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JEL Classification: N/A

Keywords: product strategy, pricing strategy, second degree price discrimination, search, vertical differentiation, retail competition

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Product Lines and Price Discrimination in Markets with Information Frictions

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July 27, 2020

Abstract

A well known principle in economics is that firms differentiate their product offerings in order to relax competition. However, in this paper we show that information frictions can invalidate this principle. We build a duopolistic model of second-degree price competition with information frictions in which (i) there always exists an equilibrium with overlapping qualities, whereas (ii) the equilibrium with non-overlapping qualities exists only when both information frictions and the costs of providing high quality are small enough. As a consequence, reasons other than the attempt to soften competition should be used to explain why firms in some cases carry non-overlapping product lines.

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1 Introduction

Different reasons have been advanced to explain why in some markets competing firms carry overlapping product lines, while not in others. Pervasive among the latter is the well known Chamberlinian principle that firms seek to differentiate their products in order to relax competition (Chamberlin, 1933). Champsaur and Rochet (1989) (CR, thereafter) formalized this principle in a model in which quality choices are followed by price competition.[1] CR showed that there exists an equilibrium in which firms choose

[1] Shaked and Sutton (1982) and Moorthy (1988) formalized the same idea in a model similar to Champsaur and Rochet's (1989), with the difference that firms are allowed to offer one quality only. Thus, in Shaked and Sutton (1982) and Moorthy (1988), there is no possibility to discriminate consumers at the firm level.

non-overlapping qualities because the incentives to soften price competition dominate over the incentives to better discriminate consumers with heterogeneous preferences for quality. They also argued that, not surprisingly, this differentiation principle should weaken as markets become less competitive.

The contribution of this paper is to show that the Chamberlinian principle of product differentiation is not robust to introducing imperfect competition due to information frictions (as we discuss below, this does not necessarily apply to other forms of imperfect competition). In contrast, we show that there always exists an equilibrium with full quality overlap at which firms compete head-to-head. Since no market is immune to information frictions, our finding provides an important lesson for applied work: if competing firms carry non-overlapping product lines, it may well be for reasons other than the Chamberlinian incentive to soften competition. Hence, researchers should rely on other models to account for asymmetries in product lines.

When consumers are not perfectly informed about firms' prices and qualities, they cannot choose their preferred option unless they incur search costs to learn and compare all options. Since the seminal work of Diamond (1971), the search literature has shown that the introduction of information frictions can have substantial effects on competition. However, unlike CR, this literature has broadly neglected the possibility that firms engage in price discrimination through quality choices.[2] The general goal of this paper is to understand the interaction between information frictions and price discrimination, and their effects on product choice and pricing by competing firms.

By introducing information frictions *à la* Varian (1980) (i.e., a fraction of consumers are *uninformed* about firms' prices and product offerings) we show that, if the costs of providing high quality are large enough, an arbitrarily small amount of uninformed consumers is all it takes to rule out an equilibrium in which firms offer non-overlapping product choices.[3] Intuitively, the presence of uninformed consumers induces the firm carrying low quality products to deviate by also carrying high quality products in order to better discriminate consumers without fear of sacrificing profits on the low qualities. If providing high quality is not too costly, a sufficiently large mass of uninformed consumers also rules out the equilibrium with non-overlapping qualities as the gains from price discrimination outweigh the gains from softening competition. Instead, the equilibrium in which firms offer overlapping qualities always exists, no matter whether there are none,

[2]Unlike the current paper, in which we model second-degree price discrimination, Fabra and Reguant (2019) allow for third-degree price discrimination in markets with search costs.

[3]In the online appendix we show that our results remain unchanged if we endogenize the fraction of uninformed consumers by explicitly accounting for search costs. For this purpose, we follow the fixed-sample approach of Burdett and Judd (1983) but allow consumers to differ in their search costs.

few or many uninformed consumers,[4] and no matter how costly it is to provide high quality. In this sense, the equilibrium with overlapping qualities is particularly robust (and for a large set of parameter values, unique),[5] whereas the equilibrium with non-overlapping qualities proposed by CR is not.[6] Another compelling reason for focusing on the equilibrium with overlapping qualities is that it naturally converges to the Bertrand equilibrium as information frictions vanish out.[7]

Beyond investigating the effects of information frictions on firms' quality choices, we also aim at understanding the effects of information frictions on equilibrium pricing in general. In this sense, we extend Varian (1980)'s equilibrium to a multi-product firm setting. In particular, we show that the incentive compatibility constraints faced by multi-product firms introduce an important departure from Varian (1980): the prices for the various goods sold within a store cannot be chosen independently from each other. This has several implications for pricing behavior. For instance, in the case in which both firms carry the two goods, if competition becomes particularly intense (which Varian refers to as periods of *sales*), firms reduce the relative price of the high versus the low quality good to the extent that the incentive compatibility constraint no longer binds. The reason is that firms' incentives to compete for the consumers with a high quality preference may dominate over their incentives to minimize the high types' information rents. Additionally, incentive compatibility considerations imply that multi-

[4]The equilibrium with overlapping qualities trivially exists in the absence of information frictions, in which case it gives rise to Bertrand pricing. CR only focus on equilibria with strictly positive profits, and hence abstract from this equilibrium. One of our contributions is to show that the overlapping equilibrium always exists while the non-overlapping equilibrium may disappear altogether even when the mass of uninformed consumers is arbitrarily small.

[5]If we introduced search costs *à la* Diamond, in which consumers search firms sequentially at some positive cost, firms would also carry overlapping product lines. However, because of the Diamond's paradox, consumers would not search and firms would not compete among themselves. Therefore, this assumption would not be appropriate to analyze the interaction between competition and price discrimination. The Varian's approach avoids this paradox, giving rise to comparative statics that replicate empirical findings regarding search behavior and price patterns.

[6]Besides, it is not difficult to show that as soon as we introduce an arbitrarily small amount of uninformed consumers into CR's model, the low-quality firm has incentives to deviate outside the quality gap by carrying a sufficiently high quality that allows it to better discriminate the uninformed consumers without inducing a price response from the high-quality firm. Given this deviation, it is not longer obvious how to differentiate the high from the low-quality firm. In contrast, if we introduce a small amount of horizontal differentiation into CR's model, the quality gaps shrinks, but just a bit, and the low-quality firm remains as such. Proofs of both claims are available from the authors upon request.

[7]As suggested by a referee, yet another reason for focusing on the equilibrium with full overlap under information frictions (no matter how small) is that the presence of more than two firms makes it the unique equilibrium, even if the costs of providing high quality are not large enough.

product firms tend to charge lower prices on average as compared to single-product firms, contrary to the analysis of pricing by single-product versus multi-product monopolists under complete information.

Related Literature Our paper is related to two strands of the literature: papers that analyze competition with search frictions, and papers that characterize quality choices under imperfect competition.[8] A vast part of the search literature assumes that consumers search for one unit of an homogeneous good, with two exceptions. Some search models allow for product differentiation across firms but, unlike ours, assume that each firm carries a single product.[9] Other search models allow firms to carry several products but, unlike ours, typically assume that consumers search for more than one ('multi-product search').[10] In these models, consumers differ in their preference for buying all goods in the same store ('one-stop shopping') rather than on their preferences for quality.[11] These differences are relevant. In the first type of search models, the single-product firm assumption leaves no scope for price discrimination within the firm. Hence, pricing is solely driven by competitive forces. In the second type of search models, the multi-product search assumption implies that discrimination is based on heterogeneity in consumers' shopping costs or the complementarity across the goods, which become the main determinants of firms' product choices (Klemperer, 1992).

Within this literature, Zhou (2014) finds that multi-product firms tend to charge lower prices than single-product firms. This is not driven by the interaction between competition and price discrimination, as in our paper, but rather by a 'joint search' effect, i.e., multi-product firms charge less because they gain more by discouraging consumers from searching competitors (see also McAfee, 1995). In Rhodes and Zhou (2019), increases in

[8] As we discuss in the next section, there is also a large empirical literature investigating price discrimination in markets where search costs matter, with a focus on price patterns.

[9] For models with horizontal product differentiation, see for instance Wolinsky (1985), Anderson and Renault (1999), Kuksov (2004) and Bar-Isaac *et al.* (2012); see Ershow (2017) for an empirical application. Wildenbeest (2011) allows for vertical differentiated products but, unlike us, assumes that all consumers have the same preference for quality; hence, there is no scope for price discrimination. He finds that all firms use the same symmetric mixed strategy in utility space, which means that firms use asymmetric price distributions depending on the quality of their product. In contrast, we find that firms might use different pricing strategies for the same product, with this asymmetry arising because of price discrimination within the store.

[10] There is a recent strand of papers in the ordered search literature that analyze obfuscation by multi-product firms (Gamp, 2019; Petrikaite, 2018). Their emphasis is on the monopoly case. See Armstrong (2017) for a discussion.

[11] One-stop shopping considerations are also the driving force behind the evidence of price dispersion across stores documented by Kaplan *et al.* (2019).

search costs imply that consumers value one-stop shopping more, thus making it more likely that the equilibrium involves multi-product firms. Unlike us, for small search costs, Rhodes and Zhou (2019) predict asymmetric market structures with single-product and multi-product firms coexisting. The driving force underlying our predictions is quite different: since in our model consumers buy a single good, the multi-product firm equilibrium is not driven by one-stop shopping considerations but rather by firms' incentives to price discriminate consumers with heterogenous quality preferences.

Our model shares some of the ingredients in Shelegia's (2012); notably, the fact that some consumers are informed about firms' prices, while others are not. However, unlike us, he assumes that consumers buy more than one good and does not analyze endogenous product choices. In the case of complements, Shelegia (2012) finds that prices are negatively correlated across goods in order to satisfy the captive consumers' willingness to pay for the bundle. In our model, instead, the positive price correlation across goods is driven by incentive compatibility considerations.

Like us, Garret *et al.* (2019) also introduce frictions in a model of price competition in which firms can carry more than one product but in which consumers buy only one.[12] The main difference between the analysis in Garret *et al.* (2019) and ours is that they let firms decide qualities and prices *simultaneously*, while we model those choices as sequential. The simultaneous timing is appropriate in settings where firms can change the product design rather quickly, or alternatively, when firms commit to prices for long periods of time; for example, under long term contracts. The sequential timing is better suited to capture the notion that in many markets firms can change prices at will, while changes in product lines, which usually involve changes in the production and/or retail facilities (Brander and Eaton, 1984), occur less often. This distinction is relevant as in simultaneous settings firms cannot affect competition by pre-committing to quality choices, which is a fundamental driving force of our results.

Last, our paper also relates to the literature that analyzes quality choices followed by imperfect competition, either quantity competition (Gal-Or, 1983; Wernerfelt, 1986; Johnson and Myatt 2003) or price competition with horizontal differentiation (Gilbert and Matutes, 1993; Stole, 1995). While one may view information frictions as equivalent

[12] Another set of related papers analyze pricing for add-ons. Ellison (2005) and Verboven (1999) consider models in which consumers are well informed about base product prices but don't know the price of the add-ons, unless they search. Critically, in these models the customers that are more likely to buy the add-ons are also less likely to search. Our model is not a model of add-on pricing because some consumers are informed (i.e., they observe all prices) and others are not (i.e., they only observe the prices of the store they visit), and this applies symmetrically for both products regardless of their quality. Furthermore, our results hold regardless of whether there is correlation or not between consumers' quality preferences and informed types.

to other forms of imperfect competition, they are not. In models of imperfect competition, for the equilibrium with overlapping (i.e., symmetric) quality choices to exist, competition has to be sufficiently weak, e.g. as shown by Gal-Or (1983), under Cournot competition, the number of firms has to be sufficiently small. The same insight also applies to models of price competition with horizontal product differentiation (Wernerfelt, 1986). In contrast, the impacts of information frictions on product choices are different. Even if the mass of uninformed consumers is arbitrarily small, firms do not have incentives to deviate from the equilibrium with overlapping product choices. The reason is that information frictions restore firms' monopoly power over the uninformed consumers, even when competition for the informed consumers (or *shoppers*) is very fierce. This conclusion remains valid regardless of whether the uninformed consumers visit one firm at random, or whether they visit the one that gives them higher ex-ante utility.[13]

The rest of the paper is organized as follows. Section 2 describes an illustrative example that conveys the main intuition of the model while providing descriptive empirical evidence. Section 3 describes the model. Section 4 shows that in the absence of information frictions firms can escape the Bertrand paradox by carrying non-overlapping product lines. In contrast, Section 5 shows that, if the costs of quality are sufficiently convex, an arbitrarily small amount of information frictions is enough to induce firms to choose overlapping product lines even if this drives prices close to marginal costs. Section 6 characterizes equilibrium pricing for all potential product choice configurations, as well as the Subgame Perfect Equilibrium product choices for all levels of information frictions. Section 7 discusses the robustness of the model to several extensions. Section 8 concludes. Selected proofs are postponed to the appendix.[14]

2 An Illustrative Example

Price discrimination is pervasive in a wide range of markets in which information or search frictions matter. In gasoline markets, consumers have the choice of paying for full-service or self-service gasoline at the same station, or of searching for competing stations (Shepard, 1991). In the airline industry, travellers can choose whether to fly in business or in economy class, or just in economy class but with certain restrictions

[13]Indeed, we show that directed search by uninformed consumers strengthens our results as firms have extra reasons to become multi-product as compared to the case when product choices are non-observable and uninformed consumers decide randomly which firm to visit. See Section 7.

