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Abstract

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JEL Classification: F1, L1, L2

Keywords: Carry-Along Trade, multi-product firms, Mode of Exporting, Collusion

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Disclaimer: No animals were harmed in the production of this paper!

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1 Introduction

The traditional view of how goods get exported has been that firms produce and export their own products. The availability of better models and better data regarding exporting firm behavior has led to a great deal of new work studying precisely how firms export. Much of this work has focused on the use of intermediaries to assist firms in their export activities. In recent work by Bernard et al. (2016) they identify and document, using Belgian data, a different form of intermediation called carry-along trade (CAT). CAT occurs when Belgian firms export products produced by other Belgian firms in addition to their own products. The difference between CAT and typical intermediation in export markets is that CAT firms produce and export their own goods in addition to exporting goods produced by other firms. Bernard et al. (2016, p. 30) show that CAT is quantitatively important: "More than 90 percent of manufacturing exporters ship at least one product that they do not make and these products represent 30 percent of export value." This finding has been replicated recently for Denmark (Abreha, Smeets and Warzynski, 2013), Italy (Di Nino, 2015), Sweden (Arnarson, 2016), and France (see Bernard et al., 2016, fn 4).

Bernard et al. (2016) think of CAT as the outcome of a "make or source" decision by multi-product firms. In their paper, firms decide on the optimal scope of products to sell in a particular market, and then determine whether these products are produced in-house or sourced from independent suppliers. In that sense, CAT allows firms to add products to their product range at lower marginal costs than if they were produced in-house. The key focus of their analysis is to rationalize an important feature in the data, namely that both the number of products and the share of CAT products in total exported products are positively correlated to firm productivity. They show that certain demand and supply-side features are necessary to produce the complementarity between firm productivity and sourcing found in the data.

In this paper, we extend their view on CAT by explicitly considering the choices of the producer of the CAT product and the strategic interactions between this producer and the CAT firm. In particular, the producer of the CAT product has a choice: It can either choose to have its product carried-along by the other firm (CAT case), or it can choose to "deliver" (i.e. export) its own goods (Delivery of Own Goods, DOG case). This DOG case creates an outside option that plays an important role in the decision on the mode of exporting. We show that the CAT versus DOG decision comes down to three factors, demand linkages, transportation cost synergies, and the relative productivities of the two firms, and that these determinants are consistent with recent empirical evidence on CAT.

We also show that CAT has a significant effect on the intensive and extensive margins of trade, and the prices of the products traded. More specifically, by comparing DOG to CAT, we find that CAT tends to increase exports and lower prices when the two goods are complements and tends to reduce the quantity of exports and increase prices when goods are substitutes. Therefore, the mode of exporting has an impact on how rents are shifted between firms and consumers. In the case of substitutes, CAT produces the same outcome as product-specific, market-specific collusion.

Our paper is related to a growing literature on intermediation in global markets. Rauch and Watson (2004) use a network approach and focus on the problem of matching buyers and sellers in the export market. They develop a model to determine which firms decide to become intermediaries and whether the amount of intermediation in the market is sufficient. Blum, Claro and Horstmann (2010) document facts regarding intermediation. They find that small exporters tend to match with large importers. Import intermediaries tend to become large by matching with large sellers and buying few products. Large intermediaries are the source of imports for both small countries and low value products. Ahn, Khandelwal and Wei (2011) argue that intermediaries are an important aspect of exporting. They develop and test a model in which firms chose their mode of export. They can export directly or through an intermediary. They show that less productive firms tend to use intermediaries more and that markets that have higher export costs have more exports handled by intermediaries.

Intermediation in international trade is the focus of an interesting paper by Antràs and Costinot (2011). They consider two ways that intermediation can become "globalized." They first look at what they call W-integration in which there is one centralized market where all of the world's trades take place. In the second type of integration, M-integration, intermediaries can locate in foreign markets if they chose, but there is no direct integration of markets. This is analogous to the difference between integrated and segmented markets. They show that the results of these different types of intermediation vary and in fact, M-integration can lower welfare in the smaller country. Akerman (2016) builds a model of intermediation based on firm heterogeneity. In his model the most productive firms export without using an intermediary and the least productive firms do not export. There exists a range of firms with intermediate productivity that use intermediaries to export. He derives a number of predictions that are then tested using Swedish export data.

This paper is also related to recent work on multi-product firms. Bernard, Jensen, Redding, and Schott (2007) show that "...engaging in international trade is an exceedingly rare activity: of the 5.5 million firms operating in the United States in 2000, just 4 percent were exporters. Among these exporting firms, the top 10 percent accounted for 96 percent of total U.S. exports....In 2000, 42.2 percent of exporting firms exported a single product abroad ...these exporters represented a small share of aggregate exports, just 0.4 percent...Firms exporting five or more products accounted for 25.9 percent of firms but 98 percent of export value." This illustrates the importance of multi-product firms in exporting activities. Bernard et al. (2016) illustrate in Figure 1 of their paper (p. 6-7) that "Except for the category of single-product exporters, firms in every other category report greater numbers of products exported than products produced. Multi-product exporters are also multi-product domestic producers but the number of exported products increases much more rapidly than the number of produced products." Our paper is related to the multi-product literature because CAT allows firms to internalize demand linkages, like in recent multi-product models by Eckel and Neary (2010) and Dhingra (2013).

We begin in section 2 by developing the model, section 3 solves for equilibrium, we characterize when CAT occurs in section 4, the trade effects of CAT are determined in section 5 and section 6 looks at price effects of CAT and section 7 concludes.

2 The Model

There are two products produced by domestic firms. Firm 1 produces product 1, and firm 2 produces product 2. For simplicity, we focus on the export market. The inverse demand functions for these two products in the export destination are given by

$$p_i = a_i - bq_i - b\theta q_j \tag{1}$$

where $i, j \in \{1, 2\}$ and $j \neq i$. The parameter $\theta \in [-1, 1]$ represents the degree of product differentiation between the two products. If the two goods are perfect substitutes (i.e. homogeneous goods) then $\theta = 1$. If the goods are perfectly differentiated then $\theta = 0$ indicating that the demand for the two products is completely independent. Finally, if $-1 \leq \theta < 0$ this indicates that the products are complements.

The parameter b is the slope of a product's residual demand and is identical across products. It can be interpreted as an inverse measure of the size of the destination market for the two products. The parameter a_i is a product specific parameter that can be interpreted as a measure for the quality of the respective product. A higher value of a_i indicates a higher quality and shifts a product's demand outwards.

To simplify the exposition, we assume that Firm 1 always transports it's own good, but can possibly transport firm 2's goods, too. We will call Firm 1 the CAT firm. Firm 2 decides whether to transport it's own goods or have Firm 1 "carry along" its exports.

The marginal costs of selling these two products in the destination market consist of two elements: Marginal production costs and per unit transportation costs. Marginal production costs are constant and product specific: c_i $(i = 1, 2)$.

Transportation costs consist of an exogenous, product specific component $\bar{t}_i$, and a variable component that can be reduced through investment. We think of these investments as investment into a firm's specific transportation infrastructure, such as building a logistics center or funding R&D on optimal packaging.

In the DOG case, where each firm sells its own product, transportation costs t_i for product i are given by

$$t_i = \bar{t}_i - 2\left(k_i\right)^{\frac{1}{2}}, \tag{2}$$

where k_i is the endogenous investment of firm i in the transportation technology of its own product.

In the CAT case, Firm 2 does not transport its own product and, thus, does not face any transportation costs. Instead, Firm 1 transports and sells both products. In this case, transportation costs for product 1 (by Firm 1, denoted by t_{11}) continue to be given by

$$t_{11} = \bar{t}_1 - 2\left(k_1\right)^{\frac{1}{2}}. \tag{3}$$

Product 2 may also benefit from the investment by Firm 1 in its transportation infrastructure. We introduce a new parameter $\xi \in [0,1]$ that captures the degree of these spillovers. Unit transport costs for Firm 1 for product 2 are then given by:

$$t_{12} = \xi\left(\bar{t}_2 - 2\left(k_1\right)^{\frac{1}{2}}\right) + (1-\xi)\,\bar{t}_2 = \bar{t}_2 - 2\xi\left(k_1\right)^{\frac{1}{2}} \tag{4}$$

If $\xi = 1$, both products benefit equally from the investment by Firm 1, and have identical transportation costs in the CAT case. If $0 < \xi < 1$, product 2 still benefits from the investment by Firm 1, but to a lesser degree. One can think of this case as a scenario were the investment by Firm 1 is tailored to its own product, and the spillovers created by the investment are incomplete. If $\xi = 0$, product 2 does not benefit at all from the infrastructure of Firm 1. In this case, the transportation costs of Firm 1 for product 2 are given by $t_{12} = \bar{t}_2$, and firm 1 actually has a transportation cost disadvantage relative to Firm 2 ($t_{12} = \bar{t}_2 \geq t_2 = \bar{t}_2 - 2\left(k_2\right)^{\frac{1}{2}}\ \forall\ k_2 > 0$).

It is reasonable to assume that it is nonverifiable by how much a single product actually benefits from the transportation infrastructure, in particular so since infrastructure always exhibits elements of limited rivalry and limited exclusivity. This creates a contractual incompleteness, and we assume that it prevents any arms-length solution where Firm 1 simply sells its transportation service to Firm 2. Consequently, we only consider cases where the CAT firm actually owns both products when it exports them.

