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## REPUTATION WITH OPPORTUNITIES FOR COASTING

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## Abstract

Reputation concerns can discipline agents and generate good outcomes. But what if outcomes are not always observed? Infrequent observation can strengthen reputation incentives and encourage effort. By exerting effort when outcomes are more likely observed, the agent can improve her reputation, and when the audience is inattentive in the future, she can "coast" on this reputation without additional effort. Such opportunities to coast in the future can in fact lead to greater overall effort than constant observation. We characterize the optimal observability structure to maximize efficient effort. This has implications for the design of review systems or performance feedback systems.

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# Reputation with Opportunities for Coasting

Heski Bar-Isaac      Joyee Deb\*

September 15, 2016

## Abstract

Reputation concerns can discipline agents and generate good outcomes. But what if outcomes are not always observed? Infrequent observation can strengthen reputation incentives and encourage effort. By exerting effort when outcomes are more likely observed, the agent can improve her reputation, and when the audience is inattentive in the future, she can "coast" on this reputation without additional effort. Such opportunities to coast in the future can in fact lead to greater overall effort than constant observation. We characterize the optimal observability structure to maximize efficient effort. This has implications for the design of review systems or performance feedback systems.

## 1 Introduction

Reputation concerns are an important driver of incentives. Examples are ubiquitous: A manager must work hard to develop a reputation for effectiveness; a chef must consistently deliver high quality meals to earn a good reputation for her restaurant; and so on. Reputation affects an agent's behavior because actions taken today affect others' beliefs about the agent's type and likely actions in the future. Indeed, economic

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agents take costly effort to generate outcomes that can improve their reputation. But, in many settings, outcomes are not always observed by the audience. This may be because of institutional details like the frequency of performance reviews or because the audience is inattentive: A manager’s performance is not evaluated every day. Every consumer does not pay attention to the quality of meal at a restaurant and rate it on Yelp. But, if the agent’s actions today do not have an impact on what is observed, then reputation concerns may no longer work as an effective disciplining device. What does changing observability mean for agents’ incentives to invest in reputation building? This is the central question we ask in this paper.

The question is important not just to get a positive description of reputation building in settings with an inattentive audience, but also to address design questions about optimal performance feedback mechanisms. There is a vast (and, in an increasingly digital age, growing) variety of ways in which information can be gathered and disseminated. Understanding how the extent and nature of available information can affect the strength of reputation incentives is, therefore, a key building block in understanding economic outcomes, making predictions for their evolution, and informing the design of institutions for information-gathering and information dissemination (including personnel review systems, credit-scoring, and consumer reviews of products, restaurants, and hotels).

An inattentive audience can surely dampen an agent’s incentive to work hard: For instance, at one extreme, if the audience never paid attention to outcomes, exerting effort to obtain better outcomes cannot be useful in building reputation. However, this need not imply that more frequent observations necessarily strengthen reputation-based incentives. Indeed, the main insight of this paper is that inattention can sharpen an agent’s incentive to invest in reputation building. We identify a novel and intuitive mechanism by which inattention can encourage effort. If the agent anticipates that the audience will be inattentive in the future this can increase the benefits from working hard today. Exerting effort in periods when her action is likely observed, allows the agent to get the future reward of “coasting” (earning a reward for this reputation even while exerting no effort) in periods when her action is likely unobserved. When the audience is inattentive in the future, the agent will be able to shirk and coast on his reputation without any additional effort. We show that such opportunities to coast or rest on one’s laurels in the future can actually lead to greater overall effort and higher efficiency than constant observation. Thus, a review system such as a consumer report magazine (e.g. Michelin or Zagat guide) that assesses and reports on quality relatively infrequently can perform better than a system with continual assessment (such as on-

line reviews on Yelp). Similarly, performance review systems in organizations that engage in periodic assessment (e.g. tenure review) can be more effective at generating effort than continual assessment.

This result may run counter to the intuition of a reader used to thinking in contract-theoretic terms. In a seminal paper, Holmström (1979) presents the informativeness principle. In a contractual setting any measure of performance that reveals information about an agent's action improves the performance of a compensation contract; loosely speaking, more information is necessarily better. Instead, in a reputation environment, this need not be the case since the rewards associated with a belief about the agent's action are not chosen but instead arise endogenously. Information, in effect plays twin roles in indicating the likelihood that an agent took one action rather than another, and in determining the rewards for that action.<sup>1</sup> In our setting, less frequent observation has these twin effects: On the one hand, it dampens the incentive to work in the current period, because working and shirking are observationally equivalent when the audience is not paying attention. On the other hand, infrequent observation can also lead to greater rewards from reputation in the future because the agent will be able to maintain her reputation without taking any costly actions for a few periods in the future when actions are not observed. We show that the second effect may dominate: Reputation-based incentives may be too weak to discipline behavior if actions are observed in every period; but, infrequent observation might impose some discipline, exactly because the agent knows that hard work today will give an opportunity to “coast” on a good reputation later.

We present a stylized and simple model that allows us to characterize when more frequent observation leads to stronger reputation-based incentives. To simplify the problem we initially suppose that outcomes are either perfectly observed or not observed at all. We refer to the former case as one where the audience can pay attention, and the latter as a situation where the audience cannot and is inattentive. In our baseline model the audience's current access to information depends only on whether he was able to observe yesterday.

In this environment, we demonstrate the following results. If efficient effort can be sustained when outcomes are observed in every period then limiting observation can (trivially) never lead to a more efficient outcome but can lead to a less efficient one. However, more generally, infrequent observation can lead to more or less effort by a strategic long-lived player. Indeed, less frequent observation can lead to a more

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<sup>1</sup>See Dewatripont, Jewitt and Tirole (1999) who nicely describe this phenomenon and the possibility that information can act in these two opposing directions in a one-period career concerns model.

efficient average level of effort than when outcomes are observed in every period, through the coasting effect described above.

The fact that such inattention can sharpen the incentive to work hard can, however, imply some perverse effects: In particular, it can encourage agents to exert costly effort even when this effort is inefficient and welfare-reducing.

We also characterize the optimal observation process (among two-state Markov processes) that maximizes effort incentives. If the agent is patient, then it is easy to provide incentives to exert effort. But away from the patient limit, constant attention may not be optimal. Instead, the coasting effect can strengthen effort incentives and the optimal information structure is such that the inattentive state occurs often, but is short-lived. This makes the agent work “extra” hard when she anticipates that she will be able to rest on her laurels later, but such inattention does not last too long so as to lower the average amount of effort in the long run, and so that its benefits are not discounted too much through time preferences.

Another subtle insight that emerges is that some persistence in attention states underlies the ability to coast. For instance, if the current attention state of the audience were not at all persistent but just independently drawn in every period, then the coasting effect would not arise. Inattention can help discipline the agent only when the agent simultaneously knows that she is being observed currently and also anticipates that the audience will be inattentive in the future. This makes her want to work hard and improve her reputation right now in order to rest on her laurels later. Persistence in attention (and inattention) allows the agent to predict inattention in the future which gives rise to the coasting effect.

