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Keywords: set-asides, auctions with endogenous entry, entry deterrence, asymmetric buyers, incumbents, government procurement

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1 Introduction

Set-asides are used extensively in government procurements and resource sales like spectrum auctions. They consist either in excluding from the auction some specific bidders (e.g. large firms or previous auction winners)[1,2] or equivalently in limiting the access to some well-chosen bidders (e.g. domestic firms, small firms, minority-owned businesses or new entrants). Specifically, set-asides are routinely used to favor small businesses in countries like the US, Canada or Japan. In Japan, for example, approximatively two thirds of civil engineering contracts are subject to set-asides (Nakabayashi, 2013), and in the US, those federal procurement contracts billing between \$3,000 and \$100,000 are reserved to SMEs (Athey, Coey and Levin, 2013).[3] Promoting small businesses is also an important goal in the EU, but the European economic law prohibits explicit discriminatory practices such as set-asides. In such cases, implicit set-asides can still be at work for example through the requirement of technological constraints that would be known to forbid the access of some potential participants or by excluding a bidder on the ground that he previously caused some disappointment in the past as promoted in the new European directives (Saussier and Tirole, 2015).

An important practical question is whether -and to what extent- set-asides policies increase the costs in public procurements.[4] The research question addressed in this paper is whether excluding some bidders could reduce the procurement cost, or equivalently boost the seller's revenue in auction contexts. When addressing this question, we have in mind a positive perspective by which we mean that we consider simple and commonly used auction formats (such as the ascending auction or the first-price auction) and we investigate the effect of set-asides within such formats, not allowing for more complex forms of discrimination.[5]

[1] When contracts are renewed periodically, we could consider whether it could beneficial to exclude the incumbent. E.g. in the Veolia Transport and Transdev case, the French antitrust authorities impose as a remedy that Transdev Group, the merged entity, commits not to bid in a bunch of cities in the south-east of France for which it was the incumbent (see paragraphs 446-450 in "Décision n^{o}10-DCC-198 du 30 décembre 2010" and also paragraph 43 in "Décision n^{o}13-DCC-137 du 1er octobre 2013" that confirms that Transdev Group has met his engagements). We thank Thibaud Vergé for bringing this antitrust case to our attention.

[2] In the auctions that determines the subsidy for the production of renewable energy in Portugal, a winning bidder is automatically excluded from participating in the subsequent auctions (del Rio (2016)).

[3] In US, another form of explicit discrimination are bid subsidies. E.g. Marion (2007) estimates that in California the five percent subsidy that accrues to small businesses in auctions for road construction projects increases the procurement costs. Note that in some sectors, only set asides are used: 14% of the timber auction dataset considered by Athey, Coey and Levin (2013) -which includes non-salvage Forest Service sales in California between 1982 and 1989- is composed of auctions with set-asides (either first-price or English auctions) while the rest are non-discriminatory auctions.

[4] There is a limited empirical literature on pro-small business set-asides (Denes (1997), Nakabayashi (2013) and Athey, Coey and Levin (2013)) which finds evidence that the induced increasing participation from small businesses more than compensates the loss from those who have been set aside. On the other hand, this literature found mixed evidence concerning the procurement costs/auction's revenue. Denes (1997) found no significant differences. Nakabayashi (2013) found that set-asides reduce government procurement costs in procurement auctions for civil engineering projects in Japan. On the contrary, Athey, Coey and Levin (2013) found in US Forest Service timber auctions that small business set-asides substantially reduces revenue.

[5] From a normative perspective, set-asides appear as a suboptimal instrument. Myerson (1981) shows under exogenous entry that the optimal auction involves discrimination against strong bidders that does not take the extreme form of set-aside but consists instead in giving them some handicaps. Under endogenous entry, Jehiel and Lamy (2015) characterize the optimal auction design in a setup with both potential entrants and with some incumbent bidders assumed to participate for sure, and show that the optimal auction involves discrimination

In the context of single-good second-price auctions with private values, when the set of participants is exogenous, excluding some bidders reduces competition, and thus set-asides can only be detrimental to the seller (excluding some bidders can only reduce the second-highest valuations among the bidders and thus the final price).[6] By contrast, when participation is endogenous, set-asides may possibly boost the participation of potential entrants in such a way that it is beneficial to the seller. In this spirit, Cramton (2013) argues that set-asides were at the core of the success of Canada's spectrum auctions for Advanced Wireless Services (AWS) because they encouraged participation from deep-pocketed new entrants thereby resulting in a push of the prices both for the blocks with set-asides and the blocks without. Relatedly, in the UK 2000's spectrum auctions, the fact that one licence was reserved to a new entrant is considered to be an important source of its success.[7] The logic behind those cases is that set-asides can be pro-competitive insofar as not allowing some bidder(s) to participate can boost the participation of other kinds of bidders and possibly be overall beneficial.

We start our analysis with the second-price auction in which the reserve price is set at the seller's valuation - a format that we refer to as the Vickrey auction. We develop an endogenous entry model in which bidders are either incumbents who participate for sure in the auction or potential entrants whose participation rate is endogenously determined to ensure that ex ante entrants get the same as their outside option. In some parts, we solve for the optimal set-aside policy determining whom from the incumbents or the groups of entrants should be banned. In other parts, we restrict the question to studying the benefit of excluding a specific incumbent or a given group of potential entrants.

Our first general insight is that irrespective of the shape of the distributions of valuations, when there is only one incumbent, it is always beneficial for revenues to exclude the incumbent in the Vickrey auction.[8] The logic for this result can be understood simply. From Jehiel and Lamy (2015a), we know that when the incumbent is out, the seller's revenue corresponds (in

against the incumbents relative to the potential entrants but no set-asides.

[6]When bidders' valuations are i.i.d., Bulow and Klemperer (1996) provide a bound on how excluding one bidder is detrimental to the seller in an efficient auction: they show that switching to the optimal mechanism (namely the second-price auction with optimally set reserve price) would not compensate the seller from losing a single bidder. For the first-price auction with independently distributed valuations, Arozamena and Cantillon (2004) show that when the valuation distribution of a bidder is upgraded then his competitors bid collectively more aggressively in equilibrium. As a corollary, it implies that the seller's revenue decreases when some bidders are excluded. Under affiliated private values, Pinkse and Tan (2005) show on the contrary that a bidder may not bid more aggressively when the number of its competitors increases. However, they do not address the incidence of increased competition on the seller's revenue.

[7]In addition to the four incumbents, eleven new entrants participated in the UK auction, which strongly contrasts with the failures in terms of both entry and revenue in the auctions for the 3G Telecom licences organized by some other European countries (see Binmore and Klemperer (2002) and Jehiel and Moldovanu (2003)), such as the Netherland where only one entrant showed up resulting in a revenue per capita that was four times smaller than in UK. Maybe the most drastic example of an harmful impact of an incumbent on competition is when a single bidder deter all potential entrants from participating as illustrated in the 1994 US spectrum auctions where a license covering all of southern California was offered and where it was publicly known that Pacific Bell, the incumbent company, had a higher valuations that its rivals and thus should win for sure in an ascending auction (see Milgrom (2004) for details and Klemperer (2002b) for other entry deterrence examples, in particular with first price auctions).

[8]We note that such a finding is of practical importance to the extent that in a dynamic perspective the incumbent can often be though as the current contractor and there is typically one of them.

expectation) to the welfare net of the entry costs of the entrants -referred to as the total welfare- and that the best possible revenue is the one obtained through the Vickrey auction. When the incumbent is in, revenues are reduced for the following reason. Making use of a fundamental property of the Vickrey auction, the incumbent gets a rent equal to his marginal contribution to the welfare. Hence, by a simple accounting argument, the seller's revenue corresponds (in expectation) to the total net welfare were the incumbent absent. Because participation rates are optimally determined when the incumbent is out (as shown in Jehiel and Lamy, 2015a), but typically not so when the incumbent participates, we conclude that excluding the incumbent is always good for revenues. Thus, when there is only one incumbent, the indirect benefit of excluding the incumbent obtained through a boost of entrants' participation always dominates the direct cost of not having the incumbent for a fixed set of participants.

Interestingly, the insight about the desirability of excluding the incumbent carries over to other auction formats, as long as the incumbent gets a payoff no smaller than his marginal contribution to the welfare. This implies for example in the context of procurement auctions that if the incumbent is better at renegotiating the contract than entrants (see Bajari, Houghton and Tadelis (2014) for a discussion of how important renegotiations are in procurement auctions) then excluding the incumbent enhances the revenues. We also note in the absence of any incumbent that if one group of entrants is advantaged in the auction format as compared to another group of entrants (maybe due to asymmetric risks of breakdown), it is not good for revenues to exclude the weak group (and it may or may not be good to exclude the strong group). Finally, we briefly discuss the desirability of exclusion when there is a single incumbent in the context of first-price auctions. When the incumbent has a right of first refusal (as sometimes observed in procurement auctions), excluding the incumbent is always good. In standard first-price auctions, we observe that excluding the incumbent is good when the incumbent and entrants' valuations are drawn independently from the same distribution (and a fortiori so when the incumbent's valuation is drawn from a weaker distribution).

When moving to multiple incumbents, the determination of the optimal set-asides policy is more complex, even in the context of the Vickrey auction. Indeed, when there are multiple incumbents, excluding an incumbent may have an extra detrimental effect not present in the one incumbent case on the rent left to the other incumbents. For a specific distribution of incumbents' valuations, we show that the exclusion of any incumbent has no effect on the rent left to other incumbents (due to the adjustment of entrants' participation decisions), and thus it is optimal to exclude all incumbents in this case. Under some other distributional assumptions, we observe that excluding an incumbent increases the rents left to the other incumbents and we observe then that excluding a sufficiently weak incumbent would be detrimental to revenues.

The rest of the paper is organized as follows. Section 2 presents our general model with endogenous entry and illustrates some of our main results through simple examples. Section 3 introduces some formal notation and shows that in the Vickrey auction, no-exclusion is always optimal from a welfare perspective. Section 4 establishes our main result when there is a single incumbent and the auction format is the Vickrey auction. Section 5 extends the insight obtained with one incumbent to other (inefficient) formats. Section 6 moves to environments with multiple incumbents. Section 7 briefly considers the case of interdependent values and the case of auction formats with entry fees. Section 8 concludes. Technical proofs are gathered in the Appendix.

2 An auction setup with set-asides

2.1 The model

A risk neutral seller S is selling a good through a given auction procedure. Her reservation value is denoted by $X_S \geq 0$. When the good is auctioned off through a second-price auction with a reserve price set at X_S, then the auction format is referred to as the Vickrey auction. We assume that there are two classes of buyers: The incumbents who participate for sure in the auction (alternatively, we can think of their participation costs as being null) and the potential entrants who can submit a bid only if they have incurred an entry cost. The decisions to participate are made simultaneously by all potential entrants. We allow entrants to come from different groups, and different groups can be characterized by different entry costs and different distributions of valuations. The number of potential entrants is assumed to be large in each group, which allows us to simplify the analysis (as explained below).

Remark. It should be mentioned that our model is framed as an auction setup in which buyers are bidding in order to acquire an object. It could obviously be phrased as a procurement in which the designer seeks to obtain a service from various potential providers. The procurement interpretation fits better some of our motivations and policy implications developed in Introduction. It can be observed that for contracts of services that are auctioned off periodically, an incumbent may be thought of as being the previous holder of the contract, which will motivate our study of the single incumbent scenario.[9]

Formally, the various possible buyers and their distributions of valuations and entry costs are described as follows. There is a finite set of incumbents $\mathcal{I}$. Each incumbent $i \in \mathcal{I}$ is characterized by a cumulative distribution $F_i^I(.|z)$, from which his valuation is drawn conditional on the realization z of some underlying variable Z. There are K groups of potential entrants $\mathcal{E} = \{1, \ldots, K\}$. Each group is composed of infinitely many potential buyers, which will justify our modelling of entry as following Poisson distributions (see below). A buyer from group $k \in \mathcal{E}$ has an entry cost $C_k > 0$ and his valuation is drawn from the cumulative distribution $F_k(.|z)$ conditional on the realization z of the underlying variable Z.[10] Conditionally on z, the valuations

[9] In an auction perspective, scare resources like spectrum may also be auctioned periodically in a way that make the incumbency status relevant.

[10] The entry cost C_k can be interpreted equivalently as the expected utility of a group k buyer if he chooses an outside option (which may consist e.g. in participating in another procurement).

of the various buyers are assumed to be drawn independently.[11] To simplify, we also assume that the supports of the distributions $F_i^I(.|z)$ and $F_k(.|z)$ are uniformly bounded by $\overline{x} > X_S$, but we make no other assumption on these distributions nor on the distribution of z.

While our general formulation puts no restriction on the number of groups,[12] some of our results require further restrictions. We say that potential entrants are symmetric if $K = 1$ (and then we drop the index k from our notation). We say that all buyers (entrants and incumbents alike) are symmetric if $F_i^I(.|z) = F_k(.|z) = F(.|z)$ for each $i \in \mathcal{I}$ and $k \in \mathcal{E}$.[13] For some results concerning auctions in which there is no weakly dominant strategy, we impose the additional restriction that valuations are drawn independently with a common support $[\underline{x}, \overline{x}]$ according to continuously differentiable distributions (over their common support), and that the sole information received by bidders is their private valuation. We refer to such environments as *Myersonian environments*.[14]

In this paper, we adopt the view that the only instrument of the seller is the *set-asides policy*. Formally,

Definition 1 *A set-asides policy is a pair (I, E) which designates the set of incumbents $I \subseteq \mathcal{I}$ and the groups of entrants $E \subseteq \mathcal{E}$ who are allowed to participate. (I, E) is announced before the buyers decide whether or not to participate.*

Special cases of set-asides policies include: 1) $(I, E) = (\mathcal{I}, \mathcal{E})$, which corresponds to no-exclusion, and 2) $(I, E) = (\emptyset, \mathcal{E})$, which corresponds to excluding all the incumbents. For some results, we derive the optimal set-asides policy as if the seller were free to choose any (I, E). For other results, we consider instead whether excluding some specific bidders can be profitable for the seller (and to motivate the latter, we have implicitly in mind that the seller would not be allowed to exclude some types of buyers).

Our main interest in the rest of the paper will be to understand for the Vickrey auction (and then other formats) the effect of (I, E) on the revenue generated by the seller. For completeness, we will also briefly discuss the (more immediate) effect of (I, E) on welfare.

The timing of the game is as follows. The auction format is exogenously given. The seller announces first her set-asides policy (I, E). Second, potential entrants decide simultaneously whether or not to participate. At that time, the only information potential entrants have is the group they come from. Third, participants learn their private valuations so that we are in a

[11] Given that we impose no specific structure on the variable Z, conditional independence is a general way to introduce some correlation between buyers' valuations.

[12] This contrasts with the structural empirical literature that has typically considered two-group cases as e.g. in Athey, Levin and Seira (2011) and Athey, Coey and Levin (2013). The group structure can thus capture the idea of pre-entry signals about valuations as in Roberts and Sweeting (2012). The special case in which $F_k(x|z) = \mathbf{1}[x > x_k]$ for any $k \in \mathcal{E}$ corresponds to the case in which potential entrants from group k know their valuation x_k before entry as in McAfee (1993).

[13] Levin and Smith (1994) consider a model with symmetric potential entrants and no incumbents.

[14] In this case we drop the variable z from our notation. Alternatively, we can consider the case where z would be common knowledge among bidders after their entry decisions. What we need when invoking the "independence" restriction is that we are in a setup in which, at the auction stage, the payoffs can be expressed as a function of the "assignment rule" as in Myerson (1981).

private value environment (see Section 7.1 for an extension to an interdependent value setup). Fourth, the incumbents and the entrants who are allowed to participate are playing the auction game. If each bidder has a weakly dominant strategy (such as bidding one own's valuation in the Vickrey auction), the auction is referred to as a *dominant-strategy auction* and we then assume that buyers use their weakly dominant strategy. In such a case, our analysis is insensitive to the extra information about others' valuations bidders may receive. We will be more explicit about the needed informational assumptions at the auction stage (for example about the number and identities of other participants) when considering formats in which there is no weakly dominant strategy as in the first-price auction.

A key aspect of our analysis concerns the impact of (I, E) on the participation decisions. Due to our assumption that there is a large number of potential entrants in each group, the realized number of participants from each group will be distributed according to some Poisson distribution with the requirement that if the participation rate is positive in a group, the ex ante utility derived from participating in this group should match the entry cost in the group.[15] That is, we assume that the probability that there are n_k entrants from group k for any $k \in \mathcal{E}$ is equal to $e^{-\sum_{k=1}^{K} \mu_k^*} \cdot \prod_{k=1}^{K} \frac{[\mu_k^*]^{n_k}}{n_k!}$ where the parameters μ_k^*, $k \in \mathcal{E}$, are determined so that $\mu_k^* > 0$ (resp. $\mu_k^* = 0$) implies that the expected payoff of an entrant from group k should be equal to (resp. be lower than) his entry cost C_k.

2.2 Simple illustrations of the exclusion principle

To get some ideas about the potential effect of set-asides policies, we develop a few examples in which we consider the effect of excluding the incumbents.

Example 1a (the Vickrey auction with a single incumbent) Consider that 1) $X_S = 0$, 2) there is a single incumbent whose valuation is denoted by $x_I > 0$, 3) potential entrants are symmetric and have all the same valuation denoted by $x_E > 0$, and 4) the auction format is the Vickrey auction. In particular, when at least two entrants participate in the auction, their payoffs are null. We also assume that $E[x_E] > C$ in order to guarantee that the problem is not degenerate (otherwise entry would always be too costly). We do not make any additional restriction on the joint distribution of (x_E, x_I) (except that we implicitly assume that $E[\max\{x_E - x_I, 0\}] > 0$, i.e. that entrants have a strictly higher valuation than incumbents with positive probability). At the time they decide whether or not to enter the auction, we assume that potential entrants have no information about the realization of (x_E, x_I).

