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Abstract

We study the consequences and optimal design of bank deposit insurance and reinsurance in a general equilibrium setting. The model involves two production sectors. One sector is financed by issuing bonds to risk–averse households. Firms in the other sector are monitored and financed by banks. Households fund banks through deposits and equity. Deposits are explicitly insured by a deposit insurance fund. Any remaining shortfall is implicitly guaranteed by the government. The deposit insurance fund charges banks a premium per unit of deposits whereas the government finances any necessary bail-outs by lump-sum taxation of households. When the deposit insurance premium is actuarially fair or higher than actuarially fair, two types of equilibria emerge: One type of equilibria supports the Pareto optimal allocation, and the other type does not. In the latter case, bank lending is too large relative to equity and the probability that the banking system collapses is positive. Next, we show that a judicious combination of deposit insurance and reinsurance eliminates all non–optimal equilibrium allocations. Our paper provides a benchmark result for policy proposals that advocate deposit insurance cum reinsurance.

JEL Classification: D53, E44, G2

Keywords: Financial Intermediation, deposit insurance, Capital Structure, General Equilibrium, reinsurance

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Deposit Insurance and Reinsurance: A General Equilibrium Perspective*

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We study the consequences and optimal design of bank deposit insurance and reinsurance in a general equilibrium setting. The model involves two production sectors. One sector is financed by issuing bonds to risk–averse households. Firms in the other sector are monitored and financed by banks. Households fund banks through deposits and equity. Deposits are explicitly insured by a deposit insurance fund. Any remaining shortfall is implicitly guaranteed by the government. The deposit insurance fund charges banks a premium per unit of deposits whereas the government finances any necessary bail-outs by lump-sum taxation of households. When the deposit insurance premium is actuarially fair or higher than actuarially fair, two types of equilibria emerge: One type of equilibria supports the Pareto optimal allocation, and the other type does not. In the latter case, bank lending is too large relative to equity and the probability that the banking system collapses is positive. Next, we show that a judicious combination of deposit insurance and reinsurance eliminates all non–optimal equilibrium allocations. Our paper provides a benchmark result for policy proposals that advocate deposit insurance cum reinsurance.

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1 Introduction

We study deposit insurance in the framework of a general equilibrium model with two sectors of production. In particular, we consider a deposit insurance scheme with two components: First, the regulator obliges banks to pay a premium per unit of deposits to a deposit insurance fund. Second, the deposit insurance fund buys reinsurance from households. We show that this deposit insurance scheme guarantees an optimal allocation of inputs to the sectors of production, thus eliminating distortions which would occur under deposit insurance without reinsurance.

Motivation

In most countries, there is some form of deposit insurance for demand deposits. Usually, deposits are insured up to some fixed amount per account or per individual. Such deposit insurance may be either implicit or explicit. During the financial crisis of 2007-2009, it was a common practice for governments to guarantee deposits implicitly by bailing out many banks and thereby keeping deposits safe. With explicit deposit insurance, banks are required to pay an insurance premium to a deposit insurance fund. This fund is used to reimburse bank depositors in case a bank fails to honor its obligations. Several countries have a long history of explicit deposit insurance schemes. In the US, for instance, federal deposit insurance started under the (Glass-Steagall) Banking Act of 1933 which created the Federal Deposit Insurance Corporation (FDIC) in charge of insuring deposits at commercial banks.[1] Calomiris and Jaremski (2016) provide a comprehensive historical account of the economic and political theories of deposit insurance. They conclude that deposit insurance generally tends to increase systemic risk, rather than reduce it.

Deposit insurance has obvious benefits. It protects small, risk-averse and potentially unsophisticated savers and it prevents bank runs, which fosters financial stability. Deposit insurance is also central for the use of bank deposits as a medium of exchange by market participants. A drawback of deposit insurance, however, is that it may lead to severe distortions. It is clear that implicit deposit insurance may encourage excessive risk-taking and excessive balance sheet expansion. It is less clear–cut how distortions may arise from explicit deposit insurance. One contribution of the present paper is to use a general equilibrium approach to demonstrate

[1] A thorough discussion of this scheme can be found in Pennacchi (2009).

how explicit deposit insurance may distort the allocation of inputs to the sectors of production. The main result of the paper will be to show that these distortions can be resolved if explicit deposit insurance is suitably combined with reinsurance.

One important challenge in designing explicit deposit insurance schemes is to price the insurance adequately. A large literature has addressed deposit insurance pricing.[2] One standard approach is to aim for an actuarially fair premium. Important contributions by Pennacchi (2006) and Acharya et al. (2010) have shown that actuarially fair pricing of deposit insurance at the level of an individual bank is insufficient since banks do not represent a pool of stochastically independent risks. The reason is that bank failures tend to occur during downturns, or are widespread in a banking crisis. These insights suggest that it might be appropriate to add a "systemic risk surcharge" to the actuarially fair premium.

Two rationales for imposing surcharges have been provided. First, as derived in the seminal paper by Pennacchi (2006), premia must exceed expected losses, since the deposit insurer bears aggregate risk. Without such charges for systemic risk, insured deposits would be subsidized compared to uninsured funding. This, in turn, may lead to excessive expansion of risky investments by commercial banks that are financed by insured deposits.

Second, as derived in Acharya et al. (2010), in a systemic crisis the deposit insurance fund faces particularly low liquidation values of assets of failing banks because of fire sales and bank interconnectedness (Allen and Gale (2000) and Kahn and Santos (2005)). The shortfall per dollar of insured deposits is larger than when only one or few banks default. Hence, to cover the expected losses, higher deposit insurance premia are required than actuarially fair premia for an individual bank in isolation would suggest. In other words, the assessment of an actuarially fair deposit insurance premium must be based on the risk of individual bank failures together with the risk of widespread bank failures.[3]

In this paper, we take a complementary view by focusing on general equilibrium

[2] On the pricing see Merton (1977, 1978), Marcus and Shaked (1984), McCulloch (1985), Ronn and Verma (1986), Pennacchi (1987), and Flannery (1991). Its limits are outlined in Chan et al. (1992) and regulatory forbearance in closing banks is addressed in Allen and Saunders (1993) and Dreyfus et al. (1994). The important paper by Freixas and Rochet (1998) indicates that pricing of deposit insurance can be undesirable in adverse selection environments, since it entails subsidization of less efficient banks by the most efficient ones.

[3] Acharya et al. (2010) further show that incentive-efficient risk premia that discourage banks to take excessive correlation risk are even higher than the actuarially fair premia.

feedback effects of deposit insurance and how deposit insurance affects capital structures of banks. Moreover, we offer a deposit insurance scheme, coupled with reinsurance, that guarantees socially optimal allocations in all equilibria. The paper thus provides a benchmark result for policy proposals that favor some form of reinsurance as a complement to deposit insurance.

Boyd et al. (2002) have highlighted that changes in deposit insurance pricing can be outweighed by changes in deposit rates of return and thus may have no impact on bank lending in a simple general equilibrium setting.[4] Standard propositions such as the desirability of actuarially fair pricing or undesirability of subsidization of banks through deposit insurance may therefore not hold.

While Boyd et al. (2002) focus on banks using debt contracts, we develop a general equilibrium model in which households choose a portfolio of debt and bank equity structures. Bank equity acts as a buffer against losses. We study how bank capital structures and associated probabilities of bank failures endogenously emerge in the presence of deposit insurance and reinsurance.

Model

We adopt a general equilibrium perspective to investigate the scope and limits of deposit insurance, possibly coupled with reinsurance. We study deposit insurance and reinsurance in the basic modeling framework of a production economy with two periods and two sectors of production.[5] The model economy's basic characteristics are as follows:

- There is a continuum of risk-averse households who invest in the capital market and banks.
- There are two technologies for real investments: One "frictionless technology" (henceforth FT) leads to safe (deterministic) returns. Investment into a "moral hazard technology" (henceforth MT) leads to risky (state–contingent) returns. Households can invest directly in FT, while investment in MT requires intermediation by banks. That is, households fund banks through

[4] In Boyd et al. (2004), they show that this line of reasoning has to be altered in a monetary setting with reserve requirements.

[5] Our model economy is a variant of the one used by Gersbach et al. (2015) in order to derive a rationale for capital requirements.

deposits and equity, and banks are free to invest in both sectors. The motivation is as follows: We think of firms in FT as well–established businesses, while firms in MT are more innovative. Entrepreneurs in MT are subject to "moral hazard" when their activities are not monitored. Banks have appropriate monitoring ability, while households do not.

- Banks pay premia to a deposit insurance fund. If banks are unable to repay depositors (even after wiping out their equity), then the deposit insurance fund compensates depositors for the shortfall. If even the deposit insurance fund is unable to cover the losses completely, then the government is committed to provide a bail-out for the remaining shortfall. Such a government bail-out is financed by taxing households. Alternatively, the deposit insurance fund is required to contract sufficient reinsurance so that it will be able to compensate depositors in all contingencies. In either case, bank deposits are safe.[6]

On purpose, in the basic model, we assume favorable manifestations of the underlying frictions and distortions.

- Banks act as delegate monitors and can eliminate moral hazard in MT at no cost.[7]
- Taxation to bail out a defaulting deposit insurance fund is lump sum and thus non-distortionary.

Given this set-up, it is *a priori* unclear whether equilibria in such an economy yield the optimal allocations that would occur in an Arrow-Debreu version of the economy. Moreover, it is unclear which form of deposit insurance is conducive for welfare and whether bail-out of defaulting deposit insurance funds or reinsurance is preferable.

[6] As discussed in the motivation, the typical rationale for making deposits safe is the protection of risk–averse households and the guarantee of bank deposits as a means of exchange.

[7] There is no moral hazard on the part of bank managers monitoring entrepreneurs in the basic model. We discuss in concluding section that the underlying logic holds more generally and how moral hazard and costs of monitoring can be incorporated in the analysis.

Main Results, Intuition and Policy Perspectives

One main insight is that only a judicious combination of deposit insurance and reinsurance can accomplish two goals: First, it guarantees an optimal equilibrium allocation. Second, it ensures that government bail-outs are not needed in case of a banking crisis. At a more detailed level, we establish the following results. With deposit insurance, two classes of equilibria exist. In one class, bank equity completely absorbs losses in bad times and deposit insurance is redundant. The allocation is optimal in the sense that it maximizes the aggregate utility of households and coincides with the allocation achieved in the Arrow-Debreu version of the model.[8]

In a second class of equilibria, banks default in a bad state and equilibrium allocations are not optimal. This class of equilibria exists with actuarially fair deposit insurance schemes and even when additional systemic surcharges on deposit insurance premia are imposed. In the event of a banking crisis, deposit insurance schemes cannot cover the entire burden to guarantee deposits and additional government bail-outs financed through taxation become necessary. Non-optimal equilibrium allocations, however, still exist in extreme cases when deposit insurance premia are set at high levels that guarantee safe bank deposits even in a banking crisis.

In order to avoid non–optimal equilibrium allocations associated with deposit insurance schemes of any type, deposit insurance must be coupled with a reinsurance provision. We show that a judicious combination of deposit insurance and reinsurance (provided via the capital market or through reinsurance firms) guarantees an optimal allocation in any equilibrium. There may still be equilibria with banking crises, but those crises are immediately resolved through deposit insurance and reinsurance without government intervention.

The intuition why the inefficient equilibria exist runs as follows. In any equilibrium, portfolio decisions of risk-averse households regarding safe deposits (and bonds) and risky bank equity must be compatible with the returns and aggregate risk generated by investments in FT and MT. In equilibrium, a portfolio of safe debt

[8] Throughout the paper, we will use the terms "optimal" and "non-optimal" in this sense. We reserve the term "efficient" in order to refer to an allocation which maximizes (expected) output in the economy. In the next section, it will be made clear that optimality and productive efficiency do not coincide in our model.

and risky bank equity is the optimal choice of households, given optimal investment decisions of banks at such deposit-equity combinations. The portfolio choice of all households can be distorted by tax risks from the bail-out of deposit insurance funds and thus additional burden in bad states. The portfolio decisions are also distorted by transfers from the deposit insurance funds in the good state when insurance is not needed. We show that only if both types of transfers from and to households are eliminated, distorted portfolio decisions regarding the mix of safe debt and risky bank equity and inefficient equilibria can be avoided.

We find that a judicious design of reinsurance of the deposit insurance fund eliminates both distortions since no transfers from and to the fund are needed in good and bad states, respectively. The key is that the amount of reinsurance contracts is linked to the capital structure (and deposit insurance premia) of banks. As a consequence, avoiding both distortions only works at a particular capital structure of banks which, in turn, is uniquely pinned down in equilibrium. As a further consequence, there exists a unique socially optimal deposit insurance scheme that supports the equilibrium.

