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Abstract

This paper proposes a rational model of voter participation by generalizing a common-value model of costless voting to include not just pivotal voting but also marginal voting incentives. A new strategic incentive for abstention arises in that case, to avoid the marginal voter's curse of pushing the policy outcome in the wrong direction. The marginal voter's curse presents a larger disincentive for voting than the swing voter's curse. Moreover, marginal motivations are shown to dominate pivotal motivations in large elections. Model predictions are confirmed in a laboratory experiment and applied in a comparative analysis of electoral rules.

JEL Classification: C72, C92, D70

Keywords: Turnout, information aggregation, Underdog effect, Experiment

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1 Introduction

Models of elections have repeatedly faced two challenges. First, almost all election models limit the impact of a vote to the rare occasion in which it is *pivotal*, changing the identity of the election winner by making or breaking a tie. This is appropriate in that, mechanically, this is the only function that a vote is meant to perform. To the typical voter, however, the notion of pivotal voting seems quite foreign. Moreover, a growing empirical literature finds that candidate behavior and policy outcomes also respond to the margin by which a party wins an election.[1] Since every vote contributes (positively or negatively) to the winning party's margin of victory, a rational voter's calculus should take this into account, and the standard pivotal voting calculus is incomplete.

The second challenge for election models is closely related to the first: namely, voter participation. A well-known paradox arises because, as recognized at least since Downs (1957), a pivotal vote is unlikely in large elections, so only a small number of citizens should be willing to pay the cost of voting. Nevertheless, in spite of the cost, voter turnout is often quite high, suggesting that for most people the cost of voting is overcome by ethical motivations such as altruism or a sense of civic duty.[2] If so, however, then it is puzzling why turnout is also often quite low. Moreover, as a seminal paper by Feddersen and Pesendorfer (1996) notes, abstention frequently occurs when voting is entirely costless. For example, voters often cast incomplete ballots (or *roll off*)—voting in some races but not others even after voting costs have already been paid, and are therefore sunk. Those authors attribute abstention to a *swing voter's curse* that arises when voters share a common interest but have heterogeneous information: a citizen abstains because he is unsure which policy or candidate he should vote for, and anticipates that his peers will make a better choice on his behalf than he can make for himself. This reasoning resonates with everyday experience, and is also consistent with growing empirical evidence that improving

[1] For example, see Conley (2001), Fowler (2005, 2006), Bernhard et al. (2008), and Faravelli, Man and Walsh (2015).

[2] For example, see Riker and Ordeshook (1968) or, more recently, Myatt (2012). For reviews of the voluminous costly voting literature, see Dhillon and Peralta (2002), Feddersen (2004), Dowding (2005), or Geys (2006).

voters' information makes them more likely to vote.[3] Formally, however, the swing voter's curse is also inextricably connected to the pivotal voting calculus: when a citizen's peers vote informatively, the party with the superior policy position is more likely to lead by one vote than to trail by one vote, so a mistaken vote for the party with the inferior policy position is more likely to be pivotal than an additional vote for the superior party, and this is why a citizen who is unsure which policy is superior should abstain from voting.

This paper proposes a simple rational model of voter participation by generalizing the common-value model of costless voting to include not just pivotal voting but also marginal voting incentives. To this end, we assume that the policy outcome may jump discontinuously when one party's vote share crosses the 50% threshold, but increases continuously with the vote share even away from this threshold, as depicted in Figure 1.

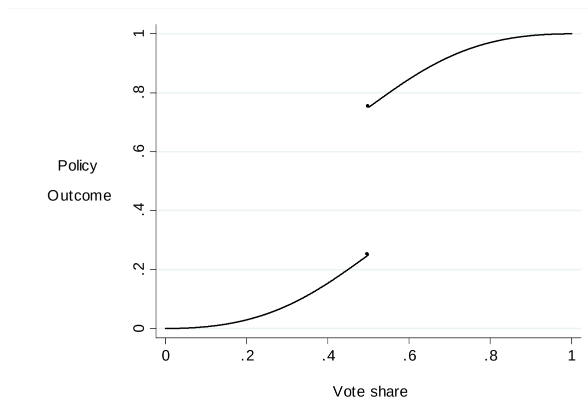

Figure 1: Mapping between vote shares and policy outcomes.

Intuitively, it might seem that this should mute the incentive to abstain, because a citizen no longer conditions his behavior solely on the presumption that his vote will be pivotal. The main result of the analysis below, however, is that the marginal voting calculus generates a new reason for abstention: namely, a *marginal voter's*

[3]For a review of this extensive empirical literature, see Triossi (2013) and McMurray (2015). In particular, turnout and roll-off are both correlated with political knowledge and with other variables associated with information, such as education and age. Evidence from the natural experiment of Lassen (2005) and the field experiments of Banerjee et al. (2010) and Hogh and Larsen (2016) suggest that improving information has a causal impact on voter participation.

curse. The latter would lead citizens to abstain from voting even if the discontinuity illustrated in Figure 1 were removed entirely, so that pivotal considerations did not matter at all to voters.

The logic of the marginal voter's curse is that a vote for the losing party has a greater impact on the vote share than a vote for the winning party has. If a voter trusts his peers to vote informatively, however, then he expects the party with the superior policy position to be ahead. Thus, if he is uncertain which party is superior, he should abstain from voting, because the benefit that his vote will generate if his private opinions are correct is smaller than the damage his vote will create if he is in error. As discussed below, the marginal voter's curse and swing voter's curse can be viewed as two manifestations of a more general *underdog effect* that has been noted in existing literature on voter participation.

Because a pivotal vote is so rare, it might seem intuitive that behavior would be altered much more dramatically by conditioning on this event than by the marginal voting calculus. It turns out, however, that the marginal voter's curse is actually stronger than the swing voter's curse, in the sense that equilibrium turnout in a pure marginal voting model (i.e. with no discontinuity at the 50% threshold) is lower than turnout in a pure pivotal voting model (i.e., in which the policy outcome shifts only at the 50% threshold, and nowhere else). An intuition for why this is the case is that, when all that matters is which side receives a majority of votes, a single mistaken vote for the political party with an inferior policy platform can be offset by a single correct vote for the party with the superior policy position. The same is not true of a pure marginal voting model, because vote shares become diluted. As a simple illustration of this, suppose that the better of two alternatives received three out of five votes, or a 60% vote share. One additional vote for the opposite party reduces this vote share to 50% (three out of six), and an additional vote of support brings it back up, but only to 57% (four out of seven). In other words, if policy outcomes move continuously with vote shares then it is not sufficient to give lots of votes to the superior side; the electorate must also give as few votes as possible to the inferior side, so that the better side not only wins, but wins by a large margin.

The analysis below uses observations such as these to derive comparative static results about how voter participation in a marginal voting environment responds to changes in the composition of the electorate, such as improvements in the distribution of expertise or changes in the number of citizens whose votes reflect partisan interests rather than objective policy opinions. The theoretical analysis also shows how incentives for voter participation change as the number of voters grows large. As the probability of a pivotal vote shrinks, the impact of conditioning on this event grows, so the swing voter's curse grows stronger. The marginal impact of a vote also shrinks in large elections, which intuitively might seem to attenuate the marginal voter's curse. To the contrary, however, the marginal voter's curse grows stronger as well, as voters increasingly delegate to purge the policy decision of noise. In fact, the pivotal impact of a vote shrinks more quickly than the marginal impact of vote, so that only the latter matters in large elections. In other words, a general model with both incentives exhibits the same levels of voter participation as a purely continuous model, with no discontinuity at all at the 50% threshold.

To test the predictions of our theoretical analysis, we conduct a laboratory experiment. Voter participation levels differ noticeably from those predicted in equilibrium, but behavioral patterns match much more closely. We also discuss an application of the insights of this paper to Proportional Rule electoral systems, which have become increasingly prevalent throughout the world in recent decades, especially in Europe and Latin America.

The remainder of this paper is organized as follows. Section 2 discusses related literature, after which Section 3 introduces the formal model. Before presenting results for the general model, we present theoretical equilibrium predictions for two polar cases in Sections 4 and 5. Section 6 compares these two polar cases and Section 7 presents theoretical equilibrium predictions for the general model. Section 7.1 then extends the original model to accommodate more general relationships between vote shares and policy outcomes, Section 8 reports the results of the laboratory experiments, and Section 9 concludes with a possible application to Proportional Rule electoral systems. Proofs of theoretical results are presented in the Appendix.

2 Literature

Since the seminal Palfrey and Rosenthal (1983, 1985), the game theoretical literature on turnout has grown extensively. Most of these papers focus on models which consider only pivotal motives in a private value settings. A more recent set of papers explore the introduction of marginal voting incentives in two-party elections (see, e.g., Castanheira (2003) and Herrera, Morelli, and Palfrey (2014), Kartal (2014a), Faravelli and Sanchez-Pages (2014), and Faravelli, Man, and Walsh (2015)). The general finding of these papers is that, unless the two parties are expected to have the same support, the magnitude of marginal voting incentives dominate in large elections. The consequence is that turnout is typically higher in these models than the one in traditional models with only pivotal motives. This contrasts with the results of sections 6 and 7 in our paper. Shotts (2006) provides a possible microfoundation for marginal voting incentives in a model where margins in a first election communicate information about voters' preference parameters to candidates in a second election. Meirowitz and Shotts (2009) show that this, too, dominates the pivotal voting incentive.

In a common-value environment, Feddersen and Pesendorfer (1999) and Krishna and Morgan (2011) analyze how the swing voter's curse is affected by preference heterogeneity. Krishna and Morgan (2012) and McMurray (2013) introduce more general distributions of private information, and the latter formulation is used in the model below. In a similar setting, Oliveros and Várdy (2015) include an initial choice of the source of (possibly biased) information. Laboratory experiments by Battaglini, Morton and Palfrey (2008, 2010), Morton and Tyran (2011) and Mengel and Rivas (2016) corroborate the basic theoretical predictions of the swing voter's curse. All of these papers, however, restrict attention to the pivotal impact of a vote. Razin (2003) proposes a model in which voting has a marginal impact of shaping candidates' policy beliefs, but does not consider abstention. McMurray (2016) demonstrates that citizens do have reason to abstain in that environment, because of a "signaling voter's curse". That result relies on candidates sharing voters' preferences, however, and learning from abstention just as they learn from votes. In contrast, the marginal voter's curse arises if the mapping from vote shares to policy outcomes is purely mechanical. Together, the swing voter's curse, signaling voter's curse, and marginal voter's curse suggest that the incentive to abstain is a robust consequence of common values and heterogeneous expertise, not an artifact of any particular political institution.

3 The Model

An electorate consists of N citizens where, following Myerson (1998), N is finite but unknown, and follows a Poisson distribution with mean n. Together, these citizens must choose a policy from an interval. There are two political parties, each with policy positions in the interval. One of these policy positions is ultimately better for society than the other. Let A denote the party with the superior position and B denote the party with the inferior position. Which party is A and which is B is determined by Nature at the beginning of the game, with equal probability. Letting 0 denote party B's position and 1 denote party A's position, $x \in [0, 1]$ can denote any policy between the two parties' positions and also the social welfare $u(x) = x$ that will be produced if that policy is implemented.

Citizens are each independently designated as one of two types. With probability $2p$, a citizen is a partisan, and with equal probability favors one party or the other, regardless of which policy position Nature designated as superior. With remaining probability $I = 1 - 2p$, a citizen is designated as *independent* or *non-partisan*. Independents prefer to do whatever is socially optimal, evaluating policy x according to the welfare function $u(x)$ given above. From an independent's perspective, each of his fellow citizens has probability p of being a partisan supporter of the superior party A and probability p of being a partisan supporter of the inferior party B.

The policy outcome is a general function $x : \mathbb{Z}_+^2 \to [0, 1]$ of the numbers a and b of votes that are cast for either party, which can be most easily described in terms of two benchmark cases. The first is the case of pure *marginal voting*, which means that the policy outcome is a weighted average of the parties' policy positions, with weights given by the parties' vote shares. That is, if a fraction $\lambda_+ = \frac{a}{a+b}$ of the electorate votes for party A and a fraction $\lambda_- = \frac{b}{a+b}$ votes for B then the policy outcome is

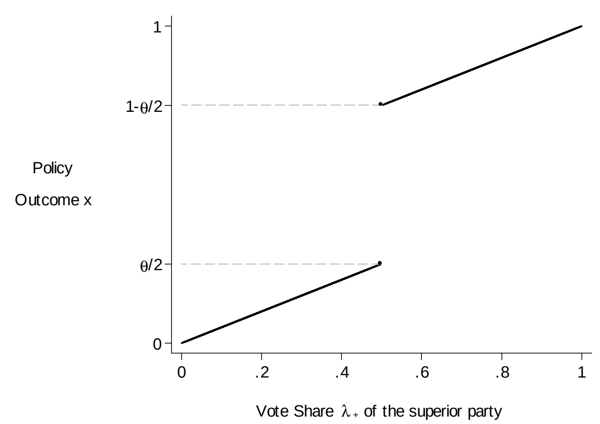

Figure 2: Mapping between vote shares and policy outcomes.

given by $x(a,b) = 0\lambda_- + 1\lambda_+ = \lambda_+$, ranging continuously from 0 to 1 are the vote share of the superior party ranges from 0% to 100%.[4] The second benchmark is the more traditional case of pure *pivotal voting.* In that case, x is simply the policy position $x_w \in \{x_A, x_B\}$ of the party w who wins the election (i.e. 0 if $b > a$ and 1 if $a > b$, breaking a tie if necessary by a fair coin toss). The general model admits both types of voting incentives: the policy outcome is a weighted average

$$x = \theta\lambda_+ + (1-\theta)\,x_w \tag{1}$$

of the benchmark cases with $\theta \in [0,1]$. With this formulation, pure marginal voting and pure pivotal voting correspond to $\theta = 1$ and $\theta = 0$, respectively. Thus, policy shifts discontinuously when one party's vote share crosses the 50% threshold but, even away from this threshold, changes in one party's vote share push the policy outcome marginally in that party's direction, as Figure 2 illustrates.

