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Abstract

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Sources of Liquidity and Liquidity Shortages*

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1 Introduction

Liquidity problems played a central role in the Global Financial Crisis. During the height of the crisis, many institutions were scrambling for liquidity. Common sources of liquidity were quickly exhausted and traditional suppliers withdrew from funding markets. Several financial institutions were on the brink of failure; some failed outright. The response by central banks was unprecedented, flooding the financial system with a sea of liquidity. Going forward, there is a growing recognition that due to the public good character of liquidity, agents in the financial system do not have the correct incentives to invest in it. As a result, regulation specifically targeted at liquidity may be needed. This view is reflected in the Liquidity Coverage Ratio (LCR) of Basel III, which stipulates minimum levels for banks to cover their short-term liabilities.

In this paper we argue that liquidity regulation requires a clear view on the possibilities for banks to generate liquidity in times of need. Where the only source of (private) liquidity is precautionary liquidity held on the balance sheet of banks, a minimum requirement may indeed be the appropriate measure. However, banks with liquidity shortages can also raise funds from other financial institutions, or from outside the financial system. The ability of banks to generate funds thus also depends on the suppliers of liquidity in the economy.

Therefore liquidity regulation must take into account general equilibrium effects. In particular, liquidity requirements at banks are likely to interact with the amount of liquidity supplied to the banking system during crisis. Taking the argument to the extreme, if total holdings of liquid assets in the economy are given, higher liquidity at banks will reduce liquidity elsewhere in the economy one-for-one, with unclear implications for welfare. The ultimate impact of liquidity regulation will also depend on the source of liquidity. Suppliers of liquidity differ not only in terms of the elasticity of their supply but also in terms of their ability to overcome informational problems in crisis times.

We develop a model in which banks interact with an "investor" sector in order to deal with uncertain future liquidity needs for projects. Banks can raise liquidity in anticipation of liquidity needs or once a need materializes.[1] There are frictions associated with ex-ante and ex-post liquidity, arising from agency and informational problems, respectively. These

[1]The model is also applicable to the liquidity needs of the corporate sector in general. The reader may prefer to think of our banks as a consolidation of banks and and their corporate loan customers. We emphasize the banking interpretation because it is most natural to consider government liquidity policy as being channeled through the financial sector.

frictions make both channels for outside liquidity costly. Ex-post a bank has various sources of liquidity: precautionary liquidity held on balance sheet, outside liquidity from investors (or other banks) and liquidations of projects. Welfare losses arise in the event of banking sector-wide liquidity shortages (the market for liquidity can take care of bank-specific problems). Such shortages can materialize for two reasons in our model: an insufficient borrowing capacity of the banking sector or an insufficient supply of liquidity by investors.

We show that the private incentives of banks and investors leads to inefficient precautionary liquidity holdings. From a welfare perspective, we find that precautionary liquidity can either be insufficient or excessive. The source of the inefficiency is a fire-sale externality: when it liquidates projects, an individual bank does not internalize the increased costs to other liquidating banks. Suppose that there are only costs to raising (outside) liquidity ex-post. In this case, holding liquidity within the banking sector – as opposed to the investor sector – maximizes the amount of liquidity that can be used at banks to cover liquidity shocks. Precautionary liquidity holdings at an individual bank then increase the bank's ability to deal with liquidity shortages, resulting in fewer liquidations in the economy. As the bank does not internalize the external effect of this, its holdings of liquidity will be insufficient.

The opposite result is obtained when the cost to liquidity arises ex-ante, for example because managers divert free liquidity. In this case, precautionary liquidity reduces the overall amount of liquidity that can be generated by banks as precautionary liquidity reduces the supply of investor liquidity ex-post by a factor larger than one. Private holdings of liquidity in the banking sector will then exceed the socially optimal amount. When there are both ex-ante and ex-post costs, we show that the bias in private liquidity holdings (insufficient or excessive) depends on the relative size of the two costs.

Liquidity regulation in the form of minimum liquidity requirements is hence not necessarily desirable. For example, financial institutions that tend to raise funds from a pool of relatively sophisticated investors (for example, investment banks or money center banks) face relatively low costs of accessing this pool in crisis times (as sophisticated investors can more easily overcome informational frictions). A unit of liquidity in the pool is thus particularly valuable. Since precautionary liquidity reduces the pool of liquidity available in a crisis, minimum liquidity requirements can lower welfare as a result. We show that in such cases efficiency can be improved through interest rate policies that encourage the private supply of liquidity ex-post, for example, by maintaining high interest rates during times of stress in order to have a wedge to the return on liquidity in normal times.

By distinguishing between different constraints banks may face when raising liquidity, our model also provides insights into the conditions under which central banks should inject liquidity in times of crisis. When the shortage of liquidity in the banking system is due to a shortage of liquidity outside the banking sector, there is a rationale for providing liquidity. However, when shortages arise because of an insufficient ability of banks to raise liquidity, standard liquidity policies are ineffective. A practical consequence of this is that the focus of the central bank should be on the price of liquidity in the economy overall – and not specifically within the banking sector.

The remainder of the paper is organized as follows. The next section discusses how the paper relates to prior literature. Section 3 describes the model. Sections 4 and 5 solve for the efficient and the private allocation of liquidity. In Section 6 we introduce public provision of liquidity. Section 7 endogenizes the ex-ante and ex-post liquidity friction. The final section concludes.

2 Relation to literature

Banks' incentives to invest in liquidity are of long-standing interest to the banking literature. A key theme is that banks have reasons to underinvest in liquidity – due to the public good character of liquidity (e.g., Bhattacharya and Gale (1986)).[2] A common assumption in most models is that it is optimal ex-ante for all resources in the economy to be channeled to banks (e.g., Diamond and Dybvig (1983)). The question of optimal liquidity investments hence boils down to the optimal share of liquid holdings in bank portfolios. Our paper, by contrast, considers the allocation of liquid resources between banks and the rest of the economy. The allocation problem is a non-trivial one since there are costs associated with precautionary liquidity holdings at banks. The different focus in our paper results in different factors that determine the efficiency of liquidity holdings (the costs of raising liquidity ex-ante and ex-post as well).

Our interest in the allocation of liquidity relates to work by Holmström and Tirole (1996) and Bolton, Scharfstein and Santos (2011). Holmström and Tirole study the ability of the economy to self-insure when consumers and firms can only imperfectly pledge incomes. They show that there is scope for public provision of liquidity as the financial

[2]There are also reasons why banks may hold excessive liquidity, arising from speculative motives or market power (see Acharya, Shin and Yorulmazer (2011), Diamond and Rajan (2011), Acharya, Gromb and Yorulmazer (2012)).

system cannot insure itself against aggregate shocks (a similar result is obtained by Gorton and Huang (2004)). Our analysis – which is also based on limits to pledgeability – by contrast focuses on inefficiencies in the private allocation of liquidity between the financial and the private sector.[3] Bolton et al. consider a setting where informational asymmetries about asset quality vary over time and use this to study the optimal timing of liquidity funding by banks. We share with them the view that banks face higher informational costs when they experience liquidity needs. Bolton et al. explicitly model how informational frictions change endogenously over time; our paper instead focuses on understanding the sources of bank liquidity and the constraints that can arise from them.

Our analysis also offers a more nuanced view on the rationale for public provision of liquidity in times of crisis. Following Walter Bagehot, the standard prescription for the central bank is to provide emergency liquidity to all solvent banks against good collateral.[4] In our model, the optimality of liquidity provision depends critically on the type of liquidity shortage. In the event of a general lack of liquidity in the economy, there is a clear role for the central bank in providing liquidity according to Bagehot's prescription. However, a shortage of liquidity that materializes only in the banking sector cannot be solved by a liquidity injection as such a shortage is due to an insufficient ability of banks to raise funds.

3 Model

The economy has three dates, $t = 0, 1, 2$. There are two types of agents, bankers and investors. Both are risk neutral and consume at date 2. Bankers (indexed with j) come from a continuum of measure 1 and are each endowed with one project that returns R at date 2.[5] Investors (indexed with i) come from a continuum of measure I and are endowed with one unit of wealth at date 0 (throughout the paper we will use subscripts i and

[3] The source of the inefficiency in our model is akin to a fire sale externality (e.g., Stein (2009)). While we focus on externalities arising from the asset side of banks, they can also arise on the liability side (see Farhi and Tirole (2012), Stein (2012) and Segura and Suarez (2017)).

[4] More generally, a rationale for central bank intervention may also arise because of co-ordination problems in the interbank market (Freixas, Parigi and Rochet (2000)), price indeterminancy (Allen, Carletti and Gale (2009)) or because it can facilitate a better distribution of liquidity among banks (Freixas, Martin and Skeie (2011)).

