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INVESTMENT DEMAND AND STRUCTURAL CHANGE

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JEL Classification: E23, E21, O41

Keywords: Transitional Dynamics, Neo-classical Growth Model, structural change

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Investment Demand and Structural Change*

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1 Introduction

The economic development of nations begins with a rise in industrial production and a relative decline of agriculture, followed by a decrease of the industrial sector and a sustained increase of services.[1] Because this structural transformation is relatively slow and associated with long time periods, the recent growth literature has studied changes in the sectoral composition of growing economies along the balanced growth path, that is to say, in economies with constant investment rates.[2]

However, within the last 60 years a significant number of countries have experienced long periods of growth that may be well characterized by transitional dynamics. For instance, Song, Storesletten, and Zilibotti (2011) and Buera and Shin (2013) document large changes in the investment rate of China and the so-called Asian Tigers over several decades after their development process started. Interestingly, these same countries experienced a sharp pattern of sectoral reallocation during the same period, which suggests that deviating from the balanced growth path hypothesis might be relevant when thinking about the causes and consequences of structural transformation.

In this paper we look into the joint determination of the investment rate and the sectoral composition of developing economies. To do so, we start by documenting three novel facts. First, using a large panel of countries from the Penn World Tables, we show that the investment rate follows a long-lasting hump-shaped profile with development, and that the peak of the hump of investment happens at a similar level of development as the peak in the hump of industry. Second, using Input-Output (IO) tables from the World Input-Output Database (WIOD), we show that the set of goods used for final investment is different from the set of goods used for final consumption. Specifically, taking the average over all countries and years, 54% of the domestic value added used for final investment comes from the industrial sector, while 43% comes from services. In contrast, only 16% of domestic value added used for final consumption comes from industry, while 79% comes from services. Therefore, investment goods are 38 percentage points more intensive in value added from the industrial sector than consumption goods. And third, we document

[1] The description of this process traces back to contributions by Kuznets (1966) and Maddison (1991). See Herrendorf, Rogerson, and Valentinyi (2014) and references therein for a detailed description of the facts.

[2] Kongsamut, Rebelo, and Xie (2001) study the conditions for structural change due to non-unitary income elasticity of demand, while Ngai and Pissarides (2007) explore the role of non-unitary price elasticity of substitution and asymmetric productivity growth. Boppart (2014) shows that both mechanisms can be combined with more general preferences. Acemoglu and Guerrieri (2008) and Alvarez-Cuadrado, VanLong, and Poschke (2018) study structural change due to capital deepening with heterogeneous production functions across sectors. They find that, while structural change is incompatible with balanced growth path in theory, the aggregate dynamics are quantitatively close to a balanced growth path.

that there is structural change within both consumption and investment goods, but that the process is more intense within consumption goods. Furthermore, the standard hump-shaped profile of industry with development is absent when looking at investment and consumption goods separately.

We show that this set of facts are consequential for macroeconomic development. First, we propose a novel mechanism of structural transformation. Sectoral reallocation can happen within consumption and within investment due to the standard income and price effects, but it will also happen through the reallocation of expenditure between consumption and investment in transitional dynamics, i.e., through changes in the investment rate. For brevity, we call *intensive margin* of structural change the reallocation that happens *within* consumption and investment goods and call *extensive margin* of structural change the reallocation that happens by shifting expenditure *between* consumption and investment goods.[3] Second, different from standard models of structural change, asymmetric productivity growth may affect the transitional dynamics of the economy because it changes the relative price of investment goods. That is, the secular increase in manufacturing productivity makes investment goods cheaper, leading to faster capital accumulation and growth.

To understand the joint determination of investment, sectoral composition, and GDP growth along the development path, we build a multi-sector neo-classical growth model with a novel ingredient: we allow for the sectoral composition of the two final goods, consumption and investment, to be different and endogenously determined through the standard mechanisms of non-unitary income and price elasticities. Exploiting data from the big panel of countries that we use to provide the three main stylized facts described above, we use the demand system of the model to estimate the parameters characterizing the sectoral composition of investment and consumption goods. We next calibrate the parameters of the model driving the dynamics of the economy and, like Cheremukhin, Golosov, Guriev, and Tsyvinski (2017b), allow for a wedge in the Euler equation of consumption to get a perfect fit for the path of investment along the development process.

Our results are as follows. First, the model reproduces well the evolution of the sectoral composition of consumption and investment. The estimated demand system recovers price elasticities within both consumption and investment that are lower than one and income elasticities of consumption demand that are lower than one for agriculture and larger than one for both manufactures and services. Interestingly, during the first third of the

[3]The terms extensive and intensive margin represent a slight abuse of standard terminology: our extensive margin is not related to a 0-1 decision —countries always invest a positive amount— but to the change in the relative importance of consumption vs. investment.

development process the income elasticity of consumption demand is substantially larger for manufactures than for services.

Second, the model also reproduces well the stylized evolution of the sectoral composition of GDP along the development path, and in particular the hump in manufacturing. We find that the extensive margin of structural change explains 1/2 of the increase and 1/2 of the fall of manufacturing with development. That is, the hump of investment rate produced by the model generates half of the hump in manufacturing. A full account of the manufacturing hump is as follows. During the first half of the development process the increase in the investment rate and an income elasticity of demand of manufactures within consumption larger than one raise the overall size of the industrial sector, despite the secular improvement in its technology and the low elasticity of substitution across goods. The decline of manufacturing in the second half of the development process is explained by the investment decline and the continued relative improvement in technology within the industrial sector, which shifts productive resources towards services.

Third, we find that the secular increase of productivity in the industrial sector relative to services accounts for most of the observed fall in the relative price of investment with development. The decline in the relative price of investment turns out to have small effects in shaping the investment rate at current prices, but it increases investment in real units, fostering capital accumulation and growth. In standard models of structural change asymmetric sectoral productivity growth is a drag for growth because it induces reallocation of production factors from manufacturing to services (the well-known Baumol (1967) cost disease). We find that, by making investment goods cheaper, asymmetric sectoral productivity growth is a net contributor to growth along the development path because the investment channel prevails over the Baumol cost disease.

Finally, a full account of the investment hump requires a wedge distorting the Euler equation of consumption. The wedge starts at 18% and declines monotonically during the first half of the development process, staying close to zero afterwards. We can think of this declining wedge as reflecting financial development that improves along the development path. The positive empirical relationship between financial and economic development is well established, see for instance a review of the empirical literature in Levine (2005). Standard explanations would be that financial development allows to diversify idiosyncratic investment risks or to lessen capital misallocation across heterogeneous producers, which in both cases could increase investment demand for a given marginal product of capital. Yet, other explanations for a declining wedge are possible. For instance, the wedge could reflect the need for a more elaborate model of saving with either more general preferences, an explicit role for demographic transitions, or declining capital gains in land's value.

There is a number of papers describing economic mechanisms that could potentially generate a hump in manufacturing for closed economies. Within the relative price effect explanations of structural change, the Ngai and Pissarides (2007) model with different and constant rates of growth in sectoral productivities may lead to humps in the sectoral composition of consumption for those sectors with intermediate rates of productivity growth. This is a mechanism also operating in our model, but with time-changing sectoral productivities for both consumption and investment. Within the income effect explanations of structural change, the model with generalized Stone-Geary preferences of Kongsamut, Rebelo, and Xie (2001) may potentially generate a hump in transitional dynamics if one moves away from the assumptions that guarantee existence of a balanced growth path with structural change. Indeed, our model featuring these type of preferences allows for a mild hump within consumption. Other ways of modelling non-homotheticicities that can generate the hump in manufacturing are for instance the hierarchic preferences in Foellmi and Zweimuller (2008), the non-homothetic CES preferences in Comin, Lashkari, and Mestieri (2020), or the intertemporally aggregable preferences in Alder, Boppart, and Muller (2019). Buera and Kaboski (2012b) combine non-homothetic demands with sectoral technologies that differ on scale. All these mechanisms require the hump of manufacturing to be strong within consumption goods. The extensive margin of structural change that we emphasize, however, allows for the share of manufacturing to be hump-shaped within GDP with mild or no hump within consumption. Our empirical evidence finds hump-shaped profiles of the share of manufacturing value added within GDP that are sharper than within consumption. We take this as evidence in favor of the extensive margin channel.

Closely to our work, the contemporaneous paper by Herrendorf, Rogerson, and Valentinyi (2020) measures the evolution of the sectoral shares within consumption and investment by use of the long time series of IO data for the US. Their results resemble our findings both in WIOD and WDI-G10S data. Both their and our paper emphasize the importance of properly accounting for the sectoral composition of investment goods when analyzing structural transformation and its macroeconomic consequences. Our paper differs from theirs in one fundamental aspect. We focus on understanding structural change in contexts where the extensive margin matters, while they concentrate on the US, whose dynamics are reasonably close to a balanced growth path for the 1947-2015 period. In that sense, we model and estimate the joint determination of the sectoral composition of the economy and the investment rate, while their paper focuses on estimating the mechanisms operating on the intensive margin only. Additionally, their focus is on characterizing the balanced growth path properties of their structural model. In particular, they show that balanced growth path definition imposes a non-linear restriction on the evolution of sectoral TFP, and find that this restriction holds for the analyzed period in the US. To our knowledge, they are also the first ones to use the terms intensive and extensive margins of structural change, which we have borrowed for this version of our paper.

The remaining of the paper is organized as follows. In Section 2 we show the key empirical facts that motivate the paper. In Section 3 we show how changes in the investment rate account for large changes in the sectorial composition of the economies in the WIOD. In Section 4 we outline the model. In Section 5 we discuss the estimation of its static demand system, the calibration of its dynamic side, and provide several counterfactual exercises to understand the joint evolution of GDP, investment, and sectoral composition of the economy. Finally, Section 6 concludes.

2 Some Facts

In this section we present empirical evidence of the three key facts that motivate the paper. As it is standard in this literature, we divide the economy in three sectors: agriculture, industry, and services, and use the term manufacturing and industry interchangeably to denote the second of them, which includes: mining, manufacturing, electricity, gas, and water supply, and construction.[4]

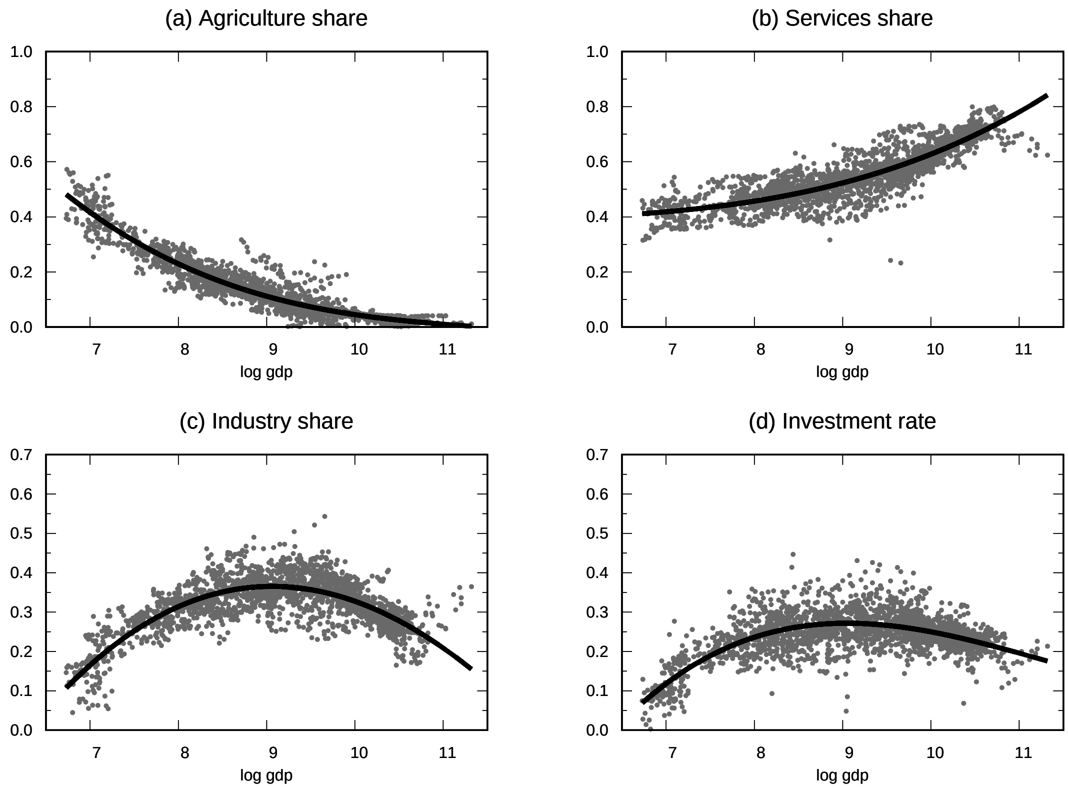
2.1 The investment rate and the sectoral composition of the economy

First, we want to characterize the evolution of investment rate with development and its relationship with the sectoral composition of the economy. To do so, we use investment data from the Penn World Tables (PWT) and sectoral data from the World Development Indicators (WDI) and the Groningen 10-Sector Database (G10S) for a large panel of countries.[5] We pool together the data of all countries and years and filter out cross-country differences in levels by regressing the investment rate or the sectoral composition of the economy against a low order polynomial of log GDP per capita in international dollars and country fixed effects. In Figure 1 we plot the resulting polynomial of log GDP (solid black line) for each variable of interest together with each country-year observation after filtering out the country fixed effects, see Appendix C for details.

[4] See Appendices A and B for details.

[5] See Section 5.1 for details on the data series and the sample construction. Feenstra, Inklaar, and Timmer (2015) and Timmer, de Vries, and de Vries (2014) provide a full description of the PWT and G10S respectively.

Figure 1: Sectoral shares, investment rate, and the level of development



Notes. Sectoral shares from G10S and WDI and investment rate from PWT —all at current prices— (dots) and projections on a cubic polynomial of log GDP per capita in constant international dollars (lines). Data have been filtered out from country fixed effects.

In Panels (a) and (b) we observe the well-known monotonically declining and rising patterns of agriculture and services, while in Panel (c) we observe the clear hump-shaped profile of the value added share of industry. Next, in Panel (d) we plot the investment rate. We observe a clear hump-shaped profile of investment with the level of development: poor countries invest a small fraction of their output, but as they develop the investment rate increases up to a peak and then it starts declining. Note that the hump is long-lived (it happens while GDP multiplies by a factor of 100), it is large (the investment rate increases by 20 percentage points), and it is present for a wide sample of countries (49 countries at very different stages of development). A hump of investment with the level of development has already been documented with relatively short country time series for the Asian Tigers, (see Buera and Shin (2013)), and Japan and OCDE countries after the IIWW (see Christiano (1989), Chen, Imrohoroğlu, and Imrohoroğlu (2007) and Antràs (2001)). Here we show this pattern to be very systematic. Furthermore, we can see

Table 1: Sectoral composition of investment and consumption goods.

	Investment			Consumption			Difference		
	Agr	Ind	Ser	Agr	Ind	Ser	Agr	Ind	Ser
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
mean	3.1	53.7	43.2	5.3	15.8	78.9	−2.2	37.9	−35.7
p_{10} *(NLD)*	0.6	40.0	59.4	0.6	9.1	90.3	−0.0	30.9	−30.8
p_{50} *(DEU)*	1.3	50.7	48.0	0.8	13.7	86.6	0.5	37.1	−37.6
p_{90} *(BRA)*	6.7	61.1	32.2	4.6	18.4	77.1	2.2	42.7	−44.9

Notes: The first row reports the average over all countries and years of the value added shares of investment goods, consumption goods, and their difference, data from WIOD. The next rows report the average over time of three particular countries (Netherlands, Germany, and Brazil). These countries are chosen as the 10th, 50th, and 90th percentiles of the distribution of the differential intensity of industrial sector between investment and consumption goods.

that the hump in industrial production in Panel (c) is very similar in size to the hump in investment in Panel (d), with the peak happening at a similar level of development (around 8,100 international dollars of 2005, this would be Japan in 1966, Portugal in 1971, South Korea in 1986, or Thailand in 1995). Indeed, the correlation between the value added share of industry and the investment rate is 0.43 in the raw data pooling all countries and years, and 0.51 when controlling for country fixed effects.

2.2 Sectoral composition of investment and consumption goods

The second piece of evidence that we put together is the different sectoral composition of the goods used for final investment and final consumption. We use the World Input Output Database (WIOD), which provides IO tables for 35 sectors, 17 years (between 1995 and 2011), and 40 (mostly developed) countries.[6] To give an example of what we do, consider how final investment goods may end up containing value added from the agriculture sector. Agriculture goods are sold as final consumption to households and as exports, but not used directly for gross capital formation. However, most of the output from the agriculture sector is sold as intermediate goods to several industries (e.g., "Textiles") that are themselves sold to other industries (e.g., "Transport Equipment") whose output goes to final investment. In short, agricultural value added is indirectly an input into investment goods. In Appendix B we explain how to obtain the sectoral composition of each final good following the procedure explained by Herrendorf, Rogerson, and Valentinyi (2013).

[6] A detailed explanation of the WIOD can be found in Timmer, Dietzenbacher, Los, Stehrer, and de Vries (2015). Our sample selection excludes 8 of the 40 countries, see Section 5.1, but results are very similar when using the full 40 country sample.

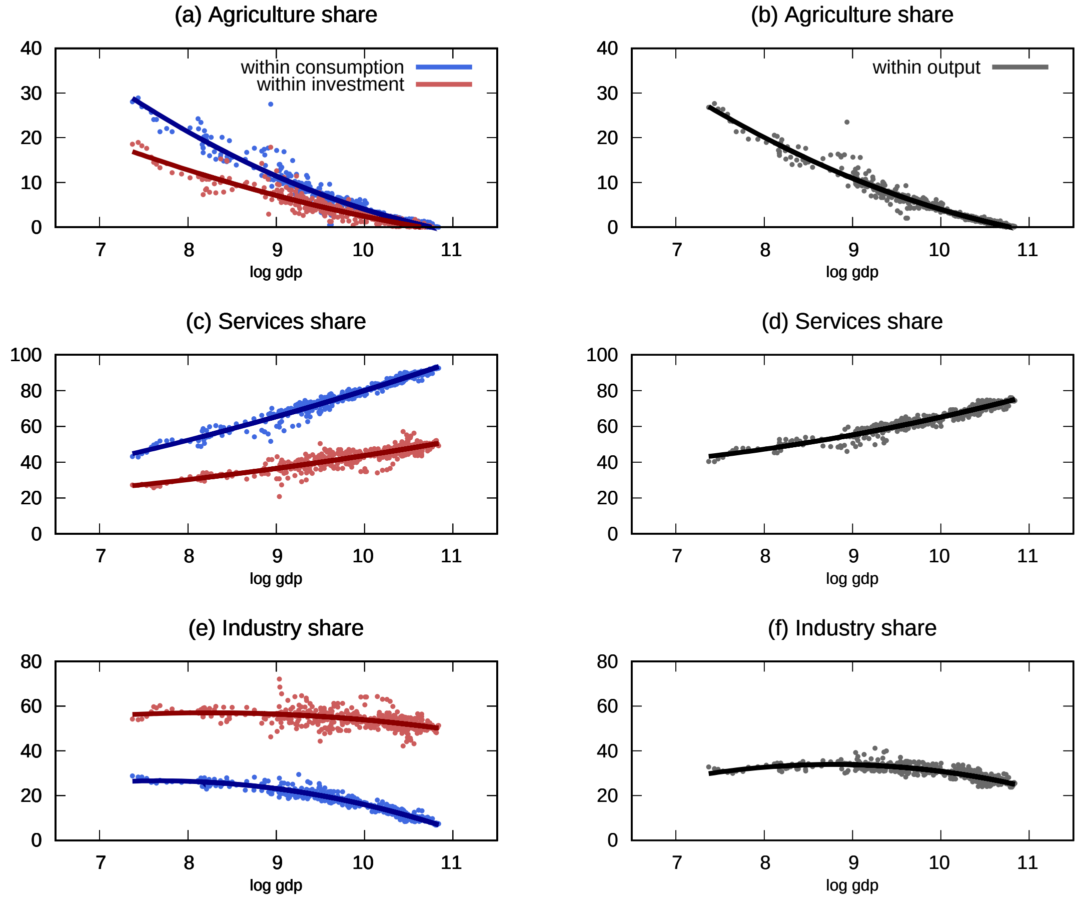
We find that investment goods are more intensive in industrial value added than consumption goods are, see Table 1. In particular, taking the average over all countries and years, the value added share of industry is 54% for investment goods (column 2) and 16% for consumption goods (column 5), a difference of 38 percentage points (column 8). The flip side of this difference is apparent in services, which represents 43% of investment goods (column 3) and 79% of consumption goods (column 6). There is some cross-country heterogeneity, but the different sectoral composition between investment and consumption goods is large everywhere. For instance, investment has 31 percentage points more of value added from manufacturing than consumption in Netherlands (the 10% lowest in the sample) and almost 43 percentage points in Brazil (the 10% highest).

2.3 Evolution of the sectoral composition of consumption and investment

The third piece of evidence we want to emphasize is the evolution of the sectoral composition of investment and consumption goods with the level of development. In particular, we show that (a) there is structural change within both investment and consumption goods, but it is stronger within consumption goods, and (b) the standard hump-shaped profile of manufacturing with development is more apparent for the whole economy than for the investment and consumption goods separately.

To document these facts we pool the WIOD data for all countries and years and exploit its within-country dimension by regressing sectoral shares against a polynomial of log GDP per capita in international dollars and country fixed effects. In Figure 2, we plot the resulting sectoral composition for investment (red), consumption (blue), and total output (black) against log GDP per capita. We first observe that the WIOD is consistent with the standard stylized facts of structural change: for the whole GDP there is a secular decline of agriculture, a secular increase in services, and a (mild) hump of manufacturing. When looking at the pattern of sectoral reallocation within each good, we observe that the share of agriculture declines faster in consumption than in investment, that the share of services increases faster in consumption than in investment, and that the share of manufacturing declines somewhat faster in consumption than in investment. These patterns imply that structural change is sharper within consumption than within investment and that the asymmetry between consumption and investment goods in terms of their content of manufacturing and services widens with development. Finally, it is important to note that the hump of manufacturing within GDP is happening neither within investment (the quadratic and higher order terms are non-significant) nor within consumption (the increasing part is missing). The comparison of the share of manufacturing within invest-

Figure 2: Sectoral shares for different goods, within-country evidence



Notes. Sectoral shares from WIOD (dots) and projections on a low-order polynomial of log GDP per capita in constant international dollars (lines). Data have been filtered out from country fixed effects.

ment and consumption with the share of manufacturing for the whole GDP is more clear in Panel (a) of Figure B.1 in Appendix B, which puts together the pics in Panel (e) and (f) of Figure 2.

3 A novel mechanism for structural change

The facts described above highlight the potential importance of the composition of final expenditure for structural change, and suggest a possible explanation for the hump in manufacturing. Standard forces of structural change like non-homotheticities and asymmetric productivity growth may explain sectoral reallocation within investment and within consumption goods. But because investment goods are more intensive in value added from manufacturing than consumption goods, the hump-shaped profile of the investment rate generates a further force of structural change. Consistent with this mechanism, the hump of manufacturing is more apparent for the whole economy than for the consumption and investment goods separately.

