DISCUSSION PAPER SERIES

DP11550

THE EFFECT OF A MERGER ON INVESTMENTS

Massimo Motta and Emanuele Tarantino

INDUSTRIAL ORGANIZATION

ISSN 0265-8003

THE EFFECT OF A MERGER ON INVESTMENTS

Massimo Motta and Emanuele Tarantino

Discussion Paper DP11550
Published 04 October 2016
Submitted 04 October 2016

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

THE EFFECT OF A MERGER ON INVESTMENTS

Abstract

It has been suggested that mergers, by increasing profitability, will also result in higher investments. To deal with this claim, we first study a general model with simultaneous cost-reducing investments and price choices. Absent scope economies, the merger is anti-competitive: it lowers both total output and investment. With sequential choices, we provide a sufficient condition in a general model for the merger to be anti-competitive. The results are confirmed in a standard Shubik-Levitan parametric model. Only if the merger entails sufficient scope economies, will it be pro-competitive. We also show that a Network Sharing Agreement (by which parties set their investment cooperatively) is preferable to a merger. Finally, we identify a class of models where the same qualitative results extend to quality-enhancing investments.

JEL Classification: N/A

Keywords: Horizontal mergers, innovation, Investments, Network-sharing Agreements

Massimo Motta - massimo.motta@upf.edu
ICREA-Universitat Pompeu Fabra and Barcelona GSE and CEPR

Emanuele Tarantino - etaranti@gmail.com
University of Mannheim and CEPR

Acknowledgements
We thank Benno Buehler, Emeric Henry, Bruno Jullien, Michele Polo, Markus Reisinger and the participants in the Northwestern University seminar and in the CEPR-JIE Conference on Applied Industrial Organization, MaCCI Summer Institute, MaCCI Annual Conference, and Economic Advisory Group plenary meeting (European Commission) for helpful comments. The usual disclaimer applies. Previous versions of this paper were written while Motta was serving as Chief Competition Economist at the European Commission. Obviously, this paper does not necessarily represent the views of that institution.

The Effect of a Merger on Investments*

Massimo Motta
ICREA-Universitat Pompeu Fabra and Barcelona GSE

Emanuele Tarantino
University of Mannheim and MaCCI

30 September 2016

Abstract

It has been suggested that mergers, by increasing profitability, will also result in higher investments. To deal with this claim, we first study a general model with simultaneous cost-reducing investments and price choices. Absent scope economies, the merger is anti-competitive: it lowers both total output and investment. With sequential choices, we provide a sufficient condition in a general model for the merger to be anti-competitive. The results are confirmed in a standard Shubik-Levitan parametric model. Only if the merger entails sufficient scope economies, will it be pro-competitive. We also show that a Network Sharing Agreement (by which parties set their investment cooperatively) is preferable to a merger. Finally, we identify a class of models where the same qualitative results extend to quality-enhancing investments.

Keywords: horizontal mergers; innovation; investments; network-sharing agreements.

*We thank Benno Buehler, Emeric Henry, Bruno Jullien, Michele Polo, Markus Reisinger and the participants in the Northwestern University seminar and in the CEPR-JIE Conference on Applied Industrial Organization, MaCCI Summer Institute, MaCCI Annual Conference, and Economic Advisory Group plenary meeting (European Commission) for helpful comments. The usual disclaimer applies. Previous versions of this paper were written while Motta was serving as Chief Competition Economist at the European Commission. Obviously, this paper does not necessarily represent the views of that institution.

1 Introduction

This paper proposes a theoretical analysis of the effects of a merger on investments. Our main motivation comes from a series of very recent high-profile mergers in the mobile telephony industry in the EU,[1] in which the industry has criticised the European Commission (which had jurisdiction on these mergers) for failing to take into account that the mergers would have led to higher investments. Mobile Network Operators (MNOs) have made two main arguments in support of this claim. The first is related to existence of scale economies of various nature, and as such it is not conceptually controversial (but it would need to be empirically verified).[2] The second argues that a merger favours investments because industry consolidation gives firms higher profits, and hence stronger incentives to invest.

Interestingly, the existing literature on competition and innovation (or investment) does not seem at first sight inconsistent with this view. The work by Aghion, Bloom, Blundell, Griffith, and Howitt (2005) identifies an inverted-U shape relationship between competition and innovation. Often building on a supposed contrast between the ideas of Schumpeter—who stress the importance of market power for enabling innovation—and those of Arrow—who show that operating in a competitive environment gives higher incentives to invest than in a monopolistic one,[3] several authors have suggested the same ambiguous relationship between competition and investment/innovation.

Vives (2008) reviews the literature on this topic and studies a range of models (incorporating different assumptions such as free and fixed entry; price and quantity choices; simultaneous and sequential moves) to identify the relationship between competition and investments. Most relevant for the question of how mergers impact upon investments is the restricted entry case (most of the mergers investigated in depth by antitrust agencies concern industries which are highly concentrated and where barriers to entry are important).[4] In Vives' baseline case (simultaneous investment and price decisions), he finds that when the number of firms, n (a proxy for competition), increases, the per-firm investment, x, decreases: competition reduces investment. This is because as n rises, two contrasting effects occur: on the one hand, the residual demand faced by the firm decreases, which tends to reduce x; on the other hand, demand elasticity increases, which tends to increase x by raising the incentive to steal business from rivals; this latter effect,

[1]See the European Commission decisions on the Hutchison/Orange (Austria), Hutchison/Telefonica Ireland, Telefonica Deutschland/EPlus, TeliaSonera/Telenor, and Hutchison 3G/Telefonica UK cases.

[2]The European Commission requires efficiencies to be verifiable, merger-specific, and beneficial to consumers, the three being cumulative conditions. This implies that efficiency claims need to be fully documented, that synergies that could be achieved through less anticompetitive means (e.g., via internal growth or via a Network-Sharing Agreement) would not be recognised, and that savings on fixed costs would not be accepted as they would not lead to lower prices.

[3]See Frontier (2014: pp.13): "On the one hand, greater market concentration resulting from a merger can be expected to increase the returns a firm might anticipate from new investments, since it will have fewer rivals to share these with or who will compete them away. This is referred to as the 'Schumpeterian effect' and leads to higher investment. On the other hand, greater market concentration may mean that a firm has a weaker incentive to 'escape competition' by investing to get ahead of its rivals. This is because the incremental benefits from such an investment compared to current returns may be smaller in a concentrated market. This suggests that less investment will take place and explains why competition authorities often claim that mergers will not incentivise companies to invest."

[4]For example, in the case of mobile telephony mergers, the number of network operators is fixed by regulation (and would in any case be limited by spectrum constraints).

however, is dominated by the former.[5] Even though competition (as proxied by the number of firms) reduces per-firm investment, Vives (2008) finds some support to the result that both total investments (nx) and R&D intensity (the ratio of investment outlays over sales) increase with the number of firms.[6] Lopez and Vives (2015) extend the setting in Vives (2008) to study the welfare consequences of symmetric cross-ownership agreements among competing firms that engage in cost-reducing investment.[7] They find that, absent spillovers, cross-ownerships reduce total and consumer surplus.[8]

In a recent paper, Shapiro (2013) has criticised the alleged contradiction between the Schumpeter and Arrow's principles. As he points out, the former principle should be read as 'appropriability matters', in the sense that *after* an innovation takes place, the successful innovator should be able to benefit from it; while the latter principle implies that *before* an innovation, firms will have more incentives to invest if they operate in a competitive environment. Authors who analyse the effects of competition on innovation, by studying the comparative statics on variables such as number of firms or product substitutability confound the ex-ante and the ex-post aspects. The reason is that they model an increase in the degree of competition as giving rise to an environment not only with more competition ex ante, but also with less appropriability ex post. But a stricter antitrust policy, for instance one that introduces an anti-cartel law, would affect the former but not the latter (which would instead be affected by the introduction of a weaker intellectual property right law).[9]

Consistent with the approach suggested by Shapiro, we argue that the comparative statics results in the abovementioned papers cannot account for the fact that, if one considers a model with symmetric firms representing the status quo, a merger will create an *asymmetry* in the market. The merging parties will combine their assets to create, say, a new firm with larger capacity, or with a larger product portfolio, or with more plants, than the rivals. Accordingly, in this paper we use a model where the merged entities become multi-product, whereas their rivals remain single-product firms. In the model, we let firms compete in prices and assume that these prices are strategic complements.[10]

[5]Vives (2008) also shows that per firm investment increases with the degree of product substitutability—another proxy for competition. However, this result holds as long as increased substitutability does not reduce average demand for product variety.

[6]Schmutzler (2013) develops on Vives (2008) by looking at a setting with asymmetric firms engaging cost reducing investment. He shows that a firm that is particularly efficient in the status quo increases its investment as competition intensifies. The presence of spillovers reduces this effect.

[7]See also Shelegia and Spiegel (2015) for a model of cost-reducing investments in an industry with partial cross-ownership.

[8]Lopez and Vives (2015) draw on the vast literature analyzing the competitive effects of cooperative R&D decisions in oligopoly. This literature was pioneered by d'Aspremont and Jacquemin (1988) and has been further developed by, among many others, Kamien, Muller, and Zang (1992), and Leahy and Neary (1997) (see Gilbert, 2006, for a survey on the topic).

[9]Shapiro (2013) also reviews the existing empirical evidence on the relationship between competition, investments and productivity. Two exhaustive surveys on this topic are in Bartelsman and Doms (2000) and Syverson (2011).

[10]Under strategic substitutability, and absent investments, the merger is generally unprofitable unless additional assumptions are made, such as there are sufficient efficiency gains or the merger involves a sufficient number of firms (see Salant, Switzer, and Reynolds, 1983). Note though that in the general model we do not impose strategic complementarity at the investment levels, and this may still raise profitability issues. However, when analysing explicit functional form examples, we shall identify conditions guaranteeing that the merger is profitable.

Both within a general model and by relying on a standard parametric model of product differentiation to illustrate our results and to sign effects when the general model gives ambiguous results, we study the effect of a merger not only on investments, but also on prices, and ultimately on consumer surplus. (Indeed, it is conceivable that, albeit the merger may increase investments, it may also lead to an increase in prices that outweighs the positive effects for consumers of reduced production costs or increased quality.)

We start by looking at a model in which prices and (cost-reducing) investments are decided simultaneously. This case replicates a situation in which the pricing stage takes place when the results of the investment efforts are not yet known. In this framework, we show that—absent economies of scope (i.e. a situation where joint ownership of assets gives rise to lower cost of investing) —the merger decreases total output and total investments, thereby unambiguously reducing consumer surplus. It is only if economies of scope are strong enough that a merger will be beneficial relative to the status quo.

In the simultaneous case, the equilibrium value of firms' investment is proportional to the final-good quantities they produce. Therefore, by raising the equilibrium value of merging parties' prices the merger yields a clearly anticompetitive outcome, since the lower quantity produced will also entail lower investment. The intuition for the increase in prices is standard: merging parties internalize the negative externality they exert on each other at the pricing stage. Strategic complementarity implies that outsiders will—other things being equal— also increase their prices, although by less than the insiders (see e.g. Deneckere and Davidson, 1985). This increases outsiders' quantities and their investments, but under regularity conditions total industry investment will fall.

We then analyze a model in which firms set first investments and then prices. First, we find that the *merging firms* increase their price (as usual) and reduce their investment: insiders internalize the impact of an increase in the level of investment on competing firms' final-good prices. Next, we analyze the impact of these results on outsiders' strategic variables. If investments were strategic complements, then the reduction in the investment by insiders would imply a reduction in outsiders' investments, and a further increase of their prices, on top of the increase due to the strategic complementarity in prices. In this case, the prices of all firms in the market increase, and the investments decrease, to make the merger clearly anti-competitive.

Throughout the paper, we call a merger "anticompetitive" if it reduces consumer surplus. This is the standard adopted by the US and European competition agencies when they screen mergers. Indeed, the US Horizontal Merger Guidelines focus on the effect of a merger on customers and efficiency gains are accepted only to the extent that they will lead to lower prices. Similarly, in assessing both agreements and mergers, the European Commission admits only cost savings that are passed on to consumers. There is a debate as to whether agencies should instead adopt a total welfare standard.[11] This discussion is beyond the scope of this paper, but for completeness we shall also indicate—when we are able to identify them—the effects of the merger on total surplus.

If instead investments are strategic substitutes, then the merger would tend to increase the investment set by outsiders, as well as the quantity they produce after the merger. In the general model, we find a sufficient condition under which the merger is unambiguously anti-competitive even if investments are strategic substitutes.

[11] See e.g. Neven and Röller (2005), Farrell and Shapiro (2006), and Pittman (2007).

We also study the sequential investment-then-price game within a standard product differentiation model based on a Shubik-Levitan linear quadratic utility function, and we find that—absent economies of scope in investments—the merger is unambiguously anti-competitive, as it increases prices and reduces investment. As a consequence, consumer and total surplus are lower than in the benchmark.[12]

Further, we also study the effects of a merger when firms undertake quality-enhancing investments. Within a general model, the results are a priori ambiguous, as we are unable to sign the net result of effects going into opposite directions. However, we show that there exists a class of models with quality-enhancing investments which turn out to be equivalent to the models with cost-reducing investments we have discussed above. For such a class,[13] the same results as above will therefore apply. Finally, the result that the merger reduces total investment and consumer surplus also arises in a vertical product differentiation model a la Shaked and Sutton. In that model, though, the merged entity will suppress one of its two products, and more differentiation will arise at equilibrium. The resulting relaxation of competition increases producer surplus so much that this effect outweighs the fall in consumer surplus.

By referring back to the arguments used by the mobile network operator industry, therefore, the claim that mergers promote investment because consolidation allows for higher profits, and hence higher incentives to invest, appears unfounded in the light of our analysis. What may be true, instead, is that if there are efficiency gains in investments that can only be achieved by means of a merger, then the merger may be beneficial. But of course whether the 'if' applies is an empirical fact that should be verified case by case, and that is already foreseen by the current rules on merger control in major jurisdictions, like the US and the EU.

The reference to the mobile telephony industry raises another related question, that is whether it is possible to benefit from savings in the investment outlays without a full-fledged merger. Indeed, several national regulatory authorities in the EU and elsewhere have often allowed MNOs to engage in Network Sharing Agreements (NSAs), whereby they share different elements of the infrastructure and possibly of the spectrum[14] while continuing to behave independently at the retail level. We model NSAs as R&D cooperative agreements (see, e.g., d'Aspremont and Jacquemin, 1988): firms participating in the agreement decide investments to maximise joint profits, but they behave non-cooperatively when setting prices.

The assumption that investment are taken cooperatively is consistent with the observation that, typically, the contract signed between NSAs' parties specifies the volume of future investments that each party is to undertake under the agreement. This practice also leaves limited room to free-riding.[15] Moreover, we assume that prices are set non-cooperatively, as otherwise the NSAs would be equivalent to full-fledged mergers.[16]

[12] In a previous version, we obtained the same qualitative results within a Salop circle model.

[13] Vertical product differentiation models with quality-adjusted prices will belong to this class. Examples include Häckner (2000) and Sutton (1997).

