for future research and conclusions.

2 The Model

Our model extends Gali & Monacelli (2005) by introducing a new policy rule based on using the exchange rate as a monetary policy instrument and adding external habit as in Campbell & Cochrane (1999), Jermann (1998), Verdelhan (2010) and De Paoli & Sondergaard (2009). Modeling assumptions are kept at a minimum to ensure that we can study the properties of the exchange rate instrument rule without introducing too many confounding factors.

2.1 Households

In each country, there is a representative household who maximizes life-time expected utility. The utility function of the household in the Home country is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t - hX_t, N_t) \tag{1}$$

where N_t is hours of labor, X_t is the level of habits defined below, and C_t is a composite consumption index defined by:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $C_{H,t}$ denotes the consumption of domestic goods by the Home consumers, $C_{F,t}$ denotes the consumption of foreign goods by Home consumers, $\eta > 0$ is the elasticity of substitution between domestic and foreign goods, and $\alpha \in [0,1]$ is the degree of openness of the country (and the inverse of home bias). C_{Ht} and C_{Ft} are aggregates of intermediate products produced by Home and Foreign combined in the following way

$$C_{Ht} = \left[\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

$$C_{Ft} = \left[\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

with ε being the elasticity of substitution between varieties, which in turn are indexed by $i \in [0, 1]$.

As in De Paoli & Sondergaard (2009) we assume that habits are external. We allow for flexibility in assessing the importance of habits by introducing in (1) the parameter $h \in [0, 1]$. When $h = 0$ the model collapses to the basic version of Gali-Monacelli, while $h = 1$ corresponds to the modeling assumptions in Campbell & Cochrane (1999) and Verdelhan (2010). The evolution of habits follows an AR(1) process with accumulation of habits based on last-period consumption:

$$X_t = \delta X_{t-1} + (1 - \delta)C_{t-1},$$

Parameter $\delta \in [0, 1]$ captures the degree of habit persistence. Again, this parameter allows us to consider various assumptions about habits with $\delta = 0$ corresponding to the assumptions in the earlier literature on habit-formation where habits are determined exclusively by the last-period consumption (e.g. Campbell (2003) and Jermann (1998)).

Consumers maximize (1) subject to the following budget constraint:

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di + \int_0^1 P_{F,t}(i)C_{F,t}(i)di + E_t \left\{ \mathcal{M}_{t,t+1}B_{t+1} \right\} \leq B_t + W_t N_t$$

where $P_{H,t}(i)$ is the price of variety i produced at home, $P_{F,t}(i)$ is the price of variety i imported from Foreign (expressed in Home currency), $\mathcal{M}_{t,t+1}$ is the stochastic discount factor, B_{t+1} is the nominal payoff in period $t + 1$ of the portfolio held at the end of period t, and W_t is the nominal wage.

The optimal allocation of expenditures within each variety gives the demand function for each product:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{Ht}} \right)^{-\varepsilon} C_{Ht};\ C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{Ft}} \right)^{-\varepsilon} C_{Ft}; \quad (3)$$

where $P_{H,t} = \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$ is the domestic price index and $P_{F,t} = \left[\int_0^1 P_{F,t}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$ is the price index of imported goods (expressed in units of Home currency). From expression (3), $P_{Ht}C_{Ht} = \int_0^1 P_{H,t}(i)C_{H,t}(i)di$ and $P_{Ft}C_{Ft} = \int_0^1 P_{F,t}(i)C_{F,t}(i)di$.

The optimal allocation of expenditures between domestic and imported goods is:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t; \quad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{4}$$

where $P_t = [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI). From the previous equations, total consumption expenditures by the domestic households is $P_t C_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$. Therefore, we can re-write the budget constraint as

$$P_t C_t + E_t\left\{\mathcal{M}_{t,t+1}B_{t+1}\right\} \leq B_t + W_t N_t \tag{5}$$

The per period utility function takes the following form

$$U(C_t, X_t, N_t) \equiv \frac{(C_t - hX_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma}$$

The first order conditions for the household's problem are

$$(C_t - hX_t)^{\sigma} N_t^{\gamma} = \frac{W_t}{P_t} \tag{6}$$

$$\beta\left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right) = \mathcal{M}_{t,t+1} \tag{7}$$

Taking expectations on both sides, we have the Euler equation:

$$R_t E_t\left\{\beta\left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right\} = 1 \tag{8}$$

with $R_t = \left[\frac{1}{E_t\{\mathcal{M}_{t,t+1}\}}\right]$ the gross return on a riskless one-period discount bond paying off one unit of domestic currency in $t+1$.

Below we elaborate on the need to use habit formation in this model but from the Euler equation it is already clear that the marginal utility of consumption increases when consumption goes down relative to the acquired level of habit. This introduces a precautionary saving motive. As we discuss below, this modeling approach generates a time-varying coefficient of relative risk aversion. This leads to a counter-cyclical risk premium driving a wedge between the interest rate differential and expected depreciation in the uncovered interest parity condition. Both Verdelhan (2010) and De Paoli & Sondergaard (2009) make this point quite forcefully.

2.2 Domestic inflation, CPI inflation, the RER, and the TOT

Before we proceed to the solution, we introduce some definitions, following Gali & Monacelli (2005).

2.2.1 Bilateral Terms of Trade (TOT)

The terms of trade, S_t, are defined as the price of Foreign good in terms of Home goods, so that

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}}$$

We define the real exchange rate as the relative price of the foreign consumption bundle in terms of the domestic consumption bundle

$$Q_t = \frac{\mathcal{E}_t P_t^*}{P_t}$$

where $\mathcal{E}_t$ is the nominal exchange rate denoted in units of domestic currency per unit of foreign currency.

Inflation is defined as the ratio of current and past CPI

$$\Pi_t = \frac{P_t}{P_{t-1}}$$

We assume that the foreign economy is large relative to the home country, so the law of one price requires that the price index in the foreign country equals the price index of foreign goods in the home economy (when converted to the same currency).

$$P_{Ft} = \mathcal{E}_t P_t^*$$

2.3 International risk sharing

Under the assumption of complete international financial markets, the rest of the world must satisfy

$$\beta \left(\frac{C_{t+1}^* - hX_{t+1}^*}{C_t^* - hX_t^*} \right)^{-\sigma} \left(\frac{P_t^* \mathcal{E}_t}{P_{t+1}^* \mathcal{E}_{t+1}} \right) = \mathcal{M}_{t,t+1} \tag{9}$$

The exchange rate reflects the fact that the security bought by the foreign households is priced in the currency of the small open economy.

Combining this expression with the one for the domestic households, we have

$$\left(\frac{C^*_{t+1} - hX^*_{t+1}}{C^*_t - hX^*_t}\right)^{-\sigma} \left(\frac{P^*_t \mathcal{E}_t}{P^*_{t+1}\mathcal{E}_{t+1}}\right) = \left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right)$$

Using the expression for the real exchange rate

$$\left(\frac{C^*_{t+1} - hX^*_{t+1}}{C^*_t - hX^*_t}\right)^{-\sigma} \left(\frac{Q_t}{Q_{t+1}}\right) = \left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t}\right)^{-\sigma} \tag{10}$$

2.4 The Uncovered Interest Parity Condition

Under complete international financial markets, $R^*_t = \left(E_t\left\{\mathcal{M}_{t,t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right\}\right)^{(-1)}$ and since $R_t = (E_t\{\mathcal{M}_{t,t+1}\})^{(-1)}$ we obtain the following equilibrium condition

$$E_t\{\mathcal{M}_{t,t+1}(R_t - R^*_t(\mathcal{E}_{t+1}/\mathcal{E}_t))\} = 0$$

Log-linearization of this condition around a perfect foresight steady state yields the standard uncovered interest parity condition:

$$r_t - r^*_t = E_t \Delta e_{t+1} \tag{11}$$

Alvarez et al. (2007) argue that assumptions leading to this simplified interest parity condition imply dynamics that are inconsistent with the data. Under assumptions of conditional log-normality of the stochastic discount factor, a time-varying risk premium emerges, as shown by Backus et al. (2010). Alternatively, a higher-order approximation of the Euler equation can also generate time-varying risk premium.

To gain some intuition on the components that drive the risk premium, assume that the stochastic discount factors, both domestic and foreign, are jointly lognormal. Under conditional log-normality, the UIP condition becomes

$$r_t - r^*_t = E_t \Delta e_{t+1} + \frac{1}{2} var_t(\Delta e_{t+1}) + cov_t(m_{t+1}, \Delta e_{t+1})$$

and it can be shown that

$$fxp_t = \frac{1}{2} var_t(\Delta e_{t+1}) + cov_t(m_{t+1}, \Delta e_{t+1}) = \frac{1}{2} var_t(m^*_{t+1}) - \frac{1}{2} var_t(m_{t+1}) \tag{12}$$

where fxp_t denotes risk premium. In good times in the Home economy, the variance of the domestic discount factor is low, relative to the variance of the foreign discount factor, and hence the risk premium that foreigners demand to hold domestic bonds in high. The time-varying risk premium will be one of the key differentiating factors between the interest rate and the exchange rate rules. Given its importance, we will elaborate further on its properties in Section 4.

2.5 Firms

We now characterize the supply side of the economy. In each country there is a continuum of monopolistic competitive firms that use labor to produce a differentiated good (each firm is associated with a different variety). Labor is the only factor of production, and we assume it to be immobile across countries.

2.5.1 Technology

Each firm operates the linear technology

$$Y_t = A_t N_t \tag{13}$$

where $a_t \equiv log(A_t)$ follows the AR(1) process

$$a_t = \rho_a a_{t-1} + u_t \tag{14}$$

The real marginal cost MC_t, denoted in units of the domestic goods, is

$$W_t = MC_t P_{Ht} A_t \tag{15}$$

2.5.2 Price setting

Prices are set as in the Calvo model, in which a measure $1 - \theta$ of randomly selected firms set new prices every period. We need to define some auxiliary variables to express the pricing decision recursively:

$$\frac{\tilde{P}_{H,t}}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{H_t}{F_t} \tag{16}$$

$$F_t = \Lambda_t Y_t + \theta \beta E_t (F_{t+1} (\Pi_{t+1})^{\varepsilon - 1})$$

$$H_t = \Lambda_t MC_t Y_t + \beta \theta E_t (H_{t+1} (\Pi_{t+1})^{\varepsilon})$$

$$\Lambda_t = (C_t - hX_t)^{-\sigma}$$

$$P_{Ht} = \left((1-\theta)\tilde{P}_{H,t}^{1-\varepsilon} + \theta P_{Ht-1}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{17}$$

This expression can be written in real terms as

$$\frac{P_{Ht}}{P_t} = \left((1-\theta)\left(\frac{\tilde{P}_{H,t}}{P_t}\right)^{1-\varepsilon} + \theta\left(\frac{P_{Ht-1}}{P_{t-1}}\right)^{1-\varepsilon}\Pi_t^{\varepsilon-1}\right)^{\frac{1}{1-\varepsilon}} \tag{18}$$

We take the price of the home consumption bundle, P_t, as a numeraire and express every variable in real terms.

2.6 The rest of the world

Because the foreign economy is exogenous to our small open economy, there is some flexibility in specifying the behavior of the foreign variables. We assume they follow an AR(1) process:

$$log\left(Y_t^*\right) = \rho_y log\left(Y_{t-1}^*\right) + u_{y^*t} \tag{19}$$

We assume that $\Pi_t^* = 0$, there is habit in consumption as in the domestic economy, $Y_t^* = C_t^*$ and the foreign interest rate R_t^* is determined by the Euler equation in the foreign economy as in the domestic economy. We assume habit persistence in the preferences of foreign consumers.

2.7 Market clearing in the goods market

In the domestic economy, the goods market clearing condition is

$$Y_t = C_{H,t} + C_{H,t}^* = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\left(\frac{P_{F,t}}{P_t^*}\right)^{-\eta} C_t^* =$$

$$= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\left(\frac{S_t P_{H,t}}{P_t}\right)^{-\eta} C_t^*$$

Using the risk-sharing condition

$$C_t^{-\sigma} = Q_t C_t^{*-\sigma}$$

we have

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\left[(1-\alpha) + \alpha Q_t^{\eta-\frac{1}{\sigma}} S_t^{-\eta}\right] = S_t^{\eta} Q_t{}^{-\eta} C_t\left[(1-\alpha) + \alpha Q_t^{\eta-\frac{1}{\sigma}} S_t^{-\eta}\right] \tag{20}$$

$$= C_t\left[(1-\alpha) S_t^{\eta} Q_t{}^{-\eta} + \alpha Q_t{}^{-\frac{1}{\sigma}}\right]$$

In the rest of the world, we have

$$Y_t^* = C_t^* \tag{21}$$

2.8 Monetary policy rules

The model is closed by specifying the monetary policy rule. First, we analyze the model under a standard Taylor rule, in which the monetary authority sets the nominal interest rate to smooth fluctuations in the output gap and CPI inflation. There is also interest rate smoothing.

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho} \left(\frac{Y_t}{\bar{Y}}\right)^{(1-\rho)\phi_y} \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{(1-\rho)\phi_m} e^{u_t} \tag{22}$$

where $\rho \in (0,1)$ is the degree of interest rate smoothing.

Second, we consider a monetary policy rule in which the central bank uses the change in the nominal exchange rate as the instrument to stabilize the output gap, CPI inflation, and movements in the nominal exchange rate. The policy is adjusted in response to anticipated deviations of these variables from their targets:

$$\frac{\mathcal{E}_{t+1}^*}{\mathcal{E}_t^*} = \frac{\bar{\mathcal{E}}_{t+1}}{\bar{\mathcal{E}}_t} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y^e} \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_m^e} e^{v_t} \tag{23}$$

with $\overline{\mathcal{E}}_{t+1}/\bar{\mathcal{E}}_t$ the depreciation required to reach the long-run equilibrium nominal exchange rate.[4] We assume that there is some smoothing in the way the nominal exchange rate adjusts to its target level

$$\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = \left(\frac{\mathcal{E}_{t+1}^*}{\mathcal{E}_t^*}\right)^{(1-\rho_e)} \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right)^{\rho_e} \tag{24}$$

The exchange rate rule and its properties have been documented, for the case of Singapore, by Khor et al. (2007). The central bank announces a path of appreciation of the exchange rate that is given by Δe_t^* and this determines the evolution of the nominal exchange rate. Note that this rule corresponds to a managed float and it is between a completely fixed exchange rate regime in which $E_t \Delta e_{t+1} = 0$ and a flexible exchange rate regime, that would correspond to the central bank using as an

[4] We follow the convention that an increase in the exchange rate implies depreciation of the domestic currency.

instrument the nominal interest rate, letting the exchange rate fluctuations be driven by market forces.

Business cycle dynamics under the two rules

The two monetary rules have different implications for business cycle fluctuations. These differences are amplified at a third order approximation, due to the presence of an endogenous and time-varying risk premium that breaks down the UIP condition.[5]

To understand how the two rules imply different business cycle dynamics, we first log-lineraize equations (22) and (24). From equation (22), we have

$$r_t = \rho r_{t-1} + (1-\rho)\phi_y \widetilde{y}_t + (1-\rho)\phi_m \widetilde{\pi}_t + u_t \tag{25}$$

where the lower case variables represent log-deviations of the variable with respect to its steady state. Under a Taylor rule, the central bank increases the nominal interest rate when inflation is higher than its target and when the output gap is positive. Instead, if inflation is low or the output gap becomes negative, the central bank stimulates the economy by lowering the nominal interest rate.

