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Abstract

We examine the effects of a novel political institution called Coalition Preclusion Contracts (CPCs) on the functioning of democracies with proportional representation. CPCs enable political parties to credibly exclude one or several parties from the range of coalitions they are prepared to envisage after elections. We consider a simple political game with a two-dimensional policy space in which three parties compete to form the government. We find that CPCs with a one-party exclusion rule defend the interests of the majority by precluding coalition governments that would include so-called extreme parties. This translates into moderation of the policies implemented and yields welfare gains for a large set of parameter values. We discuss the robustness of the results in more general settings and study how party-exclusion rules have to be adjusted when more than three parties compete in an election.

JEL Classification: D72, D82, H55

Keywords: coalition formation, political contracts, elections, government formation.

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Coalition Preclusion Contracts and Moderate Policies*

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Abstract

We examine the effects of a novel political institution called *Coalition Preclusion Contracts (CPCs)* on the functioning of democracies with proportional representation. CPCs enable political parties to credibly exclude one or several parties from the range of coalitions they are prepared to envisage after elections. We consider a simple political game with a two-dimensional policy space in which three parties compete to form the government. We find that CPCs with a one-party exclusion rule defend the interests of the majority by precluding coalition governments that would include so-called extreme parties. This translates into moderation of the policies implemented and yields welfare gains for a large set of parameter values. We discuss the robustness of the results in more general settings and study how party-exclusion rules have to be adjusted when more than three parties compete in an election.

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"How many times do I have to repeat myself, Mr. Markwort? You will not receive any other answer tonight than the one I have given over during the past weeks and months. There will be no cooperation whatsoever with the [party] Die Linke [the Left]." (Andrea Ypsilanti, SPD, 2008)[1]

1 Introduction

Motivation

In democracies with proportional representation, coalition formation is essential for government since one single party rarely obtains an outright majority of seats in parliament. On occasion, government coalitions are made up of conventional parties plus small parties perceived by a large majority of voters to be extremist and undesirable. Governments that include so-called extreme parties have been formed in several European countries in the last few decades, e.g. Denmark (SF in 1964 and 1966), Italy (PRC in 1996), and Sweden (VP in 2002), with recent examples in Austria (ÖVP + FPÖ in 2000) or the Netherlands (VVD + CDA + PVV in 2010).[2]

The conditions enabling the formation of coalition governments of this type do not seem to have disappeared in recent years. On the contrary, events such as the financial crisis in 2008/2009, the debt crisis in the Eurozone, the refugee crisis, and Brexit have led to a rapid increase in electoral support for extreme parties in many European countries. Given this development, there is concern that conventional, non-extreme parties may be tempted to secure power by relying on the support of extreme parties, which in turn may undermine welfare in those societies. In this paper, we introduce a new political institution that aims to provide proportional representation democracies with greater resilience against the influence of extreme parties in government formation and policy-making. Regardless of any other considerations, a party shall be considered *extreme* in this context if it argues for a significant policy shift only supported by a minority of voters.

It is true that in political campaigns prior to an election conventional parties usually try to persuade voters that they will not form a coalition government with specific parties, typically with extreme parties. They do not however always stick to that promise. A case in point is illustrated by the above quote from Andrea Ypsilanti, who after the election in Hesse in 2008 was prepared to go back on her pledge. Such a breach of promise may not only be undesirable in itself, but it may also affect

[1] Andrea Ypsilanti was the SPD candidate for the position of minister-president (Ministerpräsident) in the state elections in Hesse, Germany, in 2008. See `http://www.focus.de/magazin/tagebuch/tagebuch-schauspielerin-ypsilanti_aid_263564.html`, retrieved on 6 March 2013.

[2] Source: Database "Elections, parties, governments" of the Research unit "Democracy: Structures, Performance, Challenges" as part of the research area "Civil Society, Conflicts, and Democracy" at the Social Science Research Center Berlin (WZB). SF stands for Socialistisk Folkeparti, RC for Partito della Rifondazione Comunista, and VP for Vänsterpartiet. Technically, People's Party for Freedom and Democracy (VVD) and Christian Democratic Appeal (CDA) entered into a "tolerance agreement" with the Party for Freedom (PVV)—see `http://www.theguardian.com/commentisfree/2010/oct/08/geer-wilders-netherlands-holland-editorial`, retrieved on 27 March 2016.

welfare as a government coalition with extreme parties could eventually lead to policies that do not square with the preferences of a large majority of voters.

In other cases, however, conventional parties have kept their promises regarding coalition formation.[3] Whether a party sticks to such promises or not largely depends on the trade-off between the reputation costs of breaking an electoral promise and the expected benefits of forming a government. Either possibility can be attractive for parties in current parliamentary democracies because promises regarding government formation can only be based on a party's or a candidate's reputation, a variable that differs greatly across cases. Accordingly, what would be the consequences for government formation and the policies implemented if, before the election, parties could bindingly commit *not* to form a government with a particular set of parties? Would welfare be improved? These are the central questions this paper poses.

To address these questions, we examine a novel political institution that we call *Coalition Preclusion Contracts* (henceforth simply *CPCs*). They represent a new type of political contract, as surveyed in Gersbach (2012). In a CPC, a party specifies a list of parties that it commits itself not to form a government coalition with after the next elections. Such promises are certified by a public authority.[4] If the party violates the CPC, i.e. if it forms a government with a party listed in the contract, the party is severely punished.[5] For instance, it may not be allowed to nominate candidates for cabinet positions, or its public funding may be considerably reduced. We assume that violations of CPCs are punished so gravely that they will never be infringed, making such political contracts a commitment enabling a party to credibly promise not to form certain coalition governments.[6]

Model and results

To examine the consequences of allowing parties to write CPCs, we consider a political game in which three parties compete first for votes in an election and second to form the government after the elections. There are two *conventional* parties with platforms on the moderate left and right with regard to political issues such as tax policy and public-good provision. Additionally, there is an *extreme* party whose defining characteristic is that it advocates a substantial policy change in a second policy dimension orthogonal to the conventional policy dimension mentioned earlier. Examples of such a policy change would be leaving NATO or the monetary union, closing the

[3]For instance, the German Social Democratic Party (SPD) stated before the general election in 2005 that it would not form a coalition government involving the party "Die Linke" and kept its promise.

[4]Either a new authority would be created or an existing authority would be entrusted with certification of a CPC. In Germany, for instance, the Federal President could act as a CPC certifier.

[5]The same punishment would also apply to a party attempting to form a minority government by counting on the votes of parties excluded in the contract.

[6]An attempt to make campaign promises regarding coalition partners credible occurred in Catalonia, Spain, in the regional elections that took place in 2006. In an unprecedented move previous to the elections, CiU stated in a document signed before a notary that it would not establish any kind of agreement with PP—the document can be found (in Catalan) at http://www.ciu.cat/media/9890.pdf (the information was retrieved on 26 February 2014). While such moves do increase the credibility of promises, they are not legally binding and hence lack the credibility potential of CPCs. CiU stands for Convergència i Unió, PP stands for Partido Popular.

borders to immigrants, or nationalizing the banking system. Such a shift away from the status quo in the second, orthogonal policy dimension is preferred by only a minority of voters. These voters feel strongly about the policy change advocated by the extreme party and will always vote for it. In the first policy dimension, the extreme party has moderate preferences.

As a consequence, the median voter in the conventional policy dimension, who does not support the extreme policy change, would prefer a *grand coalition* of the conventional parties over a coalition government formed by one conventional party and the extreme party, if the latter were then able to impose the policy shift in the coalition agreement. Conventional parties, however, are often tempted to engage in coalitions with the extreme party as the latter is willing to offer substantial power and perks in return for the implementation of such a policy change. Even though conventional parties may have an incentive to attract more voters by ruling out a coalition with the extreme party before the election, they may be tempted to break their promises after the election if they cannot form a single-party government and thus require a coalition partner. Accordingly, a natural idea for preventing coalitions between conventional and extreme parties would be to enable parties to credibly commit themselves before the election not to form a coalition government with the specified parties after the election.[7]

We analyze the conditions under which CPCs will be written in a parliamentary democracy and what their welfare implications are. For this purpose, we study a simple model with the three parties introduced previously and three voters, each of them with political leanings towards one of the parties. While the extreme voter will always vote *sincerely* for the extreme party, conventional voters may think about proceeding *strategically*. This means that although conventional voters will never vote for the extreme party, they may consider coordinating their votes on one of the conventional parties. We shall focus on constellations of voter and party preferences such that *(i)* in the absence of CPCs, coalitions between a conventional party and the extreme party will occur, and *(ii)* non-empty contracts may be written if CPCs are available. In other constellations of preferences, CPCs would be redundant.

Our main insights from the formal model are as follows: First, as just stated, voters will vote sincerely in the situation without CPCs, leading to a coalition government between a conventional party and the extreme party. Such a government will implement the policy shift in the second policy dimension.

Second, when CPCs are introduced, whether the conventional parties will exclude the extreme party from a government coalition or not will depend crucially on the expected probability of conventional voters coordinating their votes on the party that ruled out the extreme party—assuming that only one conventional party has done so. We shall characterize the equilibria depending on this probability.

[7] A two-round election like the one used in France can be an effective way of precluding extreme parties from holding executive offices in a presidential system. Coalition Preclusion Contracts are fundamentally different from such multi-round procedures in that they do not restrict the voters' action space in the final election, but rather increase the parties' action space by offering them the possibility of credible exclusion of coalitions. CPCs can thus be applied in democracies with proportional representation, in which seats are allocated in terms of vote shares.

On the one hand, we show that both conventional parties will exclude the extreme party if this probability is sufficiently large. In such cases, a grand coalition will result, which will not carry out the shift in the second policy dimension. On the other hand, no CPC will be written when the coordination probability in favor of the only conventional party excluding the extreme party is low. Our analysis also covers all intermediate cases regarding the exact value of the coordination probability.

Third, we find that with the possibility of writing CPCs another equilibrium may also occur, where conventional parties exclude any coalition government whatsoever. As conventional voters will react to the CPCs written by coordinating their votes on one conventional party, this will make a single-party government inevitable. These additional equilibria are socially undesirable if preferences are polarized in the first policy dimension and citizens are sufficiently risk averse. Such equilibria are however eliminated when each party can exclude at most one other party in its CPC (one-party exclusion rule).

Fourth and last, with the one-party exclusion rule, the introduction of CPCs will involve welfare gains for a large set of parameter values. These gains stem from the avoidance of an extreme policy shift in the second policy dimension and from implementation of a moderate policy in the first dimension. Moreover, the introduction of a law permitting CPCs will be supported at the constitutional level by both conventional parties, the reason being that, with CPCs, conventional parties can eliminate the risk of the other conventional party forming a government with the extreme party.

Intuition

Although further intuition for our results will be given later on, it will be useful at this point to summarize here how CPCs affect choices of parties and voters in our two-dimensional policy space. Conventional parties will prefer to be either in a single-party government or in a coalition with the extreme party rather than entering into a grand coalition, as in the former they will obtain higher perks than in the latter. Conventional voters, on the other hand, prefer moderate policies in the first dimension and dislike the extreme shift in the second dimension. The latter desire is shared by each of the conventional parties, which, however, would like to implement a partisan (either leftist or rightist) policy in the first dimension. In negotiations to form a government, conventional parties are willing to trade off policies against power and perks. This will, in turn, enable the extreme party to form a government with one conventional party. The corresponding government will carry out the extreme shift in the second dimension and will implement the median policy in the first dimension to attract the support of the conventional voters. In exchange, the extreme party will renounce all perks to the benefit of its partner in the government. CPCs will disrupt this equilibrium behavior by taking away the extreme party's power to trade off policies against perks.

To sum up, we can think of our model as the sum of two models, a first Downsian model with two parties and two "median" voters, and a second, orthogonal model with a status-quo policy, which is the Condorcet winner against a discrete shift. Simultaneous convergence towards the median position in both models will only be possible with the aid of Coalition Preclusion Contracts.

Extensions, robustness, organization of the paper

As already emphasized, in the main body of the paper we shall restrict our attention to an admittedly stylized model. Not only is this useful in terms of exposition, it also comes at almost no cost, since the analysis of such a model already conveys the fundamental strategic and welfare implications of CPCs. For the sake of completeness, we nonetheless explore a number of extensions of the model in Appendix A (see supplementary material). For instance, we allow for asymmetry with regard to conventional parties, a continuum of voters, and uncertainty about the election outcome for the extreme party. These extensions add to the robustness of our findings. In particular, introducing the model with a continuum of voters enables us to investigate whether the insights from the baseline model can be replicated in a more micro-founded setting. We shall prove that a link can be established between equilibria of both models. Lastly, we also discuss how exclusion rules for CPCs need to be adapted when more than three parties participate in an election.

The paper is organized as follows: In Section 2 we relate our work to the existing literature. In Section 3 we present our simple model for elections in a parliamentary democracy. Then, in Sections 4 and 5 respectively, we consider two different rules specifying how contracts have to be written and characterize the set of equilibria in each case. In Section 6 we study the welfare implications of CPCs in the light of the results from the previous section and discuss the predicament posed by the actual implementation of CPCs in a parliamentary democracy. Section 7 concludes. In Appendix A we discuss the robustness of our results by studying various extensions of the model. All the proofs of the paper can be found in Appendices B and C (see supplementary material).

2 Relation to the Literature

We consider a political game with underlying conflicts *(i)* between voters and parties with regard to policies and *(ii)* between parties with regard to policies and coalitions. Accordingly, our paper is related to several strands in the literature.

Coalition formation

The formation of coalitions is central to many real-world phenomena, including the arrangement of governments, cartels, or bidding rings. There are different approaches to examining coalition formation. On the one hand, the axiomatic approach in cooperative game theory typically assumes the formation of a coalition of all agents (Rosenmüller, 1981). It is useful for the understanding of coalition formation properties relating to fairness and stability. This approach is supplemented by the literature about the implementation of cooperative game theory solution concepts. This literature is very extensive—see e.g. Serrano (1993) or Pérez-Castrillo and Wettstein (2001)—and allows for strategic considerations by the players. On the other hand, the extensive-game form approach tends to be generally applied in answering questions like *(i)* what coalitions will form? and *(ii)* how will the worth that a coalition can attain be divided among its members? Consult Bandyopadhyay and Chatterjee (2006) for a survey about games of this latter type, with an emphasis on political economy. Most models in this strand of literature assume that offers for coalition formation are made sequentially (Selten, 1988; Chatterjee et al., 1993), with discounting *à la* Rubinstein (Rubinstein, 1982).[8] Our paper adds to this literature by studying agents' incentives to preclude certain coalitions before they form.

Coalitions in politics

In a multi-party system with proportional seat allocation, it is very common for parties to be unable to form a government alone, so after the elections a coalition of parties emerges and takes over. Two aspects of this phenomenon have come in for particular attention in the literature, *(i)* how parties bargain in order to form a coalition government for an entire term, and *(ii)* how this information—and other information regarding coalitions—is anticipated by the electorate.

The problem in *(i)* boils down to a bargaining situation among members with some opposing preferences that have to decide about policies and perks (or cabinet positions). Our approach in this paper is twofold. On the one hand, it involves the conventional parties as *formateurs* and the Nash bargaining solution with bargaining power proportional to the share of seats in parliament as a bargaining rule within the grand coalition (see Roemer et al. (2001)). Hence, Gamson's rule (Gamson, 1961) applies.[9] Gamson's proposition is in sharp contrast with the formal literature on coalition bargaining pioneered by Baron and Ferejohn (1989), which predicts bargaining outcomes that diverge from the principle of proportionality. In our model, only negotiations within the grand coalition adhere to the latter principle.

On the other hand, we assume that the negotiations between a conventional party and the extreme party are based on the impossibility for the latter to compromise on its ideological stance. Ideology in the sense of Benabou (2008) acts as a constraint for the mobility of such a party: accepting a policy that is far away from its ideological stance would threaten its very existence. In exchange for not being mobile in one dimension, however, the extreme party is willing to renounce both policy compromises in other dimensions and perks. Depending on the CPCs drafted, either no, one, or two conventional parties will be able to bargain with the extreme party after the elections. In the latter case, conventional parties will compete *à la* Bertrand to form a coalition with the extreme party, with intra-coalition negotiations between a conventional party and the extreme party following the rules previously outlined. Only when both conventional parties have excluded the extreme party in their CPC will they form a grand coalition.

[8]There is empirical evidence that such models accurately reflect free-form bargaining (Bolton et al., 2003).

[9]For an empirical assessment of situations where the rule predicts the composition of governments, see Falcó-Gimeno and Indridason (2013).

With respect to *(ii)*, voters are typically assumed to estimate the formation probability for each coalition and anticipate the policy compromises that will result (see e.g. Austen-Smith and Banks (1988)). There is some empirical support for the view that voters are influenced by potential government coalitions emerging after the elections. For instance, Blais et al. (2006) and Meffert and Gschwend (2010) show that voters are responsive to coalition signals. We assume that voters can completely foresee the government formation process, in particular the effect that CPCs have on it.

Pre-electoral commitments

Without CPCs parties try other ways of making campaign promises credible. Aragonès et al. (2007) consider a model for repeated elections with completely informed and ideological voters and conclude that, in equilibrium, the degree to which promises are credible increases with the reputation of the candidate. Debus (2009) shows that pre-electoral (non-binding) announcements of possible coalitions can influence the outcome of the government formation game.

CPCs offer a more powerful, complementary way for parties to make credible promises regarding possible government coalitions. They work in situations in which reputation concerns are weak and campaign promises are thus cheap talk. Moreover, they do not rely on punishment threats by voters. With CPCs, parties can invariably make credible promises regarding possible partners in a coalition government.

3 A Simple Model of Elections

As already mentioned, a *Coalition Preclusion Contract (CPC)* enables a party to credibly commit itself before an election to forgoing a government coalition with any party specified in the contract. Will this possibility of excluding another party from a coalition government be used by parties? How will such a possibility affect welfare? To answer these questions, we analyze a simple model of political competition and government formation in a democracy with proportional representation. We initially assume a *one-party exclusion rule*, i.e., parties can exclude no more than one other party. In Section 5 we consider the case of CPCs in which an arbitrary number of parties can be excluded.

3.1 Two political competition games

We analyze two games. First, we consider a game in which parties can write a CPC before the elections and denote it by $\mathcal{G}^+$. Second, we consider a game in which parties can write no CPC and denote it by $\mathcal{G}^-$. In both cases, we consider two *conventional parties*, L and R, an *extreme party*, E, two *conventional voters*, l and r, and an *extreme* voter, e. Citizens each have one vote, which they cast in the election. We assume that due to their ideological orientation e and E will not act *strategically*: e will always vote for E, while E will only care about the implementation of a specific policy. We also assume that conventional voters will only vote for conventional parties and that no abstention will occur. Thus, $\mathcal{N} = \{L, R, l, r\}$ will effectively be the player set for both $\mathcal{G}^-$ and $\mathcal{G}^+$. For the sake of notation, we will henceforth denote a conventional party by K, with $K \in \{L, R\}$, and a conventional voter by k, with $k \in \{l, r\}$. For notational convenience, we will also assume that $k = r$ if and only if $K = R$, and that $\left\{K, \hat{K}\right\} = \{L, R\}$ and $\left\{k, \hat{k}\right\} = \{l, r\}$. For simplicity, we shall assume that there is no discounting.

3.2 The timeline of the games

Beyond the possible draft of CPCs simultaneously by both conventional parties at the beginning of game $\mathcal{G}^+$, the timeline of events of both games will include the following stages: First, before the election, all voters and parties will observe a public signal. This signal can be of any kind, from a performance assessment of previous governments or polls in the media to personal scandals involving politicians. As signing a CPC may involve more TV coverage for the party involved, the public signal may depend on the parties' contract choices. Because the signal will be publicly observed by all voters, it can be used by the conventional voters to coordinate their votes on one of the conventional parties.

Second, on election day, each conventional voter will cast a vote for one conventional party. Subsequently, the government will be formed. If a conventional party has received two votes, it will form the government. If no party has obtained a majority, coalitions will be formed following the lead of one of the conventional parties acting as a *formateur*. As a tie-breaking rule, we assume that whenever each of the conventional parties has one vote, the tie will be fairly broken. This means that with probability 1/2 either conventional party will be the formateur. The extreme party, on the other hand, is indifferent between forming a coalition with one or the other conventional party as long as they offer the same policies in government. We will show later on that from a strategic viewpoint E's indifference is reasonable. The identity of the formateur can then be used by the extreme party as a tie-breaking rule to decide which party it wants to form a coalition with.