[14] There exist several types of NSAs, from passive NSAs where the firms just share the site (say, each firm has its own equipment but they put it on the same mast), to active NSA where active elements are shared, which in their more extreme form also include sharing the spectrum.

[15] It is of course possible, nonetheless, that the value of the investment levels set in the contract accounts for the possibility of free-riding on the infrastructure created within the NSA.

[16] Our approach to modelling NSAs also implicitly assumes that NSA members set access prices to respective facilities at marginal cost.

We show that, in terms of consumer welfare, a NSA is more efficient than the merger and the status quo with independent firms in the model with simultaneous moves. Instead, in the model with sequential choices, a NSA presents in general the following trade-off. *For given investments*, it does not distort retail prices—whereas a merger increases prices and thus reduces quantities of merging parties; in turn, since the marginal profit from investing rises with sales, this price effect tends to reduce investments in a merger but not in a NSA. But *for given prices*, they give lower incentives to invest than a merger. This is because when choosing investment levels jointly, it is anticipated that a firm's investment reduces prices and profits of its partner, hence reducing incentives to invest.

Whereas in a general model with sequential moves we are not able to establish whether a NSA leads to more or less investment than a merger, we show that in a horizontal product differentiation model based on a Shubik-Levitan linear-quadratic utility function the NSA leads to lower total investments than the merger, but it nonetheless leads to lower prices and higher consumer and total welfare than the merger.[17] Our analysis shows therefore that when spillovers are nil or weak enough, a merger is dominated by the status quo, whereas when spillovers are sufficiently high, the merger is dominated by the NSA.[18]

A complementary perspective to the analysis of the relationship between competition and investment is offered by the literature studying dynamic oligopoly games.[19] For instance, Mermelstein, Nocke, Satterthwaite, and Whinston (2015) study the impact of mergers on the evolution of an industry, and derive the optimal dynamic merger review policy in a model with capital accumulation and economies of scale. Differently from our model, in their setting two firms bargain over a merger to monopolize the industry. These firms invest to accumulate capital and exploit scale economies. Post-merger, an entrant appears in the market with zero capital. (Apart from having a different aim, their assuming efficiency gains and free entry clearly differentiates their environment from ours.) They find that the antitrust authority should depart from the myopic policy suggested by Nocke and Whinston (2010), and instead undertake a more restrictive policy.

The paper continues thus. We first describe a general model with cost-reducing investments (Section 2) and then study (Section 3) the effects of the merger within it, considering first a simultaneous and then a sequential game. (These games will also be studied within a parametric model.) We also consider the effects of Network-Sharing Agreements. We then analyse the case of quality enhancing investments within a general model in Section 4. Finally, Section 5 will conclude the paper. All proofs will be relegated to the Appendix.

2 A model of price competition

We use a model of Bertrand oligopoly with differentiated goods and $n \geq 3$ firms (three being the minimum number of firms for which we can look at the effects on both insiders and outsiders). Demand for the good produced by firm i is given by $q_i(p_i, \bar{p}_{-i})$, where p_i is the price of firm i

[17] In a previous version of the paper, we have reached the same conclusion by relying on a Salop circle model.

[18] Of course, it is conceivable that for contractual or other reasons the NSA may not allow partner firms to achieve the scale economies that can be obtained when there is a full merger. But this is again a factual and case-specific claim that firms should substantiate.

[19] See, among others, Ericson and Pakes (1995), Gowrisankaran (1999), Fershtman and Pakes (2000), and, more recently, Mermelstein, Nocke, Satterthwaite, and Whinston (2015).

and $\bar{p}_{-i}$ is the $(n-1) \times 1$ vector of prices set by firms $-i \neq i$. The number of independent firms, n, is exogenous, reflecting barriers to entry, although it changes with the merger.

Each firm i sets its price p_i and its cost-reducing investment x_i to maximize its profits, given rivals' choices. We denote by $c(x_i) \leq c$ firm i's marginal cost as function of x_i, with $c' < 0$, $c'' \geq 0$ and $c(0) = c_0 \geq 0$. Moreover, $F(x_i)$ is the cost borne by firm i to invest x_i, with $F', F'' > 0$ and $F(0) = 0$. 1 If firms i and k are merged or joined a Network Sharing Agreement (NSA), they generate economies of scope such that an insider's marginal cost of production decreases with the own and the other insider's investment: $c_i(x_i, x_k) = c(x_i + \lambda x_k)$, with $\lambda > 0$. Parameter λ captures the importance of scope economies triggered by the merger or the NSA.[20]

Function $q_i(p_i, \bar{p}_{-i})$ is symmetric,[21] strictly decreasing and twice continuously differentiable whenever $q_i > 0$. We employ the following standard assumptions:

$$\begin{aligned}
&(A1)\text{: } \partial_{p_i} q_i(p_i, \bar{p}_{-i}) < 0,\ \partial^2_{p_i p_i} q_i(p_i, \bar{p}_{-i}) < 0, \quad \forall p_i,\\
&(A2)\text{: } \partial_{p_j} q_i(p_i, \bar{p}_{-i}) \geq 0,\ |\partial_{p_i} q_i(p_i, \bar{p}_{-i})| > (n-1)\partial_{p_j} q_i(p_i, \bar{p}_{-i}), \quad \forall p_i, p_j, j \neq i,
\end{aligned}$$

where ∂_{p_i} and $\partial^2_{p_i p_i}$ denote, respectively, the first- and the second-order derivative with respect to p_i, for all $i = 1, ..., n$. These assumptions imply that the demand of firm i decreases and is concave in p_i (*A1*), that goods are substitutes and own price effects are larger than cross price effects (*A2*).

We also assume that

$$\begin{aligned}
&(A3)\text{: } \partial^2_{p_i p_j} \pi_i > 0\ \forall p_i, p_j, j \neq i,\\
&(A4)\text{: } \partial^2_{p_i p_i} \pi_i + (n-1)\partial^2_{p_i p_j} \pi_i < 0\ \forall p_i, p_j, i \neq j,
\end{aligned}$$

meaning that retail prices are strategic complements (*A3*), and the first-order conditions for p_i have a unique solution (*A4*). As we will show below, (*A4*) implies that $dp_i/dp_j < 1$, with $j \neq i$.

Parametric model To illustrate the impact of the merger on price, investment and welfare via closed-form solutions, we will also employ a parametric model in which $c(x_i) = c - x_i$, with $c > 0$, $F(x_i) = kx_i^2/2$, with $k = 1$, [22] and, as in the tradition of the Shubik and Levitan (1980), consumers have the following utility function:

$$U(q_1, ..., q_n, I) = \sum_{i=1}^{n} \alpha_i q_i - \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{j \neq i} q_i q_j\right) + I,$$

[20] This formulation allows us to find tractable solutions in our model with explicit functional forms. By taking this approach to model scope economies, we also follow the literature on R&D cooperation (e.g., d'Aspremont and Jacquemin, 1988, among many others). In that literature, though, the investment by a company may involuntarily reduce the cost of another company, to represent a situation where R&D may have feature of a public good. In this paper, instead, we assume away involuntary spillovers, consistent with a setting where a firm is able to fully appropriate its own investments.

[21] That is, the demand of firm i when it sets a price equal to p and all the other firms set a price equal to z in vector $\bar{z}$ is the same as the quantity of a firm j setting p given that all other firms set a price equal to z in vector $\bar{z}$ (i.e., $q_i(p, \bar{z}) = q_j(p, \bar{z})$) for all i, j.

[22] The restriction to $k = 1$ is only to simplify the exposition of our results. Our main outcomes are confirmed in a model featuring $k \neq 1$.

where $\gamma \in (0,1)$ is a parameter that measures products' substitutability and I is consumers' income. For $\gamma = 0$, products are independent and firms act as monopolists. For $\gamma = 1$, instead, firms' products become perfect substitutes. In Section 4 of the paper, we also deal with quality-increasing investments: Following Häckner (2000), we shall then interpret $\alpha_i > 0$ as measuring a product i's quality in a vertical sense, so that the marginal utility of consuming good i increases in α_i.

Consumers' utility maximization under the budget constraint implies that, with Bertrand competition, a firm i's demand is given by:

$$q_i = \frac{(\alpha_i - p_i)[\gamma(n-2)+1] - \gamma \sum_{j \neq i}(\alpha_j - p_j)}{(1-\gamma)[\gamma(n-1)+1]}, \ i = 1,..,n,$$

which implies that all our assumptions above hold true in this parametric model. It also follows that $\partial q_i/\partial \alpha_i > 0$, $\partial q_j/\partial \alpha_i < 0$ and $\partial Q/\partial \alpha_i > 0$, with $i, j = 1,..,n$, $j \neq i$, and $Q = \sum_{i=1}^{n} q_i$. As in standard models of vertical product differentiation, an increase in the quality of a firm's good increases own demand and decreases rivals' demand. Finally, industry quantity (Q) increases with the quality of a firm i's good.

3 Equilibrium analysis

In this section, we analyze the effects of the merger. First, in our base model with simultaneous choices and then in a sequential move game where firms first invest and then set prices. The simultaneous case will give us a useful starting point, to see the possible forces at work before analyzing the strategic effects taking place in the sequential-moves game.

3.1 Simultaneous choices

To begin with, we study the case in which each firm i simultaneously sets its price p_i and its cost-reducing investment x_i. First, we analyze the benchmark (or status-quo) case, where there are n independent firms. Then, we study the effects of the merger, where two out of the n firms merge. In this context, we solve for the Nash equilibrium in prices and investment under the condition that

$$(A5)\text{: } (\partial^2_{p_i p_i}\pi_i)(\partial^2_{x_i x_i}\pi_i) > \left(\partial^2_{p_i x_i}\pi_i\right)^2 \ \forall p_i, x_i,$$

which implies the stability of the equilibrium values of p_i and x_i as resulting from the first-order conditions. All proofs are in the appendix.

3.1.1 Benchmark with independent firms

In the benchmark, each firm i solves the following maximization problem:

$$\max_{p_i, x_i} \ \pi_i(p_i, \bar{p}_{-i}, x_i) = (p_i - c(x_i))q_i(p_i, \bar{p}_{-i}) - F(x_i), \ i = 1,..,n.$$

The associated first-order conditions are:

$$\partial_{p_i}\pi_i = q_i(p_i, \bar{p}_{-i}) + \partial_{p_i} q_i(p_i, \bar{p}_{-i})(p_i - c(x_i)) = 0, \quad (1)$$
$$\partial_{x_i}\pi_i = -c'(x_i)q_i(p_i, \bar{p}_{-i}) - F'(x_i) = 0. \quad (2)$$

Our assumptions guarantee that the second-order conditions with respect to p_i and x_i are satisfied. The first-order condition with respect to x_i is independent of rivals' investment x_{-i}; that is, $dx_i/dx_j = 0$. As a consequence, the investment made by a firm i (x_i) increases with the value of its own quantity q_i—i.e., (2) uniquely determines $x_i^b = x_i(p_i, \bar{p}_{-i})$. The pre-merger results are summarized in Lemma 1.

Lemma 1. *In the benchmark with n independent firms and simultaneous moves, the unique equilibrium is symmetric and features each firm setting a price $p_i = p^b$ and investing $x_i = x^b$, with $i = 1, ..., n$.*

We proceed by deriving some standard comparative-statics properties of Bertrand equilibria that will be useful to determine the competitive consequences of the merger. First, consider the effect of an exogenous increase in the price of $(n-1)$ rival firms on the price of firm i.

Lemma 2. *Let the price of $(n-1)$ firms j marginally increase (with $dp_j = dp_k$ for all $j, k \neq i$), then, also the price of firm i increases but by a lower amount: $dp_i/dp_j \in (0, 1)$.*

As remarked above, this property directly follows from assumption ($A4$). Second, we show how a change in a firm i's investment affects its price and the price of a competing firm j.

Lemma 3. *At the equilibrium, the price of firm i decreases in its own investment ($dp_i/dx_i < 0$) and in the investment of a rival firm $j \neq i$ ($dp_i/dx_j < 0$). Moreover, $|dp_i/dx_i| > |dp_j/dx_i|$.*

Lemma 3 implies that an increase in the value of investment by a firm i renders the industry more price-competitive, by reducing the price of firms i and j, with $j \neq i$. The results in Lemmas 2 and 3, together with the observation that $dx_i/dx_j = 0$, will allow us to establish the consequences for aggregate output of a marginal change in p_i and x_i. We now proceed with the analysis of the merger.[23]

3.1.2 Merger between firm i and firm k

We now analyze the merging firms' problem. The merger may generate economies of scope at the investment stage. Such economies will be represented by the value $\lambda \geq 0$.

Merging firms i and k solve:

$$\max_{p_i, p_k, x_i, x_k} \pi_{i,k}(p_i, p_k, \bar{p}_{-i,-k}, x_i, x_k) = (p_i - c(x_i + \lambda x_k))q_i(p_i, p_k, \bar{p}_{-i-k}) \\ + (p_k - c(x_k + \lambda x_i))q_k(p_k, p_i, \bar{p}_{-i-k}) - F(x_i) - F(x_k); \quad i, k = 1, ..., n, \ i \neq k.$$

The first-order conditions with respect to p_i and x_i follow (we omit those for p_k and x_k, which are symmetric):

$$\partial_{p_i}\pi_{i,k} = q_i(p_i, p_k, \bar{p}_{-i-k}) + \partial_{p_i}q_i(p_i, p_k, \bar{p}_{-i-k})(p_i - c(x_i + \lambda x_k)) \\ + \partial_{p_i}q_k(p_k, p_i, \bar{p}_{-i-k})(p_k - c(x_k + \lambda x_i)) = 0, \tag{3}$$

$$\partial_{x_i}\pi_{i,k} = -\partial_{x_i}c(x_i + \lambda x_k)q_i(p_i, p_k, \bar{p}_{-i-k}) \\ - \lambda\partial_{x_i}c(x_k + \lambda x_i)q_k(p_i, p_k, \bar{p}_{-i-k}) - F'(x_i) = 0. \tag{4}$$

[23] Note that qualitative implications of the results in Lemmas 2 and 3 extend to the merger case.

Moreover, an outsider firm $j \neq i, k$ solves the following problem:

$$\max_{p_j, x_j} \ \pi_j(p_j, \bar{p}_{-j}, x_j) = (p_j - c(x_j))q_j(p_j, \bar{p}_{-j}) - F(x_j),$$

so that its first-order conditions are isomorphic to those of a firm in the benchmark, independently of the value of the economies of scope λ, which affect the merging firms:

$$\begin{aligned} \partial_{p_j}\pi_j &= q_j(p_j, \bar{p}_{-j}) + \partial_{p_j}q_j(p_j, \bar{p}_{-j})(p_j - c(x_j)) = 0, && (5) \\ \partial_{x_j}\pi_j &= -c'(x_j)q_j(p_j, \bar{p}_{-j}) - F'(x_j) = 0. && (6) \end{aligned}$$

We discuss these conditions, and the consequences for equilibrium allocations, by distinguishing between the cases with and without economies of scope. Before proceeding, note that, as in the benchmark, the second-order conditions are satisfied under our assumptions.