Let μ_{in}^* (resp. μ_{out}^*) denote the equilibrium entry rate when the incumbent is not excluded (resp. is excluded). Note that if the entry rate is μ, the probability to be the sole entrant in the

[15] The Poisson distribution corresponds to the limit distribution of the number of entrants of each group of a model with a finite number buyers per group taking independent decisions and as the number of buyers in each group goes to infinity (and assuming every individual entrant of a given group follows the same participation strategy). Jehiel and Lamy (2015a) gives a proper foundation of the Poisson equilibrium model as the limit of equilibria with a finite number of potential entrants when the number of potential entrants in each group goes to infinity.

auction is $e^{-\mu} > 0$. The equilibrium condition requiring that the expected payoff of an entrant in the auction is equal to his entry cost leads to:

$$e^{-\mu_{in}^*} \cdot E[\max\{x_E - x_I, 0\}] = C \tag{1}$$

if $E[\max\{x_E - x_I, 0\}] > C$ and $\mu_{in}^* = 0$ otherwise, and[16]

$$e^{-\mu_{out}^*} \cdot E[x_E] = C. \tag{2}$$

When the incumbent is not excluded, the expected revenue for the seller, denoted by $R_{with-1-Inc}$, is then

$$e^{-\mu_{in}^*} \cdot 0 + \mu_{in}^* e^{-\mu_{in}^*} \cdot E[\min\{x_E, x_I\}] + (1 - e^{-\mu_{in}^*} - \mu_{in}^* e^{-\mu_{in}^*}) \cdot E[x_E] \tag{3}$$

where the first term corresponds to the case without any entrant, the second term to the case with a single entrant and the third term to the case with at least two entrants. Plugging (1) into (3) we get the alternative expression

$$R_{with-1-Inc} = E[x_E] - \Big(\frac{E[x_E]}{E[\max\{x_E - x_I, 0\}]} + \mu_{in}^*\Big) \cdot C \tag{4}$$

if $\mu_{in}^* > 0$ (otherwise we have $R_{with-1-Inc} = 0$).

When the incumbent is excluded, the expected revenue of the seller, denoted by $R_{without-Inc}$, is then $(1 - e^{-\mu_{out}^*} - \mu_{out}^* e^{-\mu_{out}^*}) \cdot E[x_E]$ and plugging (2) into this expression, we obtain that

$$R_{without-Inc} = E[x_E] - (1 + \mu_{out}^*) \cdot C. \tag{5}$$

From (1) and (2), we have $e^{\mu_{out}^* - \mu_{in}^*} = \frac{E[x_E]}{E[\max\{x_E - x_I, 0\}]} > 1$. Since $e^x > 1 + x$ if $x > 0$, we obtain finally that $1 + \mu_{out}^* - \mu_{in}^* < \frac{E[x_E]}{E[\max\{x_E - x_I, 0\}]}$ or equivalently that

$$R_{without-Inc} > R_{with-1-Inc}. \tag{6}$$

Interestingly, this inequality does not depend on the relative strengths of the entrants and the incumbent. We will show that the benefit of excluding a single incumbent is actually very general and goes beyond this simple framework with ex-post symmetric entrants. ⋄⋄

In the next example, we illustrate that the advantage of excluding the incumbent is not unique to the Vickrey format and that it applies to the first-price auction case in which the incumbent would have a right to match the best offer of the other participating bidders.

Example 1b (the first price auction with a right-to-match for the incumbent) Reconsider Example 1 if the Vickrey auction is replaced by the first-price auction and the incumbent is allowed to submit his bid after observing the bids of the entrants (giving the incumbent an

[16] The assumption $E[x_E] > C$ guarantees that a solution exists.

obvious second-mover advantage). Assume also for simplicity that the set of entrants is publicly known so that entrants bid x_E if they know they face another entrant and the incumbent bids zero if he knows he is the only bidder in the auction. Let $\beta(x_E)$ denote the equilibrium bid of an entrant when he knows there is no other entrant and his only competitor is the incumbent. We have naturally $\beta(x_E) \leq x_E$. The incumbent matches the bid of the entrants if his own valuation is higher. The equilibrium entry condition can be written as

$$e^{-\mu_{in}^*} \cdot E[(x_E - \beta(x_E)) \cdot 1[x_I < \beta(x_E)]] = C \tag{7}$$

if $E[(x_E - \beta(x_E)) \cdot 1[x_I < \beta(x_E)]] > C$ and $\mu_{in}^* = 0$ otherwise.[17] A similar calculation leads to the analog of (4) for the seller's expected revenue when $\mu_{in}^* > 0$:

$$R_{with-1-Inc} = E[x_E] - \Big(\frac{E[x_E]}{E[(x_E - \beta(x_E)) \cdot 1[x_I < \beta(x_E)]]} + \mu_{in}^* \cdot \gamma\Big) \cdot C \tag{8}$$

where $\gamma = \frac{E[x_E - \beta(x_E)]}{E[(x_E - \beta(x_E)) \cdot 1[x_I < \beta(x_E)]]} \geq 1$. Since $\gamma \geq 1$, the previous argument leading to (6) holds a fortiori. Overall, we obtain that the benefit of excluding the incumbent is somehow reinforced in this auction. Intuitively, a mechanism that advantages the incumbent, or equivalently disadvantages the entrants, reduces the attractiveness for potential entrants (here formally we have $E[(x_E - \beta(x_E)) \cdot 1[x_I < \beta(x_E)]] \leq E[\max\{x_E - x_I, 0\}]$) but also increases the rents of the incumbent for any given number of entrants (here the expected rents of the incumbent is $E[(x_I - \beta(x_E)) \cdot 1[x_I \geq \beta(x_E)]]$ which is larger than the corresponding rate $E[(x_I - x_E) \cdot 1[x_I \geq x_E]]$ in the Vickrey auction).[18] ◇◇

The next two examples explore the effect of set-asides in the presence of multiple incumbents. For this, we consider again the Vickrey auction.

Example 1c (the Vickrey auction with multiple symmetric incumbents) Reconsider Example 1a but with two (or more) incumbents who have the same valuation ex-post. From the potential entrants' perspective, it does not make any difference such that the equilibrium entry rate is equal to the same rate μ_{in}^* as in the case with a single incumbent. The expression of the seller's revenue, denoted by $R_{with-2-Inc}$, is then

$$R_{with-2-Inc} = e^{-\mu_{in}^*} \cdot E[x_I] + \mu_{in}^* e^{-\mu_{in}^*} \cdot E[x_I] + (1 - e^{-\mu_{in}^*} - \mu_{in}^* e^{-\mu_{in}^*}) \cdot E[\max\{x_E, x_I\}]. \tag{9}$$

After some calculus and plugging (1), we obtain that

$$R_{with-2-Inc} = E[\max\{x_E, x_I\}] - (1 + \mu_{in}^*) \cdot C \tag{10}$$

if $\mu_{in}^* > 0$, and $R_{with-2-Inc} = E[x_I] \geq E[\max\{x_E, x_I\}] - C$ otherwise. Since $\mu_{in}^* < \mu_{out}^*$, we obtain that

[17] We use the notation $1[A]$ where $1[A] = 1$ (resp. $1[A] = 0$) if statement A is true (resp. false).

[18] An interesting feature here is that we can conclude that the exclusion of the incumbent is good even without solving the equilibrium strategy of the entrants (which need not be straightforward).

$$R_{with-2-Inc} > R_{without-Inc}. \tag{11}$$

While it was beneficial to exclude the incumbent when there was only one, we see that the picture is completely different when there are multiple incumbents. ◇◇

However, contrary to the result with a single incumbent, the impact of excluding one incumbent is ambiguous in general when there are multiple incumbents. In particular, the inequality (11) relies crucially on the implicit assumption that incumbents have no informational rents since their valuation are perfectly correlated.

Example 1d (the Vickrey auction with asymmetric incumbents) To deepen the discussion with multiple incumbents, consider a case with only two incumbents. We now introduce some asymmetry between the two incumbents: one incumbent, designated as a weak incumbent, has the valuation x_I with probability ϵ (i.e. the same valuation as the other -strong- incumbent) and a null valuation with the remaining probability $1-\epsilon$. It is straightforward to see that excluding the weak incumbent is always detrimental since his presence or absence does not modify the entry rate while he may reduce the rent of the other incumbents. Whether it is profitable to exclude the strong incumbent and/or both incumbents depends on the strength of the weak incumbent: the weaker he is, the closer we are to a situation as if there were only one incumbent so that exclusion is profitable. To illustrate how rich the situation can be with multiple incumbents and contrasting with Example 1c, we provide a specification in the Appendix where it is detrimental to exclude each incumbent in isolation but would be good to exclude both incumbents. ◇◇

3 Preliminaries

3.1 Some definitions

In this section, we assume that the auction format is the Vickrey auction. We let $N = (n_1, \dots, n_K) \in \mathbb{N}^K$ denote a realization of the profile of entrants, $N_{-k} = (n_1, \dots, n_{k-1}, n_k - 1, n_{k+1} \dots, n_K)$ and $N_{+k} = (n_1, \dots, n_{k-1}, n_k + 1, n_{k+1} \dots, n_K)$. For a given (nonempty) set of incumbents $I \subseteq \mathcal{I}$ and $i \in I$, we let $I_{-i} = I \setminus \{i\}$. The following notation will be useful to present the analysis.

- $F^{(1:N\cup I)}(x) := E_Z[\prod_{k=1}^K [F_k(x|z)]^{n_k} \cdot \prod_{i\in I} F_i^I(x|z)]$ denotes the CDF of the first order statistic among the set of entrants N and the set $I \subseteq \mathcal{I}$ of incumbents. If $N = (0, \dots, 0)$ and $I = \emptyset$, then we adopt the convention that $F^{(1:N\cup I)}(x) = 1$.
- $P(N|\mu) = e^{-\sum_{k=1}^K \mu_k} \cdot \prod_{k=1}^K \frac{[\mu_k]^{n_k}}{n_k!}$ denotes the probability of the realization N when the entry rate vector is μ, namely when the Poisson distribution of group k buyer has mean $\mu_k \geq 0$ for any $k \in \mathcal{E}$.

- $V_k^{ent}(N, I)$ [resp. $V_i^{inc}(N, I)$] denotes the expected (interim) gross payoff (i.e. without taking into account the entry costs) of a buyer from group k [resp. the incumbent i] when the set of participants consists of the profile of potential entrants $N \in \mathbb{N}^K$ with $n_k \geq 1$ [resp. $N \in \mathbb{N}^K$] and the set of incumbents $I \subseteq \mathcal{I}$.

- $\Pi_k^{ent}(\mu, I) = \sum_{N \in \mathbb{N}^K} P(N|\mu) \cdot V_k^{ent}(N_{+k}, I) - C_k$ denotes the expected (ex ante) payoff of a group k buyer net of the entry cost C_k when the profile of entry rate is $\mu \in \mathbb{R}_+^K$ and the set of incumbents is $I \subseteq \mathcal{I}$.[19]

- $\Pi_i^{inc}(\mu, I) = \sum_{N \in \mathbb{N}^K} P(N|\mu) \cdot V_i^{inc}(N, I)$ denotes the expected (ex ante) payoff of the incumbent i when the profile of entry rate is $\mu \in \mathbb{R}_+^K$ and the set of incumbents is $I \subseteq \mathcal{I}$.

Given the criteria we are interested in, we define for every realization (N, I) of participants, the expected (interim) gross welfare (i.e. the sum of all agents' utilities excluding the entry costs):

$$W(N, I) := X_S \cdot F^{(1:N \cup I)}(X_S) + \int_{X_S}^{\overline{x}} x dF^{(1:N \cup I)}(x)$$

and the expected (interim) seller's payoff (consisting of X_S when the good is not sold and the revenue otherwise):

$$\Phi(N, I) := W(N, I) - \sum_{k=1}^{K} n_k \cdot V_k^{ent}(N, I) - \sum_{i \in I} V_i^{inc}(N, I).$$

With some abuse of terminology, we refer to $\Phi(N, I)$ as revenue in the rest of the paper.

We can now state more formally the conditions for a profile of entry rates μ to be an equilibrium. For a given set-asides policy (I, E), we say that an entry profile μ is part of an equilibrium if $\mu_k = 0$ if $k \notin E$, and for any $k \in E$,

$$\Pi_k^{ent}(\mu, I) \underset{(resp. \leq)}{=} 0 \text{ if } \mu_k \underset{(resp. =)}{>} 0. \tag{12}$$

Let $J(I, E) \subseteq \mathbb{R}_+^K$ denote the set of entry profiles compatible with equilibrium behavior for the set-asides policy (I, E), and let $\mu^*(I, E) \in J(I, E)$ refer to one such equilibrium.[20]

For a given participation profile μ and a set of incumbents $I \subseteq \mathcal{I}$, we define the total expected (ex ante) net welfare (net of the expected entry costs) by

$$TW(\mu, I) := \sum_{N \in \mathbb{N}^K} P(N|\mu) \cdot W(N, I) - \sum_{k=1}^{K} \mu_k \cdot C_k \tag{13}$$

[19] Note that from the perspective of any entrant no matter what his group k is, the probability that he faces the set of entrants N (excluding himself) is also equal to $P(N|\mu)$. This fundamental property of Poisson games is referred to as environmental equivalence in Myerson (1998).

[20] We will establish in the proof of Lemma 3.1 that $J(I, E) \neq \emptyset$ for any pair (I, E) so that such an equilibrium exists. The participation rates cannot be infinite as it would result in negative expected net payoffs when participating (since the entry costs C_k are assumed to be strictly positive).

and the corresponding expected seller's payoff (that from now on we will refer as revenue) by $R(\mu, I) := \sum_{N \in \mathbb{N}^K} P(N|\mu) \cdot \Phi(N, I)$.

From the equilibrium condition (12), the revenue of the seller given the set-asides policy (I, E) can be rewritten as

$$R(\mu^*(I,E), I) = TW(\mu^*(I,E), I) - \sum_{i \in I} \Pi_i^{inc}(\mu^*(I,E), I). \tag{14}$$

In the rest of the paper, we are interested in how $TW(\mu^*(I,E), I)$ and $R(\mu^*(I,E), I)$ vary with the set-asides policy (I, E). Clearly, $TW(\mu, I)$ and $R(\mu, I)$ are increasing in the set of incumbents I for a given μ because, in the Vickrey auction, the good is allocated to the agent who values the good most and the payment when the good is sold can only increase when there are more participants. Forbidding the access of some bidders say in I would a priori boost the participation rate of the potential entrants, which would be favorable both to TW and R. The question is how the two effects aggregate.

3.2 Welfare criterion

When the criterion is welfare, we show that it is never good to exclude incumbents or entrants. The following lemma is a key property used repeatedly in our analysis. For any set I of allowed incumbents, the equilibrium entry profile must be one that maximizes the welfare given I. While Jehiel and Lamy (2015a) establish this result in environments without incumbents, the extension to the case with incumbents is straightforward. [21]

Lemma 3.1 *$\mu^*(I, \mathcal{E}) \in Arg\max_{\mu \in \mathbb{R}_+^K} TW(\mu, I) \neq \emptyset$ for any $I \subseteq \mathcal{I}$.*

To help understand some of the following results, we now provide a sketch of the key steps in the proof (while for completeness a full proof appears in the Appendix). A fundamental property of the Vickrey auction is that the ex post utility of each bidder corresponds exactly to his marginal contribution to the welfare. Applied to a potential entrant from group k, this implies (from an interim perspective) that

$$W(N_{+k}, I) - W(N, I) = V_k^{ent}(N_{+k}, I). \tag{15}$$

From an ex ante perspective and given the Poisson model, such a property translates into:

$$\frac{\partial TW(\mu, I)}{\partial \mu_k} = \sum_{N \in \mathbb{N}^K} P(N|\mu) \cdot V_k^{ent}(N_{+k}, I) - C_k = \Pi_k^{ent}(\mu, I). \tag{16}$$

That is, the entry profiles resulting from equilibrium behavior correspond to local maxima of the total welfare function. The final step in the proof consists in showing that the function $\mu \rightarrow$

[21] The result holds true for all I because from the viewpoint of entrants, the presence of incumbents is similar to that of having stochastic reserve prices in the setup without incumbents.

$TW(\mu, I)$ is globally concave, thereby ensuring that any local maximum is a global maximum.[22]

Since $TW(\mu, I)$ is non-decreasing in I, it follows that $\max_{\mu\in\mathbb{R}_+^K} TW(\mu, I)$ is also non-decreasing in I. From Lemma 3.1, we obtain then that excluding incumbents can only reduce the net welfare. Hence, the first-best welfare $\max_{\mu\in\mathbb{R}_+^K} TW(\mu, \mathcal{I})$ can be implemented with the no-exclusion policy.

Proposition 3.2 *In the Vickrey auction, the welfare-optimal set-asides policy involves no exclusion.*

While this subsection is concerned with welfare, we make the following two observations concerning revenues. If there are no incumbents, the net welfare and the seller's revenue coincide as shown in (14). As a corollary of Proposition 3.2, we have that

Corollary 3.3 *In the Vickrey auction without incumbents, the revenue-optimal set-asides policy involves no exclusion.*[23]

We refer to cases in which the rents of incumbents are null, i.e., $V_i^{inc}(N, \mathcal{I}) = 0$ for any $N \in \mathbb{N}^K$ and $i \in \mathcal{I}$, as situations with "*full competition among incumbents*". Obviously, several incumbents are required for this to be true and it typically arises as illustrated in Example 1c when there are several incumbents with the same highest valuations. In the full competition among incumbent case, we get from (14) that $\max_{\mu\in\mathbb{R}_+^K} TW(\mu, \mathcal{I})$ is an upper bound on the seller's revenue. Since Proposition 3.2 has established that this bound is reached by the seller under the no exclusion policy, we can extend an observation made in Example 1c:

Corollary 3.4 *In the Vickrey auction with "full competition among incumbents", the revenue-optimal set-asides policy involves no exclusion.*

4 Excluding a single incumbent boosts revenues

In the presence of incumbents, revenue maximization involves a trade-off between welfare maximization and the minimization of the rents of the incumbent (as reflected by (14)). On the one hand, from Proposition 3.2, set-asides impact negatively the revenue through the welfare term. On the other hand, excluding incumbents can serve the purpose of eliminating the rents of these. Although it is in general unclear what the direction of the trade-off is, it turns out that when there is a single incumbent, excluding the incumbent is always good for revenues in the Vickrey auction.

