# DISCUSSION PAPER SERIES

DP11970

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**FINANCIAL ECONOMICS and INTERNATIONAL MACROECONOMICS AND FINANCE**

ISSN 0265-8003

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Discussion Paper DP11970
Published 19 April 2017
Submitted 19 April 2017

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# THE QUANTO THEORY OF EXCHANGE RATES


## Abstract

We present a new, theoretically motivated, forecasting variable for exchange rates that is based on the prices of quanto index contracts, and show via panel regressions that the quanto forecast variable is a statistically and economically significant predictor of currency appreciation. We also test the quanto variable's ability to forecast differential currency appreciation out of sample, and find that it outperforms predictions based on uncovered interest parity, on purchasing power parity, and on a random walk.



JEL Classification: G12, G15, F31, F37, F47

Keywords: exchange rate forecast, Exchange rate, currency, Forecasting, predictability, carry trade, quanto contracts



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Acknowledgements

# The Quanto Theory of Exchange Rates

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April, 2017

**Abstract**

We present a new, theoretically motivated, forecasting variable for exchange rates that is based on the prices of quanto index contracts, and show via panel regressions that the quanto forecast variable is a statistically and economically significant predictor of currency appreciation. We also test the quanto variable's ability to forecast differential currency appreciation out of sample, and find that it outperforms predictions based on uncovered interest parity, on purchasing power parity, and on a random walk.

*Both authors are at the London School of Economics. We thank the Systemic Risk Centre at the LSE for their support, and for providing access to data sourced from Markit under license. We are grateful to Christian Wagner, Tarek Hassan, John Campbell and Mike Chernov for their comments, and to Lerby Ergun for research assistance.

It is notoriously hard to forecast movements in exchange rates. A large part of the literature is organized around the principle of uncovered interest parity (UIP), which predicts that expected exchange rate movements offset interest-rate differentials and therefore equalise expected returns across currencies. Unfortunately many authors, starting from Hansen and Hodrick (1980) and Fama (1984), have shown that this prediction fails: returns have historically been larger on high-interest-rate currencies than on low-interest-rate currencies.[1]

Given its empirical failings, it is worth reflecting on why UIP represents such an enduring benchmark in the FX literature. The UIP forecast has (at least) three appealing properties. First, UIP forecasts are determined by asset prices alone rather than on, say, infrequently updated and imperfectly measured macroeconomic data. Second, the UIP forecast has no free parameters; with no coefficients to be estimated in-sample or "calibrated," it is perfectly suited to out-of-sample forecasting. Third, the UIP forecast has a straightforward interpretation: it is the expected exchange rate movement that must be perceived by a risk-neutral investor. Put differently, UIP holds if and only if the *risk-neutral* expected appreciation of a currency is equal to its *real-world* expected appreciation, the latter being the quantity relevant for forecasting exchange rate movements.

There is, however, no reason to expect that the real-world and risk-neutral expectations should be similar. On the contrary, the modern literature in financial economics has documented that large and time-varying risk premia are pervasive across asset classes, so that risk-neutral and real-world distributions are very different from one another: in other words, the perspective of a risk-neutral investor is not useful from the point of view of forecasting. Thus, while UIP has been a useful organizing principle for the empirical literature on exchange rates, its predictive failure is no surprise.[2]

In this paper we propose a new predictor variable that also possesses the three appealing properties mentioned above, but which does not require that one takes the

[1]Some studies (e.g. Sarno et al., 2012) find that currencies with high interest rates appreciate on average, exacerbating the failure of UIP; this has become known as the forward premium puzzle. Others, such as Hassan and Mano (2016), find that exchange rates move in the direction predicted by UIP, though not by enough to offset interest-rate differentials.

[2]Various authors have fleshed out this point in the context of equilibrium models: see for example Verdelhan (2010), Hassan (2013), and Martin (2013). On the empirical side, authors including Menkhoff et al. (2012), Barroso and Santa-Clara (2015) and Della Corte et al. (2016a) have argued that it is necessary to look beyond interest-rate differentials to explain the variation in currency returns.

perspective of a risk-neutral investor. This alternative benchmark can be interpreted as the expected exchange rate movement that must be perceived by a risk-averse investor with log utility whose wealth is invested in the stock market. (To streamline the discussion, this description is an oversimplification and strengthening of the condition we actually need to hold for our approach to work, which is based on a general identity that is new to this paper.) Related perspectives are adopted by Martin (2017) and by Martin and Wagner (2017) to forecast returns on the stock market and on individual stocks, respectively.

It turns out that such an investor's expectations about currency returns can be inferred directly from the prices of so-called *quanto contracts*. Consider, for example, a quanto contract whose payoff equals the level of the S&P 500 index at time $T$, denominated in euros. The value of such a contract is sensitive to the correlation between the S&P 500 index and the dollar/euro exchange rate. If the euro is strong relative to the dollar at times when the index is high, and weak when the index is low, then this quanto contract is more valuable than a conventional, dollar-denominated, claim on the index.[3] We show that the relationship between (currency $i$) quanto and conventional forward prices on the S&P 500 index reveals the risk-neutral covariance between currency $i$ and the index. Quantos therefore allow us to determine which currencies are risky—in that they tend to depreciate in bad times, i.e., when the stock market declines—and which are hedges; it is possible, of course, that a currency is risky at one point in time and a hedge at another. Intuitively, one expects that a currency that is (currently) risky should, as compensation, have higher expected appreciation than predicted by UIP, and that hedge currencies should have lower expected appreciation. Our framework formalizes this intuition. It also allows us to distinguish between variation in risk premia across currencies and variation over time (a distinction emphasized by Lustig et al. (2011)): according to the theory, the relative importance of the two should be revealed by the behavior of quanto prices.

It is worth emphasizing various assumptions that we do *not* make. We do not require that markets are complete (though our approach remains valid if markets are complete). We do not assume the existence of a representative agent, nor do we assume that all economic actors are rational: the forecast in which we are interested reflects the

[3] A different type of quanto contract—specifically, quanto CDS contracts—is used by Mano (2013) to estimate risk-neutral expectations of currency depreciation conditional on sovereign default.

beliefs of a rational investor, but this investor may coexist with investors with other, potentially irrational, beliefs. And we do not assume lognormality, nor do we make any other distributional assumptions: our approach allows for skewness and jumps in exchange rates. This is an important strength of our framework, given that currencies often experience crashes or jumps (as emphasized by Brunnermeier et al. (2008), Jurek (2014), Della Corte et al. (2016b), Chernov et al. (2016) and Farhi and Gabaix (2016), among others), and are prone to structural breaks more generally.

Related to this, Burnside et al. (2011) argue that the attractive properties of carry trade strategies in currency markets may reflect the possibility of peso events in which the SDF takes very large values. Our approach is well adapted to this view of the world: investor concerns about peso events should be reflected in the forward-looking asset prices that we exploit, and thus our quanto predictor variable should forecast high appreciation for currencies vulnerable to peso events, even if no such events turn out to happen in sample.

We test our approach by running panel currency-forecasting regressions, and find that the quanto predictor variable is strongly significant in both statistical and economic terms. In fact, the estimated coefficient on the quanto variable is even larger than predicted by our theory. We also show that the quanto predictor variable—equivalently, risk-neutral covariance—substantially outperforms lagged realized covariance as a forecaster of exchange rates; and that it is a strongly significant predictor of future realized covariance.

We conclude by testing the out-of-sample predictive performance of the quanto variable. In a recent survey of the literature, Rossi (2013) emphasizes that the exchange-rate forecasting literature has struggled to overturn the frustrating fact, originally documented by Meese and Rogoff (1983), that it is hard even to outperform a random walk forecast out of sample. Since our data span a relatively short period (from 2009 to 2014) over which the dollar strengthened against almost all the other currencies in our dataset, we focus on forecasting differential returns on currencies. This allows us to isolate the cross-sectional forecasting power of the quanto variable in a dollar-neutral way, in the spirit of Lustig et al. (2011), and independent of what Hassan and Mano (2016) refer to as the dollar trade anomaly. Our out-of-sample forecasts exploit the fact that our theory makes an *a priori* prediction for the coefficient on the predictor variable. When the coefficient on the quanto predictor is fixed at the level implied by the theory, we end up with a forecast of currency appreciation that has no free parameters, and which is therefore—like the UIP forecast—perfectly suited for out-of-sample forecasting. Following Meese and Rogoff (1983) and Goyal and Welch (2008), we compute mean squared error and mean absolute error for the forecasts made by the quanto theory and by three competitor models: UIP, which predicts currency appreciation through the interest-rate differential; PPP, which uses past inflation differentials (as a proxy for expected inflation differentials) to forecast currency appreciation; and a random-walk forecast. The quanto theory outperforms all three competitors on both metrics.

# 1 Theory

We start with the fundamental equation of asset pricing,

$$\mathbb{E}_t \left( M_{t+1} \widetilde{R}_{t+1} \right) = 1, \tag{1}$$

since this will allow us to introduce some notation. Today is time $t$; we are interested in assets with payoffs at time $t+1$. We write $\mathbb{E}_t$ for (real-world) expectation conditional on all information available at time $t$, and $M_{t+1}$ for a stochastic discount factor (SDF) that prices assets denominated in dollars. (We will always "think in dollars," so $M_{t+1}$ will always be the relevant SDF for us. We do not assume complete markets, so there may well be other SDFs that also price assets denominated in dollars. But all such SDFs must agree with $M_{t+1}$ on the prices of the payoffs in which we are interested, since they are all tradable.) In equation (1), $\widetilde{R}_{t+1}$ is the gross return on some arbitrary dollar-denominated asset or trading strategy. If we write $R^{\$}_{f,t}$ for the gross one-period dollar interest rate, then the equation implies that $\mathbb{E}_t\, M_{t+1} = 1/R^{\$}_{f,t}$, as can be seen by setting $\widetilde{R}_{t+1} = R^{\$}_{f,t}$; thus (1) can be rearranged as

$$\mathbb{E}_t\, \widetilde{R}_{t+1} - R^{\$}_{f,t} = -R^{\$}_{f,t} \operatorname{cov}_t \left( M_{t+1}, \widetilde{R}_{t+1} \right). \tag{2}$$

Consider a simple currency trade: take a dollar, convert it to foreign currency $i$, invest at the (gross) currency-$i$ riskless rate, $R^{i}_{f,t}$, for one period, and then convert back to dollars. We write $e_{i,t}$ for the price in dollars at time $t$ of a unit of currency $i$, so that the gross return on the currency trade is $R^i_{f,t} e_{i,t+1}/e_{i,t}$; setting $\widetilde{R}_{t+1} = R^i_{f,t} e_{i,t+1}/e_{i,t}$ in (2) and rearranging,[4] we find that

$$\mathbb{E}_t \, \frac{e_{i,t+1}}{e_{i,t}} = \underbrace{\frac{R^{\$}_{f,t}}{R^i_{f,t}}}_{\text{UIP forecast}} \underbrace{- R^{\$}_{f,t} \operatorname{cov}_t \left( M_{t+1}, \frac{e_{i,t+1}}{e_{i,t}} \right)}_{\text{residual}}. \tag{3}$$

From an empirical point of view, the challenging aspect of the identity (3) is the presence of the unobservable SDF $M_{t+1}$. It would be extremely convenient if, say, $M_{t+1}$ were constant conditional on time-$t$ information, for then the irritating—because hard to measure—covariance term would drop out and we would recover the prediction of uncovered interest parity (UIP) that $\mathbb{E}_t \, e_{i,t+1}/e_{i,t} = R^{\$}_{f,t}/R^i_{f,t}$, according to which high interest-rate currencies are expected to depreciate. Thus, if the UIP forecast is used to predict exchange rate appreciation, the implicit assumption being made is that the covariance term can indeed be neglected.

Equation (3) can also be expressed using the risk-neutral expectation $\mathbb{E}^*_t$, in terms of which the time $t$ price of any payoff, $X_{t+1}$, received at time $t+1$ is

$$\text{time } t \text{ price of a claim to } X_{t+1} = \frac{1}{R^{\$}_{f,t}} \, \mathbb{E}^*_t \, X_{t+1} = \mathbb{E}_t \left( M_{t+1} X_{t+1} \right). \tag{4}$$

The first equality is the defining property of the risk-neutral probability distribution. The second equality (which can be thought of as a dictionary for translating between risk-neutral and SDF notation) can be used to rewrite (3) as

$$\mathbb{E}^*_t \left( \frac{e_{i,t+1}}{e_{i,t}} \right) = \frac{R^{\$}_{f,t}}{R^i_{f,t}}. \tag{5}$$

Unfortunately, as is well known, the UIP prediction fares poorly empirically: the assumption that the covariance term is negligible in (3) (or, equivalently, that the risk-neutral expectation in (5) is close to the corresponding real-world expectation) is unduly optimistic. This is hardly surprising, given the existence of a vast literature in financial economics that emphasizes the importance of risk premia, and hence shows

[4] Unlike most authors in this literature, we prefer to work with true returns, $\widetilde{R}_{t+1}$, rather than with log returns, $\log \widetilde{R}_{t+1}$, as the latter are only "an approximate measure of the rate of return to speculation," in the words of Hansen and Hodrick (1980).

in particular that the SDF $M_{t+1}$ is highly volatile. In practice, therefore, the risk adjustment term in (3) cannot be neglected: expected currency appreciation depends not only on the interest-rate differential, but also on the covariance between currency movements and the SDF. Moreover, it is plausible that this covariance varies both over time and across currencies. We therefore take a different approach that exploits the following observation:

**Result 1.** *We have the identity*

$$\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} = \underbrace{\frac{R^{\$}_{f,t}}{R^{i}_{f,t}}}_{\textit{UIP forecast}} + \underbrace{\frac{1}{R^{\$}_{f,t}} \operatorname{cov}^*_t \left( \frac{e_{i,t+1}}{e_{i,t}}, R_{t+1} \right)}_{\textit{quanto risk premium}} - \underbrace{\operatorname{cov}_t \left( M_{t+1} R_{t+1}, \frac{e_{i,t+1}}{e_{i,t}} \right)}_{\textit{residual}}, \tag{6}$$

*where $R_{t+1}$ is an arbitrary gross return. The asterisk on the first covariance term in* (6) *indicates that it is computed using the risk-neutral probability distribution.*

*Proof.* Setting $\widetilde{R}_{t+1} = R^i_{f,t} e_{i,t+1}/e_{i,t}$ in (1) and rearranging, we have

$$\mathbb{E}_t \left( M_{t+1} \frac{e_{i,t+1}}{e_{i,t}} \right) = \frac{1}{R^i_{f,t}}. \tag{7}$$

We can use (4) and (7) to expand the risk-neutral covariance term that appears in the identity (6) and express it in terms of the SDF:

$$\begin{aligned} \frac{1}{R^{\$}_{f,t}} \operatorname{cov}^*_t \left( \frac{e_{i,t+1}}{e_{i,t}}, R_{t+1} \right) &\overset{(4)}{=} \mathbb{E}_t \left( M_{t+1} \frac{e_{i,t+1}}{e_{i,t}} R_{t+1} \right) - R^{\$}_{f,t}\, \mathbb{E}_t \left( M_{t+1} \frac{e_{i,t+1}}{e_{i,t}} \right) \\ &\overset{(7)}{=} \mathbb{E}_t \left( M_{t+1} \frac{e_{i,t+1}}{e_{i,t}} R_{t+1} \right) - \frac{R^{\$}_{f,t}}{R^i_{f,t}}. \end{aligned} \tag{8}$$

Note also that

$$\operatorname{cov}_t \left( M_{t+1} R_{t+1}, \frac{e_{i,t+1}}{e_{i,t}} \right) = \mathbb{E}_t \left( M_{t+1} R_{t+1} \frac{e_{i,t+1}}{e_{i,t}} \right) - \mathbb{E}_t \left( \frac{e_{i,t+1}}{e_{i,t}} \right). \tag{9}$$

Subtracting (9) from (8) and rearranging, we have the result. □

As (3) and (6) are identities, each must hold for all currencies $i$ in any economy that does not permit riskless arbitrage opportunities. The identity (6) generalizes (3), however, since it allows $R_{t+1}$ to be an arbitrary return. To make (6) useful for empirical work, we want to choose a return $R_{t+1}$ with two goals in mind. First, the residual term should be small. Second, the middle term (which we label the quanto risk premium for reasons that will become clear in the next section) should be easy to compute.