3 Equilibrium

3.1 Solving for DOG Equilibrium

In the DOG case, both firms produce and sell their own product.[1] Solving for the equilibrium is then relatively straightforward. Essentially it is a standard duopoly Nash equilibrium with endogenous investments. Profits

[1] We disregard the possibility of shipping goods through intermediaries since this is not the focus of this study.

for firm i are

$$\pi_i^{DOG} = (p_i - c_i - t_i)\, q_i - k_i. \tag{5}$$

Taking first order condition with respect to output and simplifying we get the following equation for Firm $i \neq j$

$$2bq_i = a_i - c_i - t_i - b\theta q_j. \tag{6}$$

The first order condition with respect to transportation investment yields

$$k_i = q_i^2 \tag{7}$$

which means that transportation costs are

$$t_i = \bar{t}_i - 2q_i. \tag{8}$$

Solving (6) and (8) we get the aggregated best response function for Firm $i \neq j$

$$2\,(b-1)\,q_i = a_i - c_i - \bar{t}_i - b\theta q_j \tag{9}$$

Next, we solve for the two outputs. Solving the two best response functions (9) we get

$$q_i = \frac{2\,(b-1)\,(a_i - c_i - \bar{t}_i) - b\theta\,(a_j - c_j - \bar{t}_j)}{4\,(b-1)^2 - b^2\theta^2}. \tag{10}$$

Existence of an equilibrium in the DOG case requires that $4\,(b-1)^2 > b^2\theta^2$. This holds if $\theta \in (-2\,[b-1]\,/b, 2\,[b-1]\,/b)$. If $b < 2$, $2\,(b-1)\,/b < 1$, and this condition is no longer satisfied for large (positive or negative) values of θ. Thus, throughout this analysis we will assume that $b > 2$ in order to ensure existence of the DOG equilibrium for all $\theta \in [-1, 1]$.

Even if $b > 2$, a DOG equilibrium does not necessarily guarantee positive outputs for both products. This depends on whether $2\,(b-1)\,(a_i - c_i - \bar{t}_i) \gtreqless b\theta\,(a_j - c_j - \bar{t}_j)$. For the moment, we will simply assume that both outputs are positive, but this will be discussed in depth in our section below on the extensive margin of trade.

Substituting the above into the profit functions we get

$$\pi_i^{DOG} = (b-1)\left[\frac{2\,(b-1)\,(a_i - c_i - \bar{t}_i) - b\theta\,(a_j - c_j - \bar{t}_j)}{4\,(b-1)^2 - b^2\theta^2}\right]^2, \tag{11}$$

where $i = 1, 2$ and $j \neq i$.

3.2 Solving for CAT Equilibrium

In the CAT equilibrium, the CAT firms buys product 2 from the other firm and sells both products in the destination market. Thus, the CAT firm acts like a multi-product monopolist instead of a single-product duopolist. The price that the CAT firm demands for product 2 in the destination market depends not only on the transportation costs, but also on the price it has to pay to Firm 2, ρ. The profit functions of the two firms are

$$\begin{aligned} \pi_1^{CAT} &= (p_1 - c_1 - t_{11})\, q_1 + (p_2 - \rho - t_{12})\, q_2 - k_1 \\ \pi_2^{CAT} &= (\rho - c_2)\, q_2 \end{aligned}$$

Since this is a bilateral monopoly, the price ρ for the CAT product 2 is most likely the outcome of a bargaining game. For simplicity, we assume that bargaining between these two firms is efficient, so that the equilibrium outcome can be described by joint profit maximization. In this case, the price paid by Firm 1 to Firm 2 cancels out, and joint profits from CAT are given by

$$\pi^{CAT} = (p_1 - c_1 - t_{11})\, q_1 + (p_2 - c_2 - t_{12})\, q_2 - k_1 \tag{12}$$

It should be pointed out that the assumption of efficient bargaining is a strong assumption, and that there are some good reasons to believe that bargaining inefficiencies might exist. These inefficiencies will reduce aggregated profits in the CAT case and make CAT less attractive. Thus, by assuming efficient bargaining our results here must be interpreted as an upper bound for the profitability of CAT.

Maximizing profits under CAT gives rise to the following first order conditions for output

$$2b\,(q_1 + \theta q_2) = a_1 - c_1 - t_{11} \tag{13}$$

$$2b\,(\theta q_1 + q_2) = a_2 - c_2 - t_{12} \tag{14}$$

We next compute the optimal investment in technology to reduce transportation costs

$$\frac{d\pi^{CAT}}{dk_1} = (k_1)^{-\frac{1}{2}}\, q_1 + \xi\, (k_1)^{-\frac{1}{2}}\, q_2 - 1 = 0 \tag{15}$$

which implies,

$$k_1 = (q_1 + \xi q_2)^2 \tag{16}$$

Equation (16) shows that the optimal investment into transportation infrastructure in the CAT case depends on the outputs of both products. The influence of the output of the CAT product on infrastructure investment depends on the degree of spillover ξ and is increasing in ξ: $dk_1^2/\,(dq_2 d\xi) > 0$. If these spillovers are larger, the marginal benefits of lower transportation costs are larger, and this leads to higher investments in transportation infrastructure.

Next substitute the expression in (3), (4) and (16) into (13) and (14) to obtain

$$2(b-1)q_1 + 2(b\theta - \xi)q_2 = a_1 - c_1 - \bar{t}_1 \quad (17)$$

$$2(b\theta - \xi)q_1 + 2\left(b - \xi^2\right)q_2 = a_2 - c_2 - \bar{t}_2 \quad (18)$$

Solving (17) and (18) we have

$$q_1 = \frac{\left(b-\xi^2\right)\left(a_1 - c_1 - \bar{t}_1\right) - (b\theta - \xi)\left(a_2 - c_2 - \bar{t}_2\right)}{2\left[(b-1)\left(b-\xi^2\right) - (b\theta-\xi)^2\right]} \quad (19)$$

$$q_2 = \frac{(b-1)\left(a_2 - c_2 - \bar{t}_2\right) - (b\theta - \xi)\left(a_1 - c_1 - \bar{t}_1\right)}{2\left[(b-1)\left(b-\xi^2\right) - (b\theta-\xi)^2\right]} \quad (20)$$

Again, existence of an equilibrium requires that the best response functions are well behaved. In the CAT case, this requires that $(b-1)\left(b-\xi^2\right) > (b\theta - \xi)^2$. This condition holds if

$$-1 < \frac{1}{b}\left[\xi - \sqrt{\left(b-\xi^2\right)(b-1)}\right] < \theta < \frac{1}{b}\left[\xi + \sqrt{\left(b-\xi^2\right)(b-1)}\right] \leq 1, \quad (21)$$

where $-1 < \frac{1}{b}\left[\xi - \sqrt{\left(b-\xi^2\right)(b-1)}\right] < 0$ and $0 < \frac{1}{b}\left[\xi + \sqrt{\left(b-\xi^2\right)(b-1)}\right] \leq 1$ follow from $\xi \in [0,1]$ and $b > 2$. This condition imposes a tighter restriction on θ and implies that an equilibrium does not exist if products are perfect substitutes or perfect complements. The restriction is tighter relative to the DOG case because the CAT firm internalizes the linkages between the two products. In the case of perfect substitutes ($\theta = 1$) and imperfect spillovers ($\xi < 1$), products 1 and 2 are essentially identical, but product 2 has higher transportation costs. In this case, the CAT firm takes product 2 off the market entirely. In the case of perfect complements ($\theta = -1$), marginal revenues are no longer falling in output, and best response functions do not intersect. Both cases make little sense in economic terms, and we focus in our analysis on cases where θ lies within this interval (21).

By substituting (13), (14) and (16) into (12), CAT profits can be written as a linear expression in outputs (in contrast to the familiar quadratic term in the competitive DOG case):

$$\pi^{CAT} = \frac{1}{2}\left(a - c_1 - \bar{t}_1\right)q_1 + \frac{1}{2}\left(a - c_2 - \bar{t}_2\right)q_2 \quad (22)$$

Substituting (19) and (20) into the above and simplifying we have

$$\begin{aligned}\pi^{CAT} = {} & \frac{\left(b-\xi^2\right)\left(a-c_1-\bar{t}_1\right)^2}{4\left[(b-1)\left(b-\xi^2\right)-(b\theta-\xi)^2\right]} - \frac{2(b\theta-\xi)\left(a-c_1-\bar{t}_1\right)\left(a-c_2-\bar{t}_2\right)}{4\left[(b-1)\left(b-\xi^2\right)-(b\theta-\xi)^2\right]} \\ & + \frac{(b-1)\left(a-c_2-\bar{t}_2\right)^2}{4\left[(b-1)\left(b-\xi^2\right)-(b\theta-\xi)^2\right]}\end{aligned} \quad (23)$$

Now, we can turn to necessary and sufficient conditions for the occurrence of CAT.

4 When is CAT chosen?

For the determination of whether CAT occurs we can distinguish between a necessary and a sufficient condition. The necessary condition looks at a CAT firm (assuming that CAT is the relevant mode) and addresses the question of whether this firm actually has an incentive to sell positive amounts of the (potential) CAT product. We will see that this is not necessarily the case. The sufficient conditions then compares aggregate CAT profits to aggregate DOG profits and determines under what conditions CAT is the dominant mode. Put differently, the necessary condition assumes that the outside option for CAT is zero, while the sufficient condition acknowledges that DOG profits provide a positive outside option.

4.1 Necessary Condition for CAT

For the necessary condition we look at the expression for the optimal output of the CAT product 2. If this is non-positive, the CAT firm has no incentives to sell the CAT product, irrespective of the outside option. Given (20), a positive output $q_2 > 0$ requires that

$$\varphi > \frac{b}{b-1}\theta - \frac{1}{b-1}\xi, \tag{24}$$

where

$$\varphi \equiv \frac{(a_2 - c_2 - \bar{t}_2)}{(a_1 - c_1 - \bar{t}_1)} > 0 \tag{25}$$

This parameter φ captures the (exogenous) profitability of the potential CAT product 2 relative to product 1. It takes into account exogenous differences in quality (a), marginal production costs between the two firms (c), and transportation costs ($\bar{t}$).