[5] We focus the analysis on the ex-ante/ex-post liquidity trade-off, and hence take the number of projects undertaken as fixed. The model delivers fairly standard results regarding the incentives to invest in projects (banks will overinvest in illiquid projects).

j to denote individual quantitities; omitting the subscript will indicate the corresponding economy-wide aggregates.)

There exists a storage technology (available to everybody) which turns one unit of wealth at date t into one unit of wealth at date $t+1$. Our modeling of liquidity demand follows the framework of Holmstöm and Tirole (1998). Projects require no outside funding at $t = 0$, but may be hit by a (common) liquidity shock at $t = 1$: with probability π ($\pi \in (0,1)$) projects require a liquidity injection of $\lambda > 0$, which is returned at $t = 2$. If the liquidity is not provided, the project becomes worthless. There also exists a technology for (partial) liquidation of projects at $t = 1$. This technology turns $1+\gamma$ units of date-2 project return into 1 unit at date 1. The liquidation cost γ depends on the total amount liquidated in the economy, denoted by l. We assume that $\gamma'(l) > 0$ and $\gamma''(l) \geq 0$. Increasing costs can be thought of as arising from fire-sales of projects or other types of liquidation externalities (in Section 5.1 we explicitly model the cost as coming from outsiders who require increasing discounts when purchasing more assets).

The ability of bankers to raise funds from investors is subject to frictions. First, only a fraction α ($\alpha \in (0,1)$) of the date-2 return of the project can be pledged to outsiders. Limited pledgeability may for example arise because bankers have project-specific skills that allow them to renegotiate promises to outsiders (as in Hart and Moore, 1994). Second, there is a cost to transferring funds from investors to banks. Specifically, in order for a banker to have one unit of funds at $t = 1$, he needs to raise $1 + \delta_0$ ($\delta_0 \in (0,1)$) funds from investors at $t = 0$. Similarly, there is a cost of transferring funds at $t = 1$, δ_1 ($\delta_1 \in (0,1)$). We assume that the liquidation technology is inferior to raising funds from investors:

Assumption 1 $\gamma(0) > \max[\delta_0, \delta_1]$.

We allow for $\delta_0 \neq \delta_1$ as the ex-ante and ex-post costs to liquidity may not be identical. For example, liquidity raised ex-post may be subject to higher adverse selection costs. Ex-ante liquidity, by contrast, may suffer from agency problems. In particular, managers may divert free liquidity between $t = 0$ and $t = 1$ in order to enjoy private benefits. To capture this, we assume that a banker enjoys (non-monetary) benefits $\phi_0 > 0$ per unit of ex-ante liquidity. In Section 7 we discuss sources of cost differences in more detail and argue that the relative size of ex-ante to ex-post cost in practice is likely to depend on the type of financial intermediary considered.

4 Efficient allocation

We consider a social planner who maximizes (utilitarian) welfare subject to a participation constraint. The constraint is that investors cannot be made worse off compared to autarky.[6] Welfare is given by total expected date-2 consumption in the economy, plus any private benefits accruing to bankers. If there were neither liquidations, funding costs nor private benefits (i.e., $l = 0$ $\delta_0 = \delta_1 = 0$ and $\phi_0 = 0$), welfare would simply be given by the sum of the date-2 project returns and investors's endowments: $R + I$. However, in our analysis welfare will differ because of *i)* losses from project liquidations at $t = 1$, *ii)* the cost of transferring funds to bankers at $t = 0$ and $t = 1$, *iii)* private benefits ϕ_0.

We denote by y_0 the amount of liquidity transferred from investors to banks at $t = 0$ (net of the cost δ_0) and by y_1 the (net) amount transferred at date 1 when the shock hits (when there is no shock, there is no need for liquidity transfer). Welfare can then be expressed as

$$W(y_0, y_1, l) = R + I - (\delta_0 - \phi_0)y_0 - \pi\delta_1 y_1 - \pi\Gamma(l), \tag{1}$$

where $\Gamma(l) := \gamma(l)l$ denotes total liquidation costs. The term $-(\delta_0 - \phi_0)y_0$ is the loss from transferring liquidity at $t = 0$ (net of private benefits) which is incurred with certainty. The term $-\pi\delta_1 y_1$ is the expected cost of transferring liquidity at $t = 1$, materializing when the shock arrives. The last term, $-\pi\Gamma(l)$, is the expected liquidation costs if, when the shock arrives, l unit of assets are liquidated at a cost of $\gamma(l)$ each.

The social planner faces the following constraints. First, the date-1 liquidity at banks (when the shock hits) has to be at least as high as the liquidity need λ. Given liquidity holdings from date 0, y_0, new liquidity transferred, y_1, and liquidation proceeds, l, this condition can be written:

$$y_0 + y_1 + l \geq \lambda. \tag{2}$$

Second, the liquidity transferred to the banks cannot exceed investors' endowments:

$$(1 + \delta_0)y_0 + (1 + \delta_1)\, y_1 \leq I. \tag{3}$$

Third, the borrowing capacity of bankers is restricted. This means that the total amount of date-2 funds promised at date 0 and date 1 cannot exceed the pledgeable return αR. Given that investors have to earn at least the autarky return, this condition is

$$(1 + \delta_0)y_0 + (1 + \delta_1)y_1 \leq \alpha R. \tag{4}$$

[6] Without this assumption the pledgeability constraint becomes meaningless as the social planner can circumvent it by transferring endowments from investors to bankers.

The planner's problem can therefore be stated as

$$\max_{y_0, y_1, l} W(y_0, y_1, l), \text{ subject to (2), (3), (4) and} \tag{5}$$
$$y_0, y_1, l \geq 0.$$

We turn to the solution of the problem. We restrict attention to parameter values for which when the liquidity shock hits, at least one of the liquidity constraints (the supply constraint (3) or the borrowing capacity constraint (4)) binds. A sufficient condition for this is that the liquidity shock λ exceeds the total amount banks can raise from investors if they use the cheapest liquidity channel:

Assumption 2 $\lambda - \frac{H}{1+\min[\delta_0, \delta_1]} > 0,$

where $H := \min[I, \alpha R]$.

When the shock hits at $t = 1$, liquidity holdings at banks should never exceed the liquidity requirement; otherwise, welfare could be improved by raising less at $t = 0$ or at $t = 1$ (and hence saving on liquidity cost δ_0 or δ_1) or by liquidating less (and saving on the cost γ). Thus equation (2) holds with equality

$$y_0 + y_1 + l = \lambda. \tag{6}$$

How much liquidity is ex-post raised from investors, y_1, and how much from liquidations, l? Consider first the case that the supply constraint is the binding one: $I \leq \alpha R$. The supply constraint, equation (3), holds then with equality. Solving it for y_1 we obtain

$$y_1 = \frac{H - (1 + \delta_0) y_0}{1 + \delta_1}. \tag{7}$$

The remaining liquidity is generated through liquidations. Using (6) and (7) we obtain

$$l = \lambda - \frac{H + (\delta_1 - \delta_0) y_0}{1 + \delta_1}. \tag{8}$$

Consider next that the borrowing constraint is the binding one: $I > \alpha R$. In this case, liquidity will be transferred to bankers up to their borrowing capacity; the remaining liquidity will again be generated through liquidations. It is optimal to undertake such liquidations on the project's non-pledgeable part in order not to reduce the borrowing capacity. A condition that guarantees that the non-pledgeable part is large enough to generate the required liquidity is given by:

Assumption 3 $(1 + \gamma(\lambda - \frac{H}{1+\min[\delta_0, \delta_1]})) \cdot (\lambda - \frac{H}{1+\min[\delta_0, \delta_1]}) \leq (1 - \alpha) R.$

Liquidity transfers y_1 and liquidations l are then still given by equations (7) and (8).

At $t = 0$, the social planner has to decide upon how much liquidity, y_0, to transfer to bankers. Substituting y_1 and l in equation (1), we obtain welfare as a function of y_0 only:

$$W(y_0) = R + I - (\delta_0 - \phi_0)y_0 - \pi\delta_1 y_1(y_0) - \pi\Gamma(l(y_0)). \tag{9}$$

What determines optimal holdings of precautionary liquidity? By differentiating (9) with respect to y_0 we obtain the (marginal) benefits of precautionary liquidity:

$$W'(y_0) = -(\delta_0 - \phi_0) - \pi\delta_1 y_1'(y_0) - \pi\Gamma'(l)l'(y_0). \tag{10}$$

The first two terms represent the trade-off that arises due to the cost of investor liquidity. More ex-ante liquidity means that costs of $\delta_0 - \phi_0$ are incurred at date 0. This provides benefits ex-post when the liquidity shock arises as less liquidity then has to be transferred ($y_1'(y_0) < 0$) and hence less of the δ_1 cost is incurred. Precautionary liquidity thus trades off incurring δ_0 at $t = 0$ for sure against incurring δ_1 at $t = 1$ with probability π.