The optimal policy cannot be observed directly, but independent voters observe private signals s_i that are informative of Nature's choice.[5] These signals are of heterogeneous quality, reflecting the fact that citizens differ in their expertise on

[4] An alternative assumption that would lead to identical analysis is that policy 1 is implemented with λ_+ and policy 0 is implemented with probability λ_-, and that independent voters are risk neutral. This could result from probabilistic voting across independent legislative districts, as in Levy and Razin (2015).

[5] Partisans could receive signals as well, of course, but would ignore them in equilibrium.

the issue at hand. Specifically, each citizens is endowed with information quality $q_i \in \mathcal{Q} = [0,1]$, drawn independently according to a common distribution F which, for simplicity, is continuous and has full support. Conditional on $q_i = q$, a citizen's signal correctly identifies the party whose policy position is truly superior with the following probability.

$$\Pr(s_i = A|q) = \frac{1}{2}(1+q) \tag{2}$$

With complementary probability, a citizen mistakes the inferior party for the superior party.

$$\Pr(s_i = B|q) = \frac{1}{2}(1-q) \tag{3}$$

With this specification, q_i can be interpreted as the correlation coefficient between a voter's private opinion and the truth. That is, a citizen with $q_i = 1$ is perfectly informed about Nature's choice, while a signal with $q_i = 0$ is completely uninformative. An independent can vote (at no cost) for the party that he perceives to be superior, or can abstain.[6] Let $\sigma : \mathcal{Q} \to [0,1]$ denote a (mixed) participation strategy, where $\sigma(q)$ denotes the probability of voting for an individual with expertise $q \in \mathcal{Q}$, and let Σ denote the set of such strategies. By Bayes' rule, (2) and (3) can be reinterpreted as a voter's posterior belief that he has correctly voted for the superior party or mistakenly voted for the inferior party, respectively.

Given a participation strategy, the probabilities with which a citizen votes for party A and party B, respectively, are given by the following.

$$v_+ = p + I\int_0^1 \sigma(q)\frac{1}{2}(1+q)\,dF(q) \tag{4}$$

$$v_- = p + I\int_0^1 \sigma(q)\frac{1}{2}(1-q)\,dF(q) \tag{5}$$

These include the probability p of favoring either party for partisan reasons, as well as the probabilities of voting as an independent with any level of expertise. Together, (4) and (5) also determine the level $v_\tau = v_+ + v_-$ of voter turnout.

If every citizen follows the same participation strategy, (4) and (5) can be interpreted as the expected vote shares of the superior and inferior parties, respectively.

[6] A strategy of voting against one's signal could be allowed but would not be used in equilibrium.

By the decomposition property of Poisson random variables (Myerson 1998), the numbers a and b of votes for the superior and inferior parties, respectively, are independent Poisson random variables with means $n_+ = nv_+$ and $n_- = nv_-$. Thus, the probability of exactly a votes for the superior party and b votes for the inferior party is the product

$$\Pr(a,b) = \frac{e^{-n_+} n_+^a}{a!} \frac{e^{-n_-} n_-^b}{b!} \tag{6}$$

of Poisson probabilities. Similarly, the expected total number of votes can be written as $n_\tau = nv_\tau$.

By the environmental equivalence property of Poisson games (Myerson 1998), an individual from within the game reinterprets a and b as the numbers of correct and incorrect votes cast by his peers; by voting himself, he might add one to either total. When there are a votes for the superior party and b votes for the inferior party, the change in utility $\Delta_+ x(a,b)$ from contributing one additional vote for the superior party and the change in utility $\Delta_- x(a,b)$ from accidentally adding one vote for the inferior party are given by the following.

$$\Delta_+ x(a,b) = x(a+1,b) - x(a,b) \tag{7}$$

$$\Delta_- x(a,b) = x(a,b+1) - x(a,b) \tag{8}$$

The magnitudes of these utility changes depend on the numbers of votes cast for either side by a citizen's peers; averaging over all possible voting outcomes, the expected benefit of voting is given by

$$\Delta Eu(q) = E_{a,b}\left[\frac{1}{2}(1+q)\,\Delta_+ x(a,b) + \frac{1}{2}(1-q)\,\Delta_- x(a,b)\right] \tag{9}$$

which depends on a citizen's expertise q. Implicitly, the expectation in (9) depends on the voting strategy adopted by a citizen's peers. If his peers all follow the strategy $\sigma \in \Sigma$, a citizen's best response is to vote if his q is such that (9) is positive and to abstain otherwise. A strategy σ^* that is its own best response constitutes a (symmetric) Bayesian Nash equilibrium of the game.

Sections 4 and 5 analyze incentives for equilibrium behavior under the two benchmark cases of pure marginal voting ($\theta = 1$) and pure pivotal voting ($\theta = 0$), respectively. Section 6 then compares equilibrium levels of participation for these two benchmarks. Section 7 next analyzes equilibrium behavior for the general case, along with social welfare, and extends the model to a more general relationship between vote totals and the policy outcome.

4 Marginal Voting

If $\theta = 1$ then the general model of Section 3 reduces to the benchmark case of pure marginal voting. In that case, the policy outcome $x = \lambda_+$ is simply the vote share of the party with the superior policy position. Changes in utility (7) and (8) from an additional vote for the superior party and from an additional vote for the inferior party can then be written as

$$\Delta_+ x(a,b) = \frac{a+1}{a+b+1} - \frac{a}{a+b} = \Delta\lambda_+ \tag{10}$$

$$\Delta_- x(a,b) = \frac{a}{a+b+1} - \frac{a}{a+b} = -\Delta\lambda_- \tag{11}$$

in terms of the increases $\Delta\lambda_+ = \frac{a+1}{a+b+1} - \frac{a}{a+b}$ and $\Delta\lambda_- = \frac{b+1}{a+b+1} - \frac{b}{a+b}$ in these vote shares that an additional correct vote or incorrect vote cause, respectively.

Since $\Delta\lambda_+$ and $\Delta\lambda_-$ are both positive, (9) is increasing in q, and is therefore positive for all q above some threshold $\bar{q}$. In other words, as Theorem 1 states below, the best response to any strategy σ can be characterized as a *threshold strategy* $\sigma_{\bar{q}}$, meaning that a citizen votes if his expertise exceeds a threshold $\bar{q}$, but abstains otherwise. Specifically, (9) is positive if and only if q exceeds $\bar{q}_M^{br}$, defined as follows.

$$\bar{q}_M^{br} = \frac{E_{a,b}(\Delta\lambda_-) - E_{a,b}(\Delta\lambda_+)}{E_{a,b}(\Delta\lambda_-) + E_{a,b}(\Delta\lambda_+)} \tag{12}$$

From (4) and (5) it is clear that $v_+ > v_-$ for any strategy in which a positive fraction of the electorate votes. Because of this, the number a of votes for the party with the superior policy position is likely to exceed the number b of votes for the opposing party. This is good for welfare, but since vote shares respond more strongly to an additional vote for the party that is behind than to an additional vote for the party that is ahead, as explained in Section 1, it also implies that $E_{a,b}(\Delta\lambda_-) > E_{a,b}(\Delta\lambda_+)$, and therefore that $\bar{q}_M^{br} > 0$.[7] In other words, the best response for citizens with the lowest levels of expertise is to abstain from voting—even though voting is costless and the swing voter's curse is not relevant—to avoid the *marginal voter's curse* of pushing the policy outcome in the wrong direction. Equilibrium existence follows from standard fixed point arguments.

Theorem 1 (Marginal Voter's Curse) *If $\theta = 1$ then $\sigma^* \in \Sigma$ is a Bayesian Nash equilibrium only if it is a threshold strategy $\sigma_{\bar{q}_M^*}$ with $\bar{q}_M^* > 0$. Moreover, such an equilibrium exists.*

The result that independent voters each receive informative private signals but not all report their signals in equilibrium implies that valuable information is lost. Intuitively, this may seem to justify efforts to increase voter participation, for example by punishing non-voters with stigma or fines. To the contrary, however, McLennan (1998) shows that, in common-value environments such as this, whatever is socially optimal is also individually optimal, implying that equilibrium abstention in this setting actually improves welfare. To see how it can be welfare improving to throw away signals, note that citizens actually have not one but two pieces of private information: their signal realization s_i and their expertise q_i. In an ideal electoral system, all signals would be utilized, but would be weighted according to their underlying expertise. Here, votes that are cast are instead weighted equally. Abstention provides a crude mechanism whereby citizens can transfer weight from the lowest quality signals to those that reflect better expertise.

Theorem 1 applies for a population of any size n. As actual electorates tend to be extremely large, however, the rest of this section now derives the limit of equilibrium behavior as n grows large. Such asymptotic results are made possible by Lemma 1, which offers an algebraic simplification of the formulas obtained previously, in terms of the expected fractions v_+ and v_- of the electorate who vote for the parties with

[7] Section 7.1 uses this *underdog property* to generalize the marginal voting model to allow non-linear functions of the vote totals.

the superior and inferior policy positions, respectively, and the total fraction v_τ who turn out to vote.

Lemma 1 *The following hold for any n and for any threshold strategy $\sigma_{\bar{q}}$.*

$$E_{a,b}\left(\Delta\lambda_+\right) = \frac{v_-}{nv_\tau^2} + \frac{n\left(v_+^2 - v_-^2\right) - 2v_-}{2nv_\tau^2}e^{-nv_\tau} \tag{13}$$

$$E_{a,b}\left(\Delta\lambda_-\right) = \frac{v_+}{nv_\tau^2} + \frac{n\left(v_-^2 - v_+^2\right) - 2v_+}{2nv_\tau^2}e^{-nv_\tau} \tag{14}$$

With Lemma 1, an equivalent condition to equation (12) is that $\bar{q}_M^{br}$ solves the following.

$$\frac{1+\bar{q}}{1-\bar{q}} = \frac{2v_+ + \left[n\left(v_+^2 - v_-^2\right) - 2v_-\right]e^{-nv_\tau}}{2v_- + \left[n\left(v_-^2 - v_+^2\right) - 2v_+\right]e^{-nv_\tau}} \tag{15}$$

As n grows large, the right-hand side of this expression converges simply to the likelihood ratio $\rho\left(\bar{q}\right) = \frac{v_+}{v_-}$ of correct to incorrect votes. Since (15) is continuous both in $\bar{q}$ and in n, therefore, the limit $q^M = \lim_{n\to\infty}\bar{q}_n^*$ of any sequence of equilibrium threshold must solve the following simpler equation,

$$\rho\left(\bar{q}\right) = \frac{1+\bar{q}}{1-\bar{q}} \tag{16}$$

which can also be written in terms of the vote share λ_+:[8]

$$\lambda_+ = \frac{1}{2}\left(1+\bar{q}\right) \tag{17}$$

For a threshold strategy, v_+ and v_- reduce from (4) and (5) to the following,

$$\begin{aligned} v_+ &= p + I\int_{\bar{q}}^{1}\frac{1}{2}\left(1+q\right)dF\left(q\right) \\ &= p + \frac{1}{2}I\left[1-F\left(\bar{q}\right)\right]\left[1+m\left(\bar{q}\right)\right] \end{aligned} \tag{18}$$

[8]The convergence of (15) to (16) is not trivial—because v_+, v_-, and v_τ change with $\bar{q}_n^*$, which changes with the population size—but nevertheless holds, as the proof of Proposition 1 demonstrates formally.

$$
\begin{aligned}
v_{-} &= p + I \int_{\bar{q}}^{1} \frac{1}{2} (1-q) \, dF(q) \\
&= p + \frac{1}{2} I \left[1 - F(\bar{q})\right] \left[1 - m(\bar{q})\right] \\
v_{\tau} &= 2p + I \left[1 - F(\bar{q})\right]
\end{aligned}
\tag{19}
$$

where $m(\bar{q}) = E(q|q > \bar{q})$ denotes the average expertise among citizens who actually vote, and the left-hand side of (16) can therefore be rewritten as

$$
\rho(\bar{q}) = \frac{K + \left[1 - F(\bar{q})\right] \left[1 + m(\bar{q})\right]}{K + \left[1 - F(\bar{q})\right] \left[1 - m(\bar{q})\right]} \tag{20}
$$

in terms of the ratio $K = \frac{2p}{I}$ of partisans to independents.

The proof of Proposition 1 shows that equation (16) has a unique solution q^M. Uniqueness in the limit does not imply a unique equilibrium in any game with finite size parameter n. But if there are multiple equilibrium participation thresholds then the implication of Proposition 1 is that these all converge to each other in the limit. A unique limiting participation threshold of course translates into a unique limiting level $v_\tau = v_+ + v_-$ of expected voter participation, and actual turnout in large elections converges to its expectation. The margin of victory $\mu = \frac{v_+ - v_-}{v_\tau}$ in a large election is determined by the same threshold.

