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JEL Classification: E4, E5, F3, F4, G11, G15

Keywords: fixed exchange rate, managed float, exchange rate stabilization, uncovered interest parity, currency returns

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Currency Manipulation*

Tarek A. Hassan[†] Thomas M. Mertens[‡] Tony Zhang[§]

August 2019

Abstract

We develop a novel, risk-based theory of the effects of currency manipulation. In our model, the choice of exchange rate regime allows policymakers to make their currency, and by extension, the firms in their country, a safer investment for international investors. Policies that induce a country's currency to appreciate when the marginal utility of international investors is high lower the required rate of return on the country's currency and increase the world-market value of domestic firms. Applying this logic to currency stabilizations, we find a small economy stabilizing its bilateral exchange rate relative to a larger economy can increase domestic capital accumulation, domestic wages, and even its share in world wealth. In the absence of policy coordination, small countries optimally choose to stabilize their exchange rates relative to the currency of the largest economy in the world, which endogenously emerges as the world's "anchor currency." Larger economies instead optimally choose to float their exchange rates. The model therefore predicts an equilibrium pattern of exchange rate arrangements that is remarkably similar to the one in the data.

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Two thirds of all countries in the world manipulate the stochastic properties of their exchange rates by stabilizing their currency relative to the US dollar. Such stabilizations take on many different forms, including pegs, moving bands, stabilized arrangements, and managed floats. Their common feature is that they set an upper bound for the volatility of the real or nominal exchange rate, without necessarily manipulating its mean. Why do so many countries stabilize their exchange rates relative to the US dollar?

In this paper, we develop a novel, risk-based theory of the effects of currency manipulation in general, and currency stabilization in particular. In this model, the choice of exchange rate regime affects economic outcomes because it allows policymakers to make their currency, and by extension, the firms based in their country, a safer investment from the perspective of international investors. Policies that induce a country's currency to retain value or even appreciate when the marginal utility of international investors is high (in "bad times") lower the required rate of return on the country's currency and thus also lower domestic interest rates and increase the world-market value of domestic firms. Policies that change the variances and covariances of real exchange rates can thus, via their effect on interest rates and asset returns, affect the allocation of capital across countries.

This approach, linking a country's exchange rate regime to the value of domestic firms, yields three main insights. First, in canonical models of exchange rate determination, a direct link exists between the stochastic properties of a country's exchange rate, the expected return on its currency, and the world-market value of firms producing country-specific (or nontraded) goods. The safer a country's currency is from the perspective of international investors, the higher the world-market value of its firms, and the higher domestic investment and wages. Second, the choice of target currency is key to the effects of any exchange rate stabilization. A country that stabilizes its exchange rate relative to a "safe" currency that appreciates when marginal utility is high inherits some or all of the stochastic properties of that target currency. Through its effect on risk premia, a stabilization relative to the safest currency in the world thus offers a maximal boost to the value of domestic firms and to domestic investment and wages. Third, stabilizations are generally cheaper to implement for smaller countries whose actions have little or no effect on the price of traded goods in world markets.

Taken together, these insights shed new light on otherwise puzzling features of exchange rate arrangements we see in the data today. Since the demise of the Bretton-Woods system of fixed exchange rates in 1975, individual countries have been largely free to choose their own exchange rate regime. Despite this lack of centralized coordination, recent research by Ilzetzki, Reinhart, and Rogoff (2018) has shown surprising regularity in the choices made by individual countries. Table 1 shows small economies tend to stabilize, whereas only the largest economies in the world float their exchange rate. The larger the economy, the softer the stabilizations tend to be. Moreover, there is remarkable agreement in the choice of target country: The vast majority of stabilizations target the currency of the largest economy in the world, the US dollar, making it the "anchor" currency of the world monetary system. Almost all exceptions to this rule instead target the currency of the largest market in the world, the euro. We argue these patterns can be understood as attempts to manage risk: They arise as the optimal non-cooperative equilibrium in a parsimonious model where currency risk premia affect the allocation of capital across countries. In other words, the US dollar may be the anchor of the world monetary system because smaller countries are optimally trying to reduce the risk associated with their currencies.

Table 1: 2010 Exchange Rate Arrangements According to Ilzetzki, Reinhart, and Rogoff (2018)

Panel A	*Exchange rate arrangement*		
GDP Decile	$1-5$ (smallest)	$6-9$	10 (largest)
Floating	0%	0%	29%
Stabilized	100%	100%	71%
soft peg	41%	60%	65%
hard peg	59%	40%	6%
Panel B	*Target currency*		
	Dollar	Euro	Other
Number of Countries	124	39	11

Notes: Countries are divided into deciles by GDP in 2010. Deciles 1-9 each contain 18 countries, the tenth 17 countries. The "floating" category refers to exchange rates classified as "freely floating" in Ilzetzki, Reinhart, and Rogoff (2018) (fine classification code 13), the "soft peg" category includes currencies with any form of crawling peg, crawling band, or managed float. the "hard peg" category includes currency unions, pre-announced pegs, pre-announced bands, and de facto pegs (codes 1 - 4).

Our work builds on a growing literature that links highly persistent differences in interest rates, currency returns, and capital-output ratios across countries to the stochastic properties of their currencies (Lustig and Verdelhan, 2007; Lustig, Roussanov, and Verdelhan, 2011; Hassan and Mano, 2019; Hassan, Mertens, and Zhang, 2016). This literature has explored various potential drivers of heterogeneity in the stochastic properties of countries' exchange rates, ranging from differences in country size (Martin, 2012; Hassan, 2013) and financial development (Maggiori, 2017) to trade centrality (Richmond, 2019) and differential resilience to disaster risk (Farhi and Gabaix, 2015; Colacito et al., 2018).[1] The common theme across these "risk-based" theories is that whatever makes countries different from each other results in differential sensitivities of their exchange rates to various shocks, such that some currencies (typically the US dollar) tend to appreciate systematically when marginal utility is high.

In this paper, we go one step further and argue the stochastic properties of exchange rates are themselves subject to policy intervention. To formalize this idea, we solve for the effect of currency manipulation on risk premia within an otherwise standard model of exchange rate determination. In the model, households consume a freely traded good and a country-specific nontraded good. The nontraded good is produced by domestic firms, the shares of which are the only assets traded in an international stock market. In equilibrium, the real exchange rate may fluctuate in response to country-specific shocks to productivity, preferences, or money supply.

As a stand-in for the various potential sources of heterogeneity in the stochastic properties of countries' exchange rates mentioned above, we allow countries to differ in size. That is, we assume all shocks are common within countries, and some countries account for a larger share of world GDP than others. In the absence of policy intervention, this heterogeneity in country size endogenously generates differences in expected currency returns, because shocks that raise the price of consumption in a larger country spill over more into the world-market price of traded goods. As a result, the currencies of larger countries tend to appreciate when the marginal utility of international investors is high. Larger countries therefore have lower risk-free interest rates, more valuable firms, and higher capital-output ratios in equilibrium.

Within this standard economic environment, we study the effects of policies that lower the variance of one "stabilizing" country's real exchange rate relative to a "target" country's currency, while leaving the mean of the exchange rate unaffected. To this end, we assume each country has a central bank that issues and controls the supply of domestic currency, and that the nominal price of the traded good is sticky in that domestic currency, giving the central bank the means to affect allocations and the real exchange rate.

Because nontraded goods cannot be shipped internationally, stabilizing the real exchange rate requires driving a wedge between the domestic and world-market prices of traded goods. For example, when the target country appreciates, the stabilizing country must artificially raise the domestic relative price of traded goods to increase the price of the domestic consumption

[1] Other papers in this literature have studied heterogeneity in the volatility of shocks affecting the nontraded sector (Tran, 2013), factor endowments (Ready, Roussanov, and Ward, 2017; Powers, 2015), and risk aversion in combination with country size (Govillot, Rey, and Gourinchas, 2010). Also see Gourinchas and Rey (2007), Campbell, Serfaty-De Medeiros, and Viceira (2010), Menkhoff et al. (2012), David, Henriksen, and Simonovska (2016), and Verdelhan (2017).

bundle and match the appreciation. We show these stabilizing wedges between the domestic and international prices of traded goods arise naturally from a simple nominal stabilization regime where the central bank exchanges domestic for foreign currency at a predetermined rate. In this sense, a nominal stabilization implements a real stabilization. (More generally, a real stabilization could also be implemented with state-contingent taxes or tariffs.)

We first consider the case in which the stabilizing country is small and thus only affects its own price of consumption. A small country that stabilizes its exchange rate relative to a larger country inherits the stochastic properties of the larger country's exchange rate, so that the stabilized exchange rate now also tends to appreciate when marginal utility is high. A safer currency, in turn, comes with a lower risk-free interest rate, a higher world-market value of domestic firms, and increased domestic capital accumulation.

By raising the domestic price of traded goods whenever the target country appreciates, the stabilizing country effectively reduces domestic consumption and thus exports additional traded goods in these states of the world. If the target country is large, these states tend to be those in which the world-market price of traded goods is high, so that the stabilizing country effectively sells traded goods when they are expensive and buys them when they are cheap. Stabilizing relative to a larger target country thus generates an insurance premium in the form of additional seigniorage. (Effectively, a stabilizing country provides more insurance to the target country than it would under freely floating exchange rates, and thus increases the volatility of its own consumption.) If the target country is sufficiently large, this insurance premium may be so large that the stabilization generates a positive net present value of revenues. In this sense, stabilizations relative to a larger country increase, rather than deplete, the central bank's resources.

However, this revenue-generating effect diminishes when the stabilizing country itself becomes larger, because the stabilization increases the variation in the stabilizing country's own demand for traded goods and therefore its price impact. When the stabilizing country is large enough to affect the equilibrium price of traded goods, the stabilization thus induces an unfavorable change in the state-contingent prices of traded goods. The larger the stabilizing country, the more resources are required to maintain a stable exchange rate.

Although the allocation is Pareto efficient if all central banks float their exchange rates, the model nevertheless produces a consistent rationale for currency stabilization. The reason is our assumption that households can transact in an international stock market, but do not have access to a full set of state-contingent (Arrow-Debreu) securities. Because of this restriction on the asset space, changes in the value of an asset that even a small country has pricing power over (the relative value of its own firms) can translate into shifts in relative wealth across countries. In particular, a small country that announces a stabilization relative to a larger country not only raises the world-market value of its firms, but also increases its households' share in world wealth. We show this valuation effect can be large enough to compensate for all domestic distortions caused by the stabilization.

The model therefore predicts an equilibrium pattern of exchange rate arrangements that is remarkably similar to the one in the data: In the absence of policy coordination, it is optimal for a small country to stabilize its exchange rate; larger countries optimally adopt "softer" stabilizations (due to the rising costs of implementing the stabilization); and the countries with the largest economies find it optimal to float. The optimal target currency for all stabilizations is the currency of the largest country in the world, endogenously rendering this currency the "anchor" currency of the world.

Because the allocation of resources under freely floating exchange rates is Pareto efficient, any utility gain accruing to a stabilizing country must come at the expense of households in another country. Interestingly, these costs of stabilization are typically not borne by the target country, but instead by other economies that float their exchange rate but are not the target of the stabilization. The reason is that all countries with floating exchange rates suffer from the valuation effect and some distortion of their consumption plans, whereas only the target country receives something in return: targeted consumption insurance, courtesy of the fact that stabilizing countries export additional traded goods whenever the target country appreciates. In this sense, the model also reflects the general intuition that being at the center of the world monetary system provides some benefit.

In various robustness checks, we show this broad set of conclusions arises regardless of whether variation in exchange rates are driven primarily by supply or demand shocks, regardless of whether the stabilization regime is fully credible, and that the positive conclusions of our analysis also extend to a model with segmented financial markets.

We make four main caveats to our interpretation. First, we focus on differences in country size only in the interest of parsimony. Variations of the model where differences in interest rates also result from differences in financial development or some of the other microfoundations mentioned above should yield similar interpretations—with the US dollar and the euro emerging as the safest currencies in the world. Second, although we solve for optimal stabilizations, we do not attempt to answer the broader question of whether other, more complicated patterns of currency manipulation might produce superior results. Similarly, we do not consider strategic interactions or optimal retaliations. Third, as in most models with standard preferences, risk premia are quantitatively small in our framework, so that a quantitative application would need additional ingredients. Finally, in our model, currency manipulation manifests itself as a wedge between the domestic and world-market prices of traded goods. In richer models, currency manipulations could also operate by changing allocations within countries, such as the sectoral allocation of labor or the distribution of wealth across households.

To our knowledge, our paper is the first to link exchange rate policy to currency risk premia. A large literature studies the effects of nominal stabilizations in New Keynesian models, where they affect the level of production by altering markups (e.g., Kollmann, 2002; Devereux and Engel, 2003; Ottonello, 2015; Fornaro, 2015).[2] More closely related to our own work, Fanelli and Straub (2019) and Gabaix and Maggiori (2015) characterize the effects of real and nominal exchange rate interventions under segmented markets. Another, largely empirical literature investigates the effects of currency stabilizations on the level of trade flows.[3] We add to these literatures in two ways. First, we study a novel effect of currency stabilization on risk premia that can operate even in a frictionless economy in which money is neutral, and in parallel to the various other effects documented in the existing literature. Second, this approach enables us to analyze how the effects of currency stabilization vary with the choice of the target currency and may endogenously give rise to a dominant anchor currency.

In this sense, our work also relates to a recent literature that argues for a special role of the US dollar in world financial markets. Branches of this literature have focused on the emergence of a dominant currency for debt issuance (Chahrour and Valchev, 2019; Farhi and Maggiori, 2017; He, Krishnamurthy, and Milbradt, 2019; Gopinath and Stein, 2019) and on the transmission of monetary shocks (Boz et al., 2019; Miranda-Agrippino and Rey, 2015; Zhang, 2019).

More broadly, our paper also relates to a large literature on capital controls.[4] Similar to Costinot, Lorenzoni, and Werning (2014), who argue that capital controls may be thought of as a manipulation of intertemporal prices, we show that currency stabilizations and other policies altering the stochastic properties of exchange rates may be thought of as a manipulation of state-

[2] One strand of the literature analyzes optimal monetary policy in small open economies with fixed exchange rates (Kollmann, 2002; Parrado and Velasco, 2002; Gali and Monacelli, 2005; Auclert and Rognlie, 2014), whereas another deals with the choice of the exchange rate regime in the presence of nominal rigidities (Helpman and Razin, 1987; Bacchetta and van Wincoop, 2000; Corsetti, Dedola, and Leduc, 2010; Schmitt-Grohé and Uribe, 2012; Bergin and Corsetti, 2015) or collateral constraints (Ottonello, 2015; Fornaro, 2015).

[3] See Hooper and Kohlhagen (1978), Kenen and Rodrik (1986), and Frankel and Rose (2002).

[4] See, for example, Calvo and Mendoza (2000), Jeanne and Korinek (2010), Bianchi (2011), Farhi and Werning (2014, 2013), Schmitt-Grohé and Uribe (2012), Korinek (2013), Korinek and Simsek (2016), and Bocola and Lorenzoni (2019).

contingent prices. The key difference between the two concepts is that capital controls affect allocations through market power and rents, whereas currency manipulation affects allocations through risk premia, even when the country manipulating its exchange rate has no effect on world market prices.

The remainder of this paper is structured as follows: Section 1 outlines the effects of currency manipulation on risk premia in their most general form. Section 2 analyzes the effects of stabilizations of the real exchange rate in the context of a baseline international real business cycle model. Section 3 generalizes the results from this analysis to stabilizations of the nominal exchange rate when prices are sticky. Section 4 discusses floating bands and partially credible stabilizations. Section 5 considers more general economic environments where exchange rates are driven by monetary or preference shocks.

1 Effects of Currency Manipulation in Reduced Form

We begin by deriving the main insights of our analysis in their most general form. Consider a world economy in which international assets are priced by a unique stochastic discount factor that depends only on the realization of a world-wide shock, λ_T. Households consume a country-specific final good, the price of which (accounted for in some common unit) depends on this world-wide shock and a country-specific shock, x^n,

$$p^n = a\lambda_T + bx^n, \tag{1}$$

where $\lambda_T \sim N(0, \sigma^2_{\lambda_T})$ and $x^n \sim N(0, \sigma^2_x)$ are normally distributed, not necessarily independent, shocks and a and b are constants greater than zero. As we show in later sections, this structure arises naturally from a microfounded model where the country-specific shock interchangeably may stem from a supply, demand, or monetary shock; in other words, it is a stand-in for any factor that affects the price of consumption in one country more than in others. The higher x^n, the higher the price of domestic consumption.

The real exchange rate between two countries is the relative price of their respective final goods. In logs,

$$s^{f,h} = p^f - p^h.$$

The risk-based view of differences in currency returns applies some elementary asset pricing to this expression. Using the Euler equation of an international investor, one can show the log expected return to borrowing in country h and to lending in country f is

$$r^f + \Delta \mathbb{E} s^{f,h} - r^h = cov\left(\lambda_T, p^h - p^f\right), \tag{2}$$

where r^n is the risk-free interest rate in country n and the log stochastic discount factor is equated to λ_T for simplicity.[5] This statement means a currency that tends to appreciate when λ_T is high pays a lower expected return and, if $\Delta \mathbb{E} s^{f,h} = 0$, also has a lower risk-free interest rate. That is, a currency that appreciates in bad times (when consumption goods are expensive everywhere in the world) provides a hedge against worldwide consumption risk and must pay lower returns in equilibrium.

Equations (1) and (2) are the main ingredients of risk-based models of unconditional differences in interest rates across countries, where different approaches model differences in the stochastic properties of p^n as the result of heterogeneity in country size, the volatility of shocks, trade centrality, financial development, factor endowments, etc.

We make a simple point relative to this literature: If this risk-based view of currency returns has merit, policies that alter the covariance between a country's exchange rate and λ_T can alter interest rates, currency returns, and the allocation of capital across countries. In particular, a country that adopts a policy that increases the price of domestic consumption in states of the world where λ_T is high can lower its risk-free interest rate relative to all other countries in the world.

As an example, consider a "manipulating" country (indexed by m) that levies a tax on domestic consumption of traded goods that is proportional to the realization of λ_T, such that

$$p^m = a\lambda_T + bx^m + \pi\lambda_T,$$

where π is some positive constant. The taxation scheme increases the tendency of p^m to appreciate when λ_T is high and thus, according to (2), lowers its interest rate relative to all other countries in the world by $\pi\sigma^2_{\lambda_T}$.

If interest rates play a role in allocating capital across countries (as is the case in our fully specified model), manipulations of the stochastic properties of exchange rates can thus divert capital investment to the country that conducts the manipulation, and, more broadly, alter the equilibrium allocation of capital across countries.

[5] $\Delta \mathbb{E} s^{f,h}$ is defined as the logarithm of the ratio of the countries' expected real price changes. See Appendix A for a formal derivation.

The remainder of this paper fleshes out this argument in the context of a general equilibrium model of exchange rate determination and applies it to one of the most pervasive policies in international financial markets: currency stabilization.

2 Stabilizing the Real Exchange Rate

We begin by studying the effect of stabilizing the real exchange rate in the most parsimonious environment, where money is neutral and the allocation of capital across countries, as well as the stochastic properties of real exchange rates, is determined solely as a function of productivity shocks (Backus and Smith, 1993). Within this canonical international real business cycle model, one country, labeled the stabilizing country, deviates from the competitive equilibrium by stabilizing its real exchange rate relative to a target country.

Our purpose in beginning our analysis in this parsimonious environment is to lay bare the main mechanisms as clearly and concisely as possible and to contrast them with the existing literature. We emphasize that none of our main insights depend on monetary neutrality, and that they continue to hold when we add more realistic frictions to the model that also address some of the well-known empirical shortcomings of the international real business cycle model. The intuition from this baseline model continues to apply when we consider stabilizations of the nominal exchange rate, monetary frictions, preference shocks, and other generalizations in the following sections.

2.1 Economic Environment

Two discrete time periods exist: $t = 1, 2$. There exists a unit measure of households $i \in [0, 1]$, partitioned into three subsets Θ^n of measure θ^n. Each subset represents the constituent households of a country. We label these countries $n = \{m, t, o\}$ for the stabilizing (manipulating), target, and outside country, respectively. Households make an investment decision in the first period. All consumption occurs in the second period.

Households derive utility from consuming an index composed of a country-specific nontraded good, $C_{N,2}$, and a freely traded good, $C_{T,2}$, where

$$C_2(i) = C_{T,2}(i)^{\tau} C_{N,2}(i)^{1-\tau} \tag{3}$$

and $\tau \in (0, 1)$. Each household exhibits constant relative risk aversion according to

$$U(i) = \frac{1}{1-\gamma}\mathbb{E}\left[(C_2(i))^{1-\gamma}\right], \tag{4}$$

where $\gamma > 0$ is the coefficient of relative risk aversion.