We permit the non-formateur party to bargain with other parties before the negotiations conducted by the formateur party are terminated. This is a justified assumption, as simultaneous negotiations between some parties often occur after elections and before the government has been formed by the formal parliamentary procedure. A recent example of this off-parliament bargaining occurred in the 2015 elections in Spain, mainly involving the Socialist Party (PSOE), Ciudadanos, and Podemos. The possibility that government coalition proposals are not strictly sequential undermines the bargaining power of the formateur party, which no longer has a first-mover advantage.[10]

More specifically, we will assume that once a formateur is determined, the government formation

[10] In particular, the formateur party is no longer able to make small adjustments to the proposal the non-formateur party would raise in a second stage in order to construct a proposal that will be implemented.

process in the case where no party has the majority of votes will consist of the following steps:

1. The formateur makes a proposal for a government coalition that has to honor the CPCs of all parties involved (if any).

2. Upon observing the previous proposal, the non-formateur conventional party makes another proposal for a government coalition that also has to honor the CPCs of all parties involved (if any).[11]

3. Each party (R, L, and E) votes for one of the two government coalitions previously proposed, at most. Abstention is allowed.

4. If a winning government coalition is agreed upon by all its members, a vote of confidence takes place, and the coalition forms. In any other case, a caretaker government will be appointed.

Whatever the outcome of both the election and the government formation process, the government formed thereafter will implement a policy, which we denote by p. We assume that negotiations between parties obey the following principles: First, the negotiations for government formation between the two conventional parties are based on a proportionality principle, i.e., the more seats, the more power. This assumption plays a particularly important role in Appendix A (see supplementary material). In the baseline model, it ensures that the policy implemented by the grand coalition is moderate in all policy dimensions.[12] Second, the negotiations between a conventional party and the extreme party depend entirely on the latter's *bargaining power* and its wish to dictate a certain policy in exchange for renouncing perks. A caretaker government consists of bureaucrats ensuring that operations in the executive branch go on running. The possible policies a government will implement are denoted as follows:

- A single-party government of conventional party K will implement policy p_K.

- A government formed by the two conventional parties will implement policy p_{LR}.

- If any kind of coalition between a conventional party and party E is feasible, a coalition government involving extreme party E and conventional party K will implement policy p^s_{KE}. Furthermore, we assume that $p^s_{RE} = p^s_{LE} =: p_E$.[13]

[11]The results of the paper would not change if we assumed that the two proposals are simultaneous and thus the non-formateur cannot observe the first proposal.

[12]In this simple model, a "grand coalition" between L and R will have the same number of votes/parliamentary seats as a coalition between either L or R and E. Typically, the vote share for extreme parties is less than that of the large conventional parties, as implied in our reference to a coalition between L and R as the "grand coalition". However, by considering only three voters, each leaning towards one party, we capture the situation where under sincere voting the set of all minimal coalitions coincides with the set of two-party coalitions. Only in such cases is the analysis of CPCs interesting.

[13]The rationale for this assumption is discussed below.

- If the coalition between K and party E is feasible and the coalition between $\hat{K}$ and party E is not, a coalition government involving party E and conventional party K will implement policy $p^w_{KE} =: p_{KE}$.

- A caretaker government will implement p_{ct}.

The policies referred to are solutions to the bargaining problem that occurs at the government formation stage. Due to the assumptions that we will impose, policies in other end-nodes of the political game will not occur in equilibrium. Party and voter preferences over the above policies will be specified below. At this stage, it is only important that policies differ depending on the government coalition and on the conventional parties that the extreme party can form a coalition with. The latter distinction reflects the bargaining power of the extreme party. For later use, we say that E has *strong bargaining power* if it can form a coalition with both conventional parties. We assume in this case that the same policy p_E will be implemented by a government coalition of either conventional party plus the extreme party. This reflects the assumption that conventional parties are competing head-to-head for a coalition with E. When the latter party can form a coalition government with only one conventional party, we say that it has *weak bargaining power*. In such cases, policies will depend on the preferences of the conventional party that can form a coalition with E, and thus p_{LE} and p_{RE} will be different policies.

To sum up, we consider a political game that involves at most four main sequential stages:

Stage 1: Coalition Preclusion Contracts (if available),

Stage 2: Coordination Signal,

Stage 3: Elections,

Stage 4: Government Formation.

Game $\mathcal{G}^-$, which consists of Stages 2–4, describes current democracies with proportional representation. Game $\mathcal{G}^+$, which consists of Stages 1–4, describes the potential situation of those democracies where CPCs could be written. We will henceforth denote the set of possible policy outcomes by $\mathcal{P} = \{p_L, p_R, p_{LR}, p_{LE}, p_{RE}, p_E, p_{ct}\}$.

3.3 The players' utility profile

Voters and parties will derive utility from a policy $p \in \mathcal{P}$ implemented by the government. The utility of party K when policy p is implemented will be denoted by $V_K(p)$, and the utility of voter k when policy p is implemented will be denoted by $v_k(p)$. Moreover, conventional parties will also derive utility from the perks associated with being in government. Their utility will be separable in policies and perks. The latter include all sources of utilities beyond p for parties in power, such as exerting power, ego rents, administrative or leadership position of party members, or public expenditures targeted at the interest groups supporting the parties. Some of these perks use public funds and thus lower utility for voters. We assume that due to the possibility of extracting perks, a conventional party in both a single-party government or a government coalition with the extreme party will obtain an additional utility B and that each conventional party in a grand-coalition government will obtain an additional utility b, where $B > b > 0$ and

$$2b \geq B. \tag{1}$$

That is, each conventional party will obtain higher utility from perks in both a single-party government and a government with E than in a government with the other conventional party. However, the difference between the two utility levels will not be higher than two-fold. We stress that only conventional parties will extract perks. This is justified when both conventional parties are better connected to powerful groups than the extreme party or, as we assume in the paper, when party E is only interested in implementing the extreme policy. The extreme party will always prefer trading perks for implementing such a policy. In Catalonia, Spain, the deal concluded between CUP and *Junts pel Sí (Together for Yes)* is an instance of such a phenomenon.[14] Moreover, we assume that the voters' disutility from perks is the same regardless of the government coalition. The above assumptions do not affect the results, but they make exposition simpler.

The parties' von Neumann-Morgenstern expected utility before the elections is also separable. It consists of two terms. One term captures the expected utility from the policy implemented, the other term captures expected perks. Accordingly, Condition (1) can be interpreted in the following sense: Conventional parties are risk-averse regarding perks, as they value perks obtained with certainty in a grand coalition at least as much as expected perks obtained when there is equal probability that a single-party government by either conventional party will be formed. When Condition (1) holds as equality, we will say that conventional parties are *risk-neutral* regarding perks.

Both with and without the possibility of writing CPCs, the coalition government that will be formed and the policy it will carry out will hinge on the parties' and voters' preferences over policies in $\mathcal{P}$ and perks. Such preferences will be common knowledge. In the following, we will impose a number of assumptions on such preferences that are designed in the first place to guarantee the following two conditions: *(i)* if CPCs are not available, the government coalition will include party E, and *(ii)* CPCs might be written in equilibrium if they are available. These assumptions are of great importance in focusing our analysis on the circumstances where CPCs make a difference. When such conditions are not met, CPCs would be redundant. Below, we discuss the rationale for the most critical assumptions in detail.

[14]CUP stands for *Candidatura d'Unitat Popular*.

3.3.1 Party preferences

We initially specify how parties rank policies in $\mathcal{P}$. First, the most preferred policy of a conventional party will be the one it will implement, if it is able to form a single-party government. The rationale is obvious, since a party having a majority of seats in parliament can implement its most preferred policy. That is,

$$\{p_K\} = \arg\max_{p \in \mathcal{P}} V_K(p). \tag{2}$$

Second, when a conventional party receives one vote and no CPC has been written, party E will have strong bargaining power. We assume that the policy implemented in this case by a coalition between a conventional party and party E will yield lower utility than the policy implemented by the grand coalition. That is,

$$V_K(p_{LR}) > V_K(p_E). \tag{3}$$

Condition (3) says that, from the perspective of a conventional party, the policy compromise that needs to be made with the extreme party to form a coalition government yields lower utility than the compromise that needs to be made with the other conventional party in the grand coalition. Recall that when each party receives one vote and no CPC has been written, there will be a probability of 1/2 that each conventional party will be in a government together with E. Condition (3) also implies that from an *ex-ante* viewpoint, i.e., before elections have been held, any conventional party will prefer the policy outcome of a grand-coalition government to the outcome of a coalition government formed by a conventional party and the extreme party if both conventional parties have the same chance to form such a coalition ex-post. Using Conditions (1) and (3), we obtain

$$V_K(p_{LR}) + b > V_K(p_E) + \frac{1}{2}B. \tag{4}$$

By contrast, from an *ex-post* viewpoint, i.e. once elections have been held, any conventional party will prefer to form a government with the extreme party, as the latter party will be able to offer higher perks in return for implementing its desired policy shift. More specifically, we will assume that

$$V_K(p_E) + B > V_K(p_{LR}) + b. \tag{5}$$

The above inequality sets a lower bound for $B - b$.

Third, when the extreme party has weak bargaining power, it will also be more attractive *ex-post* for the conventional party that did not exclude party E in its CPC to form a coalition with it than with the other conventional party. The reason is that whatever the utility loss associated with the policy implemented in a coalition with the extreme party, it will be offset by higher perks. That is, we will assume that

$$V_K(p_{KE}) + B > V_K(p_{LR}) + b. \tag{6}$$

Condition (6) imposes another lower bound on $B - b$ and follows from Condition (5) if

$$V_K(p_{KE}) > V_K(p_E). \tag{7}$$

It will suffice to impose the latter condition.

Fourth, utility from perks will be high enough for any conventional party to prefer *ex-ante* any outcome in which there is a significant probability that it will take part in the government to any outcome in which it will not do so with certainty. It will be sufficient to assume that

$$V_K(p_E) + \frac{1}{2}B > V_K(p_{\hat{K}}). \tag{8}$$

The above condition imposes a lower bound on B.

Fifth, we assume that a conventional party is better off when the other conventional party is in a single-party government than when such a party forms a coalition with the extreme party, the latter with weak bargaining power. Thus,

$$V_K(p_{\hat{K}}) > V_K(p_{\hat{K}E}). \tag{9}$$

As we have seen, when the extreme party has weak bargaining power and forms a coalition with a conventional party, each of the two parties will be able to impose part of their policy agenda. This will result in a very unfavorable outcome for the other conventional party.

Sixth and last, the policy implemented by a caretaker government is the worst possible outcome for both conventional parties, i.e.,

$$\{p_{ct}\} = \arg\min_{p \in \mathcal{P}} V_K(p). \tag{10}$$

Summing up, from Conditions (2), (4), and (6)–(10) we derive the following ex-ante complete preferences over policy and perks:

$$V_L(p_L) + B > V_L(p_{LE}) + B > V_L(p_{LR}) + b > V_L(p_E) + \frac{B}{2} > V_L(p_R) > V_L(p_{RE}) > V_L(p_{ct}), \tag{11}$$

$$V_R(p_R) + B > V_R(p_{RE}) + B > V_R(p_{LR}) + b > V_R(p_E) + \frac{B}{2} > V_R(p_L) > V_R(p_{LE}) > V_R(p_{ct}). \tag{12}$$

If we assume that parties mainly care about the extent of the perks they can obtain in power, the preference orderings in (11) and (12) impose mild requirements on the conventional parties' preferences with regard to policies. Our analysis also encompasses all intermediate cases with regard to the extent to which parties are office-oriented. The complete preference orderings specified in (11) and (12) reduce the complexity of the analysis of political games $\mathcal{G}^-$ and $\mathcal{G}^+$. Indeed, under such assumptions—and under Condition (5)—, both the government coalition formation and the policy implemented will follow directly from the CPCs written and from the election outcome—as indicated below by Table 1—, with the formateur being randomly chosen by nature in certain cases. This, in turn, will enable us to focus on the strategic interactions between conventional parties and conventional voters that occur before Stage 4.

<table>
<tr><td></td><td></td><td colspan="3">(σ_l, σ_r)</td></tr>
<tr><td></td><td></td><td>(R, R)</td><td>(R, L) & (L, R)</td><td>(L, L)</td></tr>
<tr><td rowspan="6">$(\sigma_K, \sigma_{\hat{K}})$</td><td>$(\emptyset, \emptyset)$</td><td rowspan="6">$p_R$ (R)</td><td rowspan="3">p_E (RE/LE)</td><td rowspan="6">p_L (L)</td></tr>
<tr><td>$(\emptyset, \{K\})$</td></tr>
<tr><td>$(\{\hat{K}\}, \{K\})$</td></tr>
<tr><td>$(\emptyset, \{E\})$</td><td rowspan="2">p_{KE} (KE)</td></tr>
<tr><td>$(\{\hat{K}\}, \{E\})$</td></tr>
<tr><td>$(\{E\}, \{E\})$</td><td>p_{LR} (LR)</td></tr>
</table>

Table 1: Policy implemented under the one-party exclusion rule and government coalition (in parentheses).

In particular, under the one-party exclusion rule, a strategy σ_K for party $K \in \{L, R\}$ will choose an element of $\{L, R, E\} \setminus \{K\}$, and a strategy for voter $k \in \{l, r\}$ will be a function that assigns an element of $\{L, R\}$ to each pair (σ_L, σ_R). A strategy profile will be denoted by $\sigma = (\sigma_L, \sigma_R, \sigma_l, \sigma_r)$. We point out that we are referring to game $\mathcal{G}^+$, game $\mathcal{G}^-$ being simply the subgame of $\mathcal{G}^+$ that starts at the node after $(\sigma_L, \sigma_R) = (\emptyset, \emptyset)$ has been chosen. Table 1 then shows the coalitions precluded by CPCs will only affect the policy implemented when each conventional party receives one vote in the election—i.e., when no party can form a government on its own. In such cases, CPCs helps selecting among p_E, p_{RE}, p_{LE}, and p_{LR}.

3.3.2 Voter preferences

The central tension in our model will arise from some major differences in the preferences of conventional voters with respect to the preferences of the parties on the same political side (i.e., left or right). First, conventional voters will rank at the top of their preferences both the policy that the conventional party on the same side will implement and the policy that the grand coalition will implement. That is,

$$v_k(p) \geq \min\{v_k(p_{LR}), v_k(p_K)\} \text{ with } p \in \mathcal{P} \Longrightarrow p \in \{p_{LR}, p_K\}. \tag{13}$$

Second, the above two policies will be preferred by both conventional voters to the policy that a government of a conventional party plus the extreme party will implement when the latter has strong bargaining power, i.e.,

$$\min\{v_k(p_{LR}), v_k(p_K)\} > v_k(p_E). \tag{14}$$

Third, conventional voters will nonetheless prefer the policy that any government of a conventional party in coalition with the extreme party will implement when the latter has strong bargaining power to the policy that a single-party government formed by the conventional party on the other side will implement, i.e.,

$$v_k(p_E) > v_k(p_{\hat{K}}). \tag{15}$$

Fourth, a conventional voter will prefer the ideal policy of the conventional party on the other side to the policy that would be carried out by any coalition made up of a conventional party and the extreme party if the latter has weak bargaining power. That is, we impose

$$v_k(p_{\hat{K}}) > \max\left\{v_k(p_{KE}), v_k(p_{\hat{K}E})\right\}. \tag{16}$$

Fifth and last, the policy implemented by a caretaker government will be the worst possible outcome also for both conventional voters, i.e.,

$$\{p_{ct}\} = \arg\min_{p \in \mathcal{P}} v_k(p). \tag{17}$$

The above assumptions regarding voter preferences need to be discussed in more detail. This is done in Section 3.4. We first point however that when voter preferences are single-peaked in each component of a multi-dimensional policy space, these assumptions have immediate implications on the *dimension* of $\mathcal{P}$. This is shown in the following result, the proof of which can be found in Appendix B (see supplementary material):

Lemma 1
Assume that $\mathcal{P} \subseteq \mathbb{R}^n$, that Conditions (14)–(16) hold, and that the conventional voter and the conventional party preferences are single-peaked in each dimension. Then, if the policy resulting from the bargaining between any two parties is in each dimension a convex combination of their most preferred policies, $\mathcal{P}$ cannot be embedded in a one-dimensional space, so $n > 1$.

The above result demonstrates that the conditions imposed on the voters' preferences can only be fulfilled in a spatial model. For the illustration of the results, it will come in hand to consider that

$$\mathcal{P} \subseteq \mathcal{T} \times \mathcal{D}.$$

We will interpret the policy space $\mathcal{T} \times \mathcal{D}$ as consisting of *(i)* a first policy dimension, $\mathcal{T} \subsetneq \mathbb{R}$, which represents a choice that can be varied continuously (e.g. a tax rate) and *(ii)* a second, binary policy dimension $\mathcal{D} = \{0, \bar{d}\}$ that represents an indivisible choice (e.g. membership of a monetary union). We use $d = 0$ to denote the status quo in $\mathcal{D}$ and assume that

$$p_K = (t_K, 0), p_{KE} = (t_K, \bar{d}), p_k = (t_k, 0), p_{LR} = (t_m, 0), p_E = (t_E, \bar{d}), \text{ and } p_e = (t_e, \bar{d}), \tag{18}$$

where p_k denotes k's ideal policy, with $k \in \{l, r, e\}$.

3.4 Rationale for the model and main assumptions

We have now imposed a number of assumptions on voter and party preferences. We have already discussed the assumptions about party preferences and shown that they are not very demanding when parties are mainly office-oriented and perks-seekers. Next, we discuss the assumptions on conventional voter preferences together with a few other assumptions of our model. Although such a discussion provides a rationale for our modeling assumptions, it is lengthy and is not required for the understanding of the results of Sections 4—6. Hence, readers who are mainly interested in the main insights when CPCs are introduced can skip this part and proceed to Section 3.5.

First, Condition (13) ensures that conventional voter k will lean more towards the position of conventional party K. This renders party and voters labels meaningful. Second, Condition (14) requires that both conventional voters prefer the median policy in the usual left-right dimension to the discrete shift in the orthogonal dimension. Both assumptions are standard. Third, Condition (17) can be interpreted together with Condition (10) in the sense of the Nash bargaining: p_{ct} specifies the disagreement point when conventional voters do not coordinate their votes and no coalition is allowed by CPCs. Now we emphasize the following, more contentious assumptions:

(A) There are only three voters and three parties. Moreover, only voters and parties labeled *conventional* are effectively players in the game: The extreme party E and the extreme voter e are "non-strategic".

(B) The bargaining power of the extreme party in the government formation process—which is determined by CPCs—is key in determining the policy outcome, provided that Conditions (15) and (16) hold.

Let us discuss (A) and (B). Regarding (A), the assumption that there are few voters is not essential, provided that voters can coordinate. This is shown in Appendix A (see supplementary material). A one-to-one correspondence is established there between equilibria in both our simple model and a micro-founded model with a continuum of voters. We can thus interpret this as saying that voter l (or r) represents the mass of voters leaning more towards the political position of party L (or R) but might consider voting for the other conventional party for strategic reasons.

By contrast, both the extreme voter e (as representative of a certain share of voters) and the extreme party E care mainly about the implementation of $d = \bar{d}$. On the one hand, $d = \bar{d}$ is party E's defining characteristic and acts as an ideological constraint on its mobility (see Mueller (2003); Benabou (2008)). This implies that accepting $d = 0$ would lead to a "collapse" of E, as the party would lose its reputation and its *raison d'être*. Thus, party E will only agree to form a government coalition if this coalition will bring about the policy shift, i.e., only if one of the following policies is chosen: p_E, p_{LE}, and p_{RE}. The assumption that the second policy dimension, $\mathcal{D}$, is binary thus captures the nature of E's mobility in the negotiations.[15] On the other hand, e will always vote for E. This is reasonable, as doing so is a necessary condition for implementation of $d = \bar{d}$. Additionally, e might vote for E because he attaches some utility to casting such a vote.

[15]It would actually suffice for $\mathcal{D}$ to be discrete. Assuming that $\mathcal{D}$ is binary simply facilitates the interpretation of our model and can be done without any loss of generality.

Finally, the situation with more than three parties is interesting in itself. It is addressed in Appendix A (see supplementary material). There we show that in the presence of more than one extreme party the one-party exclusion rule needs to be adjusted.

