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WHO GETS THE URBAN SURPLUS?

Abstract

High productivity in cities creates an economic surplus relative to other areas. How is this divided between workers and land-owners? Simple models with homogenous labour suggest that it accrues largely – or entirely – in the form of land-rents. This paper shows that heterogeneity of labour in two main dimensions (productivity differentials and housing demands) radically changes this result. Even a modest amount of heterogeneity can drive the land share of surplus down to 2/3rds or lower, as high productivity and/or low housing demand individuals receive large utility gains. In a system of cities the sorting of workers across cities mean that the land-rent share of surplus is lowest in the largest and most productive cities.

JEL Classification: R1, R10, R2

Keywords: Land rent, cities, productivity, wages, sorting

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1. Introduction

Who benefits from the high productivity levels of workers in cities? A standard answer is that the surplus goes principally to land owners. The limiting case is that there is an infinitely elastic supply of homogenous workers so – with this assumption fixing their reservation utility levels and with no other claimants on the surplus – land owners must take 100%.

This note is an exploration of what happens when workers are heterogeneous, focusing on two salient dimensions of heterogeneity. One is the productivity differential the worker receives by locating in the city; the other is the amount of housing the worker consumes. The idea is simple. Workers with a large productivity differential and small housing demand will do better than those with the opposite characteristics, and will therefore capture some of the urban surplus. Fundamentally, the presence of sufficient heterogeneity means that the assumption of an infinitely elastic supply of workers becomes untenable. If there were infinitely elastic supply the city would be occupied entirely by the type of worker that gets the greatest return (bankers with a preference for small apartments). We know that – while sorting takes place – this is not the case.

The questions then are: moving away from homogeneity how much of the surplus still goes to landlords? Is the change in their share large or small, and what does it depend on? We find that the effect is large. Modest amounts of heterogeneity appear to reduce the share received by landlords to between two-thirds and one-half. The relationship is convex, so that even a small amount of heterogeneity brings a relatively large reduction in rent share. Heterogeneity in either the productivity differential or housing demand reduces the rent share, and the effect is largest when there is a negative correlation between the two attributes of individuals. Looking at systems of cities, the share of rent is lowest in those cities that offer relatively large productivity differentials; these are the largest cities, but also those in which sorting has the most extreme effect.

These answers are important for a number of issues. Analytically, they matter for our understanding of urban models. Empirically, much work rests on the assumed homogeneity of workers, for example recent work by Albouy (2016) deriving the value of cities from data on urban wages. Evidently, answers matter for income distribution. They also matter for public finance and the extent to which land taxation can capture the value created by urban agglomerations, as studied in 'Henry George' theorems.

Existing literature recognises the critical nature of the homogeneity assumption, as succinctly stated by Arnott and Stiglitz (1979) who state 'the conceptual basis of capitalization studies is sound only when marginal individuals are very similar to infra-marginal individuals...' (p496). Duranton and Puga (2015) discuss the issues that arise as heterogeneous individuals sort into different locations, and the technical complexities that follow. The recent literature on heterogeneity focuses on constructing large scale models that analyse both the sources of heterogeneity and their implication for city systems, as exemplified by Behrens et al. (2014) (see also the survey by Behrens and Robert-Nicoud 2015). The approach of this paper is, by contrast, to focus tightly on the question posed in the title, and to derive a quantitative sense of the answer to the question. We take spatial productivity differentials and housing demands as exogenous, drawing out their implications rather than modelling their causes.

The remainder of the paper is as follows. Section 2 looks at an economy containing a single city, and at individuals who decide whether or not to enter the city and where to live in the city. Section 2.1 presents some analytical results (solving the assignment problem for the simplest case), and section 2.2 presents the core quantitative results on the division of urban surplus between landowners and individuals, based on numerical analysis of the model. Section 3 moves to a multi-city variant of the model in which individuals sort between cities, and demonstrates how the share of surplus that is captured by rent varies across city types, and is lowest in the largest and most productive cities.

2. Model 1: A single city

We start by considering an economy with a single city and an 'outside' location. Each worker has a productivity differential between working in the city and outside, denoted q, and demands h units of housing. For each worker the attributes q and h are exogenous, as is the population density function $f(q,h)$ over $\{q, h\}$ space. Sources of these productivity gains have been extensively analysed elsewhere.[1] In equilibrium some workers will choose to live in the city, the remainder outside. Urban workers commute to jobs in the CBD, and a worker living at distance x faces commuting costs tx and pays price differential (relative to living outside the city) $p(x)$ for a unit of housing.[2] We initially assume one unit of housing per unit land, so $p(x)$ can be thought of as differential rent per unit land. An individual's utility from living in the city at distance x from the CBD, as compared to living outside the city, is therefore

$$u(q,h;x) = q - hp(x) - tx\,. \qquad (1)$$

Later in the paper we allow for housing construction within the city (i.e. an endogenous number of houses per unit land), and for price elastic individual demand for housing.[3]

Individuals choose to reside in the place that yields highest utility. Given the price function $p(x)$, an individual with characteristics $\{q, h\}$ makes choices:

[1] See Duranton and Puga (2004) for the definitive survey.

[2] We use the simplest form of the Alonso-Mills-Muth open city model, see Duranton & Puga (2014).