At the start of the first period, each household owns a firm that produces the local, country-specific, nontraded good using a Cobb-Douglas production technology that employs capital and labor. Each household supplies one unit of labor inelastically to its own firm and, in addition, owns one unit of capital, which it can sell to its own firm or to any other firm in the world. Each firm's output of nontraded goods is

$$Y_{N,2}(i) = \exp(\eta^n)\,(K(i))^{\nu} \tag{5}$$

where $0 < \nu < 1$ is the capital share in production, $K(i)$ is the (per capita) stock of capital, and η^n is a country-specific productivity shock realized at the start of the second period,

$$\eta^n \sim N\left(-\frac{1}{2}\sigma_N^2, \sigma_N^2\right).$$

Capital can be freely shipped in the first period, at the end of which it is invested for use in the production of nontraded goods in the second period. In the second period, each household is also endowed with one unit of the traded consumption good.

At the end of the first period, firms trade units of capital and households trade claims to the output of their firms (stocks) in an international stock market. Throughout, we use the traded consumption good as the numéraire, such that all prices and returns are accounted for in the same units. To simplify the derivation, we also assume households receive a country-specific transfer in the first period, κ^n, which equates the marginal utility of wealth in the first period across all households in the world. Finally, because all households and firms within a given country are identical and consumption only occurs in the second period, we henceforth drop the household index i as well as the time subscript t whenever appropriate and write the per-capita capital stock, output, and consumption of traded and nontraded goods in country n as K^n, $Y_{N,2}^n$, C_T^n, and C_N^n, respectively.

In sum, the economic environment of our baseline model is identical to that of a standard international real business cycle model. Our only, somewhat subtle, departure from this frictionless benchmark is that we confine households to trading stocks in international markets, and do not allow them to trade a full set of state-contingent claims. We prefer adding this modest restriction on the asset space both for realism and because it gives rise to a model-consistent rationale for stabilization which we discuss in detail in section 2.5.

In the meantime, however, note that because households can trade a unique set of stocks for each country and shock, financial markets are "first-order complete" (Coeurdacier and Rey, 2013), in the sense that the payoffs of the available assets span all states of the world in the log-linear solution to the competitive equilibrium. As a result, the allocation of goods across households (given a distribution of wealth) in our log-linearized solution is efficient in the absence of government interventions and coincides with the solution to the Social Planner's problem with unit Pareto weights. As a result, all the positive predictions of our baseline model are invariant to whether or not we impose the aforementioned restriction on the asset space.[6] The restriction is relevant only for the normative analysis.

Currency Stabilization We define a real exchange rate stabilization as any policy that decreases fluctuations of the stabilizing country's log real exchange rate with the target country by a fraction $\zeta \in (0, 1]$ relative to the freely floating regime, without distorting the conditional mean of the log real exchange rate. Denoting the real exchange rate that would arise under freely floating exchange rates with an asterisk, a stabilization is thus a policy such that

$$\text{var}\left(s^{t,m}\right) = (1-\zeta)^2 \text{var}\left(s^{t,m*}\right) \tag{P1}$$

and

$$\mathbb{E}\left[s^{t,m}|\{K^n\}\right] = \mathbb{E}\left[s^{t,m*}|\{K^n\}\right]. \tag{P2}$$

We refer to $\zeta \in (0, 1]$ as a stabilized real exchange rate and $\zeta = 1$ as a "hard" peg.

The stabilizing country's government has two policy instruments available to achieve (P1) and (P2): It has the ability to pay a lump-sum transfer, $\bar{Z}$, to each household in its country in the first period and to levy a state-contingent tax on the domestic consumption of traded goods in the second period ($Z(\omega)$). (When we introduce sticky prices and money into the model, these parts will be taken over simply by the central bank's control of monetary policy.)

The per-capita cost of implementing exchange rate stabilization is thus

$$\Delta Res = \bar{Z} - \mathbb{E}\left[\left(\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\right)(Z(\omega)-1)\, C_T^m(\omega)\right], \tag{6}$$

[6] See Appendix B.4 for details.

where $\Lambda_T(\omega)$ represents the (world market) shadow price of one unit of traded consumption in state ω of the second period and $\Lambda_{T,1} = \mathbb{E}[\Lambda_T(\omega)]$ is the marginal utility of wealth in the first period.

We begin by assuming the government can finance this cost using an independent supply of traded goods (currency reserves) that absorbs any surpluses or deficits generated by the taxation scheme (ΔRes). We prefer this specification mainly because it simplifies the exposition and also allows us to cleanly separate the effects of stabilizations from the (well-studied) effects of over- or under-valuations of the real exchange rate. However, we stress that none of the positive predictions of the model depend on this assumption. When we analyze the welfare effects of exchange rate stabilization in section 2.5, we set $\Delta Res = 0$, so that the cost of the stabilization is fully borne by the households in the stabilizing country. In this case, any stabilization also distorts the level of the real exchange rate, and thus violates (P2).

Interestingly, we also show below that, under a range of relevant parameters, the cost of currency stabilization is negative, so that many exchange rate stabilizations (achieving both (P1) and (P2)) are implementable even if the government has no access to currency reserves.

The market clearing conditions for traded, nontraded, and capital goods are

$$\int_{i\in[0,1]} C_{T,2}(i,\omega)di = 1 + \theta^m \Delta Res, \tag{7}$$

$$\int_{i\in\theta^n} C_{N,2}(i,\omega)di = \theta^n Y^n_{N,2}(\omega), \tag{8}$$

and

$$\sum_n \theta^n K^n = 1. \tag{9}$$

The economy is in an equilibrium when all households maximize utility taking prices and taxes as given, firms maximize profits, and goods markets clear.

2.2 Solving the Model

Appendix B.1 formally derives the conditions of optimality characterizing the equilibrium allocation. The first-order conditions with respect to C^n_T equate the shadow price of traded consumption across the target and outside countries:

$$\tau \left(C^n(\omega)\right)^{1-\gamma} \left(C^n_T(\omega)\right)^{-1} = \Lambda_T(\omega),\ n = o, t. \tag{10}$$

In the stabilizing country, the state-contingent tax that implements the currency stabilization appears as a wedge on that shadow price

$$\tau \left(C^m(\omega)\right)^{1-\gamma} \left(C_T^m(\omega)\right)^{-1} = Z(\omega)\Lambda_T(\omega). \tag{11}$$

In all countries, marginal utilities with respect to $C^n_{N,2}$ define the shadow prices of nontraded goods

$$(1-\tau) \left(C^n(\omega)\right)^{1-\gamma} \left(C_N^n(\omega)\right)^{-1} = \Lambda_N^n(\omega). \tag{12}$$

In addition, households' portfolio problem and the firm's capital demand function jointly imply

$$K^n = \frac{\nu}{\Lambda_{T,1} Q_K} \mathbb{E}\left[\Lambda_N^n(\omega) Y_N^n(\omega)\right], \tag{13}$$

where Q_K denotes the first-period price of a unit of capital. This Euler equation defines the level of capital accumulation in country n as a function of first-period prices and the expected (utility) value of its nontraded goods, $\mathbb{E}\left[\Lambda_N^n(\omega) Y_N^n(\omega)\right]$. This latter term will differ across countries and reflect any precautionary motives for capital accumulation, including those that arise as a function of the stochastic properties of the country's exchange rate.[7]

Finally, the (redundant) first-order conditions with respect to the consumption index C^n pin down the shadow prices of overall consumption in each country:

$$\left(C^n(\omega)\right)^{-\gamma} = \Lambda^n(\omega), \tag{14}$$

so that $P^n(\omega) = \Lambda^n(\omega)/\Lambda_T(\omega)$ is the price of the consumption bundle country n. The real exchange rate between two countries h and f equals the ratio of these prices,

$$S^{f,h}(\omega) = P^f(\omega)/P^h(\omega).$$

In equilibrium, the resource constraints (7)-(9) and the conditions of optimality (10)-(13) jointly determine the endogenous variables $\left\{C_N^n(\omega), C_T^n(\omega), K^n, \Lambda_N^n(\omega)\right\}_{n\in\{p,t,o\}}, \Lambda_T(\omega)$, and Q_K. To study the model in closed form, we log-linearize around the deterministic solution — the point at which the variances of shocks are zero ($\sigma_{N,n} = 0$) and all firms have a capital stock fixed at the deterministic steady-state level. To simplify the exposition, we thus ignore the feedback effect of

[7] Because households freely trade stocks and capital across borders, (13) holds in all countries, including in the stabilizing country, even though the government's intervention drives a wedge between Λ_T and the marginal utility of traded consumption in the stabilizing country. See Appendix B.1 for a formal derivation.

differential capital accumulation on the size of risk premia, studying the *incentives* to accumulate different levels of capital across countries, while holding the capital stock fixed. Appendix E.3 shows that all propositions in this section continue to hold when we allow for this feedback effect. Throughout, lowercase variables continue to refer to natural logs.

2.3 The Freely Floating Regime

We begin by showing that, in the absence of currency manipulation, the model predicts that large countries should have lower real interest rates (Hassan, 2013) and accumulate higher capital-output ratios (Hassan et al., 2016). If $\zeta = 0$, equilibrium consumption of traded goods is given by

$$c_T^{n*} = \frac{(1-\tau)(\gamma-1)}{(1-\tau)+\gamma\tau}\left(\bar{y}_N - y_N^n\right), \tag{15}$$

where $\bar{y}_N = \sum_n \theta^n y_N^n$ is the average log per-capita output of nontraded goods across countries. The expression shows that households use shipments of traded goods to insure themselves against shocks to the output of nontraded goods. If $\gamma > 1$, households receive additional traded goods whenever they have a lower-than-average output of nontraded goods, and vice versa.[8]

This risk-sharing behavior generates a shadow price of traded goods of the form,

$$\lambda_T^* = -(\gamma-1)(1-\tau)\sum_n \theta^n y_N^n, \tag{16}$$

where each country's weight is proportional to its size: shocks to the productivity of larger countries affect a larger measure of households and thus tend to spill over to the rest of the world in the form of higher shadow prices of traded goods. If $\gamma > 1$, the shadow price of traded goods falls with the average output of nontraded goods across countries. Thus, λ_T tends to be low in good states of the world when countries, on average, experience positive productivity shocks.

The real exchange rate between two countries f and h is

$$s^{f,h*} = p^{f*} - p^{h*} = \frac{\gamma(1-\tau)}{(1-\tau)+\gamma\tau}\left(y_N^h - y_N^f\right), \tag{17}$$

[8]The condition $\gamma > 1$ (more generally, γ multiplied by the elasticity of substitution between traded and nontraded goods > 1) ensures that the cross-partial of marginal utility from traded consumption with respect to the nontraded good is negative; that is, the relative price of a country's nontraded good falls when its supply increases. Because most empirical applications of international asset pricing models find a relative risk aversion significantly larger than 1 and an elasticity of substitution around 1, most authors assume this condition holds (see Coeurdacier (2009) for a detailed discussion). We show in section 5 that this condition is not needed if variation in exchange rates is driven predominantly by monetary or preference shocks.

showing that the country with the lower per-capita output of nontraded goods appreciates because its consumption index is relatively more expensive. (The literature often criticizes this somewhat counter-intuitive prediction of the real business cycle model. However, none of our conclusions depend on this prediction. Instead, the crucial feature of the model is merely that whatever shock causes a country's real exchange rate to appreciate also prompts this country to demand higher imports of traded goods, as we show formally in section 5.)

Inspecting λ_T^* and $s^{f,h*}$ shows that currencies of larger countries are "safer" in the sense that they tend to appreciate when the shadow price of traded goods is high: Whenever a country suffers a low productivity shock, its real exchange rate appreciates. For a given percentage decline in productivity, this appreciation occurs independently of how large the country is (note $s^{f,h*}$ is independent of θ). However, a shock to a larger country has a larger impact on the shadow price of traded goods (λ_T). It then immediately follows from (2) that larger countries have a lower risk-free rate:

$$r^{f*} + \Delta \mathbb{E} s^{f,h*} - r^{h*} = \operatorname{cov}\left(\lambda_T^*, p^{h*} - p^{f*}\right) = \frac{(\gamma-1)\gamma(1-\tau)^2}{1+(\gamma-1)\tau}\left(\theta^h - \theta^f\right)\sigma_N^2. \tag{18}$$

To see that these differences in interest rates across countries translate into differential incentives to accumulate capital, we can rearrange the Euler equation for capital accumulation (13) and derive an expression that links differences in capital to differences in interest rates[9]

$$k^{f*} - k^{h*} = \frac{\gamma}{\tau(\gamma-1)^2}\left(r^{h*} - \Delta \mathbb{E} s^{f,h*} - r^{f*}\right). \tag{19}$$

Firms based in larger countries thus have a lower cost of capital, which increases their value in world markets and prompts them to invest more. It is efficient to accumulate more capital in the larger country because a larger capital stock in a larger country represents a good hedge against global consumption risk: Households around the world fear states of the world in which the large country receives a bad productivity shock. Although households cannot affect the realization of productivity shocks, they can partially insure themselves against low output in large countries by accumulating more capital in these countries. This precautionary behavior raises expected output in these countries and dampens the negative effects of a low productivity shock.

[9]For a derivation, see Appendix B.5.

2.4 Effects of Currency Stabilization

Under freely floating exchange rates, larger (safer) countries thus have lower risk-free rates and higher capital-output ratios. With this result in mind, we now analyze how a country can influence interest rates and the allocation of capital by stabilizing its currency.

Whereas currency stabilization ((P1) and (P2) with $\zeta < 1$) can, in principle, be achieved with a range of different nonlinear policies, such as intervening only in response to shocks smaller or larger than some critical value, we focus our discussion on the unique linear policy that entails a proportional intervention in each state. The advantage of focusing on this case is simply that it preserves the Gaussian structure of the problem and thus lends itself to closed-form solutions. In section 4, we discuss issues that arise when the government cannot credibly commit to stabilizing shocks larger or smaller than some critical value and show that our main conclusions do not change in that case.

The following lemma characterizes the unique linear form of state-contingent taxes that implements the exchange rate stabilization:

Lemma 1

A tax on the consumption of traded goods in the stabilizing country of the form

$$z(\omega) = \zeta \frac{\gamma\tau + (1-\tau)}{\gamma\tau}\left(p^{t*}(\omega) - p^{m*}(\omega)\right)$$

implements a real exchange rate stabilization of strength ζ*.*

The cost of implementing the stabilization equals the change in the world-market cost of traded goods consumed by households in the stabilizing country,

$$\Delta Res = \mathbb{E}\left[\left(\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\right) C_T^m(\omega)\right] - \mathbb{E}\left[\left(\frac{\Lambda_T^*(\omega)}{\Lambda_{T,1}^*}\right) C_T^{m*}(\omega)\right]. \tag{20}$$

Proof. See Appendix B.6. ■

The intuition for both results is simple and quite general: When the target country appreciates (p^{t*} increases), the stabilizing country must increase its own price level to keep pace. Because the number of nontraded goods in the country is fixed, the only way it can do so is by artificially increasing the relative price of traded goods in the stabilizing country, driving a wedge between the domestic and world-market price of traded goods ($z(\omega)$). When the target country appreciates, the stabilizing country thus reduces the domestic consumption of traded goods relative to what it would have been in the freely floating regime and exports additional traded goods to the rest of the world.[10]

$$c_T^m - c_T^{m*} = -\zeta \frac{(1-\theta^m)}{\tau\gamma} \left(p^{t*} - p^{m*}\right). \tag{21}$$

Conversely, when the target country depreciates, the stabilizing country subsidizes imports of traded goods, resulting in higher imports of traded goods than under the freely floating regime. The cost of implementing the stabilization, therefore, is simply the change in the world-market cost of traded goods consumed by households in the stabilizing country.

We start by analyzing the effect of this stabilization policy on allocations, prices, and currency reserves in the stabilizing country. Afterwards, we analyze its impact on the target country.

2.4.1 Internal Effects of Currency Stabilization

The most immediate effect of currency stabilization is that the price level in the stabilizing country becomes more correlated with the price level in the target country:

$$p^m = p^{m*} + (1-\theta^m)\zeta(p^{t*} - p^{m*}).$$

Because larger countries tend to appreciate when λ_T is high, a stabilization relative to a larger country ($\theta^t > \theta^m$) naturally also makes the stabilizing country appreciate in these states; that is, stabilization increases the covariance between the stabilizing country's price level, p^m, and the shadow price of traded goods, λ_T, similar to the intervention considered in section 1. As a result, a risk-free asset that pays one unit of the stabilizing country's consumption bundle with certainty becomes a better hedge against consumption risk, increasing its value in the world market, and lowering the stabilizing country's risk-free interest rate.

Moreover, stabilizing relative to a larger country increases domestic capital accumulation because it raises the world-market value of domestic firms by increasing the covariance of their dividends with the larger country's price level, and thus with λ_T :

$$p_N^m + y_N^m = (p_N^{m*} + y_N^{m*}) + \zeta \frac{(\theta^m + (\gamma-1)\tau)}{\tau\gamma} \left(p^{t*} - p^{m*}\right), \tag{22}$$

where $p_N^m = \lambda_N^m - \lambda_T$ is the price of nontraded goods in country m.

[10] Note the relative prices of nontraded goods are no longer a sufficient statistic for the real exchange rate, because the state-contingent tax drives a wedge between the domestic and world-market prices of traded goods.

Proposition 1

If $\gamma > 1$, a country that stabilizes its real exchange rate relative to a target country sufficiently larger than itself lowers its risk-free interest rate and increases the world-market value of domestic firms, domestic capital accumulation, and domestic wages relative to the target country.

Proof. The interest rate differential with respect to the target country is

$$r^m + \Delta\mathbb{E}s^{m,t} - r^t = r^{m*} + \Delta\mathbb{E}s^{m,t*} - r^{t*} - \zeta\frac{\gamma(1-\tau)^2\left((\theta^t - \theta^m)(\gamma-1)\tau + 2\theta^m(1-\zeta)\right)}{\tau(1+(\gamma-1)\tau)}\sigma_N^2.$$

See Appendix B.7 for details and the corresponding proof for capital accumulation, which requires that the target country be sufficiently large. ■

Aside from these effects on interest rates and capital accumulation, the stabilization policy also affects the level of currency reserves. From (20), we already know the cost of implementing the stabilization is simply the cost of altering the stabilizing country's purchases of traded goods in world markets. Moreover, we also know the stabilization induces the stabilizing country to sell additional traded goods in response to an appreciation of the target country, and to buy additional traded goods in response to a depreciation. If the target country is larger than the stabilizing country, this policy amounts to selling traded goods when they are expensive and buying them when they are cheap. In other words, stabilization induces the stabilizing country to provide insurance to the world market against the (larger) target country's shocks, so that it pockets an insurance premium.

Proposition 2

If $\gamma > 1$ and the stabilizing country is small, $\theta^m = 0$, the cost of stabilization globally decreases with the size of the target country and locally increases with the size of the stabilizing country. Additionally, the cost of stabilization (ΔRes) is negative if and only if

$$\theta^t > \frac{\zeta + (\gamma-1)\tau}{(\gamma-1)^2\tau^2}.$$

Proof. See Appendix B.8. ■

If the target country is sufficiently large relative to the stabilizing country, this insurance premium can be so large that the stabilization generates positive net revenues, so that the stabilization increases, rather than decreases, currency reserves.[11]

[11] That is, the portfolio of stocks that pays exactly the cost of the stabilization policy in each state of the world has negative cost in the first period. See Appendix B.6 for details on the form of this portfolio.

When the stabilizing country itself is large ($\theta^m > 0$), its purchases and sales of traded goods also affect the equilibrium shadow price of traded goods, λ_T. This price impact generally increases the cost of stabilization. The reason is that stabilization effectively induces the stabilizing country to "do more" of what it would have done under freely floating exchange rates: Even under freely floating exchange rates, all countries increase their exports of traded goods when a large country appreciates. Stabilization then induces the stabilizing country to export even more than it ordinarily would have (compare equations (15) and (21)). The larger the stabilizing country is (i.e., the more price impact it has), the more costly it therefore is to maintain the stabilization. This increasing cost of stabilization will be key to our finding below that stabilization relative to the largest country in the world tends to be an optimal policy for small but not large countries.

2.4.2 External Effects of Currency Stabilization

If the stabilizing country is large ($\theta^m > 0$), its actions also have external effects on consumption and prices in the rest of the world. The shadow price of traded goods is

$$\lambda_T = \lambda_T^* - \frac{(1 + (\gamma - 1)\tau)}{\gamma\tau}\zeta\theta^m \left(p^{t*} - p^{m*}\right).$$

The second term on the right-hand side shows that stabilization by a large country reduces the covariance between the target country's price level and λ_T. By selling insurance against the target country's shocks, the stabilizing country dampens the effect of these shocks on the world-market price of traded goods. It follows immediately that becoming the target of a stabilization raises the target country's interest rate and lowers its capital accumulation.

Proposition 3

If $\gamma > 1$, a country that becomes the target of a stabilization of any strength $\zeta > 0$ imposed by a large country experiences an increase in its risk-free interest rate, a decrease in capital accumulation, and a decrease in average wages relative to all other countries. If the stabilizing country is smaller than the target country ($\theta^m < \theta^t$), the stabilization also lowers the volatility of consumption in the target country.