We consider variations of our baseline setting. We find that the coasting intuition is robust to the introduction of imperfect monitoring, or to changes in the information structure (specifically, we characterize outcomes if the agent were not aware of the current attention state). Further, it may be reasonable to assume that the attention state does not evolve exogenously, but rather depends on the current reputation of the agent. For instance, a customer may be unlikely to pay attention when the reputation of the agent is very high or very low, thinking that there is not much to be learnt from observing another outcome, but may be much more attentive at intermediate beliefs. We show that the coasting effect is still robust in this setting.

## Related Literature

The literature on reputation in economics is extensive.<sup>2</sup> Much of this work, following Fudenberg and Levine (1989, 1992) focuses on payoff bounds for a patient long-run player facing a sequence of short-run players when there is uncertainty regarding the long-run player's type. These papers take the mapping from actions to observations as constant through time. What distinguishes this paper from existing literature is our focus on monitoring that changes over time, while maintaining the classic perspective that observations are commonly observed by all interested parties.

Closer in spirit are papers by Liu (2011), Liu and Skrzypacz (2014), Ekmekci (2009) in which all information is not passed on directly between different generations of short-run players, but rather only recent outcomes are observed. This can sustain uncertainty about the long-run agent's type and, thereby, sustain reputation incentives.<sup>3</sup> This is an important feature in the context of the work of Cripps, Mailath and Samuelson (2004) which shows that under imperfect monitoring, reputation effects cannot be sustained in perpetuity. In our baseline model, as in the classical work of Kreps and Wilson (1982) and Milgrom and Roberts (1982), we consider perfect monitoring (or no monitoring at all) so that reputation effects can play a role even in the long-run. The perfect monitoring structure gives us tractability and allows us to develop intuition that extends under imperfect monitoring.

Our work is substantively related to recent work on the design of feedback systems.<sup>4</sup> Dellarocas (2006) considers the effect of infrequent reviews in a pure moral hazard setting. The effects are quite different: In Dellarocas (2006) infrequent review is useful for obtaining a signal that reduces noise. In our setting, infrequent review leads to less information collected and may be optimal when monitoring is perfect whereas in Dellarocas (2006) it would be optimal to update in every period.<sup>5</sup> In recent contemporaneous work, Hörner and Lambert (2016) characterize the rating system that maximizes effort provision in a standard career concerns framework, and find that

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<sup>2</sup>Mailath and Samuelson (2006), Mailath and Samuelson (Forthcoming), Cripps (2006), MacLeod (2007) and Bar-Isaac and Tadelis (2008) provide useful and complementary surveys of the broad literature on reputation.

<sup>3</sup>The literature has described other mechanisms to sustain such uncertainty, including exogenous type changes, Holmström (1999); name-trading, Tadelis (2002) and Mailath and Samuelson (2001); and organizational design, Bar-Isaac (2007).

<sup>4</sup>See Dellarocas, Dini and Spagnolo (2006) for a broad overview and, notably, Ekmekci (2009) on how much information should be passed on to new consumers.

<sup>5</sup>See also Fuchs (2007) who considers optimal review period in a relational contracting environment (that is with pure moral hazard and an ongoing relationship rather than a series of short-lived agents) with private imperfect monitoring.

a mixture two-state Markov process is optimal. While their question is substantively similar to ours, their model and results are quite different. They focus on characterizing the optimal rating system, while we identify a new mechanism through which infrequent observation can strengthen incentives.

The effect of monitoring structure on reputation is also related to recent work by Bohren (2016), Board and ter Vehn (2013), Board and ter Vehn (2015), and the related one period model of Fleckinger, Glachant and Moineville (2015), though they consider the effect of different information structures in an environment where the “type” of the long-run player is not constant throughout the game, but the result of an exogenous constant stochastic process and the player’s actions. A nascent literature is beginning to consider endogenous monitoring structures in environments where incentives arise through reputation or similar forces; in particular, Garcia (2014) considers endogenous monitoring where reputation is for the extent to which an advisor has aligned preferences with a principal; Halac and Prat (2016) consider an environment with two-sided learning; and Dana and Spier (2015*a,b*) consider firm strategies that affect market monitoring as a means of commitment to investing in quality.

The broader insight that in non-contractual environments less information might be helpful to a principal unable to commit in advance to rewards is well articulated in Cremer (1995), Dewatripont and Maskin (1995) and Kessler (1998), for example.<sup>6</sup> Dewatripont, Jewitt and Tirole (1999) illustrate in a one period career concerns model with mult-tasking that information may vary in the extent to which it reflects effort and the extent to which it reflects an agent’s type and, so, affects the rewards to particular observations; instead, we consider a one-dimensional environment and a mechanism that relies on more than two possible periods of interaction.

The rest of the paper is structured as follows. In Section 2, we present the model. In Section 3, we characterize all of the (Markov perfect) equilibria, whether in mixed or pure strategies, and demonstrate the coasting effect. Section 4 discusses welfare and characterizes the optimal inattention process to maximize effort. In Section 5, we discuss the intuition behind the coasting effect, and Section 6 discusses robustness.

## 2 Model and Preliminaries

There is a long-lived firm that faces a continuum of short-lived consumers. The firm can be one of two types:  $C$ , the commitment type or  $O$ , the opportunistic or normal

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<sup>6</sup>That less information might be useful to blunt incentives that might be too strong has been discussed in Holmström (1999), for example.

type. Opportunistic firms have two possible action choices: high effort and low effort;  $a \in \{H, L\}$  where  $L$  is costless and taking action  $H$  costs  $c > 0$ .  $C$ -firms are committed to exerting high effort always, or find it costless to take action  $H$ . We let  $\phi_t$  denote the belief at the start of period  $t$  that the firm is the  $C$  type. The firm's action can produce one of two possible outcomes - good or bad  $y \in \{g, b\}$ . High effort is more likely to produce a good outcome than low effort. In fact, until Section 6 we suppose that (as long as consumers can observe) there is perfect monitoring of actions; that is, action  $H$  produces outcome  $g$  with probability 1, and action  $L$  ensures an outcome of  $b$ . Consumers value a good outcome at 1 and a bad outcome at 0.

At the start of any period  $t$ , consumers find themselves in one of two attention states: Attentive and inattentive; i.e.,  $\omega_t \in \{A, I\}$ . When  $\omega_t = A$ , they all observe the outcome, and when  $\omega_t = I$ , they do not. The consumers' attention state follows a Markov process: If the consumers are in the inattentive state ( $\omega_t = I$ ), then with probability  $\iota$ , the state remains inattentive in the next period. If consumers are attentive ( $\omega_t = A$ ) then with probability  $\alpha$  the state transitions to being inattentive in the next period. Note that  $\iota = \alpha$  would mean independent state transitions. Further, note that this transition process implies that the stationary distribution of attention states is given by  $(\frac{\alpha}{1-\iota+\alpha}, \frac{1-\iota}{1-\iota+\alpha})$ ; i.e.,  $\frac{\alpha}{1-\iota+\alpha}$  and  $\frac{1-\iota}{1-\iota+\alpha}$  represent the proportion of time consumers spends in the inattentive and attentive states respectively.

The timing of the game is as follows.

1. At the start of any period  $t$ ,  $\phi_t \in [0, 1]$  denotes the current belief that firm is a  $C$ -type, and  $\omega_t \in \{A, I\}$  denotes the current attention state.
2. The firm first collects the payment from the consumer  $p_t$ .
3. Then the firm makes its effort choice,  $a_t \in \{H, L\}$ , and an outcome,  $y_t \in \{g, b\}$ , is realized.
4. The outcome is observed or not by consumers, depending on the current attention state.
5. Consumers update their belief about the firm's type to  $\phi_{t+1}$ .
6. An attention state  $\omega_{t+1}$  is realized.
7. Play proceeds to next period.

