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PATENT POOLS IN INPUT MARKETS

Abstract

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JEL Classification: K11, L41, M2

Keywords: Complementary Patents, Patent Pools and Joint Marketing Agreements, Vertical Integration and Restraints, FRAND, Antitrust Policy

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Patent Pools in Input Markets*

Markus Reisinger† Emanuele Tarantino‡

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1 Introduction

A patent pool is an agreement among patent owners to license a bundle of patents to each other or to third parties (Quint, 2008). From the 1890s to the 1940s, most of the important manufacturing industries in the U.S. had a patent pooling arrangement (Lerner and Tirole, 2007): From automobiles and movie projectors, to hydraulic pumps and TV equipment. Following a number of unfavourable Antitrust rulings, though, pools essentially vanished between the mid-1950s and the mid-1990s. This changed in 1995, after the release of new guidelines on the licensing of Intellectual Property by the Department of Justice (DOJ) and Federal Trade Commission (FTC). In the years that followed, American authorities approved patent pools tied to major technologies in electronics, information technology and medicine.[1]

A lenient approach to patent pools is in line with the observation that products in these industries build on a large number of patented inputs, which are often owned by several licensors. This forces licensees to "navigate a patent thicket:" A web of overlapping claims that may preclude the commercialization of a new product because licensees need to get patents from multiple sources. The economic literature has shown that patent thickets trigger a problem of horizontal double marginalization: When multiple licensors sell complementary patents, each licensor does not take into account that lowering its price has a positive effect on other licensors' profit, due to an increase in the demand for the bundle. Thus, prices of complementary patents are optimally set by a patent pool (Shapiro, 2001)—a result that develops on Cournot's (1838, 1897) insight that a single monopoly raises welfare when products are perfect complements.[2]

This theoretical literature has been assuming that licensors hold patents on final goods.[3] This is problematic because patent pools are ubiquitous in vertically related industries, where patent owners deal with manufacturers competing on the product market rather than final consumers. Moreover, as we document for the set of pools in information technology in Table 1, and shown by Layne-Farrar and Lerner (2011), vertically integrated firms account for a significant share of patent pool members:[4] Although in some pools none of the licensors is affiliated with a licensee, others feature close to 80% of integrated licensors.

[Table 1 about here.]

[1] A non-exhaustive list includes the MPEG, DVD, Bluetooth, Firewire, 3G-Mobile and laser eye surgery technologies.

[2] Lerner and Tirole (2004) build on the Cournot-Shapiro analysis to determine the policies leading to the break-up of anticompetitive pools, in settings where it is difficult for a court to establish whether technologies are complements or substitutes. Another thorny consequence of patent thickets is the danger that the new product unintentionally infringes on existing patents. Then, a pool of complementary patents limits the risk of infringement litigation (Choi and Gerlach, 2015).

[3] Among others, this assumption has been made by Shapiro (2001), Lerner and Tirole (2004), Rey and Tirole (2013), Quint (2014), Choi and Gerlach (2015), and Boutin (2016).

[4] Table 1 reports the information available on the institutional websites of the companies that manage pools' licensing activities (e.g., price setting and revenue management). It reports the number of licensors and licensees, and the fraction of licensors that is affiliated with a licensee. The information is grouped based on the company that manages a pool's licensing business (for example, the eleven pools in the first group are all served by MPEG4LA).

We propose a theory of patent pools in which constrained monopoly licensors sell their complementary patents on input goods to manufacturers.[5] Our goal is to provide a model that is able to establish the consequences on welfare of the formation of patent pools in vertically related markets. We also give guidance on the policies that are best suited at scrutinizing anticompetitive pools. Following Lerner and Tirole (2004), we consider a policy requiring that pool members keep ownership of their patents—a policy followed by American, European or Japanese authorities—and thus be able to grant individual licenses. We study whether the adoption of this "independent licensing requirement" is able to eliminate the worry that pools reduce consumer welfare.

Another feature of interest in the recent pools approved by American authorities is that licensors are not allowed to discriminate manufacturers when setting their rates. This non-discriminatory policy is often imposed within the enforcement of Fair Reasonable And Non-Discriminatory (FRAND) licensing agreements (see, e.g., Layne-Farrar, Padilla and Schmalensee, 2007; Layne-Farrar, 2010; Gilbert, 2011; Ganglmair, Froeb and Werden, 2012; Lemley and Shapiro, 2013; Layne-Farrar, Llobet and Padilla, 2014; among many others). We provide the conditions under which a strictly non-discriminatory rule eliminates the concerns raised by Antitrust agencies.

To develop our model of pools in vertically related markets, we build on two main theoretical pillars. First, we assume that firms can engage in interlocking vertical relationships. This assumption allows for the formation of an interconnected web of trades between licensors and manufacturers at equilibrium, thus capturing a salient feature of patent thickets. Within the 3G Patent Platform Partnership (the pool on the 3G-Mobile technology), Samsung, LG and Sony, among others, cooperated with vertically specialized licensors like Bosch on the pricing of the bundle of patents, and, at the same time, competed on the product market.

Second, in our model patent owners can use non-linear two-part tariffs to license their patents to manufacturers. This allows us to isolate the impact of the pool's formation on welfare from the role of contractual inefficiencies. As noted by Gilbert (2011), the joint use of fixed fees and linear royalties has desirable efficiency properties, by providing incentives for licensees to produce more and take advantage of declining total costs. As a consequence of this assumption, an equilibrium of our model can feature licensors asking for the payment of a per-unit price and a fixed fee, or a per-unit price only. This is consistent with the available anecdotal evidence: The HDMI Licensing program requires a fixed payment of 10,000USD per year next to a per-unit rate of (up to) 0.15USD.[6] Another example is the pool on the Digital Video Broadcasting-Terrestrial (DVB-T) technology, which only required the payment of a per-unit price for the first generation of the technology, and both a fixed fee and a per-unit price for the second generation.

Our main result is that a patent pool formed by licensors of perfectly complementary input goods is anticompetitive if at least one of the pool's members is integrated with a manufacturer. Instead, the pool does not pose anticompetitive concerns if licensors stand alone. When a licensor is integrated downstream, the pool serves as a coordination device

[5] Specifically, the market power of a monopoly licensor is constrained by the presence of a less efficient status-quo technology.

[6] Similarly, the Wi-Fi Joint Licensing Program asks for a yearly payment of 5,000 Euro and a per-unit price of (up to) 0.30 Euro.

that patent owners use to restrict supplies to the final-good market, at the advantage of the manufacturer(s) affiliated to the pool. In this way, licensors soften product market competition, and then share the larger profit gained by the integrated manufacturer. Before discussing in greater detail our main theoretical mechanism, it is important to point out that our conclusions are robust to the employment of observable and unobservable licensing offers, more general non-linear contracts, the number of firms in the licensing and manufacturing markets, and the degree of final-good differentiation.

These results go against the wisdom, guiding the decisions of major Antitrust agencies, that pools between complementary patents are procompetitive. In what follows, we explain why the formation of a pool allows patent holders to raise larger profits, and reduce consumer welfare, than a situation of independent (or noncoordinated) licensing. The question we ask is: What is it that prevents patent holders to replicate the pool outcome when pricing independently? We distinguish between the analysis with unobservable (or secret) and the one with observable (or public) offers.

Absent the pool, the use of unobservable offers implies that a nonintegrated licensor lacks the commitment to monopolize the product market (e.g., Hart and Tirole, 1990; Rey and Tirole, 2007). Since a manufacturer does not observe the offer made to rival manufacturers, when the licensor contracts with each manufacturer it acts as if the two are integrated. This pairwise maximization problem entails a bilateral efficient contract, in which the unit price is equal to the licensor's marginal cost.[7] The same outcome occurs when nonintegrated licensors form a patent pool. Thus, if none of the pool members is integrated downstream, the pool is welfare neutral (Proposition 1). Instead, if a pool member is vertically integrated, the pool is a way for licensors to achieve monopolization, and is therefore anticompetitive (Proposition 2). Specifically, once in the pool, the stand-alone licensor follows a foreclosure strategy at the expense of independent manufacturers, and then shares with the other pool members the gains of the affiliated manufacturer. An analogous result arises if each licensor is integrated with a manufacturer, as in this case the pool can implement the unconstrained monopoly outcome (Proposition 3).

The assumption of secret contracts reflects the lack of systematic empirical evidence on patent pricing. However, there are several examples of pools that, in an effort to make contract terms transparent, publish their licensing offer on institutional websites.[8] Then, we ask: How does observability of offers change our results? The main difference with respect to the setting with secret offers is that, with public offers, a manufacturer reacts to a change in the unit price offered by a licensor to rival manufacturers. This means that, when formulating the unit-price offer in the tariff, each licensor takes into account the impact of this decision on the value of the relationships with all manufacturers. Observability then allows licensors to commit to raising unit prices above marginal cost: On the one hand, this allows them to soften product market competition. On the other hand, it brings back the problem of horizontal double marginalization.

With public offers, we find that the pool is procompetitive if none of the licensors is vertically integrated (Proposition 4): The pool allows for improved pricing coordination among

[7] As remarked by Rey and Tirole (2007), this outcome is related to the phenomenon behind the Coasian conjecture on the pricing policy of a durable good monopolist that lacks commitment to future prices.

[8] Yet, many other programs, like the pools on the MPEG-Audio, LBS, ATSS, WSS, TOP Teletext, DECT, HEVC and Telemetry technologies, keep their licensing terms confidential.

licensors, which leads to a reduction in the unit prices paid by manufacturers. Although this conclusion is in line with the Cournot-Shapiro argument, the mechanism is different. With input goods, licensors lower unit prices to reduce manufacturers' bargaining threat, thereby allowing them to demand a larger fixed component of the tariff. If instead a licensor is integrated with a manufacturer, the pool is again anticompetitive (Proposition 5): In this case, improved licensors' coordination means that, once in the pool, patent owners jointly raise their unit price to the independent manufacturers, so to shift output at the advantage of the licensor's downstream affiliate. Although the incentive to lower unit prices to reduce manufacturers' bargaining threat is still present, it is dominated by the possibility to share the larger profit of the affiliated manufacturer. Therefore, the pool is anticompetitive with vertical integration.

We then focus on policies that solve the anticompetitive result. First, we find that, in our setting, the "independent licensing requirement" is not sufficient to break anticompetitive patent pools. Instead, authorities should require that pool members set their licenses independently and unbundle the claims on their patents from the pool (the "unbundling and pass-through requirement"). Essentially, this alternative requirement boils down to adding a ban on monetary transfers among pool members to the independent licensing prescription. This result is in line with the literature that shows that the "independent licensing requirement" is not sufficient if pools include more than two patents (Boutin, 2016), or pool members engage in tacit collusion (Rey and Tirole, 2013).

Finally, we analyze the effects of the pool on welfare under a mandated non-discriminatory policy enforcing FRAND commitments. A ban on discrimination eliminates the pool's incentive to privilege integrated manufacturer(s). Moreover, it preserves the pool's merit in coordinating prices, which leads to lower unit prices. We find that a non-discriminatory policy yields an unambiguously procompetitive outcome if some of the licensors stand-alone. Instead, pools can still be anticompetitive if all licensors are integrated with a manufacturer. As remarked above, this happens because, in this case, the pool allows the licensors to implement the unconstrained monopoly outcome.

The empirical literature provides evidence showing that pools may limit innovation (Lampe and Moser, 2016) and that, absent regulation, pools may refuse to license their technologies (Lampe and Moser, 2014). There is also case evidence on pool members' use of provisions restricting downstream competition (Gilbert, 2004), and on the practice of granting affiliated manufacturers with a privileged access to the pool's patent bundle (Harris, 2003; Flamm, 2013).[9] Our theory is the first that can explain why coordinated pricing decisions and the use of vertical restraints harm welfare, even within pools selling complementary patents.

The other two papers considering the role of pools in vertically related industries are Kim (2004) and Schmidt (2014). Kim (2004) employs a model featuring unconstrained monopolists selling complementary patents via public linear tariffs. His main result is that

[9] Remarkably, via the decisions that halted the formation of patent pools in the mid-1950s, the American Supreme Court resolution was to allow only those pools that "contained no restrictions as the quantity of goods to be produced, or the price to be charged, or the territory in which they might be sold by the licensee" (*Baker-Cammack Hosiery Mills v. Davis*).

pools raise welfare by eliminating double marginalization.[10] Schmidt (2014) extends the analysis in Kim (2004) to a more general setting: His main take-away remains that pools perform an important welfare increasing function.[11]

We also contribute to the literature on vertical integration and restraints. Most of this literature has looked at stylized market structures, where upstream firms offer perfectly substitute products, or deal with manufacturers in conditions of exclusivity. Two exceptions are Rey and Vergé (2010) and Nocke and Rey (2014), who consider settings featuring non-linear tariffs and multiple interlocking bilateral relationships. While these papers propose models in which suppliers offer imperfectly substitute intermediate goods, our contribution is to study the case with complementary inputs.[12]

2 The Model

There are two patented inputs A and B, each produced by a licensor L_j, $j = A, B$. Licensors supply two manufacturers, M_1 and M_2, that are Cournot rivals in a downstream market.[13] Inputs A and B are in a relationship of perfect complementarity. Thus, manufacturers need a unit of each input to produce one unit of the homogeneous final product.[14] Licensors are symmetric and each one bears cost c to produce one unit of its input.[15]

When contracting with manufacturer M_i, $i = 1, 2$, each licensor L_j, $j = A, B$, makes a take-it-or-leave-it offer of a two-part tariff contract consisting of a fixed component, F_{ji}, and a unit price, w_{ji}. If it accepts, manufacturer M_i's total marginal cost is $w_{Ai} + w_{Bi}$. If a manufacturer rejects the offer of a licensor, then it can use the status quo (nonpatented) technology and bear a marginal cost of production of $\hat{c}_j = \hat{c} > c$ on the input produced by j, with $j = A, B$. We use $e = \hat{c} - c$ to capture the degree of patent's essentiality.[16]

Absent a patent pool, either firms are not integrated (vertical separation) or a licensor L_j is integrated with one of the two manufacturers (vertical integration). Given vertical separation, the game proceeds as follows:

[10] Gallini (2014) studies the interplay between patent pools and technology standards in a model with exclusive vertical chains. She finds that the procompetitive result in Kim (2004) gets reverted when accounting for the role of pools in coordinating standard adoption.

[11] There are two more complementary literatures that we touch upon. The first studies the role of pools in settings with essential patents (Quint, 2014), and how a policy of price commitments can prevent licensors from setting unreasonable royalties in standard setting (Llanes and Poblete, 2014; Lerner and Tirole, 2015). The second analyzes the incentives to enter cross-licensing agreements (Galasso, 2012) and the relationship between pools and cross-licensing (Choi, 2010). However, these literatures do not study the impact vertical integration and restraints on welfare.

[12] Other models analyze the role of input complementarity within vertically related markets in settings with inefficient bargaining or specialized chains (see, among others, Laussel, 2008; Laussel and Van Long, 2012; Matsushima and Mizuno, 2012; Hermalin and Katz, 2013).

[13] In Section 5.1, we show that our results extend beyond the case of this simple two-by-two industry.

[14] In Section 5.2, we explain why our results are robust to the assumption of downstream differentiated products.

[15] The focus on symmetric licensors allows us to considerably ease the exposition and is without loss of generality. Indeed, the analysis with asymmetric upstream firms produces qualitatively analogous results (Reisinger and Tarantino, 2013).

[16] In Section 5.3, we show that the use of quantity-forcing contracts raises a problem of equilibrium multiplicity, without changing our conclusions on the competitive consequences of the pool.

1. Each licensor L_j secretly offers to each manufacturer M_i a two-part tariff $\{w_{ji}, F_{ji}\} \equiv T_{ji}$. Manufacturers simultaneously and secretely accept or reject the contract offer.

2. Manufacturers order a quantity of the input goods and pay the tariff. Then, they transform the inputs into the final good and bring output to the market.

Afterwards, purchases of final consumers are made, and profits are realized.

The second stage of the game implies that manufacturers are Cournot competitors. We denote by q_i, $i = 1, 2$, the quantity sold by each manufacturer M_i, so that aggregate output is given by $Q = q_1 + q_2$. The (inverse) demand function for the final good is $p = P(Q)$. It is strictly decreasing and twice continuously differentiable whenever $P(Q) > 0$. Moreover, we employ the standard assumption that $P'(Q) + QP''(Q) < 0$, which guarantees that the profit functions are (strictly) quasi-concave and that the Cournot game exhibits strategic substitutability (Vives, 1999). This ensures concavity of the licensors' profit functions.

We solve for the perfect Bayesian Nash equilibrium that satisfies the standard "passive beliefs" refinement (Hart and Tirole, 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Tirole, 2007; Arya and Mittendorf, 2011). With passive beliefs, a manufacturer's conjecture about the contract offered to the rival is not influenced by an out-of-equilibrium contract offer it receives. This is a natural restriction on the potential equilibria of a game with secret offers and supply to order because, from the perspective of the upstream licensors, under these two assumptions manufacturers M_1 and M_2 form two separate markets (Rey and Tirole, 2007).[17]

With vertical integration, the licensor and its downstream affiliate maximize joint profits. The game proceeds as laid out above, with the exception that, as is natural and in line with Hart and Tirole (1990) and Rey and Tirole (2007), the downstream affiliate M_i of the integrated firm L_j-M_i knows the terms of its upstream unit's offer to the rival manufacturer M_{-i}. We also assume that the downstream affiliate knows the acceptance decision of its rival.[18]

Our modelling strategy borrows from Lerner and Tirole (2004) the focus on all-or-nothing pools, which include all the licensors in the industry, and the assumptions that (i) the pool forms at equilibrium if it raises total profits as compared to independent licensing, and that (ii), if licensors join in a pool, they maximize and then share joint profits.[19] Specifically, the pool sets secret two-part tariffs T_{pi} that a manufacturer M_i outside the pool needs to pay to obtain the right to use the bundle of patented inputs. Otherwise, the manufacturer will have to resort to the status-quo technologies and pay $2\hat{c}$ to produce.

[17] All our results hold true under the alternative assumption that retailers hold wary beliefs (McAfee and Schwartz, 1994; Rey and Vergé, 2004). Moreover, we show in Section 6 that our results survive when assuming observable contracts.

[18] We also assume that, were the rival manufacturer M_{-i} to reject the offer of L_j, the integrated manufacturer M_i believes that the rival is still accepting the offer of L_{-j}, as it does at equilibrium. We show in Online Appendix B that our results are robust to the use of alternative formulations of this assumption.

[19] The assumption is that there are no failures in coordination or bargaining among pool members (Rysman and Simcoe, 2008; Gallini, 2014).

We denote by q_i^m the monopoly quantity produced by manufacturer M_i when it alone obtains the inputs at marginal cost (so that $w_{ji} = c$ for $j = A, B$),

$$q_i^m \equiv \arg\max_q \; (P(q) - 2c)q,$$

whereas π_i^m denotes M_i's monopoly profit when producing q_i^m:

$$\pi_i^m \equiv \max_q \; (P(q) - 2c)q.$$

The Cournot equilibrium is the solution to the system

$$q_1 = \arg\max_q \; (P(q + q_2) - w_{A1} - w_{B1})q \quad \text{and} \quad q_2 = \arg\max_q \; (P(q_1 + q) - w_{A2} - w_{B2})q. \tag{1}$$

If m_i is the sum of the margins paid by manufacturer M_i on inputs A and B, with $m_i = m_{Ai} + m_{Bi}$ and $m_{ji} = w_{ji} - c$, then we denote by $q_1(m_1, m_2)$ and $q_2(m_2, m_1)$ the Cournot-Nash equilibrium solution of (1), which is unique given the assumed properties of inverse demand. This solution depends on the margin paid by M_i, given the margin paid by its rival M_{-i}. For convenience, we denote $q_i(0, 0)$ by q^c, then

$$\pi^c \;\equiv\; (P(2q^c) - 2c)\, q^c \tag{2}$$

is M_i's equilibrium profit when both manufacturers produce (q^c). More generally, we use $\pi_i(m_i, m_{-i})$, with $i = 1, 2$, for the Cournot-Nash equilibrium profit when M_i pays margins of m_i and its rival M_{-i} pays m_{-i}.