[14]Proofs of the characterization of the pricing equilibria in subgames with asymmetric product choices are relegated to the online appendix. They follow a similar logic as the proofs contained in the appendix of the main text.

(Borenstein and Rose, 1994). Other examples in which price discrimination, competition and information frictions coexist include coffee shops (McManus, 2000), cereals (Nevo and Wolfram, 2002), theaters (Leslie, 2004), Yellow Pages advertising (Busse and Rysman, 2005), mobile telephony (Miravete and Röller, 2004), cable TV (Crawford and Shum, 2007), or markets in which competing firms offer advanced-purchase discounts (Möller and Watanabe, 2016; Nocke *et al.*, 2011), among others.

To build intuition on the main forces underlying our model, we focus on a market that fits well our modeling framework: the market for online books.[15] While previous empirical papers have analyzed search in these markets (De Los Santos *et al.*, 2012; Hong and Shum, 2006), their focus has been on estimating buyers' search behavior for given product choices and prices. Rather, our focus here is simply to motivate and illustrate the predictions of the model by exploring firms' product choices and prices given consumers' search behavior. For this purpose, we have collected daily book prices at Amazon and Barnes & Noble, the two leading online booksellers, from December 2016 to March 2017, for each of the 2012-2016 #1 New York Times fiction and non-fiction best-sellers.

2.1 Theoretical intuition

To develop intuition, let us think of two online stores competing to sell books to consumers with heterogenous preferences for quality. Before choosing prices, booksellers must decide whether to offer both the hardcopy and the paperback versions of each book, or just one of the two, if any. Since the hardcover version is generally thought of as being of better quality than the paperback, we will sometimes refer to the two as the high and low quality goods, respectively. In turn, we will refer to those consumers who are willing to pay the extra cost of producing the hardcover as the high types, and the remaining consumers as the low types.

In the absence of frictions, there are two types of equilibria. On the one hand, if the two stores offer the two versions of the book, Bertrand competition would drive prices down to marginal costs. Because of the Bertrand reasoning, stores cannot deviate from this equilibrium by dropping one of the two versions, as their profits would be zero in any event. On the other hand, CR's prediction is that firms can escape the Bertrand paradox by differentiating their product offerings. Indeed, there also exists one

[15]Admittedly, our simple theoretical model does not capture all the ingredients of real-world online markets. Notably, it assumes that the shares of loyal consumers are symmetric across firms, which is not likely to hold in reality. Nevertheless, even with asymmetric customer basis, the key properties of our equilibrium would be preserved: there would be price dispersion, and at least one of the two firms would make monopoly profits out of its loyal consumers. See Narasimhan (1988) for an analysis of Varian's model with asymmetric shares of loyal consumers.

equilibrium at which one store offers the hardcover, and the other one the paperback. If one of the bookstores deviates from this equilibrium by carrying an additional format, competition would drive its price down to marginal costs, thereby making such a deviation unprofitable. Furthermore, if the cost difference between the hardcover and the paperback is not too large, the store would have to give a discount on the other format to stop consumers from buying the one priced at marginal costs, further reducing the profitability of such a deviation.

Is CR's prediction robust to adding information frictions? To shed light on this question, suppose that an arbitrarily small fraction of consumers visit one of two sites at random without searching any further. These consumers are uninformed as they only observe the version(s) of the book and price(s) of the site they have visited.[16] If the site offers the two versions of the book, the uninformed consumers buy the one that gives them higher utility (if positive) given their quality preferences. If the site only offers one of the two versions, they buy it as long as it gives them positive utility.

Following CR's prediction, suppose that the two stores offer different versions of the book. Now, the one carrying the paperback might have incentives to also carrying the hardcover. By doing so, it would make more profits from the uninformed consumers high types as they are willing to pay more for the hardcover. In turn, if the costs of the hardcover relative to the paperback are sufficiently high, the low types would not be willing to buy the hardcover even if it was sold at cost. Hence, since carrying the hardcover would not intensify competition for the low types, the profits made on the paperback would remain unchanged. It follows that, when the costs of providing high quality are sufficiently high, even an infinitesimally small amount of information frictions would be enough to rule out the equilibrium with non-overlapping products. This would also hold true under smaller quality differences as long as the mass of uninformed consumers is sufficiently large.

Alternatively, consider the Bertrand-like equilibrium at which both stores sell the hardcover and the paperback. Now, bookstores face a trade-off when setting prices: on the one hand, they want to set high prices to maximize the profits from selling the books to the uninformed consumers; on the other, they want to set low prices to compete for the informed consumers who have visited the two sites. These countervailing incentives imply that the equilibrium must be in mixed strategies, with the bookstores choosing random prices between the monopoly level and somewhere above marginal costs. Therefore, in

[16] In the context of online books, De los Santos *et al.* (2012) show that, within a 7 days window, 76% of consumers only visit one store. They also report the presence of loyal consumers: 24% of consumers engage in multiple transactions but only buy from one store, even if it exhibits a higher price, thus suggesting the presence of specific store preferences, independently of prices.

equilibrium, the bookstores' profits are the same *as if* they were monopolists over the uninformed consumers, but also *as if* competition washed away all the profits from the informed consumers.

This has a key implication: the bookstores' product choices are only driven by their incentives to better discriminate the uniformed consumers. Accordingly, they do not have incentives to drop neither format as doing so would not enhance their market power over the informed consumers, but would rather reduce the rents they can extract from the uninformed consumers . While this incentive structure mimics the monopoly case, there is a fundamental difference with respect to the duopoly case: product overlap among competitors reduces their profits, to the extent that these go down to zero as the mass of uninformed consumers vanishes out.

2.2 Evidence in the data

The product choice and pricing patterns observed in the online books' data are in stark contrast with the predictions of the CR model, but can be rationalized when accounting for information frictions, as explained above. First, we find that the two stores sell both the hardcover and the paperback version whenever these versions exist.[17] Second, we find that the prices of the hardcover and paperback versions of the same title do not remain constant. Consistently with our model, the presence of uninformed consumers makes it profitable for the stores to carry overlapping versions, which generates equilibrium price dispersion.[18]

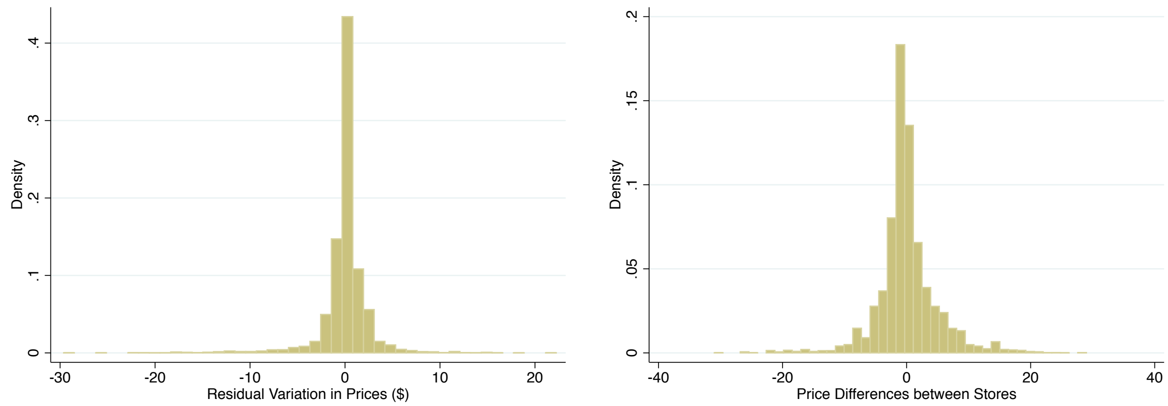
Figure 1 provides evidence of price dispersion at the book-format level. In panel 1a one can see that prices fluctuate substantially for each title even after partialling out book-store-format means.[19] As shown in panel 1b, this dispersion is not explained by common fluctuations, e.g., fluctuations for particular books over time that are common across stores. Indeed, there are price differences between stores, which also fluctuate

[17] Out of the 200 books that we consider, some of them are only available in paperback or hardcover. For those available in both versions, all of them are served by Amazon and Barnes & Noble at the same time, except in one instance in which Amazon does no longer offer the hardcover version (which is only sold by other sellers). Given the almost complete overlap, we focus our analysis in cases in which both stores sell both versions.

[18] Seim and Sinkinson (2016) provide evidence of mixed strategy pricing in online markets. Arguably, other reasons could also explain price dispersion in these markets, e.g. the use of algorithmic pricing (Chen *et al.*, 2016).

[19] The figure shows prices for both Amazon and Barnes & Noble, hardcover and paperback. Separate figures for each book format and store exhibit similar distributions. Note that average prices for hardcover and paperback are roughly \$20 and \$10, respectively. Therefore, variation of a few dollars can imply substantial variation in prices.

Figure 1: Patterns in prices for online bookstores



(a) Residual price dispersion

(b) Residual price differences between stores

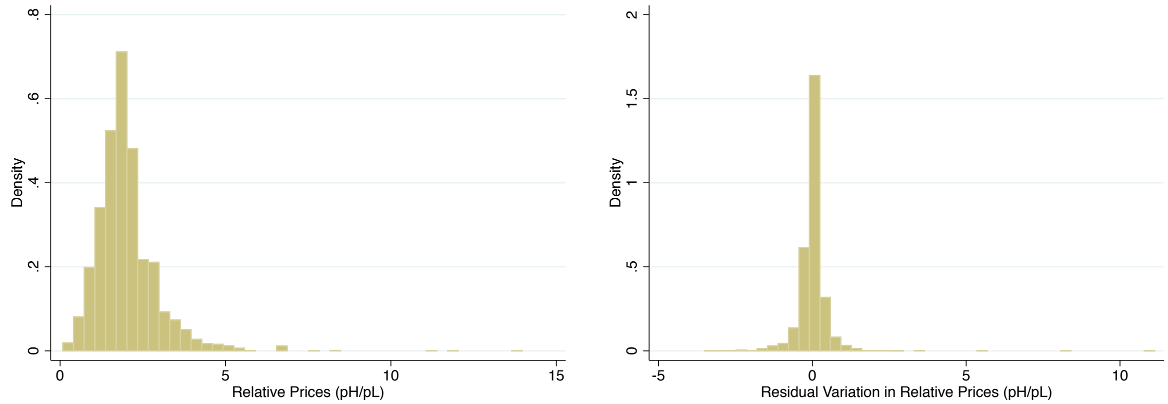
Notes: This figure shows patterns in online book prices. Panel (a) shows price dispersion after partialling out book-store-format means. Panel (b) takes the difference in prices at a given date between stores for the same book-format.

substantially even after taking out constant mean differences by book.

The interaction between price discrimination and information frictions can also explain the dispersion in the relative prices of the two formats. Whereas existing search models cannot capture fluctuations in relative prices because they do not allow for price discrimination, our model predicts that information frictions not only lead to price differences across stores, but also to price differences across different versions of the book sold within a store, and over time. Figure 2 summarizes patterns in relative prices. Panel 2a shows the distribution of relative prices between the hardcover and the paperback of a given title. One can see that there is substantial variation in relative prices, partly due to differences across different titles, and partly due to variation in such relative prices over time. Panel 2b shows the variation in relative prices after partialling out book-store means. One can see that the relative prices between the hardcover and the paperback versions also move over time, and that such variation is not just due to variation across books, but also to variation within book titles.

In sum, the market for online books is characterized by a series of stylized facts. First, the norm is that all booksellers offer the hardcopy and the paperback versions of the same book, whenever available, even if this triggers intense competition for almost identical goods (up to the horizontal differences that consumers may perceive across stores). Additionally, book prices fluctuate substantially, both at the book-store level but, more importantly, also across stores, thus making search meaningful. Relative prices between book versions also exhibit substantial dispersion, indicating that this is another

Figure 2: Patterns in relative prices for online bookstores



(a) Relative prices

(b) Price dispersion in relative prices

Notes: This figure shows patterns in relative prices. Panel (a) shows the distribution of relative prices, with a peak around 2, i.e., the hardcover version of a book is about twice as expensive as the paperback version on average. Panel (b) shows residual variation in relative prices after partialling out book-store means.

dimension that firms use when sorting out consumers and attracting them from rivals. The model that we present next is capable of generating these predictions by highlighting the impact of information frictions on equilibrium product choices and price patterns.