We restrict attention to Markov perfect equilibria.<sup>7</sup>

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<sup>7</sup>This is a standard notion of equilibrium to explore the role of reputation in infinite horizon games; without such a restriction repeated game considerations and bootstrapping equilibria can arise even if consumers are certain that the agent is opportunistic. See, for example, Mailath and Samuelson (2001) for a brief discussion.

**Definition 1.** A Markov perfect equilibrium is a tuple  $(\tau(\phi, \omega), p(\phi, \omega), \phi_g, \phi_b, \phi_I)$  such that:

- At any belief and attention state  $(\phi, \omega)$ , it is optimal for the firm to play  $H$  with probability  $\tau(\phi, \omega)$ .
- The customer is willing to pay  $p(\phi, \omega)$  which is the expected utility of the consumer given  $\phi, \omega$  and  $\tau(\phi, \omega)$ ; i.e.,  $p(\phi, \omega) = \phi + (1 - \phi)\tau(\phi, \omega)$ .
- The posterior belief of customers, after observing outcomes  $g, b$  or nothing (in the inattentive state) are calculated using Bayes Rule and given the strategy  $\tau(\phi, \omega)$ ; i.e.,

$$\phi_g(\phi, \omega) = \frac{\phi}{\phi + (1 - \phi)\tau(\phi, \omega)} \quad \phi_b(\phi, \omega) = 0 \quad \phi_I(\phi, \omega) = \phi.$$

We start with two basic observations. First, in any MPE, the firm never exerts effort at degenerate beliefs. To see why, note that the only reason the firm takes effort is to affect the consumers' belief about his type. Therefore, there is no incentive to exert effort at degenerate beliefs. Second, given the timing within every period, in any MPE the firm never exerts effort in the inattentive state.

### 3 Inattention and the Opportunity to Coast

We characterize all (pure and mixed) MPE in this environment under perfect monitoring. There are two types of pure strategy equilibria: one in which the firm never exerts effort and another in which he exerts effort at interior beliefs in the attentive state. We characterize the conditions for existence of these two classes of equilibria. We then show that inattention can be good for effort incentives in Section 3.1, before completing the characterization of all MPE in Section 3.2.

We start by characterizing the equilibrium with no effort.

**Proposition 1** (No effort equilibrium). *The necessary and sufficient condition for the existence of an MPE with low effort at every belief and in both attentive and inattentive states is given by*

$$\frac{\delta}{1 - \delta} < c.$$

*Proof.* The firm's payoff from exerting no effort in the attentive state is given by  $\phi_t + 0$ . If the firm deviates and exerts effort in the attentive state, then his payoff is given by

$\phi_t - c + \frac{\delta}{1-\delta}$ . It follows that, no effort in all states is an equilibrium if and only if  $\frac{\delta}{1-\delta} < c$ .  $\square$

The proposition above tells us that if the firm is patient (for  $\delta$  large enough), never exerting effort cannot be an equilibrium. The intuition is straightforward. A patient firm will be tempted to deviate and exert costly effort today because this single deviation will convince consumers that he is the commitment type, and will yield the best possible payoffs forever in the future. It is also worth noting that the condition for existence of this equilibrium with no effort does not depend on the extent of attention (independent of  $\alpha$  or  $\iota$ ). The next result presents the condition for existence of an equilibrium with effort at every interior belief in the attentive state. (Recall that there is never any effort exertion in equilibrium in the inattentive state).

**Proposition 2** (Full effort equilibrium). *The necessary and sufficient condition for the existence of an MPE with high effort at every interior belief in attentive states and low effort in the inattentive states, given an initial reputation  $\phi$ , is given by:*

$$c < \delta \left( 1 - \alpha \frac{1 - \delta - \phi}{1 - \iota \delta} \right). \quad (1)$$

*Proof.* First, consider necessity. For an equilibrium with full effort in the attentive state at any interior belief  $\phi$ , we can write down the value functions, given belief  $\phi$  and attention state as follows:

$$V(\phi, A) = 1 - c + \delta \alpha V(\phi, I) + \delta(1 - \alpha)V(\phi, A)$$

and

$$V(\phi, I) = \phi + \delta \iota V(\phi, I) + \delta(1 - \iota)V(\phi, A).$$

We can solve the above to obtain:

$$V(\phi, A) = \frac{(1 - c)(1 - \iota \delta) + \alpha \delta \phi}{(1 - \delta)(1 + \alpha \delta - \iota \delta)} \text{ and } V(\phi, I) = \frac{\delta(1 - c)(1 - \iota) + \phi(1 - \delta(1 - \alpha))}{(1 - \delta)(1 + \alpha \delta - \iota \delta)}$$

For effort to be optimal, the firm must prefer working hard to exerting no effort, i.e., the firm will not deviate to shirking as long as

$$\delta \alpha V(\phi, I) + \delta(1 - \alpha)V(\phi, A) > c.$$

Substituting in the expressions for the value functions  $V(\phi, I)$  and  $V(\phi, A)$ , we obtain:

$$\delta \left[ 1 - \alpha \frac{1 - \delta - \phi}{1 - \iota \delta} \right] > c. \quad (2)$$

For sufficiency, the proof is similarly straightforward. Suppose that the condition in the proposition holds. Then, from the algebra above, we know that the firm would prefer working hard to shirking. Therefore, exerting effort at every interior belief in the attentive state, and shirking otherwise is an MPE.  $\square$

### 3.1 Coasting Effect

Notice that the condition (1) for a full effort equilibrium to exist *does* depend on the level of inattention. In particular, when there is no inattention (i.e.,  $\alpha = \iota = 0$ ), then this condition reduces to  $c < \delta$ . At the other extreme, when inattention is likely and almost absorbing ( $\alpha = \iota \approx 1$ ), the condition reduces to  $c < \frac{\delta\phi}{1-\delta}$ . This implies that inattention can be good for effort incentives: When  $\phi > 1 - \delta$ , the full effort equilibrium is easier to sustain under inattention than with full attention.<sup>8</sup> The intuition behind this is the “coasting” effect. When the firm knows that the attentive state is likely to transition to a persistent inattentive state, then it has a greater incentive to exert effort in the attentive state because this will allow it to maintain her reputation and enter the inattentive state with this high reputation. If the firm enters the inattentive state with a high reputation, it can later shirk in equilibrium but continue to coast on her reputation and reap the benefits. Further, the rewards from coasting are monotonic in the current reputation level. Formally, note that the effort incentive is easier to satisfy when  $\phi$  is higher.