Finally, $\overline{\pi}_i$ denotes the profit of manufacturer M_i that pays $c + \hat{c}$ to buy patents, given that its rival produces the Cournot quantity $q_{-i}(e, 0)$. Instead, $\overline{\overline{\pi}}_i$ denotes the profit of manufacturer M_i that pays $2\hat{c}$ to buy the patents, given that its rival produces the Cournot quantity $q_{-i}(0, e)$. Hence,

$$\overline{\pi}_i \;\equiv\; \max_q \{(P(q + q_{-i}(e, 0)) - c - \hat{c})\, q\} \tag{3}$$

and

$$\overline{\overline{\pi}}_i \;\equiv\; \max_q \{(P(q + q_{-i}(0, e)) - 2\hat{c})\, q\}. \tag{4}$$

Throughout the analysis, we assume that the status quo technology is effective in constraining the market power of L_j, $j = A, B$, i.e., e is low enough. This implies that the quantity of M_i is positive despite using the nonpatented technologies ($q_i(2e, 0) > 0$). Moreover, to establish the consequences of the pool on consumer welfare, we employ the following property of Cournot games:

PROPERTY 1. *The sum of the first-order conditions of M_1 and M_2 is given by:*

$$2P(Q) + P'(Q)Q = w_{A1} + w_{A2} + w_{B1} + w_{B2}. \tag{5}$$

Therefore, the industry quantity (Q) is determined by the sum of the unit prices paid by manufacturers.

Since the left-hand side of (5) is decreasing in Q, by Property 1, to prove that a pool yields an anticompetitive outcome, it is enough to show that it rises the sum of licensors' unit prices. Proofs of the results obtained under the assumption of unobservable contracts can be found in Appendix A.[20]

3 Contracting with Unobservable Contracts

In this section, we present the equilibrium analysis under the assumption of secret offers. We first consider an industry featuring no vertically integrated firm, then the one with a single integrated firm and, finally, the one with two integrated firms.

3.1 Vertical Separation

Assume first that neither manufacturer is vertically integrated. We will show that, within the vertically separated industry, the pool is welfare neutral. This result follows from the assumption of contract unobservability: As we will show, this means that a necessary condition for the pool to raise anticompetitive concerns is that (at least) one of its members is vertically integrated.

3.1.1 Independent Licensing by L_A and L_B

Consider first the case in which licensors L_A and L_B set their tariffs noncooperatively.

LEMMA 1. *In a symmetric equilibrium, each independent licensor (L_j) offers each manufacturer a two-part tariff with the unit price equal to L_j's marginal cost (c) and the fixed component equal to half of licensors' joint contribution to the manufacturer's resulting profit on the product market; that is, the equilibrium tariff offered by licensor L_j to manufacturer M_i is $T = \{c, (\pi^c - \max_q\{(P(q + q^c) - 2\hat{c})q\})/2\}$.*

With secret offers, L_A and L_B lack the commitment to monopolize the product market. (Hart and Tirole, 1990; Rey and Tirole, 2007). Due to contracts' unobservability, a manufacturer's decisions cannot change if L_j deviates in its offer to the rival manufacturer. Therefore, when the monopoly licensor contracts with each manufacturer, it acts as if the two are integrated. This pairwise maximization problem requires that the contractual arrangements between L_j and M_i maximize bilateral profits. This entails a unit price equal to the monopoly licensor's marginal cost (c).[21]

As far as the fixed component of the tariff is concerned, in the symmetric equilibrium licensors equally share the difference between a manufacturer's Cournot profit and the profit that the same manufacturer obtains when rejecting both licensors' offers. Indeed, were each

[20] All the remaining proofs are in Online Appendixes B and C.

[21] As we demonstrate later, the equilibrium allocations differ with public offers. In that case, a manufacturer reacts to a change in the contract offered by L_j to the rival manufacturer. The equilibrium per-unit price of an independent licensor is then above marginal cost.

licensor L_j to offer a value of the fixed component equal to its contribution to the profit of the manufacturer (and equal to $\pi^c - \max_q\{(P(q+q^c) - c - \hat{c})q\}$), then the manufacturer would find it optimal to reject both offers and resort to the status quo technologies.

Although we focus on the symmetric equilibrium, a continuum of asymmetric equilibria exist in which each licensor sets a per-unit price equal to its marginal cost (c) and a value of the fixed component such that $F_{Ai} + F_{Bi} = \pi^c - \max_q\{(P(q+q^c) - 2\hat{c})q\}$. Therefore, the product market equilibrium allocation in all these asymmetric equilibria coincide with the one implied by the symmetric equilibrium. In this respect, our restriction to the symmetric equilibrium outcome is without loss of generality.

3.1.2 Patent Pool between L_A and L_B

If L_A and L_B join in a pool, they set licensing contracts cooperatively. The following lemma gives the equilibrium results in this case.[22]

LEMMA 2. *In equilibrium, the pool charges a two-part tariff with the unit price equal to* ($2c$) *and the fixed component equal to licensors' joint contribution to each manufacturer's product market profit; that is, the equilibrium tariff offered by the pool to manufacturer* M_i *is* $T_p = \{c, \pi^c - \max_q\{(P(q+q^c) - 2\hat{c})q\}\}$.

The result in the lemma follows from the simple observation that the pool faces the same commitment problem as the one of a stand-alone licensor. The immediate consequence of this result is that a pool formed by independent licensors is welfare neutral, as it does not change the sum of licensors' unit prices with respect to independent licensing.

PROPOSITION 1. *The pool formed by independent licensors* L_A *and* L_B *is welfare neutral.*

The proposition shows that the standard argument on the procompetitive effects of patent pools does not hold true if licensors offer input goods and contracts are bilaterally efficient. As we will see in Section 6, it applies when contracts are observable instead. In what follows, we will take the vertically separated industry as our benchmark framework, and analyze how results change in an industry featuring the presence of integrated firm(s).

3.2 Single Integration

Assume that L_A is integrated with M_1, and L_B and M_2 remain independent. We first analyze the framework without pool and then proceed studying how the pool changes patent pricing.

3.2.1 Independent Licensing by L_A-M_1 and L_B

Absent the pool, licensors L_A-M_1 and L_B market their patents independently.

LEMMA 3. *In equilibrium, the independent licensor* (L_B) *offers each manufacturer* M_i *a two-part tariff with the unit price equal to its marginal cost* (c)*. Instead, the integrated licensor* (L_A-M_1) *offers to manufacturer* M_2 *a unit price equal to* $\hat{c}$ *and sets the internal transfer price equal to marginal cost. Accordingly, the integrated licensor's fixed component*

[22] The proof follows the same lines as the proof of Lemma 1, and is therefore omitted.

is equal to zero, and L_B's fixed components are equal to its contribution to the manufacturers' profit in the ensuing Cournot competition. Thus, $T_{A2} = \{\hat{c}, 0\}$, $T_{B1} = \{c, \pi_1(0, e) - \overline{\pi}_1\}$ and $T_{B2} = \left\{c, \pi_2(e, 0) - \overline{\overline{\pi}}_2\right\}$.

The intuition to the first part of the lemma is analogous to the one developed below Lemma 1, and implies that both licensors offer to each M_i the bilateral efficient contract featuring $w_{ji} = c$. As for the second part of the lemma, it follows from the simple intuition that, different from L_B, L_A internalizes the effect selling to the rival manufacturer has on the profits made by its affiliate. Therefore, the temptation of opportunism vanishes and L_A can credibly commit to reducing supplies to the rival manufacturer by setting a unit price equal to $\hat{c}$. This is the marginal cost that a manufacturer pays when using the status quo and nonpatented technology.

3.2.2 Patent Pool between L_A-M_1 and L_B

Assume now that L_A-M_1 and L_B join in a patent pool. We proceed as follows: First, we show that firms have incentive to form the pool and characterize the equilibrium contract that the pool offers to M_2. Then, we compare the outcome of a pool in which members set a tariff on the bundle of the technologies with that in the absence of a pool.

LEMMA 4. *In equilibrium, L_A-M_1 and L_B join the pool. The pool sets the internal transfer price equal to marginal cost and makes to manufacturer M_2 the following offer: $T_{p2} = \{2\hat{c}, 0\}$.*

With the pool, the independent manufacturer M_2 pays $2\hat{c}$ for the bundle of technologies. The reason is that, once it joins the pool, L_B behaves as if it is integrated with L_A-M_1. Therefore, it will give M_1 a privileged access to its patent. This strategy leads to an increase of the joint profits of L_B and L_A-M_1, so that the pool will always form at equilibrium.

PROPOSITION 2. *With independent licensing, industry output is equal to $Q^{il} = q_1(0, e) + q_2(e, 0)$, it is equal to $Q^p = q_1(0, 2e) + q_2(2e, 0) < Q^{il}$ with the pool. Thus, the pool between L_B and L_A-M_1 is anticompetitive.*

The claim in the proposition directly follows from Property 1: Since it increases the sum of unit prices paid by M_1 and M_2, the pool implies a reduction of industry quantity and, thus, consumer welfare. This result goes against the wisdom, guiding the decisions of all major Antitrust agencies, that pools between complementary patents are procompetitive (see, among others, Shapiro, 2001; Lerner and Tirole, 2004). Lemma 4 shows that when the pool includes patents on input goods, and one of the pool's members is integrated downstream, the pool acts to monopolize the final good market. Proposition 2 shows that this equilibrium outcome causes a reduction of consumer welfare with respect to an industry with independent licensing.

In our framework, licensors of perfectly complementary technologies use the pool to restrict supplies to manufacturers and soften product-market competition. With secret offers, this happens because, absent the pool, a non-integrated licensor L_B lacks the commitment to monopolize the product market. The pool is a way for L_B to achieve monopolization. Indeed, once in the pool, L_B undertakes a foreclosure strategy at the expense of M_2 and then shares with L_A-M_1 the higher profits gained by M_1. We will show in Section 6 that this result arises even with observable contracts.

To further the analysis of the relationship between industry structure and pool's pricing decisions, we show below that the pool is anticompetitive also in an industry featuring pairwise integration.

3.3 Pairwise Integration

Assume now that L_A is integrated with M_1, and L_B is integrated with M_2. The results in the previous section suggest that, with independent licensing, integrated licensors will raise the unit price on their own patented input to $\hat{c}$ and foreclose the rival manufacturer. As we will see, this does not necessarily happens at equilibrium. In what follows, we first analyze the framework without pool and then proceed studying how the pool changes patent pricing.

3.3.1 Independent Licensing by L_A-M_1 and L_B-M_2

LEMMA 5. *In the unique symmetric equilibrium, each integrated licensor offers a contract consisting of a per-unit price equal to* $w = c - P'(Q)q$*, where* $Q = 2q(w-c, w-c)$*, if* $P(Q)/2 \le \hat{c}$*, and* $w = \hat{c}$ *if* $P(Q)/2 > \hat{c}$*. The associated fixed component of the tariff is*

$$\begin{aligned} F &= [P(2q(w-c,w-c)) - c - w]q(w-c,w-c) \\ &\quad -[P(q(e,w-c)+q(w-c,e)) - c - \hat{c}]q(e,w-c) \\ &\quad -(w-c)[q(w-c,e) - q(w-c,w-c)], \end{aligned} \tag{6}$$

if $P(Q)/2 \le \hat{c}$*, and* $F = 0$ *if* $P(Q)/2 > \hat{c}$*.*

The lemma shows that in an industry featuring pairwise integration, integrated firms can find it optimal to depart from a foreclosure strategy. The intuition behind this result is as follows: Since an integrated firm L_A-M_1 sets the internal transfer price equal to marginal cost, M_1 receives input A at a lower per-unit price than M_2 (and, likewise, M_2 receives input B at a lower per-unit price than M_1.) This implies that each manufacturer is more efficient than the rival with respect to the input of its integrated licensor. L_A then faces a trade-off when increasing the unit price to the rival manufacturer M_2. By raising the per-unit price the licensor softens competition. However, from a cost perspective, L_A benefits if the quantity produced by the manufacturer that is more efficient on input B increases, at the expense of M_1.[23] How can the licensor achieve this, given that it cannot commit to restricting its downstream affiliate's quantity? As, with vertical integration, M_1 observes L_A's offer to M_2, L_A needs to lower the unit price w_{A2}: In this way, M_2's quantity increases and M_1 responds by reducing its output. L_A then extracts part of the larger profit of M_2 via the fixed component of the tariff.

The fixed component of the tariff consists of two main parts. First, the contribution of a licensor L_A to manufacturer M_2's profit (which is given by the first two terms in (6)). The second part reflects the following mechanism. When setting the fixed component of

[23] This effect is similar to the output shifting effect of vertical integration, which arises if downstream firms bear different costs of production (Reisinger and Tarantino, 2015).

the tariff, a licensor L_A needs to take into account that, as it increases the value of F_{A2}, M_2's temptation to reject T_{A2} rises. With pairwise integration, this rejection induces the downstream unit of L_A (M_1) to expand its quantity, because M_1 is informed of M_2's rejection. At the same time, the expansion of q_1 allows L_B-M_2 to raise the per-unit revenue it obtains from M_1. Overall, this mechanism makes rejecting a licensor's offer more profitable for an integrated rival manufacturer rather than for an independent rival manufacturer.[24]

The equilibrium value of the fixed component of the tariff implies that each licensor takes into account the impact of changing the unit price on both manufacturers' profits and both licensors' per-unit margins. As a consequence, each integrated firm acts "as if" it maximizes the industry profit, under the constraint that it cannot set a unit price above $\hat{c}$ because of the presence of the status-quo technologies. This consideration is crucial to assess the competitive consequences of the pool in this setting.

Although there is a unique symmetric equilibrium, as it is standard in frameworks with complementary inputs a continuum of asymmetric equilibria exist in the first stage of the game. In all these equilibria the sum of the unit prices set by the licensors is the same and equal to $w_{A1} + w_{B2} = 2c - P'(Q)Q$.[25] By Property 1 these equilibria all give rise to the same aggregate quantity Q; thus, our focus on the symmetric equilibrium simplifies the exposition without loss of generality.[26]

3.3.2 Patent Pool between L_A-M_1 and L_B-M_2

In what follow, we first show that L_A-M_1 and L_B-M_2 form the pool and characterize the tariff set by the pool in equilibrium. Then, we compare the market allocation with the pool to the one without the pool.

LEMMA 6. *In equilibrium, L_A-M_1 and L_B-M_2 join the pool. The pool sets the internal transfer price equal to $\overline{w}$, where $\overline{w}$ is implicitly defined by*

$$\frac{q^m}{2} = \arg\max_q \left(P(q + q(2(\overline{w} - c), 2(\overline{w} - c))) - 2\overline{w}\right)q. \tag{7}$$

In contrast to the case with single integration, the licensors in the pool do not set the internal transfer price equal to marginal cost. The intuition is that licensors cannot control the quantities of their downstream units directly, but only via the inputs' per-unit prices. The reason is that, at the quantity-setting stage of the game, each manufacturer maximizes its profit. By contrast, licensors within a pool seek to maximize their joint profit. To achieve this outcome, they set unit prices above marginal cost so that each manufacturer sells half

[24]The results in the proposition do not rely on the assumption of contract secrecy: Indeed, each manufacturer is integrated with a licensor, it receives the offer of the only other licensor, and is informed by its upstream affiliate of the offer formulated to the rival manufacturer.

[25]Technically, this happens because the derivative of the per-unit price best-response function, dw_{-ji}/dw_{j-i}, is equal to -1.

[26]To further support the selection of the symmetric equilibrium, note that, in the presence of (even an arbitrarily small degree of) differentiation between manufacturers, a unique equilibrium exists in which licensors charge symmetric unit prices. Thus, in the limiting case in which product differentiation vanishes, the equilibrium in Lemma 5 will be selected.

of the monopoly quantity. This strategy yields the monopoly profit, which means that the integrated licensors will join the pool in equilibrium.

PROPOSITION 3. *If $\hat{c}$ is sufficiently small, industry output with independent licensing is $Q^{il} = 2q(e,e)$, whereas it is equal to $Q^p = q^m < Q^{il}$ with the pool. Thus, the pool formed by L_A-M_1 and L_B-M_2 is anticompetitive. Instead, if $\hat{c}$ is sufficiently large, the industry output is the same with independent licensing and with the pool. Hence, the pool is welfare neutral in this case.*

This proposition shows that a patent pool between licensors of complementary inputs is not welfare improving. Instead, it reduces industry output if the status-quo technology is relatively efficient. As in Section 3.2.2, the intuition is that the pool acts as a coordination device. Without the pool, both integrated licensors would like to have manufacturers jointly producing the unconstrained monopoly quantity. They cannot achieve this if the status-quo technology is sufficiently efficient. The reason is that each manufacturer only cares about its own profit. Thus, if the licensor of an input sets its per-unit price above $\hat{c}$, the manufacturer reverts to the status-quo technology. After the pool forms, its members ignore the status-quo technologies, as each manufacturer also cares about the effect of its action on the profit of the rival manufacturer. Finally, were the status-quo technology relatively inefficient, independent licensors are able to set their per-unit prices to replicate the unconstrained monopoly outcome. In this case, the pool is welfare neutral—the reason is that, as discussed after Lemma 5, each independent licensor acts as if it maximizes industry profits.

To clarify the relevance of the neutrality result, we provide two further results: First, it is easy to show that the pool is anticompetitive if the demand function is linear. In that case, licensors always set $w_{A1} = w_{B2} = \hat{c}$ under independent licensing. Second, the neutrality result is due to the information transmission protocol between integrated units. As we show in Online Appendix B, the pool is always anticompetitive under the assumption that an integrated manufacturer is not informed about the decision taken by the rival on the offer made by its upstream unit.

This section closes the analysis of the competitive consequences of the pool. In the next section, we first summarize the implications of these results for public policy formulation and then provide a screening device of procompetitive patent pools.

4 Unbundling and Pass-through as Screening Device

The equilibrium analysis in Section 3 shows a patent pool involving licensors of input goods is anticompetitive if at least one of the licensors is integrated with a manufacturer (Propositions 2 and 3). Instead, the pool is welfare neutral if its members are vertically specialized (Proposition 1). The intuition that we develop throughout the analysis is that, when a licensor is integrated downstream, the pool serves as coordination device: It allows licensors to soften product market competition and raise the profits of the affiliated manufacturer(s). These results stand in sharp contrast to the message arising from the theoretical literature studying pools between firms selling final goods (see, e.g., Shapiro, 2001; Lerner and Tirole, 2004; Rey and Tirole, 2013). In this literature, a patent pool is anticompetitive if its members offer substitute final products. Instead, pools are procompetitive whenever their products are complementary. In what follows, we discuss the implications of our results for public policy formulation.

To break anticompetitive pools, Lerner and Tirole (2004) suggest that pool-members must also be allowed to license their patents independently ("independent licensing requirement"). As shown by Boutin (2016), this requirement suffices when pools offer two patents, however it is not enough to break larger pools.[27] We now demonstrate that, in our model with input goods, the independent licensing requirement is insufficient even though the pool offers only two patents.

Under the independent licensing requirement, the game unfolds as follows: In the first stage the pool sets its tariffs and, in the second stage (or continuation stage), licensors simultaneously and non-cooperatively offer their individual contracts to the manufacturers. Manufacturers then can choose among not buying at all, buying the package from the pool, or buying individual patents from L_A and L_B. We denote the share that L_A(-M_1) obtains from the pool's profit by α and L_B's share by $1-\alpha$. Let us consider the case of single integration: By Lemma 4, the pool offers $T_{p2} = \{2\hat{c}, 0\}$ to M_2 and transfers the patents internally at marginal cost. The value of α is set in such a way that licensors obtain a (weakly) higher profit than with independent licensing, so that they are willing to form the pool.

Since it is integrated with M_1, at the continuation stage, L_A's dominant strategy features setting $w_{A2} = \hat{c}$. This means that $L_A - M_1$ cannot raise larger profits than in the pool, independently of what L_B offers to M_2 in the continuation stage. This is not necessarily true for L_B. Based on the analysis in Lemma 3, one would expect that L_B's best response at the continuation stage is to offer $w_{Bi} = c$, with $i = 1, 2$, and the respective value of tariff's fixed component. However, as we show in Online Appendix B, if $\hat{c}$ is sufficiently large, L_B has no incentive to deviate from the pool pricing equilibrium outcome, and thus sets $w_{B2} = \hat{c}$ and $w_{B1} = c$. The reason is that its profit from the pool is larger than the one it can raise by licensing non-cooperatively. As a consequence, the pool is anticompetitive and stable under the independent licensing requirement.[28]

This shows that the independent licensing requirement is an imperfect screening device of welfare-enhancing pools. The reason is that pool members are free to share the pool's profit to ensure firms' participation. To get around this problem, Rey and Tirole (2013) show that an alternative requirement, featuring unbundling and pass-through, is able to break anticompetitive pools in their setting. This policy recommendation is valid in our framework, too. Specifically, under the "unbundling and pass-through requirement:"

1. The pool sets the nonlinear tariffs T^p_{ji} at which manufacturers can acquire individual patents from the pool (instead of nonlinear tariffs for the bundle of patents T^p_i). In addition, manufacturers can acquire patents directly from L_A and L_B.

2. A pool member's profit corresponds to the revenue generated by its technology.

The requirement of unbundling and pass-through does no longer allow the pool members to share their profits on an arbitrary basis. Instead, each pool member's revenue cannot be

[27] Rey and Tirole (2013) show that the requirement is also insufficient if pool members engage in tacit collusion.