3 The Model

3.1 Model Description

Consider a market served by two competing firms (which we sometimes refer to as *stores*), which carry either one or two goods: either a good with high quality q^H and high costs c^H, or another one with lower quality q^L and lower costs c^L, or both.[20] We use $\Delta q \equiv q^H - q^L > 0$ and $\Delta c \equiv c^H - c^L > 0$ to denote the quality and cost differences across goods.[21]

[20]Without substantial effort, our model could be interpreted as one of quantity discounts, with firms offering the different quantities of the same product to consumers with either low or high demands. Results would go through as long as costs are not linear in the quality; for instance, if bigger bundles require costly product design features, such as packaging.

[21]We can think of these costs as the wholesale prices at which retailers buy the products from either competitive manufacturers, or from a monopoly manufacturer. Endogenizing the qualities of the products or the costs faced by the retailers is out of the scope of this paper.

There is a unit mass of consumers who buy at most one good. Consumers differ in their preferences over quality. A fraction $\lambda \in (0,1)$ of consumers have a low valuation for quality θ^L, while the remaining $(1-\lambda)$ fraction have a high valuation for quality θ^H, with $\Delta\theta \equiv \theta^H - \theta^L > 0$. As in Mussa and Rosen (1978), a consumer of type $i = L, H$ who purchases good $j = L, H$ at price p^j obtains net utility $u^i = \theta^i q^j - p^j$. We assume that the gross utility of a low type (high type) from consuming the low (high) quality product always exceeds the costs of producing it, i.e., $c^i < \theta^i q^i$ for $i = L, H$. Therefore, for a consumer of type θ^i to be willing to buy good of quality q^i, the following incentive compatibility constraints have to be satisfied

$$\theta^i q^i - p^i \geq \theta^i q^j - p^j, \tag{IC^i}$$

for $i, j \in \{L, H\}$ and $i \neq j$, which can also be re-written as

$$p^i \leq \theta^i q^i - \left(\theta^i q^j - p^j\right).$$

The second term on the right-hand side of the inequality represents consumers' *information rents*, i.e., the minimum surplus a consumer of type i needs to obtain to be willing to buy good i instead of good j.

The timing of the game is as follows. First, firms simultaneously decide which product(s) to offer for sale (or "product line"). Once chosen, firms observe the product line of the rival but consumers do not. Second, firms simultaneously choose the prices for the product(s) they carry and consumers visit the stores in order to learn firms' product choices and their respective prices. We will write (ϕ_i, ϕ_j) to denote firms' product choices, with $\phi_i \in \{\varnothing, L, H, LH\}$, and use $\Pi(\phi_i, \phi_j)$ to denote the profits of firm i at the pricing stage given those product choices.

Following Varian (1980), we assume that there is a fraction $\mu \leq 1$ of consumers who always visit the two stores and are therefore *informed* about where to find the cheapest product of each quality type. Since the remaining $1-\mu$ fraction of consumers only visit one store (with equal probability),[22] they are *uniformed* about the products and prices offered at the rival store. Hence, they can only compare the prices of the goods sold *within* the store they have visited, but not *across* stores. The fractions μ and λ are uncorrelated.[23] Once consumers have visited the store(s), they buy the product that gives them higher utility, provided it is non-negative. In case of indifference, low (high)

[22] In some settings it may be reasonable to assume that these consumers are uninformed about prices, but not about the product lines. Accordingly, we have also considered the case in which these consumers visit the store that gives them higher expected utility (and split randomly between the two stores in case of symmetry). The main results of the paper are strengthened. See Section 7.

[23] As we discuss in Section 7, our main results do not change if we allow μ and λ to be correlated.

type consumers buy the low (high) quality product. In what follows, we will use the fraction of uninformed consumers $1-\mu$ as a proxy for information frictions. Accordingly, the higher μ the lower the information frictions, with $\mu = 1$ representing a frictionless market.[24]

Assumptions In order to make the analysis meaningful, we rely on two assumptions that are standard in models of second-degree price discrimination (Tirole, 1988). The first one guarantees that a monopolist carrying both goods finds it optimal to sort consumers out. For the multi-product monopolist, the incentive compatible (i.e., constrained monopoly) prices are thus

$$\begin{aligned} p^L &= \theta^L q^L \text{ and} \\ p^H &= \theta^H q^H - \Delta\theta q^L \\ &= \theta^L q^L + \theta^H \Delta q. \end{aligned}$$

The alternative for the monopolist is to only sell good H to the high types at the (unconstrained) monopoly price $\theta^H q^H$, thus avoiding to leave information rents to the high types but also giving up the profits on good L.[25] To guarantee that this alternative is indeed less profitable than selling the two goods requires that the profit from selling good L to the low types is enough to compensate for the information rents that must be left with the high types:

$(A1)$ $\lambda(\theta^L q^L - c^L) \geq (1-\lambda)\Delta\theta q^L$.

Note that $(A1)$ is evaluated at monopoly prices. Assuming that a monopolist prefers to carry all qualities does not necessarily imply that the same holds true when competition drives prices below the monopoly level.

Our second assumption guarantees that there is no 'bunching' at the competitive solution. This requires marginal cost pricing to be incentive compatible, which is equivalent to assuming that the high types are willing to pay for the extra cost of high quality, whereas the low types are not:

$(A2)$ $\Delta c \in \left(\theta^L \Delta q, \theta^H \Delta q\right)$.

[24] In the online appendix we endogenize μ following the fixed-sample search model of Burdett and Judd (1983) but allowing consumers to differ in their search costs.

[25] Note that this alternative assumes that serving the high types with product H is more profitable than serving all consumers with product H at price $\theta^L q^H$. This is guaranteed by our assumption $(A3)$ below.

Implicit in $(A2)$ is the standard property that the cost of providing quality must be strictly convex in quality, i.e., $c^H/q^H > c^L/q^L$; otherwise, either type would buy the high quality product or nothing at all (CR adopt a similar assumption).[26]

Last, in order to reduce the number of cases we need to consider without affecting our results, we will assume that a monopolist that only carries good H prefers to extract all the surplus from the high types, even if it implies not selling to the low types, which would require reducing the price to $\theta^L q^H$:[27]

$(A3)$ $(1-\lambda)(\theta^H q^H - c^H) \geq \theta^L q^H - c^H$.

It follows that the single-product monopoly prices are $\theta^H q^H$ for the firm carrying good H and, as implied by $(A1)$, $\theta^L q^L$ for the firm carrying good L. In what follows we will denote the single-product monopoly profits as $\pi^i \equiv \theta^i q^i - c^i$ for $i \in \{L, H\}$.

Minmax profits Inspection of assumption $(A2)$ above allows to obtain useful expressions for the analysis of the model. As implied in $(A2)$, the maximum profits that can be made out of product $i \in \{L, H\}$ when good $j \neq i$ is priced at marginal costs are strictly positive. Since firms would never sell their products below marginal costs, these constitute *minmax profits*. In particular, if good L is sold at c^L, good H can at most be sold at the highest price that satisfies the high types' incentive compatibility constraint, i.e., $p^H \leq c^L + \theta^H \Delta q$. This gives per unit profits of

$$\varphi^H \equiv \theta^H \Delta q - \Delta c > 0.$$

The minmax profits for good H are always strictly below monopoly profits π^H given that, for all values of c^L, good L imposes a competitive constraint on good H.

In turn, if good H is sold at c^H, good L can at most be sold at the highest price that satisfies the low types' participation and incentive compatibility constraints, i.e., $p^L \leq \min\left\{\theta^L q^L, c^H - \theta^L \Delta q\right\}$. This gives per unit profits of

$$\varphi^L \equiv \min\left\{\pi^L, \Delta c - \theta^L \Delta q\right\} > 0.$$

For $c^H \geq \theta^L q^H$, the participation constraint binds first, so that good L can be sold at the monopoly price even when good H is priced at marginal cost, i.e., $\varphi^L = \pi^L$. Alternatively, for $c^H < \theta^L q^H$, the incentive compatibility constraint binds first, so that the minmax profits for good L are strictly below monopoly profits, i.e., $\varphi^L = \Delta c - \theta^L \Delta q < \pi^L$.

[26]Note also that convexity ensures that there is a non-empty region of λ values for which $(A1)$ and $(A2)$ are valid.

[27]Note that this assumption is redundant when the costs of providing high quality are large enough, i.e., $c^H \geq \theta^L q^H$, but it does imply that these costs cannot be much lower than $\theta^L q^H$.

In sum, the per unit profits that a firm that monopolizes good $i \in \{L, H\}$ loses when product $j \neq i$ is made available at marginal cost equal $\pi^i - \varphi^i \geq 0$ (with equality only for good L when $c^H \geq \theta^L q^H$).[28]

We are now ready to solve the game. We start by analyzing the case in which all consumers are informed, $\mu = 1$, then move on to introducing an arbitrarily small fraction of uninformed consumers, $\mu \to 1$, and finish by providing a full equilibrium characterization for all $\mu \in [0, 1)$.

4 Escaping the Bertrand Paradox

In this section we characterize the Subgame Perfect Equilibrium (SPE) of the game with no information frictions.

Proposition 1 *Assume all consumers are informed, $\mu = 1$. There exist two (pure strategy) Subgame Perfect Equilibria (SPE):*

(i) The "overlapping" equilibrium (LH, LH), at which both firms make zero profits.

(ii) The "specialization" equilibrium (L, H), at which both firms make strictly positive profits.[29]

Proof. See the appendix. ■

In the absence of information frictions, there exist two types of equilibria: (i) a Bertrand equilibrium in which firms carry both products and make zero profits ("overlapping equilibrium"), and (ii) an equilibrium in which firms carry non-overlapping product lines and each makes a strictly positive profit ("specialization equilibrium"). Hence, in the absence of search costs, simultaneous quality choices followed by price competition allow firms to escape the Bertrand paradox. [30]

To understand why the latter equilibrium exists, first note that under product choices (L, H), there does not exist a pure strategy equilibrium at the pricing stage. This stems from an important result: in equilibrium, firms' prices must satisfy incentive compatibility. Otherwise, the firm selling good H would sell nothing and would thus be better off

[28] We will sometimes express profit expressions as functions of φ^H and φ^L. The following equalities will be particularly useful throughout the analysis: $\pi^H - \Delta\theta q^L = \pi^L + \varphi^H$, and if $c^H < \theta^L q^H$ then $\theta^L q^H - c^H = \pi^L - \varphi^L$.

[29] If $c^H < \theta^L q^H$, there also exists a symmetric mixed strategy equilibrium with positive profits such that firms choose L and H with positive probability. On the contrary, if $c^H \geq \theta^L q^H$, this equilibrium does not exist as it is dominated by playing LH.

[30] Firms would also escape the Bertrand paradox if one of them carries both products and the other none. This equilibrium is not only little interesting but also irrelevant in our analysis as it disappears as soon as we introduce information frictions.

reducing its price to satisfy incentive compatibility. However, if the high types' incentive compatibility constraint is binding, the firm carrying good L could in turn attract all customers by slightly reducing its own price. Since these opposing forces destroy any candidate price choice in pure strategies, the equilibrium has to be in mixed strategies. Furthermore, all prices in the support of the mixed strategies must be strictly above marginal costs.

This has meaningful implications for equilibrium product choices. First, since at (L, H) product L is priced *above* marginal costs, profits on good H are strictly above its minmax. If firm H deviated to also carrying good L, p^L would be driven down to marginal costs. Hence, the profits on good L would be zero and the profits on good H would be driven down to its minmax, making such a deviation unprofitable. Similarly, since at (L, H) product H is priced *above* marginal costs, profits on good L are (weakly) above its minmax. If firm L deviated to also carrying good H, it would make no profits on good H and would (weakly) reduce its profits on good L as competition for good H becomes fiercer.[31] Last, if either firm deviated so that the two products overlapped, leading to (L, L) or (H, H), they would both make zero profits. In sum, since firms do not gain by deviating from (L, H), the "specialization equilibrium" constitutes a SPE of the game with no information frictions.

CR disregard the "overlapping equilibrium" by requiring that both firms make strictly positive profits in equilibrium. One way to justify this choice would be to assume that firms face (even infinitesimally small) fixed costs of carrying a product. Another one would be to rely on the Pareto criterion, as both firms make strictly higher profits at the "specialization equilibrium".[32] CR's focus on the "specialization equilibrium" has been very influential in spreading the view that firms can soften competition by differentiating their product choices. However, in the next section, we show that CR's prediction is not robust to introducing information frictions.

5 Back to the Bertrand Paradox

Before solving the game for all $\mu \in [0, 1)$, in this section we show that an arbitrarily small amount of information frictions $\mu \to 1$ is enough to result in positive profits under the

[31] If $c^H \geq \theta^L q^H$, the firm carrying good L makes the same profits at (L, H) as at (LH, H) since good H does no impose a competitive constraint on good H. In any event, firm L could increase its profits to also carrying good H.