Equation (24) compares the relative profitability of the CAT product φ to a weighed average of the two parameters θ and ξ. The parameter $\xi \in [0, 1]$ measures the size of the spillovers in the transportation infrastructure. Not surprisingly, larger spillovers make CAT more attractive. The parameter θ captures the demand linkages between the two products and can be positive or negative depending on whether the two products are substitutes or complements. If they are complements ($\theta < 0$), inequality (24) always holds, and optimal CAT output is always positive. The reason is straightforward: In the case of complements, sales of one product boost sales of the second product, and it is always profitable to add a complementary product to the product range. But if the two products are substitutes ($\theta > 0$), inequality (24) can be binding. If the profitability of the CAT product φ is low and spillovers ξ are weak, the CAT firm might find it unprofitable to add the CAT product to its product range. The reason for this is the cannibalization effect (Eckel and Neary, 2010; Dhingra, 2013): Adding a substitute to the product range diverts demand away from product 1, and if this effect is strong, the CAT firm has no incentive to do so.

Even if the outside option is zero, the CAT firm will not necessarily add the CAT product to its product range. This result is summarized in proposition 1:

Proposition 1 *The optimal output of the CAT product is not necessarily positive, even in the absence of any outside option for the CAT product. The CAT firm has an incentive to sell the CAT product only if its own relative profitability and the transportation costs synergies exceed demand divergence through cannibalization. This is always the case if the two product are complements or unrelated in demand, but can be binding in the case of substitutes.*

The necessary condition can also be rewritten as a condition on the degree of substitution:

$$\theta < \frac{\xi}{b} + \frac{b-1}{b}\varphi, \tag{26}$$

The necessary condition is binding relative to the stability condition (21) if $\varphi < \sqrt{\left(b-\xi^2\right)/\left(b-1\right)}$. Since $\sqrt{\left(b-\xi^2\right)/\left(b-1\right)} > 1$, it is binding when $\varphi = 1$, an important benchmark case for our analysis.

Of course, if φ is very high, implying that product 1 is relatively unprofitable, the CAT firm might consider taking product 1 off the market. We will discuss this later in our section on the extensive margin of trade.

4.2 Sufficient Condition for CAT

The sufficient condition takes into account that the delivery of own goods (DOG) is a viable alternative to carry-along trade (CAT) and that the outside option of CAT might be positive. Thus, CAT is only chosen when CAT profits exceed profits in the DOG case. We define

$$\Delta\Pi \equiv \frac{1}{\left(a_1 - c_1 - \bar{t}_1\right)^2}\left(\pi^{CAT} - \sum \pi^{DOG}\right) \tag{27}$$

as our measure for the relative profitability of CAT.

Given (11), (23), and (25) $\Delta\Pi$ can be rewritten as

$$\begin{aligned} \Delta\Pi\left(b,\varphi,\xi,\theta\right) \quad = \quad & \frac{\left(b-1\right)\varphi^2 - 2\left(b\theta-\xi\right)\varphi + \left(b-\xi^2\right)}{4\left[\left(b-1\right)\left(b-\xi^2\right) - \left(b\theta-\xi\right)^2\right]} \\ & - \frac{\left(b-1\right)\left[\left(4\left(b-1\right)^2 + \theta^2 b^2\right)\left(\varphi^2+1\right) - 8\theta b\left(b-1\right)\varphi\right]}{4\left(b-1\right)^2 - b^2\theta^2} \end{aligned} \tag{28}$$

The sufficient condition for CAT requires that

$$\Delta\Pi\left(b,\varphi,\xi,\theta\right) > 0 \tag{29}$$

If $\Delta\Pi$ is larger than zero, CAT profits are larger than the sum of DOG profits. If this is the case, CAT is the dominant mode of supplying the market. If $\Delta\Pi$ is smaller than zero, DOG profits exceed CAT profits, and DOG is the dominant mode.

Naturally, the necessary condition has to be satisfied. Thus, depending on φ, the restriction on θ may be tighter than (21), as discussed in the previous section. We assume that

$$\theta \in \left[\theta^{\min}, \theta^{\max}\right], \tag{30}$$

where $\theta^{\min} \equiv \frac{1}{b}\left[\xi - \sqrt{\left(b-\xi^2\right)(b-1)}\right]$ and $\theta^{\max} \equiv \min\left\{\frac{1}{b}\left[\xi + \sqrt{\left(b-\xi^2\right)(b-1)}\right]; \frac{\xi}{b} + \frac{b-1}{b}\varphi\right\}$. Note that $\theta^{\min} < 0$ and $\theta^{\max} > 0$.

The expression $\Delta\Pi$ is a complex function of b, φ, ξ and θ. Some special cases provide first insights into how these parameters influence the relative profits of CAT and DOG:

1. $\theta = \xi = 0$: $\Delta\Pi\left(b, \varphi, 0, 0\right) = -\frac{\varphi^2}{4b(b-1)} < 0$

   Without supply or demand linkages, CAT is never profitable. If products are completely unrelated, and there are no transportation costs synergies, CAT really does not bring any advantages. In fact, it has a clear disadvantage because transportation costs are higher in the CAT case, so it will never be chosen.

2. $\theta = 0$ and $\xi = 1$: $\Delta\Pi\left(b, \varphi, 1, 0\right) = \frac{\varphi^2 + 2\varphi(b-1)+1}{4b(b^2-3b+2)} > 0$

   If products are unrelated, but transportation cost synergies are perfect, CAT is always profitable. Thus, demand linkages are not a necessary condition for the profitability of CAT.

3. $\xi = 0$ and $\theta \to \theta^{\min} = -\sqrt{(b-1)/b}$: $\lim_{\theta \to \theta^{\min}+} \Delta\Pi = +\infty > 0$

   If transportation costs synergies do not exist (and CAT actually has a transportation costs disadvantage), but products are sufficiently strong complements, CAT is profitable. This shows that transportation costs synergies are not a necessary condition, either. Finally:

4. $\varphi = 1$ and $\theta \to \theta^{\max} = (b-1+\xi)/b$: $\lim_{\theta \to \theta^{\max}} \Delta\Pi = \frac{1}{4}\frac{(b-1)^2 + [6(b-1)+\xi]\xi}{(b-1)(3b+\xi-3)^2} > 0$

   If the two products are of equal (exogenous) profitability and sufficiently good substitutes, CAT is always profitable. This last point illustrates that CAT does not require that the two products are complements but can be a dominant strategy in the case of substitutes, too.

These four cases illustrate that CAT requires some synergies from joint sales, either through demand linkages or through transportation cost synergies, but that neither of these synergies is a necessary condition.

We will proceed by first analyzing the role of θ and ξ in the case of equally profitable products ($\varphi = 1$), and then study how changes in φ affect the profitability of CAT.

4.2.1 The Case of Equally Profitable Products

In the case of equally profitable products ($\varphi = 1$) our measure for the relative profitability of CAT can be simplified to

$$\Delta\Pi(b, 1, \xi, \theta) = \frac{1}{4}\frac{(b-1) - 2(b\theta - \xi) + \left(b - \xi^2\right)}{(b-1)\left(b - \xi^2\right) - (b\theta - \xi)^2} - \frac{2(b-1)}{\left[2(b-1) + \theta b\right]^2} \tag{31}$$

Figure 1 illustrates the $\Delta\Pi(b, 1, \xi, \theta) = 0$ locus in a $\theta - \xi$ diagram.

The $\Delta\Pi(b, 1, \xi, \theta) = 0$ locus in figure 1 is highly stylized for expositional purposes. In particular, the shape of the locus is not necessarily symmetric. In the appendix we provide a detailed discussion of the shape of the $\Delta\Pi(b, 1, \xi, \theta) = 0$ locus. In particular, we can prove the following features of this locus:

Lemma 1 *In a $\theta - \xi$ diagram, the $\Delta\Pi(b, 1, \xi, \theta) = 0$ locus has two intersections (θ_1, θ_2) with the θ-axis, where $\theta^{\min} < \theta_1 < 0 < \theta_2 < \theta^{\max}$. As ξ increases, the distance between θ_1 and θ_2 decreases, and there exists a $\xi^{\max} \in (0, 1)$ where the $\Delta\Pi(b, 1, \xi, \theta) = 0$ locus has a maximum in this diagram.*

Proof. See appendix ■

In the area surrounded by $\Delta\Pi(b, 1, \xi, \theta) = 0$ around the origin, DOG is the dominant form of supplying the market $[\Delta\Pi(b, 1, \xi, \theta) < 0]$. In the area outside of this space, CAT is more profitable $[\Delta\Pi(b, 1, \xi, \theta) > 0]$. This figure illustrates for the case where $\varphi = 1$ that if demand linkages and/or transportation cost synergies are sufficiently large, the two firms will choose CAT as the more profitable form of supplying the market. But if demand linkages and/or transportation cost synergies are too low, the two firms will choose to supply the market independently (DOG). The following proposition summarizes our insights from figure 1:

Proposition 2 *CAT is dominated by DOG if transportation cost synergies are low AND demand linkages are weak. CAT dominates DOG if transportation costs synergies OR demand linkages are sufficiently high. For the demand linkages it does NOT matter whether products are complements or substitutes.*

In particular this last sentence is important: What matters are the size of the demand linkages, not the type of linkages. Technically speaking: The value of θ has to be sufficiently large, the sign is only of secondary importance. On first sight, this might come as a surprise because one might have thought that carrying-along a complementary product is more attractive than carrying-along a substitute to one's own products. This is, in fact, true: Profits are indeed higher in the case of complements than in the case of substitutes. But this is not the relevant comparison. Profits in the DOG case are also higher in the case of complements than in the case of substitutes.