Similarly, if we put together equations (23) and (24), and we log-linearize the expression, we have

$$\Delta e_t = -(1-\rho_e)\phi_y^e \widetilde{y}_t - (1-\rho_e)\phi_m^e \widetilde{\pi}_t + \rho_e \Delta e_{t-1} + v_t \tag{26}$$

Under an exchange rate rule, the central bank stimulates the economy by depreciating the currency when inflation is low or the output gap is negative. In this case, the nominal exchange rate goes up (depreciates) to increase demand from the rest of the world. Notice that, unlike in the Taylor rule, when lower inflation and the output gap lead to a lower interest rate, the gradual depreciation under the exchange rate rule will lead to an increase in the interest rate, through the UIP condition.[6] When UIP does not hold, there is an additional effect on the nominal interest rate, which is driven by a precautionary saving motif.

We then evaluate how the two rules behave after a positive domestic productivity

[5] At a first order approximation of the model, there is no risk premium; at a second order approximation, there is a risk premium but it is constant; time varying fluctuations of the risk premium are captured at order 3 and introduce deviations from UIP.

[6] This is somewhat counterintuitive as "fighting inflation" implies a lower interest rate. The communication from the Monetary Authority of Singapore is quite clear in terms of using appreciation of the currency to lower inflation. Anecdotally, one can see the decline in interest rates, e.g. the announced appreciation of the Singapore dollar in October 2007, led to a decline in the one-month rates from 2.5% to 2.38% within a month and eventually to 1.13% before the following announcement in April 2008.

shock. A central bank that follows a Taylor rule decreases the nominal interest rate (see equation (25)) and the domestic currency depreciates. When UIP holds, non-arbitrage determines a future appreciation of the domestic currency, which leads to an overshooting of the nominal exchange rate. Under an exchange rate rule, however, the overshooting does not happen. The reason is that, after the shock, the central bank reacts by announcing a slow depreciation of the currency (see equation (26)). As forward-looking consumers expect that the currency will continue to depreciate, there is an excess supply of domestic bonds, which lowers the price of these bonds, hence increasing the nominal interest rate. When UIP holds, the increase in the nominal interest rate equals the expected future depreciation (see equation (11)). Because there is no overshooting, this increase is lower than the decrease in the domestic interest rate under the Taylor rule, and hence the exchange rate rule generates less fluctuations of domestic variables. Some degree of exchange rate smoothing in the rule is key to avoid overshooting under the exchange rate rule. Therefore, the two rules imply differences in business cycle fluctuations due to the different response of the exchange rate depreciation and the nominal interest rate.

External habits introduce deviations of the UIP condition at higher order approximations through the existence of a risk premium, which is endogenous and time-varying. An endogenous risk premium amplifies the differences between the interest rate rule and the exchange rate rule on business cycle fluctuations. After a positive domestic productivity shock, there is excess demand for foreign goods, and the domestic currency depreciates. Furthermore, the precautionary savings of the foreign consumers increase relative to that of the domestic investors and so the risk premium increases, both under the exchange rate rule and the Taylor rule. However, when the monetary authority is managing the exchange rate, the fluctuations of the risk premium are lower under the exchange rate rule. The precautionary savings motif amplifies the effect on interest rates under the exchange rate rule—albeit not significantly since the risk premium increases less in this case—and it counteracts it under the Taylor rule. This effect makes the Taylor rule less effective at stabilizing inflation with the interest rate, and it amplifies differences in economic variables under the two rules.

3 Quantitative exercise

We calibrate the model and perform a quantitative analysis to compare the performance of the two rules in terms of smoothing economic fluctuations. To simplify the analysis, we assume that the central bank adjusts its instrument (i.e., the short-term nominal interest rate or the nominal exchange rate) to smooth fluctuations of inflation, and we leave aside stabilization of the output gap. We analyze extended monetary rules in Appendix B. We report and analyze impulse responses, second moments, and the evolution of the risk premium. At a first order approximation of the model, differences between the two rules depend on the actual implementation: the fact that the central bank announces a gradual path of appreciation or depreciation of the currency. At a third order approximation, the performance of the policy rules will depend on how they affect the dynamics of the risk premium, which is time-varying (see Van Binsbergen, Fernandez-Villaverde, Koijen & Rubio-Ramirez (2012)). As in De Paoli & Sondergaard (2009), we use the log-linear version of the demand and supply conditions, while taking a third order approximation of the equations that depend on the risk premium directly. In that way, our non-linear model isolates the role of a time-varying risk premium. We call this the **hybrid** model. In appendix B, we present the simulation results after solving for a third order approximation of the full model. We use Dynare for our numerical exercises. In all of them, we perform 10,000 simulations and use pruning for the third order approximation.

3.1 Calibration

The calibrated parameters are reported in Table 1. We follow closely the parametrization of De Paoli & Sondergaard (2009) and Gali & Monacelli (2005).

Table 1: Calibrated Parameters

Parameter	Description	Value
h	Habit	0.85
δ	Degree habit	0.97
ε	Elast.subst.imports	6
η	Elast.subst.interm.	1
γ	Labor supply elast.	3/(1-h)
α	Openness	0.08
β	Discount Factor	0.99
θ	Price stickiness	0.75
σ	Intertemp. elast.	5

The parameters of habit persistence are set to $h = 0.85$ and $\delta = 0.97$. The elasticity of substitution across traded goods, ε equals 6, and across intermediate goods is $\eta = 1$. We set the Frisch labor supply parameter, γ equal to $\frac{3}{1-h}$.[7] The degree of openness, α, is set to 0.08 (see Lubik & Schorfheide (2007)). The discount

[7] In Appendix B, we report results for different values of the Frish labor supply parameter.

factor is set at $\beta = 0.99$, which implies a steady-state interest rate of 4% in a quarterly model. We assume the degree of price stickiness to be $\theta = 0.75$, which is consistent with the average period of price adjustment of one year, and the inverse of the intertemporal elasticity of substitution to be $\sigma = 5$, which is within the range found by the empirical literature of [2, 10].

As in Gali & Monacelli (2005), domestic productivity is assumed to have a standard deviation of 0.71%, while the foreign productivity shock has a standard deviation of 0.78%. The shocks are assumed to be positively correlated with correlation $\rho(\sigma_a, \sigma_a^*) = 0.3$.

For simplicity, and to illustrate our mechanism, we perform first the analysis with a simple rule in which the monetary authority adjusts its instrument, either the nominal interest rate or the nominal exchange rate, to react to fluctuations of inflation, with a certain degree of smoothing in the instrument. In particular, for the Taylor rule, we set $\phi_m = 1.5$, $\phi_y = 0$ and $\rho = 0.85$ as in Lubik & Schorfheide (2007). For the exchange rate rule, there is not a clear value for ϕ_m^e that allows us to compare the two rules exactly. Therefore, we consider two values, $\phi_m^e = \{1, 3\}$. We consider different parameter values to prevent the differences between the two rules from being driven by an arbitrary choice of the exchange rate rule parameters.[8] We assume the same degree of smoothing of the instrument, that is, $\rho = \rho^e = 0.85$. Later, we compare both rules under a wide range of alternative values of the policy parameters, to check whether there exists any combination of the parameters for which the two rules deliver identical business cycle dynamics. As a robustness check, we also extend the policy rules by allowing the monetary authority to react to the output gap in addition to inflation.

We analyze two cases. First, we study a first order approximation of the model, so that there are no deviations from UIP. Then, we do a third order approximation to introduce the effect of the time-varying risk premium that generates deviation from UIP.

3.2 Impulse response functions

We compare the qualitative performance of the two rules by computing impulse response functions to one-standard-deviation shocks to domestic productivity and foreign output. Figures 1 to 4 show the results.[9] In these exercises, we define the FX premium as the excess return on investing on domestic currency, that is, $fxp_t =$

[8] Several previous studies on optimal monetary policy tend to restrict $\phi_m \leq 3$ (see Schmitt-Grohé & Uribe (2007)). We do the same in our simulations.

[9] In the Figures, ERR(1) and ERR(3) correspond to an exchange rate rule in which the coefficient of inflation, ϕ_m^e, is set to 1 and 3, respectively.

$r_t - (r_t^* + E_t \Delta e_{t+1})$.

Uncovered Interest Parity Condition holds (order=1)

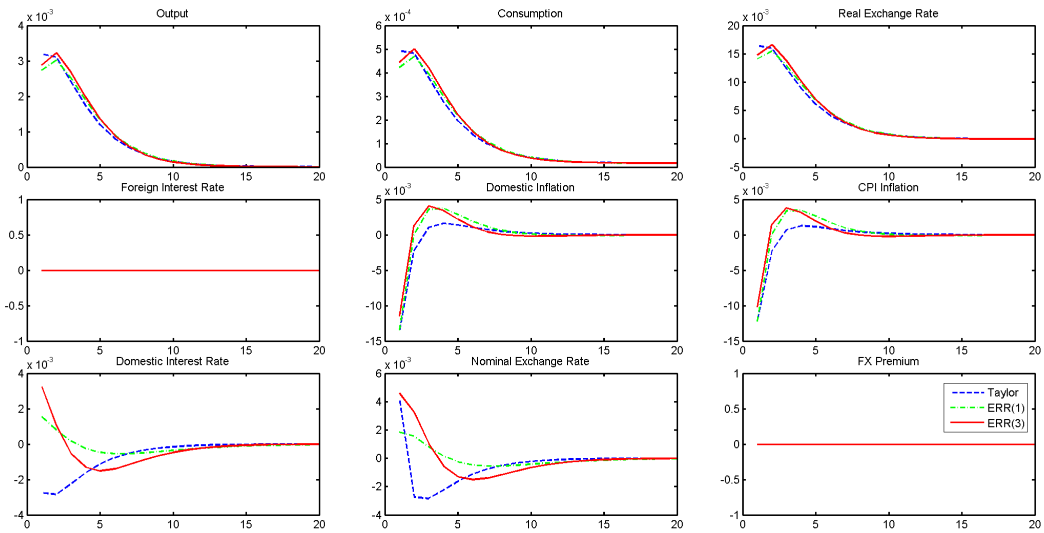
We start by reporting impulse responses for the case when UIP holds, hence there is no risk premium. This case corresponds to taking a first order approximation of the model.

After a positive domestic productivity shock (Figure 1), both output and consumption increase. Domestic inflation decreases because the economy is now more productive. The reaction of the central bank to these fluctuations is different under the two rules. If the central bank follows a Taylor rule, it will decrease interest rates (see equation (22)) and the domestic currency depreciates. The initial depreciation is followed by a future appreciation, since UIP holds and the initial increase in interest rate must be compensated for by a future appreciation of the currency. There is overshooting of the nominal exchange rate. CPI inflation decreases because the initial decrease in domestic inflation dominates the depreciation of the currency.

If, instead, the central bank follows an exchange rate rule, after the positive domestic TFP shock, the central bank reacts to the fall in inflation by announcing a gradual depreciation of the exchange rate. In this case, the nominal interest rate increases, since, as households expect a future depreciation of the currency, there is an excess supply of domestic bonds.

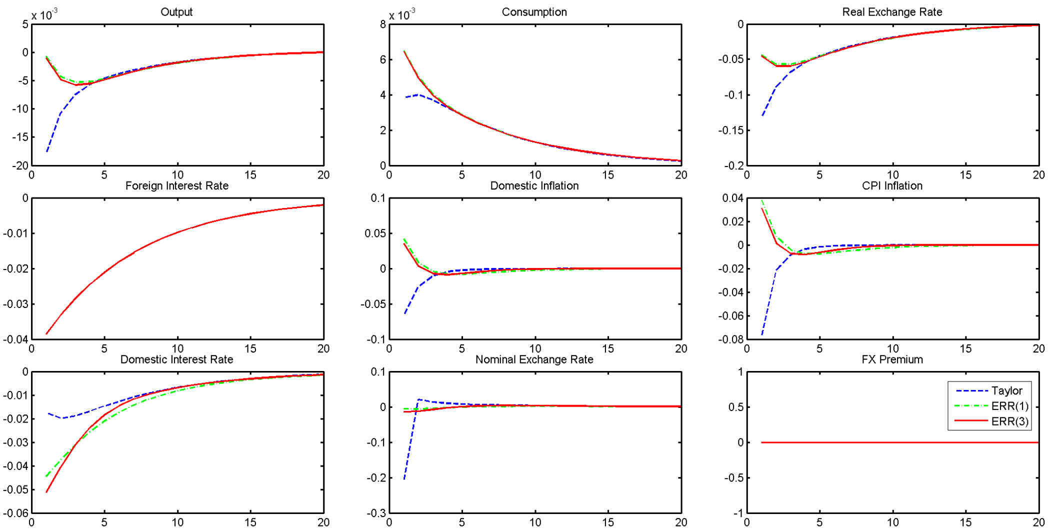
Therefore, the response of the nominal exchange rate and interest rate is different under the two rules, which translates into differences in business cycle dynamics.

Figure 1: Domestic productivity shock (order=1)



After a positive foreign shock (Figure 2), differences in business cycle dynamics caused by the two monetary rules become even more pronounced. The positive foreign shock induces excess demand for domestic goods by foreigners, which causes an appreciation of the domestic currency when the central bank follows a Taylor rule as exchange rates are flexible and determined by market forces. The appreciation causes a decrease in output and domestic inflation. Under the Taylor rule, the central bank lowers the interest rate. Under an exchange rate rule, however, the exchange rate is determined by the central bank. After the positive foreign shock, domestic inflation increases as there is an increase in demand from foreigners, and the central bank announces a path of appreciation of the currency. CPI inflation also increases because the announced appreciation does not compensate for the increase in domestic inflation. From the UIP condition, the foreign interest rate decreases substantially and, since the appreciation of the exchange rate is small, the domestic interest rate follows closely the dynamics of the foreign interest rate. Contrary to the case of a Taylor rule, output and consumption go up under the exchange rate rule; and, as with the Taylor rule, there is a larger appreciation of the domestic currency as a result of the decrease of the nominal interest rate.

Figure 2: Foreign productivity shock (order=1)



Our impulse response analysis shows that there exists qualitative differences in business cycle dynamics that are driven by the implementation of the monetary rule, especially when the small open economy experiences foreign shocks. The differences arise because with the exchange rate rule there is no overshooting of the nominal exchange rate, which helps stabilize the nominal variables directly affected by exchange rate fluctuation without increasing the volatility of other economic variables such as output and consumption.

Deviations from the Uncovered Interest Parity Condition (order=3)

We now show that differences in business cycle dynamics between the two rules are amplified at a third order approximation, due to the existence of a time-varying risk premium. When there is a time-varying risk premium, a new channel is introduced—a precautionary saving channel motive—that may cancel or reinforce the overshooting effect.