[22] The entry game presents some analogy with the theory of potential games (Monderer and Shapley, 1996) where the potential function here would be the total welfare. Technically, there is still a difference since the Poisson model involves implicitly a continuum of players. We know from the theory of potential games that any global maximum of the potential function constitutes an equilibrium and that the converse is true if the potential function is concave (Neyman, 1997).

[23] Jehiel and Lamy (2015a) adopt a mechanism design approach à la Myerson (1981) and show the much stronger result that the Vickrey auction is the optimal mechanism when there are no incumbents.

In words, the argument is as follows. In the Vickrey auction, for any realization N of the profile of entrants, the expected rent of the incumbent $V_i^{inc}(N, I)$ coincides exactly with his marginal contribution to the welfare $W(N, I) - W(N, I_{-i}) = V_i^{inc}(N, I)$ (this is the fundamental property of the Vickrey auction already used for entrants to prove Lemma 3.1). We obtain then (from eq. (14)) that the expected revenue of the seller coincides with the (hypothetical) total welfare were the incumbent to be absent. Since the latter welfare is maximized when the incumbent is excluded and all entrants whatever their group are allowed to participate, we obtain:

Theorem 1 *When there is a single incumbent, the revenue-optimal set-asides policy in the Vickrey auction consists in excluding the incumbent and allowing entrants whatever their group to participate. That is, the revenue-maximizing set-asides policy is* $(I, E) = (\emptyset, \mathcal{E})$.

Proof Since the ex post utility of the incumbent coincides with his marginal contribution to the welfare, we obtain the analog of (15) but for an incumbent $W(N, I) - W(N, I_{-i}) = V_i^{inc}(N, I)$ and then from an ex ante perspective

$$\Pi_i^{inc}(\mu, I) = TW(\mu, I) - TW(\mu, I_{-i}) \tag{17}$$

which can be viewed as the analog of (16). Plugged into (14) and for environment with a single incumbent, we obtain that for any set-asides policy (I, E),[24] that the revenue of the seller is given by

$$R(\mu^*(I, E), I) = TW(\mu^*(I, E), \emptyset) \leq TW(\mu^*(\emptyset, E), \emptyset) = R(\mu^*(\emptyset, E), \emptyset) \tag{18}$$

where the middle inequality comes from Lemma 3.1. **Q.E.D.**

It should be highlighted that the conclusion of Theorem 1 holds true irrespective of the shape of the distributions of valuations. In particular it covers the case where there is correlation between bidders' valuations and/or bidders can receive information about others' valuations at the auction stage. Theorem 1 can thus be viewed as being immune to the Wilson critique (see Wilson, 1987).

In some cases, it may be that the seller is not allowed to exclude the incumbent. One may then be interested in whether or not it is good for revenues to exclude some groups of entrants from the auction. Obviously, if there is only one group of entrant, this cannot be good as excluding the entrants would reduce the revenues to X_S (and the seller is bound to get more than X_S in the Vickrey auction with positive participation). When there are several groups of entrants, one might have thought based on Theorem 1 that it is not a good idea to exclude any group of entrants. This turns out be incorrect as illustrated in the following example.

Example 2 Consider one incumbent having for sure valuation x_I. Consider two groups of potential entrants. With probability $q \in (0, 1)$, all entrants have the high valuation $\overline{x}_E > x_I$.

[24] Namely, either if $I = \{i\}$ or if $I = \emptyset$.

With probability $1-q$, entrants from group 1 (resp. 2) have the low valuation $\underline{x}_E^1 < x_I$ (resp. $\underline{x}_E^2 < x_I$). We assume that $\underline{x}_E^1 < \underline{x}_E^2$ and that their entry costs, denoted by C_k for $k = 1, 2$, are such that $C_1 < C_2$ with the difference $C_2 - C_1$ being small enough as made precise below. We also assume that $\overline{x}_E - x_I > C_2$, which guarantees that some entry arises in equilibrium. The expected gross profit of an entrant is the same whether he comes from group 1 or group 2. However, since the entry cost is bigger in group 2 than in group 1, when there are no set-asides, then only bidders from group 1 participate. Formally, the profile of equilibrium entry rates (μ_1, μ_2) without set-asides is such that

$$q \cdot e^{-\mu_1}(\overline{x}_E - x_I) = C_1$$

and $\mu_2 = 0$. By contrast, when bidders from group 1 are excluded then the equilibrium entry rate of group 2 is given by $\widetilde{\mu}_2$ such that

$$q \cdot e^{-\widetilde{\mu}_2}(\overline{x}_E - x_I) = C_2.$$

As the difference $C_2 - C_1$ gets small, we have that $\widetilde{\mu}_2 \approx \mu_1$. The welfare is smaller with exclusion (as we already know from Proposition 3.2), but the difference becomes negligible here. However, given that $\underline{x}_E^1 < \underline{x}_E^2$, the seller's revenue is larger when group 1 is excluded (and the difference, which is approximately equal to $(1 - e^{-\mu_1}) \cdot (1-q) \cdot (\underline{x}_E^2 - \underline{x}_E^1) > 0$, is non negligible when $C_2 - C_1$ gets small).

In the above example, there is a discrepancy between the ability to reduce the rents of the incumbent and the ability to increase the welfare. Group 2's bidders are more effective for the former while group 1's bidders are more effective for the latter. It is then intuitive that it can be profitable to exclude those potential participants who have a greater ability to increase the welfare (so that they will enter without exclusion and discourage the other group to participate) but less ability to reduce the incumbent's rents. $\diamond\diamond$

Understanding further which groups of entrants should be excluded when the incumbent is present is not straightforward given the potential complex effect of E on the participation rates. To illustrate some counter-intuitive effects, Example 3 in Appendix shows that it may be good to exclude a group that is dominated both in terms of entry costs and in terms of the distribution of valuations by another group.[25]

Simple extensions: 1) Theorem 1 relies on our large market assumption that has led us to adopt the Poisson formulation. Alternatively, we could consider a model with a finite number of potential entrants in each group (assumed to be sufficiently large so that the interim

[25]The intuition is the following: The example relies on a "strong group" where valuations among bidders are perfectly correlated and a "weak group" where valuations are drawn independently of each other. Due to the perfect correlation in the strong group, bidders do not enter much in equilibrium (because their payoff are null when two of them enter). By contrast, the equilibrium level is higher in the weak group (which is useful to reduce the rents of the incumbent when he wins the auction but have to pay the highest valuation among his competitors) which counterbalances at the end the fact that for a given set of competitors, bidders from the strong group increase more the revenue than those from the weak group.

expected payoff of any entrant from group k whould still be equal to C_k) and we would obtain the same result provided that different entrants from the same group participate with the same probability (symmetry assumption) and that the seller is able to guarantee that the equilibrium entry probability without the incumbent is the one she prefers (among equilibrium ones).[26] 2) The optimality of the pivot mechanism when there are no incumbents extends to any assignment problem under private values when the seller can coordinate potential entrants on the entry equilibrium she prefers as developed in Jehiel and Lamy (2015b) in a public good perspective. Similarly, Theorem 1 holds for general assignment problems when the pivot mechanism is used, including multi-object generalized Vickrey auction. 3) Assuming the seller cannot use reserve prices, Theorem 1 extends straightforwardly to the extent that the incumbent's valuation is always positive. In the second price auction without reserve price, the seller's revenue corresponds to her revenue as if her true valuation were 0 plus the probability that the good remains unsold times X_S. As a direct application of Theorem 1, the first term is maximized when the incumbent is excluded. We conclude after noting that the second term is also maximized when the incumbent is excluded given that when the incumbent's valuation is positive, the good is always sold when he is in. [27]

5 Beyond the Vickrey auction

In some procurement applications, bidders are not treated symmetrically. For example, some bidders may be better at renegotiating the terms of the contracts, thereby giving them an advantage at the auction stage (winning at the same official price would translate in a lower effective price for such bidders). Or bidders may be asymmetric in the risk of breakdown. In other cases, some bidders may be allowed to bid after seeing the offers of others, thereby giving them a second-mover advantage. In all such cases in which bidders may be treated asymmetrically, the auction format cannot be viewed as being equivalent to the Vickrey auction, and it is of interest to analyze whether and when excluding the incumbent or some groups of entrants may be beneficial to the seller.

Our first insight in this Section is derived from the following observation. A simple inspection of the argument used to prove Theorem 1 reveals that the benefit of excluding the incumbent carries over to situations in which the rent obtained by the incumbent is no smaller than his marginal contribution to the welfare (in the Vickrey auction, it is just equal) and the good would be allocated efficiently among entrants. We identify classes of auction formats in which this would be the case. Roughly, they correspond to formats which give an advantage to the incumbent,

[26] This would also correspond to an equilibrium selection that is popular in game theory when one deals with a potential game (as it is the case here and where the potential function is the welfare net of the entry costs, see footnote 22): it consist of selecting the equilibrium that maximizes globally (and not only locally) the potential (see Haufbauer and Sorger (1999) and Carbonell-Nicolau and McLean (2014)). Such a discussion appears in Jehiel and Lamy (2015a).

[27] More generally, the argument extends as long as the reserve price is below the seller's valuation while the incumbent's valuation is always above it.

and we extent the insight about the exclusion of the incumbent to such classes of mechanisms. In order to cover also the exclusion of groups of entrants, we also consider mechanisms that favor one group of entrants and disadvantage another one. After deriving our results within an abstract class of mechanisms in which bidders have a (weakly) dominant strategy, we show how these results can be used in more concrete applications, such as ones resulting from asymmetric abilities to renegotiate the contracts or the right of first refusal. We also discuss the implications of our results when first-price auctions are used.

5.1 Preliminaries: some fundamental properties

An assignment rule, denoted by ϕ, is a function which maps any realization of bidders' valuations to a vector of probabilities characterizing the probability that each participant receives the good. We say that an assignment rule is deterministic if the vectors of probabilities are composed only of 0 and 1. E.g. the assignment rule ϕ^{eff} associated to the Vickrey auction is one that assigns the probability one to the participant (including the seller herself) who has the highest valuation.[28]

Definition 2 *We say that an assignment rule advantages (resp. disadvantages) a bidder if this bidder gets the good with probability 1 [resp. 0] if (ex post) efficiency dictates to assign (resp. not to assign) it to him.*

Relatedly, we will say that an auction advantages (or disadvantages) a given bidder if in equilibrium the associated assignment rule advantages (or disadvantages) the given bidder. For auctions in which bidders have a dominant strategy, we will use the assignment rule ϕ induced by the auction mechanism to designate the auction format.[29]

Let us illustrate the previous definition with a class of assignment rules when there is at most one incumbent. Consider $r \in \mathbb{R}_+$ and $b : [r, \infty) \to [X_S, \infty)$ an increasing and continuous function with $b(r) = X_S$. Let us define an assignment rule $\phi(b, r)$ in the following way:[30]

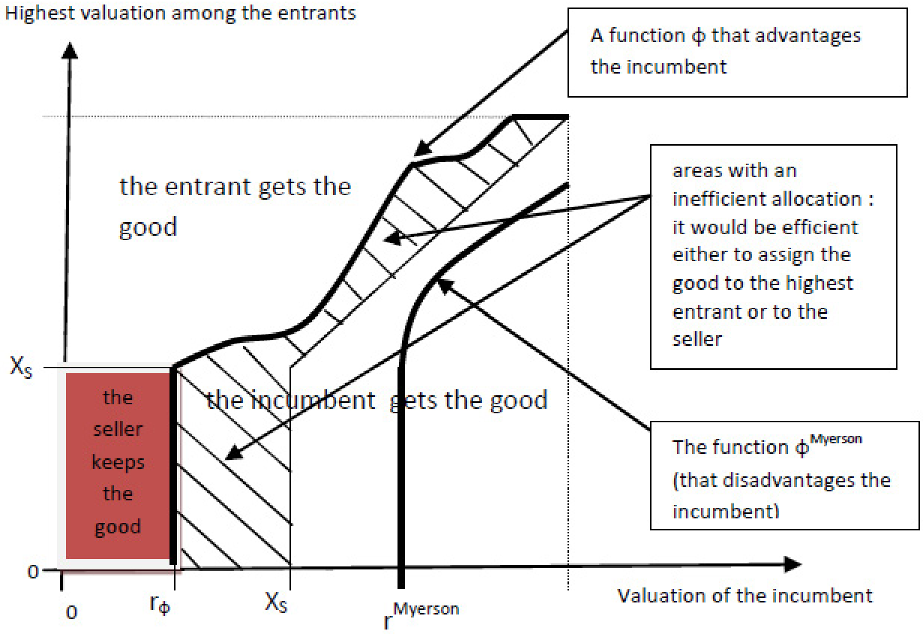
- When all entrants have a valuation below X_S and the incumbent has a valuation below r, the seller keeps the good,
- When the incumbent has a valuation below r and there is at least one entrant with a valuation strictly above X_S, the good is assigned to the entrant with the highest valuation,
- When the incumbent has a valuation strictly above r and there is no entrant with a valuation strictly above X_S, the good is assigned to the incumbent,

[28] When several bidders have the same valuation in the Vickrey auction, it does not matter how to break ties in terms of bidders payoffs.

[29] In general, this terminology is abusive because it relies on the endogenous equilibrium behavior induced by the auction rules.

[30] If the incumbent is excluded then it corresponds to say that he has a null valuation so that he never gets the good. Similarly, if there is no entrants we adopt the convention that the highest valuation among the entrants is null.

Figure 1: Examples of assignment rules



- When the incumbent has a valuation x^I strictly above r and the highest valuation among the entrants x^E is strictly above X_S, the good is assigned to the incumbent if $b(x^I) \geq x^E$ and to the entrant with the highest valuation otherwise.

It is readily verified that a $\phi(r, b)$-assignment rule advantages (resp. disadvantages) the incumbent if $b(x) \geq x$ and $r \leq X_S$ (resp. $b(x) \leq x$ and $r \geq X_S$). The assignment rule associated to the Vickrey auction is the knife-edge case where $r = X_S$ and $b(x) = x$.

This construction is illustrated in Figure 1 in which two assignment rules are depicted in bold. The assignment rule $\phi(r, b)$ delineates three areas: in the red rectangle the seller keeps the good, above the bold line associated to b and above the red rectangle the entrant with the highest valuation gets the good, below the bold line associated to b and on the right to the red rectangle the incumbent gets the good. This assignment rule is inefficient. More precisely, inefficiencies occur when we are in the shaded area: the incumbent gets the good although efficiency would dictate to put it in the hands of another agent (an entrant or the seller herself). This assignment rule advantages the incumbent and on the contrary disadvantages the entrants.

The assignment rule $\phi^{Myerson}$ depicts the one associated to the optimal mechanism in a Myersonian environment as characterized in Jehiel and Lamy (2015a). It disadvantages the incumbent and advantages the entrants. [31]

It is well-known in mechanism design that bidders' expected rents are characterized (either for dominant-strategy mechanisms or for general mechanisms in Myersonian environments) by

[31] More precisely, we have $\phi^{Myerson} = \phi(r^{Myerson}, b^{Myerson})$ where $r^{Myerson}$ denotes the solution to $r^{Myerson} - \frac{1-F^I(r^{Myerson})}{f^I(r^{Myerson})} = X_S$ and the function $b^{Myerson}(x) = x - \frac{1-F^I(x)}{f^I(x)} \leq x$ where F^I (resp. f^I) denotes the CDF (resp. PDF) of the incumbent's valuation.

the assignment rule and the expected monetary transfer of a bidder with the lowest type. In the next results (up to Section 5.4), we consider so-called "dominant-strategy auctions without fees" where a bidder with a zero valuation is always null.

Lemma 5.1 *In a dominant-strategy auction without fees that advantages (resp. disadvantages) incumbent i and that always assigns the good efficiently among the remaining bidders and the seller, the payoff of incumbent i is larger (resp. smaller) than his marginal contribution to the welfare.*

In other words, for any realization of bidders' valuations, incumbent i if advantaged (resp. disadvantaged) grabs more (resp. less) than the ex post surplus he brings by his presence. Formally, after adapting our notation about the welfare and payoff functions to make them depend on the assignment rule ϕ associated to the given dominant-strategy auction, we obtain $W(N, I; \phi) - W(N, I_{-i}; \phi) \leq V_i^{inc}(N, I; \phi)$ (resp. $W(N, I; \phi) - W(N, I_{-i}; \phi) \geq V_i^{inc}(N, I; \phi)$) if the incumbent i is advantaged (resp. disadvantaged) and if the auction always assigns the good efficiently among the remaining bidders and the seller. From an ex ante perspective, when the incumbent is advantaged, we obtain for any entry profile μ that

$$TW(\mu, I; \phi) - TW(\mu, I_{-i}; \phi) \leq \Pi^{inc}(\mu; \phi). \tag{19}$$

As an illustration, when there is a single incumbent (I is a singleton), in a dominant strategy auction without fees that implements a $\phi(b, r)$-assignment rule and if we let $G_\mu(.)$ denote the CDF of the highest valuation among the entrants when the entry profile is $\mu \in \mathbb{R}_+^K$, when the incumbent's valuation is distributed independently of the entrants' valuation, standard calculation leads to (see the Appendix for details)

$$\begin{aligned} TW(\mu, I; \phi(b,r)) - TW(\mu, \emptyset; \phi(b,r)) = \Pi^{inc}(\mu, I; \phi(b,r)) + (r - X_S) \cdot G_\mu(X_S) \cdot (1 - F^I(r)) \\ + \int_r^\infty (x - b(x)) \cdot (1 - F^I(x)) \cdot d[G_\mu(b(x))]. \end{aligned} \tag{20}$$

It is clear from the previous expression that the more the incumbent is advantaged (when $b(x)$ gets larger and r smaller), the wider is the discrepancy between his payoff and his marginal contribution to the welfare.