In addition to stating uniqueness in the limit, Proposition 1 derives some comparative static implications of the limiting equilibrium condition (16). Intuitively, it might seem that the marginal voter's curse should attenuate as n grows large, because the damage caused by one mistaken vote shrinks, so citizens should be less worried about making mistakes. If so, abstention should decline as the electorate grows large, and turnout should tend toward 100% in the limit. Contrary to this intuition, however, the first part of Proposition 1 states that q^M is strictly positive, meaning that a positive fraction of the electorate continue to abstain no matter how large the electorate grows. In fact, if there are no partisans then q^M equals one, meaning that—in sharp contrast with the intuition above—turnout tends to 0% in the limit.

Proposition 1 *There exists a unique* q^M *such that, for any sequence* $\bar{q}^*_M(n)$ *of equilibrium thresholds under pure marginal voting,* $\lim_{n\to\infty} \bar{q}^*_M(n) = q^M$. *Moreover,* q^M *exhibits the following properties:*

(i) $q^M > \frac{1}{2}$ *for any partisan share* p. *If* $p = 0$ *then* $q^M = 1$.

(ii) q^M *decreases strictly with* p.

(iii) If $p > 0$ *then improvements in the distribution* F *of expertise that satisfy the monotone likelihood ratio property increase* q^M.

Intuitively, the reason why the incentive to abstain does not vanish in large elections is that the policy outcome is a weighted average of the two extremes, with weights corresponding to vote shares. Citizens wish to vote as unanimously as possible in favor of the superior side, and this is accomplished by limiting participation to those who are the least likely to err. That $q^M = 1$ when there are no partisans is a consequence of the assumption that the distribution F of expertise has full support; more generally, if the maximum level of expertise is $q_{\max}$ then, when there are no partisans, $q^M = q_{\max}$. Either way, the entire electorate defers to the vanishing segment of the electorate who are least likely to dilute the electoral outcome with incorrect votes.

The limiting level of equilibrium participation can be most easily compared with that of pure pivotal voting by solving the limiting equilibrium condition (16) for K, as follows.

$$\frac{1 - F(\bar{q})}{\bar{q}} \left[m(\bar{q}) - \bar{q} \right] = K \tag{21}$$

This also facilitates numerical examples, as the abstention rate $F(\bar{q})$ and conditional mean $m(\bar{q})$ can easily be computed for specific distributions of expertise. The simplest example of this is a uniform distribution, for which $F(\bar{q}) = \bar{q}$ and $m(\bar{q}) = \frac{1+\bar{q}}{2}$, so that (21) reduces to

$$q^M = (K + 1) - \sqrt{K(K + 2)}. \tag{22}$$

Using the uniform distribution, Figure 3 plots the left- and right-hand sides of (16) for various levels p of partisanship.

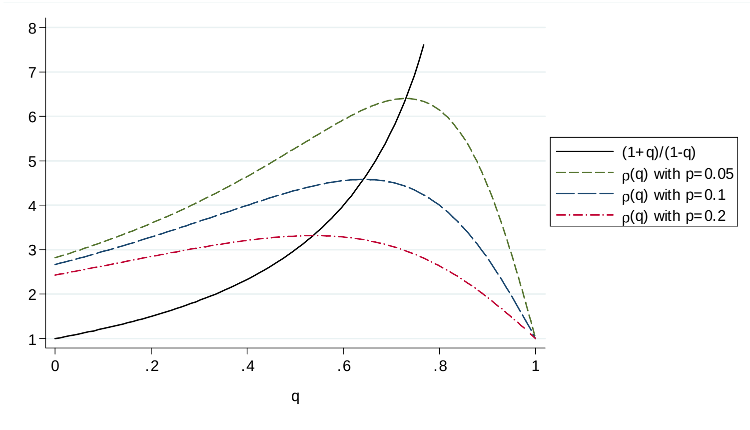

Figure 3: Left and right-hand side of equation (16) when the distribution F of expertise is uniform for various levels of partisans.

Evidently, the left-hand side of (16) is maximized precisely at the intersection of the two. Indeed, in demonstrating uniqueness, the proof of Proposition 1 shows that this must always be the case. The intuition for Proposition 1 is related to the intuition for this phenomenon. To see this, first note that $\rho(\bar{q})$ represents the likelihood ratio of a correct vote to an incorrect vote, for a randomly chosen voter. The objective of independent voters is precisely to make this ratio as large as possible, so that the policy outcome will be as close as possible to whatever is truly optimal. The right-hand side of (16) is the likelihood ratio of a correct vote to an incorrect vote for the marginal independent voter—that is, one whose expertise is right at the participation threshold. Equilibrium equates the average and the margin.[9] Equating the average and marginal likelihood ratios serves to maximize the average, just as equating the average and marginal costs of a firm's production serves to minimize the average: if the marginal voter's likelihood ratio is not as good as the average voter's, increasing the participation threshold removes votes of below-average quality, thus improving the average; if the marginal voter's likelihood ratio is better than the average voter's, raising the participation threshold removes votes of above-average quality, thus making things worse.

[9]Equivalently, as is clear from (17), equilibrium equates the vote share of the superior party with the posterior beliefs of the marginal voter.

The second part of Proposition 1 states that the marginal voter's curse is most severe when there are fewer partisans. Since partisans always vote, this implies that turnout is lower with fewer partisans, as well. With no partisans at all, average quality always exceeds marginal quality, because the marginal voter is precisely the one with the lowest expertise—except at the very top of the domain of expertise; thus, the equilibrium threshold rises all the way to $q^M = 1$. From an independent's perspective, however, adding partisans adds noise to the electoral process. For any participation threshold $\bar{q}$, therefore, a higher level of partisanship reduces the average accuracy of a vote, as Figure 3 makes clear. The accuracy of the marginal voter is unchanged, however, and strictly improves with $\bar{q}$, implying that the solution q^M to (16) is lower, as stated in the second part of Proposition 1. Since partisans always vote, this has the clear effect of raising turnout.

The last part of Proposition 1 states that improving the distribution of expertise has the effect of raising the limiting participation threshold q^M. The intuition for this merely complements that of increasing partisanship: improving expertise improves the correct-to-incorrect vote ratio for any participation threshold $\bar{q}$, so the solution to (16) is higher than before. In the case of partisanship, lowering q^M unambiguously raises voter participation. Holding fixed the distribution of expertise, increases in q^M correspond to decreases in voter turnout. If changes in q^M are the result of changes in the distribution of expertise, however, then the effect of these changes on participation are ambiguous: on one hand, raising citizens above the participation threshold increases turnout by transforming non-voters into voters, but on the other hand, raising the participation threshold lowers turnout, by transforming voters into non-voters.

5 Pivotal Voting

If $\theta = 0$ then the general model of Section 3 reduces to the benchmark case of pure pivotal voting. In that case, the policy outcome is a random variable x_w that equals 0 if $a < b$, 1 if $a > b$, and 0 or 1 with equal probability if $a = b$. A single vote for the superior party increases its probability of winning by the following amount,

$$\Pr(\mathcal{P}_+) = \frac{1}{2}\Pr(a = b) + \frac{1}{2}\Pr(a = b + 1) \tag{23}$$

which is the standard probability of being *pivotal* (event $\mathcal{P}_+$). Similarly, the probability with which an incorrect vote is pivotal (event $\mathcal{P}_-$) is given by the following.

$$\Pr(\mathcal{P}_-) = \frac{1}{2}\Pr(a = b) + \frac{1}{2}\Pr(b = a + 1) \tag{24}$$

A pivotal vote for the party with the superior policy position increases utility from zero to one (a change of 1) and a pivotal vote for the inferior party decreases utility from one to zero (a change of -1). Outside of these pivotal events, a vote does not change the policy outcome, and so does not impact utility; accordingly, the expected benefit (9) of voting reduces to the following.

$$\Delta Eu(q) = \frac{1}{2}(1 + q)\Pr(\mathcal{P}_+) - \frac{1}{2}(1 - q)\Pr(\mathcal{P}_-) \tag{25}$$

Since pivot probabilities are positive, (25) increases in q, implying that the best response to any strategy is again for a citizen to vote if his expertise is sufficiently high but abstain otherwise. In other words, as Theorem 2 states below, best-response voting can be characterized as a threshold strategy, just as in the case of pure marginal voting. Specifically, the best-response threshold $\bar{q}_P^{br}$ is given by the following.

$$\bar{q}_P^{br} = \frac{\Pr(\mathcal{P}_-) - \Pr(\mathcal{P}_+)}{\Pr(\mathcal{P}_-) + \Pr(\mathcal{P}_+)} \tag{26}$$

As noted above, $v_+ > v_-$ for any strategy in which a positive fraction of the electorate votes. Because of this, the number a of votes for the party with the superior policy position is likely to exceed the number b of votes for the opposing party. This is again good for welfare, since the party with the superior policy position is more likely to win the election by one vote than to lose by one vote, and an additional vote for the inferior party is more likely to change the election outcome than an additional vote for the superior party. In other words $\Pr(\mathcal{P}_-) > \Pr(\mathcal{P}_+)$, implying that $\bar{q}_P^{br} > 0$. For citizens with the lowest levels of expertise, then, the best response is to abstain from voting as in Feddersen and Pesendorfer (1996), to avoid the *swing voter's curse* of overturning an informed electoral decision. Thus, pure marginal voting and pure pivotal voting elicit the same type of voting behavior from citizens. As before, equilibrium existence follows from standard fixed point arguments. Also, the logic of McLennan (1998) again implies that equilibrium voter abstention improves social welfare.

Theorem 2 (Swing voter's curse) *If $\theta = 0$ then $\sigma^* \in \Sigma$ is a Bayesian Nash equilibrium only if it is a threshold strategy $\sigma_{\bar{q}_P^*}$ with $\bar{q}_P^* > 0$. Moreover, such an equilibrium exists.*

Next, we consider how the equilibrium threshold changes as n grows large for the case of pure pivotal voting (i.e., $\theta = 0$). Myerson (2000) provides a useful preliminary result, which is that pivot probabilities in large elections can be written as follows, where $h_1(n)$ and $h_2(n)$ both approach one as n grows large.

$$\Pr(\mathcal{P}_+) = \frac{e^{-n\left(\sqrt{v_+}-\sqrt{v_-}\right)^2}}{4\sqrt{n\pi\sqrt{v_+v_-}}} \frac{\sqrt{v_+}+\sqrt{v_-}}{\sqrt{v_+}} h_1(n) \tag{27}$$

$$\Pr(\mathcal{P}_-) = \frac{e^{-n\left(\sqrt{v_+}-\sqrt{v_-}\right)^2}}{4\sqrt{n\pi\sqrt{v_+v_-}}} \frac{\sqrt{v_+}+\sqrt{v_-}}{\sqrt{v_-}} h_2(n) \tag{28}$$

Using these formulas, the equilibrium condition (26) converges to the following,

$$\bar{q} = \frac{\sqrt{v_+}-\sqrt{v_-}}{\sqrt{v_+}+\sqrt{v_-}}$$

which is equivalent to the following.

$$\rho(\bar{q}) = \frac{v_+}{v_-} = \left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2 \tag{29}$$

The limit q^P of any sequence $\bar{q}_P^*(n)$ of equilibrium thresholds must be a solution to this equation.[10] As before, this can be rewritten using (20), and solved for K, as

[10] This approximation actually requires that the number of votes tend to infinity, not just the

follows, which makes it easy to compute q^P for specific example distributions.

$$\frac{1-F(\bar{q})}{\bar{q}}\left(\frac{(1+\bar{q}^2)}{2}m(\bar{q})-\bar{q}\right)=K. \tag{30}$$

Proposition 2 now states that a solution to (29) exists and, if the distribution F of expertise is well-behaved, this solution is unique.[11] As in the case of pure marginal voting, uniqueness in the limit implies that if multiple equilibria exist then they all converge to the same behavior, determined by the partisan share p and the distribution F of expertise. As before, these uniquely determine expected voter turnout and the expected margin of victory, as well, and in large elections actual turnout and margins of error converge to their expectations. Uniqueness also facilitates the derivation of comparative statics, which according to Proposition 2 match those of pure marginal voting: a higher partisan share leads to a lower q^P (i.e., higher participation) and a better-informed electorate leads to a higher q^P (i.e., lower participation).

Proposition 2 *If f is log-concave then there exists a unique q^P such that, for any sequence $\{\bar{q}_n^*\}$ of equilibrium thresholds under pure pivotal voting,* $\lim_{n\to\infty}\bar{q}_n^* = q^P$. *Moreover, q^P exhibits the following properties:*

(i) $0 < q^P < 1$

(ii) q^P strictly decreases in p

(ii) Improvements in the distribution F of expertise that satisfy the monotone likelihood ratio property increase q^P.

In stating that q^P is strictly positive, the first part of Proposition 2 implies that, for any level of partisanship, a positive fraction of the electorate abstain from voting, no matter how large the electorate grows. This was also true of pure marginal voting,

number of citizens, but this is guaranteed by the result below that $q^M < 1$ no matter what fraction of the electorate is partisan.

[11] Bagnoli and Bergstrom (2005) show that log-concavity is satisfied by many of the most standard density functions, but log-concavity is actually stronger than necessary for uniqueness: one can easily construct examples that exhibit unique equilibria but are not log-concave. The important thing, as the proof of Proposition 2 indicates, is that raising the participation threshold $\bar{q}$ should not increase the average expertise $m(\bar{q})$ of citizens above the threshold too quickly. This will be the case as long as the distribution of expertise does not include atoms, or "spikes" of probability (see also McMurray 2013).

as stated in Proposition 1. The result that q^P is also strictly less than one implies that a positive fraction of the electorate continues to vote, no matter how large the electorate grows. Under pure marginal voting, Proposition 1 states the same result for any $p > 0$ but if there are no partisans then everyone abstains in the limit but the infinitesimal fraction who are perfectly informed. Section 6 further emphasizes this difference between electoral rules.