[32] However, as we will see in the next section, one compelling reason for focusing on the "overlapping equilibrium" is that it naturally converges to the Bertrand equilibrium as information frictions vanish out, while the "specialization equilibrium" may not.

equilibrium with overlapping product lines. Furthermore, we show that if the costs of providing high quality are high enough, the "specialization" equilibrium no longer exists.

To explore this in more detail, let us first analyze pricing incentives at the subgame with "overlapping" product choices (LH, LH). Information frictions, no matter how small, imply that marginal cost pricing is not an equilibrium as firms could make positive profits out of the uninformed consumers. Similarly, setting prices at the (constrained) monopoly level is not an equilibrium either as firms would have incentives to charge slightly lower prices so as to attract the informed consumers. More generally, information frictions rule out any equilibrium candidate in pure strategies as firms face a trade off between charging high prices to exploit the uninformed consumers versus charging low prices to attract the informed consumers. Since firms must be indifferent between charging any price vector in the support, expected equilibrium profits can be computed by summing the profits of each good at the upper bound, where firms optimally serve their share of uninformed consumers at (constrained) monopoly prices,

$$\Pi(LH, LH) = \frac{1-\mu}{2}\left[\lambda\pi^L + (1-\lambda)(\pi^H - \Delta\theta q^L)\right]. \tag{1}$$

Importantly, each firm's equilibrium profits are a fraction $(1-\mu)/2$ of the multiproduct monopolist's profits because at the upper bound firms only make profits out of the uninformed consumers. For prices below the upper bound, firms make the same profits in expectation: the positive profits they obtain from the informed consumers compensate for the lower profits they obtain from the uninformed consumers. As μ approaches 1 and all customers become informed, the equilibrium price distributions concentrate around marginal costs, and firms' profits are driven down to (almost) zero. The Bertrand outcome is thus restored.

Could firms escape from the Bertrand paradox by having one of them drop one product, either L or H?[33] Let us first analyze the incentives of moving from (LH, LH) to (H, LH). Since a pure strategy equilibrium does not exist, and firms have to be indifferent across all prices in the support, expected profits for product H equal those of serving the uninformed consumers at the upper bound. Since firm H is not constrained by incentive compatibility, by $(A3)$, its optimal price at the upper bound is the (unconstrained) monopoly price. Its expected profits become

$$\Pi(H, LH) = \frac{1-\mu}{2}(1-\lambda)\pi^H. \tag{2}$$

Since firm H's profits are a fraction $(1-\mu)/2$ of monopoly profits, comparing (1) and (2) is equivalent to assessing the monopolist's incentives to carry the high quality good

[33] No firm has incentives to drop both products altogether as they both make positive profits at (LH, LH).

only versus carrying both goods. Assumption $(A1)$ guarantees that (1) exceeds (2) as the losses from not selling the low quality product exceed the information rents left to the high types. Thus, even though product L erodes the rents made on product H, the firm is better off carrying it.

The alternative is for one of the two firms to drop product H, thus moving from (LH, LH) to (L, LH). Now, the expected profits of firm L must be equal to the profits of serving all the uninformed consumers at the unconstrained monopoly price,[34]

$$\Pi\left(L, LH\right) = \frac{1-\mu}{2}\pi^L,$$

again a fraction $(1-\mu)/2$ of monopoly profits. By $(A2)$, this payoff is strictly less than (1) since the firm gives up the extra profit that firm L could make by selling the high quality good to the uninformed high types, who are willing to pay for the extra cost of providing quality.

In sum, firms' profits are the same *as if* they exploited their monopoly power over the uninformed consumers and competed fiercely for the informed, obtaining no profits out of the latter. Hence, firms' incentives to price discriminate through product choice mimic those of the monopolist. Consequently, in the presence of arbitrarily small information frictions, there exists a SPE with overlapping product lines (LH, LH) in which firms make strictly positive profits, in contrast with CR's prediction.

To assess whether this equilibrium is unique or not, let us first note that the "specialization" equilibrium of Proposition 1 is ruled out when the cost of providing high quality is sufficiently large, $c^H \geq \theta^L q^H$ (or equivalently, when the costs of providing quality is sufficiently convex). Starting at (L, H), firm L is strictly better off adding product H given that under (LH, H) it can now price discriminate the uninformed consumers without eroding its profits on good L. Indeed, the firm would be able to increase its profits by $(1-\mu)(1-\lambda)\varphi^H/2 > 0$ from selling the high rather than the low quality product to the uninformed high types, while it would still make profits $\lambda\pi^L$ out of the low types. Intuitively, the low types would never like to buy the high quality good even if it was sold at cost.

In contrast, if the costs of high quality are sufficiently low, the addition of good H erodes the rents of good L, making firm L worse off: the rents on product H are infinitesimally small while the profits on good L would go down by $\lambda\left(\pi^L - \varphi^L\right) > 0$. Similarly, firm H does not want to add product L as its profits would fall by $(1-\lambda)\left(\pi^H - \varphi^H\right) > 0$. Thus, the "specialization" equilibrium survives the introduction of infinitesimally small

[34]Note that in this case the firm would serve both the low and the high-types, since the latter are also willing to buy the low quality product at the (unconstrained) monopoly price for product L.

information frictions but *only* when the costs of providing high quality are sufficiently low.

Our second Proposition summarizes these results.

Proposition 2 *Assume that the mass of uninformed consumers is infinitesimally small, $\mu \to 1$.*

(i) The "overlapping" equilibrium (LH, LH) constitutes a SPE for all parameter values. Equilibrium prices approximate marginal costs.

(ii) The "specialization" equilibrium (L, H) constitutes a SPE if and only if $c^H < \theta^L q^H$. Equilibrium prices are strictly above marginal costs.

Proof. See the discussion above. A formal derivation can be found as a particular case of the proof to Proposition 7. ■

The addition of even infinitesimally small information frictions implies that, when the costs of providing high quality are high enough (or when low type consumers do not value high quality enough), firms can no longer escape the Bertrand paradox by differentiating their product lines. Indeed, the "specialization" equilibrium no longer exists, making the Bertrand-like "overlapping" equilibrium the unique SPE of the game.

In our model, the "overlapping" equilibrium always exists. This is in contrast to previous papers analyzing quality choices followed by imperfect competition (Gal-Or, 1983; Gilbert and Matutes, 1993; Johnson and Myatt, 2003; Stole, 1995; Wernerfelt, 1986), in which the "overlapping" equilibrium exists *only if* the rents created by imperfect competition are high enough (e.g., few firms competing *à la* Cournot). In those papers, just as in CR, there is a tension between competition and price discrimination: competition reduces the rents on the overlapping products at the same time as it enlarges consumers' information rents, thus reducing the gains from price discrimination.

In this paper, under the "overlapping" equilibrium that arises with information frictions, such a tension is not present because firms only care about the profits made out of the uninformed consumers, from whom they obtain monopoly profits (in expectation). Thus, firms' product choices are solely driven by their incentives to discriminate consumers, leading them to carry the full product range even when the rents created by information frictions are arbitrarily small. This shows that the impact of information frictions on product choices, and through these on prices, may be different from other forms of imperfect competition.

6 Equilibrium Product Lines and Prices

In this section we characterize equilibrium product and price choices for all values of $\mu < 1$. We show that the "overlapping" equilibrium is robust to introducing information frictions, no matter how big or small. In contrast, the "specialization" equilibrium fails to exist when the mass of informed consumers μ is sufficiently small or, for all μ, when the cost of providing high quality c^H is sufficiently high. In general, the "overlapping" equilibrium is more likely to be unique the smaller the mass of informed consumers and/or the higher the costs of providing high quality.

We again proceed by backwards induction by first analyzing equilibrium pricing behavior and then product choices. The pricing subgames will also serve to understand pricing decisions for non-overlapping product configurations, which may prove relevant to cases in which product choices are constrained by factors outside our model (e.g., fixed costs of carrying a product).

6.1 Pricing Behavior

We first provide an important property of pricing behavior by multi-product firms.

Lemma 1 *In equilibrium, multi-product firms choose incentive compatible prices for their products, i.e.,* $\Delta p \in \left[\theta^L \Delta q, \theta^H \Delta q\right]$.

Proof. See the appendix. ■

Lemma above shows that it is always optimal for a multi-product firm to choose prices that satisfy incentive compatibility. The intuition is simple. If the price of the high quality product is too high so that all consumers buy the low quality product, it is profitable for the firm to reduce p^H, while leaving p^L unchanged, so as to attract the high types and obtain a larger profit margin. Similarly, if the price of the high quality product is too low so that all consumers buy it, it is profitable for the firm to increase p^H, while leaving p^L unchanged, so as to extract more surplus from the high types as these are willing to pay more for higher quality. This result constitutes an important departure from Varian (1980), as it implies that the price of one product cannot be picked independently from the price of another product *within* the same store.[35]

We are now ready to characterize equilibrium pricing at every possible subgame.

[35] This is in contrast to Johnson and Myatt (2015) prediction. In a model of quality choice followed by Cournot competition, they find conditions under which the equilibrium prices chosen my multi-product oligopolists are close to the single-product prices.

Full product overlap We start by considering subgames with full product overlap: (LH, LH), (L, L), and (H, H). The next Proposition characterizes (symmetric) equilibrium pricing under the former one.

Proposition 3 *Given product choices (LH, LH), there does not exist a pure strategy equilibrium. The equilibrium must be in mixed strategies, and it must satisfy the following properties:*

(i) Prices at the upper bound of the price support correspond to the (constrained) monopoly prices, $\overline{p}^L = \theta^L q^L$ and $\overline{p}^H = \theta^H q^H - \Delta\theta q^L = \theta^L q^L + \theta^H \Delta q$, so that the high types' incentive compatibility constraint is binding, $\Delta\overline{p} \equiv \overline{p}^H - \overline{p}^L = \theta^H \Delta q$.

(ii) Prices at the lower bound of the price support are strictly above marginal costs, $\underline{p}^i > c^i$ for $i = L, H$, and such that the high types' incentive compatibility constraint is not binding, $\Delta\underline{p} \equiv \underline{p}^H - \underline{p}^L < \theta^H \Delta q$.

(iii) Any pair of prices, $p^H \in [\underline{p}^H, \bar{p}^H]$ and $p^L \in [\underline{p}^L, \bar{p}^L]$, is chosen according to some joint distribution function $F^{LH}(p^H, p^L)$ that is consistent with Lemma 1: $p^H - p^L \equiv \Delta p \in [\theta^L \Delta q, \theta^H \Delta q]$.

Proof. See the appendix. ■

The non-existence of pure strategy equilibria is shared with most search models, starting with Varian (1980) (see also Burdett and Judd (1993) and McAFee (1995), among others). It stems from firms' countervailing incentives, as on the one hand they want to reduce prices to attract the informed consumers, but on the other, they want to extract all rents from the uninformed.

Despite this similarity, our analysis shows that equilibrium pricing by multi-product firms has a distinctive feature: it is constrained by incentive compatibility (Lemma 1). This comes up clearly when characterizing the upper bound of the price support: firms are not able to extract all the surplus from the uninformed consumers high types because firms have to give up information rents $\Delta\theta q^L$.[36] Hence, because of incentive compatibility, firms make lower profits on the high quality good than in the single-product case, in contrast to McAFee (1995).

Since firms make strictly positive profits at the upper bound, prices at the lower bound must be strictly above marginal costs. The reduction in prices from the upper to the lower bound is more pronounced for the high quality product than for the low quality one. Competition for the high types is fiercer because selling the high quality product is more profitable. In turn, this implies that at the lower bound, the incentive compatibility

[36] Note that in equilibrium nothing prevents $F^{LH}(p^H, p^L)$ be such that a firm plays $\bar{p}^L$ together with $p^H \in [\underline{p}^L + \theta^L \Delta q, \bar{p}^H]$, or alternatively, $\underline{p}^L$ together with $p^H \in [\underline{p}^H, \underline{p}^L + \theta^H \Delta q]$.

constraint for the high types is not binding, so that the price wedge between the two products at the upper bound is wider than at the upper bound. We can conclude that high quality products are relatively cheaper during periods of "sales" *à la* Varian, i.e., when both goods are priced at the lower bounds of the price supports. Even when firms do not price the two goods simultaneously at the lower bound, the relative price difference never exceeds the one under monopoly, $\theta^H \Delta q$, as otherwise incentive compatibility would not be satisfied (Lemma 1). Thus, competition among multi-product firms reduces the relative prices of the two goods.

Since firms have to be indifferent between charging any price in the support, including the upper bounds, expected equilibrium profits are unambiguously given by

$$\Pi(LH, LH) = \frac{1-\mu}{2}\left[\lambda\pi^L + (1-\lambda)(\pi^H - \Delta\theta q^L)\right]. \quad (3)$$

Just as we noted in the previous section, these profits are a fraction $(1-\mu)/2$ of the (constrained) monopoly profits.