While the WIOD data may not be ideal to study structural change because of the short time dimensions and the small number of developing countries, we can still use it to have a first assessment of our mechanism. To do so we start by using National Accounts identities to note that the value added share of sector i within GDP can be written as,

$$\frac{\mathrm{VA}_i}{\mathrm{GDP}} = \left(\frac{\mathrm{VA}^x}{\mathrm{GDP}}\right)\left(\frac{\mathrm{VA}_i^x}{\mathrm{VA}^x}\right) + \left(\frac{\mathrm{VA}^c}{\mathrm{GDP}}\right)\left(\frac{\mathrm{VA}_i^c}{\mathrm{VA}^c}\right) + \left(\frac{\mathrm{VA}^e}{\mathrm{GDP}}\right)\left(\frac{\mathrm{VA}_i^e}{\mathrm{VA}^e}\right) \tag{1}$$

which is a weighted average of the sectoral share within investment $\mathrm{VA}_i^x/\mathrm{VA}^x$, within consumption $\mathrm{VA}_i^c/\mathrm{VA}^c$, and within exports $\mathrm{VA}_i^e/\mathrm{VA}^e$. The first two are the objects that we have documented in Table 1 and in Panel (a), (c), and (e) of Figure 2. The weights are the domestic investment rate $\mathrm{VA}^x/\mathrm{GDP}$, the domestic consumption rate $\mathrm{VA}^c/\mathrm{GDP}$, and the domestic exports rate $\mathrm{VA}^e/\mathrm{GDP}$. The domestic investment rate (and analogously the domestic consumption and export rates) is the ratio over GDP of the domestic valued added that is used for final investment. This is different from the investment spending over GDP of National Accounts, X/GDP, because part of the investment spending buys imported valued added (either directly by importing final investment goods, or indirectly by importing intermediate goods that will end up in investment through the IO structure of the economy). Indeed, one can write:

$$\frac{\mathrm{VA}^x}{\mathrm{GDP}} = \frac{\mathrm{VA}^x}{X}\frac{X}{\mathrm{GDP}}; \quad \text{and} \quad \frac{\mathrm{VA}^c}{\mathrm{GDP}} = \frac{\mathrm{VA}^c}{C}\frac{C}{\mathrm{GDP}}; \quad \text{and} \quad \frac{\mathrm{VA}^e}{\mathrm{GDP}} = \frac{\mathrm{VA}^e}{E}\frac{E}{\mathrm{GDP}};$$

where X, C, and E are the expenditure in investment, consumption, and exports. While by construction the domestic investment rate will be weakly smaller than the actual investment rate, the evolution of both magnitudes presents a similar hump with the level of development, see Panel (b) of Figure B.1 in Appendix B. Hence, structural change can happen because there is a change in the sectoral composition of investment, consumption or export goods (the intensive margin) or because there is a change in the investment, consumption or export demand of the economy (the extensive margin).

To decompose the evolution of sectoral shares into the intensive and extensive margins, we do two complementary exercises. In both exercises we build two counterfactual series for each sectoral share of the economy, in which only the intensive or extensive margin are active. In the first exercise, which we call "open economy", the intensive margin counterfactual holds the $\mathrm{VA}^j/\mathrm{GDP}$ ($j = \{x, c, e\}$) terms of the right hand side of equa-

Table 2: Decomposition of structural change.

	Data	Open economy			Closed economy		
	Data	All	Int	Ext	All	Int	Ext
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Agriculture	-25.3	-25.3	-23.0	-2.3	-25.4	-22.4	-3.0
Industry	-6.8	-6.8	-17.9	11.0	-8.4	-17.5	9.2
Increase	2.2	2.2	-3.3	5.5	4.5	-2.9	7.4
Decrease	-9.0	-9.0	-14.6	5.6	-12.8	-14.7	1.8
Services	32.1	32.1	40.9	-8.7	33.8	39.9	-6.2

Notes: rows "Agriculture", "Industry", and "Services" show the change in percentage points of the corresponding sectoral share for the entire development process. Rows "Increase" and "Decrease" refer to the changes in the size of "Industry" during the increasing and decreasing parts of the development process respectively (in terms of the share of industrial sector). The Data column reports the change implied by the polynomial of log GDP in Panel (b), (d), and (f) of Figure 1. The other columns report the same statistic for several counterfactual series, see text and footnote 7.

tion (1) equal to their country averages, while the extensive margin counterfactual holds constant the $\mathrm{VA}_i^j/\mathrm{VA}^j$ ($j = \{x, c, e\}$) terms. In the second exercise, which we call "closed economy", we first build counterfactual sectoral shares omitting exports and imports as follows,

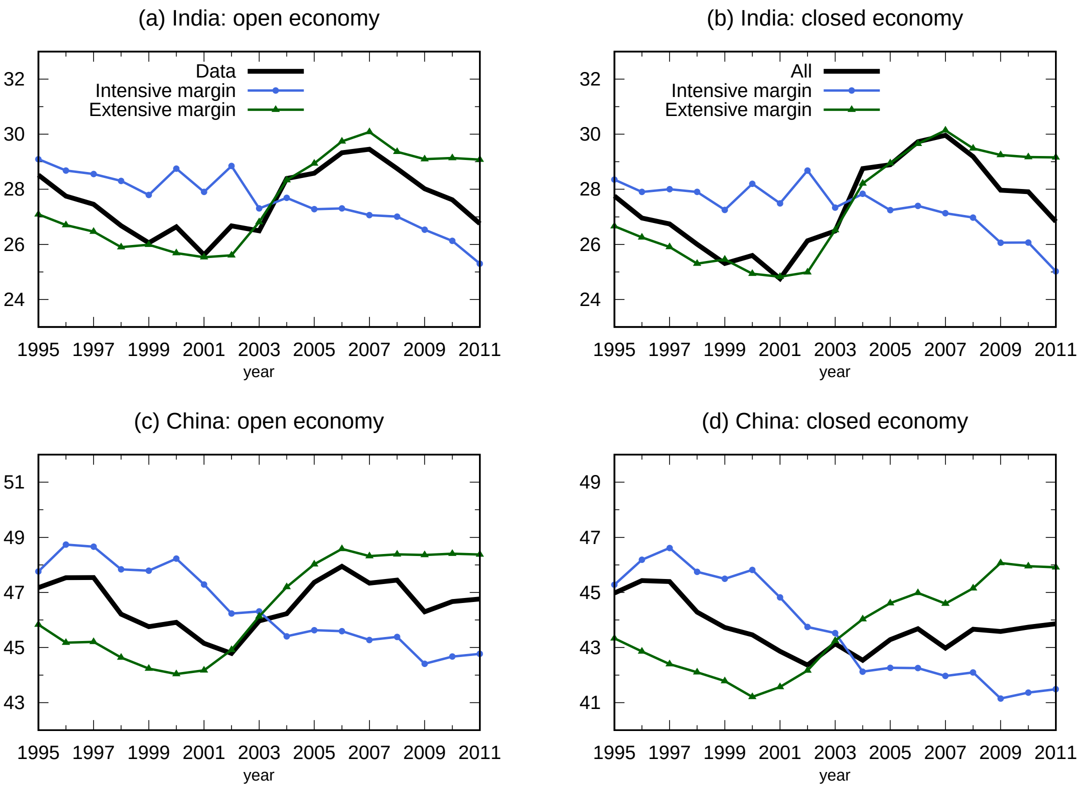
$$\frac{\widehat{\mathrm{VA}_i}}{\mathrm{GDP}} = \frac{X}{X+C}\left(\frac{\mathrm{VA}_i^x}{\mathrm{VA}^x}\right) + \frac{C}{X+C}\left(\frac{\mathrm{VA}_i^c}{\mathrm{VA}^c}\right) \tag{2}$$

Then, we build the intensive margin counterfactual by holding the $\frac{X}{X+C}$ and $\frac{C}{X+C}$ terms in equation (2) equal to their average and the extensive margin counterfactual by holding constant the $\mathrm{VA}_i^j/\mathrm{VA}^j$ ($j = \{x, c\}$) terms.

We report in Table 2 the average importance of the intensive and extensive margin of structural change across the 32 countries and 17 years. In the first column, we report the average change in the share of Agriculture (decline of 25.3 percentage points), Industry (decline of 6.8 percentage points, which comes from an initial increase of 2.2 followed by a decline of 9.0 percentage points), and Services (increase of 32.1 percentage points) across all countries and years as described in Figure 2. In the third and fourth columns, we report the change accounted for by the intensive and extensive margins in the "open economy" exercise.[7] We find that the extensive margin is important for the evolution of the industrial and service sectors. For instance, sectoral reallocation within consumption, investment, and exports would have implied a decline of industry value added of 17.9 percentage points, a fall 11 percentage points larger than what we observe. Instead, the

[7]These changes comes from treating the counterfactual series as the actual data: we pool all years and countries together and keep the relationship between sectoral share and log GDP after filtering out country fixed effects.

Figure 3: Industrial share of GDP: India and China



Notes. The black lines correspond to the actual share of industrial value added in GDP in Panels (a) and (c), while they correspond to the counterfactual series according to equation (2) in Panels (b) and (d). See text for the extensive and intensive margin decomposition.

variation in investment, consumption, and export rates pulled the demand for industrial value added upwards for those 11 percentage points. In the fifth column, we report the changes in sectoral shares implied by the "closed economy" through equation (2). We see that the sectoral shares of the closed economy pose a good approximation to the actual ones, with the implied changes in the relative size of sectors differing from the actual ones in less than two percentage points for industry and services and less than one percentage point for agriculture. In the sixth and seventh columns, we report the decomposition in the "closed economy" exercise, which abstracts from movements of imports, exports, and of their composition. The results still show the importance of the extensive margin in the evolution of the services and manufacturing shares.

Not all countries have experienced large changes in the investment rate over the short period covered by the WIOD. To highlight the importance of the extensive margin of structural change for some countries and years, we analyze the evolution of the share of the industrial sector in India and China. In Figure 3 we report the counterfactual exercises for the "open economy" —panels (a) and (c)— and the "closed economy" —panels (b) and (d)— exercises. We can see that in both countries and for both exercises the intensive margin (blue line) predicts a steady decline of manufacturing of around 4 percentage points in the space of 17 years. However, the actual sectoral evolution in these countries has no trend (black line) as both countries experienced a sharp increase in manufacturing between 2002 and 2006, which is completely explained by the extensive margin (green line).

4 The Model

In the previous Section we have seen how changes in the investment rate can account for a big fraction of the observed sectoral changes with development. In order to understand where these changes in the investment rate come from and how they interact with the standard income and price effects of structural change, we build a multi-sector neoclassical growth model for a closed economy with one distinct characteristic.[8] Namely, we allow for the sectoral composition of the two final goods, consumption and investment, to be different and endogenously determined. This is needed to have an operative extensive margin of structural change and an endogenous relative price of investment driving the dynamics of the investment rate.

4.1 Set up

The economy consists of three different sectors that produce intermediate goods: agriculture, manufacturing, and services, indexed by $i = \{a, m, s\}$. Output y_{it} of each sector can be used both for final consumption c_{it} and for final investment x_{it}. An infinitely-lived representative household rents capital k_t and labor (normalized to one) to firms, and chooses how much of each good to buy for consumption and investment purposes while satisfying the standard budget constraint:

$$w_t + r_t k_t = \sum_{i=\{a,m,s\}} p_{it} \left(c_{it} + x_{it} \right) \tag{3}$$

[8]We study a closed economy where the investment rate equals the savings rate. This equality does not hold in the data for every country and year but it is a reasonable approximation: Feldstein and Horioka (1980) famously documented a very strong cross-country correlation between investment and savings, Aizenman, Pinto, and Radziwill (2007) showed that capital accumulation of developing economies is mainly self-financed through internal savings, and Faltermeier (2017) shows that the decline of the marginal product of capital with development is unrelated to capital flows.

where p_{it} is the price of output of sector i at time t, w_t is the wage rate, and r_t is the rental rate of capital faced by firms. Capital accumulates with the standard law of motion

$$k_{t+1} = (1-\delta)\, k_t + x_t \tag{4}$$

where $0 < \delta < 1$ is a constant depreciation rate, and $x_t \equiv X_t(x_{at}, x_{mt}, x_{st})$ is the amount of efficiency units of the investment good produced with a bundle of goods from each sector. The period utility function is defined over a consumption basket $c_t \equiv C(c_{at}, c_{mt}, c_{st})$ that aggregates goods from the three sectors. We specify a standard CES aggregator for investment, whereas we also allow for non-homotheticities in consumption:

$$C(c_a, c_m, c_s) = \left[\sum_{i\in\{a,m,s\}} (\theta_i^c)^{1-\rho_c} (c_i + \bar{c}_i)^{\rho_c}\right]^{\frac{1}{\rho_c}} \tag{5}$$

$$X_t(x_a, x_m, x_s) = \chi_t \left[\sum_{i\in\{a,m,s\}} (\theta_i^x)^{1-\rho_x} \quad x_i^{\rho_x}\right]^{\frac{1}{\rho_x}} \tag{6}$$

with $\rho_j < 1$, $0 < \theta_i^j < 1$ and $\sum_{i\in\{a,m,s\}} \theta_i^j = 1$ for $j \in \{c, x\}$, $i \in \{a, m, s\}$. These two aggregators differ in several dimensions. First, we allow the sectoral share parameters in consumption θ_i^c to differ from the sectoral share parameters in investment θ_i^x. Second, we introduce the terms $\bar{c}_i$ in order to allow for non-homothetic demands for consumption. Much of the literature has argued that these non-homotheticities are important to fit the evolution of the sectoral shares of GDP, and non-unitary income elasticities have been estimated in the micro data of household consumption. We omit similar terms in the investment aggregator partly due to the difficulty to separately identify them from $\bar{c}_i$ in the data and partly due to the lack of micro-evidence.[9] Third, we allow the elasticity of substitution, given by $1/(1-\rho_j)$, to differ across goods. Finally, χ_t captures exogenous investment-specific technical change, a feature that is shown to be quantitatively important in the growth literature, see Greenwood, Hercowitz, and Krusell (1997) or Karabarbounis and Neiman (2014). Note that the literature of structural change has

[9] Agricultural goods are typically modelled as a necessity because of the strong decline in the share of agriculture with development. Emphasizing this non-homotheticitiy within consumption goods is also consistent with the micro data evidence showing that the budget share for food decreases as household income increases. See for instance Deaton and Muellbauer (1980), Banks, Blundell, and Lewbel (1997), or Almås (2012). Services instead are typically modelled as luxury goods because their share increases with development. A typical interpretation is that services have easy home substitutes and households only buy them in the market after some level of income. See for instance Rogerson (2008) and Buera and Kaboski (2012a).

typically assumed that either the aggregators for consumption and investment are the same, that the investment goods are only produced with manufacturing value added, or that the investment good is a fourth type of good produced in a fourth different sector.[10]

4.2 Household problem

Households have a CRRA utility function over the consumption basket c_t,

$$u\left(c_t\right) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} \tag{7}$$

The optimal household plan is the sequence of consumption and investment choices that maximizes the discounted infinite sum of utilities.[11] Whenever the inequality constraints $c_{it} \geq 0$, $x_{it} \geq 0$ are not binding the problem can be split into (a) the static optimal composition of consumption and investment expenditure, and (b) the dynamic choice of consumption *vs* investment. In particular, the optimal composition of consumption and investment expenditures are given by,

$$\frac{p_{it}c_{it}}{\sum_{j=a,m,s} p_{jt}c_{jt}} = \left[\sum_{j=a,m,s} \frac{\theta_j^c}{\theta_i^c}\left(\frac{p_{it}}{p_{jt}}\right)^{\frac{\rho_c}{1-\rho_c}}\right]^{-1}\left[1 + \frac{\sum_{j=a,m,s} p_{jt}\bar{c}_j}{\sum_{j=a,m,s} p_{jt}c_{jt}}\right] - \frac{p_{it}\bar{c}_i}{\sum_{j=a,m,s} p_{jt}c_{jt}} \tag{8}$$

$$\frac{p_{it}x_{it}}{\sum_{j=a,m,s} p_{jt}x_{jt}} = \left[\sum_{j=a,m,s} \frac{\theta_j^x}{\theta_i^x}\left(\frac{p_{it}}{p_{jt}}\right)^{\frac{\rho_x}{1-\rho_x}}\right]^{-1} \tag{9}$$

where it is apparent that the sectoral shares within investment only depend on relative prices, while the sectoral shares within consumption depend on both relative prices and the overall level of expenditure. The value of the consumption and investment expenditure are related to the baskets by,

$$\sum_{i=a,m,s} p_{it}c_{it} = p_{ct}c_t - \sum_{i=a,m,s} p_{it}\bar{c}_i \tag{10}$$

$$\sum_{i=a,m,s} p_{it}x_{it} = p_{xt}x_t \tag{11}$$

[10] An example of the first case is Acemoglu and Guerrieri (2008), examples of the second case are Echevarría (1997), Kongsamut, Rebelo, and Xie (2001) or Ngai and Pissarides (2007), while examples of the third case are Boppart (2014) or Comin, Lashkari, and Mestieri (2020). Instead, García-Santana and Pijoan-Mas (2014) and Herrendorf, Rogerson, and Valentinyi (2020) already allow for a different composition of investment and consumption goods. The former paper measures this different composition in a calibration exercise with Indian data, while Herrendorf, Rogerson, and Valentinyi (2020) estimates it with Input-Output data for the U.S.

[11] See Appendix E for the full derivation of the model solution.

where the implicit prices for the consumption and investment baskets are given by,

$$p_{ct} \equiv \left[\sum_{i=a,m,s} \theta_i^c \, p_{it}^{\frac{\rho_c}{\rho_c-1}}\right]^{\frac{\rho_c-1}{\rho_c}} \tag{12}$$

$$p_{xt} \equiv \frac{1}{\chi_t}\left[\sum_{i=a,m,s} \theta_i^x \, p_{it}^{\frac{\rho_x}{\rho_x-1}}\right]^{\frac{\rho_x-1}{\rho_x}} \tag{13}$$

Finally, the Euler equation driving the dynamics of the model is given by,

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \left(1+\tau_t\right)^{-1} \frac{p_{xt+1}}{p_{ct+1}} \frac{p_{ct}}{p_{xt}} \left[\frac{r_{t+1}}{p_{xt+1}} + (1-\delta)\right] \tag{14}$$

This states that the value of one unit of consumption today must equal the value of transforming that unit into capital, renting the capital to firms, and consuming the proceeds next period. The term in square brackets in the right-hand-side is the investment return in units of the investment good. When divided by the increase in the relative price of consumption it becomes the investment return in units of the consumption good, which is the relevant one for the Euler equation. The introduction of the wedge τ_t follows Cole and Ohanian (2002), Chari, Kehoe, and McGrattan (2007) and Cheremukhin, Golosov, Guriev, and Tsyvinski (2017b) to capture in reduced form time-varying misalignment between the data and the intertemporal Euler equation. As it is well-known, the standard one sector neo-classical growth model with Cobb-Douglas production, time-separable CRRA utility, and constant productivity growth cannot generate a hump-shaped path of investment along the transitional dynamics, see Barro and Sala-i-Martin (1999). Our model with non-homothetic consumption demands for different sectors and time-changing productivity trajectories has the potential for non-monotonic investment paths, but it is an empirical matter whether these forces are strong enough to capture the increasing investment rate during the first half of the development process. Chari, Kehoe, and McGrattan (2007) show how capital or investment wedges, standing in for different types of distortions, appear from first principles in the Euler equation of the one-sector neoclassical growth model.[12]

[12] Indeed, these authors show how popular models of financial frictions, like Bernanke, Gertler, and Gilchrist (1999) or Carlstrom and Fuerst (2006) appear in the Euler equation as investment wedges.

4.3 Production

There is a representative firm in each sector $i = \{a, m, s\}$ that combines capital k_{it} and labor l_{it} to produce the amount y_{it} of the good i. The production functions are CES with identical share $0 < \alpha < 1$ and elasticity $\epsilon < 1$ parameters. There is a labour-augmenting common technology level B_t and a sector-specific Hicks-neutral technology level B_{it}:

$$y_{it} = B_{it}\left[\alpha k_{it}^{\epsilon} + (1-\alpha)\left(B_t l_{it}\right)^{\epsilon}\right]^{1/\epsilon}$$

Assuming CES production functions with Hicks-neutral sector-specific technical progress extends the canonical Cobb-Douglas multi-sector growth model by allowing for non-unitary elasticity of substitution between capital and labour while retaining the analytical tractability of equal capital to labor ratio across sectors.[13] We obtain the FOC,

$$r_t = p_{it} \quad \alpha \quad B_{it}^{\epsilon}\left(\frac{y_{it}}{k_{it}}\right)^{1-\epsilon} \tag{15}$$

$$w_t = p_{it}\,(1-\alpha)\,B_t^{\epsilon}\,B_{it}^{\epsilon}\left(\frac{y_{it}}{l_{it}}\right)^{1-\epsilon} \tag{16}$$

4.4 Equilibrium

Let $i \in \{a, m, s\}$ indicate sector. Given k_0, an equilibrium for this economy is a sequence of exogenous productivity and wedge paths $\{B_t, \chi_t, B_{it}, \tau_t\}_{t=0}^{\infty}$; a sequence of aggregate allocations $\{c_t, x_t, y_t, k_t\}_{t=0}^{\infty}$; a sequence of sectoral allocations $\{k_{it}, l_{it}, y_{it}, x_{it}, c_{it}\}_{t=0}^{\infty}$; and a sequence of equilibrium prices $\{r_t, w_t, p_{it}, p_{ct}, p_{xt}\}_{t=0}^{\infty}$ such that (a) households optimize, (b) firms optimize, and (c) markets clear: $\sum_i k_{it} = k_t$, $\sum_i l_{it} = 1$, $y_{it} = c_{it} + x_{it}$ for $t = \{0, 1, 2, ..., \infty\}$. We define GDP y_t from the production side as as $y_t \equiv \sum_{i=a,m,s} p_{it} y_{it}$. Note that the market clearing conditions and equations (10) and (11) imply that the GDP from the expenditure side is given by $y_t = p_{xt}x_t + \sum_{i=a,m,s} p_{it}c_{it} = p_{xt}x_t + p_{ct}c_t - \sum_{i=a,m,s} p_{it}\bar{c}_i$.

In order to determine the equilibrium prices, note that the FOC of the firms imply that the capital to labor ratio is the same across all sectors and equal to the capital to labor ratio in the economy $k_{it}/l_{it} = k_t$. Hence, the relative sectoral prices are given by relative sectoral productivities:

$$\frac{p_{it}}{p_{jt}} = \frac{B_{jt}}{B_{it}} \tag{17}$$

[13] With CES production functions and Hicks-neutral technical progress there is no Balanced Growth Path, see Uzawa (1961) and Appendix E for details. For this reason, in order to solve the model, we will assume that the only source of growth in the very long run is the common labour-augmenting technical progress.

