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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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Abstract

This paper revisits the debate on ruling out speculative hyperinflations in monetary models. Although apparently a narrow issue, studying these extreme economies turns out to be quite illuminating in understanding the fundamentals of price level determination. It is also relevant in evaluating the broader claims that advocates of the fiscal theory of the price level have made. In Obstfeld and Rogoff (1983, 1986) we show that in pure fiat money models, where the government gives no backing whatsoever to currency, there is in fact no reasonable way to rule out speculative hyperinflations where the value of money goes to zero, even if the money supply itself is exogenous and constant. Such perverse equilibria are ruled out, however, if the government provides even a very small real backing to the currency – a fiscal mechanism, but one that comes into play only as a backstop. Indeed that backing does not have to be certain. Cochrane (2011, 2019), however, argues that this result is wrong, and that fractional currency backing is a Maginot line that is insufficient to rule out hyperinflation. We show here why, in fact, his analysis involves a subtle change in model specification that adds a distinct monetary fragility to our model. Our baseline analysis uses a canonical money-in-the-utility-function setup due to Brock (1974, 1975), but following Wallace (1981), we show the same results go through in an overlapping-generations model of money.

JEL Classification: E31, E41, E52, E63

Keywords: Asset bubbles, Fiscal theory of the price level, Hyperinflation, money demand

Maurice Obstfeld - obstfeld@econ.berkeley.edu
University of California, Berkeley and CEPR

Kenneth Rogoff - krogoff@harvard.edu
Harvard University, Department of Economics, Littauer Center 216, Cambridge, MA 02138 USA

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In recent years, economists have faced considerable challenges in understanding price level dynamics. Major central banks have struggled to raise inflation to target despite recurrent episodes of apparently massive monetary stimulus. However, given the sharp rise in government debt levels post COVID-19, together with continuing heroic central bank measures, some express concern that over the long run, inflation, not deflation, will again be the problem. Once seemingly a settled issue, understanding inflation dynamics has emerged as a central topic of research in modern macroeconomics. There has rightly been broad discussion of alternative paradigms, particularly the fiscal theory of the price level, which emerged in the 1990s but now has acquired particular relevance as the interplay between monetary and fiscal policy has become much more salient (see, for example, Leeper 1991; Woodford 1995; Sims 1994, 2016; Canzoneri, Cumby, and Diba 2011; and Cochrane 1999, 2011, 2019).

This paper focuses on one corner of the debate, which has nevertheless become a touchstone in discussions of overthrowing the classical *ancien régime* of monetary conventional wisdom. Is classical monetary theory adequate even to ensure that stable monetary policies produce stable inflation? Pure speculative hyperinflations, in which the price level grows explosively but the money supply is stationary, would certainly be a sharp contradiction to Milton Friedman's dictum that "Inflation is always and everywhere a monetary phenomenon" (Friedman 1970) – but such outcomes seem possible in a variety of macroeconomic models. Failure to offer convincing theoretical foundations to rule out speculative hyperinflations would constitute a major failing of classical monetary theory, showing it as unable to explain even the kind of extreme situations where it should, in principle, be best suited.

In this paper, we will argue that ruling out pure speculative hyperinflations is one important issue that has been correctly solved, and which renders classical monetary predictions broadly correct in many settings. In itself, therefore, the theoretical possibility of a pure speculative hyperinflation is not a reason to seek alternative frameworks. In this paper we take up a fundamental challenge to monetarist orthodoxy posed in by Cochrane (2011, 2019), who argues that even the partial contingent scheme for backing currency that we suggested in Obstfeld and Rogoff (1983, 1986) is logically insufficient. Cochrane's critique has much broader ambitions than just refuting our suggested mechanism: he seeks to show that *only* the fiscal theory of the price level provides a coherent theory of price determinacy. Disputing our paper is a key element of that program, since it has been cited by numerous influential macroeconomists as providing a basis for ruling out multiple monetary equilibria in many settings (for example, Woodford 2003). While we will make the case that Cochrane's criticisms are mistaken, we do point out the obvious, that our monetary backing is a fiscal backstop, albeit one triggered by extreme price movements and not independent of them. Our view has always been that some form of government support is necessary to make government-issued fiat money different from privately-produced fiat money, like Bitcoin. However, our suggested mode of support is one that can stay in the background in equilibrium and leave many of the predictions of standard monetary models intact. We note also that while our framework assumes flexible prices, the same issues of determinacy arise in more recent sticky-price models (as Woodford 2003 and Cochrane 2011 discuss) and our mechanism rules out speculative hyperinflations in that setting too.

Section 1 of this paper reviews the basic argument from our earlier work. Section 2 shows the flaw in Cochrane's charge that our backing scheme is in reality a Maginot line that would crumble in the face of speculative inflation expectations, admitting self-fulfilling hyperinflations. We show that implicit in this claim is a subtle alteration of the original assumptions of our model, and that the candidate hyperinflation equilibrium collapses otherwise, exactly as we originally claimed. This observation suffices to refute Cochrane's argument, but there are broader reasons to question his interpretation. We believe that these broader issues, which involve the social function of money and the government's tools to support acceptance of fiat currency, are key reasons why purely self-fulfilling hyperinflationary equilibria are rarely if ever seen in the real world. In section 3 we extend the discussion to consider issues of credibility (as considered formally by Bassetto 2002, and informally in Cochrane 2019). Section 4 shows that our main conclusions do not depend on the money-in-utlity framework, but hold up equally well in an overlapping generations model inspired by Wallace (1981).

In our conclusions, we underscore that even though our analysis does indeed rule out the extreme case of explosive speculative hyperinflations, multiple stable equilibria likely remain a problem in any reasonable monetary model. We question the generality of the fiscal theory of the price level as conventionally applied, and we speculate more broadly on important price puzzles that remain and have become more pressing in light of recent events.