The logic for the result that q^P decreases in p is analogous to the corresponding result for q^M: when the fraction of partisans is low, an uninformed independent worries that he will cancel the vote of a better-informed independent, but when the fraction of partisans is high, it is more likely that he is canceling the vote of a partisan; in the former case he wishes to abstain, but in the latter case he wishes to vote. Mathematically, an increase in p lowers the average vote quality for any participation threshold, and therefore the correct-to-incorrect vote ratio $\rho(\bar{q})$, which is the left-hand side of (29). Since this ratio increases in $\bar{q}$, this implies a solution that is lower than before. Similar logic underlies the last part of Proposition 2, because improving the distribution of expertise raises $\rho(\bar{q})$ for any $\bar{q}$, so the solution to (29) is higher than before.

6 Comparison

Sections 4 and 5 emphasize the similarities between the comparative static implications of the marginal voter's curse for pure marginal voting and the swing voter's curse for pure pivotal voting. Maintaining a focus on large electorates, this section now compares the levels of equilibrium voter participation under the two regimes (assuming a log-concave density of expertise, so that equilibrium behavior under either electoral system is unique). Such a comparison is surprisingly unambiguous, because of the strong similarity between the limiting equilibrium conditions (16) and (29) for proportional and pure pivotal voting.

Intuitively, it might seem that conditioning on the event a pivotal vote should have a much greater impact on behavior than conditioning on the marginal impact of a nudge in one direction or the other—especially in large elections, where a pivotal vote

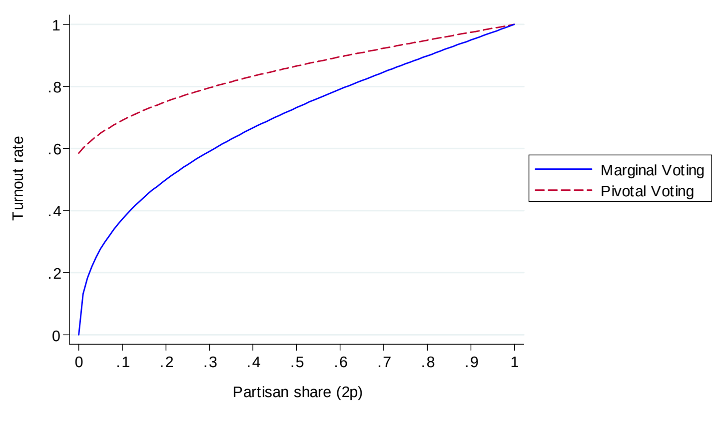

Figure 4: Turnout among independent voters as a function of the partisan share ($2p$) when the distribution F of expertise is uniform.

is so extremely rare, and where the magnitude of the nudge is vanishingly small. If so, abstention should be much higher—and turnout much lower—under pure pivotal voting than under pure marginal voting. As Theorem 3 now states, however, the opposite is true: q^M exceeds q^P, meaning that voter participation is highest under pure pivotal voting.

Theorem 3 *If f is log-concave then $q^M > q^P$.*

In stating that $q^M > q^P$, Theorem 3 leaves open the possibility that the two thresholds are quite close to one another, so that the difference is negligible. For specific distributions, this is straightforward to investigate. Suppose, for example, that F is uniform and that partisans comprise one third of the electorate (i.e. $p = \frac{1}{6}$, and therefore $K = \frac{1}{2}$). From (22), this implies that $q^M \approx 0.38$, and therefore that approximately 75% of the electorate vote in large pure marginal voting elections (i.e. 62% of independents and 100% of partisans). From (30), $q^P \approx 0.19$, implying approximately 87% turnout in large majoritarian elections (i.e. 81% among independents and 100% among partisans). Similar computations can be made for any level of partisanship, and corresponding turnout levels are displayed in Figure 4. Evidently, there is a substantial gap between q^M and q^P for all but the highest levels of partisanship.

The difference in turnout for the two electoral rules is most notable when there are no partisans. In that case, as Section 4 explains, turnout under pure marginal voting tends toward 0%, because of strategic unraveling: citizens with below-average expertise abstain, so as to not bring down the average vote quality, but then the average among those who are still voting is higher, and citizens below this average abstain, and so on. Since the marginal citizen is always below average, this unraveling continues until only the infinitesimal fraction of the most expert citizens remain. Intuitively, it might seem that turnout should unravel under pure pivotal voting, as well, because regardless of the electoral system, the marginal citizen always has less expertise than the average citizen, and so should eventually abstain, to get out of the way. The implication of Proposition 2, however, is that this does not occur: a substantial fraction of the electorate continue to vote, no matter how large the electorate grows. As McMurray (2013) explains, this reflects a trade-off between the quantity of information and the quality of information: holding the number of voters fixed, electoral outcomes are better when the expertise behind those votes is higher, which would lead people to vote, but holding expertise fixed, increasing the number of votes also improves election accuracy, just as in the classic "jury theorem" of Condorcet (1785), which gives citizens a motivation for participation. For a citizen with below-average expertise, voting decreases the average quality of information but increases the quantity. These competing considerations balance in the limit so that turnout remains substantial. In pure marginal voting, by contrast, the quality of information matters but the quantity of information does not: fixing the distribution of expertise, there is no advantage to having more votes. An expected outcome of winning by 50 to 40 is just as good as one of winning by 500 to 400, for example, because the policy outcomes $\frac{50}{90} = \frac{500}{900}$ are the same, whereas under pure pivotal voting, the latter outcome is much more robust to small deviations that would reverse the election outcome. Thus, quality considerations dominate, and turnout unravels in that case.

An alternative intuition for the discrepancy between turnout levels under majority and pure marginal voting makes use of the optimality arguments above. As Section 4 notes, the limiting equilibrium condition (16) coincides with the first-order condition for maximizing $\rho(\bar{q}) = \frac{v_+}{v_-}$—or equivalently, for maximizing $\frac{v_+}{v_+ + v_-}$. The latter is essentially an expected vote share, but in large elections this also specifies independent

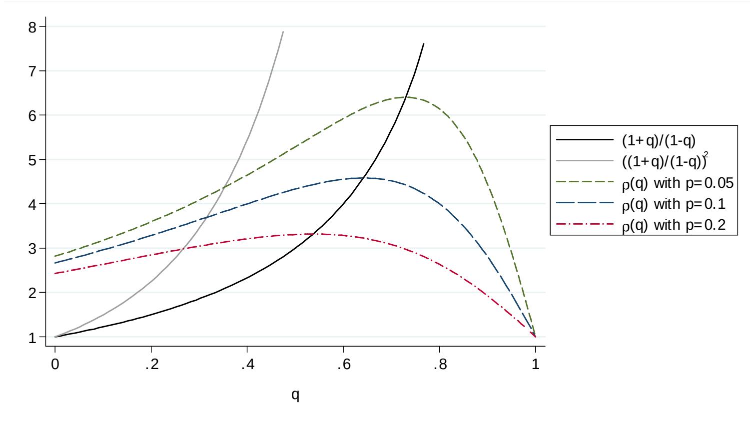

Figure 5: Left and right-hand sides of equations (16) and (29) when the distribution F of expertise is uniform for various levels of partisanship.

voters' utility, since actual vote shares converge to their expectations. In other words, $\rho(\bar{q})$ can be viewed as a monotonic transformation of voters' objective function (in large elections), and the threshold adjusts in equilibrium to the level q^M that maximizes this objective. The condition (29) for pure pivotal voting equates $\rho(\bar{q})$ to $\left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2$ instead of to $\frac{1+\bar{q}}{1-\bar{q}}$. Since the latter maximizes $\rho(\bar{q})$, the former does not, as Figure 5 illustrates for a uniform distribution. The figure also illustrates how $\left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2 > \frac{1+\bar{q}}{1-\bar{q}}$ guarantees that $q^P < q^M$, which is the crux of the proof of Theorem 3.

That q^M maximizes the objective function for pure marginal voting but q^P does not begs the question of whether q^P maximizes the objective function for pure pivotal voting. Indeed, this turns out to be the case—a feature that seems not to have been noted in existing literature on majority rule. As Myerson (2002) shows, the probability with which a Poisson variable with mean nv_- exceeds an independent Poisson variable with mean nv_+ with $v_+ > v_-$ is of order $e^{-n\left(\sqrt{v_+}-\sqrt{v_-}\right)^2}$, where the exponent $\left(\sqrt{nv_+}-\sqrt{nv_-}\right)^2$ is defined by Myerson (2000) as the *magnitude* of the event.[12] The first-order condition for minimizing this magnitude is none other than the limiting equilibrium condition (29). Thus, just as q^M maximizes utility under pure marginal voting, q^P maximizes utility under pure pivotal voting. Intuitively,

[12]In addition to measuring the magnitude of the event of a win, this quantity measures the magnitude of the event of a tie, which is the smallest deviation from the expected outcome.

what matters is not only that the expectation of a exceeds the expectation of b, but also that the variances of a and b are small relative to their expectations, so that accidents in which $b > a$ do not occur.[13] The standard deviation of a Poisson random variable is the square root of its mean, so $\sqrt{nv_+}$ and $\sqrt{nv_-}$ represent the expected numbers of correct and incorrect votes, measured in standard deviations instead of in numbers of votes.

The results that equilibrium participation thresholds for both pure marginal and pure pivotal voting maximize their respective objective functions, but generate different levels of voter participation, begs the question of which system is better for social welfare. To answer this, first let x_n^P and x_n^M denote any equilibrium policy outcomes under majority and pure marginal voting, respectively, for a population size parameter n, and let $u^P = \lim_{n\to\infty} E\left(x_n^P\right)$ and $u^M = \lim_{n\to\infty} E\left(x_n^M\right)$ denote the limits of expected utility under either regime. u^P and u^M are well-defined because, as shown above, equilibrium behavior under either electoral rule converges to the same limiting behavior as n grows large.

Intuitively, the observation above that equilibrium abstention improves welfare, together with the result of Theorem 3 that participation in large elections is higher under a majoritarian regime than under pure marginal voting, might seem to suggest that welfare should be higher under pure marginal voting than under pure pivotal voting.

Proposition 3 $u^P = 1$ *for all* $p < \frac{1}{2}$. u^M *decreases in* p, *with* $u^M = 1$ *when* $p = 0$ *and* $u^M = \frac{1}{2}$ *when* $p = \frac{1}{2}$.

The comparison here of welfare has little to do with the comparison of turnout from Theorem 3. What drives the result is that, under pure pivotal voting, A partisans and B partisans negate one another's influence, so that the majority decision is determined entirely by the behavior of independent voters, no matter how small this group is. In a large election, a majority of these almost surely identify the true state of the world. If there are no partisans then pure marginal voting delivers the same outcome in the

[13] These two considerations correspond to the considerations of quality and quantity, discussed above.

limit: as participation is limited to increasingly elite voters, the election outcome tends toward unanimity, and the policy outcome converges to the desired extreme. A positive mass of partisan votes for either side, however, bounds the policy outcome away from 0 and 1, implying some utility loss, which is increasing in p. If the domain of F were bounded such that even the maximum level of expertise were some $q_{\max} < 1$ then pure marginal voting would fall short of pure pivotal voting even for the case of no partisans, because independent voters would be unable to coordinate unanimously on the desired side.

Another way of describing the result of Proposition 3 is to say that pivotal voting leads to *full information equivalence*, as Feddersen and Pesendorfer (1996, 1999) emphasize, meaning that large elections produce the same policy outcomes that would occur if information were made perfect. The same is not true of marginal voting, because full information equivalence requires that citizens unanimously rally around the correct extreme. This is impossible unless there are no partisans *and* the best informed independent voters are essentially infallible. Otherwise, the damage caused by mistaken votes and partisan votes is irreversible.

7 General Model

If θ is strictly between 0 and 1 then the electoral rule is a hybrid of the marginal voting and pivotal voting benchmarks, as given by equation (1). In that case, the expected benefit of voting is merely the weighted average of the expected benefits derived in Sections 4 and 5.

$$\Delta Eu\left(q\right) = \frac{1+q}{2}\left[\theta\left(\Delta\lambda_{+}\right) + \left(1-\theta\right)\Pr\left(\mathcal{P}_{+}\right)\right] - \frac{1-q}{2}\left[\theta\left(\Delta\lambda_{-}\right) + \left(1-\theta\right)\Pr\left(\mathcal{P}_{-}\right)\right]$$

This difference is positive if and only if q exceeds the following threshold.

$$\bar{q}_G^{br} = \frac{\theta\left(\Delta\lambda_{-} - \Delta\lambda_{+}\right) + \left(1-\theta\right)\left[\Pr\left(\mathcal{P}_{-}\right) - \Pr\left(\mathcal{P}_{+}\right)\right]}{\theta\left(\Delta\lambda_{+} + \Delta\lambda_{-}\right) + \left(1-\theta\right)\left[\Pr\left(\mathcal{P}_{+}\right) + \Pr\left(\mathcal{P}_{-}\right)\right]} \tag{31}$$

For any p, this threshold lies strictly between (12) and (26). Since $\bar{q}_P^{br}$ and $\bar{q}_M^{br}$ are both positive for any strategy in which a positive fraction of the electorate votes, (31) is positive as well, implying that the best response for citizens with the lowest levels of expertise is to abstain. Theorems 1 and 2 make no claim of equilibrium uniqueness, but if the distribution of expertise is such that there is a unique equilibrium participation threshold for each of these extreme electoral rules then the fact that (31) is between (12) and (26) implies that any equilibrium threshold $\bar{q}_G^*$ in the general model must be strictly between $\bar{q}_P^*$ and $\bar{q}_M^*$. As in the special cases of pure marginal or pure pivotal voting, the logic of McLennan (1998) implies that equilibrium abstention is good for social welfare.