The last term is the effect on the liquidation costs and depends on the costs of δ_1 relative to δ_0 (we have that $sign(\Gamma'(l)l'(y_0)) = -sign(\delta_1 - \delta_0)$). What is the reason for this? Consider first the case of the supply constraint. When $\delta_0 < \delta_1$, ex-ante transfers are more *effective* than ex-post transfers in the sense that they maximize the amount of investor liquidity that can be made available to banks at $t = 1$. They thus lower liquidations and hence liquidation costs. When $\delta_1 < \delta_0$, exactly the opposite argument arises and ex-post liquidity lowers liquidations. The case of the borrowing constraint follows the same logic. This constraint limits the amount that can be pledged to outsiders, which translates into a maximum amount of liquidity that can be transferred to banks. So using the more effective means of liquidity alleviates the borrowing constraints and lowers liquidations.

Optimal liquidity holdings thus minimize the cost from transferring liquidity to banks and the amount of liquidations. A trade-off arises when one channel of liquidity is more costly (in terms of the liquidity costs δ) but makes liquidations less likely. For example, when $\delta_0 - \phi_0 > \pi\delta_1 |y_1'(y_0)|$ and $\delta_0 < \delta_1$, ex-ante liquidity is more costly but reduces liquidations. To see the effects more precisely, the following proposition examines the comparative statics of interior solutions when the average liquidation cost function is linear:

Proposition 1 *For linear liquidation costs $\gamma(l) = \gamma_0 + \frac{\gamma_1}{2}l$ ($\gamma_0, \gamma_1 > 0$) efficient precautionary liquidity y_0^{eff}*

(i) increases (decreases) in the size of the liquidity shock λ when $\delta_1 > \delta_0$ ($\delta_1 < \delta_0$);

(ii) decreases (increases) in the size of the investor sector I and the pledgeable return αR when $\delta_1 > \delta_0$ ($\delta_1 < \delta_0$);

(iii) increases (decreases) in the cost parameters γ_0 and γ_1 when $\delta_1 > \delta_0$ ($\delta_1 < \delta_0$).

(iv) increases in the private benefits ϕ_0.

Proof. *See appendix.* ■

The results *(i)-(iii)* all follow the same intuition. A parameter change that increases the liquidation problem (an increase in the shock λ, a tightening of the liquidation constraints through a reduction in I or αR, or an increase in the costs γ_0 and γ_1) will increase (unit) liquidation costs γ. This, in turn, makes avoiding liquidations more beneficial. The consequence of this will be that the more effective channel of liquidity becomes more valuable, making it optimal to hold more ex-ante liquidity when $\delta_0 < \delta_1$, and more ex-post liquidity when $\delta_0 > \delta_1$. Part *(iv)* of the propositions shows, as to be expected, that higher private benefits from ex-ante liquidity make holding more of it optimal.

5 Equilibrium

We start by deriving the optimization problems for bankers and investors. A banker has to decide how much precautionary liquidity to raise from investors (y_{0j}). At $t = 1$, when the liquidity shock hits, he also has to decide how much liquidity to raise ex-post (y_{1j}) and how much to liquidate (l_j). In his optimization, the banker takes as given the interest rate on (two-period) borrowing from investors at date 0, r_0, the interest rate at date 1 when the liquidity shock hits, r_1, as well as the liquidation cost γ.

A banker faces two constraints, which are similar to the one of the social planner. First, when the shock materializes, his liquidity (after borrowing and liquidations) has to be at least as high as the liquidity need λ:

$$y_{0j} + y_{1j} + l_j \geq \lambda. \tag{11}$$

Second, his borrowing capacity cannot be exceeded:

$$(1 + \delta_0)(1 + r_0)y_{0j} + (1 + \delta_1)(1 + r_1)y_{1j} \leq \alpha R. \tag{12}$$

The expected utility for the banker can be written as the project return R, minus costs arising from ex-ante and ex-post liquidity, as well as liquidations:

$$W^B(y_{0j}, y_{1j}, l_j) = R - ((1+\delta_0)(1+r_0) - \phi_0 - 1)y_{0j} - \pi((1+\delta_1)(1+r_1) - 1)y_{1j} - \pi\gamma(l)l_j. \tag{13}$$

The term $-((1+\delta_0)(1+r_0)-\phi_0-1)y_{0j}$ is the cost of precautionary liquidity, arising because the liquidity cost is higher than the private benefit and because interest has to be paid. The term $-\pi((1+\delta_1)(1+r_1)-1)y_{1j}$ is the expected cost of ex-post liquidity, due to liquidity costs and interests. The final term $-\pi\gamma(l)l_j$ is the banker's expected cost due to liquidations. This cost depends on both the banker's liquidations l_j and aggregate liquidations l as the latter determine the unit costs γ.

Investors have to decide how much to lend at $t=0$ and when the shock hits at $t=1$. They will store any unused liquidity for the next period. Denoting lending at $t=0$ and $t=1$ with s_{0i} and s_{1i}, respectively, an investor's expected utility can be written as:

$$W^I(s_{0i},s_{1i})=1+r_0s_{0i}+\pi r_1 s_{1i}. \tag{14}$$

Definition 1 *A competitive equilibrium consists of interest rates r_0^* and r_1^*, bank liquidity and liquidation choices $\{y_{0j}^*,y_{1j}^*,l_j^*\}_{j\in[0,1]}$ and investor lending choices $\{s_{0i}^*,s_{1i}^*\}_{i\in[0,I]}$ such that*

(i) for each bank j: y_{0j}^*,y_{1j}^*,l_j^* maximize expected profits W^B subject to (11), (12) and $y_{0j},y_{1j},l_j\geq 0$;

(ii) for each investor i: s_{0i}^*,s_{1i}^* maximize expected profits W^I, subject to $s_{0i},s_{1i}\geq 0$ and $s_{0i}+s_{1i}\leq 1$;

(iii) the markets for liquidity clear at $t=0$ and $t=1$: $(1+\delta_0)y_0^*=s_0^*$ and $(1+\delta_1)y_1^*=s_1^*$.

Lemma 1 *In equilibrium, the difference between a banker's and the social (marginal) benefits of precautionary liquidity is given by*

$$W^{B\prime}(y_{0j})-W'(y_0)=\pi\gamma'(l)l'(y_0)l(y_0). \tag{15}$$

Proof. *See appendix.* ■

Private and social benefits from liquidity differ due to an externality. An individual banker does not take into account that his liquidity decisions affect aggregate liquidations l and hence the unit liquidation costs γ for all other bankers in the economy. The sign of the externality depends again on the relative costs. In particular, when $\delta_0<\delta_1$, precautionary liquidity reduces liquidations ($l'(y_0)<0$) and hence creates a positive externality from precautionary liquidity, while when $\delta_0>\delta_1$, the externality is a negative one.

The following proposition derives the efficiency properties of the equilibrium.[7]

[7] Note that in equilibrium only aggregate precautionary liquidity, y_0, is determined – but not the individual holdings at each bank (each bank is indifferent to the amount of own precautionary liquidity).

Proposition 2 *The equilibrium amount of precautionary liquidity y_0^* differs from the (constrained) efficient one y_0^{eff} whenever $\delta_0 \neq \delta_1$. In particular:*

(i) When $\delta_0 < \delta_1$, equilibrium liquidity is less than the (constrained) efficient one: $y_0^* < y_0^{eff}$;

(ii) When $\delta_0 > \delta_1$, equilibrium liquidity exceeds the (constrained) efficient one: $y_0^* > y_0^{eff}$.

Proof. *See appendix.* ■

The inefficiency depends on the relative size of δ_0 and δ_1, regardless of which liquidity constraint binds. In the case of the supply constraint, using the more effective channel for raising liquidity (that is, ex-ante liquidity when $\delta_0 < \delta_1$ and ex-post liquidity when $\delta_0 > \delta_1$) increases the total liquidity available to banks at $t = 1$. This in turn lowers liquidations and liquidation costs, with positive effects for all liquidating banks. There is thus a positive externality associated with using the (privately) most efficient way of raising liquidity. In the case of the borrowing constraint, using the more effective channel means that for given borrowing capacity banks maximize the amount of liquidity they can raise without exceeding the constraint, again lowering liquidations and unit liquidation costs. However, the level of aggregation at which the externality operates differs. In the case of constrained liquidity supply, individual liquidity holdings at $t = 0$ affect *aggregate* liquidity available at $t = 1$, and through this affect liquidations. In the case of borrowing constraints, individual liquidity holdings at a bank affect all banks' *individual* constraints at $t = 1$, and through this liquidations.