[3] Throughout, we maintain the assumption that real incomes of all individuals remaining outside the city are constant, unaffected by city size.

live in city at $x^*(q,h) = \arg\max_x u(q,h;x)$ if $u(q,h;x^*(q,h)) \geq 0$, (2)

live outside city if $u(q,h;x^*(q,h)) < 0$.

We define $\pi(q,h;x)$ as the indicator function, equal to unity if type $\{q, h\}$ lives in the city at distance x and equal to zero otherwise, so the number of type $\{q, h\}$ people choosing to live at x is

$$N(q,h;x) = \pi(q,h;x) f(q,h). \tag{3}$$

The number of housing units at distance x is denoted $s(x)$, and market clearing for housing at each distance is

$$\int_h h \int_q N(q,h;x) dq dh = s(x). \tag{4}$$

The left hand side is demand for housing at x, and the right hand side is housing supply. We initially assume a linear city in which supply at each distance is exogenous and constant, $s(x) = s$. The price of housing, $p(x)$, adjusts to clear the market, and the city edge (denoted $\tilde{x}$) is where this price equals zero,

$$p(\tilde{x}) = 0. \tag{5}$$

Equilibrium is characterised by these five equations giving utility, location choice, numbers of individuals of each type at each place, the house price function $p(x)$ and the city edge.[4] From this we derive the variables we are interested in.

Total differential land rent: $R = \int_0^{\tilde{x}} s(x) p(x) dx$.

The total number of people living at x: $N(x) = \int_q \int_h N(q,h,x) dh dq$.

Total commuting costs: $C = \int_0^{\tilde{x}} N(x) tx dx$.

The total productivity differential (summing over all workers):

$$PS = \int_0^{\tilde{x}} \int_q q \int_h N(q,h;x) dh dq dx.$$

The total utility differential accruing to urban workers:

$$U = \int_0^{\tilde{x}} \int_q \int_h u(q,h;x) N(q,h;x) dh dq dx.$$

The total urban surplus we define as $R + U$, and our primary question is, how is this surplus divided between the two components? Much of the remainder of the paper is a numerical investigation of this, based on the model above and various extensions of it. The following subsection presents some analytical results for the simplest case.

[4] We will often refer to differential utility, house prices, rent and productivity simply as utility, house prices, rent, and productivity.

2.1 Analysis

Homogenous population: The benchmark is when workers are homogenous and the common values of their productivity differentials and housing demands are $\bar{q}$, $\bar{h}$. Utility is $u(\bar{q},\bar{h};x)=\bar{q}-\bar{h}p(x)-tx=0$, giving house price schedule $p(x)=(\bar{q}-tx)/\bar{h}$, and city edge $\tilde{x}=\bar{q}/t$. Values of other variables are

$$R=s\bar{q}^2/2\bar{h}t,\quad N=\tilde{x}s/\bar{h}=s\bar{q}/\bar{h}t,\quad C=s\bar{q}^2/2\bar{h}t,\quad PS=N\bar{q}=s\bar{q}^2/\bar{h}t,\quad U=0. \quad (6)$$

The entire urban surplus goes in rent, $R/(R+U)=1$.[5]

Heterogeneous productivity differential: If individuals all have the same demand for housing $\bar{h}$, then those who enter the city are indifferent as to where they live. Heterogeneous productivity differentials means that individuals with productivity greater than or equal to some (endogenous) cut-off value $\tilde{q}$ will enter the city, and the marginal worker has $u(\tilde{q},\bar{h};x)=\tilde{q}-\bar{h}p(x)-tx=0$ for all $x\geq\tilde{x}$. This sets the house price schedule as $p(x)=(\tilde{q}-tx)/\bar{h}$, with boundary value $p(\tilde{x})=0$, so $\tilde{x}=\tilde{q}/t$. The marginal worker is such that the city is filled when occupied by all individuals with $q\geq\tilde{q}$, i.e. $\tilde{q}$ satisfies

$$\bar{h}\int_{\tilde{q}}^{\infty}f(q,\bar{h})dq=s\tilde{x}. \quad (7)$$

For present purposes, the point is simply that the level of rent is set by the productivity differential of the marginal worker, $\tilde{q}$, and intra-marginal workers each capture surplus $q-\tilde{q}>0$. The total size of this, relative to city rents, is greater the larger is the dispersion of *q*, as will be shown in section 2.2.

Heterogeneous housing demand: The converse case is where individuals have different housing demands *h* but the same productivity differentials, $q=\bar{q}=1$. They choose different locations within the city, with lower *h* individuals choosing higher rent (closer to the CBD) places. The choice is given by first order condition for maximisation of eqn. (1), $\partial p(x^*(\bar{q},h))/\partial x=-t/h$. Characterisation of the equilibrium involves constructing the price function $p(x)$ that satisfies this equation across a population of individuals with different values of *h*. To do this, note that the city out to distance *x* will be occupied by individuals with housing demand less than or equal to some value, $h(x)$, defined by $\int_0^{h(x)}hf(\bar{q},h)dh=sx$. This gives a relationship between *h* and *x* which can be used to turn the first order condition into a simple differential equation in *x*. This, together with the boundary condition that at the

[5] Notice also that $U+R=PS$, a property that holds if housing supply *s* and individuals' housing demand *h* are perfectly price inelastic. It does not hold once these assumptions are relaxed and producer and consumer surplus triangles come into play.