Proof. The interest rate differential between the target and outside country is

$$r^t + \Delta\mathbb{E}s^{t,o} - r^o = \left(r^{t*} + \Delta\mathbb{E}s^{t,o*} - r^{o*}\right) + \zeta\frac{\theta^p(1-\tau)^2\gamma}{\tau\left(1 + (\gamma - 1)\tau\right)}\sigma_N^2.$$

See Appendix B.9 for details and the remainder of the proof. ■

Currency stabilization can thus divert capital from the target country to the stabilizing country even though it has no effect on the level of the real exchange rate. This finding is particularly interesting because it sheds new light on recent public controversies, for example, between Chinese and US officials (Levy, 2011), which usually focus on the idea that an undervaluation of the Chinese real exchange rate favors Chinese workers at the expense of U.S. workers. By contrast, our model suggests that even a currency stabilization that manipulates the variance but not the level of the real exchange rate can have this effect.

On the flip side, currency stabilization by a large country decreases the volatility of consumption in the target country, because it effectively prompts the stabilizing country to provide consumption insurance to the target country. We show below that this positive effect of insurance provision can dominate, so that stabilization is associated with utility gains in both the stabilizing and the target country, at the expense of the outside country.

2.5 Welfare and the Rationale for Stabilization

Having characterized the positive effects of currency stabilization, we next study why a country might find it optimal to stabilize its currency. The existing literature has shown currency stabilization can be a second-best policy response in the presence of monetary and other frictions.[12] Perhaps surprisingly, we show that even in the absence of such frictions, stabilization relative to a larger country may increase welfare in the stabilizing country through a valuation effect that increases its share in world wealth.[13]

So far, we have defined a currency stabilization as reducing the variance of the log real exchange rate (P1) while not distorting its level (P2). Achieving both objectives simultaneously requires that the government has the ability to add and subtract resources from the economy by accumulating or depleting currency reserves. For the purposes of assessing the welfare effects of currency stabilization, we now drop objective (P2) and assume that, instead, the government rebates the cost of stabilizing the exchange rate back to households using the lump-sum transfer, so that $\Delta Res = 0$ and (7) becomes $\int_{i\in[0,1]} C_{T,2}(i,\omega)di = 1$. That is, households in the stabilizing country directly bear the financial cost or benefit of stabilizing the exchange rate, shifting the level of their traded consumption in all states of the world, and thus also affecting the level of their real exchange rate. Closing the model in this way does not interfere with the intuition

[12] For a recent example see Fanelli and Straub (2019).

[13] We define the valuation effect as the (log) difference in the value of the household's traded consumption from its value under freely floating exchange rates.

of the positive results derived above but increases the complexity of the solution, so that we relegate the mathematical details to Appendix C.

Within this closed model, solving for the effect of an exchange rate stabilization on the utility of a household in a small stabilizing country ($\theta^m = 0$) yields:[14]

$$\Delta u^m = \underbrace{\frac{(-\zeta^2 + \zeta\Theta^t(\gamma-1)\tau)(1-\tau)^2}{\tau(1+(\gamma-1)\tau)^2}\sigma_N^2}_{\Delta K, Revenues} - \underbrace{\frac{(\zeta\Theta^t + \zeta^2)(\gamma-1)(1-\tau)^2}{(1+(\gamma-1)\tau)^2}\sigma_N^2}_{\Delta Var[c^m]} + \underbrace{\frac{(\zeta\Theta^t + \zeta^2)\tau(\gamma-1)^2(1-\tau)^2}{(1+(\gamma-1)\tau)^2}\sigma_N^2}_{\text{Valuation Effect}}, \qquad (23)$$

where Δu^m is measured as the percentage increase of the household's certainty-equivalent consumption attributable to the stabilization, and $\Theta^t = \theta^t(\gamma-1)\tau - 1$ is positive and monotonically increasing in θ^t if $\gamma > 1$ and the target country is sufficiently large.

The first term on the right-hand side reflects changes in the expected level of consumption that result from changes in the level of domestic capital accumulation and the cost of implementing the stabilization. We have already seen that a stabilization relative to a larger country can increase capital accumulation and generate positive revenue, so that this term is positive if θ^t is sufficiently large. However, stabilization also increases the variance of consumption because the stabilizing country effectively provides insurance to the world market against shocks that affect the target country. This increase in the volatility of consumption strictly reduces expected utility, as shown in the second term.

One can show that the second term is always larger than the first term so that stabilizing would never be welfare increasing if not for the third term: the effect of the stabilization on the stabilizing country's share in world wealth.[15] This term reflects the fact, already shown above, that stabilizations relative to a larger country increase the world-market value of firms in the stabilizing country. Households in the stabilizing country are the monopoly suppliers of domestic firms so that, even if the country is small and a price-taker in international markets, it is always large enough to affect the world-market price of its own firms relative to the world-market price of foreign firms.[16] Because we have assumed households and governments can only trade stocks

[14] In keeping with the solution method outlined above, we solve for the equilibrium valuation change in households' portfolios using a second-order approximation around the point at which the marginal utility of wealth of households in all countries is equalized.

[15] See Appendix C for a formal proof of this statement.

[16] For a similar result, where small countries benefit from deviating from policy coordination, see Chari and Kehoe (1990).

in these firms in international markets, but not a full set of state-contingent claims, this valuation effect effectively enables the stabilizing government to shift wealth from the rest of the world to its own country by announcing a stabilization relative to a larger country.[17]

Proposition 4

If $\gamma > 1$ and all households own the portfolio of stocks that decentralizes the Pareto-efficient allocation of consumption under freely floating exchange rates at the time of the announcement of the stabilization policy, then there exists a $\overline{\theta} > 0$ such that a small stabilizing country ($\theta^m = 0$) strictly increases the welfare of its households by stabilizing relative to a target country with $\theta^t > \overline{\theta}$.

Proof. See Appendix C. ■

In other words, the positive effect of the stabilization on the valuation of domestic firms can be large enough to make stabilization relative to a larger country a welfare-increasing policy for the stabilizing country.

Panel (a) of Figure 1 illustrates this result graphically by plotting the three terms of (23) over the size of the target country for a typical numerical example where $\theta^m = 0$, $\zeta = 1$, $\tau = 1/3$, and $\gamma = 7$.[18] If the target country is small, all three terms are negative, but as the size of the target increases, both the first and the third term monotonically increase and become positive. The sum across the three lines represents the total change in the stabilizing country's welfare. This net effect is positive for all $\theta^t > \overline{\theta} = (\zeta + (1-\zeta)\tau^2(\gamma-1))/(\tau^3(\gamma-1)^2)$. *If it is optimal for a small country to stabilize relative to any target country, that country is thus always the largest country in the world.*

This increase in welfare through stabilization is, for a given set of parameters, easier to achieve for a small country than for a large country. As we have already seen above, a stabilization implemented by a large stabilizing country manipulates state-prices of traded goods in an unfavorable direction, which increases the cost of implementing the stabilization. The welfare benefits of stabilization thus tend to decrease with the size of the stabilizing country. Panel (b) of Figure 1 shows the utility gain from stabilization is smaller for larger stabilizing countries. The figure

[17] One can show the same result holds if instead households are confined to trading international bonds, because, again, stabilizing relative to a larger country increases the world-market value of the stabilizing country's bonds. See Appendix C.1 for details.

[18] Because the consumption index (3) has a unit elasticity of substitution between traded and nontraded goods, the portfolio of stocks that decentralizes the Pareto-efficient allocation of consumption under freely floating exchange rates is naturally home biased, in the sense that a given country's households own a relatively larger share of their own country's firms. As a result, an increase in the relative valuation of the stabilizing country's firms shifts wealth from foreign to domestic agents. Appendix C gives analytical solutions.

also shows the optimal stabilization need not be a hard peg: In the example shown, the largest stabilizing country ($\theta^m = 0.2$) maximizes its utility gains with a soft peg ($\zeta = 0.2$).

Taken together, these findings provide a rich set of predictions for a stabilizing country's optimal choice of exchange rate regime ($\zeta \in (0, 1]$) as a function of its own size (θ^m) and the size of the target country (θ^t). Panel (a) of Figure 2 shows a graphical representation of this optimal choice for the same numerical example as above. If a sufficiently large target country exists ($\theta^t > \bar{\theta}$), a small stabilizing country finds it optimal to impose a hard peg relative to that country. As the size of the stabilizing country increases, the optimal stabilization becomes looser. Finally, stabilizing countries above a certain size find it optimal to float their exchange rates.

Because the allocation under freely floating exchange rates is Pareto efficient, any utility gains from exchange rate stabilization accruing to households in a stabilizing country with positive mass ($\theta^m > 0$) must be causing losses to households somewhere else in the world. Interestingly, this collateral damage typically does not fall on the target country, but rather on the outside country (which, on the surface, has no relation to the stabilization). The reason is that although both the target and outside countries suffer from distortions to the state prices of traded goods, and from the relatively higher prices of firms in the stabilizing country, the target country also receives a benefit: The stabilizing country provides tailor-made insurance against shocks that are specific to the target country.

Panel (b) of Figure 2 shows the same triangular region as in Panel (a) (the area where stabilization is welfare improving for the stabilizing country), but now highlights the area where the target country also receives a net utility gain (the lower shaded area). In this subset of the parameter space, stabilization is thus welfare increasing for residents of both the stabilizing and the target country, and goes exclusively to the detriment of residents in the outside country (which always loses when it is optimal for the stabilizing country to stabilize).[19]

In the upper-left triangular region, the target country would also prefer the stabilizing country to float its exchange rate and not stabilize. However, given a stabilization, the welfare loss in the target country is less than the welfare loss of the outside country ($\Delta u^t > \Delta u^o$). In this sense, the model generates the intuitive result that for a large country that cannot gain from stabilizing itself, being the target country of choice can be beneficial: Given that other countries stabilize, being the target of that stabilization is preferable to being the outside country. (See Appendix C.2 for a formal proof.)

[19] We believe these statements hold quite generally. However we were unable to prove them formally as the analytical expressions are quite complex. See Appendix C.2.

In sum, our simple model endogenously produces a potential rationale, based on the traditional welfare criterion, for the patterns of stabilizations we see in the data, where (i) small countries find it optimal to stabilize their exchange rates, (ii) larger countries instead find it optimal to maintain looser stabilizations, (iii) the largest countries float their currencies, and (iv) all stabilizations are relative to the largest economy in the world (the United States). These insights on the optimal choice of exchange rate regime rely crucially on the interaction of two forces. The first is the fact that exchange rate stabilization makes domestic firms safer investments from the perspective of international investors, and thus increases their world-market value. The second is our assumption that households do not have access to a full set of Arrow-Debreu securities (the prices of which a small country would not be able to influence), but instead transact only in an international stock market. Because of this restriction on the asset space, changes in the value of an asset that even small countries have pricing power over (the relative value of their own firms) translate into shifts in relative wealth across countries.[20]

Having studied the positive and normative implications of exchange rate stabilization in this canonical and (modulo our restriction on the asset space) frictionless environment, we now show how the insights from this analysis continue to hold in more general settings.

3 Nominal Stabilization and Monetary Policy

We begin by showing that the insights of our baseline model carry over directly to a standard "new open economy" framework in which the prices of traded goods are sticky and stabilizing wedges arise naturally from a simple nominal stabilization regime.[21] To this end, we extend the setup of our model in section 2.1 by assuming each country has a central bank that issues and controls the supply of the domestic currency. The nominal price of the traded good in terms of

[20] Maybe as relevant in practice as these welfare considerations, our model also lends itself to a political economy rationalization for the same patterns: A large literature argues that policymakers trying to win elections have an interest in raising wages (e.g., if the median voter is a worker, Persson and Tabellini (2002)) and often prefer generating revenue through central bank or currency board operations to direct taxation, even if these operations are distortionary, because they are less visible to the public and easier to control (Cukierman et al., 1992; Bates, 2005). Currency stabilizations relative to the largest economy in the world achieve both of these objectives and may thus be politically attractive. For example, a stabilization relative to the largest economy in the world may be optimal even in the absence of valuation effects if policymakers in a stabilizing country maximize a function of the form

$$EU^n + \mu_1 K^n - \mu_2 \Delta Res,$$

where μ_1 and μ_2 are constants that may reflect the political influence of workers, externalities from capital accumulation, or a motive for generating revenues in a way that avoids direct taxation of households or firms.

[21] A large body of empirical work documents such rigidity, which creates a wedge in the prices of traded goods across borders, that is, failures in the law of one price (Mussa, 1986; Engel, 1999; Cavallo et al., 2014).

this currency is set before shocks are realized at the beginning of period 1, where $\tilde{P}_T^n$ denotes the (fixed) number of units of domestic currency needed to buy one unit of the traded good in country n. Households face a cash-in-advance constraint denominated in their domestic currency. That is, they must use their domestic currency when buying stocks in period 1 and when buying consumption goods in period 2.[22]

Having introduced money into the model, we can write the log nominal exchange rate as

$$\tilde{s}^{f,h} = p^f - p^h + \tilde{p}_T^f - \tilde{p}_T^h. \tag{24}$$

In keeping with our convention above, we define a stabilization of the *nominal* exchange rate of strength $\tilde{\zeta}$ as a set of policies that decreases the variance of this log nominal exchange rate between the stabilizing and target countries, $\text{var}\,(\tilde{s}^{t,m}) = (1-\tilde{\zeta})^2 \text{var}\,(\tilde{s}^{t,m*})$, while keeping the conditional mean of the log nominal exchange rate unchanged, $\mathbb{E}\,[\tilde{s}^{t,m}|\{K^n\}] = \mathbb{E}\,[\tilde{s}^{t,m*}|\{K^n\}]$.

Each central bank controls the growth rate of its own money supply, where Δm_1^n and $\Delta m^n(\omega)$ denote the growth rate of the money supply in the first period and state ω of the second period, respectively. We assume the central banks in the target and outside countries use their control of money supply to recover the efficient allocation of resources, taking as given the actions of the stabilizing country's central bank. By contrast, the central bank in the stabilizing country uses its control of monetary policy to stabilize the nominal exchange rate.

Although the actors and policymakers have different names in this extended version of the model, the equilibrium is, in fact, identical to the one already discussed above. To see this result, note first that because the price of traded goods is fixed in the domestic currency, a nominal stabilization automatically also implements a stabilization of the real exchange rate of equal strength. (The term $\tilde{p}_T^f - \tilde{p}_T^h$ in (24) is fixed so that the real exchange rate is simply proportional to the nominal exchange rate.) In other words, if the price of traded goods is sticky, a central bank that stabilizes the nominal exchange rate relative to some target currency implicity also stabilizes the real exchange rate relative to that same target country.

Second, through its control of money supply, the stabilizing country's central bank effectively has the same ability to drive a state-contingent wedge between λ_T and the domestic price of traded goods in the second period (and pay a lump-sum transfer in the first period) as the stabilizing government in section 2. Solving the extended model therefore yields identical first order conditions to those in section 2.2, except that $Z(\omega)$ is now replaced with the growth rate

[22] Appendix D gives formal details and additional notation.

of the money supply, $\exp[-\Delta m^m(\omega)]$. The condition pinning down the shadow price of traded consumption in the stabilizing country now reads

$$\tau \left(C^m(\omega)\right)^{1-\gamma} \left(C_T^m(\omega)\right)^{-1} = \exp[-\Delta m^m(\omega)]\Lambda_T(\omega). \tag{25}$$

Because the nominal price of the traded good cannot adjust in the second period, expansions and contractions of the money supply thus again drive a wedge between the domestic and world-market price of traded goods.

Therefore, intuitively, in order to stabilize its real and nominal exchange rates relative to a given target country, the stabilizing central bank must contract the domestic money supply whenever the target country appreciates,

$$-\Delta m^m(\omega) = z(\omega) = \zeta \frac{\gamma\tau + (1-\tau)}{\gamma\tau}(p^{t*}(\omega) - p^{m*}(\omega)).$$

The only difference to our baseline model is that this policy is now much easier to map to the real-world nominal exchange rate stabilization policies that central banks typically follow: When the target country appreciates, the central bank in the stabilizing country decreases the domestic money supply by buying domestic currency and selling foreign currency, matching the nominal appreciation. Because the price of traded goods is sticky in domestic currency, this reduction in domestic money supply increases the real price of traded goods in the stabilizing country, prompting domestic households to consume fewer traded goods whenever the target country appreciates. A conventional nominal stabilization thus automatically replicates the effect of stabilizing state-contingent taxes: The stabilizing country exports additional traded goods whenever the target country appreciates, and vice versa.

Proposition 5

If the price of the traded good is rigid in terms of the stabilizing country's currency,

1. *a nominal stabilization implements a real stabilization of equal strength* $\zeta = \tilde{\zeta}$, *and*

2. *the seigniorage from stabilization is equal to* $-\Delta Res$,

$$seigniorage = \mathbb{E}\left[\frac{\Lambda_T^*(\omega)}{\Lambda_{T,1}^*} C_T^{m*}(\omega)\right] - \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}} C_T^m(\omega)\right] = -\Delta Res.$$

Proof. See Appendix D. ■

If households need domestic currency to buy consumption goods and prices are sufficiently sticky to give the central bank some leverage over real allocations, we thus conclude that stabilizations of the real exchange rate can be implemented with a simple rule that commits the central bank's control of the money supply to enforce a nominal stabilization. That is, even if prices are only partially rigid, a nominal peg, where the central bank commits to exchanging currency at a predetermined rate, implements some real exchange rate stabilization, entailing all the positive and normative effects on real allocations discussed in the previous section. Appendix D.2 shows similar results for an economy where prices are flexible and monetary policy instead affects real allocations because financial markets are segmented.

4 Partially Credible Stabilizations and Floating Bands

A major issue in the study of policies that manipulate the first moment of exchange rates (under- or over-valuations), is the depletion of reserves and the credibility of such manipulations in the face of potential speculative attacks (Krugman, 1979; Garber and Svensson, 1995). By contrast, we have already shown that stabilizations of the real exchange rate relative to a large target country may generate, rather than deplete, reserves, assuaging some potential concerns about the policy's credibility. (The portfolio of stocks that finances the stabilization policy in each state has a negative cost in period 1.)

Nevertheless, it is worthwhile to consider the effects of only partially credible stabilizations. Suppose the government, either by choice or necessity, abandons the stabilization in a subset of states $\Omega_{-s} \subset \Omega$ (where Ω is the set of all possible states). Assuming the government continues to stabilize state-by-state within $\Omega_s = \Omega \backslash \Omega_{-s}$, and that this limited stabilization continues to leave the mean of the real exchange rate undistorted (e.g., the partition of Ω into Ω_s and Ω_{-s} is symmetric around the mean), we can show that[23]

$$\text{var}(s^{m,t}) = \left(\text{Prob}\left[\omega \in \Omega_s\right](1-\zeta)^2 + \text{Prob}\left[\omega \in \Omega_{-s}\right]\right) \text{var}\left[s^{*m,t}|\Omega_{-s}\right] < \text{var}(s^{m,t*})$$

and

$$r^m + \Delta\mathbb{E}[s^{m,t}] - r^t = -\left(\text{Prob}\left[\omega \in \Omega_s\right](1-\zeta) - \text{Prob}\left[\omega \in \Omega_{-s}\right]\right) \text{cov}\left[\lambda_T, s^{*m,t}|\Omega_s\right].$$

In contrast to partially credible manipulations of the level of the real exchange rate, partially

[23]See Appendix E.1 for a formal derivation.

credible manipulations of its variance are thus still effective: They reduce the variance of the real exchange rate and affect interest rates and other outcomes in the same way as characterized above — only less so than a fully credible stabilization. In this sense, we may simply think of partially credible stabilizations as "weaker" credible stabilizations.

Additionally, the two expressions above also directly describe the effects of a variety of non-linear stabilization policies, such as floating bands, that allow a freely floating exchange rate between some upper and lower limit, and intervene state by state only when the real exchange rate departs this band.

Similarly, Appendix E.2 shows that our analysis above also extends directly to stabilizations relative to a basket of currencies, where stabilizing relative to a basket of currencies has effects akin to a stabilization relative to a (hypothetical) country with a weighted average size of the basket's constituents.

5 Segmented Markets and Preference Shocks

So far, we have based our analysis of currency stabilization on a conventional international real business cycle model, where productivity shocks are the only drivers of variation in real exchange rates (Backus and Smith, 1993). Although an important benchmark, this framework has a number of well-known empirical shortcomings. First, it predicts a perfectly negative correlation between appreciations of the real exchange rate and aggregate consumption growth — a currency appreciates when the country's aggregate consumption decreases. Second, the model predicts consumption should be more correlated across countries than output, whereas the opposite is true in the data (Backus, Kehoe, and Kydland, 1994). Third, real exchange rates and terms of trade seem much too volatile to be rationalized by real (productivity) shocks alone (Chari, Kehoe, and McGrattan, 2002).

In this section, we argue the conclusions from our analysis of exchange rate stabilizations do not rely on any of these counterfactual features of the international real business cycle model. Instead, they depend solely on two, more general, features of conventional approaches to modeling variation in exchange rates: First, whatever shock causes a country's real exchange rate to appreciate also prompts this country to demand higher imports of traded goods. Second, shocks that raise the price of consumption in a larger country spill over more into the world-market price of traded goods. Both of these forces are common features of a broader class of models where real exchange rates may also fluctuate in response to shocks to preferences or money supply.