What matters for the mode decision are relative profits of CAT: CAT profits relative to DOG profits. And these relative profits are affected by the size of the demand linkages because the CAT firm behaves like a multi-product firm that internalizes these linkages. The larger these linkages the more the CAT firm can benefit from their internalization (relative to the DOG case where firms compete non-cooperatively). Whether these linkages are positive or negative is not that important. What matters is their size, not their sign. This is why carry-along trade is profitable for both complements and substitutes.

4.2.2 The Role of Product Heterogeneity

In the previous section we assumed that the products are essentially identical in their exogenous profitability ($\varphi = 1$). Now we want to analyze how (small) differences in these products affect the incentive to engage in carry-along trade. Since each of the products is produced by a unique firm, differences in the profitability of products is isomorphic to differences in firm productivity in our framework.

Changes in product profitability φ affect both profits in the CAT case and in the DOG case in a similar way. However, when evaluated at $\varphi = 1$ and $\Delta\Pi(b, 1, \xi, \theta) = 0$, the derivative of $\Delta\Pi$ with respect to φ can be clearly signed (see appendix for proof):

$$\frac{\partial}{\partial\varphi}\Delta\Pi(b, 1, \xi, \theta)\bigg|_{\Delta\Pi=0} = \frac{1}{4}\frac{\xi^2 - 1}{(b-1)\left(b-\xi^2\right) - (b\theta - \xi)^2} < 0 \tag{32}$$

This result implies that the $\Delta\Pi(b, 1, \xi, \theta) = 0$ locus is shifted inwards when φ falls and CAT becomes a dominant strategy for a larger range of parameter constellations (see figure 2).

Proposition 3 *Compared to a situation where products are equally profitable, CAT is relatively more profitable if the CAT product is relatively less profitable.*

This proposition provides two interesting insights: First, it states that product heterogeneity tends to favor CAT. Heterogeneity tends to raise aggregate profits in oligopolistic markets in general, and the CAT firm can benefit from an asymmetry even more because it internalizes the demand linkages. Second, this proposition implies that it is more profitable for a more productive firm to carry-along a less productive firm's product than the other way around. This prediction is in line with evidence presented by Bernard et al. (2016) on carry-along trade that shows that CAT firms tend to be on average more productive firms.

5 Trade Effects of CAT

Does carry-along trade matter for the volume of trade? In this section we analyze how the mode of trading affects the quantities traded. We differentiate between how CAT affects the extensive margin of trade and the intensive margin of trade, beginning with the latter.

5.1 CAT and the Intensive Margin of Trade

Because the CAT firm internalizes the demand linkages between the two products, a change in the mode of trading will also affect equilibrium quantities and will thus have an effect on the intensive margin of trade.

Define $\Delta q_i \equiv \left(q_i^{CAT} - q_i^{DOG}\right) / \left(a - c_i - \bar{t}_i\right)$ as a measure for the change in equilibrium quantities when comparing the CAT equilibrium to the DOG equilibrium. We obtain

$$\Delta q_1 \left(b, \varphi, \xi, \theta\right) = \frac{\left(b - \xi^2\right) - \left(b\theta - \xi\right)\varphi}{2\left[\left(b-1\right)\left(b - \xi^2\right) - \left(b\theta - \xi\right)^2\right]} - \frac{2\left(b-1\right) - b\theta\varphi}{\left(2\left(b-1\right) - b\theta\right)\left(2\left(b-1\right) + b\theta\right)} \tag{33}$$

and

$$\Delta q_2 \left(b, \varphi, \xi, \theta\right) = \frac{\left(b-1\right)\varphi - \left(b\theta - \xi\right)}{2\left[\left(b-1\right)\left(b - \xi^2\right) - \left(b\theta - \xi\right)^2\right]} - \frac{2\left(b-1\right)\varphi - b\theta}{\left(2\left(b-1\right) - b\theta\right)\left(2\left(b-1\right) + b\theta\right)}. \tag{34}$$

The change in aggregate output is given by

$$\begin{aligned} \Delta Q \left(b, \varphi, \xi, \theta\right) &\equiv \Delta q_1 + \Delta q_2 & (35)\\ &= \frac{\left(b - \xi^2\right) - \left(b\theta - \xi\right) + \left[\left(b-1\right) - \left(b\theta - \xi\right)\right]\varphi}{2\left[\left(b-1\right)\left(b - \xi^2\right) - \left(b\theta - \xi\right)^2\right]} - \frac{\varphi + 1}{2\left(b-1\right) + b\theta}. & (36) \end{aligned}$$

We begin by characterizing the changes in the intensive margin for the case of symmetric products ($\varphi = 1$) and then discuss how changes in φ affects the results.

5.1.1 Symmetric Products: Aggregate Quantity

When $\varphi = 1$, ΔQ reduces to

$$\Delta Q \left(b, 1, \xi, \theta\right) = \frac{\left(b - \xi^2\right) - \left(b\theta - \xi\right) + \left(b-1\right) - \left(b\theta - \xi\right)}{2\left[\left(b-1\right)\left(b - \xi^2\right) - \left(b\theta - \xi\right)^2\right]} - \frac{2}{2\left(b-1\right) + b\theta}. \tag{37}$$

In order to focus on the intensive margin, we assume that

$$\theta \in \left(\left[\xi - \sqrt{\left(b - \xi^2\right)\left(b-1\right)}\right]/b, \left(\xi + b - 1\right)/b\right) \tag{38}$$

in order to guarantee non-negative outputs for both products in both scenarios [see (26)].

For the case of symmetric products, we establish the following:

Proposition 4 *Whether CAT raises or lowers the aggregate quantities traded depends on the degree of substitution. There exists a critical degree of substitution θ^Q such that CAT reduces trade at the intensive margin if $\theta > \theta^Q$ and increases trade if $\theta < \theta^Q$. If CAT is profitable, this critical level is larger than zero $\left(\theta^Q > 0\right)$.*

Proof. Define $\Omega \equiv \Delta Q / \left[2\left(\left(b-1\right)\left(b - \xi^2\right) - \left(b\theta - \xi\right)^2\right)\left(2\left(b-1\right) + b\theta\right)\right]$, so that the roots of Ω are also the roots of ΔQ. This function Ω can be rewritten as a polynomial of degree two in θ:

$$\Omega \left(b, 1, \xi, \theta\right) = 2b^2\theta^2 - b\theta\left(2b + \xi^2 + 6\xi - 3\right) + 2b\left(\xi^2 + 2\xi - 1\right) + 2\left(1 - \xi\right)^2. \tag{39}$$

Since $2b^2 > 0$, the parabola $\Omega(b,1,\xi,\theta)$ opens upwards and has a minimum at $\Omega_\theta(b,1,\xi,\theta) = 0$. We can prove that this Ω-function has a positive value when evaluated at the minimum value of $\theta^{\min} = \left(\xi - \sqrt{(b-\xi^2)(b-1)}\right)/b$ and a negative value when evaluated at $\theta^{\max} = (\xi + b - 1)/b$:

$$\Omega\left(b,1,\xi,\theta^{\min}\right) = \left[2(b-2) + (\xi+1)^2\right]\sqrt{(b-1)\left(b-\xi^2\right)} + \left(2b-1-\xi^2\right)\xi + 2(b-1)^2 > 0 \qquad (40)$$

$$\Omega\left(b,1,\xi,\theta^{\max}\right) = -(1-\xi)(1+\xi)(b-1-\xi) < 0 \qquad (41)$$

Given the parabolic form of the Ω-function, this proves that there exists a $\theta^Q \in \left(\theta^{\min},\theta^{\max}\right)$ such that $\Omega\left(b,1,\theta^Q,\xi\right) = 0$. To prove that $\theta^Q > 0$ when CAT is profitable, note that $\Omega(b,1,\xi,0) \propto \Delta\Pi(b,1,\xi,0)$. Hence, if $\Delta\Pi(b,1,\xi,0) > 0$, $\Omega(b,1,\xi,0) > 0$, and $\theta^Q > 0$. ■

This proves that aggregate trade flows are higher in the CAT case when products are complements, and lower when products are close substitutes. The intuition behind this result is based on the internalization of the cannibalization effect. If the products are complements, higher sales of one product increase demand for the other product. The CAT firm internalizes these linkages, and sells higher quantities of both products. If products are substitutes, the argument goes in the opposite direction. The CAT firm internalizes that fact that higher sales of one product reduce demand for the other product, and sells smaller amounts of both.

The fact that equilibrium exports are larger in the CAT case when products are unrelated ($\theta = 0$) is due to the spillovers in the transportation infrastructure. If products are unrelated, these spillovers are the only difference between CAT and DOG. If spillovers are small, there is little investment in transportation infrastructure. Consequently, marginal costs are higher in the CAT case, leading to lower aggregate sales, and CAT is not profitable. But if spillovers are large, investment in infrastructure is higher and marginal costs are lower. Thus, aggregate output is larger making CAT the dominant strategy. This shows how changes in aggregate output and the profitability of CAT are related when $\theta = 0$, and explains why the critical level $\theta^Q > 0$.

In Figure 3, the $\Delta Q = 0$ locus is upward sloping and intersects the ξ-axis exactly at the intersection of the $\Delta\Pi = 0$ locus. To the left of the $\Delta Q = 0$ locus, ΔQ is positive, and negative to the right of this locus. If $\xi = 1$,

$$\Delta Q(b,1,1,\theta) = \frac{2-b\theta}{(b-2+b\theta)\left[2(b-1)+b\theta\right]}. \qquad (42)$$

Hence, the $\Delta Q = 0$ locus hits the $\xi = 1$ line at $\theta = 2/b < 1$.