Finally, we define average productivity in consumption B_{ct} and investment B_{xt} as,

$$B_{ct} \equiv \left[\sum_{i=a,m,s} \theta_i^c B_{it}^{\frac{\rho_c}{1-\rho_c}}\right]^{\frac{1-\rho_c}{\rho_c}} \quad \text{and} \quad B_{xt} \equiv \left[\sum_{i=a,m,s} \theta_i^x B_{it}^{\frac{\rho_x}{1-\rho_x}}\right]^{\frac{1-\rho_x}{\rho_x}} \tag{18}$$

These productivity levels are useful because they summarize all the information on sectoral productivities that is needed to describe the aggregate dynamics of the homothetic version of our economy ($\bar{c}_i = 0$), and also the aggregate dynamics around the asymptotic Balanced Growth Path. In fact, B_{ct} and $\chi_t B_{xt}$ can be thought of as the Hicks-neutral productivity levels in a two-good economy that produces consumption and investment goods with otherwise identical CES production functions in capital and labor.[14] Using the definitions of p_{ct} and p_{xt} in equations (12) and (13) we can write,

$$\frac{p_{it}}{p_{ct}} = \frac{B_{ct}}{B_{it}} \quad \text{and} \quad \frac{p_{it}}{p_{xt}} = \chi_t \frac{B_{xt}}{B_{it}} \tag{19}$$

and also

$$\frac{p_{xt}}{p_{ct}} = \frac{1}{\chi_t} \frac{B_{ct}}{B_{xt}} \tag{20}$$

Hence, the relative price of investment has two components: the exogenous investment-specific technical change χ_t and the evolution of the relative productivity of investment and consumption B_{xt}/B_{ct}, which is the result of the endogenous sectoral composition of investment and consumption plus the exogenous changes in sectoral productivities. Note also that equations (17), (19), and (20) determine relative prices but that the overall price of the economy (and its evolution) is undetermined. We will use the investment good as numeraire when we study the aggregate dynamics of the economy with hat variables. For that purpose, it will be useful to write the expressions for output and the interest rate in units of the investment good as follows:

$$y_t/p_{xt} = \chi_t B_{xt} \left[\alpha k_t^\epsilon + (1-\alpha) B_t^\epsilon\right]^{1/\epsilon} \tag{21}$$

$$r_t/p_{xt} = \alpha \left(\chi_t B_{xt}\right)^\epsilon \left(\frac{p_{xt} k_t}{y_t}\right)^{\epsilon - 1} \tag{22}$$

with the capital to output ratio given by,

$$\left(\frac{p_{xt} k_t}{y_t}\right)^{-1} = \chi_t B_{xt} \left[\alpha + (1-\alpha)\left(\frac{B_t}{k_t}\right)^\epsilon\right]^{1/\epsilon} \tag{23}$$

[14]This is analogous to Herrendorf, Rogerson, and Valentinyi (2020), see Appendix E.6 for details.

4.5 Sectoral composition of output

Using the market clearing conditions for each good and the expenditure side definition of GDP we can express the sectoral shares of GDP at current prices with the following identities:

$$\frac{p_{it}y_{it}}{y_t} = \frac{p_{it}x_{it}}{p_{xt}x_t}\frac{p_{xt}x_t}{y_t} + \frac{p_{it}c_{it}}{\sum_{j=a,m,s} p_{jt}c_{jt}}\left(1 - \frac{p_{xt}x_t}{y_t}\right) \qquad i \in \{a, m, s\} \tag{24}$$

This states that the value added share of sector i in GDP is given by the share of sector i within investment times the investment rate plus the share of sector i within consumption times the consumption rate. The sectoral shares within consumption and investment are obtained from the demand system of the static problem, see equations (8) and (9). Therefore, structural change will happen because of sectoral reallocation *within consumption* due to both income and price effects, because of sectoral reallocation *within investment* due to price effects only, and because of reallocation in expenditure *between consumption and investment* in transitional dynamics, i.e., changes in the investment rate. The first two form the intensive margin of structural change, while the third one is the extensive margin of structural change. The larger the difference in sectoral composition between investment and consumption goods, the stronger this latter effect.

4.6 Aggregate dynamics and balanced growth path

We have two difference equations to characterize the aggregate dynamics of this economy: the Euler equation of consumption in equation (14) and the law of motion of capital in equation (4). After substituting prices away they become,

$$\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} = \beta\left(1+\tau_{t+1}\right)^{-1}\left[\frac{B_{ct+1}}{B_{ct}}\frac{B_{xt}}{B_{xt+1}}\frac{\chi_{xt}}{\chi_{xt+1}}\right]\left[\alpha\left(\chi_{t+1}B_{xt+1}\right)^{\epsilon}\left(\frac{p_{xt+1}k_{t+1}}{y_{t+1}}\right)^{\epsilon-1} + (1-\delta)\right] \tag{25}$$

and

$$\frac{k_{t+1}}{k_t} = (1-\delta) + \frac{y_t}{p_{xt}k_t} - \chi_t\frac{B_{xt}}{B_{ct}}\frac{c_t}{k_t}\left(1 - \sum_{i=a,m,s}\frac{B_{ct}\bar{c}_i}{B_{it}c_t}\right) \tag{26}$$

with the capital to output ratio given by equation (23). This dynamic system is driven by the four types of exogenous time-varying forces of the model: the economy-wide labor saving technology B_t, the sector-specific Hicks neutral technology B_{it} (which enter directly, but also indirectly through the investment and consumption specific Hicks neutral technology levels B_{xt} and B_{ct}), the investment-specific technology χ_t, and the investment wedge τ_t.

We define the Balanced Growth Path (BGP) as an equilibrium in which the capital to output ratio $p_{xt}k_y/y_t$ is constant. For the general case with $\epsilon \neq 0$ a BGP requires, (i) γ_{Bt} constant, (ii) $\gamma_{Bxt} = -\gamma_{\chi t}$, (iii) γ_{Bct} constant, (iv) $\bar{c}_i$ vanish asymptotically, and (v) the wedge τ_t is constant. In the BGP, variables in units of the investment good will grow at the rate $(1+\gamma_B)$ and variables in units of the consumption good will grow at the rate $(1+\gamma_B)(1+\gamma_{Bc})$, where γ_B and γ_{Bc} are the constant rates of growth of B_t and B_{ct} in the BGP. If we are ready to dispose with the knife-edge condition for the sequence of χ_t that satisfies condition (ii) for any arbitrary sequence of sectoral productivities, condition (ii) requires that χ_t and B_{it} are constant, and hence so are B_{xt} and B_{ct}. This means that sectoral productivity has to be symmetric across sectors and labour saving (and hence captured by B_t), that there cannot be any investment-specific technical progress, and that the relative productivity of the investment good remains constant. In this BGP there cannot be structural change because relative sectoral productivities are constant, the $\bar{c}_i$ have vanished asymptotically, and the investment rate is constant. The case with $\epsilon = 0$ (Cobb-Douglas production) is different in that a BGP with $\gamma_\chi > 0$ is possible, but it is still true that sectoral productivity growth has to be symmetric and no structural change would happen in BGP, see Appendix E for a detailed discussion of both cases.

5 Bringing the model to the data

We want the model to reproduce the stylized patterns of investment and sectoral reallocation of output in the PWT and WDI-G10S described in Figure 1, as well as the stylized facts of sectoral reallocation within the investment and consumption goods in the WIOD described in Figure 2. We explain the data construction in Section 5.1. Because the inter-temporal and intra-temporal choices of the model can be solved independently, we split the parameterization in two parts. First, in Section 5.2 we estimate the demand system, which provides values for the aggregator parameters θ_i^c, θ_i^x, ρ_c, ρ_x, and $\bar{c}_i$. Next, given these estimated parameters, in Section 5.4 we use the dynamic part of the model to calibrate the remaining parameters and back out the time series for the productivity processes and the investment wedge.

5.1 Data

We estimate our model with data from a large panel of countries that represents well the process of development that we have documented in Section 2. In particular, we use data for the investment rate at current domestic prices ($p_{xt}x_t/y_t$), the implicit price deflators of consumption and investment (p_{ct} and p_{xt}), and GDP in international dollars (y_t) from the PWT; the value added shares of GDP at current domestic prices and the implicit price deflator for each sector $i \in \{a, m, s\}$ ($\frac{p_{it}y_{it}}{y_t}$ and p_{it}) from the WDI-G10S; and the value added shares at current domestic prices for each sector $i \in \{a, m, s\}$ within investment ($\frac{p_{it}x_{it}}{p_{xt}x_t}$) and within consumption ($\frac{p_{it}c_{it}}{\sum_{j=a,m,s} p_{jt}c_{jt}}$) from the WIOD.[15] The base year for all prices is 2005, and hence note that the relative prices are equal to one in all countries in 2005. All in all, we use data from 49 countries between 1950 and 2011 for the combined PWT-WDI-G10S data set and 32 countries between 1995 and 2011 for the WIOD data set.[16] To implement our estimation, we first filter out country fixed effects from each country time series. That is, in the absence of a country with a very long time series describing the entire process of development, we remove unobserved country fixed effects from our panel to exploit within country variation provided by countries observed at different stages of development. We do so because we want to abstract from possible country-specific unobservables —like abundance of natural resources in Australia or political institutions promoting capital accumulation in China— that might affect the sectoral shares and the investment rate that we see in the data and might be correlated with development but are outside the mechanisms of our model.

5.2 The demand system

For the country-years with IO data, we can build separate time series for the sectoral composition of investment and consumption, and estimate the parameters of each aggregator separately. Then, we have two estimation equations for each sector $i \in \{m, s\}$:

$$\frac{p_{it}c_{it}}{\sum_{j=a,m,s} p_{jt}c_{jt}} = g_i^c\Big(\Theta^c; P_t, \sum_{i=\{a,m,s\}} p_{it}c_{it}\Big) + \varepsilon_{it}^c \tag{27}$$

$$\frac{p_{it}x_{it}}{p_{xt}x_t} = g_i^x\left(\Theta^x; P_t\right) + \varepsilon_{it}^x \tag{28}$$

where the functions g_i^c and g_i^x are the model-implied sectoral shares within consumption and investment given by equations (8) and (9), $\Theta^c = \{\theta_i^c, \rho_c, \bar{c}_i\}$ and $\Theta^x = \{\theta_i^x, \rho_x\}$

[15] The choice of WDI or G10S for sectoral data is country-specific and based on the length of the time series available, if at all, in each data set.

[16] Our requirements for a country to make it into the sample are that the country is: (a) not too small (population in 2005 > 2M), (b) not too poor (GDP per capita in 2005 > 5% of US), and (c) not oil-based (oil rents < 10% of GDP on average). In addition, for estimation purposes, we need that (d) all countries in WIOD are also available in the combined PWT-WDI-G10S data set —as this data set provides the relative sectoral price data— and (e) countries that only appear in PWT-WDI-G10S have data since at least 1980 and countries that appear in both data sets have data since at least 1996.

are the vectors of parameters relevant for the consumption and investment aggregators, P_t is the vector of relative sectoral prices at time t, and the terms ε^x_{it} and ε^c_{it} are the econometric errors that can be thought of as measurement error in the sectoral shares reported in the WIOD database. Non-linear estimators that exploit moment conditions like $E[\varepsilon^x_{it}|P_t] = 0$ and $E[\varepsilon^c_{it}|P_t, \sum_i p_{it}c_{it}] = 0$ deliver consistent estimates of the model parameters. This empirical strategy is analogous to Herrendorf, Rogerson, and Valentinyi (2013), who apply it to consumption for US postwar data, and to the contemporaneous work of Herrendorf, Rogerson, and Valentinyi (2020), who apply it to investment as well as to consumption.

For the country-years without IO data, an alternative approach is to use time series for the sectoral composition of the whole GDP and estimate the model parameters by use of equation (24), which relates the sectoral shares for aggregate output with the investment rate and the unobserved sectoral shares within goods. In particular, we get one estimation equation for each sector $i \in \{m, s\}$:

$$\frac{p_{it}y_{it}}{y_t} = g^x_i\left(\Theta^x; P_t\right)\frac{p_{xt}x_t}{y_t} + g^c_i\left(\Theta^c; P_t, \sum_i p_{it}c_{it}\right)\left(1 - \frac{p_{xt}x_t}{y_t}\right) + \varepsilon^y_{it} \tag{29}$$

where ε^y_{it} is measurement error in the aggregate sectoral share reported in PWT-WDI. The covariance between the investment rate and the sectoral composition is critical for identification. As an example, consider the simplest case where $\rho_c = \rho_x = 0$ and $\forall i\ \bar{c}_i = 0$. In this situation, the shares of sector i in consumption and investment are just given by θ^c_i and θ^x_i. Consequently, the value added share of sector i in GDP is given by,

$$\frac{p_{it}y_{it}}{y_t} = \theta^x_i\frac{p_{xt}x_t}{y_t} + \theta^c_i\left(1 - \frac{p_{xt}x_t}{y_t}\right) + \varepsilon^y_{it} = \theta^c_i + \left(\theta^x_i - \theta^c_i\right)\frac{p_{xt}x_t}{y_t} + \varepsilon^y_{it}$$

This expression shows that with homothetic demands and unit elasticity of substitution between goods, the standard model delivers no structural change under balanced growth path —that is to say, whenever the investment rate is constant. However, the model allows for sectoral reallocation whenever the investment rate changes over time and $\theta^x_i \neq \theta^c_i$. A simple OLS regression of the value added share of sector i against the investment rate of the economy identifies the two parameters, with the covariance between investment rate and the share of sector i identifying the differential sectoral intensity $(\theta^x_i - \theta^c_i)$ between investment and consumption. In the general setting described by equation (29), a non-linear estimator that exploits moment conditions like $E[\varepsilon^y_{it}|P_t, \sum_i p_{it}c_{it}, p_{xt}x_t/y_t] = 0$ will deliver consistent estimates of the parameters. This means that conditional on sectoral

Table 3: Demand system

Panel A: Estimated parameters

Consumption						Investment		
ρ_c	θ_m^c	θ_s^c	$\bar{c}_a$	$\bar{c}_m$	$\bar{c}_s$	ρ_x	θ_m^x	θ_s^x
−65.64	0.19	0.79	−0.02	1872.4	7608.7	−0.94	0.55	0.42
(11.142)	(0.002)	(0.002)	(.)	(68.2)	(288.4)	(0.062)	(0.002)	(0.002)

Panel B: Stone-Geary terms

	$\bar{c}_a$	$\bar{c}_m$	$\bar{c}_s$
$\lvert p_{it}\bar{c}_i\rvert / \sum_i p_{it}c_{it}$ at $t = 0$	0.00	3.87	8.03
$\lvert p_{it}\bar{c}_i\rvert / \sum_i p_{it}c_{it}$ at $t = T$	0.00	0.02	0.13

Notes: Panel A reports the parameters estimated with the demand system in Section 5.2, GMM robust standard errors reported in parenthesis. Panel B reports the (absolute) value of the $\bar{c}_i$ relative to the value of consumption expenditure, that is, $\lvert p_{it}\bar{c}_i\rvert / \sum_i p_{it}c_{it}$, for the first and last period of the development process.

prices P_t and consumption expenditure ($\sum_i p_{it}c_{it}$) —which together determine the sectoral composition of consumption and investment goods— the covariance between the investment rate and the sectoral composition of GDP allows to estimate our model without IO data.[17]

In practice, we combine both approaches and use a two-sample GMM estimator that optimally exploits valid moment conditions of: (a) the sectoral share within consumption and investment in equations (27) and (28) using IO data from WIOD and (b) the sectoral shares of GDP in equation (29) using data from WDI-G10S. Note that, because the poorest and richest countries in the WDI-G10S-PWT panel are not available in the WIOD data set, we do not have IO data for very early and very late levels of development and hence only sectoral shares of GDP from WDI-G10S and equation (29) can be used at those levels of development.[18]

We report the parameter estimates and their GMM robust standard errors in Table 3. We find $\rho_x = -0.94$ and $\rho_c = -65.63$. These values imply that the elasticity of substitu-

[17]Note that conditioning on P_t and $\sum_i p_{it}c_{it}$ still leaves several sources of exogenous variation to identify our parameters. In particular, different combinations of the exogenous processes χ_t and B_t and transitional dynamic forces given by the predetermined value of k_t imply different values of the investment rate for a given set of sectoral prices and total consumption expenditure.

[18]The sectoral composition of GDP from WIOD and WDI-G10S align well for the country and years present in both samples. However, once we filter out country fixed effect they become misaligned. This is because after regressing out country fixed effects we add to each country-year observation the average country fixed effect in the corresponding data set. Because the countries and years in each data set are different, the constants we add to the sectoral composition of consumption and investment in WIOD are inconsistent with the one we add to the sectoral composition of GDP in WDI-G10S. For this reason, we add a constant α_i to the estimation equation (29).

tion for sectoral value added is 0.52 within investment and only 0.02 within consumption, making the value added from different sectors less substitutable than in a Cobb-Douglas aggregator in both cases.[19] This means that changes in relative sectoral prices generate changes in sectoral shares in the same direction and, for the case of consumption, of similar size.[20] We find that both $\bar{c}_m$ and $\bar{c}_s$ are positive, while $\bar{c}_a$ is negative and very close to zero, hitting the estimation constraint $\bar{c}_a < 0$. Table 3 also reports the value of these parameters relative to the value of the consumption expenditure at the beginning and at the end of the sample. The terms associated to manufacturing and services are large at the beginning of the sample and the term associated to services is still sizable at the end. All in all, these estimates imply that the income elasticity at the beginning of the development process is less than one for agriculture and more than one for manufacturing and services. Indeed, for the first third of the development process the income elasticity for manufacturing is substantially larger than for services, see Appendix D for details.

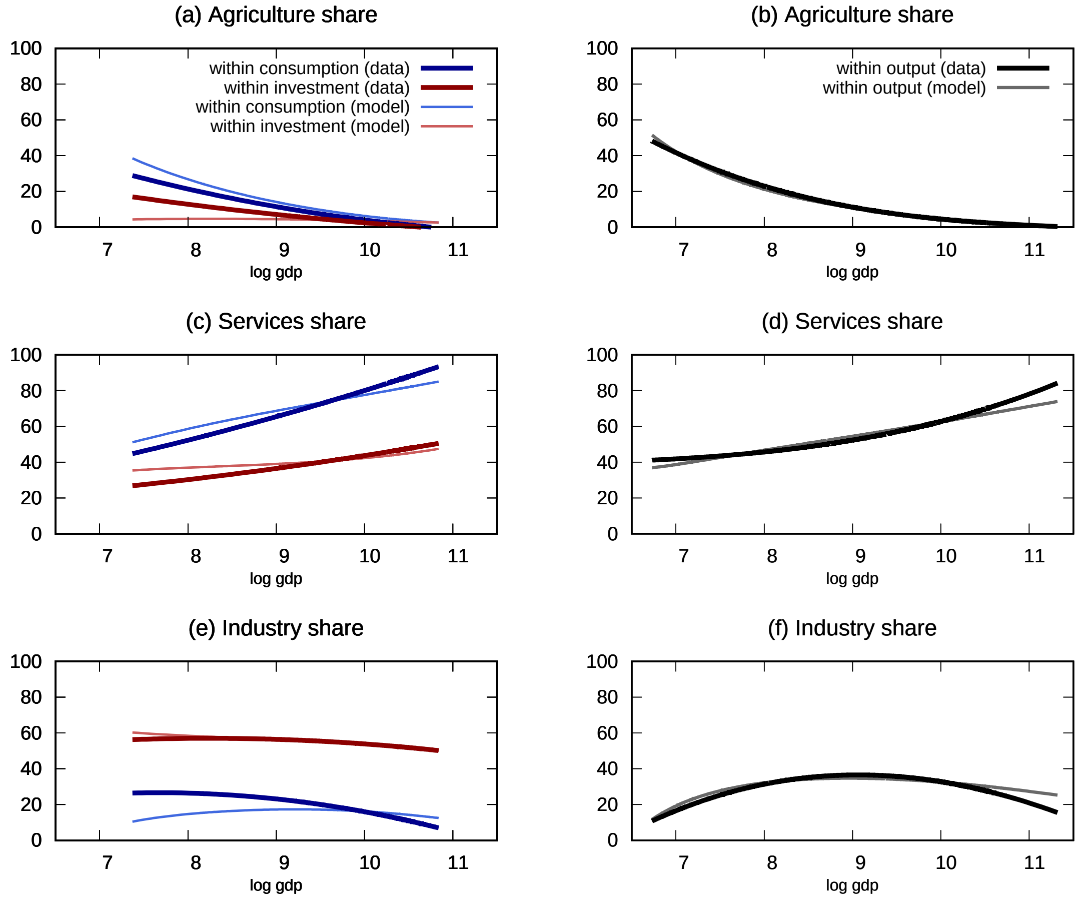
The model fit is displayed in Figure 4.[21] We see that the model reproduces the sectoral composition of GDP extremely well during the whole development process. Looking at the sectoral composition of investment and consumption goods, we see that the model also does quite well. First, the model matches the average sectoral composition of consumption and investment. Second, it predicts well the decline of agriculture within consumption but misses the decline of agriculture within investment. Third, it predicts the increase of services within both consumption and investment, although quantitatively it misses part of it. And fourth, it matches perfectly the fall of manufacturing within investment and slightly understates the decline of manufacturing within consumption, creating a small hump of manufacturing within consumption that is absent in the WIOD. The reason for this latter result is that the increase of manufacturing within GDP in the early stages of development measured in the WDI-G10S data set is very sharp and cannot be completely accounted by the observed increase in the investment rate. Hence, the estimation requires a slight increase of manufacturing within consumption and/or investment, which is achieved by an income elasticity of manufacturing within consumption larger than one at the beginning of the development process.

[19]The elasticity of substitution within investment is given by $1/(1-\rho_x)$. However, $1/(1-\rho_c)$ is only the asymptotic elasticity of substitution of sectoral value added within consumption when all $\bar{c}_i/c_{it} = 0$.

[20]Herrendorf, Rogerson, and Valentinyi (2020) find elasticities of substitution between goods and services for consumption and investment that are much closer to zero for the 1947-2015 period in the US.

[21]In particular, we plot the projections of the sectoral shares in the data against the level of development and the predicted sectoral shares when we fit the estimation equations with the projections against the level of development of relative sectoral prices, investment rate, and consumption expenditures.