The two goals are in tension. If we set $R_{t+1} = R^{\$}_{f,t}$, for example, then (6) reduces to (3), which achieves the second of the goals but not the first. Conversely, one might imagine setting $R_{t+1}$ equal to the return on an elaborate portfolio exposed to multiple risk factors and constructed in such a way as to minimise the volatility of $M_{t+1}R_{t+1}$: this would achieve the first but not necessarily the second (as will become clear in the next section).

To achieve both goals simultaneously, we want to pick a return that offsets a substantial fraction of the variation in $M_{t+1}$; but we must do so in such a way that the risk-neutral covariance term can be measured empirically. For the remainder of the paper, we will take $R_{t+1}$ to be the return on the S&P 500 index. It is highly plausible that this return is negatively correlated with $M_{t+1}$, as dictated by the first goal; in fact we provide conditions below under which the residual is exactly zero. We will now show that the second goal is also achieved with this choice of $R_{t+1}$ because we can calculate the quanto risk premium directly from asset prices, thereby avoiding the need to estimate it within an inevitably imperfect model.

## 1.1 Quantos

An investor who is bullish about the S&P 500 index might choose to go long a *forward contract* at time $t$, for settlement at time $t+1$. If so, he commits at time $t$ to pay $F_t$ for the index at time $t+1$. This trade will be profitable if the spot price of the index at time $t+1$, which we will call $P_{t+1}$, is high. The payoff on the investor's long forward contract is $P_{t+1} - F_t$ at time $t+1$. Market convention is to choose $F_t$ to make the market value of the contract equal to zero, so that no money needs to change hands initially. This requirement implies that

$$F_t = \mathbb{E}^*_t \, P_{t+1}. \tag{10}$$

A *quanto forward contract* is closely related. The key difference is that the quanto forward commits the investor to pay $Q_{i,t}$ *units of currency* $i$ at time $t+1$, in exchange for $P_{t+1}$ *units of currency* $i$. (At each time $t$, there are $N$ different quanto prices indexed by $i = 1, \ldots, N$ for each of the $N$ currencies in our data set. The underlying asset is always the S&P 500 index, whatever the currency.) The payoff on a long position in a quanto forward contract is therefore $P_{t+1} - Q_{i,t}$ units of currency $i$ at time $t+1$; this is equivalent to a time $t+1$ dollar payoff of $e_{i,t+1}(P_{t+1} - Q_{i,t})$. As with a conventional forward contract, the market convention is to choose the quanto forward price, $Q_{i,t}$, in such a way that the contract has zero value at initiation. It must therefore satisfy

$$Q_{i,t} = \frac{\mathbb{E}_t^* \, e_{i,t+1} P_{t+1}}{\mathbb{E}_t^* \, e_{i,t+1}}. \tag{11}$$

(We converted to dollars because $\mathbb{E}_t^*$ is the risk-neutral expectations operator that prices *dollar* payoffs.) Combining equations (5) and (11), the quanto forward price can be written

$$Q_{i,t} = \frac{R_{f,t}^i}{R_{f,t}^\$} \, \mathbb{E}_t^* \, \frac{e_{i,t+1} P_{t+1}}{e_{i,t}},$$

which implies, using (5) and (10), that the gap between the quanto and conventional forward prices captures the conditional risk-neutral covariance between the exchange rate and stock index,

$$Q_{i,t} - F_t = \frac{R_{f,t}^i}{R_{f,t}^\$} \operatorname{cov}_t^* \left( \frac{e_{i,t+1}}{e_{i,t}}, P_{t+1} \right). \tag{12}$$

We will make the simplifying assumption that dividends earned on the index between time $t$ and time $t+1$ are known at time $t$ and paid at time $t+1$. It then follows from (12) that

$$\frac{Q_{i,t} - F_t}{R_{f,t}^i P_t} = \frac{1}{R_{f,t}^\$} \operatorname{cov}_t^* \left( \frac{e_{i,t+1}}{e_{i,t}}, R_{t+1} \right). \tag{13}$$

Thus the quanto risk premium term that appears in (6) is directly revealed by the gap between quanto and conventional index forward prices.

We still need to deal with the troublesome final term, however. We will initially do so by taking the perspective of an unconstrained, rational investor with log utility whose wealth is fully invested in the S&P 500 index. We can then exploit the fact that from the perspective of such an investor, $M_{t+1} = 1/R_{t+1}$, so that the residual term is *exactly* zero (Martin, 2017). We emphasize, however, that we are adopting this perspective for simplicity: all we really need is that the second covariance term in (6) can reasonably be neglected. The key advantage of our approach relative to the UIP benchmark is that it is much more reasonable to do so than to neglect the covariance term in (3). (In Section 2.3, we dig deeper into the properties of the covariance term by conducting a principal component analysis of the residuals in our regressions and relating the first principal component to the average forward discount introduced by Lustig et al. (2014).)

The following result summarizes the above discussion.

**Result 2.** *If we take the perspective of a rational investor with log utility whose wealth is fully invested in the index then*

$$\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} - 1 = \underbrace{\frac{R^{\$}_{f,t}}{R^{i}_{f,t}} - 1}_{IRD} + \underbrace{\frac{Q_{i,t} - F_t}{R^{i}_{f,t} P_t}}_{QRP}, \tag{14}$$

*where $R_{t+1}$ is the return on the S&P 500 index, $P_t$ is the spot price of the index, $F_t$ is the forward price of the index, and $Q_{i,t}$ is the quanto forward price of the index (where currency $i$ is the quanto currency).*

*Rearranging, the expected excess return on speculation in currency $i$ equals the quanto risk premium:*

$$\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \frac{Q_{i,t} - F_t}{R^{i}_{f,t} P_t}.$$

Equation (14) splits expected currency appreciation into two terms. The first is the UIP prediction which, as we have seen in equation (5), equals *risk-neutral* expected currency appreciation. We will often refer to this term as the *interest-rate differential* (IRD); and as above we will generally convert to net rather than gross terms by subtracting 1. (Note that a high interest-rate currency will have a negative interest-rate differential, i.e. a negative UIP forecast.) The second is a risk adjustment term: by taking the perspective of the log investor, we have converted the general form of the residual that appears in (3) into a quantity that can be directly observed using the gap between a quanto forward and a conventional forward. We refer to this term as the *quanto risk premium* (QRP). Lastly, we refer to the sum of the two terms as the *quanto forecast*, or as *expected currency appreciation* (ECA).

## 2 Empirics

We obtained forward prices and quanto forward prices on the S&P 500, together with domestic and foreign interest rates, from Markit; the maturity in each case is 24 months. The data is monthly and runs from December 2009 to November 2014 for the Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), Danish krone (DKK), Euro (EUR), British pound (GBP), Japanese yen (JPY), Korean won (KRW), Norwegian krone (NOK), Polish zloty (PLN), and Swedish krona (SEK). After accounting for missing entries in our panel (notably in DKK, KRW, and PLN: see Figure 1) we have 555 currency-month observations.

The two building blocks of our empirical analysis are the currencies' interest-rate differentials vis-à-vis the US dollar (IRD, which would measure expected exchange rate appreciation if UIP held) and their quanto risk premia (QRP, which measure the risk-neutral covariances between each currency and the S&P 500 index, as shown in equation (13)). Our measure of expected currency appreciation (the quanto forecast, or ECA) is equal to the sum of IRD and QRP, as shown in equation (14) of Result 2.

Figure 1 shows the evolution over time of ECA (solid) and of the UIP forecast (dashed) for each of the currencies in our panel. The gap between the two lines for a given currency is that currency's quanto risk premium, which varies over time and across currencies and whose magnitude is economically significant for all currencies. The quanto risk premium is negative for JPY and positive for all other currencies (with the partial exception of EUR, for which we observe a sign change in QRP toward the very end of our time period). Table 1 reports summary statistics of ECA. The penultimate line of the table averages the summary statistics across currencies; the last line reports summary statistics for the pooled data. Table 2 reports the same statistics for the constituent parts of ECA, namely IRD and QRP. The last two lines of each panel report the statistics averaged across currencies and summary statistics of the pooled data.

The volatility of quanto risk premia is similar to that of interest-rate differentials, both currency-by-currency and in the panel. There is considerably more variability in IRD and QRP when we pool the data than there is in the time series of a typical currency: this reflects substantial dispersion in IRD and QRP across currencies that is captured in the pooled measure but not in the averaged measure.

Table 3 reports volatilities and correlations for the time series of individual currencies' ECA, IRD, and QRP. The table also shows three aggregated measures of volatilities and correlations. The row labelled "Time series" reports time-series volatilities and correlations for a typical currency, calculated by averaging time-series volatilities and correlations across currencies. Conversely, the row labelled "Cross section" reports cross-sectional volatilities and correlations on a typical day, that is, reports cross-currency volatilities and correlations of time-averaged ECA, IRD, and QRP. Lastly, the row labelled "Pooled" averages on both dimensions: it reports volatilities and correlations for the pooled data.

All three variables (ECA, IRD, and QRP) exhibit substantially more volatility in the cross section than in the time series. This is particularly true of interest-rate differentials, which exhibit far more dispersion across currencies than over time.

The correlation between interest-rate differentials and quanto risk premia is negative when we pool our data ($\rho = -0.691$). Given the sign convention on IRD, this indicates that currencies with high interest rates (relative to the dollar) tend to have high risk premia; thus the predictions of the quanto theory are consistent with the carry trade literature and the findings of Lustig et al. (2011). The average time-series (i.e., within-currency) correlation between interest-rate differentials and quanto risk premia is more modestly negative ($\rho = -0.178$): a typical currency's risk premium tends to be higher, or less negative, at times when its interest rate is high relative to the dollar, but this tendency is fairly weak. The disparity between these two facts is accounted for by the strongly negative cross-sectional correlation between interest-rate differentials and quanto risk premia ($\rho = -0.800$). According to the quanto theory, therefore, the returns to the carry trade are more the result of persistent cross-sectional differences between currencies than of a time-series relationship between interest rates and risk premia. This prediction is consistent with the empirical results documented by Hassan and Mano (2016). (These results also help to illustrate an important advantage of our approach. When, for example, Lustig et al. (2011) assess the relative importance of cross-sectional and time-series effects, they are forced to split their sample in order to estimate cross-sectional effects without using in-sample information. In contrast, our theory suggests that quanto risk premia should reveal both time-series and cross-sectional dispersion in currency risk premia in a forward-looking way.) Figure 2 makes the same point graphically by plotting each currency's quanto risk premium over time;

for clarity, the figure drops two currencies for which we have highly incomplete time series (PLN and DKK).

We see a corresponding pattern in the time-series, cross-sectional, and pooled correlations of ECA and QRP. The time-series (within-currency) correlation of the two is substantially positive ($\rho = 0.628$), while the cross-sectional correlation is negative ($\rho = -0.308$). In the time series, therefore, a rise in a given currency's quanto risk premium is associated with a rise in its expected appreciation; whereas in the cross-section, currencies with relatively high quanto risk premia on average have relatively *low* expected currency appreciation on average (reflecting relatively high interest rates on average). Putting the two together, the pooled correlation is close to zero ($\rho = 0.024$). That is, the quanto theory predicts that there should be no clear relationship between currency risk premia and expected currency appreciation; again, this is consistent with the findings of Hassan and Mano (2016).

These properties are illustrated graphically in Figure 3. We plot confidence ellipses centred on the means of QRP and IRD in panel (a), and of QRP and ECA in panel (b), for each currency. The sizes of the ellipses reflect the volatilities of IRD and QRP (or ECA): under joint Normality, each ellipse would contain 50% of its currency's observations in population.[5] The orientation of each ellipse illustrates the within-currency time series correlation, while the positions of the different ellipses reveal correlations across currencies. The figures refine the discussion above. QRP and IRD are negatively correlated within currency (with the exceptions of CAD, CHF, and KRW) and in the cross-section. QRP and ECA are positively correlated in the time series for every currency, but exhibit negative correlation across currencies; overall, the pooled correlation between the two is close to zero.

## 2.1 Return forecasting

We run two sets of panel regressions in which we attempt to forecast, respectively, currency excess returns and currency appreciation. (We report the results of regressions for individual currencies in Tables 10 and 11.) The literature on exchange rate forecasting finds it substantially more difficult to forecast pure currency appreciation than currency excess returns, so the second set of regressions should be considered more

[5]We are interested in the relative sizes of the ellipses, so choose 50% to make the figure readable.

empirically challenging. In each case, we test the prediction of our theory as expressed in Result 2. In order to relate to the existing literature on the forward premium puzzle that uses the interest-rate differential to forecast exchange rates (in line with the UIP prediction) and currency excess returns, we benchmark our predictions against the two forecasting regressions of Fama (1984). We run both pooled panel regressions and panel regressions with fixed effects; the latter regressions isolate the effect of within-currency variation.