Case $\lambda = 0$ (Absence of economies of scope) Were the value of the scope economies to be nil, the first-order conditions above trigger the following outcomes.

First, the value of insiders' and outsiders' investments as function of the vector of prices is the same as in the benchmark with independent firms—i.e., $x_i^{in}(\cdot) = x_i^b(\cdot)$ and $x_j^{out}(\cdot) = x_j^b(\cdot)$, *ceteris paribus*. Moreover, the price set by the insiders (for given investment) as resulting from (3) will generally be larger than in the benchmark (since $p_i > c(x_i)$ and $p_k > c(x_k)$ at equilibrium). The intuition is standard: each insider internalizes the impact of its price choice on the revenue of the other merging party.

As a consequence of this increase in the price of insiders, and the consequent reduction in their output (for $\lambda = 0$ one can see immediately from equation (4) that the marginal profitability of investments increases with the quantity sold), the value of insiders' investments will decrease with respect to the benchmark, *ceteris paribus*.

Therefore, with respect to the benchmark, the merger will yield an increase in the level of firms' prices, because of the combination of a direct and an indirect effect. First, the increase in the price set by insiders directly implies that also outsiders' prices increase, although by a lower amount, by strategic complementarity (Lemma 2). Second, the decrease in the investments set by the merging firms implies an indirect increase in insiders' and outsiders' prices (Lemma 3). In Proposition 1, we recap the consequences of the merger for firms' prices and investment in the model with simultaneous choices and without economies of scope.

Proposition 1. *With simultaneous moves and absent merger-induced scope economies, after the merger insiders raise the equilibrium price and outsiders follow suit due to strategic complementarity. Moreover, insiders decrease their investments, which causes their price to increase further.*

Given the results in Proposition 1, we ask: what is the impact of the merger on industry investment and output? Is the merger profitable? To answer these questions, *we first analyze the impact of a "marginal" merger within our general demand model.* Specifically, given the symmetric equilibrium in the benchmark without merger, we analyze how industry quantities, and insiders' profits, react to a marginal increase in an insider firm's price (p_i) and a marginal decrease of the same firm's investment (x_i), under the condition that the other firms' prices and investments are set at the equilibrium value of the benchmark. This approach allows us to study the marginal effects of the merger, i.e., the implications of a merger within an industry with a large number of firms. To complement this analysis, in our parametric model, we look at a merger within an industry with a small number of firms.

In the following lemma, we study how industry quantity $Q = q_i(p_i(\bar{x}), \bar{p}_{-i}(\bar{x})) + (n-1)q_j(p_j(\bar{x}), \bar{p}_{-j}(\bar{x}))$, with $j \neq i$, reacts to a marginal increase in the price set by firm i (p_i).

Lemma 4. *Consider a marginal increase in firm i's price p_i, at the optimal level of x_i. Under our assumptions, firm i's output falls and firm j's output increases. However, aggregate output moves in the same direction as firm i's output.*

Not surprisingly, a softening in industry competition, as captured by the increase in a firm's price, reduces the aggregate quantity. We proceed by looking at the impact of a marginal change in firm i's investment (x_i) on Q. Indeed, we know from Lemma 3 that a change in x_i has an impact on the prices set by the same firm i and its rivals $-i$; thus, it will eventually affect the value of the industry quantity.

Lemma 5. *Consider a marginal decrease in firm i's investment x_i, at the optimal level of p_i. Under our assumptions, firm i's output falls and firm j's output increases. However, aggregate output moves in the same direction as firm i's output.*

Lemmas 4 and 5 show that, starting from a symmetric equilibrium, the combined effect of a marginal increase in firm i's price and a marginal decrease in the same firm's investment is to reduce aggregate quantity. Since the value of a firms' investment is in a direct relationship with firms' quantity, the reduction in industry output implies that industry investment falls, too. Overall, then, the combined effects of the "marginal" merger act to reduce consumer surplus.

Proposition 2. *With respect to the benchmark (no merger), a marginal increase in firm i's price (p_i), combined with a marginal decrease in firm i's investment (x_i), reduces total investment and consumer surplus.*

Proposition 2 gives us the effects of the "marginal" merger on consumer surplus. In our setting, the analysis of the "marginal" merger requires considering the impact of the marginal changes in the price and investment of merging parties on output and investments, given the symmetric equilibrium in the benchmark without merger. We will analyze the "total" effects of the merger within our parametric demand model, including the equilibrium adjustment of outsiders. We conclude this section with an analysis of the profitability of the "marginal" merger to insiders. Specifically, starting from the symmetric equilibrium of the benchmark (Lemma 1), we show that a marginal increase in merging firms' price, or a marginal reduction in their investment, raises insiders' profits:

Lemma 6. *Given the symmetric equilibrium in the benchmark with independent firms, a marginal increase in firm i's price p_i, or a marginal decrease in firm i's investment x_i, increases firm i's profit.*

Therefore, in a model with simultaneous moves that abstracts from merger-induced scope economies, the "marginal" merger is profitable and anticompetitive. These results are in line with the intuition in Deneckere and Davidson (1985) that, in a model with Bertrand competition, both insiders and outsiders gain from the insiders' price increase due to the merger. Our analysis extends this intuition in a setting in which, after the merger, firms can simultaneously adjust prices and investments within an industry with a large number of firms.

In what follows, we analyze the impact of the merger within our parametric model. We first derive the equilibrium outcomes with the merger, and then compare these outcomes with those in the benchmark (no merger). This will allow us to extend the analysis considering the "marginal" merger to a framework in which the effects of the merger are considered in an industry with 3 firms.

Case $\lambda > 0$ (Presence of economies of scope) If the merger generates economies of scope in investments, so that $\lambda > 0$, a trade-off arises. On the one hand, for given level of the merging parties' investment, the merger tends to raise the price with respect to the status quo, as we have seen above. On the other hand, for given prices, the investment set by merging parties rises due to economies of scope. The magnitude of this increase, and whether it can yield higher investments than in the benchmark, depends on the value of λ.

Were the investment to increase after the merger, then we know from Lemma 3 that firms would set prices more aggressively, *ceteris paribus*. This effect can make the merger procompetitive only if it offsets the negative impact of the increase in price on the industry quantity. Otherwise, the same results as in Proposition 1 arise.

3.1.3 The merger in the parametric model with simultaneous choices

To provide a closed-form illustration of our results, we employ the parametric demand model in Section 2 and assume that, to simplify the analysis, $n = 3$ and $\alpha_i = \alpha_j = \alpha > c$ for all i, j.[24]

In the benchmark, all firms set the following values of price and investment:[25]

$$p^b = \frac{c(1+3\gamma) - 2(\alpha - c)\gamma^2}{1+3\gamma}, \ x^b = \frac{(\alpha - c)(1+\gamma)}{1+3\gamma}.$$

Therefore, the values of a firm profit and consumer surplus are equal to:

$$\pi^b = \frac{3(\alpha - c)^2(1+\gamma)[1+\gamma(1-4\gamma)]}{2(1+3\gamma)^2}, \ C^b = \frac{3(\alpha - c)^2(1+\gamma)^2(1+2\gamma)}{2(1+3\gamma)^2}.$$

Assume now firms 1 and 2 merge. We then then find that insiders set the following values of price and investment:[26]

$$\begin{aligned} p^{in} &= \frac{\alpha\gamma(1-2\gamma)(1-3\gamma^2) + c(1+\gamma)(1+2\gamma)(1-\gamma-\gamma^2)}{1+3\gamma-4\gamma^2[1+(2-\gamma)\gamma]}, \\ x^{in} &= \frac{(\alpha - c)(1-3\gamma^2)}{1+3\gamma-4\gamma^2[1+(2-\gamma)\gamma]}. \end{aligned}$$

Outsiders, instead, set

$$p^{out} = \frac{c(1+3\gamma-2\gamma^2-8\gamma^3) - 2\alpha(1-\gamma^4)}{1+3\gamma-4\gamma^2[1+(2-\gamma)\gamma]}, \ x^{out} = \frac{(\alpha - c)(1+\gamma)(1-2\gamma^2)}{1+3\gamma-4\gamma^2[1+(2-\gamma)\gamma]}.$$

[24] Recall that, with $n = 3$, we have the minimum number of firms that allows us to study the effects of the merger on insiders and outsiders, i.e., not to analyze a merger to monopoly.

[25] Note that the second-order conditions and the stability condition in ($A5$) are satisfied. Moreover, the values of firms' investment and marginal cost are positive for all $\alpha < [2c(1+2\gamma)]/(1+\gamma)$ and $\gamma \in (0, 1)$.

[26] The second-order conditions and the stability condition in ($A5$) are satisfied, and insiders' and outsiders' marginal costs and investments are positive, for all $\gamma \in (0, 1/\sqrt{3}]$ and $\alpha < 2c(1+2\gamma)[1-(3-\gamma)\gamma^2]/(1+\gamma)(1-2\gamma^2)$.

Finally, the value of profits and consumer surplus is:

$$\begin{aligned}
\pi^{in} &= \frac{(\alpha-c)^2(1-3\gamma^2)^2(1+2\gamma-4\gamma^2)}{2\left[1+3\gamma-4\gamma^2[1+(2-\gamma)\gamma]\right]^2}, \\
\pi^{out} &= \frac{(\alpha-c)^2(1+\gamma)(1-2\gamma^2)^2[1+\gamma(1-4\gamma)]}{2\left[1+3\gamma-4\gamma^2[1+(2-\gamma)\gamma]\right]^2}, \\
C^m &= \frac{(\alpha-c)^2(1+\gamma)(1+2\gamma)[3-2(1-\gamma)\gamma(2+7\gamma)]}{2\left[1+3\gamma-4\gamma^2[1+(2-\gamma)\gamma]\right]^2}.
\end{aligned}$$

In the following corollary, we compare the equilibrium outcomes with and without the merger.

Corollary 1. *In the parametric model with three firms and simultaneous moves, the merger is profitable for all values of $\gamma \leq 0.479$. Moreover, for any profitable merger, we find that, with respect to the case in which firms stand alone:*

- *Insiders' investment decreases, and outsider's investment increases.*
- *Each merging firm reduces its own quantity, whereas the quantity of firm 3 increases with the merger.*
- *Total investment, consumer surplus and total welfare fall with the merger.*

First note that the merger is profitable only for values of γ that are sufficiently small; thus, as products become closer substitutes, the merger becomes unprofitable. The intuition is the following: We know that, after the merger, an outsider investment increases and an insider investment decreases. This will lead to a cost advantage for the outsider, which results in higher market share lost by the insider the more substitutable the products are. Note that this result differs from the one we obtain in the analysis of the profitability of the "marginal" merger (Lemma 6) in which we find that the "marginal" merger is generally profitable. The reason is that, in Lemma 6, we study the marginal impact of the merger on prices and investment. In the parametric model, instead, we analyze the effects of the merger including the equilibrium adjustment of firm 3's price and investment.

The results on firms' quantity follow from the combined effect of the change in the level of investment and the level of prices set by the firms with and without the merger. At equilibrium, while we find that for the merging parties the investments decrease and the prices tend to increase, for the outsider the opposite hold true: it increases its investment and decreases its price after the merger. Note that, in this model with prices and investments, the price set by an outsider can go either way after the merger. Strategic complementarity implies that the increase in the price set by merging parties moves outsider's price upward. At the same time, though, the increase in the investment set by the insider with the merger moves the outsider's price downward. The results in the corollary show that the former effects prevail at equilibrium.

What are the implications we can draw from this analysis for welfare? We find that whenever the merger is profitable the value of consumer surplus with the merger is lower than the value of consumer surplus in the benchmark. (This conclusion also holds if one looks at total surplus.) This shows that the merger is overall anti-competitive in this setting.

Case $\lambda > 0$ (Presence of economies of scope) Let the merger generate economies of scope, we solve for the equilibrium outcomes within our parametric model and find that, for a given γ, the merger raises total investment for $\lambda \geq \overline{\lambda}_I(\gamma)$ and consumer surplus for $\lambda \geq \overline{\lambda}_C(\gamma)$, with $\lambda_C(\gamma) > \lambda_I(\gamma)$. That is, it takes larger values of the economies of scope to render the merger procompetitive, despite the increase in total investment. Moreover, as products become more differentiated (γ falls), it takes smaller values of the scope economies parameter for the merger to raise consumer welfare.

For example, if $\gamma = 0.4$, the merger decreases total investment and consumer surplus for $\lambda < 0.08$, it increases total investment but not consumer surplus for $0.08 \leq \lambda < 0.144$, and increases both consumer surplus and total investment for $\lambda \geq 0.144$. Therefore, consumer welfare rises with the merger only if about 15% of the marginal increase in the cost-reducing investment undertaken by an insider is absorbed by the other merging party.

3.1.4 Network-Sharing Agreement between firm i and firm k

Before looking at the sequential case, we study the consequences of a NSA on prices and investment. In a NSA, parties maximize joint profits when setting investment, yet behave non-cooperatively when setting the price.[27] Therefore, the first-order conditions of a NSA-member firm i are (we omit those for k and for the outsiders):

$$\partial_{p_i}\pi_{i,k} = q_i(p_i, p_k, \bar{p}_{-i-k}) + \partial_{p_i}q_i(p_i, p_k, \bar{p}_{-i-k})(p_i - c(x_i + \lambda x_k)) = 0, \tag{7}$$

$$\begin{aligned}\partial_{x_i}\pi_{i,k} = &-\partial_{x_i}c(x_i + \lambda x_k)q_i(p_i, p_k, \bar{p}_{-i-k}) \\ &-\lambda\partial_{x_i}c(x_k + \lambda x_i)q_k(p_i, p_k, \bar{p}_{-i-k}) - F'(x_i) = 0.\end{aligned} \tag{8}$$

The first-order condition for the investment of firm i is the same as in the merger. Instead, the first order condition for the price is as in the benchmark (no merger). This implies that, in the simultaneous moves case, the NSA (weakly) dominates the benchmark with independent firms and the merger in terms of the consumer welfare it generates—and this conclusion holds for any value of $\lambda \geq 0$. Compare the NSA with the benchmark case first. Consider $\lambda = 0$. The first-order conditions are all identical, so the equilibrium solutions must coincide. When $\lambda > 0$, the NSA-members internalize the effect of the scope economies when setting their investment. This raises the value of investments, for given prices. At the same time, due to the scope economies, the prices set by the members of the NSA are lower than in the benchmark. Consider now the comparison with the merger. Whenever $\lambda \geq 0$ the first-order conditions of the investments will be identical, leading to identical investments other things (notably the price levels) being equal. But under the NSA the price externality is not taken into account when NSA members set their prices, leading to lower prices and hence higher quantities sold - which in turn will result in higher investments.

Overall, then, for whatever level of economies of scope a NSA is (weakly) more efficient than both the merger and the benchmark with independent firms in our model with simultaneous moves and cost-reducing investment.