1 Speculative Hyperinflations and Partial Backing: Reprise

We begin by reprising our argument that by backing currency, a government can foreclose the possibility of hyperinflationary equilibria. We will use the slightly simpler version of the model in Obstfeld and Rogoff (1983, 1986), also employed in Cochrane (2011), which excludes physical capital. Importantly, in this setup the government can always make good any commitment to redeem money partially by levying lump-sum taxes.

Individuals receive y units of the perishable consumption good each period. Let c_t denote an individual's consumption rate at time t, M_t her nominal money holdings, and $\beta \in (0,1)$ her subjective discount factor. The infinitely lived representative consumer maximizes

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) + v\left(M_t/P_t\right)\right], \tag{1}$$

where P_t is the price level at time t, subject to

$$M_t - M_{t-1} = P_t(y - c_t) + H_t, \quad M_{-1} \text{ given},$$

with H_t denoting transfers from the government that the individual takes to be exogenous. We assume that $u(.)$ and $v(.)$ are increasing and strictly concave, with the usual smoothness and Inada properties. The above assumption of separability between the utility from consumption and the derived utility from holding cash balances is quite important, as Obstfeld and Rogoff (1983, 1986) emphasize: otherwise, even though there may not exist divergent speculative paths, there can still be multiple stable equilibrium paths converging to the same monetary equilibrium (as shown in Obstfeld 1984), a fundamental point to which we return later. The consumer's first-order necessary conditions imply

$$\frac{u'(c_t)}{P_t} = \frac{v'(M_t/P_t)}{P_t} + \beta \frac{u'(c_{t+1})}{P_{t+1}} \tag{2}$$

for $t \geq 0$.

Defining real balances as $m_t = M_t/P_t$, and assuming purely for simplicity that the money supply is constant at M, the last equation above can be rewritten as the simple difference equation

$$\beta u'(y) m_{t+1} = m_t[u'(y) - v'(m_t)], \tag{3}$$

where we have imposed the equilibrium condition $c_t = y$. Following the diagrammatic technique of Brock (1974, 1975), we define $A(m) = m[u'(y) - v'(m)]$ and $B(m) = \beta u'(y)m$, in which case the equilibrium can be illustrated graphically as in Figure 1, replicated below from Obstfeld and Rogoff (1983, Figure 2).[1]

[1]Figure 1 assumes that

$$\lim_{m \to 0} m v'(m) = 0,$$

so that the utility level from money balances is bounded from below even as real balances become

Figure 1 illustrates how, unless the price level starts out exactly at the unique stationary equilibrium level $\bar{P}$, then the price will either implode (if it starts out below the equilibrium level), or grow exponentially (if it starts out above the stationary equilibrium level). As we showed, in a pure fiat money system, it is not possible to rule out explosive bubbles where $m_{T+1} = 0$ (that is, where the price level reaches infinity at time $T+1$) if the prior path of real balances $\{m_t\}_{t=0}^{T}$ ends at the point $m_T = \widetilde{m}$ where

$$u'(y) = v'(\widetilde{m}). \tag{4}$$

Even though money becomes worthless at $T+1$, it still gives just enough marginal benefit at time T to compensate for the (small) loss of consumption the individual must forgo to hold on to it, which is why P_T can remain finite even when everyone knows $P_{T+1} = \infty$ is imminent.

We went on to show, however, that speculative price bubbles can be ruled out if the government gives even a very small backing ϵ to the currency, sufficient to cap the price level at $\overline{\overline{P}} = 1/\epsilon < \bar{P}$ through the simple arbitrage argument that money can never trade at a price below the value at which the government is willing to redeem it.[2] (Whether the government will back the currency and the value of backing can both be uncertain.) It is immediately obvious from Figure 1 that such a price ceiling (a floor on the value of money) implies that all the aforementioned equilibrium hyperinflationary paths must unravel backward, because the price level cannot go to infinity as the equilibrium paths would require.

It should be obvious that if there were no contingent backing initially in place and the economy embarked on a speculative hyperinflation, the government could crush it by subsequently promising a floor on the value of money, however low. This last scenario is,

very small (Obstfeld and Rogoff 1983, Theorem 1). We argued that the case where $v(m)$ becomes unboundedly negative as $m \to 0$ is implausible.

[2]In this case, currency incorporates a perpetual put option (an option that never expires) to sell each unit of money for $1/\overline{\overline{P}} = \epsilon$ units of goods.

however, different from the one we have modeled, in which having a contingent guarantee in place ex ante will prevent speculative hyperinflationary equilibria even from forming.

2 Cochrane's Critique

Cochrane (2011) argues that even if there is a ceiling $\overline{\overline{P}}$ on the price level (a floor $1/\overline{\overline{P}}$ on the price of money), there will still be an equilibrium path leading to a steady state with $P = \infty$ *and* with $M = 0$. He contemplates the possibility of a date-T Nash equilibrium in which all individuals trade their entire money balances to the government at the support price $1/\epsilon$, leaving the economy demonetized thereafter. He writes: "Here is how the hyperinflationary equilibrium actually ends, with the buyback guarantee in place: $P_{T+1} = \infty$" (Cochrane 2011, p. 613).

To support this ultimate equilibrium, Cochrane argues that an individual deciding money demand at the start of penultimate period T will balance the marginal utility of consuming another \$1 against the marginal flow utility from, instead, adding \$1 to money balances over period T *plus* consuming the \$1 at the end of T by trading it to the government at the redemption price $1/\overline{\overline{P}}$. This possibility gives a terminal Euler equation for period T that differs from the one set out above:

$$\frac{u'(c_T)}{P_T} = \frac{\upsilon'(M/P_T)}{P_T} + \frac{u'(c_T)}{\overline{\overline{P}}}. \tag{5}$$

Cochrane (2011, p. 613) concludes that in the equilibrium with $c_T = y$, the price level just prior to the government's absorption of the money supply will be slightly below $\overline{\overline{P}}$, and defined implicitly by

$$P_T = \left[1 - \frac{\upsilon'(M/P_T)}{u'(y)}\right]\overline{\overline{P}} < \overline{\overline{P}}. \tag{6}$$

He contends that it is an equilibrium for everyone to trade in their money to the government at the end of period T, with the economy remaining in a non-monetized steady state for all periods after T.