Theorem 4 *If* $0 < \theta < 1$ *then* $\sigma^* \in \Sigma$ *is a Bayesian Nash equilibrium only if it is a threshold strategy* $\sigma_{\bar{q}^*}$ *with* $\bar{q}_G^* > 0$*. Moreover, such an equilibrium exists.*

The fact that $q_G^*(n)$ lies between $\bar{q}_P^*(n)$ and $\bar{q}_M^*(n)$ implies that the limit q^G of a sequence of equilibrium thresholds under a general model lies weakly between q^P and q^M. As noted above, however, the distance between these thresholds may be rather large. The goal of the rest of the section is to generate more specific predictions about the location of q^G, relative to the other two thresholds.

Equation (31) gives the equilibrium condition for a general model and a finite n. This equation depends on the marginal changes $\Delta\lambda_+$ and $\Delta\lambda_-$ in policy associated with additional votes for the superior and inferior parties, respectively, and the probabilities $\Pr(\mathcal{P}_+)$ and $\Pr(\mathcal{P}_-)$ of such votes being pivotal. The former can be rewritten as (13) and (14), which each have one term that decreases linearly in n and another that decreases exponentially with n, and the latter can be written as (27) and (28), which decrease exponentially with n. (31) is therefore equivalent to

$$\frac{1+\bar{q}}{1-\bar{q}} = \frac{\theta\left[\frac{v_+}{nv_\tau^2} + \frac{n\left(v_-^2 - v_+^2\right) - 2v_+}{2nv_\tau^2} e^{-nv_\tau}\right] + (1-\theta)\left[\frac{e^{-n\left(\sqrt{v_+} - \sqrt{v_-}\right)^2}}{4\sqrt{n\pi\sqrt{v_+ v_-}}} \frac{\sqrt{v_+} + \sqrt{v_-}}{\sqrt{v_-}}\right]}{\theta\left[\frac{v_-}{nv_\tau^2} + \frac{n\left(v_+^2 - v_-^2\right) - 2v_-}{2nv_\tau^2} e^{-nv_\tau}\right] + (1-\theta)\left[\frac{e^{-n\left(\sqrt{v_+} - \sqrt{v_-}\right)^2}}{4\sqrt{n\pi\sqrt{v_+ v_-}}} \frac{\sqrt{v_+} + \sqrt{v_-}}{\sqrt{v_+}}\right]}$$

which reduces simply to

$$\frac{1+\bar{q}}{1-\bar{q}} = \frac{v_+}{v_-} = \rho(\bar{q}) \tag{32}$$

because exponential terms vanish more quickly than linear terms.[14]

The limit of any sequence of equilibrium thresholds $q_G^*(n)$ must be a solution to (32). By the arguments of Section 4, such a solution q^G exists and is unique. In fact, as Theorem 5 now states, it coincides exactly with q^M, because (32) is identical to the limiting equilibrium condition (16) for pure pure marginal voting. In other words, the general model allows the possibility of a discrete policy jump when parties' vote shares cross the 50% threshold, but nevertheless in equilibrium in large elections voters act as if there is no such jump. Put differently, a mixture of pure pivotal voting and pure marginal voting (i.e. $0 < \theta < 1$) elicits voting behavior in large elections that is the same as what is elicited by pure pure marginal voting (i.e. $\theta = 1$). The behavior described in Section 5 arises only in the knife-edge case of pure pivotal voting (i.e. $\theta = 0$).

Theorem 5 *If $\theta \in (0, 1]$, then $q^G = q^M$.*

The general model analyzed in this section has only a single discontinuity, at the 50% threshold. In some applications, it might be reasonable to assume additional discontinuities. In the U.S. Senate, for example, a party that obtains at least 2/3 of the power can prevent the opposing party from using filibusters to block legislative actions. In cases like this, it might be reasonable to assume that there is a large discontinuity at the 50% threshold and smaller discontinuities at the $\frac{1}{3}$ and $\frac{2}{3}$ thresholds. Another example might be Proportional Representation (PR) systems, as discussed in Section 9, where seats in the legislature are apportioned in a way that is proportional to vote shares. An ideal PR system would be continuous, like the model above, but in practice the number of seats is quite finite, so the smooth

[14]Castanheira (2003) and Faravelli, Man, and Walsh (2015) argue that this slower convergence rate of marginal voting can also help explain turnout in costly voting environments. In private value settings, pivot probabilities decrease exponentially only when the electoral split is uneven, and is exactly known: with a small amount of uncertainty regarding the exact electoral split, pivot probabilites can decrease at a linear rate instead (see Myatt 2012). The key ingredient for that result, however, is that either side might actually have the electoral advantage. Here, small uncertainty regarding the distribution of expertise or other parameters of the game would not change the result that pivot probabilities decrease exponentially with n, because every citizen with $q_i > 0$ is more likely to vote for the superior party. The average level of expertise $E(q_i|i \text{ votes})$ might not be known precisely, but is unambiguously positive.

transition from 0% to 100% of the vote share translates into a step-function of power, jumping up at discrete intervals.

The logic that pivotal events become exponentially less likely relies in no way on there being a unique pivotal event; with two or more pivotal events, each should becoming exponentially less likely as n grows large, so that in the limit, any marginal changes away from the pivotal events will be all-important, and the combined set of pivotal discontinuities will be irrelevant.

7.1 Extension: Nonlinear Policy Functions

The model of Section 3 assumes that the policy outcome $x(a,b) = \frac{a}{a+b} = \lambda_+$ in the marginal voting component of the general model is simply equal to the vote share of the superior party. This section considers a more general *policy function* $\psi : \mathbb{Z}_+^2 \to [0,1]$ and shows that the marginal voter's curse still arises as long as $x = \psi(a,b)$ satisfies the following three conditions.

Condition 1 (Monotonicity) *$\psi(a,b)$ increases in a and decreases in b.*

Condition 2 (Symmetry) *For any $a, b \in \mathbb{Z}_+$, $\psi(b,a) = 1 - \psi(a,b)$.*

Condition 3 (Underdog property) *$|\psi(a,b+1) - \psi(a,b)| - |\psi(a+1,b) - \psi(a,b)|$ has the same sign as $a - b$.*

The monotonicity condition merely states that A and B votes push the policy outcome toward 1 and 0 and therefore increase and decrease utility, respectively. Symmetry implies that reversing the numbers of votes that each party receives exactly reverses the parties' power, for example implying that $\psi(a,b) = \frac{1}{2}$ when $a = b$. The underdog property states that the impact of one additional vote for the party that has fewer votes is greater than the impact of one additional vote for the party with a majority. If ψ is a monotonic and symmetric function of the vote share $\lambda_+ = \frac{a}{a+b}$ then Condition 3 is implied if ψ is *S-shaped*—that is, convex for vote shares in $\left[0, \frac{1}{2}\right]$ and concave for vote shares in $\left[\frac{1}{2}, 1\right]$, meaning that the vote share has a diminishing marginal impact on the majority party's power. Other examples of policy functions that satisfy Conditions 1 through 3 include any "contest" function of the form

$$\psi(a,b) = \frac{a^z}{a^z + b^z}$$

with positive z. Such contest functions are S-shaped for $z > 1$ but an inverted S-shape (i.e. concave and then convex) for $z < 1$. Theorem 6 now generalizes Theorem 4 to state that a marginal voter's curse arises for any policy function that satisfies Conditions 1 through 3.

Theorem 6 *If $\psi : \mathbb{Z}_+^2 \to [0,1]$ satisfies Conditions 1 through 3 then $\sigma^* \in \Sigma$ is a Bayesian Nash equilibrium only if it is a threshold strategy $\sigma_{\bar{q}^*}$ with $\bar{q}^* > 0$. Moreover, such an equilibrium exists.*

Condition 3 highlights an important similarity between the swing voter's curse and the marginal voter's curse, which is that these curses arise from two manifestations of the same phenomenon, namely that an electoral rule exhibits an underdog property. In pivotal voting environments, a vote for the losing party is more likely to be pivotal than a vote for the winning party; in marginal voting environments, a vote for the losing party induces a larger policy shift than a vote for the winning party. Ex ante, parties in this model are symmetric: there is no winning party and no losing party. When citizens vote informatively, however, the party with the superior policy platform receives more votes (in expectation) than the inferior party. Thus, a common-value environment translates either manifestation of the underdog property into an incentive for poorly informed citizens to abstain from voting, in deference to those with better expertise.

As noted above, the underdog property—which is so central to the logic of the marginal voter's curse—is satisfied by a large class of continuous policy functions. Asymptotically, the condition actually holds for *any* increasing and differentiable function $\psi(\lambda_+)$ of the vote share $\lambda_+ = \frac{a}{a+b}$. To see this, note that, given monotonicity, the difference $|\psi(a,b+1) - \psi(a,b)| - |\psi(a+1,b) - \psi(a,b)|$ can be rewritten as follows.

$$-[\psi(a,b+1) - \psi(a,b)] - [(a+1,b) - \psi(a,b)]$$

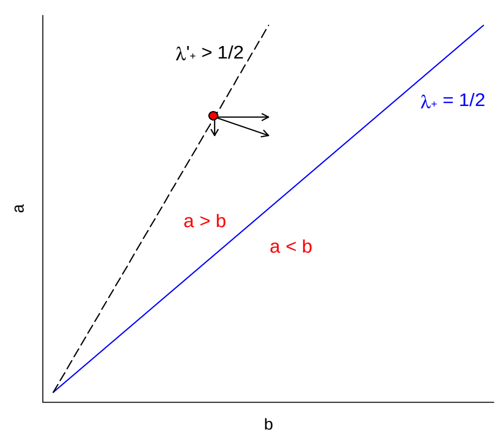

Figure 6: Isolevels of $\psi(a,b)$

As a and b grow large (keeping the vote share λ_+ fixed), this converges to the following,

$$-\frac{\partial}{\partial b}\psi(a,b) - \frac{\partial}{\partial a}\psi(a,b) = -\psi'(\lambda_+)\frac{-a}{(a+b)^2} - \psi'(\lambda_+)\frac{b}{(a+b)^2}$$

which has the same sign as $a-b$.

An intuition for the asymptotic genericity of Condition 3 can be aided by Figure 6, which plots isolevels of $\psi(a,b)$ for vote totals a and b. When ψ depends on a and b only through the vote share λ_+, sets of vote totals that generate the same policy outcome, the isolevels, correspond to rays from the origin. For example, the 45-degree line corresponds to exact ties, or a vote share of $\lambda_+ = \frac{1}{2}$. The steeper dashed line corresponds to a higher vote share $\lambda'_+ > \frac{1}{2}$.

For large a and b, the vector of changes to the policy outcome generated by increases in a and b is proportional to the gradient, which by definition is perpendicular to the isolevel. In the region where $a > b$, therefore, the policy reaction to a B vote exceeds the reaction to an A vote, while the opposite is true in the region where $a < b$.

8 Experiment

The analytical results above give sharp comparative static predictions for pure marginal and pure pivotal voting electoral systems. To test these predictions, one might hope for a natural experiment in which a single group of voters decide two issues for which the distributions of expertise are the same, but the mechanisms (e.g. voting rules or legislative bargaining protocols) that translate votes into policy outcomes differ. Such a natural experiment seems unlikely to occur, however, so we instead turn for evidence from a laboratory experiment. This presents challenges of its own: first, financial and space constraints limit the number of participants, making it difficult to test asymptotic results for large elections. Second, technical features of the model such as the Poisson population uncertainty and the continuum of possible types, while elegant and convenient for theoretical derivations, are impractical to explain to experimental subjects. To circumvent these challenges we implement a simplified version of the model above, and compute equilibrium predictions numerically. While simple relative to the model above, this preserves the key comparative static implications, as explained below.

8.1 Experimental Design and Procedures

Experiments were conducted at the Experimental Economics Laboratory at the University of Valencia (LINEEX) in November 2014, with 360 students as subjects. Subjects were randomly assigned into groups of twelve, and each group was assigned to one of six treatments, described below. Before the experiment, instructions were provided in writing and also read aloud by an instructor, and subjects answered a questionnaire to check their full understanding of the experimental design.[15] Each session consisted of fourty rounds. The instructions for each round were the same. In each of the rounds, the group of twelve was randomly divided into two subgroups of six participants, and payoffs in that round depended only on the choices of participants in the subgroup. All interactions among participants took place only via computer

[15] Instructions for treatments P25 and M25 descrived later can be found in Appendix A3.

terminals.[16] Each participant could earn between 0 and 100 points in each round, and at the end of the experiment, five periods were randomly selected for payment and points were converted to Euros at the rate of 2.5€ per 100 points.[17]

At the beginning of each round, the color of a triangle was chosen randomly to be either blue or red with equal probability. Subjects were not told the color of the triangle, but were told that the goal of the subgroup was to guess the color of the triangle. Independently, each would observe one ball (analogous to the *signal* modeled in Section 3) drawn randomly from an urn with 20 blue and red balls. With 40% probability, a participant would be designated as a *high* type H, meaning that 19 of the 20 balls in the urn would be the same color as the triangle. With 60% probability, a participant would be designated as a *low* type L, meaning that only 13 of the 20 balls would be the same color as the triangle. Individuals were told their own types, but did not know the types of the other five team members.