Figure 4: Model fit, sectoral composition

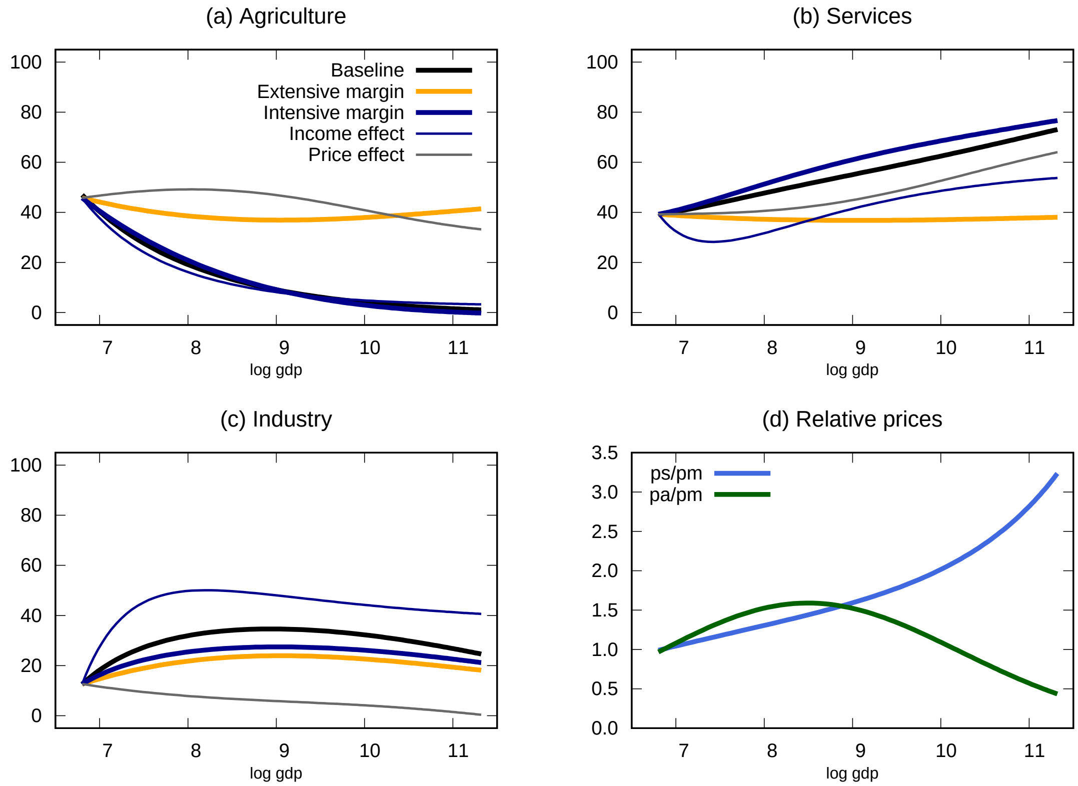


Notes. Panel (a), (c) and (e) report data from WIOD (thick dark lines) and model predictions (thin light lines) for the sectoral composition of consumption and investment. Panel (b), (d) and (f) report data from WDI-G10S (thick dark lines) and model predictions (thin light lines) for the sectoral composition of GDP. Data are projections on a quadratic polynomial of log GDP per capita in constant international dollars. Series have been filtered out from country fixed effects.

5.3 Counterfactual exercises with the demand system

In order to assess the relative importance of the different elements of the demand system, we re-evaluate equation (24) in a series of counterfactual exercises that we plot in Panels (a) to (c) of Figure 5. First, we set the sectoral composition within consumption and investment constant (and equal to the first period) and hence the only source of structural change is the change in the investment rate, that is, the extensive margin (see the thick yellow lines). Second, we instead set the investment rate constant (and equal to the first period) such that we isolate the structural change coming from the intensive margin (thick dark blue lines). These two exercises show how the overall trends in agriculture and services are roughly well captured by the standard mechanisms operating in the intensive

FIGURE 5: Sectoral composition of GDP: counterfactual exercises



Notes. In Panels (a), (b), and (c) "Baseline" refers to the sectoral share predictions of GDP with the estimated parameters. "Extensive margin" and "Intensive margin" refer to the counterfactual predictions when only one of the two is operative. "Income elasticity" refers to the case with $\rho_x = \rho_c = 0$ *and constant investment rate, while "Price elasticity" refers to the case with* $\bar{c}_i = 0$ *and constant investment rate. See test for details.*

margin. However, when looking at the evolution of the share of manufacturing in GDP we see that both the intensive and the extensive margins matter to generate the hump. With the sectoral composition of investment and consumption goods held constant, the change in the investment rate produces an increase in the share of manufactures of 11 percentage points (as compared to 22 in the data) and a decline afterwards of 6 (as compared to 10 in the data). With the investment rate held constant, the change in the sectoral composition within consumption and investment produces a hump in manufacturing similar in shape and size to the one produced by the changes in the investment rate, although with a peak 3 percentage points higher.

Finally, we perform two more exercises to separate the different channels operating in the intensive margin. First, we set $\rho_x = \rho_c = 0$ and hold the investment rate constant such that we produce structural change coming from income effects only (thin dark blue lines), and second we set $\bar{c}_i = 0$ also holding the investment rate constant such that we isolate changes in sectoral composition coming from relative price effects only (thin gray lines).[22] We note that the price of services relative to the price of manufactures increases monotonically over the development process, while the price of agricultural goods increases relative to the price of manufactures in the first third of the development process but starts to decline afterwards, see Panel (d) of Figure 5. We find that the decline in the share of agriculture is mostly driven by the income effect, while the relative decline in the price of agriculture generates little action. Regarding services, both channels matter: the increase in the relative price of services increases the service share of the economy in 25 percentage points (34 in the data), while the increase in GDP increases the service share of the economy in 14 percentage points. Finally, these two forces have opposite effects for the hump of manufacturing. We see that the income effect generates a large increase of manufacturing with development, indeed larger than in the data, followed by a small decline. Instead, we see that the decline in the price of manufactures relative to services moves the share of manufacturing downwards, partly offsetting the desired increase of manufactures due to income effects in the first half of the development process and helping create the overall decline of manufacturing in the second half.[23]

5.4 The intertemporal side

After estimating the static demand system, we want the model to reproduce the observed dynamics of output, investment, and sectoral composition along the development path. To do so, we use the projections of our panel data on a low-order polynomial on the level of development filtered from country fixed effect and think of these projections as describing the development process of a synthetic country whose log GDP per capita goes from an initial level of 6.80 (or 900 international dollars of 2005, which corresponds to China in 1952) to a final level of 11.32 (or 82,454 international dollars of 2005, which corresponds to Norway in 2010). Note that these projections coincide with the thick black lines in Figure 1 describing the evolution of the sectoral shares of GDP and the investment rate, and the thick red and blue lines in Panels (a), (c), and (e) of Figure 2 describing the sectoral evolution of consumption and investment. The stylized evolution of relative sectoral prices is constructed likewise and reported in Panel (d) of Figure 5, while the stylized evolution of the relative price of investment to consumption is reported

[22] When we change ρ_x, ρ_c, or $\bar{c}_i = 0$ we re-calibrate θ_i^x and θ_i^c to match the average sectoral shares within investment and consumption in the first period.

[23] Note that for the case with $\rho_x = \rho_c = 0$ services and manufacturing decline at the start and at the end of the development process respectively, which seems at odds with the larger than one income elasticity. The reason is that, despite $\rho_x = \rho_c = 0$, sectoral prices do affect sectoral shares within consumption because they interact with the $\bar{c}_i$ driving the income effects, see equation (8).

Table 4: Calibrated Parameters

Economy	*A. Calibrated parameters*							*B. Sources of growth (%)*					
	ϵ	σ	$\hat{k}_0/\hat{k}^*$	γ_B	α	δ	β	E_0	E_0-E_1	E_1-E_2	E_2-E_3	E_3-E_4	E_4
Benchmark	0	2	0.20	0.02	0.33	0.03	0.96	4.87	-0.02	0.06	3.70	0.05	1.08
Lower ϵ	-0.25	2	0.18	0.02	0.45	0.03	0.96	4.87	-0.02	0.03	3.76	0.02	1.07
Higher σ	0	4	0.20	0.02	0.33	0.03	1.00	4.87	-0.05	0.04	3.21	0.14	1.53
Higher k_0	0	2	0.54	0.02	0.33	0.03	0.96	4.87	-0.02	0.05	4.06	0.05	0.74

Notes: Panel A reports the calibrated parameters for the Benchmark economy plus four other economies with, respectively, lower elasticity of substitution between capital and labor (0.8 instead of 1), higher initial capital stock (capital to output ratio twice as big), lower intertemporal elasticity of consumption (0.25 instead of 0.5), and no sectoral reallocation (income and price elasticity of demand for each good equal to one). Panel B reports the average growth rate of GDP in consumption units between $t = 0$ and $t = T$ for these economies. Column E_0 refers to the calibrated economy, column E_0-E_1 isolates the effect of investment-specific technical change, column E_1-E_2 isolates the effect of asymmetric productivity growth across sectors, column E_2-E_3 isolates the effect of symmetric productivity growth, column E_3-E_4 isolates the effect of the investment wedge, and in column E_4 only the effect of low initial capital remains.

in Panel (b) of Figure 6. Finally, we use data on output growth along the development path (see Panel (c) in Figure 6) to put all these projections against time, see Appenidx C.

We ask our model to fit these projections. This requires solving numerically the full model from $t = 0$ to the BGP. For a BGP to exist, we assume that at some time $t = \hat{T} > T$, $B_{at}, B_{mt}, B_{st}, \chi_t, \tau_t$ remain constant and B_t grows at the constant rate γ_B, which will be the rate of growth of the economy in the BGP. Hence, the capital in efficiency units defined as $\hat{k}_t = k_t/B_t$ will be constant in BGP. In order to solve the model, we need time paths for the different productivity sequences and for the wedge, $\{B_t, B_{at}, B_{mt}, B_{st}, \chi_t, \tau_t\}_{t=0}^{\infty}$; values for the parameters σ, β, δ, ϵ, α, γ_B; and a value for the initial condition $\hat{k}_0$. We start by setting $\epsilon = 0$ to focus on the Cobb-Douglas case, set $\gamma_B = 0.02$, $\sigma = 2$, and choose α, β, and δ to match a capital share, a capital to output ratio, and an investment rate of 0.33, 3 and 0.15 respectively in BGP. We choose $\hat{k}_0$ to match the capital to output ratio of 0.68 in China in 1952 by means of equation (23).[24] All values are reported in the first row of Panel A in Table 4.

Next, we use our data between $t = 0$ and $t = T$ to recover values for the exogenous sequences $\{B_t, B_{at}, B_{mt}, B_{st}, \chi_t, \tau_t\}_{t=0}^{T}$. We normalize $B_{mt} = 1\ \forall t$. Given B_{mt}, equation

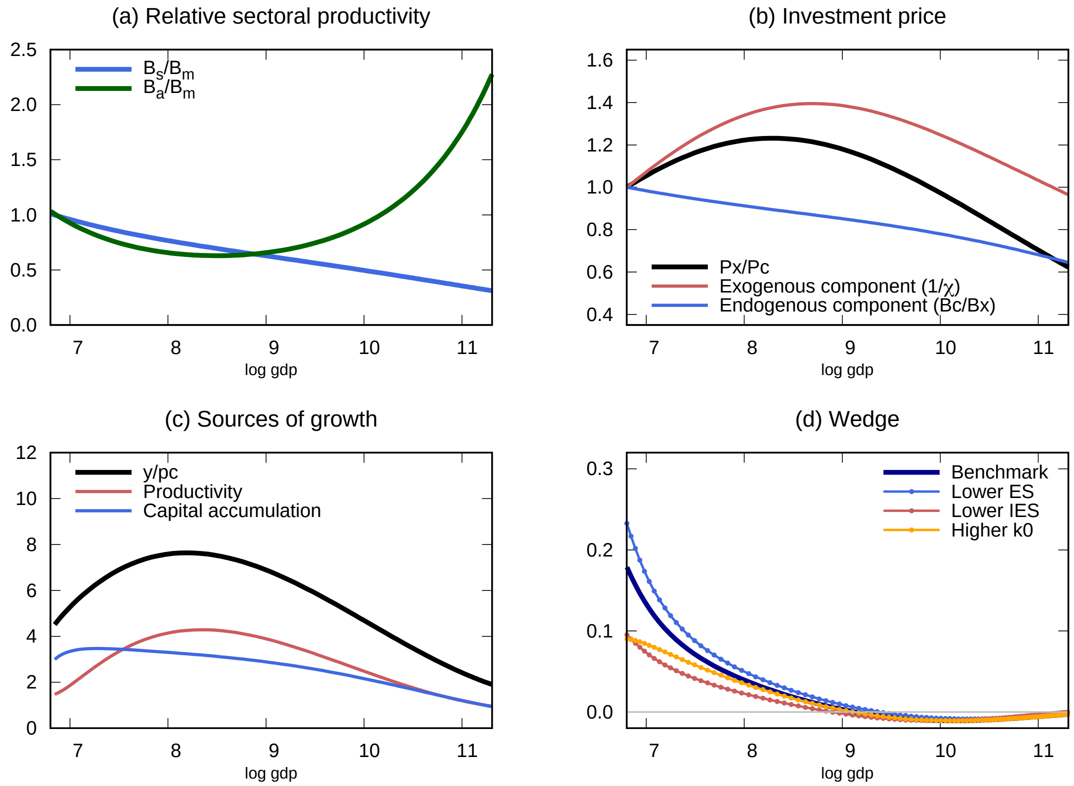
[24] We take China 1952 as the initial period of our development process, although the poorest country-year in our sample is China in 1961. However, this is a peculiar year for China as GDP per capita declined sharply in 1961 and 1962, bringing it below its 1952 level and consequently leaving a capital to income ratio much larger than in 1952. In particular, Cheremukhin, Golosov, Guriev, and Tsyvinski (2017a) report that the capital stock and GDP in China were 52,580 and 77,330 million 1978 yuans respectively in 1952 and 150,230 and 115,000 in 1961 (Tables 25 and 23 of the online appendix). This gives a capital to output ratio of 0.68 in 1952 and 1.30 in 1961. We take the value for 1952 and look at the results with the value for 1961 in Section 5.6.

(17) allows to recover B_{at} and B_{st} from sectoral price data, equation (18) allow to build B_{ct} and B_{xt}, and equation (20) allows to recover χ_t from data on the relative price of investment. We recover B_t from the production function (21) and our data on output and investment accumulated into capital through the law of motion for capital (26). Finally, we need to recover the path for the wedge τ_t. We do so by use of the Euler equation in (25), with c_t coming from the consumption aggregator in (5) with the parameters and sectoral consumption sequences obtained from the estimation of the demand system. Note that we have $T+1$ observations of consumption and only T wedges, which is the same to say that the wedges allow to fit the consumption growth data but leave free the consumption level c_T. But matching c_T is straightforward. As discussed by Cheremukhin, Golosov, Guriev, and Tsyvinski (2017b), there are infinite different combinations of the unobserved sequences $\{B_t, B_{at}, B_{mt}, B_{st}, \chi_t, \tau_t\}_{t=T+1}^{\infty}$ that are consistent with the observed c_T while keeping the economy in the stable arm towards the BGP.[25]

Looking at the calibrated economy, we see that the development process starts relatively far from the BGP, with the initial capital in efficiency units being 20% of its BGP level. Starting from an initial log GDP of 6.80, it takes 96 years for the model economy to cover the distance to log GDP of 11.32 for an average growth rate of 4.87%. The recovered productivity series $\{B_t, B_{at}, B_{mt}, B_{st}, \chi_t\}_{t=0}^{T}$ can be found in Figure 6. In Panel (a) we see how, mirroring relative price data in Figure 5, manufactures become more productive relative to services along the whole development process and also more productive relative to agriculture during the first third of the development process, while agriculture becomes more productive than manufactures afterwards. Panel (b) displays the evolution of the relative price of investment p_{xt}/p_{ct} in the data, together with its decomposition between the exogenous and endogenous investment-specific technical change, that is, the $1/\chi_t$ and B_{ct}/B_{xt} components in equation (20). We see that the relative price of investment declines 38% over the development process, although this decline is not monotonic: it increases 23% during the first third and declines 50% afterwards. The relative decline in the price of manufactures coupled with the larger importance of manufactures within investment generates a monotonic decline in B_{ct}/B_{xt}, making investment goods 36% cheaper at the end of the development process, with a 10% decline during the first third and a 28% decline afterwards. Hence, structural change explains the overall decline in the relative price of investment and 1/2 of it during the last 2/3 of the development process. The full

[25] Our choices for these sequences are as follows. First, wee choose $\hat{T} = T + 50$ and set the exogenous sequences $\forall t \geq \hat{T}$ as discussed above to guarantee existence of a BGP. Second, for $t \in \left[T+1, \hat{T}-1\right]$ we linearly interpolate them with the values in T and $\hat{T}$, while imposing $\tau_T = 0$. Finally, we add a small lump sum transfer in the law of motion for capital between $T+1$ and $\hat{T}-1$ to match the investment rate at T.

FIGURE 6: Exogenous series



Notes. Panel (a) plots the recovered sequences of relative sectoral productivities. In Panel (b) we decompose the relative price of investment into its exogenous and endogenous components, in Panel (c) we decompose the rate of growth of the economy into productivity growth and capital accumulation. The black lines in Panels (b) and (c) refer to our filtered data from PWT. Panel (c) reports the investment wedge τ_t for the benchmark and the alternative calibrations.

shape of p_{xt}/p_{ct} is recovered residually through the investment specific technical change, with $1/\chi_t$ increasing by 37% in the first third of development and declining declining 30% afterwards.[26] Next, in Panel (c) we plot the data series for the annual rate of growth of output in consumption units. We see that it is hump-shaped with development, with the growth rate starting at 4.5%, peaking at about 7.6%, and slowly converging to the 2% rate for rich economies. We decompose growth of output in consumption units into productivity growth and capital accumulation.[27] We see that capital accumulation is relatively more important in the first periods of development, when the capital to output ratio is

[26] The decline in $1/\chi_t$ during the last two thirds of the development process is consistent with the idea of faster technical change in the production of investment goods. The increase in $1/\chi_t$ during the first third of development could be associated to faster technical change in the production of consumption goods or to mounting distortions in the production of investment goods, see Restuccia and Urrutia (2001).

[27] Using equation (21) for output in investment units and equation (20) for the relative price of investment, we can write output in consumption units for the case $\epsilon = 0$ as $y_t/p_{ct} = \left[B_{ct}B_t^{1-\alpha}\right] k_t^{1-\alpha}$.

low and the transitional dynamics matter relatively more, while productivity growth is relatively more important afterwards.

Finally, the solid dark blue line in Panel (d) of Figure 6 displays the wedge τ_t needed to match the investment path. We see that the wedge is largest at the beginning of the development process and that it declines monotonically during the first half of development and stays around zero afterwards. The starting value is not too large: the equivalent of a 18% tax in the Euler equation of consumption. The wedge τ_t allows to account for forces outside our model that may shape the investment rate along the development path. As discussed in the Introduction, we can think of this wedge as a stand-in for financial development. The positive empirical relationship between financial development and growth is well established, see for instance a review in Levine (2005). There is a variety of mechanisms through which this may happen. Financial intermediation facilitates the diversification of idiosyncratic entrepreneurial risk, which implies a higher capital demand for a given interest rate, see for instance Townsend (1978) or Castro, Clementi, and MacDonald (2004). Alternatively, collateral constraints may generate an inefficient allocation of capital across heterogeneous entrepreneurs and a lower aggregate demand of capital as in Buera and Shin (2013) or Song, Storesletten, and Zilibotti (2011). The fact that financial development increases with GDP can arise endogenously through a variety of mechanisms, see Benhabib, Rogerson, and Wright (1991), Greenwood and Jovanovic (1990), Zilibotti (1994), or Acemoglu and Zilibotti (2001). However, our empirical exercise allows for other interpretations for the declining wedge. For instance, the wedge could reflect the need for a more elaborate model of saving with either more general preferences, an explicit role for demographic transitions, or declining capital gains in land's value. An example of the former would be Stone-Geary utility functions like Christiano (1989) and King and Rebelo (1993) or preferences with habit formation as Carroll, Overland, and Weil (2000) and Álvarez Cuadrado, Monteiro, and Turnovsky (2004). The potential role of declining fertility and increasing life expectancy on savings was first advocated by Coale and Hoover (1958), and has been recently explored by Higgins (1998) or Imrohoroğlu and Zhao (2018) among others. See Laitner (2000) for the saving rate in transitions from Malthusian to modern growth with declining capital gains of land.

5.5 Counterfactual exercises with the full model

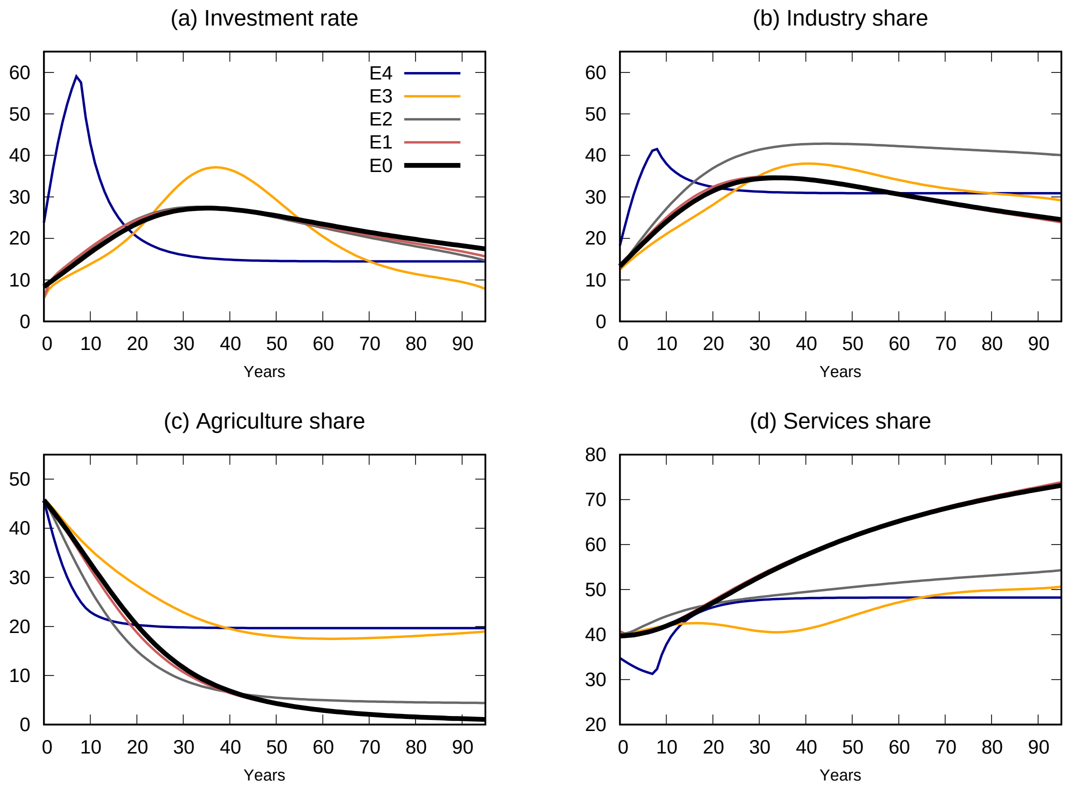
We want to understand the joint determination of the investment rate and the sectoral composition of the economy along the development path. Our model has three exogenous sources of technology change: aggregate productivity, asymmetric sector-specific productivity, and investment-specific technical level. In addition, it features endogenous transitional dynamics arising from the low initial capital stock and suffers an implicit tax in capital accumulation. All these elements can potentially shape the paths of output, investment, and sectoral composition of the economy. First, aggregate productivity growth and transitional dynamics make the economy richer and drive structural change in the intensive margin through the non-unitary income elasticities in the consumption demand for the different sectoral goods. They also affect the investment rate through the interplay of intertemporal income and substitution effects generated by the simultaneous increase in output and decline in the interest rate, and hence drive the extensive margin of structural change. Second, the asymmetric sector-specific productivity growth affects the intensive margin of structural change through the non-unitary elasticity of substitution across goods both within consumption and within investment. It also affects the investment rate through the induced changes in the endogenous component of the relative price of investment, and hence the extensive margin of structural change. Finally, the investment-specific technical change and the investment wedge affect the investment rate, and because of this they affect the extensive margin of structural change. They also have a (negligible) effect on the intensive margin, as changes in the investment rate change total consumption expenditure for a given income level and hence interact with the non-homotheticities within consumption.