Regarding (B), we focus separately on Conditions (15) and (16). It will be convenient to consider Figure 1, which depicts some party and voter preferences in a two-dimensional policy space. For better understanding of the figure, $\mathcal{D}$ takes on discrete values and p_{ct} is not depicted.

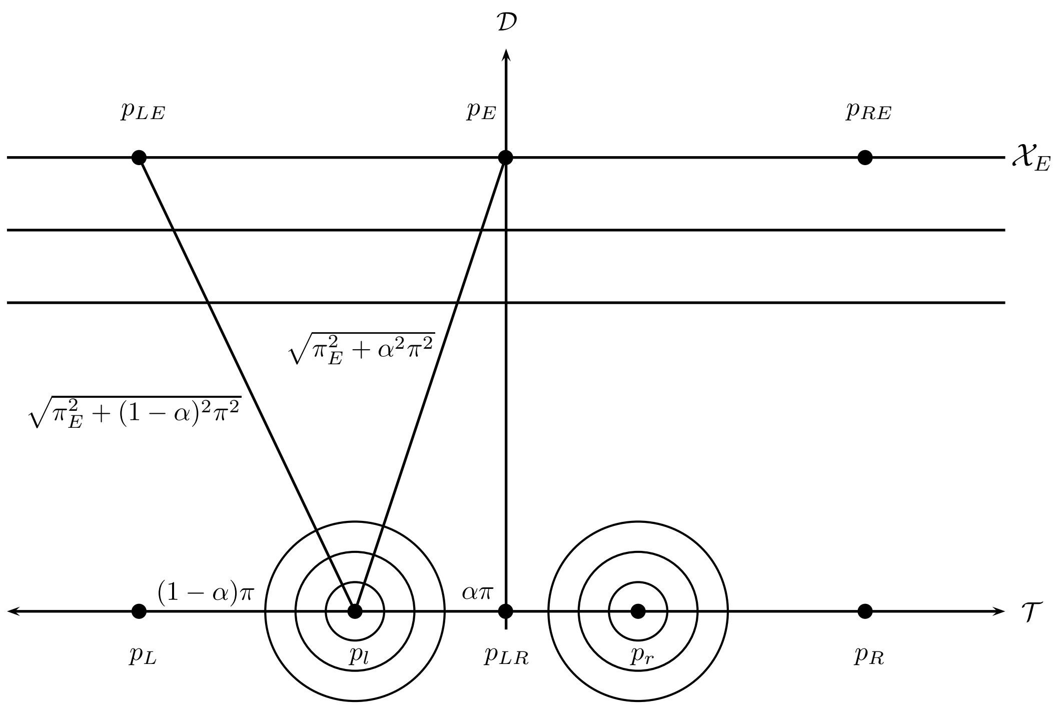

Figure 1: A two-dimensional policy space and the voters' indifference curves ("quadratic" for both conventional voters with bliss points p_l and p_r, and "lexicographic" for the extreme voter and the extreme party with bliss point p_E).

We start with Condition (15), i.e. we assume that party E has strong bargaining power. On the one hand, the policy choice when E is in the government coalition may reflect the true preferences of its members (or of its constituency) given its bargaining power and that of its partner. In this approach, p_E would be selected when E has strong bargaining power—and both conventional parties compete *à la* Bertrand—simply because this gives maximum utility to party E. On the other hand, if E's only concern were to implement the aforementioned shift, its bliss point could simply derive from strategic reasons. By choosing p_E, the extreme party would thus try to ensure that Condition

(15) holds for both conventional voters. In Figure 1, we observe that

$$\{p_E\} = \arg\max_{p \in \mathcal{X}_E} \{\min\{v_l(p), v_r(p)\}\} .$$

The policy choice when the extreme party has strong bargaining power is hence polarized only in the second policy dimension.

We now analyze Condition (16). When the extreme party has weak bargaining power and K is the only conventional party with whom it can form a coalition, the latter party will have greater power in dictating policy, provided the discrete shift is carried out. Accordingly, party K will solve the following problem:[16]

$$\{p_{KE}\} = \arg\max_{p \in \mathcal{X}_E} \{V_K(p)\} .$$

The policy choice when the extreme party has weak bargaining power is thus polarized in the two policy dimensions.

In real-world examples, ideology and strategy concerns are not mutually exclusive reasons for extreme parties to aim for the median position in the policy dimension that does not define its *raison d'être*. For instance, in the 2015 regional elections of Catalonia, Spain, the coalition *Junts pel Sí* referred to earlier advocated a discrete shift (independence of Catalonia) while maintaining a close-to-median position with regard to the usual left-right policy issues.

In no way we claim that the aforementioned conditions hold in every political system. However, we will see below that, if CPCs are not available and Condition (15) did not hold, conventional voters would always find it convenient to coordinate their votes on one conventional party. This would prevent the extreme party from entering government. In this paper we limit ourselves to situations where conventional parties form government coalitions with an extreme party. Hence, whatever the reason, Condition (15) has to hold in situations of this type. We will also see below that if Condition (16) does not hold, then no CPCs will be offered in equilibrium, which renders this new political institution redundant. Accordingly, if CPCs brought about welfare improvements when the aforementioned condition holds, we could say that in general CPCs *weakly* bring about welfare improvements. This justifies restricting our analysis to a setting where Condition (16) is satisfied.

To illustrate the kind of policy spaces that derive from our assumptions, we conclude this section with a more in-depth analysis of Figure 1. In such a figure, $t_L < t_l < t_m < t_r < t_R$ and preferences are Euclidean for conventional parties and conventional voters. Let $d(p_1, p_2)$ denote the Euclidean distance between $p_1 \in \mathcal{P}$ and $p_2 \in \mathcal{P}$. Then, $\pi = d(p_{LR}, p_L) = d(p_{LR}, p_R)$, $\pi_E = d(p_{LR}, p_E)$, and $\alpha = \frac{d(p_l, p_{LR})}{\pi} = \frac{d(p_r, p_{LR})}{\pi}$ are the parameters indicated. π captures the parties' degree of polarization in the left-right dimension. π_E refers to the degree of polarization between party positions in the

[16] There exist intermediate bargaining protocols for the two- or three-party bargaining problem, respectively. However, the exact details of the bargaining procedure are not crucial for our results, provided that they do not reverse the voters' preference orderings.

orthogonal policy dimension. α is a measure for the degree of polarization between the conventional parties' positions and the conventional voters' positions. Conditions (14)–(16) imply respectively that

$$\max\{\alpha, 1-\alpha\}\pi < \sqrt{\pi_E^2 + \alpha^2\pi^2}, \tag{19}$$

$$\sqrt{\pi_E^2 + \alpha^2\pi^2} < (1+\alpha)\pi, \tag{20}$$

$$(1+\alpha)\pi < \sqrt{\pi_E^2 + (1-\alpha)^2\pi^2}. \tag{21}$$

A necessary condition for Conditions (19)–(21) to be compatible is that $\alpha < \frac{1}{2}$. This means that the electorate is much less polarized than the parties. In such a case, the above conditions reduce to

$$\frac{1}{\sqrt{1+2\alpha}} < \frac{\pi}{\pi_E} < \min\left\{\frac{1}{2\sqrt{\alpha}}, \frac{1}{\sqrt{1-2\alpha}}\right\}. \tag{22}$$

Hence, from the viewpoint of conventional voters, party preferences are polarized in two orthogonal dimensions with a similar degree. If the degree of polarization were very unequal, i.e. Condition (22) did not hold, there would be no room for fruitful negotiations between a conventional party and the extreme party. For instance, a conventional party would never accept installing a dictatorship, if the latter were backed by an extreme party.

3.5 Equilibrium concept

Before we explain our notion of equilibrium, we introduce some further notation. First, let $\mathcal{Y} := \mathcal{Y}(\sigma_L, \sigma_R)$ denote the random variable that represents the signal of Stage 2. We allow parties to hold different beliefs on the probability distribution induced by $\mathcal{Y}$. We denote conventional party K's beliefs by $\mathcal{S}^K := (\mathcal{S}_L^K, \mathcal{S}_R^K)$, with $\mathcal{S}_L^K = Pr(\mathcal{Y} = L)$ and $\mathcal{S}_R^K = Pr(\mathcal{Y} = R)$. It will be common knowledge that the conventional parties' priors are given by $\mathcal{S}^L$ and $\mathcal{S}^R$ and that L (or R) can be interpreted as the recommendation to coordinate on party L (or R). We further assume that

$$(\mathcal{S}_L^K, \mathcal{S}_R^K) = \begin{cases} (q_K, 1-q_K) & \text{if } (\sigma_L, \sigma_R) = (\{E\}, \emptyset), \\ (1-q_K, q_K) & \text{if } (\sigma_L, \sigma_R) = (\emptyset, \{E\}). \end{cases} \tag{23}$$

That is, q_K is conventional party K's prior that, before the election takes place, the signal will prompt coordination on the only conventional party that excluded party E in its CPC—if only one party does so. At this point, it is worth noting that with the one-party exclusion rule, we do not need to specify priors for other combinations of CPCs, as following the coordination signal will not be a best response for voters in those cases. However, such distributions are also part of the game, and they will need to be specified in Section 5. Unless otherwise stated, we will assume that both conventional parties have the same priors, i.e. $\mathcal{S} := \mathcal{S}^L = \mathcal{S}^R$. In such a case, we say that parties hold *rational expectations* and denote $q := q_L = q_R$. When no confusion is possible, we refer to q

as the signal's prior distribution instead of $\mathcal{S}$. The assumption of rational expectations is relaxed in Appendix A (see supplementary material), where we show that it is not crucial for our results.

Second, we let $\mathsf{f} := \mathsf{f}(\sigma_L, \sigma_R, \sigma_l, \sigma_r)$ denote the mapping that specifies the policy implemented by the government, given the election outcome and the CPCs written. It turns out that under the one-party exclusion rule, the CPCs written in Stage 1, the realization of $\mathcal{S}$ in Stage 2, and the votes cast by the voters in Stage 3 will uniquely determine the policies and perks chosen in Stage 4 (up to the identity of the formateur). Thus, f is well-defined. This can be verified in Table 1.

To analyze the two games of political competition and government formation, $\mathcal{G}^-$ and $\mathcal{G}^+$, we use the concepts of correlated equilibrium and sequential rationality. More precisely, we refer to a profile of pure strategies $(\sigma_L^*, \sigma_R^*, \sigma_l^*, \sigma_r^*)$ as an *equilibrium* of $\mathcal{G}^+$ if the following two conditions are satisfied:

(a) Conventional voters vote according to the signal when doing so is beneficial, i.e.,

$$v_k(\mathsf{f}(\sigma_K^*, \sigma_{\hat{K}}^*, \mathcal{Y}, \mathcal{Y})) \geq v_k(\mathsf{f}(\sigma_K^*, \sigma_{\hat{K}}^*, \sigma_k, \mathcal{Y})) \text{ for all } \sigma_k \in \{L, R\} \Longrightarrow \sigma_k^* = \mathcal{Y}.$$

(b) Strategies are sequentially rational. That is,

(b.1) for conventional voter k,

$$\sigma_k^* \in \underset{\sigma_k \in \{L,R\}}{\arg\max}\, v_k(\mathsf{f}(\sigma_L^*, \sigma_R^*, \sigma_k, \sigma_{\hat{k}}^*)).$$

(b.2) for conventional party K,

$$\sigma_K^* \in \underset{\sigma_K \in \{\emptyset, \{\hat{K}\}, \{E\}\}}{\arg\max} \mathbb{E}_{\mathcal{S}} \left[V_K \left(\mathsf{f}(\sigma_K, \sigma_{\hat{K}}^*, \sigma_l^*, \sigma_r^*) \right) + \mathcal{B}_K(\sigma_K, \sigma_{\hat{K}}^*, \sigma_l^*, \sigma_r^*) \right],$$

where $\mathcal{B}_K(\sigma_K, \sigma_{\hat{K}}^*, \sigma_l^*, \sigma_r^*)$ denotes K's perks.

The operator $\mathbb{E}_{\mathcal{S}}[\cdot]$ denotes expectation before the signal $\mathcal{S}$ is known. An equilibrium is *in weakly undominated strategies* if beyond assuming the above definition, parties also eliminate weakly dominated pure strategies. We refer to a profile of pure strategies (σ_l^*, σ_r^*) as an *equilibrium* of $\mathcal{G}^-$ if conditions (a) and (b.1) hold.

Lastly, we note that in voting games, voters are sometimes confronted with a coordination problem, typically voting for the same second-best in order to prevent a Condorcet loser from arising as an outcome (see e.g. Andonie and Kuzmics (2012) or Ekmekci (2009)). We have assumed that coordination among voters can occur without friction if voters' strategies are best responses. However, selecting one equilibrium when there are two or more equilibria that are not Pareto comparable from the voters' perspective remains a problem. Signal $\mathcal{S}$ will be decisive in such selection problems.

4 Solving the Game

We now solve our game of political competition and government formation—with and without the possibility of drafting CPCs. We start with the former case.

4.1 Framework with CPCs

Consider game $\mathcal{G}^+$. We use backward induction to solve it.

4.1.1 Government formation (Stage 4)

For completeness, we recall the equilibrium dynamics of Stage 4. According to Table 1, a single-party government will only come about at the government formation stage if one of the conventional parties obtains both votes from the two conventional voters as a result of one conventional voter having acted *strategically*. By contrast, if voters vote *sincerely*, i.e., if each of them votes for the party whose most preferred policy is closer to his own, a coalition between a conventional party and the extreme party will result, given that at least one of the conventional parties has not excluded the extreme party in its CPC. Only if both L and R have excluded E will a grand coalition come about.

4.1.2 Elections and coordination signal (Stages 2–3)

At the election stage, conventional voters have to decide whether to vote sincerely or strategically. We distinguish three cases, depending on the CPCs written in Stage 1.

Case I: $(\sigma_L, \sigma_R) = (\emptyset, \emptyset)$

When no party has excluded E, the correlation device's suggestion to coordinate the votes on one conventional party is not a best response for the voter who is prompted to vote strategically. Indeed, due to Condition (15), both conventional voters will prefer a coalition between a conventional party and E implementing policy p_E to a single-party government by their less preferred conventional party. This is depicted in Table 2, where the unique equilibrium outcome is marked in gray.

σ_l \ σ_r	L	R
L	$v_k(p_L)$	$v_k(p_E)$
R	$v_k(p_E)$	$v_k(p_R)$

Table 2: Election game when $(\sigma_L, \sigma_R) = (\emptyset, \emptyset)$.

Case II: $(\sigma_K, \sigma_{\hat{K}}) = (\emptyset, \{E\})$

Now assume that only one conventional party, say $\hat{K}$, has excluded the extreme party in its CPC and that the correlation device signals coordination on one conventional party—either K or $\hat{K}$. In either case, suppose that one conventional voter does not follow the suggestion implied by the correlation device. Then each conventional party will obtain one vote, K will form a coalition with the extreme party, and p_{KE} will be implemented. Following the signal of the correlation device, a single-party government of either L or R will form. According to Condition (16), both conventional voters prefer a single-party government of their less preferred conventional party to a coalition government formed by a conventional party and party E implementing p_{KE}. Consequently, the correlation signal is a best response in the case where only one conventional party has excluded E from a coalition government. This is depicted in Table 3, where the two possible equilibrium outcomes are marked in gray.

σ_l \ σ_r	L	R
L	$v_k(p_L)$	$v_k(p_{KE})$
R	$v_k(p_{KE})$	$v_k(p_R)$

Table 3: Election game when $(\sigma_L, \sigma_R) = (\emptyset, \{E\})$.

Case III: $(\sigma_L, \sigma_R) = (\{E\}, \{E\})$

When both conventional parties have excluded E, the conventional voters' decisions will depend on how they rank p_{LR}, p_R, and p_L. Providing that Condition (13) holds, the equilibrium outcome is always p_{LR}. This is depicted in Table 4, where the equilibrium outcomes are marked in gray.

σ_l \ σ_r	L	R
L	$v_k(p_L)$	$v_k(p_{LR})$
R	$v_k(p_{LR})$	$v_k(p_R)$

σ_l \ σ_r	L	R
L	$v_k(p_L)$	$v_k(p_{LR})$
R	$v_k(p_{LR})$	$v_k(p_R)$

Table 4: Election game when $(\sigma_L, \sigma_R) = (\{E\}, \{E\})$ and either $v_k(p_{LR}) > v_k(p_K)$ (left table) or $v_k(p_{LR}) < v_k(p_K)$ (right table).

To sum up, the conventional voters will follow the signal to coordinate their voting behavior if and only if one party has excluded E in its CPC. Otherwise, following the signal is not a best response for one conventional voter, and voters will vote sincerely. Moreover, since conventional voters can always coordinate their votes on either conventional party, neither a caretaker government nor a coalition government of a conventional party and the extreme party will form in equilibrium when the latter has weak bargaining power. Hence, policies p_{ct}, p_{LE} and p_{RE} cannot arise in equilibrium, only p_{LR}, p_L, p_R, and p_E. The question remains, however, which of the latter policies may arise.

4.1.3 Coalition Preclusion Contracts (Stage 1)

The discussion in the rest of Section 4.1 is devoted very largely to the analysis of the incentives for conventional parties to sign a CPC when such a possibility is available. We will assume that party

E will be excluded rather than not excluded by a conventional party when the latter is indifferent between the two options. This tie-breaking rule is not essential.[17] Under the one-party exclusion rule we can summarize the contract choice game played by the conventional parties as in Table 5, where $V_K^x(p_1,p_2) := x \cdot V_K(p_1) + (1-x) \cdot V_K(p_2)$ for $x \in [0,1]$ and $p_1, p_2 \in \mathcal{P}$.[18]

	$\emptyset$	$\{E\}$
$\emptyset$	$\frac{B}{2} + V_L(p_E), \frac{B}{2} + V_R(p_E)$	$(1-q)B + V_L^{1-q}(p_L,p_R), qB + V_R^{1-q}(p_L,p_R)$
$\{E\}$	$qB + V_L^q(p_L,p_R), (1-q)B + V_R^q(p_L,p_R)$	$b + V_L(p_{LR}), b + V_R(p_{LR})$

Table 5: Conventional parties' contract choice game. Row player is party L and column player is Party R.

From Table 5 we deduce that an equilibrium of contract choices will depend crucially on the conventional parties' beliefs regarding voter coordination, i.e., on the exact value of q. We proceed in two steps.

First, suppose without loss of generality that L has excluded E. Then we analyze R's incentives to also exclude E. On the one hand, suppose that R decides to exclude E. From the discussion in Section 4.1.2, we know that when both conventional parties exclude the extreme party, voters will vote sincerely, and a grand coalition will result. On the other hand, in the case where R does not exclude the possibility of forming a coalition with E, party R believes that with probability q a single-party government of L will be established and with the complementary probability R will be able to form a single-party government on its own. Consequently, signing a CPC excluding E given that L has excluded E is profitable ex-ante for R if and only if

$$q \cdot V_R(p_L) + (1-q) \cdot (V_R(p_R) + B) \leq V_R(p_{LR}) + b,$$

that is, if and only if

$$q \geq \frac{(B-b) + V_R(p_R) - V_R(p_{LR})}{B + V_R(p_R) - V_R(p_L)} \equiv q_R^c. \tag{24}$$

The weak inequality sign follows from the tie-breaking rule. With regard to the incentive of L to exclude E given that R does so, the symmetric condition needs to be satisfied, i.e.,

$$q \geq \frac{(B-b) + V_L(p_L) - V_L(p_{LR})}{B + V_L(p_L) - V_L(p_R)} \equiv q_L^c. \tag{25}$$

Note that the right-hand sides of inequalities (24) and (25) are strictly smaller than one, implying that both parties excluding E will be an equilibrium if, from the parties' perspectives, the probability

[17] If the parameters of party and voter preferences are drawn from a non-degenerate continuous distribution, the latter event has probability zero.

[18] We only consider conventional parties as writers of CPCs, since the extreme party has no incentive to exclude a conventional party. This would only reduce its bargaining power at the government formation stage.

that the voters will coordinate on the sole conventional party excluding E is sufficiently high. We point out that q^c_K may be smaller or larger than $\frac{1}{2}$.