To illustrate this finding, we augment our baseline model in section 2.1 with two (widely cited) demand-based sources of variation in real exchange rates. First, we allow households in each country to experience preference shocks as suggested Pavlova and Rigobon (2007):

$$U(i) = \frac{1}{1-\gamma}\mathbb{E}\left[(\exp(\chi^n)C_2(i))^{1-\gamma}\right], \tag{26}$$

where χ^n is a common shock to households' preference for consumption goods in country n,

$$\chi^n \sim N\left(-\frac{1}{2}\sigma_\chi^2, \sigma_\chi^2\right).$$

Second, we also allow for a direct effect of inflation on real exchange rates by assuming a measure $1-\phi$ of "inactive" households within each country exclusively hold nominal bonds denominated in their own currency, as suggested by Alvarez et al. (2002).[24] The remaining measure ϕ of ("active") households within each country trade stocks and nominal bonds in international markets as before.[25] Each country's nominal bond pays off one unit of the country's nominal consumer price index, $P_2^n e^{-\mu^n}$, where μ^n is a (monetary) shock to the growth rate of the nominal price of one unit of the traded good in the currency of country n,

$$\mu^n \sim N\left(-\frac{1}{2}\tilde{\sigma}^2, \tilde{\sigma}^2\right).$$

A higher μ^n thus implies a higher inflation rate in country n. Active households own all stocks and are short the nominal bonds owned by inactive households, so that monetary shocks effectively shift resources from inactive households (who live hand-to-mouth and are not hedged against inflation) to active agents whose marginal utilities determine exchange rates and asset prices. (See Appendix F.1 for details on the budget constraints of both kinds of households.)

As before, the government of the stabilizing country stabilizes its exchange rate with the target country using state-contingent taxes.

The punchline is that currency stabilization in this richer model of exchange rate determination works in the same way as in our baseline model with productivity shocks. First, note that larger countries continue to have lower interest rates and a lower cost of capital under freely

[24] A substantial fraction of households in the US and in other developed economies own savings accounts or bonds denominated in their domestic currency, but do not own stocks or other more sophisticated financial instruments that could hedge their portfolios against inflation (Giannetti and Koskinen, 2010; Nechio, 2010).

[25] Because this richer model has six linearly independent shocks, we need a stock and a bond for each country so that the payoffs of the available assets span all states of the world in the log-linear solution to the competitive equilibrium, as before.

floating exchange rates. Solving the model yields

$$s^{f,h*} = p^{f*} - p^{h*} = \frac{\gamma(1-\tau)}{\phi(1-\tau)+\gamma\tau}\left((1-\phi)\left(\mu^h - \mu^f\right) + \frac{\phi\left(\gamma-1\right)}{\gamma}\left(\chi^h - \chi^f\right)\right)$$

and

$$\lambda_T^* = -\gamma\left(\frac{1-\phi}{\phi}\right)\sum_n \theta^n \mu^n - (\gamma-1)\sum_n \theta^n \chi^n.$$

The structure of these expressions is identical to (16) and (17): Countries import more traded goods when they appreciate, and shocks to the price of consumption in a larger country spill over more to λ_T^*, so that larger countries tend to appreciate when λ_T^* is high. For example, a low χ in a given country increases the marginal utility of its households, appreciates its real exchange rate, and prompts it to import more traded goods. If the country is large, these higher imports also raise λ_T^*, so that a larger country's preference shocks spill over more to the rest of the world. Similarly, a low monetary shock (deflation) shifts resources away from active households (who are short the nominal bonds owned by inactive households), increases their marginal utility, and thus appreciates the country's real exchange rate – prompting it to import more traded goods. If the country is large, these higher imports again have a proportionately higher impact on λ_T^*.

As a result, larger countries again have safer currencies, lower interest rates, and more valuable firms, as they did in our baseline model. Similarly, the effects of exchange rate stabilization follow the same logic as above: A smaller country stabilizing its real exchange rate relative to a larger country increases the covariance between its exchange rate and λ_T:

$$p^m = p^{m*} + \zeta\left(\frac{\gamma\tau + \theta^m(1-\tau)\phi}{\gamma\tau}\right)(p^{t*} - p^{m*});$$

and by making its currency safer, the stabilizing government increases domestic capital accumulation and its households' share in world wealth. As before, the stabilizing government must artificially increase its exports of traded goods whenever the target country appreciates, in order to maintain the stabilization:

$$c_T^m - c_T^{m*} = -\zeta\frac{(1-\theta^m)\left(\tau + (1-\tau)\phi\right)^2 + (1-\tau)(1-\phi)(\gamma\tau + (1-\tau)\phi)}{\gamma\tau\left(\tau + (1-\tau)\phi\right)}\left(p^{t*} - p^{m*}\right).$$

Moreover, by effectively selling insurance against the target country's shocks, the stabilizing country again dampens the effect of the target country's shocks on the world-market price of traded goods:

$$\lambda_T = \lambda_T^* - \zeta\theta^m \left(\frac{\gamma\tau + (1-\tau)\phi}{\gamma\tau}\right)(p^{t*} - p^{m*}).$$

It follows directly that all of our positive predictions about the effects of currency stabilizations carry over to this richer model. (The conclusions of our normative analysis also continue to hold as long as the traditional welfare criterion is applicable ($\phi = 1$), but are harder to interpret once we have more than one class of households per country.)

Proposition 6

In the model with market segmentation, monetary shocks, and preference shocks with $\gamma > 1$, the following hold:

1. *A country that stabilizes its real exchange rate relative to a target country sufficiently larger than itself lowers its risk-free interest rate and increases the world-market value of domestic firms, domestic capital accumulation, and domestic wages relative to the target country.*

2. *If the stabilizing country is small ($\theta^m = 0$), the cost of the stabilization globally decreases with the size of the target country.*

3. *A country that becomes the target of a stabilization of any strength $\zeta > 0$ imposed by a large country experiences an increase in its risk-free interest rate, a decrease in capital accumulation, and a decrease in average wages relative to all other countries.*

Proof. See Appendix F.4. ■

In addition to reinforcing the main insights from our baseline model, this richer framework addresses the three major empirical shortcomings of the international real business cycle model outlined above: The combination of market segmentation, monetary shocks, and preference shocks loosens or even reverses the negative correlation between appreciations of the real exchange rate and aggregate consumption growth, lowers the correlation of aggregate consumption across countries, and increases the volatility of real and nominal exchange rates (Alvarez et al., 2002; Pavlova and Rigobon, 2007; Kollmann, 2012). All of our conclusions from section 2 thus carry over to this empirically more viable model of exchange rate determination.

Beyond this particular model, we believe that the results stated in Proposition 6 are quite general and hold in a wide range of models where currency manipulation transmits itself through a wedge on the price of traded goods. As noted in the introduction, more general models could also allow governments to stabilize exchange rates by manipulating additional wedges on allocations within countries, such as the sectoral allocation of labor or the distribution of wealth across households. Within this broader class of models, it is possible to construct examples where stabilization of the real exchange rate is achieved by reducing rather than increasing exports in response to an appreciation by the target country. In those examples, stabilizations relative to larger countries continue to lower domestic interest rates and increase capital accumulation, but some of the other implications highlighted above may not generalize. In this sense, the first statement in Proposition 6 is the most general, whereas the second and third statements rely on the – we believe plausible – assumption that interventions in currency markets affect allocations primarily through their effect on trade and the prices of traded goods.

Conclusion

The majority of countries in the world stabilize their real or nominal exchange rate relative to the US dollar. Although exchange rate stabilizations are possibly the most pervasive form of currency market interventions, existing theories give relatively little guidance on the effects of such stabilizations, on what might be special about the US dollar as a target currency, and on how these stabilizations might affect the target country.

Building on a growing literature that views risk premia as the main driving force behind large and persistent differences in interest rates across developed economies, we propose a novel, risk-based theory of the effects of currency manipulation: Policies that reduce the riskiness of a country's currency from the perspective of international investors reduce its risk premium in international markets, lower the country's risk-free interest rate, and increase domestic capital accumulation, domestic wages, and the world market value of domestic firms.

In particular, we show that stabilizing a country's real exchange rate relative to a larger (and safer) target economy is precisely such a policy that enables small countries to increase the world-market value of their capital stock, bonds, and firms. Moreover, if the prices of traded goods are at least partially sticky in terms of the domestic currency, such real stabilizations correspond directly to the kinds of simple stabilizations of nominal exchange rates relative to the US dollar we observe in the data, where central banks engage in a variety of open market operations to maintain pegs, moving bands, or managed floats.

In equilibrium, the effect of exchange rate stabilizations on risk premia gives rise to a pattern of optimal stabilizations that is remarkably similar to the one we see in the data: In the absence of coordination, small countries find it optimal to stabilize their exchange rates relative to the currency of the largest economy in the world, which endogenously emerges as the world's "anchor" currency. By contrast, larger countries optimally choose looser stabilizations or float their exchange rates. In other words, our model suggests that the dollar-centric pattern of exchange rate regimes that has arisen since the collapse of the Bretton-Woods system can be understood as an attempt to manage risk and attract investment.

Interestingly, our model also suggests that this (non-cooperative) equilibrium pattern of stabilization tends to benefit not only the stabilizers, but also target country, while other countries that are too large to stabilize their own exchange rates are always worse off relative to a world-wide freely-floating regime.

In sum, we believe our paper provides a novel way of thinking about the effects of currency stabilization. Along with highlighting a model-consistent rationale for stabilizing, we also give an account of the costs and benefits of important choices for the stabilization regime, such as the choice of target country, the effects of hard pegs versus floating bands, and stabilizations relative to a single country versus a basket of currencies.

Our work leaves open at least three avenues for future research. First, careful empirical work will be needed to identify the effect of currency manipulation in the data and disentangle the effects of altered risk premia from effects that may transmit themselves through more conventional channels, such as facilitating trade with the target country and establishing credibility for monetary policy. A prerequisite to making progress on these questions will be to identify (and control for) stabilizations that also involve manipulating the mean of the real exchange rate—a contentious political issue that has not been satisfactorily resolved in the empirical literature. Second, although many models have argued for risk premia as the main drivers of cross-sectional differences in interest rates, all of these papers, including our own, rely on standard preferences and thus generally imply risk premia are quantitatively small. Recent work by Govillot et al. (2010), David et al. (2016), and Colacito et al. (2018) makes progress in this dimension by studying dynamic models with heterogeneous countries and recursive preferences. However, the literature is still far from rationalizing the large differences in mean returns across currencies we see in the data in a microfounded quantitative model. Finally, our analysis has focused exclusively on a simple problem in which each country optimally chooses its own exchange rate regime, taking as given the policies of other countries. Analogous to a large literature on strategic interactions in trade policy (Bagwell and Staiger, 1999; Ossa, 2011), our prediction that exchange rate policy alters the equilibrium allocation of factors of production may also serve as the basis of a multilateral theory of strategic interactions in the choice of exchange rate regime.

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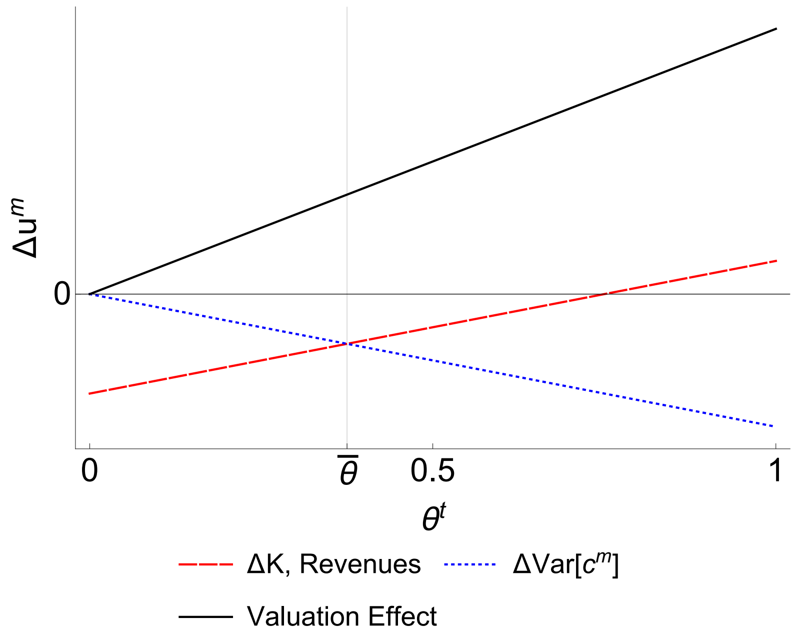
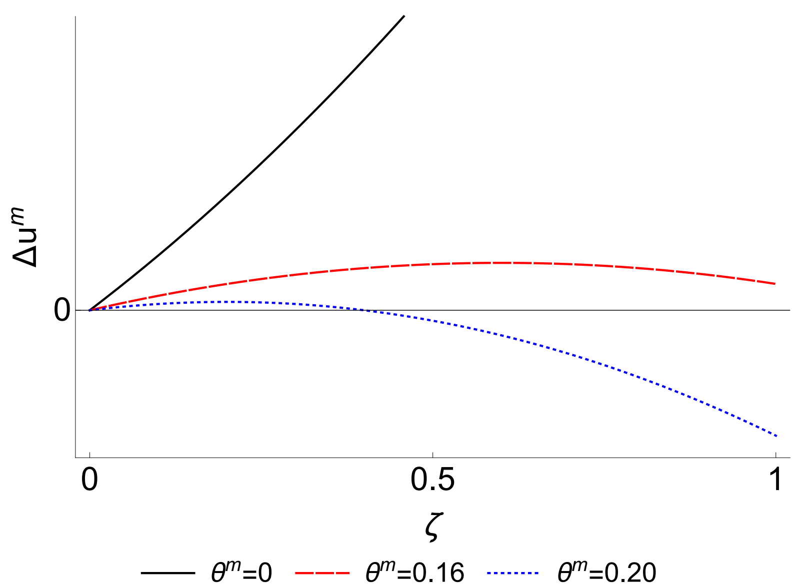
Figure 1: Effect of Stabilization on Utility in the Stabilizing Country

(a) Drivers of utility gains/losses over the size of the target country

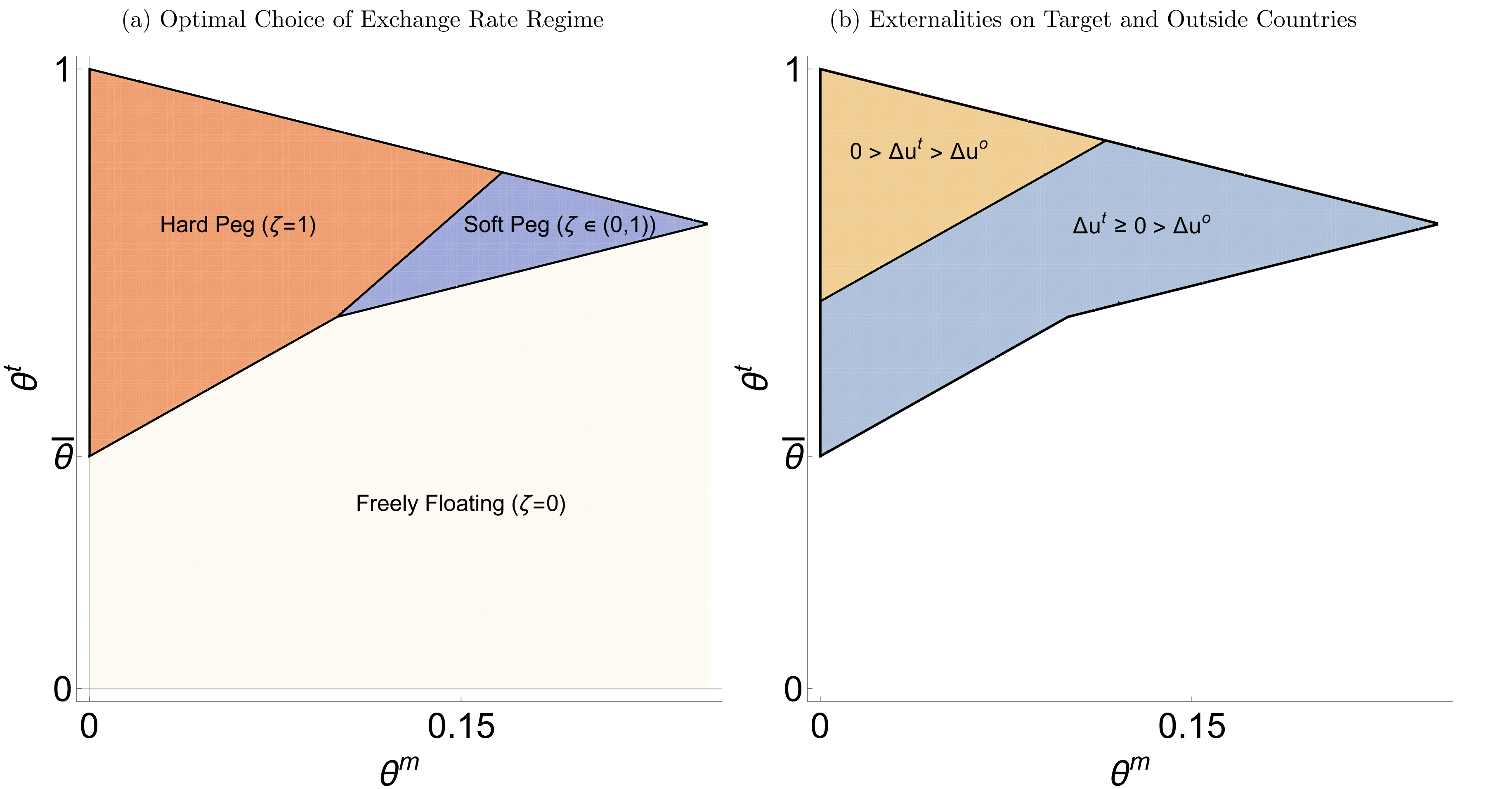


(b) Utility gains of stabilization over strength of stabilization for stabilizing countries of different sizes



Notes: Both plots show the percentage increase in the certainty-equivalent consumption of a representative household in the stabilizing country attributable to the stabilization (Δu^m) for a numerical example where $\tau = 1/3$, and $\gamma = 7$. Panel (a) shows the three components of Δu^m shown on the right hand side of (23) for a small stabilizing country ($\theta^m = 0$) and a hard peg ($\zeta = 1$). The net utility gain is the sum of the three lines. It is positive for all $\theta^t > \bar{\theta}$. Panel (b) shows the net utility gain as a function of ζ for stabilizing countries of different sizes. See Appendix C for the generalization of (23) that allows for $\theta^m > 0$.

Figure 2: Optimal Stabilizations



Notes: The triangular region shown in both figures marks the subset of the parameter space where a stabilization is strictly welfare-increasing for the stabilizing country ($\Delta u^m > 0$) for a numerical example where $\tau = 1/3$, and $\gamma = 7$. Outside of triangular region the stabilizing country maximizes its own utility by floating ($\zeta = 0$). Panel (a) marks regions where the stabilizing country's optimal choice is to impose a hard peg ($\zeta = 1$, marked in red) versus a "softer" stabilization ($\zeta \in (0, 1)$, marked in purple). Panel (b) illustrates the (pecuniary) externalities of these optimal stabilizations on the target and outside countries.

Appendix

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A Appendix to Section 1

The country n risk-free bond pays off P^n units of the traded good at maturity. We derive the value of the risk-free bond, V_P^n, by applying the asset pricing equation to the bond payoff:

$$V_P^n = \mathbb{E}\left[\Lambda_T P^n\right],$$

where Λ_T denotes the stochastic discount factor. The country n risk-free rate (in levels), R^n, is the inverse of the price of the risk-free bond:

$$R^n = \frac{1}{V_P^n}.$$

Putting the previous two equations together yields the following relationship:

$$\mathbb{E}\left[\Lambda_T P^n\right] R^n = 1.$$

As a result, the risk-free rates of countries f and h are related as follows:

$$\mathbb{E}\left[\Lambda_T P^f\right] R^f = \mathbb{E}\left[\Lambda_T P^h\right] R^h = 1$$

If the stochastic discount factor and prices are log-normal, we can perform the following calculations:

$$\begin{aligned}
& \mathbb{E}\left[\Lambda_T P^f\right] R^f = \mathbb{E}\left[\Lambda_T P^h\right] R^h \\
\Leftrightarrow\quad & \mathbb{E}\left[\exp\left[\lambda_T + p^f + r^f\right]\right] = \mathbb{E}\left[\exp\left[\lambda_T + p^h + r^h\right]\right] \\
\Leftrightarrow\quad & \mathbb{E}\left[\lambda_T + p^f\right] + \frac{1}{2}\mathrm{var}\left(\lambda_T\right) + \frac{1}{2}\mathrm{var}\left(p^f\right) + \mathrm{cov}\left(\lambda_T, p^f\right) + r^f \\
& = \mathbb{E}\left[\lambda_T + p^h\right] + \frac{1}{2}\mathrm{var}\left(\lambda_T\right) + \frac{1}{2}\mathrm{var}\left(p^h\right) + \mathrm{cov}\left(\lambda_T, p^h\right) + r^h,
\end{aligned}$$

We cancel out $\text{var}(\lambda_T)$ from both sides of the previous equation.

$$
\begin{aligned}
& \mathbb{E}\left[p^f\right] + \frac{1}{2}\text{var}\left(p^f\right) + \text{cov}\left(\lambda_T, p^f\right) + r^f = \mathbb{E}\left[p^h\right] + \frac{1}{2}\text{var}\left(p^h\right) + \text{cov}\left(\lambda_T, p^h\right) + r^h \\
\Leftrightarrow\quad & r^f + \mathbb{E}\left[p^f - p^h\right] + \frac{1}{2}\text{var}\left(p^f\right) - \frac{1}{2}\text{var}\left(p^h\right) - r^h = -\text{cov}\left(\lambda_T, p^f - p^h\right) \\
\Leftrightarrow\quad & r^f + \log\left(\mathbb{E}\left[P^f\right]/\mathbb{E}\left[P^h\right]\right) - r^h = -\text{cov}\left(\lambda_T, p^f - p^h\right)
\end{aligned}
$$

We define $\Delta\mathbb{E}\left[s^{f,h}\right] = \log\left(\mathbb{E}\left[P^f\right]/\mathbb{E}\left[P^h\right]\right)$. With this definition:

$$
r^f + \Delta\mathbb{E}\left[s^{f,h}\right] - r^h = -\text{cov}\left(\lambda_T, p^f - p^h\right).
$$

B Appendix to Sections 2.1 - 2.4

B.1 Equilibrium Conditions

In this appendix, we provide additional details for our baseline model in Section 2.1 and formally derive its equilibrium conditions. To avoid solving the optimization problem separately for households in the stabilizing country and households in the rest of the world, we generalize the notation to allow all countries to impose state-contingent taxes, $Z^n(\omega)$, and provide lump sum transfers, $\bar{Z}^n$. The governments in the target and outside countries do not use these instruments, such that $Z^t(\omega) = Z^o(\omega) = 1$ and $\bar{Z}^t = \bar{Z}^o = 0$.