After observing their signals, subjects each had to take one of three actions: vote Blue, vote Red, or Abstain from voting. Regardless of which action they chose, however, they were told that their action choice might be replaced at random, by the choice of a computer: with 10% probability, their vote choice would be changed to Abstain.[18] With some probability $2p \in \{0\%, 25\%, 50\%\}$ the voting choice would be replaced with a partisan vote—Blue or Red with equal probability (leaving a vote unchanged with probability $.9-2p$). Replacements of votes were determined independently across subjects. The partisanship parameter $2p$ was one of the two treatment variables of the experiment. The second was the *voting rule*, which determined subjects' payoff as a function of the team's votes. Under pure pivotal voting (P), the team scored 100 points if the number of votes for the color of the triangle exceeded the number of votes for the other color, 50 points in the event of a tie, and 0 points

[16] The experiment was programmed and implemented with z-Tree (Fischbacher 2007).

[17] In total, subjects earned an average of 14.21€, including a show-up fee of 4€. Each experimental session lasted approximately an hour.

[18] This form of population uncertainty follows Feddersen and Pesendorfer (1996). With a known number of voters, the swing voter's curse depends heavily on whether that number is even or odd. If it is odd, for example, there is always an equilibrium with full participation, because a vote is then pivotal only if the rest of the electorate is evenly split. In that case, an individual infers no information beyond his or her own signal the event of a pivotal vote, and therefore has a strict incentive to vote.

otherwise. Under pure marginal voting (M), the team's score was the percentage of non-abstention votes that had the same color as the triangle—or 50 points, if everyone abstained. At the end of each round, each subject was told the true color of the triangle, their own original and final vote, and the total numbers of final Blue votes, Red votes, and abstentions in their team (though they were not told whether these were the intended votes of the other participants, or computer overrides). In pure marginal voting treatments, they also observed the percentage of votes that matched the color of the triangle; in pure pivotal voting treatments, they instead were told whether the color of the Triangle received more, equal, or fewer votes than the other color.

The six possible combinations of the partisanship share and the electoral rule constituted the six treatments for the experiment. Five twelve-member groups were assigned to each treatment, producing 30 independent observations of voting behavior. For each treatment, equilibrium behavior can be derived numerically. Empirical voting behavior for voter types in each of the six treatments is discussed in the following section.

8.2 Experimental Results

Figure 7 displays voter abstention for high- and low-type voters, across the six treatments. The most basic behavioral feature evident here is that voters respond to both the swing voter's curse and the marginal voter's curse: under both electoral rules, a positive fraction of subjects seek to improve the election outcome by *not* voting. Moreover, consistent with the predictions of Propositions 1 and 2, subjects with higher levels of expertise were more willing to vote. Indeed, the payoffs used in the experiment are such that high types should never abstain, and consistent with this, the level of abstention is quite low across treatments, ranging from 0.3% to 3.2%.

The payoffs of this experiment were chosen to make as stark equilibrium predictions as possible. Under pure pivotal voting, the equilibrium incentive for low types under pure pivotal voting is to abstain if there are no partisans, but vote if the partisan share is 25% or 50%. Under pure marginal voting, low types should abstain if the partisan share is 0% or 25% but vote if the partisan share is 50%. Figure 7

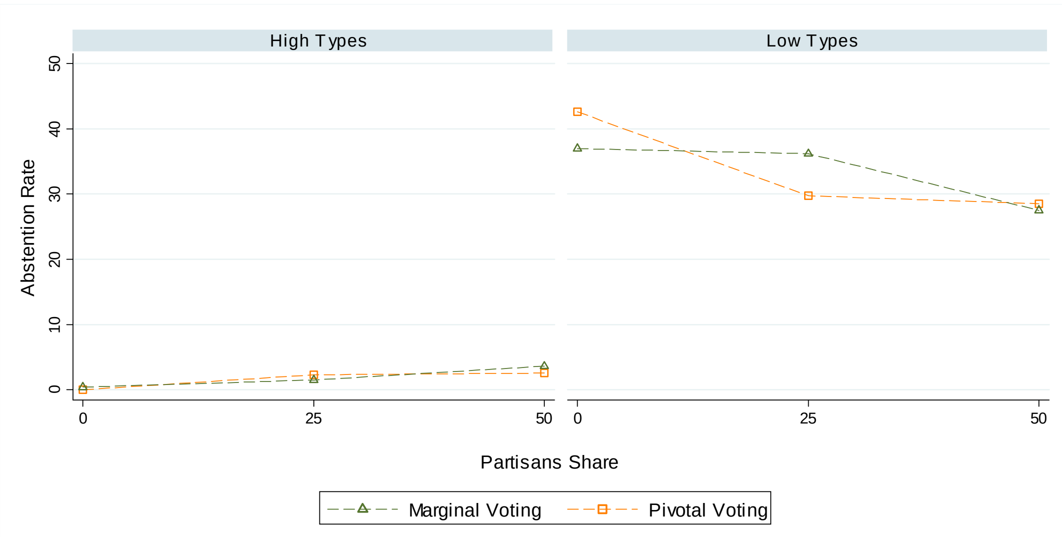

Figure 7: Observed abstention for each treatment, by voter type, averaged across round and groups.

makes clear that empirical levels of abstention do not closely match the equilibrium predictions, but do follow the same pattern, dropping dramatically (from 42.6% to 29.7%) between treatments M0 and M25 but then staying about the same (28.5%) in M50, and staying about the same (37.0% and 36.2%) in P0 and P25 but then dropping dramatically (to 27.5%) in P50. Consistent with Theorem 3, then, abstention with moderate levels of partisanship is higher under pure marginal voting than pure pivotal voting.

The statistical significance of these patterns can be tested by using a random effects GLS regression of the frequency of abstention on dummies for each possible combination of voter type and treatment (excluding high types in treatment M0 as the reference category). Coefficient estimates for such a regression are presented in Table 1 in the appendix.[19] Using this regression, the hypothesis that abstention levels are the same for all three partisan levels can rejected ($\chi^2 = 58.75$, $p < 0.001$) for pure pivotal voting but is marginally insignificant ($\chi^2 = 2.17$, $p = 0.338$) for pure marginal

[19] To ease interpretation of the coefficients, this regression specification does not include temporal variables, and therefore ignores potential evolution in behavior throughout the experiment. However, none of the main results from the basic regression analysis are sensitive to the inclusion of temporal variables.

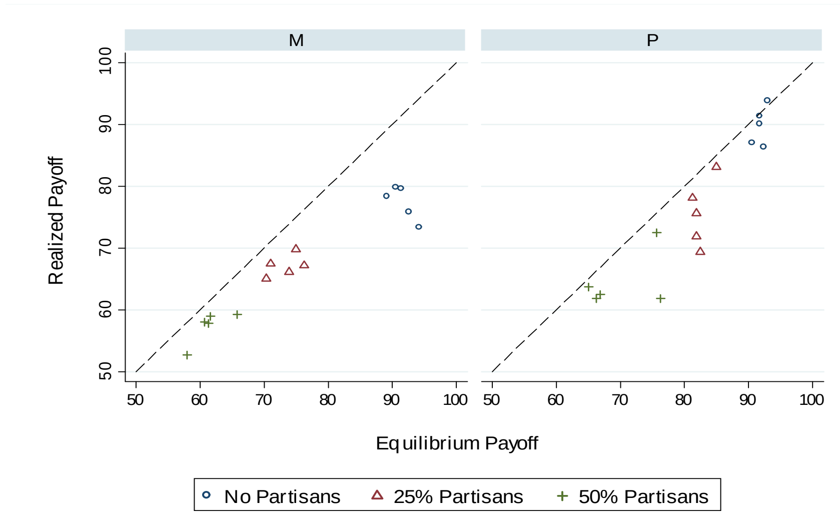

Figure 8: Realized payoff vs equilibrium payoff in each independent group.

voting. Pure pivotal and pure marginal voting generate abstention levels that do not differ significantly when the level of partisans is 0% or 50% ($\chi^2_1 = 0.43$, $p = 0.514$ and $\chi^2_1 = 0.04$, $p = 0.844$ respectively), but pure marginal voting generates significantly higher abstention when the partisan share is 25% ($\chi^2_1 = 16.76$, $p < 0.001$).

In addition to comparing behavior across electoral rules, the experiment above allows us to evaluate voter welfare. For each treatment, the average payoff across groups is displayed in Figure 8. Consistent with the prediction of Theorem 3, welfare is lower under pure marginal voting than under pure pivotal voting, for any level of partisanship ($\chi^2 = 93.83$, $p < 0.001$ for 0% partisan share, $\chi^2 = 9.71$, $p < 0.01$ for 25% partisan share, and $\chi^2 = 13.96$, $p < 0.001$ for 50% partisan share), because the mal-effects of partisan behavior and non-partisan mistakes are harder to negate.[20] Figure 8 also compares realized utility with the utility subjects would have obtained if they had followed equilibrium predictions. Across treatments, actual utility is lower than equilibrium utility, although the average loss is only 8.3%, which is relatively minor. The loss is slightly higher under pure marginal voting than pure pivotal voting (10.3% as opposed to 6.3%), but is not as severe when the partisan share is

[20]These test results come from a regression similar to that of Table 1, but with the payoff as the independent variable, rather than abstention.

highest (since voter behavior tends not to influence utility in that case).

With payoffs as they are, subjects should never vote against their private signals: they should either vote in line with their signals or abstain. Empirically, 4.5% of high-type voters and 16.5% of low-type voters did vote against their signals.[21] Additionally, these frequencies seems to increase with the level of partisans. This is consistent with models of quantal response equilibrium, where mistakes are less prevalent when the payoff difference across actions is smaller.[22]

9 Discussion

This paper has analyzed a common-value model of voter participation in which the full impact of a vote includes the marginal effect of adding or subtracting one from the winning party's margin of victory in addition to the standard pivotal impact of pushing one party's vote share above the 50% threshold. Just as pivotal voting considerations generate a swing voter's curse, marginal voting considerations generate a marginal voter's curse that dissuades citizens from voting unless they are sufficiently confident in their policy opinions. Both curses can be viewed as manifestations of an underdog effect, but the marginal voter's curse is actually stronger. Moreover, pivotal voting considerations become irrelevant in large elections, so the marginal voter's curse alone determines equilibrium levels of voter participation.

Across the world, plurality rule is probably the most prevalent electoral system, which explains the emphasis on pivotal voting in extant literature. Increasingly, however, nations and other electoral bodies have adopted alternative electoral rules. In legislative elections, the most common alternative to majority rule is Proportional Representation (PR).[23] In an ideal PR system, a party that receives 37% of the

[21] This anomaly has been found systematically in experimental studies on information aggregation. See, for instance, Bouton, Castanheira and Llorente-Saguer (2015) or Bouton, Llorente-Saguer and Malherbe (2016).

[22] See Guarnaschelli, McKelvey and Palfrey (2000) and Holt and Goeree (2005) for applications of QRE to voting.

[23] According to the ACE Electoral Knowledge Network, Proportional Representation is now used for national legislative elections in 53.3% of countries, including much of Europe and Latin America. See http://aceproject.org/epic-en/CDTable?question=ES005&set_language=en (accessed 9/1/2015).

popular vote receives 37% of the seats in parliament, and thus (depending on the bargaining protocol) 37% of the power in determining policy. Accordingly, efforts to study pure marginal voting formally have often modeled policy outcomes as linear (or at least continuous) functions of parties' vote shares.[24] As a practical matter, of course, this is simplistic, as actual legislatures have limited numbers of seats, so PR systems cannot match legislative seats to vote shares precisely.[25] To the extent that the continuous model approximates an actual PR electoral system, however, the analysis above can be applied to this increasingly popular institution.

With this application in mind, the main result of the analysis above implies that, even though the swing voter's curse does not apply, citizens who lack information should tend to abstain from voting or to cast incomplete ballots in PR elections just as they do in majoritarian elections. This is intuitively appealing, since the basic logic of abstaining in deference to those with better information does not seem highly sensitive to the institutional details of the voting rule. Empirically, Sobbrio and Navarra (2010) indeed find that voter participation in PR elections is correlated with voter information, just as it is in majoritarian systems.[26] A clear example of roll off in PR elections is the Peruvian national election. In 2011, for example, PR elections for the Peruvian Congress and for the Andean Parliament were conducted on the same day as the first round of a presidential election.[27] Out of 20 million registered voters 89% turned out to vote. Of these, only 88% cast valid votes in the presidential election, 77% cast valid votes in the PR elections for Congress, and 61% cast valid votes for the Andean Parliament. Evidently, the incentive to roll off is just as robust in PR elections as it is in the presidential election, or in the decision of whether to turn out at all.[28] Moreover, information seems plausibly to be the most important

[24]See Ortuño-Ortín (1997), Llavador (2006, 2008), Iaryczower and Mattozzi (2013), Herrera, Morelli and Palfrey (2014), Kartal (2014a and 2014b), Faravelli, Man, and Walsch (2015), Faravelli and Sanchez-Pages (2015), Herrera, Morelli, and Nunnari (2015), and Matakos, Troumpounis, and Xefteris (2015a, 2015b).