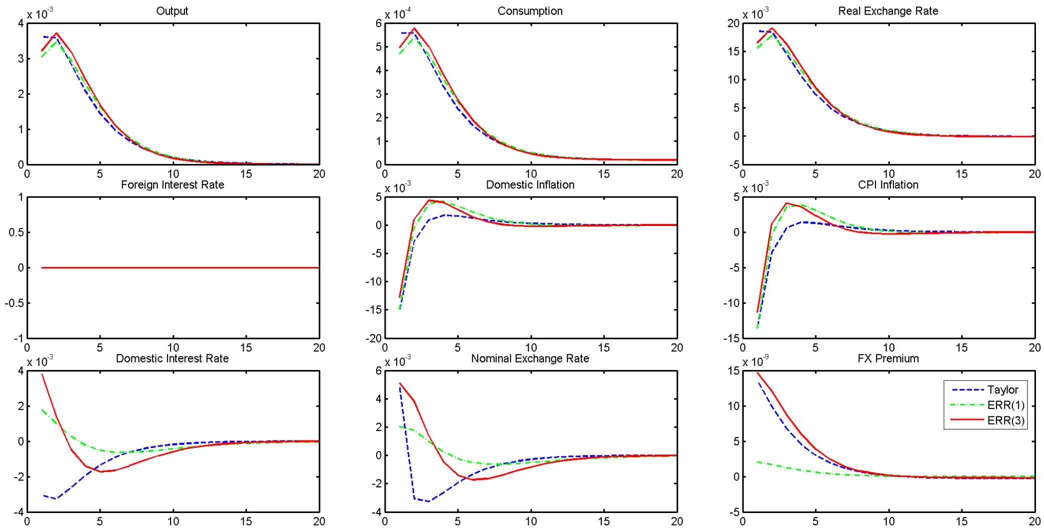
After a positive domestic productivity shock, the business cycle dynamics under the two rules are identical only to a first order approximation of the model (see Figure 3). The existence of a time-varying risk premium, however, introduces differences that are visible only at a higher order approximations. These differences are introduced through a precautionary saving motive.

After a positive domestic productivity shock, precautionary savings of the foreign consumers increase relative to those of the domestic investors, which increases the risk premium of foreigners for holding domestic bonds. This effect introduces a tendency for domestic interest rates to increase, under both rules. However, the final effect on the interest rate depends on whether the central bank follows an exchange rate rule or a Taylor rule, since interest rates move in opposite directions at a first order approximation. Under an exchange rate rule, the central bank announces a path of depreciation of the currency, which causes an increase in the domestic interest rate at a first order approximation. Therefore, the precautionary saving motif reinforces the increase in the domestic interest rate. As a result, under an exchange rate rule, the interest rate increases more if there is a time-varying risk premium than when UIP holds. Nevertheless, this additional increase is not very large because the risk premium does not increase significantly under the exchange rate rule.

When the central bank follows a Taylor rule, however, after a positive productivity shock, the central bank wants to decrease the domestic interest rate to stabilize inflation. The tendency of the interest rate to increase due to precautionary savings partially compensates for the decrease that the central bank would like to engineer to stabilize inflation, absent a risk premium. As a result, with a Taylor rule, interest rates increase by less than when UIP holds, and the central bank is less effective at stabilizing the economy. Hence, the presence of an endogenous risk premium amplifies business cycle fluctuations due to the effect of shocks in the precautionary savings of the consumers.

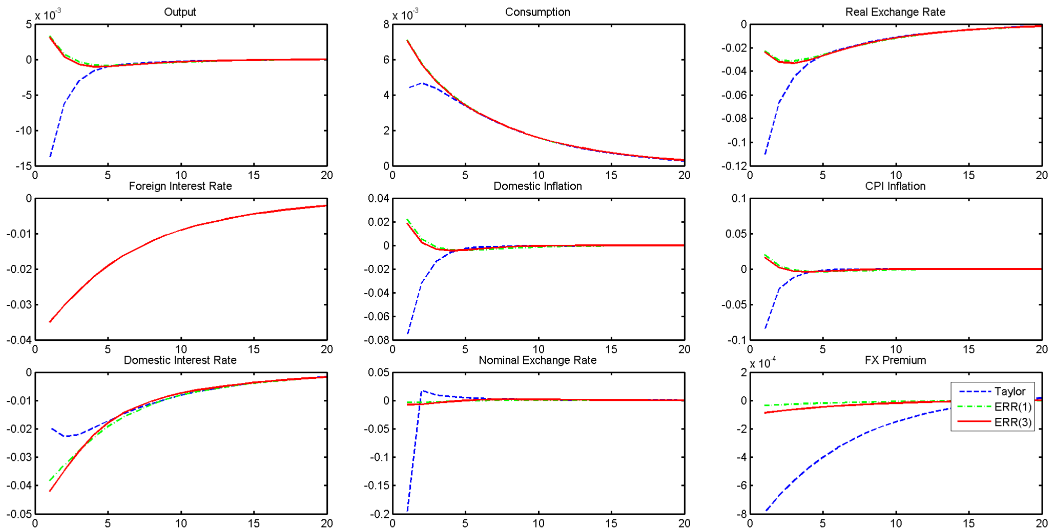
These results are valid for different values of ϕ_m^e. However, when the central bank reacts very intensively to fluctuations of inflation, i.e. $\phi_m^e = 3$, the business cycle dynamics look more similar to those of the Taylor rule. In this case, the central bank announces a steeper path of depreciation of the domestic currency, which features overshooting of the nominal exchange rate.

Figure 3: Domestic productivity shock (order=3)



After a positive foreign shock (see Figure 4), foreign consumers experience a decrease in their risk aversion relative to domestic consumers, and their risk premium for holding domestic currency decreases. In this case, the foreign interest rate has a tendency to increase. At a first order approximation, the domestic interest rate decreases under the two rules. The existence of the time-varying risk premium introduces an additional effect. In the case of the Taylor rule, interest rates will decrease more than when UIP holds. In the case of the exchange rate rule, the effect on the domestic interest rate will be similar to the case in which UIP holds, as the risk premium is less volatile. The lower volatility of the risk premium is driven by how the exchange rate rule is implemented, that is, by announcing a gradual appreciation or depreciation of the currency that does not cause overshooting of the exchange rate.

Figure 4: Foreign productivity shock (order=3)



Our qualitative results show that the two monetary policy rules generate different business cycle dynamics when UIP holds (i.e., at a first order approximation). These differences are amplified by deviations from UIP. The exchange rate rule generates less fluctuations than the Taylor rule, especially when the economy is exposed to foreign shocks. That is, small open economies that are exposed to shocks originating in the rest of the world may benefit from rules that use the exchange rate as their instrument to stabilize the economy.

3.3 Moments

In this section, we report moments for the main economic variables under the two monetary policy rules. We do that for first and third (hybrid model) order approximations of the model, so that we can isolate the role of the risk premium in accounting for quantitative differences in the volatility of key macroeconomic variables.

Table 2 reports moments of key variables for a first order approximation of the model. In this case, differences in the volatility of economic variables between the two rules are entirely driven by how they are implemented: the exchange rate rule with smoothing does not lead to overshooting as the central bank can manage expectations better. We find that the exchange rate rule generates lower volatility of inflation, output, and the real and nominal exchange rates, at the cost of increasing fluctuations in the nominal interest rate and consumption. Differences between the two rules at a first order approximation are driven by whether or not the central bank generates an overshooting of the nominal exchange rate. There is no role for the risk premium in this case. Indeed, as the bottom panel of Table 2 shows, the volatility of the risk premium is zero.

Table 2: Moments: No risk premium

	Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	1.89%	0.48%	0.51%
$\sigma_{\Delta C}$	0.42%	0.69%	0.69%
$\sigma_{\Delta Q}$	13.73%	4.53%	4.73%
σ_{R}	4.72%	7.86%	8.11%
σ_{R^*}	7.40%	7.40%	7.40%
$\sigma_{\Delta E}$	21.02%	1.02%	2.13%
σ_{π_H}	7.75%	4.29%	3.61%
σ_{π}	8.61%	3.91%	3.22%
σ_{fxp}	0.00%	0.00%	0.00%
$\sigma_{\Delta E^e}$	2.89%	0.88%	1.68%
$\sigma_{fxp,\Delta E^e}$	0.00%	0.00%	0.00%

Note: No risk premium corresponds to an approximation of order=1

Table 3 reports moments for the hybrid model. In this case, the different performance between the two rules will be driven by the interaction between the implementation of the rule and the existence of a time-varying risk premium. We find that, in this case, the exchange rate rule is even more successful than the Taylor rule at reducing fluctuations of inflation. Indeed, under the Taylor rule, inflation volatility increases when there are fluctuations of the risk premium. However, under the exchange rate rule, inflation volatility is reduced by half, as the monetary authority is more effective at stabilizing the economy by introducing less fluctuations in the risk premium, The volatility of the risk premium is 0.01% under an exchange rate rule versus 0.15% under a Taylor rule. Therefore, the different effect that the two rules have on the risk premium contributes to amplifying differences in business cycle dynamics between the two rules. As we described in the impulse response analysis, precautionary savings motives counteract the desired effect on interest rates by a central bank with a Taylor rule, making it less effective at stabilizing inflation.

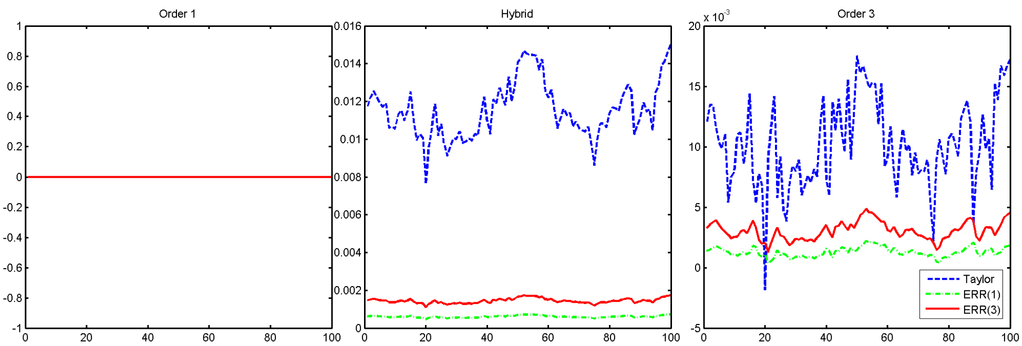
Table 3: Moments: Time-varying risk premium

	Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	1.62%	0.60%	0.61%
$\sigma_{\Delta C}$	0.48%	0.76%	0.76%
$\sigma_{\Delta Q}$	12.19%	2.58%	2.71%
σ_R	5.66%	7.40%	7.55%
σ_{R^*}	7.15%	7.15%	7.15%
$\sigma_{\Delta E}$	20.26%	0.58%	1.22%
$\sigma_{\pi H}$	9.26%	2.45%	2.07%
σ_π	9.94%	2.23%	1.85%
σ_{fxp}	0.15%	0.01%	0.01%
$\sigma_{\Delta E^e}$	2.10%	0.50%	0.97%
$\sigma_{fxp,\Delta E^e}$	-0.00245%	0.00001%	0.00005%

Note: Time-varying risk premium corresponds to an approximation of order=3

Finally, we compute the simulated risk premium for the small open economy under the two rules. Consistent with our previous results, we find that the risk premium is lower and less volatile under an exchange rate rule than under an interest rate rule (see Figure 5).[10]

Figure 5: Risk premium



Note that, when $\phi_m^e = 3$, the business cycle dynamics become more similar under the two rules. In this case, there is overshooting of the nominal exchange rate even

[10]In Figure 5, hybrid corresponds to a model in which the demand and supply conditions are linearized, but the equations that contain the risk premium are non-linear. **Order 3** corresponds to the full third order approximation of the model and we discuss further about this case in Appendix B.

under the exchange rate rule, and the business cycle properties of the risk premium looks closer to Taylor.

3.4 The importance of openness in selecting the monetary policy rule

Small open economies are more exposed to foreign shocks; hence, exchange rate fluctuations affect these economies more than closed economies. Therefore, as the degree of openness of an economy increases, stabilizing inflation through the exchange rate may be a more important objective of the central bank. In our model, the degree of openness of the economy is captured by α. In Table 4, we report moments for the hybrid model for a larger value of α. In particular, we set it to 0.3, while in our baseline calibration it was set to 0.08.

We find that for a larger degree of openness, the differences between the Taylor rule and the exchange rate rule become even more striking. Inflation becomes more volatile under a Taylor rule (from 9% when $\alpha = 0.08$ to around 14% when $\alpha = 0.3$), whereas the exchange rate rule is quite successful at stabilizing inflation (when $\alpha = 0.08$ from 2.23% to 0.82% when $\alpha = 0.3$). The role of the risk premium becomes more evident in this case. Under an exchange rate rule, the volatility of the risk premium is almost zero. Furthermore, the UIP coefficient in a regression testing for deviations from UIP ($\hat{\beta}_{UIP}$ in the table) reveals that, with an exchange rate rule, we cannot reject the hypothesis of UIP holding. However, in the case of the Taylor rule, the UIP coefficient is much lower, around 0.5323.

These results suggest that, as the economy becomes more exposed to foreign shocks, a monetary rule that uses the nominal exchange rate as its instrument will be more successful at reducing business cycle fluctuations.

Table 4: Moments: Time-varying risk premium and $\alpha = 0.3$

	Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	3.24%	0.59%	0.60%
$\sigma_{\Delta C}$	0.67%	0.79%	0.79%
$\sigma_{\Delta Q}$	5.27%	0.93%	0.95%
σ_R	7.05%	7.21%	7.23%
σ_{R^*}	7.15%	7.15%	7.15%
$\sigma_{\Delta E}$	18.64%	0.18%	0.38%
σ_{π_H}	12.79%	1.21%	1.07%
σ_π	14.38%	0.82%	0.68%
σ_{fxp}	0.18%	0.00%	0.00%
$\sigma_{\Delta E^e}$	0.73%	0.14%	0.26%
$\sigma_{fxp,\Delta E^e}$	-0.00037%	0.00000%	0.00000%
$\hat{\beta}_{uip}$	0.5323	0.9854	0.9804
$\sigma_{\hat{\beta}_{uip}}$	0.2654	0.0081	0.0105

Note: Time-varying premium corresponds to an approximation of order=3

3.5 The importance of the intertemporal elasticity of substitution

Another important parameter in our calibration is the intertemporal elasticity of substitution, $1/\sigma$, which captures the willingness of households to smooth consumption over time and their risk premium. This parameter determines the elasticity of real variables to changes in the real interest rate: the larger the σ, the less elastic consumption is to changes in the real interest rate.

We perform our simulations for two extreme values of the parameter: $\sigma = 2$ and $\sigma = 10$. Table 5 reports the results. For lower values of σ, differences between the Taylor rule and the exchange rate rule in terms of fluctuations in output and the real exchange rate are lower than for larger values. For $\sigma = 10$, instead, real variables become much less volatile under the exchange rate rule than under the Taylor rule. The main reason is that the effectiveness of the Taylor rule relies on the household's willingness to smooth consumption over time in response to changes in the real interest rate. A lower intertemporal elasticity of substitution (i.e., a higher σ), however, decreases the responsiveness of real variables to interest rate changes and thereby the effectiveness of a monetary policy that uses the interest rate as its instrument.