[28] Similarly, it is possible to show that the anticompetitive pool in Lemma 6, with pairwise integration, survives the independent licensing requirement.

greater than the revenue accruing from the sale of its patent; thus, the requirement boils down to a ban on monetary transfers within the pool. By that, the requirement restores effective competition between licensors. Consider the case of single integration and assume that $\hat{c}$ is large enough. Under the independent licensing requirement, neither licensor has the incentive to deviate from the pool outcome at the continuation stage. With the unbundling and pass-through requirement, this is no longer true for L_B. Why? Within the pool, L_B sells the patent at a rate of $\hat{c}$ to M_2 and at marginal cost to M_1. However, we know from Lemma 3 that, when taking its pricing decisions noncooperatively, L_B sells its technology at marginal cost to both manufacturers. That is, when choosing its prices maximizing own profits, L_B has the incentive to deviate from the pool outcome and, by revealed preference, this deviation is profitable.[29]

To sum up, we find that the pool is unstable under the unbundling and pass-through requirement. Therefore, with the proposed policy in place, we should observe patent pools forming only if they generate efficiency gains (like, for instance, the reduction of transaction costs). In this sense, the unbundling and pass-through requirement is a screening device of (weakly) procompetitive patent pools. In Section 7, we will also analyze whether the imposition of a non-discriminatory policy enforcing FRAND commitments can make pools procompetitive.

5 Robustness of the Results with Unobservable Offers

In this section, we show that the results obtained with the model in Section 2 are robust to the employment of alternative assumptions regarding the number of firms in the industry, the degree of final product differentiation, and the tariffs' structure.

5.1 Patent Pools and Firm Entry

As we prove below, our results in Section 3 survive the entry of independent firms in the simple vertically related industry we consider.

5.1.1 Vertical Separation and Single Integration

Let stand-alone firms enter either the licensing or the final good market, we find that the equilibrium unit prices with independent licensing and the pool remain the same as those in Lemmata 1–4 and Propositions 1–3. The reason why those results do not depend on the number of independent licensors is that, by the assumption of secret offers, the unit prices set by licensors do not interact strategically. At the same time, the presence of independent manufacturers does not alter the problem of a stand-alone licensor because, with secret offers, each licensor-manufacturer pair maximizes the value of the bilateral relationship. Finally, an integrated licensor's dominant strategy features foreclosing the access of all rival manufacturers to its technology, independently of the number of manufacturers in the market.

[29] By the same token, the unbundling and pass-through requirement breaks the anticompetitive pool with pairwise integration.

5.1.2 Pairwise Integration

With pairwise integration, our results are robust to the entry of independent licensors who then join the pool with the vertically integrated firms. The reason is that, with independent licensing, each stand-alone licensor would set the bilaterally efficient contract featuring the unit price equal to marginal cost. Moreover, once in the pool, these independent licensors will set the transfer price inside the pool to monopolize the product market. Instead, as we show below, an increase in the number of independent manufacturers does not change the anticompetitive nature of the pool. However, it certainly mitigates the magnitude of the pool's anticompetitive impact. Let an independent manufacturer compete with M_1 and M_2: The pool can no longer implement the unconstrained monopoly outcome in Lemma 6, because the independent manufacturer can resort to the status-quo technologies and outperform pool-member manufacturers.

Consider the case with linear demand ($P(Q) = 1 - Q$), with $n > 2$ total manufacturers and $m = n - 2$ stand-alone manufacturers. The equilibrium with independent licensing features integrated firms setting their transfer price equal to marginal cost and a unit price equal to $\hat{c}$ to all the other $n-1$ manufacturers. The pool, instead, sets a unit price equal to $2\hat{c}$ to the m manufacturers outside the pool and $2\overline{w}$ with $\overline{w} = \min[(1+2(n+1)c+2m\hat{c})/4n, 2\hat{c}]$ internally.[30] Comparing the sum of unit prices demonstrates that the pool is anticompetitive.

5.2 Competition between Differentiated Goods

In the main analysis, we study an industry in which manufacturers sell homogeneous products. In what follows, we explain why our results are robust to the introduction of product differentiation.

Consider first the case with vertical separation. As explained above, both with independent licensing and with a patent pool, licensors maximize bilateral profits. They achieve this

[30] Specifically, the pool sets $\overline{w}$ to induce its affiliated manufacturers (M_1 and M_2) to produce half of the quantity each of them would produce when purchasing the technologies at marginal cost, given that the other m manufacturers pay $2\hat{c}$, instead. That is, $\overline{w}$ is implicitly defined by

$$\frac{q_p}{2} = \arg\max_q \left(P\left(q + q(2(\overline{w}-c), 2(\overline{w}-c); 2e) + \sum_{k\neq 1,2} q_k \right) - 2\overline{w} \right) q,$$

where $q(2(\overline{w}-c), 2(\overline{w}-c); 2e)$ is the quantity produced by a pool-affiliated manufacturer, given that the other manufacturer in the pool pays $2(\overline{w}-c)$ and all the other manufacturers outside the pool pay $2\hat{c}$, and q_p is given by

$$q_p = \arg\max_q \left(P\left(q + \sum_{k\neq 1,2} q_k \right) - 2c \right) q$$

with

$$q_k = \arg\max_q \left(P\left(q + 2q_i(2(\overline{w}-c), 2(\overline{w}-c); 2e) + \sum_{l\neq k} q_l \right) - 2\hat{c} \right) q, \text{ with } i = 1, 2.$$

by setting a per-unit price equal to marginal cost. This result is independent of the degree of product differentiation; thus, the pool is welfare neutral with differentiated goods.

With single integration, the presence of product differentiation implies that the integrated firm L_A-M_1 will no longer necessarily foreclose the rival manufacturer's access to the input good: The reason is that M_2 generates additional value. Specifically, if $\hat{c}$ is large enough, the integrated firm will set a per-unit price between c and $\hat{c}$. Otherwise, the same results as in the homogeneous goods case apply and $w_{A2} = \hat{c}$. With the pool, we find the same results as with homogeneous goods: Licensors coordinate pricing decisions and set w_p between $2c$ and $2\hat{c}$ provided $\hat{c}$ is large enough. They set $w_p = 2\hat{c}$ otherwise. Thus, the pool's anticompetitive result remains if $\hat{c}$ is relatively small: In this case, with independent licensing and the pool, the unit prices are exactly the same as with homogeneous goods.[31]

Finally, with pairwise integration, when pricing their inputs independently licensors are constrained by the status-quo technology. They are unconstrained when they join in the pool. It follows that the impact of the pool on welfare with homogeneous and differentiated goods is the same: The pool is anticompetitive if $\hat{c}$ is small enough, and neutral if otherwise.

5.3 Quantity Forcing Contracts

Assume licensors offer more general non-linear contracts than two-part tariffs. In our setting, the most general class of non-linear contracts features quantity-forcing arrangements of this kind: $T_{ji} \equiv \{q_i, F_{ji}\}$.[32] The main difference with respect to two-part tariffs is that, if a manufacturer wants to produce a larger volume of output than the one specified in the contract by L_j, it has to use the status-quo technology. As we will show below, the use of two-part tariffs is justified by two main arguments: Quantity forcing contracts yield the same equilibrium outcomes as two-part tariffs, at the cost of generating a problem of equilibrium multiplicity.

Consider the vertically separated industry, and let L_B offer $T_{Bi} = \{q^c, F_{Bi}\}$, $i = 1, 2$. Due to perfect complementarity, the optimal response of L_A is to offer a tariff with a quantity of q^c. Moreover, in a symmetric equilibrium licensors require manufacturers to pay $F_{Ai} = F_{Bi} = (\pi^c - \max_q\{(P(q + q^c) - 2\hat{c})q\})/2$, with $i = 1, 2$. Hence, the market allocation with two-part tariff in Lemma 1 coincides with the one obtained under quantity-forcing contracts.

However, the one above may not be the only equilibrium with quantity-forcing contracts. The reason is that, due to input complementarity, a coordination problem can arise regarding the value of the quantity specified in the tariff. Developing on the example given above, if L_B offers $q' < q^c$ in T_{Bi}, L_A's best-response contract may also feature q'. Let L_A offer a tariff specifying a quantity of $q'' > q'$, the manufacturer can only produce this q'' by purchasing the additional units it needs of input B at a cost of $\hat{c}$. However, if $\hat{c}$ is sufficiently large, this is not optimal for the manufacturer. In fact, for q' slightly smaller than q^c, an equilibrium exists in which both licensors offer q' in their tariffs. Thus, with quantity-forcing contracts, multiple equilibria exist, each featuring a different value of the quantity in the tariff.

[31] The pool is instead welfare neutral in the complementary case in which $\hat{c}$ is relatively large.

[32] Since there is no asymmetric information in the negotiations between licensors and manufacturers, licensors can do no better in extracting manufacturers' revenues by using menus of contracts instead of one quantity forcing contract.

6 Contracting with Observable Contracts

In this section, we solve the game in Section 2 under the assumption that licensors' offers are public.[33] We maintain the assumption that, with vertical integration, the licensor and its downstream affiliate maximize joint profits, and the licensor informs its manufacturer about the offer formulated to the rival manufacturer and its acceptance decision. Moreover, if licensors form a pool they maximize, and then share, joint profits by setting the public two-part tariffs T_{pi} that a manufacturer M_i outside the pool needs to pay to obtain the right to use the bundle of patented inputs. Finally, both with vertical integration and the pool, internal transfer prices are not observable to outsiders.

Differently from the setting with secret offers, with public offers a manufacturer M_{-i} reacts to a change in the unit price offered by L_j to its rival M_i. Therefore, when offering the unit price in the tariff, a licensor L_j will not only take into account the impact of this decision on the value of the bilateral relationship with M_i (as it happens with unobservable contracts), but also on the profit it raises from M_{-i}. This consideration makes it crucial to determine how licensors' share the rents of each manufacturer. To this end, we show in Online Appendix B that, with public offers, it does not exist a marginal contribution equilibrium in which each licensor obtains its contribution to a manufacturer profit.

Denote the marginal contribution of L_j to M_i's profits by μ_{ji}, and by $\hat{\pi}_i$ the profit of M_i when using the status quo technologies. The result in the lemma reflects the fact that licensors' marginal contributions to M_i's profit are not superadditive (Laussel and Le Breton, 2001; Bergemann and Välimäki, 2003): $\mu_{Ai} + \mu_{Bi} > \hat{\pi}_i$.[34] Then, if L_j offers $F_{ji} = \mu_{ji}$, M_i prefers using both status-quo technologies to accepting an offer by L_{-j} featuring $F_{-ji} = \mu_{-ji}$.[35] This result has two main implications. The first is that, despite being otherwise symmetric (so that $\mu_{Ai} = \mu_{Bi} = \mu_i$), each licensor will set a different unit price to M_i, depending on the specific share of M_i's profit it extracts via the fixed component of the tariff. The second, and related implication is that $\hat{\pi}_i$ corresponds to M_i's profit (i.e., its bargaining threat when dealing with licensors).

Before proceeding, note that the analysis of the pairwise integration case is the same as in Section 3.3, because both manufacturers are integrated and thus informed about the offer made by their upstream unit to the rival. Therefore, contracts are "as if" they were observable in that case. Finally, note that the proofs of the propositions and lemmata in this section are in Online Appendix C.

6.1 Vertical Separation

Assume that neither manufacturer is vertically integrated and offers are public. Differently from the model with secret offers, we find that the pool can raise efficiency in this context.

[33] We also require that licensors cannot renegotiate their offers to manufacturers. Were renegotiation possible, then we would obtain the same results as with secret offers.

[34] As we explain in the proof, the result directly follows from the property that a manufacturer's product-market profit function is convex in the marginal costs of production.

[35] This result applies also to the vertical separation case with private offers. However, since secrecy implies that each licensor-manufacturer pair maximizes the value of the bilateral relationship, unit prices do not depend on how licensors' share a manufacturer's rents. Thus, at equilibrium, licensors' unit prices are all the same, and equal to marginal cost.

6.1.1 Independent Licensing by L_A and L_B

First, let licensors L_A and L_B set their tariffs noncooperatively.

LEMMA 7. *In any equilibrium with vertical separation, all unit prices (i.e., w_{A1}, w_{A2}, w_{B1}, and w_{B2}) are larger than c. Moreover, the sum of the unit prices is strictly below $4\hat{c}$ for all $\hat{c} > c$. Finally, given the equilibrium unit prices, the fixed components of the tariffs are such that $F_{ji} + F_{-ji} = \pi_i(m_i, m_{-i}) - (P(q_i(2e, m_{-i}) + q_{-i}(m_i, m_{-i})) - 2\hat{c})q_i(2e, m_{-i})$, with $i = 1, 2$ and $i \neq -i$.*

First, the lemma shows that the per-unit prices set by L_A and L_B are larger than c. Differently from the case with secret contracts, raising w_{ji} affects what licensor L_j can extract from M_i and M_{-i}. In particular, by increasing w_{j-i} above c, the costs of M_{-i} increase, implying a lower q_{-i}. This is beneficial to M_i, thereby allowing L_j to demand a higher fixed component of the tariff from this manufacturer. Therefore, contract observability allows the (constrained) upstream monopolists (L_A and L_B) to restrict their supplies to manufacturers. Second, we find that the sum of the per-unit prices is lower than $4\hat{c}$. M_i's bargaining threat is given by the profit it raises when using both status-quo technologies ($\hat{\pi}_i$). This rent increases in the per-unit prices paid by M_{-i}; thus, it does not pay off for L_j to raise w_{j-i} to $\hat{c}$, because this raises the value of $\hat{\pi}_i$ and reduces the fixed component of the tariff that L_j can claim from M_i.[36]

In what follows, we study the impact of the pool on licensors' pricing choices. Were the pool to change the value of the equilibrium tariffs, then, by a revealed preferences argument, it also raises licensors' profits.

6.1.2 Patent Pool between L_A and L_B

Let now L_A and L_B form a pool and cooperatively set the pool's tariffs.

LEMMA 8. *The pool sets $w_{pi} > 2c$, with $i = 1, 2$. In addition, the sum of the unit prices ($w_{p1} + w_{p2}$) is strictly below $4\hat{c}$ for all $\hat{c} > c$. Finally, given the equilibrium unit prices, the pool sets the fixed components of the tariffs so that $F_{pi} = \pi_i(m_i, m_{-i}) - (P(q_i(2e, m_{-i}) + q_{-i}(m_i, m_{-i})) - 2\hat{c})q_i(2e, m_{-i})$, with $i = 1, 2$ and $i \neq -i$.*

The per-unit prices set by the pool, w_{p1} and w_{p2}, are larger than $2c$, and their sum is lower than $4\hat{c}$. While the intuition is analogous to the one developed with independent licensing, it is worth remarking that the pool maximizes the *sum* of licensors' profits. This implies that the pool should be more effective in coordinating licensors' pricing decisions. Then we ask: Does the pool form at equilibrium? What are its consequences for consumer welfare?

PROPOSITION 4. *With observable contracts, at equilibrium L_A and L_B form the pool and this pool is procompetitive.*

[36] A similar effect is also pointed out by Rey and Tirole (2007) in a model with a single input good. They note that, in the presence of alternative sources, a manufacturer offering observable contracts may lower input prices to reduce the rents that accrue to downstream firms.

Without the pool, each licensor L_j ignores that a reduction in the unit prices it sets reduces manufacturers' bargaining threat, and thus increases L_{-j}'s fixed component of the tariffs. As a consequence, L_j raises the value of w_{ji} above the level that is optimal under coordinated pricing. This generates a problem of horizontal double marginalization that the pool solves by reducing unit prices. The result in Proposition 4 extends the argument put forward by Shapiro (2001) to a setting with input goods and nonlinear pricing: A pool formed by licensors is welfare improving in an economy with observable contracts and vertical separation. Although this conclusion is analogous to the one reached by the literature on pools formed by final good providers, the mechanism is different. Indeed, with input goods, licensors reduce unit prices to reduce manufacturers' bargaining threat.

Finally, since the pool changes the equilibrium value of tariffs, it also raises licensors' profits with respect to independent licensing. With coordinated pricing, pool members could set the same tariffs as with independent licensing. However, at equilibrium they decide to lower their tariffs; thus, by revealed preferences, coordinated pricing raises licensors' profits and the pool forms at equilibrium.

6.2 Single Integration

In this section, we study how the presence of an integrated firm (L_A-M_1) influences pricing decisions with and without the pool. We will show that, as with secret offers, the pool is anticompetitive.

6.2.1 Independent Licensing by L_A and L_B

First, let firms L_A-M_1 and L_B set their tariffs noncooperatively.

LEMMA 9. *With independent licensing and observable contracts, in the unique stable equilibrium the integrated firm sets the internal price at marginal cost and* $w_{A2} = \hat{c}$. *Instead,* L_B *sets* $w_{B1}, w_{B2} > c$, *with* $w_{B1} + w_{B2} < 2\hat{c}$. *At equilibrium, the fixed component of the tariff set by* L_A-M_1 *is equal to zero. Instead, the fixed components of the tariffs set by* L_B *are* $F_{B1} = \pi_1(m_{B1}, m_2) - (P(q(e, m_2) + q(m_2, m_{B1})) - c - \hat{c})q(e, e + m_{B2})$ *and* $F_{B2} = \pi_2(m_2, m_{B1}) - (P(q(2e, m_{B1}) + q(m_{B1}, e + m_{B2})) - 2\hat{c})q(2e, m_1)$.

The intuition is similar to the one supporting the results in Lemma 3. The integrated licensor cares about the profit loss of its downstream affiliate when setting the tariff offered to the rival manufacturer. It will therefore raise the unit price up to $\hat{c}$ and set the internal transfer price equal to marginal cost c. In contrast to Lemma 3, the non-integrated licensor L_B sets w_{B1} and w_{B2} above marginal costs. By doing so, L_B softens product-market competition and obtains larger profits.

We note that the equilibrium in the lemma is the only stable equilibrium of the licensing game. As we show in the proof, the slope of the reaction functions at any interior equilibrium (i.e., $w_{A2} < \hat{c}$) is larger than 1 in absolute value. Hence, any interior equilibrium is unstable. We then show that there is a unique extremal equilibrium at which $w_{A2} = \hat{c}$.

6.2.2 Patent Pool between L_A-M_1 and L_B

Assume now that L_A-M_1 and L_B join in a pool and cooperatively set their tariffs on the bundle of patents.

LEMMA 10. *With observable contracts, the pool sets the internal transfer prices equal to marginal cost and makes to manufacturer M_2 the following offer: $T_{p2} = \{2\hat{c}, 0\}$.*

As in the setting with secret offers (Lemma 4), the pool has a clear interest in raising the profit of the integrated licensor's downstream affiliate (M_1). In this setting, the competitive consequences of the pool are not obvious, though, because, with respect to independent licensing, the pool reduces w_{B1} and raises w_{B2}, for given w_{A2}. The next proposition establishes that, with respect to the equilibrium in Lemma 9, the pool is clearly anticompetitive.

PROPOSITION 5. *With observable contracts, the vertically integrated licensor L_A-M_1 and the stand-alone licensor L_B join the pool and this pool is anticompetitive.*

The proposition has two main take-aways: The first is that the pool raises firms profits, thus L_A-M_1 and L_B join the pool in equilibrium. The second is that, with the pool, w_{B2}'s increase to $\hat{c}$ is larger than w_{B1}'s fall to c; thus, the pool is anticompetitive. This result suggests that, under independent licensing, w_{B1} and w_{B2} are only slightly larger than marginal cost. The reason is that, with independent licensing, increasing w_{Bi} has only an indirect impact on L_B's profit, due to the larger fixed component of the tariff that L_B can then extract from M_{-i}. By contrast, once in the pool, L_B shares with L_A the profit of M_1, thus it directly benefits from raising the per-unit price to M_2.

7 Non-Discriminatory Licensing Policy

In what follows, we solve the model in Section 2 under the assumption that licensors cannot discriminate between manufacturers.[37] Our aim is to capture the impact on equilibrium market allocations of the imposition of a strictly Non-Discriminatory policy enforcing FRAND commitments by a court. Therefore, we assess the impact on consumer welfare of the formation of a pool within an industry in which, under independent licensing, the contracts offered by licensors are non-discriminatory. Our results suggest that the introduction of a strictly non-discriminatory policy makes a pool procompetitive with vertical separation and single integration. We also show the conditions under which the pool remains anticompetitive in the case with pairwise integration. The proofs of the propositions in this section are relegated to Online Appendix C.

7.1 Vertical Separation

We begin with the case of a vertically separated industry.

[37]Formally, the main difference with respect to the model with secret offers is that agents hold symmetric beliefs, rather than passive beliefs.

