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Abstract

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Credit Ratings and Market Information*

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Abstract

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1 Introduction

Credit rating agencies (CRAs) assess credit risk. One way they learn about credit risk is from a fundamental analysis of information conveyed to them by issuers. Other sources of information abound - the bond market, the credit default swap (CDS) market, media announcements, even equity analysis. How does this market information affect credit ratings? How do credit ratings affect the informativeness of the market? In this paper, we analyze the interaction between credit ratings and the market for credit risk.

In the model, a CRA receives an imperfect private signal about the quality of an asset. It decides how to rate this investment given that it will make more profits now from a higher rating, but may diminish its reputation if the investment proves to be of poor quality. This rating is released to the public, and investors may purchase the asset. A market for credit risk, which may represent the credit default swap market or a secondary market for the asset, then establishes a market price à la Kyle (1985): a speculator may acquire information to profit off of liquidity traders, and a market maker clears the market. Lastly, the asset payoffs are realized, leading to monetary payoffs for investors and a reputational payoff for the CRA.

The interaction between the CRA and the market price has two contrasting effects. More-accurate ratings decrease the informativeness of market trading since they diminish the speculator's incentives to acquire information (by decreasing mispricing). From this perspective, information revelation by the CRA and the speculator are strategic substitutes. On the other hand, more-informative trading increases the CRA's incentives to be accurate by increasing transparency about whether the CRA inflated ratings and, thus, augmenting the CRA's reputational costs. This demonstrates that information revelation by the CRA and the speculator are also strategic complements. This leads to a unique equilibrium.

If strong enough, the negative effect of accurate ratings on information acquisition in the market can lead to a perverse result - policies such as reputational sanctions (e.g., increased liability standards) that make ratings more accurate can reduce the total amount of information produced. From a different angle, our result that the market disciplines CRAs suggests that policies to increase market informativeness (such as increasing due diligence standards) reap informational benefits beyond the market itself.

We extend the model to allow for the CRA to react to the information produced by the market (and receive new information) by revising its ratings. This leads to several interesting results. First, the public market information must be sufficiently negative for the CRA to fully incorporate it. When it outweighs the CRA's best possible private signals, any other reaction would automatically reveal that the CRA had been acting strategically and would hurt the CRA's reputation. This also implies that the market does not always learn something from a rating revision.

Second, the CRA may suppress negative private information that it receives by not downgrading. If it inflated the initial rating, downgrading with probability one will make the CRA more likely to suffer a reputational loss. The CRA is more likely to not downgrade when others expect the investment to be good, implying that the CRA panders to protect its reputation. This could be problematic for investors - exactly when their beliefs are incorrect, the CRA provides less information. Lastly, the more rating inflation that investors expect, the less likely the CRA is to downgrade a bad-quality investment. This means that more initial rating inflation brings about more subsequent rating inflation. In other words, rating inflation is persistent.

The result that more-informative market trading leads to more-informative ratings has support in the literature. Ederington and Goh (1998) find that most bond downgrades are preceded by decreases in actual and forecast (by stock analysts) earnings. Fong, Hong, Kacperczyk, and Kubik (2014) show that ratings become less informative about defaults and future downgrades when equity analysis provides less information. Gopolan, Gopolan, and Koharki (2017) find that ratings of unlisted firms in India are higher, less sensitive to financial conditions, and contain less information about subsequent defaults than ratings of listed firms. Badoer and Demiroglu (2017) find that regulation that mandated disclosure about price and volume information on over-the-counter transactions of corporate debt makes ratings downgrades more sensitive to changes in credit spreads. Hull, Predescu, and White (2004), Norden and Weber (2004), Norden (2014), Finnerty, Miller, and Chen (2013), and Chava, Ganduri, and Orthanalai (2015) show that CDS spreads anticipate credit rating downgrades. Norden and Weber (2004) find that rating downgrades are anticipated by decreases in stock prices. Dilly and Mahlmann (2014) find that ratings for firms where there are actively traded CDS are more strongly correlated with bond spreads at issuance, are adjusted more quickly, and are better predictors of defaults. They also examine whether CDS markets discipline CRAs or provide additional information for them to use. They define issues based on whether measures of conflicts of interest are either high or low. CDS benchmarks have a larger positive impact on rating quality when measures of conflicts of interest are high. They find similar results when they compare the impact of CDS on issuer-pays ratings (i.e., S&P, Moody's, and Fitch) to investor-pays (Egan-Jones) ratings.

The result that more-informative ratings lead to less-informative market trading is less common in the literature. Chava, Ganduri, and Orthanalai (2015) find that ratings downgrades have a smaller impact on stock price (i) when CDS begin trading on a firm's debt; and (ii) for firms with traded CDS contracts versus those without traded CDS contracts. There are also findings that there is little relation between the two sources of information. Hull, Predescu, and White (2004) find that CDS spreads don't change in response to downgrades, while Norden and Weber (2004) find that only Moody's downgrades affect CDS spreads.

The model views CRA reputational concerns as arising from reduced future profits. Do CRAs suffer reputational losses? In the case of the structured finance market, the market (and the need for ratings) dried up as the crisis hit, and stock market valuations for Moody's fell significantly.[1] One might view Standard and Poors' recent settlement with the U.S. government and states for $1.5 billion as a reputational sanction.[2] In addition, a regulatory environment that is more or less friendly to the CRAs may also affect the CRAs' reputational incentives. Lastly, Arthur Andersen's implosion represents a severe punishment to a certification intermediary in a similar line of business (auditing).

Theoretical Literature

The link between ratings quality and reputation is key for our results. Mathis, McAndrews, and Rochet (2009) examine how a CRA's concern for its reputation affects its ratings quality. They present a dynamic model of reputation in which a monopolist CRA may mix between lying and truth-telling to build up/exploit its reputation. Strausz (2005) is similar in structure to Mathis et al. (2009), but examines information intermediaries in general.

Several papers have studied how a firm's disclosure policy affects information aggregation when there is a market composed of sophisticated investors and uninformed liquidity traders (e.g. Verecchia (1982), Diamond (1985), and Kim and Verrecchia (1991)). Goldstein and

[1] Moody's is the only stand-alone public company of the three major CRAs, and thus the only one that has a public market price.

[2] http://www.bloomberg.com/news/articles/2015-02-03/s-p-ends-legal-woes-with-1-5-billion-penalty-with-u-s-states

Yang (2017) review this literature in depth. Gao and Liang (2013) focus on the real impacts of this interaction. In these models, disclosure reduces gains to information acquisition by speculators, as in our paper. However, we also examine information being produced by a strategic rating agency and the two-way interaction between market information and the rating agency's information.

The interaction between market information and the rating agency's information is a type of feedback effect. There is a substantial literature on feedback effects of market prices, which examines how markets guide real decisions and the feedback loop between the two that results - see Bond, Edmans, and Goldstein (2012) for a review. Our paper examines the feedback between two information providers rather than between one information provider (the market) and a real decision maker. Bond and Goldstein (2015) look at both the real and informational feedback between government interventions and market information. When the intervention consists of disclosing information, they also find a crowding out effect of speculators' information acquisition.

There are few papers which discuss rating revisions. Boot, Milbourn, and Schmeits (2006) have downgrades of initial ratings. However, the CRA in their model passively rates a project and monitors a moral hazard problem, serving as a coordination device. We have a CRA that maximizes its payoffs and receives (and interacts with) multiple sources of information. Mariano's (2012) model has some similarities with our model of rating revisions. She considers how reputation disciplines a CRA's use of private information when public information is also available. In our game, the CRA at this stage is constrained by its previous choices and the market for public information is endogenous.

The rest of the paper is organized as follows. Section 2 describes the basic model. We then proceed to characterize the market for credit risk and the rating process. In Section 3, we describe the equilibrium trading strategies in the market for risk, the speculator's information acquisition decision, and how this is affected by the CRA's rating strategy. In Section 4, we describe the equilibrium rating strategy and how this depends on the precision of the speculator's signal. This allows us to solve for the unique equilibrium and describe its properties in Section 5. In Section 6 we extend the model to analyze the strategic choice of rating revision by the CRA and how this is affected by the realization of the market for risk. Section 7 discusses several assumptions and results from the model in greater detail. Finally, Section 8 concludes. Detailed proofs are presented in the Appendix and the Internet

Appendix.

2 The Model

Our model has two distinct elements, the ratings process and the market for credit risk. We first present them separately and then analyze the strategic interactions between them. The market for ratings takes place first, and is followed by the market for credit risk.

2.1 The Ratings Process

The ratings process takes place at time $t = 1$, and consists of three types of agents: an issuer, a monopoly credit-rating agency (CRA), and investors. All agents in the model are rational. We consider a risk-neutral CRA and use a reduced-form approach towards investors and issuer's preferences, introduced in Section 2.3. Both risk neutrality and risk aversion fit this approach.

The issuer is endowed with a risky investment project that he wishes to sell to the investors.[3] There are two possible outcomes for an investment: outcome $y \in \{S, F\}$, where S (F) represents *Success* (*Failure*). The quality of the investment is denoted by $\theta \in \{B, G\}$, where $B(G)$ stands for *Bad*(*Good*), and relates to its probability of default. A good investment fails with some probability $f_G \in (0, 1)$; a bad investment always fails - i.e., $f_B = 1$.[4] Therefore, whenever the investment fails, there is still uncertainty about its quality. The realization of y is common knowledge. The investment quality is *a priori* unknown, including to the issuer himself. Good and bad investments have ex-ante probability $\frac{1}{2}$ of occurring.

The CRA acts as an information intermediary between the issuer and investors. If approached by the issuer, the CRA observes a private signal about the quality of the investment, $\sigma \in \{g, b\}$. This (noisy) signal represents the CRA's processing of the information that an issuer would provide.

We assume that a good signal is indicative but not conclusive for a good investment -

[3] In Internet Appendix IA.3, we consider a different setting, where the issuer seeks financing for a risky investment project and the bad investment has negative NPV. We show that our results hold in full here as well. We also discuss this in Section 7. We thank Uday Rajan (our discussant) for suggesting this.

[4] The assumption that $f_B = 1$ is not important to the model and is made to simplify the exposition. We just need $f_B > f_G$ for our results to hold.

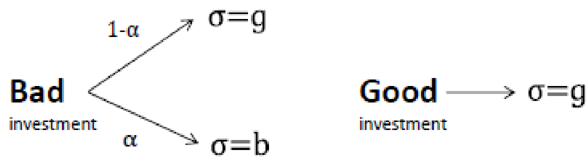

Figure 1: Structure of the CRA's private signal.

i.e.,

$$\Pr(G \mid \sigma = g) = \frac{1}{2-\alpha}, \tag{1}$$

where $\alpha \in (0,1)$ parametrizes the precision of $\sigma = g$. This signal is imprecise, due to the possibility of errors in risk assessment or incomplete information.

Regarding the bad signal, we assume the following:

Assumption 1: A bad signal b is conclusive for a bad investment: $\Pr(B \mid \sigma = b) = 1$.

This assumption simplifies the exposition but does not affect any of the results in the baseline model.[5]

Figure 1 characterizes the possible signals generated by each investment quality θ and their probabilities. Having observed σ, the CRA offers the issuer a credit report, which can be either *high* or *low*. The issuer either pays the rating fee and has the report publicized or refuses to purchase it. This allows for *"rating shopping"* by the issuer. The issuer chooses to buy the credit report only if it is high. The outcome of the rating process, as observed by the investors, is thus $m \in \{H, \varnothing\}$, where H signifies a high rating and $\varnothing$ signifies that there is no rating. If the issuer refuses to buy the CRA's report and goes on the market as *unrated*, that in itself is a signal to the investors. Given m, the issuer sets a uniform price p and sells the investment to the investors.[6]

The investment outcome - i.e. $y \in \{S, F\}$ - realizes at time $t = 3$.

[5] When we consider the extension in Section 6 that allows for rating revision, Assumption 1 eliminates incentives to upgrade ratings. This isolates the effects of downgrades in the game. However, the empirical literature finds that upgrades have no strong informational effect, which reduces their interest.

[6] We discuss the CRA's rating strategy, the market for investment, and the way p is determined in Section 2.3.

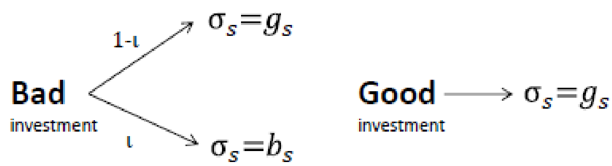

Figure 2: Structure of the speculator's private signal.

2.2 The Market Price of Risk

At time $t = 2$ (after the rating m is observed and the investment is sold to the investors), a market for credit risk takes place. This may represent the credit default swap (CDS) market or the secondary market for the asset. We will describe the case for the CDS market below. The same setting can be used to model the secondary market for the asset when the speculator is endowed with some amount of the investment.[7]

Let p^{cds} denote the price for a CDS contract[8] and x the net volume of trades. A CDS contract is formalized as follows: at time $t = 2$, the contract is signed, and the buyer of the swap pays an amount p^{cds} to the swap's seller. In return, the seller agrees that in the event of default at time $t = 3$, the seller will pay the buyer an amount 1.[9]

Trading occurs among liquidity/noise traders, one speculator and a competitive market maker, and p^{cds} is formed in a simplified model à la Kyle (1985). We now describe the agents in detail:

Speculator: Having observed the rating m, the speculator decides whether to enter the CDS market. If she does enter, she can gather information about the asset. She privately chooses the precision of her signal $\iota \in [0, 1]$ at a cost $c(\iota)$. The possible signals are $\sigma_s \in \{b_s, g_s\}$. We use the same structure as in the model for the CRA's information acquisition,

[7]As will be seen below, it is necessary for the speculator to be able to hide her trades to make a profit. This means she must be able to take either long or short positions. The easiest way to model this in a secondary market is to endow the speculator with some of the asset. Allowing for shorting would be equally good, of course.

[8]In a CDS contract, a protection buyer pays a premium to the protection seller, in exchange for a payment from the latter if a credit event (usually bankruptcy) occurs on a given reference entity within a predetermined time period. The protection buyer does not need to hold the reference entity ("naked" CDS). The amount that the protection seller has to pay in case of the credit event is called the notional amount. The premium is quoted in basis points per year of the contract's notional amount and is called the CDS spread.

[9]We normalize the notional amount *per contract* to 1 and let p^{cds} represent the CDS spread.

depicted in Figure 2:

$$\Pr\left(G \mid \sigma_s = g\right) = \frac{1}{2-\iota}. \tag{2}$$

This is all common knowledge to all agents in the model, and we assume:

$$c' \geq 0;\ c'' > 0;\ c\left(0\right) = c'\left(0\right) = 0;\ c\left(1\right) = \infty. \tag{3}$$

The more she spends on ι, the more precise is her signal. The speculator tries to use her superior information to profit from risk mispricing in the market. Let x^s denote her demand. We use the convention $x^s < 0$ when she is selling protection, and $x^s > 0$ when she is buying protection.

Noise traders: Aggregate demand from noise traders is $x^n \in \{-n; +n\}$, with both realizations equally likely.

Market maker: The market maker observes the trade orders - i.e., $\{x^s, x^n\}$, but not the identity of the trader submitting each order.[10] Having observed $\{x^s, x^n\}$ and m, he sets a price p^{cds} and clears the market. We assume that he makes zero profit, which implies $p^{cds} = E_\theta\left(f_\theta \mid m, \{x^s, x^n\}\right)$, where the expectation takes into account equilibrium beliefs about the CRA's rating strategy and the speculator's choice of precision and trading strategy.[11]

Let $x = x^s + x^n$ denote the total order flow. The informativeness of market trading is defined by the speculator's choice of precision ι. The other agents in the model do not directly observe the speculator's choice of ι; we denote as ι^e their expectation about it.

[10] As in the discrete setup of Faure-Grimaud and Gromb (2000), to ensure existence of a Perfect Bayesian Equilibrium we allow the market maker to observe trade orders (but not the identity of those trade orders).

[11] We implicitly assume (i) that the speculator on the CDS market does not participate in the initial investment market and (ii) the investors who purchased in the initial investment market choose not to participate in the CDS market. Regarding (i), this is consistent with recent empirical work that shows that speculative trading concentrates in the CDS market, due to its relative liquidity advantage: see Oehmke and Zawadowski (2016). We discuss this further in Section 7. Regarding (ii), we show in Section 7 that risk-neutral investors would not want to participate in the CDS market and that while risk-averse investors may want to participate in the CDS market, our results still hold.

2.3 The CRA's Reputation and Rating Strategy

The CRA can be of two different types: *committed* or *strategic*. Let $\tau \in \{\mathcal{S}, \mathcal{C}\}$ denote a realized type, where $\mathcal{S}$ ($\mathcal{C}$) stands for *Strategic* (*Committed*).[12] The realization of τ is the CRA's private information. Investors' prior beliefs about τ are given by

$$\Pr(\tau = \mathcal{C}) = q \in (0,1); \quad \Pr(\tau = \mathcal{S}) = 1 - q. \tag{4}$$

A committed CRA is always honest in its assessment of credit risk given the information available. A strategic CRA maximizes a weighted sum of its profits from selling the rating and its expected reputational payoff; this captures the tension between reputational concerns and profits from selling high ratings. The reputational payoff is the CRA's reputation for being a committed type. We represent this by the CRA's posterior reputation for being a committed type $\psi^{(m,x,y)}$, given the rating m, the realization of the CDS market x,[13] and the observable realization of the investment y. We let γ denote the weighting factor, which represents the relative importance of reputational payoffs to time $t = 1$ profits. The weighting γ can be potentially larger than one (as, for example, in Laffont and Tirole (1993)), as future payoffs may arrive over a long time horizon.

The CRA's private information refers to the signal it has observed and its type. Hence, we can denote the CRA's overall type by a pair (τ, σ).

Rating Strategy The committed CRA always offers a high (low) rating for a good (bad) signal. The strategic CRA chooses its rating m to maximize a weighted sum of its profits from selling the rating and its expectation over its reputational payoffs at $t = 3$, $\psi^{(m,x,y)}$, with γ as the weighting factor. The CRA knows that offering a low rating to the issuer is equivalent to making zero profit at $t = 1$, since the issuer will not purchase it: this makes room for the possibility of rating inflation. Let ε be the probability with which a strategic CRA chooses to *inflate* a bad initial signal - i.e., offers a high rating after having observed

[12] This follows the approach of Fulghieri, Strobl, and Xia (2014) and Mathis, McAndrews, and Rochet (2009) (who, in turn, follow the classic approach of modeling reputation of Kreps and Wilson (1984) and Milgrom and Roberts (1984)).

[13] As we will see, p^{cds} in equilibrium is an invertible function of the realized x. Hence, it is not important for our analysis whether the investors directly observe the amount of trades or the corresponding price when updating their beliefs about the CRA's type, since these are observationally equivalent.

a bad signal. The CRA can also under-report the signal, offering a low rating after having observed a good signal. This could allow it to build up reputation by appearing to be tough to investors. Let δ be the probability with which a strategic CRA chooses to *deflate* a good signal. The rating strategy is characterized by the following probabilities:

$$\Pr(m = H \mid \sigma = g) = 1 - \delta; \;\; \Pr(m = H \mid \sigma = b) = \varepsilon. \tag{5}$$

Let ε^e and δ^e denote the investors' conjectures about ε and δ, respectively. Let $p(m)$ denote the equilibrium price that the issuer charges investors for the investment project, for given ε^e, δ^e, and m. Making a take-it-or-leave-it offer, the CRA then charges the issuer the increase in p induced by $m = H$. The equilibrium rating fee is then:

$$\varphi = p(H) - p(\varnothing). \tag{6}$$

We make the following assumption about the investment market for the asset after the rating m:

Assumption 2: Assume that the difference $p(H) - p(\varnothing)$ is a non-increasing function of ε^e and is strictly positive at $\varepsilon^e = 0$.

Assumption 2 is consistent with any sensible modeling of the investment market since, as expectations about rating inflation ε^e increase, $m = H$ becomes a less reliable signal for $\theta = G$ in the eyes of the investors (and $m = \varnothing$ becomes a weakly less reliable signal for $\theta = B$).[14]

2.4 Timing of the Model

The timing of the model is written below and summarized in Figure 3:

Time $t = 0$:

[14] This makes the analysis robust to many possible ways of modeling of the investment market. Let us give an example: consider a competitive investment market with risk-neutral investors; the investment pays 1 in the case of success and 0 otherwise. In this case, we have

$$p(H) = \Pr(G \mid m = H)(1 - f_G); \;\; p(\varnothing) = \Pr(G \mid m = \varnothing)(1 - f_G).$$

Thus, $p(H) - p(\varnothing)$ is decreasing in ε^e since $\Pr(G \mid m = H)$ is decreasing and $\Pr(G \mid m = \varnothing)$ is non-decreasing in ε^e. In equilibrium, $\Pr(G \mid m = \varnothing) = 0$, so it will not change with ε^e.

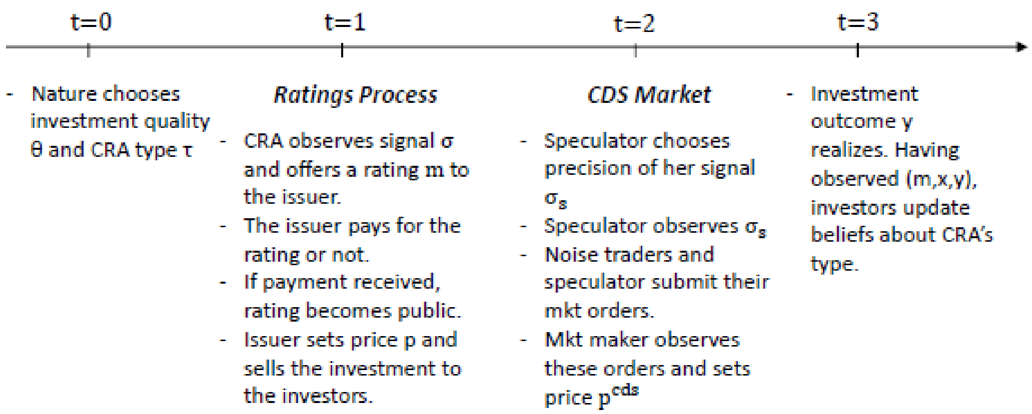

Figure 3: The timeline.