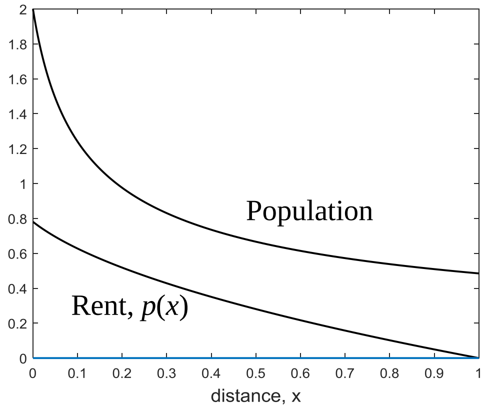
city edge (where $p(\tilde{x}) = 0$) marginal workers have zero utility differential, $u(\bar{q}, h(\tilde{x}); x^*(\bar{q}, h(\tilde{x}))) = 0$, defines the price function. If h has uniform distribution with lower support h_0 and density f_h then the price function takes the form

$$p(x) = (\bar{q} - tx)2 / \left\{ \left[2s\bar{q} / tf_h + h_0^2\right]^{1/2} + \left[2sx / f_h + h_0^2\right]^{1/2} \right\}. \tag{8}$$

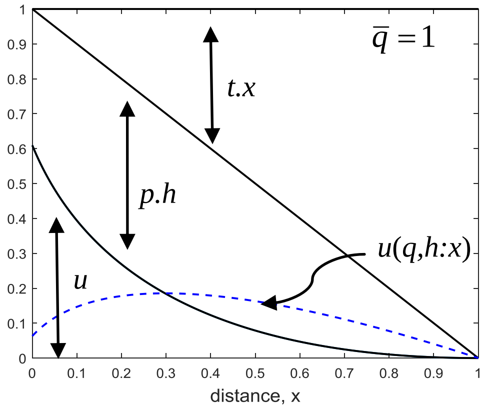
(see appendix). This decreases with distance, is convex, and takes value zero at the city edge. However, rent does not capture the entire city surplus, except in the limiting case where the distribution collapses to a point. The division of surplus is illustrated in Fig. 1 (parameters given in appendix). Panel (a) gives the rent (house price) function and population density as a function of distance *x* from the CBD; in this example equilibrium city size is unity and population density varies by a factor of 4:1, reflecting heterogeneity of *h*. Panel (b) gives the distribution of the surplus. All workers have the same productivity differential, $\bar{q} = 1$, but its division between commuting costs, housing expenditure and utility depends on their housing demand and chosen location. Working down from the top of Fig 1(b), part goes to commuting costs, *t.x*, part goes to rent, *p.h*, and the remainder is differential utility. Low *h* individuals live near the CBD and pay low commuting costs and high rent, but since they consume little housing they receive a net utility benefit. Integrating across the population of the city gives total differential rent *R* and utility *U* (areas *p.h* and *u*, each weighted by population). In the example of fig. 1 two-thirds of the urban surplus is captured by land rent, $R/(R + U) = 0.66$.[6]

Figure 1: Heterogeneous housing demand

(a) Rent and population by distance



(b) Commuting cost, housing expenditure, utility, per person by distance



[6] The dashed line on Fig. 1b gives the utility of a worker of given type as a function of *x*, its maximum indicating the worker's location choice. The locus of such maxima is the curve *u*.

2.2 Numerical exploration:

We now provide insight into the relative magnitudes of urban surplus captured by landowners and by workers. We look at different joint distributions of worker attributes, $f(q,h)$, and extend the model to include elastic individual housing demand and house supply. Answers are derived numerically, centred around a base case in which mean values of the productivity differential and housing demand are set unity, $\bar{q}=1$, $\bar{h}=1$, total population (urban and non-urban) at 100, and $t = 1$, $s = 50$. It follows that if workers are homogenous then the city edge is $\tilde{x}=\bar{q}/t=1$ and urban population is $N=\tilde{x}s/\bar{h}=50$, i.e. ½ of the total population occupy the city. With homogeneity all workers are indifferent between being in or out of the city, so supply at the margin is perfectly elastic. Values of other variables come from (6) and are: R =25, C = 25, PS = 50, $PS – C$ = 25, U = 0. As is standard, the entire city surplus goes to landowners.