At the lower bound, each firm attracts all the informed consumers plus its share of the uninformed consumers of each type. Hence, expected profits can also be expressed as a function of the lower bounds,

$$\Pi(LH, LH) = \frac{1+\mu}{2}\left[\lambda(\underline{p}^L - c^L) + (1-\lambda)(\underline{p}^H - c^H)\right]. \quad (4)$$

Since there are two goods, and only one profit level, as defined in equations (3) and (4), the problem has an extra degree of freedom: there are potentially many price pairs $\underline{p}^L > c^L$ and $\underline{p}^H > c^H$ satisfying $\Delta\underline{p} < \theta^H \Delta q$ that yield the same equilibrium profits. This implies that, even though equilibrium profits are unique and well defined, there might be multiplicity of mixed strategy equilibria.[37]

Last, there could also be full overlap among single-product firms, (L, L) and (H, H). Since single-product firms selling the same product are not constrained by incentive compatibility, they play a mixed strategy equilibrium with an upper bound equal to the (unconstrained) monopoly price, as in Varian (1980). However, the presence of heterogenous consumers adds a small twist to Varian's pricing. In particular, there can now be a gap in the price support between the prices at which firms are indifferent between serving the high types only (at a high price) versus serving both types of consumers (at a lower price). Note that, when this is the case, the low types are left out of the market with some positive probability. Other than this, since equilibrium profits are fully determined by the upper bound, equilibrium profits are as in Varian (1980).

[37] See McAfee (1995) and Shelegia (2012) for a similar result. Appendix B characterizes one equilibrium of this subgame.

Partial product overlap Let us now characterize equilibrium pricing in the subgames with partial overlap: (L, LH), (H, LH). Interestingly, even though the single-product firm does not face an incentive compatibility constraint *within* its store, its pricing is nevertheless affected by incentive compatibility considerations through the effect of competition *across* stores.

The following Proposition characterizes the price equilibrium at the (L, LH) subgame.

Proposition 4 *Given product choices (L, LH):*

(i) A pure strategy equilibrium does not exist.

(ii) At the unique mixed-strategy equilibrium, firm LH charges $p^H = p^L + \theta^H \Delta q$, and both firms choose p^L in $\left[\underline{p}^L, \theta^L q^L\right]$, with firm L putting a probability mass at the upper bound.

Proof. See the online appendix. ■

In equilibrium, the two firms choose random prices for the low quality product over a common support. In turn, given its price choice for the low quality good, the multi-product firm prices the high quality product to just comply with incentive compatibility for the high types. Hence, unlike the previous case, the price difference between the two products remains constant at $\theta^H \Delta q$ over the whole support, and the density of prices for the high quality product is the same as that for the low quality product (just shifted out to the right by $\theta^H \Delta q$). It follows that, whenever the multi-product firm has the low price for the low quality product, all the informed consumers (both the low or the high types) buy from it. Otherwise, if the single-product has the low price for the low quality product, it serves all the informed consumers, including the low and the high types.[38] Its profits are nevertheless determined by its upper-bound price. As before, they are a fraction $(1 - \mu)/2$ of its monopoly profits

$$\Pi(L, LH) = \frac{1 - \mu}{2} \pi^L.$$

Now we turn to characterizing the price equilibrium at the (H, LH) subgame.

Proposition 5 *Given product choices (H, LH), there exists $\hat{\mu} \in (0, 1)$ such that:*

[38] It is worth pointing out that the multi-product firm charges lower prices on average as compared to the single-product firm. The reason is that, when it has the low price, its ability to discriminate between the low and the high types allows the multi-product firm to make extra profits $\mu(1 - \lambda)\varphi^H$ out of the informed high type consumers. Since the multi-product firm has stronger incentives to undercut its rival's price, the single product firm has to put a probability mass at the upper-bound. In turn, since the two firms cannot put a mass at the same price, it follows that when the single-product firm is pricing at the upper bound it is only selling to the uninformed consumers with probability one.

(i) For $\mu \leq \hat{\mu}$, there exists a unique pure strategy equilibrium: firm H chooses the (unconstrained) monopoly price $p^H = \theta^H q^H$, and firm LH chooses the (constrained) monopoly prices, $p^H = \theta^H q^H - \Delta\theta q^L$ and $p^L = \theta^L q^L$.

(ii) For $\mu > \hat{\mu}$, there does not exist a pure strategy equilibrium. In the mixed-strategy equilibrium, firm LH chooses prices p^H in $\left[\underline{p}^H, \theta^H q^H - \Delta\theta q^L\right]$ with a mass on its upper bound, and $p^L = \min\left\{\theta^L q^L, p^H - \theta^L \Delta q\right\}$. Firm H chooses prices p^H in $\left\{\left[\underline{p}^H, \theta^H q^H - \Delta\theta q^L\right], \theta^H q^H\right\}$ with a (strictly) positive mass on its upper bound.

Proof. See the online appendix. ■

There now exists a pure strategy equilibrium as long as the fraction of the informed consumers μ is small enough. At this equilibrium, the multi-product firm charges the (constrained) monopoly prices, while the single-product firm charges the (unconstrained) monopoly price for the high quality product.

When the fraction of informed consumers is higher, the above is no longer an equilibrium as it now pays the single-product firm to fight for the informed consumers. In this case, the equilibrium must be in mixed strategies.[39,40] The precise shape of the mixed strategy equilibrium depends on whether it pays firm H to serve the low types or not.

If $c^H \geq \theta^L q^H$, it never pays firm H to serve the low types because the costs of high quality exceed their willingness to pay for it. Thus, the two firms compete for the informed high type consumers only, while the low quality product is still priced at the monopoly level, $\theta^L q^L$. Since the incentive compatibility constraint of the multi-product firm is not binding, its profits are the same as if the two products were sold independently. In contrast, when $c^H < \theta^L q^H$, the low types might be tempted to buy the high quality good when its price is sufficiently low. In this case, the price of the low quality good has to be reduced below the monopoly price to achieve separation.

Regarding the single-product firm, since $\theta^H q^H - \Delta\theta q^L$ is the highest price that the multi-product firm would ever charge for the high quality good, the firm will play either the (unconstrained) monopoly price, $\theta^H q^H$, or something less than the (constrained) monopoly price, $\theta^H q^H - \Delta\theta q^L$. Any price in between is unprofitable, either because it doesn't extract enough from the uninformed high types or because it doesn't attract the informed consumers when the multi-product firm happens to price the good at or below

[39] Interestingly, there is continuity between the pure and the mixed-strategy equilibrium. The two firms charge the upper bounds of their price supports, $\theta^H q^H - \Delta\theta q^L$ and $\theta^H q^H$, with positive and identical mass. This mass fades away as μ grows larger—from one, when $\mu \to \widehat{\mu}$ towards zero, when $\mu \to 1$.

[40] Unlike in subgame (LH, LH), the equilibrium is now unique: since one firm only has one product, there are no longer two degrees of freedom as in the symmetric two product case.

$\theta^H q^H - \Delta\theta q^L$. In either case, profits remain as in the pure strategy equilibrium because $\theta^H q^H$ always belongs to the price support. Therefore, for all μ,

$$\Pi(H, LH) = \frac{1-\mu}{2}(1-\lambda)\pi^H,$$

again a fraction $(1-\mu)/2$ of the monopoly profits.[41]

Non-overlap Let us now move to characterizing equilibrium pricing in the subgames with no product overlap: $(\varnothing, L)$, $(\varnothing, H)$, $(\varnothing, LH)$ and (L, H). The first three correspond to the single-product monopoly solution. Hence, here we turn our attention to the more interesting subgame with specialized firms, (L, H).

Proposition 6 *Given product choices (L, H), there exists $\tilde{\mu} \in (\hat{\mu}, 1)$ such that:*

(i) For $\mu \leq \hat{\mu}$, there exists a unique pure strategy price equilibrium: firms charge the (unconstrained) monopoly prices $p^H = \theta^H q^H$ and $p^L = \theta^L q^L$.

(ii) For $\mu > \hat{\mu}$ there does not exist a pure strategy equilibrium. At the unique mixed-strategy equilibrium, firm L chooses prices p^L in $\left[\underline{p}^L, \theta^L q^L\right]$ with a mass on the upper bound. If $\mu \in (\hat{\mu}, \tilde{\mu})$ firm H chooses prices p^H in $\left\{\left[\underline{p}^H, \theta^H q^H - \Delta\theta q^L\right], \theta^H q^H\right\}$ with a mass on the upper bound that falls to zero as $\mu \to \tilde{\mu}$; if $\mu \geq \tilde{\mu}$, $\theta^H q^H$ is not part of firm H's support.

Proof. See the online appendix. ■

Equilibrium pricing at subgames (L, H) and (H, LH) share some similarities. In particular, just as in Proposition 5, if the mass of informed consumers μ is small enough, there exists a pure strategy equilibrium as the firm selling the high quality product is better off serving the uninformed high types at the (unconstrained) monopoly price than competing for the informed high types.[42] Furthermore, there is continuity between the pure and the mixed strategy equilibrium in that the probability mass that the high quality firm puts on the (unconstrained) monopoly price fades away as μ grows larger.

The main difference between the two subgames is that, under (L, H), the high quality firm chooses not to include the unconstrained monopoly price in the support when μ is very large. The reason is that the profits from serving a small fraction of uninformed consumers become lower than the profits from fighting for the informed consumers.[43]

[41] Just as in the previous subgame, the equilibrium price distribution used by the multi-product firm for the high quality good (weakly) first-order stochastically dominates that of the single-product firm. It follows that, on average, the price charged by the single-product firm for the high quality product exceeds the one charged by the multi-product firm.

[42] Note that the threshold for the existence of a pure-strategy equilibrium is the same under both subgames.

[43] At subgame (H, LH), competition for good H is more intense given that both firms carry it. This

6.2 Product Line Choices

We are now ready to analyze product line decisions given the continuation equilibria characterized above. For this purpose, it is useful to implicitly define the threshold μ^* as

$$(1-\mu^*)(1-\lambda)\left(\pi^H-\pi^L\right)=(1+\mu^*)\lambda\left(\pi^L-\varphi^L\right). \tag{5}$$

Note that μ^* is increasing in c^H, and that $\mu^*=1$ for $c^H \geq \theta^L q^H$. The following Proposition characterizes the Subgame Perfect Equilibrium (SPE) product choices.

Proposition 7 *(i) If $\mu<\mu^*$, the "overlapping" equilibrium (LH, LH) constitutes the unique SPE of the game. (ii) Otherwise, both the "overlapping" equilibrium (LH, LH) and the "specialization" equilibrium (L, H) constitute SPE of the game.*

Proof. See the appendix. ■

In Proposition 2 we showed that a SPE with overlapping product lines exists in the presence of an arbitrarily small amount of uninformed consumers. Proposition 7 now shows that this prediction remains valid for all values of μ. The underlying logic remains the same: the existence of the "overlapping" equilibrium hinges upon the incentives of firms to mimic those of a monopolist, regardless of whether μ is large or small.

Regarding the existence of the "specialization" equilibrium, Proposition 2 showed that it exists for $\mu\to 1$ as long as $c^H<\theta^L q^H$. Thus, the equilibrium with overlapping qualities is the unique one when information frictions are arbitrarily small, as long as the costs of providing high quality are large enough. Proposition 7 now shows that the presence of uninformed consumers relaxes the condition for the uniqueness of the equilibrium with overlapping qualities. In particular, whereas $c^H<\theta^L q^H$ is still needed to guarantee the existence of the specialization equilibrium, it is no longer sufficient: additionally, information frictions have to be low enough for the gains from softening competition to exceed the costs of giving up profitable opportunities to discriminate. To see this in more detail, consider the incentives to deviate from the "specialization" equilibrium by the firm carrying product L. Adding product H would allow the firm to better discriminate the high types, thus making extra profits $\left(\pi^H-\pi^L\right)$ from selling product H to the uninformed high types with probability $(1-\mu)(1-\lambda)/2$. In contrast, adding product H would also intensify competition for product L, forcing the firm to give up rents $\left(\pi^L-\varphi^L\right)$ on all the low types (excluding the uninformed low types that visit the rival's store) with probability $(1+\mu)\lambda/2$. The magnitude of the two effects coincides at $\mu=\mu^*$, as implicitly defined in (5). In turn, since in expectation firms only

explains why in that case firm H always puts mass at the unconstrained monopoly price, while at subgame (H, L) firm H eventually decides not to include it in its price support.

benefit from discriminating the uninformed consumers, the softening of competition effect dominates the incentives to discriminate only when the mass of uninformed consumers $(1-\mu)$ is sufficiently small, i.e., when $\mu \geq \mu^*$. Therefore, for $\mu < \mu^*$ the "specialization" equilibrium breaks down, making the "overlapping" equilibrium the unique SPE of the game.