Alas, this line of argument, albeit clever, falls afoul of one fundamental difficulty: The last Euler equation requires money to have no value on date $T+1$, that is, the price level jumps from a finite P_T to $P_{T+1} = \infty$. But the government's promise to redeem money remains good on date $T+1$. Any individual who deviates from the proposed equilibrium and instead carries \$1 into period $T+1$ will be able to sell it on the market to other agents at any real price less than or equal to $1/\overline{\overline{P}} = \epsilon$ because they, in turn, can then sell the \$1 to the government for ϵ in output. That simple arbitrage argument implies that the market price of money on date $T+1$ simply cannot be zero. It will be at least ϵ, and so the true price level, measured in terms of money, will be at most $\overline{\overline{P}}$, not ∞.

In this case, we need to ask why people would in fact all turn in their money at the end of T, and collectively revert thereafter to barter, when they know the price level will continue to be finite. Cochrane essentially argues that on a hyperinflationary path, everyone expects that starting at time $T+1$, no one will accept currency any longer, even though the government fully backs the currency with real purchasing power and even though everyone was accepting currency a period before. The fact that a dollar bill has real value in terms of goods does not matter, he claims, since no one in the private sector will accept dollars for goods once they all agree that money will remain useless forever, notwithstanding the government's buyback guarantee, making it preferable to trade all of their dollars away to the government and forget that money ever existed.

In making, this argument, however, Cochrane departs from our basic Brock model in a fundamental way. In doing so, we will argue, he conflates two distinct fragilities that afflict monetary equilibria in more general theoretical models. It is therefore worthwhile to spend time to break down his argument and make explicit the way that he (implicitly) alters the model. That discussion will help us assess (i) why he needs to change the basic Brock model to reach his conclusion and (ii) whether his modification would realistically lead to the equilibrium he claims to have discovered.

Starting with the model: the analytical framework we employ specifically assumes that the derived transactions services one gets from holding real balances depends only on the *individual's* holdings, and does not depend on the real balances that others are holding. So even if all other agents in the economy decide to dispose of their money balances, the individual can continue to enjoy transactions benefits as long as the price level is finite, which it always will be under our backing scheme.

Even in this simple model, hyperinflationary equilibria potentially can arise due to an inescapable externality: other people's decisions on money demand affect the aggregate price level, which affects individual utility through its impact on the real value of currency. In Brock's model, however, this is the *only* way that others' monetary decisions affect an individual. Brock (1974, p. 776) felicitously refers to this externality as "expectation pollution." Within the boundary that this formal assumption sets, our proposed partial backing scheme prevents multiplicity owing to expectation pollution (by putting a ceiling on the price level) and thereby rules out speculative hyperinflations, full stop. There is no alternative equilibrium.

Cochrane implicitly adds a new element to the model. It is the assumption that if no one else holds money, the individual can get no utility from holding money. This externality is different from expectation pollution: it is a *network externality.* One way to formalize this is to assume that for an individual i of infinitesimal mass in the population, the utility from holding money is a function $v(M^i/P, \bar{M}/P)$ of own currency holdings M^i and aggregate holdings $\bar{M}$ such that

$$\frac{\partial v\left(M^i/P, 0\right)}{\partial M^i} = 0$$

for all M^i if P is finite (at the level $\overline{\overline{P}}$, the backstop price, or any other value). But this is a fundamental change in our model. Cochrane relies on this additional externality (a network externality, which creates a strategic complementarity) to construct his proposed equilibrium.

A model where monetary utility is given $v(M^i/P, \bar{M}/P)$ will always carry the theoretical possibility of a monetary extinction event in which each consumer simply decides not to accept or proffer money and dumps all of their money for goods, driving the price of money to zero and the economy to a demonetized state. But this very suboptimal Nash equilibrium is made possible by the network externality, not by expectation pollution (even though the jump to this equilibrium results in a zero value for money).[3,4]

In the modified model, and with our fiscal backstop in place, people could still decide to sell all their money to the government at the price $\overline{\overline{P}}$, reverting to a non-monetary equilibrium, just because everyone expects everyone else to do the same. The government can prevent the price level from rising above $\overline{\overline{P}}$, but without some instruments of coercion, it may not be able to force people to use money rather than barter.[5] Nonetheless, realism argues strongly against the likelihood of this kind of equilibrium.

First, all the infrastructure for using cash is not going to disappear overnight, nor will the custom of using cash, as Cochrane's proposed equilibrium assumes. In real-world hyperinflations, people continue to carry out monetary exchanges but switch to a foreign money, as in the experience of Zimbabwe and other hyperinflating countries. A credibly "backed" domestic currency would similarly find a ready market. Given that the currency can be

[3] The fact that P enters the utility function is not the driver of this mechanism of coordination failure. In a model with a real good subject to a similar network externality – where the good's price does not enter the utility function – if everyone were to decide not to demand the good, its price would likewise fall to zero. Kiyotaki and Wright (1989) present a formal model in which an equilibrium with positive-value fiat money will collapse if everyone loses faith in money's acceptability.

[4] Even if one grants that a fiscal theory of the price level eliminates price-level indeterminacy due to expectation pollution, it seems a bigger stretch that consideration of the government's budget constraint could prevent a jump in P to $+\infty$ owing to a network-driven flight from money.

[5] Strictly speaking, one might assume that the government is willing to sell back some small amount of currency (as well as buy any amount), which ensures that the price level P is well defined and that $P \leq \overline{\overline{P}}$.

traded for $1/\epsilon$ units of the good, there is no reason a merchant should refuse it outright. The fact that governments invariably make their currencies legal tender reinforces the point that any merchant would accept currency as payment, on some terms, if it had a guaranteed real value.