[27] While this appears natural in a sequential move game, it may appear difficult to interpret in a simultaneous move game. However, NSAs are often structured in such a way that investment decisions are fully delegated to a joint venture whose managers are to behave independently from the managers of the parent companies. Independence between investment decisions and price decisions is also often a requirement of competition agencies to authorize the NSA. Note that if the NSA were to maximize joint profits with respect to both investments and prices, it would simply be identical to a merger.

Proposition 3. *With respect to both the benchmark and the merger, the NSA yields an increase in consumer welfare for any value of $\lambda \geq 0$, because it reduces industry prices and raises investment.*

Since the results are clearcut, we omit the analysis of the NSA using the parametric model within the simultaneous-moves case.

3.2 Sequential choices

The simultaneous case replicates a model in which the investments in cost reductions cannot be observed by rivals when firms take pricing decisions. In this section, we look at the case in which firms know the investment set by their rivals by the pricing stage.

We assume that first firms simultaneously set the value of their cost-reducing investment x_i and then simultaneously choose their price p_i, $i = 1, ..., n$. We solve this game using the sub-perfect Nash equilibrium concept, and proceed by backward induction. As in the previous subsection, we begin by looking at the benchmark (status quo) case with n symmetric independent firms, and then at the merger between two out of these n firms. We will assume throughout the general model that $\lambda = 0$, since we already know from the analysis in the previous section that economies of scope will move the results toward a more favorable effect of both the merger and the NSA.[28]

3.2.1 Benchmark with independent firms

If firms act independently, in the second stage each solves the following maximization problem:

$$\max_{p_i} \ \pi_i(p_i, \bar{p}_{-i}, x_i) = (p_i - c(x_i))q_i(p_i, \bar{p}_{-i}) - F(x_i), \ i = 1, ..., n.$$

The first-order conditions are:

$$\partial_{p_i}\pi_i = q_i(p_i, \bar{p}_{-i}) + \partial_{p_i}q_i(p_i, \bar{p}_{-i})(p_i - c(x_i)) = 0. \tag{9}$$

Under our assumptions, the system of n first-order conditions is uniquely solved by the vector of symmetric equilibrium prices in $\bar{p}^b(\bar{x})$, as function of the vector of cost-reducing investments $\bar{x}$. Before proceeding with the analysis of the firms' investment choices, note first that standard calculations imply that, as in the simultaneous choice model, $dp_i/dp_j \in (0, 1)$, with $j \neq i$.[29]

In the first stage, firms maximize π_i with respect to x_i, which, invoking the Envelope theorem, yields the following system of first-order conditions:

$$\partial_{x_i}\pi_i = -c'(x_i)q_i(\bar{p}(\bar{x})) - F'(x_i) + (n-1)\partial_{p_j}q_i(\bar{p}(\bar{x}))\frac{dp_j}{dx_i}(p_i(\bar{x}) - c(x_i)) = 0, \tag{10}$$

for all $i = 1, ..., n$ and $j \neq i$. The conditions in (10) define the equilibrium level of investment in the sequential choice game with independent firms.[30] The difference between (10) and the first-order condition with respect to x_i in (2) is that, with sequential moves, each firm i takes into

[28] We will analyze within our parametric model how economies of scope affect our results.

[29] The proof goes along the same lines as in Lemma 2, and is therefore omitted.

[30] For the stability and the unicity of the equilibrium at the investment stage in the benchmark and with the merger, we invoke the conditions derived in Kolstad and Mathiesen (1987). We will check that they are satisfied within the parametric model that we use to illustrate our results.

account that raising its investment reduces the prices set by its rivals, and hence will impact negatively own profits since it will make price competition more fierce. This effect is reflected by the last term in (10).

Lemma 7. *As in the model with simultaneous choices, at equilibrium the price of firm i decreases in its own investment ($dp_i/dx_i < 0$) and in the investment of a rival firm $j \neq i$ ($dp_i/dx_j < 0$). Moreover, $|dp_i/dx_i| > |dp_j/dx_i|$.*

As a consequence of Lemma 7, the symmetric equilibrium investment values in $\bar{x}^b(\bar{p})$, as set by each firm i solving conditions (10) in the benchmark model with sequential moves, is lower than in the simultaneous-moves case, *ceteris paribus*.

3.2.2 Merger between firm i and firm k

We proceed with the analysis of the merging firms' problem. In the second stage, firms i and k solve:

$$\begin{aligned}\max_{p_i, p_k} \ \pi_{i,k}(p_i, p_k, \bar{p}_{-i,-k}, x_i, x_k) \ &= \ (p_i - c(x_i))q_i(p_i, p_k, \bar{p}_{-i-k}) \\ &\quad +(p_k - c(x_k))q_k(p_k, p_i, \bar{p}_{-i-k}) - F(x_i) - F(x_k),\end{aligned}$$

for all $i, k = 1, .., n$ and $i \neq k$. The first-order condition with respect to p_i (we omit that for p_k) is:

$$\begin{aligned}\partial_{p_i}\pi_{i,k} = q_i(p_i, p_k, \bar{p}_{-i-k}) + \partial_{p_i}q_i(p_i, p_k, \bar{p}_{-i-k})(p_i - c(x_i)) \\ +\partial_{p_i}q_k(p_k, p_i, \bar{p}_{-i-k})(p_k - c(x_k)) = 0.\end{aligned} \tag{11}$$

Since $\partial_{p_i}q_k(p_k, p_i, \bar{p}_{-i-k}) \geq 0$ and $p_k > c(x_k)$ at the equilibrium, and for given investments, the merger increases the price set by each insider with respect to the benchmark. The intuition is the same as in the simultaneous-choice case, and as in any merger.

An outsider firm $j \neq i, k$ solves the following problem:

$$\max_{p_j} \ \pi_j(p_j, \bar{p}_{-j}, x_j) = (p_j - c(x_j))q_j(p_j, \bar{p}_{-j}) - F(x_j),$$

so that its first-order condition with respect to p_j is isomorphic to those of the firms in the benchmark with independent firms:

$$\partial_{p_j}\pi_j = q_j(p_j, \bar{p}_{-j}) + \partial_{p_j}q_j(p_j, \bar{p}_{-j})(p_j - c(x_j)) = 0. \tag{12}$$

In the first stage, insiders maximize joint profits $\pi_{i,k}$ with respect to x_i and x_k. Using the Envelope theorem, the associated first-order conditions are as follows:

$$\begin{aligned}\partial_{x_i}\pi_{i,k} \ &= \ -c'(x_i)q_i(\bar{p}(\bar{x})) - F'(x_i) \\ &\quad +(n-2)\frac{dp_j}{dx_i}\left[\partial_{p_j}q_i(p(\bar{x}))(p_i(\bar{x}) - c(x_i)) + \partial_{p_j}q_k(p(\bar{x}))(p_k(\bar{x}) - c(x_k))\right] = 0,\end{aligned} \tag{13}$$

for all $j \neq i, k$.[31]

[31] The first-order condition of an outsider is isomorphic to the one of a firm in the benchmark (10), and therefore not reported.

For given prices, the merger reduces the investment of merging parties for two reasons: the first is the same as in the simultaneous-moves case, and stems from the reduction in insiders' quantities. The second is related to the fact that, with sequential moves, insiders internalize the impact of an increase in the level of investment x_i on the profits of both insiders.

So far, by means of a simple comparison between the first-order conditions in the benchmark (9)-(10) and with the merger (11)-(13) we find that, for given investments, insiders raise prices and, for given prices, they choose a lower level of investments. In Proposition 4, we analyze the total impact of the merger on the equilibrium values of prices and investments in the industry.

Proposition 4. *With sequential moves and absent merger-induced economies, after the merger insiders raise the equilibrium price and outsiders follow suit due to strategic complementarity. Moreover, under the assumption that the model is stable, merging firms decrease their investments, which causes their price to further increase, and total investments decrease.*

In the proposition, we show that total investments decrease under a sufficient condition on the stability of the investment game (which implies that $(n-1)|dx_i/dx_k| < 1$). The intuition is simple: the increase in insiders' prices after the merger reduces a merging party's investment. If investments are strategic complements, then this implies a contraction in all firms' investments. If investments are strategic substitutes and $(n-1)|dx_i/dx_k| < 1$, then the increase in x_{-i} does not compensate for the fall in x_i. The fall in firms' investments leads to higher prices, as $dp_l/dx_i < 0$ for all $l = 1, ..., n$, which reinforces the increase in prices due to strategic complementarity ($dp_i/dp_j > 0$).

What is the impact of a "marginal" merger on aggregate quantity? As in the simultaneous-choice case, industry quantity Q falls as a consequence of a marginal increase in prices.[32] What about the impact of a change in the value of firm i's investment (x_i) on Q? The analysis in this case differs from the model with simultaneous moves, because firms' investment choices interact strategically with sequential moves. Thus, we need to take into account this additional effect when assessing the competitive consequences of the "marginal" merger.

Lemma 8. *Consider an exogenous marginal decrease in firm i's investment x_i, at the optimal level of p_i. Aggregate output moves in the same direction as firm i's output if the following condition holds true:*

$$\frac{dx_l}{dx_i} > -\frac{\frac{dp_i}{dx_i} + (n-1)\frac{dp_j}{dx_i}}{(n-1)\left(\frac{dp_i}{dx_l} + (n-1)\frac{dp_j}{dx_l}\right)} \in (-1, 0), \tag{14}$$

with $i \neq j$ and $l \neq i$.

Condition (14) requires that the slope of the reaction functions of the investment game is either positive (or not "too" negative). Essentially, if investments are strategic complements, or if they are strategic substitutes but such that the sufficient condition in (14) is satisfied, then a marginal fall in firm i's investment reduces industry quantity. Instead, were condition (14) violated, outsiders would increase their investment to the point of raising the total output on the final-good market. In this case, the "marginal" merger is not unambiguosly anticompetitive. The analysis of our parametric model below will allow us to sign the net effect of the merger in a standard model with sequential choices.

[32] The proof is analogous to the proof of Lemma 4, and is therefore omitted.

We conclude this section with the analysis of the profitability of the "marginal" merger. Specifically, starting from the symmetric equilibrium arising in the benchmark with independent firms, we show the consequences on firms' profits of a marginal increase in merging firms' price and a marginal reduction in their investment.

Lemma 9. *Given the symmetric equilibrium in the benchmark with independent firms, a marginal increase in firm i's price p_i increases the profit of firm i. A marginal decrease in firm i's investment x_i, instead, decreases firm i's profit. Overall, then, following an increase in p_i and a decrease in x_i, firm i's profit rise if*

$$\frac{dx_l}{dx_i} > \max\left[\frac{\frac{dp_j}{dp_i}}{\frac{dp_j}{dx_j} + (n-2)\frac{dp_j}{dx_l}}, -1\right], \tag{15}$$

with $i \neq j$ and $l \neq i$.

Like condition (14), condition (15) has to do with the slope of the reaction functions of the investment game: it requires that the slope is either positive (or not "too" negative) for the "marginal" merger to be profitable. More specifically, the expression in (15) provides a lower bound on the degree of strategic substitutability in investments for the profit of a firm i to increase following an increase in its price and a decrease in its investment. The intuition for this result is analogous to the one developed in Salant, Switzer, and Reynolds (1983), and is that strategic substitutability in investment dampens a merging firm's profit. If following the reduction in firm i's investment, the other firms react by all raising their investments by a comparable amount, then, first, they become more efficient than firm i, and, second, competition intensifies. This explains why the increase in a firm i's price may not trigger an increase in profits that compensates for the reduction in the same profits caused by the reduction in x_i.[33]

In what follows, we assess the impact of the merger on equilibrium outcomes when moves take place sequentially by means of our parametric demand model.

3.2.3 The merger in the parametric model with sequential choices

Consider now our parametric demand model with $n = 3$, sequential choices and $\alpha_i = \alpha_j = \alpha > c$ for all i, j. In the benchmark, we find that all firms set the following values of price and investment:[34]

$$p^b = \frac{c(1+\gamma)(1+2\gamma)(2+3\gamma) - \alpha\gamma^2(3+5\gamma)}{2+\gamma(1+\gamma)(9+\gamma)}, \ x^b = \frac{(\alpha - c)(1+\gamma)[2+(3-\gamma)\gamma]}{2+\gamma(1+\gamma)(9+\gamma)}.$$

Instead, the value of a firm profit and consumer surplus is equal to:

$$\begin{aligned}
\pi^b &= \frac{(\alpha-c)^2(1+\gamma)[4+16\gamma+9\gamma^2-\gamma^3(29+31\gamma+\gamma^2)]}{2[2+\gamma(1+\gamma)(9+\gamma)]^2}, \\
C^b &= \frac{3(\alpha-c)^2(1+\gamma)^2(1+2\gamma)(2+3\gamma)^2}{2[2+\gamma(1+\gamma)(9+\gamma)]^2}.
\end{aligned}$$

[33] In the framework with simultaneous, we find the unambiguous result that an increase in p_i and a reduction in x_i raise firm i's profit because investment choices do not interact strategically there.

[34] In the benchmark, the second-order conditions and the stability conditions are satisfied for all $\gamma \in (0, 0.733)$. Moreover, the values of firms' investment and marginal cost are positive for all $\alpha < 2c[2+\gamma(7+6\gamma)]/(1+\gamma)[2+(3-\gamma)\gamma]$.