5.1.2 Symmetric Products: Individual Quantities

We now take a closer look at the changes in individual outputs as we move from DOG to CAT. First of all, note that $\Delta q_1 (b, 1, \xi, \theta) > \Delta q_2 (b, 1, \xi, \theta)$ if $\xi < 1$. To see this result, consider the case in which the two products are perfectly symmetric if $\varphi = 1$. In the DOG case, output should be the same in both industries. In the CAT case, however, marginal costs of exporting the CAT product depend on the spillovers in the transportation technology. If they are incomplete ($\xi < 1$), the CAT product has higher marginal costs of exporting, and thus lower equilibrium quantities, than the CAT firm's own product.

One implication of this result is that if $\theta = \theta^Q$ and $\Delta Q = \Delta q_1 + \Delta q_2 = 0$,

$$\Delta q_1 \left(b, 1, \xi, \theta^Q\right) > 0 > \Delta q_2 \left(b, 1, \xi, \theta^Q\right). \tag{43}$$

Hence, if $\theta \approx \theta^Q$, CAT has asymmetric effects on the intensive margins of trade. Exports of the CAT product fall, while exports of the CAT firm's own product rise.

Let us now take a more general look at how the changes in output depend on the degree of substitution. It turns out that changes in the individual output of the CAT product q_2 are qualitatively similar to changes in aggregate output Q. First note that

$$\lim_{\theta \to \theta^{\min}} \Delta q_2 (b, 1, \xi, \theta) = \infty > 0 \tag{44}$$

and

$$\Delta q_2 (b, 1, \xi, \theta^{\max}) = - [3 (b - 1) + \xi]^{-1} < 0. \tag{45}$$

Then note that the CAT output is continuously falling in the degree of substitution θ:

$$\frac{1}{(a - c_1 - \bar{t}_1)} \frac{dq_2}{d\theta} = -\frac{b}{2} \frac{(b - 1 + \xi - b\theta)^2 + (b - 1)\left(1 - \xi^2\right)}{\left[(b - 1)\left(b - \xi^2\right) - (b\theta - \xi)^2\right]^2} < 0. \tag{46}$$

If the products are complements ($\theta < 0$), the CAT firm internalizes the positive effect that the output of one product has on the demand for the other product. Thus, it sells more of both products in the CAT case. This effect becomes smaller as θ rises and eventually switches signs, so that the CAT firm restricts the output of the CAT product to be lower than in the DOG case [$\Delta q_2 (b, 1, \xi, \theta) < 0$] (see Figure 4).

Interestingly, the CAT firm's own product (q_1) will vary in a much different way. Even though the limit of Δq_1 is positive when θ approaches $\theta^{\min}$, Δq_1 is also *positive* when evaluated at $\theta^{\max}$:

$$\lim_{\theta \to \theta^{\min}} \Delta q_1 (b, 1, \xi, \theta) = \infty > 0, \tag{47}$$

$$\Delta q_1 (b, 1, \xi, \theta^{\max}) = \frac{1}{2 (b - 1)} \frac{b - 1 + \xi}{3 (b - 1) + \xi} > 0. \tag{48}$$

Interestingly, q_1 is not a monotonic function of θ.

$$\frac{1}{(a-c_1-\bar{t}_1)}\frac{dq_1}{d\theta}=-\frac{b}{2}\frac{\left(b-\xi^2+\xi-b\theta\right)^2-\left(b-\xi^2\right)\left(1-\xi^2\right)}{\left[(b-1)\left(b-\xi^2\right)-(b\theta-\xi)^2\right]^2} \tag{49}$$

This derivative is zero ($dq_1/d\theta=0$) at

$$\tilde{\theta}=\theta^{\max}-\frac{1}{b}\left[\sqrt{\left(b-\xi^2\right)\left(1-\xi^2\right)}-\left(1-\xi^2\right)\right]\in(0,\theta^{\max}] \tag{50}$$

Note that $\tilde{\theta}<\theta^{\max}$ requires that $\xi<1$.

If the two products are complements, the outputs of both products react very similarly to changes in the degree of substitution θ. As θ rises (the degree of complementarity falls), outputs in both cases, CAT and DOG, fall (see Figure 4.) However, because the CAT firm internalizes the demand linkages in the CAT case, and a reduction in the value of θ implies smaller demand linkages, the outputs fall by more in the CAT case, so that Δq_i (and ΔQ) fall as θ rises.

However, if the two products are substitutes, things work very differently. As the degree of substitution increases (θ rises), the cannibalization effect becomes stronger. If the two products were perfectly symmetric, we would just observe a continuation in the fall of the outputs of both products (like in the DOG case). But if $\xi<1$, the two products are *not symmetric* even if $\varphi=1$. If $\xi<1$, spillovers from investment in transportation technology are incomplete, and the marginal costs of exporting one unit of the CAT product (q_2) is systematically higher than the marginal costs of exporting one unit of the CAT firm's own product (q_1). As a consequence, the marginal profit of the CAT firm's own product is higher than the marginal profit of the CAT product, and if the cannibalization effect is sufficiently strong, the CAT firm finds it profitable to shift market share away from the CAT product in order to expand the market share of its (more profitable) own product.

The critical level of the degree of substitution is given by $\tilde{\theta}$ (see Figure 4.) At this level, the motivation of the CAT firm switches from supplying both products to taking the CAT product off the market. If $\theta<\tilde{\theta}$, the CAT firm restricts the output of its own product in order to generate enough demand for the CAT product despite its higher transportation costs. If $\theta>\tilde{\theta}$, the CAT firm expands the output of its own product at the expense of the CAT product. This creates another parameter range where a switch from DOG to CAT creates asymmetric effects at the intensive margin. At $\theta=\theta^{\max}$, we have

$$\Delta q_1\left(b,1,\xi,\theta^{\max}\right)>0>\Delta q_2\left(b,1,\xi,\theta^{\max}\right). \tag{51}$$

The following proposition summarizes our findings regarding the changes of individual quantities.

Proposition 5 *(1) If the two products are complements, CAT leads to an increase in trade at the intensive margin for both products. (2) If the two products are substitutes, CAT tends to reduce trade at the intensive* *margin for both products for low values of θ. As θ increases, eventually the CAT firm reduces the exports of the CAT product while increasing the exports of its own product. (3) If spillovers in transportation technology across products are incomplete, asymmetric effects on the intensive margin are possible.*

These results are illustrated in Figure 4. When the products are complements moving to CAT increases both outputs. For the substitute case outputs are reduced for lower values of θ, but as θ increases if $\xi < 1$, the CAT firm starts increasing the output of its product q_1 while reducing the output of q_2.

5.1.3 Asymmetric Products

If $\varphi \neq 1$, products are asymmetric not only because of differences in transportation costs, but also because of differences in production costs. This adds a second source of heterogeneity across products, which has implications for the trade effects of CAT.

First, let us look at how changes in φ affect aggregate trade flows. Note that $\Delta Q\left(b,\varphi,\xi,\theta\right)$ can be rewritten as $\Delta Q\left(b,\varphi,\xi,\theta\right) = \Delta q_1\left(b,1,\xi,\theta\right) + \varphi \Delta q_2\left(b,1,\xi,\theta\right)$. Thus, we have

$$\frac{d\Delta Q}{d\varphi} = \Delta q_2\left(b,1,\xi,\theta\right) \tag{52}$$

When evaluated at $\Delta Q = 0$, we know from our analysis above that $\Delta q_2\left(b,1,\xi,\theta\right) < 0$ if $\xi < 1$ and $\Delta q_2\left(b,1,\xi,\theta\right) = 0$ if $\xi = 1$. Thus, $d\Delta Q/d\varphi|_{\Delta Q=0} \leq 0$. In Figure 3, this implies that the $\Delta Q = 0$ locus rotates counterclockwise around the point $(\xi,\theta) = (1, b/2)$. Consequently, a switch from DOG to CAT is trade increasing at the intensive margin for a larger range of parameter values.

Regarding the effect of φ on individual outputs we are particularly interested in how changes in φ affect the critical level $\tilde{\theta}$ where the function of q_1 has a minimum. Calculating $dq_1/d\theta = 0$, $\tilde{\theta}$ is implicitly given by

$$2\left(b\theta - \xi\right)\left(b - \xi^2\right) = \varphi\left[\left(b-1\right)\left(b-\xi^2\right) + \left(b\theta - \xi\right)^2\right] \tag{53}$$

By totally differentiating this equation we obtain

$$\frac{d\tilde{\theta}}{d\varphi} = \frac{\left(b-1\right)\left(b-\xi^2\right) + \left(b\theta-\xi\right)^2}{2b\left[\left(b-\xi^2\right) - \left(b\theta - \xi\right)\varphi\right]} > 0 \tag{54}$$

Hence, if φ falls, $\tilde{\theta}$ also falls, and the CAT firm reduces the exports of the CAT product even further. As mentioned earlier, the fall in φ adds a second source of heterogeneity. The CAT product is now relatively more costly to transport and more costly to produce. This second source of heterogeneity adds to the incentive to reallocate market shares away from the CAT product and towards the CAT firm's own product. Naturally, if φ rises, the CAT product becomes more competitive and the incentive is to increase market share of the CAT product.

This is summarized in our last proposition on the trade effects of CAT:

Proposition 6 *A fall in the profitability of the CAT product tends to boost exports at the intensive margin. However, it also increases the incentive to take the CAT product off the market.*

5.2 CAT and the Extensive Margin of Trade

CAT can be trade creating at the extensive margin if the output of one of the two products is zero in the DOG mode and positive in the CAT mode. Alternatively, CAT can be trade reducing at the extensive margin if both goods are produced under DOG but one of them is taken off the market under CAT.