In the second period, all households maximize their utility (4) subject to their budget constraint:

$$
Z^n(\omega)C_T^n(\omega) + P_N^n(\omega)C_N^n(\omega) \leq \sum_l A_l^n P_N^l(\omega)Y_N^l(\omega) + Y_T \tag{27}
$$

where $P_N^n(\omega)$ is the price of the nontraded good in the stabilizing country in state ω, A_l^n is number the stocks a country n household owns in the firm in country l, and $Y_T = 1$ is the unit endowment of the traded good.

In the first period, households choose their portfolio of stocks to maximize expected utility in the second period. The first-period budget constraint reads:

$$
\sum_l A_l^n Q_N^l + Q_K K_N^n \leq W_0^n. \tag{28}
$$

where W_0^n represents initial household wealth in terms of traded goods in the first period.

$\Lambda_T^n(\omega)$ denotes the Lagrange multiplier on the budget constraint for the country n household in state ω in the second period. The first-order conditions are:

$$\frac{\tau\left(C^n(\omega)^{1-\gamma}\right)\left(C_T^n(\omega)\right)^{-1}}{Z^n(\omega)} = \Lambda_T^n(\omega) \tag{29}$$

$$(1-\tau)\left(C^n(\omega)^{1-\gamma}\right)\left(C_N^n(\omega)\right)^{-1} = \Lambda_T^n(\omega) P_N^n(\omega). \tag{30}$$

The consumption tax drives a wedge between the marginal utility of consumption of traded goods and its shadow price, as equation (29) shows. Equations (12) and (30) jointly imply $P_N^n(\omega) = \Lambda_N^n(\omega)/\Lambda_T^n(\omega)$.

Next, we derive equilibrium conditions that determine first-period investment in stocks and capital. Since the final consumption bundle is a Cobb-Douglas aggregate of traded and nontraded goods, households spend a fraction τ of their second-period wealth on traded consumption and a fraction $1-\tau$ on nontraded consumption:

$$C_T^n(\omega) = \tau\left(\frac{\sum_l A_l^n P_N^l(\omega) Y_N^l(\omega) + Y_T}{Z^n(\omega)}\right) \text{ and } C_N^n(\omega) = (1-\tau)\left(\frac{\sum_l A_l^n P_N^l(\omega) Y_N^l(\omega) + Y_T}{P_N^n(\omega)}\right).$$

In the first period, households choose their portfolio of stocks and firms decide on their capital investment, K_N^n. We plug the consumption of traded and nontraded goods into equations (3) and (4) and take first-order conditions to obtain:

$$Q_N^l = \mathbb{E}\left[\left(\frac{\tau^\tau(1-\tau)^{1-\tau}}{\Lambda_{T,1}^n\left(Z^n(\omega)\right)^\tau\left(P_N^n(\omega)\right)^{1-\tau}}\right)\left(C^n(\omega)\right)^{-\gamma} P_N^l(\omega) Y_N^l(\omega)\right],$$

where $\Lambda_{T,1}^n$ denotes the Lagrange multiplier on the first-period budget constraint for a household in country n.

Divide through by Q_N^l and apply the definition of the price index $P^n(\omega)$ given by equation (36) in Appendix B.2 to obtain

$$\mathbb{E}\left[\frac{\Lambda_T^n(\omega)}{\Lambda_{T,1}^n}\frac{P_N^l(\omega) Y_N^l(\omega)}{Q_N^l}\right] = 1. \tag{31}$$

Firms invest in capital to maximize the expected discounted value of profits:

$$\max_{K^n} \mathbb{E}\left[\left(\frac{\Lambda_T^n(\omega)}{\Lambda_{T,1}^n}\right) P_N^n(\omega)\exp\left(\eta^n\right)\left(K^n\right)^\nu\right] - Q_K\left(K^n - 1\right).$$

Their first-order condition with respect to K^n yields

$$Q_K = \nu \mathbb{E}\left[\frac{\Lambda_T^n(\omega)}{\Lambda_{T,1}^n} P_N^n(\omega) \exp\left(\eta^n\right) \left(K^n\right)^{\nu-1}\right].$$

Multiply both sides of the previous equation by K^n, divide by Q_K, substitute $Y_N^n = \exp(\eta^n)\left(K^n\right)^{\nu}$, and apply the definition of $P_N^n(\omega)$ to get (13).

Equations (31) and (13) show $\Lambda_T^n(\omega)/\Lambda_{T,1}^n$ are the stochastic discount factors used to price assets that pay off in traded goods in the second period. Since stocks and capital are freely traded in international markets, all households must be marginal to investing in all stocks and all firms must be marginal to purchasing an additional unit of capital. As a result, the stochastic discount factors are equal in equilibrium across countries,

$$\frac{\Lambda_T^n(\omega)}{\Lambda_{T,1}^n} = \frac{\Lambda_T^m(\omega)}{\Lambda_{T,1}^m} \quad \forall\ n, m, \tag{32}$$

even though the government's intervention drives a wedge between $\Lambda_T(\omega)$ and the marginal utility of traded consumption in the stabilizing country, as equation (29) shows.

As a final step, we derive the equations that pin down the first and second-period Lagrange multipliers. Household wealth in the first period is:

$$W_0^n = Q_N^n + Q_K + \kappa^n + \bar{Z}^n,$$

Recall that households are endowed with a unit of stock and a unit of capital. κ^n is the transfer that equalizes the marginal utility of wealth across households when countries do not manipulate the exchange rate, and the transfer $\bar{Z}^m$ ensures the same is true under a stabilization, so that

$$\Lambda_{T,1}^n = \Lambda_{T,1} \quad \forall\ n. \tag{33}$$

As a result, (32) implies

$$\Lambda_T^n(\omega) = \Lambda_T(\omega) \quad \forall\ n, \omega.$$

Hence, we drop the country index on the Lagrange multipliers, and interpret $\Lambda_T(\omega)$ as the shadow price of traded consumption in the target and outside countries in the second period. This result implies equations (10), (12) and (13).

Equation (33) shows the first-period Lagrange multipliers are equal to each other, but it does not determine the level of the Lagrange multipliers. Without loss of generality, we normalize the first period Lagrange multiplier:

$$\Lambda_{T,1} = \mathbb{E}\left[\Lambda_T^n(\omega)\right]. \tag{34}$$

B.2 Deriving the Price Index

The cost of one unit of consumption in country n is given by the price index

$$P^n = \arg\min C_T^n + P_N^n C_N^n \text{ s.t. } \left(C_T^n\right)^\tau \left(C_N^n\right)^{1-\tau} = 1. \tag{35}$$

First-order conditions imply $C_N^n = (1-\tau)/(P_N^n \tau)\, C_T^n$. We plug this expression for C_N^n into the constraint $\left(C_T^n\right)^\tau \left(C_N^n\right)^{1-\tau} = 1$, and solve for C_T^n:

$$C_T^n = \left(\frac{\tau}{1-\tau} P_N^n\right)^{1-\tau}.$$

We plug the expressions for C_T^n and C_N^n back into equation (35) to derive the optimal price index:

$$P^n = \frac{\left(P_N^n\right)^{1-\tau}}{\tau^\tau (1-\tau)^{1-\tau}}. \tag{36}$$

The total value of consumption for households in country n is

$$P^n C^n = \left(\frac{\left(P_N^n\right)^{1-\tau}}{\tau^\tau (1-\tau)^{1-\tau}}\right)\left(\left(C_T^n\right)^\tau \left(C_N^n\right)^{1-\tau}\right) = \frac{C_T^n}{\tau}.$$

Similarly, we use the expression $P_N^n = \frac{1-\tau}{\tau}\frac{C_T^n}{C_N^n}$ to show that

$$C_T^n + P_N^n C_N^n = \frac{C_T^n}{\tau} = P^n C^n.$$

B.3 Log-linearized System of Equations

This appendix derives the log-linearized first-order conditions. To reiterate, we log-linearize around the deterministic solution — the point at which the variances of shocks are zero ($\sigma_{N,n} = 0$) and all firms have a capital stock fixed at the deterministic steady-state level.

We have shown in Appendix B.1 that the stochastic discount factor $\Lambda_T^n(\omega)/\Lambda_{T,1}^n$ is equalized across all households in all states. It is convenient to write the logarithm of this stochastic discount factor as:

$$q = \lambda_T^n - \lambda_{T,1}^n.$$

We can then write the log-linear first-order conditions for the second period as

$$(1-\gamma)\left(\tau c_T^n + (1-\tau)c_N^n\right) - c_T^n + \log\tau = z^n + q + \lambda_{T,1}^n$$
$$(1-\gamma)\left(\tau c_T^n + (1-\tau)c_N^n\right) - c_N^n + \log(1-\tau) = p_N^n + q + \lambda_{T,1}^n,$$

and the log-linear resource constraints are:

$$c_N^n = y_N^n$$
$$\sum_{n=m,t,o} \theta^n c_T^n = 0$$

where $z^m = \log\left(Z^m(\omega)\right)$ and $z^t = z^o = 0$. Note that ΔRes is a second-order term (linear in σ^n) and consequently does not show up in the log-linear resource constraint. This set of ten linear equations (two first order conditions for each country and four resource constraints) allows us to solve for the endogenous variables $\{c_N^n, c_T^n, p_N^n\}_{n=m,t,o}$ and q. Keeping in mind the log-linear relationship between each country's output and its respective productivity shock (5), it is convenient to solve for these endogenous variables in terms of each country's output $\{y_N^m, y_N^t, y_N^o\}$, and the Lagrange multipliers $\left\{\lambda_{T,1}^m, \lambda_{T,1}^t, \lambda_{T,1}^o\right\}$.

Solving the system yields:

$$c_T^m = \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^m\right) - \frac{1-\theta^m}{1+(\gamma-1)\tau} z^m + \frac{\bar{\lambda}_{T,1} - \lambda_{T,1}^m}{1+(\gamma-1)\tau}$$
$$c_T^t = \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^t\right) + \frac{\theta^m}{1+(\gamma-1)\tau} z^m + \frac{\bar{\lambda}_{T,1} - \lambda_{T,1}^t}{1+(\gamma-1)\tau}$$
$$c_T^o = \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^o\right) + \frac{\theta^m}{1+(\gamma-1)\tau} z^m + \frac{\bar{\lambda}_{T,1} - \lambda_{T,1}^o}{1+(\gamma-1)\tau}$$
$$c_N^n = y_N^n \quad \forall n$$

where $\bar{\lambda}_{T,1} = \sum_n \theta^n \lambda^n_{T,1}$. In addition, we have the equilibrium prices:

$$
\begin{aligned}
q &= -(1-\tau)(\gamma-1)\bar{y}_N - \theta^m z - \bar{\lambda}_{T,1} \\
p^m_N &= \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma}{1+(\gamma-1)\tau}y^m_N + \frac{\theta^m+(\gamma-1)\tau}{1+(\gamma-1)\tau}z^m + \frac{\bar{\lambda}_{T,1}-\lambda^m_{T,1}}{1+(\gamma-1)\tau} + \log[\frac{1-\tau}{\tau}] \\
p^t_N &= \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma}{1+(\gamma-1)\tau}y^t_N + \frac{\theta^m}{1+(\gamma-1)\tau}z^m + \frac{\bar{\lambda}_{T,1}-\lambda^t_{T,1}}{1+(\gamma-1)\tau} + \log[\frac{1-\tau}{\tau}] \\
p^o_N &= \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma}{1+(\gamma-1)\tau}y^o_N + \frac{\theta^m}{1+(\gamma-1)\tau}z^m + \frac{\bar{\lambda}_{T,1}-\lambda^o_{T,1}}{1+(\gamma-1)\tau} + \log[\frac{1-\tau}{\tau}]
\end{aligned}
$$

It is also useful to keep track of the shadow prices of total consumption in each country. To this end, we also use the log-linear expression $\lambda^n = -\gamma\left(\tau c^n_T + (1-\tau)c^n_N\right)$ along with the solutions for traded and nontraded consumption above to obtain:

$$
\begin{aligned}
\lambda^m &= -\frac{(\gamma-1)(1-\tau)\gamma\tau}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma(1-\tau)}{1+(\gamma-1)\tau}y^m_N + \frac{(1-\theta^m)\gamma\tau}{1+(\gamma-1)\tau}z^m - \frac{\gamma\tau\left(\bar{\lambda}_{T,1}-\lambda^m_{T,1}\right)}{1+(\gamma-1)\tau} \\
\lambda^t &= -\frac{(\gamma-1)(1-\tau)\gamma\tau}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma(1-\tau)}{1+(\gamma-1)\tau}y^t_N + \frac{\theta^m\gamma\tau}{1+(\gamma-1)\tau}z^m - \frac{\gamma\tau\left(\bar{\lambda}_{T,1}-\lambda^t_{T,1}\right)}{1+(\gamma-1)\tau} \\
\lambda^o &= -\frac{(\gamma-1)(1-\tau)\gamma\tau}{1+(\gamma-1)\tau}\bar{y}_N - \frac{\gamma(1-\tau)}{1+(\gamma-1)\tau}y^o_N + \frac{\theta^m\gamma\tau}{1+(\gamma-1)\tau}z^m - \frac{\gamma\tau\left(\bar{\lambda}_{T,1}-\lambda^o_{T,1}\right)}{1+(\gamma-1)\tau}
\end{aligned}
$$

Finally, the following log-linear equations determine the first-period Lagrange multipliers. Again, recall the stabilizing country's government uses $\bar{Z}^m$ to add and subtract resources from the economy to achieve (P2), which equalizes the marginal utility of initial wealth across households. As a result:

$$\lambda^m_{T,1} = \lambda^t_{T,1} = \lambda^o_{T,1} = \lambda_{T,1},$$

so that the remaining endogenous term drop out of the solutions above, $\bar{\lambda}_{T,1} - \lambda^m_{T,1} = 0$. Next, we normalize $\lambda_{T,1}$ using the second-order approximation of equation (34):

$$\lambda_{T,1} = \mathbb{E}\left[\lambda_T\right] + \frac{1}{2}var\left[\lambda_T\right].$$

B.4 Equilibrium Asset Portfolio

In the log-linear solution, all prices and quantities are a linear combination of $\{y^m_N, y^t_N, y^o_N\}$. In particular, household expenditure, $p^n + c^n$, in each state of the world is a linear combination of $\{y^m_N, y^t_N, y^o_N\}$. All asset payoffs are also linear combinations of $\{y^m_N, y^t_N, y^o_N\}$. Any set of assets with the same rank as the set of household expenditures will thus be able to span the space of household expenditure. Therefore, given the appropriate set of assets, we can write household expenditure in each state of the world as a linear combination of these assets.

It is straightforward to verify that the set of log-linear stock payoffs spans the space of log-linear household wealth.

Lemma 2

Households in the freely floating exchange rate equilibrium hold levered positions in their own country's stocks and hold short positions in other countries' stocks,

$$A_n^n = \frac{1-\theta^n\tau}{1-\tau} \text{ and } A_l^n = -\frac{\theta^n\tau}{1-\tau} \text{ for } l \neq n$$

Proof. The household budget constraint (27) can be re-written as:

$$P^n(\omega)C^n(\omega) = \sum_{l=m,t,o} A_l^n P_N^n(\omega)Y_N^n(\omega) + Y_T^n$$

The log-linear approximation of household expenditure of the left-hand side is:

$$\frac{1}{\tau}\left(p^n + c^n + \log\left[\tau\right]\right) = \frac{(\gamma-1)(1-\tau)}{\tau\left(1-(\gamma-1)\tau\right)}\left(\bar{y}_N - y_N^n\right)$$

The log-linear approximation the stock portfolio payoff (right-hand side) is:

$$\sum_{l=m,t,o} A_l^n \frac{1-\tau}{\tau}\left(p_N^{l*} + y_N^l - \log\left[\frac{1-\tau}{\tau}\right]\right).$$

where:

$$p_N^{l*} + y_N^l = \frac{(\gamma-1)(1-\tau)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^l\right) + \log\left[\frac{1-\tau}{\tau}\right].$$

We equate household expenditures in each state of the world with the portfolio payoff:

$$\frac{(\gamma-1)(1-\tau)}{(1-(\gamma-1)\tau)}\left(\bar{y}_N - y_N^n\right) = \sum_{l=m,t,o} A_l^n \frac{1-\tau}{\tau}\left(p_N^{l*} + y_N^l - \log\left[\frac{1-\tau}{\tau}\right]\right)$$

Since this equation holds state-by-state, we solve for the shares, A_l^n, by matching the coefficients on y_N^l in the portfolio payoff with the coefficients on y_N^l in household expenditure. ■

B.5 Derivation of Equation (19)

To derive (19), we use the following second-order approximation of equation (13):

$$\lambda_{T,1} + q_K + k^n = \log[\nu] + \mathbb{E}\left[\lambda_N^n + y_N^n\right] + \frac{1}{2}\text{var}\left(\lambda_N^n + y_N^n\right),$$

Next, we substitute $\lambda_N^n = p_N^n + \lambda_T$, and take differences across two arbitrary countries f and h to obtain:

$$k^f - k^h = \frac{1}{2}\text{var}\left(p_N^f + y_N^f\right) - \frac{1}{2}\text{var}\left(p_N^h + y_N^h\right) + \text{cov}\left(p_N^f + y_N^f - p_N^h - y_N^h, \lambda_T\right). \tag{37}$$

For any country n:

$$p_N^{n*} + y_N^{n*} = \frac{(1-\tau)(\gamma-1)}{1+(\gamma-1)\tau}\left(\bar{y}_N - y_N^n\right).$$

Plugging this expression for $p_N^{n*} + y_N^{n*}$ into the right-hand side of equation (37) shows:

$$k^{f*} - k^{h*} = \frac{(\gamma-1)^3(1-\tau)^2\tau}{1+(\gamma-1)\tau}\left(\theta^f - \theta^h\right)\sigma_N^2.$$

Combine this equation with equation (18) to derive (19),

B.6 Proof of Lemma 1

First, we solve for the state-contingent taxes that implement the real exchange rate stabilization. Afterwards, we derive an expression for the cost of stabilizing the exchange rate. We guess a tax of the form $Z(\omega) = \left(Y_N^m(\omega)/Y_N^t(\omega)\right)^a$ stabilizes the exchange rate for a constant a and solve for the coefficient a that stabilizes the real exchange rate. In logs, this tax is: $z = a\left(y_N^t - y_N^m\right)$. We plug this expression into the solution of the model derived in Appendix B.3 and solve for the log real exchange rate:

$$s^{m,t} = \frac{\gamma(1-\tau)}{1+(\gamma-1)\tau}\left(y_N^t - y_N^m\right) + a\frac{\gamma\tau}{1+(\gamma-1)\tau}\left(y_N^m - y_N^t\right).$$

Choose a such that $s^{m,t} = (1-\zeta)s^{m,t*}$. This yields $a = \zeta(1-\tau)/\tau$. Finally, we use the expression for $s^{f,h*}$ given by equation (17) to write z as a function of p^{m*} and p^{t*}.