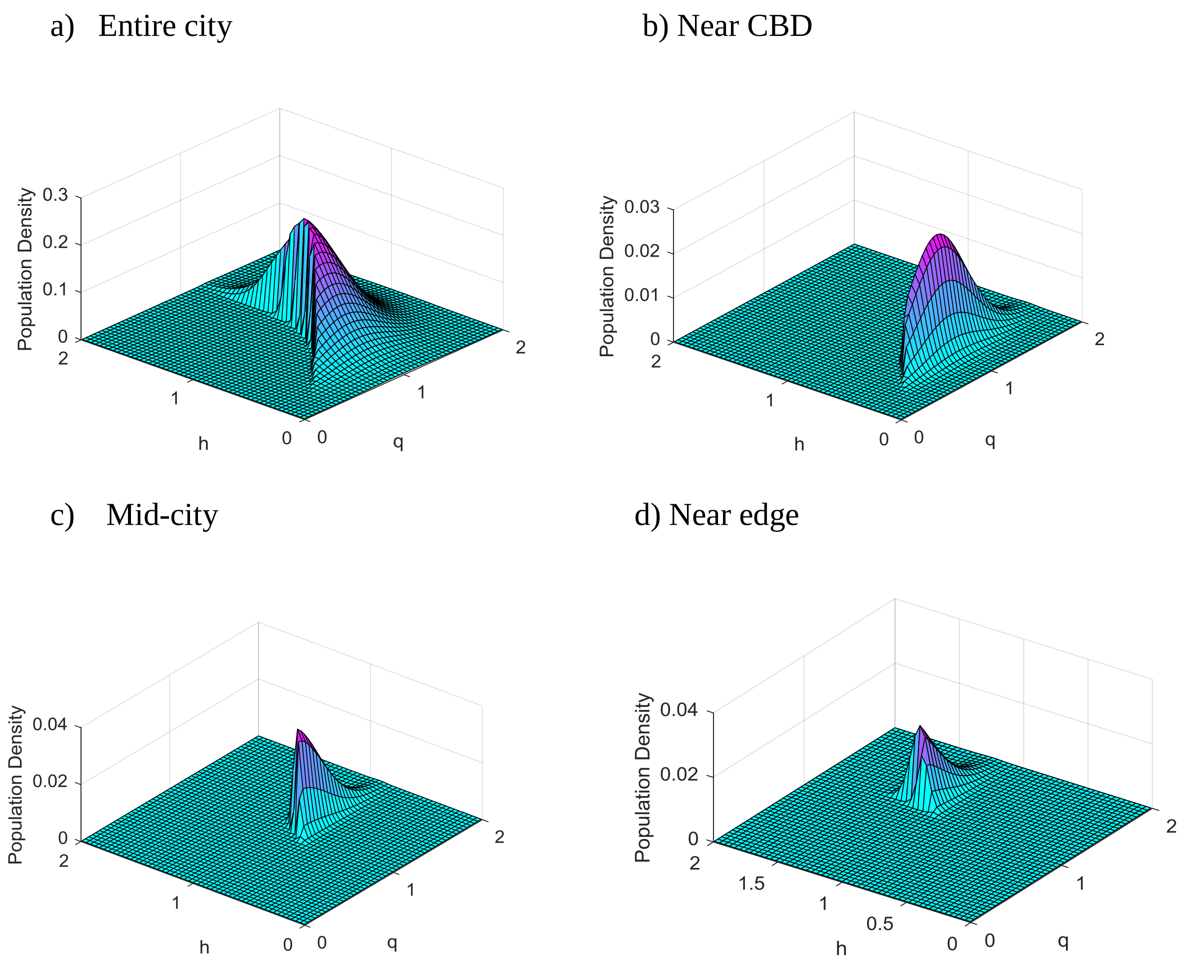
We start exploring heterogeneity assuming that the distribution of characteristics $f(q,h)$ is bivariate normal with $\bar{q}=1$, $\bar{h}=1$ and variances and covariance $\sigma_q^2, \sigma_h^2, \sigma_{qh}^2$. For computational reasons we smooth location choice using the logit function so the probability of individual of type $\{q, h\}$ living at x is

$$\pi(q,h;x)=\exp[\mu u(q,h:x)]\Big/\left\{\exp[\mu u_0]+\int_0^{\tilde{x}}\exp[\mu u(q,h:x)]dx\right\}. \qquad (9)$$

The logit parameter μ is set at a high value, so the function focuses probability quite tightly on the location that gives the highest utility, as will be clear from the figures that follow. This and other details of computation are given in the appendix.

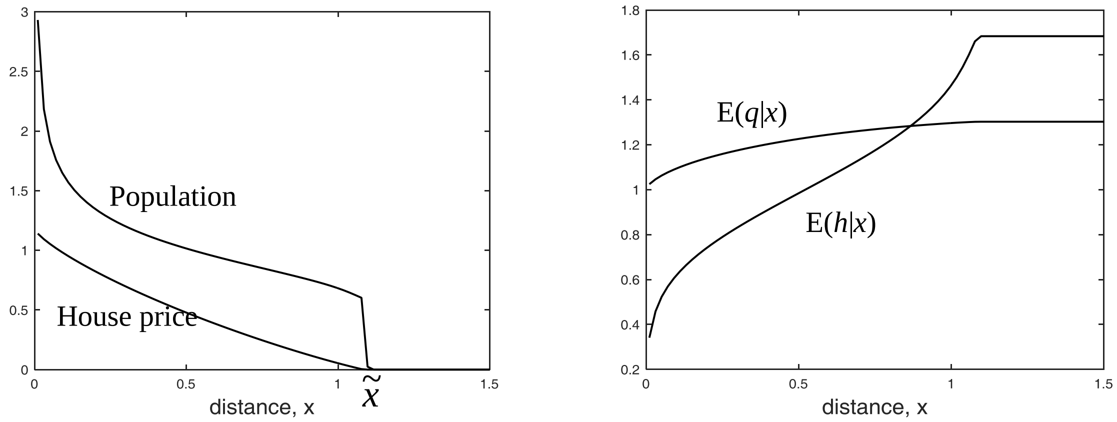
Individuals sort into different locations, and visualising this is assisted by Fig. 2, constructed with $\sigma_q=\sigma_h=0.1, \sigma_{qh}=0$. The horizontal plane is $\{q, h\}$ space, the vertical axis gives the density of individuals and the volume under the surface is number of people. Panel (a) of the figure gives the city population, and is the part of the normal surface corresponding to individuals who gain utility from being in the city, i.e. those with relatively low h and/or high q. Other panels give population at different points in the city. Panel (b) is population close to the CBD. As would be expected, given the urban rent gradient these are people with very low h, although covering a wide range of productivity differentials. Panel (c) is mid-city and (d) is near the edge, these areas picking up individuals with greater demand for housing. Panels (b) – (d) each appear like a fin across the surface. With an infinitely fine grid and exact location choice they would be lines giving the combinations $\{q, h\}$ for which a particular x maximises, $u(q,h;x)$, given equilibrium house prices.