As will prove useful when considering the exclusion of (some groups of) entrants, a similar argument can be developed to compare a given bidder's payoff to his marginal contribution to the welfare in the class of loser neutral assignment rules that require that a losing bidder should not influence (through his bid) how the good is assigned to the remaining bidders.

Formally, letting $\phi_i(x_1, \cdots, x_n)$ denote the probability that bidder i (with valuation x_i) gets the good in the assignment rule ϕ, loser neutrality is defined as:

Definition 3 *An assignment rule is loser neutral if for any vector X and any bidder i such that $\phi_i(x_1, \cdots, x_n) < 1$, we have $\phi_j(x_1, \cdots, x_n) = \frac{\phi_j(x_{-i})}{1-\phi_i(x_1,\cdots,x_n)}$ for any $j \neq i$.*

We have:

Lemma 5.2 *In a loser neutral dominant-strategy auction without fees that advantages (resp. disadvantages) a given bidder, the payoff of the given bidder is larger (resp. smaller) than his marginal contribution to the welfare.*

Remark. From standard arguments in mechanism design, it is well-known that the more a bidder gets the good the greater his payoff. This in turn yields Lemma 5.1 when the incumbent is advantaged given that in such a case $W(N, I; \phi) - W(N, I_{-i}; \phi)$ is less than the corresponding amount in the Vickrey auction. For either Lemma 5.1 when the the incumbent is disadvantaged or Lemma 5.2, the argument is less straightforward.

5.2 Set-asides in dominant-strategy auctions

Excluding the incumbent when advantaged:

As an application of Lemma 5.1, the payoff of the incumbent is always larger than his marginal contribution to the welfare in auctions that favor the incumbent. This implies that the seller's revenue is still bounded from above by the total welfare maximizing solution in the absence of the incumbent (specifying that the good is allocated to the agent -entrant or seller- with highest valuation and participation rates are defined to maximize total expected welfare). Since such an upperbound on revenues is reached by excluding the incumbent and keeping all groups of entrants, we obtain the following result, which constitutes a generalization of Theorem 1.

Theorem 2 *Consider an environment with a single incumbent and a dominant-strategy auction without fees which advantages the incumbent and always assigns the good efficiently among the remaining bidders and the seller. The revenue-optimal set-asides policy consists in excluding the incumbent and allowing all groups of entrants to participate.*

Keeping the disadvantaged entrants:

Turning to entrants, we observe making use of Lemma 5.2 that in the absence of incumbents and with two groups of entrants, if the auction format advantages one group and disdvantages the other, it is detrimental for revenues to exclude the disadvantaged group.

Theorem 3 *Consider environments without incumbents and with two groups of entrants ($K = 2$) and assume that the equilibria played are the ones that are most preferred by the seller. Consider a loser neutral dominant strategy auction without fees. If the auction disadvantages (resp. advantages) bidders from group 1 (resp. 2), then it can only be detrimental to exclude bidders from group 1.*

5.3 Applications of Theorems 2 and 3

Theorems 2 and 3 can be used to shed light on a number of applications in which bidders are not treated alike. We first consider the case of procurements in which the terms of the contracts can be renegotiated after the auction stage and the incumbent would be better than the entrants at such renegotiations. We next consider the possibility that bidders may go bankrupt after the auction in which case the contract promised at the auction stage would not be honored, and we allow for assymetries in the risk of bankrupcy. Such considerations are of primary importance in the context of procurement auctions as reported among others in Spulber (1990). Finally, we consider first-price auctions and the possibility that the incumbent would have the option to match the best offer of the entrants, as sometimes observed in procurement auctions (see for example, Lee (2008)).

5.3.1 Asymmetric renegotiation facilities

Renegotiation is an important dimension of procurements as emphasized by Bajari, Houghton and Tadelis (2014). There is typically an important discrepancy between initial bids and final payments,[32] where the discrepency comes from the fact that contracts are renegotiated due to unforseen contingencies that require adaptation costs. Bajari et al. (2014) argue that adaptation costs are of larger magnitude than the losses due to imperfect competition at the auction stage. Importantly, firms are not on equal footing to obtain good deals at the renegotiation stage (presumably incumbents who have more familiarity are better at obtaining good deals). Such asymmetries at the renegotiation stage lead to asymmetric bidding behaviors at the auction stage despite the fact that the auction format seems to be treating all bidders in the same way.

Let us consider a very simple model of renegotiation in the Vickrey auction. Each bidder i has some ability to renegotiate the final price, which is modeled as follows. Let $\beta_i : [r_i, \infty) \to [X_S, \infty)$ denote an increasing function such that if bidder i wins the auction and is supposed to pay $p \geq X_S$ then the effective price after renegotiation is $\beta_i^{-1}(p) \geq r_i \equiv \beta_i^{-1}(X_S)$.[33] Note that r_i can be interpreted as the effective reserve price bidder i faces. Due to the renegotiation stage and assuming that bidders perfectly anticipate the discrepancy between the final bid and the effective price they will pay, bidders no longer have an incentive to bid their valuation: it is now a weakly dominant strategy for bidder i to bid $\beta_i(x)$ for any valuation $x \geq r_i$ and not to participate otherwise. If all bidders have the same ability to renegotiate (i.e. the same function β) and if the corresponding effective reserve price is $r = X_S$, then the second-price auction with the reserve price X_S implements the true Vickrey auction payoffs, namely the ones corresponding to bidders marginal contribution to the welfare, and we can apply Theorem 1. But, when bidders differ in

[32] One of the most spectacular example is Sydney opera house budgeted at an initial cost of $7 million and ended up costing more than $100 million (see Flyvbjerg (2005) for practical elements on cost overruns).

[33] It does not matter that the effective price paid ex post is a deterministic function of the final auction price. We only need to interpret $\beta_i^{-1}(p)$ as the *expected* price that bidder i would pay if the final price in the auction is p.

terms of ability to renegotiate, there is now some inefficiency at the auction stage, and it is of interest to analyze what these inefficiencies imply in terms of the desirability of the exclusion of some bidders.

Consider two bidders i and j. We say that bidder i is a better renegotiator than bidder j if $\beta_i(x) \geq \beta_j(x)$ for any $x \geq r_j$ and if $r_i \leq r_j$. Consider first environments in which all entrants have a (common) ability to renegotiate that may differ from that of the incumbent. Specifically, let β_{ent} (resp. β_{inc}) denote the associated renegotiation function for entrants (resp the incumbent). If $\beta_{inc}(x) \neq \beta_{ent}(x)$ for some x on bidders valuation distribution, then the second price auction (with the reserve price X_S) is no longer ex post efficient. If $\beta_{inc}(x) \geq \beta_{ent}(x)$ and $r_{inc} \leq X_S = r_{ent}$, the incumbent is advantaged while the auction is efficient among entrants and the seller, and we can then apply Theorem 2.

Corollary 5.3 *If the incumbent is a better renegotiator ex-post ($\beta_{inc}(x) \geq \beta_{ent}(x)$, for any $x \geq r_{ent}$, and $r_{inc} \leq r_{ent}$) and if $r_{ent} = X_S$, then the revenue-optimal set-asides policy consists in excluding the incumbent in the second-price auction with the reserve price X_S.*

Consider next environments without incumbents and with two groups of entrants ($K = 2$) having asymmetric abilities to renegotiate. Let β^i_{ent} denote the function that characterizes the ability to renegotiate of entrants from group $i = 1, 2$. If $\beta^2_{ent}(x) \geq \beta^2_{ent}(x)$ (for any $x \geq r^1_{ent}$) and $r^2_{ent} \leq X_S \leq r^1_{ent}$, then entrants from group 1 (resp. 2) are disadvantaged (resp. advantaged) and we can apply Theorem 3.

Corollary 5.4 *Suppose there are two groups of entrants, entrants from group 2 are better renegotiators than entrants from group 1 and if $r^2_{ent} \leq X_S \leq r^1_{ent}$. Then it is detrimental to exclude bidders from group 1 in the second-price auction with the reserve price X_S.*[34]

Comment. In some procurements, bid subsidies[35] are used to favor some kinds of bidders, e.g. domestic bidders or small businesses. In practice, it typically takes the form of linear bid subsidies: it means that if a favored bidder wins the auction at price p, he will pay only $(1-\alpha)p$ where $\alpha \in (0, 1)$. A bid subsidy is then analogous to an ex-post renegotiation as formalized above and the above analysis can equally be applied to such contexts.

5.3.2 B. Asymmetric risks of bankruptcy

Aside from renegotiation, another major concern in procurement auctions is the risk of bankrupcy, and different firms may have different risks of bankrupcy (see Spagnolo (2012) and Saussier and Tirole (2015)).[36] Several works have formalized the risk of failure (see e.g. Zheng

[34] We still need an equilibrium selection. To alleviate the presentation we omit it here as in our subsequent corollaries of Theorem 3 .

[35] See Athey, Coey and Levin (2013), Krasnokutskaya and Seim (2011) and Marion (2007).

[36] Abnormally low bids are often perceived as irregular and are discarded on the ground that the risk of failure is too high.

(2001), Board (2007), Burguet et al. (2012)) through models in which the winner has the option not to realize the project ex post at a stage where the firms get better informed about it cost. In that literature, the risk of failure is endogenous to the contract to the extent that the exit option is exerted differently depending on the terms of the contract at the auction stage. By contrast we consider below a simpler model in which the risk of bankruptcy is unrelated to what happens at the auction stage. Our model fits better situations in which the risks of breakdown are not driven by the considered procurement assumed to be small in regard of the activity of the firm. For simplicity, we also assume that there are no monetary transfers if bankruptcy occurs in which case the seller is assumed to keep the good. In terms of welfare, this means that when a bidder with valuation x has a probability p to go bankrupt, then the corresponding effective valuation to be counted for the contribution to welfare, referred to as the "correct valuation", is $(1-p)\cdot x + p \cdot X_S$.

In an auction where payments occur only when there is no bankruptcy, the risk of bankruptcy does not play any role in the bidding incentives. In particular, in the second-price auction, for each bidder it is still a dominant strategy to bid his valuation. If all bidders have the same probability of default, then the second-price auction with the reserve price X_S corresponds to the Vickrey auction with respect to the correct valuation and we can apply Theorem 1. We now discuss cases where bidders differ in terms of risks of bankrupcy.

Consider first environments without incumbents and with two groups of entrants ($K = 2$). The probability to go bankrupt for entrants from group $i = 1, 2$ is denoted by $p^i_{ent} \in [0, 1]$. If $p^1_{ent} \neq p^2_{ent}$ the second price auction (with $r = X_S$) is not ex post efficient. If $p^1_{ent} < p^2_{ent}$, entrants from group 1 (resp. 2) are disadvantaged (resp. advantaged) and we can apply Theorem 3.

Corollary 5.5 *If there are two groups of entrants and if the probability of bankruptcy is lower for group 1, then it is detrimental to exclude bidders from group 1 in the second-price auction with the reserve price X_S.*

Consider next environments in which all entrants have a (common) probability $p_{ent} \in [0, 1]$ to go bankrupt, and the (single) incumbent has a probability $p_{inc} \in [0, 1]$ to go bankrupt . If $p_{inc} \neq p_{ent}$, then the second price auction (with $r = X_S$) is not ex post efficient. If $p_{inc} > p_{ent}$, the incumbent is advantaged (according to the correct valuations) and we can apply Theorem 2.

Corollary 5.6 *If the risk of failure is larger for the incumbent ($p_{inc} \geq p_{ent}$), then the revenue-optimal set-asides policy consists in excluding the incumbent in the second-price auction with the reserve price X_S.*

Comments: 1) In a number of instances, it may not be that plausible that the incumbent would have a larger risk of breakdown than entrants so that Corollary 5.6 should be read as saying that excluding the incumbent enhances revenues to the extent that the bankruptcy risks of the incumbent and the entrants are not too dissimilar. 2) In our environment with risks of breakdown, the correct Vickrey auction would assign the good to the bidder with the highest correct valuation. More precisely, the bidder with the highest correct valuation $(1-p)\cdot x + p \cdot X_S$ should win the good (provided that $x \geq X_S$) and pay his contribution to the welfare $x' + \frac{(p-p')}{(1-p)}(x' - X_S)$[37] where x' and p' correspond respectively to the valuation and probability of bankruptcy of the bidder with the second highest correct valuation (provided $x' \geq X_S$). For such an efficient auction, we could apply Theorem 1 directly. More generally, if the auction does not take into account appropriately the relative risk of bankruptcy beyond the point where it advantages the incumbent (resp. disadvantages the entrants from group 1), then Theorem 2 (resp. 3) extends. 3) Our environment with risk failure is actually equivalent to the auction models used for advertisement slots on Internet (as used by Google and most publishers) under a pay-per-click system (Agarwal, Athey and Yang, 2009). Given that there is a huge heterogeneity in terms of the probability of clicks (or conversion rates), winners are no loner ranked according to their bid per click as it used to be but rather their bid times the estimated probability that they receive a click. The evolution of the mechanism can be roughly interpreted as a move from the (inefficient) Vickrey auction to the Vickrey auction with respect to the correct valuation.

5.4 General mechanisms in Myersonian environments

So far we have considered auction formats in which bidders had a weakly dominant strategy. The analysis extends straightforwardly to situations in which the distributions of valuations are independent across bidders (Myersonian environment) and the equilibrium allocation is the same as the one considered in our mechanisms relying on weakly dominant strategies. In particular, Theorem 1 extends straightforwardly to any mechanism which is payoff-equivalent to the Vickrey auction from an ex ante perspective and for any possible entry profile μ and any vector of incumbents I such that the expected profit of the incumbent i (resp. a group k entrant) is still equal to $\Pi_i^{inc}(\mu, I)$ (resp. $\Pi_k^{ent}(\mu, I)$). On the one hand, the revenue will be the same as in the Vickrey auction for any entry profile. On the other hand, since the equilibrium free entry conditions are determined by the payoffs of the entrants whose expression remains unchanged, the set of equilibrium entry profiles will be the same as in the Vickrey auction. From the well-known "payoff equivalence Theorem" (see e.g. Milgrom, 2004), in a Myersonian environment, any mechanism which assigns the good efficiently and leaves no rents to buyers with null valuation is payoff-equivalent to the Vickrey auction. As shown by Jehiel and Lamy (2015a), an example of such a mechanism is the first-price auction with the reserve price X_S in a Myersonian setup with symmetric buyers but also under the extra assumption that the set of entrants is publicly observed before the bidding stage. In particular, with a single incumbent, we still get that it would profitable to exclude him.[38]

[37]Indeed, the payment occurs only when the winner is not bankrupt so that the expected payment is the previous figure multiplied by $(1-p)$.

[38]If the set of entrants is not observed, then it creates an asymmetry between the incumbent and the entrants: an entrant expect to face one more competing bidder than the incumbent and thus bid more aggressively than the incumbent which induces inefficiencies. Formally, if $G_\mu(.)$ (resp. $G_\mu^I(.)$) denote the CDF of the highest bid

More generally, in a Myersonian environment, we know that the rents of the various agents are fully determined (up to some constants) by the assignment rule. If a given auction induces (in equilibrium) an assignment rule that can be implemented in dominant strategy,[39] and if we can apply Theorems 2 or 3 to the payoff equivalent dominant strategy auctions, then the results apply to our given auction. For example, in environments with a single incumbent and if the auction induces a $\phi(b,r)$-assignment rule with $b(x) \geq x$ and $r \leq X_S$, then the optimal set aside policy consists in excluding the incumbent and keeping the entrants.[40] We apply this principle to the study of first-price auctions.

First-price auctions with or without the right-of-first-refusal

A form of explicit discrimination sometimes encountered in procurement auctions is the right-of-first-refusal: it consists in letting a special (or preferred) bidder to match the final highest bid as it is analyzed in Burguet and Perry (2006) in a procurement setup.[41] In a first price auction with the reserve price X_S assuming that entrants simultaneously choose to enter and submit a bid,[42] if the incumbent has a right-of-first-refusal, then the equilibrium assignment rule is distorted from the efficient assignment by assigning the good too often to the incumbent. More precisely, if we let $\beta : [X_S, \infty) \to [X_S, \infty)$ denote the equilibrium bid function of the entrants (where $\beta(x) < x$ for $x > X_S$ as a result of bid shading), when the valuation of the incumbent is in the interval $(\beta(x), x)$ where x is the highest valuation among the entrants, then the incumbent will exert his right-of-first-refusal and win the good, which is inefficient. Thus, the equilibrium allocation is such that the incumbent is advantaged, and, in a Myersonian environment, we obtain as corollary of Theorem 2 that the result found in Example 1b applies more generally:

Corollary 5.7 *In a Myersonian environment, if the incumbent has a right-of-first-refusal in the first-price auction with the reserve price X_S, then the revenue-optimal set-asides policy consists in excluding the incumbent.*

among the entrants (resp. the bid of the incumbent), then in the Poisson model the distribution of the highest competing bid of any entrant entrant is $b \to G_\mu(b) \cdot G^I_\mu(b)$, which first-order stochastically dominates $b \to G_\mu(b)$ the distribution of the highest competing bid of the incumbent. If bidders do not receive extra information additional to their valuation, then entrants should bid more aggressively than the incumbent in the auction and the incumbent is thus disadvantaged which is a countervailing force against his exclusion.

[39] From Mookherjee and Reichelstein (1992), in our simple single-good auction setup: dominant strategy implementation is feasible if and only if the probability that a bidder wins the good is non-decreasing in his valuation for any set of valuation of his opponents. E.g. $\phi(b,r)$ assignment rules are implementable in dominant strategy.