To provide a sense of the data before turning to our regression results, Figures 4 and 5 represent our baseline univariate regressions graphically in the same manner as in Figure 3. Figure 4 plots realized currency excess returns (RXR) against QRP and against IRD. Excess returns are strongly positively correlated with QRP both within currency and in the cross-section, suggesting strong predictability with a positive sign. The correlation of RXR with IRD is negative in the cross-section but close to zero, on average, within currency. Figure 5 shows the corresponding results for realized currency appreciation (RCA). Panel (a) suggests that the within-currency correlation with the quanto predictor ECA is predominantly positive (with the exceptions of AUD and CHF), as is the cross-sectional correlation. In contrast, panel (b) suggests that the correlation between realized currency appreciation and interest-rate differentials is close to zero both within and across currencies, consistent with the view that interest-rate differentials do not help to forecast currency appreciation.

We follow Fama (1984) in running regressions with both currency excess returns ($e_{i,t+1}/e_{i,t} - R^{\$}_{f,t}/R^{i}_{f,t}$) and with currency appreciation ($e_{i,t+1}/e_{i,t} - 1$) as the dependent variable. We first run a horse race between the quanto risk premium and interest-rate differential as predictors of currency excess returns:

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \beta \frac{Q_{i,t} - F_t}{R^{i}_{f,t} P_t} + \gamma \left( \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} - 1 \right) + \varepsilon_{i,t+1}. \tag{15}$$

We also run two univariate regressions. The first of these,

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \beta \frac{Q_{i,t} - F_t}{R^{i}_{f,t} P_t} + \varepsilon_{i,t+1}, \tag{16}$$

is suggested by Result 2. The second uses interest-rate differentials to forecast currency excess returns, as a benchmark:

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R_{f,t}^{\$}}{R_{f,t}^{i}} = \alpha + \gamma \left( \frac{R_{f,t}^{\$}}{R_{f,t}^{i}} - 1 \right) + \varepsilon_{i,t+1}. \tag{17}$$

We also run all three regressions with currency fixed effects $\alpha_i$ in place of the shared intercept $\alpha$.

Table 4 reports the results. The top panel shows coefficient estimates for each regression with and without fixed effects; standard errors (computed by block bootstrap) are shown in parentheses.[6] The quanto risk premium is positive, economically large, and strongly individually significant in every specification in which it occurs. Moreover, the $R^2$ values are substantially higher in the two regressions (15) and (16) that feature the quanto risk premium than in the regression (17) in which it does not occur.

The bottom three panels show the $p$-values associated with Wald tests of various hypotheses on the regression coefficients. In two respects, the regression results are inconsistent with Result 2. The first is that the estimated coefficient $\beta$ is statistically significantly larger than 1 in regressions (15) and (16) when we include currency fixed effects; and even without fixed effects, we come close to rejecting the null hypothesis $\beta = 1$ in the regression (16) at conventional significance levels. That is, the predictive power of the quanto risk premium is even stronger than our theory predicts. This is a relatively minor disappointment; a second, more irritating, inconsistency is that the constant $\alpha$ is significantly smaller than zero. For these two reasons, when we test the joint hypotheses suggested by Result 2 that $\alpha = 0$, $\beta = 1$, and $\gamma = 0$ in (15) and that $\alpha = 0$ and $\beta = 1$ in (16), we are able to reject with $p$-values of 0.035 and 0.015, respectively.

The significantly negative intercept $\alpha$ indicates that the currencies in our panel underperformed across the board, relative to the prediction of the model: that is, it reflects an unexpectedly strong dollar over our sample period. To remove this dollar

[6]Details of the stationary bootstrap methodology are provided in Politis and White (2004) and Patton et al. (2009). We resample the data by drawing with replacement blocks of 24 time-series observations from the panel and ensure this time-series resampling is synchronized in the cross-section. The resulting panel is then resampled with replacement in the cross-sectional dimension by drawing blocks of uniformly distributed width (between 2 and 8). We then compute the point estimates of the coefficients from the two-dimensionally resampled panel and repeat this procedure 100,000 times. The standard errors are then computed as the standard deviations of the respective coefficients across the bootstrap repetitions.

effect, we also conduct a joint test of the hypotheses that $\beta = 1$ and $\gamma = 0$ in (15) (and that $\beta = 1$ in (16)) without also testing $\alpha = 0$, and thereby test whether our model forecasts *differential* currency returns in the manner implied by Result 2. At the conventional 5% or even 10% significance level, we cannot reject the hypothesis that the slope coefficients take the theory-implied values. Once we include currency fixed effects, however, we find, as previously, that the quanto risk premium is an even stronger predictor of currency returns than our theory implies.

The bottom panel of the table reports $p$-values for tests of null hypotheses that the right-hand-side variables are useless, $\beta = \gamma = 0$. We are able to reject the null of no predictability with some confidence for the pooled regressions (15) and (16) (with $p$-values on the null of 0.030 and 0.005, respectively). In contrast, there is only weak evidence of predictability for the interest-rate differential in the pooled regression (17) ($p$-value of 0.099). Once we include fixed effects, we can strongly reject the null of no predictability for each of the specifications.

We can also test how our theory fares at predicting currency appreciation. To do so, we run the regression

$$\frac{e_{i,t+1}}{e_{i,t}} - 1 = \alpha + \beta \frac{Q_{i,t} - F_t}{R^i_{f,t} P_t} + \gamma \left( \frac{R^{\$}_{f,t}}{R^i_{f,t}} - 1 \right) + \varepsilon_{i,t+1}. \tag{18}$$

The coefficient estimates in (18) are mechanically related to those of regression (15): the only difference is that the coefficient estimate on $\gamma$ is exactly 1 greater in (18) than in (15). Correspondingly, we find identical $p$-values on the Wald tests of the joint hypotheses implied by Result 2 in Tables 4 and 5 (up to very small deviations that can be attributed to randomness in the bootstrap). We therefore run (18) not because we are interested in the resulting coefficient estimates, but because we are interested in the $R^2$.

To explore the relative importance of the quanto risk premium and interest-rate differentials for forecasting currency appreciation, we also run univariate regressions of currency appreciation onto the quanto risk premium,

$$\frac{e_{i,t+1}}{e_{i,t}} - 1 = \alpha + \beta \frac{Q_{i,t} - F_t}{R^i_{f,t} P_t} + \varepsilon_{i,t+1}, \tag{19}$$

and onto interest-rate differentials,

$$\frac{e_{i,t+1}}{e_{i,t}} - 1 = \alpha + \gamma \left( \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} - 1 \right) + \varepsilon_{i,t+1}. \tag{20}$$

As previously, we also run the three regressions (18)–(20) with fixed effects.

The regression results are shown in Table 5, which is structured similarly to Table 4. There are two important conclusions that emerge from the table that are not directly implied by Table 4. First, there is little evidence that the interest-rate differential helps to forecast currency appreciation on its own, consistent with the large literature that documents the failure of UIP. In the pooled panel, the estimated $\gamma$ in regression (20) is close to 0 and the $p$-value on the null of no predictability by IRD is 0.924, and the $R^2$ is essentially zero (0.02%). Even with fixed effects, we marginally fail to reject the null of no predictability ($p$-value of 0.057).

Second, the quanto risk premium makes a substantial difference in terms of $R^2$ and is on the margin of individual significance in the pooled regressions (18) and (19) (and is very strongly significant when fixed effects are included). When we include the quanto risk premium in our pooled regressions instead of the interest-rate differential, $R^2$ increases from roughly zero to 11.04%; and when the two variables are included together, $R^2$ increases to 19.94%. Moreover, the coefficient estimate on $\gamma$ increases, in the presence of the quanto risk premium, toward its theoretically predicted value of 1.

## 2.2 Risk-neutral covariance outperforms realized covariance

If we take the perspective of the log investor then we can conclude (using (3)) that

$$\mathbb{E}_t \frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = R^{\$}_{f,t} \operatorname{cov}_t \left( \frac{e_{i,t+1}}{e_{i,t}}, -\frac{1}{R_{t+1}} \right). \tag{21}$$

Note that it is the risk-neutral covariance that appears in (13), and the real-world covariance that appears in (21); the advantage of the former is that it is unambiguously observable (via quanto prices) at time $t$. Both expressions capture the intuition that a currency that tends to depreciate when the S&P 500 declines is risky and hence should appreciate on average, as compensation for this risk.

Various authors have pursued a similar approach in exploring the relationship between currency risk premia and currency betas (see, for example, Lustig and Verdelhan (2007), Campbell et al. (2010) and Burnside (2011)). Motivated by this earlier literature, and by equation (21), we define an empirical proxy for currency beta that is based on lagged realized covariance:[7]

$$\mathrm{RPCL}_{i,t} = R^{\$}_{f,t}\left(\sum\left[\frac{e_{i,s+1}}{e_{i,s}}\left(-\frac{1}{R_{s+1}}\right)\right] - \frac{1}{T-t}\sum\left(-\frac{1}{R_{s+1}}\right)\sum\frac{e_{i,s+1}}{e_{i,s}}\right).$$

The summation is over daily returns on trading days $s$ preceding $t$ over a time-frame corresponding to the forecasting horizon, $T-t$, so that $\mathrm{RPCL}_{i,t}$ is observable at time $t$. We also define a corresponding measure, $\mathrm{RPC}_{i,t}$, that is identical to the above definition except that the summation is over daily returns on trading days $s$ *following* $t$ over the appropriate time-frame, and which is therefore not observable at time $t$.

To compare the predictive performance of lagged realized covariance relative to the quanto risk premium (which equals risk-neutral covariance), we regress currency excess returns onto lagged realized covariance in the panel regression

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \gamma \mathrm{RPCL}_{i,t} + \varepsilon_{i,t+1}. \tag{22}$$

As shown in Table 6, lagged realized covariance is a marginally significant predictor of currency excess returns. (The table also contains results for the corresponding regression with currency fixed effects, which delivers very similar conclusions.) We also run a horse-race with the quanto risk premium,

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \beta\frac{Q_{i,t} - F_t}{R^{i}_{f,t}P_t} + \gamma \mathrm{RPCL}_{i,t} + \varepsilon_{i,t+1}, \tag{23}$$

and find that lagged realized covariance is driven out by the quanto risk premium, which is individually significant.[8] We can strongly reject the hypothesis that $\beta = \gamma = 0$ ($p$-value of 0.013), but not the hypothesis that $\beta = 1$ and $\gamma = 0$ ($p$-value of 0.147). Moreover, $R^2$ more than doubles when we move from (22) to (23); this high $R^2$ is almost entirely due to the predictive success of the quanto risk premium, since the $R^2$

[7]The results are almost identical if we replace $-1/R_{s+1}$ with $R_{s+1}$ in the definition of beta.

[8]The quanto risk premium is even competitive with (future) realized covariance, although this is obviously not observable in real time. Results available on request.

of regression (23) is hardly any higher than in the univariate regression (16) of currency excess returns onto the quanto risk premium alone.

We can next ask whether the predictive success of risk-neutral covariance can be attributed to its ability to forecast realized covariance, $\mathrm{RPC}_{i,t}$. We run pooled regressions of (unlagged) realized covariance $\mathrm{RPC}_{i,t}$ onto the quanto risk premium (that is, risk-neutral covariance):

$$\mathrm{RPC}_{i,t} = \alpha + \beta \frac{Q_{i,t} - F_t}{R^i_{f,t} P_t} + \varepsilon_{i,t+1}. \tag{24}$$

The estimated coefficient on the quanto risk premium, $\beta = 0.432$, is strongly significant. We also run regressions with fixed effects, and find that the resulting estimate of $\beta$ is positive, though not significant at conventional levels. Thus the success of the quanto risk premium in forecasting realized covariance is due in part to its ability to forecast persistent differences in covariances across currencies.

## 2.3 Examining the residuals

The identity (6) decomposes expected currency appreciation into the UIP forecast, the quanto risk premium, and a conditional covariance term $(-\operatorname{cov}_t(M_{t+1}R_{t+1}, e_{i,t+1}/e_{i,t}))$. Thus far, we have neglected this last term. Now we investigate whether its presence is detectable in realized currency returns.

To do so, we calculate two measures of residuals $\varepsilon_{i,t+1}$ based on the regression (16). These realized residuals reflect both the ex ante residual from the identity (6) and the ex post realizations of unexpected currency returns. The identity (6) implies that the predictable component of the realized residuals—if there is one—reveals the covariance term, $-\operatorname{cov}_t(M_{t+1}R_{t+1}, e_{i,t+1}/e_{i,t})$. We compute (realized) *theory residuals* by imposing the coefficients implied by Result 2, that is, $\alpha = 0$ and $\beta = 1$; and we compute (realized) *regression residuals* using the estimated coefficients from the fixed effects specification of regression (16). The time series of theory and regression residuals are shown in Figure 6 for each currency.

We decompose the theory and regression residuals into their respective principal components. Table 7 shows the principal component loadings.[9] The first principal

[9]To minimise the impact of missing observations, we drop DKK, KRW, and PLN from the panel

component, which explains just over 66% of the variation in residuals, can be interpreted as a level factor since it loads positively on all currencies (with the exception of GBP when using regression residuals).[10] It is therefore reminiscent of the 'dollar' factor $\overline{\text{IRD}}_t$ constructed by Lustig et al. (2014) as the cross-sectional average of IRD, and in fact Figure 7 shows that $\overline{\text{IRD}}_t$ does indeed correlate reasonably strongly ($\rho = -0.57$) with the first principal component. This suggests that $\overline{\text{IRD}}_t$ may capture the variation in the conditional covariance term that we have chosen to neglect thus far.

To assess this possibility, we will add $\overline{\text{IRD}}_t$ to our baseline specification (15) and run the regression

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \beta \frac{Q_{i,t} - F_t}{R^{i}_{f,t} P_t} + \gamma \left( \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} - 1 \right) + \delta \left( \overline{\text{IRD}}_t \right) + \varepsilon_{i,t+1}. \tag{25}$$

For reference, we first run the regression

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \gamma \left( \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} - 1 \right) + \delta \left( \overline{\text{IRD}}_t \right) + \varepsilon_{i,t+1}. \tag{26}$$

The results are reported in Table 8. The estimated coefficient on $\overline{\text{IRD}}_t$ in regression (26) has the expected sign: it is negative, indicating that, all else equal, if the dollar interest rate is currently low relative to average foreign-currency interest rates, a typical foreign currency should be expected to appreciate against the dollar or, equivalently, that low dollar interest rates go hand-in-hand with a *currently* strong dollar. This is consistent with the findings of Lustig et al. (2014). The coefficients $\gamma$ and $\delta$ are on the margins of joint significance ($p$-values on the joint hypothesis $\gamma = \delta = 0$ are 0.094 in the pooled regression and 0.053 with fixed effects). The $R^2$ for regression (26) is 17.52% (pooled) and 21.39% (with fixed effects). These are lower than the corresponding $R^2$ for regression (15) in which $\overline{\text{IRD}}_t$ is replaced by the quanto risk premium (24.50% and 36.69% for the pooled and fixed-effects regressions, respectively).