PROPOSITION 6. *With vertical separation, the equilibrium outcomes with independent licensing and with the pool are the same as in the model with observable contracts. Therefore, the formation of a pool increases consumer welfare.*

If discrimination is banned, then, after receiving an offer by a licensor, each manufacturer knows that the rival manufacturer must have received the same proposal by that licensor. This means that contracts are "as if" observable; thus, as much as in the model with observable contracts, the enforcement of a non-discriminatory policy allows licensors to raise their unit prices above marginal cost (Rey and Tirole, 2007). However, as discussed below Proposition 4, the equivalence with observable contracts also means that the pool improves pricing coordination among licensors.

7.2 Single Integration

In this section, we study how a non-discriminatory policy affects input prices with and without the pool, when M_1 is integrated upstream with L_A.

PROPOSITION 7. *With single integration and under $P''(Q) \approx 0$, the implementation of a non-discriminatory policy renders the pool procompetitive.*

As we show in the proof of Proposition 7, the ban to discrimination means that the integrated licensor L_A-M_1 fixes its unit price above marginal cost: Were L_A to set the internal transfer price to M_1 equal to marginal cost, it would then be legally compelled to do the same with the rival manufacturer. This is costly to L_A-M_1, by intensifying product-market competition, and causing industry profitability to fall. Thus, the enforcement of a non-discriminatory policy pushes licensors to raise the unit prices above marginal cost to soften product-market competition, but stay below the cost of the status-quo technology to reduce the manufacturers' bargaining threat. The same reasoning applies to the decisions on the unit prices set by L_A-M_1 and L_B after they form a pool, with the difference that the pool is better suited at coordinating licensors' pricing choices.[38]

The proposition shows that, with a non-discrimination policy in place, the standard Cournot-Shapiro result that the prices of complementary goods are optimally set by a single entity holds true, even though one of the licensors is integrated with a manufacturer.

7.3 Pairwise Integration

We now turn to the situation in which both licensors are integrated with a manufacturer.

PROPOSITION 8. *Let $P''(\cdot) \approx 0$. With pairwise integration, a pool formed by L_A-M_1 and L_B-M_2 is anticompetitive if $\hat{c}$ is lower than a threshold, procompetitive otherwise.*

[38] We assume that consumer demand is (close to) linear. Under the non-discriminatory constraint, if, off equilibrium, M_i rejects the offer of a licensor, this offer still applies to M_{-i}. This makes manufacturers asymmetric out of equilibrium, and complicates the comparison between unit prices. These comparisons are clearcut under the condition that $P''(Q) \approx 0$.

The proposition shows that the competitive consequences of the pool are less clear-cut than in the cases with vertical separation and single integration, where a pool is unambiguously procompetitive. First note that, as shown in Lemma 6, the unit prices set by the pool are independent of $\hat{c}$. By contrast, the unit prices with independent licensing are strictly increasing in $\hat{c}$. Now, for $\hat{c}$ sufficiently small, the pool is anticompetitive because the constraint imposed by the presence of the status-quo technologies on the pricing decisions of the independent licensors is binding. Instead, for $\hat{c}$ large, the pool is procompetitive because, differently from independent licensors, it coordinates pricing decisions. The argument is as follows: When taking licensing decisions independently, L_j-M_i does not take into account the per-unit revenue obtained by L_{-j} on input $-j$. This revenue is the larger the smaller w_j, because manufacturers' quantities are decreasing in w_j. Instead, the pool takes the per-unit revenues of both inputs into account, and therefore sets lower per-unit prices.

8 Conclusions, Case Evidence and Policy Discussion

In this paper, we deliver the following message: A patent pool formed by licensors of complementary patents is anticompetitive if at least one of the pool's licensors is integrated downstream. With vertical integration, the pool serves as a coordination device that licensors use to soften competition, and share the larger profit raised by the integrated manufacturer. Our conclusions are robust to the number of licensors and manufacturers, and to the contracting environment. Although we focus on the case of patent pools, these results apply to all joint marketing agreements—including joint ventures and mergers—among suppliers of complementary input goods.

Our theory is the first that can explain why the use of vertical restraints by patent pools of complementary patents harms welfare. The U.S. International Trade Commission investigation of the CD-R pool reports evidence in line with the theoretical mechanism we put forward. Specifically, Judge Harris (2003) establishes that "manufacturers who sell CD-RRW discs to Philips [...] pay no royalty on those discs to the pool members." That is, Phlilips (the pool administrator) gave to affiliated manufacturers a privileged access to the pool's patent bundle. Moreover, the evidence in Flamm (2013) suggests that an analogous form of price discrimination was carried out by Philips within the 3C DVD pool. Interestingly, this pool was approved by the Department of Justice (DOJ) based on the assurance that the pool administrator would have licensed on a non-discriminatory basis to all interested parties.

Gilbert (2004) gives a number of cases in which patent pools employ other forms of vertical restraints (like exclusive dealing and territories or resale price maintenance), the use of which is consistent with our theoretical mechanism. For example, in *United States v. New Wrinkle, Inc.*, the Supreme Court alleged that the patent licenses granted by New Wrinkle were used to fix downstream prices.[39] The terms of the licensing agreements specified the minimum prices and other clauses on the conditions at which wrinkle finish products could be sold. *United States v. Holophane Co.* involved a network of agreements among holders of patents on the manufacturing process of prismatic glassware, which granted the exclusive

[39]New Wrinkle was founded by the companies holding complementary patents on the manufacturing process of wrinkle finishes to license these patents out.

right to serve a territory to different companies. Finally, in *United States v. Associated Patents*, patent holders pooled their patents and then granted companies the exclusive right to manufacture particular types of machines.

To guide policy formulation by Antitrust authorities and court decisions in these cases, we consider two policies. First, we show that the unbundling and pass-through requirement eliminates the pool's anticompetitive nature. This policy appends the unbundling of patent claims to the independent licensing requirement in Lerner and Tirole (2004). In line with Rey and Tirole (2013) and Boutin (2016), under this augmented requirement, a licensor's dividend is equal to the revenue generated by its technology. This corresponds to the imposition of a ban on monetary transfers between pool members. Second, we show that the imposition of a mandated non-discriminatory policy that enforces FRAND commitments makes the pool procompetitive only with vertical separation and single integration. Instead, we show that the pool has still the potential to raise anticompetitive concerns when all licensors are integrated with a manufacturer.

Throughout the analysis, we focused on the pricing implications of patent pools. Another important question in the literature on innovation concerns how patent pools affect future incentives to innovate in the industry. Such an analysis could be conducted in a model with sequential innovation (Denicolò, 2002; Hopenhayn, Llobet and Mitchell, 2006). At the same time, in this paper we abstract from the impact of the pool on the development of technological standards. To this end, one could build on Gallini (2014) to analyze whether a formation of a pool spurs the process of standard setting. We leave these questions to future research.

A Appendix

In this appendix, we report the proofs of the results in the model with secret offers (Section 3)—that is, Proposition 3 and Lemmata 1, 3–6.

Proof of Lemma 1. We solve the game by backward induction. In the last stage, manufacturer M_i produces $q_i(m_i, m_{-i})$ as defined by (1). Accordingly, one-to-one production technology with perfect input complementarity implies that M_i orders $q_i(m_i, m_{-i})$ from each licensor L_j.

We first determine licensors' per-unit price offers. With passive beliefs, the equilibrium contract offered by L_j to each manufacturer M_i must maximize their joint profits (McAfee and Schwartz, 1994). Therefore, L_j's first-stage maximization problem can be written as

$$\max_{w_{ji}} \; q_i(m_i, m_{-i})(w_{ji} - c) + \left(P(q_i(m_i, m_{-i}) + q_{-i}) - w_{ji} - w_{-ji}\right) q_i(m_i, m_{-i}).$$

Taking the first-order condition with respect to w_{ji} and invoking the Envelope Theorem, we obtain (functional notation is dropped, for simplicity)

$$(w_{ji} - c)\frac{\partial q_i}{\partial w_{ji}} = 0.$$

Because $\partial q_i / \partial w_{ji} < 0$, at the equilibrium $w_{ji} = c$.

As far as the computation of the equilibrium fixed components of the tariffs are concerned, each licensor L_j needs to account for two possible cases.

In the first case, M_i could accept the offer of L_{-j} and reject L_j's offer. To avoid this, the value of F_{ji} must satisfy the following constraint:

$$\pi^c - F_{ji} - F_{-ji} \;\; \geq \;\; \max_q\{(P(q + q^c) - c - \hat{c})q\} - F_{-ji},$$

so that

$$F_{ji} \;\; \leq \;\; \pi^c - \max_q\{(P(q + q^c) - c - \hat{c})q\}. \tag{A-1}$$

If this constraint is fulfilled for both licensors, then M_i accepts the offers of L_A and L_B.

In the second case, M_i can reject the offers of L_A and L_B, and use the status quo technology for both inputs. Hence, F_{ji} must satisfy

$$\pi^c - F_{ji} - F_{-ji} \;\; \geq \;\; \max_q\{(P(q + q^c) - 2\hat{c})q\}. \tag{A-2}$$

Let licensor L_{-j} ask for its contribution to M_i's profits,

$$F_{-ji} \;\; = \;\; \pi^c - \max_q\{(P(q + q^c) - c - \hat{c})q\}.$$

Plugging this value of F_{-ji} into (A-2), we obtain that F_{ji} must satisfy

$$F_{ji} \;\; \leq \;\; \max_q\{(P(q + q^c) - c - \hat{c})q\} - \max_q\{(P(q + q^c) - 2\hat{c})q\}. \tag{A-3}$$

In equilibrium, one of the two constraints (A-1) and (A-3) must be binding, because otherwise L_j could raise its profits by increasing F_j. Comparing the right-hand sides of (A-1) and (A-3), we obtain that (A-3) is tighter if and only if

$$\frac{1}{2}\pi^c + \frac{1}{2}\max_q\{(P(q+q^c) - 2\hat{c})q\} \quad > \quad \max_q\{(P(q+q^c) - c - \hat{c})q\},$$

which holds true by the convexity of the profit function with respect to marginal costs.[40] Since the relevant constraint is (A-3), in equilibrium the sum of F_{Ai} and F_{Bi} must be such that $F_{Ai} + F_{Bi} = \pi^c - \max_q\{(P(q+q^c) - 2\hat{c})q\}$, with equality because licensors hold full bargaining power.

In the unique symmetric equilibrium, then, $F_{Ai} = F_{Bi} \equiv F$, so that $F = (\pi^c - \max_q\{(P(q+q^c) - 2\hat{c})q\})/2$. Thus, the symmetric equilibrium value of the tariffs is $T_{Ai} = T_{Bi} \equiv T = \{c, (\pi^c - \max_q\{(P(q+q^c) - 2\hat{c})q\})/2\}$.

Q.E.D.

Proof of Lemma 3. To begin with, note that the maximization problem of licensor L_B is isomorphic to the one of a licensor L_j in the proof of Lemma 1. Therefore, following the same steps yields that $w_{Bi} = c$, for all $i = 1, 2$. In addition, L_A-M_1 sets the internal transfer price equal to marginal cost ($w_{A1} = c$).[41] Then, the maximization problem of L_A-M_1 with respect to w_{A2} and F_{A2} is

$$\max_{w_{A2}, F_{A2}} \max_q\{(P(q + q_2(m_{A2}, 0)) - c - \hat{c})q\} + (w_{A2} - c)q_2(m_{A2}, m_{B1}) + F_{A2},$$

subject to $F_{A2} = \max_q\{(P(q + q_1(0, m_{A2})) - c - \hat{c})q\} - \max_q\{(P(q + q_1(0, e)) - \hat{c} - c)q\}$. Plugging F_{A2} into the maximization problem above, and taking the derivative with respect to w_{A2}, we obtain the following first-order condition:

$$(P'(Q)q_1 + w_{A2} - c)\frac{\partial q_2}{\partial w_{A2}} + P'(Q)q_2\frac{\partial q_1}{\partial w_{A2}} = 0.$$

In what follows, we show that, at equilibrium $w_{A2} = \hat{c}$. First note that the manufacturers' first-order conditions in the downstream market are given by

$$P(Q) - 2c + P'(Q)q_1 = 0 \quad \text{and} \quad P(Q) - w_{A2} - c + P'(Q)q_2 = 0.$$

They can be combined to get $P'(Q)q_1 = c - w_{A2} + P'(Q)q_2$. Inserting this expression into the first-order condition for w_{A2} yields

$$P'(Q)q_2\left(\frac{\partial q_2}{\partial w_{A2}} + \frac{\partial q_1}{\partial w_{A2}}\right) = 0. \tag{A-4}$$

[40] Because $\pi(C)$ is equal to $\max_q\{(P(Q) - C)q\}$, differentiating $\pi(C)$ twice with respect to C yields $\partial\pi/\partial C = -q < 0$ and $\partial^2\pi/\partial C^2 = -\partial q/\partial C > 0$.

[41] We refer to Reisinger and Tarantino (2015) for the proof of this result.

Since $P'(Q) < 0$ and $\partial q_2/\partial w_{A2} + \partial q_1/\partial w_{A2} < 0$, it follows that (C-12) is fulfilled if and only if $q_2 = 0$; thus, L_A-M_1 sets its unit price as high as possible, which, given the presence of the status-quo technology, implies that $w_{A2} = \hat{c}$. As a consequence, $F_{A2} = 0$.

We now proceed with the determination of the fixed components of the tariffs. Specifically, the values of F_{B1} and F_{B2} are such that

$$F_{B1} \leq \max_q\{(P(q + q_2(e,0)) - 2c)q\} - \max_q\{(P(q + q_2(e,0)) - c - \hat{c})q\} = \pi_1(0,e) - \overline{\pi}_1$$

and

$$F_{B2} \leq \max_q\{(P(q + q_1(0,e)) - c - \hat{c})q\} - \max_q\{(P(q + q_1(0,e)) - 2\hat{c})q\} = \pi_2(e,0) - \overline{\overline{\pi}}_2,$$

with $e = \hat{c} - c$. These constraints account for the option that each manufacturer has to substitute the technology of the efficient licensors with the status quo technology, and bear a marginal cost of $\hat{c}$. Moreover, the value of F_{A2} set by L_A is such that

$$F_{A2} \leq \max_q\{(P(q + q_1(0,e)) - c - \hat{c})q\} - \max_q\{(P(q + q_1(0,e)) - c - \hat{c})q\} = 0.$$

Since licensors hold full bargaining power, in equilibrium the constraints above bind. Thus, $F_{B1} = \pi_1(0,e) - \overline{\pi}_1$, $F_{B2} = \pi_2(e,0) - \overline{\overline{\pi}}_2$ and $F_{A2} = 0$. Q.E.D.

Proof of Lemma 4. Under the pool, L_B and L_A-M_1 act as if they are integrated. This means that, different from the case of independent licensing, L_B internalizes the impact of its pricing decisions on the profits of M_1. As a consequence, in equilibrium the pool sets the internal transfer price equal to marginal cost, and offers to M_2 the following tariff: $T_{p2} = \{2\hat{c}, 0\}$. Thus, the combined profits raised by the pool members are $\pi_1(0,2e) + 2eq_2(2e,0)$. From Lemma 3, the sum of the profits of L_A-M_1 and L_B is instead equal to $\overline{\pi}_1 + q_2(e,0)e + F_{B1} + F_{B2}$. Then, for the firms to be willing to participate in the pool, the following condition must hold true:

$$\begin{aligned}
\pi_1(0,2e) + 2eq_2(2e,0) &\geq F_{B1} + F_{B2} + \overline{\pi}_1 + eq_2(e,0) \\
&= \pi_1(0,e) - \overline{\pi}_1 + \pi_2(e,0) - \overline{\overline{\pi}}_2 + \overline{\pi}_1 + eq_2(e,0) \\
&= eq_2(e,0) + \pi_1(0,e) + \pi_2(e,0) - \overline{\overline{\pi}}_2. \qquad \text{(A-5)}
\end{aligned}$$

From (4),

$$\overline{\overline{\pi}}_2 = \max_q\{(P(q + q_1(0,e)) - 2\hat{c})\, q\}.$$

Thus, (A-5) can be rewritten as

$$\begin{aligned}
(P(q_1(0,2e) + q_2(2e,0)) - 2c)\, q_1(0,2e) \geq\ & eq_2(e,0) - 2eq_2(2e,0) \\
&+ (P(q_1(0,e) + q_2(e,0)) - 2c)\, q_1(0,e) \\
&+ (P(q_1(e,0) + q_2(0,e)) - c - \hat{c})\, q_2(e,0) \\
&- (P(q_1(0,e) + q_2(2e,0)) - 2\hat{c})\, q_2(2e,0).
\end{aligned}$$

Using $e = \hat{c} - c$ and symmetry, the last condition becomes

$$\begin{aligned}(P(q_1(0,e)+q_2(e,0))-2c)\,(q_1(0,e)+q_2(e,0)) \;\;\le\;\; & (P(q_1(0,2e)+q_2(2e,0))-2c)\,q_1(0,2e) \\ & +(P(q_1(0,e)+q_2(2e,0))-2c)\,q_2(2e,0).\end{aligned}$$

Moreover, since $q_1(0,2e) \ge q_1(0,e)$ implies that $P(q_1(0,2e)+q_2(2e,0)) \le P(q_1(0,e)+q_2(2e,0))$, a sufficient condition for licensors to join the pool requires that

$$\begin{aligned}&(P(q_1(0,e)+q_2(e,0))-2c)\,(q_1(0,e)+q_2(e,0)) \le \\ &\qquad\qquad (P(q_1(0,2e)+q_2(2e,0))-2c)\,(q_1(0,2e)+q_2(2e,0)).\end{aligned}$$

By a standard property of Cournot-Nash equilibria, $|\partial q_i/\partial C_i| > |\partial q_{-i}/\partial C_i|$ (the direct effect of a firm's increase in own costs C_i is larger than the reaction of its competitor). Thus, $q_2(e,0) - q_2(2e,0) \ge q_1(0,2e) - q_1(0,e)$ and $q_1(0,e)+q_2(e,0) \ge q_1(0,2e)+q_2(2e,0)$. As a consequence, the industry quantity with independent licensing is larger than the industry quantity with the pool. This means that, with respect to independent licensing, the pool raises industry profits. Q.E.D.

Proof of Lemma 5. First note that, by standard arguments, each integrated licensor L_j-M_i sets the internal transfer price equal to marginal cost, so that $m_{ji} = m_{-j-i} = 0$. The optimization problem of L_j-M_i is then

$$\max_{w_{j-i},F_{j-i}} \pi_i(m_{-ji}, m_{j-i}) + q_{-i}(m_{j-i}, m_{-ji})(w_{j-i} - c) + F_{j-i} - F_{-ji}. \tag{A-6}$$

Since M_i is informed by L_j of M_{-i}'s rejection of L_j's offer, M_i will set its quantity taking into account the margin faced by M_{-i} on input j. Therefore, when deciding whether to accept or reject L_j-M_i's offer, M_{-i} takes into consideration the impact of that decision on M_i's output, as this changes its own downstream profit and the revenue that its upstream unit L_{-j} extracts from M_i. The fixed fee set by L_j-M_i is then

$$\begin{aligned}F_{j-i} \;=\; & \pi_{-i}(m_{j-i}, m_{-ji}) - \pi_{-i}(m_{j-i}, m_{-ji}) \\ & + q_i(m_{-ji}, m_{j-i})(w_{-ji} - c) - q_i(m_{-ji}, e)(w_{-ji} - c).\end{aligned} \tag{A-7}$$

The first and third terms correspond to the difference in L_{-j}-M_{-i}'s downstream profit when accepting and when rejecting the offer, while the second and fourth terms correspond to the difference in L_{-j}'s revenues from sales to M_i.

We now rewrite the optimization program of L_j-M_i by using only those terms that directly depend on w_{j-i}:

$$\max_{w_{j-i}} \pi_i(m_{-ji}, m_{j-i}) + \pi_{-i}(m_{j-i}, m_{-ji}) + q_{-i}(m_{j-i}, m_{-ji})(w_{j-i} - c) + q_i(m_{-ji}, m_{j-i})(w_{-ji} - c).$$

The associated first-order condition with respect to w_{j-i} is (in what follows, we use $q_i = q_i(m_{-ji}, m_{j-i})$ and $q_{-i} = q_{-i}(m_{j-i}, m_{-ji})$):

$$(P'(Q)q_i + w_{j-i} - c)\,\frac{\partial q_{-i}}{\partial w_{j-i}} + (P'(Q)q_{-i} + w_{-ji} - c)\,\frac{\partial q_i}{\partial w_{j-i}} = 0, \tag{A-8}$$

with $j = A, B$ and $i = 1, 2$.[42] Taking the total derivative of the first-order conditions for final good quantities, which are given by $P(Q) - c - w_{-ji} + P'(Q)q_i = 0$ and $P(Q) - c - w_{j-i} + P'(Q)q_{-i} = 0$, we can solve for $\partial q_i / \partial w_{-ji}$ and $\partial q_{-i} / \partial w_{-ji}$ to get that

$$\frac{\partial q_{-i}}{\partial w_{j-i}} = \frac{2P'(Q) + q_i P''(Q)}{P'(Q)\left(3P'(Q) + QP''(Q)\right)}, \text{ and } \frac{\partial q_i}{\partial w_{j-i}} = -\frac{P'(Q) + q_i P''(Q)}{P'(Q)\left(3P'(Q) + QP''(Q)\right)}.$$

Inserting these expressions into the first-order conditions in (A-8), and solving for w_{j-i} and w_{-ji}, we obtain:

$$w_{j-i} = c - P'(Q)q_i, \quad w_{-ji} = c - P'(Q)q_{-i}, \tag{A-9}$$

with $j = A, B$ and $i = 1, 2$.