Over the last decades, several authors have advocated reinsurance as a complement to deposit insurance in order to avoid or reduce government bail-outs. Plaut (1991) and Sheehan (2003) have outlined possible reinsurance solutions, and Madan and Unal (2008) have examined the pricing of reinsurance contracts.[9] We provide a general equilibrium benchmark result and suggest that a judicious combination of deposit insurance and reinsurance eliminates all non–optimal equilibrium allocations and avoids government bail-outs — although banking crises can still occur. These crises, however, are resolved quickly and anticipating them does not distort the investment allocation in the economy.

Key Assumptions

The preceding intuition clarifies the reasons why our benchmark result holds and points to the key assumptions driving our results. While the results will be derived in a general equilibrium setting, with assumptions about technologies and preferences, we discuss in the concluding section that underlying logic holds more general for economies with banks and aggregate risk.

[9] Also the establishment of a European Deposit insurance scheme (EDIS) involves reinsurance in particular phases (EU Directive 2014/49/EU, 2014).

The paper is organized as follows. In the next section, we outline the detailed set-up of our model and characterize the Arrow-Debreu equilibrium. In Section 3, we provide alternative characterizations of the first-best allocation. In Section 4, we introduce banks and deposit insurance and develop the corresponding equilibrium concept. In Section 5, we characterize these equilibria and identify the set of equilibria supporting optimal and non-optimal allocations, respectively. In Section 6, we introduce reinsurance and show that the ensuing equilibrium allocation is unambiguously optimal.

2 An economy without financial intermediation

2.1 Production side

We start with the description of the production side of our two-period model. The periods are denoted by $t = 1, 2$. There is a continuum of measure one of identical households initially endowed with an amount $\omega > 0$ of an investment good. The investment good cannot be consumed or stored. For convenience, we use the variable ω for both the per capita endowment of households and for the aggregate endowment with the investment good in the economy.

There are two technologies by which the investment good at $t = 1$ can be transformed into a consumption good at $t = 2$. The first technology transforms y units of investment good into $f(y)$ units of consumption good irrespective of the realization of the state $s = g, b$. The production function $f(y)$ is twice continuously differentiable, strictly increasing, strictly concave, and satisfies the upper and lower Inada conditions $\lim_{y\downarrow 0} f'(y) = +\infty$ and $\lim_{y\uparrow\omega} f'(y) = 0$. Henceforth, we call this technology the frictionless technology (FT). The returns of the second technology are risky and depend on the state of the world. The state of the world at $t = 2$ belongs to the state space $\{g, b\}$, that is, the state of the world can be "good" or "bad." It is common knowledge that state g occurs with probability σ, and state b occurs with complementary probability $1 - \sigma$.

The second technology transforms y units of investment good into $y\overline{R}$ units of consumption good in state g, and into $y\underline{R}$ units of consumption good in state b, where $\overline{R} > \underline{R} \geq 0$. This technology is called the moral hazard technology (MT).

The expected return of investing one unit of the investment good in MT is given by

$$R_M := \mathbb{E}[\widetilde{R}] = \sigma\overline{R} + (1-\sigma)\underline{R}.$$

An important remark is in order. Since households are risk–averse, the optimal allocation is not characterized by an equal expected marginal product in both sectors. We will give a detailed characterization of the optimal allocation in Section 6.

In the sequel, we denote by (y_M, y_F) the factor demands in the MT and FT sectors, respectively. Several remarks are in order. First, FT is supposed to represent established business while MT stands for risky new businesses. Second, the Inada conditions imposed on the production function $f(y)$ ensure existence and interior solutions. However, the assumption is more stringent than needed. For instance, the upper Inada condition may be replaced by $f'(w) < \underline{R}$.[10]

In each sector of production, there is competition among a continuum of small and identical firms who maximize profits while taking all aggregate economic variables as given.[11] Therefore, it is appropriate to focus the analysis on a representative firm for each sector. One can interpret the representative firm in FT as an established company while the representative MT firm may be a small or medium-sized company or a start-up.

We assume complete contingent commodity markets — or, equivalently, complete asset structures. For this purpose, we introduce the price vector $(1, p_g, p_b)$, where the price of the investment good has been normalized to one. The price at $t = 1$ for obtaining one unit of the consumption good in the good state and nothing in the bad state is denoted by p_g. The price at $t = 1$ for obtaining one unit of the consumption good in the bad state and nothing in the good state is denoted by p_b. The profit function of the representative FT and MT firms, respectively, can

[10] We refer to Gersbach et al. (2015) for a detailed discussion on how Inada conditions can be weakened in this type of model.

[11] In the case of FT, one often assumes that each firm operates a project of size one and productivities are project-specific. The distribution of firm productivities generates the function $f(y_F)$.

be written as:

$$\Pi_F(y_F, p_g, p_b) = (p_g + p_b) f(y_F) - y_F,$$
$$\Pi_M(y_M, p_g, p_b) = (p_g \overline{R} + p_b \underline{R} - 1) y_M.$$

Since the representative firm in either sector is a price–taker, it considers p_g and p_b as given and treats its factor demand as its decision variable. Indeed, choosing y_F and y_M in order to maximize the representative firms' profits leads to the conditions

$$(p_g + p_b)^{-1} = f'(y_F), \tag{1}$$
$$p_g \overline{R} + p_b \underline{R} = 1. \tag{2}$$

Observe that competitive markets and constant returns to scale in the MT sector imply that $\Pi_M = 0$ in equilibrium. If condition (2) did not hold, the demand for the investment good of the representative firm would be zero or infinite, which cannot occur in equilibrium. In the FT sector, however, the profit $\Pi_F > 0$ is strictly positive despite perfect competition because of decreasing returns to scale.

2.2 Consumer side

All households have identical preferences over consumption pairs (c_g, c_b). These preferences are represented by a utility function $U(c_g, c_b)$ which is additively separable across states and exhibits constant relative risk aversion. Formally, we assume that

$$\begin{aligned} U(c_g, c_b) &= \sigma u(c_g) + (1 - \sigma) u(c_b), \\ u(c_s) &= (1 - \theta)^{-1} \, c_s^{1-\theta}, \; s = g, b, \end{aligned}$$

with $\theta > 0$ and $\theta \neq 1$.

All households are equally endowed with ownership of the FT and MT firms. Due to market completeness, we need not model any trade in the ownership shares of the firms. Under these assumptions, we can proceed as if there was a single representative household with utility function u and an initial endowment $\omega > 0$

of investment good. The profits of firms in both sectors are denoted by Π_F and Π_M, where we have already argued that $\Pi_M = 0$. Profits are distributed equally to all households, so that the representative household has a budget set

$$\mathcal{B}(\Pi_F, p_g, p_b) = \{(c_g, c_b) \in \mathbb{R}^2_+ | w + \Pi_F \geq p_g c_g + p_b c_b\}.$$

The household seeks to maximize utility over this budget set, which leads to the first-order condition

$$\left(\frac{c_g}{c_b}\right)^\theta = \left(\frac{p_b}{p_g}\right)\left(\frac{\sigma}{1-\sigma}\right). \tag{3}$$

2.3 Equilibrium without financial intermediation

Before discussing frictions and the role of banks, it is helpful to consider a simple economy without financial intermediation as a benchmark. In that economy, households can directly finance the representative firms in both sectors of production, and all agents trade in complete contingent commodity markets. In Appendix A, we provide a detailed characterization of the Arrow–Debreu equilibrium for this economy. We refer to this notion of equilibrium as an *equilibrium without financial intermediation.* Here, we only summarize two results from Appendix A that are important for understanding and benchmarking the subsequent results.

Theorem 1.

(i) The tuple $(c_g^*, c_b^*, y_F^*, y_M^*) \gg 0$ is part of an equilibrium without financial intermediation if it solves the following system of equations:

$$\begin{aligned} y_M &= \omega - y_F, \\ c_g &= f(y_F) + y_M \overline{R}, \\ c_b &= f(y_F) + y_M \underline{R}, \\ \left(\frac{c_g}{c_b}\right)^\theta &= \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\overline{R} - f'(y_F)}{f'(y_F) - \underline{R}}\right). \end{aligned}$$

(ii) An equilibrium without financial intermediation involves the optimal allocation.

The second result is of course the manifestation of the first welfare theorem for a production economy with risk. Note that the equilibrium in Theorem 1(i) specifies

the input allocation and the consumption allocation, respecting the production plans, and thus the complete commodity allocation.

In the sequel, the allocation characterized by Theorem 1 serves as a benchmark as we discuss economies with frictions caused by financial intermediation as well as deposit insurance.

3 An economy with banks and deposit insurance

3.1 Model description

In the previous subsection, we have considered a frictionless economy in which households directly invest in the two technologies. In that economy, the equilibrium allocation of the investment good to the two production technologies is optimal. From now on, we will add two new features to the model: First, we assume that households can invest directly in the FT sector, but any investment in the MT sector requires financial intermediation by banks. One interpretation is to think of the MT sector as an industry where firms are subject to moral hazard, and the necessary monitoring and contract enforcement can only be done by banks and not by households.[12]

There is a continuum of banks. Contrary to households, banks are able to monitor MT entrepreneurs, thus dealing with the moral hazard problem in that sector. Contrary to the earlier benchmark economy without financial intermediation, households do not invest directly in MT but provide funds to banks as deposits and as equity. Banks act in the interest of their shareholders who have limited liability for losses. Moreover, the government is committed to ensuring the viability of the banking system. More specifically, depositors of a failing bank are reimbursed by a deposit insurance fund and by a government bail-out. This assumption introduces a distortion into the model economy: Since shareholders benefit from any profits of the banking sector, but can externalize part of the losses to the households, the banks have an incentive to over-invest in the risky MT sector.

In addition, we assume that there is a deposit insurance fund to which banks must

[12]See Freixas and Rochet (2008) for a comprehensive account of the microeconomic foundations of financial intermediation.

contribute a certain share of their deposits as a premium. The deposit insurance fund invests the premium in the FT sector. If a bank fails to repay its depositors, then, as a first step, bank equity is used to honor the payment obligation. Once all equity has been wiped out, the deposit insurance fund reimburses depositors. If even the deposit insurance fund does not have sufficient means to compensate the depositors, then the government repays deposits and finances this bail-out by a lump–sum tax on all households. If the deposit insurance fund is not depleted in the second period, then the remaining funds are distributed equally to all households.

Observe that this deposit insurance scheme leads to a second distortion in the model economy because lump–sum refunds in the good state are the result of the portfolio decisions of all households, but are taken as given in the portfolio choice of an individual household. The former distortion also persists unless the deposit insurance premia are sufficiently high to cover the entire shortfall in the bad state.

Provided that the deposit insurance fund and the government are committed to reimbursing depositors, financial intermediation and deposit insurance introduce two distortions into the model economy compared to the economy without financial intermediation. We will show that these distortions may lead to non–optimal equilibrium allocations.

In order to focus on the aforementioned distortions, we make three additional assumptions to rule out other sources of friction: First, we assume that banks can monitor the representative MT firm perfectly at zero cost — any moral hazard is eliminated. Consequently, we can think of MT as simply a risky technology banks can invest in. Second, we assume that the bank acts in the best interest of its shareholders/equity–holders. In particular, there is no friction between the interests of the bank managers as agents and the shareholders as principals. In the last section, we outline how moral hazard of bank managers can be integrated in our analysis. Third, we only consider lump–sum taxation of households, thus eliminating any tax distortions.

As before, all individual actors in the economy (households, firms, and banks) are atomic and, therefore, ought to be treated as price (or contract) takers. Consequently, an individual decision maker in the economy takes all aggregate values of economic variables as given. We will now discuss in turn the relevant optimization

problems of firms, banks, and households. This discussion prepares the ground for the definition of the equilibrium concept for the economy with banks and deposit insurance. Ultimately, we will show that both the optimal allocation and a multitude of non–optimal allocations can be supported by such equilibria.

3.2 Firms

Recall that in the current model setup, households still invest directly in the FT sector. More formally, we will say that FT firms issue *bonds* to households. The bond obliges the firm to pay an amount R_F to the household in the second period, while its purchase price is normalized to one. FT firms maximize profits by an appropriate choice of the factor demand y_F. That is, an FT firm solves the optimization problem

$$\max_{y_F} f(y_F) - R_F y_F.$$

It is straight–forward that at the solution of this problem, we have

$$R_F = f'(y_F).$$

Verbally, given the profit–maximizing behavior of FT firms, the rate of return on bonds is equal to the marginal product in FT production. Now consider the MT sector. For the representative MT firm, the only channel of funding is the financial intermediation by banks.