In order to assess the relative importance of these mechanisms, we solve for the following four counterfactual economies. First, starting from the calibrated economy —which we call E_0— we remove the exogenous investment-specific technical change (ISTC) by setting $\gamma_{\chi,t} = 0 \ \forall t$ and call this economy E_1. Next, we remove the asymmetry in sectoral productivity growth by setting $\gamma_{Bat} = \gamma_{Bst} = \gamma_{Bmt} = \tilde{\gamma}_{Bmt} \ \forall t$ and choose $\tilde{\gamma}_{Bmt}$ equal to the rate of growth of the Hicks-neutral technical change of GDP in economy E_1.[28] We call this economy E_2. Next, we remove total factor productivity (TFP) growth by setting $\tilde{\gamma}_{Bmt} = 0 \ \forall t$ and call the resulting economy E_3. Finally, we remove the investment wedge in economy E_4.

Growth. The first result to highlight is the contribution of each exogenous series to the overall growth of the economy, see Panel B in Table 4. The calibrated economy grows

[28] We can define the Hicks-neutral technical level of GDP B_{yt} as the weighted average of the Hicks-neutral technical level in investment and consumption, $B_{yt} \equiv B_{xt}\chi_t \left(p_{xt}x_t/y_t\right) + B_{ct}\left(1 - p_{xt}x_t/y_t\right)$. Keeping the same investment rate as in economy E_1 we can recover the time path of $\tilde{\gamma}_{Bmt}$ that replicates the growth of B_{yt} in economy E_1. To the extent that the investment rate in this counterfactual economy will differ from the one in economy E_1 the final process of B_{yt} will be different, but it will be so for endogenous reasons.

Figure 7: Dynamic model: counterfactual exercises



Notes. Each panel reports a different model outcome for the calibrated economy (E_0) plus some counterfactual economies. E_1 removes ISTC, E_2 additionally removes the asymmetry in sectoral productivity growth, E_3 additionally removes neutral productivity growth, E_4 additionally removes the investment wedge.

at an average annual rate of 4.87%. We find that the exogenous ISTC has a negligible effect in growth as χ_t displays zero average growth along the development path. Next we find that the asymmetry in sectoral productivity growth explains 0.06% of annual growth. This is an important result. The so-called Baumol disease states that asymmetric productivity growth, by reallocating production factors towards sectors with slow-growing productivity, should decrease overall productivity growth in the economy, see for instance Ngai and Pissarides (2007) or Duernecker, Herrendorf, and Valentinyi (2019). However, we find that when one considers the different sectoral composition of investment and consumption goods, asymmetric productivity growth also has a positive effect in the growth of the economy in transitional dynamics by making investment goods cheaper and hence fostering capital accumulation in real units. Overall, we find that the two effects almost offset each other and the Baumol disease becomes inconsequential for economic development. Third, TFP growth accounts for the bulk of the growth along the development path, accounting for 3.70% of average growth. Finally, the transitional dynamics in economy E_3 are also an important source of growth, accounting for an anual rate of 1.13%. It is interesting to note that the investment wedge has a negligible contribution to the overall growth in transitional dynamics, with a 0.05% average annual growth. As we will see below, this is because the investment wedge does not reduce overall investment it just delays it. Indeed, the investment wedge removes 0.5% annual growth during the first third of development but adds 0.3% of growth afterwards due to the unexploited investment opportunities.

Investment. We report the results for the investment rate in Panel (a) of Figure 7. We find that neither the exogenous nor the endogenous components of the ISTC are quantitatively important in shaping the path of investment at current prices. In particular, adding both exogenous ISTC and asymmetric sectoral productivity growth to economy E_2 (thin grey line) to produce economy E_0 (thick black line), we see that the only difference is that the decline in the investment rate in the second half of the development process is reduced by 3 percentage points. Instead, TFP growth, transitional dynamics, and the investment wedge do matter. To understand the role of TFP growth, we can compare economy E_3 (thin yellow line) —featuring transitional dynamics with the investment wedge— to economy E_2 —featuring also TFP growth. We see that economy E_3 without TFP growth displays a sharper hump of investment. As economies grow, the investment rate is determined by the interplay of the intertemporal substitution effect —the evolution of the after tax marginal product of capital in consumption units— and the intertemporal income effect —the growth of GDP, which mitigates the former because of the desire to smooth consumption intertemporally. GDP grows much less in Economy E_3 than in Economy E_2, which weakens the intertemportal income effect in economy E_3 and makes the investment dynamics more reliant on the movements of the (hump-shaped) after-wedge marginal product of capital. Finally, it is important to note that removing the investment wedge from economy E_3 to produce economy E_4 does not remove the hump in investment, it just makes it happen earlier and be shorter-lived. To understand why, we first need to recall that a realistically calibrated standard one-sector neo-classical growth model with Cobb-Douglas production and CRRA utility predicts a large investment rate at the start of development —when the capital to output ratio is low and the marginal product of capital is large— that declines monotonically afterwards, with the intertemporal substitution effect dominating the intertemporal income effect throughout the process, see King and Rebelo (1993). The investment wedge captures in reduced form the distortions in the capital accumulation process offsetting this mechanism. Yet, our multi-sector economy E_4 features transitional dynamics without the wedge and does not adhere to this logic. The reason for this are the static non-unitary income elasticities of sectoral consumption demand that turn out to have dynamic implications at low levels of development for the economy without the investment wedge. At the start of development, when resources are scarce and the marginal product of capital is large, the household problem hits the inequality constraint $c_{mt} \geq 0$ as households would like to sell its endowment of non-tradable home produced manufactures, $\bar{c}_m$, to finance profitable investment without giving up highly-valued agricultural consumption.[29] As the economy gets richer and the constraint is still binding, c_{mt} does not change as it is held fixed to $\bar{c}_m$, while c_{at} and c_{st} grow very little due to the strong complementarity between goods. Hence, the investment rate grows despite declining marginal product of capital. When the inequality constraint does not bind anymore, the investment rate starts to decline monotonically as in the standard one-sector model. In this sense, the aggregate dynamics in our multi-sector growth model resemble the one-sector model with Stone-Geary utility function along the lines of Christiano (1989) or King and Rebelo (1993). Finally, note that the role of the wedge is not to diminish overall investment but to shift its timing. When adding the wedge to economy E_4 to produce economy E_3, there is little investment at the beginning of the development process, which keeps the marginal product of capital high. As the wedge diminishes with development, a strong investment process starts enocuraged by the unexploited large marginal product of capital.

Structural change. Regarding the sectoral composition of the economy, Panels (b) to (d) of Figure 7 report the evolution of the share of industry, agriculture, and services in GDP. The first thing to note is that the exogenous ISTC plays no role in structural change as ISTC does not operate at the intensive margin and it has only negligible effects in the investment rate (the sectoral paths of economy E_1 are indeed indistinguishable from the ones in economy E_0). Next, we see that asymmetric sectoral productivity growth plays a minor role in agricultural decline but it has an important role in the reallocation between manufacturing and services. In particular, comparing economies E_2 and E_0 we see that asymmetric sectoral productivity growth is responsible for only 3.5 out of 44.7 percentage points secular decline in agriculture. This result is consistent with the findings in Section 5.2 that the decline in agriculture is mostly driven by income effects and that relative price effects do not matter much. Instead, asymmetric sectoral productivity growth generates an increase in the share of services of 13.6 out of a total 33.5 percentage points and removes 16.2 percentage points of the increase in manufacturing generated by income effects (see comparison of economies E_2 and E_0). It is important to note

[29]See Appendix E.7 for details on how to solve the model with binding inequality constraints.

that the effects of asymmetric sectoral productivity growth operate mostly through the intensive margin because asymmetric sectoral productivity growth plays a very small role in shaping investment. Transitional dynamics and TFP growth are important for structural change both at the intensive and extensive margins. In agriculture we see that transitional dynamics and TFP growth explain a decline of 26.8 and 14.2 percentage point respectively (see economy E_3 and the difference between economy E_2 and economy E_3 respectively). In services transitional dynamics and TFP growth account for increases of 10.1 and 3.5 percentage point respectively. In manufacturing, transitional dynamics accounts for a 16.7 percentage points increase and a sharper hump than in the data, while TFP growth accounts for 10.7 percentage points increase. Finally, note that the income effects of transitional dynamics are larger than the ones of TFP growth despite the latter providing a larger contribution to income growth. The reason is that the heterogeneity in income elasticities across goods is larger in the first third of the development process, when growth due to transitional dynamics is more important than growth due to technology improvement. Finally, note that without the investment wedge (economy E_4) the rise of manufactures and the decline in agriculture would accelerate in the first periods of the development process, while the share of investment would decrease, all these changes coming from the extensive margin of structural change.

5.6 Robustness exercises

In this Section we examine how our results change as we allow for slightly different parameterizations of the dynamic model. The main take is that different parameterizations require different investment wedges for the model to reproduce the investment path, but the main counterfactual exercises turn out to be little affected.

We start by lowering the elasticity of substitution between capital and labor (ES) to 0.8 ($\epsilon = -0.25$).[30] An ES below one prevents the marginal product of capital from being too large at low levels of capital. Because of the weakening of the intertemporal substitution effect at early stages of development, this can result in lower initial investment than under Cobb-Douglas and even hump-shaped investment paths, see Antràs (2001) and Smetters (2003). However, in our setting allowing for an ES<1 requires a larger not lower investment wedge at the start of development, see blue line in Panel (d) of Figure 6. The

[30]Estimates of the ES below 1 are relatively common in the literature, see for instance Antràs (2004), Klump, McAdam, and Willman (2007) or Leon-Ledesma, McAdam, and Willman (2010) for US time series. Using firm-level data, Oberfield and Raval (2014) estimate the aggregate ES to be 0.7 for the US, 0.8 for Chile and Colombia and 1.1 for India. Villacorta (2018) exploits country panel data from EU KLEMS and finds that most (but not all) countries in the EU have ES less than one. In contrast, exploiting cross-country variation, Karabarbounis and Neiman (2014) find an elasticity larger than 1.

reason for this is that the calibration exercise with $\epsilon = -0.25$ requires a much higher α and somewhat lower $\hat{k}_0$ for the economy to be consistent with the long run capital share of 0.33 and the initial capital to output ratio of 0.68, see the second row in Table 4. The main results remain unchanged.

Next, we examine the role of the intertemporal elasticity of substitution of consumption (IES) by setting $\sigma = 4$. The IES is a fundamental ingredient to shape the path of investment in transitional dynamics because it drives the strength of the income effect, see Barro and Sala-i-Martin (1999). The economy with a lower IES makes the income effect stronger —households do not want to invest too much when they are poor— and hence our calibrated economy recovers a smaller investment wedge, see red line in Panel (d) of Figure 6. Overall, however, the main results are little affected. For instance, Panel B of Table 4 shows how the growth decomposition is very similar as in the benchmark calibration, with a somewhat larger role for the transitional dynamics (adding up the last two columns, the annual growth rate due to transitional dynamics is 1.67%, as opposed to 1.13% in the benchmark calibration).

Finally, the choice of initial capital is an important determinant for the strength of transitional dynamics in the development process. We try with an initial capital to income ratio of 1.30, which is about twice as big as the 0.68 in the benchmark economy, see footnote 24 for details. Using equation (23) we recover an initial capital in efficiency units relative to its BGP level of 0.54, which is 2.7 times larger than the 0.20 value in the benchmark economy. We recover a smaller wedge at the start of the development process because, with larger initial capital the desired initial investment is smaller, see yellow line in Panel (d) of Figure 6. The rest of results are relatively similar to the benchmark calibration, with the exception of the relative importance of transitional dynamics: with higher initial capital, transitional dynamics account of 0.79% annual growth instead of 1.13%.

6 Conclusions

The structural transformation process of developing economies described by Kuznets (1966) has become one of the most investigated empirical regularities in modern macroeconomics. We emphasize that, empirically, the development process is often not consistent with BGP, and hence accounting for the aggregate dynamics of the economy is crucial when thinking about the causes and consequences of structural transformation. In this paper, we provide a novel analysis of the development process of nations using a framework in which the investment rate and the sectoral composition of the economy are endogenously determined.

A new channel of structural change emerges within our framework: because investment and consumption goods are different in terms of their value added composition, changes in the investment rate shift the sectoral composition of the economy. We document three novel facts that suggest this channel to be quantitatively relevant: (i) the investment rate follows a long-lasting hump-shaped profile with development, and the peak of the hump of investment happens at a similar level of development as the peak in the hump of manufacturing; (ii) investment goods are 38 percentage points more intensive in value added from the industrial sector than consumption goods; (iii) the standard hump-shaped profile of manufacturing with development is absent when looking at investment and consumption goods separately.

When estimating a multi-sector model embedding these features with a panel of countries at different stages of development, we find that this novel channel of structural change explains 1/2 of the increase and 1/2 of the fall of manufacturing with development. We also find that the different sectoral composition of investment and consumption goods results in important aggregate implications for productivity growth that is asymmetric across sectors. In particular, the secular productivity increase that is faster in manufacturing than in services leads to a large decline in the relative price of investment, which in turn increases capital accumulation and promotes growth.

An important aspect for further research is the fact that our multi-sector growth model demands a declining wedge in the Euler equation to account for the large increase in the investment rate during the first half of the development process. A candidate explanation for this wedge is the decline of financial frictions at the early stages of development. However, we note that a proper microeconomic foundation of the financial frictions captured by the wedge may also shape the productivity paths in the model, see for instance Jeong and Townsend (2007), Erosa and Hidalgo-Cabrillana (2008), Buera and Shin (2013), or Moll (2014).

Finally, we want to stress that our mechanism is more general. As shown by equation (1), changes in the export rate and in the fraction of investment and consumption goods that are imported can also have first order effects on the sectoral composition of the economy. These are important questions for future research.

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Appendix A: Data sources and sector definitions

We use four different data sources: the three described in this Section and the WIOD described in Appendix B.

A.1 World Development Indicators (WDI)

We use the WDI database to obtain value added shares at current and at constant prices for our three sectors. The WDI divides the economy in 3 sectors: Agriculture (ISIC Rev 3.1 A and B), Industry (C to F), and Services (G to Q), which are the one that we use.[31]

In addition, we also use the variables for population and oil rents as a share of GDP in order to drop countries that are too small in terms of population and countries whose GDP is largely affected by oil extraction.

A.2 Groningen 10-Sector Database (G10S)

We use the G10S database to obtain value added shares at current and at constant prices for our three sectors. The G10S divides the economy in 10 industries, which we aggregate into our three main sectors as described in Table A.1.

Table A.1: G10S industry classification

Industry	Assigned Sector	ISIC 3.1 Code	Description
Agriculture	Agr	A,B	Agriculture, Hunting, Forestry and Fishing
Mining	Ind	C	Mining and Quarrying
Manufacturing	Ind	D	Manufacturing
Utilities	Ind	E	Electricity, Gas and Water Supply
Construction	Ind	F	Construction
Trade Services	Ser	G,H	Wholesale and Retail Trade; Repair of Motor Vehicles, Motorcycles and Personal and Household Goods; Hotels and Restaurants
Transport Services	Ser	I	Transport, Storage and Communications
Business Services	Ser	J,K	Financial Intermediation, Renting and Business Activities (excluding owner occupied rents)
Government Services	Ser	L,M,N	Public Administration and Defense, Education, Health and Social Work
Personal Services	Ser	O,P	Other Community, Social and Personal Service Activities, Activities of Private Households

[31] For some countries and years it also provides a breakdown of the Industry category with the Manufacturing sector (D) separately.

A.3 Penn World Tables (PWT)

We use the 9.0 version of the PWT to obtain the series for consumption, investment, export, and import shares of GDP in LCU at current prices. We also use the series for GDP per capita in constant LCU and the per capita GDP in constant international dollars.

Appendix B: The World Input-Output tables

In this section we provide more details on how we use the 2013 Release of the *World Input-Output Database* (WIOD) to construct some of the variables that we use in the paper. In particular, we explain (a) how we construct sectoral value added shares for consumption, investment, and exports for all countries and years, (b) how we aggregate from these sectoral value added shares by type of final good to sectoral value added shares of GDP, and (c) how we approximate the aggregation of sectoral value added shares without IO data.

B.1 Sectoral value added shares in consumption, investment, and exports

The 2013 Release of the WIOD provides national IO tables disaggregated into 35 industries for 40 countries and 17 years (the period 1995-2011). We aggregate the 35 different industries into agriculture, industry, and services according to table B.1. Total production in each industry is either purchased by domestic industries (intermediate expenditure) or by final users (final expenditure), which include domestic final uses and exports. To measure how much domestic value added from each sector goes to each final use we have to follow three steps. This procedure follows closely the material present in the Appendix of Herrendorf, Rogerson, and Valentinyi (2013).

First, we build $(n \times 1)$ vectors $\mathbf{e}_C$, $\mathbf{e}_X$, $\mathbf{e}_E$ with the final expenditure in *consumption* (final consumption by households plus final consumption by non-profit organisations serving households plus final consumption by government), *investment* (gross fixed capital formation plus changes in inventories and valuables), and *exports* coming from each of the n sectors. Note that, in our case, the number of sectors $n = 3$.

Second, we build the $(n \times n)$ Total Requirement ($\mathbf{TR}$) matrix linking sectoral expenditure to sectoral production. In particular, the IO tables provided by the WIOD assume that each industry j produces only one commodity, and that each commodity i is used in only one industry.[32] Let $\mathbf{A}$ denote the $(n \times n)$ transaction matrix, with entry ij showing the dollar amount of commodity i that industry j uses per dollar of output it produces. Let $\mathbf{e}$ denote the $(n \times 1)$ final expenditure vector, where entry j contains the dollar amount of final expenditure coming from industry j. Note that $\mathbf{e} = \mathbf{e}_C + \mathbf{e}_X + \mathbf{e}_E$. Let $\mathbf{g}$ denote the $(n \times 1)$ industry gross output vector, with entry j containing the total output in dollar amounts produced in industry j. Let $\mathbf{q}$ denote the $(n \times 1)$ commodity gross output vector.

[32]Notice that this structure is similar to the IO provided by the BEA prior to 1972.

The following identities link these three matrices with the ($\mathbf{TR}$) matrix:

$$\begin{aligned}\mathbf{q} &= \mathbf{A}\mathbf{g} + \mathbf{e} \\ \mathbf{q} &= \mathbf{g}\end{aligned}$$

We first get rid of $\mathbf{q}$ by using the second identity. We then solve for $\mathbf{g}$:

$$\mathbf{g} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{e} \tag{B.1}$$

where $\mathbf{TR} = (\mathbf{I} - \mathbf{A})^{-1}$ is the total requirement matrix. Entry ji shows the dollar value of the production of industry j that is required, both directly and indirectly, to deliver one dollar of the domestically produced commodity i to final uses. Note that in this matrix rows are associated with industries and columns with commodities.

Finally, we combine the $\mathbf{TR}$ matrix with the final expenditure vectors $\mathbf{e}_C$, $\mathbf{e}_X$, $\mathbf{e}_E$ to obtain:

$$\begin{aligned}\mathbf{VA}_X &= <\mathbf{v}>\mathbf{TR}\,\mathbf{e}_I \\ \mathbf{VA}_C &= <\mathbf{v}>\mathbf{TR}\,\mathbf{e}_C \\ \mathbf{VA}_E &= <\mathbf{v}>\mathbf{TR}\,\mathbf{e}_X\end{aligned} \tag{B.2}$$

where the $(n \times n)$ matrix $<\mathbf{v}>$ is a diagonal matrix with the vector $\mathbf{v}$ in its diagonal. The vector $\mathbf{v}$ contains the ratio of value added to gross output for each sector n. $\mathbf{VA}_X$, $\mathbf{VA}_C$, and $\mathbf{VA}_E$ are our main objects of interest. They contain the sectoral composition of value added used for investment, consumption, and exports. To compute the shares, we simply divide each element by the sum of all elements in each vector,

$$\begin{aligned}\frac{\mathrm{VA}_i^x}{\mathrm{VA}^x} &= \frac{\mathbf{VA}_X(i)}{\sum_{i=1}^n \mathbf{VA}_X(i)} \\ \frac{\mathrm{VA}_i^c}{\mathrm{VA}^c} &= \frac{\mathbf{VA}_C(i)}{\sum_{i=1}^n \mathbf{VA}_C(i)} \\ \frac{\mathrm{VA}_i^e}{\mathrm{VA}^e} &= \frac{\mathbf{VA}_E(i)}{\sum_{i=1}^n \mathbf{VA}_E(i)}\end{aligned} \tag{B.3}$$

B.2 Aggregation

We start with 4 national accounts identities. First, from the expenditure side GDP can be obtained as the sum of expenditure in investment X, consumption C, exports E minus imports M:

$$\mathrm{GDP} = X + C + E - M \tag{B.4}$$

Second, from the production side GDP can be obtained as the sum of value added VA_i produced in different sectors i,

$$\mathrm{GDP} = \sum_i \mathrm{VA}_i \tag{B.5}$$

Third, the value added of sector i can be expressed as:

$$\text{VA}_i = \text{VA}_i^x + \text{VA}_i^c + \text{VA}_i^e \tag{B.6}$$

where VA_i^x, VA_i^c, and VA_i^e are the valued added produced in sector i used for final investment, final consumption, and final exports respectively and are obtained from equation (B.2) above. Note that summing up equation (B.6) across sectors gives us:

$$\text{GDP} = \text{VA}^x + \text{VA}^c + \text{VA}^e \tag{B.7}$$

And fourth, the expenditure in investment X (or analogously consumption C and exports E) equals the sum of value added domestically produced that is used for investment VA^x and the imported value added that is used for investment (either directly or indirectly through intermediate goods), M^x:

$$\begin{aligned} X &= \text{VA}^x + M^x && \text{(B.8)} \\ C &= \text{VA}^c + M^c && \text{(B.9)} \\ E &= \text{VA}^e + M^e && \text{(B.10)} \end{aligned}$$

Note that summing equations (B.8)-(B.10) gives us equation (B.4) as $M = M^x + M^c + M^e$.