Second, given that the other conventional party does not sign a CPC, we obtain the following necessary and sufficient conditions for a party to exclude the extreme party from a coalition:

$$q \geq \frac{\frac{B}{2} + V_R(p_E) - V_R(p_L)}{B + V_R(p_R) - V_R(p_L)} \equiv q^n_R, \tag{26}$$

$$q \geq \frac{\frac{B}{2} + V_L(p_E) - V_L(p_R)}{B + V_L(p_L) - V_L(p_R)} \equiv q^n_L. \tag{27}$$

As indicated, we use q^c_R, q^c_L, q^n_R, and q^n_L to denote the critical values that make the parties indifferent between excluding E and not signing a contract. Using Conditions (24)–(27), we can then directly infer that both parties excluding the extreme party will be an equilibrium whenever $q \geq q^c_L$ and $q \geq q^c_R$. Moreover, neither conventional party signing a CPC will be an equilibrium if $q < q^n_L$ and $q < q^n_R$.

In our analysis, we will further assume that conventional parties are symmetric with respect to all preference parameters. This will simplify the exposition but will not affect the main thrust of our results. Moreover, it will enable us to concentrate on effects of CPCs that do not arise due to the asymmetry of conventional parties. A possibility of this kind is investigated in Appendix A (see supplementary material).

Symmetry Condition 1 (SC1): $q^n_L = q^n_R := q^n$ and $q^c_L = q^c_R := q^c$

In terms of the primitives of the model, this condition is equivalent to

$$\frac{(B-b) + V_R(p_R) - V_R(p_{LR})}{(B-b) + V_L(p_L) - V_L(p_{LR})} = \frac{\frac{B}{2} + V_R(p_E) - V_R(p_L)}{\frac{B}{2} + V_L(p_E) - V_L(p_R)} = \frac{B + V_R(p_R) - V_R(p_L)}{B + V_L(p_L) - V_L(p_R)}.$$

We note that it suffices for SC1 to hold for us to impose the following standard conditions:

$$V_R(p_R) - V_R(p_L) = V_L(p_L) - V_L(p_R), \tag{28}$$

$$V_R(p_E) - V_R(p_L) = V_L(p_E) - V_L(p_R), \tag{29}$$

$$V_R(p_R) - V_R(p_{LR}) = V_L(p_L) - V_L(p_{LR}). \tag{30}$$

For a complete characterization of the equilibria, the relations between q^n_K and q^c_K play a crucial role. We distinguish two cases depending on the latter relation.[19]

Symmetry Condition 2A (SC2A): $q^n_K < q^c_K$ for $K \in \{L, R\}$

From conditions (24)–(27) it follows that SC2A can be written as

$$(V_K(p_K) - V_K(p_{LR})) - (V_K(p_{LR}) - V_K(p_{\hat{K}})) > \left(b - \frac{B}{2}\right) + (V_K(p_E) - V_K(p_{LR}))\,. \tag{31}$$

[19]We do not consider the case $q^n_K > q^c_K$ and $q^c_{\hat{K}} > q^n_{\hat{K}}$, since this would contradict SC1.

For given b, Condition (31) sets a lower bound on the difference in the relative utilities derived from the single-party governments with respect to the grand coalition. This bound is given by the utility difference between a coalition government with the extreme party and a grand coalition. We emphasize that SC2A requires certain symmetry properties *within* each party's utility profile but does not imply a statement about the relation between the parties' utility profiles, e.g. about the relation of q_L^c and q_R^c. Under SC1 and SC2A, the equilibria outcomes depending on parties' beliefs on voter coordination are

$$(\sigma_L, \sigma_R) = \begin{cases} (\emptyset, \emptyset) & \text{if } 0 \leq q < q^n, \\ (\emptyset, \{E\}), (\{E\}, \emptyset) & \text{if } q^n \leq q < q^c, \\ (\{E\}, \{E\}) & \text{if } q^c \leq q \leq 1. \end{cases} \tag{32}$$

Symmetry Condition 2B (SC2B): $q_K^n \geq q_K^c$ for all $K \in \{L, R\}$

The interpretation of SC2B is similar to that of SC2A. Under SC1 and SC2B, the equilibria outcomes depending on parties' beliefs about voter coordination are

$$(\sigma_L, \sigma_R) = \begin{cases} (\emptyset, \emptyset) & \text{if } 0 \leq q < q^c, \\ (\emptyset, \emptyset), (\{E\}, \{E\}) & \text{if } q^c \leq q < q^n, \\ (\{E\}, \{E\}) & \text{if } q^n \leq q \leq 1. \end{cases} \tag{33}$$

A comprehensive description of the CPCs written in each equilibrium is given when SC2A holds (or when SC2B holds) by expression (32) (or expression (33)). To sum up, we have proved the following result:

Theorem 1

In the sequential game $\mathcal{G}^+$, any equilibrium under SC1 can be described for each coordination prior $q \in [0, 1]$ as follows:

(i) conventional parties will choose CPCs according to expression (32) (when SC2A holds) and expression (33) (when SC2B holds),

(ii) voters will follow the recommendation of the correlation device if and only if only one conventional party has excluded E and vote sincerely otherwise,

(iii) government formation will result in a grand coalition implementing policy p_{LR} if $(\sigma_L, \sigma_R) = (\{E\}, \{E\})$, in a coalition government between one conventional party and E implementing policy p_E if $(\sigma_L, \sigma_R) = (\emptyset, \emptyset)$, and in single-party government of one conventional party, say K, implementing its preferred policy p_K, otherwise.

4.2 Framework with no CPCs

The situation without CPCs is simple and has been implicitly analyzed in the previous section, since $\mathcal{G}^-$ is a subgame of $\mathcal{G}^+$. For the sake of completeness, however, we also describe it here. If citizens vote sincerely, none of the parties will be able to form a single-party government. Moreover, since both conventional parties will prefer a coalition with the extreme party to a grand coalition government, they will compete for E as a coalition partner. Consequently, E will have strong bargaining power, thereby leading to policy p_E within a coalition government with one of the conventional parties. Does a conventional voter have an incentive to follow the coordination signal, deviate, and vote strategically? The answer is negative. If one of the conventional voters votes strategically, the result will be a single-party government run by the conventional party they do not support. According to Condition (15), both conventional voters will prefer a coalition between L or R and E, who will implement policy p_E, to a single-party government by the conventional party they do not support. Hence, the following result holds:

Theorem 2
In the sequential game $\mathcal{G}^-$, any equilibrium of the game under SC1 can be described as follows:

(ii) voters vote sincerely,

(iii) government formation results in a coalition of one conventional party and the extreme party, implementing policy p_E.

In the light of Theorem 2, E's choice of p_E and e's choice to vote for E when CPCs are not available are thus rational. The reason is that p_E is implemented, as desired by both E and e.

5 General Coalition Preclusion Contracts

So far we have discussed a particular specification of CPCs, where each party was allowed to preclude only one other party from forming a coalition. If there is no restriction on the number of parties that can be precluded in a CPC, we may encounter additional equilibria. This is shown here by the analysis of the contract choice game for the conventional parties without the one-party exclusion rule, as indicated in Table 6. Recall that $V_K^x(p_1, p_2) := x \cdot V_k(p_1) + (1 - x) \cdot V_k(p_2)$ for $x \in [0, 1]$ and $p_1, p_2 \in \mathcal{P}$. Additionally, we let B_K^x be such that $B_L^x = B \cdot x$ and $B_R^x = b \cdot (1 - x)$ for $x \in [0, 1]$.

		σ_R: $\emptyset$	$\{E\}$	$\{L, E\}$
σ_L	$\emptyset$	$\frac{B}{2} + V_K(p_E)$	$B_K^{1-q} + V_K^{1-q}(p_L, p_R)$	$B_K^{1-q} + V_K^{1-q}(p_L, p_R)$
	$\{E\}$	$B_K^q + V_K^q(p_L, p_R)$	$b + V_K(p_{LR})$	$\frac{B}{2} + V_K^{\frac{1}{2}}(p_L, p_R)$
	$\{R, E\}$	$B_K^q + V_K^q(p_L, p_R)$	$\frac{B}{2} + V_K^{\frac{1}{2}}(p_L, p_R)$	$\frac{B}{2} + V_K^{\frac{1}{2}}(p_L, p_R)$

Table 6: Conventional parties' contract choice game without the one-party exclusion rule.

Some remarks are required. First, for conventional party K, strategy σ_K now selects an element of $\{\emptyset, \{\hat{K}\}, \{E\}, \{\hat{K}, E\}\}$. Since $\sigma_K = \emptyset$ and $\sigma_K = \{\hat{K}\}$ yield the same payoffs to party K for every strategy of party $\hat{K}$, we have only included in Table 6 those strategies that are not payoff-equivalent. Second, without the one-party exclusion rule, priors of the coordination signal for cases where $(\sigma_K, \sigma_{\hat{K}}) \neq (\emptyset, \{E\})$ now *do* matter. In Table 6, we have assumed that priors for the coordination signal are the same, namely q, whenever party K has excluded at least E and party $\hat{K}$ has not excluded any party. When no coalition is feasible at all, however, we have also assumed in Table 6 that the signal will prompt coordination on either conventional party with probability $\frac{1}{2}$. Recall that when no government coalition can be formed at all both conventional voters will always find it profitable to coordinate their votes on one conventional party. When conventional parties are symmetric regarding the CPCs offered, there is no reason to expect higher coordination on one conventional party than on the other. In that case, either conventional party will receive a majority of votes and will form a government with probability $\frac{1}{2}$.

Before we study the possible equilibria of the game defined in Table 6, it will be convenient to specify another condition.

Risk Condition (RC): $V_K(p_{LR}) \geq V_K^{\frac{1}{2}}(p_L, p_R)$

This condition on the conventional parties' utilities has the following appeal: If the policy outcomes of a single-party government are distributed symmetrically in the policy space around the grand-coalition policy outcome and parties have a concave utility function regarding policies, then RC will hold. A concave utility function for the parties implies that they are risk-averse in connection with the policy outcome. Since conventional parties are also risk-averse regarding perks—see Condition (1)—and parties' utility is separable in perks and policies, RC implies that

$$b + V_K(p_{LR}) \geq \frac{B}{2} + V_K^{\frac{1}{2}}(p_L, p_R). \tag{34}$$

A consequence of Condition (34) is that strategy $\{\hat{K}, E\}$ is weakly dominated by strategy $\{E\}$ for conventional party K. As a tie-breaking rule, we assume that in the case of indifference between excluding only E or any other possible contract choice, strategy $\{E\}$ will be chosen. The following result is straightforward:

Proposition 1

Under SC1 and RC, if conventional parties play no weakly dominated strategy, the equilibria of game $\mathcal{G}^+$ (or $\mathcal{G}^-$) with the one-party exclusion rule are the same as the equilibria of game $\mathcal{G}^+$ (or $\mathcal{G}^-$) without the one-party exclusion rule—as given by Theorem 1 (or Theorem 2).

Proposition 1 implies that if conventional parties play no weakly dominated strategies the one-party exclusion rule is not needed. However, if conventional parties were to consider playing weakly dominated strategies further equilibria would arise without the one-party exclusion rule. Without the one-party exclusion rule, all coalitions may be ruled out if coordination probability q is large enough.

Not imposing the one-party exclusion rule when parties may be considering weakly dominated strategies can thus have consequences on welfare (see next section). The reason is that the conventional parties will be able to force single-party governments to come about more often than with the one-party exclusion rule. This may bring about less utility for the voters than policy p_E. We derive all additional equilibria with weakly dominated strategies without the one-party exclusion rule in Appendix B (see supplementary material).

6 Welfare Analysis and Implementation

In this section we study *(i)* the welfare implications of CPCs and *(ii)* whether and how CPCs could be implemented.

6.1 Welfare analysis

Any analysis of welfare will hinge on four key elements. These are whether the one-party exclusion rule applies or not, how likely we regard each of the possible multiple equilibria to be, the criterion used to measure welfare, and the information available when we apply this criterion. The last-named element is important because the level of knowledge about the likelihood of coordination occurring in favor of a conventional party that excludes E depends on the stage at which we measure welfare. To analyze the different timing possibilities, it is useful to add a *Stage 0* to the political games $\mathcal{G}^-$ and $\mathcal{G}^+$, in which nature selects the exact value of q from a given distribution. We can therefore analyze the impact of CPCs on welfare *ex-ante*, i.e. before the game starts at Stage 0 or *ex-interim*, i.e. before parties write the CPCs at Stage 1, but as soon as the realization of the exact value of q—but not of the signal itself—is common knowledge to all players. Throughout this section, we only consider rational beliefs, i.e. $q = q_L = q_R$.

Next, we introduce a welfare function $W(\cdot)$ which will be used throughout this section and assume that it satisfies

$$W(p_{LR}) > W(p_E). \tag{35}$$

Condition (35) demands that, from a social perspective, the shift embodied in the extreme policy should not be desirable. In the following we show by means of an example that such welfare functions can be defined in a standard way and discuss the extent of the restrictions imposed by Condition (35). Since we have established that voters' disutility from perks is the same whatever the government coalition looks like, we further make the normative assumption that a measure of aggregate welfare should depend solely on the policy implemented, regardless of the exact distribution of perks among parties.

Example 1

Let us consider a utilitarian approach which only takes voters (and not parties) into account. Then, we need to specify two additional elements: (i) the extreme voters' utility function, namely $v_e(\cdot)$, and (ii) the population share α_k that voter $k \in \{l, r, e\}$ represents, with $\alpha_l + \alpha_r + \alpha_e = 1$. Utilitarian welfare is then given as follows:

$$W^u(p) = \alpha_l \cdot v_l(p) + \alpha_r \cdot v_r(p) + \alpha_e \cdot v_e(p).$$

Under the equivalence of Conditions (28)–(30) for the voters' utilites and assuming that p_E is voter e's preferred policy, we obtain that Condition (35) holds for $W^u(\cdot)$ if and only if

$$\frac{v_k(p_{LR}) - v_k(p_E)}{v_e(p_E) - v_e(p_{LR})} > \frac{\alpha_e}{1 - \alpha_e} \text{ for } k \in \{l, r\}.$$

Hence, it is sufficient that the share of extreme voters be not too large for Condition (35) to be satisfied.

In Section 4 we showed that either with or without CPCs, policies that differ from p_{LR}, p_L, p_R, and p_E cannot arise in equilibrium. From the point of view of the two conventional voters, any pair from p_L, p_R, and p_E cannot be ranked according to the Pareto criterion, while p_{LR} and p_E can be. How society as a whole ranks these different policies is obviously relevant in assessing the impact that CPCs may have on welfare. The following condition on $W(\cdot)$ turns out to play a crucial role in determining whether CPCs are welfare-improving or not:

$$\frac{1}{2}W(p_L) + \frac{1}{2}W(p_R) > W(p_E). \tag{36}$$

The left-hand side of the above inequality contains expected societal utility when there is an equal probability that either conventional party will form a single-party government, while the right-hand side contains the societal valuation of p_E. When interpreted within our two-dimensional space with the usual left-right policy plus the possibility of a discrete shift orthogonal to it, Condition (36) ensures that the society is not too risk-averse in the continuous policy component. Indeed, under symmetry of the conventional parties' preferred policy with respect to the policy implemented by the grand coalition, the expected policy when p_L and p_R are equally likely is p_{LR}. Condition (36) thus requires that the loss of societal utility due to uncertainty in the continuous policy be offset by the societal loss of carrying out the extreme policy with certainty. We further note that if $W(\cdot)$ is concave with regard to policy dimension $\mathcal{T}$ and the aforementioned symmetry assumptions hold, Condition (36) is more restrictive than Condition (35).

Finally, we assume that the exact value of q is distributed with full support on $[0, 1]$ according to a non-degenerate probability distribution with cumulative distribution $H(\cdot)$. The main result regarding ex-ante welfare, $\mathbb{E}_H[W(\cdot)]$, i.e., welfare before q is realized and becomes common knowledge, is now stated.

Theorem 3

Under SC1 and Condition (35), we obtain the following results:

(a) *CPCs with the one-party exclusion rule are ex-ante welfare-improving when SC2B holds.*

(b) *CPCs with the one-party exclusion rule are ex-ante welfare-improving when SC2A holds if Condition (36) applies.*[20]

Moreover, assume that Condition (1) holds tight, i.e. $B = 2b$. Then

(c) *CPCs with the one-party exclusion rule are ex-ante welfare-improving if b is large enough.*

(d) *CPCs without the one-party exclusion rule may not be ex-ante welfare-improving even if b is large enough.*

A formal proof can be found in Appendix B (see supplementary material), but the intuition of the theorem can be summarized with the help of expressions (32) and (33). First, consider Part (a) of the theorem. If SC2B holds, rational expectation equilibria with the one-party exclusion rule imply that either both conventional parties will not exclude any other party, in which case the introduction of CPCs is devoid of bite, or both conventional parties will exclude the extreme party. In the latter case, a grand coalition occurs which will implement p_{LR}. The latter outcome will be preferable to p_E, which is the outcome without CPCs. Consequently, the expected welfare gain from introducing CPCs will be positive if the probability that $q > q^c$ is positive.

Regarding Part (b) of Theorem 3, if SC2A holds instead of SC2B, a single party government by a conventional party will be formed with equal probability for L and R when the coordination probability q is in the interval $[q^n, q^c)$. Condition (36) says that the expected welfare from single-party governments by conventional parties is higher than the welfare from the policy of a small coalition with strong bargaining power on the part of the extreme party, p_E. Then we can apply the same line of argument as before. If CPCs have bite, they lead to more favorable policy outcomes in expectation than the policy that will be implemented without them. Hence the expected welfare gain from introducing CPCs will be positive if the probability that $q > q^n$ is larger than zero.

However, if the expected welfare from equally likely single-party governments is smaller than that obtained from p_E, i.e. if Condition (36) does not hold, it will depend on the probability that $q \in [q^n, q^c)$ whether the expected welfare gain from the introduction of CPCs will be positive or negative. Both probabilities q^n and q^c converge to $\frac{1}{2}$ if $b \to \infty$ and $\frac{b}{B} \to \frac{1}{2}$. Consequently, we can find a sufficiently large amount of perks b such that the interval $[q^n, q^c)$ becomes small enough for

[20] We assume that, for a given value of q, all possible equilibrium outcomes are equally likely ex-ante, i.e. before the signal is realized. This means that the probability of L excluding E is the same as the probability of R excluding E. Without this assumption, it suffices to replace Condition (36) by the stronger Condition (37) for Part (b) to hold.

the expected welfare gain associated with the introduction of CPCs to be positive. This describes Part (c) of Theorem 3.[21]

Lastly, we explain Part (d) of Theorem 3. In cases where parties play weakly dominated strategies and the one-party exclusion rule does not apply, the conventional parties may force single-party governments to come about whenever $q > q^n$, while if $q \leq q^n$ CPCs are not used to exclude another party. If Condition (36) holds, single-party governments are socially preferred to small coalitions with strong bargaining power on the part of the extreme party. Hence, the introduction of CPCs will again lead to expected welfare gains. However, if Condition (36) does not hold, CPCs will lead to lower social welfare. This will be true even for large amounts of perks, as q^n will converge to $\frac{1}{2}$, implying that single-party governments will come about if $q > \frac{1}{2}$.

In the remainder of this section we analyze ex-interim welfare, i.e., welfare evaluated after the realization of the exact value of q is known. We obtain a result similar to Theorem 3. In this case, however, it is the following condition—which is stronger than Condition (36)—that plays a role:

$$\min\{W(p_L), W(p_R)\} > W(p_E). \tag{37}$$

The interpretation of the theorem below—the proof of which can be found in Appendix B (see supplementary material)—is similar to that of Theorem 3.

Theorem 4

Under SC1 we obtain the following results:

(a) *CPCs with the one-party exclusion rule are weakly ex-interim welfare-improving when SC2B holds.*

(b) *CPCs with the one-party exclusion rule are weakly ex-interim welfare-improving when SC2A holds if Condition (37) also holds.*

Moreover, assume that Condition (1) holds tight, i.e. $B = 2b$. Then

(c) *CPCs with the one-party exclusion rule are ex-interim welfare-improving if b is large enough and $q > \frac{1}{2}$.*

(d) *CPCs without the one-party exclusion rule may not be ex-interim weakly welfare-improving even if b is arbitrarily large and $q > \frac{1}{2}$.*

[21] The reason why $q^n - q^c$ converges to zero if $B = 2b$ and b approaches infinity is as follows: For conventional party K, the difference in expected utility from perks associated with p_E and p_K, as reflected in the numerator of q^n, is the same as the difference between p_K and p_{LR} as captured in the numerator of q^c. Consequently, the difference between q^n and q^c originates from the differences regarding policy. This latter part becomes less important if perks increase, in particular negligible if perks go to infinity. That q^n and q^c must converge to $\frac{1}{2}$ then follows from the fact that expected utility from perks associated with p_E and with p_{LR} are half the size of those associated with p_{LR}. If we changed these assumptions, the critical probabilities would converge to different values.