If inattention can help provide the incentive to exert effort, a natural question is whether inattention then improves welfare. The answer is *prima facie* ambiguous. On the one hand, inattention can be welfare enhancing because it induces effort exertion even when effort cannot be sustained by full attention. On the other hand, inattention can be bad for welfare by inducing effort in equilibrium even when effort is socially inefficient (say  $c \gg 1$ ). Moreover, a welfare analysis needs to take into account the average effort taken over time. In the next section, we provide a complete welfare

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<sup>8</sup>Note that the early theoretical literature on reputation focuses on limiting results as  $\delta \rightarrow 1$ , and views commitment types (sometimes termed “crazy types”) as perturbations to the complete information model with only “normal types.” This literature therefore naturally thinks of commitment types as occurring with low probability. Instead, in our setting,  $\phi$ , the probability of the commitment type is better understood as the fraction of good rather than potentially fraudulent sellers in a market. The extent to which the fractions of honest traders is significant in online environments such as eBay, Alibaba or Amazon market place is an open empirical question, and so we do not restrict ourselves to thinking of  $\phi$  being small.

analysis. First, we complete the characterization of equilibria.

### 3.2 Mixed strategy equilibria

Above, we characterized all pure strategy Markov-perfect equilibria. There are two types of pure equilibria: one with no effort and the other will effort at interior beliefs in the attentive state. It is easy to show that both types of pure equilibria cannot exist simultaneously. Further, pure strategy MPE do not always exist: in particular, there are no pure MPE in the range  $\delta \left(1 - \alpha \frac{1-\delta-\phi}{1-\iota\delta}\right) \leq c \leq \frac{\delta}{1-\delta}$ . For completeness, we look for mixed strategy equilibria.

**Proposition 3.** *Suppose that*

$$\delta \left(1 - \frac{\alpha(1-\delta-\phi)}{1-\iota\delta}\right) < c < \delta \left(1 + \frac{\delta\alpha}{1-\iota\delta}\right).$$

*Then, there exists an MPE in which the firm mixes between high and low effort in the first attentive period (given an interior belief) and then (in case of high effort) switches to exerting high effort in all attentive states (at interior beliefs).*

We relegate the proof to the appendix. It turns out that this is the only type of mixing that is possible in equilibrium, as shown in Proposition 4. Thus, together, Propositions 1, 2, and 4 together characterize all possible MPE that can arise.

**Proposition 4.** *There is no MPE in which the firm*

1. *Mixes in the first attentive state and then shifts to low effort forever, or*
2. *Mixes in two consecutive periods of attention.*

From the propositions above and noting that  $\delta \left(1 + \frac{\delta\alpha}{1-\iota\delta}\right) < \frac{\delta}{1-\delta}$ , we can see that, even allowing for mixing, the existence of MPE is still not guaranteed everywhere.<sup>9</sup>

## 4 Welfare: Optimal inattention

Next, we ask the question of whether inattention can improve welfare. We will look at the planner's problem. As long as effort is efficient, ( $c < 1$ ), the planner wants

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<sup>9</sup>This arises, in part, since we have made the somewhat stark (if common) assumption that at the degenerate belief  $\phi = 1$  there is no updating in the (off-equilibrium) case that bad outcomes are observed. This is key for the threshold in Proposition 1. Relaxing this assumption can allow for multiplicity at some parameters, though the qualitative effects described in this paper and, in particular, the characterization of Proposition 2 still apply.

high effort as often as possible.<sup>10</sup> Looking at the stationary distribution of states, we know that the proportion of time spent in the attentive state is  $\frac{1-\iota}{1-\iota+\alpha}$ . We want to find the level of inattention ( $\alpha$  and  $\iota$ ) that results in the maximum effort exertion while satisfying incentive compatibility (the firm must prefer effort to no effort in all attentive states). Formally, given some initial reputation,  $\phi$ , we want to solve the following optimization problem:

$$\max_{\alpha, \iota} \frac{1-\iota}{1-\iota+\alpha} \quad \text{s.t. } c \leq \delta \left( 1 - \alpha \frac{1-\delta-\phi}{1-\iota\delta} \right).$$

Notice that if  $1 - \delta - \phi > 0$ , then a lower  $\alpha$  and lower  $\iota$  make both IC easier to satisfy. Moreover, the objective function is decreasing in  $\alpha$  and  $\iota$ . So,  $\alpha = 0$  and  $\iota = 0$  is optimal; that is, if the firm is sufficiently patient then it is optimal to be fully attentive. In particular, if  $c \leq \delta$  this will ensure that the firm always exerts high effort.

Next suppose that  $1 - \delta - \phi < 0$ . Then, if  $c \leq \delta$ , then we should set again  $\alpha = \iota = 0$ . Finally consider the case of  $1 - \delta - \phi < 0$  and  $c > \delta$ . Then, it is no longer optimal to be fully attentive; instead, the optimal choice will be  $\iota = 0$  and  $\alpha = \frac{\delta-c}{\delta(1-\delta-\phi)}$  as long as this  $\alpha < 1$ . Here, the inattentive state never lasts more than a single period—as soon as the state becomes inattentive it is guaranteed to become attentive in the next period; instead there is some persistence in the attentive state. In case  $\frac{\delta-c}{\delta(1-\delta-\phi)} > 1$  then the optimal solution hits a different corner of the parameter space and sets  $\alpha = 1$  (so that there is an immediate transition from the attentive to inattentive state) and  $\iota = \frac{\delta(\delta+\phi)-c}{\delta(\delta-c)}$ . This is always feasible because  $0 < \frac{\delta(\delta+\phi)-c}{\delta(\delta-c)} < 1$  as long as  $c \in (\delta, 1)$ .

This discussion can be summarized as follows:

**Proposition 5.** *Inattention improves welfare relative to the full attention benchmark whenever  $c > \delta$  and  $1 - \delta - \phi < 0$ . In particular, the optimal information structure has  $\iota = 0$  and  $\alpha = \frac{\delta-c}{\delta(1-\delta-\phi)}$  whenever feasible, and  $\iota = \frac{\delta(\delta+\phi)-c}{\delta(\delta-c)}$  and  $\alpha = 1$  otherwise.*

To see the intuition, notice that if the firm is patient, then it is easy to provide incentives to exert effort even in the full attention. So, the benefit of coasting can arise only when the discount factor  $\delta$  is not too high. Further, when the coasting effect can arise, the optimal information structure is such that the inattentive state occurs often, but is short-lived. This makes the firm work “extra” hard when she anticipates that she will be able to coast later, but inattention does not last too long so that the benefits of resting on one’s laurels are not discounted too much, but instead arise more frequently.

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<sup>10</sup>The case where effort is inefficient ( $c > 1$ ) is trivial. Optimality can be ensured by setting  $\alpha = \iota = 1$  so that the audience is never attentive.

## 5 Importance of persistence and observability

A useful insight is that some persistence in the attention states underlies the ability to coast. To see this clearly, consider the baseline setting in this paper. If  $\alpha = \iota = \frac{1}{2}$ , then transitions between states are independent, and the current state is not helpful in predicting the attention state in the future. In this case, the condition for full-effort equilibria to be sustained (Condition (1)) reduces to  $c < \frac{\delta(1-\phi)}{2-\delta}$ . On the other hand, under full attention (for  $\alpha = \iota = 0$ ), the condition for full effort to be sustained in equilibrium is given by  $c < \delta$ . Notice that the first threshold is lower than the second threshold, which means that inattention per se cannot improve the incentive to exert effort. Inattention can help discipline the firm only when the firm can predict it, and therefore wants to work hard and improve her reputation and rest on her laurels later. Therefore, the persistence in the attention states is necessary for the coasting effect.