5.1 Asset sales and liquidation costs

We have asssumed liquidation costs that are increasing in the total amount of liquidations in the economy. One interpretation of this is asset sales. When liquidations take the form of asset sales, higher selling in the economy results in lower prices ("fire-sale" prices), thus increasing liquidation costs for all banks liquidating. In Appendix B we model this channel explicitly. We consider non-bank buyers who may purchase assets but require a discount for doing so. The compensation they require for taking on assets increases in the total amount of asset already purchased because of risk-aversion: the marginal compensation for a unit of risk increases when the buyer is already exposed to this risk. We show that this results in an increasing liquidation cost function and that, as a result, the same wedge as in Proposition 2 arises between efficient and equilibrium liquidity holdings.

6 Public provision of liquidity

In this section we introduce liquidity supply by a central bank (CB). Public liquidity has two potential functions our model. First, it may lower liquidity shortages at $t = 1$ and reduce costly liquidations. Second, it may change the equilibrium allocation of liquidity in the economy – this may be desirable since this allocation is inefficient (Proposition 2).

We model the CB as follows. The CB has a total stock of liquidity of $\overline{m}$.[8] The CB can lend this stock to banks at date 0 and date 1 by setting up borrowing facilities. A borrowing facility is characterized by its interest rate r_t^{CB} and the size m_t. The CB's objective is to choose these parameters such that welfare is maximized. CB liquidity is subject to the same date-0 and date-1 frictions as liquidity from investors. To focus on (pure) liquidity policies, we assume that the CB cannot lend at an expected return that is less than the return on storage ($r_t^{CB} \geq 0$). Finally, to avoid ambiguity, when different liquidity policies obtain the same level of welfare, we assume that the CB chooses the one that results in the lowest expected amount of CB liquidity injected into the economy (this can be justified by a small deadweight cost of providing liquidity).

We denote with y_t^I the amount of private liquidity (investor liquidity) borrowed by banks, with y_t^{CB} the public (CB) liquidity borrowed and with y_t ($= y_t^I + y_t^{CB}$) total liquidity borrowed at t. Welfare is identical to equation (1) amended for the stock of public liquidity $\overline{m}$. Denoting $\widetilde{I} := I + \overline{m}$ we have

$$W = R + \widetilde{I} - (\delta_0 - \phi_0)y_0 - \pi\delta_1 y_1 - \pi\Gamma(l). \tag{16}$$

The new market clearing conditions for liquidity are given by

$$\begin{aligned} (1+\delta_0)y_0 &= s_0 + m_0, && (17)\\ (1+\delta_1)y_1 &= s_1 + m_1. && (18) \end{aligned}$$

A banker's date-1 borrowing constraint is determined by the sum of private and public liquidity raised and is given by

$$(1+\delta_0)(1+r_0)y_{0j}^I + (1+\delta_0)(1+r_0^{CB})y_{0j}^{CB} + (1+\delta_1)(1+r_1)y_{1j}^I + (1+\delta_1)(1+r_1)y_{1j}^{CB} \leq \alpha R, \tag{19}$$

[8] Limits to liquidity provision capture the idea that central bank liquidity is not costless (in particular: it can cause inflation) or that the central bank lacks credibility. Note that the case where the CB can generate unlimited liquidity (at zero cost) can be accommodated by letting $\overline{m}$ become arbitrarily large.

and his utility is

$$\begin{aligned} W^B &= R-((1+\delta_0)(1+r_0)-\phi_0-1)y_{0j}^I-((1+\delta_0)(1+r_0^{CB})-1)y_{0j}^{CB} \\ &\quad -\pi((1+\delta_1)(1+r_1)-1)y_{1j}^I+((1+\delta_1)(1+r_1^{CB})-1)y_{1j}^{CB}-\pi\gamma(l)l_j. \end{aligned} \tag{20}$$

We obtain the following modified definition of an equilibrium:

Definition 2 *An equilibrium with CB lending* $\{m_0, r_0^C, m_1, r_1^{CB}\}$ *consists of interest rates* r_0^* *and* r_1^**, bank liquidity and liquidation choices* $\{y_{0j}^{I*}, y_{0j}^{CB*}, y_{1j}^{I*}, y_{1j}^{CB*}, l_j^*\}_{j\in[0,1]}$ *and investor lending choices* $\{s_{0i}^*, s_{1i}^*\}_{i\in[0,I]}$ *such that*

(i) for each bank j*:* $y_{0j}^{I*}, y_{0j}^{CB*}, y_{1j}^{I*}, y_{1j}^{CB*}, l_j^*$ *maximize expected profits* W^B *subject to (11), (19) and* $y_{0j}^I, y_{0j}^{CB}, y_{1j}^I, y_{1j}^{CB}, l_j \geq 0$*;*

(ii) for each investor i*:* s_{0i}^*, s_{1i}^* *maximize expected profits* W^I*, subject to* $s_{0i}, s_{1i} \geq 0$ *and* $s_{0i}+s_{1i} \leq 1$*;*

(iii) the market for liquidity clears at $t=0$ *and* $t=1$*:* $(1+\delta_0)y_0^* = s_0^* + m_0$ *and* $(1+\delta_1)y_1^* = s_1^* + m_1$*;*

(iv) the borrowing facility is not exceeded at $t=0$ *and* $t=1$*:* $y_0^{CB*} \leq m_0$ *and* $y_1^{CB*} \leq m_1$*.*

The equilibrium conditions for banks and investors (*(i) and (ii)*) are unchanged – except for banks who can now also borrow from the CB. Condition *(iii)* is the new market clearing for liquidity at $t=0$ and $t=1$, whereas condition *(iv)* states that banks cannot borrow more from the CB than the CB offers them.

An analysis similar to that in Section 5 can be used to solve for the equilibrium, with CB lending. In turn this equilibrium outcome can be used to determine welfare W^*; it will be a function of the CB policy choice $(m_0, r_0^C, m_1, r_1^{CB})$. The CB's problem is then to set the parameters of the borrowing facilities such that the equilibrium with the highest welfare results:

$$\max_{m_0, r_0^{CB}, m_1, r_1^{CB}} W^*, \text{ subject to } m_0+m_1 \leq \overline{m} \text{ and } r_0^{CB}, r_1^{CB} \geq 0. \tag{21}$$

Proposition 3 summarizes the efficiency implications of public liquidity provision (the underlying optimal policies are derived in the proof in the appendix).

Proposition 3 *Liquidity policies can implement the (constrained) efficient solution unless the borrowing constraint binds* ($\alpha R < \widetilde{I}$ *and* $\lambda - \frac{H}{1+\min[\delta_0,\delta_1]} > 0$)*. In the latter case,* *precautionary liquidity will be insufficient under an optimal liquidity policy when $\delta_0 < \delta_1$ and excessive when $\delta_0 > \delta_1$.*

Proof. *See appendix.* ■

The reason why liquidity policies can implement efficiency when the liquidity constraint (but not the borrowing constraint) binds is that (market) interest rates are then positive. This allows the CB – by varying the size of its facilities – to fully control liquidity take-up at banks by lending at below market rates. If the borrowing constraint binds, market interest rates are zero and hence the CB cannot undercut the market without subsidizing banks. It can thus no longer control liquidity take-up at banks.

If we also allow the CB to subsidize banks by lending at a negative expected return, the allocation can be improved along two dimensions. The CB (or the government) can then provide liquidity to banks without requiring repayment (essentially, a transfer of resources as in a bail-out). CB lending is hence not longer included in the borrowing constraint, relaxing the constraint. This has the consequence that the amount of liquidations in the economy can be reduced. In addition, the CB can now also replace the market at either $t = 0$ or $t = 1$ (as investors will never lend at negative returns). The CB will hence also be able to correct any misallocation of liquidity that occurs when $\delta_0 \neq \delta_1$.

6.1 Discussion

The analysis has interesting implications for optimal (ex-post) liquidity policies. Suppose that a CB observes a liquidity shortage in the banking system. If the shortage arises from an insufficient *supply* of liquidity, the CB should inject liquidity into the system in order to alleviate the shortage (see the proof of Proposition 3) However, when liquidity shortages are due to an insufficient *ability* to raise funds by banks, pure liquidity policies are ineffective. In the latter case the CB optimally remains passive even though there are asset discontinuations arising from a lack of liquidity in the economy. In practical terms, these two cases can be distinguished by looking at prevailing (market) interest rates. If asset liquidations are observed and interest rates are high at the same time, we are in the case of insufficient liquidity supply (as shown in Section 5). The CB can then follow standard liquidity policies, for example, through the discount window. However, if interest rates are not elevated when asset liquidations are observed, other policies are needed (e.g., government bail-outs).

There are also implications for whether a CB should pursue active ex-ante policies.

Again, the scope for such a policy depends on the type of shortage. If a banking system tends to experience problems because of a limited ability to borrow, there is no role for influencing ex-ante interest rates. However, when problems tend to occur because of liquidity supply problems, there is a role for setting interest rates. The optimal policy requires "to lean against" the bias created by the externalities (see the proof of Proposition 3). When $\delta_0 < \delta_1$, this implies that the CB may want to stimulate precautionary liquidity by reducing interest rates in normal times (ex-ante). By contrast, when $\delta_0 > \delta_1$, the CB should try to drain liquidity from the banking system. It can do so by keeping ex-ante interest rates high to discourage borrowing, while at the same time promising low interest rates in crisis (which increases the benefits from relying on ex-post liquidity).