Assume firms 1 and 2 merge. Insiders set the following values of price and investment:[35]

$$\begin{aligned}
p^{in} &= \frac{c(1+\gamma)^2(1+2\gamma)[2+(2-\gamma)\gamma][2-2\gamma-(2-\gamma)\gamma^2]}{4+2\gamma[10+6\gamma-28\gamma^2(1+\gamma)+20\gamma^4+13\gamma^5-\gamma^6(11-2\gamma)]} \\
&\qquad + \frac{\alpha\gamma[2-2\gamma-\gamma^2(5-2\gamma)][2+2\gamma(1+\gamma)-\gamma^2(1+\gamma)(8-3\gamma)]}{4+2\gamma[10+6\gamma-28\gamma^2(1+\gamma)+20\gamma^4+13\gamma^5-\gamma^6(11-2\gamma)]}, \\
x^{in} &= \frac{2(\alpha-c)[1+(1-\gamma)\gamma][2+2\gamma(1+\gamma)-\gamma^2(1+\gamma)(8-3\gamma)]}{2+\gamma[10+6\gamma-28\gamma^2(1+\gamma)+20\gamma^4+13\gamma^5-\gamma^6(11-2\gamma)]}.
\end{aligned}$$

Outsiders, instead, set

$$\begin{aligned}
p^{out} &= \frac{c(1+2\gamma)[2+(2-\gamma)\gamma][1+(2-\gamma)\gamma](1-2\gamma^2)}{2+\gamma[10+6\gamma-28\gamma^2(1+\gamma)+20\gamma^4+13\gamma^5-\gamma^6(11-2\gamma)]} \\
&\qquad - \frac{\alpha\gamma^2[3+(3-2\gamma)\gamma][1+\gamma(1+\gamma)-\gamma^2(1+\gamma)(3-\gamma)]}{2+\gamma[10+6\gamma-28\gamma^2(1+\gamma)+20\gamma^4+13\gamma^5-\gamma^6(11-2\gamma)]}, \\
x^{out} &= \frac{2(\alpha-c)(1+2\gamma-\gamma^3)[1+\gamma(1+\gamma)-\gamma^2(1+\gamma)(3-\gamma)]}{2+\gamma[10+6\gamma-28\gamma^2(1+\gamma)+20\gamma^4+13\gamma^5-\gamma^6(11-2\gamma)]}.
\end{aligned}$$

Finally, the value of profits and consumer surplus is:

$$\begin{aligned}
\pi^{in} &= \frac{(\alpha-c)^2[2+2\gamma(1+\gamma)-\gamma^2(1+\gamma)(8-3\gamma)]^2}{2\left[2+\gamma[10+6\gamma-28\gamma^2(1+\gamma)+20\gamma^4+13\gamma^5-\gamma^6(11-2\gamma)]\right]^2} \\
&\qquad \times \frac{2+8\gamma+2\gamma^2-\gamma^3(1+\gamma)[16-\gamma(11-2\gamma)]}{2\left[2+\gamma[10+6\gamma-28\gamma^2(1+\gamma)+20\gamma^4+13\gamma^5-\gamma^6(11-2\gamma)]\right]^2}. \\
\pi^{out} &= \frac{(\alpha-c)^2(1+\gamma)[1+\gamma(1+\gamma)-\gamma^2(1+\gamma)(3-\gamma)]^2}{\left[2+\gamma[10+6\gamma-28\gamma^2(1+\gamma)+20\gamma^4+13\gamma^5-\gamma^6(11-2\gamma)]\right]^2} \\
&\qquad \times \frac{2+6\gamma-2\gamma^2-\gamma^3(14+\gamma-7\gamma^2+2\gamma^3)}{\left[2+\gamma[10+6\gamma-28\gamma^2(1+\gamma)+20\gamma^4+13\gamma^5-\gamma^6(11-2\gamma)]\right]^2} \\
C^m &= \frac{(\alpha-c)^2(1+\gamma)(1+2\gamma)[2+(2-\gamma)\gamma]^2}{4\left[2+\gamma[10+6\gamma-28\gamma^2(1+\gamma)+20\gamma^4+13\gamma^5-\gamma^6(11-2\gamma)]\right]^2} \\
&\qquad \times \frac{6+10\gamma-\gamma[26+46\gamma-32\gamma^2-64\gamma^3+11\gamma^4-\gamma^5(24-7\gamma)]}{4\left[2+\gamma[10+6\gamma-28\gamma^2(1+\gamma)+20\gamma^4+13\gamma^5-\gamma^6(11-2\gamma)]\right]^2}.
\end{aligned}$$

We now compare the results with and without the merger.

Corollary 2. *In the parametric model with three firms and sequential choices, the merger is profitable for all values of* $\gamma \leq 0.503$. *Moreover, for any profitable merger, we find that, with respect to the benchmark with independent firms:*

- *Insiders' investment decreases, and outsider's investment increases.*

[35] After the merger, the second-order conditions and the stability condition of the price and investment games are satisfied under $\gamma \in (0, 0.5958)$. Moreover, the values of firms' investment and marginal cost are positive for all

$$\alpha < \frac{c(1+2\gamma)[2+(2-\gamma)\gamma][2+2\gamma(1-3\gamma)-(4-\gamma)(1-\gamma)\gamma^3]}{2(1+\gamma)[1+(1-\gamma)\gamma][1+\gamma(1+\gamma)-\gamma^2(1+\gamma)(3-\gamma)]}.$$

- *Each merging firm reduces its own quantity, whereas the quantity of the outsider firm 3 increases with the merger.*
- *Total investment, consumer surplus and total welfare fall.*

The intuition to these results goes along the same lines as in the case with simultaneous choices (Corollary 1). Note that, as in the simultaneous case, we find that the prices of both insiders and outsiders increase with the merger.

Case $\lambda > 0$ (Presence of economies of scope) Before proceeding, note that solving the parametric demand model with sequential moves and economies of scope, we again find that, for a given γ, the merger raises total investment for $\lambda \geq \overline{\lambda}_I(\gamma)$ and consumer surplus for $\lambda \geq \overline{\lambda}_C(\gamma)$, with $\lambda_C(\gamma) > \lambda_I(\gamma)$. For example, if $\gamma = 0.4$, the merger increases total investment for $\lambda \geq 0.1$. However, it raises consumer surplus only for values of λ larger than 0.16. Therefore, we obtain in the sequential model the same qualitative results as in the simultaneous model.

3.2.4 Network-Sharing Agreements between firm i and firm k

In this section, we analyze how a NSA affects the equilibrium outcomes in the context of the sequential-choice game. Consistent with the analysis of the merger developed in the section above, we analyze the case in which the NSA does not generate economies of scope. At the pricing stage, the maximization problem and the relative first-order conditions are the same as in the benchmark with independent firms. This implies that, *for given investment*, the NSA will set the same prices.

Turning to the first stage, the maximization problem is analogous to the one solved by firms i and k in the case of the merger—the two firms choose investments to maximize joint profits. However, the first-order conditions of a NSA-insider differ, and the reason is that the first-order conditions with respect to the product-market prices differ. Invoking the Envelope theorem, the first-order condition with respect to x_i is given by:

$$\begin{aligned}\partial_{x_i}\pi_{i,k}^{NSA} &= -c'(x_i)q_i(\bar{p}(\bar{x})) - F'(x_i) \\ &\quad +(n-1)\frac{dp_j}{dx_i}\left[\partial_{p_j}q_i(\bar{p}(\bar{x}))(p_i(\bar{x}) - c(x_i)) + \partial_{p_j}q_k(\bar{p}(\bar{x}))(p_k(\bar{x}) - c(x_k))\right] = 0, \quad (16)\end{aligned}$$

for all $j = 1, ..., n$.

Since parties do not set their prices cooperatively, at the investment stage they only care about the effect that changing x_i and x_k has on their own prices and quantities. In the case of the merger, instead, they also care about the effect of changing their own investment on the prices and quantities of the rivals. In other words, if $\Phi(\bar{x})$ denotes the right-hand side of (13) at the same level of prices, the right-hand side of (16) is given by:

$$\Phi(\bar{x}) + \partial_{p_k}q_i(\bar{x})\frac{dp_k}{dx_i}(p_i(\bar{x}) - c(x_i)) + \partial_{p_i}q_k(\bar{x})\frac{dp_i}{dx_i}(p_k(\bar{x}) - c(x_k)),$$

where the second and third terms are negative,[36] implying that the incentive to invest is reduced (for given prices). The following lemma summarizes by formally showing that, for given level of prices, the NSA reduces NSA-insiders' investment.

[36] These terms do not appear in (13) because they are part of the first-order conditions with respect to p_i and p_k, respectively.

Lemma 10. *In the sequential case and for given value of second-stage prices, the level of the NSA-insiders' investment is lower than the level of the investment set by merging parties.*

As a consequence, the NSA triggers a trade-off with respect to the merger. On the one hand, for given value of investments, prices are the same as in the benchmark and thus lower than in the merger. On the other hand, for given prices, the reduction in the value of a firm's investment is larger than with the merger.

Although the comparison with the merger is ambiguous, the comparison with the benchmark with independent firms is clear-cut. The reduction of the NSA-insiders' investment level triggers an increase of their price and, consequently, a reduction of their quantity that is not compensated by the consequent increase of the competing firms' quantity. Thus, in the absence of NSA-induced economies the agreement is clearly anticompetitive.

3.2.5 The effects of the NSA in the parametric model

Assume firms 1 and 2 join in a NSA within our parametric model with sequential moves and absent economies of scope. We find that insiders set the following values of price and investment:[37]

$$
\begin{aligned}
p^{in}_{NSA} &= \frac{\alpha\gamma(1-\gamma-4\gamma^2)[\gamma(1+\gamma)(8\gamma-3)-2]+c(2+3\gamma)^2(7\gamma^3+5\gamma^4-1-3\gamma)}{\gamma(1+\gamma)[(37\gamma^2+\gamma^3(82+13\gamma)-20\gamma)-26]-4}, \\
x^{in}_{NSA} &= \frac{2(\alpha-c)(1+\gamma)[1+(1-\gamma)\gamma][\gamma(1+\gamma)(8\gamma-3)-2]}{\gamma(1+\gamma)[(37\gamma^2+\gamma^3(82+13\gamma)-20\gamma)-26]-4}.
\end{aligned}
$$

Outsiders, instead, set

$$
\begin{aligned}
p^{out}_{NSA} &= \frac{\alpha\gamma^2(3+5\gamma)[2+\gamma(1+\gamma)(4-7\gamma)]}{\gamma(1+\gamma)[(37\gamma^2+\gamma^3(82+13\gamma)-20\gamma)-26]-4} \\
&\qquad -\frac{c(1+\gamma)[2+3\gamma)(2+\gamma(8+\gamma 3-17\gamma^2-16\gamma^3)]}{\gamma(1+\gamma)[(37\gamma^2+\gamma^3(82+13\gamma)-20\gamma)-26]-4}, \\
x^{out}_{NSA} &= \frac{(\alpha-c)(1+\gamma)[2+(3-\gamma)\gamma][\gamma(1+\gamma)(7\gamma-4)-2]}{\gamma(1+\gamma)[(37\gamma^2+\gamma^3(82+13\gamma)-20\gamma)-26]-4}.
\end{aligned}
$$

[37] The values of firms' investment and marginal cost are positive for all

$$
\alpha < \frac{2c(2+3\gamma)[2+8\gamma+4\gamma^2-\gamma^3(1+\gamma)(16+\gamma)]}{(1+\gamma)[2+(3-\gamma)\gamma][2+\gamma(1+\gamma)(4-7\gamma)]}
$$

and $\gamma \in (0, 0.6225)$.

Finally, the value of profits and consumer surplus is:

$$\pi_{NSA}^{in} = \frac{2(\alpha-c)^2(1+\gamma)[2+\gamma(1+\gamma)(3-8\gamma)]^2}{[\gamma(1+\gamma)[(37\gamma^2+\gamma^3(82+13\gamma)-20\gamma)-26]-4]^2} \times \frac{2+10\gamma+11\gamma^2-9\gamma^3-2\gamma^4(8+\gamma)}{[\gamma(1+\gamma)[(37\gamma^2+\gamma^3(82+13\gamma)-20\gamma)-26]-4]^2},$$

$$\pi_{NSA}^{out} = \frac{2(\alpha-c)^2(1+\gamma)[2+\gamma(1+\gamma)(4-7\gamma)]^2}{2[\gamma(1+\gamma)[(37\gamma^2+\gamma^3(82+13\gamma)-20\gamma)-26]-4]^2} \times \frac{4+16\gamma+9\gamma^2-\gamma^3[29+\gamma(31+\gamma)]}{2[\gamma(1+\gamma)[(37\gamma^2+\gamma^3(82+13\gamma)-20\gamma)-26]-4]^2},$$

$$C_{NSA}^{m} = \frac{(\alpha-c)^2(1+\gamma)^2(2+3\gamma)^2}{2[\gamma(1+\gamma)[(37\gamma^2+\gamma^3(82+13\gamma)-20\gamma)-26]-4]^2} \times \frac{12+64\gamma+62\gamma^2-\gamma^3[214+453\gamma-4\gamma^2-\gamma^3(573+352\gamma)]}{2[\gamma(1+\gamma)[(37\gamma^2+\gamma^3(82+13\gamma)-20\gamma)-26]-4]^2}.$$

First note that, comparing π_{NSA}^{in} with $2\pi^b$, we find that the NSA is profitable if and only if $g \leq 0.4127$. We now analyze the impact of a (profitable) NSA on investment and consumer surplus with respect to the benchmark and the merger.

Corollary 3. *In the parametric model with three firms and sequential moves, and in the absence of economies of scope ($\lambda = 0$), the NSA triggers the following outcomes:*

- *With respect to the benchmark, it increases the investment of the outsider, but reduces the insiders' and total investment. Finally, it reduces consumer and total surplus.*
- *With respect to the merger, it reduces total investment, as both outsider's investment and insiders' investment shrink. However, it increases consumer and total surplus.*

These results confirm the main insights arising from the analysis of the NSA first-order conditions (16): while the NSA is generally bad for investments, it increases consumer welfare with respect to the merger. However, we also find that the NSA reduces consumer welfare when compared to the benchmark with independent firms. The picture below illustrates these results, and it also shows that when economies of scope in investments are large enough, then the merger (and a fortiori the NSA) could lead to higher investment and consumer surplus than the benchmark.

[Figure 1 about here.]

In both figures, the x-axis measures λ, the value of economies in investments achieved by the merger and the NSA; the solid black line corresponds to the benchmark, the dotted line to the merger and the dashed line to the NSA. The left-hand side panel illustrates the value of total investments in the three industry configurations. It confirms the result that the merger typically produces a higher value of investments than the NSA, independently of λ. The right-hand side panel, instead, shows that the NSA is generally preferable to the merger in terms of consumer surplus. Indeed, given γ, for the merger to result in a pro-competitive outcome, it requires a larger value of λ than the NSA. Overall, though, for $\lambda = 0$ or sufficiently small, the benchmark produces higher consumer welfare than both the merger and the NSA.[38]

[38]The figure with total surplus is not reported because it is similar to the one of the consumer surplus.

4 Quality-increasing investments

In this section, we discuss the implications of a model in which the investments carried out by firms increase the quality of their good, rather than decreasing their cost of production. Specifically, we let the quantity of a firm depends on its own and rivals' prices (p) and quality (x) level: $q_i = q_i(p_i, \bar{p}_{-i}, x_i, \bar{x}_{-i})$, with $\partial_{x_i} q_i \geq 0$ and $\partial_{x_i} q_k \leq 0$ (that is, an increase in the quality of firm i implies that q_i rises and q_k reduces, with $i \neq k$, as standard in models of quality differentiation). Assume further that, for simplicity, the price- and quality-setting stages take place simultaneously (for simplicity), the investment in quality does not generate any scope economies and each firm bears a marginal cost of production equal to c.