Due to perfect monitoring, banks can enforce the terms of the loan contract and make loan repayment rates contingent on the state. We claim that equilibrium repayment of loans by the representative firm is given as follows: In the good state, the firm repays $\overline{R}$ per unit, and repayment in the bad state is $\underline{R}$. If the pair of repayment rates was different from $(\overline{R}, \underline{R})$, then an MT firm would either demand an infinite amount of funds or no funds from banks. The former case occurs if at least one contingent repayment rate is below the equilibrium rate. The latter occurs if both repayment rates are higher than the equilibrium rate and one is strictly higher.[13] With equilibrium repayment rates $(\overline{R}, \underline{R})$ the representative

[13] One could consider a scenario when the MT firm accepts one or two higher contingent repayment rates and would default if it cannot repay. If bankruptcy imposes no costs on firms or does not reduce the investment returns, such a constellation would lead to the same effective repayment rates $\overline{R}$ and $\underline{R}$, respectively.

MT firm makes zero profits in equilibrium, reflecting the outcome under perfect competition with a constant returns to scale technology.

3.3 Banks and deposit insurance

There is a continuum of identical banks that are financed by (outside) equity and interest bearing deposits. The total amounts of debt and equity in the economy are equally distributed to all banks, and denoted by D and E. Therefore, we proceed as if there is only one representative bank receiving deposits D and equity E. The representative bank is passive regarding the choice of the capital structure and lets households decide about the amounts of deposits and equity. However, the optimal and non-optimal equilibria we will derive continue to exist if banks actively choose their capital structure.[14]

We will make use of an equilibrium concept which requires equilibrium variables, including D and E, to be strictly positive. In particular, this implies that we do not consider full equity banking (zero deposits) because then the problem of deposit insurance would be vacuous. Moreover, we do not consider cases with zero equity because otherwise the maximization of equity holders' payoff would not be well–defined.

In this subsection, we will therefore assume D and E to be strictly positive. Given that the price of the investment good has been normalized to one, and D and E are expressed in terms of investment good, one can alternatively think of D and E as the number of debt and equity contracts in the economy. In the sequel, we conduct the analysis for a representative bank that is acting competitively.

Let R_D be the rate of return on deposits. That is, for every unit of deposits, the bank pays the depositor R_D in the second period. In the presence of deposit insurance and government bail-out guarantees, bank deposits are a risk–free asset. We can use a standard arbitrage argument to show that the return on all risk–free assets in the economy must be equal if households make an optimal portfolio

[14] As in the corporate finance literature, we could consider a sequential issuance process. For instance, banks could be financed by equity first and then decide how many deposits to accept. In addition, one could allow banks to raise additional equity before they receive deposits. All equilibria we derive continue to exist if banks choose their capital structure in this way. Details are available upon request.

choice. Formally, we have

$$R_D = R_F,$$

where we recall that R_F is the rate of return on bonds. In the previous subsection, we have shown that profit maximization by the representative FT firm implies $R_F = f'(y_F)$. In what follows, we will often write R_F when we mean the risk–free return in the economy, whether it is in the context of bonds or of deposits.

The deposit insurance fund works as follows: The regulator obliges a bank to contribute an amount δD to the deposit insurance fund (DIF), where $0 \leq \delta \leq 1$. The DIF invests this amount in safe assets, i.e., in the FT sector. If the bank is able to honor its obligations towards depositors at $t = 2$ (possibly by wiping out equity), then the funds of the DIF are distributed to households. If banks cannot honor their obligations towards depositors, then the DIF reimburses the depositors. If the funds of the DIF are insufficient to repay all obligations towards depositors, then the government carries out a bail-out financed by taxing the households.

We consider the optimization problem of the representative bank. We abstract away from any principal–agent problem between the shareholders and managers of the bank. Given that the representative bank has obtained investment goods in the form of deposits and equity, of quantities D and E, respectively, the objective of the bank is to maximize the payoff to its shareholders. The choice variable of the bank is the share $\alpha \in [0, 1]$ which the bank invests in MT, while the complementary share $1 - \alpha$ is invested in FT. Due to the market completeness, the choice of α has no effect on the household's portfolio problem. Moreover, due to perfect competition in the banking sector, the representative bank is a price–taker: There is no feedback effect from the choice of α to the prices prevailing in the economy.

As a result of these considerations, the variables D and E and R_F can be taken as given when maximizing the payoff to bank shareholders. Moreover, this payoff is given by the following expressions, which take into account that shareholders are not liable for losses.

$$\overline{\pi}(\alpha) = \max\{0, \left(\alpha\overline{R} + (1-\alpha)R_F\right)((1-\delta)D + E) - R_F D\},$$
$$\underline{\pi}(\alpha) = \max\{0, (\alpha\underline{R} + (1-\alpha)R_F)((1-\delta)D + E) - R_F D.\}$$

We use the notation $\overline{R}_E(\alpha) = \overline{\pi}(\alpha)/E$ and $\underline{R}_E(\alpha) = \underline{\pi}(\alpha)/E$ for the return on

equity in either state. Since E is taken as given in the bank's optimization problem, maximizing the expected shareholder payoff is equivalent to maximizing the expected return on equity.

We want to show that the representative bank chooses $\alpha = 1$, provided that households invest sufficiently in FT.

Lemma 1.
Suppose that $R_F < \sigma\overline{R} + (1-\sigma)\underline{R}$. Then, it is optimal for the bank to choose $\alpha = 1$.

The proof of Lemma 1 is given in Appendix B. The bottom line of the analysis of the banking sector is that banks want to invest fully in the MT sector when the marginal product in FT is sufficiently low. In the sequel of the paper, we will work with equilibrium concepts which require that the marginal product in FT is indeed low enough.

3.4 Consumer choice problem

In this subsection, we discuss the consumer choice problem at the level of the individual household. Each household chooses a portfolio which consists of direct investment in FT, deposits, and equity. As discussed in the previous subsection, the rate of return on deposits is the same as that on direct investment in FT sector. Hence, the household portfolio problem can be thought of as one–dimensional: The household chooses to invest some share of its funds in risk–free assets with payoff structure (R_F, R_F), and the complementary share in equity, which is a risky asset with payoff structure $(\overline{R}_E, \underline{R}_E)$. One unit of either asset can be exchanged for one unit of the other asset; their price is normalized to one.

We denote by (t_g, t_b) the state–contingent fixed effect of the deposit insurance fund on consumption. The pair (t_g, t_b) is determined by the aggregate portfolio decisions of all households, but taken as given by any individual household. Recall that we are looking at a model where banks are obliged to pay a share δ of their deposits to a deposit insurance fund. We require this fund to invest the premium in the FT sector. In the good state, the entire deposit insurance fund is distributed to households. In the bad state, the deposit insurance fund is used to repay depositors, and any remaining shortfall is financed by a lump–sum tax on all households. Formally, the deposit insurance fund leads to the following fixed

effects (t_g, t_b) on household consumption:

$$t_g = \delta D R_F, \tag{4}$$

$$t_b = [(1-\delta)D + E]\underline{R} + \delta D R_F - D R_F. \tag{5}$$

In addition to the state–contingent fixed effect (t_g, t_b), there is a fixed effect of FT profits on household consumption. More specifically, recall that FT firms produce an amount $f(y_F)$, and need to pay their bond–holders an amount $y_F R_F$, so that they are left with a profit

$$\begin{aligned} \Pi_F &= f(y_F) - y_F R_F \\ &= f(y_F) - y_F f'(y_F) > 0, \end{aligned}$$

which is distributed to households in either state.

Since we are considering a continuum economy, the individual household not only takes (t_g, t_b) and Π_F as given, but also the allocation y_F, the risk–free rate R_F, and the returns on equity $(\overline{R}_E, \underline{R}_E)$. Moreover, the individual household also takes as given the aggregate choice of equity E. In what follows, it will be necessary to consider the choice of an individual household while holding the aggregate behavior of households fixed. This is in contrast to the "representative household" approach which is used extensively in this paper as well. The reason why one needs to switch to a different approach is that in the economy with deposit insurance cum reinsurance in the sequel of the paper, heterogeneous choices by households may occur in equilibrium.

To allow for heterogeneity of household behavior, we introduce a variable η to denote the choice of an individual household. This variable η stands for the amount of equity chosen by an individual household relative to the equity choice of the average household. More specifically, if some household chooses $\eta > 1$, then that household chooses to invest more in equity than the "average" household, while if $\eta < 1$, then this particular household invests less than "average" in equity. It will be sufficient to consider the range $\eta \in [0, \omega/E]$. The portfolio choice problem of

an individual household can now be stated formally as follows:

$$\max_{\eta\in[0,\omega/E]} \left(\frac{\sigma}{1-\theta}\right)\left(\eta E\overline{R}_E + (\omega-\eta E)R_F + t_g + \Pi_F\right)^{1-\theta} + \left(\frac{1-\sigma}{1-\theta}\right)\left(\eta E\underline{R}_E + (\omega-\eta E)R_F + t_b + \Pi_F\right)^{1-\theta}.$$

One clarification is in order: Recall that the household can distribute its funds to bank equity, bank deposits, and direct investment in the FT sector. The latter two are both risk–free and offer the same rate of return. The solution to the above maximization problem determines how much the household invests in bank equity. It leaves unspecified how the remaining funds will be divided between bank deposits and direct investment in FT. We are going to see later on that, in equilibrium, this division of funds will be pinned down by the price of deposit insurance.

Recalling that $E > 0$, the first–order condition for the consumer choice problem can be written as

$$\left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\overline{R}_E - R_F}{R_F - \underline{R}_E}\right) = \left(\frac{\eta E\overline{R}_E + (\omega-\eta E)R_F + t_g + \Pi_F}{\eta E\underline{R}_E + (\omega-\eta E)R_F + t_b + \Pi_F}\right)^{\theta}. \tag{6}$$

The left-hand side is independent of η. For $\overline{R}_E > R_F > \underline{R}_E$, the right-hand side is strictly increasing in η. That is, only one value of $\eta \in [0, \omega/E]$ can solve this first–order condition. Hence, all households must choose the same portfolio. But then it follows that $\eta = 1$. More formally, we have the following proposition:

Proposition 1. *Suppose that each individual household chooses an optimal portfolio and that $\overline{R}_E > R_F > \underline{R}_E$. Then, all households choose the same portfolio, and $\eta = 1$.*

Household consumption can now be written as

$$c_g = E\overline{R}_E + (\omega - E)R_F + \Pi_F + t_g, \tag{7}$$

$$c_b = E\underline{R}_E + (\omega - E)R_F + \Pi_F + t_b. \tag{8}$$

Using the notation $\psi = D/E$ for the debt–equity ratio, we can write return on

equity in the two states as

$$\overline{R}_E = (1 + \psi - \delta\psi)\overline{R} - \psi R_F,$$
$$\underline{R}_E = \max\{0, (1 + \psi - \delta\psi)\underline{R} - \psi R_F\}.$$

Substituting for $t_g, t_b, \Pi_F, \overline{R}_E$, and $\underline{R}_E$ into Eqns. (7) and (8), and using the condition $\omega = y_F + y_M$ yields

$$c_g = f(y_F) + y_M\overline{R},$$
$$c_b = f(y_F) + y_M\underline{R}.$$

Verbally, households are the ultimate recipients of the FT profit and any "profit" of the deposit insurance fund, and must finance the bail-out in case the deposit insurance fund is insufficient to pay off depositors. Deriving the above equations for c_g and c_b, we have verified that the entire amount of consumption goods produced by both technologies will find its way back to households.

This is a crucial step in the preparation for the equilibrium definition. In the next subsection, however, we take a closer look at the deposit insurance scheme. In particular, we will discuss a deposit insurance fund where the premium δ per unit of deposits is chosen such that the deposit insurance is actuarially fair, which seems to be a natural starting point for the analysis.

3.5 Pricing deposit insurance

Recall that $\delta \in (0, 1)$ denotes the fraction of its deposits that a bank has to pay as insurance premium. Thus, the bank can freely choose how to invest its equity as well as a share $1 - \delta$ of the deposits, while the share δ of the deposits is paid to the DIF. Due to Lemma 1, the bank invests the entire amount $E + (1 - \delta)D$ in the MT sector. Suppose that $D > 0$ and the bank defaults in the bad state. In that case, the shortfall can be written as follows:

$$\Lambda = DR_F - (E + (1 - \delta)D)\underline{R}.$$

This is the amount which the bank fails to repay to its depositors, and which would have to be compensated by the DIF. However, the funds available to the

DIF equal $\delta D R_F$, which is the insurance premium compounded by the risk–free rate. Let

$$\mu = \mu(\delta) = \frac{\delta D R_F}{D R_F - (E + (1-\delta)D)\underline{R}}$$

be the *cover ratio* of the deposit insurance fund. Clearly, there is a one–to–one correspondence between choosing the cover ratio μ of the deposit insurance and choosing its premium δ. More precisely, we find that

$$\delta = \frac{R_F - \left(1 + \frac{E}{D}\right)\underline{R}}{\left(\frac{1}{\mu}\right) R_F - \underline{R}}. \tag{9}$$

Recall that we are considering equilibria with $D, E > 0$, so we have the inequality

$$\delta < \frac{R_F - \underline{R}}{\left(\frac{1}{\mu}\right) R_F - \underline{R}}, \tag{10}$$

and we can see that $\delta < 1$ for any $\mu \leq 1$.