With these elements in place, note that the value added share of sector i in GDP can be expressed as:

$$\frac{\text{VA}_i}{\text{GDP}} = \left(\frac{\text{VA}^x}{\text{GDP}}\right)\left(\frac{\text{VA}_i^x}{\text{VA}^x}\right) + \left(\frac{\text{VA}^c}{\text{GDP}}\right)\left(\frac{\text{VA}_i^c}{\text{VA}^c}\right) + \left(\frac{\text{VA}^e}{\text{GDP}}\right)\left(\frac{\text{VA}_i^e}{\text{VA}^e}\right) \tag{B.11}$$

That is, the value added share of sector i in GDP is a weighted average of the value added share of sector i within investment $\frac{\text{VA}_i^x}{\text{VA}^x}$, consumption $\frac{\text{VA}_i^c}{\text{VA}^c}$, and exports $\frac{\text{VA}_i^e}{\text{VA}^e}$. These terms are the ones we have built in Appendix B.1 and that we describe in Table 1 and Panel (a), (c), and (e) of Figure 2. The weights are the share of domestic value added that is used for investment $\frac{\text{VA}^x}{\text{GDP}}$, for consumption $\frac{\text{VA}^c}{\text{GDP}}$ and for exports $\frac{\text{VA}^e}{\text{GDP}}$. Note that these weights are not the investment $\frac{X}{\text{GDP}}$, consumption $\frac{C}{\text{GDP}}$ and export $\frac{E}{\text{GDP}}$ rates as commonly measured in National Accounts because not all the expenditure in final investment, final consumption, and final exports comes from domesticaly produced value added. In particular,

$$\begin{aligned} \frac{\text{VA}^x}{\text{GDP}} &= \left(\frac{X}{\text{GDP}}\right)\left(\frac{\text{VA}^x}{X}\right) \\ \frac{\text{VA}^c}{\text{GDP}} &= \left(\frac{C}{\text{GDP}}\right)\left(\frac{\text{VA}^c}{C}\right) \\ \frac{\text{VA}^e}{\text{GDP}} &= \left(\frac{E}{\text{GDP}}\right)\left(\frac{\text{VA}^e}{E}\right) \end{aligned}$$

where the terms $\frac{\text{VA}^x}{X}$, $\frac{\text{VA}^c}{C}$, $\frac{\text{VA}^e}{E}$ denote the fraction of total expenditure in investment, consumption, and exports that is actually produced domestically, and which according to equations (B.8)-(B.10) must be weakly smaller than 1. Finally, note that in a closed economy the terms $\frac{\text{VA}^x}{X}$, $\frac{\text{VA}^c}{C}$, $\frac{\text{VA}^e}{E}$ will need to be one by construction and hence equation (B.11) would become,

$$\frac{\text{VA}_i}{\text{GDP}} = \left(\frac{X}{\text{GDP}}\right)\left(\frac{\text{VA}_i^x}{\text{VA}^x}\right) + \left(\frac{C}{\text{GDP}}\right)\left(\frac{\text{VA}_i^c}{\text{VA}^c}\right) \tag{B.12}$$

Equation (B.12) corresponds to equation (24) in the model.

B.3 Approximation

In order to perform decompositions of extensive and intensive margin structural change with equation (B.11) one needs IO tables for both the extensive and intensive margin terms. We can get an approximation to equation (B.11) that is less demanding in terms of data. Note that using equation (B.8) we can rewrite the term $\frac{\text{VA}^x}{X}$ as

$$\frac{\text{VA}^x}{X} = \left[\frac{\text{VA}^x + M^x}{\text{VA}^x}\right]^{-1} = \left[1 + \frac{M}{\text{GDP}}\frac{M^x/M}{\text{VA}^x/\text{GDP}}\right]^{-1} \tag{B.13}$$

and analogous expressions obtain for $\frac{\text{VA}^c}{C}$ and $\frac{\text{VA}^e}{E}$. Note that if

$$\frac{M^x/M}{\text{VA}^x/\text{GDP}} = \frac{M^c/M}{\text{VA}^c/\text{GDP}} = \frac{M^e/M}{\text{VA}^e/\text{GDP}} = 1$$

then equation (B.11) can be written as,

$$\frac{\text{VA}_i}{\text{GDP}} = \left[1 + \frac{M}{\text{GDP}}\right]^{-1}\left[\left(\frac{X}{\text{GDP}}\right)\left(\frac{\text{VA}_i^x}{\text{VA}^x}\right) + \left(\frac{C}{\text{GDP}}\right)\left(\frac{\text{VA}_i^c}{\text{VA}^c}\right) + \left(\frac{E}{\text{GDP}}\right)\left(\frac{\text{VA}_i^e}{\text{VA}^e}\right)\right] \tag{B.14}$$

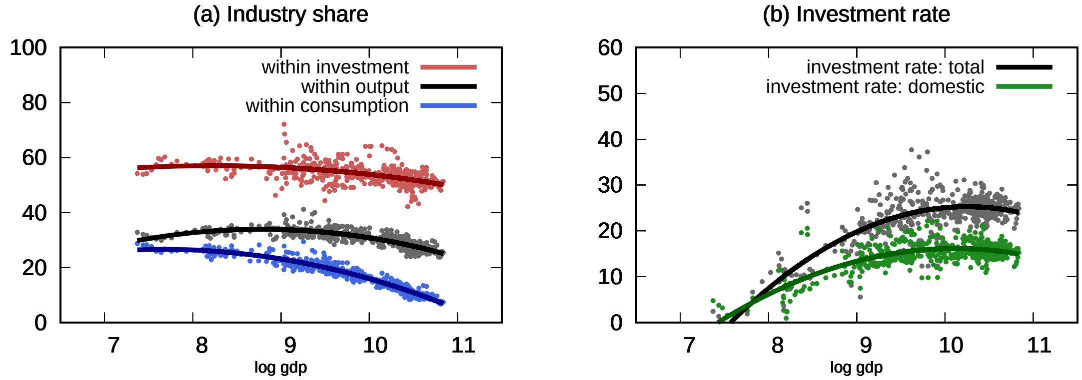
or

$$\frac{\text{VA}_i}{\text{GDP}} = \left(\frac{X}{\text{GDP}+M}\right)\left(\frac{\text{VA}_i^x}{\text{VA}^x}\right) + \left(\frac{C}{\text{GDP}+M}\right)\left(\frac{\text{VA}_i^c}{\text{VA}^c}\right) + \left(\frac{E}{\text{GDP}+M}\right)\left(\frac{\text{VA}_i^e}{\text{VA}^e}\right) \tag{B.15}$$

with this approximation one can estimate the intensive margin terms as we do in Section 5 and use national accounts to obtain the extensive margin terms, hence no IO data is needed.

The question here is: how good is this approximation? To answer this question we compute the approximated value added shares for each sector, country and year in the WIOD using equation (B.14) and compare them to the actual ones. In Table B.2 we provide a few statistics to compare the actual with the approximated series pooling all countries and years of data. Panel (a) shows that both the mean and dispersion of the actual and approximated sectoral shares are very similar. It also shows that the correlation between the actual and approximated series are over 0.99 in all three sectors, both when pooling all the data and when controlling for country fixed effects. Panel (b) reports the results of regressing the actual shares against a polynomial of log GDP and country fixed

Figure B.1: Sectoral shares for Industry and investment rate, within-country evidence



Notes. Sectoral shares and investment rates from WIOD (dots) and projections on a quadratic polynomial of log GDP per capita in constant international dollars (lines). Data have been filtered out from country fixed effects.

effects, together with the R^2 partialling out the country fixed effects.[33] Again, we see that the variation of the actual and approximated series with the level of development are very similar. The reason for this approximation being quite good is that the evolution of the terms VA^x/GDP and X/GDP (and the same for consumption and exports) are not so different after all, see Panel (b) in Figure B.1 for the case of investment.

[33] The regressions with the actual data are the ones used to construct the trends in Panel (b), (d), and (f) of Figure 2 in the paper.

Table B.1: WIOT industry classification

Industry	Assigned Sector (s)	Industry (j) Code	IO position
Agriculture, Hunting, Forestry and Fishing	Agriculture	AtB	c1
Mining and Quarrying	Industry	C	c2
Food, Beverages and Tobacco	Industry	15t16	c3
Textiles and Textile Products	Industry	17t18	c4
Leather, Leather and Footwear	Industry	19	c5
Wood and Products of Wood and Cork	Industry	20	c6
Pulp, Paper, Paper , Printing and Publishing	Industry	21t22	c7
Coke, Refined Petroleum and Nuclear Fuel	Industry	23	c8
Chemicals and Chemical Products	Industry	24	c9
Rubber and Plastics	Industry	25	c10
Other Non-Metallic Mineral	Industry	26	c11
Basic Metals and Fabricated Metal	Industry	27t28	c12
Machinery, Nec	Industry	29	c13
Electrical and Optical Equipment	Industry	30t33	c14
Transport Equipment	Industry	34t35	c15
Manufacturing, Nec; Recycling	Industry	36t37	c16
Electricity, Gas and Water Supply	Industry	E	c17
Construction	Industry	F	c18
Sale, Maintenance and Repair of Motor	Services	50	c19
Vehicles and Motorcycles; Retail Sale of Fuel			
Wholesale Trade and Commission Trade,	Services	51	c20
Except of Motor Vehicles and Motorcycles			
Retail Trade, Except of Motor Vehicles and	Services	52	c21
Motorcycles; Repair of Household Goods			
Hotels and Restaurants	Services	H	c22
Inland Transport	Services	60	c23
Water Transport	Services	61	c24
Air Transport	Services	62	c25
Other Supporting and Auxiliary	Services	63	c26
Transport Activities; Activities of Travel Agencies	Services		
Post and Telecommunications	Services	64	c27
Financial Intermediation	Services	J	c28
Real Estate Activities	Services	70	c29
Renting of M&Eq and Other Business Activities	Services	71t74	c30
Public Admin and Defense, Compulsory Social Security	Services	L	c31
Education	Services	M	c32
Health and Social Work	Services	N	c33
Other Community, Social and Personal Services	Services	O	c34
Private Households with Employed Persons	Services	P	c35

TABLE B.2: Sectoral composition: data *vs.* approximation

	Agr		Ind		Ser	
	Data	Appr	Data	Appr	Data	Appr
PANEL (A): STATISTICS						
mean	4.8	4.8	29.7	30.5	65.4	64.7
sd	4.6	4.6	6.7	6.7	9.6	9.6
corr	0.999		0.996		0.998	
corr (fe)	0.999		0.990		0.995	
PANEL (B): REGRESSION						
log GDP	-25.7	-26.6	40.6	42.0	-14.9	-15.3
log GDP $\times$ log GDP	1.0	1.1	-2.3	-2.4	1.3	1.3
R^2 (%)	60.3	60.4	19.4	18.8	45.9	45.1

Notes: Panel (a) reports mean, standard deviation, and correlation of the actual and approximated sectoral shares pooling all countries and years. It also provides the correlation of the differences with respect to country means to control for country fixed effects. Panel (b) regresses the sectoral shares, data and approximation, against country fixed effects, log GDP and log GDP squared. The coefficients are all significant at the standard 1% significance level and the R^2 corresponds to the regression of differences with respect to country means.

Appendix C: Filtering and projecting the panel data

Dots and thick dark lines in Figures. The thick dark lines in Figure 1, Figure 2, and Panels (b) and (c) in Figure 6 have been built as follows. First, we regress the desired variable z_{it} on a low order polynomial of $\log y_{it}$ and country fixed effects α_{zi}:

$$z_{it} = \alpha_{zi} + \alpha_{z1} \log y_{it} + \alpha_{z2} (\log y_{it})^2 + \alpha_{z3} (\log y_{it})^3 + \varepsilon_{zit} \tag{C.1}$$

and next we use the prediction equation,

$$\hat{z}_{it} = \alpha_z + \hat{\alpha}_{z1} \log y_{it} + \hat{\alpha}_{z2} (\log y_{it})^2 + \hat{\alpha}_{z3} (\log y_{it})^3 \tag{C.2}$$

with the arbitrary α_z intercept equal to the unweighted average of country fixed effects α_{zi}. The $\hat{z}_{it}$ form the thick dark lines in the Figures, while the clouds of points in these same figures are obtained by adding the estinated error $\hat{\varepsilon}_{zit}$ from regression equation (C.1) to the predicted series $\hat{z}_{it}$.

Data for the estimation of the demand system. We use the $\hat{z}_{it} + \hat{\varepsilon}_{zit}$ obtained from (C.1) and (C.2) as our data points. Note that this is analogous to using the actual data filtered from country fixed effects, that is, the differences between the data and the country means.

Data for the calibration of the dynamic side of the model. For the calibration of the dynamic side of the model, we first want to create time series for a synthetic country that follows a stylized process of development extracted from our panel data set. We proceed as follows.

1. Obtain the prediction functions for the variables of interest with regression (C.1).

2. Do the same for the growth of per capita GDP:

$$\Delta \log y_{it+1} = \alpha_{yi} + \alpha_{y1} \log y_{it} + \alpha_{y2} (\log y_{it})^2 + \alpha_{y3} (\log y_{it})^3 + \varepsilon_{yit}$$

3. Create a time series for GDP per capita:

   (a) Initialize the synthetic country: $\hat{y}_0 = \min \{y_{it}\}$

   (b) Fill the whole time series for $\hat{y}_t$ between $t = 1$ and T using,

$$\Delta \log \hat{y}_{t+1} = \alpha_y + \hat{\alpha}_{y1} \log \hat{y}_{it} + \hat{\alpha}_{y2} (\log \hat{y}_{it})^2 + \hat{\alpha}_{y3} (\log \hat{y}_{it})^3$$

   where $\hat{\alpha}_{y1}$, $\hat{\alpha}_{y2}$, and $\hat{\alpha}_{y3}$ are the estimated values and α_y is an arbitrary intercept that we choose such that $\Delta \log \hat{y}_T = 0.02$, which is arguably the long run rate of growth of the US economy, which we see as the economy at the technology frontier. T is determined by the number of periods it takes the synthetic country to reach the maximum income per capita in out panel, that is, T is the maximum s such that $\hat{y}_s \leq \max \{y_{it}\}$. In our exercise we find $T = 96$.

4. Create the time series for the variables of interest $\hat{z}_t$ between $t = 0$ and T using

$$\hat{z}_t = \alpha_z + \hat{\alpha}_{z1} \log \hat{y}_t + \hat{\alpha}_{z2} \left(\log \hat{y}_t\right)^2 + \hat{\alpha}_{z3} \left(\log \hat{y}_t\right)^3 \tag{C.3}$$

where $\hat{\alpha}_{z1}$, $\hat{\alpha}_{z2}$, and $\hat{\alpha}_{z3}$ are the estimated values in equation (C.1), and α_z is an arbitrary intercept equal to the unweighted average of all the country fixed effects $\hat{\alpha}_{zi}$.

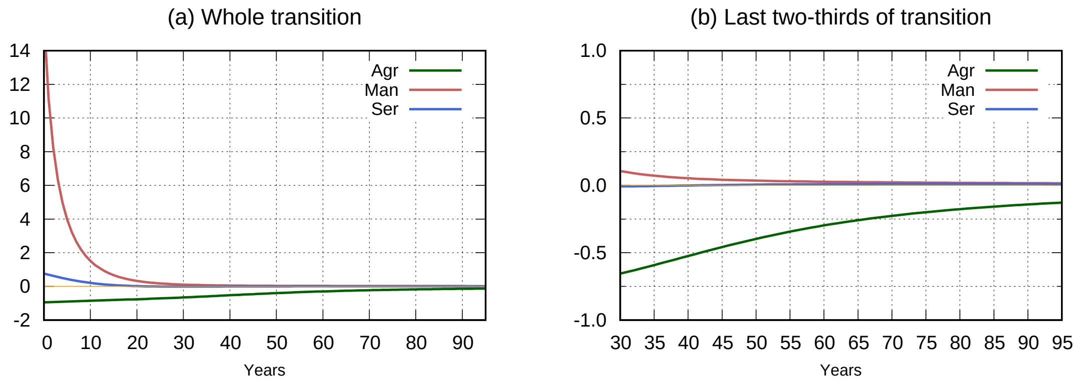
Appendix D: Estimation of the demand system: income elasticity

Our demand system generates nice closed-form solutions for the expenditure elasticity of each good. In particular, it can be shown that,

$$\left[\frac{d\left(p_{it}c_{it}/\sum_j p_{jt}c_{jt}\right)}{d\sum_j p_{jt}c_{jt}}\right]\left[\frac{\sum_j p_{jt}c_{jt}}{\left(p_{it}c_{it}/\sum_j p_{jt}c_{jt}\right)}\right] = \frac{\bar{c}_i}{c_{it}} - \theta_i^c \left(\frac{p_{ct}}{p_{it}}\right)^{\frac{\rho_c}{1-\rho_c}} \left(\frac{\sum_j p_{jt}\bar{c}_j}{p_{it}c_{it}}\right)$$

When all $\bar{c}_i$ are zero the demand system is homothetic: the expenditure shares do not change with total expenditure. Luxury goods (necessities) display a positive (negative) expenditure elasticity. Note that it is not a necessary condition to have $\bar{c}_a < 0$ for agriculture good to be necessity as the second term in the r.h.s. can be positive and larger in absolute value than $\bar{c}_a$.

Figure D.1: Expenditure elasticities



In Figure D.1 we report the expenditure elasticities implied by our estimates. We see how agriculture is a necessity and both manufacturing and services are luxury goods. This is especially important at early stages of development because as the economies become richer the $\bar{c}_i$ vanish relative to c_{it} and relative to total expenditure. Note that the expenditure elasticity is larger for manufactures than for services during the early stage of development.

Appendix E: Further model details

In order to obtain the optimality conditions in Section 4 we write the Lagrangian as,

$$\sum_{t=0}^{\infty} \beta^t \{ u(c_t) \quad + \quad \lambda_t \Big[w_t + r_t k_t - \sum_{i=\{a,m,s\}} p_{it}(c_{it} + x_{it}) \Big]$$

$$+ \quad \eta_t \Big[(1-\delta) k_t + x_t - k_{t+1} \Big] + \sum_{i=\{a,m,s\}} \tilde{\nu}_{it} p_{it} c_{it} \Big\}$$

where λ_t and η_t are the shadow values at time t of the budget constraint and the law of motion of capital respectively, and $\tilde{\nu}_{it}$ are the multipliers of the inequality constraints $p_{it}c_{it} \geq 0$. There is no need to place such inequality constraints for the amounts spent in investment as the marginal value of each investment good goes to infinity when the quantity goes to zero. Likewise, within consumption, those goods with $\bar{c}_i \leq 0$ (agriculture) will never have a binding inequality constraint because as c_{it} tends to $|\bar{c}_i|$ the marginal utility of that good goes to infinity.

Taking prices as given, the standard first order conditions with respect to goods c_{it} and x_{it} are:

$$\frac{\partial u_t(c_t)}{\partial c_t} \frac{\partial c_t}{\partial c_{it}} = \lambda_t \left(1 - \frac{\tilde{\nu}_{it}}{\lambda_t}\right) p_{it} \qquad i \in \{a, m, s\} \tag{E.1}$$

$$\eta_t \quad \frac{\partial x_t}{\partial x_{it}} = \lambda_t \, p_{it} \qquad i \in \{a, m, s\} \tag{E.2}$$

while the FOC for capital k_{t+1} is given by,

$$\eta_t = \beta \, \lambda_{t+1} r_{t+1} + \beta \, \eta_{t+1} (1-\delta) \tag{E.3}$$

In what follows, and throughout the main text, we assume that the constraints $p_{it}c_{it} \geq 0$ are not binding and hence $\tilde{\nu}_{it} = 0$. Indeed, this is the case for all the economies we solve, with the exception of counterfactual economy E_4 (where we remove the investment wedge). We defer to Section E.7 the discussion on how to solve the constrained model.

Sectoral composition of consumption expenditure. Using the utility function in equation (7) and the consumption aggregator in equation (5), the FOC of each good i described by equation (E.1) can be rewritten as:

$$c_t^{-\sigma} \left(\theta_i^c \frac{c_t}{c_{it} + \bar{c}_i} \right)^{1-\rho_c} = \lambda_t p_{it} \tag{E.4}$$

We can aggregate them (raising to the power $\frac{\rho_c}{\rho_c - 1}$ and summing them up) to obtain the FOC for the consumption basket,

$$c_t^{-\sigma} = \lambda_t p_{ct} \tag{E.5}$$

where p_{ct} is the implicit price index of the consumption basket defined in (12). Adding up the FOC for each good i we obtain equation (10) stating that total expenditure in consumption goods is equal to the value of the consumption basket minus the value of the non-homotheticities. Finally, using equations (E.4) and (10) we obtain the consumption expenditure share of each good i given by,

$$\frac{p_{it}c_{it}}{\sum_{j=a,m,s} p_{jt}c_{jt}} = \theta_i^c \left(\frac{p_{ct}}{p_{it}}\right)^{\frac{\rho_c}{1-\rho_c}} \left[1 + \frac{\sum_{j=a,m,s} p_{jt}\bar{c}_j}{\sum_{j=a,m,s} p_{jt}c_{jt}}\right] - \frac{p_{it}\bar{c}_i}{\sum_{j=a,m,s} p_{jt}c_{jt}} \tag{E.6}$$

Finally, substituting the expression for p_{ct} in equation (12) into (E.32) we obtain the sectoral consumption shares as function of sectoral prices as in equation (8).

Sectoral composition of investment expenditure. Using the aggregator in equation (6), the FOC of each good i described by equation (E.2) can be rewritten as:

$$\eta_t \chi_t^{\rho} \left(\theta_i^x \frac{x_t}{x_{it}}\right)^{1-\rho_x} = \lambda_t p_{it} \tag{E.7}$$

Following similar steps as for consumption we get the FOC for total investment,

$$\eta_t = \lambda_t p_{xt} \tag{E.8}$$

where the price of the investment basket is given by equation (13) and the value of the investment basket equals investment expenditure as stated by equation (11). Finally, combining equations (E.7) and (11) the actual composition of investment expenditure is given by

$$\frac{p_{it}x_{it}}{p_{xt}x_t} = \theta_i^x \left(\frac{\chi_t \, p_{xt}}{p_{it}}\right)^{\frac{\rho_x}{1-\rho_x}} \tag{E.9}$$

Finally, substituting the expression for p_{xt} in equation (13) into (E.9) we obtain the sectoral investment shares as function of sectoral prices as in equation (9).