In the pooled regression (25), both the quanto risk premium and $\overline{\text{IRD}}_t$ are individually significant. We can strongly reject the hypothesis that $\beta = \gamma = \delta = 0$ ($p$-value

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when calculating the principal components.

[10]The second principal component, which explains about 24% of the variation, is, loosely speaking, a yen factor.

of 0.010) and marginally reject the hypothesis that $\beta = 1$ and $\gamma = \delta = 0$ ($p$-value of 0.056). There is a modest increase in $R^2$ (to 30.97% from 24.50%) relative to regression (15) in which $\overline{\text{IRD}}_t$ is absent.

When we add fixed effects, there is an even smaller increase in $R^2$ (to 38.50% from 36.69%) relative to regression (15). The quanto risk premium remains strongly significant (and as before the estimated coefficient $\beta$ is even larger than our theory predicts) but both $\gamma$ and $\delta$ are statistically insignificant.

# 3 Out-of-sample prediction

We now test the predictive success of the quanto theory out-of-sample. Since the dollar strengthened strongly over the relatively short time period spanned by our data (as reflected in the significantly negative estimated intercept in our pooled panel regression (18)), we focus on forecasting differential currency appreciation: that is, we seek to predict, for example, the relative performance of dollar-yen versus dollar-euro.

In the previous section, we estimated the loadings on the quanto risk premium, $(Q_{i,t} - F_t)/(R^i_{f,t} P_t)$, and interest-rate differential, $R^\$_{f,t}/R^i_{f,t} - 1$, via panel regressions. These deliver the best in-sample coefficient estimates in a least-squares sense. But for an out-of-sample test we must pick the loadings *a priori*. Here we can exploit the distinctive feature of Result 2 that it makes specific quantitative predictions for the loadings: each should equal 1, as in the formula (14). We therefore compute out-of-sample currency forecasts by fixing the coefficients that appear in (18) at their theoretical values: $\alpha = 0$, $\beta = 1$, $\gamma = 1$. We compare these predictions to those of three natural competitor models that also make a priori predictions, and so do not require estimation of parameters: UIP (which predicts that currency appreciation should offset the interest-rate differential, on average), a random walk (which makes the constant forecast of zero currency appreciation), and relative purchasing power parity (which predicts that currency appreciation should offset the inflation differential, on average).

To compare the forecast accuracy of the model to those of the benchmarks, we define a dollar-neutral $R^2$-measure similar to Goyal and Welch (2008):

$$R^2_{OS} = 1 - \frac{\sum_i \sum_j \sum_t (\varepsilon^Q_{i,t+1} - \varepsilon^Q_{j,t+1})^2}{\sum_i \sum_j \sum_t (\varepsilon^B_{i,t+1} - \varepsilon^B_{j,t+1})^2},$$

where $\varepsilon^Q_{i,t+1}$ and $\varepsilon^B_{i,t+1}$ denote forecast errors (for currency $i$ against the dollar) of the quanto theory and the benchmark, respectively, so our measure compares the accuracy of differential forecasts of currencies $i$ and $j$ against the dollar. We hope to find that the quanto theory has lower mean-squared error than each of the competitor models, that is, we hope to find positive $R^2_{OS}$ versus each of the benchmarks.

The results of this exercise are reported in Table 9. Given the data of 11 currencies sampled, we form predictions on the $N(N-1)/2 = 55$ dollar-neutral currency differentials. The quanto theory outperforms each of the three competitors: when the competitor model is UIP, we find that $R^2_{OS} = 12.64\%$; and when it is relative PPP, we find $R^2_{OS} = 27.37\%$. In our sample, the toughest benchmark is the random-walk (i.e., constant) forecast, consistent with the findings of Rossi (2013). Nonetheless, the quanto theory easily outperforms it, with $R^2_{OS} = 11.39\%$.

To address the concern that these successes may merely reflect a small number of outliers, we repeat the exercise using a measure of fit based on mean *absolute* error,

$$1 - \frac{\sum_i \sum_j \sum_t \left| \varepsilon^Q_{i,t+1} - \varepsilon^Q_{j,t+1} \right|}{\sum_i \sum_j \sum_t \left| \varepsilon^B_{i,t+1} - \varepsilon^B_{j,t+1} \right|}.$$

Again, we find that the quanto theory outpredicts both benchmarks: the above quantity equals 5.08% when comparing to IRD, 5.33% when comparing to the random walk forecast, and 14.52% when comparing to relative PPP.

Lastly, we split the above measures by currency, defining

$$R^2_{OS,i} = 1 - \frac{\sum_j \sum_t (\varepsilon^Q_{i,t+1} - \varepsilon^Q_{j,t+1})^2}{\sum_j \sum_t (\varepsilon^B_{i,t+1} - \varepsilon^B_{j,t+1})^2}$$

and the analogous measure based on mean absolute error; these results are also shown in Table 9. These decompose the $R^2$ measure currency-by-currency, in order to test whether our positive results are driven by a small subset of the currencies. We find that the quanto theory outperforms all three benchmarks, on both metrics, and for all 11 choices of currency $i$.

# 4 Conclusion

The literature on exchange rate forecasting can be understood through the lens of an identity (3) that equates expected currency appreciation to an interest-rate differential (the UIP forecast) plus a residual term that depends on the covariance between the currency in question and the stochastic discount factor. UIP holds if and only if this residual term is negligible. In reality, however, there is considerable evidence that currencies with high interest rates earn high risk premia, so the residual term is *not* negligible and UIP does not hold.

In this paper, we have derived a more general identity that opens a new line of attack for empirical work on currency forecasting. Our identity (6) features an arbitrary dollar return, $R_{t+1}$, that can be flexibly chosen to optimize the identity's usefulness for empirical work. One wants to choose $R_{t+1}$ such that the second and third components of expected currency appreciation in (6) are each either observable or plausibly negligible. (The first component, the UIP forecast, is manifestly observable.) There is a tradeoff in doing so, because some natural choices of $R_{t+1}$ deal with—in the sense of making observable or negligible—one of the two terms but not the other.

In order to deal with both terms simultaneously, we set $R_{t+1}$ equal to the return on the S&P 500 index. We argue that it is then much more reasonable to view the residual in (6) as negligible than to view the residual in (3) as negligible. In particular, the residual term in (6) is exactly zero if one adopts the perspective of a rational investor with log utility who chooses to hold the S&P 500 index, as we do for much of the paper. (In contrast, the residual in (3) is zero under the much less reasonable assumption of risk-neutrality.) Moreover, the remaining terms can be observed given interest rates, S&P 500 forward prices, and S&P 500 quanto forward prices. The fact that the predictive variables are based only on asset prices has the further benefit that, in principle, we can generate currency forecasts at high frequency.

We do not assume that *all* investors have log utility, nor that all investors are rational, nor that all investors are unconstrained; but we do implicitly take the view that it is reasonable to take the perspective of such an investor as a benchmark. Strictly speaking, though, for our approach to be empirically useful, we only require that the final term in the identity (6) can reasonably be neglected. Moreover, our approach does not require us to make any assumptions about the stochastic processes followed by currencies: we can accommodate lognormality, but we can also allow for jumps in currencies. Our approach could even be used, in principle, to compute expected returns for currencies that are currently pegged but that have some probability of jumping off the peg. To the extent that skewness and jumps are empirically relevant, this fact will be embedded in the asset prices we use as forecasting variables.

The quanto theory decomposes currency forecasts into a component based on interest-rate differentials (the term that arises in a UIP forecast) and a risk-based component inferred from quanto prices (the term that, according to our theory, quantifies the failure of UIP). The theory captures a very simple intuition: a currency that tends to depreciate in bad times—represented, for us, by declines in the S&P 500 index—is risky and therefore requires a higher risk premium.

In the data, the quanto-implied risk adjustments are correlated with interest-rate differentials, so the quanto theory is consistent with the presence of the carry trade. Overall, taking into account the combination of interest-rate differentials and risk adjustments, the quanto forecasts indicate that the returns to high interest currencies are partially attenuated by their tendency to depreciate against low interest currencies. We also find that our measure of currency risk premia exhibits substantially more cross-currency variation than within-currency variation, consistent with the evidence presented by Hassan and Mano (2016).

We test our theory by estimating currency-forecasting panel regressions, and find in most of our specifications that the quanto forecasting variable is significant both in statistical terms (the standard error is small) and in economic terms (the point estimate is large). In fact, beyond its implication that the quanto risk premium should forecast currency excess returns, the theory also makes the more aggressive and unusual prediction that the coefficient on the quanto risk premium should take a *specific numerical value* (namely, 1); in several of our specifications, the estimated coefficient on the quanto variable is even larger than our model predicts, to the extent that we are formally able to reject it.

As the quanto forecasting variable (for a given currency) measures the risk-neutral covariance between that currency and the S&P 500 index, our results are related to a literature that has used lagged realized covariances as a measure of currency risk (for example Lustig and Verdelhan, 2007; Campbell et al., 2010). We show that the quanto variable successfully forecasts future realized covariances, and drives out lagged realized covariances in multivariate specifications.

Next, we investigate whether we are missing anything by assuming that the residual covariance term is negligible. To do so, we conduct a principal component analysis of the error residuals in realized currency returns. The first principal component explains around two thirds of the variation in residuals, and loads roughly equally on all currencies. It is also reasonably strongly correlated with the average forward discount variable of Lustig et al. (2014), which is based on average interest rate differentials. We therefore add the average forward discount to our baseline specification. The quanto variable continues to be significant both with and without fixed effects; the average forward discount variable itself is significant in the pooled regression but not with fixed effects, and in either case it only contributes a modest increase in $R^2$.

We conclude by returning to the theoretical prediction that the coefficient should equal one, since this constitutes an a priori benchmark. When we constrain the coefficient on the quanto risk premium to equal one, we end up with a formula for expected currency appreciation that has no free parameters, and which is therefore perfectly suited to out-of-sample prediction. We test the formula's ability to forecast differential currency appreciation out of sample and find that it outperforms UIP, PPP, and a random walk.

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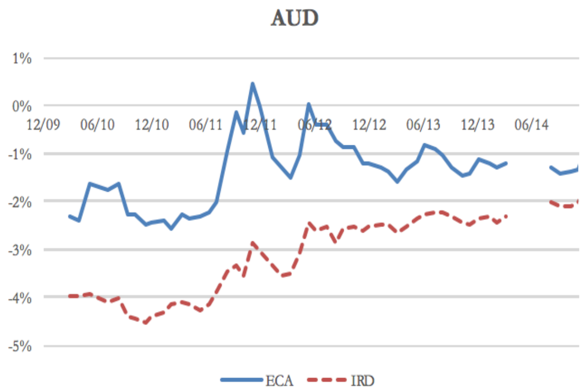
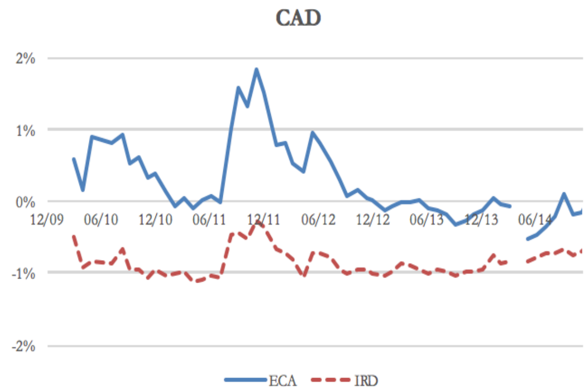
# A Appendix

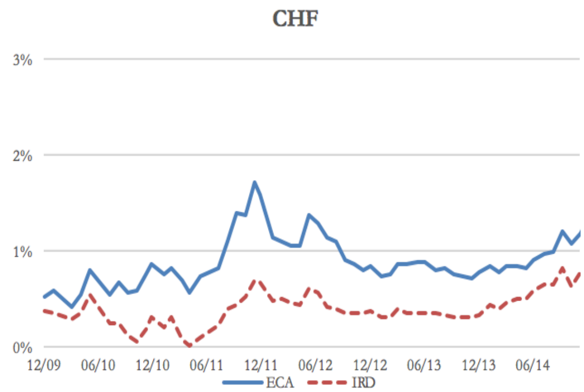
Table 1: Summary statistics of ECA

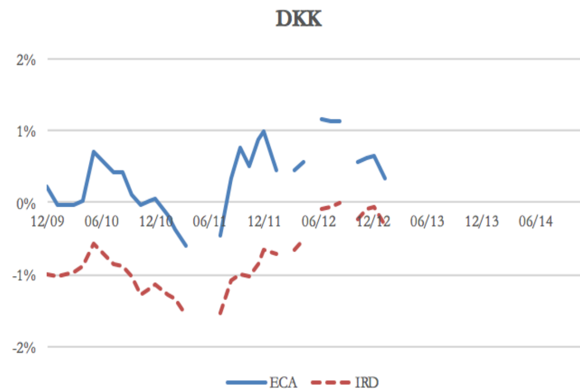
This table reports annualized summary statistics (in %) of quanto-based expected currency appreciation (ECA).