One pricing mechanism for deposit insurance schemes is actuarial fairness. A deposit insurance premium is *actuarially fair* if the DIF expects to break even, that is, the expected shortfall is equal to the insurance premium compounded at the risk–free rate.[15] In our model, an actuarially fair deposit insurance is characterized by a cover ratio of $1 - \sigma$. Choosing a cover ratio $\mu > 1 - \sigma$ is tantamount to imposing a surcharge on the actuarially fair premium. Observe that for any $\mu < 1$, the deposit insurance fund is more than exhausted in the bad state, so that a government bail-out is needed. In particular, since $1 - \sigma < 1$, an actuarially fair deposit insurance does not make government bail-outs dispensable. If $\mu = 1$, then the DIF perfectly insures the financial system; we refer to this case as *full insurance.* In the sequel, we are going to demonstrate that non–optimal equilibrium allocations are possible in economies with banks and deposit insurance as long as the cover ratio is strictly less than one.

[15] One might also consider an alternative definition of actuarial fairness which takes the perspective of the insured party. In particular, one could define an insurance as being actuarially fair if the premium equals the expected loss, that is, if $\delta D = (1 - \sigma)\Lambda$. Mutatis mutandis, the conclusions of the equilibrium analysis in Section 5 would remain valid.

3.6 Market clearing for investment good

Recall that households allocate their funds to deposits, equity, and FT investment. Denote their direct investment in FT by $y_{F,h}$, so that we have the identity

$$\omega = E + D + y_{F,h}.$$

In the presence of deposit insurance, the FT sector is funded not only by households directly but also by the DIF. We continue to use the notation y_F for the factor demand in the FT sector. Thus, the factor market clears if the following condition is satisfied:

$$y_F = \delta D + y_{F,h}.$$

Using the above identity, we can rewrite the market–clearing condition as

$$\omega = E + (1 - \delta)D + y_F.$$

Since we require household investment in FT to be non–negative, the market–clearing condition implies the inequality $y_F \geq \delta D$. Intuitively, this means that the factor demand in the FT sector must be sufficiently large so as to absorb the total deposit insurance fund. Define

$$\mathcal{D}_\mu(E) = \frac{(\omega - y_F)(R_F - \underline{R}\mu) - ER_F}{(1 - \mu)R_F}. \tag{11}$$

Recall that optimal consumer choice fixes the amount of funds invested by households in the two risk–free assets (bank deposits and direct FT investment), but does not specify the distribution of funds between those two assets. Combining Eqn. (9) and the market–clearing condition above, we can see that $\mathcal{D}_\mu(E)$ determines the equilibrium amount of deposits as a function of the equilibrium amount of equity in an economy where the deposit insurance covers the share μ of the shortfall in the bad state. Now we are ready for the equilibrium definition.

3.7 Equilibrium with banks and deposit insurance

We next introduce the equilibrium definition for arbitrary deposit insurance schemes.

Definition 1.
An equilibrium with banks and δ–deposit insurance *is a tuple* $(c_g^*, c_b^*, y_M^*, y_F^*, \psi^*, E^*, \overline{R}_E^*, \underline{R}_E^*, R_F^*) \gg 0$ *which solves the following system of equations:*

$$\left(\frac{c_g}{c_b}\right)^\theta = \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\overline{R}_E - R_F}{R_F - \underline{R}_E}\right), \tag{12}$$

$$c_g = f(y_F) + y_M \overline{R}, \tag{13}$$

$$c_b = f(y_F) + y_M \underline{R}, \tag{14}$$

$$y_F = \omega - y_M, \tag{15}$$

$$y_M = E + (1-\delta)\psi E, \tag{16}$$

$$\overline{R}_E = (1 + \psi - \delta\psi)\overline{R} - \psi R_F, \tag{17}$$

$$\underline{R}_E = \max\left\{0, (1+\psi-\delta\psi)\underline{R} - \psi R_F\right\}, \tag{18}$$

$$R_F = f'(y_F), \tag{19}$$

and, moreover, satisfies the inequalities $f'(y_F) < \sigma\overline{R} + (1-\sigma)\underline{R}$ *and* $(1+\psi)E \leq \omega$.

Equivalently, we could have included the variables (t_g, t_b, Π_F) and Eqns. (4) through (8) in the equilibrium definition. As we have argued in the previous subsection, either the pair of Eqns. (13) and (14) or the pair of Eqns. (7) and (8) would then become redundant. As a result, this alternative equilibrium definition would consist of three extra variables and three extra independent equations.

Let us discuss the equations in turn. Eqn. (12) is the condition emanating from optimal portfolio choice of the households. Eqns. (13) through (15) are simple market–clearing conditions. Eqn. (16) is a consequence of Proposition 1: Banks invest as much as possible of their equity and their deposits in MT, but due to the deposit insurance this is subject to the restriction that the share δ of the deposits must be paid to the fund as an insurance premium. Given such investment behavior by banks, and given that return on deposits equals the bond return R_F, it follows that return on equity in both states is given by Eqns. (17) and (18). Finally, Eqn. (19) states that the rate of return on bonds corresponds to the marginal product in the FT sector.

Moreover, the equilibrium definition includes an inequality which says that household investment in deposits and equity cannot exceed the funds ω available to the household. Importantly, this inequality combined with Eqn. (16) implies that in any equilibrium with banks and δ–deposit insurance, we have $y_F \geq \delta\psi E = \delta D$. Verbally, this means that the factor demand in the FT sector is sufficiently high to absorb all the funds of the deposit insurance.

Let us focus for a moment on the first four equations in the equilibrium definition. They are analogous to the definition of an equilibrium allocation without financial intermediation except that the returns $(\overline{R}_E, \underline{R}_E)$ have taken the place of the returns $(\overline{R}, \underline{R})$. The reason is obvious: In an equilibrium without financial intermediation, households invest directly in the risky MT technology, whereas in the equilibrium with banks and δ–deposit insurance, all MT investment is mediated by banks. The risky asset is bank equity, while the household perceives bank deposits as risk–free due to the deposit insurance.

Consider the special case of the above equilibrium definition where $\delta = 0$, we call it *equilibrium with banks and* 0*–deposit insurance.* This case can be interpreted in two ways: First, it can be seen as the equilibrium concept for an economy with banks, actuarially fair deposit insurance, and government bail-out guarantees in which no default occurs. Alternatively, it can also be thought of as an equilibrium concept for an economy with banks in which deposits are guaranteed by the government but there is no deposit insurance fund.

4 Equilibrium analysis

In this section, we establish two results: First, in the economy with banks and deposit insurance, the optimal allocation can be supported by a multitude of equilibria. Second, a multitude of non–optimal allocations is also consistent with equilibrium.

4.1 Equilibria supporting the optimal allocation

Proposition 2. *Suppose that the tuple* $(c_g^*, c_b^*, y_M^*, y_F^*)$ *is an equilibrium allocation without financial intermediation. Then, there exists a continuum of equilibria with banks and* 0*–deposit insurance which supports* $(c_g^*, c_b^*, y_M^*, y_F^*)$.

The proof of Proposition 2 is given in Appendix B. We now define a special case of an equilibrium with banks and 0–deposit insurance in which banks are "on the brink of default" in the bad state. That is, no default occurs in the bad state but the return on equity is equal to zero. Since the deposit insurance does not need to refund any depositors, its premium is zero. We call this equilibrium the *critical leverage equilibrium.* It supports the optimal allocation, but it will be important as a reference point for the construction of equilibria which support a non–optimal allocation.

Definition 2.
A critical leverage equilibrium *is a tuple*
$(c_g^*, c_b^*, y_M^*, y_F^*, \psi^*, E^*, \overline{R}_E^*, R_F^*) \gg 0$ *which solves the following system of equations:*

$$\left(\frac{c_g}{c_b}\right)^\theta = \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\overline{R}_E - R_F}{R_F}\right), \tag{20}$$

$$c_g = f(y_F) + y_M\overline{R}, \tag{21}$$

$$c_b = f(y_F) + y_M\underline{R}, \tag{22}$$

$$y_F = \omega - y_M, \tag{23}$$

$$y_M = E(1+\psi), \tag{24}$$

$$\overline{R}_E = (1+\psi)\overline{R} - \psi R_F, \tag{25}$$

$$R_F = f'(y_F), \tag{26}$$

$$\psi = \frac{\underline{R}}{R_F - \underline{R}}, \tag{27}$$

and, moreover, satisfies the inequalities $R_F^* < \sigma\overline{R} + (1-\sigma)\underline{R}$ *and* $E(1+\psi^*) \leq \omega$.

The critical leverage equilibrium is a special case of the equilibrium with banks and δ–deposit insurance. In particular, it is that equilibrium with banks and δ–deposit insurance in which all equity is wiped out in the bad state but bank default is just avoided.

4.2 Non–optimal equilibrium allocations

We have shown that an equilibrium with banks and δ–deposit insurance can replicate the optimal allocation of the equilibrium without financial intermediation. Next, we show that there also exist equilibria with banks and δ–deposit insurance which support non–optimal allocations. In Appendix A, we have described the optimal allocation as involving an investment $\widehat{y}_F$ in FT as well as a complementary investment $\omega - \widehat{y}_F$ in MT. We are going to show in this subsection that an allocation involving a slight under–investment in FT can be supported by an equilibrium with banks and δ–deposit insurance. More formally, let

$$Y_F' = \{y_F \in (0, \widehat{y}_F) |\ f'(y_F) < \sigma\overline{R} + (1-\sigma)\underline{R}\}.$$

We will show that any allocation in Y_F' is supported by an equilibrium for sufficiently small insurance premia.[16]

Theorem 2. *For every allocation $y_F' \in Y_F'$ and every $\delta' < \frac{\overline{R} - f'(y_F')}{\overline{R}}$, there exists an equilibrium with banks and δ'–deposit insurance which supports the allocation y_F'.*

The proof of Theorem 2 is given in the Appendix. One implication of the above theorem is that a range of non–optimal allocations can be supported by equilibria if there is no deposit insurance, that is, if $\delta = 0$. Moreover, a range of non–optimal allocations can also be supported by equilibria as long as the premium δ per unit of deposits is sufficiently small. More specifically, we see from the statement of the above theorem that a non–optimal allocation y_F' is consistent with equilibrium if $\delta < (\overline{R} - f'(y_F'))/\overline{R}$. Using the inequality $f'(y_F') < \sigma\overline{R} + (1-\sigma)\underline{R}$, this implies the following proposition.

Proposition 3. *Non–optimal equilibrium allocations exist if the deposit insurance premium satisfies*

$$\delta < (1-\sigma)\left(\frac{\overline{R} - \underline{R}}{\overline{R}}\right).$$

So far in this subsection, we have shown that non–optimal equilibrium allocations arise for sufficiently small values of δ. Now we are going to derive a condition on the cover ratio μ such that the corresponding premium δ is "sufficiently small" in

[16] In Appendix A, we argue that $f'(\widehat{y}_F) < \sigma\overline{R} + (1-\sigma)\underline{R}$. Due to the concavity of f, it follows that Y_F' is non–empty.

that sense. We have shown before that a cover ratio of $\mu \leq 1$ corresponds to a premium

$$\delta = \frac{f'(y_F) - \left(\frac{1+\psi}{\psi}\right)\underline{R}}{\left(\frac{1}{\mu}\right)f'(y_F) - \underline{R}}.$$

Since the term $(1+\psi)/\psi$ takes values in the interval $(1, \infty)$, we have

$$\frac{f'(y_F) - \left(\frac{1+\psi}{\psi}\right)\underline{R}}{\left(\frac{1}{\mu}\right)f'(y_F) - \underline{R}} < \frac{f'(y_F) - \underline{R}}{\left(\frac{1}{\mu}\right)f'(y_F) - \underline{R}}.$$

Now suppose that

$$\frac{f'(\widehat{y}_F) - \underline{R}}{\left(\frac{1}{\mu}\right)f'(\widehat{y}_F) - \underline{R}} < (1-\sigma)\left(\frac{\overline{R} - \underline{R}}{\overline{R}}\right).$$

If this inequality holds, then the previous proposition implies that we can find a non–optimal allocation in the neighborhood of the optimal allocation $\widehat{y}_F$ which can be supported by an equilibrium. Suitably rearranging the above inequality, we obtain the following proposition.