From the final good market first-order condition in the downstream market, we know that $P'(Q)q_i = -P(Q) + c + w_{-ji}$. Inserting this into (A-9) yields $w_{-ji} + w_{j-i} = P(Q)$. Hence, since Q is a function of $w_{-ji} + w_{j-i}$, we need to solve for the following fixed-point problem,

$$w_{-ji} + w_{j-i} = P(Q),$$

and check whether its solution is unique. Using once again the first-order conditions in the final good market, we can determine how $Q = q_i + q_{-i}$ changes as $w_{-ji} + w_{j-i}$ changes. Specifically, as $w_{-ji} + w_{j-i}$ increases by one unit, then $P(Q)$ changes by $P'(Q)/(3P'(Q) + P''(Q)Q)$, which, according to our working assumption $P'(Q) + P''(Q)q_i < 0$, is smaller than one. This implies that the slope of the right-hand side of $w_{-ji} + w_{j-i} = P(Q)$ is smaller than the slope of the left-hand side. Therefore, there is a unique solution to the equation in question, which determines the value of $w_{-ji} + w_{j-i}$.

At the unique symmetric solution, $w \equiv w_{-ji} = w_{j-i}$. Moreover, taking into account that w cannot be larger than $\hat{c}$ (as otherwise manufacturers would switch to the status-quo technology), we find the following: If $P(Q)/2 \leq \hat{c}$, where $Q = q_1(w-c, w-c) + q_2(w-c, w-c)$, then $w = c - P'(Q)q$. By contrast, if $P(Q)/2 > \hat{c}$, then $w = \hat{c}$. Turning to the fixed fees, from (A-7), we obtain that, at the symmetric equilibrium, we have that, if $P(Q)/2 \leq \hat{c}$, $F_{j-i} = F_{-ji} = F$ (with F defined in (6)). Instead, the fixed components of the tariffs are equal to zero if $P(Q)/2 > \hat{c}$. Q.E.D.

Proof of Lemma 6. Under the pool, both integrated firms act as if they are integrated. Since each licensor is integrated with one manufacturer, the pool maximizes joint profits. The aim of the pool is to induce the manufacturers to set the monopoly quantity, knowing that manufacturers choose their quantities as in (1). When setting $w_{A1} = w_{A2} = w_{B1} = w_{B2} = \overline{w}$, with $\overline{w}$ defined as in (7), each manufacturer will supply half of the monopoly quantity, implying that the aggregate quantity is equal to the monopoly quantity. Therefore, with wholesale prices equal to $\overline{w}$, the licensors monopolize the industry. Since manufacturers are part of the pool, they will adopt the pool technologies at any $\overline{w}$ that maximizes industry profits.

[42] Our regularity assumptions on the shape of the demand function ensure that the second-order conditions are satisfied.

At these conditions, both integrated firms are willing to participate in the pool. Pool members split π^m, and, since π^m is the largest profit that can be reaped in the industry, firms benefit from forming the pool. Q.E.D.

Proof of Proposition 3. With a patent pool, the per-unit prices are set so that the industry output is equal to the unconstrained monopoly one. The maximization problem of a monopolist is $\max_Q (P(Q) - 2c)Q$, implying that the monopoly quantity is implicitly defined by $Q = -(P(Q) - 2c)/P'(Q)$.

We now turn to the independent licensing case. Let us first consider the case in which $w = c - P'(Q)q$, where $Q = 2q(w - c, w - c)$. The product-market first-order condition for q is $P(Q) - c - w + P'(Q)q = 0$. Inserting $w = c - P'(Q)q$, and rearranging, yields that q is implicitly defined by $q = -(P(Q) - 2c)/2P'(Q)$. Since the equilibrium is symmetric, the aggregate quantity is $-(P(Q) - 2c)/P'(Q)$, which is the same as with the pool. Hence, the pool is welfare neutral.

Turning to the case with $w = \hat{c}$, we know from Lemma 5 that this outcome occurs only if $\hat{c} < P(Q)/2$ or $\hat{c} < c - P'(Q)q$. Therefore, per-unit prices with independent licensing are lower than in the previous case. Because the pool was neutral in the previous case, it must be anticompetitive in case $w = \hat{c}$: Unit prices with pool are unchanged, whereas they are lower with independent licensing. Q.E.D.

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B For Online Publication: Additional Results

In this appendix, we report additional results that complement the analysis of the main model.

B.1 Stability of the Patent Pool under the Independent Licensing Requirement

In this section, we prove the claim we make in Section 4 that, if $\hat{c}$ is sufficiently large, the independent licensing requirement is not enough to break the pool's formation formed by L_A-M_1 and L_B.

LEMMA B-1. *If $\hat{c}$ is large enough, the pool composed of L_A-M_1 and L_B is weakly stable to the independent licensing requirement.*

Proof. The profit of L_B under the noncooperative equilibrium tariffs in Lemma 3 is given by $\pi_1(0,e) - \overline{\pi}_1 + \pi_2(e,0) - \overline{\overline{\pi}}_2$. Instead, given the cooperative equilibrium tariffs in Lemma 4, the pool raises a profit of $\pi_1(0,2e) + 2(\hat{c}-c)q_2(2e,0)$, and L_B obtains a share $1-\alpha$ of it. It follows that L_B joins the pool if and only if

$$1-\alpha \geq \frac{\pi_1(0,e) - \overline{\pi}_1 + \pi_2(e,0) - \overline{\overline{\pi}}_2}{\pi_1(0,2e) + 2(\hat{c}-c)q_2(2e,0)}. \tag{B-1}$$

Assume that, at a candidate equilibrium of the continuation stage, L_B deviates from the patent pool's pricing in Lemma 4 by offering $w_{B2} = c$.[43] M_2 will then reject the pool's offer, and buy input B at marginal cost and input A from the status-quo technology. Manufacturer M_1 is informed of M_2's rejection of the pool's offer by L_A; thus, M_1 expects M_2 to pay c on input B (as in the candidate equilibrium),[44] and adjust its quantity accordingly. All this means that the fixed component of the tariff that L_B sets on M_2 is $F_{B2} = \pi_2(e,0) - \overline{\overline{\pi}}_2$.

Under this candidate equilibrium, the profit of the pool is then given by M_1's profit, which is $\pi_1(0,e)$. Because L_B obtains a share of $1-\alpha$ of the pool's profit, L_B's overall profit at this candidate equilibrium is

$$\pi_2(e,0) - \overline{\overline{\pi}}_2 + (1-\alpha)\pi_1(0,e).$$

Therefore, the candidate equilibrium we are considering breaks down if and only if

$$(1-\alpha)(\pi_1(0,2e) + 2(\hat{c}-c)q_2(2e,0)) \geq \pi_2(e,0) - \overline{\overline{\pi}}_2 + (1-\alpha)\pi_1(0,e). \tag{B-2}$$

Assume $1-\alpha$ is as in (B-1) (as L_A would not accept any lower value of α). Simplifying (B-2), and using $e = \hat{c} - c$, yields

$$(\pi_1(0,e) - \overline{\pi}_1)\left(\pi_1(0,2e) + 2eq_2(2e,0) - \pi_1(0,e)\right) - \pi_1(0,e)\left(\pi_2(e,0) - \overline{\overline{\pi}}_2\right) \geq 0. \tag{B-3}$$

[43] L_B cannot profitably offer an independent contract to M_1, since this manufacturer already obtains input B at marginal cost via the pool.

[44] The result that this candidate equilibrium breaks down is even stronger under the assumption that M_1 expects M_2 to pay $\hat{c}$ on input B.

To derive a (sufficient) condition under which (B-1) is fulfilled, we let $P(Q) = 1 - Q$ and $c = 0$. Then, (B-3) holds true if and only if

$$\hat{c}(35 - 58\hat{c} - 10\hat{c}^2) - 4 \geq 0 \iff \hat{c} \geq \bar{\hat{c}} \equiv 0.1554,$$

with $\bar{\hat{c}}$ lower than the upper bound of 0.25, which ensures that our parametric condition requiring $q_2(2e, 0) \geq 0$ is satisfied. As a consequence, L_B will not deviate from the pool's pricing in Lemma 4 if $\hat{c}$ is large enough. Q.E.D.

B.2 Alternative Assumption on the Information-Sharing Protocol Between Integrated Units

In our main model (Section 2), we assume that, if integrated with a manufacturer M_i, licensor L_j communicates to M_i both the terms of its offer to M_{-i} and M_{-i}'s decision on the same offer. This assumption implies that we need to formulate conjectures regarding the out-of-equilibrium beliefs of M_i on the tariff paid by M_{-i} when the latter rejects L_{-j}'s offer. Specifically, we assume that, were M_{-i} to reject the offer of L_j, M_i believes that the rival is still accepting the offer of L_{-j}, as it does at equilibrium.

There are two alternatives to this approach. The first features M_i believing that M_{-i} rejects both offers, and yields the same results as in our main model. With single integration, it is easy to show (along the lines of the proof in Rey and Tirole, 2007) that the integrated licensor L_j has even stronger incentives to foreclose M_{-i} than under the assumption in Section 2. The reason is that its integrated manufacturer believes that the rival obtains the complementary input at a unit price of $\hat{c} > c$. As a consequence, L_j sets $w_{j-i} = \hat{c}$ in equilibrium. With pairwise integration, the analysis remains the same as in Section 3.3, because each manufacturer will always be supplied by its integrated licensor.

The second alternative is that an integrated manufacturer is not informed by the upstream affiliate of the rival's acceptance decision. This approach is followed by Nocke and Rey (2014) and eliminates the need to specify conjectures on the out-of-equilibrium tariff paid by M_{-i} on input $-j$. Conducting the analysis with this alternative assumption affects only our results with pairwise integration. With single integration, the integrated firm's optimal strategy still features setting its unit price to M_{-i} equal to $\hat{c}$. Any out-of-equilibrium action of M_{-i} (i.e., M_{-i}'s rejection of L_j's offer) leads to the same outcome and is irrelevant for the computation of w_{j-i}.

In what follows, we show that, with pairwise integration, the pool is anticompetitive even within a model without information transmission.

LEMMA B-2. *If manufacturers are not informed about their rival's decision on the offer made by the affiliated licensor, with independent licensing the unit prices set by the integrated firms are weakly lower than in the model in Section 2.*

Proof. Assume that licensor L_j does not inform the affiliated manufacturer M_i about M_{-i}'s decision on its offer T_{j-i}. The maximization problem of L_j-M_i with respect to w_{j-i} and F_{j-i} is

$$\max_{w_{j-i}, F_{j-i}} \max_q \{(P(q + q_{-i}(m_{j-i}, m_{-ji})) - w_{-ji} - c)q\} + q_{-i}(m_{j-i}, m_{-ji})(w_{j-i} - c) + F_{j-i} - F_{-ji}.$$

Since L_{-j}-M_{-i} sets the internal transfer price equal to marginal cost c, the fixed component of the tariff set by L_j-M_i is

$$F_{j-i} = \pi_{-i}(m_{j-i}, m_{-ji}) - \max_{q}\{(P(q + q_i(m_{-ji}, m_{j-i})) - c - \hat{c})q\}. \tag{B-4}$$

Since M_i is not informed about M_{-i}'s decision on L_j's offer, M_i cannot adjust its quantity based on the decision of M_{-i}. This explains the difference between (B-4) and (A-7). Accordingly, M_i sets its quantity assuming that M_{-i} accepts the offer made by L_j.

Thus, we can rewrite the optimization program of L_j-M_i by using only the terms that directly depend on w_{-ji}:

$$\begin{aligned} \max\nolimits_{w_{j-i}} \quad & \pi_i(m_{-ji}, m_{j-i}) + \pi_{-i}(m_{j-i}, m_{-ji}) + q_{-i}(m_{j-i}, m_{-ji})(w_{j-i} - c) \\ & - \max_{q}\{(P(q + q_i(m_{-ji}, m_{j-i})) - c - \hat{c})q\}. \end{aligned}$$

The first-order condition with respect to w_{j-i} is (in what follows, $q_i = q_i(m_{-ji}, m_{j-i})$ and $q_{-i} = q_{-i}(m_{j-i}, m_{-ji})$):[45]

$$\left(P'(Q)q_i + w_{j-i} - c\right)\frac{\partial q_{-i}}{\partial w_{j-i}} + \left(P'(Q)q_{-i} - P'(\widehat{Q})\widehat{q}_{-i}\right)\frac{\partial q_i}{\partial w_{j-i}} = 0, \tag{B-5}$$

with $Q = q_i + q_{-i}$, $\widehat{Q} = \widehat{q}_{-i} + q_i$, and $\widehat{q}_{-i} = \max_q\{(P(q + q_i(m_{-ji}, m_{j-i})) - c - \hat{c})q\}$.[46]

We now evaluate the first-order condition (B-5) at the values of w_{j-i} and w_{-ji} obtained in the model with information transmission, given in (A-9). This yields

$$-\left(P'(\widehat{Q})\widehat{q}_{-i} - P'(Q)q_{-i}\right)\frac{\partial q_i}{\partial w_{j-i}}. \tag{B-6}$$

Since $\partial q_i/\partial w_{j-i} > 0$, the sign of (B-6) depends on the term in bracket. Morever, $q_{-i} > \widehat{q}_{-i}$ and $Q > \widehat{Q}$, together with the condition $P'(Q) + P''(Q)q_{-i} < 0$, imply that $|P'(Q)q_{-i}| > \left|P'(\widehat{Q})\widehat{q}_{-i}\right|$. Hence, the term in bracket is positive. It follows that evaluating (B-5) at the unit-prices in (A-9) is negative. As a consequence, the per-unit prices in the model without information transmission are lower than those with information transmission. Q.E.D.

The per-unit prices set by the pool are not affected by the assumption on the information transmission between integrated units. Thus, the pool sets the same unit price as in Lemma 6. Hence, we can confirm the result in Proposition 3 that the pool is anticompetitive. In fact, the anticompetitive result is stronger in the context without information transmission, due to the lower unit prices set under independent licensing.

[45] Note that, although M_i is not informed of M_{-i}'s acceptance decision, it knows the terms of the offer formulated by its upstream affiliate. Thus, q_i changes with w_{j-i}.

[46] The second-order condition is satisfied under our assumptions on the demand function.

B.3 Existence of a Marginal Contribution Equilibrium with Observable Offers

In this appendix, we provide the proof of the claim that a marginal contribution equilibrium does not exist in our common agency game with observable offers.

LEMMA B-3. *A marginal contribution equilibrium in pure strategies does not exist in the game featuring licensors L_A and L_B negotiating with a non-integrated manufacturer M_i with public offers.*

Proof. To begin with, we consider the incentive constraints that the fixed component of the tariffs offered by licensors must satisfy to be accepted by a manufacturer.

First, L_j must take into account that M_i will reject any offer at which L_j's fixed component of the tariff is larger than L_j's marginal contribution to the profit of the manufacturer:

$$\begin{aligned}\pi_i(m_i, m_{-i}) - F_{ji} - F_{-ji} &\geq \max_q\{(P(q + q_i(m_i, m_{-i})) - w_{-ji} - \hat{c})q\} - F_{-ji},\\ \pi_i(m_i, m_{-i}) - F_{ji} &\geq \max_q\{(P(q + q_i(m_i, m_{-i})) - w_{-ji} - \hat{c})q\},\end{aligned}$$

implying that

$$F_{ji} \leq \pi_i(m_i, m_{-i}) - \max_q\{(P(q + q_i(m_i, m_{-i})) - w_{-ji} - \hat{c})q\}. \tag{B-7}$$

Moreover, L_j must take into account that M_i can reject both licensors' offers and use the two status-quo technologies:

$$\pi_i(m_i, m_{-i}) - F_{ji} - F_{-ji} \geq \max_q\{(P(q + q_i(m_i, m_{-i})) - 2\hat{c})q\},$$

or

$$F_{ji} \leq \pi_i(m_i, m_{-i}) - \max_q\{(P(q + q_i(m_i, m_{-i})) - 2\hat{c})q\} - F_{-ji}. \tag{B-8}$$

We proceed by deriving the condition for the existence of a unique marginal contribution equilibrium. Specifically, let

$$F_{-ji} = \pi_i(m_i, m_{-i}) - \max_q\{(P(q + q_i(m_i, m_{-i})) - w_{ji} - \hat{c})q\}.$$

We plug this value of F_{-ji} into (B-8) to get:

$$F_{ji} \leq \max_q\{(P(q + q_i(m_i, m_{-i})) - w_{ji} - \hat{c})q\} - \max_q\{(P(q + q_i(m_i, m_{-i})) - 2\hat{c})q\}.$$

Therefore, for a marginal contribution equilibrium to be unique, constraint (B-7) must be more binding than (B-8). Using licensors' symmetry:

$$\begin{aligned}\pi_i(m_i, m_{-i}) \leq\ & \max_q\{(P(q + q_i(m_i, m_{-i})) - 2\hat{c})q\}\\ &+2\max_q\{(P(q + q_i(m_i, m_{-i})) - w_{ji} - \hat{c})q\}.\end{aligned} \tag{B-9}$$

Condition (B-9) is a superadditivity condition, it corresponds to the condition guaranteeing that truthful equilibria are marginal contribution equilibria in Laussel and Le Breton (2001) and Bergemann and Välimäki (2003), and is satisfied if a firm's profit function is concave in its marginal costs. However, as we show in the proof of Lemma 1, profits are convex in costs, which implies a violation of (B-9). Q.E.D.

C For Online Publication: Additional Proofs

In this appendix, we report the proofs of the results obtained under the assumption that contracts are observable contracts (Section 6), and non-discriminatory offers (Section 7)—that is, Propositions 4–8 and Lemmata 7–9.

Proof of Lemma 7. By the proof of Lemma B-3 in the Online Appendix B, the constraint on the fixed component of the tariff offered by L_j to manufacturer M_i can be written as

$$F_{ji} \leq \pi_i(m_i, m_{-i}) - F_{-ji}$$
$$-\max\left[\max_q\{(P(q + q_i(m_i, m_{-i})) - w_{-ji} - \hat{c})q\} - F_{-ji}, \max_q\{(P(q + q_i(m_i, m_{-i})) - 2\hat{c})q\}\right].$$

By the same token, the constraint to F_{-ji} is

$$F_{-ji} \leq \pi_i(m_i, m_{-i}) - F_{ji}$$
$$-\max\left[\max_q\{(P(q + q_i(m_i, m_{-i})) - w_{ji} - \hat{c})q\} - F_{ji}, \max_q\{(P(q + q_i(m_i, m_{-i})) - 2\hat{c})q\}\right].$$

To determine how licensors split rents in the negotiation with a manufacturer M_i, we first show that, in any equilibrium, only one licensor obtains its marginal contribution to the profits of a given manufacturer. Indeed, there can never be an equilibrium in which the first term in the squared bracket is larger than the second term for both licensors. The reason is that this would imply that both licensors extract respective marginal contribution, which would be unacceptable for M_i (by Lemma B-3). Moreover, it cannot exist an equilibrium in which none of the licensors obtains its marginal contribution to M_i's profits. The reason is that either manufacturer, say L_{-j}, can claim its marginal contribution to M_i's profits and be better off.

We then need to consider two cases. The first case is the one in which L_j obtains its marginal contribution on both manufacturers M_i and M_{-i}. The second is the one in which L_j obtains its marginal contribution to M_i's profits, and L_{-j} its marginal contribution to M_{-i}'s profits.