A quick glance at the expressions for outputs in (10) for the DOG case and (19) and (20) for the CAT case already provides a first important insight: Since changes at the extensive margin require (by definition) that outputs in one mode are zero, there can be no effects at the extensive margin in the case of complements or unrelated products. If $\theta \leq 0$, outputs of both products in both modes are always positive, and there are no effects at the extensive margin.

A second important result is that there are no effects at the extensive margin if products are symmetric ($\varphi = 1$), either. In this case, DOG outputs and the output of product 1 in the CAT mode are always positive. Hence, for effects at the extensive margin to exist, products have to be sufficiently heterogeneous in their profitability.

Let's first look at trade reducing CAT. There are two cases to consider, one in which the CAT product (good 2) is taken off the market and the second case in which the CAT firm takes its' own product (good 1) off the market when switching to CAT. The case in which the CAT product is taken off the market was discussed in the previous section. The CAT firm decides to take its own product off the market if φ and θ are in intermediate values: If $1 < \left(b - \xi^2\right) / \left(b - \xi\right) < \varphi < \left(b + \xi^2 - 2\right) / \xi$ and if $\left(b - \xi^2\right) / \left(b\varphi\right) + \xi / b \leq \theta < 2\left(b - 1\right) / \left(b\varphi\right)$, then $q_1^{DOG} > 0$ and $q_1^{CAT} = 0$. The motivation behind this result is similar to the incentive to take the CAT product off the market as discussed above. If the CAT product is relatively more profitable than product 1 ($\varphi > 1$), and the two products are substitutes, it may be more profitable to monopolize the market by taking product 1 off the market. Note however, that neither of these cases would show up in the data as CAT. In the case that the CAT product is taken off the market this would not show up in the data as CAT since the CAT product is not actually exported. In the case in which the CAT firm takes it own product off the market it does not fit the definition of CAT since the CAT firm does not export a product that it produces.

If the CAT product is relatively unprofitable, CAT can be trade creating. If $\varphi < \xi / \left(b - 1\right) < 1$ and $\theta \geq 2\left(b - 1\right)\varphi / b$, then $q_2^{DOG} = 0$ and $q_2^{CAT} > 0$. This means that there will be no exports if both firms sell their products independently (DOG case), but positive exports in the CAT case. Similarly, if the CAT firm's own product is relatively unprofitable [$\varphi > \max\left\{2\left(b - 1\right)/b; \left(b - 2 + \xi^2\right)/\xi\right\} > 1$] and $\theta \geq 2\left(b - 1\right) / \left(b\varphi\right)$, then CAT can be trade creating for product 1: $q_1^{DOG} = 0$ and $q_1^{CAT} > 0$. The economic intuition for these two results is based on the cannibalization effect. Because the CAT firm internalizes the demand linkages between the two products, it can create a market for a less profitable product by reducing the output of the more profitable product.

6 Price Effects of CAT

We can illustrate the impact of carry-along trade on prices in Figure 5. To do this we compute the change in price that occurs when going from DOG to CAT and plot it on Figure 1. So, Figure 5 is the same as Figure 1 with the addition of two $\Delta p = 0$ loci, $\Delta p_1 (b, 1, \xi, \theta)$ for the DOG product and $\Delta p_2 (b, 1, \xi, \theta)$ for the CAT product.

$$\Delta p_1 (b, 1, \xi, \theta) = -\frac{\left(b-\xi^2\right) - (b\theta - \xi) + \theta\left((b-1) - (b\theta-\xi)\right)}{2\left((b-1)\left(b-\xi^2\right) - (b\theta-\xi)^2\right)} + \frac{1+\theta}{2(b-1)+b\theta}$$

$$\Delta p_2 (b, 1, \xi, \theta) = -\frac{(b-1) - (b\theta - \xi) + \theta\left(\left(b-\xi^2\right) - (b\theta-\xi)\right)}{2\left((b-1)\left(b-\xi^2\right) - (b\theta-\xi)^2\right)} + \frac{1+\theta}{2(b-1)+b\theta}$$

We begin by discussing some special cases before characterizing the two loci more generally.

We begin with $\xi = \theta = 0$. In this case, $\Delta p_1 (b, \varphi, 0, 0) = 0$ and $\Delta p_2 (b, \varphi, 0, 0) = \frac{1}{2b(b-1)} > 0$. Hence, in the absence of both demand linkages and transportation cost synergies, the price of the CAT product clearly rises while the price of the product produced by firm 1 is unaffected. This is intuitive since in this case neither the output decision nor the investment decision of firm 1 is affected by carry-along trade so it makes sense that the price of good 1 is unchanged. Since firm 1 has higher transportation costs for product 2, it follows that the price of the CAT product, good 2, rises. Given how CAT affects the two prices it is not surprising that in this case CAT is never profitable.

Next, let us discuss the case where $\theta = 0$ but $\xi = 1$. In this case, $\Delta p_1 (b, 1, 1, 0) = \Delta p_2 (b, 1, 1, 0) = -\frac{1}{2(b-2)(b-1)} < 0$. Thus, if transportation costs synergies are perfect, but there are no demand linkages, both product prices are clearly lower with CAT. This is also very intuitive because these synergies change the investment behavior of the potential CAT firm. If firm 1 sells not only its own product but also the CAT product it has a larger marginal benefit from investing in lower transportation costs. Thus, it will invest more in reducing transportation costs, thereby lowering the marginal sales costs and hence, the prices of both products.

Next, we look at how prices are affected if demand linkages are large. First, let $\theta \to \theta^{\min}$ where $\theta^{\min} = \xi/b - \sqrt{\left(b-\xi^2\right)(b-1)}/b$. In the limit, both prices fall: $\lim_{\theta\to\theta^{\min}} \Delta p_1 (b, 1, \xi, \theta) = \lim_{\theta\to\theta^{\min}} \Delta p_2 (b, 1, \xi, \theta) = -\infty < 0$. This result holds independent of transportation cost synergies. Hence, if the two products are complements, prices tend to be lower. This is different if products are substitutes. To see what happens when products are substitutes let $\theta \to \theta^{\max} = \xi/b + (b-1)/b$. In this case, $\Delta p_1(b, 1, \xi, \theta^{\max}) > 0$ and $\Delta p_2(b, 1, \xi, \theta^{\max}) > 0$. If products are substitutes, prices are higher with CAT than if sold by separate firms. Hence, CAT leads to lower prices when the products are complements and increased prices if they are substitutes. The reason for this lies in the cannibalization effect. Carry-along trade essentially makes the CAT firm a multi-product firm that internalizes demand linkages. This internalization pushes prices in different directions depending on whether outputs of the two products are strategic substitutes or complements. If they are strategic substitutes ($\theta > 0$), a higher output of one product reduces the optimal output of the other product. The CAT firm internalizes this negative externality and sells less of both products at higher prices. If the two outputs are strategic complements, a higher output of one product generates a positive externality on the other product. If the CAT firm internalizes this positive externality it sells more of both products, and prices of both products are lower.

This result shows that carry-along trade can have very different effects on prices depending on whether the products carried-along are substitutes or complements. While we don't model welfare explicitly, this result suggests an interesting perspective on the welfare effects of CAT.

For the source country, these welfare effects are very complicated because of transportation costs, profit shifting and domestic consumer effects. For the destination country, these welfare effects are more straightforward. Our results show that if the CAT products are complements, CAT lowers prices and raises consumer rents relative to the DOG case. For the case of substitutes, prices increase and consumer rents are lower. This implies that CAT may not be benign for the destination market. This leads to the following remark:

Remark 1 *In the case of substitutes, CAT produces outcomes that are identical to product-specific, market-specific collusion.*

7 Conclusion

We have developed a model that focusses on the strategic aspects of CAT. In our framework, two firms decide on the mode of servicing an export market. They can either choose to export their products independently (DOG case) or have one firm "carry-along" the product of the other firm (CAT case). We have shown how the decision on the mode of exporting depends on demand linkages, transportation cost synergies, and the relative productivities of the two firms.

First, we can show that for CAT to occur there must be sufficiently large demand linkages between the products involved. For this result it does not matter whether the products are substitutes or complements. Second, we show that if the products are complements, CAT leads to larger quantities traded at lower prices, and if the products are substitutes, CAT leads to lower exports and higher prices. Our interpretation of this latter result is that CAT can produce outcomes that are identical to *product-specific, market-specific collusion.* Therefore, it is not clear that CAT is benign from the destination market point of view.

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8 Appendix

8.1 Proof of lemma 1

Lemma 1 consists of two parts. First we want to prove that the $\Delta\Pi(b,1,\xi,\theta)=0$ locus has two intersections $(\theta_1,\ \theta_2)$ with the θ-axis, where $\theta^{\min}<\theta_1<0<\theta_2<\theta^{\max}$. Then we need to prove that as ξ increases, the distance between θ_1 and θ_2 decreases, and there exists a $\xi^{\max}$ where the $\Delta\Pi(b,1,\xi,\theta)=0$ locus has a maximum in this diagram.

For both proofs it will be useful to define a new function $\Gamma(b,\xi,\theta)$ as a transformation of $\Delta\Pi(b,1,\xi,\theta)$:

$$\Gamma(b,\xi,\theta)\equiv 4\left[(b-1)\left(b-\xi^2\right)-(b\theta-\xi)^2\right][2(b-1)+\theta b]^2\,\Delta\Pi(b,1,\xi,\theta)$$

Since $\theta\in\left(\theta^{\min},\theta^{\max}\right)$ implies that $(b-1)\left(b-\xi^2\right)>(b\theta-\xi)^2$, the sign of $\Gamma(b,\xi,\theta)$ is the same as the sign of $\Delta\Pi(b,1,\xi,\theta)$, and $\Gamma(b,\xi,\theta)=0$ if and only if $\Delta\Pi(b,1,\xi,\theta)=0$.