Another useful insight is that along with persistence, some observability of states is also necessary for coasting. To appreciate the importance of observability, let us ask whether the coasting effect arises in our baseline setting if the firm *never* observed the attention state. Recall that the coasting effect implies that the firm is encouraged to exert effort in an attentive state, because effort in the attentive state is observed and boosts the firm's reputation, and moreover the firm anticipates that the state will likely change to inattention, at which point the firm can benefit from its high reputation with no effort. If the attention state is not observed at all, then the firm does not know that it is being observed and so does not have an additional incentive to exert effort today.<sup>11</sup> With unobservable states, the firm's behavior would depend only on its prior belief about states, and could not condition on the current state. Indeed, it is straightforward to show that in this case, inattention can only make effort exertion harder to sustain in equilibrium. The interested reader may refer to the Appendix A.2 for the details.

## 6 How robust is the coasting effect?

### 6.1 Can coasting arise under imperfect monitoring?

The reader may wonder whether our results are robust to the monitoring structure: If we consider a setting with imperfect monitoring is it still the case that inattention can be helpful for reputational incentives? It turns out that this is indeed the case. Rather

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<sup>11</sup>Although our baseline model assumes that the current attention state is perfectly observed, the discussion in Section 6.2 suggests that perfect observation is not necessary. The coasting effect would arise even if the firm had some information about the current state.

than present a complete analysis of the imperfect monitoring setting, we develop an example to establish this. In particular we show that no effort can be an equilibrium under full attention when it cannot be an equilibrium with some inattention.<sup>12</sup> In this sense, inattention can be good for incentives.

**Proposition 6.** *There exist parameter ranges in which no effort can be sustained as an equilibrium under full attention, but is not an equilibrium with some inattention.*

The idea is as follows. We start with a range of parameter values such that no effort is sustainable as an equilibrium under full attention ( $\alpha = \iota = 0$ ). We then compare this with a situation in which  $\iota = \alpha \approx 1$ ; i.e., inattention is an absorbing state, and the attentive state transits to the inattentive state almost surely. The firm may not want to shirk: The firm can deviate to working today to increase the reputation and coast on it indefinitely for the future. We provide the formal proof in the Supplementary Appendix.

## 6.2 What if the firm does not observe the current attention state before taking action?

In our setting, when the firm makes an action choice, it knows the current attention state of the customer. Indeed, this drives why the firm never exerts effort in the inattentive state. However, in many applications it may be more reasonable to assume that the firm cannot observe whether the audience will pay attention in the current period but can observe the attention states of the past. As we show in the Supplementary Appendix B.2, the qualitative results are unchanged. The coasting effect still arises in the sense that, there are equilibria with no effort that are sustainable under full attention but not sustainable with some inattention. To see why the timing does not make a difference recall that the coasting effect is primarily driven by some persistence in the attention states. When the firm can predict that there will be a chance for her to shirk in equilibrium in the future, she has an extra incentive to work hard in the present. The timing of actions within a period is not crucial to this intuition.

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<sup>12</sup>It turns out that full effort equilibria (effort at all interior beliefs) do not exist under imperfect monitoring. This is why we demonstrate the coasting effect under imperfect monitoring with no effort equilibria.

### 6.3 What if consumer attention depends on the reputation of firm?

In many applications, it is reasonable to assume that the attention state depends on the current reputation of the firm. For instance, a customer may be unlikely to pay attention when the reputation of the firm is very high or very low, thinking that there is little to learn from observing another outcome. The customer may be much more attentive at intermediate beliefs. Would coasting still arise?

As a final extension, we analyze a model in which the customer's attention does not evolve as a Markov process, but rather depends on the current reputation of the firm. In particular, we assume that the observability state is a function of reputation: The customer pays attention if the current reputation  $\phi_t$  is such that  $\phi_t \in \Phi_A \subset [0, 1]$ , and does not pay attention otherwise. The interested reader may refer to the online Supplementary Appendix for the formal model and analysis. It turns out that, once again, inattention can be helpful for effort incentives. There are equilibria in which full effort can be sustained under inattention, but not under full attention. The coasting intuition is robust: The firm has a stronger incentive to work hard at certain beliefs when she knows that the consumer is likely to become inattentive in the future.

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## A Appendix: Proofs of results

### A.1 Mixed Strategies

#### Proof of Proposition 3

*Proof.* For the firm to work in the attentive for ever more from next period, we require that

$$\delta \left[ 1 - \alpha \frac{1 - \delta - \phi_g}{1 - \iota \delta} \right] > c = \delta \alpha \left( \frac{1}{1 - \delta \iota} \phi_g + \frac{\delta}{1 - \delta \iota} (1 - \iota) V(\phi_g, A) \right) + \delta (1 - \alpha) V(\phi_g, A)$$

If the firm works forever, we have  $V(\phi_g, A) = \frac{(1-c)(1-\iota\delta) + \alpha\delta\phi_g}{(1-\delta)(1+\alpha\delta-\iota\delta)}$ . Substituting this above, we get

$$c = \delta \alpha \left( \frac{1}{1 - \delta \iota} \phi_g + \frac{\delta}{1 - \delta \iota} (1 - \iota) \frac{(1 - c)(1 - \iota \delta) + \alpha \delta \phi_g}{(1 - \delta)(1 + \alpha \delta - \iota \delta)} \right) + \delta (1 - \alpha) \frac{(1 - c)(1 - \iota \delta) + \alpha \delta \phi_g}{(1 - \delta)(1 + \alpha \delta - \iota \delta)},$$

which simplifies to

$$c = \frac{\delta - \alpha \delta - \iota \delta^2 + \alpha \delta^2 + \alpha \delta \phi_g}{1 - \iota \delta}.$$

Recall  $\phi_g = \frac{\phi}{\phi + (1 - \phi)\tau}$ . Substituting into the expression for  $c$ , we get

$$c = \frac{\delta - \alpha \delta - \iota \delta^2 + \alpha \delta^2 + \alpha \delta \left( \frac{\phi}{\phi + (1 - \phi)\tau} \right)}{1 - \iota \delta}.$$

Therefore, we have

$$\tau(\phi) = \frac{\phi}{1 - \phi} \left( \frac{\alpha \delta}{\alpha \delta (1 - \delta) + \iota \delta^2 + c(1 - \iota \delta) - \delta} - 1 \right)$$

For the mixing probability  $\tau$  to be well-defined (i.e.,  $\tau \in (0, 1)$ ), we need

$$\delta \left( 1 - \frac{\alpha(1 - \delta - \phi)}{1 - \iota \delta} \right) < c < \delta \left( 1 + \frac{\delta \alpha}{1 - \iota \delta} \right).$$

Also, for effort forever from the next period to be optimal, we need

$$c \leq \delta \left( 1 - \alpha \frac{1 - \delta - \phi_g}{1 - \iota \delta} \right)$$

It turns out that that this does not pose an additional constraint, because if we substitute

in the value of  $c$  in terms of  $\phi_g$ , we see that the above expression holds at equality.  $\square$

#### Proof of Proposition 4

*Proof.* 1. Suppose that there was an MPE in which the firm mixed in the attentive state and then switched to shirking forever. Mixing requires

$$\phi + (1 - \phi)\tau - c + \delta\alpha V(\phi_g, I) + \delta(1 - \alpha)V(\phi_g, A) = \phi + (1 - \phi)\tau,$$

which implies

$$c = \delta\alpha V(\phi_g, I) + \delta(1 - \alpha)V(\phi_g, A)$$

Since the firm always shirks under inattention, we have

$$V(\phi_t, I) = \frac{1}{1 - \delta\iota}\phi_t + \frac{\delta}{1 - \delta\iota}(1 - \iota)V(\phi_t, A).$$