Proposition 4. *A non–optimal allocation in a sufficiently small neighborhood of the optimal allocation $\widehat{y}_F$ can be supported by an equilibrium in the presence of a deposit insurance with any cover ratio μ such that:*

$$\mu < \frac{(1-\sigma)f'(\widehat{y}_F)}{\left(\frac{f'(\widehat{y}_F)-\underline{R}}{\overline{R}-\underline{R}}\right)\overline{R} + (1-\sigma)\underline{R}}.$$

The right–hand side of the inequality in the above proposition depends only on the primitive model parameters. For any configuration of the parameters, there is a critical cover ratio below which deposit insurance cannot rule out non–optimal equilibrium allocations. We want to argue that there can be economies in which this critical cover ratio is close to one, so that even very comprehensive deposit insurance cannot rule out non–optimal equilibrium allocations. Intuitively, consider a sequence of economies such that, along this sequence, the term $\overline{R} - \underline{R}$ goes to zero. This implies that the term $f'(\widehat{y}_F) - \underline{R}$ tends to zero as well. Since $f'(\widehat{y}_F) < \overline{R}$, the latter convergence is "faster" than the former. Consequently, the

entire fraction $\frac{f'(\widehat{y}_F)-\underline{R}}{\overline{R}-\underline{R}}$ tends to zero along the sequence. Moreover, the entire inequality becomes $\mu < 1$, irrespective of the value of σ, provided that $1-\sigma > 0$.

In the next subsection, we will state two formal results: In Proposition 5, we are going to show that non–optimal equilibrium allocations are possible in the presence of actuarially fair deposit insurance. Moreover, we are going to provide a numerical example to demonstrate that non–optimal equilibrium allocations can persist even in the presence of a deposit insurance with a 99% cover ratio.

4.3 Unavoidable non–optimalities

Proposition 5 below shows that non–optimal equilibrium allocations are possible under an actuarially fair deposit insurance. In order to prepare for the proof of this proposition, we need to make sure first that there can be economies in which the actuarially fair deposit insurance premium does not exceed the factor demand in FT. We provide two examples of economies which have this property. In the second example below, we allow that the upper Inada condition is weakened. We explore the fact that with $\mu = 1-\sigma$, Eqn. (9) implies $\delta D \leq \omega \cdot (R_F - \underline{R})(1-\sigma)/(R_F - \underline{R}(1-\sigma))$.

Example 1: Let $\omega = 1$, $f(y_F) = 2\sqrt{y_F} - y_F$, $\theta = 2$, $\sigma = 2/3$, $\underline{R} = 1/2$, $\overline{R} = 2$. In this example, $\widehat{R}_F = 1$ and $\widehat{y}_F = 1/4$. Further, since $\omega = 1$ and $(\widehat{R}_F - \underline{R})(1-\sigma)/(\widehat{R}_F - \underline{R}(1-\sigma)) = 1/5$, it can be achieved that the entire deposit insurance premium is invested in FT when an equilibrium with riskless rate $\widehat{R}_F$ or close to $\widehat{R}_F$ is realized.

Example 2: We set $\omega = 1$, $\sigma = 1/2$, $\theta = 1/2$, $\underline{R} = 0$, $\overline{R} = 2$, and assume that $f(y_F) = 2(y_F - \frac{y_F^2}{2})$. In this example, $\widehat{R}_F = 2/(1+\sqrt{2})$ and $\widehat{y}_F = 2-\sqrt{2}$. Since $\omega = 1$ and $(\widehat{R}_F - \underline{R})(1-\sigma)/(\widehat{R}_F - \underline{R}(1-\sigma)) = 1/2 < 2-\sqrt{2}$, it can be achieved that the entire deposit insurance premium is invested in FT in equilibria with riskless rates close to $\widehat{R}_F$.

We next provide a proof that non–optimal equilibrium allocations can arise under actuarially fair deposit insurance.[17] This will help prove that non–optimal

[17] Without deposit insurance, Gersbach et al. (2015) have already shown that non–optimalities can arise. We extend this proof to the case of deposit insurance.

equilibrium allocations can occur under any deposit insurance scheme.

Proposition 5. *Suppose the Arrow-Debreu equilibrium allocation satisfies $\widehat{R}_F \leq 1$. Suppose that the deposit insurance is actuarially fair. Then there exist equilibria with financial intermediation where the investment in FT is strictly smaller than $\widehat{y}_F$, the investment in the risky technology is $E + (1-\delta)D$, banks only invest in the risky technology and default in the bad state. In addition to coverage by deposit insurance, government bail-out of banks is necessary in the bad state. The resulting equilibrium allocation is non–optimal.*

The proof of Proposition 5 is given in the Appendix. Notice that the hypothesis of the proposition and the assumption that the deposit insurance premium does not exceed the factor demand of the risk-free sector are satisfied for certain model specifications as we have seen in Examples 1 and 2.

The argument in the proof of Proposition 5 relies on the fact that households (a) are taxed in the bad state and (b) receive a refund in the good state. Thus, the argument still applies when deposit insurance is slightly actuarially unfair. Actually, the validity of (a) or (b) suffices. Therefore, the proof of the proposition works even when deposit insurance covers a huge fraction of the banks' deficit in case of default, provided that (c) the deposit insurance can invest all its premium revenue in the risk-free sector, that is the factor demand of the risk-free sector is high enough and (d) Eqn. (11) can be applied. The right-hand side of (11) is ill defined and (11) is not applicable if $\mu = 1$. While (c) does not hold in general, it does hold with $\mu = 0.99$ in some model specifications as the following example demonstrates.

Example 3: Let $n > 1$ be a natural number. Put $\omega = 1$, $\underline{R} = (n+1)^{1/2} - 1$, $\overline{R} = (n+1)^{1/2}$, $y_F^+ = 1 - 1/(n+1)$, $y_F^0 = 1 - 1/(n+2)$ and

$$f(y_F) = \begin{cases} 2n^{1/2} \cdot (y_F)^{1/2} & \text{for} \quad y_F \in [0, y_F^0]; \\ \\ 2n^{1/2} \cdot (y_F)^{1/2} - [n^{1/2} \cdot (n+2)^3/4](y_F - y_F^0)^4 & \text{for} \quad y_F \geq y_F^0. \end{cases}$$

Then $\widehat{p}_g \overline{R} + \widehat{p}_b \underline{R} = 1$ implies $(\widehat{p}_g + \widehat{p}_b)(n+1)^{1/2} > 1$ and consequently $(\widehat{p}_g + \widehat{p}_b) > (n+1)^{-1/2}$. Profit maximization in FT requires $(\widehat{p}_g + \widehat{p}_b) f'(\widehat{y}_F) = 1$. Now $(\widehat{p}_g + \widehat{p}_b) f'(y_F^+) > (n+1)^{-1/2} \cdot n^{1/2} \cdot (y_F^+)^{-1/2} = (n/(n+1))^{1/2} \cdot (n/(n+1))^{-1/2} = 1$. Hence $\widehat{y}_F > y_F^+ = 1 - 1/(n+1)$. Dividing $\underline{R}$, $\overline{R}$ and f by $(4/3)(n+1)^{1/2}$ yields a model

with identical $\widehat{y}_F$ and $(3/4)(1-(n+1)^{-1/2}) = \underline{R} < \widehat{R}_F < \overline{R} = 3/4$. Next let $n = 24$. Then $y_F^* = \widehat{y}_F > 24/25 = 0.96$, $E^* = y_M^*/(1+\psi^*) = \widehat{y}_M/(1+\psi^*) \in (0, 1/25)$ and $R_F^* = \widehat{R}_F \in (3/5, 3/4)$ at the critical leverage equilibrium. For $R_F \approx \widehat{R}_F$, we obtain $y_F \approx y_F^*$ and $0 < E \approx E^*$. If in addition, we set $\mu = 0.99$ in (11), then $\mathcal{D}_\mu(E) < 100 \cdot [(\omega - y_F) \cdot (1 - 0.99\underline{R}/\overline{R})] \leq (100/25) \cdot 0.208 = 0.832 < y_F$. This means that the entire insurance premium can be invested in FT when deposit insurance provides 99% coverage. It follows that the proof of Proposition 5 can be repeated with 99% deposit insurance coverage instead of actuarially fair deposit insurance. Hence it is possible to have equilibria where banks default in the bad state, there is 99% coverage by deposit insurance, government bail-out of banks is necessary, and the equilibrium allocation is non–optimal. The construction is independent of the probability σ, provided that $\sigma \in (0, 1)$.

In this section, we have shown how financial intermediation cum deposit insurance can change the equilibrium allocation. While the optimal allocation is still supported by a continuum of equilibria, there is also a continuum of non–optimal allocations which can be supported by equilibria of this economy. Even in the presence of deposit insurance, the incentive of banks to maximize expected return on equity can result in over–investment in the risky technology, and under–investment in the safe technology, relative to what is optimal for the households. We have shown that non–optimal equilibrium allocations can persist even in the presence of a deposit insurance with a high cover ratio. Intuitively, this means that imposing a surcharge on the actuarially fair premium need not guarantee an optimal allocation, even if this surcharge is chosen very high.

5 Deposit insurance with reinsurance

5.1 The Reinsurance scheme

In the previous section, we have seen that in the presence of a deposit insurance, it is still possible to support equilibria with default and with a non-optimal allocation of input to the two sectors. Non–optimal equilibria can exist not only when deposit insurance is actuarially fair but even when a premium above the actuarially fair level is charged, as with "systemic risk surcharges." In the present section, we introduce a deposit insurance with reinsurance, and we show that under that

deposit insurance scheme, all equilibria of the model economy support the optimal allocation, even if they may involve bank default.

A deposit insurance with reinsurance works as follows: Banks pay a share δ of their deposits to a deposit insurance fund as a premium. The deposit insurance fund writes reinsurance contracts with households: A household which is willing to pay an amount q to the deposit insurance fund if the bad state occurs at $t = 2$, receives a payment of one from the deposit insurance at time $t = 1$. In equilibrium, the amount q adjusts to clear the market for reinsurance contracts. Thus, the deposit insurance fund collects a total amount of δD from the banks as a premium, passes all these funds on to households under a reinsurance contract, and the households have a payment obligation of δDq in the bad state.[18]

Implicitly, we are assuming away the possibilities that households might strategically default.

In this section, we define a *reinsurance equilibrium.* This equilibrium concept is based on the previously defined equilibrium with banks and δ–deposit insurance, but is adapted to the economy in which the deposit insurance fund contracts reinsurance instead of investing in the FT technology. Our claim is that, for a suitable choice of δ, all reinsurance equilibria support the optimal allocation. More precisely, we claim that optimality is achieved if the deposit insurance premium is determined as follows:

$$\delta_{RIE}(\psi, y_F) = \begin{cases} 0 & \text{if } \psi f'(y_F) \leq (1+\psi)\underline{R}, \\ 1/\psi & \text{if } \psi f'(y_F) > (1+\psi)\underline{R}. \end{cases} \tag{28}$$

We discuss the two cases in turn. In the first case, the amount of deposits in the economy is sufficiently low so that banks will not default in the bad state. Hence, we have trivial deposit insurance with zero premium which never leads to any payment obligation. It is intuitive that the same logic as in Theorem 5 in the Appendix applies: The equilibrium without financial intermediation can be replicated. The interesting case is where the amount of deposits in the economy

[18] We emphasize that household insolvency is not an issue here. In equilibrium, it is ensured that households have sufficient resources to meet their payment obligations under the reinsurance contracts. All that we are implicitly assuming away is the possibility of strategic default by households. In practice, one way to exclude strategic default could be to treat households' payment obligations towards the deposit insurance fund like tax liabilities.

is sufficiently large so that banks default in the bad state. In that case, the deposit insurance receives premium payments $\delta D = (1/\psi)D = E$. That is, every unit of equity is balanced by one reinsurance contract. Next, we consider the portfolio choice problem of the household in the presence of deposit insurance with reinsurance:

$$\max_{\eta\in[0,\,\omega/E],\,\kappa\in\mathbb{R}_+} \left(\frac{\sigma}{1-\theta}\right)\left(\eta E\overline{R}_E + (\omega+\kappa E-\eta E)R_F + \Pi_F\right)^{1-\theta} + \left(\frac{1-\sigma}{1-\theta}\right)\left((\omega+\kappa E-\eta E)R_F + \Pi_F - \kappa E q\right)^{1-\theta}.$$