If firms act independently, each solves the following maximization problem:

$$\max_{p_i, x_i} \pi_i(p_i, \bar{p}_{-i}, x_i) = q_i(p_i, \bar{p}_{-i}, x_i, \bar{x}_{-i})(p_i - c) - F(x_i),\ i = 1, ..., n.$$

The associated first-order conditions are:

$$\begin{aligned} \partial_{p_i} \pi_i &= q_i(p_i, \bar{p}_{-i}, x_i, \bar{x}_{-i}) + \partial_{p_i} q_i(p_i, \bar{p}_{-i}, x_i, \bar{x}_{-i})(p_i - c) = 0, \quad (17) \\ \partial_{x_i} \pi_i &= \partial_{x_i} q_i(p_i, \bar{p}_{-i}, x_i, \bar{x}_{-i})(p_i - c) - F'(x_i) = 0. \quad (18) \end{aligned}$$

If firms i and k merge, they solve

$$\begin{aligned} \max_{p_i, p_k, x_i, x_k} \pi_{i,k}(p_i, p_k, \bar{p}_{-i,-k}, x_i, x_k) &= q_i(p_i, p_k, \bar{p}_{-i-k}, x_i, x_k, \bar{x}_{-i-k})(p_i - c) - F(x_i) \\ &\quad + q_k(p_k, p_i, \bar{p}_{-i-k}, x_i, x_k, x_{-i-k})(p_k - c) - F(x_k), \end{aligned}$$

with $i, k = 1, ..., n$, and $i \neq k$. The first-order conditions with respect to p_i and x_i follow (we omit those for p_k and x_k, which are symmetric, and those of the outsiders, which are unchanged relative to the benchmark case):

$$\begin{aligned} \partial_{p_i} \pi_{i,k} &= q_i(p_i, p_k, \bar{p}_{-i-k}, x_i, x_k, \bar{x}_{-i-k}) + \partial_{p_i} q_i(p_i, p_k, \bar{p}_{-i-k}, x_i, x_k, \bar{x}_{-i-k})(p_i - c) \\ &\quad + \partial_{p_i} q_k(p_k, p_i, \bar{p}_{-i-k}, x_i, x_k, x_{-i-k})(p_k - c) = 0, \quad (19) \\ \partial_{x_i} \pi_{i,k} &= \partial_{x_i} q_i(p_i, p_k, \bar{p}_{-i-k}, x_i, x_k, \bar{x}_{-i-k})(p_i - c) \\ &\quad + \partial_{x_i} q_k(p_k, p_i, \bar{p}_{-i-k}, x_i, x_k, x_{-i-k})(p_k - c) - F'(x_i) = 0. (20) \end{aligned}$$

When investments increase a firm's quality, the impact of the merger is a priori ambiguous. First, (19) suggests that the merger has the usual effect of raising the price set by insiders, for given investment. Second, from (20) it is clear that the increase in the price set by merging parties raises their respective marginal revenue and thus boosts their incentives to invest (first term in (20)). Depending on whether the impact of the merger on investment prevails the anticompetitive effect caused by the price increase, the merger can result in an increase in consumer welfare.

4.1 Cost-reducing and quality-increasing: two indifference results

Given the ambiguous conclusions of the general model with quality-increasing investment, we resort to more specific formulations of the demand model to show the conditions under which the results of the model with cost-reducing investments survive.

Quality-adjusted model First, let $q_i(p_i, \bar{p}_{-i}, x_i, \bar{x}_{-i}) = x_i q_i(p_i, \bar{p}_{-i})$—that is, investment enhances a firm's demand by a multiplicative factor. The utility of the representative consumer then takes the following form: $U(x_1 q_1, ..., x_n q_n)$. In this model, the solution of the utility maximization problem leads to a demand system of the type $x_i q_i = D_i(z_i, \bar{z}_{-i})$, with $z_i = p_i / x_i$ and $i = 1, .., n$. As a consequence, the gross profits of a firm i can be written as

$$\pi_i(p_i, \bar{p}_{-i}, x_i) + F(x_i) = (p_i - c_i) q_i = (z_i - c/x_i) D_i(z_i, \bar{z}_{-i}).$$

This equivalence means that all the conclusions derived in the model with cost-reducing investment extend to this model with quality-adjusted prices and investments.

Parametric model As remarked in Section 2 where we presented the paramtric model (see Häckner, 2000), in this section we interpret the parameter α_i as a measure of product i's quality in a vertical sense. Our goal is to show that, given its linear structure in prices, our parametric model gives rise to the same result independently of whether investment reduces cost $c(x_i)$ or increases quality $(\alpha_i = \alpha(x_i))$.

To illustrate this property, note that, given

$$q_i = \frac{(\alpha_i - p_i)[1 + (n-2)\gamma] - \gamma \sum_{j \neq i} (\alpha_j - p_j)}{(1-\gamma)[1 + (n-1)\gamma]}, \; i, j = 1, ..., n, \; j \neq i,$$

profit maximization with respect to prices of

$$\pi_i(p_i, \bar{p}_{-i}, x_i) + F(x_i) = (p_i - c_i) q_i \tag{21}$$

yields

$$\begin{aligned} p_i \;\; = \;\; & \frac{\alpha_i \left[2 + 3\gamma(n-2) + \gamma^2[5 + (n-5)n]\right]}{[2 + \gamma(n-3)][2 + \gamma(2n-3)]} \\ & + \frac{c_i[1 + \gamma(n-2)][2 + \gamma(n-2)]\gamma - \sum_{j \neq i} (\alpha_j - p_j)\gamma[1 + \gamma(n-2)]}{[2 + \gamma(n-3)][2 + \gamma(2n-3)]}, \end{aligned}$$

and

$$q_i \;\; = \;\; \frac{(\alpha_i - c_i)\left[2 + 3\gamma(n-2) + \gamma^2[5 + (n-5)n]\right] - \gamma \sum_{j \neq i} (\alpha_j - c_j)[\gamma(n-2) + 1]}{(1-\gamma)[2 + \gamma(n-3)][1 + \gamma(n-1)][2 + \gamma(2n-3)][1 + \gamma(n-2)]^{-1}},$$

for all $i, j = 1, ..., n$, $j \neq i$. Using these expressions, the gross profits of a firm i in (21) can be rewritten as:

$$\frac{\left[(\alpha_i - c_i)\left[2 + 3\gamma(n-2) + \gamma^2[5 + (n-5)n]\right] - \gamma \sum_{j \neq i} (\alpha_j - c_j)[\gamma(n-2) + 1]\right]^2}{(1-\gamma)[2 + \gamma(n-3)]^2[1 + \gamma(n-1)][2 + \gamma(2n-3)]^2[1 + \gamma(n-2)]^{-1}}.$$

Thus, assuming that the investment by a firm i (x_i) raises consumer valuation of the good of firm i, as in $\alpha_i = \alpha(x_i)$, with $\alpha'(\cdot) \geq 0$, or decreases firm i's marginal cost, as in $c_i = c(x_i)$, with $c'(\cdot) \leq 0$, leads to the same equilibrium outcomes as long as $\alpha(x) = -c(x)$.

Discussion Obviously, not all possible quality-enhancing settings are of the abovementioned types. There may be alternative demand models in which the indifference results above would not apply, with the consequence that our conclusions could change. However, the familiar demand models à la Shubik-Levitan or Salop fall within the two cases described above, thereby yielding an equivalence between the results with cost-reducing and quality-increasing investment. In the next section, we confirm most of the conclusions developed in the main analysis in a model of Vertical Product Differentiation à la Shaked-Sutton, which does not fall within the class of models just discussed.

4.2 A special model of vertical product differentiation

In this section, we employ a model that does not belong to the classes of models yielding the indifference results in Section 4.1. Specifically, we develop an analysis based on the model of Vertical Product Differentiation (VPD) due to Shaked and Sutton (1982,1983). In what follows, we extend the standard parametric VPD model to analyze the effect of a merger on quality-increasing investments in an oligopoly with three firms. This analysis will also allow us to show the impact of the merger on the number of varieties offered by the firms in the industry.

Consumers have utility function $U = \theta u - p$ if they buy one unit of the differentiated good, and $U = 0$, if they do not buy. We denote by u and p quality and price of the good, respectively, while θ represents the taste parameter. We assume that the distribution of tastes is uniform and that $\theta \in [0, 1]$, with unit density. We normalize the highest value in the support to 1,[39] and assume that θ can be as low as zero, which implies that the market is not covered—i.e., not all consumers buy at equilibrium.

Firm i's quality u_i can be seen as $u_i = u_i^\circ + x_i$, with u_i° being the quality level absent any investment and x_i the investment made by firm i to enhance its quality. We set $u_i^\circ = 0$ for all i, so that u_i corresponds to the investment level for any firm i. Finally, we rank qualities such as $u_1 > u_2 > u_3$.

Firms play a quality-then-price game, the cost of quality improvement falling upon fixed costs only (there are no variable costs, neither of quality nor of production for simplicity), according to the function $F(u_i) = u_i^2/2$. We compare two different configurations: the first is the *benchmark* (b) – i.e., the counterfactual to the merger – with the three independent (and ex-ante identical) firms playing the quality-then-price game; as is well known, in this model firms will choose different qualities at equilibrium. In the second, we analyse the *merger* (m), where we allow a firm the possibility to produce two qualities while the remaining firm is single-product, and the two-stage game is then played. As we will show, it turns out that—independently of which two qualities it is assigned to them—the 'merging firms' will want to sell just one quality, in order to relax competition. Therefore, the merger configuration will amount to a duopoly model where the two firms will play the usual quality-then-price game. Throughout, we assume that scope economies are absent, since we know what is their impact on the analysis.

4.2.1 Benchmark: no merger

We solve the game by backward induction. To solve for the last stage of the game, we first derive demand and profit functions for the triopoly case. By looking for indifferent consumers,

[39] See Motta (1993) for details.

it is easy to find demand functions as:

$$q_1(p_1,p_2,p_3)=\overline{\theta}-\frac{p_1-p_2}{u_1-u_2},\ q_2(p_1,p_2,p_3)=\frac{p_1-p_2}{u_1-u_2}-\frac{p_2-p_3}{u_2-u_3},\ q_3(p_1,p_2,p_3)=\frac{p_2-p_3}{u_2-u_3}-\frac{p_2}{u_2}.$$

Given firms' profit function, $\pi_i=p_iq_i(p_1,p_2,p_3)-u_i^2/2,\ i=1,2,3,$ we take the first-order conditions and solve to obtain the equilibrium prices at the last stage of the game:

$$\begin{aligned}
p_1 &= \frac{(u_1-u_2)(4u_1u_2-u_1u_3-3u_2u_3)\overline{\theta}}{u_1(8u_2-2u_3)-2u_2(u_2+2u_3)},\\
p_2 &= \frac{(u_1-u_2)u_2(u_2-u_3)\overline{\theta}}{u_1(4u_2-u_3)-u_2(u_2+2u_3)},\\
p_3 &= \frac{(u_1-u_2)u_3(u_2-u_3)\overline{\theta}}{u_1(8u_2-2u_3)-2u_2(u_2+2u_3)}.
\end{aligned}$$

By replacement, we find the profits at the first stage of the game:

$$\begin{aligned}
\pi_1 &= \frac{(u_1-u_2)(4u_1u_2-u_1u_3-3u_2u_3)^2\overline{\theta}}{[u_1(4u_2-u_3)-u_2(u_2+2u_3)]^2}-\frac{u_1^2}{2},\\
\pi_2 &= \frac{(u_1-u_2)u_2^2(u_1-u_3)(u_2-u_3)\overline{\theta}}{[u_1(4u_2-u_3)-u_2(u_2+2u_3)]^2}-\frac{u_2^2}{2},\\
\pi_3 &= \frac{(u_1-u_2)^2u_3u_2(u_2-u_3)\overline{\theta}}{[u_1(4u_2-u_3)-u_2(u_2+2u_3)]^2}-\frac{u_3^2}{2}.
\end{aligned}$$

By taking the derivatives, and solving the ensuing first-order conditions, one can find the qualities at the equilibrium of the whole game, and by substitution all the equilibrium values, which are listed in Table 1 below.

[Table 1 about here.]

As it is evident from the table, one important feature of this VPD model is that, to relax price competition, firms will sharply differentiate the qualities they offer. In the benchmark, the market leader, which at equilibrium has a market share above 50%, has a quality that is five times higher than the second firm, and more than twenty-five times the third one. These predictions clearly are not consistent with what happens in many markets, where some elements of horizontal product differentiation will allow firms to relax price competition without the need of endogenously choosing such dramatically different quality levels. The VPD version of the Shubik-Levitan model we use throughout the paper reflects the latter features: firms produce sufficiently differentiated varieties that they do not need to choose different quality levels in order to relax competition.

4.2.2 The merger

Assume that a merger occurs between two firms which have been assigned a certain quality, say u_2 and u_3. The profits of firm 1 would continue to be $\pi_1=p_1q_1(p_1,p_2,p_3)-u_1^2/2$, whereas the merged entity—which is now multi-product—has profits given by $\pi_{23}=p_2q_2(p_1,p_2,p_3)-$

$u_2^2/2 + p_3 q_3(p_1, p_2, p_3) - u_3^2/2$. Solving the first-order conditions at the second stage of the game gives the following prices

$$p_1 = \frac{2u_1(u_1 - u_2)\overline{\theta}}{(4u_1 - u_2)},\ p_2 = \frac{(u_1 - u_2)u_2\overline{\theta}}{(4u_1 - u_2)},\ p_3 = \frac{(u_1 - u_2)u_3\overline{\theta}}{(4u_1 - u_2)},$$

while profits are as in what follows:

$$\pi_1 = \frac{4u_1^2(u_1 - u_2)\overline{\theta}}{[4u_1 - u_2]^2} - \frac{u_1^2}{2},\ \pi_{23} = \frac{u_1 u_2(u_1 - u_2)\overline{\theta}}{[4u_1 - u_2]^2} - \frac{u_2^2}{2} - \frac{u_3^2}{2}.$$

At equilibrium, the merged entity will set the quality of product 3 as low as possible. However, since at $u_3 = 0$ no consumer would ever buy, this is equivalent to saying that the merged entity will have an incentive not to produce the lowest quality. In this way, competition will be reduced and profits will rise. The same reasoning brings to the conclusion that whether the merging firms are (1,2), (1,3), or (2,3), the merged entity will always want to sell just one quality, in order to decrease competition.

To analyse the merger case, we shall then look at the equilibrium in the duopoly case.

Duopoly equilibrium By finding the indifferent consumers between buying high quality u_1 and low quality u_2, one finds that te demand faced by the two products sold after the merger will be $q_1(p_1, p_2) = \bar{\theta} - \frac{p_1 - p_2}{u_1 - u_2}$ and $q_2(p_1, p_2) = \frac{p_1 - p_2}{u_1 - u_2} - \frac{p_2}{u_2}$. The profits will be given by $\pi_i = p_i q_i(p_1, p_2) - u_i^2/2$ for $i = 1, 2$.

At the price equilibrium, profits for the top and bottom quality are:

$$\pi_1^D = \frac{4u_1^2(u_1 - u_2)\bar{\theta}^2}{(4u_1 - u_2)^2} - \frac{u_1^2}{2},\ \pi_2^D = \frac{u_1 u_2(u_1 - u_2)\bar{\theta}^2}{(4u_1 - u_2)^2} - \frac{u_2^2}{2}.$$

By solving the first-order conditions, we obtain the equilibrium values reported in Table 1.

4.2.3 Comparison

From the inspection of Table 1, one can see that:

- Relative to the benchmark, if there are no synergies the quality of the merged firm, u_2, decreases, while u_1 increases. This is once again because firms try to differentiate as much as possible, so one product disappears the high quality product will tend to increase and the low quality product to decrease.

- Recall that we can interpret the quality as investments ($u_i = 0 + x_i$). It is then easy to see that the merger will imply lower total investments ($u_1^b + u_2^b + u_3^b > u_1^m + u_2^m$).

- The reduced competition from the merger will determine an increase in prices, which coupled with the lower investments will result in a fall in consumer surplus.