Second, and related, individual agents would have a strong incentive to deviate from the alleged (Nash) equilibrium without money. As is well understood, the model aims to capture in a simplified way a more complex model of exchange — say, one based on specialized endowments and search, where money reduces the costs of exchange. These frictions, and the advantages of overcoming them, do not disappear simply because the opportunity to trade money for goods with the government becomes profitable (i.e., the put option attached to money suddenly is in the money). Someone who retains some money in that situation would be able to convince others to take at least some of it for goods, with resulting gains from trade, because they might be able to trade it, in turn, to someone else and failing that, could sell it to the government and be no worse off. Thus, holding on to some money would be a dominant strategy, as would be accepting it for goods. If one explores the history of currency (for example, the literature discussed in Rogoff 2016, chapter 2), one finds that the use of currency is surprisingly robust given its seeming fragility in our theoretical models. The general benefits of monetary exchange are remarkably durable. That's why history shows that the use of transactions media can spread from relatively small beginnings as economies exploit the resulting network externalities.The enormous economic benefits of moving from barter to monetary exchange would make a non-monetary equilibrium unstable in the presence of a convenient medium of exchange. But if so, Cochrane's supposed equilibrium unravels.

Third, one of the most fundamental advantages a government has in getting its currency used is that it is a large player that can insist on using fiat currency for its own receipts and payments. In other words, governments can and do use coercion to ensure acceptance of their currencies. To formalize this idea, assume that the private utility of money is given by

$v(M^i/P, \bar{M}/P) + \gamma(M^i/P)$, where $\gamma(M^i/P)$ represents the utility of money in transactions with the government. Because the latter component of utility need not depend on aggregate behavior, this modification suffices to eliminate an equilibrium in which everyone suddenly reverts from monetary exchange to barter.

Cochrane (2019) asserts that the argument in his 2011 paper is "unnecessarily complex" and present a simplified scenario in which agents can trade money to the government at the start of a period. His fundamental assumption is the same, however: He still has to argue that, once the economy reaches a P_T satisfying equation (6) (or in the setting of Cochrane 2019, satisfying equation (4)), people will coordinate on selling their money to the government and reverting to barter, even though for any individual, retaining some money would be advantageous.

One might be concerned that once the government redeems money for output in period T, it will have exhausted its ability to redeem money in future periods—in the manner of a central bank that exhausts its reserves defending a fixed exchange rate (Krugman 1979). But that is not the case here—the government only needs very minimal reserves and taxing capacity to redeem money at a sufficiently low price, so why would it withdraw its guarantee after a single period of testing? The equilibrium is therefore not subject to being undermined by a classic speculative attack. We observe that in our model, money is fiat at price levels below $\overline{\overline{P}}$ but is redeemable for a commodity at $\overline{\overline{P}}$, so we have a hybrid fiat-commodity standard in Wallace's (1981) sense.[6,7]

[6] In Obstfeld and Rogoff (1983) we consider only a limiting official buying price for money. Wallace (1981) looks at official buying and selling prices for money. The presence of a ceiling as well as a floor on money's value precludes hyperdeflationary equilibria where $P \to 0$ as well as hyperinflationary equilibria where $P \to \infty$. In Obstfeld and Rogoff (1986) we concluded that reasonable preference restrictions suffice to rule out speculative hyperdeflationary equilibria (that is, those not driven by a falling money supply)—in sharp contrast to the case of hyperinflationary equilibria. Buiter and Sibert (2007) reaffirm this result.

[7] Cochrane (2011, p. 613) conjectures that his results may differ from ours because of a confusion (on our part) between discrete- and continuous-time modeling. He states: "The central problem is Obstfeld and Rogoff's "arbitrage" condition (685) that $\overline{\overline{P}} = P_T$ in any period that people are tendering money. That argument is not valid in this discrete-time model because people can get

It is instructive to revisit the rhetorical argument Cochrane (2011, pp. 610–611) advances to support his analysis: "How could offering one kernel of corn for a billion dollars destroy an equilibrium? Given that people were holding money at T that they knew would be worthless at $T+1$, why would a tiny residual value make any difference? It doesn't." The argument is seductive, but what the formal analysis is really saying is that equilibrium speculative hyperinflations are spectacularly fragile, and indeed require very little effort to resist.

It is important to note that we are focusing on ruling out a very special kind of candidate equilibrium, speculative hyperinflations. As Obstfeld (1984) has shown, even in microfounded models of money demand, it is necessary to impose restrictions on either individual utility functions and/or the derived utility from real money balances in order to rule out multiple *dynamically stable* equilibria. And if monetary policy is endogenous, for example, in the Taylor-type interest rate feedback rule discussed by Cochrane (2011, 2019) we fully agree with his central conclusion that the problem of indeterminacy can easily arise.[8]

3 Credibility

Our backing threat uniquely implements a non-inflationary equilibrium in Bassetto's (2005) sense: the government has made a threat that is feasible even off the economy's equilibrium path and which, if it furthermore is credible, rules out all equilibria but one. Bassetto's def-

$v(m)$ plus the redemption value. This arbitrage argument would be valid in a continuous-time version of the model, and perhaps the error comes from mixing correct continuous-time intuition with a discrete-time model." As one can see from the analysis of this section and of section 4, the question of discrete versus continuous time, as usual, is irrelevant for the substantive economic conclusions.

[8]Canzoneri, Henderson and Rogoff (1983) show that with one-period nominal wage contracts, indeterminacy of equilibrium can arise under a pure interest-rate rule unless the central bank specifies at least one point along its money supply path. More recent authors (for example, Woodford 2003) argue that in an overlapping-contracts setting, pre-existing nominal contracts are are a key element to tie down equilibrium even with a pure interest-rate rule. See also Calvo (2016), who argues that nominal contracting is what gives money value (an approach that begs the question why money is used as a unit of account). Calvo (1979) also investigates multiple equilibria within the Brock model.

inition of implementability does not require, however, that implementing threats be credible in the absence of commitment, just that they be feasible. That is, Bassetto's definition does not require that the government will find it optimal (not just feasible) to fulfill its threat if faced with a hyperinflationary price path.