Euler equation. Plugging equations (E.5) and (E.8) into (E.3) we get the Euler equation driving the dynamics of the model, see equation (14)

E.1 Dynamic system in efficiency units

It is helpful to rewrite all the model variables in units of the investment good scaled by the labor saving technology level B_t. Hence, let the hat variables be $\hat{k}_t \equiv k_t/B_t$, $\hat{x}_t \equiv x_t/B_t$, $\hat{y}_t \equiv \frac{y_t}{p_{xt}} \frac{1}{B_t} = \frac{y_t}{p_{ct}} \frac{\chi_t B_{xt}}{B_t B_{ct}}$, $\hat{c}_t \equiv \frac{p_{ct}c_t}{p_{xt}} \frac{1}{B_t} = c_t \frac{\chi_t B_{xt}}{B_t B_{ct}}$. Then, the two difference equations (25) and

(26) in terms of the hat variables are given by,

$$\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{\sigma}(1+\gamma_{Bt+1})^{\sigma} = \frac{\beta}{1+\tau_t}\left[\alpha\left(\chi_{t+1}B_{xt+1}\right)^{\epsilon}\left(\frac{\hat{y}_{t+1}}{\hat{k}_{t+1}}\right)^{1-\epsilon}+(1-\delta)\right] \left[\frac{1+\gamma_{Bct+1}}{1+\gamma_{Bxt+1}}\frac{1}{1+\gamma_{\chi t+1}}\right]^{1-\sigma} \tag{E.10}$$

$$\frac{\hat{k}_{t+1}}{\hat{k}_t}(1+\gamma_{Bt+1}) = (1-\delta)+\frac{\hat{y}_t}{\hat{k}_t}-\frac{\hat{c}_t}{\hat{k}_t}+\frac{\chi_t B_{xt}}{B_t}\frac{1}{\hat{k}_t}\sum_{i=a,m,s}\frac{\bar{c}_i}{B_{it}} \tag{E.11}$$

with the capital to output ratio given by

$$\frac{\hat{y}_t}{\hat{k}_t} = \chi_t B_{xt}\left[\alpha+(1-\alpha)\,\hat{k}_t^{-\epsilon}\right]^{1/\epsilon} \tag{E.12}$$

Note that this system of equations is not autonomous due to the presence of (a) both the level and rate of growth of the labor-saving technical change, (b) both the level and rate of growth of the exogenous investment specific technical change, (c) both the levels and rates of growth of the Hicks-neutral sector-specific technical change (the latter enter directly in the law of motion of capital through the non-homotheticities, but also indirectly through the level and growth of the average productivity levels in consumption and investment B_{ct} and B_{xt}), and (d) the investment wedge τ_t..

E.2 Balanced Growth Path

We define the Balanced Growth Path (BGP) as an equilibrium in which the capital to output ratio $p_{xt}k_y/y_t$ is constant. For a BGP to exist we need the following conditions to be met:

(i) $(1+\gamma_{Bxt})(1+\gamma_{\chi t}) = 1$,

(ii) $\gamma_{Bt} = \gamma_B$ constant,

(iii) $\gamma_{Bct} = \gamma_{Bc}$ constant,

(iv) the $\bar{c}_i$ vanish asymptotically,

(v) the wedge τ_t is constant.

Equation (E.12) shows that the capital to output ratio can only be constant if condition (i) holds and capital grows at the rate γ_{Bt} such that $\hat{k}_t$ is constant. For equation (E.11) to hold in BGP we need conditions (iv) and (ii) and constant $\hat{c}_t$. Finally, for households to choose a $\hat{c}_t$ constant in the Euler equation, equation (E.10), we need condition (iii). In the BGP also output $\hat{y}_t$ and investment $\hat{x}_t$ are constant —see the production function (21) for output, and investment shall be constant if output and consumption are. Hence, capital, investment, output and consumption in units of investment good grow all at the rate γ_B and the same variables in units of the consumption good grow at the rate

$(1+\gamma_B)(1+\gamma_{Bc})$. What does this imply for the model fundamentals? Note that condition (i) imposes a knife edge condition for the whole sequence of χ_t. If we are happy to dispose with this knife-edge condition, then the BGP requires $\gamma_{Bat} = \gamma_{Bmt} = \gamma_{Bst} = \gamma_{\chi_t} = 0$.

E.3 Characterization of the Balanced Growth Path

The BGP capital $\hat{k}$ in the model is characterized by the modified golden rule. That is, taking the Euler equation in (E.10) and imposing the BGP conditions we obtain,

$$(1+\gamma_B) = \beta^{1/\sigma}\left[\alpha\chi B_x\left[\alpha + (1-\alpha)\,\hat{k}^{-\epsilon}\right]^{\frac{1-\epsilon}{\epsilon}} + (1-\delta)\right]^{1/\sigma}(1+\gamma_{Bc})^{\frac{1-\sigma}{\sigma}} \tag{E.13}$$

Then, output $\hat{y}$ in units of the investment good is given by the aggregate production function in equation (21), which becomes

$$\hat{y} = \chi B_x\left[\alpha\hat{k}^{\epsilon} + (1-\alpha)\right]^{1/\epsilon} \tag{E.14}$$

and the law of motion for capital

$$(1+\gamma_B) \;=\; (1-\delta) + \frac{\hat{y}}{\hat{k}} - \frac{\hat{c}}{\hat{k}} \tag{E.15}$$

determines consumption $\hat{c}$ and investment $\hat{x}$. Finally, from the interest rate equation (22) and the capital to labor ratio given by equation (23) we can get an expression for the capital share,

$$\frac{r\hat{k}}{\hat{y}} = \alpha\left[\alpha + (1-\alpha)\,\hat{k}^{-\epsilon}\right]^{-1} \tag{E.16}$$

Note that with the CES production functions the whole path for the investment-specific technical change $\chi_t B_{xt}$ matters in order to determine the variables in BGP. This is because this path determines the BGP level χB_x. For instance, what happens if the exogenous investment-specific technical change grows less than in our benchmark economy? The BGP value χ will be lower, meaning that the production of investment goods is more expensive in this counterfactual economy, which leads to a BGP with less capital, less investment, less output, and higher capital to output ratio, higher capital share and higher investment rate. To see this, note that when χ is lower equation (E.13) implies that $\hat{k}$ is lower, equation (E.14) implies that output $\hat{y}$ is lower, and equation (E.15) implies that investment $\hat{x}$ is lower. Also, equation (23) shows that the capital to output ratio $\frac{\hat{k}}{\hat{y}}$ is larger and equation (E.16) shows that the capital share is larger. Finally, rewriting equation (E.15) as

$$(1+\gamma_B) \;=\; (1-\delta) + \frac{\hat{x}}{\hat{y}}\frac{\hat{y}}{\hat{k}}$$

shows that the investment rate goes up. What is the logic of all this? The production function is CES in capital and labor. A lower χ makes capital more expensive relative to labor. This means that less capital is used in BGP (lower $\hat{k}$), but with ES less than one more is spent in capital, that is the capital share goes up. The lower capital level requires a lower amount of investment to be sustained in the BGP and, because output falls more than capital, both the capital to output and investment to output ratios increase. Why does output fall more than capital? Because it suffers the direct effect of the fall in χ and the indirect effect of the fall in the capital stock.

E.4 BGP with Cobb-Douglas production functions

In the Cobb-Douglas case ($\epsilon = 0$) the capital to output ratio is given by

$$\left(\frac{p_{xt}k_t}{y_t}\right)^{-1} = \chi_t B_{xt}\left(\frac{B_t}{k_t}\right)^{(1-\alpha)}$$

which is constant if capital k_t grows at the rate γ_t given by

$$1+\gamma_t = (1+\gamma_{Bt})\left[(1+\gamma_{\chi t})(1+\gamma_{Bxt})\right]^{\frac{1}{1-\alpha}}$$

Hence, it will be helpful to rewrite the model variables in units of the investment good scaled by the productivity level $B_t(\chi_t B_{xt})^{\frac{1}{1-\alpha}}$, which grows at the rate γ_t. Let the hat variables be:

$$\begin{aligned}
\hat{k}_t &\equiv k_t \frac{1}{B_t(\chi_t B_{xt})^{\frac{1}{1-\alpha}}} \\
\hat{x}_t &\equiv x_t \frac{1}{B_t(\chi_t B_{xt})^{\frac{1}{1-\alpha}}} \\
\hat{y}_t &\equiv \frac{y_t}{p_{xt}}\frac{1}{B_t(\chi_t B_{xt})^{\frac{1}{1-\alpha}}} = \frac{y_t}{p_{ct}}\frac{1}{B_t B_{ct}(\chi_t B_{xt})^{\frac{\alpha}{1-\alpha}}} \\
\hat{c}_t &\equiv \frac{p_{ct}c_t}{p_{xt}}\frac{1}{B_t(\chi_t B_{xt})^{\frac{1}{1-\alpha}}} = c_t\frac{1}{B_t B_{ct}(\chi_t B_{xt})^{\frac{\alpha}{1-\alpha}}}
\end{aligned}$$

Then, the production function in equation (21) becomes $\hat{y}_t = \hat{k}_t^{\alpha}$ and the two difference equations are:

$$\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{\sigma}(1+\gamma_{t+1})^{\sigma} = \frac{\beta}{1+\tau_t}\left[\alpha\hat{k}_{t+1}^{\alpha-1}+(1-\delta)\right]\left[\frac{1+\gamma_{Bct+1}}{1+\gamma_{Bxt+1}}\frac{1}{1+\gamma_{\chi t+1}}\right]^{1-\sigma} \tag{E.17}$$

$$\frac{\hat{k}_{t+1}}{\hat{k}_t}(1+\gamma_{t+1}) = (1-\delta)+\hat{k}_t^{\alpha-1}-\frac{\hat{c}_t}{\hat{k}_t}+\frac{1}{B_t(\chi_t B_{xt})^{\frac{\alpha}{1-\alpha}}}\frac{1}{\hat{k}_t}\sum_{i=a,m,s}\frac{\bar{c}_i}{B_{it}} \tag{E.18}$$

In the Cobb-Douglas production case the BGP requires the same conditions (iii), (iv), and (v) as in the CES case, condition (ii) is unneeded as with Cobb-Douglas B_t can be subsumed into the B_{it}, and condition (i) is replaced by

(i′) $(1+\gamma_{Bxt})(1+\gamma_{\chi t})$ constant

Again, we can dispose with the knife edge condition such that the sequence $\gamma_{\chi t}$ equals the sequence of γ_{Bxt} and we concentrate on the case with $\gamma_{\chi t}$ constant. Then, conditions (i′) and (iii) require B_{ct} and B_{xt} to grow at constant rates, which in general cannot happen because B_{ct} and B_{xt} are time-changing weighted averages of the different B_{it}. Equation (18) clearly shows that the two options for B_{xt} and B_{ct} to grow at constant rates are that either (i) $\rho_x = 0$ and $\rho_c = 0$ (unit elasticity of substitution) and the sectoral productivities grow at constant but possibly different rates, or (ii) the rate of growth of B_{it} are constant and equal in all sectors (symmetric productivity growth across sectors). Of course, there is no structural change within investment goods in neither case. Hence, in the BGP output in units of the investment good, y_t/p_{xt}, investment x_t, and consumption in units of the investment good $p_{ct}c_t/p_{xt}$ (see the law of motion for capital) grow all at the same rate γ_t, while the same variables in units of the consumption good grow at the rate $\tilde{\gamma}_t$ given by,

$$1+\tilde{\gamma}_t = (1+\gamma_{Bt})\,(1+\gamma_{Bct})\,[(1+\gamma_{\chi t})\,(1+\gamma_{Bxt})]^{\frac{\alpha}{1-\alpha}}$$

E.5 A few particular cases

Our model specification nests a few well-known cases in the literature. In particular, setting $\epsilon = 0$ and $B_t = 1\ \forall t$ to have a Cobb-Douglas production function:

Kongsamut, Rebelo, Xie. Assume that $\chi_t = 1$, $\tau_t = 0$, $\theta_a^x = \theta_s^x = 0$, $\bar{c}_m = 0$, $\rho_c = 0$, and that B_{it} grow all at the same rate γ_B. This is the Kongsamut, Rebelo, and Xie (2001) model, which assumes that there is no investment-specific technical change, that investment goods come only from manufacturing, and that productivity growth is the same in all sectors. In this model structural change happens due to the non-homotheticicities in demand. By construction conditions (i'), (iii), and (v) for BGP are met. Because the B_{it} all grow at the same constant rate, condition (iv) can only be met with structural change if

$$-\frac{\bar{c}_a}{\bar{c}_s} = \frac{B_{a0}}{B_{s0}}$$

Then, the aggregate dynamics are given by the following system of two difference equations,

$$\begin{aligned}
\frac{\hat{c}_{t+1}}{\hat{c}_t}(1+\gamma_B) &= \beta^{\frac{1}{\sigma}}\left[\alpha\,\hat{k}_{t+1}^{\alpha-1} + (1-\delta)\right]^{\frac{1}{\sigma}} \\
\frac{\hat{k}_{t+1}}{\hat{k}_t}(1+\gamma_B) &= (1-\delta) + \hat{k}_t^{\alpha-1} - \frac{\hat{c}_t}{\hat{k}_t}
\end{aligned}$$

which produce the same aggregate dynamics as the one-sector Ramsey model of growth.

Ngai, Pissarides. Assume that $\chi_t = 1$, $\tau_t = 0$, $\theta_a^x = \theta_s^x = 0$, and $\bar{c}_i = 0\ \forall i \in \{a, m, s\}$. This is the Ngai and Pissarides (2007) model. As in Kongsamut, Rebelo, and Xie (2001) there is no investment specific technical change and investment goods are produced only with value added from manufacturing. Structural change is the result of asymmetric productivity growth across sector and non-unit elasticity of substitution across goods. Conditions (iv) and (v) for BGP are met by construction. Condition (i') is met by assuming that investment goods only come from manufacturing. Condition (iii) is not met, but with $\sigma = 1$ is not needed. This gives again the same aggregate dynamics as in the one-sector Ramsey model of growth. The system of difference equations becomes:

$$\begin{aligned}
\frac{\hat{c}_{t+1}}{\hat{c}_t}(1+\gamma_{Bm}) &= \beta\left[\alpha\,\hat{k}_{t+1}^{\alpha-1} + (1-\delta)\right] \\
\frac{\hat{k}_{t+1}}{\hat{k}_t}(1+\gamma_{Bm}) &= (1-\delta) + \hat{k}_t^{\alpha-1} - \frac{\hat{c}_t}{\hat{k}_t}
\end{aligned}$$

Greenwood, Hercowitz, Krusell. Assume that $\tau_t = 0$, $\theta_i^x = \theta_i^c$ and $\bar{c}_i = 0\ \forall i \in \{a, m, s\}$. This implies that the investment and consumption goods are identical in terms of sectoral composition. If we further assume that B_{it} grow all at the same rate γ_{Bt} there is no sectoral reallocation. This is the Greenwood, Hercowitz, and Krusell (1997) model (without capital structures), whose aggregate dynamics are described by,

$$\begin{aligned}
\frac{\hat{c}_{t+1}}{\hat{c}_t}\left[(1+\gamma_{\chi t})(1+\gamma_{Bt})\right]^{\frac{1}{1-\alpha}} &= \beta^{\frac{1}{\sigma}}\left[\alpha\,\hat{k}_{t+1}^{\alpha-1} + (1-\delta)\right]^{\frac{1}{\sigma}}(1+\gamma_{\chi t})^{\frac{1-\sigma}{\sigma}} \\
\frac{\hat{k}_{t+1}}{\hat{k}_t}\left[(1+\gamma_{\chi t})(1+\gamma_{Bt})\right]^{\frac{1}{1-\alpha}} &= (1-\delta) + \hat{k}_t^{\alpha-1} - \frac{\hat{c}_t}{\hat{k}_t}
\end{aligned}$$

The model meets all required conditions for BGP whenever $\gamma_{\chi t}$ and γ_{Bt} are constant.

Herrendorf, Rogerson, Valentinyi. Assume $\tau_t = 0$, $\bar{c}_i = 0$, and $\sigma = 1$. This is the Herrendorf, Rogerson, and Valentinyi (2020) model that allows for structural change within consumption and investment as ours but which differs in that there are no income effects within consumption, and the utility is logarithmic. Then, conditions (iv) and (v) are met by construction, condition (iii) is unneeded due to the log utility, and for condition (i') to be satisfied these authors allow for time changing sectoral rates of growth γ_{Bit} that are offset by $\gamma_{\chi t}$. The aggregate dynamics are characterized by the equations,

$$\begin{aligned}
\frac{\hat{c}_{t+1}}{\hat{c}_t}\left[(1+\gamma_{\chi t})(1+\gamma_{Bxt})\right]^{\frac{1}{1-\alpha}} &= \beta\left[\alpha\hat{k}_{t+1}^{\alpha-1} + (1-\delta)\right] \\
\frac{\hat{k}_{t+1}}{\hat{k}_t}\left[(1+\gamma_{\chi t})(1+\gamma_{Bxt})\right]^{\frac{1}{1-\alpha}} &= (1-\delta) + \hat{k}_t^{\alpha-1} - \frac{\hat{c}_t}{\hat{k}_t}
\end{aligned}$$

E.6 A two-good representation of the economy

This model economy can be rewritten as model with two final goods, investment and consumption, whose production has hicks-neutral productivity $\chi_t B_{xt}$ and B_{ct} respectively.

Two-stage household problem. The household problem can be described as a two stage optimization process in which the household first solves the dynamic problem by choosing the amount of spending in consumption $p_{ct}c_t$ and investment $p_{xt}x_t$, and then solves the static problem of choosing the composition of consumption and investment given the respective spendings. In this situation, the first stage is described by the following Lagrangian

$$\sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + \lambda_t \Big[w_t + r_t k_t - \Big(p_{ct}c_t - \sum_{i=a,m,s} p_{it}\bar{c}_i \Big) - p_{xt}x_t \Big] + \eta_t \Big[(1-\delta) k_t + x_t - k_{t+1} \Big] \right\}$$

that delivers the FOC for c_t and x_t described by equations (E.5) and (E.8) and the Euler equation (E.3). Plugging equations (E.5) and (E.8) into (E.3) we get the Euler equation (14). In the second stage, at every period t the household maximizes the bundles of consumption and investment given the spending allocated to each:

$$\max_{\{c_{at},c_{mt},c_{st}\}} C(c_{at}, c_{mt}, c_{st}) \qquad \text{s.t.} \sum_{i=\{a,m,s\}} p_{it}c_{it} = p_{ct}c_t - \sum_{i=a,m,s} p_{it}\bar{c}_i$$

$$\max_{\{x_{at},x_{mt},x_{st}\}} X_t(x_{at}, x_{mt}, x_{st}) \qquad \text{s.t.} \sum_{i=\{a,m,s\}} p_{it}x_{it} = p_{xt}x_t$$

leading to the FOC for each good:

$$\frac{\partial C(c_{at}, c_{mt}, c_{st})}{\partial c_{it}} = \mu_{ct}\, p_{it} \qquad i \in \{a, m, s\} \tag{E.19}$$

$$\frac{\partial X_t(x_{at}, x_{mt}, x_{st})}{\partial x_{it}} = \mu_{xt}\, p_{it} \qquad i \in \{a, m, s\} \tag{E.20}$$

where μ_{ct} and μ_{xt} are the shadow values of spending in consumption and investment, which correspond to $1/p_{ct}$ and $1/p_{xt}$ in the full problem.

Production. There is a representative firm in each good $j = \{c, x\}$ combining capital k_{jt} and labor l_{jt} to produce the amount y_{jt} of the final good j. The production functions are CES with identical share $0 < \alpha < 1$ and elasticity $\rho < 1$ parameters. There is a labour-augmenting common technology level B_t and a sector-specific hicks-neutral technology level $\tilde{B}_{jt}$:

$$y_{jt} = \tilde{B}_{jt} \big[\alpha k_{jt}^{\epsilon} + (1-\alpha)(B_t l_{jt})^{\epsilon} \big]^{1/\epsilon}$$

The objective function of each firm is given by,

$$\max_{k_{jt},l_{jt}} \{ p_{jt}y_{jt} - r_t k_{jt} - w_t l_{jt} \}$$

Leading to the standard FOC,

$$r_t = p_{jt} \quad \alpha \quad \tilde{B}_{jt}^{\epsilon} \left(\frac{y_{jt}}{k_{jt}}\right)^{1-\epsilon} \tag{E.21}$$

$$w_t = p_{jt} (1-\alpha) B_t^{\epsilon} \tilde{B}_{jt}^{\epsilon} \left(\frac{y_{jt}}{l_{jt}}\right)^{1-\epsilon} \tag{E.22}$$

Finally, note that we can define total output of the economy y_t as the sum of value added in all sectors,

$$y_t \equiv p_{ct} y_{ct} + p_{xt} y_{xt}$$

Equilibrium. An equilibrium for this economy is a sequence of exogenous paths $\left\{B_t, \tilde{B}_{ct}, \tilde{B}_{xt}\right\}_{t=1}^{\infty}$ a sequence of allocations $\{c_t, x_t, k_t, k_{ct}, k_{xt}, l_{ct}, l_{xt}, y_{ct}, y_{xt}\}_{t=1}^{\infty}$, and a sequence of equilibrium prices $\{r_t, w_t, p_{xt}, p_{ct}\}_{t=1}^{\infty}$ such that

- Households optimize: equations (E.5), (E.8) and (E.3) hold
- Firms optimize: equations (15), (16) hold
- All markets clear: $k_{ct} + k_{xt} = k_t$, $l_{ct} + l_{xt} = 1$, $y_{ct} = c_t$ and $y_{xt} = x_t$

Note that in equilibrium the FOC of the firms imply that the capital to labor ratio is the same for both goods and equal to the capital to labor ratio in the economy $\frac{k_{ct}}{l_{ct}} = \frac{k_{xt}}{l_{xt}} = k_t$,

$$k_t = \left(\frac{\alpha}{1-\alpha} \frac{w_t}{r_t} B_t^{-\epsilon}\right)^{\frac{1}{1-\epsilon}} \tag{E.23}$$

and that relative prices are given by

$$\frac{p_{xt}}{p_{ct}} = \frac{\tilde{B}_{ct}}{\tilde{B}_{xt}} \tag{E.24}$$

Hence, we can write total output and the interest rate in units of the investment good as a function of capital per capita in the economy,

$$y_t / p_{xt} = \tilde{B}_{xt} \left[\alpha k_t^{\epsilon} + (1-\alpha) B_t^{\epsilon}\right]^{1/\epsilon} \tag{E.25}$$

$$r_t / p_{xt} = \alpha \tilde{B}_{xt} \left(\frac{y_t / p_{xt}}{k_t}\right)^{1-\epsilon} \tag{E.26}$$

Finally, we can characterize the equilibrium aggregate dynamics of this economy with the laws of motion for c_t and k_t

$$\begin{aligned}\left(\frac{c_{t+1}}{c_t}\right)^{\sigma} &= \beta\left[\frac{\tilde{B}_{ct+1}}{\tilde{B}_{ct}}\frac{\tilde{B}_{xt}}{\tilde{B}_{xt+1}}\right]\left[\alpha\,\tilde{B}_{xt+1}\left[\alpha+(1-\alpha)\left(\frac{B_{t+1}}{k_{t+1}}\right)^{\epsilon}\right]^{\frac{1-\epsilon}{\epsilon}}+(1-\delta)\right] \\ \frac{k_{t+1}}{k_t} &= (1-\delta)+\tilde{B}_{xt}\left[\alpha+(1-\alpha)\left(\frac{B_t}{k_t}\right)^{\epsilon}\right]^{1/\epsilon}-\frac{\tilde{B}_{xt}}{\tilde{B}_{ct}}\frac{c_t}{k_t}+\frac{\sum_{i=a,m,s}\frac{p_{it}}{p_{xt}}\bar{c}_i}{k_t}\end{aligned}$$

Analogy. Note that if we set $\tilde{B}_{ct}=B_{ct}$ and $\tilde{B}_{xt}=\chi_t B_{xt}$ the two economies are identical.