Figure 2: Population mix at different locations



The intra-city distribution of population is illustrated further in Fig. 3. Panel (a) gives city population $N(x)$ and house prices $p(x)$ by distance from the CBD. Rent is higher near the CBD, as expected, and so too is population density since low h individuals choose to live there. Panel (b) gives average housing demand per person and productivity differential at each distance (i.e. averaging over the characteristics of individuals living at each x). Sorting means that housing demand increases with distance, and so too does average productivity differential; even at the lower rents near the city edge high h individuals only choose to live in the city if they are also high q.

Figure 3: (a) Rent and population by distance. (b) Average *h* and *q* by distance.



What of our main question; who captures the urban surplus? Answers are given in Figs. 4(a) and 4(b), each of which has the rent share of city surplus, $R/(R + U)$, on the vertical axis, and a measure of heterogeneity on the horizontal axis. The lowest line on Fig. 3(a) has equal standard deviation of q and h with value given by the horizontal axis, $\sigma_q = \sigma_h = \sigma$; covariance is zero. The line illustrates that as σ goes to zero so the rent share goes to unity, as it must. Higher values of σ lead to a much lower rent share, and convexity of the relationship indicates that even a small amount of heterogeneity has a large impact on the rent share. The share drops below 50% if $\sigma > 0.17$. To interpret this, $\sigma > 0.17$ means that the proportion of total population with characteristics $\{q, h\}$ outside a circle with radius 0.5 around the mean of $\{1, 1\}$ is greater than 45%.[7] This does not seem an excessive amount of heterogeneity, yet it halves the rent share.

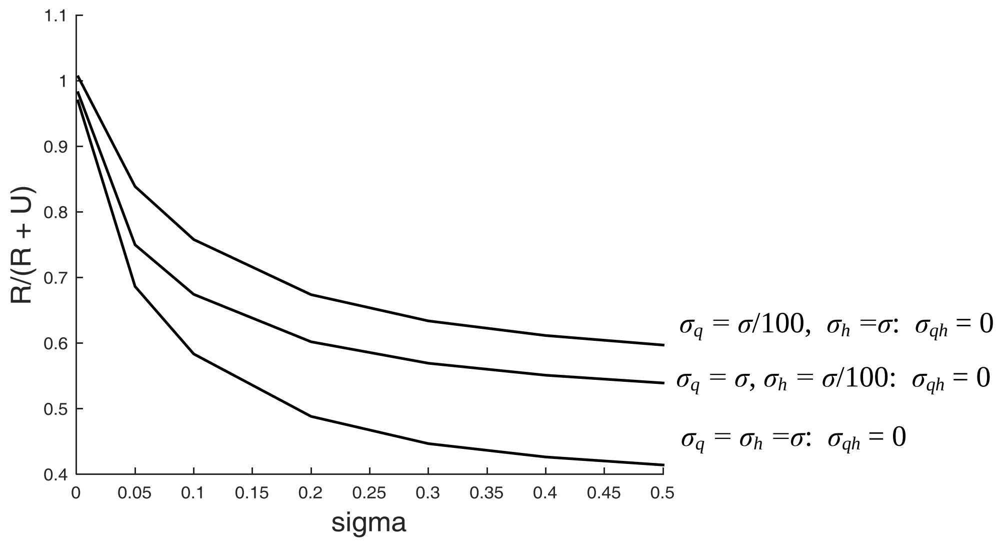
We use this as our reference case, and look at variations in other parameters in turn. The upper two lines in Fig. 4a switch off variance of q and h in turn (or more precisely, set each equal to $\sigma/100$). This approximately halves the effect of heterogeneity, the effect being similar whichever characteristic remains heterogeneous. The main message is that heterogeneity in a single dimension is sufficient to reduce the rent share, in line with the analysis of section 2.1.

Figure 4b looks at covariance. The central line reproduces the reference case of Fig. 4a (zero covariance) and the upper line gives positive covariance, $\sigma_{hq} = 0.95\sigma$. This raises the rent share, the intuition being that there is little variation in housing demand per unit productivity differential, and it is this relativity that is crucial. Conversely, negative covariance (bottom line) reduces the rent share still further. Intuitively, negative covariance means that

[7] From numerical investigation of the bivariate normal $\sigma_q = \sigma_h = 0.17$, $\sigma_{hq} = 0$.

individuals with the highest productivity differential also demand the least housing, and these are the individuals who capture the most surplus.