7 Liquidity frictions

In this section we model sources of the ex-ante and ex-post liquidity cost, δ_0 and δ_1, and discuss what determines their relative size in practice.

We modify the setup in Section 3 as follows. At date 0, banks raise liquidity from investors without incurring any cost at first. Banks are run by a manager who can appropriate a fraction $\frac{\delta_0}{1+\delta_0}$ of the (free) liquidity between date 0 and date 1, providing the manager with private benefits ϕ_0/δ_0 per unit of appropriated funds. It follows that in order for a bank to arrive at date 1 with y_{0j} units of liquidity, the bank has to raise $(1+\delta_0)y_{0j}$ units of funds from investors at date 0.

In addition, we assume that at date 1 a mass of δ_1 "bad" banks appear who pretend to be ordinary banks. Bad banks operate socially worthless projects: they require an input of l at $t = 1$ but only provide private benefits ϕ_1 ($\phi_1 > 0$) to the managers of these banks. We distinguish between two cases, depending on whether investors are *sophisticated* or *unsophisticated*. Sophisticated investors can identify the bad banks. They will hence never lend to them. Unsophisticated investors, however, cannot distinguish among banks. When lending, there is hence a likelihood of $\frac{\delta_1}{1+\delta_1}$ of the funds being channelled to a bad bank, in which case the liquidity is lost. As in the baseline model, this implies that in order to bring y_{1j} of ex-post liquidity into banks, $(1+\delta_1)y_{1j}$ of investor liquidity is needed at date 1.

Our modelling of ex-ante and ex-post frictions aims to capture that in "crisis times" (when banks are hit by negative shocks) it is particularly difficult for outsiders to make sure they invest in viable banks with good projects. Uncertainty about asset quality is likely to be a major obstacle to raising funds in such situations.[9] By contrast, in "normal times" (date 0 in our model), concerns about asset quality are less paramount. In such times, inefficiencies due to agency problems within the bank are of higher relevance.

Appendix C shows that in the case of unsophisticated investors, the model is identical to the one of Section 3. Proposition 2 still holds: there is a bias in liquidity holdings – with the direction of the bias depending on the relative size of ex-ante and ex-post costs. The case of sophisticated investors is straightforward. These investors have no problems discerning good banks when lending at $t = 1$. The presence of bad banks is hence irrelevant. The problem is thus identical to case of unsophisticated investors and no bad banks ($\delta_1 = 0$). From Proposition 2 we then have that sophisticated investors will lend inefficiently high amounts to banks ex-ante.

7.1 Discussion

The analysis in this section helps us to understand for which types of financial institutions – in practice – we would expect precautionary liquidity to be too low or too high. First, the direction of the liquidity externality will depend on the characteristics of the investors an institution deals with. Consider for example a traditional commercial bank, financing itself with retail deposits. This bank largely deals with unsophisticated investors. For retail investors the cost of providing liquidity in times of crisis when informational asymmetries are pronounced is high. We are thus likely to have the case of high δ_1 relative to δ_0 and precautionary liquidity is probably too low. Consider, alternatively, a Money Center Bank that finances itself in the commercial paper or the interbank market. Its funding base comes from informed investors who are better able to deal with asset quality problems that are key in a crisis. The relative costs of raising funds ex-post from these investors is thus low (δ_1 is small) and pre-cautionary liquidity holdings are likely to be excessive.[10]

Our analysis also suggests that the direction of the liquidity externality depends on the scope for asset substitution by managers (proxied by δ_0 in the model). For institutions where managers have large discretion and where there is little market discipline, it is

[9] This feature is also key in the model of Bolton et. al (2010), where informational assymmetries increase in liquidity crises. It also played a significant role during the crisis of 2007 and 2008, when it became difficult to ascertain the risk profile and asset quality of many financial institutions (see Gorton, 2009a and 2009b).

[10] Put more informally, liquidity supplied by such investors is very valuable in a crisis – it should hence not be "wasted" by using it at banks in normal times where it may be depleted because of agency problems.

wasteful to have spare liquidity (as this would lead to a significant reduction in the liquidity that is available to banks in crises); instead liquidity should only be raised when liquidity problems materialize. The potential for asset substitution is more significant at institutions that engage in opaque activities – such as proprietary trading and investment in OTC derivatives. This would point to large and complex banks as well as investment banks as institutions where precautionary liquidity may be inefficiently high.

We can also expect the liquidity externality to depend on the frequency with which liquidity shortages occur. If liquidity problems do not arise often, a longer period of time will (on average) pass between the raising of precautionary liquidity and the arrival of a liquidity need. This would worsen the asset substitution problem as managers have more time to divert funds (δ_0 is high), suggesting a higher potential for a negative liquidity externality.[11] We would thus expect institutions that encounter liquidity shortages infrequently to raise an inefficiently high amount of liquidity ex-ante, while institutions that regularly experience significant liquidity problems may underinvest in liquidity.

Finally, the direction of the liquidity externality also depends on how one interprets "ex-ante". If liquidity is raised at the time of the initial investment in projects, potential future liquidity needs may be far ahead (and hence δ_0 large). By contrast, if banks have the possibility to raise funds in direct anticipation of an impending liquidity shock, δ_0 will be low (as in this case there is little time for the manager to misuse funds). This suggests that the potential for excessive precautionary liquidity arises with respect to liquidity that is raised in normal times and when there is no immediate threat of a shortage. By contrast, liquidity raised just before a potential shortage may be insufficient from the social perspective.

8 Summary and conclusions

We have developed a model in which banks can raise liquidity in anticipation of liquidity needs, but also once liquidity shocks materialize. We have allowed for a relatively complete description of liquidity sources and their limits. A consequence of this is that the intuition that banks tend to hold insufficient amounts of liquidity (due to its public good character) no longer holds, as banks can still raise liquidity once hit by a shock. In particular, in sit-

[11] This could be viewed in our model as making the number of periods that pass until a liquidity shortage arises random and giving the manager the opportunity to appropriate a share of (uninvested) funds in *every* period.

uations where ex-ante liquidity is subject to squandering, precautionary liquidity holdings can undermine the banking sector's ability to generate funds in crises. As individual banks do not internalize the systemic nature of this effect, liquidity in an unregulated banking sector may hence be inefficiently high.

Our model provides several key insights. First, while the focus of the banking literature has mostly been on the optimal allocation of resources within the banking sector, our paper shows that inefficiencies can also arise across sectors. Banking policies (such as liquidity ratios or capital requirements) should hence take into account their impact on the cross-sector allocations of funds. Second, optimal liquidity policies may differ by type of financial institution. Precautionary liquidity holdings may have to be discouraged at institutions that have relatively high ex-ante costs from raising liquidity and at institutions that tend to raise liquidity from a confined pool of informed investors. At other institutions, there is a rationale for regulators to encourage precautionary liquidity holdings. Third, the optimal liquidity policy of a central bank depends on the type of liquidity shortage in a crisis. If there is a general shortage of liquidity in the economy, the standard prescription applies that the central bank should inject liquidity in response to such a shortage. However, when there is only a shortage of liquidity in the banking sector, there is no role for provision of public liquidity as such shortages are driven by limits to the borrowing capacity of banks.

Our analysis has abstracted from various issues. For one, we have taken as given banks' investment in illiquid activities. Dispensing with this assumption will create an additional inefficiency (arising from excessive investment in illiquid projects) but does not modify the liquidity results. Second, we have assumed that the amount of liquidity available in the investor sector in times of crisis falls one-to-one with the amount investors supplied to the banking sector in normal times. A more complete model of the world would consider intermediate consumption choices of investors and/or allow investors to make illiquid investments as well. The link between ex-ante and ex-post liquidity supply would then be no longer one-to-one (and possibly either smaller or larger) but our main insights should nonetheless continue to apply, since we can still expect a negative relationship between ex-ante and ex-post liquidity supply. Finally, our model focuses on liquidity needs that arise from banks. Investors, however, may also be subject to liquidity shocks, in which case the interaction between the two sectors becomes a two-way one. This is an interesting problem for future research.