This portfolio choice problem is best understood when compared to the analogous problem in the previous section. As before, the choice variable η measures the individual household's equity purchases in relation to the equity purchase of the "average" household. Contrary to the previous section, the deposit insurance does not lead to a restitution t_g nor to a loss t_b for all households – participation in the reinsurance contract is a decision of the individual household. Hence, there is a second choice variable κ. While the "average" household receives $\delta D = (1/\psi)D = E$ in the first period under the reinsurance contract, the individual household receives κE. The amount which the individual household can invest in equity or in risk–free assets is thus $\omega + \kappa E$ rather than just ω. In the bad state, however, the household is obliged to pay $\kappa E q$. The profit Π_F remains a fixed effect which is paid to every household in both states, regardless of the household's individual choices. Now let us consider the first–order conditions emanating from the above portfolio problem:

$$\left(\frac{\eta E\overline{R}_E + (\omega+\kappa E-\eta E)R_F + \Pi_F}{(\omega+\kappa E-\eta E)R_F + \Pi_F - \kappa E q}\right)^{\theta} = \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\overline{R}_E - R_F}{R_F}\right), \tag{29}$$

$$\left(\frac{\eta E\overline{R}_E + (\omega+\kappa E-\eta E)R_F + \Pi_F}{(\omega+\kappa E-\eta E)R_F + \Pi_F - \kappa E q}\right)^{\theta} = \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{R_F}{R_F - q}\right). \tag{30}$$

These portfolio conditions admit multiple optimal portfolios for an individual household. The model economy includes two states, so two independent assets suffice for market completeness. The reinsurance contracts are therefore a redundant asset. As a result, the individual household is indifferent between a continuum of possible portfolio choices. All these portfolio choices, however, lead to the same consumption bundle. Let us now turn to that consumption bundle by considering

the aggregate consumption demanded by all households. In the good state, we find

$$\begin{aligned} c_g &= E\overline{R}_E + \omega R_F + \Pi_F \\ &= ((1-\delta)D + E)\overline{R} - DR_F + \omega R_F + f(y_F) - y_F R_F \\ &= D\overline{R} + f(y_F) + R_F(y_M - D) \\ &= y_M \overline{R} + f(y_F). \end{aligned}$$

In order to understand this chain of equalities, we need the following considerations: Banks can choose how to invest an amount $(1-\delta)D + E$, and as by Proposition 1, they invest this entire amount in MT. Since only the banks can invest in MT, this implies $(1-\delta)D + E = y_M$. Moreover, since $\delta = 1/\psi = E/D$, we have $y_M = D$. Finally, the above chain of equalities uses the market–clearing conditions for investment good so that $y_M = \omega - y_F$. Using the same considerations, we can also show that

$$c_b = y_M \underline{R} + f(y_F).$$

Analogously to the previous section, the market–clearing condition for investment good combined with appropriate definitions of return on equity and of q implies market–clearing for the consumption good in both states. This observation allows us to rewrite Eqns. (29) and (30) above as

$$\begin{aligned} \left(\frac{c_g}{c_b}\right)^\theta &= \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\overline{R}_E - R_F}{R_F}\right), \\ \left(\frac{c_g}{c_b}\right)^\theta &= \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{R_F}{R_F - q}\right). \end{aligned}$$

We are now ready for the statement of the equilibrium definition.

5.2 Reinsurance equilibrium

Definition 3.
A reinsurance equilibrium *is a tuple* $(c_g^*, c_b^*, y_M^*, y_F^*, E^*, \psi^*, \overline{R}_E^*, \underline{R}_E^*, R_F^*, q^*, \delta^*)$ *which solves the following system of equations:*

$$\left(\frac{c_g}{c_b}\right)^\theta = \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\overline{R}_E - R_F}{R_F - \underline{R}_E}\right), \tag{31}$$

$$c_g = f(y_F) + y_M\overline{R}, \tag{32}$$

$$c_b = f(y_F) + y_M\underline{R}, \tag{33}$$

$$y_F = \omega - y_M, \tag{34}$$

$$y_M = (1+\psi-\delta\psi)E, \tag{35}$$

$$\overline{R}_E = (1+\psi-\delta\psi)\overline{R} - \psi R_F, \tag{36}$$

$$\underline{R}_E = \max\{0, (1+\psi-\delta\psi)\underline{R} - \psi R_F\},, \tag{37}$$

$$R_F = f'(y_F), \tag{38}$$

$$\delta\psi q = -\max\{0, (1+\psi-\delta\psi)\underline{R} - \psi R_F\}, \tag{39}$$

$$\delta = \delta_{RIE}(\psi, y_F), \tag{40}$$

as well as the additional restriction

$$\left(\frac{c_g}{c_b}\right)^\theta = \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\overline{R}_E}{q}\right) \tag{41}$$

in case $\delta\psi q > 0$, *and moreover, satisfies the inequalities* $R_F^* < \sigma\overline{R} + (1-\sigma)\underline{R}$ *and* $E(1+\psi) \leq \omega$.

Eqns. (31) through (37) correspond to the equations in the earlier definition of the equilibrium with banks and δ–deposit insurance. Eqn. (39) simply says that the payment obligation of households under the reinsurance contract corresponds exactly to the amount of the shortfall in the bad state. Eqn. (40) reiterates the above construction of the deposit insurance premium. Finally, Eqn. (41) only becomes relevant if a shortfall does occur in the bad state, and a non–zero deposit insurance is concluded. In that case, the portfolio decision of the household is no longer one–dimensional. Instead, the household can be thought of as choosing two portfolio variables. As before, in equilibrium, the household should have no

incentive to move an infinitesimal amount of its investment from bank equity to bank deposits (or to FT investment). This requirement is already formalized in Eqn. (31) above. In addition, the household should not have an incentive to contract an infinitesimal extra amount of reinsurance and invest the proceeds at time $t = 1$ into bank equity. This requirement is represented by Eqn. (41).

5.3 Optimality of the reinsurance equilibrium

Theorem 3 below is the main result of the present paper. It claims that in an economy where the deposit insurance fund contracts reinsurance rather than invest in risk–free assets, non–optimal allocations can no longer be consistent with equilibrium.

Theorem 3. *All reinsurance equilibria support the optimal allocation.*

The proof of Theorem 3 is given in Appendix B. The existence of a reinsurance equilibrium can be shown by a similar construction as the existence of an equilibrium with banks and δ–deposit insurance which supports the optimal allocation. Moreover, a reinsurance equilibrium with default also exists, but it is unique. That is, only one debt–equity ratio ψ is consistent with such an equilibrium.

Corollary 1. *All reinsurance equilibria with default (if any) involve the same debt–equity ratio.*

The proof of Corollary 1 is given in Appendix B. The uniqueness result can also be derived from the uniqueness of the Arrow-Debreu price system of the form $(1, \widehat{p}_g, \widehat{p}_b)$. In the equilibrium with default, one unit of equity provides $\overline{R}_E^* = \psi \cdot (\overline{R} - R_F)$ units of consumption in the good state and nothing in the bad state. The reinsurance contract delivers q units of consumption in the bad state, where $Eq = \Lambda = D \cdot (R_F - \underline{R})$, hence $q = \psi \cdot (R_F - \underline{R})$. But then $1/\widehat{p}_g = \overline{R}_E^*$ and $1/\widehat{p}_b = q$. Each of these two equations fixes ψ.

As a next step, we show constructively that a reinsurance equilibrium with default exists. Indeed, let a tuple $(c_g^\star, c_b^\star, y_M^\star, y_F^\star, E^\star, \psi^\star, \overline{R}_E^\star, \underline{R}_E^\star, q^\star, \delta^\star)$ be defined as follows:

$$
\begin{aligned}
\psi^\star &= \frac{\widehat{R}_F}{\widehat{R}_F - \underline{R}} + \frac{\widehat{R}_F}{\overline{R} - \widehat{R}_F}, \\
y_F^\star &= \widehat{y}_F, \\
y_M^\star &= \widehat{y}_M, \\
\overline{R}_E^\star &= \psi^\star(\overline{R} - R_F^\star), \\
\underline{R}_E^\star &= 0, \\
q^\star &= \psi^\star(R_F^\star - \underline{R}), \\
E^\star &= y_M^\star/\psi^\star, \\
\delta^\star &= 1/\psi^\star.
\end{aligned}
$$

Proposition 6. *The tuple* $(c_g^\star, c_b^\star, y_M^\star, y_F^\star, E^\star, \psi^\star, \overline{R}_E^\star, \underline{R}_E^\star, q^\star, \delta^\star)$ *is a reinsurance equilibrium.*

The proof of Proposition 6 is given in Appendix B.

Theorem 4. *A reinsurance equilibrium with default exists.*

Several remarks are in order. First, all reinsurance equilibria involve the optimal allocation, even if banks default. The reason is that neither in the good nor in the bad state, portfolio decisions of households are impacted by transfers from governments — refunds from the deposit insurance fund or taxes to bail out banks. Second, as regards the constructed excessive risk taking: While bank shareholders still would like to leverage as much as possible - and to benefit from limited liability - the banking system receives the socially optimal aggregate amount of investment goods in the form of deposits and equity and the banking system cannot increase the scale further.

Third, while a continuum of possible capital structures and associated non–optimal equilibria with default deposit insurance exist, only one equilibrium capital structure under reinsurance exists in which banks default. The reason is as follows. Since all reinsurance equilibria involve the optimal allocation, the amount of risky assets for households is equal to the amount in the Arrow-Debreu world. A particular amount of bank equity contracts and thus a particular capital structure

requires a particular combination of deposit insurance and reinsurance contracts to avoid government bail-out. Only for one particular capital structure will the ensuing portfolio of risky assets (bank equity and reinsurance contracts) mimic the amount of risky assets in the Arrow-Debreu setting. Thus, the equilibrium capital structure in the reinsurance equilibrium with default is unique.

Fourth, we have assumed that households honor their obligations when they (freely) choose to acquire reinsurance contracts. One might be concerned about strategic default of households in this context. This is no concern in our representative agent set-up, as households never have an incentive to default since they would be pushed to a minimal consumption level if they chose to go bankrupt.[19]

Fifth, one may wonder how our main results would carry over to a model with an arbitrary (finite) number of states of nature. Indeed, the existence of non-optimal equilibrium allocations requires only that there is (at least) one state in which the cover ratio of the deposit insurance is strictly less than one. Suppose that there are n states of nature, and the cover ratio of the deposit insurance fund is strictly less than one in $m < n$ of those states. Then, we can guarantee that the optimal allocation obtains in any equilibrium by allowing for a set of m independent reinsurance contracts.

6 Conclusion and Extensions

We have performed a simple general equilibrium analysis of deposit insurance and have suggested that a combination of deposit insurance and reinsurance will avoid non–optimal equilibria. Indeed only a judicious combination of deposit insurance and reinsurance promises that societies can avoid distortions associated with insured deposits.

We have derived two main results: First, deposit insurance in itself does not guarantee optimal equilibrium allocations, not even with very high premia. Second, deposit insurance cum reinsurance does guarantee that the optimal allocation is achieved in equilibrium. It is useful to stress that the main logic behind both of these results would apply in many other general equilibrium models with banks.

[19] With heterogeneous households – rich and poor – such concerns are accurate and may require some wealth or collateral thresholds for households to qualify for the acquisition of reinsurance contracts.

Deposit insurance cannot avoid that portfolio decisions of households are distorted regarding the socially optimal mix of risky and safe assets since either it involves taxation in bad states to cover bail-out costs or additional transfers in good states since the insurance fund was not needed.

A judicious combination of deposit insurance and reinsurance avoids both taxation and transfers from the government since the amount of reinsurance contracts is linked to the capital structures of banks (and deposit insurance scheme). This, in turn, pins down the equilibrium capital structure of banks. This logic could be applied to any other general equilibrium model with banks and aggregate risk.

The model allows some important extensions. One could, for instance, introduce costs of monitoring or moral hazard at the level of bank managers. In the former case, the analysis carries over readily to economies in which monitoring is costly and costs per loan are constant. In the latter case, in the spirit of Holmström and Tirole (1997), the bank manager must receive an incentive pay that motivates him to monitor firms. Then, one can only achieve a second–best allocation. Given this second–best allocation, one can perform the parallel exercise and a judicious combination of deposit insurance and reinsurance implements the second–best allocation.[20]

Of course, numerous other extensions deserve further scrutiny. For instance, the reinsurance scheme can be viewed as a form of catastrophe bond. Since the risk of banking crises is notoriously difficult to assess, reinsurance of deposit insurance might need professional expertise. One might thus ask whether the same role for reinsurance as in our main theorem could be performed by reinsurance companies which households finance by equity contracts. One might also consider circumstances when there is ambiguity about the occurrence of banking crises and how reinsurance has to be designed in such circumstances. These and other extensions will further enrich the socially valuable dual role that deposit insurance and reinsurance can have in insuring the financial system.

[20] Details are available upon request.