Figure 4: (a) Rent share of surplus: by variance



(b) Rent share of surplus: by variance and covariance

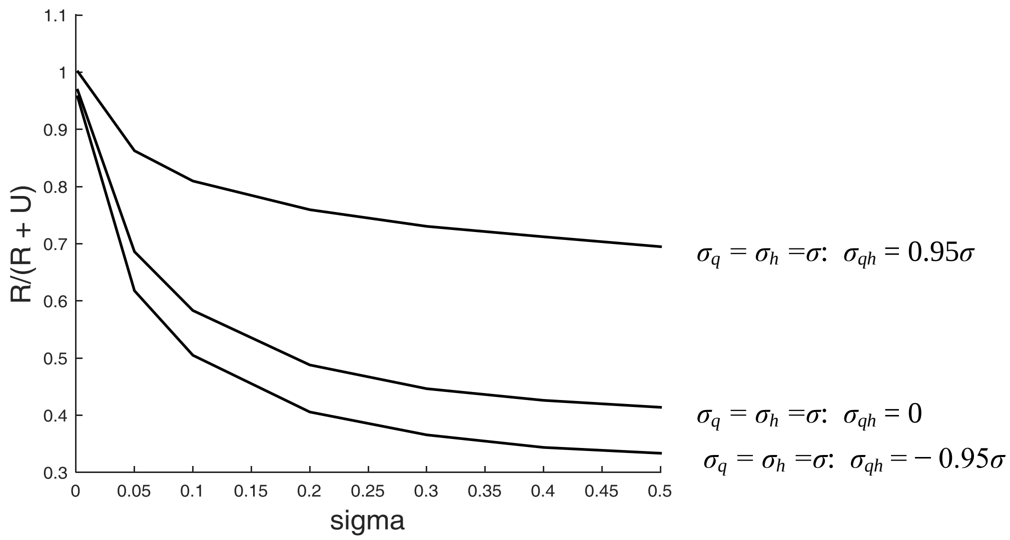


Table 1 looks at how outcomes vary with other parameters, also adding price elasticity to individuals' housing demand and to housing supply (while holding standard deviations and covariance the same in all cases, at 0.1 and zero respectively). The reference case is as in Figs. 4. Making the city circular reduces its population (since the radius is quite small) and has virtually no effect on the rent share of surplus. Log-normality makes very little difference. Changing the total population of the economy shifts the cut across the normal distribution in Fig. 2a, with larger total population rotating the cut clockwise around the origin. This has a modest impact on overall city size and raises the rent share, consistent with the idea that labour supply to the city is more elastic in a larger economy.

Remaining rows allow for positive elasticity of housing supply, η, and of individual's demand for housing, ε (see appendix for details). [8] As expected, positive elasticities increase city population and reduce the share of rent in surplus, this going to less than 40% in the most extreme case reported.

Table 1: Sensitivity: $\sigma_q = \sigma_h = 0.1$, $\sigma_{hq} = 0$.

	Rent/ city surplus	City population, (%)
Base: Linear city, normal distribution, total population = 100	0.583	58.8
Circular city	0.583	40.2
Lognormal distribution	0.589	57.3
Total population = 200	0.659	75.1 (37.5%)
Total population = 50	0.441	41.6 (83.2%)
Elasticity of demand for housing, ε = 0.5	0.529	66.3
Elasticity of supply for housing, η = 0.5	0.536	65.5
Both; $\varepsilon = \eta = 0.5$	0.476	73.0
Elasticity of demand for housing, ε = 1.5	0.375	83.0
ε =1.5, η = 0.5, population =200	0.390	149 (74%)
ε =1.5, η = 0.5, population =1000	0.432	447 (45%)

3. Model 2: a system of cities

Do these or similar results obtain with a system of cities? We investigate this in a restatement of the model used above. We now assume that cities have no internal geography and reinterpret x as an index over a set of cities, $x \in (0,1]$. The utility derived from living and working in city x (relative to being 'outside', i.e. not in any city) is

$$u(q,h;x) = qx - hp(x)\,. \tag{1'}$$

[8] Note that this is the elasticity of supply per unit land and elasticity of each individual's demand. Total supply of housing in the city is assumed endogenous throughout, since we use an open city model with endogenous city edge.

Comparing this with eqn. 1, there are no commuting costs but heterogeneity across cities is introduced by supposing that the productivity differential of a type q worker is qx, i.e. low x cities deliver little additional productivity, by construction. Many mechanisms could be posited to generate this cross-city heterogeneity but, in the spirit of this paper, we give it the simplest possible form and investigate its implications.

Equations (2) and (3) are as before, with $N(q,h;x) = \pi(q,h;x)\,f(q,h)$ now interpreted as the number of people of type $\{q, h\}$ living in city x. In equilibrium all cities will operate, since there is a positive (if small) productivity differential for all cities, $qx > 0$, relative to 'outside'.