[40] The class of $\phi(b,r)$-allocation rules excludes allocation rules that depend on the realization of the set of entrants (e.g. the number of entrants). Nevertheless, the argument would also extend if a different function b is used (to characterize the assignment rule) depending on the set of entrants (e.g. the number of entrants).

[41] In a procurement, it corresponds to a right to match the lowest bid. This right is often observed in procurements either through an explicit right or an implicit one (see Lee (2008) for examples of industries where it is a common practice). Note also that corruption in procurements has also been modeled as a bribery auction where the winner obtains a right of first refusal (Compte et al., 2005). In procurements for public transportation contracts in London, after reviewing the bids, the regulator can ask the incumbent for a second offer (if his offer is close to the winning bid) for him to win the bid (Amaral et al. (2009)). In the English auctions for cricket players in the Indian Premier League, the team that owns the auctioned player in the previous season has an equivalent "right-to-match" the winning bid (Lamy et al. (2016)). Those two examples illustrate that the right of first refusal is typically attributed to a bidder than can be viewed as an incumbent according to our framework.

[42] This guarantees that all entrants from the different groups use the same bidding function as detailed in Jehiel and Lamy (2015a).

Remark. The right-of-first-refusal can also be used in second-price/English auctions as analyzed in Bikhchandani et al. (2005). The equilibrium analysis is then straightforward: it is a dominant strategy for the bidders who do not have this right to bid up to their valuation, while the bidder with this right matches the final price if it is below his valuation. The corresponding assignment obviously advantages the incumbent while the mechanism is a dominant-strategy auctions without fees so that we can apply Theorem 2 directly.

To conclude this section, we discuss informally what happens to our exclusion insights in standard first-price auctions (with no right of first refusal). In a (standard) first-price auction where the set of entrants is disclosed, whether the incumbent is advantaged or disadvantaged depends on the relative strength of the incumbent and of the entrants' valuation distributions. From Maskin and Riley (2000) and Lebrun (1999), we know that a bidder who has a "weaker"[43] valuation distribution bids more aggressively, and thus such a weak bidder would be considered as advantaged according to our terminology. As a result, we obtain that it is always beneficial to exclude the incumbent if he is weak. At the other extreme, note that a very strong incumbent would completely discourage entry, which would then make exclusion profitable. For other cases in which the incumbent is stronger than entrants but to a more moderate extent, our analysis does not allow to conclude whether excluding the incumbent would be good for revenues.

Concerning the exclusion of entrants (when there are only two groups of entrants) and assuming there are no incumbents and the set of entrants is disclosed (not only the number of entrants but also their group identities), we obtain still as a corollary of Maskin and Riley (2000) and Lebrun (1999) that it is always detrimental to exclude the entrants from the stronger group (who are disadvantaged).

6 The Vickrey auction with multiple incumbents

In order to understand better the multiple incumbents case, we come back to the Vickrey auction, and we consider the case of symmetric potential entrants. Given the restriction to a single group of entrants, we alleviate notation, and simplify the equilibrium entry rate $\mu^*(I, E)$ into $\mu^*(I)$. With multiple incumbents, if we plug (17) for $i \in I$ into the expression of the revenue (14), we get

$$R(\mu^*(I), I) = TW(\mu^*(I), I_{-i}) - \sum_{j \in I_{-i}} \Pi_j^{inc}(\mu^*(I), I) \tag{21}$$

and the revenue effect of excluding incumbent i can be written as:

[43] Maskin and Riley (2000) and Lebrun (1999) need a stronger notion of dominance than first-order stochastic dominance: it is reverse hazard rate dominance. The incumbent is said to be weaker than the entrants if $\frac{f^I(x)}{F^I(x)} \geq \frac{f^E(x)}{F^E(x)}$ for any $x \in (\underline{x}, \overline{x}]$.

$$R(\mu^*(I_{-i}), I_{-i}) - R(\mu^*(I), I) = \overbrace{TW(\mu^*(I_{-i}), I_{-i}) - TW(\mu^*(I), I_{-i})}^{\geq 0} - \sum_{j \in I_{-i}} \underbrace{\left[\Pi_j^{inc}(\mu^*(I_{-i}), I_{-i}) - \Pi_j^{inc}(\mu^*(I), I)\right]}_{\geq \text{ or } \leq 0 \ \ ?}. \tag{22}$$

There are two conflicting effects of excluding incumbent i. On the one hand, it increases the first term in (21) since $TW(\mu^*(I), I_{-i}) \leq TW(\mu^*(I_{-i}), I_{-i})$. In words, it increases the total welfare minus the rents of incumbent i. This is the analog of the (positive) effect of exclusion that we identified previously with a single incumbent. On the other hand, there is a novel effect at work. Excluding incumbent i can also have an impact on the rents of the remaining incumbents. The sign of this second effect is ambiguous without additional restrictions. Ceteris paribus (without taking into account the effect on the entry rate), the presence of incumbent i has a negative impact on the rents of the other incumbents. However, excluding incumbent i has also an impact on the entry rate. Intuitively, without incumbent i, the participation rate of entrants should be larger, which should attenuate if not counterbalance the previous negative impact on the seller's revenue. The rest of this Section elaborates on this intuition.

6.1 A special class of incumbents

In a simple class of incumbents' valuation distributions, we are able to show that the revenue advantage of excluding the incumbents extends to the multi-incumbent case. Specifically, consider the following assumption on incumbents' valuation distributions.

Assumption A 1 *Potential entrants are symmetric with valuations distributed according to $F(\cdot|\cdot)$ and for each incumbent $i \in \mathcal{I}$, there exists λ_i such that $F_i^I(x|z) = e^{\lambda_i \cdot (1-F(x|z))}$ for each x, z.*

An intrepretation of this class of distributions is that the valuation of incumbent i can be viewed as being the highest valuation among potential entrants entering according to the Poisson distribution with λ_i entrants on average. This implies that if the incumbent i is substituted by an average of λ_i extra entrants, then the distribution of the highest competitors remains the same from the perspective of any new entrant contemplating whether or not to enter. Formally, we have then

$$\Pi^{ent}(\mu + \lambda_i, I_{-i}) = \Pi^{ent}(\mu, I). \tag{23}$$

This further implies that $\mu^*(I_{_{-i}}) = \mu^*(I) + \lambda_i$ if $\mu^*(I) > 0$. Combined with the analog of the equality (23) but for the incumbents $j \neq i$, we obtain that $\Pi_j^{inc}(\mu^*(I_{-i}), I_{-i}) = \Pi_j^{inc}(\mu^*(I), I)$ for any $j \neq i$ so that it is profitable to exclude bidder i. Given that $\mu^*(\mathcal{I}) \leq \mu^*(I)$ for any $I \subseteq \mathcal{I}$, if $\mu^*(\mathcal{I}) > 0$, our argument can be repeated for all incumbents, thereby allowing us to conclude:

Proposition 6.1 *Under Assumption 1, if $\mu^*(\mathcal{I}) > 0$, then the revenue-optimal set-asides policy in the Vickrey auction consists in excluding all incumbents.*

Remark. Note that the welfare loss from full exclusion is equal to $(\mu^*(\mathcal{I}) - \mu^*(\emptyset)) \cdot C$. However, this loss is compensated by the rents of the incumbents. For incumbent i, his equilibrium rent is always larger than $\lambda_i \cdot C$ (provided that the entry rate of potential entrants is strictly positive). The interpretation here is that an incumbent can be viewed as a ring of an average of λ_i buyers (and where the size of the ring follows a Poisson distribution). The ring sends to the auction only the buyer with the highest valuation. By contrast, if the various buyers in the ring would have behaved competitively then those buyers would be exactly in the same situation as the potential entrants and their excepted gross payoff in the auction would be C in equilibrium (it is the cooperative behavior through the ring that explains why the incumbent is making extra rents).

6.2 Asymmetric bidders

To cover more general distributions among incumbents, we consider the following set of assumptions:

Assumption A 2

1. *We are in a Myersonian environment with symmetric entrants ($K = 1$) whose valuations are distributed according to $F(\cdot)$,*
2. *The function $x \rightarrow \frac{1}{F_j^I(x)} \cdot \frac{1-F_j^I(x)}{1-F(x)}$ is (strictly) decreasing on $(\underline{x}, \overline{x})$ for any $j \in I_{-i}$,*
3. *The function $x \rightarrow -\frac{\log[F_i^I(x)]}{1-F_i^I(x)} \cdot \frac{1-F_i^I(x)}{1-F(x)}$ is (strictly) decreasing on $(\underline{x}, \overline{x})$.*

Remarks. Assumption 2 holds in the special case in which the valuation distribution of the incumbents is also F. A general simple sufficient condition guaranteeing that the monotonicity conditions hold in Assumption 2 is given in the Appendix. It works if the distribution of the incumbents can be interpreted as (possibly different) mixtures of ring of entrants of various sizes (i.e. such that the valuation is distributed according to F^k if the ring is of size $k \geq 1$). Observe that when Assumption 1 holds, Assumption 2 is violated just at the margin insofar as the monotonicity in the third requirement fails to be strict.[44]

Proposition 6.2 *Under Assumption 2 and in the Vickrey auction , the rents of the non-excluded incumbents I_{-i} increase when incumbent i is excluded.*

Hence, under the conditions of Proposition 6.2, there is a non-trivial trade-off on the effect of revenues of excluding incumbent i. On the one hand, excluding incumbent i enhances the

[44]More precisely it is the third requirement that is violated under Assumption 1 given that $-\frac{\log[F_i^I(x)]}{1-F_i^I(x)} \cdot \frac{1-F_i^I(x)}{1-F(x)} = \lambda_i$ which does not depend on x. One can check that the second requirement in A2 holds since $x \rightarrow \frac{e^{-\lambda(1-F(x))}-1}{1-F(x)}$ is (strictly) decreasing (by using the inequality $e^{1+x} \geq x$).

welfare net of incumbent i's rents as in our single incumbent case. On the other, it increases the rents of incumbents $-i$. While in general it is not clear in which direction the trade-off might go, we note now that when incumbent i is sufficiently small/weak (everything else being equal, including in particular the size of the other incumbents), it is not good to exclude i.

To see this, observe that $\mu^*(I_{-i}) - \mu^*(I)$ must be small if incumbent i is sufficiently weak (thereby winning very rarely the auction and thus affecting only marginally the entry decisions). Developing (22) reveals that the first (beneficial) effect is approximately equal to $\frac{\partial TW(\mu^*(I_{-i}),I_{-i})}{\partial \mu} \cdot [\mu^*(I_{-i}) - \mu^*(I)] + \frac{\partial^2 TW(\mu^*(I_{-i}),I_{-i})}{(\partial \mu)^2} \cdot \frac{[\mu^*(I_{-i})-\mu^*(I)]^2}{2} = \frac{\partial^2 TW(\mu^*(I_{-i}),I_{-i})}{(\partial \mu)^2} \cdot \frac{[\mu^*(I_{-i})-\mu^*(I)]^2}{2}$ since $\mu^*(I_{-i}) \in \operatorname{Arg\,max}_{\mu \in \mathbb{R}_+} TW(\mu, I_{-i})$. Hence, the first channel is of second order due to the welfare-maximizing entry rate when i is absent. By contrast, the second (detrimental) effect on other incumbents' rents is of first order,[45] thereby allowing us to conclude:

Proposition 6.3 *Let us parameterize the distribution of incumbent i by the parameter λ such that $F_i^I(x) = [G(x)]^\lambda$ and assume that Assumption 2 holds.*[46] *If λ is small enough then the revenue in the Vickrey auction shrinks when incumbent i is excluded.*

This proposition extends the insight developed in Example 1d illustrating that small/weak incumbents should not be excluded.

7 Further insights

7.1 Beyond private values: the case of an informed incumbent

In interdependent value contexts, rational bidders tend to bid more cautiously as the number of participants increases so as to internalize the winner's curse. Bulow and Klemperer (2002) suggest that it can then be beneficial for the seller to limit the total number of potential bidders when bidders are symmetric or to exclude a strong bidder whose presence would reduce the competition by exacerbating others' response to the winner's curse.[47] As we now illustrate, our exclusion principle is typically reinforced if the information held by the incumbent affects the valuation of other participants due to an informational advantage of the incumbent. Roughly, the intuition as to why it is still good to exclude the incumbent in this case is as follows. Entrants tend to bid more cautiously as compared with the situation in which they would know the signal of the incumbent so as to internalize the winner's curse. This in turn ensures that the incumbent gets a rent that is larger than his marginal contribution to the welfare. The conclusion follows then by using arguments similar to those developed above in the private value case.

To formalize this, consider the same model as in Section 2 but with the difference that what we call valuations are now private signals. For the incumbent, we still assume that his signal, denoted by x_I, coincides with his valuation. However, for a given entrant, we assume that if he

[45] For this, we use that the monotonicity in the third requirement is strict.
[46] Note that if A2 holds (with respect to incumbent i) for a given $\lambda > 0$ then it will be so for each $\lambda > 0$.
[47] Compte and Jehiel (2002) shows that even the welfare may increase when some bidders are excluded.

receives the signal x_E then his valuation is now $H(x_E, x_I)$ where $H(.,.)$ is increasing in both arguments, strictly increasing in its first argument with the derivative with respect to its second argument being less than one. Then without loss of generality, we can normalize the entrants' signals so that $H(x, x) = x$.[48]

To simplify the formalization, we consider next the ascending (button) auction with no reserve price in which the price raises continuously, bidders may quit at any time, and the auction stops when there is one bidder left who has then to pay the current price. We also assume that bidders observe the identity of the remaining active bidders and $X_S = 0$. The (weakly dominant) equilibrium strategy of the incumbent consists in remaining active up to his valuation x_I. For the entrants, the decision to exit or to remain active at price p depends on whether the incumbent is still active or not. If the incumbent has dropped from the auction thereby revealing his valuation x_I through his exit time, an entrant with signal x remains active up to $H(x, x_I)$. If the incumbent is still active, which only reveals that $x_I \geq p$, an entrant with the signal x remains active up to x.[49] Such strategies constitute an ex-post equilibrium such that the good is assigned efficiently. Without the incumbent, we are back to the Vickrey auction in a private value setting. However, it is no longer the case that the payoff of the incumbent corresponds to his contribution to welfare, i.e. (17) no longer holds in the interdependent value case. The rent (resp. the marginal contribution to the welfare) of the incumbent with valuation x_I when the largest signal among entrants is x_E can be written as $x_I - x_E$ (resp. $x_I - H(x_E, x_I)$) if $x_I > x_E$ and 0 otherwise (and obviously both the rent and the contribution to welfare coincide when there are no entrants). Since $H(x_E, x_I) \geq H(x_E, x_E) = x_E$ if $x_E \leq x_I$, the expected rent of the incumbent outweighs his expected contribution to welfare, which in turn allows us to extend Theorem 1 to this interdependent value setting:

Proposition 7.1 *In an English auction with no reserve price, when the seller's reservation value is null and when valuations may depend on the incumbent's information as described above, the revenue-optimal set-asides policy consists in excluding the incumbent and allowing all groups of entrants to participate.*

Remark. The fact that a single special bidder benefits from superior information that would be useful to his competitors in assessing their valuation arises in various applications much beyond the case when the incumbent possesses information because he was the holder of the previous contract. The seminal example in the literature is oil and gas leases auctions where a firm that owns an adjacent tract is known to benefit from a huge informational advantage (Hendricks and Porter, 1988). More recently, Abraham et al. (2013) develop a model with such information asymmetries for auctions for ad slots on Internet.

[48] As in Milgrom and Weber (1982), we consider a model involving both private and common values components. However we depart from their symmetry assumption and as in Engelbrecht et al. (1983) we consider that only one bidder is informed.

[49] This follows from usual marginal considerations as developed in Milgrom and Weber (1982).

7.2 Split-awards versus set-asides

As advocated by Milgrom (2004), split-awards constitute alternative tools to promote entry in situations with asymmetric competitors.[50] Instead of selling the good as a single lot, the seller can split the good into several lots (possibly of different sizes) typically requiring that a given bidder would be allowed to win one lot at most. While split-awards may have good properties in terms of reducing the rents left to the incumbents, we note in the following result that when there is a single incumbent, excluding the incumbent is always preferable to using even cleverly designed split-awards.

Formally, a split-award is characterized by $\alpha = (\alpha_1, \cdots, \alpha_K) \in (0,1]^K$ with $\sum_{k=1}^{K} \alpha_k = 1$ specifying that the good is split into K lots of sizes $\alpha_1, \cdots, \alpha_K$ with the requirement that a given bidder cannot be assigned more than one lot. Buyers' valuations are assumed to be linear in the size of the lot. That is, a buyer with valuation x attaches a valuation $\alpha_k \cdot x$ to the lot k with size α_k. For any split-award α, we can define an associated generalized Vickrey auction (Edelman et al., 2007) in which buyers get their marginal contribution to the welfare given the allotment constraints imposed by the split-award α (for example, if the split-award involves symmetric lots, i.e. $\alpha_1 = \cdots = \alpha_K = \frac{1}{K}$, then the associated generalized Vickrey auction is the $K+1^{th}-$ price auction).

Jehiel and Lamy (2015a) show that the property that equilibrium entry rates must maximize the total welfare (here Lemma 3.1) extends to split-award generalized Vickrey auctions. As argued earlier, we obtain then that Theorem 1 remains valid in such environments, namely that the revenue can only increase if the incumbent is excluded.[51] But then once the incumbent is excluded, Jehiel and Lamy (2015a) show that the standard Vickrey auction is optimal among all possible mechanisms and thus outperforms any split-award auction. We conclude that

Proposition 7.2 *When there is a single incumbent, the revenue in any split-award generalized Vickrey auction with the incumbent is dominated by the Vickrey auction in which the incumbent is excluded.*

Remark. We note that one cannot apply our analysis with multiple incumbents to split-award generalized Vickrey auctions: in particular, facing an incumbent with the distribution $F_i^I(x|z) = e^{\lambda_i \cdot (1-F(x|z))}$ is no longer equivalent to facing an average of λ_i entrants with the distribution $F(.|.)$ (following a Poisson distribution) because the later could get several lots.