- However, the increase in profits (slightly) outweighs the negative effect on consumer surplus, resulting in a higher total surplus.

4.2.4 Conclusion from this VPD example

The VPD model analysed here is an extension to a three-firm setting of a VPD parametric model which has been widely used in the literature in Industrial Organization. Despite being relatively standard, it is worth noting that it has some particular features, among others the fact that—in order to relax price competition (apart from possibly different qualities, the products are perfectly homogenous)—firms select sharply different quality levels at equilibrium. When analyzing mergers within such a model, we have also seen that two merging firms will always want to suppress one product, another peculiar feature of the model.

Despite having very different characteristics from the models that we have analysed in the rest of the paper, the VPD example shares the main results seen above. In particular, total investments decrease with the merger, and consumer surplus decreases, too. However, due to the suppression of a product and the readjustment of qualities in order to relax competition, the merger raises profits of both insiders and outsiders resulting in an overall increase in total surplus.

5 Summary, and some policy implications

It has been suggested that mergers, by increasing profitability, will also result in higher investments. To deal with this claim, we have first studied a general model with simultaneous cost-reducing investments and price choices and found that—absent scope economies in investments—the merger is anti-competitive: it lowers both total output and investment. With sequential choices, we have provided a sufficient condition (which basically consists in requiring that the outsiders will not increase "too much" their investments after the merging parties reduce theirs) in a general model for the merger to be anti-competitive.

In order to resolve ambiguities as to the effects of the merger, we have studied a standard Shubik-Levitan parametric model with strategic complementarities, and found that, if there are no economies in the investment function, a merger does reduce industry investments and consumer surplus. Only if the merger entails sufficient scope economies, will it be pro-competitive. We have also showed that a Network Sharing Agreement (by which parties set their investment cooperatively, and therefore similar to a R&D cooperative agreement) is preferable to a merger. Finally, we have identified a class of models where the same qualitative results extend to quality-enhancing investments.

In the light of these results, we find no support whatsoever for the claim that a merger—by relaxing competition and hence raising profitability—will increase incentives to invest. Rather, we have showed that both in the general model and in all the (standard) parametric models analysed, the merging firms will always reduce their investments. Furthermore, in the general model under certain assumptions about the strategic interaction between insiders and outsiders, and in all parametric models we have studied, the merger will reduce aggregate investments.

Therefore, absent cost reductions in the investment function, the negative effects on investments will compound the well-known detrimental effects of the merger on prices. This does not mean, of course, that a merger will always be anti-competitive. Indeed, it is possible that by combining their assets two firms will be able to be more cost-effective in their investments: if these economies of scope are large enough, they will increase investments, and may outweigh the standard detrimental effect of the merger on prices.

However, two remarks are in order. First, we have showed that to the extent that the same economies of scope can be achieved by a Network-Sharing Agreement (or, if we were talking about innovations rather than investments, by a R&D cooperative agreement), then such an agreement would be superior to the merger from the welfare point of view. This implies that the merging parties should prove not only that the merger will lead to dynamic efficiencies, but also that such efficiencies are merger-specific (that is, they cannot be reached by a less anti-competitive agreement). Second, the assessment of the antitrust authorities should also consider a temporal dimension that is completely absent from our treatment in this paper: while the merger will exercise its detrimental effect on prices from the moment it will be implemented, dynamic efficiencies—if they exist—will typically need a certain number of years before realising. This will require trading off short-run consumer losses with long-run gains, further complicating a balancing exercise which is already complex.

Appendix

Proof of Lemma 2. We take the total derivative of the first-order condition of firm i with respect to p_j, with $j \neq i$, and solve for dp_i/dp_j, with $j \neq i$. We obtain the following expression:

$$\frac{dp_i}{dp_j} = -\frac{(n-1)\partial^2_{p_i p_j}\pi_i}{\partial^2_{p_i p_i}\pi_i} = -(n-1)\frac{\partial_{p_j} q_i + \partial^2_{p_i p_j} q_i (p_i - c(x_i))}{2\partial_{p_i} q_i + \partial^2_{p_i p_i} q_i (p_i - c(x_i))}. \tag{A-1}$$

The numerator is positive due to assumption (*A3*) and the denominator is negative by assumption (*A1*). Overall, dp_i/dp_j is positive and lower than one by assumption (*A4*). This result implies that a given increase in the price of $(n-1)$ firms increases the price of firm i by a lower amount. Q.E.D.

Proof of Lemma 3. Totally differentiating (1) leads to:

$$\begin{aligned} &\underbrace{\left[2\partial_{p_i} q_i(p_i, \bar{p}_{-i}) + \partial^2_{p_i p_i} q_i(p_i, \bar{p}_{-i})(p_i - c(x_i))\right]}_{=\partial^2_{p_i p_i}\pi_i} dp_i + \underbrace{\left[-c'(x_i)\partial_{p_i} q_i(p_i, \bar{p}_{-i})\right]}_{=\partial^2_{p_i x_i}\pi_i} dx_i \\ &+(n-1)\underbrace{\left[\partial_{p_j} q_i(p_i, \bar{p}_{-i}) + \partial^2_{p_i p_j} q_i(p_i, \bar{p}_{-i})(p_i - c(x_i))\right]}_{=\partial^2_{p_i p_j}\pi_i} dp_j = 0, \end{aligned} \tag{A-2}$$

for all $j \neq i$. Repeating the same exercise for a firm $j \neq i$ we obtain:

$$\begin{aligned} &\underbrace{\left[2\partial_{p_j} q_j(p_j, \bar{p}_l) + \partial^2_{p_j p_j} q_j(p_j, \bar{p}_l)(p_j - c(x_j))\right]}_{=\partial^2_{p_j p_j}\pi_j} dp_j + \underbrace{\left[-c'(x_j)\partial_{p_j} q_j(p_j, \bar{p}_l)\right]}_{=\partial^2_{p_j x_j}\pi_j} dx_j \\ &+(n-1)\underbrace{\left[\partial_{p_l} q_j(p_j, \bar{p}_l) + \partial^2_{p_j p_l} q_j(p_j, \bar{p}_l)(p_j - c(x_j))\right]}_{=\partial^2_{p_j p_l}\pi_j} dp_l = 0, \end{aligned} \tag{A-3}$$

for all $l \neq j$.

Consider first an exogenous change in the investment of firm i, so that $dx_j = 0$ for all $j \neq i$. Solving for dp_i/dx_i and dp_i/dx_j, we obtain that, in a symmetric equilibrium,

$$\frac{dp_i}{dx_i} = -\frac{\partial^2_{p_i x_i}\pi_i[\partial^2_{p_j p_j}\pi_j + (n-2)\partial^2_{p_j p_l}\pi_j]}{\partial^2_{p_i p_i}\pi_i[\partial^2_{p_j p_j}\pi_j + (n-2)\partial^2_{p_j p_l}\pi_j] - (n-1)\partial^2_{p_i p_j}\pi_i \partial^2_{p_j p_i}\pi_j} < 0. \tag{A-4}$$

and

$$\frac{dp_j}{dx_i} = \frac{\partial^2_{p_i x_i}\pi_i \partial^2_{p_j p_i}\pi_j}{\partial^2_{p_i p_i}\pi_i[\partial^2_{p_j p_j}\pi_j + (n-2)\partial^2_{p_j p_l}\pi_j] - (n-1)\partial^2_{p_i p_j}\pi_i \partial^2_{p_j p_i}\pi_j} < 0. \tag{A-5}$$

First, (*A3*) and (*A4*), together with the conditions for the concavity of the Hessian matrix imply that the sign of the denominator of both expressions is positive. The numerator of (A-4) is negative because $-\partial^2_{p_i x_i}\pi_i > 0$ and the term in squared brackets is negative by (*A4*). This shows that the price of a firm i decreases in its own investment x_i. The numerator of (A-5), instead, is negative because $\partial^2_{p_i x_i}\pi_i < 0$ and $\partial^2_{p_j p_i}\pi_j > 0$ by (*A3*). Finally, comparing the expressions for dp_i/dx_i and dp_i/dx_j yields $|dp_i/dx_i| > |dp_i/dx_j|$, for all $i \neq j$. Q.E.D.

Proof of Lemma 4. Given the symmetric equilibrium characterized in Lemma 1, an increase in the price set by firm i implies the following for the industry quantity $Q = q_i + (n-1)q_j$ (for simplicity, in the rest of this proof we drop functional notation):

$$\begin{aligned}
\frac{dQ}{dp_i} &= \frac{\partial q_i}{\partial p_i} + (n-1)\frac{\partial q_i}{\partial p_j}\frac{dp_j}{dp_i} + (n-1)\left[\frac{\partial q_j}{\partial p_j}\frac{dp_j}{dp_i} + \frac{\partial q_j}{\partial p_i} + (n-2)\frac{\partial q_j}{\partial p_l}\frac{dp_l}{dp_i}\right], &\text{(A-6)}\\
&= \frac{\partial q_i}{\partial p_i}\left[1 + \frac{dp_j}{dp_i}(n-1)\right] + (n-1)\frac{\partial q_i}{\partial p_j}\left[1 + \frac{dp_j}{dp_i}(n-1)\right] &\text{(A-7)}\\
&= \left[\frac{\partial q_i}{\partial p_i} + (n-1)\frac{\partial q_i}{\partial p_j}\right]\left(1 + \frac{dp_j}{dp_i}(n-1)\right) < 0, &\text{(A-8)}
\end{aligned}$$

with $i \neq j$ and $l \neq j$. The equality in the second line uses the assumption of symmetric demand, which implies that $\partial q_j/\partial p_i = \partial q_i/\partial p_j$ and $\partial q_j/\partial p_j = \partial q_i/\partial p_i$ at a symmetric equilibrium. The inequality follows from the fact that the term in parentheses is strictly positive and the term in squared brackets is negative by assumption (*A2*), which implies that $\partial q_i/\partial p_i < -(n-1)\partial q_i/\partial p_j$. Q.E.D.

Proof of Lemma 5. Given the symmetric equilibrium characterized in Lemma 1, a change in the investment set by firm i implies the following for the industry quantity $Q = q_i + (n-1)q_j$ (for simplicity, we drop functional notation in the rest of this proof):

$$\begin{aligned}
\frac{dQ}{dx_i} &= \left[\frac{\partial q_i}{\partial p_i} + (n-1)\frac{\partial q_j}{\partial p_i}\right]\frac{dp_i}{dx_i} + (n-1)\left[\left(\frac{\partial q_j}{\partial p_j} + \frac{\partial q_i}{\partial p_j}\right)\frac{dp_j}{dx_i} + (n-2)\frac{\partial q_j}{\partial p_l}\frac{dp_l}{dx_i}\right]\\
&= \left[\frac{\partial q_i}{\partial p_i} + (n-1)\frac{\partial q_i}{\partial p_j}\right]\frac{dp_i}{dx_i} + (n-1)\left[\frac{\partial q_j}{\partial p_j} + (n-1)\frac{\partial q_j}{\partial p_l}\right]\frac{dp_l}{dx_i} > 0, &\text{(A-9)}
\end{aligned}$$

with $i \neq j$ and $l \neq j$. The equality at the second line follows from the property that the demand function is symmetric (and so are the cross-effects of a change in the investment on the price of a firm $l, j \neq i$). Moreover, the first and the second term of (A-9) are negative by assumption (*A2*). The inequality follows from the results in Lemma 3 that $dp_i/dx_i, dp_j/dx_i < 0$. We then obtain that a marginal decrease in firm i's investment (x_i) reduces the industry quantity by $-dQ/dx_i < 0$. Q.E.D.

Proof of Lemma 6. Given the symmetric equilibrium characterized in Lemma 1, and by the Envelope theorem, a marginal increase in the price of firm i implies the following for firm i's profits (to ease the exposition, we drop the functional notation):

$$\frac{d\pi_i}{dp_i} = (p_i - c(x_i))\frac{\partial q_i}{\partial p_j}\frac{dp_j}{dp_i}(n-1) > 0. \tag{A-10}$$

Therefore a marginal increase in firm i's price raises firm i's profits.

Consider now a marginal increase in the investment of firm i, given the symmetric equilibrium characterized in Lemma 1. Invoking the Envelope theorem, we find that

$$\frac{d\pi_i}{dx_i} = (p_i - c(x_i))\frac{\partial q_i}{\partial p_j}\frac{dp_j}{dx_i}(n-1) < 0. \tag{A-11}$$

Since a decrease in the investment of firm i increases p_j, we obtain that as firm i raises x_i the profit of firm i falls. As a consequence, a marginal decrease in x_i makes firm i's profit increase by $-d\pi_i/dx_i > 0$. Q.E.D.

Proof of Lemma 7. Note that, with sequential choices, the value of $\partial^2_{p_i x_i}\pi_i$ as resulting from the derivative of (10) with respect to p_i is as follows:

$$\partial^2_{p_i x_i}\pi_i = -c'(x_i)\partial_{p_i} q_i(\bar{p}(\bar{x})) + (n-1)\left[\partial^2_{p_j p_i} q_i(\bar{p}(\bar{x}))(p_i(\bar{x}) - c(x_i)) + \partial_{p_j} q_i(\bar{p}(\bar{x}))\right]\frac{dp_j}{dx_i}. \quad \text{(A-12)}$$

The term in squared brackets is positive because, by (*A3*),

$$\partial^2_{p_i p_j}\pi_i = \partial_{p_j} q_i(\cdot) + \partial^2_{p_i p_j} q_i(\cdot)(p_i - c(x_i)) > 0.$$

Thus, since the first term in (A-12) is negative and the term in brackets is multiplied by $dp_j/dx_i < 0$, the all expression is negative (as much as in the model with simultaneous moves). All the rest of the proof is as for Lemma 3. Q.E.D.

Proof of Proposition 4. Comparing (9) with (11) yields that, at the pre-merger investments in $\bar{x}$, insiders raise their prices with respect to the benchmark: $p_i^m(\bar{x}), p_k^m(\bar{x}) > p_i^b(\bar{x}), p_k^b(\bar{x})$. As a consequence, $\bar{p}^m(\bar{x}) > \bar{p}^b(\bar{x})$. Moreover, from (10) and (13) it follows that, at the pre-merger prices in $\bar{p}$, insiders lower their investments $(x_i^m(\bar{p}), x_k^m(\bar{p}) < x_i^b(\bar{p}), x_k^b(\bar{p}))$ so that $\bar{x}^m(\bar{p}) < \bar{x}^b(\bar{p})$.

The total effect imparted by the merger-induced increase in p_i, p_k on the investments of the insiders is captured by:

$$\left(\frac{dx_i}{dp_i} + \frac{dx_i}{dp_k}\right)\left(1 + \frac{dx_k}{dx_i}\right) + \left(\frac{dx_k}{dp_i} + \frac{dx_k}{dp_k}\right)\left(1 + \frac{dx_i}{dx_k}\right). \quad \text{(A-13)}$$

To study the sign of this expression, we proceed in two steps. First, we prove that $x_i^b(\bar{p}), x_k^b(\bar{p}) > x_i^b(\bar{p}'), x_k^b(\bar{p}')$, where $\bar{p}' > \bar{p}$ denotes the price vector post-merger. Specifically, this requires showing that $dx_i/dp_i + dx_i/dp_k < 0$ (and, thus, that $dx_k/dp_i + dx_k/dp_k < 0$).