We report results for the case in which there are 20 cities, $\sigma_q = \sigma_h = 0.1, \sigma_{qh} = 0$ and η = 0.33.[9] Fig. 5a illustrates the equilibrium distribution of population, analogous to the intra-city distribution of Fig. 3a. As expected, high index (so high productivity differential) cities are larger and have higher house prices. Fig. 5b illustrates that this is driven by high productivity differential individuals sorting very strongly to cities that value this – high x cities. The consequent high house prices in these cities induce further sorting according to individuals' housing demand.

What about the share of urban surplus going to rents? With $\sigma_q = \sigma_h = 0.1, \sigma_{qh} = 0$ the share across all cities is 59%, virtually identical to that reported for model 1 in Fig. 4 and Table 1. The share falls with variance, as before. More interesting, is to see how the share varies across the city size distribution, and this is illustrated in Fig.6. The dark bars give the rent share by city (for the top 16 of the 20 cities), indicating that the share of rent in total surplus is *smaller* for larger (high city index) cities.[10] The high productivity differential gives these cities an absolute advantage for all individuals, and this bids up house prices. Comparative advantage makes these cities attractive for high q and low h individuals, and these are precisely the individuals who capture a larger share of the surplus. The city with the highest index has rent share of just 39%.

A corollary of this is that if there are more cities – i.e. a finer slicing of x in the interval (0, 1] – then the composition of the highest productivity city becomes even more concentrated with high q and low h individuals, and the rent to surplus ratio falls further. If there are 50 cities, the one with the highest index has a rent share of just 33%. Fig. 6 illustrates the point by comparison of the dark barks with the light ones, these giving the rent share when there are just 5 cities. Thus, the light bars in the range x ε [0.85 – 1] all represent the city with the

[9] η = 0.33 implies that 25% of housing costs are construction, and 75% land rent, see appendix. In this example the total urban population is 60% of the entire population.

[10] Total land rent is higher in these cities, although it is a smaller share of total surplus.

highest index, so have the same rent share of surplus, this lying in the range of the corresponding dark (20 city) bars.

Figure 5: Heterogeneous cities:

(a) Rent and population by city index. **(b) Average *h* and *q* by city index**

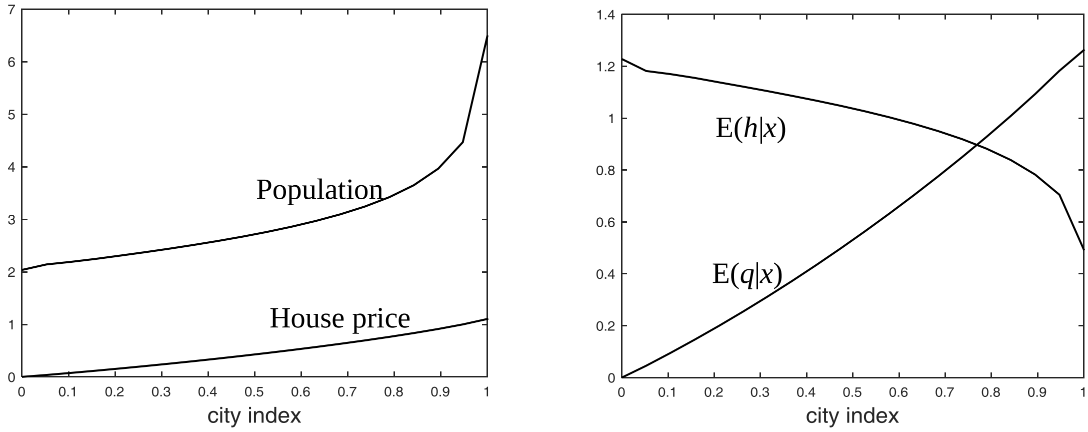
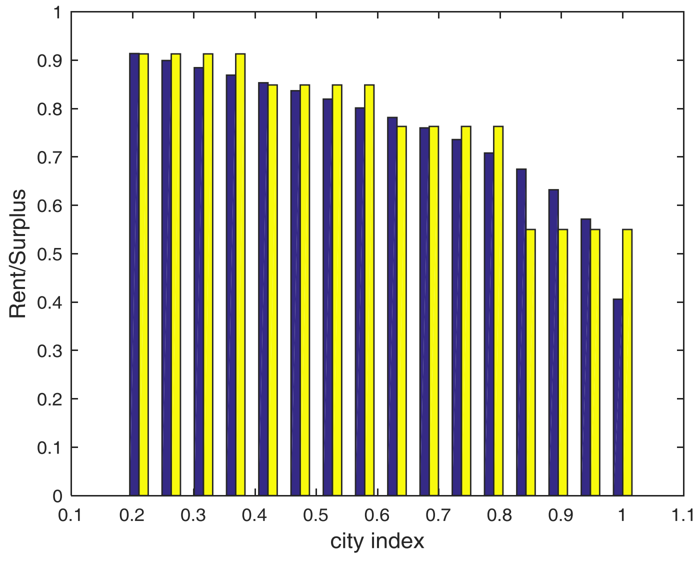


Figure 6: Rent share of surplus: by city index. $\sigma_q = \sigma_h = 0.1$, $\sigma_{hq} = 0$, $\eta = 0.33$.