The fact that μ^* is increasing in c^H means that, as the cost of high quality provision increases up to $\theta^L q^H$, the set of μ values for which the "overlapping" equilibrium is unique is enlarged; beyond that level, the "overlapping" equilibrium is the unique SPE for all μ. If $c^H \geq \theta^L q^H$, $\mu^* = 1$, implying that for high c^H, the "specialization" equilibrium (L, H) never exists (in the presence of information frictions) because if firm L adds product H it does not give up any rents on product L.

We remain agnostic as to which equilibrium firms are more likely to play when there exist multiple equilibria (i.e., for parameter values $c^H < \theta^L q^H$ and $\mu \geq \mu^*$). Still, we want to stress that there are theoretical reasons, beyond its empirical relevance, to believe that the "overlapping" equilibrium is compelling. First, the Pareto criterion does not allow to rule out the "overlapping" equilibrium in general, despite the fact that it results in lower prices. In particular, the firm that carries product H at the "specialization" equilibrium is not necessarily better off than at the "overlapping" equilibrium as at the former it fails to capture the profits from serving the uninformed low types. Furthermore, some authors have documented path dependency in equilibrium choices (Romero, 2015). In this setting, this suggests that the existence of the "overlapping" equilibrium for all $\mu < 1$ (in contrast to the "specialization" equilibrium, which only exists for $\mu \geq \mu^*$), together with low μ as an initial condition, might create inertia at (LH, LH) all the way down to $\mu \to 1$.[44] Last, but not least, the equilibrium with overlapping qualities naturally converges to the Bertrand equilibrium as information frictions vanish out, while the same is not true for the specialization equilibrium.

[44] Consider for instance a simple repetition of our two-stage game and allow for information frictions to gradually fall. If, as initial condition, there are strong information frictions so that the "overlapping" equilibrium is unique, hysteresis would lead firms to keep on playing the same equilibrium even if the reduction in information frictions implies that the "specialization" equilibrium eventually arises. The same would apply if the costs of quality are initially high and declining. In contrast, if there are initially no search costs and the costs of quality provision are low, the market could remain at the "specialization" equilibrium as either search costs or quality costs go up, eventually giving rise to the "overlapping" equilibrium when the "specialization" equilibrium ceases to exist. However, given the overall current trend towards lower search costs, this possibility does not seem empirically relevant.

6.3 Comparative Statics

Combining the results in Propositions 1, 3, and 7, our last Lemma evaluates how the mass of uninformed consumers affects expected market prices and expected consumer surplus at the SPE product choices. Results are illustrated in Figures 3 and 4.

Lemma 2 *The comparative statics of expected prices and expected consumer surplus with respect to μ are as follows:*

(i) At the "overlapping" equilibrium, expected prices monotonically decrease in $\mu \in [0,1]$. Similarly, expected consumer surplus increases in $\mu \in [0,1]$.

(ii) If, in case of equilibrium multiplicity, firms always play the "specialization" equilibrium, expected prices jump upwards at $\mu = \mu^*$. Similarly, expected consumer surplus jumps downwards at $\mu = \mu^*$.

Proof. See the appendix. ■

The conventional wisdom that milder information frictions lead to lower prices applies in this model, but only when the reduction in information frictions does not change equilibrium product lines.[45] Indeed, when lower information frictions induce firms to switch from the "overlapping" to the "specialization" equilibrium (i.e., at $\mu = \mu^*$), expected prices jump up as firms manage to mitigate competition by differentiating their product choices.

Similarly, as information frictions go down, consumer surplus goes up with a discontinuity when firms switch from the "overlapping" to the "specialization" equilibrium. The discontinuity in consumer surplus is more pronounced than the discontinuity in expected prices because not only expected prices jump up, but also gross consumers' surplus jumps down because of incomplete price discrimination at the "specialization" equilibrium (meaning that some high types fail to buy their preferred good while some low types might fail to consume at all).

7 Extensions and Variations

In the preceding sections we characterized product and price choices in a model (i) with two possible quality levels and two consumer types, in which (ii) search cannot be condi-

[45] In general, search costs are thought to relax competition, thus leading to higher retail prices, although not as intensively as the Diamond paradox would have anticipated (Diamond, 1971). There are some exceptions to this general prediction. Some recent papers have shown that search costs can lead to lower retail prices, particularly so when search costs affect the types of consumers who search. For instance, see Janssen and Shelegia (2015), Moraga-González *et al.* (2017) and Fabra and Reguant (2019).

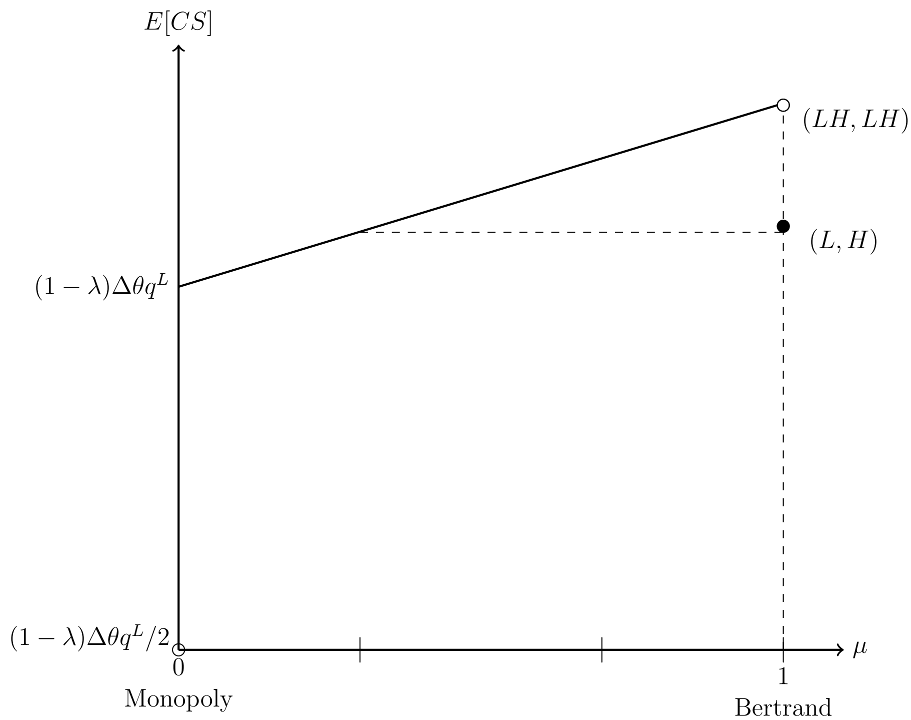


Figure 3: Expected consumer surplus as a function of μ at the SPE product choices for $c^H > \theta^L q^H$.

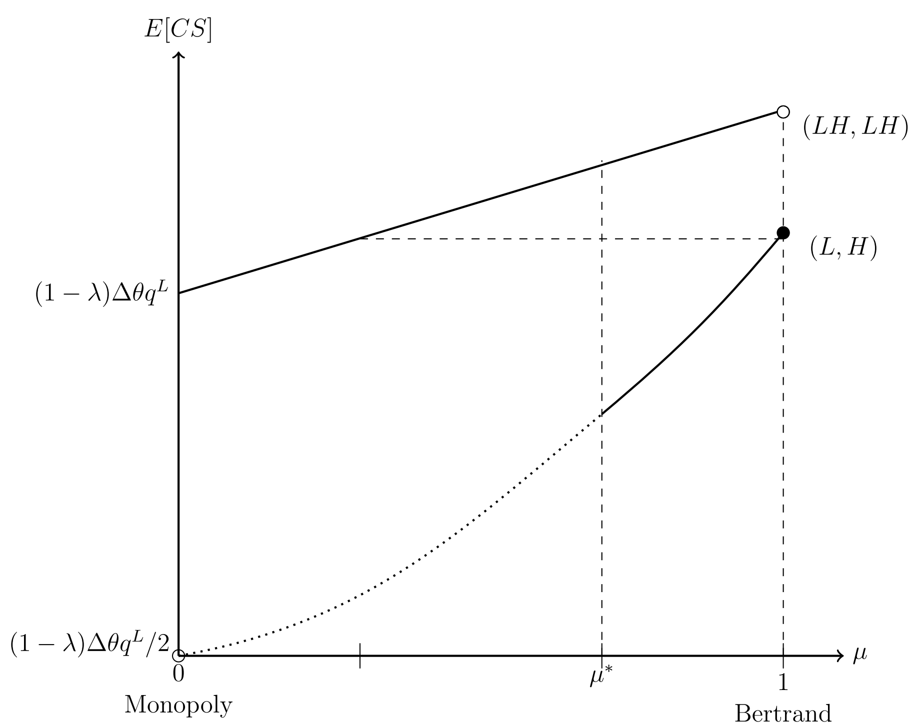


Figure 4: Expected consumer surplus as a function of μ at the SPE product choices for $c^H < \theta^L q^H$.

tioned on product choices (as these were assumed non-observable prior to search), and in which (iii) consumers' information frictions and quality preferences are uncorrelated. In this section we discuss how one can relax these assumptions while preserving our main results. Our focus is on the existence of the "overlapping" equilibrium.

Observable product choices and directed search by the uninformed consumers In the main model we assumed that consumers do not observe product lines prior to visiting the stores. In particular, we assumed that the uninformed consumers visit one of the two stores with equal probably, regardless of their product choices. Instead, suppose now that the uninformed consumers visit the store that gives them higher expected utility, given firms' (observable) product choices and expected prices (in case of indifference, uninformed consumers visit the store that carries their preferred product).[46] Allowing search to be conditioned on product choices would strengthen our main result: when directed search is allowed, carrying multiple products would allow firms to not only better discriminate, but also to attract more uninformed consumers.

Directed search by the uninformed consumers only affects pricing when firms have chosen asymmetric product lines (with symmetric product lines, expected prices are also symmetric so it is irrelevant whether search is directed or random). Let us consider subgame (L, LH). Now, all the uninformed high types visit the multi-product firm given that (i) the expected utility of buying product L is the same across the the two stores, and (ii) at store LH they are indifferent between buying L or H. In turn, the prices for product L have to be such that the uninformed low types are indifferent between visiting one store or the other (otherwise, they would all visit the one charging lower prices, but this cannot constitute an equilibrium as the high-priced firm would make no sales). From our previous analysis, we know that with an even split of uninformed consumers between the two stores, the multi-product firm charges lower prices. Hence, to rebalance firms' pricing incentives, more than one half of the uninformed low-types must visit store LH until their expected prices converge. Thus, since the market share of the single-product firm is lower, it makes lower profits than when product lines are non-observable, as we had assumed in the main model. In turn, this implies that firms have no incentives to deviate from (LH, LH) to (L, LH) - their incentives to deviate are weaker than in the main model, under which (LH, LH) already constituted an equilibrium for all $\mu < 1$ (Proposition 7). A similar reasoning applies to subgame (H, LH).

In sum, our main conclusion —namely, that the "overlapping" equilibrium is robust for all $\mu < 1$— remains valid regardless of whether product lines are observable (and

[46]This interpretation of the uninformed consumers as sophisticated buyers is closer to that in the clearing-house model *à la* Baye and Morgan (2001).

there is directed search by the uninformed consumers) or not. The conclusion that multi-product firms tend to charge lower expected prices would have to be qualified, as with directed search firms charge the same expected prices even though multi-product firms make higher profits as they attract more customers.

Correlation between information frictions and quality preferences Last, we have so far assumed that the informed and the uninformed consumers are equally likely to be either high or low types. However, this may not hold in practice. For instance, if the low types are lower income consumers with more time to search, then the uninformed consumers are more likely to be high types. Alternatively, if high types enjoy shopping for their preferred (high quality) product, then the uninformed consumers are more likely to be low types. Ultimately, this is an empirical question whose answer may vary depending on the type of product or context considered. However, as far as the predictions of the model are concerned, it is inconsequential whether the correlation between information frictions and quality preferences is positive, negative or non-existent.

To formalize this, one can introduce the parameters λ^I and λ^{NI}, representing the fraction of low types among the informed and uninformed consumers, respectively, i.e., $\lambda^I \mu + \lambda^{NI}(1-\mu) = \lambda$. If $\lambda^I > \lambda^{NI}$ there is positive correlation between information frictions and quality types as the fraction of low types is higher among the informed than among the uninformed consumers. In line with the main text, we assume that this correlation is not too strong so that monopoly profits on the uninformed are still maximized by selling both products, as in $(A1)$.

The analysis of product and price choices without information frictions remains intact since all consumers are informed by definition. As for the analysis with information frictions, profits on good H are proportional to $\left(1-\lambda^{NI}\right)$ and those on good L are proportional to λ^{NI}, thus implying that the incentive structure remains unchanged. As such, the "overlapping" equilibrium always exists just as in the case with no correlation between search and quality preferences.

8 Conclusions

In this paper we have analyzed the impact of information frictions on quality choices followed by price competition. We have found that the equilibrium in which firms carry overlapping product lines always exists, with or without information frictions. In contrast, we have also shown that the equilibrium with non-overlapping quality choices, as originally proposed in CR's influential paper, is not particularly robust: it only exists under mild information frictions if the costs of providing high quality are sufficiently small.