Therefore, we can rewrite the indifference condition as

$$c = \delta\alpha\left(\frac{1}{1 - \delta\iota}\phi_g + \frac{\delta}{1 - \delta\iota}(1 - \iota)V(\phi_g, A)\right) + \delta(1 - \alpha)V(\phi_g, A).$$

Shirking forever from the next period can be unstained in equilibrium only if  $\frac{\delta}{1 - \delta} < c$ . But note that  $V(\phi_g, A) \leq \frac{1}{1 - \delta}$ . So we need

$$\frac{\delta}{1 - \delta} < \delta\alpha\left(\frac{1}{1 - \delta\iota}\phi_g + \frac{\delta}{1 - \delta\iota}(1 - \iota)\frac{1}{1 - \delta}\right) + \delta(1 - \alpha)\frac{1}{1 - \delta}$$

or

$$0 < \delta\alpha\left(\frac{1}{1 - \delta\iota}\phi_g + \frac{\delta}{1 - \delta\iota}(1 - \iota)\frac{1}{1 - \delta}\right) + \delta(1 - \alpha)\frac{1}{1 - \delta} - \frac{\delta}{1 - \delta} = -\alpha\delta\frac{1 - \phi_g}{1 - \iota\delta}$$

which is a contradiction.

2. Suppose that there was an MPE in which the firm mixed in two consecutive periods. Then, as in the proof of Proposition 4, we must have

$$c = \delta\alpha\left(\frac{1}{1 - \delta\iota}\phi_g + \frac{\delta}{1 - \delta\iota}(1 - \iota)V(\phi_g, A)\right) + \delta(1 - \alpha)V(\phi_g, A)$$

and that

$$c = \delta\alpha\left(\frac{1}{1 - \delta\iota}\phi_{gg} + \frac{\delta}{1 - \delta\iota}(1 - \iota)V(\phi_{gg}, A)\right) + \delta(1 - \alpha)V(\phi_{gg}, A),$$

where  $V(\phi_g, A) = \phi_g$  and  $V(\phi_{gg}, A) \geq \phi_{gg} > \phi_g$

This gives a contradiction since the RHS of second equation is strictly greater than the RHS of the first, but the LHS of both is  $c$ , so it cannot be that both equalities hold simultaneously. □

## A.2 Importance of observability of states

Observability of attention is necessary for the coasting effect. To see this, let us suppose that attention followed a Markov process just as described in the baseline model, but that the firm never observed the state. In this case, even though the states are persistent, the firm cannot condition its action on the current (or past) attention state. In this case, inattention can never be better for incentives. Suppose the state at  $t = 0$  is  $A$ . Then at any period  $t > 0$ , the belief of the firm about the state is given by

$$(0, 1) \begin{pmatrix} \iota & 1 - \iota \\ \alpha & 1 - \alpha \end{pmatrix}^t$$

In the steady-state, beliefs are given by  $\left(\frac{\alpha}{1-\iota+\alpha}, \frac{1-\iota}{1-\iota+\alpha}\right)$ . Conditions for a full effort stationary MPE are given by

$$V(\phi) = \frac{1-c}{1-\delta},$$

and

$$\frac{1-c}{1-\delta} > 1 + \delta \frac{\delta \alpha V(\phi)}{1-\iota+\alpha} \iff c < \delta \frac{1-\iota+\alpha(1-\delta)}{1-\iota+\alpha(1-\delta^2)}.$$

It is clear that inattention makes full effort equilibria harder to sustain (the upper bound is decreasing in both  $\alpha$  and  $\iota$ ). Similarly, for no effort to be an equilibrium we need

$$V(\phi) = \frac{\phi}{1-\delta},$$

and

$$\frac{\phi}{1-\delta} \geq \phi - c + \frac{1-\iota}{1-\iota+\alpha} \frac{\delta}{1-\delta} + \frac{\alpha}{1-\iota+\alpha} \frac{\delta \phi}{1-\delta} \iff c \geq \frac{\delta}{1-\delta} \frac{(1-\iota)(1-\phi)}{1-\iota+\alpha}$$

In this case it is again clear that inattention makes no effort easier to sustain. So, inattention cannot help incentives.

## B Supplementary Appendix

### B.1 Coasting under imperfect monitoring

First, it is convenient to introduce a little more notation to allow for imperfect monitoring. Specifically, let  $\eta := P(y = g|H)$  and  $\nu := P(y = g|L)$ , where  $\eta > \nu$ . Note that the case of  $\eta = 1$  and  $\nu = 0$  represents “perfect monitoring,” that is characterized in the paper.

#### Proof of Proposition 6

*Proof.* As mentioned in the main text, the coasting effect is robust to the introduction of imperfect monitoring. It is easiest to see this if we consider equilibria with no effort. We can show that no effort equilibria may be sustained under full attention but not under inattention. We do not provide a complete analysis here, but rather demonstrate that there exist ranges of parameter values for which the no effort equilibrium can be sustained under full attention but not under inattention.

We provide the proof below. To see the intuition, think of a setting in which  $\iota \rightarrow 1$  and  $\alpha \rightarrow 1$ . This is an environment in which the attentive state is sure to transit to the inattentive state, and moreover the inattentive state is absorbing. A firm in this environment has a strong incentive to work hard in the attentive state, because by exerting effort just once she can improve her reputation and enjoy the benefits of this increased reputation forever. Therefore, no effort is hard to sustain as an equilibrium with this type of inattention, but may be sustainable under full attention.

Under full attention, the value function in a no effort equilibrium is given by  $V(\phi) = \phi\eta + (1 - \phi)\nu + \nu\delta V(\phi_s) + (1 - \nu)\delta V(\phi_f)$ , where  $\phi_s = \frac{\phi\eta}{\phi\eta + (1 - \phi)\nu}$  and  $\phi_f = \frac{\phi(1 - \eta)}{\phi(1 - \eta) + (1 - \phi)(1 - \nu)}$ . Therefore, the condition for no effort to be an equilibrium is given by

$$c > (\eta - \nu)\delta [V(\phi_s) - V(\phi_f)].$$

Consider  $\eta = 1$  and  $\nu = \frac{1}{2}$ . Then,  $\phi_f = 0$  and  $V(0) = \frac{1}{2} \frac{1}{1 - \delta}$  and  $\phi_s = \frac{2\phi}{1 + \phi}$ . Now,  $V(\phi_s) - V(\phi_f) = V(\frac{2\phi}{1 + \phi}) - V(0)$ . Using the expression for the value function above and simplifying we obtain

$$V(\phi_s) - V(\phi_f) = \frac{\phi}{\phi + 1} + \frac{\delta}{2}(V(\phi_{ss}) - V(0))$$

. Note that  $V(\phi_s) - V(\phi_f) = V(\phi_s) - V(0) < (V(\phi_{ss}) - V(0))$ . This follows from the fact that  $\phi_{ss} > \phi_s$  and the monotonicity of  $V$ . This implies  $V(\phi_s) - V(\phi_f) >$