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Appendix A: Proofs

Proof of Proposition 1. We derive the comparative statics using the implicit function theorem (the model can also be solved explicitly for linear costs, however, tackling the problem more generally allows to better get the intuition behind the various effects). The first-order condition for efficient liquidity can be written as $W'(y_0(\tau), \tau) = 0$, where τ represents one of the models parameters. Totally differentiating with respect to τ and solving for $y_0'(\tau)$ we obtain for the comparative statics for τ:

$$y_0'(\tau) = \frac{\frac{\partial W'(y_0)}{\partial \tau}}{-W''(y_0)}. \tag{22}$$

For linear costs, $W'(y_0)$ is given by (from equation (10)):

$$W'(y_0) = -(\delta_0 - \phi_0) + \pi\delta_1 \frac{1+\delta_0}{1+\delta_1} + \pi(\gamma_0 + \gamma_1(\lambda - \frac{\min[I, \alpha R] + (\delta_1 - \delta_0)y_0^{eff}}{1+\delta_1}))\frac{\delta_1 - \delta_0}{1+\delta_1}. \tag{23}$$

We have that the optimization problem is concave as the second derivative of welfare is negative:

$$W''(y_0) = -\pi\gamma_1 \frac{(\delta_1 - \delta_0)^2}{(1+\delta_1)^2} < 0. \tag{24}$$

It follows that at an interior solution the sign of $y_0'(\tau)$ is given by the sign of $\frac{\partial W'(y_0)}{\partial \tau}$. We focus on the comparative statics for which clear results can be obtained (for the ex-ante and ex-post costs as well as the the likelihood of a liquidity shock the comparative statics depend on the whole set of parameters).

Impact of the size of the investor sector I: The size of the investor sector (and hence the total supply of liquidity in the economy) matters when the supply constraint is the binding one ($I \leq \alpha R$). From differentiating (23) with respect to I (assuming $I \leq \alpha R$), we obtain:

$$\frac{\partial W'(y_0)}{\partial I} = -\pi\gamma_1 \frac{\delta_1 - \delta_0}{(1+\delta_1)^2}. \tag{25}$$

An increase in I will hence reduce efficient liquidity holdings y_0^{eff} when $\delta_0 < \delta_1$, and raise them when $\delta_0 > \delta_1$. What is the intuition for this result? A higher supply of liquidity by investors means that less assets have to be liquidated when the liquidity shock hits (that is, l falls). The unit-liquidation costs γ will hence be smaller. This has the consequence that the benefits from lowering liquidity shortages decline. As a result, it becomes less important to choose the more effective means of raising liquidity. In particular, when precautionary liquidity is the more effective one, its optimal amount of it falls.

Impact of the pledgeable return αR: This case matters when $\alpha R > I$ and its analysis mirrors the prior one of $\alpha R \leq I$. From differentiating (23) with respect to αR ($\alpha R > I$), we obtain:

$$\frac{\partial W'(y_0)}{\partial(\alpha R)} = -\pi\gamma_1 \frac{\delta_1 - \delta_0}{(1+\delta_1)^2}, \tag{26}$$

identical to (25). It follows that when the tightness of the liquidity constraint falls because of higher borrowing capacity, the more effective means of raising liquidity becomes less attractive.

Impact of the liquidity shock λ: The derivative with respect to λ is:

$$\frac{\partial W'(y_0)}{\partial\lambda} = \pi\gamma_1 \frac{\delta_1 - \delta_0}{1+\delta_1}. \tag{27}$$

The impact depends again on the relative costs, but with reversed sign. A higher liquidity demand at $t = 1$ means, ceteris paribus, that unit liquidation costs will be higher as more assets are liquidated. This means that the more effective mean of raising liquidity becomes more valuable, hence y_0^{eff} increases when $\delta_0 < \delta_1$ and falls when $\delta_0 > \delta_1$.

Impact of the cost parameters γ_0 and γ_1: A higher γ_0 or γ_1 raises the unit liquidation costs. Avoiding liquidations thus becomes more valuable, favouring again the more effective means of raising liquidity. This is confirmed by the relevant derivatives:

$$\frac{\partial W'(y_0)}{\partial\gamma_1} = \pi\frac{\delta_1 - \delta_0}{1+\delta_1}, \tag{28}$$

$$\frac{\partial W'(y_0)}{\partial\gamma_1} = \pi l\frac{\delta_1 - \delta_0}{1+\delta_1}. \tag{29}$$

Impact of the private benefits ϕ_0: Higher private benefits increase optimal precautionary liquidity: the derivative is given by

$$\frac{\partial W'(y_0)}{\partial\phi_0} = 1. \tag{30}$$

■

Proof of Lemma 1. We solve for the equilibrium starting with $t = 1$. We consider first the case where the liquidity supply constraint is binding ($I \leq \alpha R$). The state where the liquidity shock does not arrive can again be ignored as the the banker then has no need for funds and will hence neither borrow nor liquidate. When the liquidity shock hits, a banker will generate a sufficient liquidity to just meet the liquidity requirement (equation (11) is fulfilled with equality):

$$y_{0j} + y_{1j} + l_j = \lambda. \tag{31}$$

In equilibrium, bankers have to be indifferent between generating liquidity through borrowing and through liquidations. The (net) cost of raising one unit of liquidity from investors is $(1+\delta_1)(1+r_1)-1$, whereas the cost of raising it through liquidations is $\gamma(l)$. Setting both equal and solving for the date-1 interest rate we obtain:

$$r_1 = \frac{\gamma(l)-\delta_1}{1+\delta_1}. \tag{32}$$

The interest rate is positive (follows from Assumption 1), reflecting that there is a shortage of liquidity supply. Investors will hence supply their entire date-1 liquidity, $I-(1+\delta_0)y_0$, to banks. It follows that the total date-1 liquidity raised by banks, y_1, and total liquidations, l, are still characterized by equations (7) and (8).

We consider next $t=0$. Equilibrium requires that investors are indifferent between lending to banks and storing funds. From the equation for investors' profits, (14), we obtain that this requires

$$r_0 = \pi r_1, \tag{33}$$

that is, the excess return on liquidity at $t=0$ (earned for sure) has to equal the return at $t=1$ when the liquidity shortage materializes, times the likelihood of a liquidity shortage.

We next derives banker's profit as a function of precautionary liquidity, y_0^i. Substituting r_0 and r_1 in (13) and using that $y_{1j}=\lambda-y_{0j}-l_j$ (equation (31)) we obtain:

$$W^B(y_{0j}) = R-((1+\delta_0)(1+\pi\frac{\gamma(l)-\delta_1}{1+\delta_1})-\phi_0-1)y_{0j}-\pi(\lambda-y_{0j})\gamma(l). \tag{34}$$

We consider next the case where the borrowing constraint is the binding one ($I>\alpha R$). In the case of no liquidity shock at $t=1$, there are again no liquidity dealings between banks and investors as banks have no liquidity requirements. When the liquidity shock hits, banks will now be constrained by their borrowing capacity. Each bank will hence borrow up to its borrowing capacity, equation (12), and raise the remaining liquidity through liquidations. Since now the liquidity demand is the constraining factor, investor liquidity is in excess supply. The equilibrium interest rate at $t=1$ hence has to be $r_1=0$, in order to make investors indifferent between lending and storage.

Investors thus do not earn excess return on liquidity at $t=1$. This means that the interest rate at $t=0$ has to be zero as well (in order to make investors indifferent to lending at $t=0$ and $t=1$). The borrowing constraint hence simplifies to:

$$(1+\delta_0)y_{0j}+(1+\delta_1)y_{1j}\leq \alpha R, \tag{35}$$

Using that (35) will be fulfilled with equality at $t = 1$ when the shock hits, one can see that total ex-post liquidity, y_1, and total liquidations, l, are still given by equation (7) and (8). Setting $r_t = 0$ in (13), we get for banker's profits:

$$W^B(y_{0j}) = R - (\delta_0 - \phi_0)y_{0j} - \pi\delta_1 y_{1j} - \pi\gamma(l)l_j. \tag{36}$$

Differentiating (34) and (36) we obtain equation (15). ■

Proof of Proposition 2. When $\delta_0 = \delta_1$, the social planner and a banker's benefit from (precautionary) liquidity are identical ($W'(y_0) = W^{B\prime}(y_0)$). They face hence the same optimization problem and, therefore, we obtain efficiency of the equilibrium: $y_0^* = y_0^{eff}$. When $\delta_0 < \delta_1$, for all y_0 the social gains from liquidity exceed the banker's one: $W'(y_0) > W^{B\prime}(y_0)$. Given that we have assumed interior solutions for the planer's problem and given concavity of $W(y_0)$ and $W^B(y_0)$:

$$W''(y_0) = -\pi[l'(y_0)^2(2\gamma'(l) + \gamma''(l))] < 0, \tag{37}$$

$$W^{B\prime\prime}(y_0) = -\pi l'(y_0)^2\gamma'(l) < 0, \tag{38}$$

we thus have that $y_0^* < y_0^{eff}$. Similarly, for $\delta_0 > \delta_1$, we have that $W'(y_0) < W^{B\prime}(y_{0j})$ and hence $y_0^* > y_0^{eff}$. ■

Proof of Proposition 3. We note first that for the (contrained) efficient allocation, as analyzed in Section 4, it does not matter whether the stock of liquidity is private or public. The efficient allocation in an economy with investor liquidity I and public liquidity m is thus identical to the one of an economy with investor liquidity $\widetilde{I} = I + m$. In the following we denote with $y_0^{eff}(m)$ the efficient ex-ante liquidity in an economy with liquidity $I + m$ (as determined in Section 4) and with $y_1^{eff}(m)$ ($= \frac{\widetilde{I} - (1+\delta_0)y_0^{eff}}{1+\delta_1}$) the corresponding ex-post liquidity. Similarly, we denote with $y_0^*(m)$ and $y_1^*(m)$ (Section 5) the equilibrium amounts of liquidity when total liquidity is $\widetilde{I} = I + m$.