- **Quality of the investment and CRA type**: Nature chooses the quality of the investment $\theta \in \{B, G\}$ and the CRA type $\tau \in \{\mathcal{S}, \mathcal{C}\}$.

Time $t = 1$ (Ratings Process):

- **Rating:** If approached, the CRA observes signal $\sigma \in \{g, b\}$ and offers a rating to the issuer. If the report is high, the issuer pays the fee and the rating is publicized. The investment goes on the market as unrated otherwise.

- **Investment Market:** The investors learn the rating $m \in \{H, \varnothing\}$. The issuer sets a price $p(m)$ and sells the investment to the investors.

Time $t = 2$ (CDS Market):

- **Entry decision and information acquisition by the speculator**: Having observed m, the speculator decides whether to enter the CDS market. If she does enter, she privately chooses a level of precision ι and observes σ_s.

- **Market orders:** The noise traders and the speculator (if she has entered the market) submit their orders $\{x^s, x^n\}$. Demand x realizes.

- **Market Clearing:** Having observed $(m, \{x^s, x^n\})$, the market maker sets a price p^{cds} and clears the market.

Time $t = 3$**:**

- **Investment Outcome and Reputational Payoffs:** The investment outcome $y \in \{S, F\}$ realizes. Having observed (m, x, y), the investors update their beliefs about the CRA's type.

We use *Perfect Bayesian Equilibrium* as the solution concept and refine it using the Cho-Kreps (1987) intuitive criterion where possible.

3 The CDS Market Equilibrium

We work our way backwards by first characterizing the CDS market equilibrium for given conjectures about the CRA's rating strategy.

Here, we discuss the case in which the CRA is not expected to *deflate* a good signal. As we will see, this is consistent in equilibrium. This implies that $m = \varnothing$ conclusively reveals that the investment is bad, and the market maker then sets $p^{cds} = 1$ when he observes $m = \varnothing$. There is no mispricing of risk in this case, and so the speculator does not enter the market. Hence, we will focus on the case in which $m = H$ in what follows. The case in which there is positive expected deflation - i.e., $\delta^e > 0$ - and thus the speculator may want to enter the CDS market also when $m = \varnothing$, is discussed in Appendix A.3.

Proceeding by backward induction, we first solve for the equilibrium trading strategies and then, given these strategies, characterize the speculator's choice of precision.

3.1 Market Equilibrium

As in a standard Kyle-type setting, the speculator needs to camouflage her information-based trades with the liquidity trades. She is, therefore, constrained to trade an amount $x^s \in \{+n, -n\}$.[15] This implies that, if the speculator has entered the market, the total order flow x can take only three values - i.e., $\mathcal{X} = \{-2n, 0, +2n\}$. If $x \in \{-n, +n\}$, it must be instead that the speculator is not trading, and so did not enter the market. The following

[15] If x^s and x^n were different in absolute values, the market maker could always tell them apart, and so extract the speculator's private information. Expected profits from these trades would then be zero for the speculator.

Lemma characterizes the speculator's trading strategy and the market maker's inference from the order flow.

Lemma 1 *Given $m = H$ and for given ι^e and ε^e, the unique equilibrium of the CDS market is characterized as follows:*

1. *If the speculator enters the market, her trading strategy is $x^s(b_s) = +n$ and $x^s(g_s) = -n$;*

2. *The market maker infers $\sigma_s = b_s$ when $x^s = x^n = +n$ (i.e., $x = +2n$), $\sigma_s = g_s$ when $x^s = x^n = -n$ (i.e., $x = -2n$), and nothing when $x \in \{-n, 0, +n\}$.*

Lemma 1 is quite intuitive. The speculator buys protection when receiving negative information about the investment, and sells otherwise.[16] Given this trading strategy, the market maker's inference is also straightforward. When $x^s = x^n = +n$, both the speculator and liquidity traders are buying; when $x^s = x^n = -n$, both are selling. In these first two cases, the speculator's private information is revealed by the trade orders, so her expected profits from trading are zero (no mispricing). However, for $x^s = -x^n$ (i.e., $x = 0$), the market maker is unable to infer the direction of the speculator's order, and so her private signal; the speculator's expected profits are positive in this case. This justifies costly information acquisition by the speculator. When $x \in \{-n, +n\}$, it must be that the speculator is not trading (only liquidity traders are trading), and therefore x is not informative about θ.

Notice that in equilibrium, all other agents also learn σ_s for any $x \in \{-2n, +2n\}$ and nothing about the investment quality when $x \in \{-n, 0, +n\}$. The equilibrium price is characterized as follows:

$$\begin{aligned} \widehat{p}^{cds}(x) &= \begin{cases} \mu^{(H,g_s)} + \left(1 - \mu^{(H,g_s)}\right) f_G & \text{if } x = -2n; \\ 1 & \text{if } x = +2n. \end{cases} \\ \widehat{p}^{cds}(x) &= \mu^H + \left(1 - \mu^H\right) f_G \ \text{ for } x \in \{-n, 0, +n\}, \end{aligned} \tag{7}$$

where $\mu^H = \Pr(B \mid m = H)$ and $\mu^{(H,g_s)} = \Pr(B \mid m = H, \sigma_s = g_s)$.

[16] The market maker knows that $c(0) = c'(0) = 0$. This means that, when the speculator has entered the market, the market maker always expects a strictly positive level of precision, meaning that the speculator observes $\sigma_s = b_s$ with positive probability.

3.2 Informativeness of Market Trading

We need to evaluate the speculator's expected profits. Her expected profits are zero conditional on $x \neq 0$ since, in this case, the trade orders reveal the speculator's private information, and there is no informational advantage over the market maker. In what follows, let $\widehat{p}^{cds}$ be the equilibrium price when $x = 0$; let Π^s denote the speculator's expected profits. We have

$$\Pi^s = \frac{n}{2}\mu^H \iota \left(1 - \widehat{p}^{cds}\right) + \frac{n}{2}\left[\mu^H (1-\iota)\left(\widehat{p}^{cds} - 1\right) + \left(1 - \mu^H\right)\left(\widehat{p}^{cds} - f_G\right)\right] - c(\iota). \qquad (8)$$

The ex-ante probability of observing signal $\sigma_s = b_s$ is $\mu^H \iota$. For $\sigma_s = b_s$, the speculator learns $\theta = B$ and, thus, buys protection - i.e., $x^s = +n$. Trading profits are positive only if $x = 0$, which means that x^n has to be equal to $-n$; this occurs with probability $\frac{1}{2}$. She trades n units and the profit per unit is $1 - \widehat{p}^{cds}$. This is because the speculator pays the premium $\widehat{p}^{cds}$ at $t = 2$, but at $t = 3$, the investment fails for sure and she receives 1.

For $\sigma_s = g_s$, she believes that the investment is more likely to be good and sells protection. To make profits, she needs $x^n = +n$ in this case, which occurs with probability $\frac{1}{2}$. If her signal is correct - i.e. the investment is indeed good - her profit is $\widehat{p}^{cds} - f_G$. This is because she is selling protection in this case: she receives $\widehat{p}^{cds}$ at $t = 2$ and with probability f_G the investment fails, meaning that she has to pay 1 at $t = 3$. This happens with ex-ante probability $1 - \mu^H$. However, the g signal might be wrong and the asset will be of bad quality; she loses $\widehat{p}^{cds} - 1$ per unit in this case. This occurs with probability $\mu^H (1 - \iota)$.

By substituting the expression for $\widehat{p}^{cds}$ from equation (7) into the expression for Π^s in equation (8), and then simplifying, we get $\Pi^s = n\iota\mu^H \left(1 - \mu^H\right)(1 - f_G) - c(\iota)$. Gross expected profits $\left(n\iota\mu^H \left(1 - \mu^H\right)(1 - f_G)\right)$ are always positive for any strictly positive level of ι. This means that the speculator always enters the market and chooses a strictly positive level of precision when $m = H$.[17] This also implies that the total order flow x can take only three values in equilibrium - i.e., $\mathcal{X} = \{-2n, 0, +2n\}$ - and so the equilibrium price, as described in equation (7), is an invertible function of the realized x.

[17] This is guaranteed by our assumptions on the characterization of the cost function - i.e., $c(0) = c'(0) = 0$. This, together with $c(1) = \infty$, allows us to avoid discussing in the text corner solutions for the choice of ι. In the Appendix, we consider the possibility of corner solutions.

Taking the derivative of the speculator's expected profits with respect to ι yields:

$$\frac{\partial \Pi^s}{\partial \iota} = \underbrace{n\mu^H \left(1 - \widehat{p}^{cds}\right)}_{\text{Marginal Benefit of } \iota} - c'(\iota). \tag{9}$$

Higher precision benefits the speculator by increasing the chances that she takes the right position on the market.

For a given level of expected rating inflation, the equilibrium level of precision $\widehat{\iota}(\varepsilon^e)$ sets equation (9) equal to zero. The existence and uniqueness of $\widehat{\iota}(\varepsilon^e) \in (0,1)$ is guaranteed by the assumptions on the shape of the cost function. All the other agents in the model do not directly observe the speculator's choice of ι. However, they form consistent conjectures about it, given common knowledge of Π^s for any given level of expected rating inflation. The effect of expected rating inflation on $\widehat{\iota}(\varepsilon^e)$ depends on how $\frac{\partial \Pi^s}{\partial \iota}$ changes with ε^e.

$$\frac{d}{d\varepsilon^e}\left(\frac{\partial \Pi^s}{\partial \iota}\right) = n\overset{>0}{\frac{d\mu^H}{d\varepsilon^e}}\left(1 - \widehat{p}^{cds}\right) - n\mu^H \overset{>0}{\frac{d\widehat{p}^{cds}}{d\varepsilon^e}}. \tag{10}$$

As expected rating inflation ε^e increases, the rating $m = H$ becomes a less reliable signal for $\theta = G$, and so it is relatively more likely that the investment is bad - i.e., μ^H increases towards the prior of $\frac{1}{2}$. [18] This has two different effects on $\frac{\partial \Pi^s}{\partial \iota}$. On the one hand, it is more likely that the speculator's precision is relevant,[19] which increases the speculator's marginal benefit and thus her incentives to acquire information. On the other hand, the market maker reacts to the rating being a less reliable signal of quality by increasing the price. This reduces profits from trade conditional on $\theta = B$ for the speculator and, therefore, her incentives to acquire information, by decreasing her marginal benefit of precision. In other words, when ε^e increases, the speculator gains from precision more often, but a bit less. Given that μ^H is small (it is less than $\frac{1}{2}$), the difference $1 - \widehat{p}^{cds}$ is large (since $\mu^H < \frac{1}{2}$, the equilibrium price is closer to f_G than to 1), and $\frac{d\mu^H}{d\varepsilon^e} > \frac{d\widehat{p}^{cds}}{d\varepsilon^e}$, we have that the effect on the probabilities - the

[18] In the text, we consider the case where bad and good assets are ex-ante equally likely so that, after a good rating, the investors believe that the asset is more likely to be good - i.e., $\mu^H < \frac{1}{2}$. This is true when the prior probability of the asset being bad is close to $\frac{1}{2}$ or below. This is the relevant case to examine, since here a good rating is meaningful and, therefore, inflated ratings induce wrong beliefs about asset quality.

[19] Precision is relevant only when the investment is bad, since it is the probability that, in this case, the speculator observes the right signal - i.e., $\iota = \Pr(\sigma_s = b_s \mid B)$. See Figure 2.

"more often" part - dominates and so the overall effect is always positive.

Lemma 2 *The equilibrium level of precision $\widehat{\iota}(\varepsilon^e)$ is increasing in the amount of expected rating inflation ε^e.*

Interestingly, from the speculator's point of view, information acquisition and rating inflation are strategic complements. That is, higher expected rating inflation increases the incentive to acquire information. This occurs through the mispricing channel, as the speculator can take advantage of wrong valuations due to more-opaque ratings.

4 The Rating Game

Having characterized the CDS market equilibrium for given initial ratings inflation ε^e and deflation δ^e, we can now solve for their equilibrium values.

Proposition 1 *The unique equilibrium initial rating strategy is:*

1. *A strategic CRA inflates a bad signal with positive probability $\widehat{\varepsilon} \in (0, 1]$;*
2. *A strategic CRA never deflates a good signal - i.e., $\widehat{\delta} = 0$.*

Some degree of rating inflation is always an equilibrium outcome. However, rating deflation never occurs in equilibrium. In the rest of this section, we present the ideas behind Proposition 1.

4.1 Equilibrium Rating Inflation

We first characterize the equilibrium level of rating inflation for a given level of expected rating deflation δ^e.

The choice of rating inflation is relevant only when, at $t = 1$, a strategic CRA observes a bad initial signal (type $(\mathcal{S}, b)$) and decides whether to inflate or truthfully report it. Therefore, we focus on the strategic choice of this type in what follows.

We can write total payoffs as follows:

$$\Pi_b = \varepsilon \left\{ \varphi + \gamma E_x \left[\psi^{(H,x,F)} \mid B \right] \right\} + (1 - \varepsilon) \gamma E_x \left[\psi^{(\varnothing,x,F)} \mid B \right]. \tag{11}$$

The expectation over realizations of x is as it is implied by the CDS market equilibrium. At time $t = 1$, both the realization of the CDS market x and the investment outcome y have not yet realized. However, having observed the signal $\sigma = b$, the CRA knows that the investment is of bad quality and will fail for sure. Therefore, the expectation over realizations of x is conditional on $\theta = B$. With probability $1 - \varepsilon$, the rating is not inflated, the rating fee is not collected and so $m = \varnothing$. The continuation reputational payoff is $E_x \left[\psi^{(\varnothing,x,F)} \mid B \right]$ in this case. With probability ε, the rating is inflated to $m = H$ and the fee is collected; the continuation payoff is $E_x \left[\psi^{(H,x,F)} \mid B \right]$ in this case.

The amount of rating inflation ε is then chosen as a best response to the conjectures ε^e and δ^e; let $\varepsilon^* (\varepsilon^e, \delta^e)$ denote this best response.

Definition 1 *Any equilibrium level of rating inflation $\widehat{\varepsilon} (\delta^e)$ has to be a value of ε^e that satisfies the following fixed point condition:*

$$\varepsilon^* (\varepsilon^e, \delta^e) = \varepsilon^e. \tag{12}$$

Taking the derivative of Π_b with respect to ε yields:

$$MBI \equiv \frac{d\Pi_b}{d\varepsilon} = \varphi + \gamma E_x \left[\psi^{(H,x,F)} - \psi^{(\varnothing,x,F)} \mid B \right]. \tag{13}$$

The derivative $\frac{d\Pi_b}{d\varepsilon}$ represents the marginal benefit of rating inflation for given ε^e and δ^e.

Lemma 3 *The marginal benefit of rating inflation ε is always positive at $\varepsilon^e = 0$, and decreasing and continuous in ε^e. This implies that $\widehat{\varepsilon} (\delta^e)$ is unique; we have:*

1. *A corner solution $\widehat{\varepsilon} (\delta^e) = 1$, whenever $MBI > 0$ at $(\varepsilon^e = 1, \delta^e)$;*

2. *An interior solution $\widehat{\varepsilon} (\delta^e) \in (0, 1]$ otherwise.*

Figure 4 depicts the possible characterizations for the equilibrium level of rating inflation.

The marginal benefit of inflation MBI is always positive at $\varepsilon^e = 0$, since the CRA can collect the fee by inflating a bad signal without incurring reputational losses. As ε^e increases, investors perceive it to be more likely that a strategic CRA inflates the rating, so reputation updating conditional on the rating $m = H$ is more severe. On the other hand, the rating

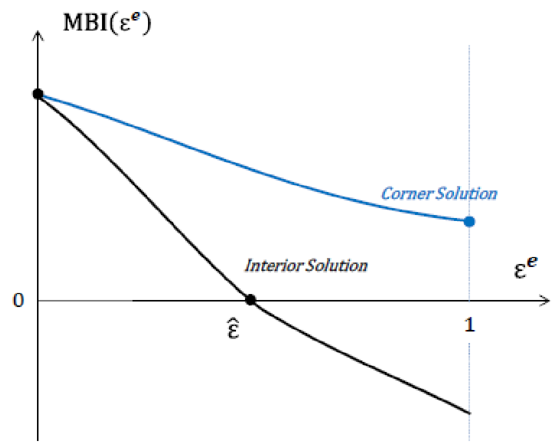

Figure 4: Characterization of MBI and equilibrium inflation.

$m = \varnothing$ becomes a stronger signal for a committed type, so forgone reputational payoffs when the CRA actually inflates are larger. Lastly, the rating $m = H$ is a less reliable signal for a good investment, which reduces the rating fee. Therefore, incentives to inflate are lower when expected rating inflation increases.

This implies that, for any given δ^e, we have a unique equilibrium level of rating inflation, which can be: a corner solution when inflating is a strictly dominant action ($MBI > 0$), even at $\varepsilon^e = 1$; or a mixed strategy when inflating is only weakly dominant ($MBI = 0$) for some interior level of expected rating inflation ε^e.

4.2 Equilibrium Rating Deflation

We can now characterize the equilibrium level of rating deflation.

The choice of rating deflation is relevant only when, at $t = 1$, a strategic CRA observes a good initial signal (type $(\mathcal{S}, g)$) and decides whether to deflate it. The analysis proceeds in the same way as for the equilibrium rating inflation and, therefore, is left to the Appendix. Here, we discuss the ideas behind the result, which is described in the following Lemma.

Lemma 4 *The marginal benefit of deflation is always strictly negative at equilibrium; therefore, a strategic CRA never deflates a good signal, setting* $\widehat{\delta} = 0$.

Deflating a good signal is a strictly dominated action for type $(\mathcal{S}, g)$ in equilibrium. Type $(\mathcal{S}, g)$ expects the investment to succeed with positive probability given its signal $\sigma = g$.

When the investment succeeds ($y = S$), investors learn that $\theta = G$ and so $\sigma = g$. Therefore, $m = \varnothing$ reveals a strategic type to the market (since the signal has been deflated), making it strictly better for type $(\mathcal{S}, g)$ not to deflate. When the investment fails ($y = F$), the marginal benefit of rating deflation is negative. This is because it is essentially the opposite[20] of the marginal benefit of rating inflation (MBI; from equation 13), which was always non-negative in equilibrium.[21] This implies that selling the rating is always attractive enough that there is no incentive to deflate.

4.3 The Effect of Trading Informativeness on Rating Inflation

The effect of a change in the expected informativeness of market trading ι^e on equilibrium rating inflation depends on its effect on the marginal benefit of rating inflation. The marginal benefit of inflation (equation 13) can be rearranged as follows:

$$MBI = \varphi + \frac{\gamma}{2}\left[\iota^e \psi^{(H,+2n,F)} + (1-\iota^e)\,\psi^{(H,-2n,F)}\right] + \frac{\gamma}{2}\psi^{(H,0,F)} - \gamma\psi^{(\varnothing,F)}. \tag{14}$$

When $x = 0$, which occurs with probability $\frac{1}{2}$, there is no learning about investment quality θ from trading. The continuation payoffs and, thus, the MBI do not depend on ι^e in this case. When $x = +2n$, investors learn that $\sigma_s = b_s$ and so $\theta = B$. When $x = -2n$, the investors learn that $\sigma_s = g_s$ which is a signal for a good investment. Taking the derivative of MBI with respect to ι^e yields:

$$\begin{aligned} \frac{dMBI}{d\iota^e} &= \frac{\gamma}{2}\left(\psi^{(H,+2n,F)} - \psi^{(H,-2n,F)}\right) \\ &\quad + \frac{\gamma}{2}\left(\iota^e \frac{d\psi^{(H,+2n,F)}}{d\iota^e} + (1-\iota^e)\frac{d\psi^{(H,-2n,F)}}{d\iota^e}\right). \end{aligned} \tag{15}$$

[20] By deflating a good signal, type $(\mathcal{S}, g)$ gives up on the fee and reputation $\psi^{(H,x,F)}$ in order to get reputation $\psi^{(\varnothing,x,F)}$ afterwards. By inflating a bad signal, type $(\mathcal{S}, b)$ gives up on $\psi^{(\varnothing,x,F)}$ in order to cash the fee and get reputation $\psi^{(H,x,F)}$ afterwards. Moreover, for type $(\mathcal{S}, b)$, the realization $x = +2n$ is more likely (as a bad signal for the CRA is more likely to be followed by a bad signal in the CDS market), so that its expected reputational costs - i.e., the difference $E_x\left[\psi^{(H,x,F)} - \psi^{(\varnothing,x,F)} \mid B\right]$, is even larger. Given that the marginal benefit of inflation for type $(\mathcal{S}, b)$ was proven to be positive, this implies the marginal benefit of deflation for type $(\mathcal{S}, g)$ is negative. A more detailed proof is in appendix A.7.

[21] For the equilibrium level of inflation, we have either a corner solution where MBI is strictly positive, or an interior solution where $MBI = 0$.