[25]These analyses also typically only include two political parties, whereas legislatures that use PR frequently have more than two major political parties.

[26]This is also consistent with Riambau (2015), which studies abstention for informational reasons in legislative elections in Austria, Germany, and Israel.

[27]Evidence of roll off in other countries is often less clear-cut, only because parliamentary elections are often conducted separately from other elections, or have different voter elligibility requirements.

[28]That abstention is higher in the PR elections than in the majoritarian election is also consistent

factor that differs across the three elections.

Normatively, proportional representation is often lauded for its protection of minorities.[29] Consider, for example, an electorate comprised solely of partisans, with no independents: under majority rule, the policy outcome corresponds with the most-preferred policy of the larger group of partisans, but this is also the least-preferred policy of the smaller group of partisans. As conceptualized here, PR instead produces a moderate compromise (i.e. the weighted average of the partisan groups' ideal policies, with weights given by vote shares). However, the analysis above suggests that this minority protection may come at the cost of informational efficiency: whereas majority rule satisfies full information equivalence and almost surely selects the party that independents truly prefer, PR suffers from a dilution problem, such that the damage caused by mistaken opinions and partisan votes cannot be undone.

The superiority of majority rule is partly a consequence of the assumption that the optimal policy α lies at one of the extremes of the policy space: if interior policies could also be optimal then there may be informational benefits to compromise in addition to the benefit of minority protection. Thus, a tractable model in which the optimal policy can lie in the interior of the policy interval would be a useful direction for future extension. In addition to altering welfare implications, this might mitigate the marginal voter's curse: as noted above, a binary state gives citizens a strong reason to abstain, to be as nearly unanimous as possible.[30] This does not seem that it would affect the basic logic of the marginal voter's curse, however: informative voting should produce vote shares that lean in the direction of the optimal policy, wherever it may be, and the underdog principle should then imply that opposing votes will have more influence than additional votes of support, so a citizen who is uncertain where the optimum lies should still abstain in deference to those who know more.

with the prediction of Theorem 3, although this is not a perfect comparison, because the runoff system for the presidential election differs slightly from the pivotal voting formulated above, and because the distribution of expertise may well differ across elections.

[29] See, for example, Lijphart (1999).

[30] An interior optimum may also balance the two parties' vote shares such that, as in Myatt (2012), pivotal voting incentives shrink to zero at the same rate (i.e., $\frac{1}{n}$) as marginal voting incentives, rather than exponentially. If so, both motivations will remain relevant in large elections.

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Appendices

Appendix A: Proofs

Proof of Theorem 1. As explained in the text, σ is an equilibrium only if it is a quality threshold strategy, with threshold given by (12). Conditional on the total number $\tau = a + b$ of votes, the precise numbers of votes a and b for either side follow binomial distributions, with probability parameters $\frac{v_+}{v_\tau}$ and $\frac{v_-}{v_\tau}$. As long as not all citizens abstain, (4) exceeds (5), implying that the distribution of the number a of correct votes first-order stochastically dominates the distribution of the number b of incorrect votes. Keeping the total $a + b$ fixed, however, $\frac{a+1}{a+b+1} - \frac{a}{a+b}$ decreases in a and increases in b, implying that the distribution of $\Delta\lambda_-$ first-order stochastically dominates the distribution of $\Delta\lambda_+$. In particular, then, $E_{a,b}(\Delta\lambda_-|a + b = \tau) > E_{a,b}(\Delta\lambda_+|a + b = \tau)$ and, taking expectations over τ, $E_{a,b}(\Delta\lambda_-) > E_{a,b}(\Delta\lambda_+)$. This implies that (12) is positive.

Equilibrium existence follows because the best response to a threshold strategy $\sigma_{\bar{q}}$ is another $\sigma_{\bar{q}_M^{br}}$. Since the best-response threshold $\bar{q}_M^{br}$ depends continuously on $\bar{q}$ and the set $[0, 1]$ of possible thresholds is compact, a fixed point $\bar{q}^* = \bar{q}_M^{br}(\bar{q}^*)$ exists by Brouwer's theorem, that characterizes a voting strategy $\sigma_{\bar{q}^*}$ that is its own best response, and therefore a Bayesian Nash equilibrium of the game. ■

Proof of Lemma 1. The expected vote share of the superior party can be written as

$$\begin{aligned} E_{a,b}\left[\lambda_+(a,b)\right] &= \sum_{a=0}^{\infty}\sum_{b=0}^{\infty} e^{-n_\tau}\frac{n_+^a}{a!}\frac{n_-^b}{b!}u\left[x(a,b)\right] \\ &= e^{-n_\tau}\sum_{a=1}^{\infty}\sum_{b=0}^{\infty}\frac{n_+^a}{a!}\frac{n_-^b}{b!}\left(\frac{a}{a+b}\right)+\frac{1}{2}e^{-n_\tau} \\ &= e^{-n_\tau}\sum_{a=1}^{\infty}\left[\frac{1}{(a-1)!}\frac{n_+^a}{n_-^a}\sum_{b=0}^{\infty}\frac{n_-^{a+b}}{b!\,(a+b)}\right]+\frac{1}{2}e^{-n_\tau} \end{aligned}$$

where the second equality follows because $x(0,0)=\frac{1}{2}$. Differentiating and integrating the innermost summand as follows,

$$\begin{aligned} \sum_{b=0}^{\infty}\frac{n_-^{a+b}}{b!\,(a+b)} &= \sum_{b=0}^{\infty}\int_0^{n_-}\frac{d}{dt}\left(\frac{t^{a+b}}{b!}\frac{1}{a+b}\right)dt \\ &= \int_0^{n_-}\sum_{b=0}^{\infty}\left(\frac{t^{a+b-1}}{b!}\right)dt \\ &= \int_0^{n_-} t^{a-1}e^t dt, \end{aligned}$$

this reduces further to the following.

$$\begin{aligned} E_{a,b}\left[\lambda_+(a,b)\right] &= e^{-n_\tau}\frac{n_+}{n_-}\int_0^{n_-}\sum_{a=1}^{\infty}\frac{\left(\frac{n_+}{n_-}t\right)^{a-1}}{(a-1)!}e^t dt+\frac{1}{2}e^{-n_\tau} \\ &= e^{-n_\tau}\frac{n_+}{n_-}\int_0^{n_-} e^{\left(\frac{n_+}{n_-}t\right)}e^t dt+\frac{1}{2}e^{-n_\tau} \\ &= e^{-n_\tau}\frac{n_+}{n_-}\int_0^{n_-} e^{\left(\frac{n_\tau}{n_-}\right)t}dt+\frac{1}{2}e^{-n_\tau} \\ &= e^{-n_\tau}\frac{n_+}{n_\tau}\left(e^{n_\tau}-1\right)+\frac{1}{2}e^{-n_\tau} \\ &= \frac{n_+}{n_\tau}+\frac{n_- - n_+}{2n_\tau}e^{-n_\tau} \end{aligned} \tag{33}$$

If a citizen votes for the party with the superior platform, this increases the expected vote share to

$$\begin{aligned}
E_{a,b}\left[\lambda_{+}\left(a+1,b\right)\right] &= \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}e^{-n_{\tau}}\left(\frac{n_{+}^{a}}{a!}\right)\left(\frac{n_{-}^{b}}{b!}\right)\frac{a+1}{a+b+1} \\
&= e^{-n_{\tau}}\sum_{a=0}^{\infty}\left[\frac{a+1}{a!}\frac{n_{+}^{a}}{n_{-}^{a+1}}\sum_{b=0}^{\infty}\frac{n_{-}^{a+b+1}}{b!\left(a+b+1\right)}\right]
\end{aligned}$$

which, differentiating and integrating as before, reduces further as follows.

$$\begin{aligned}
&= e^{-n_{\tau}}\sum_{a=0}^{\infty}\frac{a+1}{a!}\frac{n_{+}^{a}}{n_{-}^{a+1}}\int_{0}^{n_{-}}t^{a}e^{t}dt \\
&= e^{-n_{\tau}}\int_{0}^{n_{-}}\sum_{a=0}^{\infty}\left(\frac{a}{a!}\frac{n_{+}^{a}}{n_{-}^{a+1}}t^{a}+\frac{1}{a!}\frac{n_{+}^{a}}{n_{-}^{a+1}}t^{a}\right)e^{t}dt. \\
&= e^{-n_{\tau}}\int_{0}^{n_{-}}\left[\frac{n_{+}}{n_{-}^{2}}t\sum_{a=1}^{\infty}\frac{\left(\frac{n_{+}}{n_{-}}t\right)^{a-1}}{(a-1)!}+\frac{1}{n_{-}}\sum_{a=0}^{\infty}\frac{\left(\frac{n_{+}}{n_{-}}t\right)^{a}}{a!}\right]e^{t}dt \\
&= e^{-n_{\tau}}\int_{0}^{n_{-}}\left[\frac{n_{+}}{n_{-}^{2}}te^{\left(\frac{n_{+}}{n_{-}}t\right)}+\frac{1}{n_{-}}e^{\left(\frac{n_{+}}{n_{-}}t\right)}\right]e^{t}dt \\
&= \frac{n_{+}}{n_{-}^{2}}e^{-n_{\tau}}\int_{0}^{n_{-}}te^{\left(\frac{n_{\tau}}{n_{-}}\right)t}dt+\frac{1}{n_{-}}e^{-n_{\tau}}\int_{0}^{n_{-}}e^{\left(\frac{n_{\tau}}{n_{-}}\right)t}dt
\end{aligned}$$

Integrating by parts, this reduces to the following.

$$\begin{aligned}
& \frac{n_{+}}{n_{-}^{2}}e^{-n_{\tau}}\left[\frac{n_{-}^{2}e^{n_{\tau}}}{n_{\tau}}-0-\frac{n_{-}}{n_{\tau}}\int_{0}^{n_{-}}e^{\left(\frac{n_{\tau}}{n_{-}}\right)t}\right]+\frac{1}{n_{\tau}}e^{-n_{\tau}}\left(e^{n_{\tau}}-1\right) \\
=\ & \frac{n_{+}}{n_{-}^{2}}e^{-n_{\tau}}\left[\frac{n_{-}^{2}e^{n_{\tau}}}{n_{\tau}}-\left(\frac{n_{-}}{n_{\tau}}\right)^{2}\left(e^{n_{\tau}}-1\right)\right]+\frac{1}{n_{\tau}}\left(1-e^{-n_{\tau}}\right) \\
=\ & \left(\frac{n_{+}}{n_{\tau}}-\frac{n_{+}}{n_{\tau}^{2}}+\frac{1}{n_{\tau}}\right)+\left(\frac{n_{+}}{n_{\tau}^{2}}-\frac{1}{n_{\tau}}\right)e^{-n_{\tau}} \\
=\ & \frac{n_{+}\left(n_{\tau}\right)+n_{-}}{n_{\tau}^{2}}-\frac{n_{-}}{n_{\tau}^{2}}e^{-n_{\tau}}
\end{aligned} \tag{34}$$

The difference $E_{a,b}\left(\Delta\lambda_{+}\right)$ between (33) and (34) is then given by

$$E_{a,b}\left(\Delta\lambda_{+}\right)=\frac{n_{-}}{n_{\tau}^{2}}+\frac{n_{+}^{2}-n_{-}^{2}-2n_{-}}{2n_{\tau}^{2}}e^{-nv_{\tau}}$$

which is equivalent to (13). The difference $E_{a,b}\left(\Delta\lambda_{-}\right)$ is given by (14), by an analogous derivation. ■

Proof of Proposition 1. The left- and right-hand sides of the equilibrium condition (15) are continuous in n and in $\bar{q}$ (through the continuous functions v_+, v_-, and v_τ) so a sequence $\bar{q}_M^*(n)$ of solutions to (15) must converge to a solution of the limit equilibrium condition (15). If $\lim_{n\to\infty} v_\tau[\bar{q}_M^*(n)]$ is positive then the limit of (15) is given by (16), because the terms in parentheses in both the numerator and denominator of the right-hand side of (15) are bounded in absolute value by $n+2$. The limit of e^{-nv_τ} can be positive only if $\lim_{n\to\infty} v_\tau = 0$, implying that v_+ and v_- both converge to zero. But (15) converges to (16) in that case, as well.

As $\bar{q}$ increases from zero to one, the right-hand side of (16) increases from one to infinity. For any p, differentiating (18) and (19) with respect to $\bar{q}$ yields

$$\begin{aligned} v'_+ &= -\frac{I}{2}(1+\bar{q}) f(\bar{q}) \\ v'_- &= -\frac{I}{2}(1-\bar{q}) f(\bar{q}) \end{aligned}$$

and differentiating (20) therefore yields

$$\rho'(\bar{q}) = \frac{v'_+ v_- - v_+ v'_-}{v_-^2} = \frac{I}{2} f(\bar{q}) \frac{-(1+\bar{q}) v_- + (1-\bar{q}) v_+}{v_-^2}$$

which is positive if and only if the left-hand side of (16) exceeds the right-hand side. In other words, any solution to (16) also constitutes the unique maximum of $\rho(\bar{q})$; clearly, there is exactly one such solution.

If $p=0$ then (20) reduces to $\rho(\bar{q}) = \frac{1+m(\bar{q})}{1-m(\bar{q})}$ which is increasing for all $\bar{q}$, implying that the limit of equilibrium behavior is the corner solution $q^M = 1$. If $p>0$ then $\rho(0) > 1$ and $\rho(1) = 1$, so the solution q^M is strictly between 0 and 1, as claimed in (i). (20) also decreases in K (and therefore in p) for all $\bar{q}$, implying that the solution q^M to (16) strictly decreases in K (and therefore in p), as claimed in (ii).