Appendix A: Equilibria Without Financial Intermediation

An equilibrium in this economy is defined as follows:

Definition 4.
An equilibrium without financial intermediation *is a tuple* $(p_g^*, p_b^*, c_g^*, c_b^*, y_F^*, y_M^*, \Pi_F^*) \gg 0$[21] *which satisfies the following system of equations:*

$$\left(\frac{c_g}{c_b}\right)^{\theta} = \left(\frac{p_b}{p_g}\right)\left(\frac{\sigma}{1-\sigma}\right), \tag{42}$$

$$\omega = p_g c_g + p_b c_b - \Pi_F, \tag{43}$$

$$\Pi_F = (p_g + p_b) f(y_F) - y_F, \tag{44}$$

$$f'(y_F) = (p_g + p_b)^{-1}, \tag{45}$$

$$y_M = \omega - y_F, \tag{46}$$

$$c_g = f(y_F) + y_M \overline{R}, \tag{47}$$

$$c_b = f(y_F) + y_M \underline{R}. \tag{48}$$

The first equation is the optimal ratio of consumptions in both states for the household and thus represents the maximization of expected household utility. The second equation is the household's budget constraint, taking into account that the profits of FT firms are distributed to households, while MT firms make zero profits. The third equation specifies the FT profits, and the fourth equation is the condition for optimal producer choice. The remaining equations are standard market–clearing conditions for investment good, consumption good in the good state, and consumption good in the bad state, respectively.

Substitute the expressions for c_g^*, c_b^*, and Π_F^* into the budget constraint to find

$$\omega = y_F^* + y_M^* \cdot (p_g \overline{R} + p_b \underline{R}).$$

[21] Throughout the paper we use the vector notation $v_1 \gg v_2$ if vector v_1 is strictly greater in all components than vector v_2, and $v_1 > v_2$ if v_1 is weakly greater in all components than v_2 with at least one strict inequality.

Using the fact that $\omega = y_F^* + y_M^*$, we see that the condition

$$p_g \overline{R} + p_b \underline{R} = 1$$

emanating from optimal producer choice is implied by the system of equations in the definition of an equilibrium without financial intermediation.

The equilibrium allocation

A first welfare theorem

In this subsection, we characterize the optimal allocation of the investment good to the two sectors of production. We mean by an *optimal allocation* an allocation that maximizes household utility. Because of the CRRA utility function, the optimal allocation does not coincide with the *productively efficient* allocation which maximizes total expected output in the economy. More formally, we give the following definition of the optimal allocation.

Definition 5.
The input allocation $(\widehat{y}_F, \omega - \widehat{y}_F)$ *is* optimal *if it maximizes the household's utility*

$$\left(\frac{\sigma}{1-\theta}\right)\left(f(y_F) + (\omega - y_F)\overline{R}\right)^{1-\theta} + \left(\frac{1-\sigma}{1-\theta}\right)\left(f(y_F) + (\omega - y_F)\underline{R}\right)^{1-\theta}$$

over $y_F \in [0, \omega]$.

The economy without financial intermediation was previously introduced in Gersbach et al. (2015). It has been shown that the optimal allocation exists, is unique, and allocates strictly positive amounts of the investment good to both sectors. Moreover, the concomitant Arrow–Debreu equilibrium is unique up to price normalization. For later reference, we denote the Arrow–Debreu equilibrium values by $\widehat{p}_g$, $\widehat{p}_b$, $\widehat{c}_g$, $\widehat{c}_b$, $\widehat{y}_F$, $\widehat{y}_M$ and $\widehat{\Pi}_F$.

Examination of the first–order condition emanating from the optimization problem in the above definition reveals that the optimal allocation $\widehat{y}_F$ satisfies

$$\left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\overline{R} - f'(\widehat{y}_F)}{f'(\widehat{y}_F) - \underline{R}}\right) = \left(\frac{f(\widehat{y}_F) + (\omega - \widehat{y}_F)\overline{R}}{f(\widehat{y}_F) + (\omega - \widehat{y}_F)\underline{R}}\right)^{\theta}. \tag{49}$$

The following proposition is the manifestation of the first welfare theorem in the model at hand.

Proposition 7. *An equilibrium without financial intermediation involves the optimal allocation.*

Proof. In an equilibrium without financial intermediation, the market–clearing conditions imply that the right–hand side of Eqn. (49) is equal to $(c_g/c_b)^\theta$. Moreover, in such an equilibrium, the conditions for optimal producer choice imply that

$$p_g\overline{R} + p_b\underline{R} = (p_g + p_b)f'(\widehat{y}_F) = 1.$$

We can use this expression to show that

$$\left(\frac{\overline{R} - f'(\widehat{y}_F)}{f'(\widehat{y}_F) - \underline{R}}\right) = p_b/p_g.$$

We see that the first–order condition for the optimal allocation and the equilibrium condition for optimal consumer choice coincide. □

From the first–order condition describing the optimal allocation and from the inequalities $\omega > \widehat{y}_F$ and $\overline{R} > \underline{R}$, it can be inferred that if $(\widehat{y}_F, \omega - \widehat{y}_F)$ is the optimal allocation, then

$$f'(\widehat{y}_F) < \sigma\overline{R} + (1-\sigma)\underline{R}.$$

Due to the concave technology in FT, the marginal product in the FT sector is lower than the marginal product in the MT sector at the optimal allocation. Due to risk aversion, it is not optimal for households to equalize the marginal products in both sectors. Hence, as pointed out before, the optimal allocation is not the "productively efficient" allocation.

Reformulation of equilibrium

It will be useful to work with a reformulation of the equilibrium without financial intermediation that does not involve the prices (p_g, p_b) anymore. In order to achieve this reformulation, we start by considering the optimization problem of

an individual household, taking choices of all other households as given. The individual household's optimization problem is "one–dimensional": The household chooses a share $\beta \in [0,1]$ of its initial endowment to be invested in the risk–free asset (that is, in FT production), and the complementary share $1-\beta$ to be invested in the risky asset (that is, MT production). Since the individual household has zero mass, it takes the allocation y_F and the concomitant marginal product $f'(y_F)$ as given. This is in contrast to the previous subsection where we considered the optimization of the entire allocation in the economy. We can express the individual household utility as a function of β:

$$U(\beta) = \left(\frac{\sigma}{1-\theta}\right)\left(c_g^{1-\theta}(\beta)\right) + \left(\frac{1-\sigma}{1-\theta}\right)\left(c_b^{1-\theta}(\beta)\right),$$

which leads us to the first–order condition

$$\left(\frac{c_g}{c_b}\right)^{\theta} = -\left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\partial c_g/\partial\beta}{\partial c_b/\partial\beta}\right). \tag{50}$$

Equation (50) gives a general first–order condition for a one–dimensional consumer choice problem which will repeatedly be useful in the sequel. In the case at hand,

$$\begin{aligned} c_g(\beta) &= \beta(\omega+\Pi_F)f'(y_F) + (1-\beta)(\omega+\Pi_F)\overline{R}, \\ c_b(\beta) &= \beta(\omega+\Pi_F)f'(y_F) + (1-\beta)(\omega+\Pi_F)\underline{R}, \end{aligned}$$

and thus Equation (50) becomes

$$\left(\frac{c_g}{c_b}\right)^{\theta} = \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\overline{R}-f'(y_F)}{f'(y_F)-\underline{R}}\right).$$

Consider the following theorem.

Theorem 5.
The tuple $(c_g^*, c_b^*, y_F^*, y_M^*) \gg 0$ *is part of an equilibrium without financial interme-*

diation if it solves the following system of equations:

$$
\begin{aligned}
y_M &= \omega - y_F, && (51)\\
c_g &= f(y_F) + y_M \overline{R}, && (52)\\
c_b &= f(y_F) + y_M \underline{R}, && (53)\\
\left(\frac{c_g}{c_b}\right)^{\theta} &= \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\overline{R} - f'(y_F)}{f'(y_F) - \underline{R}}\right). && (54)
\end{aligned}
$$

Proof. In order to prove the theorem, we need to show that for any tuple $(c_g^*, c_b^*, y_F^*, y_M^*) \gg 0$ which satisfies Eqns. (52)–(54), one can find prices (p_g^*, p_b^*) such that $(c_g^*, c_b^*, y_F^*, y_M^*, p_g^*, p_b^*)$ is an equilibrium without financial intermediation. Indeed, let prices (p_g^*, p_b^*) be given by the equalities

$$p_b^*/p_g^* = \frac{\overline{R} - f'(y_F^*)}{f'(y_F^*) - \underline{R}}$$

and

$$(p_g^* + p_b^*)^{-1} = f'(y_F^*).$$

It is now easily verified that all the equations in the definition of an equilibrium without financial intermediation are satisfied. □

Appendix B: Proofs

Proof of Lemma 1

Consider first the case where $\overline{\Pi}(\alpha) > 0 = \underline{\Pi}(\alpha)$. Then,

$$\partial\left(\sigma\overline{\Pi}(\alpha) + (1-\sigma)\underline{\Pi}(\alpha)\right)/\partial\alpha = \sigma(\overline{R} - R_F)((1-\delta)D + E).$$

This partial derivative is independent of α. The supposition $R_F < \sigma\overline{R} + (1-\sigma)\underline{R}$ implies that $\overline{R} - R_F > 0$, and thus the above partial derivative is strictly positive, as desired. Now consider the case where $\overline{\Pi}(\alpha) > \underline{\Pi}(\alpha) > 0$. Then, the relevant partial derivative is

$$\partial\left(\sigma\overline{\Pi}(\alpha) + (1-\sigma)\underline{\Pi}(\alpha)\right)/\partial\alpha = \sigma(\overline{R}-R_F)((1-\delta)D+E)+(1-\sigma)(\underline{R}-R_F)((1-\delta)D+E).$$

Again, invoking the supposition that $R_F < \sigma\overline{R} + (1-\sigma)\underline{R}$ shows that this partial derivative is strictly positive. Indeed, the optimization problem of the representative bank has no interior solution. □

Proof of Proposition 2

The proof is constructive. Let $(c_g^*, c_b^*, y_M^*, y_F^*)$ be the optimal input allocation and concomitant consumptions, and choose any $\psi^* \in [0, \frac{\underline{R}}{R_F^* - \underline{R}}]$. Notice that any choice of ψ^* in that interval guarantees that $\underline{R}_E \geq 0$. Now define $E^*, \overline{R}_E^*$, and $\underline{R}_E^*$ as follows:

$$\begin{aligned} E^* &= \frac{y_M^*}{1+\psi^*}, \\ \overline{R}_E^* &= (1+\psi^*)\overline{R} - \psi^* R_F^*, \\ \underline{R}_E^* &= (1+\psi^*)\underline{R} - \psi^* R_F^*. \end{aligned}$$

We need to check that the tuple $(c_g^*, c_b^*, y_M^*, y_F^*, \psi^*, E^*, \overline{R}_E^*, \underline{R}_E^*)$ solves Eqns. (12) through (18) in the definition of the equilibrium with banks and δ–deposit insurance. This is immediate for Eqns. (13) through (18). To see that it is also true for Eqn. (12), observe that with 0–deposit insurance,

$$\frac{\overline{R}_E - R_F}{R_F - \underline{R}_E} = \frac{(1+\psi)\overline{R} - (1+\psi)R_F}{(1+\psi)R_F - (1+\psi)\underline{R}} = \frac{\overline{R} - R_F}{R_F - \underline{R}}.$$

□

Proof of Proposition 5

Take the critical leverage equilibrium introduced in Definition 2 where investment in FT is y_F^*, investment in the risky sector is $y_M^* = \omega - y_F^*$, deposits assume the threshold value $D^* = \dfrac{\underline{R}}{R_F^*} y_M^*$, equity assumes the value $E^* = \left(1 - \dfrac{\underline{R}}{R_F^*}\right) y_M^*$ and the equilibrium bond return R_F^* satisfies $\sigma\overline{R} + (1-\sigma)\underline{R} > R_F^*$. Similar to the earlier proof, let us fix a bond return (denoted by $\check{R}_F$) slightly above R_F^* such that $\sigma\overline{R} + (1-\sigma)\underline{R} > \check{R}_F$ and the bank chooses $\alpha = 1$. Given the higher bond return $\check{R}_F$, the representative FT firm chooses a profit maximizing input denoted by $\check{y}_F$, with $\check{y}_F < y_F^*$. The resulting profit is denoted by $\check{\Pi}_F$ and satisfies $\check{\Pi}_F < \Pi_F^*$.