Table 5: Moments: Time-varying risk premium and the intertemporal elasticity of substitution

	$\sigma = 2$			$\sigma = 10$		
	Taylor	ERR		Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	0.62%	0.63%	0.64%	3.44%	0.56%	0.57%
$\sigma_{\Delta C}$	0.45%	0.72%	0.72%	0.50%	0.77%	0.77%
$\sigma_{\Delta Q}$	5.54%	2.09%	2.19%	23.21%	3.13%	3.30%
σ_R	2.04%	3.16%	3.28%	11.83%	13.72%	13.89%
σ_{R^*}	2.97%	2.97%	2.97%	13.39%	13.39%	13.39%
$\sigma_{\Delta E}$	8.51%	0.45%	0.95%	39.59%	0.72%	1.51%
σ_{π_H}	3.59%	1.99%	1.69%	18.96%	2.96%	2.49%
σ_π	3.88%	1.81%	1.51%	20.23%	2.69%	2.23%
σ_{fxp}	0.02%	0.00%	0.00%	0.63%	0.01%	0.03%

Note: Time-varying premium corresponds to an approximation of order=3

4 Risk Premium and UIP: Understanding the mechanism

An important result of our model is that quantitative differences between the performance of the Taylor rule and the exchange-rate-based rule are amplified when we take into account dynamics of the uncovered interest parity condition. Here, we illustrate analytically the mechanics of the two rules in generating fluctuations in the risk premium that drive the differences in business cycle dynamics. To ease the exposure of the analysis, we assume an endowment economy in which all variables are conditionally and jointly log-normal. In a production economy like ours, we will have additional dynamics, but the endowment economy allows us differences in the risk premium implied by the two rules. We follow the methodology in Backus et al. (2010).

As above, the UIP condition can be written as:

$$E_t \left\{ \mathcal{M}_{t,t+1}(R_t - R_t^*(\mathcal{E}_{t+1}/\mathcal{E}_t)) \right\} = 0$$

Most models approximate the dynamics of this equation by using log-linearization, which yields the familiar expression implying that the expected rate of depreciation is equal to the interest rate differential between the home and the foreign economies:

$$r_t - r_t^* = E_t \Delta e_{t+1}$$

Under conditional log-normality, the UIP condition has a different form:

$$r_t - r_t^* = E_t \Delta e_{t+1} + \frac{1}{2} var_t(\Delta e_{t+1}) + cov_t(m_{t+1}, \Delta e_{t+1})$$

$$fxp_t = \frac{1}{2} var_t(\Delta e_{t+1}) + cov_t(m_{t+1}, \Delta e_{t+1}) = \frac{1}{2} var_t(m^*_{t+1}) - \frac{1}{2} var_t(m_{t+1}) \quad (27)$$

The additional two terms in the UIP condition capture the risk premium (possibly, time-varying), which we denote by fxp_t.[11] Verdelhan (2010) and De Paoli & Sondergaard (2009) use this setup to explain the forward premium puzzle documented by Fama (1984). The introduction of second-order terms is not sufficient in itself to explain the empirical puzzle related to the interest parity condition. Therefore, Verdelhan (2010) and De Paoli & Sondergaard (2009) introduce habit formation in their models. In order to solve the UIP puzzle, these papers generate a risk premium that co-varies negatively with the expected rate of depreciation. This negative correlation arises because models with habit formation generate a time-varying coefficient of relative risk aversion. Under the assumptions of the model, the coefficient is given by:

$$CRRA_t = \sigma \frac{C_t}{C_t - hX_t}$$

To see the mechanics, let's consider a negative domestic TFP shock. Since consumption declines, the standard intertemporal substitution effect will make domestic consumers re-optimize by shifting future consumption to the present, which will cause a decrease in saving and hence an increase in interest rates. At the same time, the drop in domestic production generates excess demand for domestic goods and appreciation of the currency. As the economy is expected to return to steady state after the temporary shock, this appreciation must be followed by expected depreciation. This is the standard UIP condition. In a model with habits, there is a second effect, however. As shown in the expression for $CRRA_t$, when consumption declines and gets closer to the level of habit, risk aversion increases. This generates a precautionary savings motif – consumers are less willing to shift as much consumption from the future to today, which alleviates the pressure on the interest rate. If the precautionary savings channel is very strong, the economy may even get to a lower interest

[11] The second equality in the definition of fxp_t comes from the international risk sharing assumption.

rate, i.e. expected currency depreciation will co-exist with a lower interest rate, as documented in the empirical literature on the forward premium puzzle. We find that when the monetary authority follows an exchange rate rule, the precautionary savings motif is weaker, and the deviations from UIP are more muted. That is why the volatility of the risk premium is lower when the instrument of monetary policy is the exchange rate.

We now derive an analytical expression for the foreign exchange risk premium as a function of the parameters of an exchange rate monetary policy rule. We follow Backus et al. (2010), but depart from them by using a utility function with habits in consumption. Once we have an expression of the domestic nominal pricing kernel, we follow De Paoli & Sondergaard (2009) to derive the risk premium.

The key is to build a model that endogenously determines inflation. In the basic setup Backus et al. (2010) use two equations for two variables (i_t, π_t):

1. Nominal interest rate as a function of the log-linear pricing kernel (which depends on inflation).
2. Taylor rule determining nominal interest rate as a function of inflation.

We solve for the case in which the monetary authority reacts to both inflation and consumption growth, that is:

$$de_{t+1} = \phi_\pi \pi_{t+1} + \phi_c E_t(c_{t+1} - c_t) \tag{28}$$

and

$$i_t = -log(\beta) + \phi_\pi E_t \pi_{t+1} + \phi_c E_t(c_{t+1} - c_t) \tag{29}$$

After deriving a solution to inflation, one can express the nominal pricing kernel as a function of exogenous variables. With this, one can derive the foreign exchange risk premium (see Appendix C). We obtain the following analytical solution for the foreign excess return under the two rules:

1. ERR

$$fxp_t = \frac{1}{2}\left[(\rho^2 - b_{\epsilon^*})\sigma^2_{\epsilon^*} - \frac{[b_\epsilon + h(b_\epsilon - b_s)]^2\sigma^2_\epsilon}{(1-h)^2}\right] - \frac{b_s h[b_\epsilon + h(b_\epsilon - b_s)]}{(1-h)^3}\sigma^2_\epsilon[(1-\phi-\rho_c)c_t + \phi x_t]$$

2. Taylor rule

$$fxp_t = \frac{1}{2}\sigma_{\epsilon^*} b^*_\epsilon - \frac{1}{2}\left[\left(\frac{(\rho+\psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right)^2 \sigma^2_\epsilon + (\psi_{\epsilon^*} + \psi_{c^*})^2 \sigma^2_{\epsilon^*}\right] \tag{30}$$

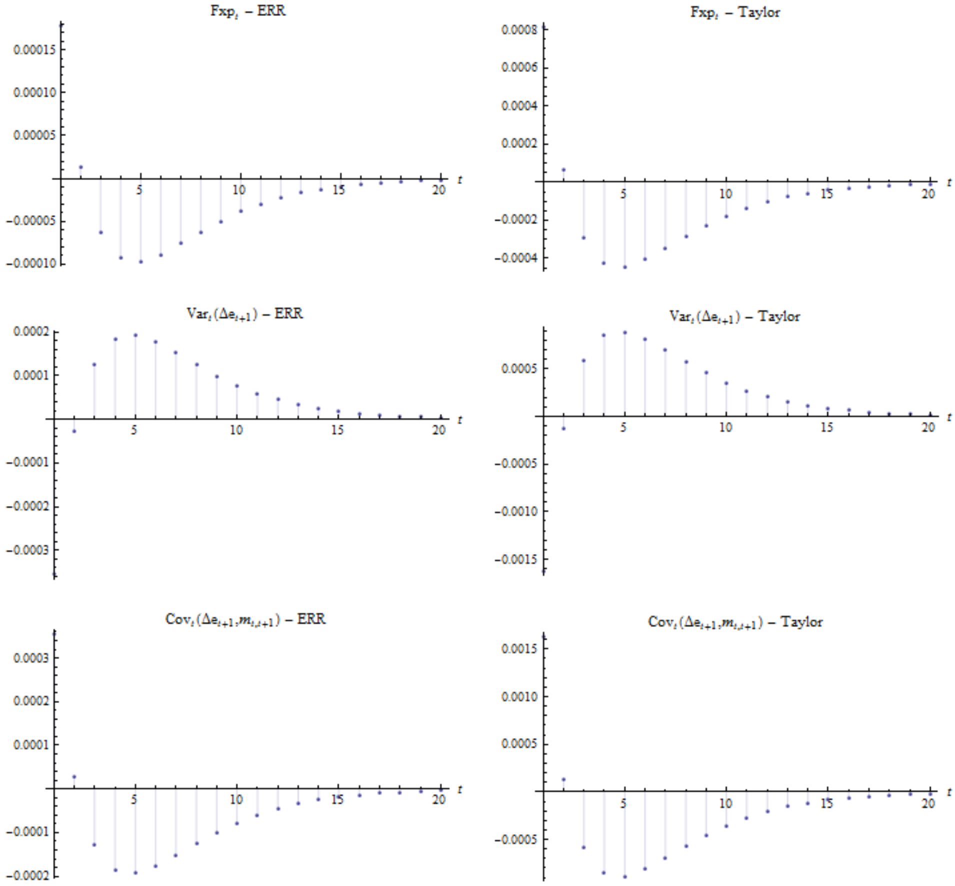
$$-\sigma_{\epsilon}^{2}\frac{(\rho+\psi_s)h}{(1-h)^2}\left[\frac{(\rho+\psi_s)h}{1-h}+\rho+\psi_{\epsilon}+\psi_c\right][(1\text{-}\phi\text{-}\rho_c)c_t+\phi x_t] \tag{31}$$

We observe that the risk premium has a different analytical expression, depending on the particular instrument of monetary policy. It is affected by changes in the volatility and persistence of the shock, the habit parameter, and the coefficients of the monetary policy rules, and these components have a different impact on the risk premium depending on whether the central bank uses the nominal interest rate or the nominal exchange rate as the instrument.

To shed light on how analytical differences in the risk premium under the two rules may drive different economic fluctuations, we perform a simple numerical analysis in which we plot the risk premium under the two rules, by setting values to the parameters in equations (30) and (31). In the top panel of Figure 6, we plot the risk premium under the two rules. For simplicity, we assume a monetary rule in which the central bank reacts to fluctuations of inflation alone–that is, we set $\phi_c = 0$ in equations (28) and (29). We find that the risk premium is lower and less volatile under the exchange rate rule. From equation (27), we can decompose the risk premium into the conditional variance of the expected depreciation rate, $var_t(\Delta e_{t+1})$, and the covariance between the stochastic discount factor and the expected rate of depreciation of the currency, $cov_t(m_{t+1}, \Delta e_{t+1})$. In the middle and bottom panels of Figure 6, we observe that both components are lower under the exchange rate rule; hence, both components contribute at lowering the risk premium in that case.[12]

[12]Santacreu (2015) shows, in an endowment economy, how the two rules generate different business cycle dynamics owing to differences in the business cycle dynamics of the risk premium.

Figure 6: Risk premium decomposition



Note that our analytical derivation of the risk premium does not take into account smoothing of the instrument in either of the rules. We have done this to ease the analytical exposition. Our results show that, in this case, the two rules deliver differences in the risk premium through its two components: the covariance of the exchange rate and the stochastic discount factor, and the volatility of the exchange rate. These differences will be larger if smoothing of the instrument were allowed since, in this case, the implementation of the rules plays a key role in generating differences in business cycle dynamics. Therefore, we can think of the differences in risk premium and its components that we obtain in this section without smoothing as a lower bound. Adding smoothing of the instrument will introduce larger differences between the rules, at the expense of complicating the algebra substantially.

5 Conclusion

The use of the exchange rate as an instrument of monetary policy was pioneered by the Monetary Authority of Singapore and has supported the economic growth of the country by ensuring low inflation since its implementation in 1981. Certainly, credit for this achievement also goes to other dimensions of economic policy. Nevertheless, this article has shown how a theoretical model based on optimizing behavior of households and producers is able to generate powerful conclusions about the desirability of the exchange rate as an instrument of monetary policy. More generally, we show that in a standard microfounded monetary model, relatively open economies can generate an improvement in reducing economic fluctuations by adopting an exchange rate rule. The improvement comes from a reduction in the volatility of key macroeconomic variables such as inflation and output, especially when households exhibit habit formation behavior.

The model reveals that there are two key sources of the reduction in volatility. First, in implementing the exchange rate rule, the central bank announces a gradual depreciation rate, which avoids the standard overshooting result. For small open economies, where a large part of the price level is determined by prices of imported goods, this policy already reduces the volatility of exchange rates and thus prices of imports. To identify the second channel, we follow recent advances in international monetary economics and asset pricing, which build into standard models countercyclical risk premia derived endogenously from habits in consumption. The time-varying risk premia drive a wedge between exchange rate movements and the interest rate differential thus further separating the implied dynamics of interest rate rules from the dynamics implied by adopting an exchange rate rule. We show that for open economies the time series properties of the risk premium differ considerably between the rules.

We have kept the model to a bare minimum in terms of its economic structure in order to identify the key factors behind the observed differences between the two rules. There are many directions in which the model can be extended to gain further insights in the desirability of exchange rate rules. First and foremost, increasing the interest rate sensitivity of key economic sectors may lead to a significant improvement in the performance of the interest rate rule. This may occur due to the importance of investment in economic fluctuations or by including a financial accelerator as in Bernanke, Gertler & Gilchrist (1999). Second, our model does not distinguish between tradable and non-tradable goods. At the same time, secular changes in relative prices due to convergence in income per capita, for example, might present interesting problems for the exchange rate rule. Finally, in the past few years, standard monetary policy rules have been put to a test by the well-known zero lower bound on nominal interest rates. This minimum bound created a problem for economies that use the interest rate as an instrument of monetary policy, forcing them to switch to quantitative easing once interest rates reached zero. For an economy operating with an exchange rate rule, the challenge is different (Amador, Bianchi, Bocola & Perri

(2017)). When the anchor currency country lowers rates to zero, while the domestic economy overheats, the response should be future appreciation of the domestic currency. To meet the no-arbitrage condition implied by UIP, domestic rates have to go below zero. And even though in the past few years negative rates have been observed, there is a limit to how low negative rates can go. These three extensions are not only interesting from a modeling point of view, but they are clearly relevant for the actual implementation of the exchange rate rule in small open economies.