The change in the expected precision ι^e has two different effects on MBI. The *Direct Effect* is how an increase in expected precision ι^e changes the expected distribution of realized order flow x given that the investment is bad; this is represented by the first line in (15). When ι^e increases, the bad signal $\sigma_s = b_s$ is expected to be more likely, and, hence, demand will be $x = +2n$ (which reveals that the investment is bad) more often. Investors put higher probabilities on the signal observed by the CRA being bad ($\sigma = b$) and, therefore, infer that the rating was more likely (relatively) to have been inflated: this reduces the CRA's posterior reputation. This implies that posterior reputation is the lowest when $x = +2n$, making the Direct Effect negative.

The *Indirect Effect* is the impact of an increase in expected precision ι^e on the CRA's posterior reputation $\psi^{(m,x,y)}$ that operates through $\Pr(B \mid x)$. This is represented by the second line in (15). Note that for $x = +2n$, everyone knows that the investment is bad and, therefore, $\Pr(B \mid x) = 1$, which does not depend on ι^e. Thus, the indirect effect is relevant only when $x = -2n$. We can write it as:

$$(1 - \iota^e) \left[\frac{\partial \psi^{(H,x,F)}}{\partial \Pr(B \mid x)} \frac{\partial \Pr(B \mid x)}{\partial \iota^e} \right]_{x=-2n} \geq 0. \tag{16}$$

The indirect effect is always positive. It is useful to think about reputation updating as a two-stage process: first, the investors use x and y to set their beliefs about the distribution of θ and then use the rating m to update their beliefs about the CRA type. The signal $x = -2n$ informs investors that a good investment $\theta = G$ is more likely. When ι^e increases, the investors put higher probabilities on this signal being correct and so on the investment being of good quality. This makes it relatively less likely that the CRA observed a bad signal and then inflated it, increasing the CRA's posterior reputation and its incentive to inflate.

We demonstrate in the following lemma that the overall effect on rating inflation is always negative, as the direct effect dominates the indirect one. The intuition for this result is that increased transparency from the speculator's information causes a *discrete* decrease in reputation when there is inflation, while the increase in reputation due to the perception that the state must have been good is *marginal*.

Lemma 5 *Equilibrium rating inflation $\widehat{\varepsilon}(\iota^e)$ is decreasing in the expected level of informativeness ι^e (weakly in the case of a corner solution).*

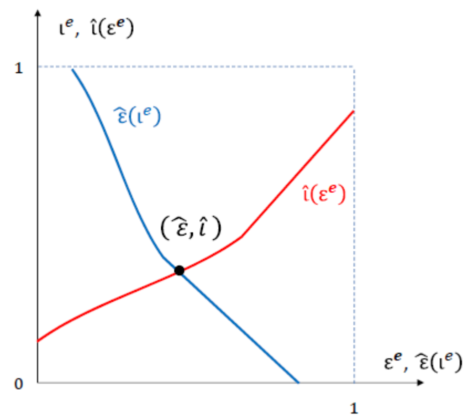

Figure 5: Rating Inflation and Trading Informativeness Equilibrium.

This indicates that from the point of view of the CRA, rating inflation and market trading informativeness are strategic substitutes. That is, more-informative market trading gives the CRA incentives to be more accurate. This arises because market transparency from informative trading makes reputational incentives more important.

5 Equilibrium and Comparative Statics

We have found that equilibrium rating inflation $\widehat{\varepsilon}(\iota^e)$ is *decreasing* in the expected level of the informativeness of market trading ι^e. On the other hand, the informativeness of market trading (choice of precision by the speculator) $\widehat{\iota}(\varepsilon^e)$ is *increasing* in the expected rating inflation, ε^e. The fact that these effects move in opposite directions implies that there is a unique equilibrium.

Proposition 2 *There exists a unique pair of rating inflation and market trading informativeness* $(\widehat{\varepsilon}, \widehat{\iota})$ *such that*

$$\widehat{\varepsilon}(\iota^e = \widehat{\iota}) = \widehat{\varepsilon};\ \widehat{\iota}(\varepsilon^e = \widehat{\varepsilon}) = \widehat{\iota}. \tag{17}$$

Figure 5 depicts the result in Proposition 2. We have $\widehat{\iota}(\varepsilon^e) \in (0,1)$ and $\widehat{\varepsilon}(\iota^e) \in (0,1]$: this implies that the two reaction functions in Figure 5 either cross (and do so only once) at some interior level $(\widehat{\varepsilon}, \widehat{\iota}) \in (0,1)^2$, or they do not cross and we have a corner solution for rating inflation - i.e., $\widehat{\varepsilon} = 1$, $\widehat{\iota} = \widehat{\iota}(1)$. For any parameter constellation, we can always find γ large enough that the corner solution is ruled out.[22] Interior solutions are the most interesting for comparative statics, so we focus on this case in what follows.

We now analyze the effect of varying the cost of precision and the weight on reputational payoffs in the CRA's objective function (γ). We characterize a cost increase as an increase in k, where $c(\iota) = k\widetilde{c}(\iota)$ and $\widetilde{c}(\iota)$ satisfies the conditions in equation (3). We first look at their effect on the equilibrium pair $(\widehat{\varepsilon}, \widehat{\iota})$. Then, we analyze their effect on a measure of informational efficiency of the two markets (the market for ratings and the CDS market).

The comparative statics on k are straightforward. When k increases, the speculator chooses a lower $\widehat{\iota}(\varepsilon^e)$ for any level of ε^e ($\widehat{\iota}(\varepsilon^e)$ shifts down). Note that the speculator's costs do not enter the CRA's objective function, so that a change in k affects the equilibrium only through its direct effect on $\widehat{\iota}(\varepsilon^e)$. This implies that the new equilibrium pair will feature a lower level of precision and higher rating inflation. Similar comparative statics results apply to a decrease in n, which parametrizes the volume of trades by noise traders. As n decreases, the speculator trades less and so makes less profit. Hence, $\widehat{\iota}(\varepsilon^e)$ shifts down.

An increase in γ moves only $\widehat{\varepsilon}(\iota^e)$, shifting it to the left. This is quite intuitive: the strategic CRA inflates less because it is more concerned about future reputation.[23] Thus, the new equilibrium pair features a lower level of both rating inflation and precision.

As a measure of informational efficiency we use the total probability that the investors find out that an investment is bad given that the CRA is strategic, after having observed the rating m and the market price $\widehat{p}^{cds}$. Let Γ denote our measure of informational efficiency. We have:

$$\Gamma = \Pr\left(m = \varnothing \cup x = +2n \mid \tau = \mathcal{S}, \theta = B\right). \tag{18}$$

The signals $m = \varnothing$ and $x = +2n$ both conclusively reveal a bad investment. The focus on conclusive signals is without loss of generality, because the signals in the model are binary:[24] the investors either learn that the investment is bad or they do not, which is interpreted as a signal for a good investment. This means that, as a conclusive signal becomes more likely (say $m = \varnothing$), its opposite (in this case, $m = H$) becomes a more reliable signal for a good

[22] Formally, we have an interior solution for any $\gamma > \overline{\gamma}$, where $\overline{\gamma}$ is such that $MBI\left(\iota^e = 0, \varepsilon^e = 1\right) = 0$.

[23] Notice that this does not affect its incentives to truthfully report good signals (i.e., deflate ratings).

[24] Strictly speaking, the order flow x is not binary, since we have $x \in \{+2n, 0, -2n\}$. However, for $x = 0$, the investors do not learn anything about θ and the probability of $x = 0$ is always $\frac{1}{2}$, which is exogenous.

investment.[25]

We can write Γ as follows:

$$\Gamma = \alpha\left(1-\widehat{\varepsilon}\right) + \frac{\left(\alpha\widehat{\varepsilon}+1-\alpha\right)\widehat{\iota}}{2}. \tag{19}$$

The first term in equation (19) is the probability that $m = \varnothing$ is observed. The second is the probability that, after $m = H$ has been observed, $x = +2n$ realizes and the investors learn $\theta = B$ from the market price.

Note that both k and γ do not directly affect the precision of the private signals that the CRA and the speculator observe, so that their effect on Γ is only indirect, through the effect on the equilibrium pair $(\widehat{\varepsilon}, \widehat{\iota})$. It is, therefore, useful to first discuss how Γ changes when $\widehat{\varepsilon}$ and $\widehat{\iota}$ change.

As $\widehat{\varepsilon}$ increases, the rating become less informative, so that Γ is always decreasing in $\widehat{\varepsilon}$. When $\widehat{\varepsilon}$ increases, $m = \varnothing$ becomes less likely and so $m = H$ more likely. When $m = H$, the investors may still learn that $\theta = B$ later on, at the market price stage. However, this happens with probability lower than one, while they always learn that $\theta = B$ when $m = \varnothing$. Therefore, $\frac{d\Gamma}{d\widehat{\varepsilon}}$ is negative.

As $\widehat{\iota}$ increases, x becomes more informative and so Γ increases.

Lemma 6 *Informational efficiency Γ is always decreasing in k and increasing in n; it may be increasing or decreasing in γ, depending on the parameters of the model.*

As we pointed out earlier, an increase in k leads to lower equilibrium precision $\widehat{\iota}$ and higher equilibrium rating inflation $\widehat{\varepsilon}$. Both of these shifts decrease informational efficiency. On the other hand, an increase in γ decreases $\widehat{\varepsilon}$ but also decreases $\widehat{\iota}$. In this case, it is ambiguous whether the total amount of information produced increases or decreases.

Informational efficiency Γ is decreasing in γ when, around the equilibrium point, $\widehat{\iota}\left(\varepsilon^e\right)$ is much more sensitive to ε^e than $\widehat{\varepsilon}\left(\iota^e\right)$ is to ι^e. In this case, the decrease in $\widehat{\iota}$ is much larger than the decrease in $\widehat{\varepsilon}$ when γ increases. In Internet Appendix IA.1, we explore the determinants of the relative slopes of $\widehat{\iota}\left(\varepsilon^e\right)$ and $\widehat{\varepsilon}\left(\iota^e\right)$. We find that $\frac{\partial\widehat{\iota}(\varepsilon^e)}{\partial\varepsilon^e}$ is large - i.e., equilibrium precision

[25] Moreover, the focus on conclusive signals allows us to disentangle the investors' learning about the investment from their learning about the CRA type (strategic or committed). In the case of non-conclusive signals, the investors' beliefs about the investment depend on their beliefs about the CRA's type, as well (which affect the perceived accuracy of its signal $m = H$).

is particularly sensitive to expected rating inflation - when the CDS market is quite liquid (n large) and the cost of precision is not very convex. In this case, trading is quite profitable and adjusting the level of precision is not too costly for the speculator, so that she reacts a lot to the degree of mispricing induced in the market by rating inflation. On the other hand, the absolute value of $\frac{\partial \widehat{\varepsilon}(\iota^e)}{\partial \iota^e}$ is small - i.e., equilibrium rating inflation is less sensitive to market informativeness - when the realization of the asset payoffs y is quite informative about the asset quality[26]. In this case, the investors put more weight on y than on x in updating their beliefs about θ - and so about the CRA's type, and therefore the CRA is less concerned about the informativeness of market trading. In Internet Appendix IA.1, we provide numerical simulations of the model showing that both cases - Γ increasing or decreasing in γ - are possible.

Lemma 6 has some interesting implications. When Γ is decreasing in γ and the CRA becomes more concerned about future reputation, rating inflation decreases, but the overall informational efficiency of the market declines. This means that policies meant to discipline the CRA's incentives, such as larger reputational sanctions (e.g., increases in liability or regulatory scrutiny), may be undesirable, because of their perverse effect on the market's incentives to acquire information.

The result that Γ is decreasing in k and increasing in n echoes the general belief that, when speculative trading is more profitable, more information is produced in equilibrium. Our paper introduces a new channel for which this is true, through the role that market transparency plays in disciplining the CRA's incentives. This also highlights a positive externality induced by policies that foster market liquidity - for example, improved governance and regulation of exchanges that instills trust and attracts trade.

Our results have also some interesting implications for the limiting cases where either the CDS market or the ratings process does not exist. On the one hand, ratings become more accurate when a secondary market on the asset is introduced - for example, when CDS start being actively traded on the asset. This is consistent with the empirical findings of Dilly and Mahlmann (2014). On the other hand, trading in the secondary market would be more informative if there were no ratings on the asset. Paradoxically, for some parameter

[26] This is the case when f_G is close to 0, so that, when the investment fails, the investors strongly believe that $\theta = B$, and therefore do not update much their beliefs upon the realization of x.

configurations, informational efficiency would be larger if the market for ratings did not exist. This is intriguing given the current regulatory debate on replacing ratings with market based measures (e.g. Flannery, Houston, and Partnoy (2010).

6 Rating Revision

In this section we present an extension to the baseline model which explores the dynamics of rating revision. Now, after the market for risk has realized, the CRA may then observe a further private signal and decide whether to revise its initial rating, taking into account the reputational repercussions. We begin by describing the additions to the model, and then solve for the equilibrium.

6.1 New Information and Possible Rating Revision

We introduce another period to the model. The rating revision stage takes place after the realization of the CDS market and before the project outcome realizes at time $t = 2.5$.

Having observed x, the CRA may want to revise the rating, which can require a new assessment of the investment. Let $\sigma_2 \in \{g, b\}$ denote the second private signal observed by the CRA, where g is once again an imprecise signal for a good project, and b conveys the information that the investment is bad for sure. Note that if the CRA observes a bad signal in the first period - let σ_1 denote its private signal at time $t = 1$ - it doesn't need a new signal, as the initial one was conclusive. We have

$$\beta_x = \Pr\left(\sigma_2 = b \mid \sigma_1 = g, x, B\right), \tag{20}$$

where β_x parametrizes the precision of σ_2, and $\beta_x \in (0, 1]\ \forall x \in \mathcal{X}$. We allow the precision of the second signal to be correlated with the public signal x.

If the initial rating is high, at time $t = 2.5$ the CRA learns x and σ_2 and chooses a rating revision action, denoted by $r \in \{D, N\}$, with $D(N)$ for *Downgrade(No rating revision)*.[27] Note that the CRA has already been paid for its rating at $t = 1$. Therefore, the rating revision action is driven purely by incentives to maximize reputation. In reality, firms pay

[27]Notice that, because of rating shopping, only a high initial rating is publicized. Therefore, the CRA can either downgrade or choose not to take a revision action.

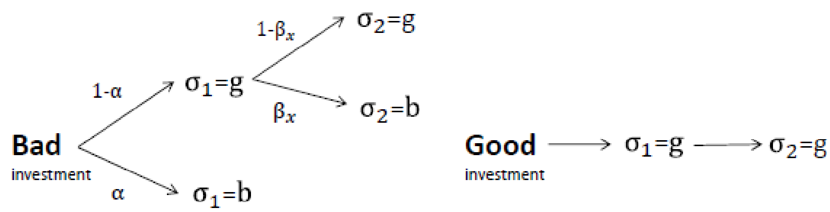

Figure 6: Structure of the CRA's private signals.

for initial ratings and subsequently pay annually for credit watches, but the initial ratings are the key margin where shopping could occur.

Let $\boldsymbol{\sigma} \in \{(g, g), (g, b), b\}$ represent a set of realized private signals for the CRA. Figure 6 characterizes the structure of the possible signals generated by each investment quality θ and their probabilities. A bad signal is conclusive about $\theta = B$ since it cannot be generated by a good investment. So, when a bad signal is observed, either at the initial signal or at the second signal, the CRA is sure that the investment is bad. For $\boldsymbol{\sigma} = (g, g)$, the CRA is still unsure about θ.

At time $t = 3$, the investment outcome realizes $y \in \{S, F\}$.

6.2 Ratings Revision Strategy

The committed CRA always offers a high (low) rating for a good (bad) initial signal. Since a bad signal is conclusive about $\theta = B$, a committed CRA would never offer the issuer a rating revision at $t = 2.5$ when it offered a low report (that went unpurchased and unpublished) at $t = 1$. Therefore, if there was no rating at $t = 1$ and a rating was issued at $t = 2.5$, this would reveal to the investors that the CRA was the strategic type.[28]

The strategic CRA chooses its initial rating m to maximize a weighted sum of its profits from selling the rating and its expectation over its reputational payoffs at $t = 3$, $\psi^{(m,x,y,r)}$, with γ as the weighting factor. At the rating revision stage, given the realization of x and its new private signal σ_2, a strategic CRA chooses the rating revision r to maximize its expected reputational payoffs at $t = 3$. Let $D_{\boldsymbol{\sigma}}$ denote the probability with which type $(\mathcal{S}, \boldsymbol{\sigma})$ downgrades, and let $D^e_{\boldsymbol{\sigma}}$ denote investors' conjecture about $D_{\boldsymbol{\sigma}}$.

[28]This revelation occurs off the equilibrium path. To ensure that there are no profitable deviations for a strategic CRA to issue a rating at $t = 2.5$ after one was not issued at $t = 1$, we apply the Cho-Kreps (1987) intuitive criterion.

We characterize the equilibrium rating revision strategy for given conjectures by investors about the initial rating inflation ε^e and deflation δ^e. The equilibrium initial rating strategy, as it is described in Proposition 1, does not change when we introduce the rating revision stage.[29] This means that a strategic CRA still always inflates a bad initial signal with positive probability and never deflates a good signal. Hence, here we discuss the case for some conjecture $\varepsilon^e \in (0,1]$ and $\delta^e = 0$.

Rating revision takes place only if an initial rating has been publicized at $t = 1$, so everything from now on will be conditional on $m = H$. A committed CRA never inflates the initial rating. Hence, we need only consider committed types that received the initial good signal, $\sigma_1 = g$: $(\mathcal{C}, (g, g))$, and $(\mathcal{C}, (g, b))$. As the strategic CRA may inflate the initial rating, the possible set of strategic types is: $(\mathcal{S}, b)$, $(\mathcal{S}, (g, g))$, and $(\mathcal{S}, (g, b))$. Given that we are considering a conjecture $\varepsilon^e \in (0,1]$ and $\delta^e = 0$, all of these types are expected to be active.[30] The strategic types, of course, will attempt to pool with the committed types so as not to be discovered. The fact that there are fewer committed types than strategic types and the strategic types may have different inferences makes this problem interesting.

The realization of x is observed by all agents and, thus, may affect the strategy of the CRA. A committed CRA that has learned about the investment being bad - i.e., $(\mathcal{C}, (g, b))$ always downgrades. As we saw earlier, when $x = +2n$ all the agents in the model learn that the investment is bad. Hence, even a committed type that has not received a bad signal $(\mathcal{C}, (g, g))$ downgrades, since it knows that its private signals are wrong. However, for $x = 0$ or $x = -2n$, this type still thinks that the investment is more likely to be good and so does not downgrade. Therefore, we write the choices of the committed CRA as:

$$\text{for } x = +2n: \begin{array}{ll} \text{Downgrade} & \text{No rating revision} \\ (\mathcal{C}, (g, b)) & \\ (\mathcal{C}, (g, g)) & \end{array} ;$$

[29] As we will see, the equilibrium play at this stage is such that no further information about the CRA's type is revealed through rating revision. This means that expected reputational payoffs at time $t = 1$ do not change when we introduce the rating revision stage, and so does the equilibrium initial rating strategy. A more detailed discussion of this result, and its proof, are in Appendix A.11.6.

[30] The set of strategic types the investors expect to play the rating revision game depends on the conjectures for rating inflation ε^e and deflation δ^e. When $\varepsilon^e = 0$, type $(\mathcal{S}, b)$ is not expected to inflate the initial rating and, therefore, would not enter the rating revision stage. When $\delta^e = 1$, a similar inference occurs for types $(\mathcal{S}, (g, b))$, $(\mathcal{S}, (g, g))$, who are always expected to have deflated the initial rating in this case. These cases are described in Appendix A.11.5.

$$\text{for } x \neq +2n : \begin{array}{cc} \text{Downgrade} & \text{No rating revision} \\ (\mathcal{C},(g,b)) & (\mathcal{C},(g,g)) \end{array} .$$

For $x = +2n$, the committed CRA types pool on the same action, namely downgrading (D). Thus, no rating revision (N) would reveal a strategic type to the market. The strategic types, then, always pool on downgrading D, as well,[31] so as not to be discovered and suffer a reputational loss, and the rating r is not informative about the type of the CRA in equilibrium (since all the CRA types play the same rating revision strategy). Therefore, the market does not learn anything from the rating revision that it doesn't already know.[32]

Rating revision becomes interesting when $x \neq +2n$: in this case, the strategic types have to decide which committed type to mimic.

6.3 Analysis

The characterization of the equilibrium is trivial when $x = +2n$ (because all types pool at downgrading ($r = D$)), so we focus on the case $x \neq +2n$ in what follows. Note that when $x \neq +2n$, both actions D and N are on the equilibrium path, as the committed type chooses them with positive probability (given its signals).

When the bad signal has been observed, at either the initial or the second signal, the CRA knows that the investment is bad and will fail for sure. This implies that types $(\mathcal{S},(g,b))$ and $(\mathcal{S},b)$ have the same payoff structure at $t = 2.5$ and, thus, the same best response function; therefore, they will always play the same strategy at equilibrium. The investors know this and conjecture a symmetric downgrading strategy $D_b^e = D_{g,b}^e \equiv D_{\mathcal{B}}^e$.

Proposition 3 *For given $\varepsilon^e > 0$ and $\delta^e = 0$ and a realized x, the unique equilibrium rating revision strategy is as follows:*

1. *For $x = +2n$, all CRA types downgrade;*

[31] Once again, Cho-Kreps (1987) pins down beliefs off the equilibrium path; observing an action N induces investors to believe that the CRA is strategic with probability 1.