ΔRes is defined by equation (6). First, we solve for $\bar{Z}$ by plugging in the equilibrium consumption of nontraded goods and $K^m = 1$ into the budget constraint (28):

$$\bar{Z} = (A_m^m - 1)\, Q_N^m + \sum_{l \neq m} A_l^m Q_N^l - \kappa^m.$$

Next, multiply equation (27) by the stochastic discount factor, $\Lambda_T(\omega)/\Lambda_{T,1}$, and take expectations to derive the present value of tax revenues:

$$\begin{aligned}
&\mathbb{E}\left[\left(\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\right)(Z(\omega) - 1)\, C_T^m(\omega)\right] \\
=&\mathbb{E}\left[\left(\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\right)\left((A_m^m - 1)\, P_N^m(\omega) Y_N^m(\omega) + \sum_{l \neq m} A_l^m P_N^m(\omega) Y_N^m(\omega) + Y_T^n - C_T^m(\omega)\right)\right] \\
=&\,(A_m^m - 1)\, Q_N^m + \sum_{l \neq m} A_l^m Q_N^l + Y_T^m \mathbb{E}\left[\left(\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\right)\right] - \mathbb{E}\left[\left(\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\right) C_T^m(\omega)\right]
\end{aligned}$$

Finally, we derive κ^m. In the freely floating exchange rate economy, $\bar{Z} = 0$ and $Z(\omega) = 1$. Plug these values into equations (27) and (28). Finally, we substitute equation (34) and use the fact that Y_T^m is a constant to show:

$$\kappa^m = \mathbb{E}\left[\left(\frac{\Lambda_T^*(\omega)}{\Lambda_{T,1}^*}\right) C_T^{m*}(\omega)\right] - Y_T^m \tag{38}$$

We plug the expressions for $\bar{Z}$, the present value of tax revenues, and κ^m in equation (6), and simplify to arrive at equation (20).

We also derive the portfolio of stocks that exactly finances the stabilization policy. This is the portfolio that pays the difference between traded consumption when stabilizing and traded consumption in the freely floating regime given by equation (21). For convenience, this equation is repeated here where $p^{t*} - p^{m*}$ is written in terms differences in nontraded output:

$$c_T^m - c_T^{m*} = \zeta \frac{(1 - \theta^m)(1 - \tau)}{\tau\,(1 + (\gamma - 1)\tau)} \left(y_N^t - y_N^m\right).$$

We use the same log-linear approximation of the stock portfolio as in Appendix B.4. Letting A_l^m denote the number of shares of country l stock the stabilizing country's central bank holds, we get

$$A_m^m = \zeta \frac{1 - \theta^m}{\gamma - \zeta(\gamma - 1)(1 - \tau)},\ A_t^m = -A_m^m,\ A_o^m = 0.$$

B.7 Proof of Proposition 1

We use the expressions from Appendix B.3 to calculate $p^t - p^m = \lambda^t - \lambda^m$ and we plug the resulting expression into equation (2):

$$\begin{aligned} r^m + \Delta\mathbb{E}s^{m,t} - r^t &= \operatorname{cov}\left(\lambda_T, p^t - p^m\right) \\ &= \left(r^{m*} + \Delta\mathbb{E}s^{m,t*} - r^{t*}\right) - \zeta\frac{(1-\tau)^2\gamma\left(2\theta^m(1-\zeta) + \left(\theta^t - \theta^m\right)(\gamma-1)\tau\right)}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2. \end{aligned}$$

When the stabilizing country is smaller than the target country, $\theta^m < \theta^t$, the right-hand side of this expression implies the stabilization decreases the risk-free rate in the stabilizing country relative to the risk-free rate in the target country.

We use equation (37) to calculate the differential incentives to accumulate capital:

$$k^m - k^t = k^{m*} - k^{t*} + \zeta\left(\frac{(\gamma-1)^2(1-\tau)^2\left((1-2\theta^m)(1-\zeta) + (\theta^t - \theta^m)(\gamma-1)\tau\right)}{\left(1+(\gamma-1)\tau\right)^2}\right)\sigma_N^2.$$

The last term of the right-hand side of this expression shows that incentives to accumulate capital in the stabilizing country increase relative to the target country as long as

$$\theta^t > \theta^m + \frac{(1-2\theta^m)(1-\zeta)}{\tau(\gamma-1)}.$$

Because firms are competitive, wages are given by the marginal product of labor. $w^n = (1-\nu)\exp\left(\eta^n\right)\left(K^n\right)^\nu$. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate stabilization increases wages in the stabilizing country relative to all other countries.

Recall, the world-market value of the country m domestic firm given by equation (31) is:

$$Q_N^m = \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}P_N^m Y_N^m\right].$$

The second-order log-linear approximation of the world-market value of the country m domestic firm is:

$$q_N^m = \mathbb{E}\left[\lambda_T - \lambda_{T,1} + p_N^m + y_N^m\right] + \frac{1}{2}var\left[\lambda_T - \lambda_{T,1} + p_N^m + y_N^m\right].$$

The spread between the value of the firm in the stabilizing and target countries yields the same expression as the right-hand side of equation (37). Hence, we have already shown the value of the firm in the stabilizing country increases relative to the target country if θ^t is large enough.

B.8 Proof of Proposition 2

Equation (20) shows the cost of the stabilization is the difference in the value of traded consumption between the freely floating regime and stabilized regime. We derive a second-order log-linear approximation of the value of traded consumption:

$$v_T^m = \mathbb{E}\left[\lambda_T - \lambda_{T,1} + c_T^m\right] + \frac{1}{2}\text{var}\left[\lambda_T - \lambda_{T,1} + c_T^m\right].$$

We plug the expressions for λ_T, $\lambda_{T,1}$, and c_T^m into the previous equation in order to derive the change in the log value of traded consumption

$$v_T^m - v_T^{m*} = \frac{\left((\zeta + (\gamma-1)\tau) - \tau^2(1-\gamma)^2\theta^t\right)(1-\tau)^2\zeta\sigma_N^2}{\tau^2\left(1+(\gamma-1)\tau\right)^2}.$$

This expression is decreasing in the size of the target country, and becomes negative if and only if the target country is large enough: $\theta^t > (\zeta + (\gamma-1)\tau) / (\tau(\gamma-1))^2$.

Next, we evaluate the derivative of $v_T^m - v_T^{m*}$ with respect to θ^m at the point where $\theta^m = 0$:

$$\left.\frac{\partial(v_T^m - v_T^{m*})}{\partial\theta^m}\right|_{\theta^m=0} = \zeta\frac{(\gamma-1)(1-\tau)^2\left(\theta^t + 2\zeta + 2(1+\theta^t)(\gamma-1)\tau\right)}{\tau\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2 > 0$$

Hence, the cost of the stabilization increases locally with the size of the stabilizing country.

B.9 Proof of Proposition 3

We use the expressions from Appendix B.3 to calculate $p^o - p^t = \lambda^o - \lambda^t$ and we plug the resulting expression into equation (2):

$$r^t + \Delta\mathbb{E}s^{t,o} - r^o = \text{cov}\left(\lambda_T, p^o - p^t\right) = \left(r^{t*} + \Delta\mathbb{E}s^{t,o*} - r^{o*}\right) + \zeta\frac{\theta^m(1-\tau)^2\gamma}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2,$$

which implies the exchange rate stabilization increases the risk-free rate in the target country relative to the risk-free rate in the outside country.

We use equation (37) to calculate the differential incentives to accumulate capital,

$$k^t - k^o = k^{t*} - k^{o*} - \frac{\theta^m(\gamma-1)^2(1-\tau)^2}{\left(1+(\gamma-1)\tau\right)^2}\zeta\sigma_N^2$$

The last term on the right-hand side shows that incentives to accumulate capital in the target country decrease relative to the outside country.

Because firms are competitive, wages are given by the marginal product of labor. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate stabilization decreases wages in the target country relative to all other countries.

Finally, we show that if the stabilizing country is smaller than the target country, $\theta^m < \theta^t$, then the stabilization lowers the volatility of consumption in the target country. The log-linear approximation of household consumption in the target country is, $c^t = \tau c_T^t + (1-\tau)c_N^t$. We use the expression for traded consumption derived in Appendix B.3 and the expression for the state-contingent tax derived in Appendix B.6 to derive the volatility of aggregate consumption in the target country:

$$\text{var}\left(c^t\right) = \text{var}\left(c^{t*}\right) - \zeta \frac{2\theta^m(1-\tau)^2\left(1-\theta^m\zeta + \left(\theta^t - \theta^m\right)(\gamma-1)\tau\right)}{\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2.$$

Therefore, $\text{var}\left(c^t\right)$ decreases when a country stabilizes its exchange rate relative to the target country as long as the stabilizing country is smaller, $\theta^t > \theta^m$.

C Appendix to Section 2.5

We derive the welfare consequences of stabilization when $\Delta Res = 0$ and households hold the portfolio of assets derived in Appendix B.4. Households continue to maximize utility subject to their budget constraints (27) and (28). However households initial wealth is:

$$W_0^n = \sum_{l\in\{m,t,o\}} A_l^{n*} Q_N^l + Q_K K_N^{n*}.$$

We plug this value of W_0^n into the budget constraint (28). Next, we multiply equation (27) by the stochastic discount factor and take expectations. After performing these calculations, the right-hand side of (27) is equal to the left-hand side of equation (28) plus the value of the traded endowment. Hence, we substitute (28) into (27) and subtract the value of nontraded consumption from both sides. We arrive at the following expression for the value of traded consumption in country n:

$$\mathbb{E}\left[\frac{\Lambda_T^n(\omega)}{\Lambda_{T,1}^n} C_T^n(\omega)\right] = \left(A_n^{n*} - 1\right) Q_N^n + \sum_{l\neq n} A_l^{n*} Q_N^l + Y_T^n. \tag{39}$$

The left-hand side represents the value of traded consumption when stabilizing. The right-hand side represents the household's wealth from its portfolio of stocks after subtracting out expenditure on nontraded consumption and capital investment.

We derive a second-order approximation for equation (39):

$$\mathbb{E}\left[\lambda_T - \lambda_{T,1} + c_T^n\right] + \frac{1}{2}\text{var}\left[\lambda_T - \lambda_{T,1} + c_T^n\right] = \frac{1-\tau}{\tau}\left(\left(A_n^{n*} - 1\right) q_N^n + \sum_{l \neq n} A_t^{l*} q_N^t\right) + q_T \tag{40}$$

where:

$$q_N^n = \mathbb{E}\left[\lambda_T - \lambda_{T,1} + p_N^n + y_N^n\right] + \frac{1}{2}\text{var}\left[\lambda_T - \lambda_{T,1} + p_N^n + y_N^n\right]$$

and

$$q_T^n = 0,$$

and the expressions for $p_N^n + y_N^n$ are given by equation (22). We solve for the Lagrange multipliers $\lambda_{T,1}^n$ that satisfy equation (40). Let us denote the set of Lagrange multipliers derived from solving equation (39) by $\lambda_{T,1,Stock}^n$. After solving the Lagrange multipliers, we obtain solutions for traded consumption by plugging the Lagrange multipliers, $\lambda_{T,1,Stock}^n$, into the log-linear expressions for c_T^n derived in Appendix B.3. These new expressions for traded consumption reflect the level shifts in traded consumption due to changes in the value of the household's stock portfolio.

To decompose changes in welfare, below, we compute traded consumption when the household's value of traded consumption post exchange rate stabilization exactly equals the value of traded consumption prior to the stabilization:

$$\mathbb{E}\left[\lambda_T - \lambda_{T,1} + c_T^n\right] + \frac{1}{2}\text{var}\left[\lambda_T - \lambda_{T,1} + c_T^n\right] = \mathbb{E}\left[\lambda_T^* - \lambda_{T,1}^* + c_T^{n*}\right] + \frac{1}{2}\text{var}\left[\lambda_T^* - \lambda_{T,1}^* + c_T^{n*}\right]. \tag{41}$$

Denote the Lagrange multipliers derived from solving equation (41) by $\lambda_{T,1,AD}^n$. Again, we plug these the Lagrange multipliers $\lambda_{T,1,AD}^n$ into the expressions from Appendix B.3 to derive expressions for traded consumption with stabilization, but without any change in the total value of traded consumption.

Next, we calculate changes in welfare using a second-order approximation of household utility. Utility in country n is:

$$u^n = \frac{1}{1-\gamma}\log\left[(1-\gamma)U^n\right] = \mathbb{E}[c^n] - \frac{\gamma-1}{2}\text{var}[c^n] \tag{42}$$

where $c^n = \tau c_T^n + (1-\tau)c_N^n$. We plug in the solutions for c_T^n, with the Lagrange multipliers derived above, into the welfare function. Define the welfare change $\Delta u^n = u^n - u^{n*}$, where u^{n*} is the value of u^n when $\zeta = 0$. The welfare change in the stabilizing country is:

$$
\begin{aligned}
\Delta u^m =& \frac{\zeta(\gamma-1)^2(\theta^m-1)(\tau-1)^2\tau((\gamma-1)\tau(\theta^m-\theta^t)+1)}{(1+(\gamma-1)\tau)^2}\sigma_N^2 \\
&+ \frac{\zeta^2(1-\tau)^2\left((\gamma-1)\left((\theta^m)^2-1\right)\tau+(\gamma-1)^2(\theta^m-1)(2\theta^m-1)\tau^2+\theta^m-1\right)}{\tau(1+(\gamma-1)\tau)^2}\sigma_N^2
\end{aligned}
$$

Equation (23) displays the welfare consequences for a small stabilizing country ($\theta^m = 0$).

The first term of (23) is calculated by plugging the expression for c_T^n with the Lagrange multipliers $\Lambda_{T,1,AD}^n$ into c^n and deriving the change in $\mathbb{E}\left[c^n\right]$ when ζ deviates from zero. The $\Delta\text{var}\left[c^m\right]$ term reflects the change captured by $\frac{\gamma-1}{2}\text{var}\left[c^m\right]$. The "Valuation Effect" is calculated by plugging the expression for c_T^n with the Lagrange multipliers $\Lambda_{T,1,Stock}^n$ into c^n, deriving the change in $\mathbb{E}\left[c^n\right]$ when ζ deviates from zero, and then subtracting out the first term of (23).

When the first term of (23) is combined with the "Valuation Effect":

$$
\Delta u^m = -\frac{\zeta^2(1-\tau)^2}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2 + \frac{\left(\zeta\Theta^t+\zeta^2\right)\theta^t\tau(\gamma-1)^2(1-\tau)^2}{\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2
$$

The first term on the right-hand side is clearly negative, which indicates the welfare losses from the increase in consumption volatility are larger than any gains from accumulating reserves.

Finally, equation (23) can be condensed to:

$$
\Delta u^m = \zeta\frac{(1-\tau)^2\left(-\zeta\left(1+(\gamma-1)\tau\right)+(\theta^t(\gamma-1)\tau-1+\zeta)(\gamma-1)^2\tau^2\right)}{\tau\left(1+(\gamma-1)\tau\right)^2}\sigma_N^2.
$$

The right-hand side of this equation is positive if:

$$
\theta^t > \bar{\theta} = \frac{1-\zeta}{(\gamma-1)\tau} + \frac{\zeta(1+(\gamma-1)\tau)}{(\gamma-1)^3\tau^3}.
$$

C.1 Equilibrium Bond Portfolio

Suppose households are confined to trading international risk-free bonds rather than stocks. The country n risk-free bond pays $P^n(\omega)$ units of the traded good in state ω of period 2. Similar to the exercise in Appendix B.4, these asset payoffs are linear combinations of the nontraded output in each country. Likewise, it is straightforward to verify the set of log-linear bond payoffs spans the space of log-linear household wealth. As a result, equilibrium outcomes in the economy are unaffected by the change in the asset space. We just need to solve for the household bond portfolios that pay the appropriate payoff in each state in the second period.

Let B_l^n denote the number of country l bonds purchased by households in country n. Hence, the log-linear approximation of the payoff received from the portfolio held by country n households is:

$$\sum_{l=m,t,o} B_l^n \frac{1}{\tau} p^l$$

Again, we solve for the portfolio weights, B_l^n, by matching the coefficients on y_N^l in the portfolio payoff with the coefficients on y_N^l in household expenditure. This procedure yields the following result:

$$B_n^n = \frac{(1-\theta^n)(\gamma-1)}{\gamma} \text{ and } B_l^n = -\frac{\theta^l(\gamma-1)}{\gamma} \text{ for } l \neq n.$$

Households thus hold levered positions in their domestic risk-free bond. Proposition 1 shows the stabilizing country's risk-free rate decreases when the target country is larger than the stabilizing country, increasing the relative value of its bonds. As a result, the same intuition from Proposition 4 shows that announcing a stabilization relative to a larger country increases the stabilizing country's share of world wealth and thus, by the same logic, can increase the welfare of its households.

C.2 Welfare Consequences in Target and Outside Countries

In this appendix, we provide expressions for the welfare consequences of stabilization on households in the target and outside countries. Analogous to the calculation of Δu^m, we plug the Lagrange multipliers derived in Appendix C into the expression of c_T^t and c_T^o derived in Appendix B.3. We again plug the value of c_T^t into the second-order approximation of household welfare given by equation (42):

$$\begin{aligned}\Delta u^t = &\frac{\zeta^2\theta^m(1-\tau)^2((\gamma-1)\tau((\gamma-1)(2\theta^m-1)\tau+\theta^m)+1)}{\tau(1+(\gamma-1)\tau)^2}\sigma_N^2 \\ &+\frac{(\gamma-1)\zeta\theta^m(1-\tau)^2((\gamma-1)\tau((\gamma-1)\tau(\theta^m-\theta^t)+1)+2)}{((\gamma-1)\tau+1)^2}\sigma_N^2.\end{aligned}$$

The analogous calculation for the outside country yields:

$$\Delta u^o = \Delta u^t - \frac{\zeta\theta^m(\gamma-1)(1-\tau)^2}{(1+(\gamma-1)\tau)^2}\sigma_N^2.$$

Households in the outside country are weakly worse off than households in the target country as a result of the stabilization.

D Appendix to Section 3

D.1 Sticky Prices

Households enter each period with a fixed quantity of the domestic currency, and all goods consumed in a given country must be purchased using the domestic currency. In the first period, households are endowed with a fixed amount of domestic currency that they use to purchase stocks. We can write the first-period budget constraint as:

$$\tilde{P}_{T,1}^{n}\left(\sum_{l} A_{l}^{n} Q_{N}^{l} + Q_{K} K_{N}^{n}\right) \leq \exp\left[\Delta m_{1}^{n}\right] \tilde{P}_{T,1}^{n}\left(Q_{N}^{n} + Q_{K} + \kappa^{n}\right).$$

In the second period, households face the cash constraint

$$\tilde{P}_{T,1}^{n} C_{T}^{n}(\omega) + \tilde{P}_{N}^{n}(\omega) C_{N}^{n}(\omega) \leq \exp\left[\Delta m^{n}(\omega)\right] \tilde{P}_{T,1}^{n}\left(\sum_{l} A_{l}^{n} P_{N}^{l}(\omega) Y_{N}^{l}(\omega) + Y_{T}^{n}\right).$$

To reiterate, Δm_1^n and $\Delta m^n(\omega)$ denote the growth rate of money supply in the first period and in state ω of the second period, respectively. The central banks in the target and outside countries use their control of money supply to recover the efficient allocation of resources, taking as given the actions of the stabilizing country's central bank. By contrast, the central bank in the stabilizing country uses its control of monetary policy to stabilize the nominal exchange rate.

We divide both sides of the first-period budget constraint above by $\tilde{P}_{T,1}^{n}$ to recover

$$\sum_{l} A_{l}^{n} Q_{N}^{l} + Q_{K} K_{N}^{n} \leq \exp\left[\Delta m_{1}^{n}\right]\left(Q_{N}^{n} + Q_{K} + \kappa^{n}\right). \tag{43}$$

We divide through the second-period budget constraint above by $\exp\left[\Delta m^{n}(\omega)\right] \tilde{P}_{T,1}^{n}$ to recover

$$\exp\left[-\Delta m^{n}(\omega)\right] C_{T}^{n}(\omega) + \exp\left[-\Delta m^{n}(\omega)\right] \frac{\tilde{P}_{N}^{n}(\omega)}{\tilde{P}_{T,1}^{n}} C_{N}^{n}(\omega) \leq \sum_{l} A_{l}^{n} P_{N}^{l}(\omega) Y_{N}^{l}(\omega) + Y_{T}^{n}. \tag{44}$$

In the second period, households maximize utility (4) subject to (44). Letting $\tilde{\Lambda}_T(\omega)$ denote the Lagrange multiplier on this budget constraint, the first-order conditions with respect to traded and nontraded consumption are:

$$\tau \left(C^n(\omega)\right)^{1-\gamma} \left(C_T^n(\omega)\right)^{-1} = \exp\left[-\Delta m^n(\omega)\right] \tilde{\Lambda}_T^n(\omega)$$
$$(1-\tau) \left(C^n(\omega)\right)^{1-\gamma} \left(C_N^n(\omega)\right)^{-1} = \exp\left[-\Delta m^n(\omega)\right] \frac{\tilde{P}_N^n(\omega)}{\tilde{P}_{T,1}^n} \tilde{\Lambda}_T^n(\omega).$$

Next, we derive the Euler equations for investment. Since utility (4) is Cobb-Douglas, consumption in country n is described by:

$$C_T^n(\omega) = \tau \left(\exp\left[\Delta m^n(\omega)\right] \left(\sum_l A_l^n P_N^l(\omega) Y_N^l(\omega) + Y_T^n \right) \right)$$
$$C_N^n(\omega) = (1-\tau) \left(\frac{\exp\left[\Delta m^n(\omega)\right] \tilde{P}_{T,1}^n \left(\sum_l A_l^n P_N^l(\omega) Y_N^l(\omega) + Y_T^n \right)}{\tilde{P}_N^n(\omega)} \right).$$

We plug these expressions into equation (4) and take first-order conditions with respect to A_l^n, subject to (43). Let $\tilde{\Lambda}_{T,1}^n$ represent the Lagrange multiplier on the household's first-period budget constraint. The first-order conditions with respect A_1^n determine the prices of stock in each country:

$$\tilde{\Lambda}_{T,1}^n Q_N^l = \mathbb{E}\left[\frac{\tilde{P}_{T,1}^n \left(C^n(\omega)\right)^{-\gamma}}{\tilde{P}^n(\omega)} P_N^l(\omega) Y_N^l(\omega) \right]$$

Next, we show that under the appropriate monetary policy, the central bank in the stabilizing country can replicate the equilibrium from the baseline model. Suppose the central banks set $\Delta m^m(\omega) = z^m(\omega)$, $\Delta m_1^m = 1 + \bar{Z}^m / (Q_N^m + Q_K + \kappa^n)$, $\Delta m^t(\omega) = \Delta m^o(\omega) = 0$ and $\Delta m_1^t = \Delta m_1^o = 1$. We plug these expressions for monetary policy into the first-order conditions above. The first-order conditions with respect to traded consumption show $\tilde{\Lambda}_T^n(\omega)$ coincides with the shadow price of traded consumption from Appendix B.1, $\tilde{\Lambda}_T^n(\omega) = \Lambda_T(\omega)$. Moreover, letting $\Lambda_N^n(\omega)$ represent the shadow price of nontraded consumption reveals relative (real) price of nontraded consumption is defined by:

$$P_N^n(\omega) = \frac{\Lambda_N^n(\omega)}{\Lambda_T(\omega)} = \exp\left[-\Delta \tilde{m}^n(\omega)\right] \frac{\tilde{P}_N^n(\omega)}{\tilde{P}_{T,1}^n}.$$

As a result, we have shown that with the appropriate monetary policy, the first-order conditions in coincide with those from Appendix B.1. Hence, the allocation of goods must also be the same.