This $\Gamma(b,\xi,\theta)$ function can be rewritten as a polynomial of degree three in θ:

$$\Gamma(b,\xi,\theta)=-2b^3\theta^3+b^2\left(2b-(\xi-1)^2\right)\theta^2+4b(b-1)\left(1-\xi^2-2\xi\right)\theta+4(b-1)\left[(\xi-1)^2-b\left(1-\xi^2-2\xi\right)\right].$$

This polynomial has (at most) three real roots $(\theta_1,\theta_2,\theta_3)$ and two local extrema $\left(\theta_1',\theta_2'\right)$ and goes to $+\infty$ when θ approaches $-\infty$ and to $-\infty$ when θ approaches $+\infty$. Thus, if they exist, θ_1' is a local minimum and θ_2' is a local maximum.

When evaluated at the corners $\theta^{\min}$ and $\theta^{\max}$, we obtain

$$\Gamma(b,\xi,\theta^{\max})=(1-\xi)(\xi+1)\left[(\xi+6(b-1))\xi+(b-1)^2\right]>0$$

and

$$\Gamma\left(b,\xi,\theta^{\min}\right)=\left(2b-1-\xi^2+2\sqrt{b-\xi^2}\sqrt{b-1}\right)\left(2b-2+\xi-\sqrt{b-\xi^2}\sqrt{b-1}\right)^2>0.$$

Since both $\Gamma(b,\xi,\theta^{\max})$ and $\Gamma\left(b,\xi,\theta^{\min}\right)$ are positive, $\theta_3>\theta^{\max}$ must hold and only two roots (θ_1 and θ_2) can be in the relevant range $\left(\theta^{\min},\theta^{\max}\right)$. Thus, we are left with three possible cases:

1. $\theta^{\min}<\theta^{\max}<\theta_1<\theta_1'$: In this case, CAT is always profitable.

2. $\theta^{\min}<\theta_1<\theta_1'<\theta_2<\theta^{\max}$: In this case, DOG is profitable if $\theta\in(\theta_1,\theta_2)$, CAT otherwise.

3. $\theta_2 < \theta^{\min} < \theta^{\max} < \theta_3$: In this case, CAT is always profitable.

The interesting case is case 2. This case requires that $\Gamma\left(b, \xi, \theta_1'\right) < 0$, i.e. that the value of the Γ function evaluated at the local minimum is negative.

To determine θ_1' we take the derivative

$$\frac{1}{2b}\Gamma_\theta\left(b, \xi, \theta\right) = -3b^2\theta^2 + b\left(2b - \left(\xi - 1\right)^2\right)\theta + 2\left(b - 1\right)\left(1 - \xi^2 - 2\xi\right).$$

This $\Gamma_\theta\left(b, 1, \xi, \theta\right)$-function has two roots, θ_1' and θ_2'. They are the local extrema of the Γ function, where θ_1' is a local minimum and θ_2' is a local maximum:

$$\begin{aligned}
\theta_1'\left(b, \xi\right) &= \frac{1}{3} - \frac{\left(\xi - 1\right)^2}{6b} - \frac{1}{6b}\sqrt{24\left(b - 1\right)\left(1 - \xi^2 - 2\xi\right) + \left(2b - \left(\xi - 1\right)^2\right)^2} \\
\theta_2'\left(b, \xi\right) &= \frac{1}{3} - \frac{\left(\xi - 1\right)^2}{6b} + \frac{1}{6b}\sqrt{24\left(b - 1\right)\left(1 - \xi^2 - 2\xi\right) + \left(2b - \left(\xi - 1\right)^2\right)^2}
\end{aligned}$$

To see whether Γ evaluated at the local minimum $\theta_1'\left(b, \xi\right)$ is negative, we begin by showing that it must be negative when $\xi = 0$:

$$\begin{aligned}
\theta_1'\left(b, 0\right) &= \frac{1}{3} - \frac{1}{6b} - \frac{1}{6b}\sqrt{24\left(b - 1\right) + \left(2b - 1\right)^2} < 0 \\
\Gamma\left[b, 0, \theta_1'\left(b, 0\right)\right] &= \frac{2b}{54}\left(188 - 98b\right) - \frac{181}{54} - \frac{1}{54}\left(\sqrt{20b + 4b^2 - 23} - 2b\right)\left(20b + 4b^2 - 23\right)
\end{aligned}$$

Since $b \geq 2$, it follows that $\Gamma\left[b, 0, \theta_1'\left(b, 0\right)\right] < 0$. Thus, if $\xi = 0$, we have case 2: $\theta^{\min} < \theta_1 < \theta_1' < \theta_2 < \theta^{\max}$.

In addition, we can prove that $\theta_1 < 0 < \theta_2$ by proving that $\Gamma\left(b, 0, \theta\right)$ evaluated at $\theta = 0$ is negative: $\Gamma\left(b, 0, 0\right) = -4\left(b - 1\right)^2 < 0$. This proves that $\theta^{\min} < \theta_1 < 0 < \theta_2 < \theta^{\max}$.

Finally, we take the derivative with respect to ξ:

$$\frac{\Gamma_\xi\left(b, \xi, \theta\right)}{8\left(b - 1\right)b} = \frac{\xi}{b} + \frac{b - 1}{b} - \theta + \xi\left(1 - \theta\right) + \frac{\left(1 - \xi\right)\theta^2 b}{4\left(b - 1\right)} > 0$$

Therefore, the DOG space becomes smaller when ξ rises.

If ξ rises, one of two things can happen. (1) The two extrema will collapse into a single inflection point. (2) The local minimum might persist as a minimum but can move above zero.

To prove that the DOG space disappears when ξ rises sufficiently, we introduce the following variable:

$$\Delta\theta' \equiv 3b\left[\theta_2'\left(b, \xi\right) - \theta_1'\left(b, \xi\right)\right] = \sqrt{\zeta\left(b, \xi\right)},$$

where $\zeta\left(b, \xi\right) \equiv 24\left(b - 1\right)\left(1 - \xi^2 - 2\xi\right) + \left(2b - \left(\xi - 1\right)^2\right)^2$.

(1) Note that $\zeta\left(b, \xi\right)$ evaluated at $\xi = 0$ is clearly positive: $\zeta\left(b, 0\right) = 20b + 4b^2 - 23 > 0$. Note also that $\zeta_\xi\left(b, \xi\right) = -4\left(b - 1\right)\left(10 + 14\xi\right) + 4\xi\left(1 - \xi\right)^2 - 4\left(\xi - 1\right)\left(\xi + 1\right) < 0$. Thus, if ξ rises, $\theta_2' - \theta_1'$ falls. If $\zeta\left(b, \xi\right)$ goes to zero, the polynomial has only an inflection point, but no local minimum. This implies that the Γ

function is monotonically decreasing and must be entirely positive in the area between $\theta^{\min}$ and $\theta^{\max}$. Thus, no DOG case exists.

Evaluate $\zeta(b,\xi)$ at $\xi = 1$ (the maximum ξ): $\zeta(b,1) = 4\left(b^2 - 12b + 12\right)$, This is negative if $b \in \left(6 - 2\sqrt{2}\sqrt{3}, 6 + 2\sqrt{2}\sqrt{3}\right)$. Thus, if $b \in \left(2, 2\sqrt{2}\sqrt{3} + 6\right)$ [note that $6 - 2\sqrt{2}\sqrt{3} = 1.101 < 2$], there exists a $\xi^{\max} \in (0,1)$ so that the Γ function is entirely positive in the area between $\theta^{\min}$ and $\theta^{\max}$ for all $\xi > \xi^{\max}$.

(2) If $b > 2\sqrt{2}\sqrt{3} + 6 = 10.899$, then the local minimum persists even if $\xi \to 1$. In this case we have to check whether the value of the Γ function evaluated at the minimum is positive. If it is, the Γ function has no roots between $\theta^{\min}$ and $\theta^{\max}$, and no DOG case exists.

Evaluate Γ at $\xi = 1$ and $\theta = \theta_1'(b,1) = \frac{1}{3}\frac{b-\sqrt{-12b+b^2+12}}{b}$:

$$\Gamma\left[b,1,\theta_1'(b,1)\right] = \frac{2}{27}\left(2b + \sqrt{-12b+b^2+12}\right)\left(24(b-1) + 2b\left(b - \sqrt{-12b+b^2+12}\right)\right) > 0.$$

Taking (1) and (2) together, we have proven that the DOG case disappears when ξ rises sufficiently.