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Online Appendix

A Equations and steady state

A.1 Model equations

We describe the equilibrium in this model. The equilibrium determines the following variables:

$\left\{C_t, X_t, C_{Ht}, C_{Ft}, A_t, W_t, \frac{P_{Ht}}{P_t}, \frac{P_{Ft}}{P_t}, \Pi_t, N_t, R_t, S_t, Q_t, C_t^*, e_t, Y_t, N_t, MC_t, Y_t^*, \Pi_t^*, R_t^*\right\}$

The equations that determine the equilibrium are:

Households

$$R_t E_t \left\{ \beta \left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\} = 1$$

$$X_t = \delta X_{t-1} + (1-\delta) C_{t-1}$$

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t$$

$$C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t$$

$$(C_t - hX_t)^{\sigma} N_t^{\gamma} = \frac{W_t}{P_t}$$

Firms

$$Y_t = A_t N_t$$

$$a_t = \rho a_{t-1} + u_t$$

$$W_t = MC_t P_{Ht} A_t$$

Price setting

$$\frac{\tilde{P}_{H,t}}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{H_t}{F_t}$$

$$F_t = \Lambda_t Y_t + \theta \beta E_t (F_{t+1} (\Pi_{t+1})^{\varepsilon - 1})$$

$$H_t = \Lambda_t MC_t Y_t + \beta \theta E_t (H_{t+1} (\Pi_{t+1})^{\varepsilon})$$

$$\Lambda_t = (C_t - hX_t)^{-\sigma}$$

$$\frac{P_{Ht}}{P_t} = \left((1 - \theta) \left(\frac{\tilde{P}_{H,t}}{P_t} \right)^{1-\varepsilon} + \theta \left(\frac{P_{Ht-1}}{P_{t-1}} \right)^{1-\varepsilon} \Pi_t^{\varepsilon - 1} \right)^{\frac{1}{1-\varepsilon}}$$

Market clearing condition

$$Y_t = C_t \left[(1 - \alpha) S_t^{\eta} Q_t^{-\eta} + \alpha Q_t^{-\frac{1}{\sigma}} \right]$$

Monetary policy rule

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\rho} \left(\frac{Y_t}{\bar{Y}} \right)^{(1-\rho)\phi_y} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{(1-\rho)\phi_m}$$

$$\frac{e_{t+1}}{e_t} = \left(\frac{e^*_{t+1}}{e^*_t} \right)^{(1-\rho_e)} \left(\frac{e_t}{e_{t-1}} \right)^{\rho_e}$$

Terms of trade and real exchange rate

$$S_t = \frac{P_{Ft}}{P_{Ht}}$$

$$Q_t = \frac{P_{Ft}}{P_t}$$

$$P_{Ft} = \mathcal{E}_t P_t^*$$

Uncovered interest parity

$$E_t \left\{ \mathcal{M}_{t,t+1} \left(R_t - R_t^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \right\} = 0$$

International risk sharing condition

$$\beta \left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = \mathcal{M}_{t,t+1}$$

Foreign economy

$$C_t^* = Y_t^*$$

$$\left(\frac{Y_t^*}{\bar{Y}} \right) = \left(\frac{Y_{t-1}^*}{\bar{Y}} \right)^{\rho_y} exp(u_t^y)$$

$$R_t^* E_t \left\{ \beta \left(\frac{C_{t+1}^* - hX_{t+1}^*}{C_t^* - hX_t^*} \right)^{-\sigma} \right\} = 1$$

A.2 The Steady State

We solve for the symmetric steady state. Here are the equations.

- $\tilde{C} = (1-h)C$
- $X = C$
- $\beta R = 1$
- $P_H = P_F = P$
- $C_H = (1-\alpha)C$
- $C_F = \alpha C$
- $Y = C$
- $\frac{W}{P} = \omega = \frac{\varepsilon - 1}{\varepsilon}$
- $Y = N = (1-h)^{\frac{-\sigma}{\sigma+\psi}} \left(\frac{\varepsilon-1}{\varepsilon} \right)^{\frac{1}{\sigma+\psi}}$
- $Q = 1$
- $MC = \frac{\varepsilon-1}{\varepsilon}$
- $F = \frac{\Lambda Y}{1-\theta\beta}$
- $H = FMC$

- $\Lambda = C^{-\sigma}(1-h)^{-\sigma}$
- $\frac{\tilde{P}_H}{P_t} = 1$

A.3 Equivalence of the monetary rules

In our numerical exercise, we studied differences in business cycle dynamics of two alternative monetary policy rules, assuming that, in both cases, the central bank reacts to exactly the same economic variables, although with a different intensity–as shown by differences in the policy parameters. That is, we have assumed that under both rules the central bank reacts to smooth only fluctuations of inflation, with a certain degree of smoothing in the respective instruments. We then showed that, regardless of the value of the policy parameters, the exchange rate rule outperforms the Taylor rule in smoothing economic fluctuations. There are two reasons for this: (i) the way in which the exchange rate rule is implemented, and (ii) the effect that the particular implementation of the rule has on the risk premium.

An interesting question to study is whether, without imposing that the two rules have to react to the exact same economic variables, there exists an interest rate rule that is able to mimic the business cycle dynamics of an exchange rate rule. In this section, we try to answer this question. We start by assuming an exchange rate rule in which the central bank reacts only to fluctuations of inflation. We then use the equations of the model to derive how the domestic nominal interest rate should adjust to generate the same economic fluctuations as the exchange rate rule that we assume.

Following our previous analysis, we log-linearize the monetary policy rules around the steady state. That is,

$$\Delta e_t = -\phi_m^e \pi_t \tag{32}$$

Our goal is to study whether there exists an interest rate rule that delivers identical business cycle fluctuations. We do that by using the main equilibrium conditions of our model.

Let's start by adding and subtracting i_t from equation (32)

$$i_t = \phi_m^e \pi_t + \Delta e_t + i_t \tag{33}$$

From the definition of the real exchange rate, we have

$$\Delta e_t = \Delta q_t + \pi_t - \pi_t^* \tag{34}$$

Then, in the case in which UIP holds, we can use the log-linearized version of equation (10)

$$\Delta q_t = \sigma \Delta \tilde{c}_t - \sigma \Delta \tilde{c}_t^* \tag{35}$$

with $\tilde{C}_t = C_t - hX_t$.

Combining (33) and (34), we have

$$i_t = \phi_m^e \pi_t + \Delta q_t + \pi_t - \pi_t^* + i_t \tag{36}$$

Plugging equation (35) into (36)

$$i_t = \phi_m^e \pi_t + \sigma \Delta \tilde{c}_t - \sigma \Delta \tilde{c}_t^* + \pi_t - \pi_t^* + i_t \tag{37}$$

Finally, using the UIP condition $i_t = \Delta e_{t+1} + i_t^*$, and equation (35) one period ahead, we have

$$i_t = \phi_m^e \pi_t + \sigma \Delta \tilde{c}_t - \sigma \Delta \tilde{c}_t^* + \pi_t - \pi_t^* + \sigma E_t \Delta \tilde{c}_{t+1} - \sigma E_t \Delta \tilde{c}_{t+1}^* + E_t \pi_{t+1} - E_t \pi_{t+1}^* + i_t^* \tag{38}$$

Rearranging,

$$i_t = (1 + \phi_m^e) \pi_t + \sigma \Delta \tilde{c}_t + \sigma E_t \Delta \tilde{c}_{t+1} + E_t \pi_{t+1} + u_t^* \tag{39}$$

with $u_t^* = -E_t \pi_{t+1}^* + i_t^* - \sigma \Delta \tilde{c}_t^* - \pi_t^* - \sigma E_t \Delta \tilde{c}_{t+1}^*$.

Therefore, from equation 39, an interest rate rule that mimics the business cycle dynamics of the exchange rate rule (32) is one in which the monetary authority adjusts the nominal interest rate whenever there are changes in current and future inflation, current and future consumption and foreign shocks (see equation (39)). Note that this rule implies that the central bank can successfully react to foreign shocks to stabilize the economy.

We then study the business cycle dynamics of the two equivalent rules as defined in equations (32) and (39). At a first order approximation, they should generate the same business cycle dynamics, since as they are implemented in the same way, both will generate an overshooting of the nominal exchange rate. At a third order approximation, the equivalence is not as clear, since there is the presence of a risk premium that moves endogenously with changes in the shocks. To isolate the effect of the risk premium in driving differences between the interest rate rule and the exchange rate rule, we simulate a third order approximation of our model under (32) and (39). We find that these differences are very small, even at a third order approximation, both for domestic and foreign shocks (see Figures (7) and (8)).

Figure 7: Domestic productivity shock for equivalent rules (order=3)

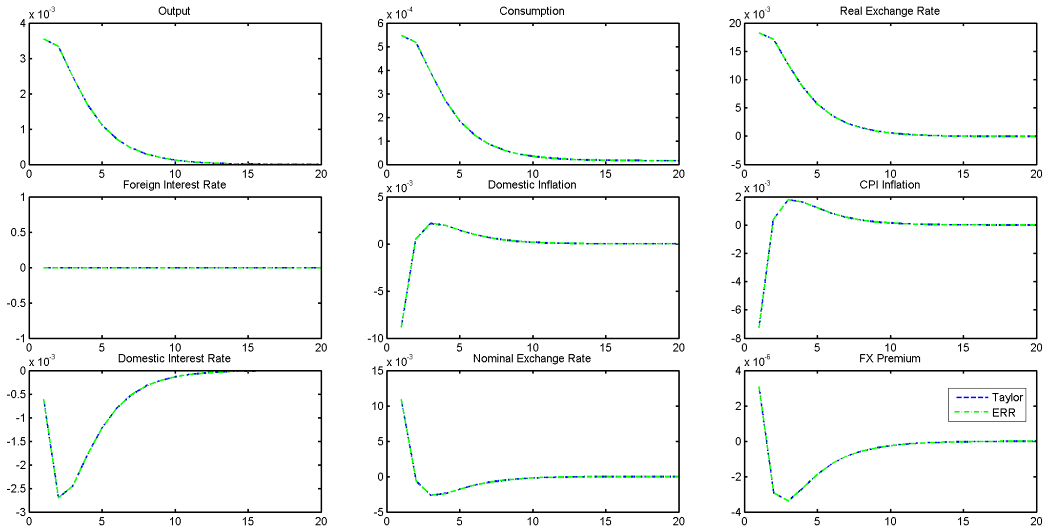
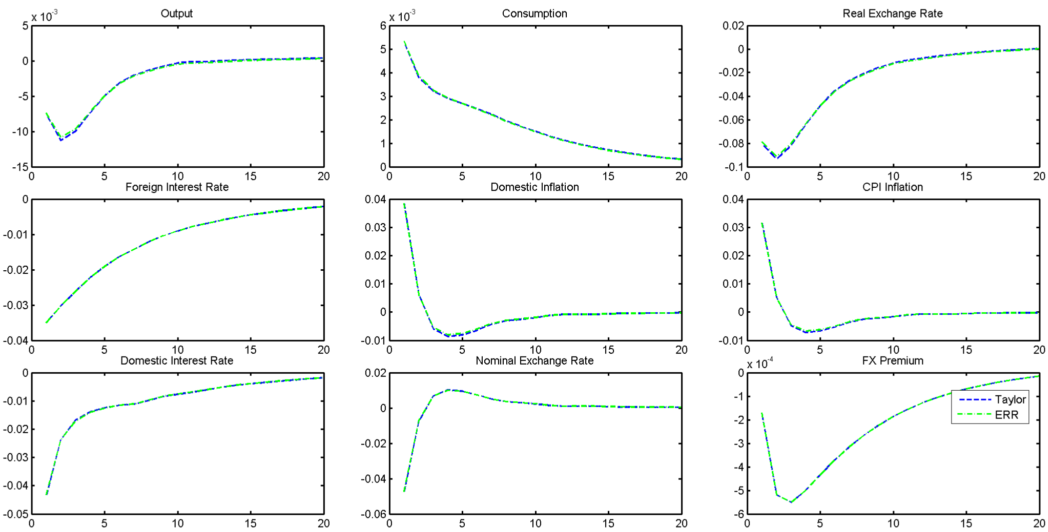


Figure 8: Foreign productivity shock for equivalent rules (order=3)



In Table 6, we report moments of key economic variables and the risk premium for a third order approximation of the model under our baseline calibration of the shocks, as well as for a larger persistence and volatility of the shocks.[13] Consistent

[13]De Paoli & Sondergaard (2009) find that models of external habit in consumption that feature higher persistence and volatility of the shocks deliver deviations from UIP that are closer to those observed in the data.

with our findings from the impulse response functions, the business cycle dynamics of the two rules are very similar, and only slight differences arise when the persistence or the volatility of the shocks is large.

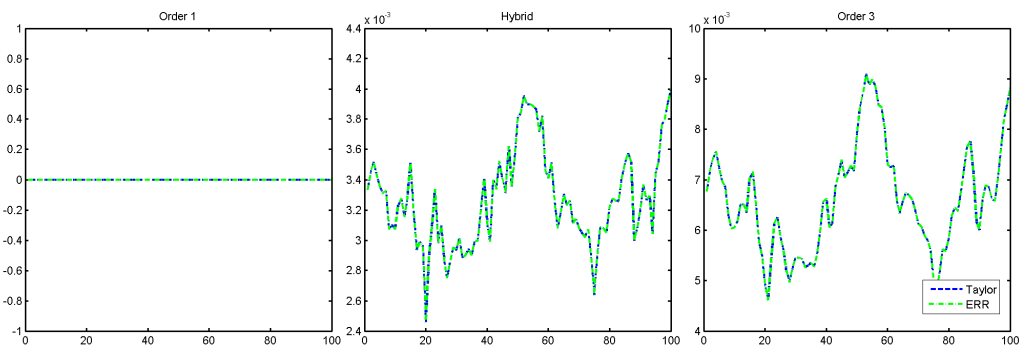
Table 6: Moments: Time-varying risk premium

	Baseline		High persistence		High volatility	
	Taylor	ERR	Taylor	ERR	Taylor	ERR
$\sigma_{\Delta Y}$	0.83%	0.80%	1.03%	1.00%	3.91%	3.69%
$\sigma_{\Delta C}$	0.63%	0.63%	0.62%	0.62%	1.11%	1.13%
$\sigma_{\Delta Q}$	7.35%	7.20%	8.01%	7.82%	26.37%	25.17%
σ_{R}	6.66%	6.63%	1.84%	1.77%	12.09%	11.74%
σ_{R^*}	7.15%	7.15%	1.10%	1.10%	12.58%	12.58%
$\sigma_{\Delta E}$	4.40%	4.32%	4.80%	4.69%	15.76%	15.10%
σ_{π_H}	3.58%	3.51%	3.91%	3.81%	12.92%	12.26%
σ_{π}	2.95%	2.88%	3.22%	3.13%	10.62%	10.07%
σ_{fxp}	0.11%	0.11%	0.31%	0.31%	0.90%	0.90%
$\sigma_{\Delta E^e}$	1.72%	1.68%	1.58%	1.52%	7.50%	7.16%
$\sigma_{fxp,\Delta E^e}$	-0.00139%	-0.00139%	-0.00079%	-0.00113%	-0.04480%	-0.04453%

Note: Time-varying premium corresponds to an approximation of order=3

Our results in this section suggest that the main driving force in generating differences in business cycle dynamics between the two rules lies in the actual implementation of the rule. When the two rules are implemented similarly, the business cycle dynamics are identical, even at a third order approximation. Under the two equivalent rules in this section there is overshooting of the exchange rate and the effect on the time-varying risk premium is similar. This finding is corroborated in Figure 9: The risk premium behaves identically under the two equivalent rules.

Figure 9: Risk premium for equivalent rules



In summary, our results imply that differences in economic fluctuations between the two rules arise at first order, and are amplified at third order, only when the rules are implemented differently–that is, when the exchange rate rule results into a gradual appreciation or depreciation of the currency, whereas the Taylor rule generates an overshooting of the exchange rate. While, technically, the central bank can replicate any exchange rate rule by adjusting the nominal interest rate to changes in inflation, output and foreign shocks, this is not the way that standard rules considered in the literature operate.