$\frac{\phi}{\phi+1} + \frac{\delta}{2}(V(\phi_s) - V(0)) > \frac{\phi}{\phi+1} \frac{2}{2-\delta}$ . Therefore, a necessary condition for the no effort equilibrium is

$$c > \frac{\delta}{2} \frac{\phi}{\phi+1} \frac{2}{2-\delta}.$$

Now consider the case of inattention (with no effort).

$$V(\phi, I) = \phi + (1-\phi)\frac{1}{2} + \delta\iota V(\phi, I) + \delta(1-\iota)V(\phi, A)$$

$$V(\phi, A) = \phi + (1-\phi)\frac{1}{2} + \delta\alpha\nu V(\phi_s, I) + \delta(1-\alpha)\nu V(\phi_s, A) + \delta\alpha(1-\nu)V(\phi_f) + \delta(1-\alpha)(1-\nu)V(\phi_f)$$

The condition for no effort to be optimal (in the attentive phase) is given by:

$$c > \delta\alpha(\eta-\nu)V(\phi_s, I) + \delta(1-\alpha)(\eta-\nu)V(\phi_s, A) - \delta\alpha(\eta-\nu)V(\phi_f, I) - \delta(1-\alpha)(\eta-\nu)V(\phi_f, A).$$

Again with  $\eta = 1$  and  $\nu = \frac{1}{2}$  we have  $\phi_f = 0$  and  $V(0, I) = V(0, A) = \frac{1}{2} \frac{1}{1-\delta}$ . Further, if we consider the case  $\iota \rightarrow 1$  then we have  $V(\phi, I) \rightarrow \frac{1}{1-\delta}(\phi + (1-\phi)\frac{1}{2})$ . Now let's think about the case  $\alpha \rightarrow 1$  and  $\iota \rightarrow 1$ . Then, the condition for no effort to be optimal becomes

$$c > \delta(\eta-\nu)(V(\phi_s, I) - V(0)) = \frac{\delta}{2} \left[ \frac{1}{1-\delta} \left( \frac{2\phi}{1+\phi} + \left(1 - \frac{2\phi}{1+\phi}\right) \frac{1}{2} \right) - \frac{1}{2} \frac{1}{1-\delta} \right].$$

Let us compare the two conditions for no effort to be optimal under full attention and with inattention respectively. We know that no effort is sustained as an equilibrium under full attention by not under inattention as long as

$$\frac{1}{1-\delta} \left( \frac{2\phi}{1+\phi} + \left(1 - \frac{2\phi}{1+\phi}\right) \frac{1}{2} \right) - \frac{1}{2} \frac{1}{1-\delta} > \frac{\phi}{\phi+1} \frac{2}{2-\delta} \iff \frac{\delta\phi}{(2-\delta)(1-\delta)(1+\phi)} > 0,$$

which is true. □

## B.2 If the firm does not observe the current attention state

Suppose that the firm did not observe the current attention state of the consumer. We can model this by altering the timing of the games as follows:

1. At the start of any period  $t$ ,  $\phi_t \in [0, 1]$  denotes the current belief that firm is a  $C$ -type, and  $\omega_t \in \{A, I\}$  denotes the current attention state.
2. The firm first collects the payment from the consumer  $p_t(\phi_t, \omega_t)$ .
3. Then the firm makes its effort choice,  $a_t \in \{H, L\}$ , and an outcome,  $y_t \in \{g, b\}$ , is realized.

4. An attention state  $\omega_{t+1}$  is realized.
5. The outcome is observed or not by consumers, depending on the current attention state.
6. Consumers update their belief about the firm's type to  $\phi_{t+1}$ .
7. Play proceeds to next period.

(i.e. 4 and 6 are reversed, relative to the baseline model)

**Proposition 7.** *There exist parameter ranges in which no effort can be sustained as an equilibrium under full attention, but is not an equilibrium with some inattention.*

As for Proposition 6, our approach is to demonstrate a range of parameter values in which no effort is an equilibrium under full attention but not under inattention. Let us consider  $\eta = 1$  and  $\nu = \frac{1}{2}$ , so that  $\phi_f = 0$ ,  $\phi_s = \frac{2\phi}{1+\phi}$  and  $\phi_{ss} = \frac{4\phi}{3\phi+1}$ .

The approach is to show that with this monitoring structure there are values of  $\phi$  and  $\delta$  such that the condition for a no effort equilibrium to exist under full attention is easier to satisfy than the condition under inattention. These conditions are respectively:

$$\begin{aligned} c &> \delta(\eta - \nu) [V(\phi_s) - V(\phi_f)], \text{ and} \\ c &> (\eta - \nu)\delta(1 - \alpha) [V(\phi_s, A) - V(\phi_f, A)]. \end{aligned}$$

Thus, it is sufficient to show that there are  $\phi, \delta$  where  $(1 - \alpha) [V(\phi, A) - V(0, A)] > V(\phi) - V(0)$ , to ensure that there exist values of  $c$  such that the first condition is satisfied, and the second is violated. Let us set  $\nu = 1$  and  $\alpha = \frac{1}{2}$ . Then, we have  $V(0) = \frac{\nu}{1-\delta} = \frac{1}{2(1-\delta)}$ .

$$\begin{aligned} V(\phi) - V(0) &= \phi\eta + (1 - \phi)\nu - \nu + \nu\delta V(\phi_s) + (1 - \nu)\delta V(\phi_f) - \delta V(0) \\ &= \frac{\phi}{2} + \frac{\delta}{2} [V(\phi_s) - V(0)] \\ &= \frac{\phi}{2} + \frac{\delta}{2} \frac{\phi_s}{2} + \frac{\delta^2}{4} [V(\phi_{ss}) - V(0)] \\ &= \frac{\phi}{2} + \frac{\delta}{2} \frac{\phi}{1+\phi} + \frac{\delta^2}{4} [V(\phi_{ss}) - V(0)] \end{aligned}$$

The first equality above follows from the definition of  $V(\phi)$ , the second by imposing  $\eta = 1$  and  $\nu = \frac{1}{2}$  and simplifying. The next two are simple substitutions.

Note that  $V(\phi_{ss}) < \frac{1}{1-\delta}$ . It follows that  $V(\phi) - V(0) < \frac{\phi}{2} + \frac{\delta}{2} \frac{\phi}{1+\phi} + \frac{\delta^2}{4} \frac{1}{1-\delta}$ .