[32] This result is reminiscent of events in July 2007 when the major CRAs massively downgraded mortgage-backed securities; enough information had been revealed to investors by that time (e.g., New Century Financial, the second largest subprime mortgage originator, had just gone bankrupt) that not responding would have certainly revealed rating inflation.

2. *For $x \neq +2n$, type $(\mathcal{S},(g,g))$ never downgrades; types $(\mathcal{S},(g,b)),(\mathcal{S},b)$ downgrade with probability* $\widehat{D}_{\mathcal{B}} = \frac{(1-\alpha)\beta_x[\varepsilon^e\mu\alpha+\mu(1-\alpha)+f_G(1-\mu)]}{[\mu(1-\alpha)+f_G(1-\mu)][(1-\alpha)\beta_x+\varepsilon^e\alpha]} \in (0,1)$*, where* $\mu = \Pr(B \mid x)$.[33]

Proposition 3 summarizes the equilibrium strategies. When $x = +2n$, all CRA types pool at downgrading. Otherwise, the strategic CRA types that know the investment is bad ($(\mathcal{S},(g,b))$ and $(\mathcal{S},b)$) mix between downgrading and not, and the strategic CRA type that thinks the investment may be good $(\mathcal{S},(g,g))$ does not downgrade.

The strategic CRA type that thinks the investment may be good $(\mathcal{S},(g,g))$ finds it to be a dominant strategy to mimic the committed type that received the good signals $(\mathcal{C},(g,g))$ and to not downgrade. When the investment succeeds ($y = S$), the investors learn that it was good (bad investments always fail) and so $\boldsymbol{\sigma} = (g,g)$. Hence, when investors observe a downgrade, they think that it came from a strategic type,[34] making it strictly better for type $(\mathcal{S},(g,g))$ not to downgrade. Conditional on the investment failing ($y = F$), type $(\mathcal{S},(g,g))$ has the same payoff structure as types $(\mathcal{S},(g,b))$ and $(\mathcal{S},b)$ (that face a bad investment and expect it to fail for sure). Downgrading is weakly dominated at equilibrium for these types and is, therefore, a weakly dominated action for type $(\mathcal{S},(g,g))$, as well.[35] Thus, type $(\mathcal{S},(g,g))$ never wants to downgrade.

This implies that types $(\mathcal{S},(g,b))$ and $(\mathcal{S},b)$ play a mixed strategy in equilibrium.[36]

6.4 Comparative Statics on Rating Revision

Here, we discuss some features of the equilibrium rating revision strategy. The following lemma summarizes the comparative statics:

[33] This is the posterior probability that the investment is bad, given a realized x, when the distribution of x is as it is implied by the CDS market equilibrium when $m = H$. See Appendix A.4, equation (31) for the characterization of this probability.

[34] Investors know that $\boldsymbol{\sigma} = (g,g)$, and so they are facing either $(\mathcal{C},(g,g))$ or $(\mathcal{S},(g,g))$. They know that $(\mathcal{C},(g,g))$ always plays N. So, using the Cho-Kreps (1987) intuitive criterion, when they observe D, they think it came from $(\mathcal{S},(g,g))$.

[35] If both types $(\mathcal{S},(g,b))$ and $(\mathcal{S},b)$ always downgrade, then downgrading will be inferred as being relatively more likely to have come from a strategic type than from a committed type, making a deviation profitable.

[36] We know that type $(\mathcal{S},(g,g))$ never downgrades at equilibrium - i.e., $D^e_{g,g} = 0$. At $D^e_{\mathcal{B}} = D^e_{g,g} = 0$, all the strategic types are expected to pool on the same action, namely N. This conjecture cannot be part of an equilibrium since, then, when D is observed, the investors think it came from a committed type, making playing D a profitable deviation. This implies that downgrading cannot be strictly dominated for either type $(\mathcal{S},(g,b))$ or $(\mathcal{S},b)$ at equilibrium. Therefore, it is only weakly dominated, and these types play a mixed strategy - i.e., $D^e_{\mathcal{B}} \in (0,1)$.

Lemma 7 *The equilibrium rating revision strategy is such that:*

1. *The probability* $\widehat{D}_{\mathcal{B}}$ *with which types* $(\mathcal{S},(g,b)),(\mathcal{S},b)$ *downgrade when* $x \neq +2n$ *is decreasing in* ε^e*;*

2. *The probability* $\widehat{D}_{\mathcal{B}}$ *is increasing in* $\Pr(B \mid x)$*;*

3. *For* $x = +2n$*, the market does not learn anything about the investment from rating revision.*

When the public signal is not very negative ($x \neq +2n$), as the investors' expected level of rating inflation ε^e increases, a strategic CRA suppresses bad news about the investment (does not downgrade) more often. In equilibrium, when observing a downgrade D, investors learn that the CRA received a bad signal ($\boldsymbol{\sigma} = b$ or (g,b)) and so the investment is bad ($\theta = B$) since types $(\mathcal{C},(g,g))$ and $(\mathcal{S},(g,g))$ are never expected to downgrade. For investors, this raises the likelihood that $\sigma_1 = b$ and the rating was inflated. Hence, an increase in ε^e is more harmful to the CRA when D is played, and so the CRA tends to downgrade less often when ε^e increases.

This result suggests that in equilibrium, rating inflation induces two sources of inefficiencies in the rating process. The first one is in the initial rating, which is less accurate when rating inflation increases. The second one is in rating revision later on, where the CRA's incentives to reveal bad news about the investment are diminished. In other words, rating inflation is *persistent*.

The strategic CRA suppresses negative information more when the market expects the investment to be good ($\Pr(B \mid x)$ small).[37] The CRA, thus, panders to investors' expectations. It does this because mimicking type $(\mathcal{C},(g,g))$ is relatively less costly in this case: the investors believe that the investment was good but failed by chance. This is quite bad from the investors' perspective, as the CRA suppresses private information more often when this information is more valuable to them, since they have formed wrong beliefs about the investment.

Lastly, for $x = +2n$, the market perfectly predicts the rating revision action (all the CRA types pool on $r = D$). Therefore, the D rating itself does not convey any additional information about the signals $\boldsymbol{\sigma}$ or investment quality θ to the investors.

[37] For example, the CRA downgrades more often when $x = 0$ than when $x = -2n$, since the former is not informative about θ, while the latter is a signal for a good investment.

7 Discussion

In this section, we discuss several assumptions and results in the model in more detail:

Allowing for fund-raising and negative NPV projects: In our model, the issuer is endowed with a risky investment project that he wishes to sell to the investors. As both types of assets (*good* and *bad*) have non-negative NPV, the issuer sells the asset regardless of the rating he receives by the CRA. Therefore, ratings provide public information about the asset quality, and so affect the pricing of the asset. Credit ratings thus help investors make better investment decisions and provide useful information to other parties (e.g. to regulators regarding the risk of financial institutions). More-accurate ratings are socially desirable. Other papers - e.g., Mathis et al. (2009), Opp, Opp, and Harris (2012), and Fulghieri, Strobl, and Xia (2014) - consider a setting where (i) the issuer seeks financing for a risky investment project and (ii) bad assets have negative NPV, so that low rated securities are not issued. In this setting, accurate-ratings have a different social value, as the CRA can screen projects such that bad projects do not receive financing and are not implemented. In Internet Appendix IA.3, we show that our results hold in full also in such a setting.

The speculator does not participate in the investment market: This is consistent with recent empirical work by Oehmke and Zawadowski (2016), who show that speculative trading concentrates in the CDS market, due to its relative liquidity advantage. If the speculator buys only a small amount of the investment initially, that will not affect her behavior in the CDS market. If, however, the speculator were to buy a large amount of the investment in the investment market, her incentives might change. Receiving a bad signal could make the purchase of insurance so valuable that the speculator would not want to mask her trades. Or if we were to model a further secondary market and liquidity shocks, the speculator with a bad signal might not want to reveal any information whatsoever, so as not to depress the price of the investment in the further secondary market.

The initial investors in the asset do not participate in the CDS market: If the investors who purchased the investment in the initial investment market were risk neutral, they would choose not to participate in the CDS market. The reason for this is that they suffer an informational disadvantage with respect to the market maker,[38] and so cannot make

[38] The signal observed by the speculator is not known to uninformed traders at the moment they submit their trade orders, while the market maker learns it with positive probability in equilibrium before setting $\widehat{p}^{cds}$.

profits (in expectation) from trading in this market. However, initial investors who are risk averse might want to participate in the CDS market, in order to buy protection for the share of the investment they purchased. We now discuss a simple variation of the model which could allow for this possibility and demonstrate that risk averse initial investors would not participate in the CDS market.

Suppose risk averse initial investors buy an amount of protection Λ.[39] In this case, the speculator's trading strategy does not change with the respect to ones in Lemma 1. The total order flow x then takes values in $\mathcal{X} = \{\Lambda - 2n,\ \Lambda,\ \Lambda + 2n\}$, where the market maker infers $\sigma_s = g_s$ when $x = \Lambda - 2n$, $\sigma_s = b_s$ when $x = \Lambda + 2n$, and nothing when $x = \Lambda$. The investors anticipate they are going to buy CDS protection when purchasing the investment at time $t = 1$. Their willingness to pay for the investment and therefore the rating fee φ depend on ι^e in this case, as ι^e moves their expectations about the equilibrium CDS price. A sufficient condition for the results in the model to still hold in this framework would be that their willingness to pay for the investment is non-increasing in ι^e.[40] This is satisfied, since as we show in Internet Appendix IA.2, the expected price of CDS protection does not change with ι^e, but its variance increases as ι^e increases. The increase in variance decreases risk averse investors' willingness to pay for the investment.

Equity Analysts and Market Information: Our analysis could translate to how other certification intermediaries interact with market information. One example that has received much attention is that of sell-side equity analysts. These analysts have a purpose and incentives similar to those of CRAs: they process information from firms and are affected by reputational concerns (Stickel, 1992) and conflicts of interest.[41] The evidence echoes our results from the rating revision game. Easterwood and Nutt (1999) provide evidence that analysts underreact to bad information, while Chen and Jiang (2006) find that analysts underweight public information. Abarbanell (1991) finds that analysts do not incorporate all of the information from changes in market prices.

[39] We could have that Λ depends on expected rating inflation - i.e., $\Lambda = \Lambda(\varepsilon^e)$, to allow for example the investors to demand more CDS protection when ε^e increases, making the rating $m = H$ a less reliable signal for a good investment. As long as the characterization of $\Lambda(\varepsilon^e)$ is common knowledge among the agents in the model, none of our results would change.

[40] This is needed for Lemma 5, which proves that $\frac{dMBI}{d\iota^e} < 0$. If φ depends on ι^e, $\frac{dMBI}{d\iota^e}$ will include the expression $\frac{\partial \varphi}{\partial \iota^e}$.

[41] While regulation is in place to prevent them from being compensated for generating underwriting business, but evidence suggests that such conflicts may remain (Brown et al. 2015).

8 Conclusion

Credit ratings have real effects. Therefore, an understanding of their dynamics is key. In this paper, we analyze a strategic CRA that trades off current profits and future reputation while incorporating information from public and private sources. More-informative ratings decrease the quality of information in market trading, but more-informative market trading increases the quality of ratings. Downgrades are not always informative and cater to the expectations of investors. It would be interesting to incorporate competition in the rating market and dynamics in market trading to create a richer environment.

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A Appendix

A.1 Posterior Reputation

In what follows, all the probability measures are for given investors' conjectures about the strategic CRA's strategy- i.e., for given $(\varepsilon^e, \delta^e)$. The distribution of x is as it is implied by the equilibrium of the CDS market for a given rating m.

Let z represent a realized information set for the investors; the CRA's posterior reputation given z is as follows:

$$\begin{aligned}
\psi^z &= \Pr(\mathcal{C} \mid z) = \frac{\Pr(z \mid \mathcal{C})\, q}{\Pr(z \mid \mathcal{C})\, q + \Pr(z \mid \mathcal{S})(1-q)} \\
&= \begin{cases} \left(1 + \frac{1-q}{q}\frac{\Pr(z|\mathcal{S})}{\Pr(z|\mathcal{C})}\right)^{-1} \text{ if } \Pr(z \mid \mathcal{C}) \neq 0; \\ 0 \text{ otherwise.} \end{cases}
\end{aligned}$$

A strategic CRA is concerned about its reputation at $t = 3$ - i.e., $\psi^{(m,x,y)}$. We have

$$\begin{aligned}
\Pr(m, x, y \mid \tau) &= \sum_{\theta \in \Theta} \Pr(\theta) \Pr(m, x, y \mid \tau, \theta) \\
&= \sum_{\theta \in \Theta} \Pr(\theta) \Pr(x \mid \theta) \Pr(m, y \mid \tau, \theta, x). \qquad (21)
\end{aligned}$$

Notice that $\Pr(\theta)\Pr(x \mid \theta) = \Pr(x)\Pr(\theta \mid x)$. We can thus write

$$\Pr(m, x, y|\tau) = \Pr(x) \sum_{\theta \in \Theta} \Pr(\theta \mid x) \Pr(m, y \mid \tau, \theta, x). \qquad (22)$$

Let us now focus on $\Pr(m, y \mid \tau, \theta, x)$. When conditioning on θ, the distribution of y does not depend on the realizations of τ and x; we have

$$\Pr(m, y \mid \tau, \theta, x) = \Pr(y \mid \theta) \Pr(m \mid \tau, \theta, x). \tag{23}$$

The investors' conjectures about the CRA's strategy are defined over the signal observed by the CRA. Hence, it is convenient to decompose $\Pr(m \mid \tau, \theta, x)$ as follows:

$$\Pr(m \mid \tau, \theta, x) = \sum_{\sigma \in \{b,g\}} \Pr(\sigma \mid \theta) \Pr(m \mid (\tau, \sigma)), \tag{24}$$

where (τ, σ) is the CRA overall type.

The conditioning over x in $\Pr(m \mid (\tau, \sigma))$ is no longer relevant, since x realizes after m is chosen. Using equations from (21) to (24), we have:

$$\Pr(m, x, y \mid \tau) = \Pr(x) \sum_{\theta \in \Theta} \Pr(\theta \mid x) \Pr(y \mid \theta) \left(\sum_{\sigma \in \{b,g\}} \Pr(\sigma \mid \theta) \Pr(m \mid (\tau, \sigma)) \right).$$

Notice that $\Pr(x)$ does not depend on τ, so it cancels out when considering $\frac{\Pr(m,x,y|\mathcal{S})}{\Pr(m,x,y|\mathcal{C})}$ in $\psi^{(m,x,y)}$.

When we consider the extension on rating revision, investors update the CRA's reputation after observing the rating revision action, as well. Hence, we have $\psi^{(m,x,y,r)}$. We need to introduce the investors' conjectures about the CRA's rating revision strategy. Let $\mathcal{R}$ denote, for a given realization of x, the set of $D_{\boldsymbol{\sigma}}$ for all types $(\mathcal{S}, \boldsymbol{\sigma} = (\sigma_1, \sigma_2))$; let $\mathcal{R}^e$ denote the investors' conjectures about $\mathcal{R}$. We therefore have:

$$\mathcal{R}^e = \left(D^e_b,\ D^e_{g,b},\ D^e_{g,g} \right).$$

The derivation of $\Pr(m, x, y, r \mid \tau)$ follows the same steps as for $\Pr(m, x, y \mid \tau)$. We can decompose $\Pr(m, r \mid (\tau, \boldsymbol{\sigma}), x)$ as

$$\Pr(m, r \mid (\tau, \boldsymbol{\sigma}), x) = \Pr(m \mid (\tau, \sigma_1)) \Pr(r \mid (\tau, \boldsymbol{\sigma}), m, x)$$

since the CRA has observed only σ_1 when m is chosen.

In this case, we have:

$$\Pr\left(m,x,y,r|\ \tau\right)=$$
$$=\Pr\left(x\right)\sum_{\theta\in\Theta}\Pr\left(\theta\mid x\right)\Pr\left(y\mid\theta\right)\left(\sum_{\boldsymbol{\sigma}\in\Sigma}\Pr\left(\boldsymbol{\sigma}\mid x,\theta\right)\Pr\left(m\mid\left(\tau,\sigma_{1}\right)\right)\Pr\left(r\mid\left(\tau,\boldsymbol{\sigma}\right),m,x\right)\right), \tag{25}$$

where $\Sigma\equiv\left\{\left(g,g\right),\left(g,b\right),b\right\}$. Once again, $\Pr\left(x\right)$ cancels out when considering $\frac{\Pr(m,x,y,r|\mathcal{S})}{\Pr(m,x,y,r|\mathcal{C})}$.

Derivative with respect to $D^{e}_{\boldsymbol{\sigma}}$

Notice that $\Pr\left(H,x,y,r|\ \mathcal{C}\right)$ does not depend on $D^{e}_{\boldsymbol{\sigma}}$. We have

$$\frac{\partial\psi^{(H,x,y,r)}}{\partial D^{e}_{\boldsymbol{\sigma}}}=-\left(\psi^{(H,x,y,r)}\right)^{2}\frac{1-q}{q\Pr\left(H,x,y,r|\ \mathcal{C}\right)}\frac{\partial\Pr\left(H,x,y,r|\ \mathcal{S}\right)}{\partial D^{e}_{\boldsymbol{\sigma}}}. \tag{26}$$

We have

$$\frac{\partial\Pr\left(H,x,y,D|\ \mathcal{S}\right)}{\partial D^{e}_{\boldsymbol{\sigma}}}\geq 0,\ \frac{\partial\Pr\left(H,x,y,N|\ \mathcal{S}\right)}{\partial D^{e}_{\boldsymbol{\sigma}}}\leq 0, \tag{27}$$

with strict inequalities whenever the investors believe that, with some positive probability, they are facing type $(\mathcal{S},\boldsymbol{\sigma})$.

This is implied by equation (25), since $\Pr\left(D\mid\left(\mathcal{S},\boldsymbol{\sigma}\right),H,x\right)$ is increasing and $\Pr\left(N\mid\left(\mathcal{S},\boldsymbol{\sigma}\right),H,x\right)$ decreasing in $D^{e}_{\boldsymbol{\sigma}}$. Therefore, we have

$$\frac{\partial\psi^{(H,x,y,D)}}{\partial D^{e}_{\boldsymbol{\sigma}}}\leq 0,\ \frac{\partial\psi^{(H,x,y,N)}}{\partial D^{e}_{\boldsymbol{\sigma}}}\geq 0, \tag{28}$$

with strict inequalities when the inequalities in (27) are strict.

A.2 Proof of Lemma 1

We prove Lemma 1 in two steps. First, we prove that the described trading strategy for the speculator is the unique possible in equilibrium. Second, we verify that, given the speculator's strategy, the market maker's inference is consistent with Bayes rule.

Step one: Speculator's trading strategy

Consider the following trading strategy for the speculator: buy when $\sigma_s = b_s$; sell otherwise. We show that this is the unique possible trading strategy in equilibrium. Notice that we have

$$E_\theta \left(f_\theta \mid H, g_s \right) \leq \widehat{p}^{cds} \leq 1. \tag{29}$$

The informativeness of x about θ is driven by the speculator's private information σ_s. The inequalities in (29) set bounds for $\widehat{p}^{cds}$: it cannot be greater than 1 ($f_B = 1$); it cannot be lower than $E_\theta \left(f_\theta \mid H, g_s \right)$, since $\sigma_s = g_s$ is a signal for a good investment. For $\sigma_s = b_s$, the speculator learns that the investment is bad and will fail for sure. If she buys (sells) protection, profit from trade is $1-\widehat{p}^{cds}$ ($\widehat{p}^{cds}-1$), which by (29) is strictly positive (negative) when x does not reveal σ_s to the market maker, and 0 otherwise. Thus, buying protection is always a dominant strategy when $\sigma_s = b_s$. For $\sigma_s = g_s$, the speculator forms the following expectation about the probability of default: $E_\theta \left(f_\theta \mid H, g_s \right)$. If she sells (buys) protection, expected profit from trade is $\widehat{p}^{cds} - E_\theta \left(f_\theta \mid H, g_s \right)$ ($E_\theta \left(f_\theta \mid H, g_s \right) - \widehat{p}^{cds}$), which is strictly positive (negative) when x does not reveal σ_s, and 0 otherwise. Hence, selling protection is always a dominant strategy when $\sigma_s = g_s$.

Notice that the described strategy is strictly dominant whenever the market maker does not always learn σ_s from x, which is the case in our setting (because of camouflage with the liquidity traders). Therefore, it is the unique possible equilibrium trading strategy.

Step two: Market maker's inference

The inference when $x \neq 0$ trivially satisfies Bayes rule. Here, we show that $\Pr\left(x = 0 \mid \theta\right) = \frac{1}{2} \forall \theta \in \Theta$, and so no inference about θ is possible from $x = 0$. Using conditioning over σ_s , we can write

$$\Pr\left(x = 0 \mid \theta\right) = \sum_{\sigma_s} \Pr\left(\sigma_s \mid \theta\right) \Pr\left(x = 0 \mid \sigma_s, \theta\right).$$

Note that $\Pr\left(x = 0 \mid \sigma_s, \theta\right) = \frac{1}{2}$ for any σ_s [42] and so does not depend on the realization

[42] When $\sigma_s = g_s$, the speculator is selling, so $x^s = -n$. For x to be 0, we need to have $x^n = +n$, which has probability $\frac{1}{2}$. The opposite is true when $\sigma_s = b_s$.

of θ. We can write, then:

$$\Pr(x=0 \mid \theta) = \frac{1}{2}\sum_{\sigma_s}\Pr(\sigma_s \mid \theta) = \frac{1}{2}\left[\Pr(b_s \mid \theta) + \Pr(g_s \mid \theta)\right].$$

For $\theta = G$, we have $\Pr(g_s \mid G) = 1$, and so $\Pr(x=0 \mid G) = \frac{1}{2}$. For $\theta = B$, we have

$$\Pr(x=0 \mid B) = \frac{1}{2}\left[\iota^e + (1-\iota^e)\right] = \frac{1}{2}.$$

A.3 CDS Market Equilibrium for $m = \varnothing$ and $\delta^e > 0$

As we mentioned before, when the CRA is not expected to deflate a good signal - i.e., $\delta^e = 0$, $m = \varnothing$ conclusively reveals that the investment is bad, and so the speculator does not enter the CDS market. This is satisfied in equilibrium, as the CRA does never deflate a good signal. However, we need to characterize the CDS market equilibrium also for $\delta^e > 0$, in order to solve for the equilibrium rating strategy later on.