If $q > \frac{1}{2}$, writing a CPC makes it more likely for a conventional party to receive the coordinated votes of the conventional voters. A more detailed look at the findings of the previous sections reveals further results regarding ex-interim welfare. First, not only CPCs are quite often weakly ex-interim welfare-improving but in many cases they yield policy p_{LR} whenever each conventional party excludes the extreme party. Such a policy may be the *first-best outcome* if conventional voters are not very partisan. Under our assumptions on parties' and voters' preferences, first-best outcomes are never attainable without CPCs. Second, if $q^c, q^n < \frac{1}{2}$, then CPCs with the one-party exclusion rule yield p_{LR}, even in cases where the probability of voters coordinating on the sole party that has excluded E, i.e. q, is lower than a half.

6.2 Implementation

Although CPCs with the one-party exclusion rule are welfare-improving, it is by no means certain that they will be introduced. And if they are introduced, they could also be manipulated. Therefore we now analyze the incentives of parties to introduce CPCs and to manipulate the rules governing their use. The critical issues are:

- Can a conventional party bypass CPCs by accepting the votes of parties that were excluded in its CPC?
- Will the possibility of writing CPCs be introduced at the constitutional stage?
- Are there incentives for newly-created governments to abolish the law regulating CPCs?

Accordingly, we develop our analysis in three steps. First, CPCs are political contracts that constrain the choices regarding government coalition formation but do not impose any direct restriction on policy outcomes. Hence, the verification whether a party honors or violates its CPC has to occur at the stage where the government is formed through a vote of confidence. Later on, when the parliament decides on particular policies, CPCs have no bite. Nevertheless, conventional parties may try to bypass a CPC by forming a minority government tolerated by the extreme party. This could occur by (secretly) supporting the conventional party when the vote of confidence takes place and by openly supporting policy compromises with the conventional party later on. As already mentioned, a recent example of such a political deal occured in Catalonia, Spain, with the CUP supporting the government of *Junts pel Sí* without explicitly being part of it. To avoid such types of manipulations, verification of CPCs has to be designed with care. In particular, one could specify that a party does not violate its CPC if the share of supporting votes obtained in the vote of confidence from parties that are not excluded in its CPC is at least 50%.[22] Otherwise, the party would be subject to the penalization associated with CPC violation.

[22] As those votes may be secret, verification can be achieved by a third person (authority or judge) or by codes in electronic ballots identifying party affiliation.

Second, we demonstrate that both conventional parties R and L would support a law introducing CPCs with the one-party exclusion rule whenever they are risk-averse or perks are high. Indeed, while the situation without CPCs yields a government formed by each conventional party and the extreme party with probability $\frac{1}{2}$, which implements p_E, the situation with CPCs yields the following outcomes: On the one hand, if SC2B holds, either there will be no change compared to the situation without CPCs or p_{LR} will be implemented by the grand coalition. It will suffice to focus on the latter situation to delineate the incentives for introducing CPCs. Recall that throughout our analysis we have assumed Condition (4), which can be rewritten as follows:

$$V_K(p_{LR}) + b > \frac{1}{2} \cdot (V_K(p_E) + B) + \frac{1}{2} \cdot (V_K(p_E) + 0)\,.$$

Hence, from a constitutional perspective both conventional parties prefer the certain possibility of a grand coalition to the stochastic possibility leading to a coalition government of a conventional party plus the extreme party with equal probability.

On the other hand, when SC2A holds, the situation is a bit more involved. In such a case either there will be no change compared to the situation without CPCs, or p_{LR} will be implemented by the grand coalition, or a single-party government of either conventional party, say K, will be formed and will implement policy p_K. The first and second possibilities are the same as in the case where SC2B holds. Accordingly, we focus on the latter possibility, which occurs whenever $q \in [q^n, q^c)$. In such a case, conventional party K's expected utility will be higher than in the case without CPCs if the following sufficient condition holds:

$$\frac{1}{2} \cdot V_K(p_K) + \frac{1}{2} \cdot V_K(p_{\hat{K}}) > V_K(p_E). \tag{38}$$

Interpreted within our two-dimensional space, the above condition ensures that party K will not be too risk-averse in dimension $\mathcal{T}$ with respect to the shift proposed by the extreme party in dimension $\mathcal{D}$. The following result is then straightforward:

Proposition 2

Both conventional parties favor the introduction of CPCs if Condition (38) holds or perks are large enough.

To sum up, before a campaign for an election starts, both conventional parties would favor the introduction of CPCs under either of the plausible assumptions—regardless of whether SC2A or SC2B holds. In such circumstances, they could successfully engineer a change in the constitution to permit this type of political contracts as they have a supermajority of members in the parliament, a threshold which is often required for constitutional changes.[23]

Third, newly-formed governments may have an incentive to abolish the law governing the use and application of CPCs—recall Conditions (4)–(5). A conventional party, say $\hat{K}$, may even be tempted

[23]Often, 2/3 or 3/4 supermajorities are required to change the constitution. Of course, the extreme party dislikes the introduction of CPCs as it may lose its influence on government formation and policies.

to form a government with the extreme party excluded in its CPC and then abolish the law governing CPCs immediately after. There are two ways to prevent such attempts. First, one could impose a clause in the constitution stipulating that laws governing CPCs cannot be abolished retroactively, so that abolishing the law is not beneficial for a conventional party once it has formed a coalition with the extreme party. Second, one could stipulate that abolition itself requires a supermajority, as it may require an amendment of the constitution. Because after the elections there is always at least one conventional party in favor of CPCs, a supermajority requirement would effectively block the elimination of the law governing CPCs. Note that from the perspective of conventional party K, with $K \neq \hat{K}$, it holds that

$$\min\{b + V_K(p_{LR}), B + V_K(p_E), B + V_K(p_{KE})\} > \max\{V_K(p_E), V_K(p_{\hat{K}E})\}. \tag{39}$$

To understand Condition (39), consider the worst possible outcome for the party which is not contemplating breaking its CPC and forming a coalition with E, namely party K, if the law is not removed. Then such an outcome yields a higher utility to K than the best possible outcome when $\hat{K}$ wants to eliminate the law (and not honor its CPC). Hence K will never support such a constitutional change after the election and before the government is formed and the supermajority to abolish the law regulating CPCs will not be achieved.

7 Conclusions and Extensions

Coalition Preclusion Contracts (CPCs) are a simple device that can affect the way democracies with multiple parties operate. As coalition formation is observable and verifiable, CPCs are easy to implement. We have suggested that, on balance, these contracts moderate policies and improve welfare when coupled with the one-party exclusion rule.[24] Beyond the normative value of the paper, our results can also be interpreted from a purely positive perspective to the extent that they provide insights with regard to when and why party announcements about possible coalitions are either credible or not. Our results also help us gain some intuition about the social value of parties' commitments regarding coalition formation.

Several extensions of our model could be investigated. First, we could include two extreme parties instead of just one. Second, we could relax the assumption of rational expectations. Third, we could consider a continuum of voters. Fourth, we could allow asymmetric conventional parties. Fifth, we could introduce uncertainty with respect to the support of the extreme party in the election. Sixth, we could limit the power of the extreme party in bilateral negotiations with conventional parties by constraining the offers acceptable for these latter parties. Seventh, we could consider that perks increase with the parliamentary support for the government coalition. Eighth and last, we could analyze the long-term costs and benefits of CPCs for parties and society by considering

[24]CPCs should under all circumstances honor constitutional rights of minorities.

multiple elections. In Appendix A (see supplementary material) we have pursued all these extensions. Overall, they reveal that our findings are quite robust and that the one-party exclusion rule has to be adapted when there are more than three parties.

Numerous further issues wait to be explored. For instance, combining CPCs with endogenous platform choices in campaigns or exploring the consequences of such contracts in systems with more than two conventional parties are obvious candidates. Our results suggest that CPCs could be introduced on an experimental basis in democracies.

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A Appendix

In this appendix we reconsider some of the assumptions made in the main body of the paper and check whether our conclusions remain valid, at least qualitatively. We study several variations, the sole condition being that we only change one feature of the model at a time.

1. We consider two extreme parties, E_1 and E_2, instead of just one.
2. We relax the assumption of rational expectations, i.e. q_L and q_R might be different.
3. We consider a continuum of voters.
4. A difference between conventional parties is introduced by allowing their corresponding ideal points not to be symmetrically located with respect to the policy a grand coalition would implement.
5. We introduce uncertainty with respect to the share of the electorate represented by the extreme voter e.
6. We limit the power of party E in its bilateral negotiations with conventional parties by constraining the offers acceptable for those parties.
7. We consider the case where the ability of the governing coalition to distribute perks increases with its parliamentary support, as does the voters' disutility from perks.
8. We analyze the long-term costs and benefits of CPCs for parties and society when more than one election is considered.

A.1 Multiple extreme parties

As in most parliamentary democracies there are several extreme parties trying to enter parliament, it is informative to examine the situation with more than one such party. This case is also interesting because even if there is only one extreme party and CPCs were in place, the extreme party might have an incentive to split into two or more parties to bypass the effect of CPCs—especially under the one-party exclusion rule.

We stay within the setup of the main body of the paper, the only modification being that we now consider two extreme parties, E_1 and E_2, instead of just one. The two extreme parties pursue an implementation of shifts in policy dimensions $\mathcal{D}_1 = \{0, \bar{d}_1\}$ and $\mathcal{D}_2 = \{0, \bar{d}_2\}$ respectively, with 0 denoting the status quo in these dimensions, respectively. Both $\mathcal{D}_1$ and $\mathcal{D}_2$ are orthogonal to $\mathcal{T}$. If $\mathcal{D}_1 = \mathcal{D}_2$, both extreme parties aim for the same shift. If $\mathcal{D}_1 \neq \mathcal{D}_2$, they aim for shifts in orthogonal dimensions. We also assume that each extreme party obtains the same share of the votes in the election, so that a coalition with either of them would be sufficient to form a government.[25] Note that if one extreme party had a higher share than the other one, the extreme party with the lowest share would be irrelevant in the model, as this extreme party would not be able to form a majority with a conventional party. In that case, the analysis set out in the main body of the paper would still be valid.

As in the case of one extreme party only, we will say that an extreme party possesses *strong bargaining power* if neither conventional party has excluded it in its CPC. By contrast, we will say that an extreme party possesses *weak bargaining power* if exactly one conventional party has excluded it in its CPC. We let both extreme parties be identical in the following senses: First, a coalition government formed by a conventional party and extreme party E_J ($J = 1, 2$) with strong bargaining power will implement policy p_{E_J}. Second, a coalition government formed by a conventional party and one extreme party with weak bargaining power

[25] To remain strictly within the setup of Section 3, we assume that the extreme voter is able to split his vote. In the case where each conventional party attempts to form a coalition with one extreme party, each coalition has a probability of 1/2 for forming the government.

will implement policy p_{KE_J}. Preferences of parties and voters carry over from the baseline setup to this new setting. In addition, each conventional party and each conventional voter dislikes $\bar{d}_1$ and $\bar{d}_2$ equally. This latter observation will enable us to simplify (and slightly abuse) notation and denote $\mathcal{D} = \mathcal{D}_1 = \mathcal{D}_2 = \{0, \bar{d}\}$, while keeping in mind that extreme parties E_1 and E_2 will either act together (when $\bar{d}_1 = \bar{d}_2$) or compete against each other (when $\bar{d}_1 \neq \bar{d}_2$). In particular, we also set $p_E = p_{E_1} = p_{E_2}$ and $p_{KE} = p_{KE_1} = p_{KE_2}$, as such policies yield the same utilities for conventional parties and voters, respectively.

For the analysis of this modified setup, we need to specify the outcome at the government formation stage when one conventional party, say K, has excluded one of the extreme parties, say E_1, while the other conventional party, $\hat{K}$, has excluded neither E_1 nor E_2. First, if $\bar{d}_1 = \bar{d}_2$, extreme party E_1 will walk away from any negotiations, thereby granting E_2 strong bargaining power in the negotiations with the conventional parties as both extreme parties have common interests. Second, consider $\bar{d}_1 \neq \bar{d}_2$. In this case, if no conventional party has received a majority of votes, there will be competition regarding a coalition with E_2, and that competition will grant E_2 strong bargaining power. Hence, if E_2 forms a government coalition with one conventional party, it will implement policy p_E. This can be justified similarly to the case of only one extreme party since conventional parties compete head to head to form a coalition with this extreme party. By contrast, as E_1 will only be able to form a coalition government with party $\hat{K}$ if E_1 forms a government coalition with conventional party $\hat{K}$, it will implement policy $p_{\hat{K}E}$.

Whether a coalition government will form with E_1 or with E_2 will depend on the particular bargaining protocol between the *four* parties, which admits of no unique, obvious solution. We shall use blue to indicate the protocol where a coalition with the extreme party possessing strong bargaining power will be formed, and green to indicate the other possibility, where the extreme party with weak bargaining power becomes part of the government. By allowing both possible outcomes, we avoid having to impose more structure on voter and party preferences. To sum up, in the case where no party has received a majority of votes, we consider the following two possibilities:

$$[\sigma_K = \{E_1\} \text{ and } \sigma_{\hat{K}} = \emptyset] \Rightarrow \begin{cases} \hat{K} \text{ or } K \text{ and } E_2 \text{ form the government, and } p_E \text{ is chosen (blue protocol),} \\ \hat{K} \text{ and } E_1 \text{ form the government, and } p_{\hat{K}E} \text{ is chosen (green protocol).} \end{cases}$$

Given the conventional voters' preferences (recall Section 3) and the assumption that voters can observe CPCs and anticipate their effect on the policies eventually carried out by the government, we find that the election outcome will lead to the following policy choices:

$$\begin{aligned} &[\sigma_K = \{E_1\} \text{ and } \sigma_{\hat{K}} = \emptyset] \Rightarrow \\ &\begin{cases} p_E \text{ is chosen (blue protocol).} \\ p_K \text{ is chosen with prob. } q \text{ and } p_{\hat{K}} \text{ is chosen with prob. } 1-q \text{ (green protocol).} \end{cases} \end{aligned} \tag{40}$$

In Expression (40) we have assumed that coordination on one conventional party from the side of the conventional voters will be prompted by the signal of Stage 2 with probability q in favor of the conventional party that excluded a larger number of extreme parties, if any.

The object of this section is to characterize the equilibrium outcomes under the above assumptions when CPCs are available. We shall assume throughout SC1, RC, rational expectations, and that parties do not use weakly dominated strategies. It is worth making the following two additional remarks: First, at the election stage of the game, conventional voters will coordinate on one of the conventional parties if the outcome under sincere voting is either policy p_{KE}, with $K \in \{L, R\}$, or a caretaker government implementing p_{ct}. Second, given that RC implies that $q^c \leq \frac{1}{2}$, if parties do not use weakly dominated strategies, we can use the table below to describe the game at the first stage when the conventional parties sign their contracts. Recall that for $x \in [0, 1]$, we denote $V_K^x(p_1, p_2) = x \cdot V_K(p_1) + (1 - x) \cdot V_K(p_2)$, where $p_1, p_2 \in \mathcal{P}$, while B_K^x is such that $B_L^x = B \cdot x$ and $B_R^x = B \cdot (1 - x)$. A strategy for party K now selects a subset of $\{E_1, E_2, \hat{K}\}$. Lastly, Table 7 below contains the parties' contract choice game for both possible bargaining protocols.

	$\emptyset$	$\{E_1\}$	$\{E_2\}$	$\{E_1, E_2\}$
$\emptyset$	$\frac{B}{2} + V_K(p_E)$	$\frac{B}{2} + V_K(p_E)$ $B_K^{1-q} + V_K^{1-q}(p_L, p_R)$	$\frac{B}{2} + V_K(p_E)$ $B_K^{1-q} + V_K^{1-q}(p_L, p_R)$	$B_K^{1-q} + V_K^{1-q}(p_L, p_R)$
$\{E_1\}$	$\frac{B}{2} + V_K(p_E)$ $B_K^{q} + V_K^{q}(p_L, p_R)$	$\frac{B}{2} + V_K(p_E)$	$\frac{B}{2} + V_K^{\frac{1}{2}}(p_L, p_R)$	$B_K^{1-q} + V_K^{1-q}(p_L, p_R)$
$\{E_2\}$	$\frac{B}{2} + V_K(p_E)$ $B_K^{q} + V_K^{q}(p_L, p_R)$	$\frac{B}{2} + V_K^{\frac{1}{2}}(p_L, p_R)$	$\frac{B}{2} + V_K(p_E)$	$B_K^{1-q} + V_K^{1-q}(p_L, p_R)$
$\{E_1, E_2\}$	$B_K^{q} + V_K^{q}(p_L, p_R)$	$B_K^{q} + V_K^{q}(p_L, p_R)$	$B_K^{q} + V_K^{q}(p_L, p_R)$	$b + V_K(p_{LR})$

Table 7: Conventional parties' contract choice game with two extreme parties. Row player is party L and column player is party R.

Note that we have written only those strategies that are payoff-different and have omitted all the strategies that are weakly dominated for both bargaining protocols. Indeed, let $h \in \{1, 2\}$. Then, it is easy to verify that for party K the following pairs of strategies are payoff-equivalent: $\{\hat{K}\}$ and $\emptyset$, $\{E_h\}$ and $\{\hat{K}, E_h\}$, $\{E_1, E_2\}$, and $\{\hat{K}, E_1, E_2\}$. Moreover, $\{\hat{K}, E_1, E_2\}$ is weakly dominated by $\{E_1, E_2\}$. We stress that, as in the case of only one extreme party, Table 7 is also obtained if we allow parties to use weakly dominated strategies but impose instead a two-party exclusion rule: i.e. in a CPC, each party can only exclude a coalition containing two other parties at the most. As a tie-breaking rule, we assume that conventional parties prefer to exclude extreme parties and not to exclude the other conventional party. Lastly, we note that in $V_k^x(p_L, p_R)$ the parameter q now denotes the probability of conventional voters coordinating their votes on the conventional party that has excluded a larger number of extreme parties in its CPC. If both conventional parties have excluded the same number of extreme parties, we assume that the coordination probability for both parties is a half.

The characterization of equilibria depending on the parties' perceived coordination probability q can be found in Appendix C. Here we state the main results regarding the welfare implications of CPCs in this modified framework in the light of the results from the setup with only one extreme party. The proposition below follows from the analysis contained in the aforementioned appendix and is proved therein. By default, we assume that no rule limits the maximum number of parties to be precluded in a CPC.

Proposition 3
Consider the framework with two extreme parties of equal size, and assume SC1, RC, rational expectations, and that parties do not use weakly dominated strategies. Then we obtain the following results:

(a) *CPCs are ex-ante welfare-improving under both protocols (blue and green) if Condition (36) holds.*

(b) *Assume that $B = 2b$. Then, if b is large enough, ex-ante welfare with the blue protocol is higher than ex-ante welfare with the green protocol if and only if Condition (36) holds.*

(c) *CPCs are ex-interim welfare-improving under both protocols if b is large enough and $q > \frac{1}{2}$.*

(d) *CPCs with the one-party exclusion rule always yield worse outcomes than without the one-party exclusion rule. Moreover, if the reverse of Condition (36) holds, CPCs with the one-party exclusion rule are (ex-ante and ex-interim) welfare-decreasing.*

In the basic setup with only one extreme party, we saw that the simple one-party exclusion rule was sufficient to prevent any equilibrium in which all coalitions are ruled out. In that setting, the same outcomes were reached without the one-party exclusion rule if conventional parties played no weakly dominated strategies. In the case with two extreme parties, we can specify a two-party exclusion rule, or, equivalently, assume that no conventional party will play a weakly dominated strategy. Limiting the number of parties that may be excluded could give parties an incentive to split into two or more parties to relax the constraints imposed by CPCs and still be able to be part of a government coalition albeit under a different party name. But parties typically have to register some time before the election. After registration, the maximum number of parties to be excluded in a contract is fixed, and an exclusion rule based on that number cannot be bypassed by party splitting.[26]

A.2 No rational expectations

Next we assume that parties' beliefs on voter coordination may not be rational, i.e., $q_R = q_L$ need not hold, so the expected impact of CPCs on vote coordination is different for both conventional parties. Throughout this subsection, we assume SC1 and that the one-party exclusion rule is in place. As for the baseline model with rational expectations, it can be shown that signing a CPC excluding E given that $\hat{K}$ has excluded E is only profitable ex-ante for K if

$$q_K \geq \frac{(B-b) + V_K(p_K) - V_K(p_{LR})}{B + V_K(p_K) - V_K(p_{\hat{K}})} \equiv q_K^c = q^c. \tag{41}$$

Similarly, given that conventional party $\hat{K}$ does not sign a CPC, we obtain the following condition for party K to exclude the extreme party from a coalition:

$$q_K \geq \frac{\frac{B}{2} + V_K(p_E) - V_K(p_{\hat{K}})}{B + V_K(p_K) - V_K(p_{\hat{K}})} \equiv q_K^n = q^n. \tag{42}$$

For a complete characterization of the equilibria, the relations between q_K^n and q_K^c, with $K \in \{L, R\}$, play a crucial role. We distinguish two cases depending on the latter relation. First, we assume SC2A. Figure 2 contains the equilibria outcomes when SC2A holds.