B Robustness

B.1 Comparing the two rules under a range of alternative policy parameters

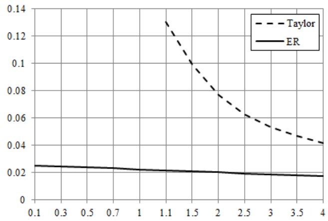
In our numerical analysis, we have compared the performance of the two monetary policy rules for an arbitrary value of the policy parameters. Our findings suggest that the exchange rate rule generates lower business cycle fluctuations than the interest rate rule. In this section, we compare the volatility of key economic variables using a wider range of policy parameters, to check whether any combination of parameters exists for which the two rules deliver identical business cycle fluctuations. We simulate the hybrid model by varying the parameter values of the two rules. In particular, we set $\phi_m(1.1, 3)$ and $\phi_m^e \in (0, 3)$. The range of the parameter values are chosen as follows: (i) To avoid indeterminacy in the Taylor rule , ϕ_m must be larger than 1. There is not a restriction in the lower bound of ϕ_m^e; (ii) we set the upper limit following Schmitt-Grohé & Uribe (2007) who, when computing optimal policy, restrict the value of the reaction to inflation to be lower than 3. For each parameter value, we compute the volatility of inflation, output growth, the nominal interest rate, the exchange rate depreciation, domestic and CPI inflation, and the risk premium.

The results are shown in Figure 10. For every combination of parameter values that we consider, we find that the exchange rate rule (solid line) outperforms the Taylor rule (dashed line), in terms of reducing the volatility of key economic variables. One exception is the nominal interest rate, which is always more volatile under the exchange rate rule. This finding is consistent with those in Table 3. The Figure also shows that, as the intensity at which the central bank reacts to inflation increases, the volatility of output growth, inflation and the exchange rate decreases for both rules. However, the volatility of the risk premium increases.

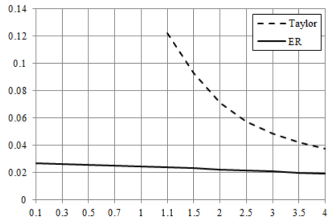
Our results show that, when the central bank reacts to CPI inflation in its monetary policy rules, with a certain degree of smoothing of the instrument, the exchange rate rule generates less fluctuations of economic variables than the Taylor rule.

Figure 10: Comparing the two rules under alternative parameter values of ϕ_m

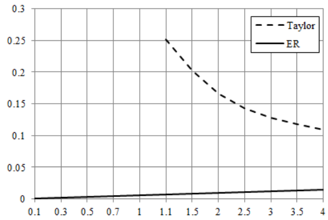
(a) CPI inflation



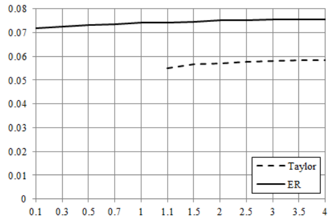
(b) Domestic inflation



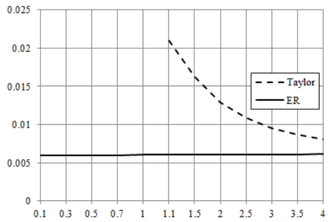
(c) Depreciation rate



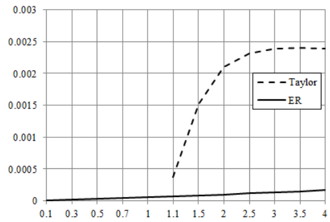
(d) Domestic interest rate



(e) Output growth



(f) Risk Premium

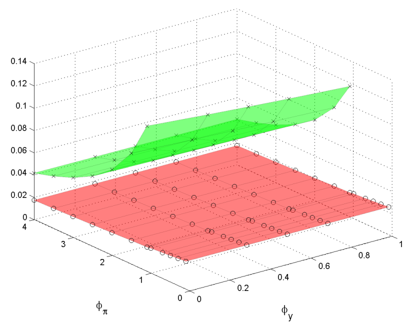


B.2 Augmented monetary rules: inflation and the output gap

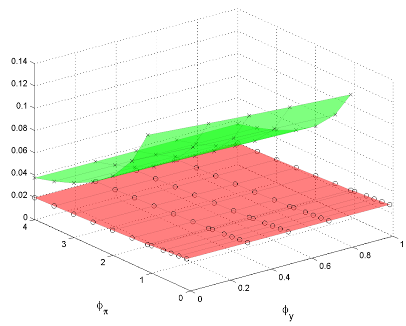
In this section, we compute the volatility of key economic variables and the risk premium when the central bank follows an augmented monetary policy rule in which it reacts to both inflation and deviations of output from its steady state, with a certain degree of smoothing of the instrument. We assume that the smoothing parameter for both rules is, as in our previous numerical analysis, $\rho = 0.85$. We then vary the parameters on inflation and deviations of output and compute the corresponding volatility of key economic variables for the Taylor rule, the exchange rate rule, and the case of the peg. Figure 11 displays the results. The exchange rate rule outperforms both the peg and the Taylor rule in reducing fluctuations of inflation and domestic inflation. As was the case before, the exchange rate rule generates larger fluctuations in the nominal interest rate and lower risk premium volatility.

Figure 11: Augmented monetary rule (Standard deviations: (red (o) = ERR, green (x) = Taylor))

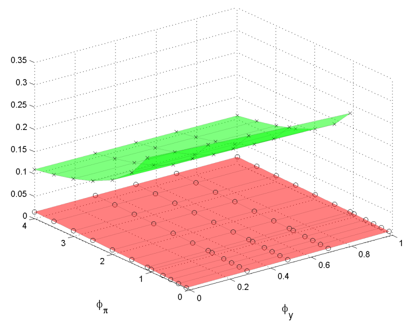
(a) CPI inflation



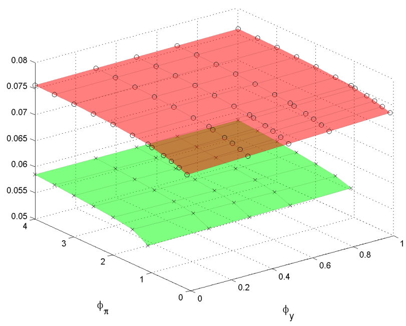
(b) Domestic inflation



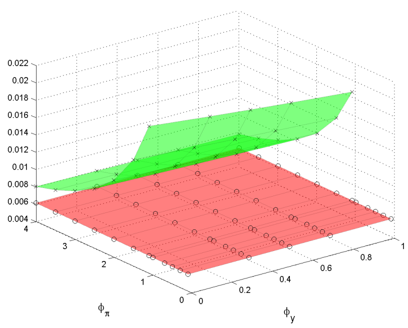
(c) Depreciation rate



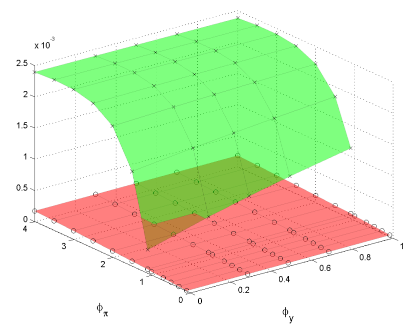
(d) Domestic interest rate



(e) Output growth



(f) Risk Premium



B.3 Alternative values of the parameters

In this section, we simulate the hybrid model for alternative values of the elasticity of substitution between intermediate goods, the labor supply elasticity and the smoothing parameter in the utility function, so that preferences become log-linear. The results are consistent with our previous findings. In the case of log-utility, the differences between the exchange rate rule and the Taylor rule are less important.

Table 7: Higher elasticity of substitution ($\eta = 2$)

	Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	2.58%	0.59%	0.60%
$\sigma_{\Delta C}$	0.57%	0.77%	0.77%
$\sigma_{\Delta Q}$	9.16%	1.72%	1.79%
σ_R	6.41%	7.28%	7.34%
σ_{R^*}	7.15%	7.15%	7.15%
$\sigma_{\Delta E}$	19.34%	0.35%	0.75%
σ_{π_H}	11.37%	1.65%	1.40%
σ_π	11.90%	1.50%	1.26%
σ_{fxp}	0.17%	0.00%	0.01%
σ_{R-R^*}	1.28%	0.29%	0.56%
$\sigma_{\Delta E^e}$	1.38%	0.29%	0.55%
$\sigma_{R-R^*,\Delta E^e}$	1.33%	0.29%	0.55%
$\sigma_{fxp,\Delta Y}$	0.00039%	0.00000%	0.00000%
$\sigma_{fxp,\Delta E}$	-0.00193%	0.00001%	0.00002%
$\sigma_{fxp,\Delta E^e}$	-0.00145%	0.00000%	0.00002%
$\hat{\beta}_{uip}$	1.0097	0.9870	0.9826
$\sigma_{\hat{\beta}_{uip}}$	0.1504	0.0071	0.0092

Table 8: Labor supply elasticity ($\psi = 3$)

	Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	2.69%	0.63%	0.65%
$\sigma_{\Delta C}$	0.30%	0.77%	0.77%
$\sigma_{\Delta Q}$	17.95%	2.12%	2.35%
σ_R	3.86%	7.52%	7.80%
σ_{R^*}	7.15%	7.15%	7.15%
$\sigma_{\Delta E}$	22.29%	0.62%	1.29%
σ_{π_H}	4.84%	1.90%	1.57%
σ_π	5.77%	1.72%	1.39%
σ_{fxp}	0.11%	0.00%	0.01%
$\sigma_{\Delta E^e}$	3.75%	0.59%	1.18%
$\sigma_{fxp,\Delta E^e}$	-0.00375%	0.00002%	0.00007%
$\hat{\beta}_{uip}$	1.0939	0.9949	0.9924
$\sigma_{\hat{\beta}_{uip}}$	0.0600	0.0035	0.0044

Table 9: Log-utility ($\sigma = 1$)

	Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	0.45%	0.64%	0.65%
$\sigma_{\Delta C}$	0.44%	0.68%	0.68%
$\sigma_{\Delta Q}$	3.23%	1.72%	1.79%
σ_R	0.93%	1.65%	1.76%
σ_{R^*}	1.50%	1.50%	1.50%
$\sigma_{\Delta E}$	4.49%	0.36%	0.76%
σ_{π_H}	1.88%	1.65%	1.40%
σ_π	1.99%	1.50%	1.26%
σ_{fxp}	0.00%	0.00%	0.00%
$\sigma_{\Delta E^e}$	0.79%	0.29%	0.57%
$\sigma_{fxp,\Delta E^e}$	-0.00003%	0.00000%	0.00000%
$\hat{\beta}_{uip}$	0.9758	0.9926	0.9881
$\sigma_{\hat{\beta}_{uip}}$	0.0562	0.0070	0.0089

B.4 The risk premium and the persistence of the shocks

In this section, we analyze the response of economic variables to domestic and foreign shocks under the two monetary policy rules, when the shocks are highly persistent. In particular, we set the persistence to 0.9977 as in De Paoli & Sondergaard (2009). They show that the effect of shocks on precautionary savings and hence the risk premium are amplified for high persistence shocks. The reason is that the precautionary saving motive and hence the volatility of the stochastic discount factor is larger the larger the persistence and volatility of the shocks.

We can see that in the bottom of Table 10. The differences in the volatility of the risk premium between the two rules are larger. Furthermore, the coefficient that determines deviations from UIP, β_{UIP} is almost 1 for the exchange rate rule and 0.66 for the interest rate rule. This suggests that, under an exchange rate rule, deviations from UIP are weaker than under a Taylor rule.

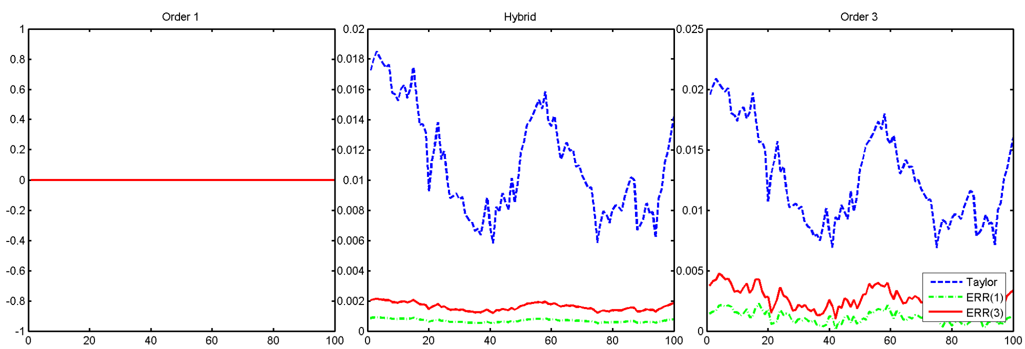
Table 10: Moments: Time-varying risk premium and high persistence

	Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	0.65%	0.61%	0.62%
$\sigma_{\Delta C}$	0.68%	0.74%	0.74%
$\sigma_{\Delta Q}$	5.35%	3.03%	3.11%
σ_R	3.38%	1.32%	1.71%
σ_{R^*}	1.10%	1.10%	1.10%
$\sigma_{\Delta E}$	6.80%	0.84%	1.58%
σ_{π_H}	2.62%	2.75%	2.23%
σ_π	2.74%	2.49%	1.98%
σ_{fxp}	0.71%	0.02%	0.06%
$\sigma_{\Delta E^e}$	2.05%	0.76%	1.33%
$\sigma_{fxp,\Delta E^e}$	0.00989%	0.00004%	0.00013%
$\hat{\beta}_{uip}$	0.6992	0.9962	0.9937
$\sigma_{\hat{\beta}_{uip}}$	0.0254	0.0045	0.0063

Note: Time-varying premium corresponds to an approximation of order=3

Finally, note that the risk premium is much more volatile with the Taylor rule when the persistence of the shock is high, relative to the volatility of the risk premium with the exchange rate rule.

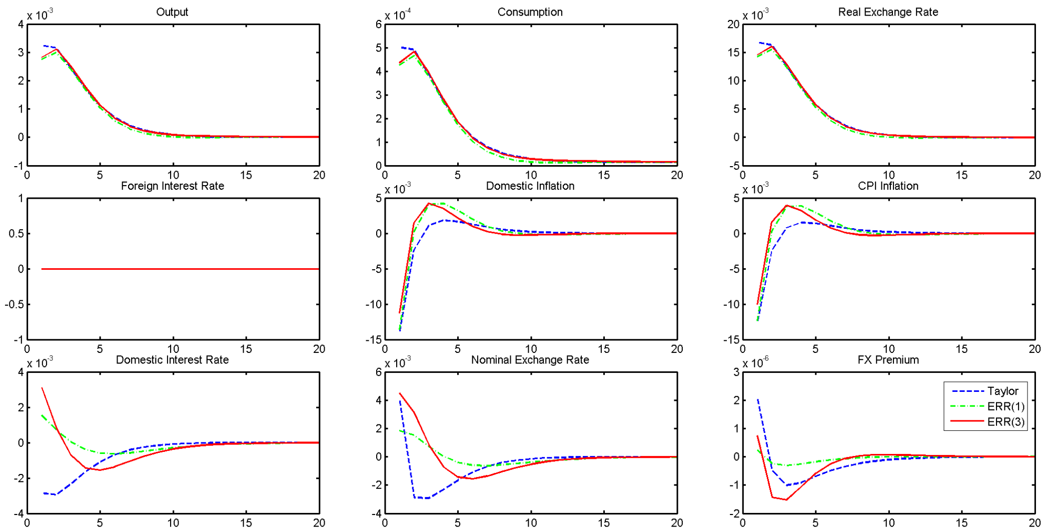
Figure 12: Risk premium



B.5 Third order approximation of the full model

In this section, we report impulse response functions and moments when we perform a third order approximation of the full model. Recall that, so far, we have used a hybrid model, in which we linearize the supply and demand equations in order to isolate the role of the time-varying risk premium. The results are consistent with our previous findings. The two rules generate qualitatively different business cycle implications.