<u>*Case 1*</u>: In the first case, L_j solves

$$\max_{w_{ji}, w_{j-i}, F_{ji}, F_{j-i}} (w_{ji} - c)q_i(m_i, m_{-i}) + (w_{j-i} - c)q_{-i}(m_{-i}, m_i) + F_{j-i} + F_{ji}. \tag{C-1}$$

under

$$\begin{aligned} F_{ji} &= \pi_i(m_i, m_{-i}) - \max_q\{(P(q + q_{-i}(m_{-i}, m_i)) - \hat{c} - w_{-ji})q\} \\ F_{j-i} &= \pi_{-i}(m_{-i}, m_i) - \max_q\{(P(q + q_i(m_i, m_{-i})) - \hat{c} - w_{-j-i})q\}. \end{aligned}$$

Licensor L_{-j}, instead, solves

$$\max_{w_{-ji}, w_{-j-i}, F_{-ji}, F_{-j-i}} (w_{-ji} - c)q_i(m_i, m_{-i}) + (w_{-j-i} - c)q_{-i}(m_{-i}, m_i) + F_{-j-i} + F_{-ji}, \tag{C-2}$$

under

$$\begin{aligned} F_{-ji} &= \pi_i(m_i, m_{-i}) - \max_q\{(P(q + q_{-i}(m_{-i}, m_i)) - 2\hat{c})q\} - F_{ji} \\ F_{-j-i} &= \pi_{-i}(m_{-i}, m_i) - \max_q\{(P(q + q_i(m_i, m_{-i})) - 2\hat{c})q\} - F_{j-i}. \end{aligned}$$

After plugging the fixed component of the tariffs into the objective functions in (C-1)-(C-2), we find that the first-order conditions with respect to w_{ji} and w_{j-i} are given by, respectively,

$$\begin{aligned} &(w_{ji} - c + P'(Q)q_{-i} - P'(\widehat{Q})\widehat{q}_{-i})\frac{\partial q_i}{\partial w_{ji}} + (w_{j-i} - c + P'(Q)q_i - P'(\widehat{Q})\widehat{q}_i)\frac{\partial q_{-i}}{\partial w_{ji}} = 0, \\ &(w_{ji} - c + P'(Q)q_{-i} - P'(\widehat{Q})\widehat{q}_{-i})\frac{\partial q_i}{\partial w_{j-i}} + (w_{j-i} - c + P'(Q)q_i - P'(\widehat{Q})\widehat{q}_i)\frac{\partial q_{-i}}{\partial w_{j-i}} = 0, \end{aligned}$$

where $\widehat{q}_i = \arg\max_q\{(P(q + q_{-i}(m_{-i}, m_i)) - w_{-ji} - \hat{c})q\}$ and $\widehat{Q} = \widehat{q}_i + q_{-i}(m_{-i}, m_i)$.

The first-order conditions with respect to w_{-ji} and w_{-j-i} are equal to, respectively,

$$\begin{aligned} &(w_{-ji} - c + P'(Q)q_{-i} - P'(\widetilde{Q})\widetilde{q}_{-i})\frac{\partial q_i}{\partial w_{-ji}} + (w_{-j-i} - c + P'(Q)q_i - P'(\widetilde{Q})\widetilde{q}_i)\frac{\partial q_{-i}}{\partial w_{-ji}} = 0, \\ &(w_{-ji} - c + P'(Q)q_{-i} - P'(\widetilde{Q})\widetilde{q}_{-i})\frac{\partial q_i}{\partial w_{-j-i}} + (w_{-j-i} - c + P'(Q)q_i - P'(\widetilde{Q})\widetilde{q}_i)\frac{\partial q_{-i}}{\partial w_{-j-i}} = 0, \end{aligned}$$

with $\widetilde{q}_i = \arg\max_q\{(P(q + q_{-i}(m_{-i}, m_i)) - 2\hat{c})q\}$ and $\widetilde{Q} = \widetilde{q}_i + q_{-i}(m_{-i}, m_i)$.[47] Recall that

$$\frac{\partial q_i}{\partial w_{ji}} = \frac{2P'(Q) + q_{-i}P''(Q)}{P'(Q)\,(3P'(Q) + QP''(Q))}, \; \frac{\partial q_{-i}}{\partial w_{ji}} = -\frac{P'(Q) + q_{-i}P''(Q)}{P'(Q)\,(3P'(Q) + QP''(Q))}. \tag{C-3}$$

Thus, solving for the unit prices set by L_j and L_{-j} yields

$$w_{ji} = c + P'(\widehat{Q})\widehat{q}_{-i} - P'(Q)q_{-i}, \;\; w_{j-i} = c + P'(\widehat{Q})\widehat{q}_i - P'(Q)q_i, \tag{C-4}$$

$$w_{-ji} = c + P'(\widetilde{Q})\widetilde{q}_{-i} - P'(Q)q_{-i}, \;\; w_{-j-i} = c + P'(\widetilde{Q})\widetilde{q}_i - P'(Q)q_i. \tag{C-5}$$

To further simplify these expressions, we use the first-order conditions for the equilibrium quantities, which imply that $P'(Q)q_i = -P(Q) + w_{ji} + w_{-ji}$, $P'(\widehat{Q})\widehat{q}_i = -P(\widehat{Q}) + \hat{c} + w_{-ji}$ and $P'(\widetilde{Q})\widetilde{q}_i = -P(\widetilde{Q}) + 2\hat{c}$. This allows us to solve for the sum of the per-unit prices charged by L_j and by L_{-j}. We obtain

$$\begin{aligned} w_{ji} + w_{j-i} &= P(Q) - P(\widehat{Q}) + c + \hat{c}, \\ w_{-ji} + w_{-j-i} &= \frac{1}{2}(P(Q) + P(\widehat{Q}) - 2P(\widetilde{Q}) + c + 3\hat{c}). \end{aligned}$$

We now show that $w_{ji} + w_{j-i} < 2\hat{c}$, $w_{-ji} + w_{-j-i} < 2\hat{c}$, and $w_{ji} > c$ for all $j = A, B$ and

[47]The second-order conditions are satisfied under our assumptions on the demand function.

$i = 1, 2$. Let us first look at $w_{ji} + w_{j-i}$. This sum is strictly lower than $2\hat{c}$ since $\hat{c} > c$ and $P(Q) < P(\widehat{Q})$ (because $\widehat{Q} < Q$).

We now show that w_{ji} (and, by symmetry, w_{j-i}) is larger than c at an equilibrium. From (C-4), $w_{ji} > c$ if and only if $P'(\widehat{Q})\widehat{q}_{-i} - P'(Q)q_{-i} > 0$. $P'(\widehat{Q})\widehat{q}_{-i}$ and $P'(Q)q_{-i}$ differ only because M_{-i} pays $\hat{c}$ for input j in $P'(\widehat{Q})\widehat{q}_{-i}$ and $w_{j-i} < \hat{c}$ in $P'(Q)q_{-i}$. Indeed, M_i cannot observe whether M_{-i} rejected L_j's offer, thus the value of its quantity is the same in Q and $\widehat{Q}$ and equal to $q_i(m_i, m_{-i})$. At the same time, the values of $m_i = m_{ji} + m_{-ji}$ and w_{-j-i} are the same in $P'(\widehat{Q})\widehat{q}_{-i}$ and $P'(Q)q_{-i}$ (by the definition of $\widehat{q}_{-i}$). Therefore, $P'(\widehat{Q})\widehat{q}_{-i} - P'(Q)q_{-i} > 0$ holds true if and only if $P'(Q)q_{-i} < 0$ is increasing in w_{j-i}. Formally, this requires that

$$\frac{\partial \left(P'(Q)q_{-i}\right)}{\partial w_{j-i}} = \left(P'(Q) + P''(Q)q_{-i}\right) \frac{\partial q_{-i}}{\partial w_{j-i}} > 0,$$

which holds true due to profit functions' (strict) quasi-concavity. Indeed, this property implies that $(P'(Q) + P''(Q)q_{-i}) < 0$ and, from the first-order condition for q_{-i}, that

$$\frac{\partial q_{-i}}{\partial w_{j-i}} = \frac{1}{2P'(Q) + P''(Q)q_{-i}} < 0.$$

Hence, $w_{ji} > c$.

We now turn to the analysis of $w_{-ji} + w_{-j-i}$, which it is lower than $2\hat{c}$ if and only if

$$P(Q) + P(\widehat{Q}) - 2P(\widetilde{Q}) < \hat{c} - c.$$

This inequality holds true because the right-hand side is positive ($\hat{c} - c > 0$), instead the left-hand side is negative as $P(Q) \leq P(\widehat{Q})$ and

$$\begin{aligned} P(\widehat{Q}) < P(\widetilde{Q}) \quad &\iff \quad \widehat{Q} > \widetilde{Q} \\ &\iff \quad q_{-i}(m_{-i}, m_i) + \widehat{q} > q_{-i}(m_{-i}, m_i) + \widetilde{q} \\ &\iff \quad \widehat{q} > \widetilde{q} \ \ \forall \hat{c} > c. \end{aligned}$$

Finally, one can use the expressions in (C-5) to show that w_{-ji} (and w_{ji}) lies above c if and only if $P'(Q) + P''(Q)q_{-i} < 0$, for all $i = 1, 2$.

Case 2: In the second case, a licensor L_j obtains its marginal contribution to M_i's profits, and L_{-j} its contribution to M_{-i}'s profits. The value of F_{ji} and F_{j-i} is then equal to

$$\begin{aligned} F_{ji} &= \pi_i(m_i, m_{-i}) - \max_q\{(P(q + q_{-i}(m_{-i}, m_i)) - \hat{c} - w_{-ji})q\} \\ F_{j-i} &= \pi_{-i}(m_{-i}, m_i) - \max_q\{(P(q + q_i(m_i, m_{-i})) - 2\hat{c})q\} - F_{j-i}. \end{aligned}$$

and those of L_{-j} are

$$\begin{aligned} F_{-ji} &= \pi_i(m_i, m_{-i}) - \max_q\{(P(q + q_{-i}(m_{-i}, m_i)) - 2\hat{c})q\} - F_{ji} \\ F_{-j-i} &= \pi_{-i}(m_{-i}, m_i) - \max_q\{(P(q + q_i(m_i, m_{-i})) - \hat{c} - w_{j-i})q\}. \end{aligned}$$

Plugging these expressions into (C-1) and (C-2), we find that the first-order conditions with respect to w_{ji} and w_{j-i} are equal to, respectively,

$$(w_{ji} - c + P'(Q)q_{-i} - P'(\widetilde{Q})\widetilde{q}_{-i})\frac{\partial q_i}{\partial w_{ji}} + (w_{j-i} - c + P'(Q)q_i - P'(\widehat{Q})\widehat{q}_i)\frac{\partial q_{-i}}{\partial w_{ji}} = 0,$$
$$(w_{ji} - c + P'(Q)q_{-i} - P'(\widetilde{Q})\widetilde{q}_{-i})\frac{\partial q_i}{\partial w_{j-i}} + (w_{j-i} - c + P'(Q)q_i - P'(\widehat{Q})\widehat{q}_i)\frac{\partial q_{-i}}{\partial w_{j-i}} = 0,$$

and those for w_{-ji} and w_{-j-i} are

$$(w_{-ji} - c + P'(Q)q_{-i} - P'(\widehat{Q})\widehat{q}_{-i})\frac{\partial q_i}{\partial w_{-ji}} + (w_{-j-i} - c + P'(Q)q_i - P'(\widetilde{Q})\widetilde{q}_i)\frac{\partial q_{-i}}{\partial w_{-ji}} = 0,$$
$$(w_{-ji} - c + P'(Q)q_{-i} - P'(\widehat{Q})\widehat{q}_{-i})\frac{\partial q_i}{\partial w_{-j-i}} + (w_{-j-i} - c + P'(Q)q_i - P'(\widetilde{Q})\widetilde{q}_i)\frac{\partial q_{-i}}{\partial w_{-j-i}} = 0.$$

Thus, the unit prices set by L_j and L_{-j} are[48]

$$w_{ji} = c + P'(\widetilde{Q})\widetilde{q}_{-i} - P'(Q)q_{-i},\ w_{j-i} = c + P'(\widehat{Q})\widehat{q}_i - P'(Q)q_i,$$
$$w_{-ji} = c + P'(\widehat{Q})\widehat{q}_{-i} - P'(Q)q_{-i},\ w_{-j-i} = c + P'(\widetilde{Q})\widetilde{q}_i - P'(Q)q_i.$$

Using the first-order conditions for the equilibrium product-market quantities, we find that $w_{ji} + w_{-ji} + w_{j-i} + w_{-j-i} = 2(c + \hat{c} + P(Q) - P(\widehat{Q}))$. The value of this sum is strictly lower than $4\hat{c}$ if and only if $P(Q) - P(\widehat{Q}) < \hat{c} - c$, which holds true because $P(\widehat{Q}) > P(Q)$ for all $\hat{c} - c >$. At the same time, as shown above, $P'(Q) + P''(Q)q_{-i} < 0$ implies that all unit prices are strictly larger than c. Q.E.D.

Proof of Lemma 8. The pool solves:

$$\max_{w_{pi}, w_{p-i}, F_{pi}, F_{p-i}} (w_{pi} - 2c)q_i(m_i^p, m_{-i}^p) + (w_{p-i} - 2c)q_{-i}(m_{-i}^p, m_i^p) + F_{p-i} + F_{pi},$$

where $m_i^p = w_{pi} - 2c$, and

$$\begin{aligned} F_{pi} &= \pi_i(m_i^p, m_{-i}^p) - \max_q\{(P(q + q_{-i}(m_{-i}^p, m_i^p)) - 2\hat{c})q\} \\ F_{p-i} &= \pi_{-i}(m_{-i}^p, m_i^p) - \max_q\{(P(q + q_i(m_i^p, m_{-i}^p)) - 2\hat{c})q\}. \end{aligned}$$

After plugging F_{pi} and F_{p-i} into the maximand, we find that the first-order condition with respect to w_{pi} is given by:[49]

$$(w_{pi} - 2c + P'(Q^p)q_{-i}^p - P'(\widetilde{Q}^p)\widetilde{q}_{-i}^p)\frac{\partial q_i^p}{\partial w_{pi}} + (w_{p-i} - 2c + P'(Q^p)q_i^p - P'(\widetilde{Q}^p)\widetilde{q}_i^p)\frac{\partial q_{-i}^p}{\partial w_{pi}} = 0,$$

[48] Also in this second case, the second-order conditions are satisfied under our assumptions on the demand function.

[49] The first-order condition with respect to w_{p-i} is analogous and therefore omitted.

where $q_i^p = \arg\max_q\{(P(q+q_{-i}(m_{-i}^p, m_i^p)) - w_{pi})\}$, $Q^p = q_{-i}(m_{-i}^p, m_i^p) + q_i^p$, $\widetilde{q}_i^p = \arg\max_q\{(P(q+q_{-i}(m_{-i}^p, m_i^p)) - 2\hat{c})\}$, and $\widetilde{Q}^p = q_{-i}(m_{-i}^p, m_i^p) + \widetilde{q}_i^p$.[50]

Since

$$\frac{\partial q_i}{\partial w_{pi}} = \frac{2P'(Q) + q_{-i}P''(Q)}{P'(Q)\,(3P'(Q) + QP''(Q))}, \frac{\partial q_{-i}}{\partial w_{pi}} = -\frac{P'(Q) + q_{-i}P''(Q)}{P'(Q)\,(3P'(Q) + QP''(Q))},$$

solving for w_{pi} and w_{p-i}, we find that

$$w_{pi} = 2c + P'(\widetilde{Q}^p)\widetilde{q}_{-i}^p - P'(Q^p)q_{-i}^p,\ \ w_{p-i} = 2c + P'(\widetilde{Q}^p)\widetilde{q}_{-i}^p - P'(Q^p)q_{-i}^p.$$

Proceeding as in the proof of Lemma 7, we can show that $P''(Q)q_{-i} + P'(Q) < 0$ implies $w_{p1}, w_{p2} > c$.

Finally, using the first-order conditions for the product market quantities, we obtain

$$\begin{aligned} w_{pi} + w_{p-i} &= 2c + 2\hat{c} + P(Q^p) - P(\widetilde{Q}^p), \\ &\leq 4\hat{c} \iff 2c + P(Q^p) \leq 2\hat{c} + P(\widetilde{Q}^p), \end{aligned}$$

which holds true for all $P(\widetilde{Q}^p) > P(Q^p)$ and $\hat{c} - c > 0$ (since $\widetilde{Q}^p < Q^p$). Q.E.D.

Proof of Proposition 4. To determine the competitive effects of the pool, we compare the unit price set to M_i under the pool (i.e., w_{pi} from Lemma 7) with the sum of the unit prices offered to M_i without the pool (i.e., $w_{ji} + w_{-ji}$ from Lemma B-3). By symmetry, the same argument applies to M_{-i}. If the unit prices paid by manufacturers are higher in one of the two cases the associated equilibrium quantities will be lower, thereby implying a less competitive industry.

From the proof of Lemma 8, w_{pi} is implicitly determined by the first-order condition

$$(w_{pi} - 2c + P'(Q^p)q_{-i}^p - P'(\widetilde{Q}^p)\widetilde{q}_{-i}^p)\frac{\partial q_i^p}{\partial w_{pi}} + (w_{p-i} - 2c + P'(Q^p)q_i^p - P'(\widetilde{Q}^p)\widetilde{q}_i^p)\frac{\partial q_{-i}^p}{\partial w_{pi}} = 0. \quad \text{(C-6)}$$

From the proof of Lemma 7, w_{ji} and w_{-ji} are also implicitly determined by their respective first-order conditions. Using the fact that, by symmetry,

$$\frac{\partial q_i}{\partial w_{-ji}} = \frac{\partial q_i}{\partial w_{ji}} \quad \text{and} \quad \frac{\partial q_{-i}}{\partial w_{-ji}} = \frac{\partial q_{-i}}{\partial w_{ji}},$$

we can sum up the first-order conditions for w_{ji} and w_{-ji}. Thus, independently of which equilibrium is played without the pool, we obtain

$$\begin{aligned} &(w_{ji} + w_{-ji} - 2c + 2P'(Q)q_{-i} - P'(\widehat{Q})\widehat{q}_{-i} - P'(\widetilde{Q})\widetilde{q}_{-i})\frac{\partial q_i}{\partial w_{ji}} \\ &\quad + (w_{j-i} + w_{-j-i} - 2c + 2P'(Q)q_i - P'(\widehat{Q})\widehat{q}_i - P'(\widetilde{Q})\widetilde{q}_i)\frac{\partial q_{-i}}{\partial w_{ji}} = 0. \quad \text{(C-7)} \end{aligned}$$

[50] The second-order conditions are satisfied under our assumptions on the demand function $P(\cdot)$.

We can now evaluate (C-7) at $w_{ji} + w_{-ji} = w_{pi}$ and $w_{j-i} + w_{-j-i} = w_{p-i}$. Then, we also need to replace q_i, q_{-i}, $\widehat{q}_i$, $\widehat{q}_{-i}$, $\widetilde{q}_i$, and $\widetilde{q}_{-i}$ with the respective quantities with the pool. Note that this also implies that $\partial q_i/\partial w_{ji}$ and $\partial q_{-i}/\partial w_{ji}$ must be replaced by $\partial q_i^p/\partial w_{-ji}$ and $\partial q_{-i}^p/\partial w_{-ji}$, respectively. Using (C-6) in (C-7) then yields

$$(P'(Q^p)q_{-i}^p - P'(\widehat{Q}^p)\widehat{q}_{-i}^p)\frac{\partial q_i^p}{\partial w_{pi}} + (P'(Q^p)q_i^p - P'(\widehat{Q}^p)\widehat{q}_i^p)\frac{\partial q_{-i}^p}{\partial w_{pi}}.$$

Because of symmetry, $q_{-i}^p = q_i^p$ and $\widehat{q}_{-i}^p = \widehat{q}_i^p$, which implies that the latter expression can be further reduced into

$$(P'(Q^p)q_i^p - P'(\widehat{Q}^p)\widehat{q}_i^p)\left(\frac{\partial q_i^p}{\partial w_{pi}} + \frac{\partial q_{-i}^p}{\partial w_{pi}}\right). \tag{C-8}$$

Using (C-3), $\partial q_i^p/\partial w_{pi} < 0$ and $\partial q_{-i}^p/\partial w_{pi} > 0$. However, by a standard property of the Cournot equilibrium, $|\partial q_i^p/\partial w_{pi}| > \left|\partial q_{-i}^p/\partial w_{pi}\right|$. It follows that the second term in (C-8) is negative. Moreover, from the proof of Lemma 7 if follows that the term in the first bracket is also negative (as $P'(Q) + P''(Q)q_{-i} < 0$). Thus, the sum of the equilibrium values of w_{ji} and w_{-ji} without pool must be larger than the unit price without pool (w_{pi}). Hence, the pool is procompetitive. Q.E.D.