As we have discussed, Cochrane (2011, 2019) contends that even a *credible* government promise to back the currency's real value would fail to stop a speculative hyperinflation. Hopefully we have convinced the reader that his argument is flawed. However, Cochrane (2019) adds a second line of criticism, suggesting that even if small partial backing were sufficient to rule out speculative hyperinflations under commitment, the mechanism we suggest is not realistic or credible.

On realism, our response would be that even the slightest whiff of a backstop would be enough to rule out speculative hyperinflations. Obstfeld and Rogoff (1986) formalize this point by showing that even if the private sector attaches a very small probability to a very small redemption value, this does the job. Ours is only one example of an institutional mechanism that would work – several authors have more recently described others. These papers suggest devices to forestall off-the-equilibrium path deviations that are perhaps more realistic than ours, though the core idea is the same.[9]

The credibility issue is more substantive, albeit outside the scope our original model. In particular, Cochrane (2019) informally argues that it is not credible for the government to commit to offer ϵ units of goods for currency, but not offer to sell an arbitrarily large amount of currency in exchange for goods at the same price. After a devastating hyperinflation, he suggests, the public would be clamoring for real balances, while the government should be eager for the seigniorage revenue it would earn by providing the money. If the government made an open-ended two-sided commitment, it would automatically satisfy that demand – a more credible promise, in Cochrane's view.[10]

[9] See, for example, Benigno (2020) and Christiano and Takahashi (2018).

[10] The time path of any seigniorage would depend on exactly how the government provided money to

Of course, the assumption of commitment is hardly unusual in macroeconomic models: a core building block of the canonical fiscal theory of the price level is the assumption that the government can commit to a real path of surpluses and deficits, even if its budget constraint does not hold off the equilibrium path (Woodford 1995 calls such policies "non-Ricardian.") To take credibility issues into account, one would need a detailed game-theoretic analysis specifying all players' incentives and constraints, as, for example, in Bassetto (2002). We are not going to attempt that here, except to note that formally abandoning government commitment greatly expands the possibility of multiple equilibria of all types.

Importantly, the government loses nothing significant by sticking to its one-sided commitment. The only plausible reason the government would defect is fear of demonetizing the economy: the utility of a representative agent is higher in an equilibrium with positive real balances. But realistically, as individuals see the government purchasing money at the backstop price, they would eventually realize that the backstop is credible and the price level would drop, expanding real balances. If individuals went all the way to zero money holdings, the government could easily restore liquidity fully by issuing a minimal amount of currency – without resorting to a full two-sided peg, which entails and an unlimited commitment to sell as well as buy money.[11]

In addition, however, an open-ended two-sided commitment would not necessarily accomplish the government's assumed goal of stopping inflation and reliquefying the economy. The argument is much the same as the one showing why rigidly fixed exchange rate regimes are vulnerable to speculative runs, which would not be a problem with our one-sided frac-

the public. In any case, because the seigniorage would presumably be rebated to the public as lump-sum transfers, it is unclear why it would make a welfare difference, and therefore, why a benevolent government should care about it.

[11] Cochrane (2019) points out that "We could rule out this [hyperinflationary] equilibrium by having monetary policy also insist that $M_{T+1} = M$. The combination of $M_T = M$ and the redemption guarantee would indeed be a policy setting for which no equilibrium can form ...." He then goes on to characterize this policy as "inconsistent," presumably because the money supply would need to fall if people were allowed to trade in money. However, what we suggest in the text is that $0 \leq M_{T+1} \leq M$ is a consistent monetary policy that would easily kill the inflationary equilibrium.

tional peg but would be in a two-sided peg.[12] This point is most easily seen in the context of Cochrane's (2019) example of a government that implements the two-sided peg by promising to go on the gold standard if prices get too high. Consistent with Cochrane's argument, a government threat to implement a gold standard if prices get too high would certainly not rule out speculative price developments, as a literature demonstrating multiple equilibria in this case illustrates (Froot and Obstfeld 1991). But Cochrane also assumes that the policy, once implemented, would bring inflation to an end, and this is not necessarily the case. Suppose that to end a speculative hyperinflation, the government makes currency convertible into gold through a fixed buying *and* selling price for money of ϵ units of gold per dollar when its total gold stock is ϵM (which just allows it to buy back the pre-existing money supply M.) Assume that at the same time, believing that lower inflation expectations will raise money demand by the public, the government wishes to replenish real money balances so as to stabilize the price level with zero inflation. If the private sector is holding large amounts of gold, as Cochrane seems to assume, then it is not clear why private liquidity would be scarce (gold is easily collateralized) and why more paper circulation would be needed. So it is more realistic to assume that private gold hoards are limited. To reliquefy the economy fully in that case, the government would have either to make monetary transfers or lend money to the public, thereby raising the money supply and thus the total potential claims on its gold above its gold holdings. If people expect the inflation to continue notwithstanding the currency reform, they can simply purchase the government's gold stock in a speculative attack, collapsing the gold peg and sending the price level upward on its expected path.[13] In principle the government could raise taxes or borrow to buy more gold, until it has backed the entire money supply, but if it is constrained in its ability to do so, its attempt to enforce

[12]See Obstfeld and Rogoff (1996) for a discussion of the speculative attacks literature.

[13]The logic is simlar to Flood and Garber's (1984) demonstration that Krugman's (1979) celebrated speculative attack equilibrium is no longer unique when there are bubbles in the post-attack floating exchange rate.

a commodity standard will likely be inherently fragile and subject to attack.[14]

Credibility is a fundamental issue. But a casual appeal to credibility no more undermines our approach to ruling out speculative hyperinflations than it does many macroeconomic models, including canonical versions of the fiscal theory of the price level. More careful analysis, moreover, suggests that a contingent commitment to buy, but not sell, money without limit is quite credible and would be preferred ex ante and ex post to a two-way unlimited promise.

4 An Alternative Model

The preceding money-in-the-utility-function framework is a crude shorthand for a much richer multi-good model in which the transactions value of money is derived from its ability to solve the problem of "double coincidence of wants" on the part of inherently heterogenous market actors who might be unable fully to realize the available multilateral gains from trade without using a commonly accepted medium of exchange. Reasonably interpreting the model as capturing a richer underlying multi-good model with heterogenous agents underscores the implausibility of a sudden rejection of a widely used—and backed—currency for no reason whatsoever.