Figure 13: Domestic productivity shock (order=3)



Finally, as the results in Table 11 show, the exchange rate rule outperforms the Taylor rule in smoothing fluctuations of inflation.

Table 11: Moments, full third order approximation

	Taylor	ERR	
		$\phi_\pi = 1$	$\phi_\pi = 3$
$\sigma_{\Delta Y}$	12.22%	1.82%	1.14%
$\sigma_{\Delta C}$	2.28%	0.61%	0.64%
$\sigma_{\Delta Q}$	58.99%	11.94%	8.43%
σ_R	27.88%	7.76%	7.98%
σ_{R^*}	7.15%	7.15%	7.15%
$\sigma_{\Delta E}$	25.27%	2.47%	3.57%
σ_{π_H}	56.02%	11.46%	6.59%
σ_π	51.34%	10.43%	5.90%
σ_{fxp}	0.36%	0.04%	0.08%
$\sigma_{\Delta E^e}$	22.02%	2.12%	2.80%
$\sigma_{fxp,\Delta E^e}$	0.04175%	-0.00037%	-0.00057%
$\hat{\beta}_{uip}$	0.9893	1.0028	1.0022
$\sigma_{\hat{\beta}_{uip}}$	0.0056	0.0061	0.0080

C Deriving the risk premium under the two rules

We derive an analytical solution for the foreign exchange risk premium as a function of the parameters of the monetary rule. Following Backus et al. (2010), we show that the foreign exchange risk premium (therefore deviations from UIP) depend on the parameters of the monetary rule. We replicate their exercise using the ERR in addition to the Taylor rule. We follow Backus et al. (2010), but depart from them in that we use a utility function with external habits in consumption (instead of Epstein-Zin preferences). Once we have an expression of the domestic nominal pricing kernel, we follow De Paoli & Sondergaard (2009) to derive the risk premium.

The key is to build a model that endogenously determines inflation. In the basic setup Backus et al. (2010) use two equations for two variables (i_t, π_t):

1. Nominal interest rate as a function of the log-linear pricing kernel (which depends on inflation).

2. Taylor rule determining nominal interest rate as a function of inflation.

After deriving a solution to inflation, one can express the nominal pricing kernel as a function of exogenous variables. With this, one can derive the foreign exchange risk premium.

Relative to the previous version, we start here first with the foreign economy.

C.1 Definitions

Backus et al. (2010) define the nominal interest differential $i_t - i_t^*$, the expected nominal depreciation $\mathbb{E}_t[de_{t+1}]$ and the exchange rate risk premium as fxp_t in terms of the domestic and foreign nominal pricing kernel, $M_{t,t+1}$ and $M^*_{t,t+1}$, respectively:

$$\begin{aligned}
i_t - i_t^* &= log\mathbb{E}_t[M^*_{t,t+1}] - log\mathbb{E}_t[M_{t,t+1}] && (40)\\
\mathbb{E}_t[de_{t+1}] &= \mathbb{E}_t[logM^*_{t,t+1}] - \mathbb{E}_t[logM_{t,t+1}] && (41)\\
fxp_t &= \frac{1}{2}\left[Var_t[logM^*_{t,t+1}] - Var_t[logM_{t,t+1}]\right] && (42)
\end{aligned}$$

Note that this follows from assuming *log-normality* of the nominal pricing kernel. Under external habit formation the nominal pricing kernel is

$$M_{t,t+1} = \beta\left(\frac{C_{t+1} - hX_{t+1}}{C_t - hX_t}\right)^{-\rho}\Pi_{t+1}^{-1} \tag{43}$$

Defining surplus consumption $S_t = \frac{C_t - hX_t}{C_t}$ this can be written solely as a product[14]

$$M_{t,t+1} = \beta \left(\frac{S_{t+1} C_{t+1}}{S_t C_t} \right)^{-\rho} \Pi_{t+1}^{-1} \tag{44}$$

As in Backus et al. (2010), De Paoli & Sondergaard (2009) and Verdelhan (2010) consumption is assumed to follow an AR(1) process.

$$logC_{t+1} = \lambda logC_t + \epsilon_{c,t+1} \tag{45}$$

Foreign consumption also follows an AR(1) process.

$$logC_{t+1}^* = \lambda^* logC_t^* + \epsilon_{c^*,t+1} \tag{46}$$

C.2 Solving for inflation

In this section we solve for inflation under (1) an exchange rate rule and (2) a standard Taylor rule.

C.2.1 Exchange rate rule

- First notice that equation (40) follows from taking expectations of the key Lucas (1984) equation relating the depreciation rate to the ratio of the nominal pricing kernels: $\frac{e_{t+1}}{e_t} = \frac{M_{t,t+1}^*}{M_{t,t+1}}$. It also holds here without the expectation terms. In log-linear form it writes:

$$de_{t+1} = logM_{t,t+1}^* - logM_{t,t+1} \tag{47}$$

- Now we need to find an expression for the log of the two pricing kernels. We assume that foreign inflation is zero (flexible prices in the rest of the world).

$$logM_{t,t+1}^* = log\left[\beta \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\rho}\right] = log\beta - \rho[log(C_{t+1}^*) - log(C_t^*)] \tag{48}$$

$$= log\beta + \rho(1-\rho_c^*)c_t^* - \rho\epsilon_{c^*,t+1} \tag{49}$$

[14]Note that this is convenient, because it allows linearizing the pricing kernel by taking logs without using approximations.

The domestic pricing kernel writes:

$$
\begin{aligned}
logM_{t,t+1} &= log\left[\beta\left(\frac{S_{t+1}C_{t+1}}{S_tC_t}\right)^{-\rho}\Pi_{t+1}^{-1}\right] && (50)\\
&= log\beta - \rho[c_{t+1} + s_{t+1} - c_t - s_t] - \pi_{t+1} && (51)\\
&= log\beta + \rho(1 - \rho_c c_t - \rho[\epsilon_{c,t+1} + s_{t+1} - s_t] - \pi_{t+1} && (52)
\end{aligned}
$$

Note that we did not use any approximation until this point.

- It follows that (47) can be written as

$$
\begin{aligned}
de_{t+1} &= log\beta + \rho(1-\rho_c^*)c_t^* - \rho\epsilon_{c^*,t+1}\\
&- [log\beta + \rho(1-\rho_c)c_t - \rho[\epsilon_{c,t+1} + s_{t+1} - s_t] - \pi_{t+1}]\\
&= \rho[(1-\rho_c^*)c_t^* - (1-\rho_c)c_t] - \rho[\epsilon_{c^*,t+1} - \epsilon_{c,t+1}] + \rho[s_{t+1} - s_t]\\
&+ \pi_{t+1} && (53)
\end{aligned}
$$

- We allow for an exchange rate rule that targets inflation and consumption growth, to make it as close as possible to the actual rule. When assuming an exchange rate rule there will be two equations in two unknowns (de_t, π_{t+1}), i.e.

$$de_{t+1} = \rho[(1-\rho_c^*)c_t^* - (1-\rho_c)c_t] - \rho[\epsilon_{c^*,t+1} - \epsilon_{c,t+1}] + \rho[s_{t+1} - s_t] + \pi_{t+1} \quad (54)$$

$$de_{t+1} = \phi_\pi\pi_{t+1} + \phi_c E_t(c_{t+1} - c_t) \quad (55)$$

The second equation is the exchange rate monetary policy rule.

- As in Backus et al. (2010), we use the **Method of undetermined coefficients**. We do not plug in the ERR rule into equation (54), but (1) guess a linear solution for inflation and (2) compare the exchange rate rule coefficients with the coefficients from equation (54). We guess that the solution of inflation has the form:

$$\pi_t = \psi_c c_t + \psi_c^* c_t^* + \psi_s(s_t - s_{t-1}) + \psi_\epsilon\epsilon_t + \psi_{\epsilon^*}\epsilon_t^* \quad (56)$$

For convenience we write this solution at $t+1$:

$$\pi_{t+1} = \psi_c\rho_c c_t + \psi_c^*\rho_c^* c_t^* + \psi_s(s_{t+1} - s_t) + (\psi_c + \psi_\epsilon)\epsilon_{c,t+1} + (\psi_c^* + \psi_{\epsilon^*})\epsilon_{c^*,t+1} \quad (57)$$

- Collecting terms, the system of equation becomes:

$$\begin{aligned} de_{t+1} &= [\rho(1-\rho_c^*) + \psi_c^*\rho_c^*]c_t^* \\ &+ [\psi_c\rho_c - \rho(1-\rho_c)]c_t + (\psi_c^* + \psi_e^* - \rho)\epsilon_{c^*,t+1} \\ &+ (\psi_c + \rho + \psi_e)\epsilon_{c,t+1} + (\rho + \psi_s)(s_{t+1} - s_t) \end{aligned} \tag{58}$$

and rearranging,

$$\begin{aligned} de_{t+1} &= [\phi_\pi\psi_c\rho_c - (1-\rho_c)\phi_c]c_t + \rho_c^*\phi_\pi\psi_c^* c_t^* + \phi_\pi\psi_s(s_{t+1} - s_t) \\ &+ [\phi_\pi(\psi_c + \psi_\epsilon) + \phi_c]\epsilon_{c,t+1} + [\phi_\pi(\psi_c^* + \psi_{\epsilon^*})]\epsilon_{c^*,t+1} \end{aligned} \tag{59}$$

- Comparing coefficients gives solutions to ψ_i and therefore for domestic inflation.

$$\begin{aligned} \rho(1-\rho_c^*) + \psi_c^*\rho_c^* &= \rho_c^*[\phi_\pi\psi_c^*] \quad \Longrightarrow \psi_c^* = \frac{-\rho(1-\rho_c^*)}{\rho_c^*(1-\phi_\pi)} \\ \psi_c\rho_c - \rho(1-\rho_c) &= [\rho_c\phi_\pi\psi_c - (1-\rho_c)\phi_c] \quad \Longrightarrow \psi_c = \frac{(\rho-\phi_c)(1-\rho_c)}{\rho_c(1-\phi_\pi)} \\ \rho + \psi_s &= \phi_\pi\psi_s \quad \Longrightarrow \psi_s = \frac{-\rho}{1-\phi_\pi} \\ (\psi_c^* - \rho + \psi_e^*) &= [\phi_\pi(\psi_c^* + \psi_{\epsilon^*})] \quad \Longrightarrow \psi_e^* = \frac{\rho}{\rho_c^*(1-\phi_\pi)} \\ (\psi_c + \rho + \psi_\epsilon) &= [\phi_\pi(\psi_c + \psi_\epsilon) + \phi_c] \quad \Longrightarrow \psi_e = \frac{\phi_c - \rho}{\rho_c\,(1-\phi_\pi)} \end{aligned}$$

- The solution therefore is:

$$\begin{aligned} \psi_c^* &= \frac{-\rho(1-\rho_c^*)}{\rho_c^*(1-\phi_\pi)} \\ \psi_c &= \frac{(\rho-\phi_c)(1-\rho_c)}{\rho_c(1-\phi_\pi)} \\ \psi_{\epsilon^*} &= \frac{\rho}{\rho_c^*(1-\phi_\pi)} \\ \psi_s &= \frac{-\rho}{1-\phi_\pi} \\ \psi_\epsilon &= \frac{\phi_c - \rho}{\rho_c\,(1-\phi_\pi)} \end{aligned}$$

- We can now use the solution for inflation to derive a closed form solution of the domestic nominal pricing kernel using equation (52):

$$
\begin{aligned}
logM_{t,t+1} &= log\beta + \rho(1-\rho_c)c_t - \rho[\epsilon_{c,t+1} + s_{t+1} - s_t] \\
&- [\psi_c \rho_c c_t + \psi_c^* \rho_c^* c_t^* + \psi_s(s_{t+1} - s_t) + (\psi_c + \psi_\epsilon)\epsilon_{c,t+1} + (\psi_c^* + \psi_{\epsilon^*})\epsilon_{c^*,t+1}] \\
&= log\beta + [\rho(1-\rho_c) - \psi_c\rho_c]c_t - (\psi_s + \rho)(s_{t+1} - s_t) - (\rho + \psi_c + \psi_\epsilon)\epsilon_{c,t+1} \\
&- \psi_c^* \rho_c^* c_t^* - (\psi_c^* + \psi_{\epsilon^*})\epsilon_{c^*,t+1}
\end{aligned}
$$

Note that the domestic pricing kernel depends on the coefficients of the ERR, because it depends on the solution to inflation (ψ_i).