When $\delta^e > 0$, $m = \varnothing$ is just an imperfect signal about $\theta = B$, and hence the speculator may want to enter the market in this case. Let $\widetilde{\iota}$ denote the speculator's choice of precision when $m = \varnothing$, and $\widetilde{\iota^e}$ the other agents' expectations about it. The following Lemma characterizes the speculator's trading strategy and the market maker's inference from the order flow, when $m = \varnothing$.

Lemma 8 *Given $m = \varnothing$ and for given $\widetilde{\iota^e}$ and a pair $(\varepsilon^e, \delta^e)$, the unique equilibrium of the CDS market is characterized as follows:*

1. *If the speculator enters the market, her trading strategy is the following:*

    (a) *if $\Pr(B \mid m = \varnothing, \sigma_s = g_s) < \frac{1}{2}$, her trading strategy is $x^s(b_s) = +n$ and $x^s(g_s) = -n$;*

    (b) *she always trades $x^s = +n$ otherwise.*

2. *The market maker's inference is as follows:*

    (a) *if $\Pr(B \mid m = \varnothing, \sigma_s = g_s) < \frac{1}{2}$, he infers $\sigma_s = b_s$ when $x = +2n$, $\sigma_s = g_s$ when $x = -2n$, and nothing when $x \in \{-n, 0, +n\}$;*

(b) *the realization of x is never informative about θ otherwise.*

Lemma 8 is quite intuitive as well. If $\Pr(B \mid m = \varnothing, \sigma_s = g_s) < \frac{1}{2}$, when the speculator observes $\sigma_s = g_s$ she thinks the investment is more likely to be good, and so does sell protection in this case. If $\Pr(B \mid m = \varnothing, \sigma_s = g_s) \geq \frac{1}{2}$, even after having observed $\sigma_s = g_s$, the speculator still thinks the investment is more likely to be bad, and so still wants to buy protection. Given these strategies, the market maker inference is straightforward.

Notice that, when $\Pr(B \mid m = \varnothing, \sigma_s = g_s) \geq \frac{1}{2}$, the speculator ignores its private information, since she always chooses to buy protection, regardless of its private signal. This means that she cannot use her informational advantage to profit from trading in the market, and so chooses not to enter. Conversely, she has positive expected profits from trades when $\Pr(B \mid m = \varnothing, \sigma_s = g_s) < \frac{1}{2}$, and so does enter the market in this case.

The sign of this inequality depends on the relative informativeness of the signals $m = \varnothing$ and $\sigma_s = g_s$, hence, on $(\varepsilon^e, \delta^e)$ and $\widetilde{\iota}$. For a given $\delta^e > 0$, whenever the speculator enters the market $\widetilde{\iota}$ is strictly greater than $\widetilde{\iota}^*$ such that $\Pr(B \mid m = \varnothing, \sigma_s = g_s, \widetilde{\iota}^*) = \frac{1}{2}$. The cost of precision is crucial in this: if precision is cheap, the speculator can afford $\widetilde{\iota} > \widetilde{\iota}^*$, and therefore enters the market; she prefers not to enter the market otherwise. The implications, in terms of the informativeness of x, are summarized in the following Remark.

Remark 1 *Given $m = \varnothing$ and for a given pair $(\varepsilon^e, \delta^e)$, where $\delta^e > 0$, we have one of the possible cases, depending on $c(\iota)$:*

1. *The speculator chooses not to enter the market; in this case, x is not informative about θ;*

2. *The speculator enters the market and chooses a level of precision $\widetilde{\iota} > \widetilde{\iota}^*$; in this case, x is informative about θ.*

A.4 Distribution of x implied by the CDS Market Equilibrium

Given the characterization of the CDS market equilibrium for any given rating m, we can now describe the implications on the distribution of x.

When $m = H$, the speculator always enters the CDS market, and is expected to choose a strictly positive level of precision ι^e. The distribution of x implied by the equilibrium trading strategies in Lemma 1, is the following:

$$\Pr\left(x \mid B, m=H\right)=\begin{cases}\frac{\iota^e}{2} \text{ for } x=+2n,\\ \frac{1-\iota^e}{2} \text{ for } x=-2n,\\ \frac{1}{2} \text{ for } x=0.\end{cases};\ \Pr\left(x \mid G, m=H\right)=\begin{cases}0 \text{ for } x=+2n,\\ \frac{1}{2} \text{ for } x=-2n,\\ \frac{1}{2} \text{ for } x=0.\end{cases}. \tag{30}$$

Let $X(\theta, H)$ denote the random variable generated by the above distribution. Therefore, x induces the following posterior beliefs about θ when $m = H$:

$$\mu \equiv \Pr\left(B \mid x,\ x \sim X(\theta, H)\right)=\begin{cases}1 \text{ for } x=+2n,\\ \frac{1-\iota^e}{2-\iota^e} \text{ for } x=-2n,\\ \frac{1}{2} \text{ for } x=0.\end{cases}. \tag{31}$$

As we discussed before, when $m = \varnothing$ the speculator may or may not enter the CDS market. When she does enter, she is expected to choose a strictly positive level of precision $\widetilde{\iota}^e$. The distribution of x in this case is:

$$\Pr\left(x \mid B, m=\varnothing\right)=\begin{cases}\frac{\widetilde{\iota}^e}{2} \text{ for } x=+2n,\\ \frac{1-\widetilde{\iota}^e}{2} \text{ for } x=-2n,\\ \frac{1}{2} \text{ for } x=0.\end{cases};\ \Pr\left(x \mid G, m=\varnothing\right)=\begin{cases}0 \text{ for } x=+2n,\\ \frac{1}{2} \text{ for } x=-2n,\\ \frac{1}{2} \text{ for } x=0.\end{cases}. \tag{32}$$

Let $X(\theta, \varnothing)$ denote the random variable generated by the above distribution. Hence, x induces the following posterior beliefs about θ when $m = \varnothing$ and the speculator enters the market:

$$\mu' \equiv \Pr\left(B \mid x,\ x \sim X(\theta, \varnothing)\right)=\begin{cases}1 \text{ for } x=+2n,\\ \frac{1-\widetilde{\iota}^e}{2-\widetilde{\iota}^e} \text{ for } x=-2n,\\ \frac{1}{2} \text{ for } x=0.\end{cases}. \tag{33}$$

When she does not enter the market, we have $x \in \{-n, +n\}$, and x is not informative about θ; therefore, the realization of x does not affect the agents beliefs about θ in this case.

A.5 Proof of Lemma 2

We need to show that $\frac{d}{d\varepsilon^e}\left(\frac{\partial \Pi^s}{\partial \iota}\right) < 0$. We have

$$\frac{\partial \Pi^s}{\partial \iota} = n\mu^H \left(1 - \widehat{p}^{cds}\right) - c'(\iota) = n\mu^H \left(1 - \mu^H\right)(1 - f_G) - c'(\iota) =$$
$$= n\left(\mu^H - \left(\mu^H\right)^2\right)(1 - f_G) - c'(\iota).$$

since $\widehat{p}^{cds} = f_G + \mu^H (1 - f_G)$.

This implies that

$$\frac{d}{d\varepsilon^e}\left(\frac{\partial \Pi^s}{\partial \iota}\right) = n(1 - f_G)\frac{d\mu^H}{d\varepsilon^e}\left(1 - 2\mu^H\right).$$

We want to prove that $\frac{d}{d\varepsilon^e}\left(\frac{\partial \Pi^s}{\partial \iota}\right) \geq 0$. First, we show that $\mu^H < \frac{1}{2}$, so that $1 - 2\mu^H > 0$. Then, we show that $\frac{d\mu^H}{d\varepsilon^e} \geq 0$.

Let ψ^H denote the updated probability that the investors assign to the CRA being committed after they observe $m = H$- i.e., $\psi^H = \Pr\left(\tau = \mathcal{C} \mid m = H\right)$. We have

$$\mu^H = \Pr(B \mid H) = \left(1 + \frac{\Pr(H \mid G)}{\Pr(H \mid B)}\right)^{-1}$$

where

$$\Pr(H \mid G) = 1; \;\; \Pr(H \mid B) = 1 - \alpha + \alpha(1 - \psi^H)\varepsilon^e < 1.$$

This implies that $\mu^H < \frac{1}{2}$ [43]. Let us focus on $\frac{d\mu^H}{d\varepsilon^e}$; we have

$$\frac{d\mu^H}{d\varepsilon^e} = \left(\frac{\mu^H}{\Pr(H \mid B)}\right)^2 \frac{d\Pr(H \mid B)}{d\varepsilon^e},$$

where

$$\frac{d\Pr(H \mid B)}{d\varepsilon^e} = \alpha\left(1 - \psi^H\right) - \underbrace{\frac{\partial \psi^H}{\partial \varepsilon^e}}_{<0}\varepsilon^e > 0.$$

[43] $m = H$ is always a signal for a good project; its accuracy depends on $(\alpha, q, \varepsilon^e)$.

This implies that $\frac{d\mu^H}{d\varepsilon^e} \geq 0$, and so $\frac{\partial \Pi^s}{\partial \iota \partial \varepsilon^e} \geq 0$.[44]

A.6 Proof of Lemma 3

We prove Lemma 3 in two steps. First, we derive the expression for $\psi^{(H,x,F)}$ and $\psi^{(\varnothing,x,F)}$, and show that $\psi^{(H,x,F)}$ is decreasing and $\psi^{(\varnothing,x,F)}$ increasing in ε^e for any given x; this implies that MBI is always decreasing in ε^e. Second, we prove that MBI is strictly positive at $\varepsilon^e = 0$ for any given δ^e.

Step One: MBI decreasing in ε^e

Let $\mu = \Pr(B \mid x,\ x \sim X(\theta, H))$;[45] let us first derive the expression for $\psi^{(H,x,F)}$. We have:

$$\frac{\Pr(H,x,F|\,\mathcal{S})}{\Pr(H,x,F|\,\mathcal{C})} = \frac{\overbrace{\mu\alpha\varepsilon^e}^{\text{prob. }(\mathcal{S},b)} + \overbrace{(\mu(1-\alpha) + (1-\mu) f_G)(1-\delta^e)}^{\text{prob. }(\mathcal{S},g)}}{\underbrace{\mu(1-\alpha) + (1-\mu) f_G}_{\text{prob. }(\mathcal{C},g)}}. \tag{34}$$

Therefore, for given ε^e and δ^e, we can write $\psi^{(H,x,F)}$ as:

$$\begin{aligned} \psi^{(H,x,F)} &= \left(1 + \frac{1-q}{q}\frac{\Pr(H,x,F|\,\mathcal{S})}{\Pr(H,x,F|\,\mathcal{C})}\right)^{-1} \\ &= \left(1 + \frac{1-q}{q}\frac{\mu\alpha\varepsilon^e + (\mu(1-\alpha) + (1-\mu) f_G)(1-\delta^e)}{\mu(1-\alpha) + (1-\mu) f_G}\right)^{-1}. \end{aligned} \tag{35}$$

Note that $\psi^{(H,x,F)}$ is decreasing in ε^e.

[44] Note that we have:
$$\frac{\partial \psi^H}{\partial \varepsilon^e} = -\left(\psi^H\right)^2 \frac{1-q}{q \Pr(H \mid \mathcal{C})} \frac{\partial \Pr(H \mid \mathcal{S})}{\partial \varepsilon^e} > 0,$$
where $\Pr(H \mid \mathcal{S}) = \frac{1}{2}(1-\alpha+\alpha\varepsilon^e) + \frac{1}{2}$.

[45] This is the posterior probability that the investment is bad, given a realized x, when the distribution of x is as it is implied by the CDS market equilibrium when $m = H$. See Appendix A.4 for the characterization of this probability.

Now we derive the expression for $\psi^{(\varnothing,x,F)}$. Let $\mu' = \Pr(B \mid x,\ x \sim X(\theta,\varnothing))$;[46] we have

$$\frac{\Pr(\varnothing,x,F|\,\mathcal{S})}{\Pr(\varnothing,x,F|\,\mathcal{C})} = \frac{\overbrace{\mu'\alpha(1-\varepsilon^e)}^{\text{prob. }(\mathcal{S},b)} + \overbrace{(\mu'(1-\alpha)+(1-\mu')f_G)\,\delta^e}^{\text{prob. }(\mathcal{S},g)}}{\underbrace{\mu'\alpha}_{\text{prob. }(\mathcal{C},b)}}.$$

This implies

$$\begin{aligned}\psi^{(\varnothing,x,F)} &= \left(1+\frac{1-q}{q}\frac{\mu'\alpha(1-\varepsilon^e)+\mu'(1-\alpha)\delta^e+(1-\mu')f_G\delta^e}{\mu'\alpha}\right)^{-1}\\ &= \frac{q\mu'\alpha}{q\mu'\alpha+(1-q)\left[\mu'\alpha(1-\varepsilon^e)+\mu'(1-\alpha)\delta^e+(1-\mu')f_G\delta^e\right]}.\end{aligned} \tag{36}$$

Note that $\psi^{(\varnothing,x,F)}$ is, instead, increasing in ε^e.

Therefore, we have

$$\frac{dMBI}{d\varepsilon^e} = \underbrace{\frac{\partial\varphi}{\partial\varepsilon^e}}_{<0\text{ by Ass.2}} + \gamma E_x\left[\underbrace{\frac{d\psi^{(H,x,F)}}{d\varepsilon^e}}_{<0} - \underbrace{\frac{\partial\psi^{(\varnothing,x,F)}}{\partial\varepsilon^e}}_{>0} \mid B\right] < 0.$$

Step Two: MBI positive at $\varepsilon^e = 0$

Here, we prove that MBI is strictly positive at $\varepsilon^e = 0$. By Assumption 2, we know that the rating fee is strictly positive at $\varepsilon^e = 0$. This implies that $E_x\left[\psi^{(H,x,F)} - \psi^{(\varnothing,x,F)} \mid B\right] \geq 0$ is a sufficient condition for MBI positive at $\varepsilon^e = 0$. We can show that $\psi^{(H,x,F)} \geq \psi^{(\varnothing,x,F)}$ for

[46] This is instead the posterior probability that the investment is bad, given a realized x, when the distribution of x is as it is implied by the CDS market equilibrium when $m = \varnothing$. See Appendix A.4 for the characterization of this probability.

any possible realization of x.[47] We have

$$\underbrace{\frac{q}{1-\delta^e(1-q)}}_{\psi^{(H,x,F)}\text{ at }\varepsilon^e=0,\text{ by (35)}} \geq \underbrace{\frac{q\mu'\alpha}{\mu'\alpha+(1-q)(\mu'(1-\alpha)+(1-\mu')f_G)\delta^e}}_{\psi^{(\varnothing,x,F)}\text{ at }\varepsilon^e=0,\text{ by (36)}}$$
$$\Leftrightarrow 1 \geq \frac{\mu'\alpha(1-\delta^e(1-q))}{\mu'\alpha+(1-q)((1-\alpha)+(1-\mu')f_G)\delta^e}$$
$$\Leftrightarrow \mu'\alpha-\mu'\alpha\delta^e(1-q) \leq \mu'\alpha+(1-q)((1-\alpha)+(1-\mu')f_G)\delta^e$$
$$\Leftrightarrow 0 \leq \mu'\alpha\delta^e(1-q)+(1-q)((1-\alpha)+(1-\mu')f_G)\delta^e.$$

This implies that $E_x\left[\psi^{(H,x,F)}-\psi^{(\varnothing,x,F)} \mid B\right] \geq 0$ and so MBI is positive at $\varepsilon^e=0$.

A.7 Proof of Lemma 4

We need to prove that deflating a good signal is a strictly dominated action in equilibrium.

A good initial signal is not conclusive about the quality of the investment; therefore, type $(\mathcal{S},g)$ has a conditional expectation over the investment quality θ, which affects the expectation over the realizations of x and y. Given a good signal, there are three possibilities: the investment is bad; the investment is good and fails; and the investment is good and successful. The expected payoffs for type $(\mathcal{S},g)$ are:

$$\Pi_g = \Pr(B \mid g)\,E_x\left[\delta\gamma\psi^{(\varnothing,x,F)}+(1-\delta)\left(\varphi+\gamma\psi^{(H,x,F)}\right) \mid B\right]+$$
$$+\Pr(G \mid g)\,f_G E_x\left[\delta\gamma\psi^{(\varnothing,x,F)}+(1-\delta)\left(\varphi+\gamma\psi^{(H,x,F)}\right) \mid G\right]+$$
$$+\Pr(G \mid g)(1-f_G)\left\{\delta\gamma\psi^{(\varnothing,S)}+(1-\delta)\left(\varphi+\gamma\psi^{(H,S)}\right)\right\}.$$

The signal is deflated with probability δ, in which case $m=\varnothing$ and the rating fee is not collected. If the signal is not deflated, the rating is $m=H$, and the rating fee is collected.[48]

[47] The space of the realizations of $x \sim X(\theta,H)$ is the same as for $x \sim X(\theta,\varnothing)$ when the speculator enters the CDS market when $m=\varnothing$: in both cases, we have $x \in \{-2n,0,+2n\}$. If the speculator does not enter the market, we have instead that $x \sim X(\theta,\varnothing)$ takes values in $\{-n,+n\}$. However, x is not informative about θ in this case, and so posterior reputation conditional on $m=\varnothing$ does not depend on x - i.e., we consider $\psi^{(\varnothing,F)}$ and not $\psi^{(\varnothing,x,F)}$ in this case.

[48] Note that the rating fee φ does not depend on the realization of x (the initial rating is assigned before x is observed); we include it into the expectation to lighten notation.

Notice that, when the investment succeeds, the investors infer that this must be good (a bad investment always fails) and so reputation updating does not depend on x in this case.

Type $(\mathcal{S}, g)$ chooses δ as the best response to the conjectures $(\varepsilon^e, \delta^e)$. Taking the derivative of Π_g with respect to δ yields:

$$\begin{aligned}\frac{d\Pi_g}{d\delta} &= \Pr(B \mid g)\, E_x\left[\gamma\psi^{(\varnothing,x,F)} - \left(\varphi + \gamma\psi^{(H,x,F)}\right) \mid B\right] + \\ &+ \Pr(G \mid g)\, f_G E_x\left[\gamma\psi^{(\varnothing,x,F)} - \left(\varphi + \gamma\psi^{(H,x,F)}\right) \mid G\right] + \\ &+ \Pr(G \mid g)\,(1 - f_G)\left\{\gamma\psi^{(\varnothing,S)} - \left(\varphi + \gamma\psi^{(H,S)}\right)\right\}. \end{aligned} \tag{37}$$

The derivative $\frac{d\Pi_g}{d\delta}$ represents the marginal benefit of rating deflation. We want to show that this is always negative in equilibrium, so that deflating is strictly dominated (by truthfully reporting the signal) and $\widehat{\delta} = 0$.

When the investment succeeds ($y = S$), investors learn that $\theta = G$ and so $\sigma = g$. Therefore, $m = \varnothing$ reveals a strategic type to the market (the signal has been deflated) - i.e., $\psi^{(\varnothing,S)} = 0$. The last term in (37) is, thus, strictly negative.

Note that we have $MBI \geq 0$ at equilibrium;[49] this implies that the first two lines of (37) are negative, as well. We have

$$MBI \geq 0 \Leftrightarrow E_x\left[\left(\varphi + \gamma\psi^{(H,x,F)}\right) - \gamma\psi^{(\varnothing,x,F)} \mid B\right] \geq 0. \tag{38}$$

This trivially implies that the first line in (37) is non-positive.

Now, we show that this also implies that the second line in (37) is negative. Notice that

[49] For the equilibrium level of inflation, we have either a corner solution where MBI is strictly positive, or an interior solution where $MBI = 0$.

$\psi^{(H,x,F)}$ is decreasing in μ, where $\mu = \Pr(B \mid x,\ x \sim X(\theta, H))$.[50] We have

$$\psi^{(H,x,F)} = \frac{1}{1 + \frac{1-q}{q} \underbrace{\frac{\mu\alpha\varepsilon^e + (\mu(1-\alpha) + (1-\mu) f_G)(1-\delta^e)}{\mu(1-\alpha) + (1-\mu) f_G}}_{\equiv A'}};$$

$$\frac{dA'}{d\mu} = \frac{\alpha f_G \varepsilon^e}{(\mu(1-\alpha) + (1-\mu) f_G)^2} > 0,$$

which implies that $\frac{\partial \psi^{(H,x,F)}}{\partial \mu} < 0$.

This means that:

$$E_x\left[\psi^{(H,x,F)} \mid B\right] < E_x\left[\psi^{(H,x,F)} \mid G\right], \tag{39}$$

since $\Pr(x \mid B, m = H)$ puts more (less) density than $\Pr(x \mid G, m = H)$ over $x = +2n$ ($x = -2n$), which is a signal for a bad (good) investment.[51] This is because, when $m = H$, the speculator always enters the CDS market, and so x is informative about the true θ.