This point is quite clear in Wallace's (1981) overlapping-generations model of money. While arguably unrealistic as a complete model of money demand, it does illustrate rigorously how potential gains from monetary trade between heterogeneous agents can underpin the demand for money and lead some individuals to deviate from supposedly nonmonetary equilibria provided the government gives some backing to the currency.[15] Here, we develop

[14] See Obstfeld (1986) for a discussion of how the government's intertemporal budget constraint determines exchange-rate sustainability, including the role of constraints on taxation. Note also that throughout the last few centuries, fractionally backed gold standards, as in our example, have been the norm.

[15] As is well known, under perfect certainty, monetary equilibrium in this model might not survive the introduction of dominating assets such as capital.

a simple example based on Gale (1973) and Brock and Scheinkman (1980).[16]

In this example a generation lives for two periods, receiving an endowment w^y when young and $w^o < w^y$ when old and maximizing

$$U_t = u(c_t^y) + u(c_{t+1}^o) \tag{7}$$

subject to the constraints

$$M_t = P_t\left(w^y - c_t^y\right) = P_{t+1}\left(c_{t+1}^o - w^o\right),\ M_t \geq 0, \tag{8}$$

where M_t is the money that a member of the generation born on date t (generation t) carries into its old age in period $t+1$. The utility function $u(c)$ is, as usual, increasing and strictly concave. The assumption that $w^y > w^o$ creates an incentive for the young to acquire money balances so as to smooth their consumption over time—an incentive that can be offset by a sufficiently high expected rate of price-level inflation.

On the tentative assumption that the nonnegativity constraint on money balances will in fact never bind, the intertemporal Euler equation for an individual who is born on date t will be

$$\frac{1}{P_t}u'(c_t^y) = \frac{1}{P_{t+1}}u'(c_{t+1}^o). \tag{9}$$

On the assumption of a fixed aggregate money supply M and again defining aggregate real money balances on date t as $m_t \equiv M/P_t$, equilibrium paths satisfy the difference equation

$$A(m_t) \equiv m_t u'(w^y - m_t) = u'(w^o + m_{t+1})\, m_{t+1} \equiv B(m_{t+1}). \tag{10}$$

There is a steady-state positive level of real balances $\overline{m}$ that satisfies $A\left(\overline{m}\right) = B\left(\overline{m}\right)$,

[16]Sims (2013), also in an overlapping-generations framework, develops the related idea that monetary equilibrium becomes unique if the government each period levies taxes to repurchase some money (in contrast to the offer of a free put option that we model here).

and therefore a finite steady-state price level given by $\overline{P} = M/\overline{m}$. Figure 2 illustrates the determination of this Pareto-optimal steady state, but also shows there are other, inefficient equilibria (for example, the speculative hyperinflationary path starting at $m = m_0$) such that money asymptotically becomes worthless. The intuition is the same as in the Brock model.[17] However, in parallel to the Brock model, a government promise to redeem money for a small amount ϵ of goods effectively caps the price level at $\overline{\overline{P}} = 1/\epsilon$, and this fact therefore rules out all paths but the steady-state path, because those paths are supported only by the self-fulfilling expectation of an ever-increasing path of prices.

An important nuance here is that absent any backing, there is a second steady-state equilibrium other than the monetary equilibrium $\overline{m}$, in which money is rejected instantly and entirely. In the nonmonetary equilibrium, people do not use money simply because no one else uses money—everyone thinks money is useless and will be forever, and so it is. Here, $P = \infty$ (permanently) and $m = 0$, so money is irrelevant: each generation is restricted to consuming its own current endowment. Instant rejection of money is a legitimate equilibrium, albeit an inefficient one. A date-t young person might *wish* for a way to transfer some savings into old age, but would never pay a positive price for money on date t if the money will be worthless on date $t + 1$. In other words, no individual young person would deviate from a date-t equilibrium with $P_t = \infty$ so long as she knows that $P_{t+1} = \infty$. And the date-$(t + 1)$ young will act the same, knowing that $P_{t+2} = \infty$, and so on, *ad infinitum*.

[17]For simplicity, the figure shows the simple special case in which $u(c) = \ln(c)$, but our main points carry over more generally. In the logarithmic case, $\overline{m} = \frac{1}{2}(w^y - w^o)$. The equilibrium of this economy when there is no money is inefficient. The government can raise every generation's welfare, however, by endowing the initial old with the stock of money M, which they immediately sell to the initial young for output, allowing the initial young to do the same next period when they are old and have lower income. The result will be efficient if the price level settles immediately at $\overline{P}$ and stays there forever. But as our diagram shows, this need not happen if the market is left to find the equilibrium on its own—and this is where the government's partial backing of money is helpful. For a lucid discussion of the welfare economics of the overlapping-generations model, see Weil (2008). Figure 2 (the main features of which are valid more generally) makes clear that the nonnegativity constraint on money holdings will never bind in any equilibrium. The figure also shows that speculative hyperdeflations cannot arise in equilibrium (because the real money balances proferred by the old can never exceed the endowment of the young).

Things are different if the government guarantees for all dates a small redemption value for money, ϵ. To see why, let's ask if there is a Cochrane-like equilibrium where on some date t, the old suddenly tender their money to the government for backing at the price $1/\overline{\overline{P}} = \epsilon$ and the price level then jumps permanently to $P = \infty$.

The easy answer is no. Any individual young person can dissuade an old person from exchanging their money with the government by offering to pay them instead the price $\epsilon + \eta$, where $\eta > 0$ is arbitrarily small. Because the autarky (nonmonetary) consumption levels over the young person's lifetime would satisfy $u'(w^o) > u'(w^y)$, she can raise her utility by purchasing money at $\epsilon + \eta$ from an old person and selling it at ϵ to the government later when she, herself, is old—provided η is small enough. The alleged equilibrium therefore collapses, just as the analogous one collapses in the Brock model.