For coordination beliefs (q_L, q_R) such that $(q_L, q_R) \in Q_{E,\emptyset} \cup Q_{\emptyset,E}$, where $Q_{E,\emptyset} = [q^n, 1] \times [0, q^n) \cup [q^c, 1] \times [q^n, q^c)$ and $Q_{\emptyset,E} = [0, q^n) \times [q^n, 1] \cup [q^n, q^c) \times [q^c, 1]$, there is a unique equilibrium which is asymmetric and where one conventional party excludes E and conventional voters coordinate their votes on one of the parties as described by the following strategy profiles:

$$(\sigma_L, \sigma_R) = \begin{cases} (\{E\}, \emptyset) & \text{if} \quad (q_L, q_R) \in Q_{E,\emptyset}, \\ (\emptyset, \{E\}) & \text{if} \quad (q_L, q_R) \in Q_{\emptyset,E}. \end{cases}$$

Moreover, when $(q_L, q_R) \in Q_{mult} \equiv [q^n, q^c) \times [q^n, q^c)$ both asymmetric contract choices are the only equilibria. All in all, asymmetric equilibria may occur simply because conventional parties have different beliefs.

Second, we assume SC2B. Figure 3 contains the equilibria outcomes when SC2B holds.

Now the area where asymmetric CPCs are chosen reduces to $Q_{E,\emptyset} := [q^n, 1] \times [0, q^c)$ and $Q_{\emptyset,E} := [0, q^c) \times [q^n, 1]$, while in the area $Q_{mult} := [q^c, q^n) \times [q^c, q^n)$ both symmetric contract choices are feasible in equilibrium. Note that rational belief equilibria are again indicated by the bisectrix in Figure 3.

We also analyze the welfare implications of CPCs when we do not impose the rational expectations assumption. To that end, we assume that (q_L, q_R) is distributed on $[0, 1] \times [0, 1]$ and that the exact value of the true probability of how voters coordinate, q, is not known by the conventional parties before they write the contracts. Hence, we should now consider the whole area in Figures 2 and 3 instead of the bisectrix. Particularly relevant is the following fact: even if $B = 2b$ and b is arbitrarily large (which implies $q^c, q^n \simeq \frac{1}{2}$), there are regions in which asymmetric equilibria occur. These regions cover those circumstances in which both conventional parties believe that being the sole party excluding E only benefits coordination on that party for one of the conventional parties.

[26] A law should not be based on the identity of parties. Hence, from a legal point of view an "extreme" party is no different from a conventional party except for its share. However, in a political system with two well-established conventional parties, the number of extreme parties is $\eta - 2$, where η is the total number of parties. A law could therefore be based on η.

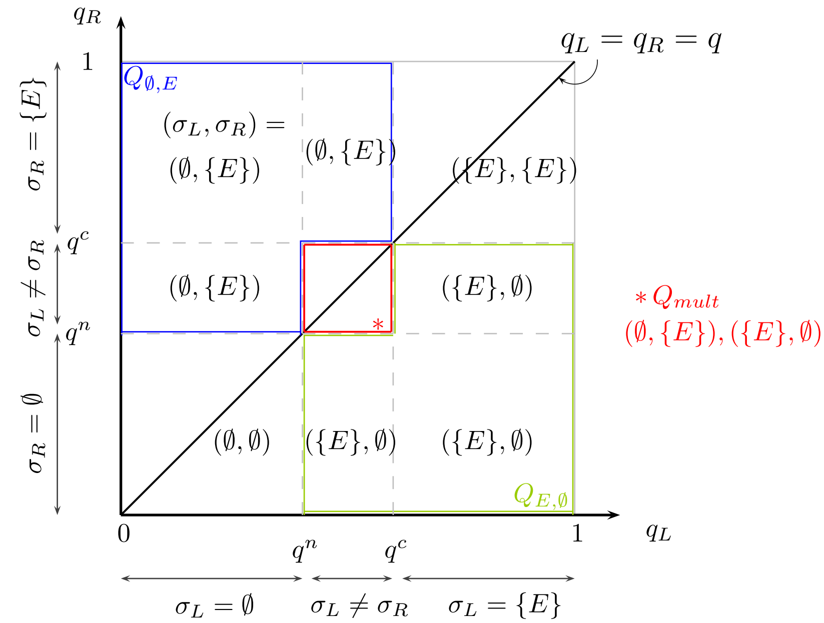

Figure 2: Equilibrium outcomes depending on parties' beliefs about voter coordination under SC2A.

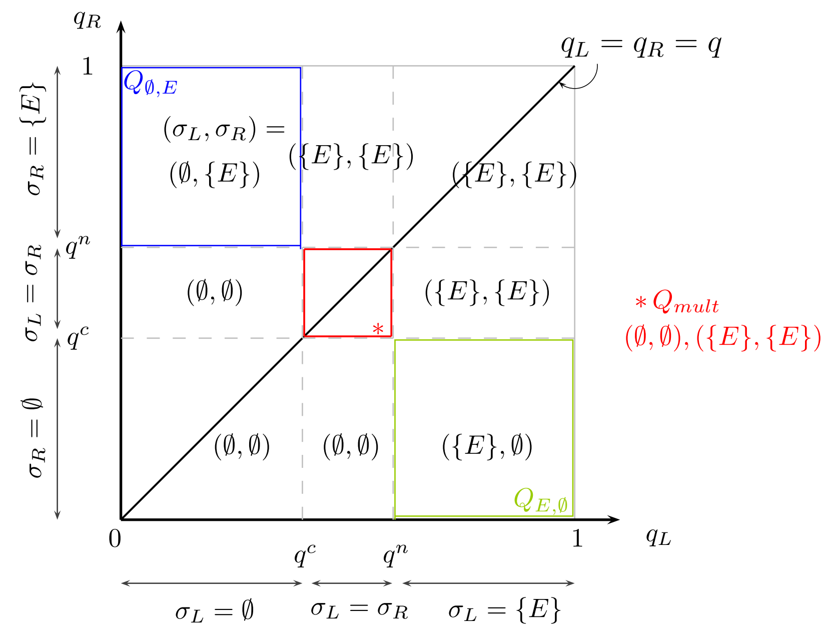

Figure 3: Equilibrium outcomes depending on parties' beliefs about voter coordination under SC2B.

A.3 A continuum of voters

In the baseline model we have assumed that conventional voters l and r represent a share of voters that are inclined towards parties L and R, respectively, but who might consider voting for the other party for strategic reasons. In the same vein, we have assumed that e represents the share of voters willing to vote for party E unconditionally. These assumptions have simplified our analysis of the implications of CPCs on policies and welfare by enabling us to leave out of account issues like vote miscoordination and to place the focus on the interaction implications among parties on the one hand, and between parties and voters on the other. However, to what extent do our conclusions from the previous sections depend on the assumption that there are two conventional voters—and one extreme voter—instead of many more voters?

To answer this question, we modify our political games ($\mathcal{G}^-$ without CPCs and $\mathcal{G}^+$ with CPCs) and assume that there is a continuum of voters, say $[0,1]$, with each of them casting a single vote in the election. We consider proportional elections, i.e. the number of seats a party obtains in parliament is proportional to its vote share. As already mentioned, the assumption that allocation of cabinet portfolios in parliamentary democracies is proportional to the legislative seat shares of the governing parties is called Gamson's Law (Gamson, 1961). We assume that this principle applies to the negotiations between conventional parties but not to the ones between a conventional party and the extreme party.

For the sake of simplicity, we assume that the share of extreme voters, α_E, is lower than $\frac{1}{3}$ and is common knowledge and that the extreme voters are uniformly distributed among $[0,1]$ and will always vote for party E.[27] Let $\Omega \subsetneq [0,1]$ denote the set of conventional voters. Considering a continuum of (conventional) voters has the following immediate consequence: The range of the vote share of parties L and R is broad. To deal with this fact, we will assume it to be common knowledge that in a grand coalition, conventional parties will receive a utility from perks that is proportional to their share and that the policy implemented by such a government will be the policy $p = (t,d) \in \mathcal{T} \times \mathcal{D}$, with $\mathcal{T} \subsetneq \mathbb{R}$ and $\mathcal{D} = \{0, \bar{d}\}$, that maximizes[28]

$$\alpha_L \cdot V_L(p) + \alpha_R \cdot V_R(p). \tag{43}$$

Note that we have assumed—and we will assume throughout this section—that parties are risk-neutral with respect to perks. This implies in particular that $B = 2b$, where b denotes the utility derived from perks that each conventional party obtains in the grand coalition when both conventional parties have the same vote share.[29] Hence, conventional parties will be indifferent between the perks obtained in a grand coalition—with equal-size parties—and the expected perks obtained by each of them in a situation where either coalition between a conventional party and the extreme party may occur with probability of $\frac{1}{2}$.

For this modified setting, a strategy profile will be a combination of CPCs (for the parties) and voting strategies (for the voters). We will refer to a profile of pure strategies as an *equilibrium* if

(a) voters vote according to the signal from the correlation device if and only if they find it profitable to do so,

(b) for each subset of (conventional) voters, $S \subseteq \Omega$, there exists no profile of strategies for voters in S such that all of them obtain a larger utility by changing their strategies, provided that voters in $\Omega \setminus S$ do not change their strategies,

(c) given (a) and (b), strategies are subgame perfect.

As in the baseline model, voters care about the policy $p = (t,d)$ implemented and derive a disutility from the (constant) level of perks received by members of the government. Conventional voter i's utility is single-peaked, with $(t_i, d_i) \in \mathcal{T} \times \mathcal{D}$ denoting his ideal policy. To simplify the exposition, we assume in the following that for each conventional voter $i \in \Omega$ and every policy $p = (t,d) \in \mathcal{T} \times \mathcal{D}$,

$$V_i((t,d), B) = u_i(|t - t_i|) - d - B,$$

[27] The assumption that extreme voters always vote for party E can be justified as in Sections 3 and 4. The notion of extreme voters is used to describe the preferences of a minority that desires a discontinuous and large change of the status quo against more than two-thirds of the electorate.

[28] This bargaining procedure yields the Nash Bargaining solution with bargaining power proportional to the share in parliament.

[29] This assumption has no qualitative effect on our results.

where $u_i(\cdot) = u(\cdot)$ and $u(\cdot)$ is decreasing and concave. In particular, $\{u_i(|t - t_i|)\}_{i\in\Omega}$ satisfies the *strong single-crossing property*, which means that for all $i, j \in \Omega$ such that $t_i < t_j$ and $t, t' \in \mathcal{T}$ with $t < t'$, we have, for each $x \geq 0$,

(a) $u_i(|t' - t_i|) - u_i(|t - t_i|) > x \Rightarrow u_j(|t' - t_j|) - u_j(|t - t_j|) > x$,

(b) $u_j(|t - t_j|) - u_j(|t' - t_j|) > x \Rightarrow u_i(|t - t_i|) - u_i(|t' - t_i|) > x$.

Although we do not explicitly express it, we will also assume that voters obtain an extra disutility when they vote strategically. The factor behind this assumption is the tension between *sincere voting* and *strategic voting* (see e.g. Austen-Smith and Banks (1988); Austen-Smith (1989)). Whereas under sincere voting voters are assumed to take only ideological information—i.e., parties' ideal points—into account when casting their vote, under strategic voting voters care about eventual policy outcomes, so their beliefs about the probability that they are pivotal will influence their vote. We assume the existence of the so-called *ideological burden* of voting strategically, which lowers (albeit perhaps very slightly) utility for a citizen when he votes strategically, i.e., when he votes for a party whose ideal point is not the closest to his own. The larger the burden of voting strategically is for a voter, the more likely it is that he votes sincerely, i.e., for the party whose ideal point is closest to his ideal point. We assume that the ideological burden for extreme voters is very large, as they are committed to the single issue of the extreme party. Hence, as already mentioned, they will always vote for the extreme party. By contrast, we assume that such a burden is small for conventional voters i, with $i \in [0, 1]$, and that it is decreasing in the distance between their ideal point $p_i = (t_i, 0)$ and the ideal point of the median voter m in dimension $\mathcal{T}$, $p_m := (t_m, 0)$. Beyond assuming that $t_L < t_m < t_R$, consider that

$$t_m - t_L = t_R - t_m. \tag{44}$$

While $t_L < t_m < t_R$ ensures that the ideal points of conventional parties are on opposite sides of the distribution $\{t_i\}_{i\in\Omega}$, Condition (44) requires those ideal points to be symmetric with respect to t_m. Accordingly, party L (or R) can be called the *left-wing* (or *right-wing*) party. Lastly, for notational convenience we introduce, for each voter $i \in \Omega$, the set

$$\mathcal{F}^i := \{j \in \Omega \mid t_i < t_j\}, \tag{45}$$

which consists of all conventional voters with an ideal point larger than t_i.[30] We use the operator $|\cdot|$ to denote the measure of a set. Then we can denote by l the conventional voter such that $\mathcal{F}^l$ is minimal with respect to inclusion among the sets $\mathcal{F}^i$ as defined in (45), where i is a conventional voter, with the property that $|\mathcal{F}^i \cup \{i\}| \geq \frac{1}{2}$. That is, l is the conventional voter i with the largest t_i such that all conventional voters j with $t_j \geq t_i$ account for at least half of the population. Analogously, we define r such that $\Omega \setminus \mathcal{F}^r$ is minimal with respect to inclusion among the sets $\Omega \setminus \mathcal{F}^i$ where i is a conventional voter, with the property that $|\Omega \setminus \mathcal{F}^i| \geq \frac{1}{2}$. That is, r is the conventional voter i with the smallest t_i such that all conventional voters j with $t_j \leq t_i$ account for at least half of the population. Since $|\Omega| = 1 - \alpha_E < 1$, the sets $\mathcal{F}^l$ and $\mathcal{F}^r$ do not overlap.

Figure 4 illustrates all the previous results and definitions graphically. The following result reveals that in our framework there is no loss of generality in assuming that there are only two conventional voters, namely l and r.

Proposition 4

(a) *All extreme voters and all conventional voters in* $\left[\Omega \setminus \left(\mathcal{F}^l \cup \{l\}\right)\right] \cup \mathcal{F}^r$ *vote sincerely.*

(b) *All conventional voters in* $\mathcal{F}^l \setminus [\mathcal{F}^m \cup m]$ *cast the same vote as* l.

(c) *All conventional voters in* $\mathcal{F}^m \setminus \mathcal{F}^r$ *cast the same vote as* r.

[30] To ensure that all concepts are well-defined, we assume that $t_i \neq t_j$ for all $i, j \in \Omega$ with $i \neq j$.

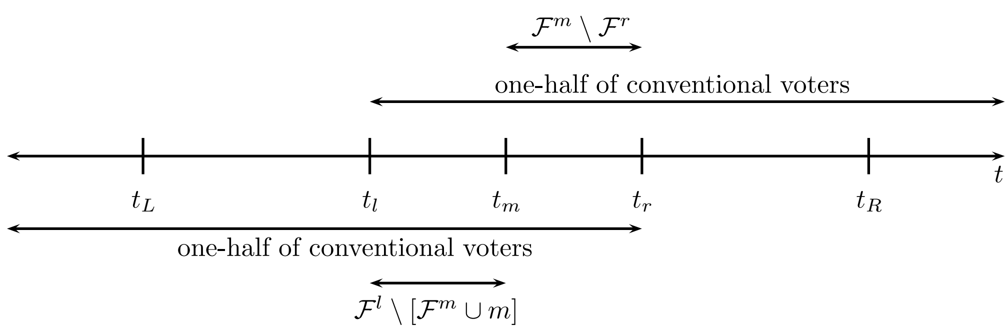

Figure 4: The critical voters.

Hence, l and r are *critical voters* in the sense that it is their vote that determines the outcome of the election. From Proposition 4 we immediately obtain the main result in this section.

Proposition 5
A strategy profile in the political game with a continuum of agents is an equilibrium if and only if the strategy profile of parties and critical voters is an equilibrium in game $\mathcal{G}^+$ (or $\mathcal{G}^-$) and the strategy profile fulfills Proposition 4.

As a consequence, the welfare implications of CPCs derived for the baseline model with three voters carry over to this more micro-founded model. Finally, we also stress that the conditions on the preferences of the conventional voters in the baseline model can be translated into the modified setting with a continuum of voters by adapting them to conditions on the preferences of the critical voters l and r.

A.4 Asymmetric ideal points for conventional parties

Let us now assume that conventional parties' ideal points, p_L and p_R, are not located symmetrically with respect to p_{LR}. Let us further assume that this implies that, under sincere voting, there will always be a unique conventional party, say L, with the higher share of votes. In a setting with many voters, this would occur because more than half of the conventional voters' ideal points would be closer to party L's ideal point than to party R's ideal point. To remain within our baseline setting with only three voters, it will be sufficient to assume different and positive weights for each party. When the asymmetry between conventional parties' vote share under sincere voting is so large that party L will obtain a vote share larger than $\frac{1}{2}$, this party would obtain a majority in parliament. In this latter case, either with or without CPC, a single-party government by L would be formed to implement its ideal point.

In all other cases, the effect of CPCs on the outcome of the elections is not so clear-cut, since both conventional parties are interested in forming a coalition with the extreme party at Stage 4. Observe that the larger the share of party L is with respect to party R under sincere voting, the "easier" it would be for voters to coordinate on L rather than R, for in the former case a smaller number of voters would be required to vote strategically.[31] As a consequence, we should not expect the probability q of coordinating on the only party that excludes E to be independent of which conventional party does so, even under rational expectations. This requires a complete analysis.

For $K \in \{L, R\}$, let x_K denote the probability that coordination occurs in favor of K when $\sigma_K = \{E\}$ and $\sigma_{\hat{K}} = \emptyset$. We assume that x_R and x_L are common knowledge among the two conventional parties. We summarize the contract choice game played by the conventional parties with the one-party exclusion rule in the following table. Recall that B_K^x is such that $B_L^x = B \cdot x$ and $B_R^x = B \cdot (1 - x)$ for $x \in [0, 1]$.

[31] A symmetric argument could be carried out for the opposite case.

	$\emptyset$	$\{E\}$
$\emptyset$	$\frac{B}{2} + V_K(p_E)$	$B_K^{1-x_R} + (1-x_R) \cdot V_K(p_L) + x_R \cdot V_K(p_R)$
$\{E\}$	$B_K^{x_L} + x_L \cdot V_K(p_L) + (1-x_L) \cdot V_K(p_R)$	$b_K + V_K(p_{LR})$

Table 8: Conventional parties' contract choice game under asymmetric ideal points for the conventional parties. Row player is party L and column player is party R.

We note that now b_K denotes the utility that party K derives from perks in a grand-coalition government when all voters vote sincerely. We assume that

$$B + V_K(p_E) > b_K + V_K(p_{LR}).$$

This inequality adapts Condition (5) to this modified setting.