Proof of Lemma 9. First note that L_A-M_1 sets the internal transfer price equal to marginal cost ($w_{A1} = c$). We now analyze the problem of L_A-M_1 when dealing with M_2, under the assumption that L_A-M_1 obtains its marginal contribution to M_2's profits. The maximization problem of L_A-M_1 is

$$\max_{w_{A2}, F_{A2}} \pi_1(m_{B1}, m_2) + (w_{A2} - c)q_2(m_2, m_{B1}) + F_{A2}, \tag{C-9}$$

subject to $F_{A2} = \pi_2(m_2, m_{B1}) - \pi_2(e + m_{B2}, m_{B1})$. Licensor L_B solves the same problem as in the second case considered in the proof of Lemma 7,[51] using $w_{A1} = c$ and

$$\begin{aligned} F_{B1} &= \pi_1(m_{B1}, m_2) - \max_q\{(P(q + q_2(m_2, m_{B1})) - \hat{c} - c)q\} \\ F_{B2} &= \pi_2(m_2, m_{B1}) - \max_q\{(P(q + q_1(m_{B1}, e + m_{B2})) - 2\hat{c})q\} - F_{ji}. \end{aligned}$$

From the proof of Lemma 7, we know that

$$w_{B1} = c - P'(Q)q_2 + P'(q_1 + \widetilde{q}_2)\widetilde{q}_2 \quad \text{and} \quad w_{B2} = c - P'(Q)q_1 + P'(\widehat{q}_1 + q_2)\widehat{q}_1, \tag{C-10}$$

where $\widetilde{q}_2 = \arg\max_q \{(P(q + q_1(m_{B1}, m_2)) - w_{A2} - \hat{c})q\}$ and

$$\widehat{q}_1 = \arg\max_q \{(P(q + q_2(m_2, m_{B1})) - c - \hat{c})q\}.$$

We now turn to the problem of L_A-M_1. Plugging F_{A2} in (C-9), and taking the derivative

[51] The reason is that L_B obtains its marginal contribution only from M_1.

with respect to w_{A2}, we obtain the following expression:

$$\left(P'(Q)q_1 + w_{A2} - c\right)\frac{\partial q_2}{\partial w_{A2}} + P'(Q)q_2\frac{\partial q_1}{\partial w_{A2}}. \tag{C-11}$$

We will first show that setting $w_{A2} = \hat{c}$ is an equilibrium (*Step 1*). Afterwards, in *Step 2*, we will prove that it is the unique stable equilibrium.

Step 1: To show that $w_{A2} = \hat{c}$ is an equilibrium, we use the product-market first-order conditions:

$$P(Q) - c - w_{B1} + P'(Q)q_1 = 0 \quad \text{and} \quad P(Q) - w_{A2} - w_{B2} + P'(Q)q_2 = 0.$$

Combining them yields $P'(Q)q_1 = c + w_{B1} - w_{A2} - w_{B2} + P'(Q)q_2$. Inserting this expression into (C-11) gives

$$\left(w_{B1} - w_{B2}\right)\frac{\partial q_2}{\partial w_{A2}} + P'(Q)q_2\left(\frac{\partial q_2}{\partial w_{A2}} - \frac{\partial q_1}{\partial w_{A2}}\right). \tag{C-12}$$

Since $|\partial q_2/\partial w_{A2}| > |\partial q_1/\partial w_{A2}|$ and $\partial q_2/\partial w_{A2} < 0$, it follows that, if $w_{B1} \leq w_{B2}$, (C-12) is strictly positive. This means that, if $w_{B1} \leq w_{B2}$, the integrated firm's profit increase in w_{A2}, and L_A-M_1 optimally sets $w_{A2} = \hat{c}$.

We now show that, given $w_{A2} = \hat{c}$, it is optimal for L_B to set $w_{B1} \leq w_{B2}$. Given $w_{A2} = \hat{c}$, we have $q_1 > q_2$ and $\widehat{q}_1 > \widetilde{q}_2$ for any $w_{Bi} \leq \hat{c}$, $i = 1, 2$.[52] In addition, since quantities are convex in costs, the difference between q_1 and $\widehat{q}_1$ and larger than the one between q_2 and $\widetilde{q}_2$ (i.e., $q_1 - \widehat{q}_1 > q_2 - \widetilde{q}_2$). Using the condition $P'(Q) + P''Q < 0$ then yields that $w_{B1} \leq w_{B2}$.

As a consequence, at equilibrium L_A-M_1 sets $w_{A2} = \hat{c}$ and L_B sets $w_{B1} \leq w_{B2}$.

Step 2: We now prove that there is no other stable equilibrium other than the one featuring $w_{A2} = \hat{c}$. First, the first-order condition for w_{A2}, given by

$$\left(P'(Q)q_1 + w_{A2} - c\right)\frac{\partial q_2}{\partial w_{A2}} + P'(Q)q_2\frac{\partial q_1}{\partial w_{A2}} = 0, \tag{C-13}$$

implies that, at an interior equilibrium, the value of w_{A2} is

$$w_{A2} = c - P'(Q)q_1 + \frac{P'(Q)q_2\left(P'(Q) + P''(Q)q_1\right)}{2P'(Q) + P''(Q)q_1}. \tag{C-14}$$

An interior equilibrium value of w_{A2} is unstable if $|dw_{A2}/dw_{B1}| \geq 1$. Since

$$\frac{dw_{A2}}{dw_{B1}} = -\frac{\frac{\partial^2 \Pi_{L_A - M_1}}{\partial w_{A2}\partial w_{B1}}}{\frac{\partial^2 \Pi_{L_A - M_1}}{\partial w_{A2}^2}},$$

[52] Given $w_{A2} = \hat{c}$ and $m_{A1} = 0$, $q_2 < q_1$ for all $w_{B2} \geq w_{B1}$. Let, instead, $w_{B2} < w_{B1}$. Specifically, assume that $w_{B1} = \hat{c}$ and $w_{B2} = c$. Then, from the definition of a quantity q_i in (1), we have that $q_1 = q_2$. However, for all $w_{B1} < \hat{c}$, we have $q_1 > q_2$. To show that $\widetilde{q}_2 < \widehat{q}_1$ for all w_{B1}, w_{B2}, we proceed analogously and let $w_{B1} = \hat{c}$ and $w_{B2} = c$: We get that $\widetilde{q}_2 = \arg\max_q \{(P(q + q_1(e,e)) - 2\hat{c})q\} < \widehat{q}_1 = \arg\max_q \{(P(q + q_2(e,e)) - c - \hat{c})q\}$.

then $|dw_{A2}/dw_{B1}| \geq 1$ if and only if $|\partial^2\Pi_{L_A-M_1}/\partial w_{A2}\partial w_{B1}| - |\partial^2\Pi_{L_A-M_1}/\partial w_{A2}^2| \geq 0$.

Given (C-11), we compute

$$\begin{aligned}\frac{\partial^2\Pi_{L_A\text{-}M_1}}{\partial w_{A2}^2} &= \left(P'(Q)\frac{\partial q_1}{\partial w_{A2}} + P''(Q)\frac{\partial Q}{\partial w_{A2}} + 1\right)\frac{\partial q_2}{\partial w_{A2}} + \left(P'(Q)q_1 + w_{A2} - c\right)\frac{\partial^2 q_2}{\partial w_{A2}^2} \\ &+ \left(P'(Q)\frac{\partial q_2}{\partial w_{A2}} + P''(Q)q_2\frac{\partial Q}{\partial w_{A2}}\right)\frac{\partial q_1}{\partial w_{A2}} + P'(Q)q_2\frac{\partial^2 q_1}{\partial w_{A2}^2} \qquad \text{(C-15)}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial^2\Pi_{L_A\text{-}M_1}}{\partial w_{A2}\partial w_{B1}} &= \left(P'(Q)\frac{\partial q_1}{\partial w_{B1}} + P''(Q)\frac{\partial Q}{\partial w_{B1}}\right)\frac{\partial q_2}{\partial w_{A2}} + \left(P'(Q)q_1 + w_{A2} - c\right)\frac{\partial^2 q_2}{\partial w_{A2}\partial w_{B1}} \\ &+ \left(P'(Q)\frac{\partial q_2}{\partial w_{B1}} + P''(Q)q_2\frac{\partial Q}{\partial w_{B1}}\right)\frac{\partial q_1}{\partial w_{A2}} + P'(Q)q_2\frac{\partial^2 q_1}{\partial w_{A2}\partial w_{B1}}. \qquad \text{(C-16)}\end{aligned}$$

Using

$$\frac{\partial q_i}{\partial w_{ji}} = \frac{2P'(Q) + q_{-i}P''(Q)}{P'(Q)\left(3P'(Q) + QP''(Q)\right)},\ \frac{\partial q_{-i}}{\partial w_{ji}} = -\frac{P'(Q) + q_{-i}P''(Q)}{P'(Q)\left(3P'(Q) + QP''(Q)\right)},$$

we can then compute $\partial^2 q_i/\partial w_{A2}^2$ and $\partial^2 q_i/\partial w_{A2}\partial w_{B1}$, for all $i = 1, 2$. Plugging the resulting expressions, together with the value of w_{A2} in (C-14), into (C-15) and (C-16), we obtain that

$$\begin{aligned}&\text{sign}\left\{\left|\frac{\partial^2\Pi_{L_A-M_1}}{\partial w_{A2}\partial w_{B1}}\right| - \left|\frac{\partial^2\Pi_{L_A-M_1}}{\partial w_{A2}^2}\right|\right\} = \\ &\qquad\qquad \text{sign}\left\{P'(Q)\left(P'(Q) + P''(Q)q_1 - \frac{P'(Q)P''(Q)q_2}{2P'(Q) + P''(Q)q_1}\right)\right\},\end{aligned}$$

which is positive for any $P''(Q) \in (-\infty, -P'(Q)/(q_1 + q_2))$. Hence, an interior equilibrium is unstable.

The only stable equilibria can then be at extreme values for w_{A2}. Ruling out below-cost pricing, the lower bound for w_{A2} is c, whereas the upper bound is $\hat{c}$. We have already shown in *Step 1* that $w_{A2} = \hat{c}$ constitutes an equilibrium. Were $w_{A2} = c$, it follows from (C-10) that $\widetilde{q}_2 = \widehat{q}_1$, which in turn leads to $w_{B1} = w_{B2}$. However, we know from above that the unique best-response of firm L_A-M_1 in this case features setting $w_{A2} = \hat{c}$, contradicting $w_{A2} = c$. Therefore, $w_{A2} = \hat{c}$ is the unique stable equilibrium.[53]

Finally, as in the proof of Lemma 7, one can also show that, at this equilibrium, $w_{B1} > c$, $w_{B2} > c$ and $w_{B2} + w_{B1} < 2\hat{c}$. Turning to the fixed components of the tariffs, the equilibrium value of F_{A2} is equal to zero. Moreover, L_B sets $F_{B1} = \pi_1(m_{B1}, e+m_{B2}) - (P(q_1(e, e+m_{B2}) + q_2(e+m_{B2}, m_{B1})) - c - \hat{c})q_1(e, e+m_{B2})$, and $F_{B2} = \pi_2(e+m_{B2}, m_{B1}) - (P(q_2(2e, m_{B1}) + q_1(m_{B1}, e+m_{B2})) - 2\hat{c})q(2e, m_{B1})$. Q.E.D.

Proof of Proposition 5. The sum of the per-unit unit prices with the pool is $2c + 2\hat{c}$ (Lemma 10). Without the pool, the equilibrium in Lemma 9 features L_A-M_1 setting the internal

[53]Proceeding in the same way, we can show that the same result holds if L_A-M_1 does not obtain its marginal contribution from M_2.

transfer price equal to marginal cost and $w_{A2} = \hat{c}$. Moreover, L_B offers:

$$w_{B1} = c - P'(Q)q_2 + P'(q_1 + \widetilde{q}_2)'\widetilde{q}_2 \quad \text{and} \quad w_{B2} = c - P'(Q)q_1 + P'(\widehat{q}_1 + q_2)\widehat{q}_1, \tag{C-17}$$

where $\widehat{q}_1$ and $\widetilde{q}_2$ are defined as in the proof of Lemma 9. The final-good market's first-order conditions of M_1 when buying from the efficient licensors and the status-quo, respectively, are

$$P(Q) - c - w_{B1} + P'(Q)q_1 = 0 \quad \text{and} \quad P(\widehat{q}_1 + q_2) - c - \hat{c} + P'(\widehat{q}_1 + q_2)\widehat{q}_1 = 0.$$

Similarly, the first-order conditions of M_2 when buying from the efficient licensors and the status quo for input B are

$$P(Q) - \hat{c} - w_{B2} + P'(Q)q_2 = 0 \quad \text{and} \quad P(q_1 + \widetilde{q}_2) - 2\hat{c} + P'(q_1 + \widetilde{q}_2)\widetilde{q}_2 = 0.$$

Plugging these expressions into (C-17), and summing up, yields

$$w_{B1} + w_{B2} = c + \hat{c} + P(Q) - \frac{P(\widehat{q}_1 + q_2) + P(q_1 + \widetilde{q}_2)}{2}.$$

It follows that

$$\begin{aligned} & w_{A2} + w_{A1} + c + \hat{c} + \left(P(Q) - \frac{P(\widehat{q}_1 + q_2) + P(q_1 + \widetilde{q}_2)}{2}\right) \\ = \quad & 2c + 2\hat{c} + \left(P(Q) - \frac{P(\widehat{q}_1 + q_2) + P(q_1 + \widehat{q}_2)}{2}\right) \\ < \quad & 2c + 2\hat{c}, \end{aligned}$$

since $P(\widehat{q}_1 + q_2) > P(Q)$ and $P(q_1 + \widetilde{q}_2) > P(Q)$.[54] Applying the Property 1 that the industry quantity is determined by the sum of unit costs, it follows that the pool is anticompetitive.

Finally, we show that L_A-M_1 and L_B join the pool. We compare the industry profit under the pool with the industry profit with independent licensing: The profit with the pool is $\pi_1(2e, 0) + 2eq_2(2e, 0)$, whereas the profit under independent licensing is $\pi_1(m_{B1}, e + m_{B2}) + \pi_2(e + m_{B2}, m_{B1}) + m_{B1}q_1(m_{B1}, e + m_{B2}) + m_{B2}q_2(e + m_{B2}, m_{B1}) + \overline{\overline{\pi}}_2^{pub}$, in which $m_{Bi} = w_{Bi} - c$, $i = 1, 2$, with $w_{Bi} > c$ and $w_{B1} + w_{B2} < 2\hat{c}$, and

$$\overline{\overline{\pi}}_2^{pub} = \max_q \{P(q + q_1(m_{B1}, e + m_{B2}) - 2\hat{c})q\}.$$

Proceeding as in the proof of Lemma 4, we find that the pool is profitable if

$$\begin{aligned} & \left(P(q_1(0, 2e) + q_2(2e, 0)) - 2c\right) q_1(0, 2e) + 2eq_2(2e, 0) \geq \\ & \left(P(q_1(m_{B1}, e + m_{B2}) + q_2(e + m_{B2}, m_{B1})) - 2c\right) \left(q_1(m_{B1}, e + m_{B2}) + q_2(e + m_{B2}, m_{B1})\right) \\ & \qquad - \left(P(q_1(m_{B1}, e + m_{B2}) + q_2(2e, m_{B1})) - 2\hat{c}\right) q_2(2e, m_{B1})). \end{aligned}$$

[54] Recall that $q_2 > \widetilde{q}_2$ and $q_1 > \widehat{q}_1$.

Moreover, since

$$(P(q_1(m_{B1}, e + m_{B2}) + q_2(2e, m_{B1})) - 2\hat{c})\, q_2(2e, m_{B1})) \geq \\ (P(q_1(m_{B1}, 2e) + q_2(2e, m_{B1})) - 2\hat{c})\, q_2(2e, m_{B1}))$$

and

$$(P(q_1(m_{B1}, 2e) + q_2(2e, m_{B1})) - 2\hat{c})\, q_2(2e, m_{B1})) \geq (P(q_1(0, 2e) + q_2(2e, 0)) - 2\hat{c})\, q_2(2e, 0)),$$

where the last inequality follows from $q_2(2e, m_{B1})) > q_2(2e, 0))$ and

$$P(q_1(m_{B1}, 2e) + q_2(2e, m_{B1}) \geq P(q_1(0, 2e) + q_2(2e, 0))$$

for all $w_{B1} > c$, a sufficient condition for the pool to be profitable prescribes that

$$(P(q_1(m_{B1}, e + m_{B2}) + q_2(e + m_{B2}, m_{B1})) - 2c)\,(q_1(m_{B1}, e + m_{B2}) + q_2(e + m_{B2}, m_{B1})) \leq \\ (P(q_1(0, 2e) + q_2(2e, 0)) - 2c)\,(q_1(0, 2e) + q_2(2e, 0)).$$

The result established above that the sum of unit prices under independent licensing, $c + \hat{c} + w_{B1} + w_{B2}$, is lower than the sum of unit prices with the pool, $2c + 2\hat{c}$, implies that the industry quantity with independent licensing $q_1(m_{B1}, e + m_{B2}) + q_2(e + m_{B2}, m_{B1})$ is larger than the industry quantity with the pool $q_1(0, 2e) + q_2(2e, 0)$. As a consequence, firms' profits increase with the pool. Q.E.D.

Proof of Proposition 6. Due to the strictly non-discriminatory rule, licensors are bound to offer the same tariff to both manufacturers. That is, at equilibrium manufacturers bear the same margin so that $m_{j1} = m_{j2} = w_j - c$, $m_1 = m_2 = m$ and $q_1 = q_2 = q$. We first consider the case of independent licensing. The maximization problem of L_j is

$$\max_{w_j, F_j} \; 2(w_j - c)q(m, m) + 2F_j. \tag{C-18}$$

Let L_j obtain its marginal contribution to the profit of both manufacturers:

$$F_j = \pi(m, m) - \max_q \{(P(q + q(m, m)) - \hat{c} - w_{-j})q\}.$$

Then, the maximization problem of L_{-j} is

$$\max_{w_{-j}, F_{-j}} \; 2(w_{-j} - c)q(m, m) + 2F_{-j},$$

with $F_{-j} = \pi(m, m) - \max_q \{(P(q + q(m, m)) - 2\hat{c})q\} - F_j$.

Solving for w_j and w_{-j}, and dropping functional notation, we obtain the following first-order conditions:

$$w_j \quad : \quad 2(w_j - c + P'(Q)q - P'(\widehat{Q})\widehat{q})\frac{\partial q}{\partial w_j} = 0 \tag{C-19}$$

$$w_{-j} \quad : \quad 2(w_{-j} - c + P'(Q)q - P'(\widetilde{Q})\widetilde{q})\frac{\partial q}{\partial w_{-j}} = 0 \tag{C-20}$$

with $\widehat{q} = \arg\max_q\{(P(q + q(m,m)) - w_{-j} - \hat{c})q\}$, $\widehat{Q} = q(m,m) + \widehat{q}$, $\widetilde{q} = \arg\max_q\{(P(q + q(m,m)) - 2\hat{c})q\}$ and $\widetilde{Q} = q(m,m) + \widetilde{q}$.[55] The conditions in (C-19)-(C-20) lead to the same equilibrium tariffs as the first-order conditions of the maximization problems in (C-1)-(C-2), after imposing symmetry.

The maximization problem of the pool is

$$\max_{w_p, F_p} \quad 2(w_p - 2c)q(m^p, m^p) + 2F_p,$$

with $m^p = w_p - 2c$ and $F_p = \pi(m^p, m^p) - \max_q\{(P(q+q(m^p, m^p)) - 2\hat{c})q\}$. The corresponding first-order condition is

$$2(w_p - 2c + P'(Q)q - P'(\widetilde{Q}^p)\widetilde{q}^p)\frac{\partial q^p}{\partial w_p} = 0, \tag{C-21}$$

where $q^p = \arg\max_q\{(P(q + q(m^p, m^p)) - w_p)\}$, $Q^p = 2q(m^p, m^p)$, $\widetilde{q}^p = \arg\max_q\{(P(q + q(m^p, m^p)) - 2\hat{c})\}$, and $\widetilde{Q}^p = q(m^p, m^p) + \widetilde{q}^p$. The condition in (C-21) leads to the same equilibrium tariffs as the first-order condition of the pool in the proof of Lemma 8, after imposing symmetry.

Then, with independent licensing as much as with the patent pool, the market allocations obtained under a non-discriminatory policy coincide with those under observable contracts. Q.E.D.

Proof of Proposition 7. First, recall that, under the non-discriminatory rule, at equilibrium manufacturers bear the same margin; thus, $m_{j1} = m_{j2} = w_j - c$, $m_1 = m_2 = m$ and $q_1 = q_2 = q$. Consider now the maximization problem of L_A-M_1, under the assumption that it obtains its marginal contribution to manufacturers' profit. Since the value of F_A set by L_A on M_1 drops out of the maximization problem, L_A-M_1 solves

$$\max_{w_A, F_A} \max_q\{(P(q + q(m,m)) - w_A - w_B)q\} + 2(w_A - c)q(m,m) + F_A$$

under $F_A = \pi(m,m) - \max_{q_2}\{(P(q_2 + q_1(m, e + m_B)) - \hat{c} - w_B)q_2\}$. Omitting functional notation, the problem's first-order condition is:

$$2\left(w_A - c + qP'(Q)\right)\frac{\partial q}{\partial w_A} - \check{q}_2 P'(\check{Q})\frac{\partial \check{q}_1}{\partial w_A} = 0, \tag{C-22}$$

[55]To derive $\partial q/\partial w_j$, we totally differentiate the first-order conditions on the product market, and get $\partial q/\partial w_j = 1/(3P'(Q) + P''(Q))$.

where $\check{q}_2 \equiv \arg\max_q\{(P(q+q_1(m,e+m_B)) - \hat{c} - w_B)q\}$ and $\check{q}_1 \equiv \arg\max_q\{(P(q+q_2(e+m_B,m)) - w_A - w_B)q\}$. In what follows, we assume $P''(\cdot) \approx 0$.