5 Concluding Remarks

We have shown that in classical monetary models, speculative hyperinflations in which the price level explodes despite a stationary money supply process can indeed be ruled out under the weak and plausible assumption that the government provides a minimal backstop to the real value of money. In particular, the hyperinflationary candidate equilibrium proposed in Cochrane (2011, 2019) unravels in the face of such a backstop, if the model is applied consistently. That said, the broader critique that it is very difficult to rule out multiple *stable* equilibria in classical monetary models remains valid.

More than half a century ago, Hahn (1965) highlighted the challenges of integrating money into general-equilibrium models. Macroeconomists have struggled to provide an answer ever since. Kydland and Prescott (1982) attempted to avoid the problem altogether by arguing that macroeconomics could model business cycles adequately as real phenomena, and thus could do completely without money. Unfortunately, this view seems wildly inconsistent with the facts and it therefore has relatively few adherents today. More recently, models emphasizing that modern central bank policy focuses on the setting the nominal interest rate path (e.g., models with Taylor-type rules) finesse the need to model money demand or supply by effectively postulating an infinitely elastic supply curve for money at the level of the policy interest rate—Woodford (2003) makes this point very explicitly. But even so, these models require a degree of inherited price inertia as a necessary ingredient of price-level uniqueness—an assumption that may not hold in the face of a major structural change—and even then, they need additional stability postulates to rule out extraneous price paths driven by self-fulfilling speculation. Thus, there is no shortcut to achieving price level determinacy without a deeper analysis of money.

We conclude with some observations on the "fiscal theory of the price level," (FTPL) which attempts to offer an alternative approach (to ours) to ruling out speculative hyperinflations, and indeed purports to be a more general overall approach to price level determinacy. Fiscal-monetary interactions gained prominence after the financial crisis of 2007-09, and they have become even more evident in the massive global policy response to the current pandemic. Some economists have recommended a fiscal approach for elevating inflation rates that have been persistently below target levels (Sims 2016), and the huge runup in government deficits and debt during the COVID-19 crisis could provide a natural experiment to test the FTPL's predictions. The theory's claims therefore have great current relevance.

In its simplest form, FTPL assumes that the government can commit to a (possibly state-contingent) path for real government spending, taxes, and transfers. Given a stock of nominal government debt, the price level then simply falls out of the government budget constraint so as to equate the real par value of government debt to the present discounted value of real primary net government surpluses. There is no scope for speculative hyperinflations. The central bank is completely passive in this setting, as opposed to models where the central bank sets the money supply path or interest rate according to some rule, and the government must adjust its fiscal plans accordingly to ensure solvency. Bassetto (2002) illustrates how the equilibrium is likely to depend in reality on the structure of the game being played by the fiscal authorities, the monetary authorities, and the private sector. In general, the role of seigniorage can complicate the link between fiscal policy and prices unless one makes strong assumptions about the reaction functions of real spending and taxes – effectively, yet another way to sweep monetary issues under the rug.

Moreover, one must account for the possibility that the government might (endogenously) choose to default outright on it nominal debt, even when it has the capacity to print money.[18] Sovereign debt models are replete with multiple-equilibrium issues (see Obstfeld and Rogoff 1996). Moreover, in addition to outright default, governments can default de facto through financial repression (see Reinhart and Rogoff, 2009; Reinhart, Kirkegaard, and Sbrancia 2011; or Chari Dovis, and Kehoe 2020). Modifying the FTPL with exogenous default probabilities does not add to our understanding of the determinants of default and inflation, and masks some fundamental problems of indeterminacy that are well known to arise.

Multiple equilibria could also arise from the government's fiscal policy rule—which, in turn, could reflect an equilibrium interaction of private-sector and government-agency objectives and constraints. In general, even with fiscal dominance, political economy factors can introduce multiple equilibrium problems, either because of governments that alternate endogenously depending (in part) on economic outcomes, or if one allows that there is no mechanism for governments to commit to future policies (including default). Any game-theoretic situation becomes vulnerable to the folk theorem.

Finally, the fiscal theory of the price level must contend with the empirical fact that for the United States, as for other industrial countries, nominal growth rates of GDP have

[18] Reinhart and Rogoff (2009) document a long history outright defaults on domestic debt, including the U.S. abbrogation of the gold clause in the 1930s; see also Edwards 2019.

consistently exceeded government nominal borrowing rates in the postwar period. Abel, Mankiw, Summers, and Zeckhauser (1989) made this point three decades ago (see also Bohn 1995), and Blanchard (2019) has reaffirmed its continuing validity. Whether the FTPL still applies in this environment depends on why government borrowing rates are so low. Bassetto and Cui (2018) show that the FTPL fails to tie down the price level uniquely if the economy is dynamically inefficient (as Geerolf 2018 argues is the case across industrial economies) and may not tie it down even when the economy is dynamically efficient, but government borrowing costs are low owing to liquidity benefits from holding government bonds. Jiang et al. (2020) contend that for the United States, the present value of expected government surpluses is negative, a finding hard to reconcile with the observed positive real value of U.S. government debt if the FTPL is a valid guide to the equilibrium price level.

The truly striking thing about monetary equilibria with government-issued fiat money is that, contrary to the tenuousness predicted by our theoretical models, they seem to be remarkably stable and robust. To set off a monetary hyperinflation, it takes a large-scale government resort to monetary finance of deficits—a fiscal theory, not of the price level in general, but of its instability. And in such circumstances, there are always broader questions about more pervasive institutional breakdown—as Lerner (1947) put it, money is a "creature of the state."[19] While it goes too far to argue that the literature on money has made no progress at all, there clearly remain difficult puzzles to be solved.

[19] Lerner (1947) suggested that a monetary equilibrium is assured if the government requires money in payment of taxes. Brock and Scheinkman (1980) explored this avenue further.