Notice also that $\psi^{(\varnothing,x,F)}$ is, instead, increasing in μ', where $\mu' = \Pr(B \mid x,\ x \sim X(\theta, \varnothing))$.[52] We have

$$\psi^{(\varnothing,x,F)} = \frac{1}{1 + \frac{1-q}{q} \underbrace{\frac{\mu'\alpha(1-\varepsilon^e) + \mu'(1-\alpha)\delta^e + (1-\mu') f_G \delta^e}{\mu'\alpha}}_{\equiv A''}};$$

$$\frac{dA''}{d\mu'} = -\frac{\delta^e f_G}{(\mu')^2 \alpha} < 0,$$

which implies that $\frac{\partial \psi^{(\varnothing,x,F)}}{\partial \mu'} > 0$.

This means that:

$$E_x\left[\psi^{(\varnothing,x,F)} \mid B\right] \geq E_x\left[\psi^{(\varnothing,x,F)} \mid G\right], \tag{40}$$

with strict inequality when the speculator chooses to enter the CDS market.

[50] As the investors think it is relatively more likely that the investment is bad, $m = H$ becomes a stronger signal for a strategic CRA, as it is relatively more likely that the CRA observed a bad signal and inflated it.

[51] See equation (30) in Appendix A.4.

[52] As the investors think it is relatively more likely that the investment is bad, $m = \varnothing$ becomes instead a stronger signal for a committed type, as it is relatively more likely that the CRA observed a bad signal and did not inflate it.

As we discussed before, conditional on $m = \varnothing$, the speculator may or may not enter the CDS market. When she does not enter, $\Pr(x \mid B, m = \varnothing)$ and $\Pr(x \mid G, m = \varnothing)$ are the same, since x is not informative about θ. However, each time she does enter the market, the realization of market trading x is informative about θ, and the same reasoning as above holds.

The inequalities in (39) and (40) imply that the second line of (37) is always negative when the inequality in (38) holds.

A.8 Proof of Lemma 5

We need to show that $DE + IE < 0$, where DE refers to the *Direct Effect* and IE to the *Indirect Effect* in equation (15).

Let $\widetilde{\mu} = \Pr(B \mid x = -2n,\ x \sim X(\theta, H))$; we can rearrange IE as follows:

$$IE = (1 - \iota^e) \underbrace{\frac{\partial \psi^{(H,-2n,F)}}{\partial \widetilde{\mu}}}_{\leq 0} \underbrace{\frac{\partial \widetilde{\mu}}{\partial \iota^e}}_{\leq 0} = -\frac{\partial \psi^{(H,-2n,F)}}{\partial \widetilde{\mu}} \frac{(1 - \iota^e)}{(2 - \iota^e)^2}.$$

since $\widetilde{\mu} = 1 - \frac{1}{2-\iota^e}$ and so $\frac{\partial \widetilde{\mu}}{\partial \iota^e} = -\frac{1}{(2-\iota^e)^2}$.

Note that we can write $\widetilde{\mu} = \frac{1-\iota^e}{2-\iota^e}$ and $(1 - \widetilde{\mu}) = \frac{1}{2-\iota^e}$. We have

$$IE = -\frac{\partial \psi^{(H,-2n,F)}}{\partial \widetilde{\mu}} \widetilde{\mu} (1 - \widetilde{\mu}).$$

DE, instead, is as follows:

$$DE = \psi^{(H,+2n,F)} - \psi^{(H,-2n,F)}.$$

The proof consists of computing $DE + IE$ as defined above, and showing that it is always negative. We have

$$\psi^{(H,-2n,F)} \underbrace{=}_{(35)\text{ at }\mu=\widetilde{\mu}} \frac{q\left(\widetilde{\mu}\left(1-\alpha-f_G\right)+f_G\right)}{f_G+\widetilde{\mu}\left(1-\alpha-f_G+\varepsilon^e\left(1-q\right)\alpha\right)};$$

$$\psi^{(H,+2n,F)} \underbrace{=}_{(35)\text{ at }\mu=1} \frac{q\left(1-\alpha\right)}{q\left(1-\alpha\right)+\left(1-q\right)\left(1-\alpha+\alpha\varepsilon^e\right)};$$

$$-\frac{\partial\psi^{(H,-2n,F)}}{\partial\widetilde{\mu}} = \frac{q\left(1-q\right)f_G\alpha\varepsilon^e}{\left(f_G+\widetilde{\mu}\left(1-\alpha-f_G+\varepsilon^e\left(1-q\right)\alpha\right)\right)^2}.$$

We can then evaluate $DE+IE$. We have

$$DE+IE=-\frac{q\left(1-q\right)\varepsilon^e f_G^2\alpha\left(1-\widetilde{\mu}\right)^2}{\left(1-\alpha+\alpha\varepsilon^e\left(1-q\right)\right)\left[f_G+\widetilde{\mu}\left(1-\alpha-f_G+\varepsilon^e\left(1-q\right)\alpha\right)\right]^2}\leq 0. \tag{41}$$

A.9 Proof of Proposition 2

The proof for Proposition 2 is straightforward. We have $\widehat{\iota}\left(\varepsilon^e\right)\in(0,1)$ and $\widehat{\varepsilon}\left(\iota^e\right)\in(0,1]$: this implies that the two reaction functions in Figure 5 either do cross (and do so only once) at some interior level $(\widehat{\varepsilon},\widehat{\iota})\in(0,1)^2$, or they do not cross, and we have a corner solution for rating inflation- i.e., $\widehat{\varepsilon}=1$, $\widehat{\iota}=\widehat{\iota}\left(1\right)$.

A.10 Proof of Lemma 6

Figure 7 describes the sequence of possible events when the CRA is strategic and the investment is bad. The measure of informational efficiency Γ corresponds to the sum of the probabilities over the paths that end in a conclusive signal- i.e., $m=\varnothing$ or $x=+2n$ (in red in Figure 7). We have:

$$\Gamma=\alpha\left(1-\widehat{\varepsilon}\right)+\frac{\left(\alpha\widehat{\varepsilon}+1-\alpha\right)\widehat{\iota}}{2}. \tag{42}$$

We prove Lemma 6 in two steps. First, we show that Γ is decreasing in $\widehat{\varepsilon}$ and increasing $\widehat{\iota}$; since an increase in k leads to lower $\widehat{\iota}$ and higher $\widehat{\varepsilon}$, the result about k follows. The result on n is similar. Then, we show that Γ may instead be increasing or decreasing in γ, depending on the parameter configuration. This second part of the proof is in Internet Appendix IA.1.

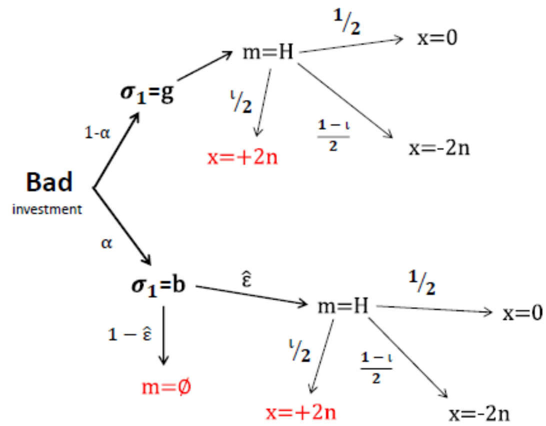

Figure 7: Sequence of Events given $\theta = B$ and $\tau = \mathcal{S}$.

Taking the derivative of Γ with respect to $\widehat{\varepsilon}$ yields:

$$\frac{d\Gamma}{d\widehat{\varepsilon}} = -\alpha + \frac{\alpha\widehat{\iota}}{2} = \frac{\alpha}{2}\left(-2 + \widehat{\iota}\right) < 0,$$

Taking the derivative of Γ with respect to $\widehat{\iota}$ yields:

$$\frac{d\Gamma}{d\widehat{\iota}} = \frac{(\alpha\widehat{\varepsilon} + 1 - \alpha)}{2} > 0.$$

A.11 Proof of Proposition 3

We characterize the equilibrium rating revision strategy for given conjectures by investors about the initial rating inflation ε^e and deflation δ^e. Then, we show that the playing of the rating revision game, in equilibrium, does not affect the CRA's incentives at the initial rating stage, so that the equilibrium initial rating strategy - as it is described in Proposition 1 - does not change.

Payoffs and Best Response Functions. At $t = 2.5$, CRA type $(\mathcal{S}, \boldsymbol{\sigma})$ downgrades with probability $D_{\boldsymbol{\sigma}}$. Its continuation payoffs are

$$\Pi_{\boldsymbol{\sigma},2.5} = D_{\boldsymbol{\sigma}} E_y \left[\psi^{(H,x,y,D)} \mid x, \boldsymbol{\sigma}\right] + (1 - D_{\boldsymbol{\sigma}})\, E_y \left[\psi^{(H,x,y,N)} \mid x, \boldsymbol{\sigma}\right]. \tag{43}$$

At $t = 2.5$, the investment outcome has not yet realized. Therefore, the CRA has an expectation over possible realizations of the outcome y, conditional on the signals observed about the type of the investment θ - i.e., x and $\boldsymbol{\sigma}$. These signals change the expectations about the realized θ and, through this, the probability distribution over y.

Let $\mathcal{R}$ denote, for a given realization of market trading x, the set of $D_{\boldsymbol{\sigma}}$ for all types $(\mathcal{S}, \boldsymbol{\sigma})$; let $\mathcal{R}^e$ denote the investors' conjectures about $\mathcal{R}$. We therefore have:

$$\mathcal{R}^e = \left(D_b^e,\ D_{g,b}^e,\ D_{g,g}^e\right). \tag{44}$$

For a given x, each type $(\mathcal{S}, \boldsymbol{\sigma})$ chooses $D_{\boldsymbol{\sigma}}$ as a best response to the conjecture $\mathcal{R}^e$; let $\mathcal{R}^*(\mathcal{R}^e)$ denote the set of best response functions for all the strategic CRA types.

Definition 2 *For given ε^e, δ^e and a realized x, an equilibrium rating revision strategy has to satisfy the following fixed point condition:*

$$\mathcal{R}^*(\mathcal{R}^e) = \mathcal{R}^e. \tag{45}$$

Let $\widehat{\mathcal{R}}$ be a conjectured strategy that satisfies (45) and, therefore, is an equilibrium rating revision strategy.

Analysis. The proof for Part 1 of Proposition 3 is trivial and, thus, omitted. Hence, we will always be considering the case for $x \neq +2n$ - i.e., $x \in \{-2n, 0\}$ - in what follows. We prove Part 2 in three steps. First, we state and prove Lemma 9, which is useful for the characterization of the equilibrium. Second, we characterize the best response function for types $(\mathcal{S}, (g, b))$ and $(\mathcal{S}, b)$ to the conjectures $\left(D_{\mathcal{B}}^e,\ D_{g,g}^e\right)$, so that we may characterize the value of $D_{\mathcal{B}}^e$ that satisfies consistency of expectations for a given $D_{g,g}^e \in [0,1]$.[53] Let $\widehat{D}_{\mathcal{B}}(D_{g,g}^e)$ denote this value. Finally, given the characterization of $\widehat{D}_{\mathcal{B}}(D_{g,g}^e)$, we show that type $(\mathcal{S}, (g, g))$ always plays N at equilibrium.

A.11.1 Lemma 9 and its proof

Lemma 9 *In the rating revision game, for any $x \neq +2n$, the following must hold:*

[53] The fixed point condition in equation (46).

1. *Any $\mathcal{R}^e$ that puts probability one on all of the strategic types playing the same action r cannot be an equilibrium of the game, for any $r \in \{D, N\}$;*
2. *In any equilibrium of the game, types $(\mathcal{S}, (g, b))$ and $(\mathcal{S}, b)$ play the same strategy;*
3. *Any $\mathcal{R}^e$ in which types $(\mathcal{S}, (g, b))$ and $(\mathcal{S}, b)$ always downgrade cannot be part of an equilibrium of the game.*

Part 1 of Lemma 9 considers the following candidate equilibrium conjectures for the rating revision strategy:

$$\mathcal{R}^{e\,\prime}: \begin{array}{ll} \text{Downgrade} & \text{No rating revision} \\ (\mathcal{C}, (g, b)) & (\mathcal{C}, (g, g)) \\ (\mathcal{S}, (g, g)) & \\ (\mathcal{S}, (g, b)) & \\ (\mathcal{S}, b) & \end{array} \quad ; \; \mathcal{R}^{e\,\prime\prime}: \begin{array}{ll} \text{Downgrade} & \text{No rating revision} \\ (\mathcal{C}, (g, b)) & (\mathcal{C}, (g, g)) \\ & (\mathcal{S}, (g, g)) \\ & (\mathcal{S}, (g, b)) \\ & (\mathcal{S}, b) \end{array}$$

Under the conjecture $\mathcal{R}^{e\,\prime}$, whenever N (no rating revision) is observed, the investors learn that the CRA is a committed type. Therefore, $r = N$ is a profitable deviation for any strategic type. The same reasoning applies to the conjecture $\mathcal{R}^{e\,\prime\prime}$.

The proof for Part 2 of Lemma 9 is trivial and, thus, omitted. Part 3 of Lemma 9 describes the following conjectures:

$$\widetilde{\mathcal{R}}^e: \begin{array}{cc} \text{Downgrade} & \text{No rating revision} \\ (\mathcal{C}, (g, b)) & (\mathcal{C}, (g, g)) \\ (\mathcal{S}, (g, b)) & \\ (\mathcal{S}, b) & \end{array} \quad ;$$

type $(\mathcal{S}, (g, g))$ downgrades with some probability $D^e_{g,g} \in [0, 1]$.

We need to show that $r = N$ is a profitable deviation for some strategic type under the conjecture $\widetilde{\mathcal{R}}^e$. For types $(\mathcal{S}, (g, b))$ and $(\mathcal{S}, b)$, that means $\psi^{(H,x,F,N)} > \psi^{(H,x,F,D)}$ (given that they know that the project is bad and will fail for sure). Let $\mu = \Pr(B \mid x,\ x \sim X(\theta, H))$;

we can write the ratio $\frac{\Pr(H,x,F,D|\mathcal{S})}{\Pr(H,x,F,D|\mathcal{C})}$ evaluated at $\widetilde{\mathcal{R}}^e$ as:

$$\frac{\overbrace{\mu\alpha\varepsilon^e}^{\text{prob. }(\mathcal{S},b)} + \overbrace{\mu(1-\alpha)\beta_x(1-\delta^e)}^{\text{prob. }(\mathcal{S},(g,b))} + \overbrace{[\mu(1-\alpha)(1-\beta_x)+(1-\mu)f_G](1-\delta^e)D^e_{g,g}}^{\text{prob. }(\mathcal{S},(g,g))}}{\underbrace{\mu(1-\alpha)\beta_x}_{\text{prob. }(\mathcal{C},(g,b))}}$$
$$= 1-\delta^e + \frac{\mu\alpha\varepsilon^e + [\mu(1-\alpha)(1-\beta_x)+(1-\mu)f_G](1-\delta^e)D^e_{g,g}}{\mu(1-\alpha)\beta_x}.$$

The ratio $\frac{\Pr(H,x,F,N|\mathcal{S})}{\Pr(H,x,F,N|\mathcal{C})}$ is, instead, as follows:

$$\frac{\overbrace{[\mu(1-\alpha)(1-\beta_x)+(1-\mu)f_G](1-\delta^e)\left(1-D^e_{g,g}\right)}^{\text{prob. }(\mathcal{S},(g,g))}}{\underbrace{\mu(1-\alpha)(1-\beta_x)+(1-\mu)f_G}_{\text{prob. }(\mathcal{C},(g,g))}} = (1-\delta^e)\left(1-D^e_{g,g}\right).$$

This implies that $\frac{\Pr(H,x,F,D|\mathcal{S})}{\Pr(H,x,F,D|\mathcal{C})} > \frac{\Pr(H,x,F,N|\mathcal{S})}{\Pr(H,x,F,N|\mathcal{C})}$ and so $\psi^{(H,x,F,N)} > \psi^{(H,x,F,D)}$. Hence, playing N is a profitable deviation under $\widetilde{\mathcal{R}}^e$.

A.11.2 Characterization of $\widehat{D}_{\mathcal{B}}(D^e_{g,g})$

Definition 3 *For a given $D^e_{g,g} \in [0,1]$ and realized x, let $\widehat{D}_{\mathcal{B}}(D^e_{g,g})$ be a value of $D^e_{\mathcal{B}}$ such that:*

$$D^*_{\mathcal{B}}\left(D^e_{\mathcal{B}} = \widehat{D}_{\mathcal{B}},\ D^e_{g,g}\right) = \widehat{D}_{\mathcal{B}}\left(D^e_{g,g}\right), \tag{46}$$

where $D^*_{\mathcal{B}}\left(D^e_{\mathcal{B}},\ D^e_{g,g}\right)$ is the best response by types $(\mathcal{S},(g,b))$ and $(\mathcal{S},b)$ to the conjectures $\left(D^e_{\mathcal{B}},\ D^e_{g,g}\right)$.

Given that a bad signal is conclusive about $\theta = B$, types $(\mathcal{S},(g,b))$ and $(\mathcal{S},b)$ are symmetric in what follows (they play the same rating revision strategy and get the same continuation payoff, denoted, respectively, by $D_{\mathcal{B}}$ and $\Pi_{\mathcal{B},2.5}$). Given that a bad investment always fails, we can write $\Pi_{\mathcal{B},2.5}$ as

$$\Pi_{\mathcal{B},2.5} = D_{\mathcal{B}}\psi^{(H,x,F,D)} + (1-D_{\mathcal{B}})\psi^{(H,x,F,N)}.$$

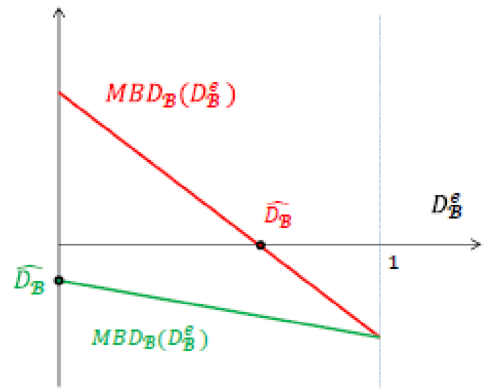

Figure 8: Characterization of $MBD_{\mathcal{B}}$.

Taking the derivative of $\Pi_{\mathcal{B},2.5}$ with respect to $D_{\mathcal{B}}$ yields:

$$MBD_{\mathcal{B}} \equiv \frac{\partial \Pi_{\mathcal{B},2.5}}{\partial D_{\mathcal{B}}} = \psi^{(H,x,F,D)} - \psi^{(H,x,F,N)}.$$

The derivative $\frac{\partial \Pi_{\mathcal{B},2.5}}{\partial D_{\mathcal{B}}}$ represents the marginal benefit of downgrading for types $(\mathcal{S},(g,b))$ and $(\mathcal{S},b)$ ($MBD_{\mathcal{B}}$ henceforth). We know that $MBD_{\mathcal{B}}$ is decreasing in $D^e_{\mathcal{B}}$ [54] and negative at $D^e_{\mathcal{B}} = 1$, for any $D^e_{g,g} \in [0,1]$. The latter is implied by Part 3 of Lemma 9: at $D^e_{\mathcal{B}} = 1$, playing N is always a profitable deviation for types $(\mathcal{S},(g,b))$ and $(\mathcal{S},b)$, which means that $MBD_{\mathcal{B}} < 0$.

Figure 8 describes the two possible characterizations of $MBD_{\mathcal{B}}$: the red line depicts the case in which $MBD_{\mathcal{B}}$ is positive for some $D^e_{\mathcal{B}} \in [0,1)$. In this case, $\widehat{D}_{\mathcal{B}}(D^e_{g,g})$ has to be an interior solution;[55] the green line depicts the case in which $MBD_{\mathcal{B}}$ is always negative. If that is the case, we have $\widehat{D}_{\mathcal{B}}(D^e_{g,g}) = 0$. In both cases, $\widehat{D}_{\mathcal{B}}(D^e_{g,g})$ is unique for any given $D^e_{g,g}$. Let us offer the following remark.

Remark 2 *$MBD_{\mathcal{B}}$ is non-positive at $D^e_{\mathcal{B}} = \widehat{D}_{\mathcal{B}}(D^e_{g,g})$, for any given $D^e_{g,g}$. This means that $MBD_{\mathcal{B}}$ is always non-positive in any equilibrium of the game.*

[54] From equation (28), we have: $\frac{\partial MBD_{\mathcal{B}}}{\partial D^e_{\mathcal{B}}} = \underbrace{\frac{\partial \psi^{(F,x,H,D)}}{\partial D^e_{\mathcal{B}}}}_{\leq 0} - \underbrace{\frac{\partial \psi^{(F,x,H,N)}}{\partial D^e_{\mathcal{B}}}}_{\geq 0} \leq 0.$

[55] Any other $D^e_{\mathcal{B}}$ would not satisfy consistency of expectations. Consider points at the left of $\widehat{D}_{\mathcal{B}}$ - i.e., $D^e_{\mathcal{B}} \in [0, \widehat{D}_{\mathcal{B}})$: here we have $MBD_{\mathcal{B}} > 0$; therefore, $D^*_{\mathcal{B}} = 1$. For $D^e_{\mathcal{B}} \in (\widehat{D}_{\mathcal{B}}, 1]$, we have $MBD_{\mathcal{B}} < 0$ and so $D^*_{\mathcal{B}} = 0$.