7.3 Set-asides versus fees/subsidies

If the seller were free to charge any fee including to incumbents, then the seller's revenue would be aligned with the total welfare assuming incumbents have no private information at the

[50] The split-award literature (Anton and Yao (1989) and Gong, Li and McAfee (2011)) has emphasized the possible benefits in terms of pre-participation investments rather than entry.

[51] More generally, we can apply our exclusion insight in any auction format that can be reinterpreted as a pivot/generalized Vickrey auction of a different problem.

time the fee is charged, and thus set-asides would not be optimal.[52]

In this section, we develop a more restrictive view on fees: we consider that the seller can only tax or subsidize entry. In some procurements, there are some funds dedicated to reimburse partially the physical participation costs of the bidders (in an attempt to reduce the barriers to entry).[53] For example, in the merger case considered in footnote 1, the French antitrust authority decided to create a fund (to be financed by the merged entity) aimed at boosting competition by reimbursing the procurements participation costs of those firms that were not incumbents. Below, we have implicitly in mind that subsidizing entry is easier to implement than imposing an entry tax (which may in some circumstances lead to the violation of participation constraints).

Similarly to set-asides, subsidies for entrants are an instrument that allows to get closer to the optimal revenue characterized in Jehiel and Lamy (2015a) to the extent that it allows the seller to reduce the incumbent's rents.

We note from (21) that the following expression

$$\max_{\mu\in\mathbb{R}_+^K} R(\mu, I) = \max_{\mu\in\mathbb{R}_+^K} \Big\{TW(\mu, I_{-i}) - \sum_{j\in I_{-i}} \Pi_j^{inc}(\mu, I)\Big\} \tag{24}$$

is an upperbound on the revenue that can be reached with the set of incumbents I containing a given incumbent i assuming the seller can charge fees/subsidies only to entrants on the top of the Vickrey auction. If the seller is free to charge any group specific fee/subsidy to entrants, such a bound can be reached (this amounts to adjusting the fees/subsidies so that the required μ are obtained).[54]

If incumbent i is excluded, the revenue is bounded from above by

$$\max_{\mu\in\mathbb{R}_+^K} R(\mu, I_{-i}) = \max_{\mu\in\mathbb{R}_+^K} \Big\{TW(\mu, I_{-i}) - \sum_{j\in I_{-i}} \Pi_j^{inc}(\mu, I_{-i})\Big\} \tag{25}$$

and this bound is reached without fees if there is a single incumbent (i.e. $I_{-i} = \emptyset$). Since $\Pi_j^{inc}(\mu, I)$ is nonincreasing in I, and more specifically $\Pi_j^{inc}(\mu, I) \leq \Pi_j^{inc}(\mu, I_{-i})$ for any $j \in I_{-i}$, we obtain the following result:

Proposition 7.3 *Consider an environment in which the seller is free to post any entry fees to the entrants in the Vickrey auction. 1) If there is a single incumbent, then fees do not outperform set-asides: the revenue-optimal set-asides and fee policy consists in excluding the incumbent, keeping all kinds of entrants and having no fees. 2) Excluding one incumbent is always (weakly) dominated by the policy that consists in imposing optimal fees and keeping the incumbent. 3)*

[52]With such fees, it is as if the seller internalizes the rents of the incumbents. However, such a solution, which outperforms Jehiel and Lamy's (2015a) optimal auction, does not seem realistic because it will stand in conflict with participation constraints.

[53]See Gal and al. (2007) for such a model that relies crucially on the fact that those participation costs are verifiable to some extent, namely that we can not have opportunistic bidders that enter the auction only to collect those subsidies.

[54]We stress that this bound is lower than the optimal revenue in Jehiel and Lamy (2015a) whose analysis allows implicitly any kinds of fees for entrants.

When entrants are homogeneous, the optimal fees take the form of a partial reimbursement.

Comments. 1. To the extent that the cost C_k can be interpreted as the outside option value of a group k bidder, it may differ from the physical participation cost. The required subsidy in result 3 of Proposition 7.3 while smaller than C_k need not be smaller than the physical participation cost. Hence, set-asides may in some cases strictly dominate subsidies required to be smaller than the physical participation costs. 2. When there is a single incumbent, we note that there are two ways of reaching the optimal revenue: either by excluding the incumbent or alternatively without excluding the incumbent but imposing a fee that implements the optimal entry rate. However, this latter policy is much more demanding in terms of needed information for the seller: setting optimal fees requires the knowledge of the primitives of the model (the underlying valuations distribution) while the rule consisting in excluding the incumbent does not require fine knowledge about the environment.

8 Conclusion

Set-asides are a common form of discrimination in procurement auctions. Optimal mechanisms either assuming the set of participants is exogenous (Myerson, 1981) or that entry is endogenous for some groups of bidders (Jehiel and Lamy, 2015a) do not deliver that set-asides are the best forms of discrimination for revenue purposes. Our main result is that in the presence of a single incumbent -a bidder who would participate no matter what if allowed to- it is best not to let this bidder participate in the second-price auction and in a number of other auction formats. The picture is more complex with multiple incumbents except that from our results it seems suboptimal in general to exclude weak/small bidders.

Our analysis has abstracted away from dynamic considerations. Two different lines of research could be pursued in relation to this. First, keeping the same underlying economic setup, we could allow the seller to employ dynamic mechanisms so as to better coordinate the entry decisions of bidders. While such mechanisms (if feasible - they may in some cases be considered to rely on illegal discrimination) could improve the efficiency (as Bergemann and Välimäki's (2010) dynamic pivot mechanism), it is not so clear what the effect on revenues would be as illustrated by the different insights obtained by Bulow and Klemperer (2009) and Roberts and Sweeting (2013). The welfare benefits from coordination could be counterbalanced by the fact that early entrants would now enjoy some rents giving them a status similar to that of the incumbents in our (static) framework . Second, we could embed the present static economic environment into a dynamic one in which bidders would make some pre-participation investments (e.g. information acquisition as in Bergemann and Välimäki (2002) or upgrade in the type distribution as in Arozamena and Cantillon (2004)) anticipating the future rents associated to those. The study of these is left for future research.

References

[1] I. Abraham, S. Athey, M. Babaioff, and M. Grubb. Peaches, lemons, and cookies: Designing auction markets with dispersed information. In *ACM Conference on Electronic Commerce (ACM-EC 2013)*, June 2013.

[2] N. Agarwal, S. Athey, and D. Yang. Skewed Bidding in Pay-per-Action Auctions for Online Advertising. *American Economic Review*, 99(2):441–47, 2009.

[3] M. Amaral, S. Saussier, and A. Yvrande-Billon. Auction procedures and competition in public services: The case of urban public transport in France and London. *Utilities Policy*, 17(2):166–175, 2009.

[4] J. J. Anton and D. A. Yao. Split awards, procurement, and innovation. *RAND Journal of Economics*, 20(4):538–552, 1989.

[5] L. Arozamena and E. Cantillon. Investment incentives in procurement auctions. *Review of Economic Studies*, 71(1):1–18, 2004.

[6] S. Athey, D. Coey, and J. Levin. Set-asides and subsidies in auctions. *American Economic Journal: Microeconomics*, 5(1):1–27, 2013.

[7] S. Athey, J. Levin, and E. Seira. Comparing open and sealed bid auctions: Theory and evidence from timber auctions. *Quarterly Journal of Economics*, 126(1):207–257, 2011.

[8] P. Bajari, S. Houghton, and S. Tadelis. Bidding for incomplete contracts: An empirical analysis of adaptation costs. *American Economic Review*, 104(4):1288–1319, April 2014.

[9] D. Bergemann and J. Välimäki. Information acquisition and efficient mechanism design. *Econometrica*, 70(3):1007–1033, 2002.

[10] D. Bergemann and J. Välimäki. The dynamic pivot mechanism. *Econometrica*, 78(2):771–789, 2010.

[11] S. Bikhchandani, S. A. Lippman, and R. Reade. On the Right-of-First-Refusal. *The B.E. Journal of Theoretical Economics*, 5(1):1–44, 2005.

[12] K. Binmore and P. Klemperer. The Biggest Auction Ever: the Sale of the British 3G Telecom Licences. *Economic Journal*, 112(478):74–96, 2002.

[13] S. Board. Bidding into the Red: A Model of Post-Auction Bankruptcy. *Journal of Finance*, 62(6):2695–2723, 2007.

[14] J. Bulow and P. Klemperer. Auctions versus negotiations. *Amererican Economic Review*, 86(1):180–94, 1996.

[15] J. Bulow and P. Klemperer. Prices and the Winner's Curse. *RAND Journal of Economics*, 33(1):1–21, 2002.

[16] J. Bulow and P. Klemperer. Why do sellers (usually) prefer auctions? *American Economic Review*, 99(4):1544–75, 2009.

[17] R. Burguet, J.-J. Ganuza, and E. Hauk. Limited liability and mechanism design in procurement. *Games and Economic Behavior*, 76(1):15–25, 2012.

[18] R. Burguet and M. K. Perry. Preferred suppliers in auction markets. *RAND Journal of Economics*, 40(2):283–295, 2009.

[19] O. Carbonell-Nicolau and R. P. McLean. Refinements of Nash equilibrium in potential games. *Theoretical Economics*, 9(3), 2014.

[20] O. Compte and P. Jehiel. On the Value of Competition in Procurement Auctions. *Econometrica*, 70(1):343–355, 2002.

[21] O. Compte, A. Lambert-Mogiliansky, and T. Verdier. Corruption and competition in procurement auctions. *RAND Journal of Economics*, 36(1):1–15, 2005.

[22] P. Cramton. Spectrum Auction Design. *Review of Industrial Organization*, 42(2):161–190, 2013.

[23] P. del Rio. Auctions for renewable support in Portugal: Instruments and lessons learnt. Report D4.1-PT, AURES (Auctions for Renewable Energy Support), 2016.

[24] T. A. Denes. Do small business set-asides increase the cost of government contracting ? *Public Administration Review*, 57:441–444, 1997.

[25] B. Edelman, M. Ostrovsky, and M. Schwarz. Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords. *American Economic Review*, 97(1):242–259, 2007.

[26] R. Engelbrecht-Wiggans, P. Milgrom, and R. J. Weber. Competitive bidding and proprietary information. *Journal of Mathematical Economics*, 11(2):161–169, 1983.

[27] B. Flyvbjerg. Design by deception: The politics of megaproject approval. *Harvard Design Magazine*, (22):50–59, June 2005.

[28] S. Gal, M. Landsberger, and A. Nemirovski. Participation in auctions. *Games and Economic Behavior*, 60(1):75–103, 2007.

[29] J. Gong, J. Li, and R. P. McAfee. Split-award contracts with investment. *Journal of Public Economics*, 96(1):188–197, 2012.

[30] K. Hendricks and R. H. Porter. An Empirical Study of an Auction with Asymmetric Information. *American Economic Review*, 78(5):865–83, 1988.

[31] J. Hofbauer and G. Sorger. Perfect foresight and equilibrium selection in symmetric potential games. *Journal of Economic Theory*, 85(1):1–23, 1999.

[32] P. Jehiel and L. Lamy. On discrimination in auctions with endogenous entry. *American Economic Review*, 105(8):2595–2643, 2015a.

[33] P. Jehiel and L. Lamy. A mechanism design approach to the tiebout hypothesis. *CEPR Discussion Papers*, 10758, 2015b.

[34] P. Jehiel and B. Moldovanu. An economic perspective on auctions. *Economic Policy*, 18(36):269–308, 2003.

[35] P. D. Klemperer. What really matters in auction design. *Journal of Economic Perspectives*, 16:169–189, 2002.

[36] E. Krasnokutskaya and K. Seim. Bid preference programs and participation in highway procurement auctions. *American Economic Review*, 101(6):2653–86, 2011.

[37] V. Krishna. *Auction Theory*. Academic Press, San Diego, 2003.

[38] L. Lamy, M. Patnam, and M. Visser. Correcting for sample selection from competitive bidding, with an application to estimating the effect of wages on performance. *CEPR Discussion Papers*, 11376, 2016.

[39] B. Lebrun. First Price Auctions in the Asymmetric N Bidder Case. *International Economic Review*, 40(1):125–42, 1999.

[40] J.-S. Lee. Favoritism in asymmetric procurement auctions. *International Journal of Industrial Organization*, 26(6):1407–1424, 2008.

[41] D. Levin and J. L. Smith. Equilibrium in auctions with entry. *American Economic Review*, 84(3):585–599, 1994.

[42] J. Marion. Are bid preferences benign? The effect of small business subsidies in highway procurement auctions. *Journal of Public Economics*, 91:1591–1624, 2007.

[43] E. Maskin and J. Riley. Asymmetric auctions. *Review of Economic Studies*, 67(3):413–38, 2000.

[44] R. P. McAfee. Mechanism design by competing sellers. *Econometrica*, 61(6):1281–1312, 1993.

[45] P. Milgrom. *Putting Auction Theory to Work*. Cambridge University Press, Cambridge, 2004.

[46] P. Milgrom and R. Weber. A theory of auctions and competitive bidding. *Econometrica*, 50:1089–1122, 1982.

[47] D. Monderer and L. S. Shapley. Potential games. *Games and Economic Behavior*, 14(1):124–143, 1996.

[48] D. Mookherjee and S. Reichelstein. Dominant strategy implementation of Bayesian incentive compatible allocation rules. *Journal of Economic Theory*, 56(2):378–399, 1992.

[49] R. B. Myerson. Population uncertainty and Poisson games. *International Journal of Game Theory*, 27(3):375–392, 1998.

[50] J. Nakabayashi. Small business set-asides in procurement auctions: An empirical analysis. *Journal of Public Economics*, 100:28 – 44, 2013.

[51] A. Neyman. Correlated equilibrium and potential games. *International Journal of Game Theory*, 26:223–227, 1997.

[52] J. Pinkse and G. Tan. The affiliation effect in first-price auctions. *Econometrica*, 73(1):263–277, 2005.

[53] J. W. Roberts and A. Sweeting. Competition versus auction design. Duke University Working Paper, 2012.

[54] J. W. Roberts and A. Sweeting. When should sellers use auctions? *Amrican Economic Review*, 5:1830–1861, 2013.

[55] S. Saussier and J. Tirole. Renforcer l'efficacité de la commande publique. *Les notes du conseil d'analyse économique*, 22, April 2015.

[56] G. Spagnolo. Reputation, competition, and entry in procurement. *International Journal of Industrial Organization*, 30(3):291–296, 2012.

[57] D. F. Spulber. Auctions and Contract Enforcement. *Journal of Law, Economics and Organization*, 6(2):325–44, 1990.

[58] R. Wilson. *Game Theoretic Analysis of Trading Processes*, volume Advances in Eco- nomic Theory. Cambridge University Press, 1987.

[59] C. Z. Zheng. High Bids and Broke Winners. *Journal of Economic Theory*, 100(1):129–171, 2001.

Appendix

Example 1d (continued)

Assume x_I and x_E are deterministic variables with $x_E > x_I + C$. We also make the normalization $C = 1$. Let $R_{with-1weak-Inc}$ denote the expected revenue of the seller if the strong incumbent (i.e. the one which has valuation x_I for sure) is excluded. Let $\alpha = \frac{x_E}{x_E - x_I} > 1$ and $\beta = \alpha + \epsilon - \alpha \cdot \epsilon \in [1, \alpha]$. Straightforward calculations lead to

$$R_{without-Inc} = x_E - \Big(1 + \ln[x_E]\Big),$$

$$R_{with-1weak-Inc} = x_E - \Big(\frac{\alpha}{(1-\epsilon)\alpha + \epsilon} + \ln[(1-\epsilon)x_E + \epsilon(x_E - x_I)]\Big)$$

and

$$R_{with-2-Inc} = x_E - \Big((1-\epsilon)\alpha + \epsilon + \ln[x_E - x_I]\Big).$$

We have then:

$$R_{with-2-Inc} - R_{with-1weak-Inc} = \frac{\alpha}{\beta} - \beta + \ln[\beta]$$

For a given $\alpha > 1$, the difference $R_{with-2-Inc} - R_{with-1weak-Inc}$ is decreasing in β or equivalently increasing in ϵ. On the whole, we obtain that starting from the two incumbents situation, it is better to exclude (resp. to keep) the strong incumbent if ϵ is small (resp. large) enough.

We have then:

$$R_{with-2-Inc} - R_{without-Inc} = 1 - \beta + \ln[\alpha]$$

For a given α, the difference $R_{with-2-Inc} - R_{without-Inc}$ is decreasing in β or equivalently increasing in ϵ. On the whole, we obtain that starting from the two incumbents situation, it is better to exclude (resp. to keep) both incumbents if ϵ is small (resp. large) enough.

This example allows us to cook situations where $R_{with-2-Inc} < R_{without-Inc}$ and $R_{with-1weak-Inc} > R_{with-2-Inc}$, i.e. such that starting from two incumbents it is detrimental to exclude each incumbents separately but would be profitable to exclude them jointly. We can check that this works if $\alpha = 20$ and $\beta = 4$. $\diamond\diamond$

Proof of Lemma 3.1

Below, we show more generally that $J(I, E) = \operatorname{Arg\,max}_{\mu \in \mathbb{R}_+^K | \mu_k = 0 \text{ if } k \notin E} TW(\mu, I) \neq \emptyset$ for any set-asides policy (I, E). As a by-product, we will also obtain the existence of our equilibrium notion.