In particular, it providing high quality is particularly costly, CR's equilibrium fails to exist even when the mass of uninformed consumers is infinitesimally small. This finding casts doubts on the prediction that strategic incentives alone induce firms to soften competition by carrying non-overlapping product lines. Our results extend to more general settings, including the case with more than two goods and consumer types, more than two firms, directed search by the uninformed consumers and the possibility that information frictions and quality tastes are positively or negatively correlated.

We have shown that, through product choice, information frictions can have important implications for market outcomes beyond their well studied price effects. In particular, we have shown that analyzing the price effects of information frictions without endogenizing product choices can sometimes lead to overestimating their anticompetitive effects. This is for instance the case when the addition of information frictions induces firms to carry overlapping products, thus creating head-to-head competition.

The multi-product nature of firms also adds important twists to the analysis of competition in the presence of information frictions. An important departure from Varian (1980) is that goods within a store cannot be priced independently from each other. In particular, the incentives to separate both consumer types impose an upper (lower) bound on the highest price that can be charged for a high (low) quality good, given the price of the low (high) quality one. This holds true even for a single-product firm competing with a multi-product one, as through competition, the effects of price discrimination by the multi-product firm affect pricing by the single product firm. In line with Varian (1980), we have also shown that information frictions give rise to price dispersion when the two competing firms carry multiple products- a possibility not considered by Varian (1980).

Admittedly, there are several determinants of firms' product choices beyond the ones studied in this paper. In particular, throughout the analysis we have assumed that firms do not incur any fixed cost of carrying a product. This modelling choice was meant to highlight the strategic motives underlying product choice. However, fixed costs of carrying a product (which could arguably be higher for high quality products),[47] could induce firms to offer fewer and possibly non-overlapping products. Our prediction is not that competitors should always carry overlapping product lines. Rather, our analysis suggests that if their product lines do not overlap in markets with information frictions,

[47] In some cases, such costs can be substantial, e.g. firms have to advertise that they are carrying an additional product, or the transaction costs of dealing with an additional provider can sometimes be high. The marketing literature has analyzed several factors explaining the limited number of products sold per firm. For instance, Villas-Boas (2004) analyzes product line decisions when firms face costs of communicating about the different products they carry to their customers. They show that costly advertising can induce firms to carry fewer products as well as to charge lower prices for their high-quality goods.

it must be for reasons other than firms' attempts to soften competition through product choice- for instance, due to the presence of fixed costs.

Appendix A: Selected Proofs

Proof of Proposition 1 [SPE under $\mu = 1$] We show that the specialization equilibrium (L, H) constitutes a SPE. First, at subgames (LH, LH), (L, L) and (H, H), both firms make zero profits. Second, at subgame (L, LH) the low quality product is priced at marginal cost c^L while the high quality product is sold at the highest price that satisfies the high types' incentive compatibility constraint, i.e., $c^L + \theta^H \Delta q$. Firm L makes zero profits while firm LH gets a payoff of $(1-\lambda)\varphi^H$, which equals its minimax. Third, at subgame (H, LH), the high quality product is priced at marginal cost c^H while the low quality product is sold at the highest price that satisfies the low types' incentive compatibility constraint and participation constraints, i.e., $\min\left\{c^H - \theta^L \Delta q, \theta^L q^L\right\}$. Firm H makes zero profits while firm LH makes profits $\lambda\pi^L$ if $c^H > \theta^L q^H$ or $\lambda\varphi^L$ otherwise, i.e., its minmax. Finally, at subgame (L, H) the equilibrium is in mixed strategies. For the purposes of this proof, it suffices to put bounds on equilibrium profits. Minmax profits for each firm are computed by characterizing the firm's best response to the rival pricing its good at marginal cost. Following our previous analysis, the minmax profits for the H firm are $(1-\lambda)\varphi^H > 0$, while the minmax profits for the L firm are $\lambda\pi^L > 0$ if $c^H > \theta^L q^H$ or $\lambda\varphi^L > 0$ otherwise. Since at the mixed strategy equilibrium firms always price above marginal costs (otherwise they would have zero profits, but this cannot be since their minmax profits are positive), equilibrium profits are *strictly above* the minimax whenever the participation constraint is not binding. The only case where above marginal cost pricing does not necessarily imply that firm L's profits are strictly above its minmax is when $c^H > \theta^L q^H$, as in this case firm L's best response is the the same regardless of whether firm H prices at c^H or above.[48] Indeed, for the case $c^H > \theta^L q^H$, equilibrium profits are exactly equal to the minmax $\lambda\pi^L$. To see this, note that at the mixed-strategy equilibrium (MSE), the upper bounds of firms' price supports are the constrained monopoly prices. Furthermore, firm L has to play a probability mass at its

[48] It is straightforward to see that in a mixed strategy equilibrium we must have $\underline{p}^H > c^H$ and $\underline{p}^L > c^L$; otherwise, each firm's profits would be zero, but this leads to a contradiction since profits cannot be below the minimax. Hence, firm H would never like to price lower than $\underline{p}^L + \theta^H \Delta q > c^L + \theta^H \Delta q > c^H$. Since at a price $\underline{p}^L + \theta^H \Delta q$ firm H would at least be serving the high types, its profits must be strictly greater than its minmax $(1-\lambda)\varphi^H$. Similarly, if $\underline{p}^H > \theta^L q^H$, firm L would be a monopolist over the low-types, so it could always secure profits of at least $\lambda\pi^L$. If $\underline{p}^H < \theta^L q^H$, firm L would never like to charge prices lower than $\underline{p}^H - \theta^L \Delta q > c^H - \theta^L \Delta q$. Since at a price $\underline{p}^H - \theta^L \Delta q$ firm L would at least be serving the low types, its profits must be strictly greater than its minmax $\lambda\varphi^L$.

upper bound. Otherwise, firm H would make zero profits at its upper bound (as all consumers would strictly prefer to buy from firm L), but this cannot be the case since its minmax is strictly positive. Last, the two firms cannot put positive mass at their upper bounds as firm L would be better off putting all its mass slightly below its upper bound (so as to attract all consumers whenever the rival plays the mass at the upper bound). It thus follows that when firm L plays its upper bound, the rival is pricing below its upper bound with probability one. Hence, at the upper bound firm L only serves the low types, thus making profits that exactly equal its minmax, $\lambda \pi^L$.

We are now ready to show that (L, H) is a SPE. Starting at (L, H), firm H does not want to carry good L as at (L, LH) its profits are equal to the minmax, while they are strictly above that level at (L, H). Similarly, firm L does not want to carry good H as at (LH, H) its profits are equal to the minmax, while at (L, H) its profits are (weakly) greater than its minmax.

Last, we characterize the MSE. Suppose that the rival chooses L with probability α and H with probability $(1-\alpha)$. Equating the profits from choosing L and H,

$$\alpha \Pi (L, L) + (1-\alpha) \Pi (L, H) = \alpha \Pi (H, L) + (1-\alpha) \Pi (H, H)$$

Since $\Pi (H, H) = \Pi (L, L) = 0$, solving for α,

$$\alpha = \frac{\Pi (L, H)}{\Pi (L, H) + \Pi (H, L)}.$$

Thus implying that equilibrium profits at the MSE are

$$\frac{\Pi (L, H) \Pi (H, L)}{\Pi (L, H) + \Pi (H, L)}.$$

This equilibrium constitutes a SPE if and only if it is not dominated to choosing LH, i.e.,

$$\Pi (L, H) [\Pi (L, L) - \Pi (LH, L)] + \Pi (H, L) [\Pi (L, H) - \Pi (LH, H)] \geq 0$$

The first term is negative, while the sign of the second term depends on c^H : (i) if $c^H \geq \theta^L q^H$, it is negative, implying that LH dominates the candidate MSE, which therefore does not exist; on the contrary, (ii) if $c^H < \theta^L q^H$, the second term is positive, implying that a MSE cannot be ruled out, particularly so for low c^H, which is when the second term is higher (note that as $c^H \to \theta^L q^H$ the second term is close to zero, so the MSE is ruled out for some $c^H < \theta^L q^H$). Therefore, for those parameter values for which the above inequality holds, a MSE exists. *Q.E.D.*

Proof of Proposition 2 [SPE under $\mu \to 1$] The results on existence and uniqueness of the "overlapping" equilibrium (LH, LH) under the assumption $\mu \to 1$ are a particular case of the proof of Proposition 7. The proof of non-existence of the "specialization" equilibrium (L, H) for $\mu \to 1$, for the case $c^H \geq \theta^L q^H$ is also contained in the proof of Proposition 7. Hence, it only remains to prove that $c^H < \theta^L q^H$ implies the existence of the "specialization" equilibrium. The proof of Proposition 1 above shows that $\Pi(L, H)$ and $\Pi(H, L)$ are *strictly* above the minmax, while the proofs of Proposition 5 and 4 show that $\Pi(LH, H)$ and $\Pi(LH, L)$ are equal to their minmax as $\mu \to 1$. Deviating to $\Pi(L, L)$ or $\Pi(H, H)$ is also not profitable given that firms would make almost zero profits. It follows that no firm wants to deviate from (L, H). *Q.E.D.*

Proof of Lemma 1 Argue by contradiction and suppose that the firm chooses $\Delta p > \theta^H \Delta q$. Hence, all buyers buy product L, and the firm makes a profit margin equal to $\left(p^L - c^L\right)$. If the firm reduced p^H so that $\Delta p = \theta^H \Delta q$, it would earns more on any high type buyers given that $p^H - c^H = p^L + \theta^H \Delta q - c^H > p^L - c^L$, where the inequality follows from $(A2)$. Furthermore, the firm would also (weakly) increase the probability that informed consumers buy from it. A similar reasoning applies to rule out $\Delta p < \theta^L \Delta q$. *Q.E.D.*

Proof of Proposition 3 [pricing at subgame (LH, LH)] The non-existence of a pure strategy equilibrium follows from standard arguments. Firms cannot tie in prices as a slight reduction in the price would allow a firm to attract all the informed consumers. Firms cannot charge different prices either. If one firm charged a higher price for a product, it would only serve the uninformed consumers and would thus be better off by either undercutting the rival's price or by charging the (constrained) monopoly prices to maximize profits out of the uninformed consumers; in turn, if the high-priced firm acts as the (constrained) monopolist, the other firm would find it profitable to slightly price below that level, thus not making it profitable any more for the rival to charge the (constrained) monopoly prices. Thus, the equilibrium must be in mixed-strategies. Since firms are symmetric, we focus on characterizing the symmetric mixed strategy equilibria. Standard arguments imply that there are no holes in the support and that firms play no mass point at any price of the support, including the upper bound (see, for instance, Narasimhan, 1998). (i) At the upper bound, firms serve the uninformed consumers only. Since profits are increasing in prices subject to $\left(IC^H\right)$, the optimal prices at the upper bounds are $\overline{p}^H = \theta^H q^H - q^L \Delta\theta$ and $\overline{p}^L = \theta^L q^L$, so that $\Delta\overline{p} = \theta^H \Delta q$.