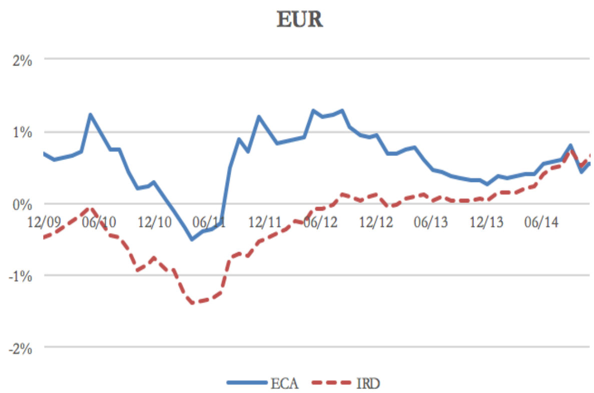
| **ECA** | **Mean** | **Std Dev.** | **Skew** | **Kurtosis** | **Min** | **Max** | **Autocorr.** |
|---|---|---|---|---|---|---|---|
| AUD | -1.375 | 0.710 | 0.257 | -0.100 | -2.550 | 0.450 | 0.837 |
| CAD | 0.291 | 0.561 | 1.079 | 0.518 | -0.526 | 1.835 | 0.852 |
| CHF | 0.893 | 0.263 | 0.993 | 1.180 | 0.422 | 1.718 | 0.857 |
| DKK | 0.331 | 0.487 | -0.097 | -0.606 | -0.587 | 1.172 | 0.762 |
| EUR | 0.567 | 0.429 | -0.561 | 0.245 | -0.493 | 1.300 | 0.880 |
| GBP | 0.334 | 0.380 | -0.154 | -0.871 | -0.444 | 1.077 | 0.895 |
| JPY | -0.449 | 0.325 | 0.593 | -0.459 | -0.978 | 0.300 | 0.859 |
| KRW | 0.639 | 0.785 | 1.667 | 2.999 | -0.182 | 3.387 | 0.772 |
| NOK | -0.596 | 0.410 | -0.289 | 0.131 | -1.474 | 0.323 | 0.732 |
| PLN | -1.593 | 0.634 | 0.572 | -0.553 | -2.554 | -0.269 | 0.839 |
| SEK | 0.414 | 0.589 | -0.006 | 0.484 | -0.907 | 1.885 | 0.842 |
| Average | -0.050 | 0.507 | 0.369 | 0.270 | -0.934 | 1.198 | 0.830 |
| Full panel | -0.050 | 0.903 | -0.519 | 0.562 | -2.554 | 3.387 | |

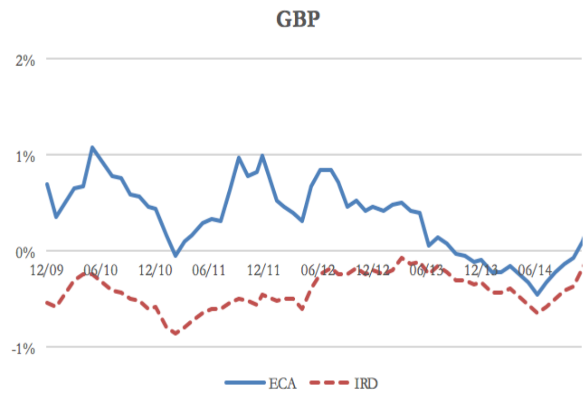

AUD
1%
0%
-1%
-2%
-3%
-4%
-5%
12/09
06/10
12/10
06/11
12/11
06/12
12/12
06/13
12/13
06/14
ECA
IRD


CAD
2%
1%
0%
-1%
-2%
12/09
06/10
12/10
06/11
12/11
06/12
12/12
06/13
12/13
06/14
ECA
IRD


CHF
3%
2%
1%
0%
12/09
06/10
12/10
06/11
12/11
06/12
12/12
06/13
12/13
06/14
ECA
IRD


DKK
2%
1%
0%
-1%
-2%
12/09
06/10
12/10
06/11
12/11
06/12
12/12
06/13
12/13
06/14
ECA
IRD


EUR
2%
1%
0%
-1%
-2%
12/09
06/10
12/10
06/11
12/11
06/12
12/12
06/13
12/13
06/14
ECA
IRD


GBP
2%
1%
0%
-1%
12/09
06/10
12/10
06/11
12/11
06/12
12/12
06/13
12/13
06/14
ECA
IRD

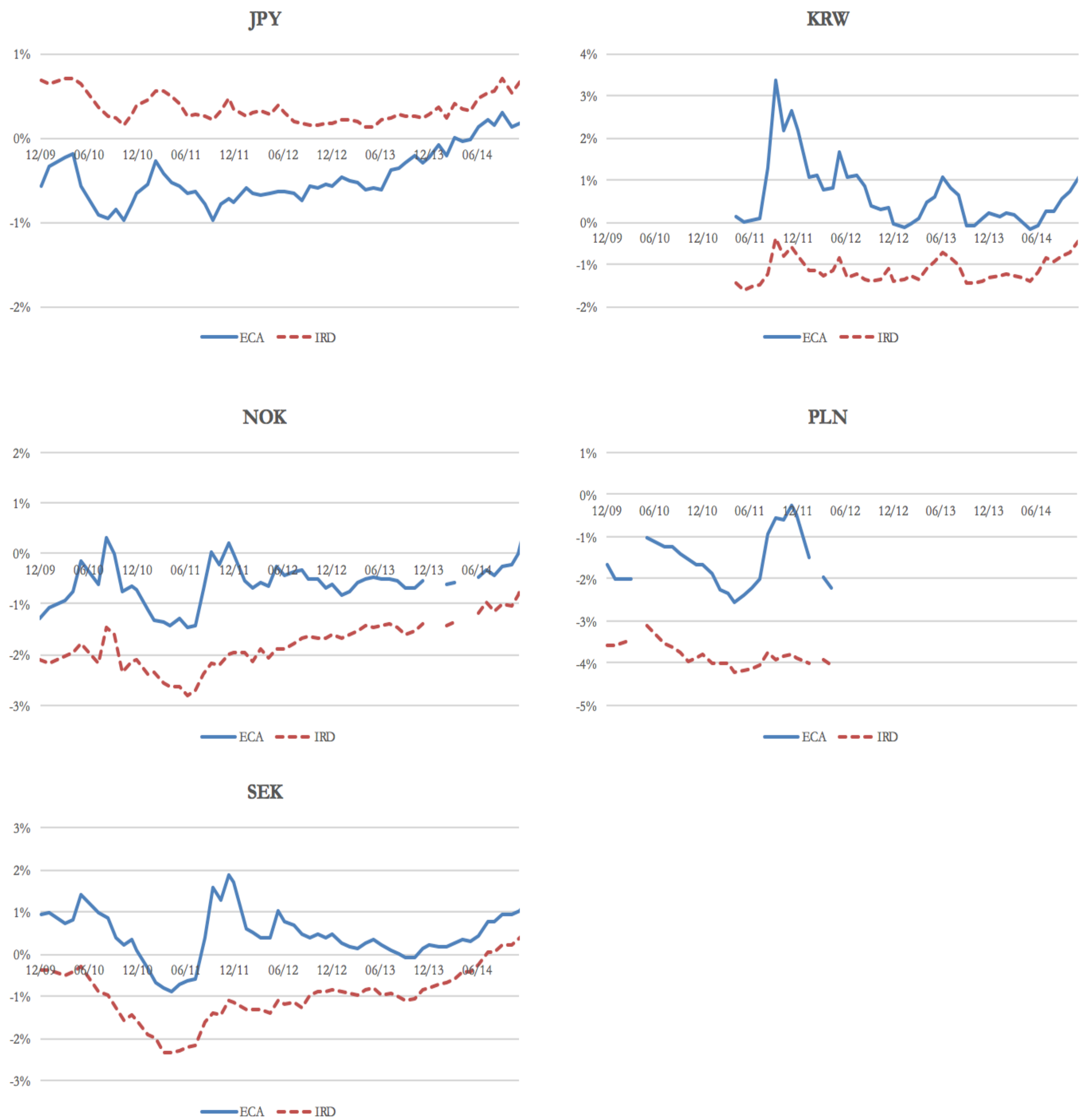



Figure 1: Time series of annualized expected currency appreciation implied by the quanto theory (ECA) and by UIP (IRD).

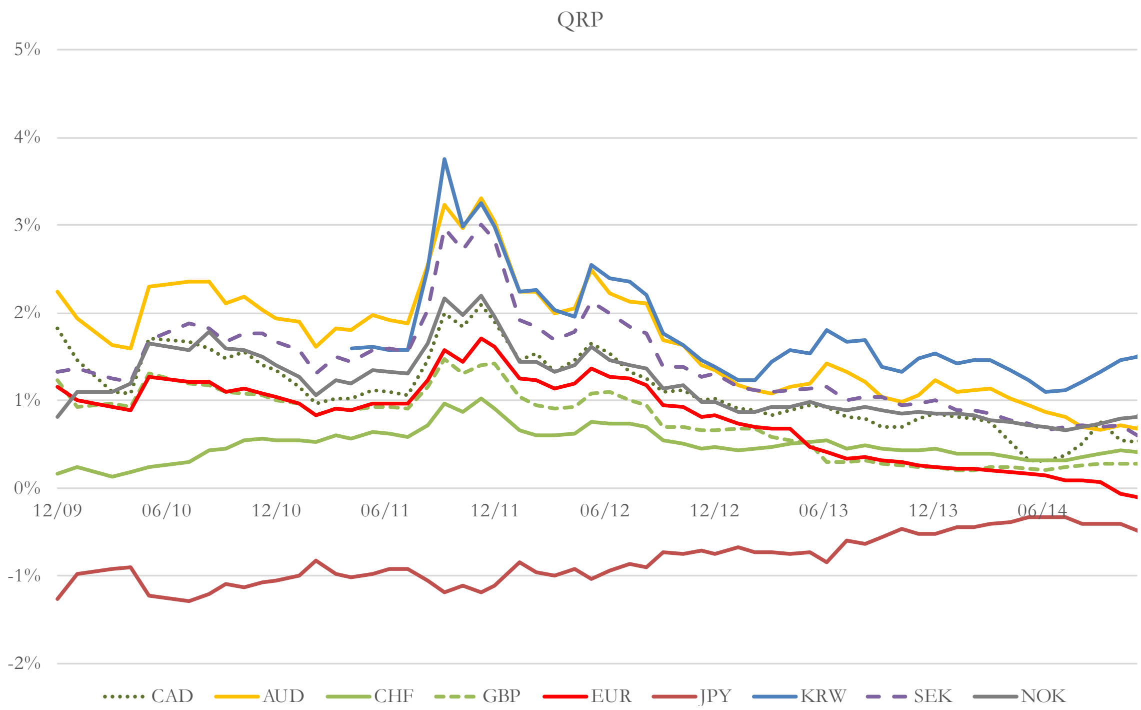



Figure 2: Currency risk premia, as measured by QRP, in the time series. For clarity, the figure drops two currencies for which we have highly incomplete time series (PLN and DKK).

Table 2: Summary statistics of IRD and QRP

This table reports annualized summary statistics (in %) of UIP forecasts (IRD, top panel), and quanto risk premia (QRP, bottom).

| **IRD** | **Mean** | **Std Dev.** | **Skew** | **Kurtosis** | **Min** | **Max** | **Autocorr.** |
|---|---|---|---|---|---|---|---|
| AUD | -3.119 | 0.820 | -0.337 | -1.490 | -4.533 | -2.000 | 0.967 |
| CAD | -0.835 | 0.215 | 1.220 | 1.228 | -1.133 | -0.168 | 0.694 |
| CHF | 0.386 | 0.173 | 0.235 | 0.132 | 0.013 | 0.815 | 0.838 |
| DKK | -0.821 | 0.470 | 0.298 | -0.794 | -1.596 | 0.005 | 0.915 |
| EUR | -0.228 | 0.521 | -0.578 | -0.288 | -1.377 | 0.739 | 0.965 |
| GBP | -0.413 | 0.188 | -0.245 | -0.574 | -0.865 | -0.078 | 0.885 |
| JPY | 0.350 | 0.166 | 0.811 | -0.431 | 0.133 | 0.706 | 0.860 |
| KRW | -1.146 | 0.297 | 0.822 | -0.044 | -1.614 | -0.366 | 0.705 |
| NOK | -1.812 | 0.470 | -0.064 | -0.480 | -2.798 | -0.812 | 0.911 |
| PLN | -3.797 | 0.306 | 1.186 | 1.554 | -4.215 | -2.936 | 0.761 |
| SEK | -1.014 | 0.634 | -0.088 | 0.126 | -2.354 | 0.399 | 0.961 |
| Average | -1.132 | 0.387 | 0.296 | -0.096 | -1.849 | -0.336 | 0.860 |
| Full panel | -1.132 | 1.250 | -1.001 | 0.442 | -4.533 | 0.815 | |

| **QRP** | **Mean** | **Std Dev.** | **Skew** | **Kurtosis** | **Min** | **Max** | **Autocorr.** |
|---|---|---|---|---|---|---|---|
| AUD | 1.744 | 0.653 | 0.384 | -0.293 | 0.666 | 3.306 | 0.922 |
| CAD | 1.125 | 0.432 | 0.252 | -0.506 | 0.309 | 2.090 | 0.909 |
| CHF | 0.507 | 0.185 | 0.571 | 0.753 | 0.131 | 1.023 | 0.909 |
| DKK | 1.153 | 0.275 | 0.400 | 0.336 | 0.643 | 1.768 | 0.788 |
| EUR | 0.795 | 0.470 | -0.237 | -1.007 | -0.106 | 1.708 | 0.968 |
| GBP | 0.747 | 0.387 | -0.061 | -1.262 | 0.207 | 1.472 | 0.952 |
| JPY | -0.800 | 0.282 | 0.190 | -1.130 | -1.287 | -0.329 | 0.851 |
| KRW | 1.786 | 0.603 | 1.507 | 1.958 | 1.097 | 3.752 | 0.846 |
| NOK | 1.216 | 0.385 | 0.700 | -0.078 | 0.665 | 2.194 | 0.891 |
| PLN | 2.204 | 0.607 | 0.932 | 0.027 | 1.435 | 3.509 | 0.852 |
| SEK | 1.428 | 0.568 | 0.943 | 0.964 | 0.606 | 3.004 | 0.919 |
| Average | 1.082 | 0.441 | 0.507 | -0.022 | 0.397 | 2.136 | 0.892 |
| Full panel | 1.082 | 0.883 | -0.239 | 0.687 | -1.287 | 3.752 | |