Analogously to the analysis in Subsection A.2, we can identify four relevant critical values. As a tie-breaking rule, we assume that parties will always exclude E. For $K \in \{L, R\}$, let

$$x^n_{\hat{K}} := \frac{\frac{B}{2} + V_K(p_E) - V_K(p_{\hat{K}})}{B + V_K(p_K) - V_K(p_{\hat{K}})} \text{ and } x^c_{\hat{K}} := \frac{(B-b) + V_K(p_K) - V_K(p_{LR})}{B + V_K(p_K) - V_K(p_{\hat{K}})}.$$

That is, given that party $\hat{K}$ has not excluded E, party K will prefer to exclude E if and only if $x_K \geq x^n_{\hat{K}}$, whereas given that party $\hat{K}$ has excluded E, party K will prefer to exclude E if and only if $x_{\hat{K}} \geq x^c_{\hat{K}}$. Note that $x^n_{\hat{K}} = q^n_K$ and $x^c_{\hat{K}} = q^c_K$.

Under SC1 we can define $x^n := x^n_L = x^n_R$ and $x^c := x^c_L = x^c_R$. There are two possible cases that lead to different equilibria in the choice game defined in Table 8.

Case I:[32] $x^n < x^c$

Figure 5 contains the equilibrium outcomes in this case.

Case II:[33] $x^c \geq x^n$

Figure 6 contains the equilibrium outcomes in this case.

In both Figures 5 and 6, there are two regions in which there exist no equilibria in pure strategies. This is in sharp contrast with the equilibria in Section A.2. From the two figures we deduce, however, that as long as the difference between x_L and x_R is not very significant—geometrically that means that we stay close to the bisectrix—, the results concerning equilibria and welfare remain essentially the same as those in the main body of the paper.

A.5 Uncertainty about the extreme party's vote share

One of the assumptions of our model is that the share of the extreme party E is perfectly foreseeable. There are many reasons why this could be the case, e.g. policy d might be an issue that is not subject to political fluctuations with a public opinion that is stable over time. However, it is interesting to speculate about the robustness of our results if the share of the extreme party is stochastic. A simple way of doing that is to assume that, prior to the game, it is common knowledge that with probability $\pi_E \in (0, 1)$ party E will get into the parliament with a share α_E and with probability $1 - \pi_E$ it will not reach the threshold of votes needed to enter the parliament, resulting in zero share. If the extreme party stayed out of the parliament, coordination on one of the conventional parties would result in a (super)majority for this party,

[32]In terms of the conditions on the parameters of the model, this case is equivalent to SC2A.
[33]In terms of the conditions on the parameters of the model, this case is equivalent to SC2B.

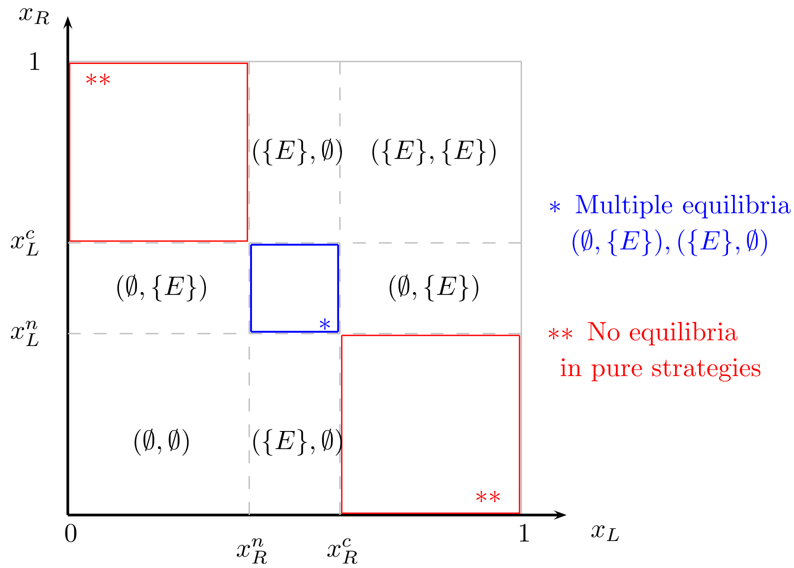

Figure 5: Equilibrium outcomes depending on parties' beliefs about voter coordination under SC2A when parties' ideal points are not symmetric.

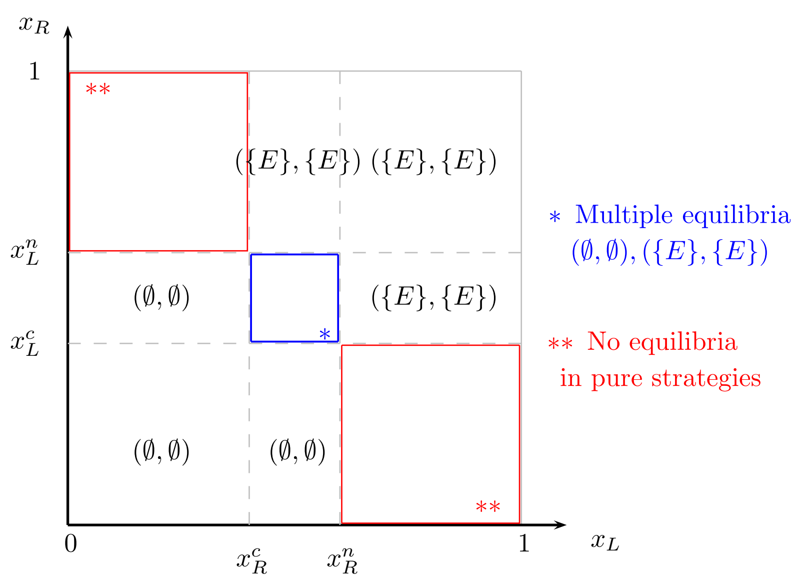

Figure 6: Equilibrium outcomes depending on parties' beliefs about voter coordination under SC2B when parties' ideal points are not symmetric.

which would implement its ideal point with certainty. Therefore, if it is certain that E will not get any seats in the parliament, no voter would find it profitable to vote strategically, so CPCs would have no effect.

In that event, the unique equilibria of the political game would consist of sincere voting and a single-party government of either L or R (each with probability $\frac{1}{2}$) to respectively implement p_L or p_R. Note that a conventional party would be able to form a government with less than half the votes but with at least half the seats in the parliament.

Under the uncertainty modeled by π_E, however, voters would need to balance the expected benefits/costs of coordinating on one conventional party when only one conventional party writes a CPC against the expected benefits/costs of voting sincerely. In particular, there will exist a threshold π_E^* such that, if and only if $\pi_E > \pi_E^*$, conventional voters l and r—as critical voters—would still find it profitable to coordinate on one conventional party if only one conventional party excludes E. Parties would anticipate voters' behavior, so all results would remain true when $\pi_E > \pi_E^*$. That is, if the variance on party E's share is small, the conclusions regarding the impact of CPCs on welfare prevail.

A.6 Less power for the extreme party

Another important assumption of the paper is that, whereas conventional parties are free to accept any possible bargaining outcome with other parties, the extreme party can only accept bargains that offer $\bar{d}$. We might also assume, however, the existence of ideological constraints on the mobility of conventional parties. A polar case is to imagine that a conventional party can only accept $\bar{d}$ in exchange for its ideal point in the policy dimension $\mathcal{T}$. That is to say that party E would never have strong bargaining power. In such a case, given the conventional parties' and voters' preferences as outlined in Section 3, no government coalition in which the extreme party is a member would form in equilibrium without CPCs.

A.7 Increasing perks

Throughout the paper, the amount of perks has been assumed to be exogenously fixed and independent of the exact composition of the government. However, there seems to be widespread evidence that very large parliamentary support for the government reduces opposition, not only at the political level but also at the media level.[34] With reduced opposition, parties in the government may be able to increase the amount of perks they get. In anticipation of such behavior, voters might not grant a super-majority to parties in the government. Nevertheless, as long as assumptions similar to those imposed on conventional parties' and voters' preferences in Section 3 obtained the thrust of our results would not change. Note that, to model varying levels of perks, we would need to extend the model to account for more than three voters.

A.8 Reputation effects

Elections are often considered a one-shot game, as they cannot be compared with each other. Candidates may change, socioeconomic circumstances may be very different, etc. However, parties are institutions that last for many years. In particular, they have long-term strategies. When more than one election is considered, writing a CPC might have an effect on the reputation of a party's commitment credibility. For instance, voters might believe the party's announcements more (or less) than before. To account for such an extension formally, a dynamic model would be required.

[34]Because of the existing links between power and the media.

B Appendix

In this appendix we do several things. First, we prove Lemma 1. Second, we study how the one-party exclusion rules affects our results by characterizing the equilibria of game $\mathcal{G}^+$ when the one-party exclusion rule does not apply. Third and last, we prove Theorems 3 and 4. Accordingly, we start with the proof of Lemma 1.

Proof of Lemma 1

We proceed by establishing a contradiction: we suppose that $\mathcal{P} \subseteq \mathbb{R}$. Without loss of generality, let $p_L < p_R$.[35] From the bargaining protocol it then follows that $p_L < p_{LR} < p_R$. We can then further assume without loss of generality that $p_{LR} < p_E$. Let p_k denote voter k's ideal policy, with $k \in \{l, r, e\}$. We distinguish two possibilities:

Case I: $p_L < p_{LR} < p_E < p_R$.

In this case, since the bargaining protocol results in a convex combination of ideal policies in each policy dimension, we have $p_E < p_{RE} < p_R$. We distinguish two subcases:

Case I.A: $p_l \leq p_E$

In this subcase we obtain $v_l(p_{RE}) \geq v_l(p_R)$, which contradicts Condition (16).

Case I.B: $p_E < p_l$

In this subcase we obtain $v_l(p_E) \geq v_l(p_{LR})$, which contradicts Condition (14).

Case II: $p_L < p_{LR} < p_R < p_E$.

We distinguish two subcases:

Case II.A: $p_l \leq p_R$

In this subcase we obtain $v_l(p_R) \geq v_l(p_E)$, which contradicts Condition (15).

Case II.B: $p_R < p_l$

In this subcase we obtain $v_l(p_R) \geq v_l(p_{LR})$, which contradicts Conditions (14)–(15).

□

Second, we analyze the case where parties can write arbitrary CPCs. More specifically, we show that under RC and rational expectations, the equilibria of game $\mathcal{G}^+$ without the one-party exclusion rule depend on the relation between q, q^n, q^c, and $\frac{1}{2}$ when conventional parties may play weakly dominated strategies. We present the different cases in Tables 10–12. Note that RC implies Condition (34), which can be rewritten as

$$q^c \leq \frac{1}{2}. \tag{46}$$

We shall focus on the draft of CPCs. From Stage 2 onwards, the equilibrium dynamics will be the same as with the one-party exclusion rule, except when all coalitions are excluded (as this case could not occur with the one-party exclusion rule). It will suffice to assume that policies are still determined by $\mathsf{f}(\cdot,\cdot,\cdot,\cdot)$, a mapping that in this case is based on the corresponding extension of Table 1—see Table 9 below.

[35] Note that, due to Conditions (14)–(16), no two policies in $\mathcal{P}$ are equally desirable according to the voters' preferences.

(σ_l, σ_r)

<table>
<tr><th></th><th></th><th>(R,R)</th><th>(R,L) & (L,R)</th><th>(L,L)</th></tr>
<tr><td rowspan="9">$(\sigma_K, \sigma_{\hat{K}})$</td><td>$(\emptyset,\emptyset)$</td><td rowspan="9">$p_R$ (R)</td><td rowspan="3">p_E (RE/LE)</td><td rowspan="9">p_L (L)</td></tr>
<tr><td>$(\emptyset,\{K\})$</td></tr>
<tr><td>$(\{\hat{K}\},\{K\})$</td></tr>
<tr><td>$(\emptyset,\{E\})$</td><td rowspan="2">p_{KE} (KE)</td></tr>
<tr><td>$(\{\hat{K}\},\{E\})$</td></tr>
<tr><td>$(\{E\},\{E\})$</td><td>p_{LR} (LR)</td></tr>
<tr><td>$(\{\hat{K},E\},\{E\})$</td><td rowspan="3">p_K (K)</td></tr>
<tr><td>$(\{E\},\{K,E\})$</td></tr>
<tr><td>$(\{\hat{K},E\},\{K,E\})$</td></tr>
</table>

Table 9: Policy implemented without the one-party exclusion rule and government coalition (in parentheses).

We distinguish three cases:

Case I: SC2A

All CPCs that may be written in equilibrium in this case are summarized in Table 10.

$0 \leq q < q^n$	$q^n \leq q < q^c$	$q^c \leq q < \frac{1}{2}$	$\frac{1}{2} \leq q \leq 1$
$(\emptyset,\emptyset)$	$(\emptyset,\{E\})$ $(\{E\},\emptyset)$ $(\{R,E\},\emptyset)$ $(\emptyset,\{L,E\})$	$(\{E\},\{E\})$ $(\{R,E\},\emptyset)$ $(\emptyset,\{L,E\})$	$(\{E\},\{E\})$ $(\{R,E\},\{L,E\})$

Table 10: CPCs under SC2A without the one-party exclusion rule.

Case II: SC2B and $q^n < \frac{1}{2}$

All CPCs that may be written in equilibrium in this case are summarized in Table 11.

$0 \leq q < q^c$	$q^c \leq q < q^n$	$q^n \leq q < \frac{1}{2}$	$\frac{1}{2} \leq q \leq 1$
$(\emptyset,\emptyset)$	$(\emptyset,\emptyset)$ $(\{E\},\{E\})$	$(\{E\},\{E\})$ $(\{R,E\},\emptyset)$ $(\emptyset,\{L,E\})$	$(\{E\},\{E\})$ $(\{R,E\},\{L,E\})$

Table 11: CPCs under SC2B and $q^n < \frac{1}{2}$ without the one-party exclusion rule.

Case III: SC2B and $q^n \geq \frac{1}{2}$

All CPCs that may be written in equilibrium in this case are summarized in Table 12.

$0 \leq q < q^n$	$q^n \leq q < \frac{1}{2}$	$\frac{1}{2} \leq q < q^n$	$q^n \leq q \leq 1$
$(\emptyset, \emptyset)$	$(\emptyset, \emptyset)$ $(\{E\}, \{E\})$	$(\emptyset, \emptyset)$ $(\{E\}, \{E\})$ $(\{R, E\}, \{L, E\})$	$(\{E\}, \{E\})$ $(\{R, E\}, \{L, E\})$

Table 12: CPCs under SC2B and $q^n \geq \frac{1}{2}$ without the one-party exclusion rule.

We note that, in all cases, if q is large enough, two different equilibria may arise: one in which any coalition with the extreme party is precluded, and another in which all coalitions are precluded. By definition, this latter equilibrium cannot arise if the one-party exclusion rule is in place.

Third and last, we analyze the welfare implications of CPCs. In particular, we prove Theorems 3 and 4.

Proof of Theorem 3:

Throughout the proof, we use W^{CPC} to denote welfare with the possibility of writing CPC and W^{NCPC} to denote welfare without such a possibility. We prove each item of the theorem sequentially. First, Parts (a) and (b) follow immediately from expressions (32) and (33) respectively. On the one hand, assume that SC2B holds. Then,[36]

$$\begin{aligned}
&\mathbb{E}_H[W^{CPC}] - \mathbb{E}_H[W^{NCPC}] = H(q^c) \cdot [W(p_E) - W(p_E)] \\
&+ (H(q^n) - H(q^c)) \cdot \left[\frac{1}{2}W(p_{LR}) + \frac{1}{2}W(p_E) - W(p_E)\right] + (1 - H(q^n)) \cdot [W(p_{LR}) - W(p_E)] \\
&= \left(1 - H(q^n) + \frac{1}{2}\left(H(q^n) - H(q^c)\right)\right) \cdot [W(p_{LR}) - W(p_E)] > 0,
\end{aligned} \tag{47}$$

where the inequality holds due to Condition (35). On the other hand, assume that SC2A holds. Then,

$$\begin{aligned}
&\mathbb{E}_H[W^{CPC}] - \mathbb{E}_H[W^{NCPC}] = H(q^n)\,[W(p_E) - W(p_E)] \\
&+ (H(q^c) - H(q^n)) \cdot \left[\int_{q^n}^{q^c} \left(\frac{1}{2}(qW(p_L) + (1-q)W(p_R)) + \frac{1}{2}((1-q)W(p_L) + qW(p_R)) - W(p_E)\right)\right] \\
&+ (1 - H(q^c))\,[W(p_{LR}) - W(p_E)] \\
&= H(q^n)\,[W(p_E) - W(p_E)] + (H(q^c) - H(q^n)) \cdot \left[\frac{1}{2}W(p_L) + \frac{1}{2} \cdot W(p_R) - W(p_E)\right] \\
&+ (1 - H(q^c))\,[W(p_{LR}) - W(p_E)] > 0,
\end{aligned} \tag{48}$$

where the inequality holds due to Conditions (35) and (36). We note that when $q^n < q < q^c$, we have assumed that with probability $\frac{1}{2}$ party L will exclude E and with probability $\frac{1}{2}$ party R will exclude E.[37] While, conditional on the first case, coordination on R occurs with probability q and coordination on L with probability $1 - q$, for the second case the conditional probabilities are reversed. It is then a matter of simple algebra to check that these observations yield the result that the occurrence of policy p_L and policy p_R each has a probability of $\frac{1}{2}$.

Second, to prove Part (c) it suffices to observe that, if $B = 2b$ and $K \in \{L, R\}$,

$$\lim_{b \to \infty} q^c = \lim_{b \to \infty} \frac{b + V_K(p_K) - V_K(p_{LR})}{2b + V_K(p_K) - V_K(p_{\hat{K}})} = \frac{1}{2}, \tag{49}$$

[36] We have assumed that whenever there are two equilibria, both will occur with the same probability. This assumption does not qualitatively affect our results. Further, recall that $H(\cdot)$ denotes the cumulative distribution function of q.

[37] If the two probabilities differ, Condition (36) can be replaced by Condition (37).

and

$$\lim_{b\to\infty} q^n = \lim_{b\to\infty} \frac{b + V_K(p_E) - V_K(p_{\hat{K}})}{2b + V_K(p_K) - V_K(p_{\hat{K}})} = \frac{1}{2}, \tag{50}$$

Third and last, Part (d) follows from Tables 10, 11, and 12. When the one-party exclusion rule does not apply and $B = 2b$, we obtain[38]

$$\begin{aligned}&\lim_{b\to\infty} \mathbb{E}_H[W^{CPC}] - \mathbb{E}_H[W^{NCPC}]\\ &= \frac{1 - H\left(\frac{1}{2}\right)}{2} \cdot [W(p_{LR}) - W(p_E)] + \frac{1 - H\left(\frac{1}{2}\right)}{2} \cdot \left[\frac{1}{2}W(p_L) + \frac{1}{2}W(p_R) - W(p_E)\right].\end{aligned}$$

The sign of the above expression may be positive or negative, provided that Condition (36) does not hold.

□

Proof of Theorem 4:

As in the proof of Theorem 3, we use W^{CPC} to denote welfare with CPCs and W^{NCPC} to denote welfare without CPC, and we prove each item of the theorem sequentially. First, Parts (a) and (b) follow from expressions (32) and (33) respectively. On the one hand, assume that SC2B holds. Then,

$$W^{CPC} - W^{NCPC} = \begin{cases} 0 & \text{if } 0 \le q < q^c, \\ 0 \text{ or } W(p_{LR}) - W(p_E) & \text{if } q^c \le q < q^n, \\ W(p_{LR}) - W(p_E) & \text{if } q^n \le q \le 1. \end{cases} \tag{51}$$

Due to Condition (35), none of the above cases yields a negative sign. On the other hand, assume that SC2A holds. Then,

$$W^{CPC} - W^{NCPC} = \begin{cases} 0 & \text{if } 0 \le q < q^n, \\ W(p_R) - W(p_E) \text{ or } W(p_L) - W(p_E) & \text{if } q^n \le q < q^c, \\ W(p_{LR}) - W(p_E) & \text{if } q^c \le q \le 1. \end{cases} \tag{52}$$

Due to Conditions (35) and (37), none of the above cases yield a negative sign. Second, to prove Part (c) it suffices to observe that, if $B = 2b$,

$$\lim_{b\to\infty} q^c = \lim_{b\to\infty} q^n = \frac{1}{2}.$$

Third and last, Part (d) follows from Tables 10, 11, and 12. When the one-party exclusion rule does not apply, $B = 2b$, and $q > \frac{1}{2}$, we obtain either

$$\lim_{b\to\infty} W^{CPC} - W^{NCPC} = W(p_{LR}) - W(p_E) > 0$$

or

$$\lim_{b\to\infty} W^{CPC} - W^{NCPC} = W(p_K) - W(p_E),$$

with $K \in \{L, R\}$. When Condition (37) does not hold the latter expression could be negative for at least one conventional party K.