Next, we derive an expression for seigniorage and relate seigniorage to ΔRes, given by equation (20). The central bank earns seigniorage by changing the money supply. The net present value of seigniorage is:

$$\text{seigniorage} = -\left(\frac{\Delta\tilde{M}_1^n}{\tilde{P}_{T,1}^n}\right) - \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\left(\frac{\Delta\tilde{M}^n(\omega)}{\tilde{P}_{T,1}^n}\right)\right],$$

where

$$\Delta\tilde{M}_1^n = \left(\exp\left[\Delta\tilde{m}_1^n\right] - 1\right)\tilde{P}_{T,1}^n\left(Q_N^n + Q_K + \kappa^n\right) \tag{45}$$

and

$$\Delta\tilde{M}^n(\omega) = \left(\exp\left[\Delta\tilde{m}^n(\omega)\right] - 1\right)\tilde{P}_{T,1}^n\left(\sum_l A_l^n P_N^l(\omega) Y_N^l(\omega) + Y_T^n\right) \tag{46}$$

are the level changes in the money supply in country n.

Next, we use the budget constraints (43) and (44) to re-write the expression for seigniorage as a function of household consumption. Plugging (44) into (46) shows:

$$\begin{aligned}\frac{\Delta\tilde{M}^n(\omega)}{\tilde{P}_{T,1}^n} &= C_T^n(\omega) + P_N^l(\omega)C_N^n(\omega) - \sum_l A_l^n P_N^l(\omega)Y_N^l(\omega) - Y_T^n \\ &= C_T^n(\omega) - \left(A_n^n - 1\right)P_N^l(\omega)Y_N^n(\omega) - \sum_{l\neq n} A_l^n P_N^l(\omega)Y_N^l(\omega) - Y_T^n.\end{aligned}$$

The second equality comes from plugging in the equilibrium condition $C_N^n = Y_N^n$. We multiply this result with the stochastic discount factor, $\Lambda_T(\omega)/\Lambda_{T,1}$, and take expectations:

$$\mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\left(\frac{\Delta\tilde{M}^n(\omega)}{\tilde{P}_T^n(\omega)}\right)\right] = \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}C_T^n(\omega)\right] - \left(A_n^n - 1\right)Q_N^n - \sum_{l\neq n} A_l^n Q_N^l - Y_T^n\mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\right].$$

Plugging equation (43) into equation (45) shows:

$$\begin{aligned}\frac{\Delta\tilde{M}_1^n}{\tilde{P}_{T,1}^n} &= \sum_l A_l^n Q_N^l + Q_K - Q_N^n - Q_K - \kappa^n \\ &= \left(A_n^n - 1\right)Q_N^n + \sum_{l\neq n} A_l^n Q_N^l - \kappa^n.\end{aligned}$$

Combining the two previous expressions yields:

$$\left(\frac{\Delta\tilde{M}_1^n}{\tilde{P}_{T,1}^n}\right) + \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\left(\frac{\Delta\tilde{M}^n(\omega)}{\tilde{P}_T^n}\right)\right] = \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}C_T^n(\omega)\right] - Y_T^n\mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\right] - \kappa^n.$$

We plug in the definition of κ^m given by equation (38) to show:

$$\begin{aligned}
\text{seigniorage} &= -\left(\frac{\Delta \tilde{M}_1^n}{\tilde{P}_{T,1}^n}\right) - \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\left(\frac{\Delta \tilde{M}^n(\omega)}{\tilde{P}_T^n(\omega)}\right)\right] \\
&= \mathbb{E}\left[\frac{\Lambda_T^*(\omega)}{\Lambda_{T,1}^*}C_T^{n*}(\omega)\right] - \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}C_T^n(\omega)\right] \\
&= -\Delta Res.
\end{aligned}$$

D.2 Model with Segmented Markets and Cash-In-Advance Constraint

This appendix analyzes an alternative monetary friction where prices are flexible and monetary policy affects real allocations because financial markets are segmented (Alvarez et al., 2002). The key takeaway from this exercise is that, even with this alternate type of monetary friction, a simple nominal stabilization can implement a real stabilization of the type discussed in Section 2 of the main text.

Each country has a central bank that issues a national currency. All goods must be paid for in the domestic currency of the country from which they originate. All households face a cash-in-advance constraint, and all prices are flexible. Within each country, only a fraction ϕ of households can trade in the international stock market, label these households 'active.' The remaining $1-\phi$ of households do not have access to financial markets. The central banks in the target and outside countries use their control of the money supply to recover the efficient allocation of resources, taking as given the actions of the stabilizing country's central bank. By contrast, the central bank in the stabilizing country uses its control of monetary policy to stabilize the nominal exchange rate.

In the second period, the cash-in-advance constraint for active households is:

$$\tilde{P}_T^n(\omega)C_T^n(\omega) + \tilde{P}_N^n(\omega)C_N^n(\omega) \leq \tilde{M}_1^n + \tilde{P}_T^n(\omega)\left(\sum_l A_l^n P_N^l(\omega)Y_N^l(\omega) + Y_T^n\right). \tag{47}$$

where $\tilde{P}_T^n$ is the nominal price of the traded good in country n and $\tilde{M}_1^n$ is the nominal money holding of the active household in terms of the national currency of its home country n that is carried over from the first period. Since inactive households do not have access to financial markets, their cash in advance constraint in period 2 is:

$$\tilde{P}_T^n(\omega)\hat{C}_T^n(\omega) + \tilde{P}_N^n(\omega)\hat{C}_N^n(\omega) \leq \hat{M}_1^n,$$

where $\hat{M}_1^n$ is the cash holding of an inactive households carried over from the first period. All households within a given country start the first period with identical cash holdings, $\tilde{M}_0^n$. The first-period constraint for active households is

$$\tilde{M}_1^n + \tilde{P}_{T,1}^n \left(\sum_l A_l^n Q_N^l + Q_K K_N^n \right) \leq \tilde{P}_{T,1}^n \left(Q_N^n + Q_K + \kappa^n \right) + \tilde{M}_0^n, \tag{48}$$

and the first-period constraint for inactive households is

$$\hat{M}_1^n \leq \tilde{P}_{T,1}^n \left(Q_N^n + Q_K + \hat{\kappa}^n \right) + \hat{M}_0^n.$$

The assumption that all goods must be paid for in the domestic currency from which they originate implies the money market clearing condition:

$$\tilde{P}_T^n(\omega) Y_T^n + \tilde{P}_N^n(\omega) Y_N^n = \bar{M}^n(\omega) \tag{49}$$

where $\bar{M}^n = \phi \tilde{M}^n + (1-\phi)\hat{M}^n$ is the aggregate money supply in country n. The central bank changes the monetary base in the second period through open market operations in the stock market:

$$\phi \tilde{P}_T^n(\omega) \left(\sum_l A_l^n P_N^l(\omega) Y_N^l(\omega) + Y_T^n \right) = \bar{M}_2^n - \bar{M}_1^n = \phi \left(\tilde{M}_2^n - \tilde{M}_1^n \right). \tag{50}$$

Inactive households split their supply of money between traded and nontraded goods. Their consumption is:

$$\hat{C}_T^n(\omega) = \tau \frac{\hat{M}_1^n}{\tilde{P}_T^n(\omega)}, \text{ and } \hat{C}_N^n(\omega) = (1-\tau) \frac{\hat{M}_1^n}{\tilde{P}_N^n(\omega)}.$$

Because prices are flexible, changes in the money supply affect the equilibrium allocation in this economy only because it affects the real purchasing power of these inactive households. That is, the central bank can affect the allocation by increasing or decreasing the purchasing power of these households. Define the shock to the real purchasing power of inactive households in country n, controlled by country n's central bank as

$$\exp(-\mu^n) = \frac{1}{P^n(\omega)} \frac{\hat{M}_1^n}{\tilde{P}_T^n(\omega)}, \tag{51}$$

so that a high μ corresponds to an expansionary monetary policy, higher inflation, and lower purchasing power of inactive households.

Given this definition, the consumption of inactive households can be re-written as:

$$\hat{C}_T^n(\omega) = \tau \exp(-\mu^n) P^n(\omega), \text{ and } \hat{C}_N^n(\omega) = (1-\tau)\exp(-\mu^n) P^n(\omega) \frac{\tilde{P}_T^n(\omega)}{\tilde{P}_N^n(\omega)}$$

Active households maximize their expected utility subject to their budget constraints (47) and (48), as well as the consumption of inactive households. We derive first-order conditions and log-linearize around the deterministic equilibrium. The real exchange rate between the stabilizing country and target country is:

$$s^{p,t} = \frac{\gamma(1-\tau)}{\gamma\tau + \phi(1-\tau)} \left(y_N^t - y_N^p\right) + \frac{\gamma(1-\tau)(1-\phi)}{\gamma\tau + \phi(1-\tau)} \left(\mu^t - \mu^p\right)$$

A positive μ^n (high inflation) shifts resources to the active households in country n and depreciates the stabilizing country's real exchange rate.

The real exchange rate under the freely floating regime is:

$$s^{m,t*} = \frac{\gamma(1-\tau)}{\gamma\tau + \phi(1-\tau)} \left(y_N^t - y_N^m\right)$$

and the variance of this exchange rate is:

$$\text{var}\left[s^{m,t*}\right] = \frac{2\gamma^2(1-\tau)^2}{\left(\gamma\tau + \phi(1-\tau)\right)^2} \sigma_N^2$$

The stabilizing country imposes a real exchange rate stabilization of strength ζ by choosing:

$$\begin{aligned} \mu^m &= \zeta \frac{1}{1-\phi} \left(y_N^t - y_N^m\right) \\ &= \zeta \frac{\gamma\tau + \phi(1-\tau)}{\gamma(1-\tau)(1-\phi)} \left(p^{m*} - p^{t*}\right) \end{aligned} \tag{52}$$

When the target country appreciates, the stabilizing country lowers its own inflation (deflates) to match the appreciation. Lower inflation shifts resources from the active household towards the inactive household, which increases the marginal utility of active households and thus the real price level in the stabilizing country. As a result, we can also recover the relationship:

$$p^m = p^{m*} + (1-\theta^m)\zeta(p^{t*} - p^{m*}).$$

Next, we solve for the monetary policy that enforces a nominal exchange rate stabilization.

The nominal exchange rate in this economy is equal to the real exchange rate plus inflation:

$$\tilde{s}^{m,t} = p^m + \mu^m - p^t - \mu^t$$

$\mu^t = \mu^o = 0$ by assumption. However, the nominal exchange rate is affected by monetary policy through μ^m. We solve for the monetary policy that implements a nominal exchange rate stabilization of strength $\tilde{\zeta}$:

$$\mu^m = \frac{\tilde{\zeta}\gamma(1-\tau)}{\gamma(1-\phi)(1-\tau) - (\gamma\tau + (1-\tau)\phi)} \left(y_N^t - y_N^m\right). \tag{53}$$

$$= \frac{\tilde{\zeta}\left(\gamma\tau + \phi(1-\tau)\right)}{\gamma(1-\phi)(1-\tau) - (\gamma\tau + (1-\tau)\phi)} \left(p^{m*} - p^{t*}\right) \tag{54}$$

Under this policy, the stabilizing country's real exchange rate is:

$$s^{m,t} = \left(1 - \frac{\tilde{\zeta}\gamma(1-\tau)(1-\phi)}{\gamma(1-\phi)(1-\tau) - (\gamma\tau + (1-\tau)\phi)}\right) s^{m,t*}$$

Hence, a policy that implements a nominal stabilization of strength of $\tilde{\zeta}$ will implement a real stabilization of strength:

$$\zeta = \tilde{\zeta}\frac{\gamma(1-\tau)(1-\phi)}{\gamma(1-\phi)(1-\tau) - (\gamma\tau + (1-\tau)\phi)}$$

If $\gamma(1-\phi)(1-\tau) > (\gamma\tau + (1-\tau)\phi)$, then a nominal stabilization implements a stronger real stabilization.

Seigniorage is a function of the present discounted value of the change in the money supply in both periods:

$$\text{seigniorage} = -\frac{\bar{M}_1^n - \bar{M}_0^n}{\tilde{P}_{T,1}^n} - \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\left(\frac{\bar{M}_2^n(\omega) - \bar{M}_1^n}{\tilde{P}_T^n(\omega)}\right)\right].$$

Following the same calculations as in Appendix D, we can show:

$$\begin{aligned}\text{seigniorage} = \mathbb{E}&\left[\left(\frac{\Lambda_T^*(\omega)}{\Lambda_{T,1}^*}\right)\left(\phi C_T^{m*}(\omega) + (1-\phi)\hat{C}_T^{m*}(\omega)\right)\right] \\ &- \mathbb{E}\left[\left(\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\right)\left(\phi C_T^{m}(\omega) + (1-\phi)\hat{C}_T^{m}(\omega)\right)\right]\end{aligned}$$

where asterisks denote an equilibrium in which the stabilizing country does not actively manipulate the variance of the exchange rate. In the segmented markets model, seigniorage is still equal to change in the value of traded consumption. However, seigniorage in the segmented markets model takes into account the consumption of both active and inactive households.

E Model Extensions

E.1 Partial exchange rate stabilization

This appendix formalizes the effects of partial exchange rate stabilization. In a first step, we use the partition defined in the main text to write the variance of exchange rates in the freely floating regime as

$$\begin{aligned}
\text{var}[s^{*m,t}] &= \int_{\Omega} \left(s^{*m,t} - \mathbb{E}[s^{*m,t}|\{K_n\}]\right)^2 g(\omega)d\omega \\
&= \int_{\Omega_s} \left(s^{*m,t} - \mathbb{E}[s^{*m,t}|\{K_n\}]\right)^2 g(\omega)d\omega + \int_{\Omega_{-s}} \left(s^{*m,t} - \mathbb{E}[s^{*m,t}|\{K_n\}]\right)^2 g(\omega)d\omega \qquad (55) \\
&= \text{Prob}\left[\omega \in \Omega_s\right] \text{var}\left[s^{*m,t}|\Omega_s\right] + \text{Prob}\left[\omega \in \Omega_{-s}\right] \text{var}\left[s^{*m,t}|\Omega_{-s}\right]
\end{aligned}$$

since the conditional means in the two subregions of the state space are identical. By the same token, partial stabilization delivers a variance of the exchange rate of

$$\begin{aligned}
\text{var}[s^{m,t}] &= \text{Prob}\left[\omega \in \Omega_s\right] \text{var}\left[s^{m,t}|\Omega_s\right] + \text{Prob}\left[\omega \in \Omega_{-s}\right] \text{var}\left[s^{m,t}|\Omega_{-s}\right] \\
&= \text{Prob}\left[\omega \in \Omega_s\right] \text{var}\left[(1-\zeta)(s^{*m,t} - \mathbb{E}[s^{*m,t}|\{K_n\}])|\Omega_s\right] + \text{Prob}\left[\omega \in \Omega_{-s}\right] \text{var}\left[s^{*m,t}|\Omega_{-s}\right] \\
&= \text{Prob}\left[\omega \in \Omega_s\right] (1-\zeta)^2 \text{var}\left[s^{*m,t}|\Omega_s\right] + \text{Prob}\left[\omega \in \Omega_{-s}\right] \text{var}\left[s^{*m,t}|\Omega_{-s}\right] \\
&< \text{var}\left[s^{*m,t}\right].
\end{aligned} \qquad (56)$$

With exchange rate stabilization of strength ζ, the interest rate differential given by equation (2) becomes

$$\begin{aligned}
r^m + \Delta\mathbb{E}[s^{m,t}] - r^t &= -\text{cov}\left[\lambda_T, s^{m,t}\right] \\
&= -\text{cov}\left[\lambda_T, (1-\zeta)s^{*m,t}\right] \\
&= -(1-\zeta)\text{cov}\left[\lambda_T, s^{*m,t}\right].
\end{aligned}$$

The effects of partial stabilization for interest rate differentials work in the same direction. Again using the fact that the conditional means are identical in the two subregions, we decompose the covariance into the following terms:

$$
\begin{aligned}
r^m + \Delta\mathbb{E}[s^{m,t}] - r^t = -\text{cov}\left[\lambda_T, s^{m,t}\right] &= -\int_\Omega \left(\lambda_T - \mathbb{E}\left[\lambda_T|\{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}\left[s^{m,t}|\{K_n\}\right]\right) g(\omega)d\omega \\
&= -\text{Prob}\left[\omega \in \Omega_s\right] \int_{\Omega_s} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_s(\omega)d\omega \\
&\quad - \text{Prob}\left[\omega \in \Omega_{-s}\right] \int_{\Omega_{-s}} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_{-s}(\omega)d\omega \\
&= -\text{Prob}\left[\omega \in \Omega_s\right] \int_{\Omega_s} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\Omega_s, \{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_s(\omega)d\omega \\
&\quad - \text{Prob}\left[\omega \in \Omega_s\right] \left(\mathbb{E}\left[\lambda_T|\Omega_s, \{K_n\}\right] - \mathbb{E}\left[\lambda_T|\{K_n\}\right]\right) \int_{\Omega_s} \left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_s(\omega)d\omega \\
&\quad - \text{Prob}\left[\omega \in \Omega_{-s}\right] \int_{\Omega_{-s}} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\Omega_{-s}, \{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_{-s}(\omega)d\omega \\
&\quad - \text{Prob}\left[\omega \in \Omega_{-s}\right] \left(\mathbb{E}\left[\lambda_T|\Omega_{-s}, \{K_n\}\right] - \mathbb{E}\left[\lambda_T|\{K_n\}\right]\right) \int_{\Omega_{-s}} \left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_{-s}(\omega)d\omega \\
&= -\text{Prob}\left[\omega \in \Omega_s\right] \int_{\Omega_s} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\Omega_s, \{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_s(\omega)d\omega \\
&\quad - \text{Prob}\left[\omega \in \Omega_{-s}\right] \int_{\Omega_{-s}} \left(\lambda_T - \mathbb{E}\left[\lambda_T|\Omega_{-s}, \{K_n\}\right]\right)\left(s^{m,t} - \mathbb{E}[s^{m,t}|\{K_n\}]\right) g_{-s}(\omega)d\omega \\
&= -\text{Prob}\left[\omega \in \Omega_s\right] \text{cov}\left[\lambda_T, s^{m,t}|\Omega_s\right] - \text{Prob}\left[\omega \in \Omega_{-s}\right] \text{cov}\left[\lambda_T, s^{m,t}|\Omega_{-s}\right],
\end{aligned}
$$

where $g_s(\omega) = \dfrac{g(\omega)}{\text{Prob}\left[\omega \in \Omega_s\right]}$ and $g_{-s}(\omega) = \dfrac{g(\omega)}{\text{Prob}\left[\omega \in \Omega_{-s}\right]}$. The second-to-last step follows from the fact that the conditional means are identical and thus $\mathbb{E}\left[s^{m,t} - \mathbb{E}\left[s^{m,t}|\{K_n\}\right]|\Omega_s\right] = 0$.
With partial exchange rate stabilization, we get

$$
\begin{aligned}
r^m + \Delta\mathbb{E}[s^{m,t}] - r^t &= -\text{cov}\left[\lambda_T, s^{m,t}\right] \\
&= -\text{Prob}\left[\omega \in \Omega_s\right] \text{cov}\left[\lambda_T, s^{m,t}|\Omega_s\right] - \text{Prob}\left[\omega \in \Omega_{-s}\right] \text{cov}\left[\lambda_T, s^{m,t}|\Omega_{-s}\right] \\
&= -\text{Prob}\left[\omega \in \Omega_s\right] (1-\zeta)\text{cov}\left[\lambda_T, s^{*m,t}|\Omega_s\right] - \text{Prob}\left[\omega \in \Omega_{-s}\right] \text{cov}\left[\lambda_T, s^{*m,t}|\Omega_{-s}\right].
\end{aligned}
$$

Rearranging the last equation to

$$
r^m + \Delta\mathbb{E}[s^{m,t}] - r^t = -\text{cov}\left[\lambda_T, s^{m,t}\right] = -\text{cov}\left[\lambda_T, s^{*m,t}\right] + \zeta\text{Prob}\left[\omega \in \Omega_s\right] \text{cov}\left[\lambda_T, s^{*m,t}|\Omega_s\right],
$$

we see that the effects of partial stabilization are a milder version of currency stabilization discussed previously. In fact, partial stabilization of strength ζ in a subset of the state space corresponds to currency stabilization of strength $\zeta\text{Prob}\left[\omega \in \Omega_s\right] \text{cov}\left[\lambda_T, s^{*m,t}|\Omega_s\right]$.