At the critical leverage equilibrium, the demand for equity is E^* when $t_b = 0$ and $\delta = 0$, the return on bonds is R_F^* and a unit of equity pays $\overline{R}(1 + \frac{D^*}{E^*}) - R_F^* \frac{D^*}{E^*}$ in the good state and zero in the bad state. If one replaced R_F^* by $\check{R}_F > R_F^*$, Π_F^* by $\check{\Pi}_F$, $\delta = 0$ by the actuarially fair rate given by (9) when $\mu = 1 - \sigma$, $E = E^*$ and $D = D^*$, and $t_b = 0$ by $\check{t}_b = (1-\delta)\check{R}_F D^* - \underline{R}(E^* + (1-\delta)D^*) > 0$, then the household would demand more of the risk-free asset.[22]

Now assume $E \in (0, E^*]$ and $D = \mathcal{D}_{1-\sigma}(E)$ given by (11). Consider the household's portfolio choice when the profit distributed is $\check{\Pi}_F$, δ is given by (9) with $\mu = 1-\sigma$, $t_b = (1-\delta)\check{R}_F D - \underline{R}(E + (1-\delta)D)$, the return on bonds is $\check{R}_F$ and a unit of equity pays $\overline{R}(1 + \frac{(1-\delta)D}{E}) - \check{R}_F \frac{D}{E}$ in the good state and zero in the bad state. There is a unique optimal $\gamma(E) \in [0,1]$ so that the household invests $\gamma(E)\omega$ in bonds and $[1 - \gamma(E)]\omega$ in equity and deposits. By Berge's maximum theorem (or the explicit solution of the household's portfolio choice problem), $\gamma(E)$ is a continuous function of E. Set $\eta(E) = [1 - \gamma(E)]\omega$. As reasoned above, $\eta(E^*) < E^*$. Like in the proof of Proposition 6 of Gersbach et al. (2015), the examination of the solution of the household's portfolio choice problem shows existence of $E_o \in (0, E^*)$ with

[22] Observe first of all that $c_g > c_b$ and homothetic preferences of the household (together with standard properties) imply that $|MRS|$ is smaller at the consumption bundle $(\check{c}_g, \check{c}_b) = (c_g - (\Pi_F - \check{\Pi}_F) + \delta D^* \check{R}_F, c_b - (\Pi_F - \check{\Pi}_F) - \check{t}_b)$ than at (c_g, c_b). Next consider normalized gradients of the form $(|MRS|, 1)$. Denote by ∇ the household's normalized gradient at (c_g, c_b) and by $\check{\nabla}$ its normalized gradient at $(\check{c}_g, \check{c}_b)$. If in the reference equilibrium situation, the household replaces one unit of the bond by one unit of equity, then consumption is changed in the direction $\upsilon = (\overline{R}(1 + \frac{D^*}{E^*}) - R_F \frac{D^*}{E^*} - R_F, -R_F)$ and at equilibrium, portfolio choice is optimal, that is $\nabla \cdot \upsilon = 0$. If in the new situation, the household replaces one unit of the bond by one unit of equity, then consumption is changed in the direction $\check{\upsilon} = (\overline{R}(1 + \frac{D^*}{E^*}) - \check{R}_F \frac{D^*}{E^*} - \check{R}_f, -\check{R}_F)$. It follows that $0 = \nabla \cdot \upsilon > \nabla \cdot \check{\upsilon} > \check{\nabla} \cdot \check{\upsilon}$. But $\check{\nabla} \cdot \check{\upsilon} < 0$ means that the household benefits from reducing its equity holding and increasing its bond holding by the same amount.

$\eta(E_o) > E_o$. By the intermediate value theorem, there exists $E \in (E_o, E^*)$ with $\eta(E) = E$. At this E and the corresponding values for D, δ and t_b, the asset market is cleared — as well as the consumption good market in both states — while the bond return is $\check{R}_F$ and FT production is less than at the Arrow-Debreu equilibrium. Hence the equilibrium allocation is non–optimal. By 3.5, bail-out is necessary if the bank defaults.

It remains to check whether the bank is actually going to default in the bad state. In the reference equilibrium, $\underline{R}(E^* + D^*) - R_F D^* = 0$. Let $\Delta = y_F^* - \check{y}_F > 0$. Then $\underline{R}(E^* + D^* + \Delta) - \check{R}_F(D^* + \Delta) < 0$. Further $E^* + D^* = \omega - y_F^*$, $E + D(1-\delta) = \omega - \check{y}_F$ and $E < E^*$. Hence $E^* + D^* + \Delta = \omega - \check{y}_F = E + D(1-\delta)$ and $D^* + \Delta = \omega - E^* - y_F^* + y_F^* - \check{y}_F = \omega - E^* - \check{y}_F < \omega - E - \check{y}_F = (1-\delta)D < D$. It follows that $\underline{R}(E + (1-\delta)D) - \check{R}_F D < \underline{R}(E^* + D^* + \Delta) - \check{R}_F(D^* + \Delta) < 0$ which means that the bank is going to default in the bad state, indeed. □

Proof of Proposition 6

It is immediate that our construction satisfies Eqns. (34) through (40). Now consider Eqn. (41). By substitution from the above construction of $c_g^\star, c_b^\star, \overline{R}_E^\star$ and $q^\star$, we can reduce Eqn. (41) to

$$\left(\frac{f(\widehat{y}_F) + \widehat{y}_M \overline{R}}{f(\widehat{y}_F) + \widehat{y}_M \underline{R}}\right)^\theta = \left(\frac{\sigma}{1-\sigma}\right)\left(\frac{\overline{R} - \widehat{R}_F}{\widehat{R}_F - \underline{R}}\right).$$

This equality is true due to the definition of $\widehat{y}_F$ and $\widehat{y}_M$. Indeed, our construction satisfies Eqn. (41). Finally, we have to verify that our construction also satisfies Eqns. (31)–(33). Given the previous step, it is sufficient to show that the following equality is satisfied:

$$\frac{\psi^\star \overline{R} - (1+\psi^\star)\widehat{R}_F}{\widehat{R}_F} = \frac{\overline{R} - \widehat{R}_F}{\widehat{R}_F - \underline{R}}.$$

Solving this expression for $\psi^\star$, we see that it is simply equivalent to the definition of $\psi^\star$. Indeed, we have now shown that our construction satisfies all the Eqns. (31) through (41). □

Proof of Theorem 2

Step 1. Fix $y'_F \in Y'_F$ and $\delta' < \frac{\overline{R}-f'(y'_F)}{\overline{R}}$. Consider $\overline{R}_E$ as a function of ψ, thus

$$\begin{aligned}\overline{R}_E(\psi) &= (1+\psi-\delta'\psi)\overline{R} - \psi f'(y'_F) \\ &= \overline{R} + \psi[(1-\delta')\overline{R} - f'(y'_F)].\end{aligned}$$

Observe that the term in brackets is strictly positive, hence we find the limit behavior $\lim_{\psi\to 0}\overline{R}_E(\psi) = \overline{R}$ and $\lim_{\psi\to\infty}\overline{R}_E(\psi) = \infty$. Since $\overline{R}_E(\psi)$ is continuous on $\mathbb{R}_+$, we can invoke the intermediate value theorem to find that for any $\rho > \overline{R}$, there exists $\psi > 0$ such that $\overline{R}_E(\psi) = \rho$.

Step 2. Now let us define the function

$$\rho(y_F) = \left(\frac{f(y_F)+(\omega-y_F)\overline{R}}{f(y_F)+(\omega-y_F)\underline{R}}\right)^{\theta}\left(\frac{1-\sigma}{\sigma}\right)f'(y_F) + f'(y_F).$$

Observe that ρ is continuous and strictly decreasing in y_F. Thus for y'_F we have $\rho' := \rho(y'_F) > \rho(\widehat{y}_F) \geq \widehat{R}$.[23] We have now established that there is ψ' such that $\overline{R}_E(\psi') = \rho'$.

Step 3. Consider the tuple $(c'_g, c'_b, y'_M, y'_F, E', \psi', \overline{R}'_E, \underline{R}'_E)$, where y'_F is as fixed in Step 1, ψ' is as defined in Step 2, $\overline{R}'_E = \overline{R}_E(\psi')$, and the remaining variables are

[23] Regarding the latter inequality: By (20), the critical leverage equilibrium with 0–deposit insurance yields $\rho(\hat{y}_F) = \overline{R}^*_E$. By (25) and (27),

$$\begin{aligned}\overline{R}^*_E &= \frac{1}{f'(\hat{y}_F)-\underline{R}}\cdot\{[\underline{R}-\underline{R}+f'(\hat{y}_F)]\cdot\overline{R} - \underline{R}\cdot f'(\hat{y}_F)\} \\ &= \frac{(\overline{R}-\underline{R})\cdot f'(\hat{y}_F)}{f'(\hat{y}_F)-\underline{R}} \geq \overline{R}\end{aligned}$$

where the last inequality follows from $\overline{R} > f'(\hat{y}_F) > \underline{R} \geq 0$. Hence $\rho(\hat{y}_F) \geq \overline{R}$. Actually, $>$ holds for $\underline{R} > 0$ and $=$ holds for $\underline{R} = 0$.

given by:

$$\begin{aligned} y'_M &= \omega - y'_F, \\ E' &= y'_M/(1+\psi'-\delta'\psi'), \\ c'_g &= f(y'_F) + y'_M\overline{R}, \\ c'_b &= f(y'_F) + y'_M\underline{R}. \end{aligned}$$

It is now easily verified that the tuple under consideration satisfies Eqns. (12)–(18). We note that $\underline{R}_E = 0$ since $y'_F \in Y'_F$ implies that $\psi' > \underline{R}/\left(R_F - (1-\delta)\underline{R}\right)$ and thus, $\underline{R}+\psi'[(1-\delta')\underline{R}-f'(y'_F)] < 0$. Moreover, since (15) and (16) are satisfied, we note that even in the extreme case with households investing almost everything in deposits, $y'_F \geq \delta D'$ still holds.

Hence, the tuple constructed is indeed an equilibrium with banks and δ'–deposit insurance, as desired.

□

Proof of Theorem 3

Comparing the system of Eqns. (31)-(41) above with the system of Eqns. (51)-(54) describing the optimal allocation, we see that a reinsurance equilibrium supports the optimal allocation if the equilibrium variables $\overline{R}^*_E$ and y^*_F satisfy

$$\frac{\overline{R}^*_E - R^*_F}{R^*_F - \underline{R}^*_E} = \frac{\overline{R} - R^*_F}{R^*_F - \underline{R}}.$$

In order to verify that this equality holds in any reinsurance equilibrium, we distinguish two cases. The first case is a reinsurance equilibrium in which $\psi^* R^*_F \leq (1+\psi^*)\underline{R}$. In such an equilibrium, $\delta^* = 0$. Then, substituting for the returns on equity from Eqns. (36) and (37), we have

$$\frac{\overline{R}^*_E - R^*_F}{R^*_F - \underline{R}^*_E} = \frac{(1+\psi^*)\overline{R}^* - (1+\psi^*)R^*_F}{(1+\psi^*)R^*_F - (1+\psi^*)\underline{R}} = \frac{\overline{R} - R^*_F}{R^*_F - \underline{R}},$$

as desired. The second case is that of a reinsurance equilibrium in which $\psi^* R^*_F > (1+\psi^*)\underline{R}$. In such an equilibrium, $\delta^* = 1/\psi^*$ and $\delta^*\psi^* q^* = q^* > 0$. Thus, by applying Eqn. (41) and by substitution for $\overline{R}^*_E$ and q^* from Eqns. (36) and (39),

we obtain the chain of equalities

$$\frac{\overline{R}_E^* - R_F^*}{R_F^* - \underline{R}_E^*} = \frac{\overline{R}_E^*}{q} = \frac{\psi^* \overline{R} - \psi^* R_F^*}{\psi^* R_F^* - \psi^* \underline{R}} = \frac{\overline{R} - R_F^*}{R_F^* - \underline{R}}.$$

Indeed, all reinsurance equilibria support the optimal allocation. □

Proof of Corollary 1

Consider a reinsurance equilibrium with default, and let $\overline{R}_E^*$ be the return on equity in the good state and ψ^* be the debt–equity ratio in that equilibrium. From the previous theorem, it follows that

$$\frac{\overline{R}_E^* - \widehat{R}_F}{\widehat{R}_F} = \frac{\overline{R} - \widehat{R}_F}{\widehat{R}_F - \underline{R}}.$$

Moreover, in a reinsurance equilibrium with default, Eqn. (36) reduces to

$$\overline{R}_E^* = \psi^*(\overline{R} - \widehat{R}_F).$$

Substituting this expression for $\overline{R}_E^*$ in the previous equation, and suitably rearranging terms yields

$$\psi^* = \frac{\widehat{R}_F}{\widehat{R}_F - \underline{R}} + \frac{\widehat{R}_F}{\overline{R} - \widehat{R}_F}.$$

□

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