8.2 Proof of Proposition 3

To prove that $\frac{\partial}{\partial\varphi}\Delta\Pi(b,1,\xi,\theta)\Big|_{\Delta\Pi=0} < 0$ we begin by taking the derivative of $\Delta\Pi(b,\varphi,\xi,\theta)$ with respect to φ:

$$\begin{aligned}\frac{\partial\Delta\Pi(b,\varphi,\xi,\theta)}{\partial\varphi}\frac{\varphi}{2} &= \frac{(b-1)\varphi^2 - (b\theta-\xi)\varphi}{4\left[(b-1)\left(b-\xi^2\right) - (b\theta-\xi)^2\right]} \\ &\quad - \frac{(b-1)\{2(b-1)\varphi[2(b-1)\varphi - b\theta] - b\theta\varphi(2(b-1) - b\theta\varphi)\}}{\left[4(b-1)^2 - b^2\theta^2\right]^2}\end{aligned}$$

$\frac{\partial}{\partial\varphi}\Delta\Pi(b,\varphi,\xi,\theta) < 0$ implies that

$$\frac{(b-1)\varphi^2 - (b\theta-\xi)\varphi}{4\left[(b-1)\left(b-\xi^2\right) - (b\theta-\xi)^2\right]} < \frac{(b-1)\{2(b-1)\varphi[2(b-1)\varphi - b\theta] - b\theta\varphi(2(b-1) - b\theta\varphi)\}}{\left[4(b-1)^2 - b^2\theta^2\right]^2}$$

Now evaluate inequality at $\Delta\Pi(b,\varphi,\xi,\theta) = 0$:

$$\frac{(b-1)\varphi^2 - 2(b\theta-\xi)\varphi + \left(b-\xi^2\right)}{4\left[(b-1)\left(b-\xi^2\right) - (b\theta-\xi)^2\right]} = \frac{(b-1)\left[(2(b-1) - b\theta\varphi)^2 + (2(b-1)\varphi - b\theta)^2\right]}{\left[4(b-1)^2 - b^2\theta^2\right]^2}$$

Then, this inequality can be reduced to

$$\frac{2(b-1)[2(b-1) - b\theta\varphi] - b\theta[2(b-1)\varphi - b\theta]}{2(b-1)[2(b-1)\varphi - b\theta] - b\theta[2(b-1) - b\theta\varphi]} < \frac{\left(b-\xi^2\right) - (b\theta-\xi)\varphi}{(b-1)\varphi - (b\theta-\xi)}$$

If $\varphi = 1$, this becomes

$$1 < \frac{\left(b - \xi^2\right) - \left(b\theta - \xi\right)}{\left(b - 1\right) - \left(b\theta - \xi\right)}$$

This clearly holds because $\xi^2 < 1$. This proves that $\frac{\partial}{\partial\varphi}\Delta\Pi\left(b, 1, \xi, \theta\right)\Big|_{\Delta\Pi=0} < 0$.

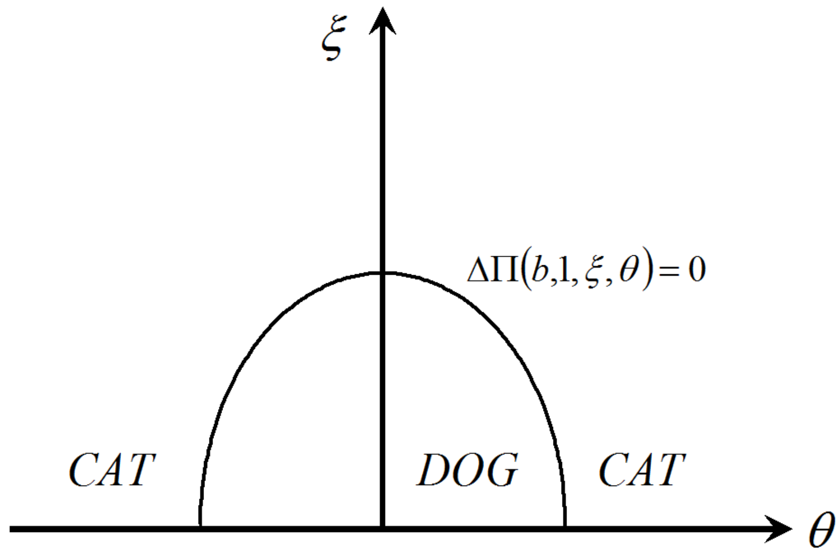

Figure 1: CAT versus DOG

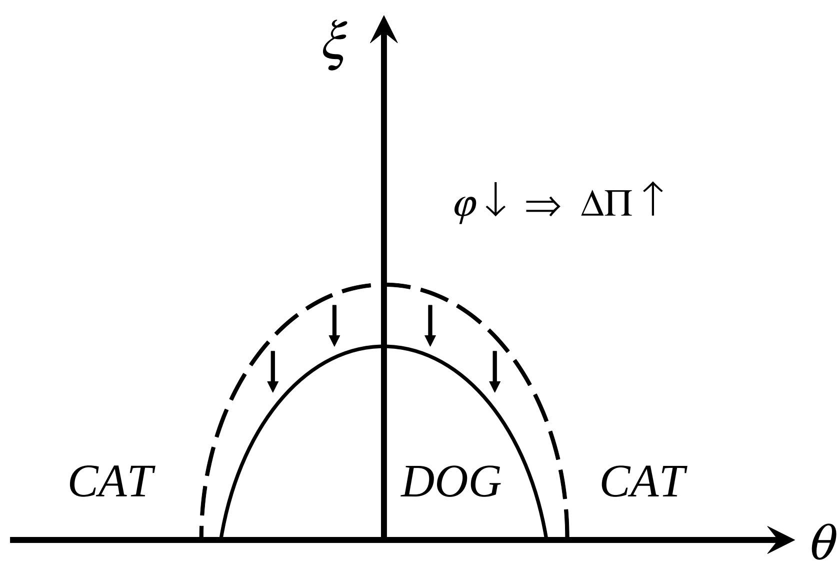

Figure 2: Effect of lower φ

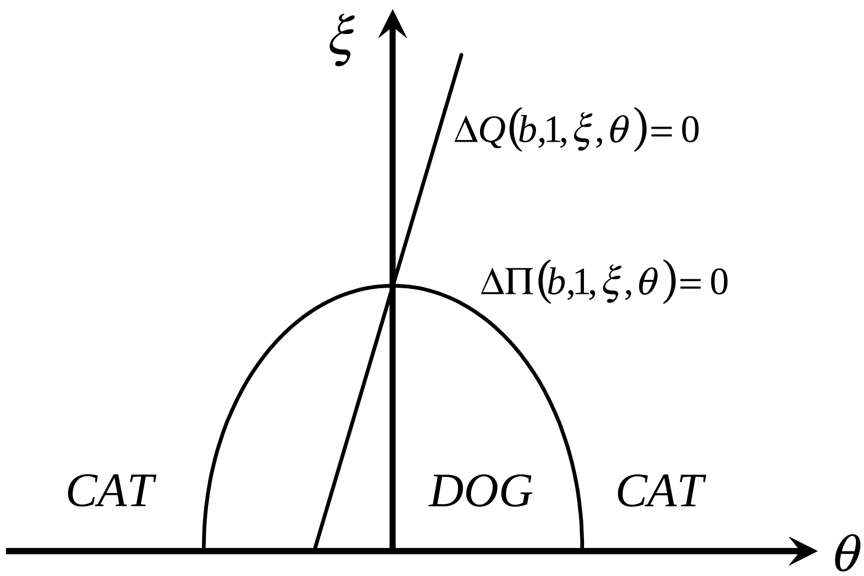

Figure 3: Changes in aggregate exports

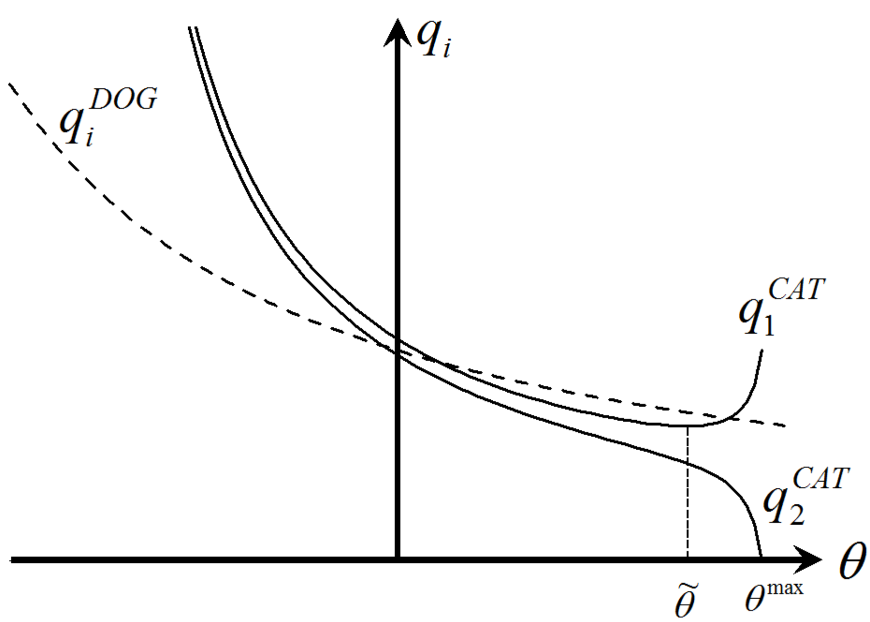

Figure 4: Changes in individual quantities

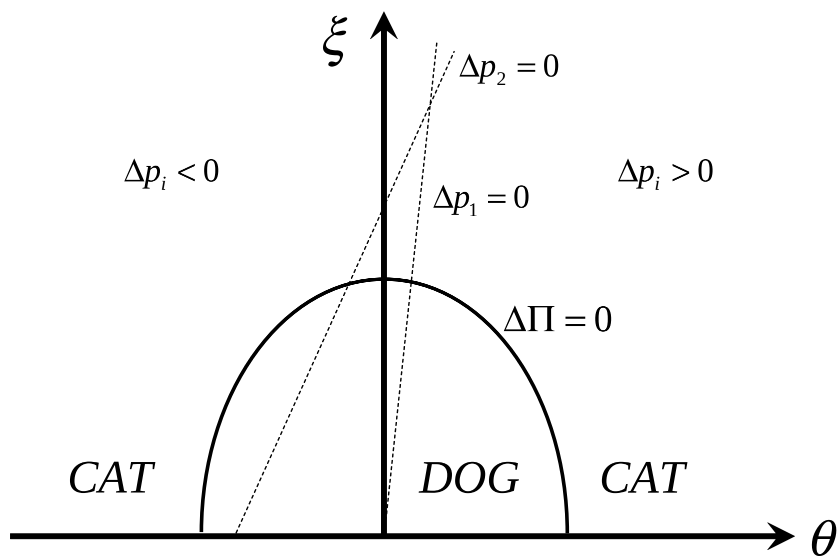

Figure 5: Prices