E.2 Stabilization Relative to a Basket of Currencies

Our analysis above also extends directly to stabilizations relative to a basket of currencies. Consider a country that wishes to stabilize its real exchange rate with the basket

$$p^b = (1-w)p^t + wp^o$$

where w is the basket's weight on the outside country and $1-w$ the weight on the target country. Using (2), it is then easy to show that stabilizing relative to a basket of currencies has effects akin to a stabilization relative to a (hypothetical) country with a weighted average size of the basket's constituents:

$$r^m + \Delta\mathbb{E}s^{m,o} - r^o = \left(r^{t*} + \Delta\mathbb{E}s^{t,o*} - r^{o*}\right) - \zeta\frac{\gamma(1-\tau)^2\left((\bar{\theta}-\theta^m)(\gamma-1)\tau + \theta^m(2-w-2\bar{w}\zeta)\right)}{\tau\left(1+(\gamma-1)\tau\right)}\sigma_N^2$$

where $\bar{\theta} = w\theta^t + (1-w)\theta^o$ is the weighted average size of the basket's constituents and $\bar{w} = 1-(1-w)w$ is a positive constant less than one.

Although clearly a less effective means of lowering domestic interest rates than stabilizations relative to the largest economy in the world, stabilizing relative to a basket may be appealing for some countries, because it reduces price impact. When stabilizing relative to a basket, the stabilizing country's exports are less sensitive to shocks affecting only one of the two other countries, decreasing the volatility of its exports and thus lowering the stabilization's impact on world-market prices. For a large country, stabilizing relative to a basket may thus be cheaper to implement than stabilizing relative to the largest economy in the world.

E.3 Feedback between Risk Premia and Capital Accumulation

In this appendix, we show Propositions 1 through 3 continue to hold when we solve explicitly for the feedback between risk premia and capital accumulation. First, note that changes in the level of capital accumulation affect the expected level of consumption, but not the conditional covariance of consumption across countries in our log-linear solution. It follows immediately that all statements in Propositions 1 through 3 that depend on the covariances between asset payoffs and the shadow price of traded goods are unchanged. That is, all statements regarding interest differentials, expected currency returns, and the world-market value of domestic firms continue to hold.

Second, to show that all statements in Propositions 1 through 3 pertaining to the capital stock itself continue to hold when we solve explicitly for the feedback between risk premia and capital accumulation. To this end, we use the second-order approximation of the Euler equation for capital accumulation (13):

$$\lambda_{T,1} + q_K + k^n = \log[\nu] + \mathbb{E}\left[\lambda_N^n + y_N^n\right] + \frac{1}{2}\text{var}\left[\lambda_N^n + y_N^n\right] \quad \forall n,$$

and the log-linear resource constraint for capital:

$$0 = \sum_n \theta^n k^n.$$

We plug in the expression for $\lambda_N^n = p_N^n + q + \lambda_{T,1}^n$ from Appendix B.3 to write λ_N^n as a function of y_N^n, and then we plug in $y_N^n = \eta + \nu k^n$ to write the y_N^n as a function of the capital stock and the productivity shock. We solve this system of four equations for k^m, k^t, k^o and q_K.

In a freely floating exchange rate economy ($\zeta = 0$), we find

$$k^{m*} - k^{t*} = \frac{(\gamma-1)^3(1-\tau)^2\tau}{(1+(\gamma-1)\tau)\left(1+(\gamma-1)(1-\tau)\nu+(\gamma-1)\tau\right)}\left(\theta^m - \theta^t\right)\sigma_N^2.$$

Comparing this expression with $k^m - k^t$ derived from solving the system of four equations above shows that allowing for feedback between risk premia and capital accumulation merely reduces the size of the difference in capital accumulation by a constant factor smaller than one, leaving the economic insights of our analysis unaffected.

The same is true for the equivalent expression under stabilized exchange rates, though this factor is too large to reproduce in print. For the special case of $\zeta = 1$, we can show

$$\left(k^m - k^t|_{\zeta=1}\right) = k^{m*} - k^{t*} + \frac{(\gamma-1)^3(1-\tau)^2\tau\left(\theta^t - \theta^m\right)\sigma_N^2}{(1+(\gamma-1)\tau)\left(1+(\gamma-1)(1-\tau)\nu+(\gamma-1)\tau\right)} = 0.$$

F Appendix to Section 5

In this appendix, we provide additional details about the model in section 5 and formally derive its equilibrium conditions. To avoid solving the optimization problem separately for households in the stabilizing country and households in the rest of the world, we generalize the notation to allow all countries to impose state-contingent taxes, $Z^n(\omega)$, and provide lump sum transfers, $\bar{Z}^n$. The governments in the target and outside countries do not use these instruments, such that $Z^t(\omega) = Z^o(\omega) = 1$ and $\bar{Z}^t = \bar{Z}^o = 0$.

F.1 Equilibrium Consumption

Inactive households in country n maximize utility, defined in equation (26), in each state of the world by splitting their wealth $\exp(-\mu^n)P^n(\omega)$ optimally between traded and nontraded consumption,

$$\max_{\hat{C}_T^n(\omega),\hat{C}_N^n(\omega)} \frac{1}{1-\gamma}\left(\exp\left(\chi^n\right)\left(\hat{C}_T^n\left(\omega\right)\right)^{\tau}\left(\hat{C}_N^n\left(\omega\right)\right)^{1-\tau}\right)^{1-\gamma}$$
$$\text{s.t. } \hat{C}_T^n(\omega) + P_N^n(\omega)\hat{C}_N^n(\omega) \leq \exp(-\mu^n)P^n(\omega),$$

where hats indicate consumption by inactive households. We solve this problem by setting up a Lagrangian and taking first-order conditions with respect to $\hat{C}_T^n(\omega)$ and $\hat{C}_N^n(\omega)$. Inactive households then optimally consume the following bundle of traded and nontraded goods,

$$\hat{C}_T^n(\omega) = \exp(-\mu^n)\tau P^n(\omega), \quad \hat{C}_N^n(\omega) = \exp(-\mu^n)\frac{(1-\tau)P^n(\omega)}{P_N^n(\omega)}.$$

Active households own all the productive assets within the country and are short the nominal bonds owned by inactive households. They maximize their utility (26) subject to their intertemporal budget constraint:

$$\begin{aligned}&\mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\left(Z^n(\omega)C_T^n(\omega) + P_N^n(\omega)C_N^n(\omega) + \frac{1-\phi}{\phi}P^n(\omega)e^{-\mu^n}\right)\right] \\ &\leq \frac{1}{\phi}\left(Q_K - Q_K K^n + \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\left(P_N^n(\omega)Y_N^n + Y_T^n\right)\right] + \kappa^n + \bar{Z}^n\right),\end{aligned} \tag{57}$$

where $(1-\phi)/\phi$ is the number of inactive households per active household in each country and endowments are adjusted by a factor $1/\phi$ because active households now own proportionally more productive assets per capita; κ^n again denotes the transfer that decentralizes the allocation corresponding to the social planner's problem with unit Pareto weights under freely floating exchange rates. In the stabilizing country, the government use the lump-sum transfer, $\bar{Z}^m$, to equalize the marginal utility of wealth between the stabilizing country and the rest of the world (P2).

The first-order conditions of the active households' problem are:

$$\frac{\tau\exp((1-\gamma)\chi^n)\left(C^n\right)^{1-\gamma}\left(C_T^n\right)^{-1}}{Z^n(\omega)} = \Lambda_{T,1}Q(\omega) \tag{58}$$

$$(1-\tau)\exp((1-\gamma)\chi^n)\left(C^n\right)^{1-\gamma}\left(C_N^n\right)^{-1} = \Lambda_{T,1}Q(\omega)P_N^n(\omega). \tag{59}$$

Analogous to Appendix (B.3), we find it convenient to denote the stochastic discout factor with $Q(\omega) = \Lambda_T(\omega)/\Lambda_{T,1}$. The first-order condition with respect to capital accumulation is

$$Q_K = \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}} P_N^n(\omega) e^{\eta^n} \nu \left(K^n\right)^{\nu-1}\right]. \tag{60}$$

F.2 Log-linearized System of Equations

We next derive log-linearized first-order conditions. Equation (7) defines the resource constraint for traded goods. Equation (8) defines the (three) resource constraints for nontraded goods in each country, and equation (9) defines the resource constraint for capital goods. Equations (58) and (59) define the three first-order conditions with respect to traded consumption and the three first-order conditions with respect to nontraded consumption. Equation (60) defines the three Euler equations for capital investment in each country. In total, we derive a system of 14 equations. To study the model in closed form, we again log-linearize around the deterministic solution — the point at which the variances of shocks are zero ($\sigma_{N,n} = 0$) and all firms have a capital stock fixed at the deterministic steady-state level. To simplify the exposition, we thus again ignore the feedback effect of differential capital accumulation on the size of risk premia, studying the *incentives* to accumulate different levels of capital across countries, while holding the capital stock fixed. The log-linear first-order conditions are:

$$(1-\gamma)\chi^n + (1-\gamma)\left(\tau c_T^n + (1-\tau)c_N^n\right) - c_T^n + \log\tau = z^n + q + \lambda_{T,1}$$
$$(1-\gamma)\chi^n + (1-\gamma)\left(\tau c_T^n + (1-\tau)c_N^n\right) - c_N^n + \log(1-\tau) = p_N^n + q + \lambda_{T,1}.$$

Similar to Appendix B.1, let $q = \lambda_T - \lambda_{T,1}$ denote the stochastic discount factor. Also recall that the transfes $\{\kappa^n\}, \bar{Z}^m$ equalize the first-period Lagrange multiplier $\lambda_{T,1}$ across active households in all countries. The log-linear approximation of equation (60) is:

$$\lambda_{T,1} + q_K + k^n = \log[\nu] + \mathbb{E}\left[\lambda_N^n + y_N^n\right] + \frac{1}{2}\text{var}\left(\lambda_N^n + y_N^n\right).$$

The log-linear resource constraints are:

$$\phi c_N^n + (1-\phi)\left(-\mu^n - \tau\left(\lambda_N^n - \lambda_T - \log\left(\frac{1-\tau}{\tau}\right)\right)\right) = \eta^n + \nu k^n = y_N^n,$$

$$\sum_{n=m,t,o} \theta^n \left[\phi c_T^n + (1-\phi)\left(-\mu^n - (1-\tau)\left(\lambda_N^n - \lambda_T - \log\left(\frac{1-\tau}{\tau}\right)\right)\right)\right] = \sum_{n=m,t,o} \theta^n y_{T,1}^n = 1,$$

$$\sum_{n=m,t,o} \theta^n k^n = 1.$$

This set of fourteen equations allows us to solve for the following fourteen unknowns $\{k^n, c_N^n, c_T^n, \lambda_N^n\}_{n=m,t,o}$, $\lambda_{T,1}$ and q.

F.3 Cost of Stabilization

First, we solve for the state-contingent taxes that implement the real exchange rate stabilization in the model in section 5, and then we derive an expression for the cost of the peg. Throughout, we can recover the results in Appendix B.6 by removing the market segmentation ($\phi = 1$), by setting the monetary shocks to zero ($\mu^n = 0$) and by setting the preference shocks to zero ($\chi^n = 0$).

Analogous to Appendix B.6, we search for a state-contingent tax of the form

$$Z(\omega) = \left(\frac{Y_N^m}{Y_N^t}\right)^{a_1}\left(\frac{\exp\left(-\mu^t\right)}{\exp\left(-\mu^m\right)}\right)^{a_2}\left(\frac{\exp\left(\chi^m\right)}{\exp\left(\chi^t\right)}\right)^{a_3}.$$

In logs, this state-contingent tax is

$$z = a_1\left(y_N^t - y_N^m\right) + a_2\left(-\mu^t + \mu^m\right) + a_3\left(\chi^t - \chi^m\right).$$

We follow the procedure from Appendix B.6 to derive the coefficients a_1, a_2 and a_3 that stabilize the exchange rate. The following lemma summarizes these results.

Lemma 3

In the model in section 5, where real exchange rates fluctuate in response to monetary shocks, preference shocks, and productivity shocks, a tax on the consumption of traded goods in the stabilizing country of the form

$$z(\omega) = \frac{\zeta(1-\tau)}{\tau\left(\tau+\phi(1-\tau)\right)}\left(y_N^m - y_N^t\right) + \frac{(1-\tau)(1-\phi)}{\tau\left(\tau+\phi(1-\tau)\right)}\left(\mu^m - \mu^t\right) + \frac{(\gamma-1)(1-\tau)\phi}{\gamma\tau\left(\tau+\phi(1-\tau)\right)}\left(\chi^m - \chi^t\right)$$

implements a real exchange rate stabilization of strength ζ.

Next, we derive the cost of the stabilization. We start with the budget constraint of the active household in the stabilizing country given by equation (57), and we identify the components of the lump-sum transfer, $\bar{Z}$. Following the same calculations as in Appendix B.6, we show:

$$\begin{aligned} \Delta Res = \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\left(\phi C_T^m(\omega) + (1-\phi)\hat{C}_T^m(\omega)\right)\right] \\ - \mathbb{E}\left[\frac{\Lambda_T(\omega)}{\Lambda_{T,1}}\left(\phi C_T^{m*}(\omega) + (1-\phi)\hat{C}_T^{m*}(\omega)\right)\right] \end{aligned} \tag{61}$$

In the model in section 5, the cost of stabilization is thus the change in the value of the stabilizing country's total consumption of traded goods by active and inactive households.

F.4 Proof of Proposition 6

We first prove results for the internal effects of a real exchange rate stabilization. We plug the log-linear expressions derived from solving the system of equations in Appendix F.2 into equation (2). We can then write the interest rate differential between the stabilizing country and the target as

$$\begin{aligned} r^m + \Delta\mathbb{E}s^{m,t} - r^t = \left(r^{m*} + \Delta\mathbb{E}s^{m,t*} - r^{t*}\right) - \zeta\frac{\gamma(1-\tau)^2\left((\theta^t-\theta^m)\tau(\gamma-\phi) + 2\phi\theta^m(1-\zeta)\right)}{\tau\phi\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_N^2 \\ - \zeta\frac{\gamma(1-\tau)(1-\phi)^2\left((\theta^t-\theta^m)\gamma\tau + 2\phi\theta^m(1-\zeta)(1-\tau)\right)}{\tau\phi\left(\gamma\tau+(1-\tau)\phi\right)}\tilde{\sigma}^2 \\ - \zeta\frac{\phi(1-\tau)(1-\gamma)^2\left((\theta^t-\theta^m)\gamma\tau + 2\phi\theta^m(1-\zeta)(1-\tau)\right)}{\tau\gamma\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_\chi^2, \end{aligned}$$

which implies the exchange rate stabilization decreases the risk-free rate in the stabilizing country relative to the risk-free rate in the target country if the target country is larger than the stabilizing country, $\theta^t > \theta^m$.

Plugging in the log-linear expressions for p_N^m, p_N^t, and λ_T into equation (37), we again find that the relative incentives to accumulate capital in the stabilizing country increase with the size of the target country. Because the closed-form solution equivalent to the one above is too large to print, it is easier to prove this statement by showing that relative incentives to accumulate capital increase linearly in θ^t:

$$\frac{d}{d\theta^t}\left[\left(k^m - k^t\right) - \left(k^{m*} - k^{t*}\right)\right] = \frac{\zeta(\gamma-1)(1-\tau)^2\tau(\gamma-\phi)^2}{(\phi+(1-\phi)\tau)\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_N^2 + \frac{\zeta(\gamma-1)\gamma(1-\tau)\tau(\gamma-\phi)(1-\phi)^2}{(\phi+(1-\phi)\tau)\left(\gamma\tau+(1-\tau)\phi\right)}\tilde{\sigma}^2 + \frac{\zeta(\gamma-1)^3(1-\tau)\tau(\gamma-\phi)\phi^2}{\gamma\left(\phi+(1-\phi)\tau\right)\left(\gamma\tau+(1-\tau)\phi\right)}\sigma_\chi^2 > 0.$$

It follows immediately that there exists some $\theta_{min} > 0$ such that stabilizing the real exchange rate relative to any country larger than θ_{min} will increase the incentives to accumulate capital in the stabilizing country. Analogous to Appendix B.7, the spread between the value of the firm in the stabilizing and target countries yields the same expression as the right-hand side of equation (37). Hence, we have already shown the value of the firm in the stabilizing country increases relative to the target country if θ^t is large enough.

Because firms are competitive, wages are given by the marginal product of labor. Hence, an exchange rate stabilization relative to a sufficiently large target country increases wages in the stabilizing country relative to all other countries, concluding the proof of the first statement in Proposition 6.

Next, we derive the cost of stabilization. We calculate changes in the log value of traded consumption in the stabilizing country given by (61). The log-linear approximation of the total traded consumption in the stabilizing country from active and inactive households is: $\phi c_T^m + (1-\phi)\hat{c}_T^m$. We calculate:

$$\log \Delta Res = v_T - v_T^*,$$

where we use the following second-order approximation of the log value of total traded consumption:

$$v_T = \mathbb{E}\left[\lambda_T - \psi_T + \phi c_T^m + (1-\phi)\hat{c}_T^m\right] + \frac{1}{2}\text{var}\left[\lambda_T - \psi_T + \phi c_T^m + (1-\phi)\hat{c}_T^m\right]$$

When the stabilizing country is small ($\theta^m = 0$), the cost of the stabilization decreases as the target country gets larger:

$$\frac{d}{d\theta^t}\left(v_T - v_T^*\right) = -\zeta\frac{(1-\tau)(1-\phi)^2(\gamma-\phi)}{\left(\phi+(\gamma-\phi)\tau\right)^2}\tilde{\sigma}^2 - \zeta\frac{(1-\tau)^2(\gamma-\phi)^2}{\left(\phi+(\gamma-\phi)\tau\right)^2}\sigma_N^2 - \zeta\frac{(\gamma-1)^2(1-\tau)(\gamma-\phi)\phi^2}{\gamma\left(\phi+(\gamma-\phi)\tau\right)^2}\sigma_\chi^2 < 0.$$

Hence, it is cheaper to stabilize relative to a larger country.

Finally, we prove results for the external effects of a real exchange rate stabilization. Using equation (2) and the solution of the model from Appendix F.2, we can write interest rate differential between the target country and the outside country as

$$r^t + \Delta\mathbb{E}s^{t,o} - r^o = \left(r^{t*} + \Delta\mathbb{E}s^{t,o*} - r^{o*}\right) + \frac{\zeta\theta^m\gamma(1-\tau)^2}{\tau\left(\gamma\tau + \phi(1-\tau)\right)}\sigma_N^2 + \frac{\zeta\theta^m\gamma(1-\tau)^2(1-\phi)^2}{\tau\left(\gamma\tau + \phi(1-\tau)\right)}\tilde{\sigma}^2 + \frac{\theta^m\zeta(\gamma-1)^2(1-\tau)^2\phi^2}{\gamma\tau\left(\gamma\tau + (1-\tau)\phi\right)}\sigma_\chi^2,$$

which implies the exchange rate stabilization increases the risk-free rate in the target country relative to the risk-free rate in the outside country.

We plug the log-linear expressions for p_N^t, p_N^o and λ_T into (37) to derive the differential incentive to accumulate capital in the target country relative to the outside countries:

$$k^t - k^o = k^{t*} - k^{o*} - \frac{\theta^m\zeta(1-\tau)^2(\gamma-\phi)^2}{\left(\gamma\tau + (1-\tau)\phi\right)^2}\sigma_N^2 - \frac{\theta^m\gamma\zeta(1-\tau)(\gamma-\phi)(1-\phi)^2}{\left(\gamma\tau + (1-\tau)\phi\right)^2}\tilde{\sigma}^2 - \frac{\theta^m(\gamma-1)^2\zeta(1-\tau)(\gamma-\phi)\phi^2}{\gamma\left(\gamma\tau + (1-\tau)\phi\right)^2}\sigma_\chi^2.$$

Incentives to accumulate capital in the target country thus decrease relative to the outside country. Since the marginal product of labor rises with the level of capital accumulation, the exchange rate stabilization decreases wages in the target country relative to all other countries.