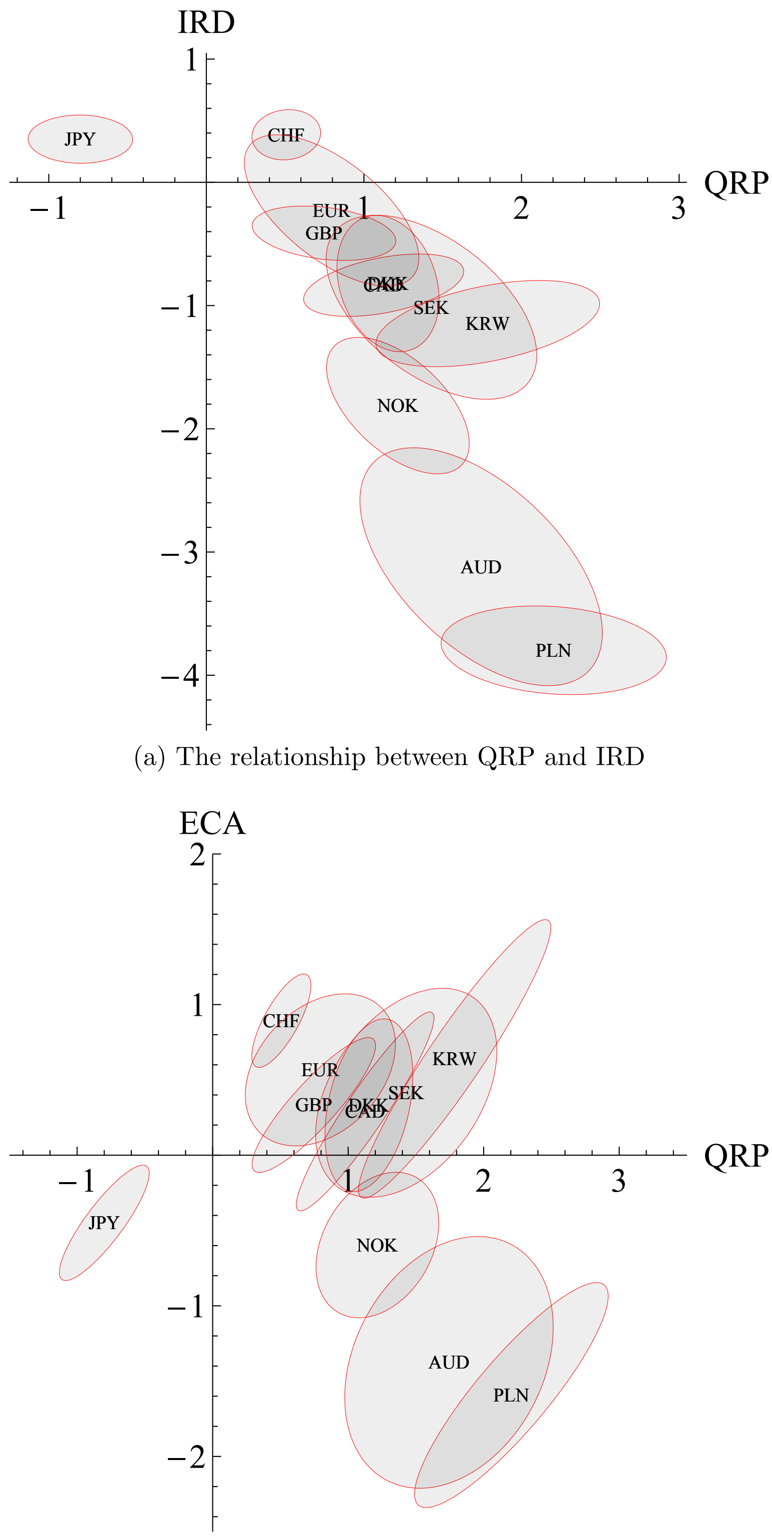
Table 3: Volatilities and correlations of ECA, IRD, and QRP

This Table presents the standard deviations (in %) of, and correlations between, the interest-rate differential (IRD), the quanto risk premium (QRP), and expected currency appreciation (ECA), calculated from (14) for each currency $i$:

$$\begin{aligned} \mathrm{IRD}_{i,t} &= \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} - 1 \\ \mathrm{QRP}_{i,t} &= \frac{Q_{i,t} - F_t}{R^{i}_{f,t} P_t} \\ \mathrm{ECA}_{i,t} &= \mathrm{QRP}_{i,t} + \mathrm{IRD}_{i,t}. \end{aligned}$$

The row labelled "Time series" reports means of the currencies' time-series standard deviations and correlations. The row labelled "Cross section" reports cross-sectional standard deviations and correlations of time-averaged ECA, IRD, and QRP. The row labelled "Pooled" reports standard deviations and correlations of the pooled data. All quantities are expressed in annualized terms.

| | $\sigma$(ECA) | $\sigma$(IRD) | $\sigma$(QRP) | $\rho$(ECA, IRD) | $\rho$(ECA, QRP) | $\rho$(IRD, QRP) |
|---|---|---|---|---|---|---|
| AUD | 0.710 | 0.820 | 0.653 | 0.644 | 0.279 | -0.556 |
| CAD | 0.561 | 0.215 | 0.432 | 0.723 | 0.939 | 0.441 |
| CHF | 0.263 | 0.173 | 0.185 | 0.713 | 0.756 | 0.079 |
| DKK | 0.487 | 0.470 | 0.275 | 0.835 | 0.342 | -0.231 |
| EUR | 0.429 | 0.521 | 0.470 | 0.524 | 0.330 | -0.631 |
| GBP | 0.380 | 0.188 | 0.387 | 0.209 | 0.881 | -0.279 |
| JPY | 0.322 | 0.166 | 0.276 | 0.515 | 0.856 | -0.002 |
| KRW | 0.785 | 0.297 | 0.603 | 0.732 | 0.942 | 0.460 |
| NOK | 0.410 | 0.470 | 0.385 | 0.626 | 0.302 | -0.555 |
| PLN | 0.634 | 0.306 | 0.607 | 0.329 | 0.879 | -0.160 |
| SEK | 0.589 | 0.634 | 0.568 | 0.572 | 0.399 | -0.524 |
| Time series | 0.506 | 0.387 | 0.440 | 0.584 | 0.628 | -0.178 |
| Cross section | 0.835 | 1.325 | 0.803 | 0.817 | -0.308 | -0.800 |
| Pooled | 0.904 | 1.250 | 0.885 | 0.706 | 0.024 | -0.691 |

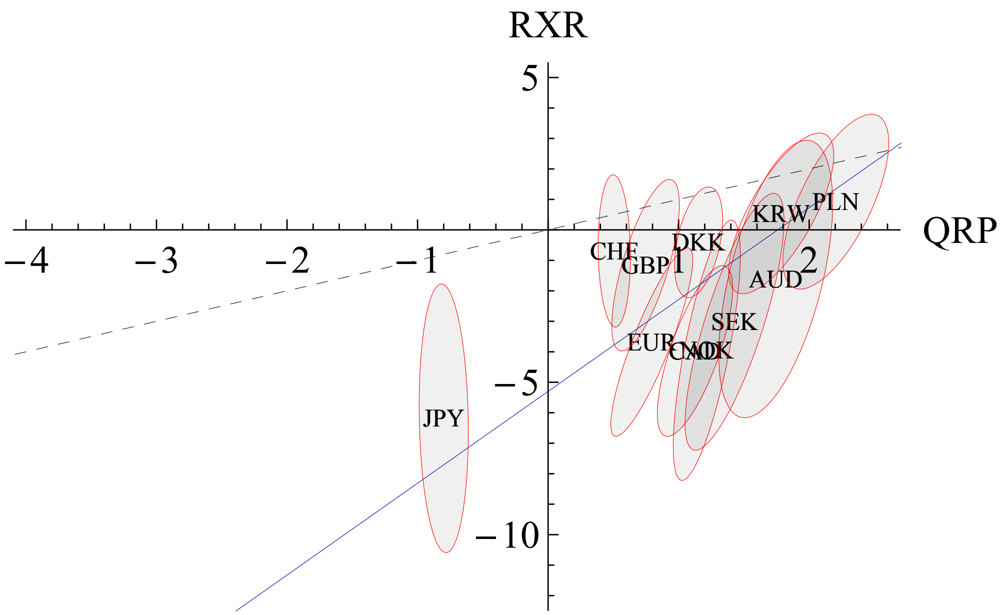



(a) The relationship between QRP and IRD

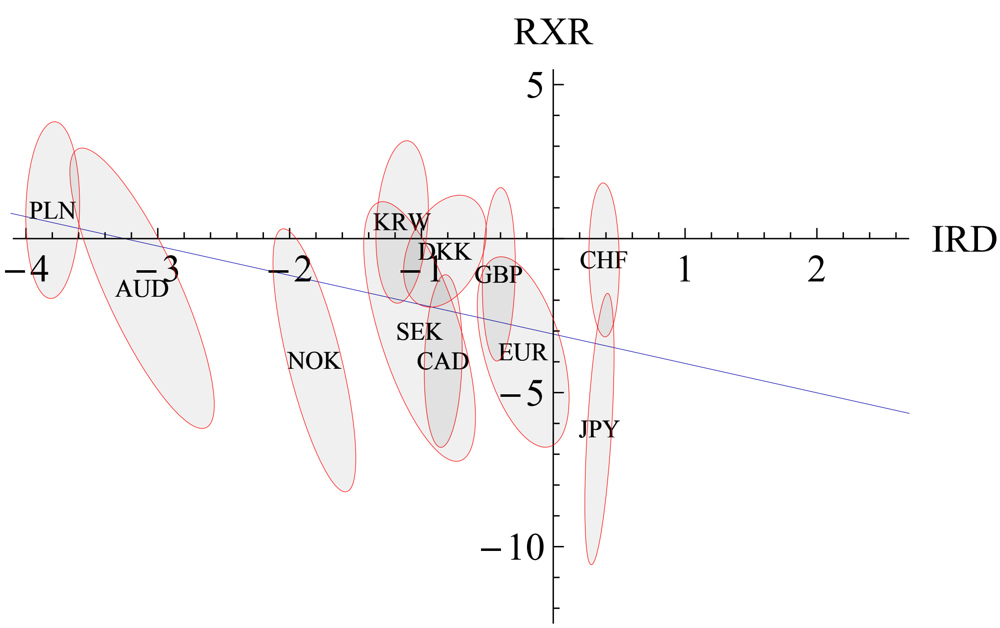


(b) The relationship between QRP and ECA

Figure 3: QRP plotted against IRD and against ECA. The figures plot each currency at its mean QRP and IRD (or ECA), surrounded by a confidence ellipse that would contain 50% of that currency's data points under Normality. The orientation of each ellipse reflects the time-series correlation between QRP and IRD (or ECA) for that currency, while the size reflects the volatilities of the two measures.

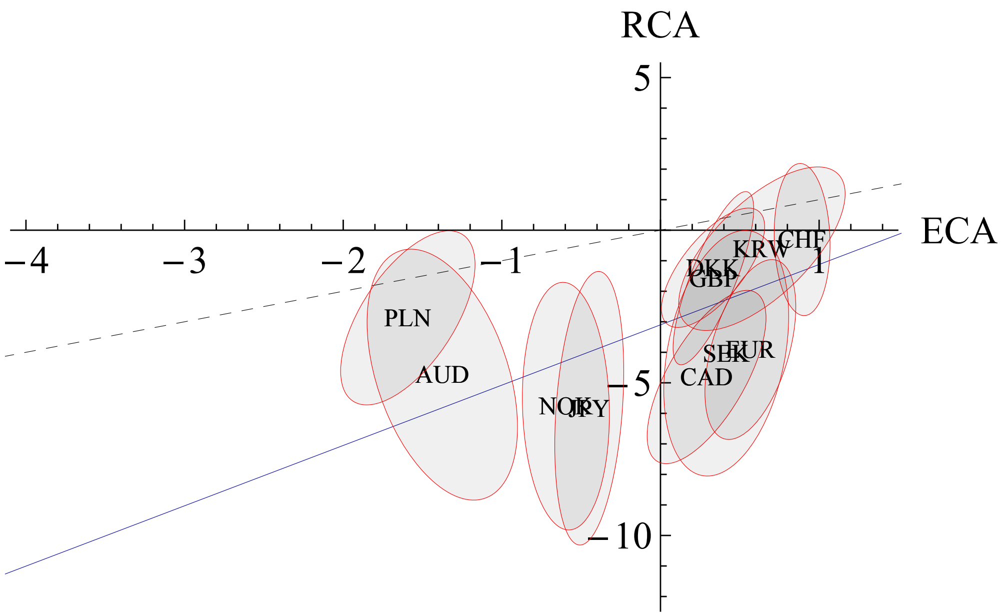



(a) Realized currency excess return against QRP, computed from (14)

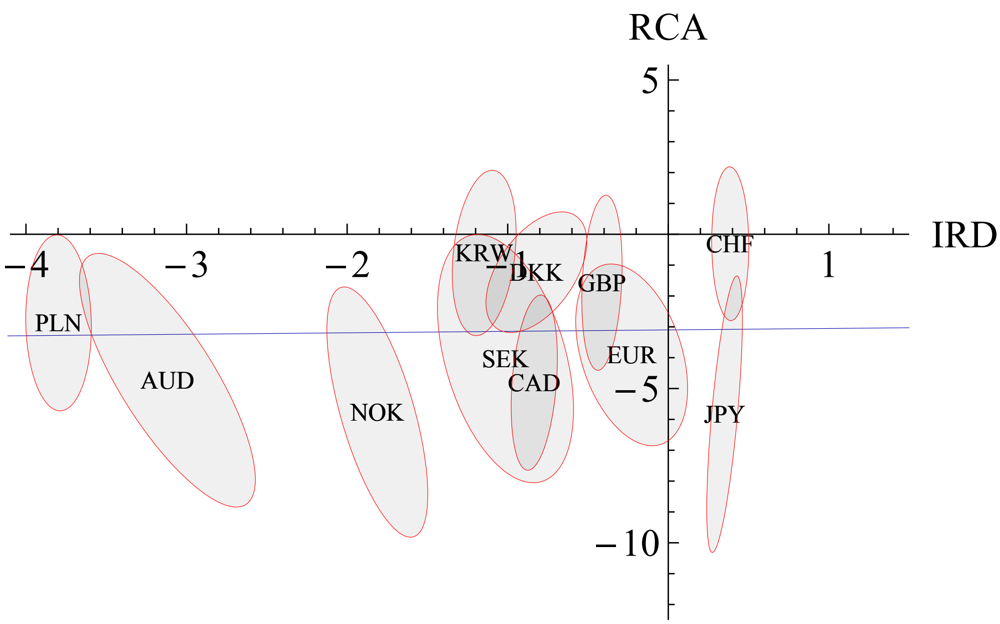



(b) Realized currency excess return against IRD

Figure 4: Realized and expected currency excess return according to (a) the quanto theory and (b) UIP. The centre of each confidence ellipse represents a currency's mean expected and realized currency excess return. In population, each ellipse would contain 20% of its currency's data points under Normality. The orientation of each ellipse reflects the time-series correlation between realized and forecast appreciation for the given currency, while the ellipse's size reflects their volatilities. The dotted 45° line in panel (a) indicates the prediction of the quanto theory.




(a) Realized currency appreciation against ECA, computed from (14)




(b) Realized currency appreciation against IRD

Figure 5: Realized and expected currency appreciation according to (a) the quanto theory and (b) UIP. The centre of each confidence ellipse represents a currency's mean expected and realized currency appreciation. In population, each ellipse would contain 20% of its currency's data points under Normality. The orientation of each ellipse reflects the time-series correlation between realized and forecast appreciation for the given currency, while the ellipse's size reflects their volatilities. The dotted 45° line in panel (a) indicates the prediction of the quanto theory.