We assume that the public stock of liquidity is large enough to implement efficient liquidity at either $t = 0$ or at $t = 1$: $\overline{m} \geq \max[y_0^{eff}(\overline{m}), y_1^{eff}(\overline{m})]$.

We next analyze optimal CB facilities and whether they can implement the efficient solution. Three different cases have to be distinguished, depending on the stock of public liquidity and the relative stringency of the date-1 constraints.

Case 1: *When all public liquidity $\overline{m}$ is injected into the economy, no constraint binds ($\lambda - \frac{\min[\widetilde{I}, \alpha R]}{1+\min[\delta_0,\delta_1]} \leq 0$).*

This is the case where public liquidity is sufficiently large such that together with investor liquidity the liquidity demand λ can be fulfilled *and* the borrowing capacity does not bind when λ is raised. In this case, the CB can avoid liquidations entirely. We can determine the smallest m that achieves this by rearranging $\lambda - \frac{\min[\widetilde{I},\alpha R]}{1+\min[\delta_0,\delta_1]} \leq 0$ for m (and using that the borrowing constraint does not bind):

$$\widehat{m} := \lambda(1 + \min[\delta_0, \delta_1]) - I. \tag{39}$$

The CB can then implement the efficient allocation by offering in total $\widehat{m}$ at the *same interest rate* as the one that prevails in the market for private liquidity. To see this, note that externalities across banks only arise because of liquidations (Section 5). In the absence of liquidations, the equilibrium amount of precautionary liquidity in an economy with liquidity $I+\widehat{m}$ will hence be equal to the efficient one: $y_0^*(\widehat{m}) = y_0^{eff}(\widehat{m})$. The CB can thus fully focus on inserting an amount of liquidity that is sufficient to resolve the liquidity shortage; it does not have to pursue an active interest rate policy in order to affect the allocation of liquidity.

Since – everything else equal – the CB prefers the policy with the lowest expected public liquidity provision, the CB will supply as much as possible of $\widehat{m}$ ex-post (as ex-post liquidity has to be provided with probability $\pi < 1$ only). It will thus set up a facility of $m_1^* = y_1^{eff}(\widehat{m})$ at date 1, and will offer the rest at date 0: $m_0^* = \widehat{m} - y_1^{eff}(\widehat{m})$. Since liquidations are absent, interest rates in the market for private liquidity will be zero. The CB can hence offer its liquidity at zero interest rates as well: $r_0^{*CB} = r_1^{*CB} = 0$.

Case 2: *When all public liquidity $\overline{m}$ is injected into the economy, the liquidity supply constraint binds ($\widetilde{I} \leq \alpha R$ and $\lambda - \frac{\min[\widetilde{I},\alpha R]}{1+\min[\delta_0,\delta_1]} > 0$).*

In this case, liquidations cannot be avoided, and take place because of an insufficient supply of liquidity. In order to minimize liquidations as much as possible, the CB will then offer its entire stock: $m^* = \overline{m}$.

Can the CB implement the efficient allocation by lending at market rates – as in the first case? The answer is no. If the CB lends at market rates, the expected (private) return on liquidity is equalized across dates – as in the case without CB (condition (33) in the proof of Lemma 1). The private incentives to hold liquidity are then inefficient (for $\delta_0 \neq \delta_1$). Consider, alternatively, that the CB lends at below market rates, but that private lending still takes place at $t = 0$ and $t = 1$ (this will be the case when the size of the borrowing facility is not too large). In this case, market borrowing will still be the marginal source of funding for banks (as banks will first take up the cheaper CB facility). The private return on liquidity will continue to be equalized across time and the incentives to hold precautionary liquidity remain inefficient.

What is the reason for that the CB cannot improve the equilibrium outcomes in these two scenarios? Precautionary liquidity is in equilibrium determined by the relative ex-ante and ex-post cost of borrowing. As market borrowing is the marginal source of funding in both scenarios, private incentives pin down the attractiveness of precautionary liquidity. As a result, the CB cannot affect the allocation of liquidity. The only possibility for the CB to improve the market allocation is hence to fully replace private lending, either ex-ante or ex-post.

Consider first $\delta_0 < \delta_1$. In this case precautionary liquidity is insufficient in a pure market equilibrium. The CB, however, can implement the efficient allocation by offering the efficient amount of liquidity ex-ante: $m_0^* = y_0^{eff}(\overline{m})$. Since there is a liquidity shortage in the economy, market interest rates at date 1, and hence also at date 0, will be larger than zero. By setting sufficiently low interest rates on its ex-ante facility, the CB can make sure that banks will only borrow public liquidity. Consider in particular $r_0^{CB*} = 0$. Banks will then borrow the facility in full (since the allocation of liquidity *across* banks has no welfare consequences, the CB can ration banks in an arbitrary way). As there is a tendency for insufficient private liquidity, the market for ex-ante private liquidity will be inactive. Total precautionary liquidity at banks will hence be $y_0^{eff}(\overline{m})$. At $t = 1$, the CB can then lend it remaining stock ($m_1^* = \overline{m} - y_0^{eff}(\overline{m})$) at market interest rates. The market interest rate is the one that makes banks ex-post indifferent between borrowing and liquidation. Equation (32) tells us that this is the case when $r_1^* = \frac{\gamma(l)-\delta_1}{1+\delta_1}$. Given that liquidations are $\widehat{l} := \lambda - \frac{\widetilde{I}-(1+\delta_0)y_0^{eff}(\overline{m})}{1+\delta_1}$, the central bank can implement a market-neutral facility by setting $r_1^{*CB} = \frac{\gamma(\widehat{l})-\delta_1}{1+\delta_1}$ on its ex-post facility.

Consider next $\delta_0 > \delta_1$. In this case precautionary liquidity is excessive in a pure market equilibrium, and there is insufficient ex-post liquidity. Consider that the CB opens a date-1 facility of $m_1^* = y_1^{eff}(\overline{m})$ at zero interest rates ($r_1^{*CB} = 0$) and lends its remaining stock ($m_0^* = \overline{m} - y_1^{eff}(\overline{m})$) at market interest rates at $t = 0$. As there is a shortage of liquidity at date 1, banks will then take up the entire public supply at this date. Since private liquidity is undersupplied ex-post, no private lending will take place. The market interest rate at date 0 is then pinned down by the condition that banks have to be indifferent between raising liquidity *ex-ante* and liquidating ex-post. Similar to equation (32), this condition is $r_0^* = \pi\frac{\gamma(l)-\delta_1}{1+\delta_1}$. The CB can thus offer its facility at a rate of $r_0^* = \pi\frac{\gamma(\widehat{l})-\delta_1}{1+\delta_1}$.

The reason for why the CB can implement efficiency is that, by lowering interest rates sufficiently, it can fully replace private liquidity at one of the two dates. Through setting the size of the lending facility, the CB can then control bank liquidity at one date, and given that overall liquidity is fixed, also at the other date.

Case 3: *When all public liquidity $\overline{m}$ is injected into the economy, the borrowing constraint binds ($\widetilde{I} > \alpha R$ and $\lambda - \frac{\min[\widetilde{I},\alpha R]}{1+\min[\delta_0,\delta_1]} > 0$).*

In this case, in order to minimize liquidations as much as possible, the CB will inject a total amount of liquidity that makes the borrowing constraint just bind: $\widehat{m} = \alpha R - I$.

Consider first $\delta_0 < \delta_1$. As equilibrium liquidity is too low, the CB would like to increase it by lending $y_0^{eff}(\widehat{m})$ at $t = 0$. However, as we have shown in Section 5, market interest rates are zero (at both dates) when the borrowing constraint binds. Thus the CB cannot lend at rates lower than the market. If it sets up a lending facility of $y_0^{eff}(\widehat{m})$ with $r_0^{CB} = 0$ at $t = 0$, banks will not take in up (in full). As the CB can then not affect the rates on marginal borrowings, the private returns on ex-ante and ex-post liquidity will remain equalized and banks will borrow precautionary liquidity $y_0^*(\widehat{m})$ regardless of the characteristics of the borrowing facility. It follows that in this case the CB has no control over the allocation of liquidity and the economy will be characterized by insufficient precautionary liquidity. The CB can in this situation focus entirely on inserting the right amount of liquidity. To minimize the expected amount of liquidity provided, the CB will offer as much as possible ex-post. This implies setting $m_1^* = y_1^*(\widehat{m})$ at $t = 1$, and offering the remaining stock, $m_0^* = \widehat{m} - y_1^*(\widehat{m})$, at date 0. The interest rates at both dates are zero ($r_0^{*CB} = r_1^{*CB} = 0$).