- The following calculations now follow closely the Appendix A in Basically now, one would need to use the definition of the pricing kernel of section 3 and solve for

$$
fxp_t = \frac{1}{2}\Big[Var_t[logM_{t,t+1}^*] - Var_t[logM_{t,t+1}]\Big] \tag{60}
$$

- To save on notation, we introduce the following coefficients

$$
\begin{aligned}
b_c &= [\rho(1-\rho_c) - \psi_c\rho_c] \\
b_s &= -(\psi_s + \rho) \\
b_\epsilon &= -(\rho + \psi_c + \psi_\epsilon) \\
b_{c^*} &= -\psi_c^* \rho_c^* \\
b_{\epsilon^*} &= -(\psi_c^* + \psi_{\epsilon^*})
\end{aligned}
$$

- and write the log-normal pricing kernel as

$$
logM_{t,t+1} = log\beta + b_c c_t + b_s(s_{t+1} - s_t) + b_\epsilon \epsilon_{c,t+1} + b_c^* c_t^* + b_{\epsilon^*}\epsilon_{c^*,t+1}
$$

- The conditional variance of the domestic nominal pricing kernel writes after dropping terms in t and constant terms:

$$
\begin{aligned}
Var_t[logM_{t,t+1}] &= Var_t[b_s s_{t+1} + b_\epsilon \epsilon_{t+1} + b_{\epsilon^*}\epsilon_{t+1}^*] \\
&= b_s^2 Var_t[s_{t+1}] + b_\epsilon^2 Var_t[\epsilon_{t+1}] + b_{\epsilon^*}^2 Var_t[\epsilon_{t+1}^*] \\
&+ 2b_s b_\epsilon Cov_t[s_{t+1}, \epsilon_{t+1}] \\
&= b_s^2 Var_t[s_{t+1}] + b_\epsilon^2 \sigma_\epsilon^2 + b_{\epsilon^*}^2 \sigma_{\epsilon^*}^2 + 2b_s b_\epsilon Cov_t[s_{t+1}, \epsilon_{t+1}]
\end{aligned} \tag{61}
$$

Note that $Cov_t[s_{t+1}, \epsilon_{t+1}^*] = 0$. The remaining terms are solved for before in using a second order approximation to surplus consumption. Therefore the conditional variance of the domestic nominal pricing kernel is

$$\begin{aligned} Var_t[logM_{t,t+1}] &= \frac{[b_\epsilon + h(b_\epsilon - b_s)]^2\sigma_\epsilon^2}{(1-h)^2} + b_{\epsilon^*}\sigma_{\epsilon^*}^2 \\ &+ \frac{2b_s h[b_\epsilon + h(b_\epsilon - b_s)]}{(1-h)^3}\sigma_\epsilon^2[(1-\phi-\rho_c)c_t + \phi x_t] \end{aligned}$$

- Finally, the **foreign exchange risk premium** has the following expression:

$$\begin{aligned} fxp_t &= \frac{1}{2}\left[Var_t[logM^*_{t,t+1}] - Var_t[logM_{t,t+1}]\right] \\ &= \frac{1}{2}\left[(\rho^2 - b_{\epsilon^*})\sigma_{\epsilon^*}^2 - \frac{[b_\epsilon + h(b_\epsilon - b_s)]^2\sigma_\epsilon^2}{(1-h)^2}\right] \\ &- \frac{b_s h[b_\epsilon + h(b_\epsilon - b_s)]}{(1-h)^3}\sigma_\epsilon^2[(1-\phi-\rho_c)c_t + \phi x_t] \end{aligned}$$

C.2.2 Taylor rule

- Notice the nominal interest rate can be written under lognormality as the following function of the nominal pricing kernel

$$\begin{aligned} i_t &= -log\mathbb{E}_t[M_{t,t+1}] \\ &= -\mathbb{E}_t[logM_{t,t+1}] - \frac{1}{2}Var_t[logM_{t,t+1}] \end{aligned}$$

- To solve for the nominal interest rate as a functin of inflation, consumption and surplus consumption, note that we can use the equation (52) for $log[M_{t,t+1}]$ and write the nominal interest rate as

$$\begin{aligned} i_t &= -log\beta - \rho(1-\rho_c)c_t + \rho\mathbb{E}_t[s_{t+1} - s_t] + \mathbb{E}_t[\pi_{t+1}] \\ &- \frac{1}{2}Var_t[-\rho[\epsilon_{c,t+1} + s_{t+1}] - \pi_{t+1}] \end{aligned}$$

- Assuming the same solution for inflation as above

$$\pi_t = \psi_c c_t + \psi_c^* c_t^* + \psi_s(s_t - s_{t-1}) + \psi_\epsilon \epsilon_t + \psi_{\epsilon^*}\epsilon_t^*$$

One-period ahead inflation is

$$\pi_{t+1} = \psi_c c_{t+1} + \psi_c^* c_{t+1}^* + \psi_s(s_{t+1} - s_t) + \psi_\epsilon \epsilon_{t+1} + \psi_{\epsilon^*}\epsilon_{t+1}^*$$

and in expectation

$$E_t \pi_{t+1} = \psi_c \rho_c c_t + \psi_c^* \rho_c^* c_t^* + \psi_s (E_t s_{t+1} - s_t)$$

From here

$$var_t(\pi_{t+1}) = E_t(\psi_c \epsilon_{t+1} + \psi_c^* \epsilon_{t+1}^* + \psi_\epsilon \epsilon_{t+1} + \psi_{\epsilon^*} \epsilon_{t+1}^*)^2$$

- When we plug this solution into the expression for the nominal interest rate, use $c_{t+1} = \rho_c c_t + \epsilon_{t+1}$ and $c_{t+1}^* = \rho_c^* c_t^* + \epsilon_{t+1}^*$ and collect terms, we find

$$\begin{aligned} i_t &= -log\beta - [\rho(1-\rho_c) - \psi_c \rho_c] c_t + (\psi_s + \rho)\mathbb{E}_t[s_{t+1} - s_t] + \psi_c^* \rho_c^* c_t^* \\ &- \frac{1}{2} Var_t[-(\rho + \psi_s) s_{t+1} - (\rho + \psi_\epsilon + \psi_c)\epsilon_{t+1} - (\psi_{\epsilon^*} + \psi_c^*)\epsilon_{t+1}^*] \end{aligned}$$

- We expand the conditional variance term $Var_t[...]$

$$\begin{aligned} i_t &= -log\beta - [\rho(1-\rho_c) - \psi_c \rho_c] c_t + (\psi_s + \rho)\mathbb{E}_t[s_{t+1} - s_t] + \psi_c^* \rho_c^* c_t^* \\ &- \frac{1}{2}\left[(\rho + \psi_s)^2 Var_t[s_{t+1}] + (\rho + \psi_\epsilon + \psi_c)^2 \sigma_\epsilon^2 (\psi_{\epsilon^*} + \psi_c^*)^2 \sigma_{\epsilon^*}^2\right] \\ &- (\rho + \psi_s)(\rho + \psi_\epsilon + \psi_c) Cov_t[s_{t+1}, \epsilon_{t+1}] \end{aligned}$$

Note that $Cov_t[\epsilon_{t+1}, \epsilon_{t+1}^*] = 0$ and $Cov_t[s_{t+1}, \epsilon_{t+1}^*] = 0$.

- As in the Exchange rate rule case, we use expressions from De Paoli & Sondergaard (2009):

$$\begin{aligned} Var_t[s_{t+1}] &= \left(\frac{h}{1-h}\right)^2 \left[\sigma_\epsilon^2 - (1-h)^{-1} 2\rho_c c_t \sigma_\epsilon^2 + 2\tilde{x}_t (1-h)^{-1} \sigma_\epsilon^2\right] \\ Cov_t[s_{t+1}, \epsilon_{t+1}] &= \frac{h}{1-h}\left[\sigma_\epsilon^2 - (1-h)^{-1} \rho_c c_t \sigma_\epsilon^2 + \tilde{x}_t (1-h)^{-1} \sigma_\epsilon^2\right] \end{aligned}$$

with $\tilde{x}_t = log(X_{t+1})$.

- Plugging these terms into the equation for the nominal interest rate gives the following

$$\begin{aligned} i_t &= -log\beta - [\rho(1-\rho_c) - \psi_c \rho_c] c_t + (\psi_s + \rho)\mathbb{E}_t[s_{t+1} - s_t] + \psi_c^* \rho_c^* c_t^* \\ &- \frac{1}{2}\left[\left(\frac{(\rho + \psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right)^2 \sigma_\epsilon^2 + (\psi_{\epsilon^*} + \psi_{c^*})^2 \sigma_{\epsilon^*}^2\right] \\ &- \sigma_\epsilon^2 \frac{(\rho + \psi_s)h}{(1-h)^2}\left[\frac{(\rho + \psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right][(1 - \phi - \rho_c)c_t + \phi x_t] \end{aligned}$$

- To solve for the coefficients of the guessed solution for inflation (ψ_i), we assume need another equation, here the Taylor rule. We assume a Taylor rule with the same targets as above:

$$i_t = -log(\beta) + \phi_\pi E_t \pi_{t+1} + \phi_c E_t(c_{t+1} - c_t)$$

- Plugging in the guessed solution for inflation into the Taylor rule gives:

$$\begin{aligned} i_t &= -log(\beta) + (\phi_\pi \psi_c \rho_c - (1-\rho_c)\phi_c)c_t + (\phi_\pi \psi_c^* \rho_c^*)c_t^* \\ &+ \phi_\pi \psi_x x_t + (\phi_\pi \psi_s + \phi_s)E_t(s_{t+1} - s_t) \end{aligned} \tag{62}$$

- As before, equation (62) is used to solve for (ψ_i) by setting the coefficients of the variables equal to:

$$\begin{aligned} -log\beta - \frac{1}{2}\left[\left(\frac{(\rho+\psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right)^2 \sigma_\epsilon^2 + (\psi_{\epsilon^*} + \psi_{c^*})^2\sigma_{\epsilon^*}^2\right] + log(\beta) &= 0 \\ \phi_\pi \psi_c^* \rho_c^* - \phi_\pi \psi_c^* - \phi_{c^*} &= 0 \\ -\sigma_\epsilon^2 \frac{(\rho+\psi_s)h}{(1-h)^2}\left[\frac{(\rho+\psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right][(1-\phi-\rho_c)] + [\rho(1-\rho_c) - \psi_c\rho_c] &= 0 \\ (\phi_\pi\psi_s + \phi_s) - (\psi_s + \rho) &= 0 \\ \sigma_\epsilon^2 \frac{(\rho+\psi_s)h}{(1-h)^2}\left[\frac{(\rho+\psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right]\phi &= 0 \end{aligned}$$

This is a system of six equations and six unknowns.

- Note first that if $\phi_\pi \neq 0$ it must follow that

$$\begin{aligned} \psi_{c^*} &= \frac{-\phi_{c^*}}{\phi_\pi(1-\rho_c)} \\ \psi_c &= \frac{\rho(1-\rho_c)}{\rho_c} \\ \psi_s &= -\psi_{c^*} \\ \psi_{\epsilon^*} &= \frac{\phi_{c^*}}{\rho_c^* - \phi_\pi} \\ \psi_\epsilon &= -\frac{(\rho+\psi_s)h}{1-h} - \psi_c - \rho \end{aligned}$$

- Therefore,

$$fxp_t = \frac{1}{2}\sigma_{\epsilon^*} b_\epsilon^* - \frac{1}{2}\left[\left(\frac{(\rho+\psi_s)h}{1-h} + \rho + \psi_\epsilon + \psi_c\right)^2 \sigma_\epsilon^2 + (\psi_{\epsilon^*} + \psi_{c^*})^2 \sigma_{\epsilon^*}^2\right]$$

$$-\sigma_\epsilon^2 \frac{(\rho+\psi_s)h}{(1-h)^2}\left[\frac{(\rho+\psi_s)h}{1-h}+\rho+\psi_\epsilon+\psi_c\right][(1\text{-}\phi\text{-}\rho_c)c_t+\phi x_t]$$

C.2.3 Comparing risk premiums for each rule

- Exchange rate rule

$$\begin{aligned} fxp_t &= \frac{1}{2}\left[(\rho^2 - b_{\epsilon^*})\sigma_{\epsilon^*}^2 - \frac{[b_\epsilon + h(b_\epsilon - b_s)]^2\sigma_\epsilon^2}{(1-h)^2}\right] \\ &- \frac{b_s h[b_\epsilon + h(b_\epsilon - b_s)]}{(1-h)^3}\sigma_\epsilon^2[(1-\phi-\rho_c)c_t + \phi x_t] \end{aligned}$$

- Taylor rule

$$fxp_t = \frac{1}{2}\sigma_{\epsilon^*} b_\epsilon^* - \frac{1}{2}\left[\left(\frac{(\rho+\psi_s)h}{1-h}+\rho+\psi_\epsilon+\psi_c\right)^2\sigma_\epsilon^2 + (\psi_{\epsilon^*}+\psi_{c^*})^2\sigma_{\epsilon^*}^2\right]$$

$$-\sigma_\epsilon^2 \frac{(\rho+\psi_s)h}{(1-h)^2}\left[\frac{(\rho+\psi_s)h}{1-h}+\rho+\psi_\epsilon+\psi_c\right][(1\text{-}\phi\text{-}\rho_c)c_t+\phi x_t]$$

C.3 Decomposition of the risk premium

We can decompose the risk premium into two components: the conditional variance of the exchange rate depreciation and the conditional covariance between the stochastic discount factor and the expected future depreciation of the currency.

The risk premium is also given by:

$$fxp_t = \frac{1}{2}var_t\left(\Delta e_{t+1}\right) + cov_t\left(\Delta e_{t+1}, log\left(M_{t,t+1}\right)\right)$$

C.3.1 Exchange rate rule

$$\begin{aligned} var_t\left(\Delta e_{t+1}\right) &= \frac{(1-h)^2\rho^2\sigma_{\epsilon^*}^2\phi_\pi^2 - \sigma_\epsilon^2((1-h)\phi_c - \rho\phi_\pi)^2}{(1-h)^2(1-\phi_\pi)^2} \\ &- \frac{2h\rho\sigma_\epsilon^2((1-h)\phi_c - \rho\phi_\pi)}{(1-h)^3(1-\phi_\pi)}\left[c_t(\rho_c+\phi-1) - \phi x_t\right] \end{aligned} \tag{63}$$

$$\begin{aligned} cov_t\left(\Delta e_{t+1}, m_{t,t+1}\right) &= -\frac{(1-h)^2\rho^2\sigma_{\epsilon^*}^2\phi_\pi - \sigma_\epsilon^2((1-h)\phi_c - \rho\phi_\pi)^2}{(1-h)^2(1-\phi_\pi)^2} \\ &+ \frac{2h\rho\sigma_\epsilon^2((1-h)\phi_c - \rho\phi_\pi)}{(1-h)^3(1-\phi_\pi)}\left[c_t(\rho_c+\phi-1) - \phi x_t\right] \end{aligned} \tag{64}$$

C.3.2 Taylor rule

$$
\begin{aligned}
var_t\left(\Delta e_{t+1}\right) &= \frac{\sigma_\epsilon^2\left(h^2\rho^4\sigma_\epsilon^2\sigma_{\epsilon^*}^2(\rho_c+\phi-1)^2+(1-h)^4\phi_c^2\right)}{\left(h\rho\sigma_\epsilon^2(\rho_c+\phi-1)-(1-h)^2\phi_\pi\right)^2} \\
&- \frac{2(1-h)^2\rho\sigma_\epsilon^2\phi_\pi\left(h\rho^2\sigma_{\epsilon^*}^2(\rho_c+\phi-1)+(1-h)\phi_c\right)}{\left(h\rho\sigma_\epsilon^2(\rho_c+\phi-1)-(1-h)^2\phi_\pi\right)^2} \\
&+ \frac{(1-h)^2\rho^2\phi_\pi^2\left((1-h)^2\sigma_{\epsilon^*}^2+\sigma_\epsilon^2\right)}{\left(h\rho\sigma_\epsilon^2(\rho_c+\phi-1)-(h-1)^2\phi_\pi\right)^2} \\
&+ \frac{2h\rho\sigma_\epsilon^2((1-h)\phi_c-\rho\phi_\pi)}{(1-h)\left((1-h)^2\phi_\pi-h\rho\sigma_\epsilon^2(\rho_c+\phi-1)\right)}\left[c_t(\rho_c+\phi-1)-\phi x_t\right]
\end{aligned}
\tag{65}
$$

$$
\begin{aligned}
cov_t\;&\left(\Delta e_{t+1}, m_{t,t+1}\right) \\
= \;& -\frac{(1-h)^2((1-h)\phi_c-\rho\phi_\pi)\sigma_\epsilon^2((1-h)\phi_c-\rho\phi_\pi)}{\left(h\rho\sigma_\epsilon^2(\rho_c+\phi-1)-(1-h)^2\phi_\pi\right)^2} \\
- \;& \frac{2h\rho\sigma_\epsilon^2((1-h)\phi_c-\rho\phi_\pi)\left((1-h)^2\phi_\pi-h\rho\sigma_\epsilon^2(\rho_c+\phi-1)\right)}{(1-h)\left(h\rho\sigma_\epsilon^2(\rho_c+\phi-1)-(1-h)^2\phi_\pi\right)^2}\left[c_t(\rho_c+\phi-1)-\phi x_t\right]
\end{aligned}
\tag{66}
$$