We now demonstrate (ii), i.e., that at the lower bound $\Delta\underline{p} < \theta^H \Delta q$. Suppose otherwise that the price gap $p^H - p^L$ is constant and equal to $\theta^H \Delta q$ at and in the neighborhood of the lower bound (or throughout the entire price support for that matter). When a firm plays $(\underline{p}^H, \underline{p}^L)$ it obtains

$$\Pi(LH, LH; \underline{p}^H, \underline{p}^L) = \left(\mu + \frac{1-\mu}{2}\right)\lambda(\underline{p}^L - c^L) + \left(\mu + \frac{1-\mu}{2}\right)(1-\lambda)(\underline{p}^H - c^H).$$

Using

$$\begin{aligned} \Pi(\cdot; \underline{p}^H, \underline{p}^L) = \bar{\pi} &\equiv (1-\mu)\left[\lambda\pi^L + (1-\lambda)\left(\pi^H - \Delta\theta q^L\right)\right]/2 \\ &= (1-\mu)\left[\pi^L + (1-\lambda)\left(\theta^H\Delta q - \Delta c\right)\right]/2, \end{aligned}$$

the payoff at the upper bound, and the assumption that $\underline{p}^H - \underline{p}^L = \theta^H \Delta q$ we obtain

$$\underline{p}^H - c^H = \frac{1-\mu}{1+\mu}(\bar{p}^H - c^H) + \lambda\frac{2\mu}{1+\mu}\varphi^H \tag{6}$$

and

$$\underline{p}^L - c^L = \frac{1-\mu}{1+\mu}(\bar{p}^L - c^L) - (1-\lambda)\frac{2\mu}{1+\mu}\varphi^H. \tag{7}$$

We now compute the marginal distribution function $F^i(p^i)$ (since Δp is fixed in the neighborhood of the lower bound there is just one distribution to consider, say $F(p^i)$). First, notice that if one firm plays something in the support, the other firm never wants to deviate and serve just the high type with a price $\theta^H q^H$, because according to $(A1)$ the payoff of doing so would be strictly lower. Thus, to obtain the cdf $F(p^H)$ around the lower bound, notice that playing any pair p^H and $p^L = p^H - \theta^H \Delta q$ around the lower bound yields an expected payoff of

$$\begin{aligned} \Pi(\cdot; p^H, p^L) = \; & (1-\lambda)(p^H - c^H)\left[\frac{1-\mu}{2} + \mu(1 - F(p^H))\right] + \\ & \lambda(p^L - c^L)\left[\frac{1-\mu}{2} + \mu(1 - F(p^H))\right] \end{aligned}$$

where $1 - F(p^H) = 1 - F(p^L = p^H - \theta^H \Delta q)$ is the probability to attract both high and low type informed consumers. Rearranging terms and using $\Pi(p^H, p^L = p^H - \theta^H \Delta q) = \bar{\pi}$ leads to

$$\frac{1-\mu}{2}(\bar{p}^H - c^H) = \left[1 - F(p^H)\right]\left[\mu(p^H - c^H) - \lambda\mu\varphi^H\right] \tag{8}$$

and

$$\frac{1-\mu}{2}\left(\bar{p}^L - c^L\right) = [1 - F(p^L)[\mu(p^L - c^L) + (1-\lambda)\mu\varphi^H]. \tag{9}$$

Evaluating $F(\underline{p}^H) = F(\underline{p}^L) = 0$ in (8) and (9) yields

$$\underline{p}^H - c^H = \frac{1-\mu}{2\mu}(\bar{p}^H - c^H) + \lambda\varphi^H$$

and

$$\underline{p}^L - c^L = \frac{1-\mu}{2\mu}(\bar{p}^L - c^L) - (1-\lambda)\varphi^H$$

which, since $\mu < 1$, are greater than (6) and (7), respectively; a contradiction.

Proof for the third item in the Proposition follows directly from Lemma 1. *Q.E.D.*

Proof of Proposition 7 [quality choices] Each firm has four potential choices: $\{\varnothing, L, H, LH\}$. On the one hand, to prove that (LH, LH) is a SPE of the game for all $\mu < 1$, just note that all equilibrium payoffs $\Pi(LH, LH)$, $\Pi(H, LH)$ and $\Pi(L, LH)$ are proportional to $(1-\mu)/2$ so that $(A1)$ allows to conclude that $\Pi(LH, LH)$ is the greatest among these, just as in the monopoly case.

On the other hand, to find the conditions under which (L, H) is an equilibrium, we need to assess firm L's deviation gains when also carrying good H (it is easy to check that this is the critical deviation; for instance, let $\mu \to 1$ and use $(A2)$ to note that firm L's deviation gains are greater than firm H's, i.e., $\Pi(LH, H) - \Pi(L, H) > \Pi(LH, L) - \Pi(H, L)$). Firm L's deviation gain is equal to

$$\Pi(L, H) - \Pi(LH, H) = \frac{1+\mu}{2}\lambda\left(\pi^L - \varphi^L\right) - \frac{1-\mu}{2}(1-\lambda)\left(\pi^H - \pi^L\right) \tag{10}$$

(See the proofs of Propositions 5 and 6 in the online appendix for the relevant payoffs). Solving for μ, the above profit difference is positive iff $\mu \geq \mu^*$, where

$$\mu^* = \frac{(1-\lambda)\left(\pi^H - \pi^L\right) - \lambda\left(\pi^L - \varphi^L\right)}{(1-\lambda)\left(\pi^H - \pi^L\right) + \lambda\left(\pi^L - \varphi^L\right)}.$$

Note that when $c^H \geq \theta^L q^H$, $\pi^L = \varphi^L$ and $\mu^* = 1$, making $c^H \geq \theta^L q^H$ a sufficient condition for the uniqueness of (LH, LH). Furthermore, taking the derivative of μ^* with respect to c^H shows that

$$\frac{\partial \mu^*}{\partial c^H} = -2\lambda(1-\lambda)\frac{\left(\pi^H - \pi^L\right) - \left(\pi^L - \varphi^L\right)}{\left((1-\lambda)\left(\pi^H - \pi^L\right) + \lambda\left(\pi^L - \varphi^L\right)\right)^2}.$$

So that

$$\begin{aligned} sign\left\{\frac{\partial \mu^*}{\partial c^H}\right\} &= -sign\left\{\left(\pi^H - \pi^L\right) - \left(\pi^L - \varphi^L\right)\right\} \\ &= -sign\left\{c^L - \theta^L q^H\right\} > 0. \end{aligned}$$

Last, there might also exist a symmetric MSE such that firms choose L and H randomly, just as we showed in the proof of Proposition 2. This equilibrium constitutes a SPE if and only if it is not dominated to choosing LH, i.e.,

$$\Pi(L, H)\left[\Pi(L, L) - \Pi(LH, L)\right] + \Pi(H, L)\left[\Pi(L, H) - \Pi(LH, H)\right] \geq 0$$

or equivalently, iff

$$\Pi(L, H) - \Pi(LH, H) \geq \frac{\Pi(L, H)}{\Pi(H, L)}\left[\Pi(LH, L) - \Pi(L, L)\right] > 0.$$

Hence, whereas the existence of the asymmetric PSE (L, H) requires the profit difference (10) to be positive, the existence of the MSE requires such a difference to be greater than a strictly positive number. If we denote with μ^{**} the critical value for the existence of the MSE, we must then have $\mu^{**} \geq \mu^{*}$. It thus follows that for $\mu < \mu^{*}$, the unique equilibrium (either pure or mixed) is (LH, LH). *Q.E.D.*

Proof of Lemma 2 [prices and consumers surplus at the SPE] It is straightforward to see that, conditional on firms playing (LH, LH), expected prices are decreasing in μ. Since there is full discrimination, total consumption of each good remains fixed so that total surplus is given by $\lambda\pi^L + (1-\lambda)\pi^H$, irrespectively of μ. Since profits in equation (3) decrease in μ, consumer surplus must increase in μ. In turn, this implies that expected prices must be decreasing in μ. For given parameter values, competition is stronger at subgame (LH, LH) than at (L, H). Hence, expected prices at the former must be lower and consumer surplus must be higher. Thus, as μ goes down, expected prices (consumer surplus) at (LH, LH) decrease continuously until they jump up when firms start playing (L, H), either at $\mu \to 1$ or at $\mu \to \mu^{*}$ depending on equilibrium selection. Similarly, as μ goes down, consumer surplus at (LH, LH) increases continuously until it jumps down when firms start playing (L, H), either at $\mu \to 1$ or at $\mu \to \mu^{*}$ depending on equilibrium selection. *Q.E.D.*

Appendix B: Additional Results

Mixed strategy pricing equilibrium at (LH, LH) Consider pricing at the subgame (LH, LH). As we argued in Section 3, there are potentially multiple mixed strategy, outcome-equivalent, equilibria. Because the incentive compatibility constraint of the high types is binding at the monopoly solution, a natural equilibrium to consider is one in which firms keep on pricing the low quality product as if they were just selling that product, but adjust their pricing for the high quality one. The following Lemma characterizes such an equilibrium:

Lemma 3 *Given product choices (LH, LH), there exists a mixed-strategy equilibrium in which firms choose p^L in $\left[\underline{p}^L, \overline{p}^L\right]$ according to the (conditional and marginal) distribution function*

$$F^L(p^L) = \frac{1+\mu}{2\mu} - \frac{1-\mu}{2\mu}\frac{(\overline{p}^L - c^L)}{(p^L - c^L)}$$

and such that, for given p^L, the price p^H is chosen in $\left[\underline{p}^H, \overline{p}^H\right]$ to satisfy

$$\frac{p^H - c^H}{p^L - c^L} = \frac{\overline{p}^H - c^H}{\overline{p}^L - c^L} \tag{11}$$

where
$$\underline{p}^i = c^i + \frac{1-\mu}{1+\mu}(\overline{p}^i - c^i) > c^i,$$
and $\overline{p}^i$ *are the (constrained) monopoly prices, for* $i = L, H$.

Proof of Lemma 3: We want to show that the equilibrium in the statement of the lemma is indeed an equilibrium. First, firms could deviate by playing the price pairs in the support with different probabilities, while still choosing price pairs that satisfy incentive compatibility. However, this is unprofitable given that all price-pairs in the support give equal expected profits. Indeed, the equilibrium has been constructed so that

$$(p^L - c^L)\left[\frac{1-\mu}{2} + \mu(1 - F^L(p^L))\right] = \frac{1-\mu}{2}(\overline{p}^L - c^L) = \frac{1+\mu}{2}(\underline{p}^L - c^L)$$

and

$$(p^H - c^H)\left[\frac{1-\mu}{2} + \mu(1 - F^H(p^H))\right] = \frac{1-\mu}{2}(\overline{p}^H - c^H) = \frac{1+\mu}{2}(\underline{p}^H - c^H)$$

with the ratio (11) derived in order for the price pair $\left(p^H, p^L\right)$ to satisfy $F^H(p^H) = F^L(p^L)$, i.e., the choice of p^L results in a choice of p^H, so that the prices satisfying that ratio are played with equal probability. Therefore, expected profits at the proposed equilibrium are as in (3).

Second, firms could deviate by choosing p^L and p^H not satisfying equation (11) while still satisfying incentive compatibility. Again, these deviations are not profitable since all the prices in the support give equal profits. Deviating to prices that do not satisfy incentive compatibility is unprofitable because of Lemma 1.

Last, firms could deviate by playing price pairs outside the support. Choosing any prices above $\left(\overline{p}^L, \overline{p}^H\right)$ as defined above is unprofitable, as at these prices the firm is only selling to the uninformed consumers and $\left(\overline{p}^L, \overline{p}^H\right)$ are the optimal monopoly prices. Choosing any prices below $\left(\underline{p}^L, \underline{p}^H\right)$ as defined above is unprofitable, as at these prices the firm is inelastically selling to all consumers with probability one and would thus gain by rasing the price up to $\left(\underline{p}^L, \underline{p}^H\right)$. *Q.E.D.*

The proposed equilibrium has several appealing features. While firms price the low quality product as if they were just selling that product (as in Varian's model), on average they choose lower prices for the high quality product than when they only sell that product. This is a direct implication of the fact that the firm cannot extract all the surplus of the uninformed high types. Indeed, the resulting distribution for p^H,

$$F^H(p^H) = \frac{1+\mu}{2\mu} - \frac{1-\mu}{2\mu}\frac{(\overline{p}^H - c^L)}{(p^H - c^L)}$$

has the same functional form as in Varian. However, since the upper bound $\overline{p}^H$ is the constrained monopoly price, the whole distribution puts higher weight on lower prices all along the support than in the independent products case.

Under this equilibrium, the choice of p^L results in a unique choice of p^H such that the relative profit margin of the two products remains constant along the whole support; see equation (11).[49] In particular, the relative markups of the two products are the same as under monopoly. That is, under this equilibrium, competition affects the *price levels* but not the *price structure* within the firm.[50] The reason why the *relative profitability of two products is kept constant* simply derives from choosing p^H so as to make the incentive compatibility constraint binding, for every p^L.

The price difference that is embodied in this price structure can be expressed as

$$\Delta p = \kappa \theta^H \Delta q + (1 - \kappa)\, \Delta c.$$

Consistently with Lemma 1, the price difference is a weighted average between $\theta^H \Delta q$ (i.e., the price difference at the monopoly solution) and Δc (i.e., the price difference at the competitive solution), where the weight $\kappa = \left(p^L - c^L\right) / \left(\overline{p}^L - c^L\right)$ represents the distance to the upper bound. At the upper bound, when the incentive compatibility constraint of the high types is binding, the price difference is maximal, $\Delta\overline{p} = \theta^H \Delta q$. As we move down the support, the incentive compatibility constraint is satisfied with slack and the price difference narrows down. The difference is minimal at the lower bound, when $\kappa = (1 - \mu) / (1 + \mu)$. Importantly, as μ approaches one, the prices at the lower bound converge to marginal costs, and the price gap approaches Δc. The equilibrium would thus collapse to the competitive solution. On the other extreme, as μ approaches zero, the prices at the lower bound converge to monopoly prices so that the price gap approaches $\theta^H \Delta q$. The equilibrium would thus collapse to the monopoly solution.

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[49] Clearly, there exists another equilibrium with the same price supports and the same price distribution for good L but in which the firm randomizes the price of good H, given the choice of p^L, such that the two prices remain incentive compatible. Again, this multiplicity is inconsequential for the purposes of this analysis as all equilibria yield equal expected profits.

[50] Note that in this equilibrium, the prices of the two products within a firm are positively correlated. This is in contrast to what the literature on multi-product loss-leading concludes. However, as discussed in the introduction, that literature applies to setups in which goods are complements and consumers buy more than one- in contrast to the assumptions made in this paper.

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