Notice also that $MBD_{\mathcal{B}}$ is positive at $D^e_{g,g} = D^e_{\mathcal{B}} = 0$, and so $MBD_{\mathcal{B}}$ looks like the red line in Figure 5 when $D^e_{g,g} = 0$. This is implied by Part 1 of Lemma 9: at $D^e_{g,g} = D^e_{\mathcal{B}} = 0$, all the strategic types are expected to pool on N; therefore, playing D is a profitable deviation-i.e., $MBD_{\mathcal{B}} > 0$.

A.11.3 Characterization of $\widehat{D}_{g,g}$

Given Remark 2, we can now sign $MBD_{g,g}$. We have

$$MBD_{g,g} = E_y\left[\psi^{(H,x,y,D)} - \psi^{(H,x,y,N)} \mid x, (g,g)\right]$$
$$= \Pr\left(S \mid x, (g,g)\right)\left(\psi^{(H,x,S,D)} - \psi^{(H,x,S,N)}\right) + \Pr\left(F \mid x, (g,g)\right)\left(\psi^{(H,x,F,D)} - \psi^{(H,x,F,N)}\right). \tag{47}$$

Conditional on the investment succeeding ($y = S$), the market learns $\theta = G$ and so $\boldsymbol{\sigma} = (g,g)$. Therefore, $r = D$ would reveal a strategic type to the market.[56] This means that $\psi^{(H,x,S,D)} = 0$, and so the first term in equation (47) is always negative.

Let us now consider $MBD_{g,g}$ evaluated at $D^e_{\mathcal{B}} = \widehat{D}_{\mathcal{B}}(D^e_{g,g})$. From Remark 2, we know that $MBD_{\mathcal{B}} = \psi^{(H,x,F,D)} - \psi^{(H,x,F,N)}$ is always non-positive at $D^e_{\mathcal{B}} = \widehat{D}_{\mathcal{B}}(D^e_{g,g})$, and so the second term in equation (47) is non-positive. It follows that $MBD_{g,g}$ is always negative in equilibrium, and so $\widehat{D}_{g,g} = 0, \widehat{D}_{\mathcal{B}} = \widehat{D}_{\mathcal{B}}(D^e_{g,g} = 0)$ is the unique equilibrium rating revision strategy. Note that, by the characterization of $MBD_{\mathcal{B}}$, we have that $\widehat{D}_{\mathcal{B}}$ is an interior solution.

A.11.4 Derivation of $\widehat{D}_{\mathcal{B}}$

Here, we derive the value of $\widehat{D}_{\mathcal{B}}$. This is such that $\psi^{(H,x,F,D)} = \psi^{(H,x,F,N)}$ when both are evaluated at the equilibrium conjecture, i.e. $D^e_{\mathcal{B}} = \widehat{D}_{\mathcal{B}}$, $D^e_{g,g} = 0$. Note that $\psi^{(H,x,F,D)} = \psi^{(H,x,F,N)}$ is equivalent to $\frac{\Pr(H,x,F,D|\mathcal{S})}{\Pr(H,x,F,D|\mathcal{C})} = \frac{\Pr(H,x,F,N|\mathcal{S})}{\Pr(H,x,F,N|\mathcal{C})}$. Let $\mu = \Pr\left(B \mid x,\ x \sim X\left(\theta, H\right)\right)$; we

[56] Investors know that $\boldsymbol{\sigma} = (g,g)$, and so they are facing either $(\mathcal{C},(g,g))$ or $(\mathcal{S},(g,g))$. They know that $(\mathcal{C},(g,g))$ always plays N. So, when they observe D, they think that it came from $(\mathcal{S},(g,g))$.

have:

$$
\begin{aligned}
\frac{\Pr\left(H,x,F,N\right|\mathcal{S})}{\Pr\left(H,x,F,N\right|\mathcal{C})} &= \frac{\overbrace{\mu\alpha\varepsilon^{e}(1-\widehat{D}_{\mathcal{B}})}^{\text{prob. }(\mathcal{S},b)}+\overbrace{\mu\left(1-\alpha\right)\beta_{x}\left(1-\delta^{e}\right)(1-\widehat{D}_{\mathcal{B}})}^{\text{prob. }(\mathcal{S},(g,b))}}{\underbrace{\mu\left(1-\alpha\right)\left(1-\beta_{x}\right)+\left(1-\mu\right)f_{G}}_{\text{prob. }(\mathcal{C},(g,g))}}+\underbrace{1-\delta^{e}}_{\frac{\text{prob. }(\mathcal{S},(g,g))}{\text{prob. }(\mathcal{C},(g,g))}};\\
\frac{\Pr\left(H,x,F,D\right|\mathcal{S})}{\Pr\left(H,x,F,D\right|\mathcal{C})} &= \frac{\overbrace{\mu\alpha\varepsilon^{e}\widehat{D}_{\mathcal{B}}}^{\text{prob. }(\mathcal{S},b)}+\overbrace{\mu\left(1-\alpha\right)\beta_{x}\left(1-\delta^{e}\right)\widehat{D}_{\mathcal{B}}}^{\text{prob. }(\mathcal{S},(g,b))}}{\underbrace{\mu\left(1-\alpha\right)\beta_{x}}_{\text{prob. }(\mathcal{C},(g,b))}}.
\end{aligned}
\tag{48}
$$

This implies that

$$
\widehat{D}_{\mathcal{B}}=\frac{\left(1-\alpha\right)\beta_{x}\left[\mu\alpha\varepsilon^{e}+\left(\mu\left(1-\alpha\right)+\left(1-\mu\right)f_{G}\right)\left(1-\delta^{e}\right)\right]}{\left[\mu\left(1-\alpha\right)+\left(1-\mu\right)f_{G}\right]\left[\varepsilon^{e}\alpha+\left(1-\alpha\right)\beta_{x}\left(1-\delta^{e}\right)\right]}. \tag{49}
$$

A.11.5 Equilibrium Rating Revision Strategy for the other cases

In what follows, we describe the equilibrium rating revision strategy for $x\neq+2n$ in the cases other than $\varepsilon^{e}>0$, $\delta^{e}<1$. Notice that, as in the general case, when $x=+2n$, all the strategic types downgrade.

Case 1. $\varepsilon^{e}>0$, $\delta^{e}=1$**.** When $\delta^{e}=1$, types $(\mathcal{S},(g,g))$ and $(\mathcal{S},(g,b))$ are always expected to deflate the initial rating and, thus, not expected to enter the rating revision stage. We need to consider only type $(\mathcal{S},b)$ as the strategic type in this case. The ideas behind the characterization of the equilibrium for the general case apply here also. We can use Part 1 of Lemma 9 to characterize the equilibrium rating revision strategy: this rules out both the conjectures $D_{b}^{e}=0$ and $D_{b}^{e}=1$.[57] Therefore, we have $\widehat{D}_{b}\in(0,1)$. The exact expression for $\widehat{D}_{b}$ is given by evaluating $\widehat{D}_{\mathcal{B}}$ in equation (49) at $\delta^{e}=1$.

Case 2. $\varepsilon^{e}=0$, $\delta^{e}<1$**.** When $\varepsilon^{e}=0$, type $(\mathcal{S},b)$ is not expected to inflate the initial rating and, thus, to enter the rating revision stage. Therefore, we need to consider only types

[57] At $D_{b}^{e}=0$, type $(\mathcal{S},b)$ is never expected to downgrade. Hence, when D is observed, the investors think that it came from a committed type. This makes playing D a profitable deviation. The same reasoning applies for $D_{b}^{e}=1$.

$(\mathcal{S},(g,g))$ and $(\mathcal{S},(g,b))$ as strategic types in this case. Again, we can apply the same ideas as in the general case. We have $MBD_{g,b}=0$ only at $D^e_{g,b}=1$ when $D^e_{g,g}=0$, and always strictly positive for any other $D^e_{g,b}\in[0,1)$. Therefore, we have $\widehat{D}_{g,b}=1$ and $\widehat{D}_{g,g}=0$. Notice that, evaluating $\widehat{D}_{\mathcal{B}}$ in equation (49) at $\varepsilon^e=1$, we have $\widehat{D}_{\mathcal{B}}=\widehat{D}_{g,b}=1$.

Case 3. $\varepsilon^e=0$, $\delta^e=1$**.** In this case, none of the strategic types is expected to enter the rating revision stage. Posterior reputation does not depend on the particular r observed and, thus, all the strategic types are indifferent between any possible rating revision strategy.

A.11.6 Equilibrium Initial Rating Strategy

Here, we show that the equilibrium initial rating strategy does not change when we introduce the rating revision stage. We do so by showing that, along the equilibrium path, reputational payoffs for the CRA do not depend on the rating revision action r. Therefore, incentives to inflate and deflate ratings for the CRA - equations (13) and (37) - do not change when we introduce the rating revision stage. This is trivially satisfied when $m=\varnothing$, since rating revision does not take place in this case. However, the rating revision game is played when $m=H$.

When $x=+2n$, all types downgrade and so the investors do not learn anything new about the CRA from observing r. We have:

$$\frac{\Pr\left(H,+2n,F,D\right|\mathcal{S})}{\Pr\left(H,+2n,F,D\right|\mathcal{C})}=\frac{\overbrace{\alpha\varepsilon^e}^{\text{prob. }(\mathcal{S},b)}+\overbrace{(1-\alpha)\,\beta_x\,(1-\delta^e)}^{\text{prob. }(\mathcal{S},(g,b))}+\overbrace{(1-\alpha)\,(1-\beta_x)\,(1-\delta^e)}^{\text{prob. }(\mathcal{S},(g,g))}}{\underbrace{(1-\alpha)\,\beta_x}_{\text{prob. }(\mathcal{C},(g,b))}+\underbrace{(1-\alpha)\,(1-\beta_x)}_{\text{prob. }(\mathcal{C},(g,g))}}$$

$$=\frac{\alpha\varepsilon^e+(1-\alpha)\,(1-\delta^e)}{(1-\alpha)}\underset{\text{by (34) at }\mu=1}{=}\frac{\Pr\left(H,+2n,F\right|\mathcal{S})}{\Pr\left(H,+2n,F\right|\mathcal{C})}. \tag{50}$$

This implies that $\psi^{(H,+2n,F,D)}=\psi^{(H,+2n,F)}$.

When $x\neq+2n$, the rating revision action depends on the private signals.[58]. However, when the investment fails ($y=F$), the equilibrium play leads investors to place the same likelihood that the CRA is committed when observing a downgrade D as when observing no rating revision N. Otherwise, there would have been a profitable deviation. This implies that

[58]Types $(\mathcal{S},b)$ and $(\mathcal{S},(g,b))$ downgrade with probability $\widehat{D}_{\mathcal{B}}$, while type $(\mathcal{S},(g,g))$ never downgrades.

the reputational payoffs for both ratings D and N are equal - i.e., $\psi^{(F,x,H,D)} = \psi^{(F,x,H,N)}$, and so still the investors do not learn anything from the rating revision action. Let $\widetilde{\mu} = \Pr(B \mid x = -2n, m = H)$; we have:

$$
\begin{aligned}
\frac{\Pr(H,-2n,F,N|\,\mathcal{S})}{\Pr(H,-2n,F,N|\,\mathcal{C})} &= \frac{\Pr(H,-2n,F,D|\,\mathcal{S})}{\Pr(H,-2n,F,D|\,\mathcal{C})} \\
&\overset{\text{by (48)}}{=} \frac{(\alpha\varepsilon^e + (1-\alpha)\beta_x(1-\delta^e))}{(1-\alpha)\beta_x} \overset{\frac{\{(1-\alpha)\beta_x\}\ \{\widetilde{\mu}\alpha\varepsilon^e + (\widetilde{\mu}(1-\alpha)+(1-\widetilde{\mu})f_G)(1-\delta^e)\}}{\{\alpha\varepsilon^e+(1-\alpha)\beta_x(1-\delta^e)\}\ \{\widetilde{\mu}(1-\alpha)+(1-\widetilde{\mu})f_G\}}}{\underbrace{}_{\widehat{D}_{\mathcal{B}}}} \\
&= \frac{\widetilde{\mu}\alpha\varepsilon^e + (\widetilde{\mu}(1-\alpha) + (1-\widetilde{\mu}) f_G)(1-\delta^e)}{\widetilde{\mu}(1-\alpha) + (1-\widetilde{\mu}) f_G} \underset{\text{by (34) at } \mu=\widetilde{\mu}}{=} \frac{\Pr(H,-2n,F|\,\mathcal{S})}{\Pr(H,-2n,F|\,\mathcal{C})}.
\end{aligned}
\tag{51}
$$

This implies that $\psi^{(H,-2n,F,D)} = \psi^{(H,-2n,F,N)} = \psi^{(H,-2n,F)}$; the same reasoning applies when $x = 0$.

When the investment succeeds ($y = S$), it must be that $\theta = G$ and so $\boldsymbol{\sigma} = (g,g)$. Therefore, the investors know they are facing either type $(\mathcal{C},(g,g))$ or $(\mathcal{S},(g,g))$. Both of them are expected to play N, and so, once again, the investors do not learn anything from the rating revision action. We have:

$$
\frac{\Pr(H,S,N|\,\mathcal{S})}{\Pr(H,S,N|\,\mathcal{C})} = (1-\delta^e) = \frac{\Pr(H,S|\,\mathcal{S})}{\Pr(H,S|\,\mathcal{C})}
$$

This means that $\psi^{(H,S,N)} = \psi^{(H,S)}$.

A.12 Proof of Lemma 7

Here, we derive the comparative statics results in Lemma 7. We consider $\widehat{D}_{\mathcal{B}}$ evaluated at the equilibrium conjecture about the initial rating strategy- i.e., $\varepsilon^e \in (0,1]$, $\delta^e = 0$. Let $\Pr(B \mid x,\ x \sim X(\theta, H))$; we have

$$
\begin{aligned}
\frac{\partial \widehat{D}_{\mathcal{B}}}{\partial \varepsilon^e} &= -\frac{\beta_x \alpha (1-\alpha)\left[\mu(1-\alpha)(1-\beta_x) + f_G(1-\mu)\right]}{\left[\beta_x(1-\alpha) + \alpha\varepsilon^e\right]^2 \left[\mu(1-\alpha) + f_G(1-\mu)\right]} < 0; \\
\frac{\partial \widehat{D}_{\mathcal{B}}}{\partial \mu} &= \frac{\beta_x \alpha (1-\alpha) f_G \varepsilon^e}{\left[\beta_x(1-\alpha) + \alpha\varepsilon^e\right]\left[\mu(1-\alpha) + f_G(1-\mu)\right]^2} > 0.
\end{aligned}
$$

Internet Appendix

IA.1 Demonstrating that Γ may increase or decrease in γ

Here, we show that the set of parameter constellations such that Γ is decreasing in γ is non-empty. First, we discuss the determinants of the relative slopes of the reaction functions in Figure 5; then, we show that Γ may decrease in γ by performing numerical simulations of the model. We assume the following functional form for the speculator's cost of precision:

$$c(\iota) = L\iota + \omega\frac{\iota^2}{2}. \tag{52}$$

We also assume that the market for the investment is competitive, and investors are risk-neutral; the investment pays R in the case of success and 0 otherwise. This means that the equilibrium rating fee is as follows:

$$\varphi = p(H) - p(\varnothing) = \left(\frac{1}{2-\alpha+\left(1-\psi^H\right)\alpha\varepsilon^e}\right)(1-f_G)\,R. \tag{53}$$

Determinants of the slopes of $\widehat{\iota}(\varepsilon^e)$ and $\widehat{\varepsilon}(\iota^e)$

Γ is decreasing in γ when $\frac{\partial\widehat{\iota}(\varepsilon^e)}{\partial\varepsilon^e}$ is much larger than $\frac{\partial\widehat{\varepsilon}(\iota^e)}{\partial\iota^e}$ around the equilibrium point, so that the decrease in $\widehat{\iota}$ is much larger than the decrease in $\widehat{\varepsilon}$ when γ increases.[59] Therefore,

[59] This is easy to see if we consider a linear approximation for $\widehat{\iota}(\varepsilon^e)$ and $\widehat{\varepsilon}(\iota^e)$. Suppose:

$$\widehat{\iota}(\varepsilon^e) = a + \frac{\partial\widehat{\iota}(\varepsilon^e)}{\partial\varepsilon^e}\varepsilon^e;\;\; \widehat{\varepsilon}(\iota^e) = b(\gamma) + \frac{\partial\widehat{\varepsilon}(\iota^e)}{\partial\iota^e}\iota^e$$

where $b(\gamma)$ $(b(\gamma) < 1)$ decreases as γ increases, so that $\widehat{\varepsilon}(\iota^e)$ shifts to the left, and $(a, b(\gamma))$ are such that the equilibrium pair $(\widehat{\varepsilon}, \widehat{\iota}) \in (0,1)^2$. We have then:

$$\widehat{\iota} = \frac{a + b(\gamma)\frac{\partial\widehat{\iota}(\varepsilon^e)}{\partial\varepsilon^e}}{1-\frac{\partial\widehat{\iota}(\varepsilon^e)}{\partial\varepsilon^e}\frac{\partial\widehat{\varepsilon}(\iota^e)}{\partial\iota^e}};\;\; \widehat{\varepsilon} = \frac{b(\gamma) + a\frac{\partial\widehat{\varepsilon}(\iota^e)}{\partial\iota^e}}{1-\frac{\partial\widehat{\iota}(\varepsilon^e)}{\partial\varepsilon^e}\frac{\partial\widehat{\varepsilon}(\iota^e)}{\partial\iota^e}}.$$

We need to have $\frac{\partial\widehat{\iota}}{\partial b}$ much larger than $\frac{\partial\widehat{\varepsilon}}{\partial b}$, so that the decrease in $\widehat{\iota}$ is much larger when γ increases. We have:

$$\frac{\partial\widehat{\iota}}{\partial b} - \frac{\partial\widehat{\varepsilon}}{\partial b} = \frac{\frac{\partial\widehat{\iota}(\varepsilon^e)}{\partial\varepsilon^e} - 1}{1-\frac{\partial\widehat{\iota}(\varepsilon^e)}{\partial\varepsilon^e}\frac{\partial\widehat{\varepsilon}(\iota^e)}{\partial\iota^e}}$$

Hence, $\frac{\partial\widehat{\iota}}{\partial b} - \frac{\partial\widehat{\varepsilon}}{\partial b}$ is large when $\frac{\partial\widehat{\iota}(\varepsilon^e)}{\partial\varepsilon^e}$ is large and $\frac{\partial\widehat{\varepsilon}(\iota^e)}{\partial\iota^e}$ approaches 0.

the relative slopes are crucial in this. Here, we discuss the determinants of the relative slopes of the reaction functions $\widehat{\iota}\left(\varepsilon^e\right)$ and $\widehat{\varepsilon}\left(\iota^e\right)$.

Given the functional form for $c\left(\iota\right)$ in equation (52), the equilibrium choice of precision $\widehat{\iota}$ solves:

$$n\mu^H\left(1-\mu^H\right)\left(1-f_G\right)=L+\omega\widehat{\iota}.$$

Therefore, we have:

$$\begin{aligned}\widehat{\iota}\left(\varepsilon^e\right) &= \frac{n}{\omega}\mu^H\left(1-\mu^H\right)\left(1-f_G\right)-\frac{L}{\omega};\\ \frac{d\widehat{\iota}\left(\varepsilon^e\right)}{d\varepsilon^e} &= \frac{n}{\omega}\underbrace{\frac{d\mu^H}{d\varepsilon^e}}_{>0}\left(1-2\mu^H\right)\left(1-f_G\right).\end{aligned}$$

The above equation characterizes the slope of the reaction function for the speculator - i.e., $\frac{d\widehat{\iota}}{d\varepsilon^e}$. This is larger when:

- n is large - i.e., when more liquidity traders participate to the CDS market, and so the speculator can make more profits;
- ω is small - i.e., when $c\left(\iota\right)$ is not very convex.

A large n and small ω characterizes an environment where speculative trading is profitable, since the CDS market is liquid - large n - and acquiring a precise signal for the speculator is not too costly. Note that, as n becomes large and ω goes to 0, $\frac{d\widehat{\iota}}{d\varepsilon^e}$ goes to infinite, while $\frac{d\widehat{\varepsilon}}{d\iota^e}$ is not affected, as neither of them enters the CRA's objective function. This means that, for any characterization of $\widehat{\varepsilon}\left(\iota^e\right)$, we can always find a pair of parameters (n,ω) such that $\frac{\partial\widehat{\iota}(\varepsilon^e)}{\partial\varepsilon^e}$ is much larger than $\frac{\partial\widehat{\varepsilon}(\iota^e)}{\partial\iota^e}$ around the equilibrium point, and so such that Γ is decreasing in γ.

The equilibrium level of rating inflation $\widehat{\varepsilon}\left(\iota^e\right)$ is such that $MBI=0$ at $\left(\varepsilon^e=\widehat{\varepsilon}\left(\iota^e\right),\iota^e\right)$. We do not have a closed form solution for $\widehat{\varepsilon}\left(\iota^e\right)$. However, we can use the sensitiveness of the CRA's incentives to inflate with respect to ι^e - i.e., $\frac{dMBI}{d\iota^e}$, as a proxy for $\frac{d\widehat{\varepsilon}}{d\iota^e}$. By equation (41), we can rearrange $\frac{dMBI}{d\iota^e}$ as follows:

$$\frac{dMBI}{d\iota^e}=-\frac{\gamma}{2}\frac{q\left(1-q\right)\varepsilon^e f_G^2\alpha\left(1-\widetilde{\mu}\right)^2}{\left(1-\alpha+\alpha\varepsilon^e\left(1-q\right)\right)\left[f_G+\widetilde{\mu}\left(1-\alpha-f_G+\varepsilon^e\left(1-q\right)\alpha\right)\right]^2}<0.$$

The term $\frac{dMBI}{d\iota^e}$ is decreasing in f_G and α - i.e., as f_G or α increase, $\frac{dMBI}{d\iota^e}$ becomes more negative. This means that $\widehat{\varepsilon}(\iota^e)$ becomes less sensitive to ι^e as f_G or α decreases.