4. Concluding comments

Homogenous labour is a useful simplifying assumption in city modelling, but we know that a key feature of cities is the distinctive skill mix of their population (e.g. Glaeser and Gottlieb 2009). This paper has argued that the assumption of homogenous labour is particularly misleading when it comes to thinking about the distribution of urban surplus between land and labour. With homogenous labour the share of land-rent in surplus takes the extreme boundary value, of unity. Heterogeneity can only move this in one direction, so the question is, by how much? This paper suggests that the answer is, a lot. Heterogeneous or productivity differentials or housing demands pull the share down, in a convex relationship so that even small amounts of heterogeneity matter. Combining both dimensions of heterogeneity suggests that, for modest degrees of population heterogeneity, rent might capture between one and two-thirds of total surplus, the rest accruing to individuals. The result extends from a single city context to multiple cities which differ in the productivity differential they offer to workers; in this case it is the largest and most productive cities that have the lowest share of rent in the urban surplus that they create. These theoretical findings matter for our thinking about income distribution and urban public finance. They also set the stage for future empirical work.

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Appendix:

The assignment problem with heterogeneous housing demand:

The first order condition for choice of location is $\partial p\left(x^*(\bar{q},h)\right)/\partial x = -t/h$. With uniform distribution of h eqn. (6) gives $\int_{h_0}^{h(x)} hf_h dh = sx$ implying that $h(x) = \left[2sx/f_h + h_0^2\right]^{1/2}$. The first order condition becomes $\partial p/\partial x = -t\left[2sx/f_h + h_0^2\right]^{-1/2}$. Integrating, this gives $p(x) = K - tf_h/s\left[2sx/f_h + h_0^2\right]^{1/2}$ where K is a constant of integration. At the city edge the land price and differential utility are both equal to zero, $p(\tilde{x}) = K - tf_h/s\left[2s\tilde{x}/f_h + h_0^2\right]^{1/2} = 0$ and $u(q,h;x) = \bar{q} - t\tilde{x} = 0$. Together these give the constant of integration $K = tf_h/s\left[2s\bar{q}/tf_h + h_0^2\right]^{1/2}$ and hence the house price function, eqn. (8),

$$p(x) = tf_h/s\left\{\left[2s\bar{q}/tf_h + h_0^2\right]^{1/2} - \left[2sx/f_h + h_0^2\right]^{1/2}\right\}.$$

This can be rearranged, denoting the terms in square brackets a, b, and noting that $a^{1/2} - b^{1/2} = (a-b)/\left(a^{1/2} + b^{1/2}\right)$, so $p(\tilde{x}) = 2(\bar{q} - tx)/\left(a^{1/2} + b^{1/2}\right)$. This gives

$$p(x) = (\bar{q} - tx)2/\left\{\left[2s\bar{q}/tf_h + h_0^2\right]^{1/2} + \left[2sx/f_h + h_0^2\right]^{1/2}\right\}.$$

Hence, if $h_0 = \bar{h}$ and $f_h \to \infty$ then $p(\tilde{x}) = \left(\bar{q} - tx\right)/h_0$.

Fig. 1 is constructed with $\bar{q} = 1$, t = 1, s = 1, $h_0 = 0.5$ and $f_h = 0.5$.

Numerical procedures:

Simulations are derived from Matlab, using a 90x90 grid of {q, h}, ranging from $q \in [0,2]$, $h \in [0.1,2]$. In section 2 there are 75 equal spaced distances that may be occupied by the city. In section 3 there are 20 cities. The logit parameter is μ = 250. All simulations used in Fig.2 and beyond use the general structure outlined below (price elastic housing demand and supply), which reduces to that of the text if elasticities ε and η are equal to zero.

Price elastic individual housing demand:

The utility an individual of type h derives from consuming H units of housing is $h^{1/\varepsilon} H^{(\varepsilon-1)/\varepsilon}\varepsilon/(\varepsilon - 1)$ so maximised utility at housing price p is represented by (indirect) utility function $u = q - hp^{1-\varepsilon}/(1-\varepsilon) - tx$, giving individual's housing demand $H = hp^{-\varepsilon}$.

Setting the price of outside housing at p_0 and normalising outside utility at zero, the more general form of equation (1) is then $u = q - h\left[p^{1-\varepsilon} - p_0^{1-\varepsilon}\right]/(1-\varepsilon) - tx$. The term in square brackets corresponds to the *differential* house price of the text. Analytical results are derived with ε equal to zero (hence $H = h$), and with $p_0 = 0$. Simulation generalises to allow ε > 0, and renormalizes using $p_0 = 1$. In section 3 utility is $u = qx - h\left[p^{1-\varepsilon} - p_0^{1-\varepsilon}\right]/(1-\varepsilon)$.

Price elastic housing supply:

The cost of building S units of housing at a particular location is $s^{-1/\eta}S^{(1+\eta)/\eta}\eta/(\eta+1)$, so the maximised profit function of a developer is $sp^{(1+\eta)}/(\eta+1)$ and housing supply is $S = sp^{\eta}$. Developer's gross revenue = $pS = sp^{1+\eta}$ and costs (given profit maximising supply) = $sp^{1+\eta}\eta/(\eta+1)$. Land rents are equal to maximised profits, and are therefore fraction $1/(\eta+1)$ of gross revenue (= household expenditure on housing). Thus, $\eta = 0.5$ implies that land rents are 2/3rds of housing expenditure, the rest being construction cost.