E.7 The constrained model

Let's now focus on the case when the inequality constraints $p_{it}c_{it}\geq 0$ are binding. It is important to note that in this case the separation between the intertemporal and intratemporal problem does not apply and the optimal savings choice needs to be solved jointly with the optimal consumption compisition.

Consumption composition. The term $\left(1-\frac{\tilde{\nu}_{it}}{\lambda_t}\right)$ in the r.h.s of equation (E.1) is the mark-down on the price of good i that would make the choice of $c_{it}=0$ an interior solution. That is, if at current price p_{it} and shadow value of income λ_t the household's unrestricted optimal choice is to sell c_{it} to obtain more income, the lower price $\left(1-\frac{\tilde{\nu}_{it}}{\lambda_t}\right)p_{it}$ would make the household choose $c_{it}=0$ as an interior solution. Let's define

$$\nu_{it}\equiv\frac{\tilde{\nu}_{it}}{\lambda_t}$$

The FOC of each good i described by equation (E.1) can be rewritten as:

$$c_t^{-\sigma}\left(\theta_i^c\frac{c_t}{c_{it}+\bar{c}_i}\right)^{1-\rho_c}=\lambda_t\left(1-\nu_{it}\right)p_{it} \tag{E.27}$$

Note that when the inequality constraint for good i is not binding $\nu_{it}=0$ and this equation determines c_{it}. Instead, if the inequality constraint binds $c_{it}=0$ and then this equation determines ν_{it}. In this case, notice that because the l.h.s is positive it must be the case that $\nu_{it}<1$. We can aggregate equations (E.27) to obtain the FOC for the consumption basket,

$$c_t^{-\sigma}=\lambda_t\left(1-\nu_{ct}\right)p_{ct} \tag{E.28}$$

where p_{ct} is the implicit price index of the consumption basket defined in (12). We can define $(1-\nu_{ct})$ as the mark-down on the price of the consumption basket that results as a weighted average of the mark-downs in each consumption good,

$$(1-\nu_{ct})\equiv\frac{\tilde{p}_{ct}}{p_{ct}} \tag{E.29}$$

where

$$\tilde{p}_{ct} \equiv \left[\sum_{i=a,m,s} \theta_i^c \left[(1-\nu_{it})\, p_{it}\right]^{\frac{\rho_c}{\rho_c-1}}\right]^{\frac{\rho_c-1}{\rho_c}} \tag{E.30}$$

Note that when the inequality binds for neither good, then $\forall i$ $\nu_{it} = 0$ and $\nu_{ct} = 0$. When the constraint binds for at least one good i, then $(1-\nu_{ct}) < 1$ and $\tilde{p}_{ct} < p_{ct}$, which will be important in the intertemporal problem because it will induce higher consumption expenditure in that period.

Adding up the FOC for each good i we obtain,

$$\sum_{i=a,m,s} (1-\nu_{it})\, p_{it} c_{it} = (1-\nu_{ct})\, p_{ct} c_t - \sum_{i=a,m,s} (1-\nu_{it})\, p_{it} \bar{c}_i \tag{E.31}$$

Finally, using equations (E.27) and (E.31) we obtain the consumption expenditure share of each good i:

$$\frac{(1-\nu_{it})\, p_{it} c_{it}}{\sum_{j=a,m,s} (1-\nu_{jt})\, p_{jt} c_{jt}} = \theta_i^c \left(\frac{(1-\nu_{ct})\, p_{ct}}{(1-\nu_{it})\, p_{it}}\right)^{\frac{\rho_c}{1-\rho_c}} \left[1 + \frac{\sum_{j=a,m,s} (1-\nu_{jt})\, p_{jt} \bar{c}_j}{\sum_{j=a,m,s} (1-\nu_{jt})\, p_{jt} c_{jt}}\right] - \frac{(1-\nu_{it})\, p_{it} \bar{c}_i}{\sum_{j=a,m,s} (1-\nu_{jt})\, p_{jt} c_{jt}} \tag{E.32}$$

and dividing (E.27) by (E.28) we can also obtain

$$\left(\theta_i^c \frac{c_t}{c_{it} + \bar{c}_i}\right)^{1-\rho_c} = \frac{(1-\nu_{it})}{(1-\nu_{ct})} \frac{p_{it}}{p_{ct}} \tag{E.33}$$

Euler equation. Plugging equations (E.28) and (E.8) into (E.3) we get the Euler equation driving the dynamics of the model.

$$c_t^{-\sigma} = \beta c_{t+1}^{-\sigma} \frac{1}{1+\tau_t} \frac{1-\nu_{ct}}{1-\nu_{ct+1}} \frac{p_{xt+1}}{p_{ct+1}} \frac{p_{ct}}{p_{xt}} \left[\frac{r_{t+1}}{p_{xt+1}} + (1-\delta)\right] \tag{E.34}$$

This is the usual equation but with one extra ingredient. The wedge $(1-\nu_{ct})\,/\,(1-\nu_{ct+1})$ captures how the intertemporal problem is distorted by the inequality constraints in the intratemporal problem. If the inequality constraints are binding neither in t nor in $t+1$ then the wedge is equal to 1 and we have the standard problem. Because the constraints bind more severely whenever the economy is poorer, we have to expect $\nu_{ct} > \nu_{ct+1}$ and hence $(1-\nu_{ct})\,/\,(1-\nu_{ct+1}) < 1$. That is to say: binding inequality constraints in the intratemporal problem will be akin to a tax on saving, pushing the household to increase consumption at t, decrease investment at t, and decrease consumption at $t+1$.

Aggregate dynamics. We have two difference equations to characterize the aggregate dynamics of this economy: the Euler equation of consumption in equation (E.34) and the law of motion of capital in equation (4). After substituting prices away the two difference equations in $\hat{k}_t$ and $\hat{c}_t$ become:

$$\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}\right)^{\sigma}(1+\gamma_{Bt+1})^{\sigma} = \frac{\beta}{1+\tau_t}\left[\frac{1-\nu_{ct}}{1-\nu_{ct+1}}\right]\left[\alpha\left(\chi_{t+1}B_{xt+1}\right)^{\epsilon}\left(\frac{\hat{y}_{t+1}}{\hat{k}_{t+1}}\right)^{-\epsilon}+(1-\delta)\right] \left[\frac{1+\gamma_{Bct+1}}{1+\gamma_{Bxt+1}}\frac{1}{1+\gamma_{\chi t+1}}\right]^{1-\sigma} \tag{E.35}$$

$$\hat{k}_{t+1}(1+\gamma_{Bt+1}) = (1-\delta)\hat{k}_t+\hat{y}_t - \hat{c}_t(1-\nu_{ct})+\frac{\chi_t B_{xt}}{B_t}\left[\sum_{i=a,m,s}\frac{\bar{c}_i}{B_{it}}-\nu_{ct}\sum_i \nu_{it}\frac{c_{it}+\bar{c}_i}{B_{it}}\right] \tag{E.36}$$

Note therefore that the aggregate dynamics of $\hat{k}_t$ and $\hat{c}_t$ depend on ν_{ct} and ν_{ct+1}, which in turn depend on the ν_{it} and ν_{it+1}. Therefore, the dynamic system in equation (E.35)-(3) needs to be solved together with equations (E.33) in t and $t+1$.

Finally, we write in efficiency units equation (E.33) determining the optimal choice of each c_{it} in the intratemporal problem:

$$\left(\theta_i^c\frac{\hat{c}_t}{\hat{c}_{it}+\frac{\chi_t B_{xt}}{B_i B_t}\bar{c}_i}\right)^{1-\rho_c} = \frac{(1-\nu_{it})}{(1-\nu_{ct})}\left(\frac{B_{ct}}{B_{it}}\right)^{\rho_c} \tag{E.37}$$

Appendix F: Solving the model in the computer

Given the paths of exogenous series $\{B_t, B_{at}, B_{mt}, B_{st}, \chi_t\}_{t=0}^{\infty}$, we use a shooting algorithm to solve numerically for the whole transition between $t=0$ to the BGP, and produce investment and output series between $t=0$ and $t=T$. In practice, this requires finding time series for $\hat{k}_t$ and $\hat{c}_t$ (given $\hat{k}_0$) that are consistent with the dynamic system described by equations (E.10) and (E.11) and that converge to the BGP, i.e., to the values implied by equations (E.13) and (E.15).

We implement two different types of shooting algorithms to make sure that we obtain the same transition path. For the case where the inequality constraints bind in economy E_4, it is very straightforward to use the backward shooting.

Forward shooting. We first run a forward shooting algorithm. Conceptually, this algorithm consists of a bisection algorithm to find the $\hat{c}_0$ that is consistent with the path from $\hat{k}_0$ to $\hat{k}^*$. We proceed as follows:

1. Initialize: set $T_{max}=2000$, $K(0)=\hat{k}_0$, $Y(0)=\hat{y}_0$, $K_{max}=\frac{(1-\delta)K(0)+Y(0)+\frac{\chi_0 B_{x0}}{B_0}\sum_i\frac{\bar{c}_i}{B_{i0}}}{(1+\gamma_{B1})}$, and $K_{min}=0$
2. Guess $K(1)=(K_{min}+K_{max})/2$ and compute the $C(0)$ implied by this guess using equation (E.11). This gives us the initial pair $C(0)$ and $K(1)$.

3. Obtain the sequence $\{C(t), K(t+1)\}_{t=1}^{T_{max}}$. In particular, given $K(t)$ and $C(t-1)$ equation (E.10) recovers $C(t)$, and given $K(t)$ and $C(t)$ equation (E.11) recovers $K(t+1)$.

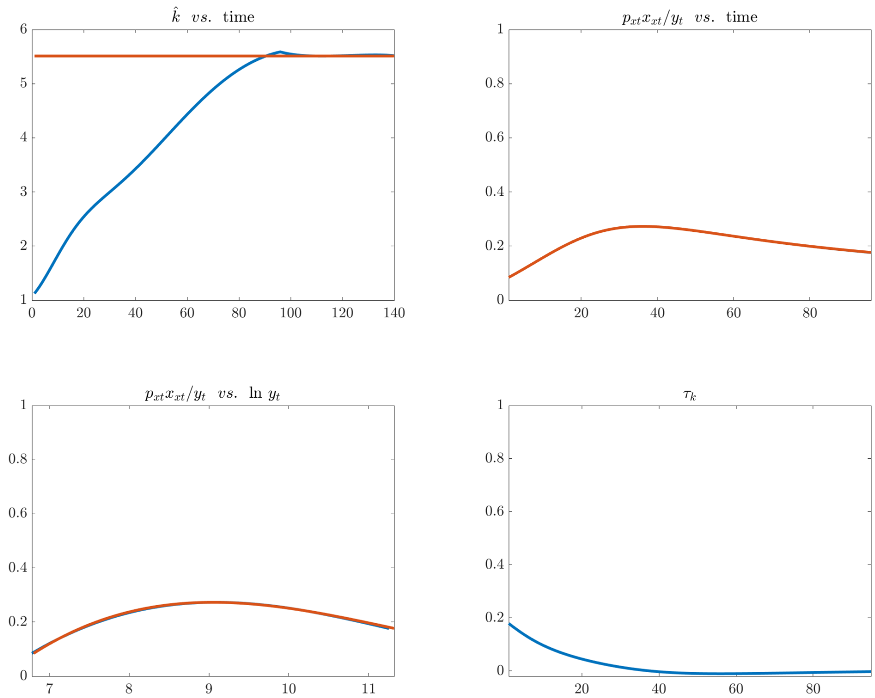
4. Evaluate the sequence $\{C(t), K(t+1)\}_{t=1}^{T_{max}}$

   (a) If $\left(\hat{k}^* - K(T_{max})\right) < 0$ set $K_{max} = K(1)$

   (b) If $\left(\hat{k}^* - K(T_{max})\right) > 0$ set $K_{min} = K(1)$

   (c) If $(K_{max} - K_{min}) < 10^{-20}$, exit. Otherwise, go back to step 2

Figure F.1: Transition from forward shooting algorithm



Notes: Figure F.1 shows the transition path that emerges as a solution from the forward shooting (the horizontal red line represents $\hat{k}$). Panel (A) shows the evolution of $\hat{k}$ over time; panel (B) shows the evolution of the investment rate against log gdp; and panel (C) shows the evolution of τ_k over time that makes our baseline economy to match the investment rate perfectly.

Figure F.1 shows the transition path that emerges as a solution from the forward shooting. The top-left Panel shows the evolution of $\hat{k}$ over time; the top-right and bottom-left Panels ashow the evolution of the investment rate against time and against log gdp respectively; finally, the bottom right Panel shows the evolution of τ_k over time that makes our baseline economy to match the investment rate perfectly. One of the advantages of the forward shooting algorithm is that one does not have to impose the time at which the economy reaches its BGP. In the case of our baseline case, that happens around $t = 120$.

Backward shooting. For all the economies that we consider, we also run a backward shooting algorithm to check that it delivers transitions that are identical to the ones delivered by the forward shooting. Conceptually, the backward shooting consists on finding the $\hat{c}_{T^*-1}$ that is consistent with the path from $\hat{k}^*$ to $\hat{k}_0$, where T^* is the period at which the economy reaches its BGP. Therefore, in order to run a backward shooting, one has to impose the value of T^*. We use the outcome of the forward shooting to have a good guess of T^*. In practise, we proceed as follows:

1. Initialize: set T^*, $K(T^*) = \hat{k}^*$, K_{max} a large number, and K_{min} that solves,

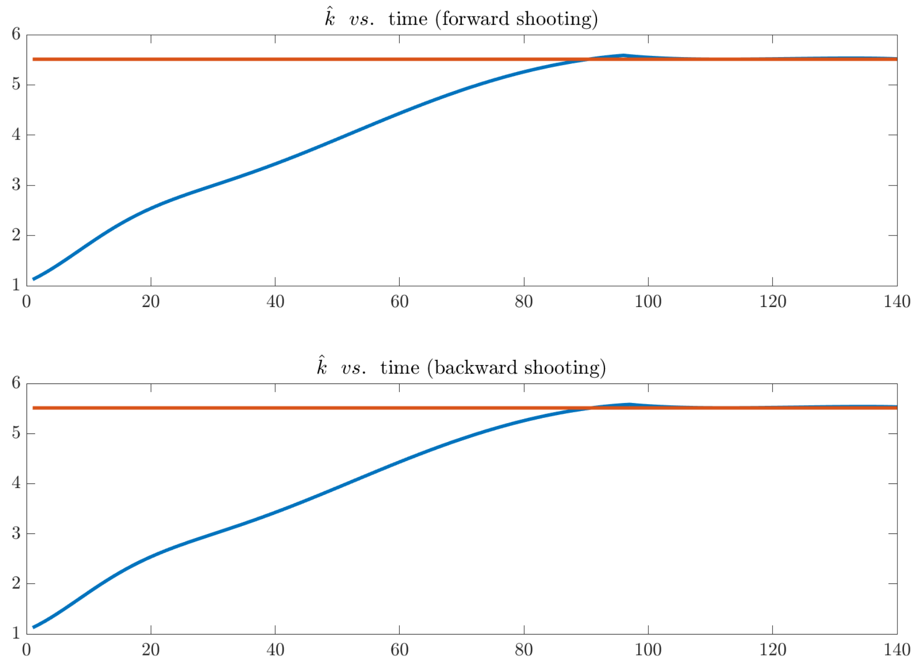
$$\begin{aligned} K(T^*)(1+\gamma_{B,T^*}) &= (1-\delta)K_{min} + \chi_{T^*-1}B_{x,T^*-1}\left[\alpha K_{min}^{\epsilon} + (1-\alpha)\right]^{1/\epsilon} \\ &+ \frac{\chi_{T^*-1}B_{x,T^*-1}}{B_{T^*-1}} \sum_{i=a,m,s} \frac{\bar{c}_i}{B_{i,T^*-1}} \end{aligned}$$

2. Guess $K(T^*-1) = (K_{min} + K_{max})/2$ and compute the $C(T^*-1)$ implied by this guess using equation (E.11). This gives us the initial pair $C(T^*-1)$ and $K(T^*-1)$.
3. Obtain the sequence $\{C(t), K(t)\}_{t=0}^{T^*-2}$. In particular, given $K(t+1)$ and $C(t+1)$ equation (E.10) recovers $C(t)$, and given $K(t+1)$ and $C(t)$ use a NLES to solve equation (E.11) for $K(t)$.
4. Evaluate the sequence $\{C(t), K(t)\}_{t=0}^{T^*-1}$
   (a) If $\left(K(1) - \hat{k}_0\right) > 0$ set $K_{max} = K(T^*-1)$
   (b) If $\left(K(1) - \hat{k}_0\right) < 0$ set $K_{min} = K(T^*-1)$
   (c) If $\left|K(1) - \hat{k}_0\right| < 10^{-3}$ exit, otherwise go back to step 2.

The transition path implied by this backward shooting algorithm is generally identical to the one generated by the forward shooting. Figure F.2 compares the two transitions for the case of our baseline parametrization.

Backward shooting for the constrained problem. As we explain in the main text of the paper, the household problem hits the inequality constraint $c_{mt} \geq 0$ for a few number of early periods, once we remove the wedges to compute the counterfactual economy E_4. To solve the constrained model, we apply a backward shooting algorithm whose logic is similar to the one presented above. As before, the backward shooting consists on finding the $\hat{c}_{T^*-1}$ that is consistent with the path from $\hat{k}^*$ to $\hat{k}_0$, where T^* is the period at which the economy reaches its BGP. Using the backward shooting to solve the constrained model is convenient since we can initialize the algorithm under the reasonable assumption that the household is rich enough at T^*-1 so that the inequality constraints are not binding ($p_{it}c_{it} \geq 0 \;\; \forall i$). We proceed as follows:

Figure F.2: Comparison transition forward vs. backward



Notes: The top panel of Figure F.2 shows the transition path that emerges as a solution from the forward shooting (the horizontal red line represents $\hat{k}$). The bottom panel shows the equivalent graph but for the solution that emerges from the backward shooting.

1. Initialize: set T^*. Assume $\nu_{i,T^*-1} = \nu_{c,T^*-1} = 0$. Set $K(T^*) = \hat{k}^*$, K_{max} a large number, and K_{min} that solves,

$$K(T^*)(1+\gamma_{B,T^*}) = (1-\delta)K_{min} + \chi_{T^*-1}B_{x,T^*-1}\left[\alpha K_{min}^{\epsilon} + (1-\alpha)\right]^{1/\epsilon} + \frac{\chi_{T^*-1}B_{x,T^*-1}}{B_{T^*-1}} \sum_{i=a,m,s} \frac{\bar{c}_i}{B_{i,T^*-1}}$$

2. Guess $K(T^*-1) = (K_{min} + K_{max})/2$ and compute the $C(T^*-1)$ implied by this guess using equation (3) under the assumption that $\nu_{i,T^*-1} = \nu_{c,T^*-1} = 0$. This gives us the initial pair $C(T^*-1)$ and $K(T^*-1)$. Use the demand system implied by equation (E.37) to recover $C_i(T^*-1)$.

3. Obtain the sequence $\{C(t), K(t), C_i(t)\}_{t=0}^{T^*-2}$ and $\{\nu_{i,t}, \nu_{c,t}\}_{t=0}^{T^*-2}$. In each t, starting from $t = T^*-2$ and approaching $t = 0$, start by assuming that $\nu_{it} = \nu_{ct} = 0$. Equation (E.10) gives $C(t)$ and equation (E.11) gives $K(t)$. Recover $C_i(t)$ from equations (E.37) and check whether the inequality constraints $p_{it}c_{it} \geq 0$ are violated.

   - If they are not violated, we know that $\nu_{it} = \nu_{ct} = 0$ and hence we have obtained the right $\{K(t), \nu_{ct}, C(t), C_i(t)\}$.

- If they are violated, solve the constrained problem. Note that equation (E.35) has two unknowns now, $\hat{c}_t = C(t)$ and ν_{ct}. Recall that ν_{ct} is a weighted average of the three ν_{it}, see equation (E.29). Hence, we have 1 equation and 4 unknowns. We need to use the 3 equations (E.37) to complete the system, but they add the three $\hat{c}_i = C_i(t)$. But we know that $\forall t\ \nu_{at} = 0$ because $\bar{c}_a < 0$, so we are left with 6 unknowns and need 2 more conditions. We proceed as follows:
  - First, if only one inequality constrain binds, say for good j, set $c_{jt} = C_j(t) = 0$ and $\nu_{-jt} = 0$ and solve the system. Verify that $c_{-jt} = C_{-j}(t) \geq 0$ if yes, done. Otherwise go to next step.
  - Second, if both inequality constraints bind, set $c_{mt} = C_m(t) = 0$ and $c_{st} = C_s(t) = 0$ and solve the system. Verify that $\nu_{mt} > 0$ and $\nu_{st} > 0$.
  - Use NLES to solve equation for $K(t)$.

  In practise, and in order to decrease the computational burden, we exploit the fact that our estimation delivers a demand system for consumption goods that is very close to a Leontief specification of the type:

$$c_t = C(c_a, c_m, c_s) \quad = \quad \min_{i \in \{a,m,s\}} \left\{ \frac{1}{\theta_i^c}(c_i + \bar{c}_i) \right\} \tag{F.1}$$

  The intra-temporal constrained problem becomes easier to solve. Imagine that it was the case that $\hat{c}_{mt} = C_m(t) < 0$. Then, we set:

$$\begin{aligned} \hat{c}_{mt} = C_m(t) &= 0 \\ \hat{c}_{st} = C_s(t) &= \left(\frac{\theta_s^c}{\theta_m^c} \bar{c}_m - \bar{c}_s \right) \frac{\chi_t B_{xt}}{B_{st} B_t} \\ \hat{c}_{at} = C_a(t) &= \left(\frac{\theta_a^c}{\theta_m^c} \bar{c}_m - \bar{c}_a \right) \frac{\chi_t B_{xt}}{B_{at} B_t} \end{aligned}$$

  The consumption basket is given by

$$\hat{c}_t = C(t) = \frac{1}{\theta_m^c} \bar{c}_m \frac{\chi_t B_{xt}}{B_{ct} B_t}$$

  Hence, once the non-negativity constraint of some good i binds at t, this solves for the consumption basket at time t without using the Euler equation as there is no interior solution to the Euler equation. We next use a NLES to solve equation (E.11) for $K(t)$ move ahead to solve the next period.

- Evaluate the sequence $\{C(t), K(t)\}_{t=0}^{T^*-1}$

  (a) If $\left(K(1) - \hat{k}_0\right) > 0$ set $K_{max} = K(T^* - 1)$

(b) If $\left(K(1) - \hat{k}_0\right) < 0$ set $K_{min} = K(T^* - 1)$

(c) If $\left|K(1) - \hat{k}_0\right| < 10^{-3}$ exit, otherwise go back to step 2.