Since the support of valuation distributions are bounded by $\overline{x}$, we have then $W(N,I) \leq \overline{x}$. If $\mu_k > \frac{\overline{x}}{C_k}$, then we have $TW(\mu,I) < 0 \leq TW((0,\ldots,0),I)$. When maximizing the continuous function $\mu \to TW(\mu,I)$ over $\{\mu \in \mathbb{R}_+^K | \mu_k = 0 \text{ if } k \notin E\}$, there is thus no loss of generality to limit ourselves to the compact subset $\{\mu \in [0, \frac{\overline{x}}{C_k}]^K | \mu_k = 0 \text{ if } k \notin E\}$. As a continuous function on a compact set reaches a maximum, we obtain then that $\operatorname{Arg\,max}_{\mu \in \mathbb{R}_+^K | \mu_k = 0 \text{ if } k \notin E} TW(\mu,I) \neq \emptyset$.

Using standard calculations from auction theory, conditional on the realization z of Z, a buyer with valuation $u \geq X_S$ who participates in the auction against the profile $N \in \mathbb{N}^K$ of entrants and the set of incumbents I will receive the expected payoff of $\int_{X_S}^{u} \prod_{k=1}^{K} [F_k(x|z)]^{n_k} \cdot \prod_{i \in I} F_i^I(x|z)dx$.[55] The corresponding (interim) payoff of a group k buyer from entering such an auction, i.e. before knowing what his valuation will be and the realization of z, is given (after simple calculations) by

$$V_k^{ent}(N_{+k}, I) = \int_{X_S}^{\overline{x}} (F^{(1:N \cup I)}(x) - F^{(1:N_{+k} \cup I)}(x))dx. \tag{26}$$

After an integration per part, the expected (interim) welfare can be expressed as

$$W(N,I) = X_S + \int_{X_S}^{\overline{x}} (1 - F^{(1:N \cup I)}(x))dx. \tag{27}$$

Combining (26) and (27), we obtain

$$W(N_{+k}, I) - W(N,I) = V_k^{ent}(N_{+k}, I). \tag{28}$$

In words, we have the fundamental property of the Vickrey auction applied to a potential entrant: his payoff corresponds to his marginal contribution to the welfare. We note that $\frac{\partial P(N|\mu)}{\partial \mu_k} = -P(N|\mu)$ if $n_k = 0$ and $\frac{\partial P(N|\mu)}{\partial \mu_k} = -P(N|\mu) + P(N_{-k}|\mu)$ if $n_k \geq 1$. For any $k \in \mathcal{E}$, we have then $\frac{\partial TW(\mu,I)}{\partial \mu_k} = \sum_{N \in \mathbb{N}^K} P(N|\mu) \cdot \left[W(N_{+k}, I) - W(N,I)\right] - C_k$. From (15), we obtain from an ex-ante perspective that

$$\frac{\partial TW(\mu,I)}{\partial \mu_k} = \sum_{N \in \mathbb{N}^K} P(N|\mu) \cdot V_k^{ent}(N_{+k}, I) - C_k = \Pi_k^{ent}(\mu, I). \tag{29}$$

As a corollary, we obtain then that having the first-order conditions (of local optimality) $\frac{\partial TW(\mu,I)}{\partial \mu_k} \underset{(resp. \leq)}{=} 0$ if $\mu_k \underset{(resp. =)}{>} 0$ for each $k \in E$ is equivalent to $\mu \in J(I,E)$ (for any μ such that $\mu_k = 0$ if $k \notin E$). Since any global maximum must satisfy the first-order conditions, we have shown that $\operatorname{Arg\,max}_{\mu \in \mathbb{R}_+^K | \mu_k = 0 \text{ if } k \notin E} TW(\mu, I) \subseteq J(I,E)$.

In order to establish the reverse inclusion and thus conclude our proof, we do establish next that the function $\mu \to TW(\mu,I)$ is globally concave. Deriving the equation (16) with respect to μ_l and using (26), we obtain that

[55] This is the integral of the (interim) probability that a bidder with valuation x wins the object as x varies from X_S to u conditional on z.

$$
\begin{aligned}
\frac{\partial^2 TW(\mu,I)}{\partial\mu_k\partial\mu_l} &= \sum_{N\in\mathbb{N}^K} P(N|\mu)\cdot\Big[V_k^{ent}([N_{+k}]_{+l},I)-V_k^{ent}(N_{+k},I)\Big]\\
&= \sum_{N\in\mathbb{N}^K} P(N|\mu)\cdot\Big[\int_{X_S}^{\overline{x}}(F^{(1:N_{+l}\cup I)}(x)-F^{(1:[N_{+k}]_{+l}\cup I)}(x)+F^{(1:N_{+k}\cup I)}(x)-F^{(1:N\cup I)}(x))dx\Big]\\
&= -\sum_{N\in\mathbb{N}^K} P(N|\mu)\cdot E_Z\Bigg[\int_{X_S}^{\overline{x}}\prod_{k=1}^{K}[F_k(x|Z)]^{n_k}\cdot\prod_{i\in I}F_i^I(x|Z)\cdot(1-F_l(x|Z))(1-F_k(x|Z))dx\Bigg]\\
&= -E_Z\Bigg[\int_{X_S}^{\overline{x}}\prod_{k=1}^{K}e^{-\mu_k\cdot[F_k(x|Z)]}\cdot\prod_{i\in I}F_i^I(x|Z)\cdot(1-F_l(x|Z))(1-F_k(x|Z))dx\Bigg]\leq 0
\end{aligned}
\tag{30}
$$

for any $k,l\in\{1,\dots,K\}$. Let $\mathbf{H}_I^\mu$ denote the Hessian matrix of the function $\mu\to TW(\mu,I)$ at the vector of participation rates μ. In order to show that $\mu\to TW(\mu,I)$ is concave on $\mathbb{R}_+^K$ (for any $I\subseteq\mathcal{I}$), it is sufficient to show that $\mathbf{H}_I^\mu$ is negative semi-definite for any μ in $\mathbb{R}_+^K$ (and any $I\subseteq\mathcal{I}$).

Let $Q(x,z):=[(1-F_1(x|z)),\dots,(1-F_K(x|z))]$. For $X\in\mathbb{R}^K$, let $X^\top$ its transpose. More generally, the notation $^\top$ is used for any matrix. We then have to show that $X^\top\cdot\mathbf{H}_I^\mu\cdot X\leq 0$ for any $X\in\mathbb{R}^K$ and any $\mu\in\mathbb{R}_+^K$. From (30), we have:

$$
X^\top\mathbf{H}_I^\mu X=-E_Z\Bigg[\int_{X_S}^{\overline{x}}\prod_{k=1}^{K}e^{-\mu_k\cdot[F_k(x|Z)]}\cdot\prod_{i\in I}F_i^I(x|Z)\cdot\underbrace{X^\top\cdot Q(x,Z)^\top Q(x,Z)\cdot X}_{=[Q(x,Z)X]^\top\cdot[Q(x,Z)X]\geq 0}dx\Bigg]\leq 0. \tag{31}
$$

In other words, (31) says that $\mathbf{H}_I^\mu$ can be viewed as a weighted sum (including integrals) with positive weights of the negative semi-definite matrices $-[Q(x,z)]^\top Q(x,z)$ and is thus also negative semi-definite.

Q.E.D.

Example 3 Consider one incumbent having for sure valuation x_I. Consider two groups of potential entrants. In group 1, all entrants are the high valuation $\overline{x}_E>x_I$. In group 2, valuations are distributed independently across entrants according to the following distribution: With probability $q\in(0,1)$ (resp. $1-q$), an entrant has valuation $\overline{x}_E$ ($\underline{x}_E=x_I$) the entry cost denoted by C is assumed to be the same in groups 1 and 2 with $C<q(\overline{x}_E-x_I)$ in order to guarantee that entry is profitable. If there are no set-asides, then only bidders from group 1 will enter. The corresponding equilibrium entry rate μ_1 is characterized by

$$
e^{-\mu_1}(\overline{x}_E-x_I)=C.
$$

On the contrary, if bidders from group 1 are excluded then the equilibrium entry rate of group 2 is now given by $\widetilde{\mu}_2$ such that

$$q \cdot e^{-\widetilde{\mu}_2 \cdot q}(\overline{x}_E - x_I) = C.$$

Let fix C, $\overline{x}_E$ and x_I such that $\frac{C}{\overline{x}_E - x_I}$ remains constant (strictly below e^{-1} in order to guarantee than $\mu_1 > 1$) while C and $\overline{x}_E - x_I$ go jointly to zero. Then the revenue without exclusion will be approximatively (up to a term that is of the same order as C) equal to $(1 - e^{-\mu_1}) \cdot x_I$ and $(1 - e^{-\widetilde{\mu}_2}) \cdot x_I$ if group 1 is excluded. Our aim is now to show that $\widetilde{\mu}_2 > \mu_1$ if q is picked adequately. Let $q = 1 - \epsilon$. From equilibrium conditions and then Taylor expansion, we have $\frac{e^{-\mu_1 q}}{e^{-\widetilde{\mu}_2 q}} = \frac{q e^{-\mu_1 q}}{e^{-\mu_1}} = 1 + \epsilon \cdot (1 - \mu_1) + \circ(\epsilon)$. If ϵ is small enough we have then $\widetilde{\mu}_2 > \mu_1$. By appropriate choice of the parameters, it is thus profitable to exclude group 1. $\diamond\diamond$

Proof of Lemma 5.1 Consider a dominant-strategy auction and take a given vector of reports $X = (x_1, \cdots, x_n)$ for the competing bidders of the given incumbent.[56] Let $P_X : \mathbb{R}_+ \to [0, 1]$ denote the function that gives the probability that the given incumbent gets the good as a function of his reported valuation. From Mookherjee and Reichelstein (1992) and taking the perspective of the given incumbent, the function $P_X(.)$ should be non-decreasing and the expected payoff of the given incumbent with type z is equal to $\int_0^z P_X(u)du$ up to a constant, and that constant is null in an auction without fees. Furthermore, if the given incumbent is advantaged (resp. disadvantaged) then the auction assigns the good with probability 1 (resp. 0) to him if it is efficient (inefficient) to do so. Formally, this means that $P_X(z) = 1$ if $z \geq \max\{\max_{i=1,\cdots,n}\{x_i\}, X_S\} \equiv x^*$ (resp. $P_X(z) = 0$ if $z \leq x^*$). This further implies that the expected payoff of the incumbent with type z is larger (resp. smaller) than $\max\{z - x^*, 0\}$, i.e. the payoff he will get in the Vickrey auction.

Since the auction assigns the good efficiently among the opponents of the given incumbent, then if the latter wins (resp. does not win) the good, his marginal contribution to the welfare is $z - x^*$ (resp. 0). If the auction advantages the given incumbent, then we get that his expect payoff is larger than his marginal contribution to the welfare. If the auction disadvantages the given incumbent, then his expected payoff and his marginal contribution to the welfare are null if he does not get the good $z \leq x^*$. By contrast, if he gets the good (in which case we have necessarily $z \geq x^*$), his expected payoff is smaller than $z - x^*$ and thus smaller than his marginal contribution to the welfare.

The previous arguments hold ex post, namely for any realization of the profile of valuations of the incumbent's competitors. It holds thus a fortiori from an interim perspective, namely for any realization of the set $N \in \mathbb{N}_+$ of competitors but before knowing their valuations. **Q.E.D.**

Proof of eq. (20) Let $TW(\mu, I; \phi(b, r))$ and $\Pi^{inc}(\mu, I; \phi(b, r))$ denote respectively the total expected net welfare and the incumbent's expected payoff in a dominant strategy auction without

[56] X is an empty list if the given bidder faces no competitors.

fees that implements a $\phi(b,r)$-assignment rule and when the entry profile is $\mu \in \mathbb{R}_+^K$ when the (single) incumbent is present. Let $G_\mu(.)$ (resp. $g_\mu(.)$) denote the CDF (resp. PDF) of the highest valuation among the entrants. In a $\phi(b,r)$-auction and if the incumbent's valuation is distributed independently of the entrants' valuations, the probability that the incumbent wins the auction if his valuation is x is given by $G_\mu(\phi(x))$ if $x \geq r$ and is null if $x < r$. The expected payoff of the incumbent if his type is $x \geq r$ is thus given by

$$\int_r^x G_\mu(b(u))du = (x-r)\cdot G_\mu(X_S) + \int_r^x (x-u)\cdot d[G_\mu(b(u))]. \tag{32}$$

So his expected payoff is the same as in a second-price auction with the reserve price r but if an entrant with valuation x bids as if his valuation were $b^{-1}(x)$. The incumbent's expected payoff (before learning his type) is then $\Pi^{inc}(\mu, I; \phi(b,r)) = \int_r^\infty [\int_r^x G_\mu(b(u))du]\cdot d[F^I(x)] = \int_r^\infty G_\mu(b(x))\cdot(1-F^I(x))dx$. With the (single) incumbent, we have by definition of a $\phi(b,r)$-auction:

$$TW(\mu, I; \phi(b,r)) = F^I(r)\cdot TW(\mu, \emptyset; \phi(b,r)) + \int_r^\infty \Big(x\cdot G_\mu(b(x)) + \int_{b(x)}^\infty ud[G_\mu(u)]\Big)d[F^I(x)].$$

After an integration per part and plugging the expression of $\Pi^{inc}(\mu, I; \phi(b,r))$, we obtain that

$$TW(\mu, I; \phi(b,r)) - TW(\mu, \emptyset; \phi(b,r)) = \Pi^{inc}(\mu, I; \phi) + \Big(r\cdot G_\mu(X_S) + \int_{X_S}^\infty xd[G_\mu(x)] - TW(\mu, \emptyset; \phi(b,r))\Big)\cdot(1-F^I(r))$$

$$+\int_r^\infty (x-b(x))\cdot(1-F^I(x))\cdot d[G_\mu(b(x))].$$

Since $TW(\mu, \emptyset; \phi(b,r)) = TW(\mu, \emptyset) = X_S G_\mu(X_S) + \int_{X_S}^{\overline{x}} xd[G_\mu(x)]$, we obtain (20). **Q.E.D.**

Proof of Lemma 5.2 Consider a dominant-strategy auction and take a given vector of reports $x_{-i} = (x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n)$ for the opponents of the given bidder i.[57] Let $P_{x_{-i}} : \mathbb{R}_+ \to [0,1]$ denote then the function that gives the probability that bidder i gets the good as a function of his reported valuation (in particular $P_{x_{-i}}(x_i) = \phi_i(X)$). From Mookherjee and Reichelstein (1992) and taking the perspective of the given bidder, the function $P_{x_{-i}}(.)$ should be non-decreasing and the expected payoff of the given bidder with type z is equal to $\int_0^z P_{x_{-i}}(u)du$ up to a constant, and that constant is null in an auction without fees. Furthermore, if bidder i is advantaged (resp. disadvantaged) then the auction assigns the good with probability 1 (resp. 0) to him if it is efficient (inefficient) to do so. Let $\overline{z}_{x_{-i}} = \inf\{z \in$

[57] X is an empty list if the given bidder faces no competitors.

$\mathbb{R}_+|\phi_i(x_1,\cdots,x_{i-1},z,x_{i+1},\cdots,x_n)=1\}$ and $\underline{z}_{x_{-i}}=\sup\{z\in\mathbb{R}_+|\phi_i(x_1,\cdots,x_{i-1},z,x_{i+1},\cdots,x_n)=0\}$. Let $\overline{w}_{x_{-i}}$ denote the welfare when bidder i is absent and $\overline{w}_X$ denote the welfare with all bidders. Note that the loser neutral property implies that $\overline{w}_X=P_{x_{-i}}(x_i)\cdot x_i+(1-P_{x_{-i}}(x_i))\cdot\overline{w}_{x_{-i}}$ or equivalently the contribution of bidder i to the welfare can be written as $P_{x_{-i}}(x_i)\cdot(x_i-\overline{w}_{x_{-i}})$. Moreover, if bidder i is advantaged (resp. disadvantaged), then $\overline{z}_{x_{-i}}\leq\overline{w}_{x_{-i}})$ (resp. $\underline{z}_{x_{-i}}\geq\overline{w}_{x_{-i}})$).

Consider first the case where bidder i is advantaged. We can restrict ourselves to the cases where the contribution of bidder i to the welfare is positive so that $x_i>\overline{z}_{x_{-i}}$ and $P_{x_{-i}}(x_i)=1$. The contribution to the welfare $x_i-\overline{w}_{x_{-i}}$ is smaller than $x_i-\overline{z}_{x_{-i}}$ which is a lower bound on bidder i's payoff.

Consider next the case where bidder i is disadvantaged. We can restrict ourselves to the cases where bidder i gets the good with positive probability so that $x_i>\underline{z}_{x_{-i}}$. The contribution to the welfare $P_{x_{-i}}(x_i)\cdot(x_i-\overline{w}_{x_{-i}})$ is larger than $P_{x_{-i}}(x_i)\cdot(x_i-\underline{z}_{x_{-i}})$ which is larger than $\int_{\underline{z}_{x_{-i}}}^{x_i}P_{x_{-i}}(u)du$ and then larger than $\int_0^{x_i}P_{x_{-i}}(u)du$, i.e. bidder i's payoff. **Q.E.D.**

Proof of Theorem 3

Let $\widehat{\mu}_1,\widehat{\mu}_2$ denote the equilibrium participation profile without set-asides. If $\widehat{\mu}_1=0$, then excluding bidders from group 1 does not have any impact. Suppose then that $\widehat{\mu}_1>0$.

For $i=1,2$, let $\mu_i^*(\mu)$ denote the equilibrium participation rate of entrants for group i given the participation rate from the other group. Formally, the function μ_1^* is characterized by $\Pi_1^{ent}(\mu_1^*(\mu),\mu)=0$ (resp. ≤ 0) if $\mu_1^*(\mu)>0$ (resp. $=0$). Similarly, the function μ_2^* is characterized by $\Pi_2^{ent}(\mu,\mu_2^*(\mu))=0$ (resp. ≤ 0) if $\mu_2^*(\mu)>0$ (resp. $=0$). It is straightforward that $\mu\to\mu_1^*(\mu)$ and $\mu\to\mu_2^*(\mu)$ are both non-increasing. If $\mu_2^*(0)=0$, then group 2 is always inactive and it is thus straightforward that it is detrimental to exclude bidders from group 1. We assume next that $\mu_2^*(0)>0$.