A similar argumentation applies to the case of $\delta_0 > \delta_1$. The CB would now like to increase ex-post lending by replacing the market at $t = 1$, However, since interest rates are already at zero, this is not possible. We thus obtain the opposite efficiency result (excessive precautionary liquidity). We can conclude that the efficiency bias in the case of the binding borrowing constraints is the same as in Section 5.[12] Optimal liquidity policies are identical to the case of $\delta_0 < \delta_1$: the central bank offers zero-interest rate facilities at date 0 and date 1 of $m_0^* = \widehat{m} - y_1^*(\widehat{m})$ and $m_1^* = y_1^*(\widehat{m})$.

The proposition follows from combining the three cases. ■

[12] In order to implement the efficient allocation, the CB would need to have more control over the liquidity holdings at banks. For example, if the CB could fully control the market for private liquidity, it could implement optimal liquidity holdings by limiting the provision of private liquidity at either $t = 0$ or $t = 1$.

Appendix B: Non-bank buyers

We now assume that – instead of discontinuations at $t = 1$ – assets can be sold to industry outsiders (Shleifer and Vishny, 1992) in order to meet liquidity shortages. Outsiders value assets less than banks, which creates costs to liquidity shortages. Asset buyers may be non-bank financial institutions (vulture funds, private equity or hedge funds) or be located outside the financial system. The risk-bearing capacity of such buyers is likely to be limited – hence assets need to be increasingly discounted in order to be acquired.

Specifically, we consider a continuum of risk-averse outsiders of measure one (while we assume risk-aversion through preferences, risk aversion may also arise indirectly because of capital or leverage constraints). Outsiders are endowed with I_O $(> l)$ units of wealth at $t = 1$ and consume at $t = 2$. Their utility takes quadratic form:

$$u(c_O) = c_O - \frac{1}{2}\rho c_O^2, \tag{40}$$

with $\rho > 0$. We assume complete segmentation between outsiders and investors: outsiders can continue bank projects at $t = 1$ but do not lend to banks, while for investors it is vice versa. We also assume that outsiders are less efficient users of assets, in their hands the date-2 return falls by an amount $\varepsilon > 0$. In addition, assets become risky when held by outsiders: in case an outsider acquires assets from banks at $t = 1$, the date-2 return will be distributed with a common variance of 1.

We first consider the efficient outcome. We assume that the social planner cannot make outsiders worse off at $t = 1$ – similar to what is assumed for investors. In addition, the social planner cannot undertake redistributions among agents at $t = 2$ (otherwise he can eliminate any risks for outsiders through ex-post transfers with other agents). We denote with q the amount of assets transferred to outsiders at $t = 1$. Since each outsider's valuation is decreasing in the amount of asset he is holding, it is optimal to distribute assets equally among outsiders. The total utility loss in the economy from outsiders taking on q units of assets is thus $\varepsilon q + \frac{1}{2}\rho q^2$.

We assume that it is optimal to solve liquidity shortages using asset sales, rather than letting assets become worthless by not providing the liquidity injection (this will be the case for ρ and ε sufficiently small). We thus have for welfare:

$$W(y_0, y_1, q) = R + I + I_O - (\delta_0 - \phi_0)y_0 - \pi\delta_1 y_1 - \pi(\varepsilon q + \frac{1}{2}\rho q^2). \tag{41}$$

This equation is identical to welfare in the baseline model (equation (1)), except that total endowment is now $I + I_O$ and the liquidation cost has become $\varepsilon q + \frac{1}{2}\rho q^2$.

Given that an outsider's utility from holding q units of assets is $Rq-\varepsilon q-\frac{1}{2}\rho q^2$, his per-unit utility is $R-\varepsilon-\frac{1}{2}\rho q$. Since outsiders cannot be made worse off, an asset can thus be transferred in return for an amount of date-1 liquidity of no more than $R-\varepsilon-\frac{1}{2}\rho q$. In order to generate one unit of liquidity for banks, hence $\frac{1}{R-\varepsilon-\frac{1}{2}\rho q}$ assets have to be transferred. Given the asset discount $\varepsilon+\frac{1}{2}\rho q$, the cost of generating one unit of liquidity is then given by $\gamma(q)=\frac{\varepsilon+\frac{1}{2}\rho q}{R-\varepsilon-\frac{1}{2}\rho q}$.

How many assets need to be transferred to investors at $t=1$ to generate l units of liquidity? The maximum liquidity that can be raised when q units are transferred is $q(R-\varepsilon-\frac{1}{2}\rho q)$. Hence to generate an amount of l, one has to transfer assets such that the condition $l=q(R-\varepsilon-\frac{1}{2}\rho q)$ is fulfilled. Solving for q gives $q=\frac{1}{\rho}\left(R-\varepsilon-\sqrt{(R-\varepsilon)^2-4\rho l}\right)$. Using this to substitute q in $\gamma(q)$ we obtain a new expression for liquidation costs:

$$\gamma(l)=\frac{R+\varepsilon-\sqrt{(R-\varepsilon)^2-4\rho l}}{R-\varepsilon+\sqrt{(R-\varepsilon)^2-4\rho l}}. \tag{42}$$

We thus have that (unit) liquidation costs increase in the total liquidity that is raised ($\gamma'(l)>0$). We also have that $\gamma(0)>0$ for sufficiently large ε and $\gamma''(l)>0$, hence this liquidation cost function can fulfill the assumption of the baseline model. Rewriting the loss from asset transfers in (41), $\varepsilon+\frac{1}{2}\rho q^2$, with $\gamma(l)l$ we obtain the same expression for welfare as in the baseline model (except that the endowment is now $I+I_O$). The constraints to the optimization problem are unchanged; it follows that the same solution is obtained as in the baseline model.

Turning to the analysis of the equilibrium, we derive the outcome in the asset market (between banks and outsiders) at $t=1$. This asset market is active when the liquidity shock arrives. Consider an equilibrium where q ($q\geq 0$) units of assets are transferred to outsiders at a price of p. Given that outsider's utility is symmetric and given that their utility is declining in the amount of the asset they are buying in the market, in an equilibrium all outsiders will acquire the same amount of assets. Since in equilibrium outsiders have to be indifferent to asset purchases, we hence have that the price equals an outsider's (unit) valuation of the asset:

$$p=R-\varepsilon-\frac{1}{2}\rho q. \tag{43}$$

This is the same discount as for the social planer. The liquidation cost function is hence also the same as in the baseline model. It follows that the equilibrium outcome is unchanged.

Since both efficient solution and equilibrium are unchanged, Proposition 2 continues to hold. That is, equilibrium liquidity holdings are inefficient (unless $\delta_0=\delta_1$). The source of the inefficiency is a standard fire-sale externality: an individual bank takes γ as given – but its liquidations (slightly) increase liquidation costs at all other banks.

Appendix C: Unsophisticated investors

We first analyze the efficient allocation. We assume that the private benefits at bad banks do not enter the planer's objective function (including them would create a hard-wired externality as ex-post liquidity supply by investors will create a benefit for the owners of bad banks). Welfare is then still given by equation (1) in the baseline model.

The liquidity constraint for ex-ante liquidity is also still given by equation (3) as in order for a bank to bring one unit of precautionary liquidity to $t = 1$ it has to raise $1 + \delta_0$ at $t = 0$. Similarly, in order to bring one unit of liquidity at $t = 1$ to good banks, investors have to lend $1 + \delta_1$ units. Hence equation (3) still applies. Consider next the borrowing capacity. In contrast to the baseline model, the return investors obtain from providing funds at $t = 1$ is now uncertain. We use $\widetilde{r}_1$ to denote the nominal (promised) return to investors and r_1 to denote investors' expected return (the latter is less than the promised one due to the presence of bad banks). The date-1 borrowing constraint is determined by the promised return:

$$(1 + \delta_0)(1 + r_0)y_0 + (1 + \widetilde{r}_1)y_1 \leq \alpha R. \tag{44}$$

We have for the relationship between nominal and expected return: $1 + r_1 = (1 + \widetilde{r}_1)(1 - \frac{\delta_1}{1+\delta_1})$. Using this relationship to substitute $\widetilde{r}_1$ in (44) we obtain equation (4) of the baseline model. We hence face the same optimization problem as in the baseline model. Similarly, the conditions for equilibrium are unchanged as well. Proposition 2 of the baseline model thus still applies.