Next, note that when  $\iota = 1$  then  $V(\phi, I) = \frac{\nu + \phi(\eta - \nu)}{1 - \delta} = \frac{1 + \phi}{2(1 - \delta)}$  and

$$\begin{aligned}
V(\phi, A) - V(0) &= \frac{1 + \phi}{2} + \delta\alpha V(\phi, I) + \frac{\delta(1 - \alpha)}{2}V(\phi_S, A) + \frac{\delta(1 - \alpha)}{2}V(\phi_f, A) - \frac{1}{2(1 - \delta)} \\
&= \frac{\phi}{2} + \delta\alpha \frac{1 + \phi}{2(1 - \delta)} + \frac{\delta(1 - \alpha)}{2}V(\phi_S, A) + \frac{\delta(1 - \alpha)}{2}V(0) - \frac{\delta}{2(1 - \delta)} \\
&= \frac{\phi}{2} + \delta\alpha \frac{\phi}{2(1 - \delta)} + \frac{\delta(1 - \alpha)}{2} [V(\phi_S, A) - V(0)] \\
&= \frac{\phi}{2} \frac{1 - \delta(1 - \alpha)}{1 - \delta} + \frac{\delta(1 - \alpha)}{2} \left[ \frac{\phi_S}{2} \frac{1 - \delta(1 - \alpha)}{1 - \delta} + \frac{\delta(1 - \alpha)}{2} [V(\phi_{ss}, A) - V(0)] \right] \\
&= \frac{\phi}{2} \frac{1 - \delta(1 - \alpha)}{1 - \delta} + \frac{\delta(1 - \alpha)}{2} \frac{1 - \delta(1 - \alpha)}{1 - \delta} \frac{\phi}{1 + \phi} + \frac{\delta^2(1 - \alpha)^2}{4} [V(\phi_{ss}, A) - V(0)]
\end{aligned}$$

Setting  $\alpha = \frac{1}{2}$  this writes as

$$V(\phi, A) - V(0) = \frac{\phi}{4} \frac{2 - \delta}{1 - \delta} + \frac{\delta}{8} \frac{2 - \delta}{1 - \delta} \frac{\phi}{1 + \phi} + \frac{\delta^2}{16} [V(\phi_{ss}, A) - V(0)] > \frac{\phi}{4} \frac{2 - \delta}{1 - \delta} + \frac{\delta}{8} \frac{2 - \delta}{1 - \delta} \frac{\phi}{1 + \phi}$$

Thus it would suffice to find values of  $\delta$  and  $\phi$  such that  $\frac{\phi}{4} \frac{2 - \delta}{1 - \delta} + \frac{\delta}{8} \frac{2 - \delta}{1 - \delta} \frac{\phi}{1 + \phi} > \frac{\phi}{2} + \frac{\delta}{2} \frac{\phi}{1 + \phi} + \frac{\delta^2}{4} \frac{1}{1 - \delta}$ . It can readily be verified that this is true for a range of values of  $\delta$  and  $\phi$  (satisfied when  $\phi$  is high). For instance,  $\delta = 0.9$  and  $\phi > 0.75$  are such values.

### B.3 Reputation-based Inattention

In this subsection, we consider an alternate model in which the consumer's attention does not evolve as a Markov process, but rather depends on the firm's current reputation: The consumer is inattentive at some beliefs and attentive at others. For instance, think of a setting in which the consumer pays attention only when the reputation is intermediate: there is little to be learnt if the reputation of the firm is already extreme, and so the consumer does not pay attention at extreme beliefs.

The remainder of the setting is as before. The long-run agent or firm is one of two types: a commitment type that always exerts effort and an opportunistic type that can exert high effort at cost  $c$  or else exert low effort at no cost. The consumer has prior  $\phi$  that the firm is the commitment type. As before, the firm's effort choice can result in one of two outcomes  $\{g, b\}$ . The outcome depends on the effort choice and the monitoring structure:  $\eta = \Pr(g|H)$  and  $\nu = \Pr(g|L)$  where  $\eta > \nu$ . Consumers value the good outcome at 1 and the bad outcome at 0. Consumers can be in one of two attention states: Attentive or Inattentive.  $\omega \in \{A, I\}$  When  $\omega = A$ , they observe the outcome, and when  $\omega = I$ , they observe nothing. The attention state is a function

of reputation in particular if  $\phi \in \Phi_A$  the state is  $A$  and if  $\phi \notin \Phi_A$  the state is  $I$

The timing of the game is as follows:

1. At the start of period the state is  $\phi_t$ , the current belief that the firm is a commitment type.
2. The firm first collects the payment from the consumer  $p_t(\phi_t)$
3. The opportunistic firm makes effort choice.
4. Outcomes are observed or not by consumers depending on whether  $\phi_t \in \Phi_A$  or not. What consumers observe depends on the action choice and the current attention state.
5. Consumers update belief about the firm type to  $\phi_{t+1}$
6. Play proceeds to next period.

For the sake of brevity, we do not provide a full analysis of the model. Rather, we provide the conditions for a no effort equilibrium and argue that it is harder to sustain under inattention than under full attention - once again establishing the coasting effect. In this setting, a firm's strategy is given by  $\tau(\phi)$  which is the probability with which a firm plays  $H$  when the customer belief is  $\phi$ . A Markov perfect equilibrium is a tuple  $(\tau(\phi), p(\phi), \phi_g, \phi_b, \phi_I)$  such that:

- At any belief  $\phi$  (and implied attention state), the firm finds it optimal to play  $H$  with probability  $\tau(\phi)$ .
- The customer is willing to pay  $p(\phi)$  which is the expected utility of the consumer given  $\phi$  and the strategy  $\tau(\phi)$ . In particular,

$$p(\phi) = \nu + (\phi + (1 - \phi)\tau(\phi))(\eta - \nu) \quad (3)$$

- The posterior beliefs of customers, after observing outcomes  $g$ ,  $b$  or nothing (in the inattentive state) are calculated using Bayes Rule and given the strategy  $\tau(\phi)$ .

$$\phi_g(\phi) = \frac{\phi\eta}{\phi\eta + (1 - \phi)\tau(\phi)\eta + (1 - \phi)(1 - \tau(\phi))\nu},$$

$$\phi_b(\phi) = \frac{\phi(1 - \eta)}{\phi(1 - \eta) + (1 - \phi)\tau(\phi)(1 - \eta) + (1 - \phi)(1 - \tau(\phi))(1 - \nu)},$$

$$\phi_I(\phi) = \phi$$

Some observations, as in the baseline model, are almost immediate. First, in any MPE the firm never exerts effort at degenerate beliefs. Second, in any MPE the firm never exerts effort in the inattentive state. Finally, with perfect monitoring and pure strategies, inattention cannot help effort incentives.

Notice that once the reputation  $\phi$  is such that  $\phi \notin \Phi_A$ , consumers are no longer updating their beliefs. Therefore, the inattentive state persists forever, and the firm does not exert any effort. Accordingly, we say that in a no-effort equilibrium, we have a “reputation trap.” Trivially, it follows that once you reach  $\phi^N \notin \Phi_A$ , there is no effort and

$$V^N(\phi^N) = \frac{p(\phi)}{1-\delta} = \frac{\nu + (\eta - \nu)\phi}{1-\delta}.$$

Consider the following illustrative example to show that the qualitative insights of the exogenous attention model apply in the reputation-based attention case. Suppose that  $\phi \in \Phi_A$  but  $\phi_g, \phi_f \notin \Phi_A$  where we suppose (for contradiction) that  $\phi_g$  and  $\phi_f$  are defined under the assumption that the firm exerts no effort (that is that  $\tau(\phi) = 0$ ). This is akin to a consumer paying more attention to intermediate rather than extreme reputations. Then the condition for no effort to be an equilibrium is given by

$$\frac{c}{\delta} > (\eta - \nu) [V^N(\phi_g) - V^N(\phi_b)] = (\eta - \nu)^2 \frac{\phi_g - \phi_b}{1 - \delta} \quad (4)$$

It is straightforward to show that there exist parameter ranges, where the above condition not satisfied with reputation based attention, and yet no effort is an equilibrium under full attention. The intuition is familiar: The firm has high stakes to enter an inattentive reputation state with a high reputation.