If the profile $(0,\mu_2^*(0))$ satisfies the equilibrium condition without set-asides, then it corresponds to the equilibrium profile if bidders from group 1 are excluded and our equilibrium selection assumption thus guarantees that it is detrimental to exclude group 1. Suppose then that $(0,\mu_2^*(0))$ is not an equilibrium profile (without set-asides). This implies that $\mu_1^*(\mu_2^*(0))>0$.

Consider the function $\mu\to\mu_1^*(\mu_2^*(\mu))$. The group 1 equilibrium profile candidates (without set-asides) corresponds to a fixed point of this function, or equivalently (given that $(0,\mu_2^*(0))$ is not an equilibrium profile) the set of μ such that $\Pi_1^{ent}(\mu_1^*(\mu_2^*(\mu)),\mu_2^*(\mu))=0$. Consider the smallest solution (as a fixed point) which we denote by $\underline{\mu}_1>0$. We have then $\mu_1^*(\mu_2^*(\mu))\geq\mu$ for any $\mu\in[0,\underline{\mu}_1]$ and thus $\Pi_1^{ent}(\mu,\mu_2^*(\mu))\geq 0$ for any $\mu\leq\mu_1^*(\mu_2^*(\mu))$.

Let $\widetilde{TW}(\mu;\phi)=TW((\mu,\mu_2^*(\mu));\phi)$ denote the net expected welfare as a function of the entry rate from group 1 given that group 2 entry rate is the equilibrium one in the auction ϕ. We show next that $\widetilde{TW}(\underline{\mu}_1;\phi)\geq\widetilde{TW}(0;\phi)$. By our equilibrium selection assumption and given that the seller's expected revenue coincides with the total welfare, if this inequality is true then it holds for the equilibrium profile which will allow us to conclude.

To show this inequality, write:

$$\frac{d\widetilde{TW}(\mu)}{d\mu} = \frac{\partial \widetilde{TW}}{\partial \mu_1}(\mu, \mu_2^*(\mu)) + \underbrace{\frac{d\mu_2^*(\mu)}{d\mu}}_{\leq 0} \cdot \frac{\partial \widetilde{TW}}{\partial \mu_2}(\mu, \mu_2^*(\mu)).$$

We apply next Lemma 5.2. If bidders from group 1 are disadvantaged in auction ϕ, then

$$W(N_{+1}; \phi) - W(N; \phi) \geq V_1^{ent}(N_{+k}; \phi) \tag{33}$$

and thus $\frac{\partial \widetilde{TW}}{\partial \mu_1}(\mu_1, \mu_2) \geq \Pi_1^{ent}(\mu_1, \mu_2)$. Hence, we obtain that $\frac{\partial \widetilde{TW}}{\partial \mu_1}(\mu, \mu_2^*(\mu)) \geq \Pi_1^{ent}(\mu, \mu_2^*(\mu)) \geq \Pi_1^{ent}(\mu_1^*(\mu_2^*(\mu)), \mu_2^*(\mu)) = 0$ for any $\mu \in [0, \underline{\mu}_1]$.

If bidders from group 2 are advantaged in auction ϕ, then

$$W(N_{+2}; \phi) - W(N; \phi) \leq V_2^{ent}(N_{+k}; \phi) \tag{34}$$

and thus $\frac{\partial \widetilde{TW}}{\partial \mu_2}(\mu_1, \mu_2) \leq \Pi_2^{ent}(\mu_1, \mu_2)$. Hence, we obtain that $\frac{\partial \widetilde{TW}}{\partial \mu_2}(\mu, \mu_2^*(\mu)) \leq \Pi_2^{ent}(\mu, \mu_2^*(\mu)) = 0$. Gathering the previous inequalities, we obtain that $\frac{d\widetilde{TW}(\mu)}{d\mu} \geq 0$ for any $\mu \in [0, \underline{\mu}_1]$, and thus $\widetilde{TW}(\underline{\mu}_1; \phi) \geq \widetilde{TW}(0; \phi)$, as needed to complete the proof. **Q.E.D.**

Technical computations for Section 6.2

We proceed by differentiating the revenue when the strength of incumbent j increase. Let $G_\lambda(.) = [F_j(.)]^\lambda$. $\lambda = 0$ corresponds to the case where incumbent j has been excluded while $\lambda = 1$ corresponds to the case where he is not excluded. Note that we have $\frac{dG_\lambda(x)}{d\lambda} = \log[F_j(x)] \cdot G_\lambda(.) \leq 0$.

Next we had the argument λ in the payoff and revenue function to indicate that the strength of the incumbent j is parameterized by λ. With respect to our previous notation, we will have in particular $\Pi^{ent}(\mu, I; \lambda) = \Pi^{ent}(\mu, I)$ (resp. $= \Pi^{ent}(\mu, I_{-j})$) if $\lambda = 0$ (resp. $= 0$) and similarly $\Pi_i^{inc}(\mu, I; \lambda) = \Pi_i^{inc}(\mu, I)$ (resp. $= \Pi_i^{inc}(\mu, I_{-j})$) if $\lambda = 0$ (resp. $= 0$).

The equilibrium entry rate $\mu(\lambda)$ as a function of the strength λ of the special incumbent j is characterized (provided that this is some entry) by the equation:

$$\Pi^{ent}(\mu(\lambda), I; \lambda) = \sum_{n=0}^{\infty} e^{-\mu(\lambda)} \frac{[\mu(\lambda)]^n}{n!} \cdot \int_{X_S}^{\infty} \prod_{i \in I_{-j}} F_i^I(x) G_\lambda(x) [F(x)]^n \cdot (1 - F(x)) dx = C$$

or equivalently

$$\int_{X_S}^{\infty} \prod_{i \in I_{-j}} F_i^I(x) G_\lambda(x) e^{-\mu(\lambda)[1-F(x)]} \cdot (1 - F(x)) dx = C.$$

We have then

$$\frac{d\mu(\lambda)}{d\lambda} = \frac{\int_{X_S}^{\infty} \prod_{i \in I_{-j}} F_i^I(x) \frac{dG_\lambda(x)}{d\lambda} e^{-\mu(\lambda)[1-F(x)]} \cdot (1 - F(x)) dx}{\int_{X_S}^{\infty} \prod_{i \in I_{-j}} F_i^I(x) G_\lambda(x) e^{-\mu(\lambda)[1-F(x)]} \cdot (1 - F(x))^2 dx}.$$

Similarly, the rent of an incumbent $i^* \in I_{-j}$ as a function of λ is given by

$$\Pi_{i^*}^{inc}(\mu(\lambda), I; \lambda) = \int_{X_S}^{\infty} \prod_{i \in I_{-j} \setminus \{i^*\}} F_i^I(x) G_\lambda(x) e^{-\mu(\lambda)[1-F(x)]} \cdot (1 - F_{i^*}^I(x)) dx = C$$

Differentiating the rent of such an incumbent rents with respect to λ, we get:

$$\begin{aligned} \frac{d\Pi_{i^*}^{inc}(\mu(\lambda), I; \lambda)}{d\lambda} &= \frac{\partial\Pi_{i^*}^{inc}(\mu(\lambda), I; \lambda)}{\partial\lambda} + \frac{d\mu(\lambda)}{d\lambda} \cdot \frac{\partial\Pi_{i^*}^{inc}(\mu(\lambda), I; \lambda)}{\partial\mu} \\ &= \int_{X_S}^{\infty} \prod_{i \in I_{-j}} F_i^I(x) \frac{dG_\lambda(x)}{d\lambda} e^{-\mu(\lambda)[1-F(x)]} \cdot (1 - F(x)) dx \cdot \end{aligned}$$

$$\left[\frac{\int_{X_S}^{\infty} \prod_{i \in I_{-j} \setminus \{i^*\}} F_i^I(x) \frac{dG_\lambda(x)}{d\lambda} e^{-\mu(\lambda)[1-F(x)]} \cdot (1 - F_{i^*}^I(x)) dx}{\int_{X_S}^{\infty} \prod_{i \in I_{-j}} F_i^I(x) \frac{dG_\lambda(x)}{d\lambda} e^{-\mu(\lambda)[1-F(x)]} \cdot (1 - F(x)) dx} - \frac{\int_{X_S}^{\infty} \prod_{i \in I_{-j} \setminus \{i^*\}} F_i^I(x) G_\lambda(x) e^{-\mu(\lambda)[1-F(x)]} \cdot (1 - F_{i^*}^I(x)) \cdot (1 - F(x)) dx}{\int_{X_S}^{\infty} \prod_{i \in I_{-j}} F_i^I(x) G_\lambda(x) e^{-\mu(\lambda)[1-F(x)]} \cdot (1 - F(x))^2 dx} \right].$$

Let $U_i(x) := \frac{1}{F_i^I(x)} \cdot \frac{1-F_i^I(x)}{1-F(x)}$, which has been assumed to be decreasing on $(\underline{x}, \overline{x})$ for any $i \in I_{-j}$.

Let h_1 denote the density function defined by $h_1(x) := \frac{\prod_{i \in I_{-j}} F_i^I(x) |\frac{dG_\lambda(x)}{d\lambda}| e^{-\mu(\lambda)[1-F(x)]} \cdot (1-F(x))}{\int_{X_S}^{\infty} \prod_{i \in I_{-j}} F_i^I(x) |\frac{dG_\lambda(x)}{d\lambda}| e^{-\mu(\lambda)[1-F(x)]} \cdot (1-F(x)) dx}$ for $x \geq X_S$ and 0 otherwise.

Let h_2 denote the density function defined by $h_2(x) := \frac{\prod_{i \in I_{-j}} F_i^I(x) G_\lambda(x) e^{-\mu(\lambda)[1-F(x)]} \cdot (1-F(x))^2}{\int_{X_S}^{\infty} \prod_{i \in I_{-j}} F_i^I(x) G_\lambda(x) e^{-\mu(\lambda)[1-F(x)]} \cdot (1-F(x))^2 dx}$ for $x \geq X_S$ and 0 otherwise.

Note that the support of the pdf h_1 and h_2 is $[\max\{X_S, \underline{x}\}, \overline{x}]$.

We have then

$$\frac{d\Pi_{i^*}^{inc}(\mu(\lambda), I; \lambda)}{d\lambda} = -\int_{X_S}^{\infty} \prod_{i \in I_{-j}} F_i^I(x) |\frac{dG_\lambda(x)}{d\lambda}| e^{-\mu(\lambda)[1-F(x)]} \cdot (1 - F(x)) dx \cdot \left[E_{x \sim h_1}[U_{i^*}(x)] - E_{x \sim h_2}[U_{i^*}(x)] \right].$$

Note also that the assumption that $x \to -\frac{\log[F_j^I(x)]}{1-F(x)}$ is decreasing on $[\underline{x}, \overline{x}]$ implies that the likelihood ratio $\frac{h_1}{h_2}$ is decreasing on the interior of its support. Likelihood ratio dominance implies first-order stochastic dominance (see Appendix B in Krishna (2003)). Since U_i is decreasing on $(\underline{x}, \overline{x})$, we obtain finally that $E_{x \sim h_1}[U_{i^*}(x)] > E_{x \sim h_2}[U_{i^*}(x)]$ and thus that $\frac{d\Pi_{i^*}^{inc}(\mu(\lambda), I; \lambda)}{d\lambda} < 0$ which further implies by integration that $\Pi_{i^*}^{inc}(\mu(1), I; 1) < \Pi_{i^*}^{inc}(\mu(0), I; 0)$ for any $i^* \in I_{-j}$.

By Taylor expansion of the expression (22) with respect to the parameter λ, the impact on the revenue to exclude the incumbent i is of the form:

$$\frac{dR(\mu(\lambda), I)}{d\lambda}|_{\lambda=0} = \underbrace{\frac{\partial TW(\mu(\lambda), I_{-i})}{\partial\mu}|_{\lambda=0}}_{=0} \cdot \underbrace{\frac{d\mu(\lambda)}{d\lambda}|_{\lambda=0}}_{\text{well defined}} + \frac{d\Pi_{i^*}^{inc}(\mu(\lambda), I; \lambda)}{d\lambda} < 0$$

as shown above.

A sufficient condition for Assumption 2

Next lemma provides a sufficient simple condition on the CDF of the incumbents that will guarantee that Assumption 2 holds. A special case of it is when incumbents and entrants are all symmetric, i.e. when $F_i^I = F$ for any incumbent i.

Lemma A1 Let $G(x) := \sum_{j=1}^{\infty} s_j \cdot [F(x)]^j$ where $s_j \in [0,1]$ for any j and $\sum_{j=1}^{\infty} s_j = 1$. Assume that the support of F is $[\underline{x}, \overline{x}]$. Then the functions $x \to \frac{1-G(x)}{G(x)(1-F(x))}$ and $x \to \frac{-\log[G(x)]}{(1-F(x))}$ are both decreasing on $(\underline{x}, \overline{x})$.

Proof

On $(\underline{x}, \overline{x})$, we have

$$\frac{(1-G(x))}{G(x)(1-F(x))} = \frac{1}{G(x)} \cdot \sum_{j=1}^{\infty} s_j \cdot \frac{1-[F(x)]^j}{1-F(x)} = \frac{\sum_{j=1}^{\infty} s_j \cdot \sum_{k=0}^{j-1} [F(x)]^k}{\sum_{j=1}^{\infty} s_j \cdot [F(x)]^j}. \tag{35}$$

Taking the derivative of the right-hand side of (35), we obtain that the derivative of $\frac{(1-G(x))}{G(x)(1-F(x))}$ has the same sign as $(\sum_{j=2}^{\infty} s_j \cdot \sum_{k=1}^{j-1} k[F(x)]^{k-1}) \cdot (\sum_{j=1}^{\infty} s_j \cdot [F(x)]^j) - (\sum_{j=1}^{\infty} s_j \cdot \sum_{k=0}^{j-1} [F(x)]^k) \cdot (\sum_{j=1}^{\infty} s_j \cdot j[F(x)]^{j-1})$.

In order to obtain that $\frac{(1-G(x))}{G(x)(1-F(x))}$ is strictly decreasing on $(\underline{x}, \overline{x})$, it is thus sufficient to check that

$$\frac{\sum_{j=1}^{\infty} s_j \cdot j[F(x)]^{j-1}}{\sum_{j=1}^{\infty} s_j \cdot [F(x)]^j} > \frac{\sum_{j=2}^{\infty} s_j \cdot \sum_{k=1}^{j-1} k[F(x)]^{k-1}}{\sum_{j=1}^{\infty} s_j \cdot \sum_{k=0}^{j-1} [F(x)]^k}$$

if $F(x) > 0$. This results from the fact that $\frac{j \cdot [F(x)]^{j-1}}{[F(x)]^j} > \frac{\sum_{k=1}^{j-1} k[F(x)]^{k-1}}{\sum_{k=0}^{j-1} [F(x)]^k}$ for any $j \geq 2$, which is equivalent to the inequalities $\sum_{k=0}^{j-1} j \cdot [F(x)]^{k-1} > \sum_{k=1}^{j-1} k \cdot [F(x)]^{k-1}$ for any $j \geq 2$, which obviously hold.

We consider now the function $x \to \frac{-\log[G(x)]}{(1-F(x))}$. On $(\underline{x}, \overline{x})$, its derivative is given by

$$\frac{\partial[\frac{-\log[G(x)]}{(1-F(x))}]}{\partial x} = \frac{f(x)}{(1-F(x))^2} \cdot \left[-\frac{\sum_{j=1}^{\infty} s_j \cdot (1-F(x)) j \cdot [F(x)]^{j-1}}{G(x)} - \log[G(x)] \right]$$

From the inequality $\log(x) > -\frac{1-x}{x}$ for any $x \in (0,1)$, we have

$$G(x) \cdot \log[G(x)] > -(1-G(x)) = \sum_{j=1}^{\infty} s_j \cdot (1-F(x)) \sum_{k=0}^{j-1} [F(x)]^k \geq \sum_{j=1}^{\infty} s_j \cdot (1-F(x)) j \cdot [F(x)]^{j-1}$$

On the whole, we obtain that $\frac{\partial[\frac{-\log[G(x)]}{(1-F(x))}]}{\partial x} < 0$ on $(\underline{x}, \overline{x})$.

Q.E.D.

Proof of Proposition 7.3

When entrants are symmetric (so that μ reduces to a scalar), the equilibrium entry profile without subsidies, denoted by μ^*_{no-fee}, satisfies $\mu^*_{no-fee} \in \max_{\mu \geq 0}\{TW(\mu, I)\}$. Let us assume that $\mu^*_{no-fee} > 0$ (which implies in particular that entrants have the highest valuations with positive probability). Since $\mu \to TW(\mu, I)$ is (strictly) concave and the functions $\mu \to \Pi_j^{inc}(\mu, I)$ are nonincreasing, we obtain that the equilibrium entry profile with the optimal subsidy, denoted by $\mu^*_{with-fee}$ and which is characterized by $\mu^*_{with-fee} \in \max_{\mu \geq 0}\{TW(\mu, I) - \sum_{j \in I} \Pi_j^{inc}(\mu, I)\}$, satisfies $\mu^*_{with-fee} > \mu^*_{no-fee}$. Since $\mu \to \Pi^{ent}(\mu, I)$ is nonincreasing with $\lim_{\mu \to \infty} \Pi^{ent}(\mu, I) = -C$, we have thus $-C < \Pi^{ent}(\mu^*_{with-fee}, I) < \Pi^{ent}(\mu^*_{no-fee}, I) = 0$ which means that the optimal fee takes the form of a partial reimbursement of the entry cost C. **Q.E.D.**