As f_G gets closer to 0, the investment outcome y becomes a more precise signal about the investment quality.[60] Therefore, the investors put relatively more weight on the realization of y and thus less weight on x, when updating their beliefs about the CRA's type. Hence, the CRA is then less concerned about ι^e, which makes $x = +2n$ - which conclusively reveals a bad investment - more likely.

As α gets closer to 0, the CRA's private signal becomes less precise. The CRA is then less concerned about the market learning that the asset was bad, as she can always hide rating inflation behind the possibility of an assessment error (which becomes more likely as α gets closer to 0).

Numerical Simulations

Figure 9 displays the reaction functions $\widehat{\iota}(\varepsilon^e)$ and $\widehat{\varepsilon}(\iota^e)$ for the following parameter configurations (Simulation 1): $q = \alpha = 0.9$, $\beta = 0.5$; $f_G = 0.6$, $R = 1.7$, $n = 3$, $L = 0.16$, $\omega = 0.0125$; the value of γ increases from $\gamma' = 1$ to $\gamma'' = 1.1$ ($\widehat{\varepsilon}(\iota^e)$ shifts down as γ increases).

The initial equilibrium pair is the following: $\widehat{\varepsilon}' \approxeq 0.74$, $\widehat{\iota}' \approxeq 0.59$; as γ increases, we have, instead, $\widehat{\varepsilon}'' \approxeq 0.73$, $\widehat{\iota}'' \approxeq 0.48$. Notice that, since $\widehat{\iota}(\varepsilon^e)$ is much steeper than $\widehat{\varepsilon}(\iota^e)$ around the initial equilibrium point, the decrease in $\widehat{\iota}$ is much larger than the decrease in $\widehat{\varepsilon}$ when γ increases. This leads Γ to decrease with γ: we have $\Gamma' \approxeq 0.459$ and $\Gamma'' \approxeq 0.424$.

In Figure 10, we consider a different parametrization for $c(\iota)$: we have $n = 3$, $L = 0.11$, $\omega = 0.125$; the rest of the parameters are the same as in Simulation 1. This new parametrization of the cost function makes $\widehat{\iota}(\varepsilon^e)$ flatter than $\widehat{\varepsilon}(\iota^e)$, so that the decrease in $\widehat{\iota}$ is lower than the decrease in $\widehat{\varepsilon}$ when γ increases. The initial equilibrium pair is the following: $\widehat{\varepsilon}' \approxeq 0.78$, $\widehat{\iota}' \approxeq 0.49$; as γ increases, we have instead $\widehat{\varepsilon}'' \approxeq 0.74$, $\widehat{\iota}'' \approxeq 0.48$. This leads Γ to increase with γ in this case: we have $\Gamma' \approxeq 0.394$ and $\Gamma'' \approxeq 0.417$.

[60] As f_G gets closer to 0, when the investment fails the investors strongly believe that $\theta = B$. Therefore, they do not update much their beliefs conditional on the realization of x.

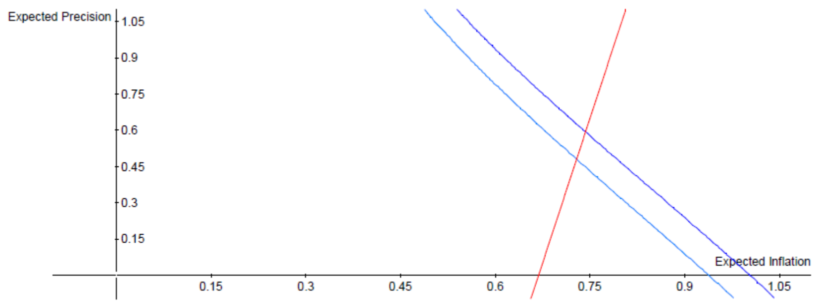

Figure 9: Rating Inflation and Trading Informativeness Equilibrium Simulation 1: Γ decreasing in γ.

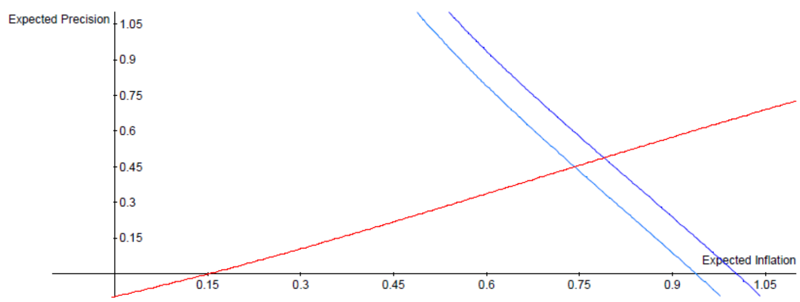

Figure 10: Rating Inflation and Trading Informativeness Equilibrium Simulation 2: Γ increasing in γ.

IA.2 Effect of ι^e on expected CDS price

Here we show that (i) the expected price of CDS protection does not depend on ι^e, but (ii) its variance increases as ι^e increases. Point (i) is implied by the *Law of Iterated Expectation*. We can write the expected CDS price as $E_x \left\{\widehat{p}^{cds}(x)\right\}$, where, by the equilibrium of the CDS market, we have $\widehat{p}^{cds}(x) = E_\theta (f_\theta \mid H, x)$. Hence, we can write:

$$E_x \left\{\widehat{p}^{cds}(x)\right\} = E_x \left\{E_\theta (f_\theta \mid H, x)\right\} = E_\theta (f_\theta \mid H),$$

where the last equality comes from the *Law of Iterated Expectation* applied on the expectation over the realization of x. This means that the expected CDS price does not depend on the realization of x, and so on its distribution - i.e., on ι^e.

We can write the expected CDS price as:

$$E_x\left\{\widehat{p}^{cds}(x)\right\} = \frac{1}{2}\mu^H\iota^e + \frac{1}{2}\left(\mu^H(1-\iota^e)+1-\mu^H\right)\widehat{p}^{cds}(x=-2n) + \frac{1}{2}\widehat{p}^{cds}(x=0). \quad (54)$$

Note that we have:

$$\widehat{p}^{cds}(x=0) = \mu^H(1-f_G)+f_G; \quad \widehat{p}^{cds}(x=-2n) = \mu^{(H,g_s)}(1-f_G)+f_G,$$

where $\mu^{(H,g_s)} = \frac{\mu^H(1-\iota^e)}{1-\mu^H\iota^e}$.

Hence, we can rearrange equation (54) as follows:

$$\begin{aligned} E_x\left\{\widehat{p}^{cds}(x)\right\} &= \frac{1}{2}\mu^H\iota^e + \frac{1}{2}\left(\mu^H(1-\iota^e)+1-\mu^H\right)\left(\frac{\mu^H(1-\iota^e)}{1-\mu^H\iota^e}(1-f_G)+f_G\right) + \frac{1}{2}\left(\mu^H(1-f_G)+f_G\right) \\ &= \mu^H(1-f_G)+f_G. \end{aligned}$$

Notice that $E_x\left\{\widehat{p}^{cds}(x)\right\}$ does not depend on ι^e.

Given that $E_x\left\{\widehat{p}^{cds}(x)\right\}$ does not depend on ι^e, showing that $VAR_x\left\{\widehat{p}^{cds}(x)\right\}$ is increasing in ι^e is equivalent to showing that $E_x\left\{\left(\widehat{p}^{cds}(x)\right)^2\right\}$ increases with ι^e. Let $\widetilde{p} \equiv \widehat{p}^{cds}(x=-2n)$; taking the derivative of $E_x\left\{\left(\widehat{p}^{cds}(x)\right)^2\right\}$ with respect to ι^e yields:

$$\begin{aligned} \frac{\partial E_x\left\{\left(\widehat{p}^{cds}(x)\right)^2\right\}}{\partial\iota^e} &= \frac{1}{2}\left(\mu^H\left(1-\widetilde{p}^2\right) + \left(1-\mu^H\iota^e\right)\frac{\partial\left(\widetilde{p}^2\right)}{\partial\iota^e}\right) \\ &= \frac{1}{2}\left(\mu^H - \mu^H\widetilde{p}^2 + \left(1-\mu^H\iota^e\right)2\widetilde{p}\frac{\partial\widetilde{p}}{\partial\iota^e}\right). \end{aligned}$$

The sign of $\frac{\partial E_x\left\{\left(\widehat{p}^{cds}(x)\right)^2\right\}}{\partial \iota^e}$ thus depends on the sign of the following expression:

$$
\begin{aligned}
&\mu^H-\widetilde{p}\left(\underbrace{\mu^H\widetilde{p}-\left(1-\mu^H\iota^e\right)\frac{\partial\widetilde{p}}{\partial\iota^e}}_{=\mu^H \text{ by } \frac{\partial E_x\left\{\left(\widehat{p}^{cds}(x)\right)\right\}}{\partial\iota^e}=0}-\left(1-\mu^H\iota^e\right)\frac{\partial\widetilde{p}}{\partial\iota^e}\right)\\
&=\mu^H-\mu^H\widetilde{p}+\left(1-\mu^H\iota^e\right)\underbrace{\frac{\partial\widetilde{p}}{\partial\iota^e}}_{<0}\underbrace{\widetilde{p}}_{\leq 1}\\
&\geq\mu^H-\mu^H\widetilde{p}+\left(1-\mu^H\iota^e\right)\frac{\partial\widetilde{p}}{\partial\iota^e}\underbrace{=}_{\text{by } \frac{\partial E_x\left\{\left(\widehat{p}^{cds}(x)\right)\right\}}{\partial\iota^e}=0}0.
\end{aligned}
$$

Therefore, we have $\frac{\partial E_x\left\{\left(\widehat{p}^{cds}(x)\right)^2\right\}}{\partial \iota^e}\geq 0$ and so $VAR_x\left\{\widehat{p}^{cds}(x)\right\}$ is increasing in ι^e.

IA.3 Allowing for fund-raising and negative NPV Projects

In our model, the issuer attempts to sell its asset. Here we show that our results hold also in a different setting, where the issuer seeks financing for a risky investment project. We assume that good projects should be financed (they have positive NPV) but that, without prior knowledge on the quality of the project, no financing takes place (ex-ante NPV is negative). Therefore, the CRA can screen projects such that good projects receive financing and are implemented. The ratings process in this new setting is similar to the one in Mathis et al. (2009). If the issuer receives financing and implements the project, a CDS market on the investment project takes place. The structure of the CDS market is the same as it is described in Section 2.2.

The project pays 1 in case of success and nothing in case of failure; the cost of the project is $I<1$. As in the basic model, good projects fail with probability $f_G\in(0,1)$, while bad projects always fail. Good projects have positive NPV - i.e., $1-f_G>I$. Regarding the ex-ante NPV of the project, we assume the following:

Assumption 3: The ex-ante NPV of the project is *slightly* negative - ie,

$$\frac{1}{2}\left(1-f_G\right)-I=-\pi,\ \text{ for some infinitesimal } \pi>0. \tag{55}$$

Assumption 3 implies that projects with a high rating have always positive NPV, and so receive financing and are implemented. This assumption is made only for ease of exposition: the results hold in full for a large π if the CRA's initial reputation is above some minimum level.[61]

Given conjectures about rating inflation ε^e and deflation δ^e, the NPV of a high-rated asset is as follows:

$$NPV^H = \Pr\left(G \mid m = H, q, (\varepsilon^e, \delta^e)\right)(1 - f_G) - I > 0, \tag{56}$$

since $\Pr\left(G \mid m = H, q, (\varepsilon^e, \delta^e)\right) > \Pr(G) = \frac{1}{2}$, as with some positive probability q the CRA is a committed type and so a high rating is always a signal for a good project.

We assume that investors are risk neutral and competitive. The equilibrium for the capital market is characterized by the zero profit condition

$$\Pr\left(G \mid m = H, q, (\varepsilon^e, \delta^e)\right) R = I + \varphi \tag{57}$$

where $R \in (0, 1]$ is the nominal return paid to investors - only paid in case of success.[62] As in Mathis et al. (2009), we assume the rating fee φ is an increasing function of the perceived rating accuracy - i.e., increasing in q, and decreasing in expected rating inflation ε^e and expected rating deflation δ^e.

The main implications of the new setting are the following. First, when the project does not receive a high rating - i.e., when $m = \varnothing$, it does not receive financing and so it is not implemented. This means that the CDS market takes place only when $m = H$. In this case, its analysis is the same as in the basic model - Section 3. Trading strategies are as described in Lemma 1; the speculator's choice of precision $\widehat{\iota}(\varepsilon^e)$ is increasing in the expected rating inflation - Lemma 2. [63]

[61] If the CRA is thought to be a committed type with a large probability, a high rating is a reliable enough signal for a good project, so that projects with a high rating always receive financing.

[62] Since the issuer is cashless, it has to finance the investment plus the rating fee. Thus the size of the issue equals the size of the investment plus the fee that the issuer has to pay for a high rating.

[63] For the purpose of the CDS market, it does not matter whether the issuer previously owned a risky asset and sold it to investors (basic model) or seeks financing from investors to implement a risky investment project (new setting). In both cases, the CDS contract takes the same structure: the buyer of the swap pays an amount p^{cds} to the swap's seller. In return, the seller agrees that in the event of default of the project at time $t = 3$, the seller will pay the buyer an amount 1. Hence, the payoff functions for the traders do not

Second, as the project it is not implemented without a high rating, its outcome $y \in \{S, F\}$ and the realization of the CDS market x are not observed when $m = \varnothing$. This implies a slight change in the expression for the incentives to inflate (MBI) and deflate (MBD) ratings. However, as we prove below, the characterization of the equilibrium rating strategy - Proposition 1 - and the reaction of rating inflation to the informativeness of the CDS market - Lemma 5 - do not change.

The proof follows the same steps as in the basic model (Section 4). First, we describe the equilibrium level of rating inflation for a given conjecture about rating deflation δ^e. Then, we show that rating deflation never occurs in equilibrium. Finally, we show that Lemma 5 holds, and so equilibrium rating inflation is decreasing in the expected level of informativeness of the CDS market.

Step 1: Equilibrium Rating Inflation. Step 1 follows closely the analysis in Section 4.1. For given rating inflation ε^e and deflation δ^e, we can write the marginal benefit of inflation MBI as follows:

$$MBI = \varphi + \gamma \left\{ E_x \left[\psi^{(H,x,F)} \mid B \right] - \psi^{\varnothing} \right\}. \tag{58}$$

The expectation over realizations of x is as it is implied by the CDS market equilibrium. Notice that, when $m = \varnothing$, the project outcome y and the realization of the CDS market x are not observed and so reputation is $\psi^{\varnothing}$. As in the basic model, a strategic CRA who has observed a bad signal and has to decide whether to inflate or not the report trades off the short term gains from inflating the report (the fee φ) with the long term loss of a diminished reputation (reputational costs). Once again, MBI is always decreasing in ε^e and positive at $\varepsilon^e = 0$, for any given δ^e. Therefore, for a given δ^e, the equilibrium level of rating inflation $\widehat{\varepsilon}(\delta^e)$ is unique; we have: a corner solution $\widehat{\varepsilon}(\delta^e) = 1$, whenever $MBI > 0$ at $(\varepsilon^e = 1, \delta^e)$; an interior solution $\widehat{\varepsilon}(\delta^e) \in (0, 1]$ otherwise. Notice that MBI is always non-negative at $\widehat{\varepsilon}(\delta^e)$.

change and neither does the characterization of the equilibrium.

Step 2: Equilibrium Rating Deflation. Step 2 follows the analysis in Appendix A.7. The marginal benefit of deflation MBD is:

$$MBD = -\varphi - \gamma \Pr(B \mid g) E_x \left[\psi^{(H,x,F)} \mid B\right] +$$
$$-\gamma \Pr(G \mid g) \left\{f_G E_x \left[\psi^{(H,x,F)} \mid G\right] + (1 - f_G) \psi^{(H,S)}\right\} + \psi^{\varnothing}.$$

Notice that when the investment suceeds ($y = S$) the investors learn that the asset was good (bad projects always fail) and so posterior reputation $\psi^{(H,S)}$ does not depend on the realization of x. [64]

We need to show that when both MBI and MBD are evaluated at $\widehat{\varepsilon}(\delta^e)$, $MBI \geq 0$ implies $MBD < 0$. A sufficient condition for this is that the following inequalities hold:

$$E_x \left[\psi^{(H,x,F)} \mid B\right] < E_x \left[\psi^{(H,x,F)} \mid G\right]; \quad E_x \left[\psi^{(H,x,F)} \mid B\right] < \psi^{(H,S)}. \tag{59}$$

The first inequality is satisfied, as $\psi^{(H,x,F)}$ is decreasing in μ, where $\mu = \Pr(B \mid x,\ x \sim X(\theta, H))$, [65] and $\Pr(x \mid B, m = H)$ puts more (less) density than $\Pr(x \mid G, m = H)$ on $x = +2n$ ($x = -2n$), which is a signal for a bad (good) investment.[66]

The second inequality is satisfied, since the following is true:

$$\psi^{(H,x,F)} < \psi^{(H,S)} \quad \text{for any } x \in \{-2n, 0, +2n\} \tag{60}$$

[64] A good initial signal is not conclusive about the quality of the investment; therefore, a strategic CRA which has observed a good signal (type $(\mathcal{S}, g)$) has a conditional expectation over the investment quality θ, which affects the expectation over the realizations of x and y. Given a good signal, there are three possibilities: the investment is bad; the investment is good and fails; and the investment is good and successful. See Appendix 4.

[65] As the investors think it is relatively more likely that the investment is bad, $m = H$ becomes a stronger signal for a strategic CRA, as it is relatively more likely that the CRA observed a bad signal and inflated it.

[66] See equation (30) in Appendix A.4. This is because, when $m = H$, the speculator always enters the CDS market, and so x is informative about the true θ.

as we have

$$\frac{\Pr\left(H,x,F\mid\mathcal{S}\right)}{\Pr\left(H,x,F\mid\mathcal{C}\right)} = \frac{\overbrace{\mu\alpha\varepsilon^{e}}^{\text{prob. }(\mathcal{S},b)}}{\underbrace{\mu\left(1-\alpha\right)+\left(1-\mu\right)f_{G}}_{\text{prob. }(\mathcal{C},g)}} + \overbrace{1-\delta^{e}}^{\frac{\text{prob. }(\mathcal{S},g)}{\text{prob. }(\mathcal{C},g)}}\ ; \tag{61}$$

$$\frac{\Pr\left(H,S\mid\mathcal{S}\right)}{\Pr\left(H,S\mid\mathcal{C}\right)} = \overbrace{1-\delta^{e}}^{\frac{\text{prob. }(\mathcal{S},g)}{\text{prob. }(\mathcal{C},g)}}\ . \tag{62}$$

When the investment suceeds ($y=S$) the investors learn that the asset was good. This means that the CRA observed a good signal ($\sigma=g$) and chose not to deflate it, which is a signal for a committed type - for any $\delta^{e}>0$. When $y=F$, there is always some positive probability (for any realization of x) that the project was bad and the CRA inflated the report: therefore, the CRA's posterior reputation following (H,x,F) is always lower than the one following (H,S).

Step 3: Effect of Trading Informativeness on Rating Inflation. Here we show that the proof of Lemma 5 - see Appendix A.8 - does not change in the new setting. Therefore, Lemma 5 still holds. Given the distribution of x implied by the CDS market equilibrium, MBI - equation (58) - can be rearranged as follows:

$$MBI=\varphi+\frac{\gamma}{2}\left[\iota^{e}\psi^{(H,+2n,F)}+\left(1-\iota^{e}\right)\psi^{(H,-2n,F)}\right]+\frac{\gamma}{2}\psi^{(H,0,F)}-\gamma\psi^{\varnothing}. \tag{63}$$

When $x=0$, which occurs with probability $\frac{1}{2}$, there is no learning about investment quality θ from trading. The continuation payoffs and, thus, the MBI do not depend on ι^{e} in this case. When $x=+2n$, investors learn that $\sigma_{s}=b_{s}$ and so $\theta=B$. When $x=-2n$, the investors learn that $\sigma_{s}=g_{s}$ which is a signal for a good investment. Taking the derivative of MBI with respect to ι^{e} yields:

$$\begin{aligned}\frac{dMBI}{d\iota^{e}} &= \frac{\gamma}{2}\left(\psi^{(H,+2n,F)}-\psi^{(H,-2n,F)}\right)\\ &\quad+\frac{\gamma}{2}\left(\iota^{e}\frac{d\psi^{(H,+2n,F)}}{d\iota^{e}}+\left(1-\iota^{e}\right)\frac{d\psi^{(H,-2n,F)}}{d\iota^{e}}\right).\end{aligned} \tag{64}$$

Notice that this is the same expression as in the basic model - equation (15), as also in the new setting the CRA's posterior reputation following $m = \varnothing$ does not depend on ι^e. [67] Therefore, the proof of Lemma does not change.

[67] In the basic model, when $m = \varnothing$ the speculator does not enter the CDS market, and so $\psi^{(\varnothing,F)}$ does not depend on x, as x is not informative about θ. In the new setting, the CDS market does not take place following $m = \varnothing$.