## Table 4: Currency excess return forecasting regressions

This Table presents results from three currency excess return forecasting regressions:

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \beta \frac{Q_{i,t} - F_t}{R^i_{f,t} P_t} + \gamma \left( \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} - 1 \right) + \varepsilon_{i,t+1} \tag{15}$$

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \beta \frac{Q_{i,t} - F_t}{R^i_{f,t} P_t} + \varepsilon_{i,t+1} \tag{16}$$

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \gamma \left( \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} - 1 \right) + \varepsilon_{i,t+1} \tag{17}$$

The top panel reports coefficient estimates for each regression, with standard errors (computed using a block bootstrap) in brackets. The second panel reports $R^2$ (in %). The bottom three panels report $p$-values of Wald tests of various hypotheses on the regression coefficients. $H_0^1$ is the hypothesis suggested by Result 2. Hypothesis $H_0^2$ drops the constraint that $\alpha = 0$, and therefore tests our model's ability to predict differences in currency returns but not its ability to predict the absolute level of (dollar) returns. Hypothesis $H_0^3$ is that the right-hand side variables are not useful for forecasting.

| | pooled | | | currency fixed effects | | |
|---|---|---|---|---|---|---|
| Regression | (15) | (16) | (17) | (15) | (16) | (17) |
| $\alpha$ (p.a.) | -0.054 (0.019) | -0.054 (0.019) | -0.035 (0.014) | | | |
| $\beta$ | 3.752 (1.656) | 2.995 (1.069) | | 5.879 (1.439) | 6.597 (1.434) | |
| $\gamma$ | 0.773 (1.057) | | -1.061 (0.643) | -2.190 (1.462) | | -4.259 (1.718) |
| $R^2$ | 24.50 | 22.90 | 5.73 | 36.69 | 33.63 | 13.06 |
| $H_0^1$: $\alpha = \gamma = 0, \beta = 1$ | 0.035 | | | | | |
| $H_0^1$: $\alpha = 0, \beta = 1$ | | 0.015 | | | | |
| $H_0^2$: $\beta = 1, \gamma = 0$ | 0.195 | | | 0.000 | | |
| $H_0^2$: $\beta = 1$ | | 0.062 | | | 0.000 | |
| $H_0^3$: $\beta = 0, \gamma = 0$ | 0.030 | | | 0.000 | | |
| $H_0^3$: $\beta = 0$ | | 0.005 | | | 0.000 | |
| $H_0^3$: $\gamma = 0$ | | | 0.099 | | | 0.013 |

## Table 5: Currency forecasting regressions

This Table presents results from three currency forecasting regressions:

$$\frac{e_{i,t+1}}{e_{i,t}} - 1 = \alpha + \beta \frac{Q_{i,t} - F_t}{R^i_{f,t} P_t} + \gamma \left( \frac{R^\$_{f,t}}{R^i_{f,t}} - 1 \right) + \varepsilon_{i,t+1} \tag{18}$$

$$\frac{e_{i,t+1}}{e_{i,t}} - 1 = \alpha + \beta \frac{Q_{i,t} - F_t}{R^i_{f,t} P_t} + \varepsilon_{i,t+1} \tag{19}$$

$$\frac{e_{i,t+1}}{e_{i,t}} - 1 = \alpha + \gamma \left( \frac{R^\$_{f,t}}{R^i_{f,t}} - 1 \right) + \varepsilon_{i,t+1} \tag{20}$$

The top panel reports coefficient estimates for each regression, with standard errors (computed using a block bootstrap) in brackets. The second panel reports $R^2$ (in %). The bottom three panels report $p$-values of Wald tests of various hypotheses on the regression coefficients. $H_0^1$ is the hypothesis suggested by Result 2 (with the constraints $\gamma = 1$ and $\beta = 1$). Hypothesis $H_0^2$ drops the constraint that $\alpha = 0$, and therefore tests our model's ability to predict differences in currency returns but not its ability to predict the absolute level of (dollar) returns. Hypothesis $H_0^3$ is that the right-hand side variables are not useful for forecasting.

| | pooled | | | currency fixed effects | | |
|---|---|---|---|---|---|---|
| Regression | (18) | (19) | (20) | (18) | (19) | (20) |
| $\alpha$ (p.a.) | -0.054 (0.019) | -0.054 (0.019) | -0.035 (0.014) | | | |
| $\beta$ | 3.752 (1.652) | 2.019 (1.096) | | 5.879 (1.440) | 6.269 (1.300) | |
| $\gamma$ | 1.776 (1.061) | | -0.061 (0.639) | -1.190 (1.465) | | -3.259 (1.712) |
| $R^2$ | 19.93 | 11.04 | 0.02 | 33.06 | 32.11 | 8.09 |
| $H_0^1$: $\alpha = 0, \beta = \gamma = 1$ | 0.034 | | | | | |
| $H_0^2$: $\beta = 1, \gamma = 1$ | 0.193 | | | 0.000 | | |
| $H_0^3$: $\beta = 0, \gamma = 0$ | 0.076 | | | 0.000 | | |
| $H_0^3$: $\beta = 0$ | | 0.065 | | | 0.000 | |
| $H_0^3$: $\gamma = 0$ | | | 0.924 | | | 0.057 |

Table 6: Realized covariance regressions

This Table presents results of regressions using the lagged realized covariance of exchange rate movements with the negative reciprocal of the S&P 500 return (RPCL):

$$\mathrm{RPCL}_{i,t} = R^{\$}_{f,t}\left(\sum\left[\frac{e_{i,s+1}}{e_{i,s}}\left(-\frac{1}{R_{s+1}}\right)\right] - \frac{1}{T-t}\sum\left(-\frac{1}{R_{s+1}}\right)\sum\frac{e_{i,s+1}}{e_{i,s}}\right),$$

where the summation is over daily returns on trading days $s$ preceding $t$ over a time-frame corresponding to our forecasting horizon $T-t$. We also define the realized covariance measure $\mathrm{RPC}_{i,t}$, which is analogous to the above definition except that the summation is over trading days *following* $t$ over the appropriate time-frame.

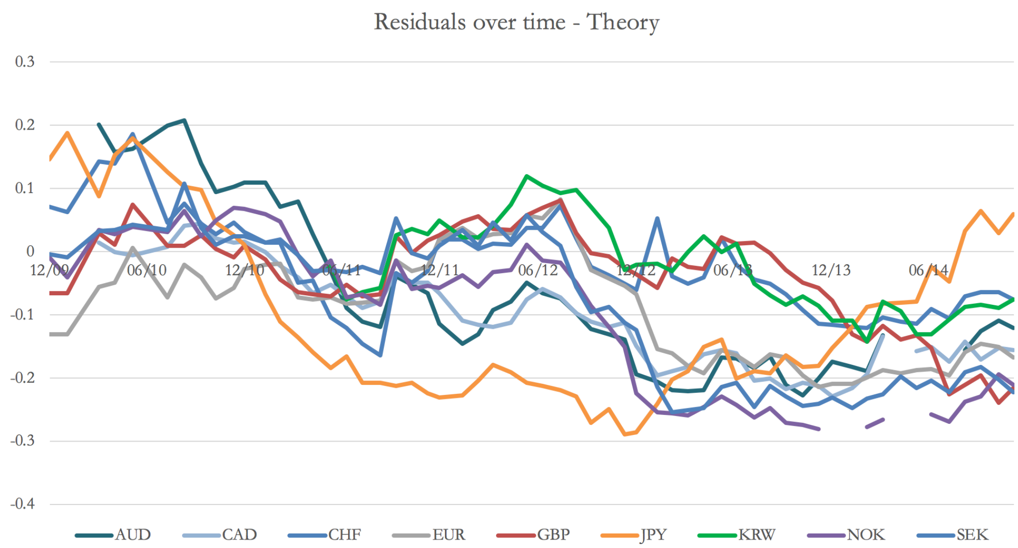
$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \gamma \mathrm{RPCL}_{i,t} + \varepsilon_{i,t+1} \tag{22}$$

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \beta\frac{Q_{i,t}-F_t}{R^{i}_{f,t}P_t} + \gamma \mathrm{RPCL}_{i,t} + \varepsilon_{i,t+1} \tag{23}$$

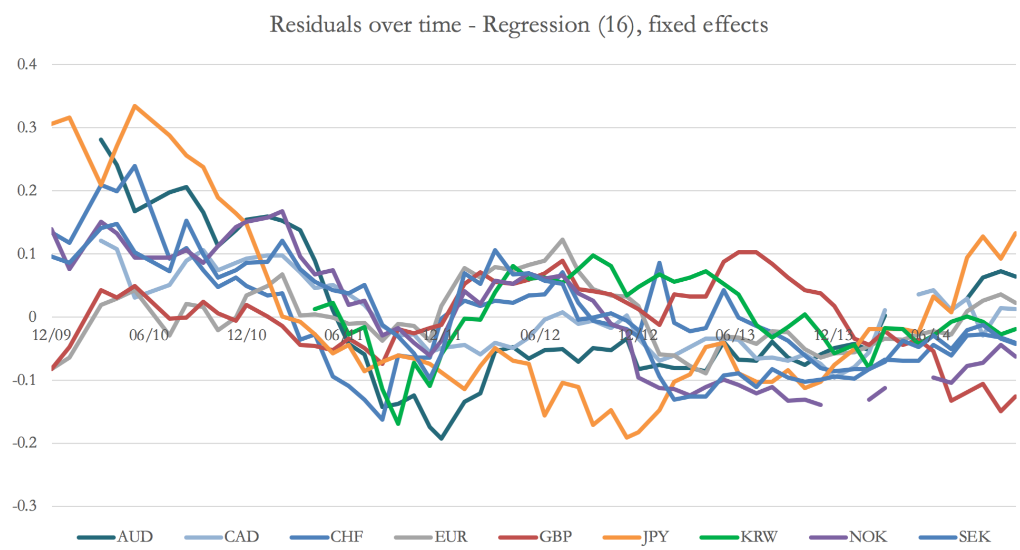
$$\mathrm{RPC}_{i,t} = \alpha + \beta\frac{Q_{i,t}-F_t}{R^{i}_{f,t}P_t} + \varepsilon_{i,t+1} \tag{24}$$

Our data runs for 59 months from December 2009 to November 2014, forming an unbalanced panel of 24-month quantos on the S&P 500 index in the Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), Danish krona (DKK), Euro (EUR), British pound (GBP), Japanese yen (JPY), Korean won (KRW), Norwegian krone (NOK), Polish zloty (PLN), and Swedish krona (SEK). The top panel reports coefficient estimates for each regression, with standard errors (computed using a block bootstrap) in brackets. The second panel reports adjusted $R^2$. The bottom three panels report $p$-values of Wald tests of various joint hypotheses on the coefficients.

| | pooled | | | currency fixed effects | | |
|---|---|---|---|---|---|---|
| Regression | (22) | (23) | (24) | (22) | (23) | (24) |
| $\alpha$ (p.a.) | -0.039 (0.016) | -0.054 (0.018) | 0.000 (0.002) | | | |
| $\beta$ | | 3.071 (1.438) | 0.432 (0.164) | | 5.743 (1.613) | 0.297 (0.190) |
| $\gamma$ | 1.578 (0.949) | -0.085 (1.175) | | 2.817 (1.612) | 1.160 (1.649) | |
| $R^2$ | 10.63 | 22.92 | 33.68 | 15.65 | 35.72 | 8.03 |
| $H_0^1$: $\alpha=\gamma=0, \beta=1$ | | 0.030 | | | | |
| $H_0^1$: $\alpha=0, \gamma=1$ | 0.033 | | | | | |
| $H_0^2$: $\beta=1, \gamma=0$ | | 0.147 | | | 0.000 | |
| $H_0^2$: $\gamma=1$ | 0.542 | | | 0.260 | | |
| $H_0^3$: $\beta=\gamma=0$ | | 0.013 | | | 0.000 | |
| $H_0^3$: $\beta=0$ | | | 0.009 | | | 0.119 |
| $H_0^3$: $\gamma=0$ | 0.096 | | | 0.080 | | |




(a) Theory residuals




(b) Regression residuals

Figure 6: The time series of residuals by currency.

Table 7: Principal components analysis of residuals

This table reports the loadings on the principal components of realized residuals obtained from the quanto theory (top panel) and the fixed-effects specification of regression (16) (bottom panel). In order to limit the impact of missing observations, the residuals are only obtained for the balanced panel of currencies (excluding DKK, KRW, and PLN).

| **Theory** | **PC1** | **PC2** | **PC3** | **PC4** | **PC5** | **PC6** | **PC7** | **PC8** |
|---|---|---|---|---|---|---|---|---|
| **AUD** | 0.515 | 0.224 | -0.059 | -0.497 | -0.343 | -0.386 | -0.234 | 0.336 |
| **CAD** | 0.306 | -0.011 | -0.153 | -0.242 | 0.011 | 0.152 | 0.895 | -0.036 |
| **CHF** | 0.216 | 0.016 | 0.697 | 0.213 | -0.514 | -0.020 | 0.097 | -0.384 |
| **EUR** | 0.233 | -0.370 | 0.034 | 0.574 | -0.111 | -0.029 | 0.110 | 0.674 |
| **GBP** | 0.132 | -0.325 | 0.609 | -0.384 | 0.562 | 0.084 | -0.063 | 0.179 |
| **JPY** | 0.308 | 0.765 | 0.137 | 0.339 | 0.406 | 0.115 | -0.001 | 0.090 |
| **NOK** | 0.484 | -0.191 | -0.227 | -0.019 | -0.082 | 0.724 | -0.341 | -0.172 |
| **SEK** | 0.440 | -0.293 | -0.213 | 0.240 | 0.343 | -0.531 | -0.059 | -0.464 |
| **Explained** | 66.33% | 23.88% | 5.64% | 2.20% | 0.90% | 0.41% | 0.36% | 0.27% |

| **Regression** | **PC1** | **PC2** | **PC3** | **PC4** | **PC5** | **PC6** | **PC7** | **PC8** |
|---|---|---|---|---|---|---|---|---|
| **AUD** | 0.514 | -0.122 | 0.024 | 0.742 | -0.197 | 0.052 | -0.216 | 0.285 |
| **CAD** | 0.229 | -0.079 | -0.226 | 0.308 | -0.136 | 0.021 | -0.590 | 0.655 |
| **CHF** | 0.273 | -0.132 | 0.688 | -0.099 | 0.275 | 0.559 | -0.020 | 0.194 |
| **EUR** | 0.058 | -0.380 | -0.034 | -0.155 | 0.701 | -0.396 | -0.379 | -0.188 |
| **GBP** | -0.053 | -0.282 | 0.620 | 0.190 | -0.436 | -0.547 | -0.077 | -0.032 |
| **JPY** | 0.608 | 0.532 | 0.082 | -0.472 | -0.108 | -0.299 | -0.129 | -0.027 |
| **NOK** | 0.366 | -0.514 | -0.224 | -0.165 | -0.416 | 0.306 | -0.200 | -0.466 |
| **SEK** | 0.314 | -0.436 | -0.181 | -0.187 | 0.029 | -0.213 | 0.632 | 0.445 |
| **Explained** | 68.87% | 14.88% | 8.21% | 3.86% | 1.91% | 1.22% | 0.67% | 0.38% |

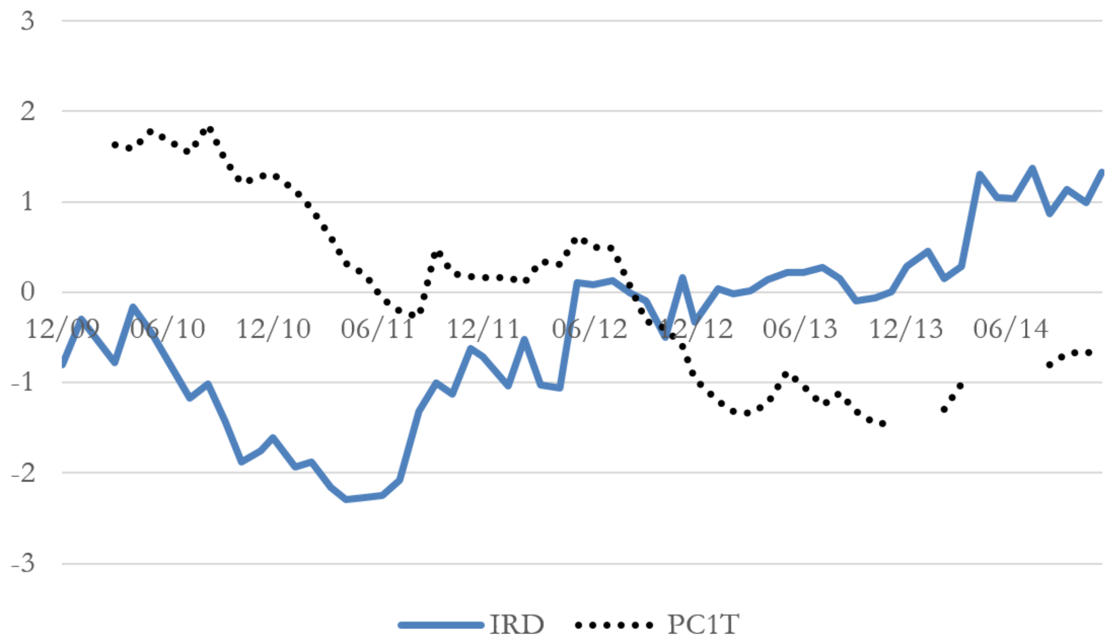



Figure 7: Time series of cross-sectional average of interest-differential ($\overline{\mathrm{IRD}}_t$) and of the first principal component of theory residuals (PC1T). All series are demeaned and scaled to have unit variance.