First, we show that the value of the unit price set by L_A-M_1, w_A, is such that $c < w_A < \hat{c}$. Using the product-market first-order conditions, we can rewrite the left-hand side of (C-22) as

$$\frac{2(w_A - c + qP'(Q))}{3P'(Q) + QP''(Q)} - \frac{\check{q}_2(2P'(\check{Q}) + \check{q}_2 P''(\check{Q}))}{3P'(\check{Q}) + \check{Q}P''(\check{Q})}. \tag{C-23}$$

Under the assumption $P''(\cdot) \approx 0$, we have that $P'(Q) = P'(\check{Q})$.

To prove that $w_A > c$, we plug $w_A = c$ into (C-23). The sign of the obtained expression simplifies to the sign of $q - \check{q}_0$. At $w_A = c$, we have $\check{q}_2 = q(e+m_B, m_B) < q = q(m_B, m_B)$. Thus, $w_A > c$ at equilibrium. To prove that $w_A < \hat{c}$, we instead plug $w_A = \hat{c}$ into (C-23). This gives us the following condition:

$$\frac{2(\hat{c} - c)}{3P'(Q)} + \frac{2}{3}(q - \check{q}_2) < 0 \iff e + qP'(Q) - \check{q}_2 P'(\check{Q}) > 0. \tag{C-24}$$

Using

$$P(Q) - w_A - w_B + P'(Q)q = 0 \quad \text{and} \quad P(\check{Q}) - \hat{c} - w_B + P'(Q)\check{q}_2 = 0,$$

condition (C-24) can be rewritten as $e + P(\check{Q}) - P(Q) > 0$, which holds true for all $e = \hat{c} - c > 0$, because $\check{Q} = Q$ at $w_A = \hat{c}$. Thus, $w_A < \hat{c}$ at equilibrium.

We now study the maximization problem of the non-integrated licensor L_B. Since L_A obtains its marginal contribution to manufacturers' profits, L_B's maximization problem is

$$\max_{w_B, F_B} \; 2(w_B - c)q(m,m) + 2F_B$$

with $F_B = \pi(m,m) - \max_{q_2}\{(P(q_2 + q_1(m, e+m_B)) - 2\hat{c})q_2\}$. The first-order condition with respect to w_B is

$$2\left(w_B - c + P'(Q)q\right)\frac{\partial q}{\partial w_B} - 2P'(\widetilde{Q})\widetilde{q}_2\frac{\partial \widetilde{q}_1}{\partial w_B} = 0, \tag{C-25}$$

where $\widetilde{q}_2 \equiv \arg\max_q(P(q+q_1(m,e+m_B)) - 2\hat{c})q$, $\widetilde{q}_1 \equiv \arg\max_q\{(P(q+q_2(e+m_B,m)) - w_A - w_B)q$, and $\widetilde{Q} = \widetilde{q}_1 + \widetilde{q}_2$.

We continue by showing that the value of the unit price set by L_B, w_B, is such that $c < w_B < \hat{c}$. Using $P''(\cdot) \approx 0$, and $P'(Q) = P'(\widetilde{Q})$, the left-hand side of (C-25) can be rewritten as

$$\frac{2(w_B - c + qP'(Q))}{3P'(Q)} - \frac{2\widetilde{q}_2}{3}. \tag{C-26}$$

Plugging $w_A = c$ into (C-26), the expression simplifies to $2(q - \widetilde{q}_2)/3$, which is positive, as, under $w_B = c$, $\widetilde{q}_2 < q = q(m_A, m_A)$.[56] Thus, $w_B > c$ at equilibrium. To prove that $w_B < \hat{c}$, we plug $w_B = \hat{c}$ into (C-26) and obtain

$$\frac{2(\hat{c} - c)}{3P'(Q)} + \frac{2}{3}(q - \widetilde{q}_2) < 0 \iff e + qP'(Q) - \widetilde{q}_2 P'(\widetilde{Q}) > 0. \tag{C-27}$$

Using the first-order conditions on the product market, (C-27) can be rewritten as $w_A - c + P(\widetilde{Q}) - P(Q) > 0$, which holds true for all $w_A - c > 0$ (as shown above) and $\widetilde{Q} < Q$.[57] Thus, $w_A < \hat{c}$ at equilibrium.

We turn now to the case in which L_A-M_1 and L_B are part of a pool, and this pool is bound to offer the same tariff to both manufacturers. The pool's maximization problem is

$$\max_{w_p, F_p} \pi(m^p, m^p) + 2(w_p - 2c)q(m^p, m^p) + F_p,$$

because the value of the fixed component of the tariff paid by M_1 drops out of the maximization problem. For the same reasons as above, the value of F_p is given by $F_p = \pi(m^p, m^p) - \max_{q_2}\{(P(q_1 + q_2(m^p, 2e)) - 2\hat{c})q_2\}$, where $m^p = w_p - 2c$. Omitting functional notation, the first-order condition of the pool is

$$2\left(w_p - 2c + 2qP'(Q)\right)\frac{\partial q}{\partial w_p} - \check{\check{q}}_2 P'(\check{\check{Q}})\frac{\partial \check{\check{q}}_1}{\partial w_p} = 0, \tag{C-28}$$

where $\check{\check{q}}_2 \equiv \arg\max_{q_2}\{(P(q_2 + q_1(m^p, 2e)) - 2\hat{c})q_2\}$, $\check{\check{q}}_1 \equiv \arg\max_{q_1}\{(P(q_1 + q_2(2e, m^p)) - w_p)q_1\}$, and $\check{\check{Q}} = \check{\check{q}}_1 + \check{\check{q}}_2$. Proceeding as in the first part of the proof, we find that $2\hat{c} > w_p > 2c$.

To conclude, we compare the value of $w_A + w_B$ under independent licensing with the value of w_p set by the pool.[58]

We first sum up the two first-order conditions for w_A and w_B, given by (C-22) and (C-25). Since $\partial q/\partial w_B = \partial q/\partial w_A$, this yields

$$\left(2(w_A + w_B) - 4c + 4qP'(Q)\right)\frac{\partial q}{\partial w_A} - 2\widetilde{q}_2 P'(\widetilde{Q})\frac{\partial \widetilde{q}_1}{\partial w_B} - \check{q}_2 P'(\check{Q})\frac{\partial \check{q}_1}{\partial w_A} = 0. \tag{C-29}$$

Rearranging the first-order condition for w_p in (C-28), we get

$$2w_p = 4c - 2qP'(Q) - \frac{\check{\check{q}}_2 P'(\check{\check{Q}})\frac{\partial \check{\check{q}}_1}{\partial w_p}}{\frac{\partial q}{\partial w_p}}.$$

[56] Indeed, under $w_B = c$, $\widetilde{q}_2 = \arg\max_q(P(q + q_1(m_A, e)) - 2\hat{c})q$ and $q = \arg\max_q(P(q + q_1(m_A, m_A)) - c - w_A)q$.

[57] To see this, note that, at $w_B = \hat{c}$, the super-additivity of Cournot quantities implies that $\widetilde{Q} = \widetilde{q}_1(m_A + e, 2e) + \widetilde{q}_2(2e, m_A + e) < 2q(e + m_A, e + m_A) = Q$.

[58] We maintain the assumption that L_A obtains its marginal contribution to manufacturers' profit. Yet, the analysis of the complementary case yields analogous results.

We now evaluate (C-29) at $w_A + w_B = w_p$. This gives

$$2qP'(Q)\frac{\partial q}{\partial w_A} + \check{\check{q}}_2 P'(\check{\check{Q}})\frac{\partial \check{\check{q}}_1}{\partial w_p} - 2\widetilde{q}_2 P'(\widetilde{Q})\frac{\partial \widetilde{q}_1}{\partial w_B} - \check{q}_2 P'(\check{Q})\frac{\partial \check{q}_1}{\partial w_A}. \tag{C-30}$$

With $P''(\cdot) \approx 0$, we obtain from the first-order conditions of the downstream market that $\partial q/\partial w_A = \partial \widetilde{q}_1/\partial w_B = 1/3P'(\cdot)$ and $\partial \check{q}_1 \partial w_A = \partial \check{\check{q}}_1/\partial w_p = 2/3P'(\cdot)$. Inserting these terms into (C-30), and simplifying, we obtain the pool sets $w_p \leq w_A + w_B$ if and only if

$$q + \check{\check{q}}_2 - \widetilde{q}_2 - \check{q}_2 \geq 0. \tag{C-31}$$

In all of the quantities in (C-31), the costs of the rival manufacturer are $w_A + w_B$. However, the costs of the manufacturer under consideration, and the expectations that the rival has about these costs, differ. In particular, $q = q(m, m)$ is the symmetric quantity that both manufacturers produce at equilibrium; $\check{q}_2 = \check{q}(e+m_B, m)$ and $\check{\check{q}}_2 = \check{q}(2e, m)$ are the quantities that M_2 produces when paying costs of $\hat{c} + w_B$ and $2\hat{c}$, respectively, and M_1 is informed about M_2's decision to purchase the status-quo technology; finally, in $\widetilde{q}_2$, M_2 pays $2\hat{c}$, but M_1 expects M_2 to pay $w_B + \hat{c}$: $\widetilde{q}_2 = \arg\max_{q_2}\{(P(q_2 + q_1(m, e + m_B) - 2\hat{c}))q_2\}$. In what follows, we show that the left-hand side of the expression in (C-31) takes a positive value.

Clearly, if $\hat{c} = c$, $q = \check{\check{q}}_2 = \widetilde{q}_2 = \check{q}_2$, so that (C-31) is equal to zero. We now take the derivative of (C-31) with respect to $\hat{c}$. It can be written as

$$\begin{aligned}&\left[\frac{\partial q}{\partial C_1} + \frac{\partial q}{\partial C_2}\right]\left(\frac{\partial w_A}{\partial \hat{c}} + \frac{\partial w_B}{\partial \hat{c}}\right) + 2\frac{\partial \check{\check{q}}_2}{\partial C_2} + \frac{\partial \check{\check{q}}_2}{\partial C_1}\left(\frac{\partial w_A}{\partial \hat{c}} + \frac{\partial w_B}{\partial \hat{c}}\right) \\ &-\frac{\partial \check{q}_2}{\partial C_2}\left(1 + \frac{\partial w_B}{\partial \hat{c}}\right) - \frac{\partial \check{q}_2}{\partial C_1}\left(\frac{\partial w_A}{\partial \hat{c}} + \frac{\partial w_B}{\partial \hat{c}}\right) - 2\frac{\partial \widetilde{q}_2}{\partial C_2} - \frac{\partial \check{q}_2}{\partial C_1}\left(\frac{\partial w_A}{\partial \hat{c}} + \frac{\partial w_B}{\partial \hat{c}}\right) - \frac{\partial \widetilde{q}_2}{\partial C_2}\Big|_{\widetilde{q}_1}.\end{aligned} \tag{C-32}$$

In (C-32), $\partial q_i/\partial C_i$, with $q_i \in \{q, \check{q}_i, \check{\check{q}}_i, \widetilde{q}_i\}$, is the derivative of M_i's quantity with respect to its own costs, given that this cost change is common knowledge. Instead, $\partial q_i/\partial C_{-i}$ is the derivative of M_i's quantity with respect to M_{-i}'s costs, given that this cost change is common knowledge. Finally, $\frac{\partial \widetilde{q}_2}{\partial C_2}|_{\widetilde{q}_1}$ is the derivative of M_2's quantity with respect to its own costs, given that the rival is not informed about this cost change and therefore cannot adjust its quantity accordingly.

Under $P''(\cdot) \approx 0$, $\partial q_i/\partial C_i = 2/3P'(\cdot)$ and $\partial q_i/\partial C_{-i} = -1/3P'(\cdot)$, with $q_i \in \{q, \check{q}_i, \check{\check{q}}_i, \widetilde{q}_i\}$. Moreover, $\frac{\partial \widetilde{q}_2}{\partial C_2}|_{\widetilde{q}_1} = \frac{1}{2P(\cdot)}$. Also, from the first-order conditions for w_A and w_B, we obtain that $\partial w_A/\partial \hat{c} = 2/5$ and $\partial w_B/\partial \hat{c} = 3/5$. Plugging all this in (C-32) yields $-9/10P'(\cdot) > 0$. Thus, (C-32) is positive for all $\hat{c} > c$. As a consequence, the first-order condition for w_p evaluated at $w_A + w_B$ is strictly positive for all $\hat{c} > c$. It follows that $w_A + w_B > w_p$ and the pool is procompetitive.[59]

Q.E.D.

Proof of Proposition 8. We start with independent licensing. Under a strictly non-discriminatory policy, licensors are bound to offer the same tariff to both manufacturers. Thus, at equilib-

[59]The result was derived under the assumption that, with independent licensing, L_A-M_1 obtains its marginal contribution to manufacturers' profits. The same result holds true under the complementary case in which it is L_B to obtain its marginal contribution to the profit of M_1 and M_2.

rium, manufacturers bear the same margin and $m_{j1} = m_{j2} = w_j - c$, $m_1 = m_2 = m$ and $q_1 = q_2 = q$.

We first consider the case of independent licensing. Specifically, the maximization problem of integrated firm L_j-M_i:

$$\max_{w_j, F_j} \ \pi(m,m) + 2(w_j - c)q(m,m) + F_j,$$

where the fixed component of the tariff can be written as

$$\begin{aligned} F_j &= \pi(m,m) - \max_q \{(P(q + q(m, m_{-j} + e)) - \hat{c} - w_{-j})q\} \\ &\quad -(w_{-j} - c)\left(q(m, m_{-j} + e) - q(m,m)\right). \end{aligned}$$

Then, the first-order condition for w_j can be written as

$$(2(w_j - c) + w_{-j} - c + 2P'(Q)q)\frac{\partial q}{\partial w_j} - \left(\check{q}_{-i}P'(\check{Q}) + (w_{-j} - c)\right)\frac{\partial \check{q}_i}{\partial w_j} = 0, \tag{C-33}$$

with $q = q(m,m)$, $\check{q}_i = q_i(m, e + m_{-j})$, $\check{q}_{-i} = q_i(e + m_{-j}, m)$ and $\check{Q} = \check{q}_i + \check{q}_{-i}$. Because this first-order condition is the same for both firms, the equilibrium unit prices must be symmetric—i.e., $w_j = w_{-j} = w$. In what follows, we use $P''(\cdot) \approx 0$, which implies $P'(\check{Q}) = P'(Q)$, and $\partial q / \partial w = 1/3P'(Q)$ and $\partial \check{q}_i / \partial w = 2/3P'(\check{Q})$.

We now show that $w > c$. Inserting $w = c$ into the left-hand side of (C-33) yields

$$\frac{2q}{3} - \frac{2\check{q}_i}{3} > 0 \iff q > \check{q}_i \ \forall \hat{c} > c.$$

As a consequence, the first-order condition for w evaluated at $w = c$ is positive, which implies that $w > c$ at equilibrium. Similarly, we can show that $w < \hat{c}$: Proceeding as above (i.e., inserting $w = \hat{c}$ into the left-hand side of (C-33), and simplifying), we obtain

$$\frac{2q}{3} - \frac{2\check{q}_i}{3} + \frac{\hat{c} - c}{3P'(\check{Q})} = \frac{\hat{c} - c}{3P'(\check{Q})} < 0 \ \forall \hat{c} > c,$$

where the equality uses the equivalence between $\check{q}_i$ and q at $w = \hat{c}$.

We now consider the case of the pool. The pool's problem is

$$\max_{\overline{w}} \ 2\left((P(2\overline{q}) - \overline{w})\overline{q} + (\overline{w} - 2c)\overline{q}\right),$$

where $\overline{q} = q(\overline{w} - 2c, \overline{w} - 2c)$. The first-order condition can be written as

$$(\overline{w} - 2c + \overline{q}P'(2\overline{q}))\frac{\partial \overline{q}}{\partial \overline{w}} = 0.$$

Rearranging gives us

$$\overline{w} = 2c - \overline{q}P'(2\overline{q}), \tag{C-34}$$

which is independent of $\hat{c}$.

We conclude the proof by showing that it exists a threshold value of $\hat{c}$ such that the pool is anticompetitive for all $\hat{c}$ smaller than this threshold. We first compare unit prices in the limiting cases featuring $\hat{c}$ small and $\hat{c}$ large.

From the analysis with independent licensing it follows that w_j is strictly between c and $\hat{c}$. This implies that as $\hat{c}$ tends to $c + \epsilon$, with $\epsilon > 0$ small, $2w$ is close to $2c$. By contrast, $\overline{w}$ is larger than $2c$ by a discrete amount (since $\overline{w}$ is independent of $\hat{c}$). It follows that for $\hat{c} = c + \epsilon$, with $\epsilon > 0$ but small, the pool is anticompetitive.

Let $\hat{c}$ be large, so that $\check{q}_{-i} \approx 0$. Under this condition, the value of the unit price solving the first-order condition of an integrated firm with independent licensing is $w = c - 2qP'(2q)$.

We can now compare (C-34) with

$$\begin{aligned} 2w &= 2c - 4qP'(2q) \\ &= 2c - 4q(2(w-c), 2(w-c))P'(2q(2(w-c), 2(w-c))). \end{aligned} \tag{C-35}$$

Suppose that $\overline{w} = 2w$. Then, the left-hand sides of both equations are the same but the right-hand side of (C-34) is strictly below the right-hand side of (C-35). This implies that $\overline{w}$ cannot be equal to $2w$, because (C-35) is not satisfied. We now determine if $\overline{w}$ is above or below $2w$. To this end, let w increase, starting from $\overline{w}$: The left-hand side of (C-35) rises, whereas the right-hand side falls since $q(2(w-c), 2(w-c))$ decreases in w—recall that $\partial q/\partial w = 1/3P(\cdot) < 0$. Because both sides are continuous, it follows that the unique solution for $2w$ must be above $\overline{w}$. As a consequence, the pool is procompetitive if $\hat{c}$ is large.

It remains to show that there is a unique threshold for $\hat{c}$ below which the pool is procompetitive, and above which the pool is anticompetitive. We know that $\overline{w}$ is independent of $\hat{c}$. Instead, from the first-order condition for w it follows that

$$\text{sign}\left\{\frac{dw_j}{d\hat{c}}\right\} = \text{sign}\left\{-P'(\check{Q})\frac{\partial \check{q}_i}{\partial w}\frac{\partial \check{q}_{-i}}{\partial \hat{c}}\right\},$$

which is strictly positive. It follows that w is strictly increasing in $\hat{c}$. Hence, there is a unique threshold for $\hat{c}$. Q.E.D.

Table 1: Patent Pools and Vertical Integration

	# Licensors	# Licensees	% Integrated Licensors
HVEC	32	105	28.1%
DisplayPort	5	27	60%
MPEG-2	27	1,227	0%
ATSC	10	128	50%
AVC/H.264	38	1,412	60.5%
MVC	19	39	26.3%
VC-1	22	323	59.1%
MPEG-4 Visual	32	718	62.5%
MPEG-2 System	10	233	40%
1394	10	134	30%
MPEG-4 System	8	34	50%
DVD-Video	9	122	77.8%
DVD-Audio	9	45	44.4%
DVD-ROM Drive	9	32	55.6%
DVD-ROM Disc	9	89	44.4%
DVD-Video Disc	9	88	44.4%
DVD-Audio Disc	9	43	33.3%
DVD-Decoder	9	21	33.3%
DVD (Video) Recorder	9	26	66.7%
DVD (Recordable Disc) Drive	9	15	33.3%
DVD Encoder	9	13	22.2%
DVD-R Disc	9	7	33.3%
DVD-RW Disc	9	7	33.3%
DVD-RAM Disc	9	6	22.2%
DVD Recordable Disc case	9	5	22.2%
+R Disc	9	3	22.2%
+RW Disc	9	3	22.2%
MPEG-Audio	5	1,142	40%
DVB-T	4	440	25%
DVB-T2	7	36	0%
ATSS	1	102	0%
WSS	8	103	0%
TOP Teletext	8	114	0%
DECT	1	21	100%
HDMI	8	1,763	0%
G.711.1	5	7	25%
G.729	3	256	33.3%
ACC	10	795	60%
AGORA-C	4	5	0%
Digital Radio Mondiale	10	16	20%
AGORA-C	4	5	0%
MPEG-2 ACC	5	790	60%