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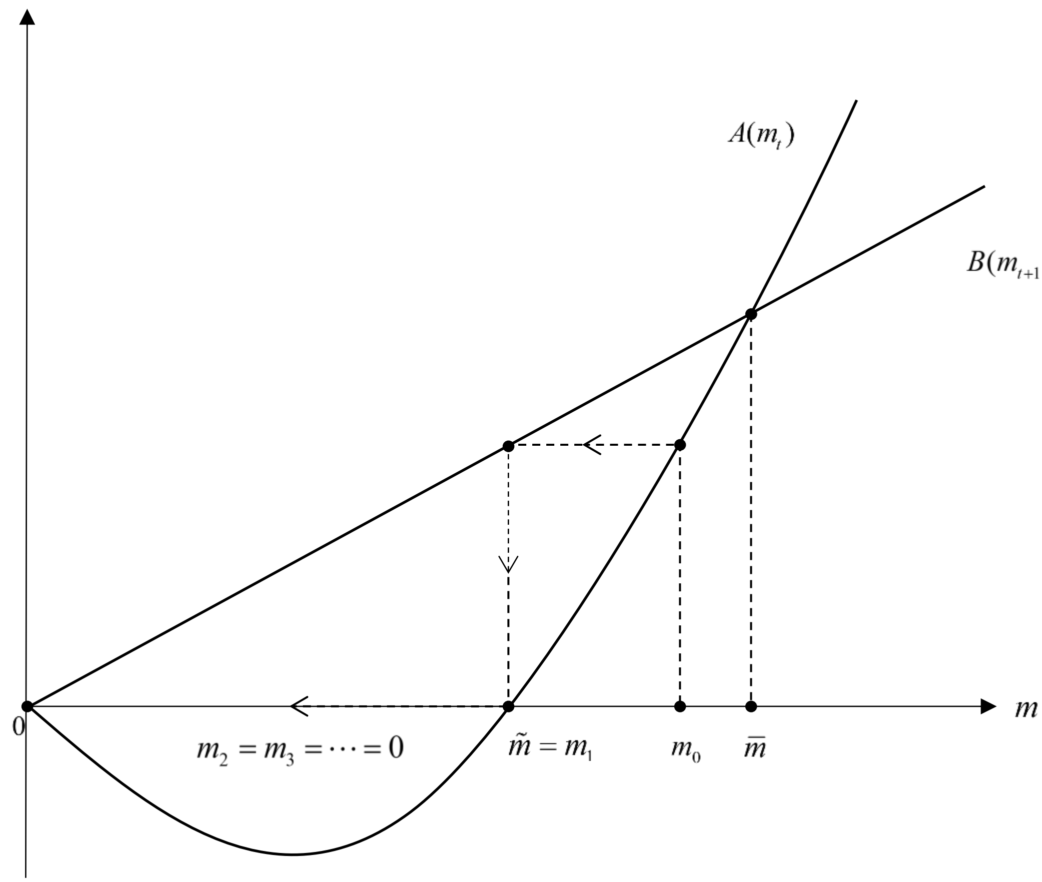

Figure 1: Speculative hyperinflation in the Brock representative-agent model

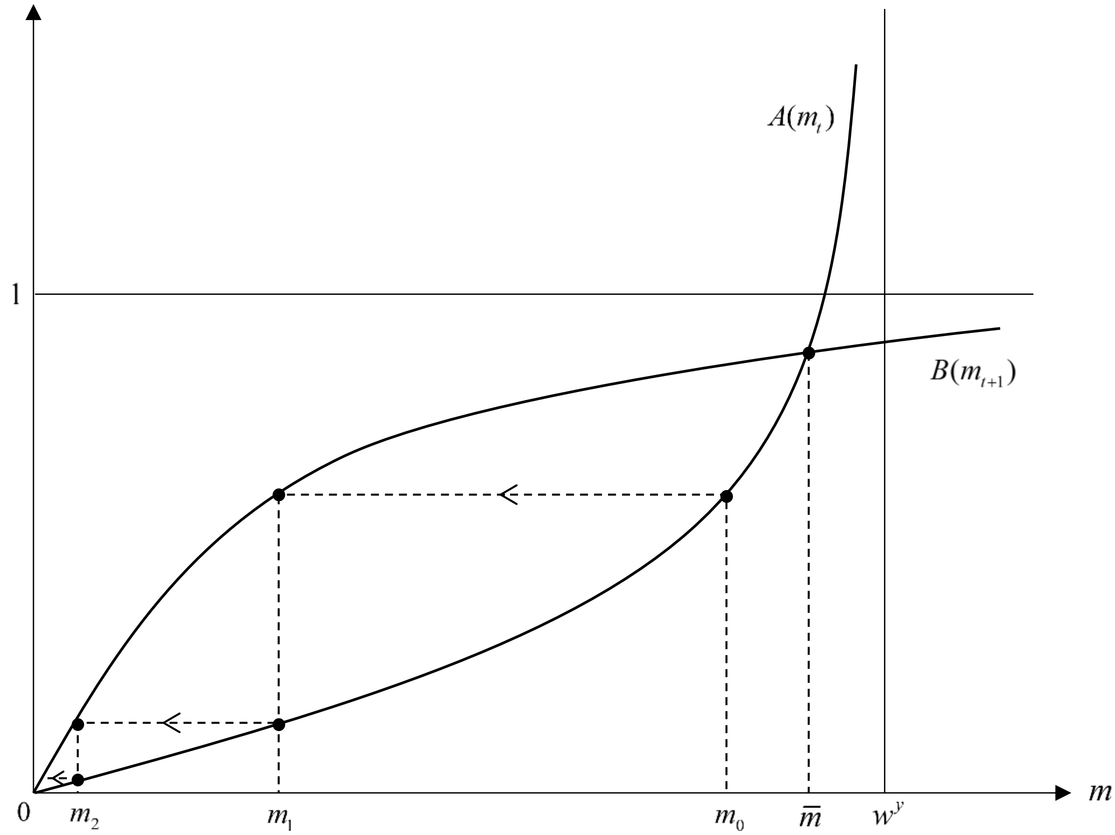

Figure 2: Speculative hyperinflation in an overlapping-generations model