Applying the Implicit Function theorem to the post-merger first-order condition for x_i in (13) yields

$$\frac{dx_i}{dp_i} = -\frac{\partial^2_{x_i p_k}\pi_{i,k}}{\partial^2_{x_i x_i}\pi_{i,k}}, \quad \frac{dx_i}{dp_k} = -\frac{\partial^2_{x_i p_k}\pi_{i,k}}{\partial^2_{x_i x_i}\pi_{i,k}}.$$

The denominator of both expressions, taken with negative sign, is positive at a (local) maximum. Thus, the sign of both ratios (and of their sum) depends on the numerators. In particular,

$$\begin{aligned}\partial^2_{x_i p_i}\pi_{i,k} &= -c'(x_i)\partial_{p_i} q_i(\bar{p}(\bar{x})) \\ &\quad + (n-2)\frac{dp_j}{dx_i}\left[\partial^2_{p_j p_i} q_i(\bar{p}(\bar{x}))(p_i(\bar{x}) - c(x_i) + \partial_{p_j} q_i(\bar{p}(\bar{x})) + \partial^2_{p_j p_i} q_k(\bar{p}(\bar{x}))(p_k(\bar{x}) - c(x_k)\right]\end{aligned}$$

and

$$\begin{aligned}\partial^2_{x_i p_k}\pi_{i,k} &= -c'(x_i)\partial_{p_k} q_i(\bar{p}(\bar{x})) \\ &\quad + (n-2)\frac{dp_j}{dx_i}\left[\partial^2_{p_j p_k} q_i(\bar{p}(\bar{x}))(p_i(\bar{x}) - c(x_i) + \partial_{p_k} q_i(\bar{p}(\bar{x})) + \partial^2_{p_j p_k} q_k(\bar{p}(\bar{x}))(p_k(\bar{x}) - c(x_k)\right].\end{aligned}$$

Since, by (*A3*),

$$\partial^2_{p_i p_j}\pi_{i,k} = \partial_{p_j} q_i(\cdot) + \partial^2_{p_i p_j} q_i(\cdot)(p_i - c(x_i)) + \partial^2_{p_i p_j} q_k(\cdot)(p_k - c(x_k)) > 0,$$

the terms in squared brackets in $\partial^2_{x_i p_i}\pi_{i,k}$ and $\partial^2_{x_i p_k}\pi_{i,k}$ are positive. Thus,

$$sign\left\{-c'(x_i)(\partial_{p_i}q_i(\bar{p}(\bar{x}))+\partial_{p_k}q_i(\bar{p}(\bar{x})))\right\} \to sign\left\{\frac{dx_i}{dp_i}+\frac{dx_i}{dp_k}\right\}.$$

Moreover, since by $(A2)$, $\partial_{p_i}q_i < \partial_{p_k}q_i$, we obtain that

$$-c'(x_i)(\partial_{p_i}q_i(\bar{p}(\bar{x}))+\partial_{p_k}q_i(\bar{p}(\bar{x}))) < 0.$$

Therefore, if investments x_i and x_k are strategic complements (so that $dx_i/dx_k > 0$), the sign of (A-13) is clearly negative. It is also negative if investments are strategic substitutes but $(n-1)|dx_i/dx_k| \in (0,1)$, which is a direct implication of the condition on the stability of the investment game, which requires that

$$\partial^2_{x_i x_i}\pi_i + (n-1)\partial^2_{x_i x_j}\pi_i < 0. \tag{A-14}$$

Moreover, the fall in the investments of merging parties i and k triggers a lower total investment insofar as investments are strategic complements, or if investments are strategic substitutes but outsiders increase their investments by less than insiders in absolute value—which is again a consequence of (A-14).

Finally, proceeding analogously to determine the total effect of the merger on equilibrium prices, we obtain that (A-14) is sufficient to guarantee that the fall in x_i and x_k further increases firms' prices, as

$$-\left[\frac{dp_l}{dx_i}\left(1+\frac{dx_i}{dx_k}\right)+\frac{dp_l}{dx_k}\left(1+\frac{dx_k}{dx_i}\right)\right] > 0,$$

if $(n-1)|dx_i/dx_k| < 1$, for all $l = 1,...,n$, since $dp_l/dx_i < 0$ and $dp_l/dx_k < 0$. This effect reinforces the increase in prices due to strategic complementarity ($dp_i/dp_j > 0$).

All this implies that prices rise and investments fall at equilibrium. Q.E.D.

Proof of Lemma 8. Given the symmetric equilibrium in the benchmark with independent firms, a change in the investment set by firm i implies the following for the industry quantity $Q = q_i + (n-1)q_j$ (for simplicity, we drop functional notation in the rest of this proof):

$$\begin{aligned}\frac{dQ}{dx_i} &= \left[\frac{\partial q_i}{\partial p_i}+(n-1)\frac{\partial q_j}{\partial p_i}\right]\left(\frac{dp_i}{dx_i}+(n-1)\frac{dp_i}{dx_l}\frac{dx_l}{dx_i}\right)+ \\ &\quad +(n-1)\frac{\partial q_i}{\partial p_j}\left(\frac{dp_j}{dx_i}+(n-1)\frac{dp_j}{dx_l}\frac{dx_l}{dx_i}\right) \\ &\quad +(n-1)^2\left[\frac{\partial q_j}{\partial p_l}\left(\frac{dp_l}{dx_i}+(n-1)\frac{dp_l}{dx_s}\frac{dx_s}{dx_i}\right)\right]\end{aligned}$$

with $i \neq j$ and $l, s \neq i$. This expression can be rewritten as

$$\begin{aligned}\frac{dQ}{dx_i} &= \left(\frac{dp_i}{dx_i}+(n-1)\frac{dp_i}{dx_l}\frac{dx_l}{dx_i}\right)\left(\frac{\partial q_i}{\partial p_i}+(n-1)\frac{\partial q_j}{\partial p_i}\right) \\ &\quad +(n-1)\left(\frac{dp_l}{dx_i}+(n-1)\frac{dp_l}{dx_s}\frac{dx_s}{dx_i}\right)\left(\frac{\partial q_i}{\partial p_l}+(n-1)\frac{\partial q_j}{\partial p_l}\right)\end{aligned}$$

with $i \neq j$ and $l, s \neq i$. Using the symmetric demand property together with assumption $(A2)$, we can simplify the expression to find that a decrease in x_i reduces industry quantity if (14) holds true. Q.E.D.

Proof of Lemma 9. Given the symmetric equilibrium in the benchmark with independent firms, a marginal increase in the price of firm i implies that the profits of firm i increase as in (A-10).

Consider now a marginal increase in the investment of firm i, given the symmetric equilibrium. Invoking the Envelope theorem, we find that (in what follows, we drop functional notation):

$$\frac{d\pi_i}{dx_i} = (p_i - c(x_i))(n-1)^2 \frac{\partial q_i}{\partial p_j} \frac{dp_j}{dx_l} \frac{dx_l}{dx_i}. \tag{A-15}$$

Specifically, following a marginal increase in x_i, the profit of firm i decreases if investments are strategic complements (so that $dx_l/dx_i > 0$), increase otherwise. This implies that a decrease in the investment of firm i reduce firm i's profit if investment are strategic substitute.

To sum up, if investments are strategic complements then the profit of a firm i increase if its price increases, or its investment reduces. In case of strategic substitutability, we find that the increase in the profit triggered by the increase in firm i's price compensates for the reduction caused by the increase in firm i's investment if (15). The expression in (15) is obtained by summing up (A-10) and (A-15). It imposes a lower bound on the degree of strategic substitutability for the profit of a firm i to increase following an increase in its price and a decrease in its investment. Q.E.D.

References

[1] Aghion, P., Bloom, N., Blundell, R., Griffith, R., and P. Howitt, 2005. Competition and Innovation: an Inverted-U Relationship. *Quarterly Journal of Economics*, 120(2), 701–728.

[2] d'Aspremont, C., and Jacquemin, A., 1988. Cooperative and Noncooperative R&D in Duopoly with Spillovers. *American Economic Review* 78, 1133–1137.

[3] Bartelsman, E., and Doms, M., 2000. Understanding Productivity: Lessons from Longitudinal Microdata. *Journal of Economic Literature*, 38, 569–94.

[4] Deneckere, R., and Davidson, C., 1985. Incentives to Form Coalitions with Bertrand Competition. *Rand Journal of Economics*, 16(4), 473–486.

[5] Ericson, R., and Pakes, A., 1995. Markov-perfect Industry Dynamics: A Framework for Empirical Work. *Review of Economic Studies*, 62, 53–82.

[6] Farrell, J., and M. L. Katz, 2006. The Economics of Welfare Standards in Antitrust, *Competition Policy International*, 2, 3–28.

[7] Farrell, J., and Shapiro, C., 1990. Horizontal Mergers: An Equilibrium Analysis. *American Economic Review*, 80(1), 107–126.

[8] Fershtman, C., and Pakes, A., 2000. A Dynamic Oligopoly with Collusion and Price Wars. *Rand Journal of Economics*, 31, 207–236.

[9] Frontier Economics, 2014. European Mobile Network Operator Mergers. A Regulatory Assessment. Report commissioned by the GSM Association.

[10] Gilbert, R., 2006. Looking for Mr. Schumpeter: Where are we in the Competition-innovation Debate?, in Innovation Policy and the Economy (A. B. Jaffe, J. Lerner and S. Stern (eds)). The MIT Press.

[11] Gowrisankaran, G., 1999. A Dynamic Model of Endogenous Horizontal Mergers. *Rand Journal of Economics*, 30, 56–83.

[12] Häckner, J., 2000. A Note on Price and Quantity Competition in Differentiated Oligopolies. *Journal of Economic Theory*, 93, 233–239.

[13] Kamien, M., Muller, E., and Zang, I., 1992. Research Joint Ventures and R&D Cartels. *American Economic Review*, 82, 1293–1306.

[14] Kolstad, C., and Mathiesen, L., 1987. Necessary and Sufficient Conditions for a Cournot Equilibrium. *Review of Economic Studies*, 54, 681–690.

[15] Leahy, D., Neary, P., 1997. Public Policy Towards R&D in Oligopolistic Industries. *American Economic Review*, 87, 642–662.

[16] Lopez, A. L., and Vives, X., 2015. Cooperation, R&D Spillovers and Antitrust Policy. Mimeo.

[17] Mermelstein, B., Nocke, V., Satterthwaite, M. A., and Whinston, M. D., 2015. Internal versus External Growth in Industries with Scale Economies: A Computational Model of Optimal Merger. Mimeo.

[18] Neven, D. J., and L.-H. Röller, 2005. Consumer surplus vs. welfare standard in a political economy model of merger control, *International Journal of Industrial Organization*, 23, 829-848.

[19] Nocke, V., and Whinston, M. D., 2010. Dynamic Merger Review. *Journal of Political Economy*, 118, 1200–51.

[20] Pittman, R., 2007. Consumer Surplus as the Appropriate Standard for Antitrust Enforcement. Economic Analysis Group D.P. 07-9, Washington: Department of Justice.

[21] Salant, S. W., Switzer, S., and Reynolds, R. J., 1983. Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium. *Quarterly Journal of Economics*, 98(2), 185–199.

[22] Schmutzler, A., 2013. Competition and Investment - A Unified Approach. *International Journal of Industrial Organization*, 31(5), 477–487.

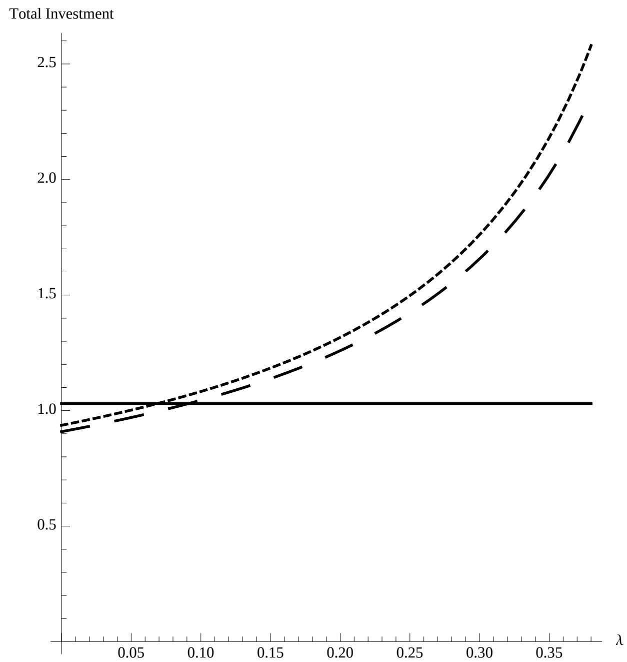
[23] Shapiro, C., 2013. Competition and Innovation: Did Arrow Hit the Bull's Eye? in The Rate and Direction of Inventive Activity Revisited (J. Lerner and S. Stern (eds)). The MIT Press.

[24] Shelegia, S., and Spiegel, Y., 2015. Cost-Reducing Investments under Partial Cross-Ownership. Mimeo.

[25] Syverson, C., 2004. Market Structure and Productivity: A Concrete Example. *Journal of Political Economy*, 112, 1181–222.

[26] Vives, X., 2008. Innovation and Competitive Pressure. *Journal of Industrial Economics*, 56(3), 419–469.

Figure 1: Merger and NSA with economies of scope



In all the figures, the solid black line corresponds to the benchmark. The dotted line to the merger and the dashed line to the NSA. The parametric values we use are $\alpha = 1.5$, $c = 1$ and $\gamma = 0.25$. The range for λ is chosen so that all parametric restrictions are satisfied.

Table 1: Equilibrium outcomes in the VPD model

	Benchmark (triopoly)	**Merger (duopoly)**
u_1	$.2527\overline{\theta}^2$	$.2533\overline{\theta}^2$
u_2	$.0497\overline{\theta}^2$	$.0482\overline{\theta}^2$
u_3	$.95\overline{\theta}^2$	—
p_1	$.1060\overline{\theta}^3$	$.1077\overline{\theta}^3$
p_2	$.0091\overline{\theta}^3$	$.0102\overline{\theta}^3$
p_3	$.0009\overline{\theta}^3$	—
q_1	$.5225\overline{\theta}$	$.5246\overline{\theta}$
q_2	$.2721\overline{\theta}$	$.2638\overline{\theta}$
q_3	$.1136\overline{\theta}$	—
π_1	$.0235\overline{\theta}^4$	$.0244\overline{\theta}^4$
π_2	$.0012\overline{\theta}^4$	$.0015\overline{\theta}^4$
π_3	$.00005\overline{\theta}^4$	—
CS	$.0443\overline{\theta}^4$	$.0432\overline{\theta}^4$
PS	$.0248\overline{\theta}^4$	$.0259\overline{\theta}^4$
W	$.06911\overline{\theta}^4$	$.06918\overline{\theta}^4$