Table 8: The connection to the average forward discount, $\overline{\text{IRD}}_t$

This Table presents results from a currency excess return forecasting regression that extends the baseline results in Table 4:

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \beta \frac{Q_{i,t} - F_t}{R^{i}_{f,t} P_t} + \gamma \left( \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} - 1 \right) + \delta \left( \overline{\text{IRD}}_t \right) + \varepsilon_{i,t+1} \tag{25}$$

and, for reference, results from the regression

$$\frac{e_{i,t+1}}{e_{i,t}} - \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} = \alpha + \gamma \left( \frac{R^{\$}_{f,t}}{R^{i}_{f,t}} - 1 \right) + \delta \left( \overline{\text{IRD}}_t \right) + \varepsilon_{i,t+1}. \tag{26}$$

In order to account for a time-varying dollar factor, we augment our baseline regressions (15) and (17) by including the time-series of the cross-sectional averages of IRD, denoted by $\overline{\text{IRD}}_t$. To limit the impact of missing observations on the cross-sectional averages, the above regression is run on the panel of 24-month quantos on the S&P 500 index in the Australian dollar (AUD), Canadian dollar (CAD), Swiss franc (CHF), Euro (EUR), British pound (GBP), Japanese yen (JPY), Norwegian krone (NOK), and Swedish krona (SEK) (i.e., excluding DKK, KRW, and PLN). The top panel reports coefficient estimates for each regression, with standard errors (computed using a block bootstrap) in brackets. The second panel reports $R^2$ (in %). The last five panels report $p$-values of Wald tests of two different hypotheses.

| | pooled | | | | currency fixed effects | | | |
|---|---|---|---|---|---|---|---|---|
| Regression | (25) | | (26) | | (25) | | (26) | |
| $\alpha$ (p.a.) | -0.104 | (0.031) | -0.091 | (0.038) | | | | |
| $\beta$ | 3.437 | (1.649) | | | 5.796 | (1.499) | | |
| $\gamma$ | 1.268 | (1.470) | -0.460 | (1.023) | -1.224 | (2.342) | -2.915 | (1.944) |
| $\delta$ | -13.274 | (6.263) | -13.631 | (8.732) | -5.765 | (5.179) | -8.867 | (7.113) |
| $R^2$ | 30.97 | | 17.52 | | 38.50 | | 21.39 | |
| $H_0^1$: $\beta = 1, \gamma = \delta = 0$ | 0.056 | | | | 0.000 | | | |
| $H_0^2$: $\beta = \gamma = \delta = 0$ | 0.010 | | | | 0.000 | | | |
| $H_0^3$: $\gamma = \delta = 0$ | | | 0.094 | | | | 0.053 | |
| $H_0^4$: $\gamma = 0$ | | | 0.653 | | | | 0.134 | |
| $H_0^5$: $\delta = 0$ | | | 0.119 | | | | 0.213 | |

## Table 9: Out-of-sample forecast performance

We define a dollar-neutral out-of-sample $R^2$ similar to Goyal and Welch (2008):

$$R^2_{OS} = 1 - \frac{\sum_i \sum_j \sum_t (\varepsilon^Q_{i,t+1} - \varepsilon^Q_{j,t+1})^2}{\sum_i \sum_j \sum_t (\varepsilon^B_{i,t+1} - \varepsilon^B_{j,t+1})^2},$$

where $\varepsilon^Q_{i,t+1}$ and $\varepsilon^B_{i,t+1}$ denote forecast errors (for currency $i$ against the dollar) of the quanto theory and the benchmark, respectively. We use the quanto theory and three competitor benchmarks to forecast currency appreciation as follows:

$$\begin{aligned}
\text{Theory: } \mathbb{E}^Q_t \frac{e_{i,t+1}}{e_{i,t}} - 1 &= \frac{Q_{i,t} - F_t}{R^i_{f,t} P_t} + \frac{R^\$_{f,t}}{R^i_{f,t}} - 1 \\
\text{UIP: } \mathbb{E}^U_t \frac{e_{i,t+1}}{e_{i,t}} - 1 &= \text{IRD}_{i,t} = \frac{R^\$_{f,t}}{R^i_{f,t}} - 1 \\
\text{Constant: } \mathbb{E}^C_t \frac{e_{i,t+1}}{e_{i,t}} - 1 &= 0 \\
\text{PPP: } \mathbb{E}^P_t \frac{e_{i,t+1}}{e_{i,t}} - 1 &= \left(\frac{\pi^\$_t}{\pi^i_t}\right)^2 - 1
\end{aligned}$$

We also report results for the following decomposition of $R^2_{OS}$ that is also dollar-neutral:

$$R^2_{OS,i} = 1 - \frac{\sum_j \sum_t (\varepsilon^Q_{i,t+1} - \varepsilon^Q_{j,t+1})^2}{\sum_j \sum_t (\varepsilon^B_{i,t+1} - \varepsilon^B_{j,t+1})^2}.$$

The last three columns report measures based on absolute rather than squared errors to diminish the impact of outliers. To further illustrate the performance of our approach relative to the benchmarks, the second panel reports the same measure for the IRD and PPP models relative to the constant benchmark. All results are reported in %.

| | Squared errors | | | Absolute errors | | |
|---|---|---|---|---|---|---|
| **Benchmark** | **IRD** | **Constant** | **PPP** | **IRD** | **Constant** | **PPP** |
| $R^2_{OS}$ | 12.64 | 11.39 | 27.37 | 5.08 | 5.33 | 14.52 |
| $R^2_{OS,AUD}$ | 10.25 | 5.09 | 12.29 | 4.02 | 1.00 | 5.40 |
| $R^2_{OS,CAD}$ | 7.38 | 7.76 | 21.84 | 2.70 | 4.21 | 10.75 |
| $R^2_{OS,CHF}$ | 1.81 | 19.68 | 11.17 | 1.78 | 8.24 | 10.32 |
| $R^2_{OS,DKK}$ | 10.01 | 7.49 | 23.18 | 4.32 | 4.65 | 14.17 |
| $R^2_{OS,EUR}$ | 9.52 | 6.42 | 25.89 | 4.11 | 4.25 | 16.08 |
| $R^2_{OS,GBP}$ | 2.57 | 10.23 | 36.49 | 0.29 | 4.38 | 21.13 |
| $R^2_{OS,JPY}$ | 22.92 | 11.64 | 35.61 | 10.49 | 4.37 | 18.23 |
| $R^2_{OS,KRW}$ | 23.16 | 18.51 | 35.97 | 13.74 | 10.55 | 21.62 |
| $R^2_{OS,NOK}$ | 4.73 | 14.19 | 19.98 | 1.11 | 7.28 | 9.53 |
| $R^2_{OS,PLN}$ | 13.54 | 9.07 | 19.31 | 4.46 | 8.93 | 11.32 |
| $R^2_{OS,SEK}$ | 8.89 | 7.58 | 29.48 | 3.19 | 2.42 | 15.27 |

Table 10: Separate return forecasting regressions using quanto predictor

In addition to the pooled regressions in Tables 4 and 5, we also run regressions (16) and (18) separately for each currency at the 24-month horizon, and at 6- and 12-month horizons for the euro. We report the OLS estimates along with Hansen-Hodrick standard errors. Adjusted $R^2$ are reported in %.

| | Regression (16) | | | | | Regression (18) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Currency | $\alpha$ | | $\beta$ | | Adj. $R^2$ | $\alpha$ | | $\beta$ | | $\gamma$ | | Adj. $R^2$ |
| AUD | -0.23 | (0.14) | 5.68 | (3.77) | 28.26 | -0.47 | (0.16) | 1.28 | (2.02) | -5.31 | (2.19) | 61.06 |
| CAD | -0.24 | (0.07) | 7.13 | (2.76) | 52.91 | -0.29 | (0.11) | 7.64 | (3.18) | -1.31 | (2.79) | 54.38 |
| CHF | 0.00 | (0.08) | -1.50 | (7.04) | -1.23 | 0.01 | (0.08) | -1.38 | (7.16) | -0.59 | (5.89) | -3.04 |
| DKK | -0.10 | (0.04) | 4.13 | (1.72) | 14.57 | -0.08 | (0.04) | 5.25 | (1.26) | 3.86 | (1.67) | 44.95 |
| EUR | -0.21 | (0.04) | 8.58 | (2.26) | 75.64 | -0.22 | (0.04) | 9.47 | (1.89) | 2.27 | (1.56) | 74.09 |
| GBP | -0.14 | (0.06) | 7.70 | (3.72) | 49.20 | -0.08 | (0.07) | 8.86 | (3.00) | 9.61 | (3.98) | 62.91 |
| JPY | -0.16 | (0.19) | -2.22 | (10.79) | -0.87 | -0.34 | (0.17) | -2.04 | (7.95) | 27.68 | (8.90) | 45.97 |
| KRW | -0.16 | (0.07) | 4.76 | (1.74) | 51.73 | -0.24 | (0.14) | 5.35 | (2.18) | -1.61 | (2.60) | 55.21 |
| NOK | -0.38 | (0.12) | 12.23 | (4.60) | 53.32 | -0.49 | (0.15) | 8.38 | (2.48) | -4.69 | (2.76) | 61.23 |
| PLN | -0.19 | (0.06) | 4.75 | (0.94) | 42.91 | -0.20 | (0.16) | 4.74 | (0.88) | 0.83 | (2.03) | 39.83 |
| SEK | -0.31 | (0.09) | 8.77 | (2.87) | 61.58 | -0.31 | (0.09) | 8.14 | (2.24) | -0.07 | (1.50) | 58.03 |
| EUR (6m) | -0.02 | (0.03) | 3.70 | (6.26) | 1.31 | -0.03 | (0.03) | 10.01 | (7.20) | 11.45 | (8.45) | 11.07 |
| EUR (12m) | -0.07 | (0.05) | 6.36 | (5.53) | 16.40 | -0.09 | (0.04) | 12.92 | (4.77) | 11.99 | (4.88) | 43.04 |

Table 11: Separate return forecasting regressions using IRD predictor

In addition to the pooled regressions in Tables 4 and 5, we also run regressions (17) and (20) separately for each currency at the 24-month horizon, and at 6- and 12-month horizons for the euro. We report the OLS estimates along with Hansen-Hodrick standard errors. Adjusted $R^2$ are reported in %.

| | Regression (17) | | | | | Regression (20) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Currency | $\alpha$ | | $\gamma$ | | Adj. $R^2$ | $\alpha$ | | $\gamma$ | | Adj. $R^2$ |
| AUD | -0.46 | (0.16) | -6.88 | (2.38) | 67.81 | -0.46 | (0.16) | -5.88 | (2.38) | 60.52 |
| CAD | -0.01 | (0.06) | 4.45 | (2.66) | 3.45 | -0.01 | (0.06) | 5.45 | (2.66) | 5.90 |
| CHF | 0.00 | (0.07) | -1.71 | (5.73) | -1.15 | 0.00 | (0.07) | -0.71 | (5.73) | -1.68 |
| DKK | 0.03 | (0.05) | 2.15 | (2.04) | 10.79 | 0.03 | (0.05) | 3.15 | (2.04) | 22.97 |
| EUR | -0.09 | (0.05) | -4.12 | (3.56) | 20.11 | -0.09 | (0.05) | -3.12 | (3.56) | 12.03 |
| GBP | 0.01 | (0.08) | 3.50 | (7.70) | 0.69 | 0.01 | (0.08) | 4.50 | (7.70) | 2.24 |
| JPY | -0.31 | (0.10) | 26.72 | (9.00) | 44.32 | -0.31 | (0.10) | 27.72 | (9.00) | 46.18 |
| KRW | 0.07 | (0.06) | 2.38 | (1.90) | 0.92 | 0.07 | (0.06) | 3.38 | (1.90) | 4.06 |
| NOK | -0.42 | (0.16) | -9.48 | (4.42) | 47.73 | -0.42 | (0.16) | -8.48 | (4.42) | 42.10 |
| PLN | -0.11 | (0.20) | -1.68 | (2.85) | -2.52 | -0.11 | (0.20) | -0.68 | (2.85) | -3.76 |
| SEK | -0.16 | (0.08) | -4.89 | (3.25) | 22.81 | -0.16 | (0.08) | -3.89 | (3.25) | 15.29 |
| EUR (6m) | 0.00 | (0.02) | 2.63 | (7.38) | -0.67 | 0.00 | (0.02) | 3.63 | (7.38) | 0.44 |
| EUR (12m) | -0.02 | (0.04) | 1.87 | (6.35) | -0.59 | -0.02 | (0.04) | 2.87 | (6.35) | 1.17 |

## Disclaimer