□

[38] Again, we have assumed that whenever there are two equilibria, both will occur with the same probability. This assumption does not qualitatively affect our results.

C Appendix

In the first part of this appendix we assume that there is no rule limiting the number of parties that can be excluded via CPCs, and we characterize the equilibria depending on the parties' perceived coordination probability $q = q_L = q_R$ in the game with two equally-sized extreme parties and two symmetric conventional parties. The critical probabilities q^c and q^n defined in Section 3 are still useful in the extended framework. In particular, it holds that

$$\begin{aligned} q \geq q^n \Leftrightarrow \frac{B}{2} + V_K(p_E) &\leq q \cdot [B + V_K(p_K)] + (1-q) \cdot V_K(p_{\hat{K}}) \\ \Leftrightarrow V_K(p_E) + \frac{B}{2} &\leq q \cdot B + V_K^q(p_K, p_{\hat{K}}). \end{aligned} \tag{53}$$

and

$$\begin{aligned} q \geq q^c \Leftrightarrow b + V_K(p_{LR}) &\geq (1-q) \cdot [B + V_K(p_K)] + q \cdot V_K(p_{\hat{K}}) \\ \Leftrightarrow b + V_K(p_{LR}) &\geq (1-q) \cdot B + V_K^{1-q}(p_K, p_{\hat{K}}). \end{aligned} \tag{54}$$

With a larger number of parties than in the baseline setup we obtain more equilibria. However, not all the different equilibria lead to different policy outcomes. As we are mainly interested in the latter, we focus our attention on the resulting equilibrium policy outcomes given coordination probability q rather than on the equilibrium strategies. In particular, we use $(q \cdot p_K, (1-q) \cdot p_{\hat{K}})$ to denote the outcome where with probability q conventional party K will form a single-party government and with the complementary probability party $\hat{K}$ will lead a single-party government of its own. In either case, the conventional party in the government will derive utility from perks amounting to B.

We distinguish three cases, depending on the relation between q^n, q^c, and $\frac{1}{2}$. More specifically, we distinguish the case where $q^c \leq q^n \leq \frac{1}{2}$ (**Case I**), the case where $q^c \leq \frac{1}{2} \leq q^n$ (**Case II**), and the case where $q^n \leq q^c \leq \frac{1}{2}$ (**Case III**). Recall that because we are assuming RC (see Section 5), it must hold that $q^c < \frac{1}{2}$.[39] Lastly, to facilitate the understanding of the proof, we also note that

$$\begin{aligned} \frac{1}{2} \geq q^n \Leftrightarrow \frac{B}{2} + V_K(p_E) &\leq \frac{1}{2} \cdot [B + V_K(p_K)] + \frac{1}{2} \cdot V_K(p_{\hat{K}}) \\ \Leftrightarrow V_K(p_E) + \frac{B}{2} &\leq \frac{B}{2} + V_K^{\frac{1}{2}}(p_K, p_{\hat{K}}) \\ \Leftrightarrow V_K(p_E) + \frac{B}{2} &\leq \frac{B}{2} + V_K^{\frac{1}{2}}(p_{\hat{K}}, p_K) \end{aligned} \tag{55}$$

and that

$$\begin{aligned} \frac{1}{2} \geq q \Leftrightarrow \frac{1}{2} \leq (1-q) \Leftrightarrow \frac{1}{2} \cdot B + V_K^{\frac{1}{2}}(p_K, p_{\hat{K}}) &\geq q \cdot B + V_K^q(p_K, p_{\hat{K}}) \\ \Leftrightarrow \frac{1}{2} \cdot B + V_K^{\frac{1}{2}}(p_K, p_{\hat{K}}) &\leq V_K^{1-q} \cdot B + V_{1-q}(p_K, p_{\hat{K}}). \end{aligned} \tag{56}$$

It turns out that for several parameter constellations, there will exist a multiplicity of equilibria. This is relevant when computing welfare, since an assumption is needed on the likelihood of each of the equilibria. By default, we make the same assumption as in the baseline model: each equilibrium is equally likely. Since not all equilibria lead to different outcomes, the latter assumption implies that some outcomes will be more likely than others. In the analysis below we will only report equilibrium outcomes. Accordingly, when there is more than one equilibrium, we express the relative frequency of each of them. More specifically, $(\times j)$ will mean that the corresponding equilibrium outcome can occur in j times more constellations than the equilibrium marked as $(\times 1)$.

Case I: $q^c \leq q^n \leq \frac{1}{2}$

All equilibrium policy outcomes for this case are summarized in Table 13:

[39] As our main goal is to calculate welfare, which depends on variable q, we neglect the cases where q coincides with either $q^c, q^n, 1-q^n$, or $\frac{1}{2}$, as they are events with zero probability.

	$0<q<q^c$	$q^c<q<q^n$	$q^n<q<\frac{1}{2}$	$\frac{1}{2}<q<1$
Blue protocol (BP)	$\left(\frac{1}{2}p_L,\frac{1}{2}p_R\right)$	$\left(\frac{1}{2}p_L,\frac{1}{2}p_R\right)$ $(\times 2)$ p_{LR} $(\times 1)$	$\left(\frac{1}{2}p_L,\frac{1}{2}p_R\right)$ $(\times 2)$ p_{LR} $(\times 1)$	p_{LR}
Green protocol (GP)	p_E	p_{LR} $(\times 1)$ p_E $(\times 1)$	$(qp_L,(1-q)p_R)$ $(\times 2)$ $((1-q)p_L,qp_R)$ $(\times 2)$ p_{LR} $(\times 1)$	p_{LR}

Table 13: Green versus blue protocol in the case of two extreme parties (Case I).

To prove the results contained in Table 13, we distinguish several subcases:

Case I.1: $0<q<q^c$

From $0<q<q^c\leq q^n\leq\frac{1}{2}<1-q$ and Equations (53)–(56) it follows that

$$q\cdot B+V_K^q(p_K,p_{\hat{K}})<\frac{B}{2}+V_K(p_E)\leq\frac{B}{2}+V_K^{\frac{1}{2}}(p_K,p_{\hat{K}})<(1-q)\cdot B+V_K^{1-q}(p_K,p_{\hat{K}}) \tag{57}$$

and

$$b+V_K(p_{LR})<(1-q)\cdot B+V_K^{1-q}(p_K,p_{\hat{K}}). \tag{58}$$

First, in the case of the blue bargaining protocol, it follows from (57) and (58) that the strategies $\emptyset$ and $\{E_1,E_2\}$ are weakly dominated for both conventional parties. Thus, the contract choice game for the conventional parties in undominated strategies reduces to

Party R

Party L	$\{E_1\}$	$\{E_2\}$
$\{E_1\}$	$\frac{B}{2}+V_L(p_E),\frac{B}{2}+V_R(p_E)$	$\frac{B}{2}+V_L^{\frac{1}{2}}(p_L,p_R),\frac{B}{2}+V_R^{\frac{1}{2}}(p_L,p_R)$
$\{E_2\}$	$\frac{B}{2}+V_L^{\frac{1}{2}}(p_L,p_R),\frac{B}{2}+V_R^{\frac{1}{2}}(p_L,p_R)$	$\frac{B}{2}+V_L(p_E),\frac{B}{2}+V_R(p_E)$

There are two equilibria, $(\{E_1\},\{E_2\})$ and $(\{E_2\},\{E_1\})$. Nevertheless, according to the voters' preferences and the fact that both extreme parties have weak bargaining power due to the contracts chosen by the parties, the policy outcomes associated with both equilibria are the same, i.e. $\left(\frac{1}{2}p_L,\frac{1}{2}p_R\right)$.

Second, in the case of the green bargaining protocol, it follows from (57) and (58) that strategy $\emptyset$ weakly dominates any other strategy for both conventional parties. Thus there is only one equilibrium, $(\emptyset,\emptyset)$. In this equilibrium, both extreme parties have strong bargaining power since no CPC has been chosen by the parties. As a consequence, the policy outcome associated with this equilibrium is p_E.

Case I.2: $q^c<q<q^n$

In this case, from Equations (53)–(56) it follows that (57) holds again, but

$$b+V_K(p_{LR})>(1-q)\cdot B+V_K^{1-q}(p_K,p_{\hat{K}}). \tag{59}$$

On the one hand, in the case of the blue bargaining protocol, it can be verified that there are three equilibria: $(\{E_1\},\{E_2\})$, $(\{E_2\},\{E_1\})$, and $(\{E_1,E_2\},\{E_1,E_2\})$. As in Case I.1, the policy outcomes associated with the first two equilibria are the same, i.e. $\left(\frac{1}{2}p_L,\frac{1}{2}p_R\right)$. Regarding the latter equilibria, however, the policy outcome is p_{LR}, as no extreme party can form a coalition with any of the conventional parties.

On the other hand, in the case of the green bargaining protocol, it can also be verified that there are two equilibria, $(\emptyset,\emptyset)$ and $(\{E_1,E_2\},\{E_1,E_2\})$. In the first equilibrium, both extreme parties have strong bargaining power due to the contracts chosen by the parties, so the policy outcome associated with it is p_E. In the second equilibrium, the policy outcome is p_{LR}, as no extreme party can form a coalition with any of the conventional parties.

Case I.3: $q^n < q < \frac{1}{2}$

In this case, from Equations (53)–(56) it follows that

$$\frac{B}{2} + V_K(p_E) < q \cdot B + V_K^q(p_K, p_{\hat{K}}) < \frac{B}{2} + V_K^{\frac{1}{2}}(p_K, p_{\hat{K}}) < (1-q) \cdot B + V_K^{1-q}(p_K, p_{\hat{K}})$$

and that (59) holds again.

On the one hand, in the case of the blue bargaining protocol it can be verified that the following are the equilibria: $(\{E_1\}, \{E_2\})$, $(\{E_2\}, \{E_1\})$, and $(\{E_1, E_2\}, \{E_1, E_2\})$, which lead respectively to policies $\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$, $\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$, and p_{LR}.

On the other hand, in the case of the green bargaining protocol, it can be verified that there are five equilibria: $(\{E_1\}, \emptyset)$, $(\{E_2\}, \emptyset)$, $(\emptyset, \{E_1\})$, $(\emptyset, \{E_2\})$ and $(\{E_1, E_2\}, \{E_1, E_2\})$. The policy outcome associated with the first four equilibria is $(qp_L, (1-q)p_R)$, $(qp_L, (1-q)p_R)$, $((1-q)p_L, qp_R)$, and $((1-q)p_L, qp_R)$ respectively, since we are considering the green bargaining protocol and in all cases there is an extreme party that possesses weak bargaining power, while the other extreme party possesses strong bargaining power. In the last equilibrium, the policy outcome is p_{LR}, as no extreme party can form a coalition with any of the conventional parties.

Case I.4: $\frac{1}{2} < q$

In this case, from Equations (53)–(56) it follows that

$$\frac{B}{2} + V_K(p_E) < \frac{B}{2} + V_K^{\frac{1}{2}}(p_K, p_{\hat{K}}) < q \cdot B + V_K^q(p_K, p_{\hat{K}}),$$

and

$$(1-q) \cdot B + V_K^{1-q}(p_K, p_{\hat{K}}) < \frac{B}{2} + V_K^{\frac{1}{2}}(p_K, p_{\hat{K}}).$$

Additionally, it also follows that

$$b + V_K(p_{LR}) > (1-q) \cdot B + V_K^{1-q}(p_K, p_{\hat{K}}).$$

With both bargaining protocols, it can be verified that the unique equilibrium is $(\{E_1, E_2\}, \{E_1, E_2\})$, which leads to policy p_{LR}.

Case II: $q^c \leq \frac{1}{2} \leq q^n$

All equilibrium policy outcomes for this case are summarized in Table 14:[40]

	$0 < q < q^c$	$q^c < q < \frac{1}{2} < q^n < 1-q$	$q^c < q < \frac{1}{2} < 1-q < q^n$	$\frac{1}{2} < q < q^n$	$q^n < q < 1$
BP	p_E	p_{LR} (×1) p_E (×1)	p_{LR} (×1) p_E (×1)	p_{LR} (×1) p_E (×1)	p_{LR}
GP	p_E	p_{LR} (×1) p_E (×1)	p_{LR} (×1) p_E (×3)	p_{LR} (×1) p_E (×3)	p_{LR}

Table 14: Green versus blue protocol in the case of two extreme parties (Case II).

Case III: $q^n \leq q^c \leq \frac{1}{2}$

All equilibrium policy outcomes for this case are summarized in Table 15:[41]

[40] A comprehensive proof of all cases can be provided by the authors upon request.

[41] As in Case II, a comprehensive proof of all cases can be provided by the authors upon request.

	$0 < q < q^n$	$q^n < q < q^c$	$q^c < q < \frac{1}{2}$	$\frac{1}{2} < q < 1$
BP	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$ $(\times 2)$ p_{LR} $(\times 1)$	p_{LR}
GP	p_E	$(qp_L, (1-q)p_R)$ $(\times 1)$ $((1-q)p_L, qp_R)$ $(\times 1)$	$(qp_L, (1-q)p_R)$ $(\times 2)$ $((1-q)p_L, qp_R)$ $(\times)$ p_{LR} $(\times 1)$	p_{LR}

Table 15: Green versus blue protocol in the case of two extreme parties (Case III).

Finally, we focus on the case where the one-party exclusion rule applies. From Table 7 it immediately follows that p_{LR} cannot arise in equilibrium as it is not an outcome of the game. Tables 16–18 below show the policies that arise in equilibrium when there are two extreme parties but the one-party exclusion rule applies. A comprehensive proof can be provided by the authors upon request. To facilitate the comparison between the two situations we subdivide the tables in the exactly same way as in the case with the one-party exclusion rule.[42]

[42]The subdivision in the tables does not correspond to the cases necessary for the computation of equilibrium outcomes. Either further subcases or fewer cases should be considered.

	$0 < q < q^c$	$q^c < q < q^n$	$q^n < q < \frac{1}{2}$	$\frac{1}{2} < q < 1$
BP	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$
GP	p_E	p_E	$(qp_L, (1-q)p_R)\ (\times 1)$ $((1-q)p_L, qp_R)\ (\times 1)$	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$

Table 16: Green versus blue protocol in the case of two extreme parties with the one-party exclusion rule (Case I: $q^c \leq q^n \leq \frac{1}{2}$).

	$0 < q < q^c$	$q^c < q < \frac{1}{2} < q^n < 1-q$	$q^c < q < \frac{1}{2} < 1-q < q^n$	$\frac{1}{2} < q < q^n$	$q^n < q < 1$
BP	p_E	p_E	p_E	p_E	p_E
GP	p_E	p_E	p_E	p_E	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$

Table 17: Green versus blue protocol in the case of two extreme parties with the one-party exclusion rule (Case II: $q^c \leq \frac{1}{2} \leq q^n$).

	$0 < q < q^n$	$q^n < q < q^c$	$q^c < q < \frac{1}{2}$	$\frac{1}{2} < q < 1$
BP	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$
GP	p_E	$(qp_L, (1-q)p_R)\ (\times 1)$ $((1-q)p_L, qp_R)\ (\times 1)$	$(qp_L, (1-q)p_R)\ (\times 1)$ $((1-q)p_L, qp_R)\ (\times 1)$	$\left(\frac{1}{2}p_L, \frac{1}{2}p_R\right)$

Table 18: Green versus blue protocol in the case of two extreme parties with the one-party exclusion rule (Case III: $q^n \leq q^c \leq \frac{1}{2}$).

Next, we prove Proposition 3, where we analyze welfare in a framework with/without CPCs and two extreme parties.

Proof of Proposition 3:

We prove each of the items sequentially. We stress that we are assuming that all equilibrium outcomes are equally likely.[43] We start with (a). Without CPCs and similarly to the case with only one extreme party, it can be verified that the outcome is p_E. Hence, if Condition (36) holds and all possible equilibrium outcomes are equally likely, it follows from Tables 13–15 that CPCs are ex-ante welfare-improving under both the green and the blue protocols. For instance, assume that $q^c \leq q^n \leq \frac{1}{2}$ (see Table 13) and let W^{CPC} (or $W^{\cancel{CPC}}$) denote welfare with CPCs (or without CPCs).[44] Recall that $H(\cdot)$ denotes the cumulative

[43]The results hold however under weaker assumptions about how society builds expected welfare when a multiplicity of equilibria exists.

[44]The other cases can be proved analogously to Case I ($q^c \leq q^n \leq \frac{1}{2}$). A complete proof of Proposition 3 can be provided by the authors upon request.

distribution function of q. In the case of the blue protocol we obtain

$$\begin{aligned}
&\mathbb{E}_H[W^{CPC}] - \mathbb{E}_H[W^{NCPC}] \\
=&H(q^c) \cdot \left[\frac{1}{2} \cdot W(p_R) + \frac{1}{2} \cdot W(p_L) - W(p_E)\right] \\
+&\left(H\left(\frac{1}{2}\right) - H(q^c)\right) \cdot \left[\frac{2}{3} \cdot \left(\frac{1}{2} \cdot W(p_R) + \frac{1}{2} \cdot W(p_L) - W(p_E)\right) + \frac{1}{3} \cdot (W(p_{LR}) - W(p_E))\right] \\
+&\left(1 - H\left(\frac{1}{2}\right)\right) \cdot [W(p_{LR}) - W(p_E)] > 0,
\end{aligned} \tag{60}$$

where the inequality holds by Conditions (35) and (36). In the case of the blue protocol we similarly obtain

$$\begin{aligned}
&\mathbb{E}_H[W^{CPC}] - \mathbb{E}_H[W^{NCPC}] \\
=&\frac{H(q^n) - H(q^c)}{2} \cdot [W(p_{LR}) - W(p_E)] \\
+&\left(H\left(\frac{1}{2}\right) - H(q^n)\right) \left[\frac{4}{5} \cdot \left(\frac{1}{2} \cdot W(p_R) + \frac{1}{2} \cdot W(p_L) - W(p_E)\right) + \frac{1}{5} \cdot (W(p_{LR}) - W(p_E))\right] \\
+&\left(1 - H\left(\frac{1}{2}\right)\right) \cdot [W(p_{LR}) - W(p_E)] > 0,
\end{aligned} \tag{61}$$

where the inequality again holds by Conditions (35) and (36). Second, part (b) holds because when $B = 2b$ and $b \longrightarrow \infty$, we have $q^n = q^c \longrightarrow \frac{1}{2}$. Then, from Equations (60) and (61) it straightforwardly follows that ex-ante welfare with the blue protocol is higher than welfare with the green protocol if and only if Condition (36) holds. Third, part (c) can be easily verified using Tables 13–15. Lastly, part (d) follows from Condition (35) by comparing Tables 13, 14, and 15 with Tables 16, 17, and 18, respectively, if we assume that $W(\cdot)$ is concave w.r.t. the policy in dimension $\mathcal{T}$ and conventional party positions are symmetric w.r.t. p_{LR}. The latter condition implies that $\frac{1}{2}W(p_L) + \frac{1}{2}W(p_R) < W(p_{LR})$.

□

Finally, we prove Proposition 4.

Proof of Proposition 4:

First, we stress that the outcome of the election depends only on the shares of the conventional parties, namely α_L and α_R. Therefore, since we are considering Strong Nash equilibria and there is a positive cost for voting strategically, the set of voters who vote strategically, denoted by S^*, will be such that either $S^* \subset \mathcal{F}^m$ or $S^* \subset \Omega \setminus \mathcal{F}^m$, and either $|S^*| = 0$ or $|S^*| = \frac{1}{2}\alpha_E$. That is, only a minimal share of voters will vote strategically. These voters will suffice to change the outcome of the election.

Second, as the above observation suggests, there may be different sets S^* that would lead to the same voting outcome. However, according to our notion of equilibrium, it will necessarily be the case that when S^* is non-empty, it will consist of the subset of measure $\frac{\alpha_E}{2}$ of conventional voters with ideal points closest to t_m. This statement follows from two facts. On the one hand, in the policy dimension $\mathcal{T}$ agents have preferences that satisfy the strong single-crossing property. Thus, if a conventional voter has no incentive to deviate from sincere voting, no other conventional voter on his political side who is farther away from the median will either. On the other hand, the closer a conventional voter's ideal point t_i is to t_m, the less he will suffer from the policy shift toward the ideal point of his least preferred conventional party.

□