To show claim (iii), first rewrite (20) as

$$\rho(\bar{q}) = \frac{K + \int_{\bar{q}}^1 (1+q) f(q)\,dq}{K + \int_{\bar{q}}^1 (1-q) f(q)\,dq} = \frac{\int_0^1 \Gamma_+(q) f(q)\,dq}{\int_0^1 \Gamma_-(q) f(q)\,dq}$$

in terms of $\Gamma_+(q) = K + I_{\bar{q}}(1+q)$ and $\Gamma_-(q) = K + I_{\bar{q}}(1-q)$, where $I_{\bar{q}}$ is the indicator function of the interval $[\bar{q}, 1]$. For any K, $\Gamma_+(q)$ and $\Gamma_-(q)$ are non-negative and respectively non-decreasing and non-increasing functions of q.

With this notation, consider a distribution g such that $\frac{g(q)}{f(q)}$ is increasing, thus satisfying the monotone likelihood ratio property MLRP. For any K and any $\bar{q} \in [0,1)$, we will show that $\rho(\bar{q})$ is higher under g than under f, implying that the solution q^M to (16) is lower under g than under f. That is, we will show the following,

$$\frac{\int_0^1 \Gamma_+(q)\, g(q)\, dq}{\int_0^1 \Gamma_-(q)\, g(q)\, dq} > \frac{\int_0^1 \Gamma_+(q)\, f(q)\, dq}{\int_0^1 \Gamma_-(q)\, f(q)\, dq}$$

which is equivalent to the following.

$$\int_0^1 \int_0^1 \Gamma_+(q)\, \Gamma_-\left(q'\right) f\left(q'\right) g(q)\, dq dq' > \int_0^1 \int_0^1 \Gamma_+(q)\, \Gamma_-\left(q'\right) f(q)\, g\left(q'\right) dq dq'$$

Equivalently, we will show that the expression

$$\Gamma = \int_0^1 \int_0^1 \Gamma_+(q)\, \Gamma_-\left(q'\right) \left[f(q)\, g\left(q'\right) - f\left(q'\right) g(q)\right] dq dq' \tag{35}$$

is positive.

Splitting the domain of integration into two symmetric parts produces the following.

$$\begin{aligned}\Gamma &= \iint\limits_{q>\tilde{q}} \Gamma_+(q)\, \Gamma_-(\tilde{q}) \left[f(\tilde{q})\, g(q) - f(q)\, g(\tilde{q})\right] dq d\tilde{q} \\ &\quad + \iint\limits_{\tilde{q}>q} \Gamma_+(q)\, \Gamma_-(\tilde{q}) \left[f(\tilde{q})\, g(q) - f(q)\, g(\tilde{q})\right] dq d\tilde{q}\end{aligned}$$

Reversing the labels of q and $\tilde{q}$ in the second double integral and collecting similar terms, this is equivalent to

$$\begin{aligned}\Gamma &= \iint\limits_{q>\tilde{q}} \left[\Gamma_+(q)\, \Gamma_-(\tilde{q}) - \Gamma_+(\tilde{q})\, \Gamma_-(q)\right] \left[f(\tilde{q})\, g(q) - f(q)\, g(\tilde{q})\right] dq d\tilde{q} \\ &= \iint\limits_{q>\tilde{q}} \Gamma_-(q)\, \Gamma_-(\tilde{q}) \left[\frac{\Gamma_+(q)}{\Gamma_-(q)} - \frac{\Gamma_+(\tilde{q})}{\Gamma_-(\tilde{q})}\right] f(q)\, f(\tilde{q}) \left[\frac{g(q)}{f(q)} - \frac{g(\tilde{q})}{f(\tilde{q})}\right] dq d\tilde{q}\end{aligned}$$

This expression is positive, as desired, because Γ_+ and $\frac{g(q)}{f(q)}$ are non-decreasing in q and Γ_- is non-increasing in q. ■

Proof of Theorem 2. As argued in the text, σ is an equilibrium only if it is a quality threshold strategy, with threshold given by (26). Conditional on the total number $a+b$ of votes, the precise numbers of votes a and b for either side follow binomial distributions, with probability parameters $\frac{v_+}{v_\tau}$ and $\frac{v_-}{v_\tau}$. As long as not all citizens abstain, (4) exceeds (5), implying from (6) that $\Pr(a = k+1, b = k)$ exceeds $\Pr(a = k, b = k+1)$ for any k and, taking averages over k, that $\Pr(a=b+1)$ exceeds $\Pr(a=b)$. (24) therefore exceeds (23), implying that (26) is positive.

Since the best response to a threshold strategy $\sigma_{\bar{q}}$ is another threshold strategy $\sigma_{\bar{q}_P^{br}}$, where the best-response threshold $\bar{q}_P^{br}$ depends continuously on $\bar{q}$ and the set $[0,1]$ of possible thresholds is compact, a fixed point $\bar{q}^* = \bar{q}_P^{br}(\bar{q}^*)$ exists by Brouwer's theorem, that characterizes a voting strategy $\sigma_{\bar{q}^*}$ that is its own best response, and therefore a Bayesian Nash equilibrium of the game. ■

Proof of Proposition 2. The left-hand side of (30) approaches infinity as $\bar{q}$ approaches 0 and approaches zero as $\bar{q}$ approaches 1. $\bar{q}=1$ is therefore a solution to (30) if and only if $K=0$. In that case, however, Theorem 3 of McMurray (2013) implies that no sequence of equilibrium thresholds can converge to one, because the assumption that f is log-concave implies that $\lim_{q\to 1}\frac{f'(q)}{f(q)} < \infty$.[31] Moreover, Theorem 4 of McMurray (2013) implies that there is a unique solution q_0^P to (30) in the open interval $(0,1)$. This implies that the left-hand side of (30) is positive on the interval $\left[0, q_0^P\right)$ and negative on the interval $\left(q_0^P, 1\right)$.

When the left-hand side of (30) is positive, it is strictly decreasing in $\bar{q}$. To see this, differentiate the left-hand side of (30) to obtain the following.

$$-f(\bar{q})\left(\frac{1+\bar{q}^2}{2}\frac{m(\bar{q})}{\bar{q}}-1\right)+\frac{1-F(\bar{q})}{2}\left(-\frac{1+q^2}{q^2}m(\bar{q})+\frac{1+q^2}{q}m'(\bar{q})\right) \tag{36}$$

Since f is assumed to be log-concave, Lemma 2 of Bagnoli and Bergstrom (2005) states that $E(q-\bar{q}|q\geq\bar{q})$ decreases with $\bar{q}$ or, equivalently, that $m'(\bar{q})<1$, implying that the second term in (36) is negative. If the left-hand side of (30) is positive then the first term of (36) is negative, as well.

That the left-hand side of (30) is decreasing whenever it is positive implies a unique solution q^P to (30) for any partisanship ratio K, thus establishing (i). This also makes clear that if K increases (or, equivalently, if p increases) then q^P decreases, thus establishing (ii). The proof of (iii) is then analogous to the proof of part (iii) of Proposition 1. ■

Proof of Theorem 3. As the proofs of Propositions 1 and 2 show, $\rho(\bar{q})$ exceeds the right-hand side of (16) if and only if $\bar{q}<q^M$ and exceeds the right-hand side of (29) if and only if $\bar{q}<q^P$. The right-hand side of (29) exceeds the right-hand side of (16) for any $\bar{q}>0$ (and when $\bar{q}=0$ both equal one, whereas $\rho(0)>1$), however, so in particular $\rho\left(q^P\right)$ exceeds the right-hand side of (16), implying that $q^P<q^M$. ■

Proof of Proposition 3. As n grows large, equilibrium strategies converge pointwise to σ_{q^P} under pure pivotal voting and to σ_{q^M} under pure marginal voting. In either regime,

[31] As that paper notes, this condition merely rules out electorates that are arbitrarily close to being perfectly informed, and is sufficient for the result but not necessary.

actual vote shares converge in probability to expected vote shares. That is, $\frac{a}{b} \rightarrow_p \frac{nv_+}{nv_-} = \frac{v_+}{v_-} = \rho$. Under pure pivotal voting, expected utility is given by $\Pr\left(\frac{a}{b} > 1\right)$, which therefore converges to $u^P = 1$ since $\frac{a}{b}$ converges to $\rho\left(q^P\right) > 1$. This logic is valid for any $p < 1$.[32] Under pure marginal voting, utility is given by $\frac{a}{a+b}$, which converges in probability to $u^M = \frac{nv_+}{nv_+ + nv_-} = \frac{\rho\left(q^M\right)}{\rho\left(q^M\right)+1}$. This decreases in p since it increases in $\rho\left(q^M\right)$, which decreases in p (as shown in Section 4). As that section explains, $p = 0$ implies that $\lim_{\bar{q} \to q^M} \rho\left(\bar{q}\right) = \infty$, therefore implying that $u^M = 1$. If $p = 1$ then from (18) and (19) it is also clear that $\rho\left(\bar{q}\right) = 1$ for any $\bar{q}$, implying that $u^M = \frac{1}{2}$ in that case. ■

Proof of Theorem 4. That a Bayesian Nash equilibrium must be a threshold strategy $\sigma_{\bar{q}^*}$ with $\bar{q}^*_G > 0$ is explained in the text. Equilibrium existence follows because the best response to a threshold strategy $\sigma_{\bar{q}}$ is another $\sigma_{\bar{q}^{br}_G}$. Since the best-response threshold $\bar{q}^{br}_G$ depends continuously on $\bar{q}$ and the set $[0,1]$ of possible thresholds is compact, a fixed point $\bar{q}^*_G = \bar{q}^{br}_G\left(\bar{q}^*\right)$ exists by Brouwer's theorem, that characterizes a voting strategy $\sigma_{\bar{q}^*}$ that is its own best response, and therefore a Bayesian Nash equilibrium of the game. ■

Proof of Theorem 6. With a generalized policy function, the expected benefit of voting (9) can be rewritten as follows,

$$\Delta E u\left(q\right) = E_{a,b}\left[\frac{1}{2}\left(1+q\right)\Delta_+\psi\left(a,b\right) + \frac{1}{2}\left(1-q\right)\Delta_- x\left(a,b\right)\right]$$

in terms of the positive difference in policy $\Delta_+\psi\left(a,b\right) = \psi\left(a+1,b\right) - \psi\left(a,b\right)$ induced by one additional vote for the superior party A and the negative policy difference $\Delta_-\psi\left(a,b\right) = \psi\left(a,b+1\right) - \psi\left(a,b\right)$ induced by one additional vote for party B. Given the symmetry condition, the latter difference can be written as

$$\begin{aligned}
\Delta_-\psi\left(a,b\right) &= \left[1-\psi\left(b+1,a\right)\right] - \left[1-\psi\left(b,a\right)\right] \\
&= \psi\left(b,a\right) - \psi\left(b+1,a\right) \\
&= -\Delta_+\psi\left(b,a\right)
\end{aligned}$$

in terms of the positive impact of a correct vote when there are b votes for party A and a votes for party B.

Since $\Delta_+\psi\left(a,b\right)$ and $\Delta_+\psi\left(b,a\right)$ are both positive, (9) is positive if and only if q exceeds the following threshold,

$$\bar{q}^{br}_M = \frac{E_{a,b}\left[\Delta_+\psi\left(b,a\right)\right] - E_{a,b}\left[\Delta_+\psi\left(a,b\right)\right]}{E_{a,b}\left[\Delta_+\psi\left(a,b\right)\right] + E_{a,b}\left[\Delta_+\psi\left(b,a\right)\right]}$$

[32] For $p = 1$, $\Pr\left(n_\alpha > n_\beta\right) = \Pr\left(n_\alpha < n_\beta\right)$ for any n, implying that $u^M = \frac{1}{2} + 0$.

which generalizes (12). The denominator of this expression is positive, and the numerator reduces as follows.

$$\begin{aligned}
& \sum_{a,b} \left[\Delta_+ \psi(b,a) - \Delta_+ \psi(a,b)\right] \frac{e^{-n_+ - n_-} n_+^a n_-^b}{a!b!} \\
= & \sum_{a>b} \left[\Delta_+ \psi(b,a) - \Delta_+ \psi(a,b)\right] \frac{e^{-n_+ - n_-} n_+^a n_-^b}{a!b!} \\
& + \sum_{a<b} \left[\Delta_+ \psi(b,a) - \Delta_+ \psi(a,b)\right] \frac{e^{-n_+ - n_-} n_+^a n_-^b}{a!b!}
\end{aligned}$$

Relabeling variables in the second summation yields the following.

$$\begin{aligned}
& \sum_{a>b} \left[\Delta_+ \psi(b,a) - \Delta_+ \psi(a,b)\right] \frac{e^{-n_+ - n_-} n_+^a n_-^b}{a!b!} \\
& + \sum_{b<a} \left[\Delta_+ \psi(a,b) - \Delta_+ \psi(b,a)\right] \frac{e^{-n_+ - n_-} n_+^b n_-^a}{a!b!} \\
= & \sum_{a>b} \left[\Delta_+ \psi(b,a) - \Delta_+ \psi(a,b)\right] \frac{e^{-n_+ - n_-} n_+^b n_-^b}{a!b!} \left(n_+^{a-b} - n_-^{a-b}\right)
\end{aligned}$$

Monotonicity and the underdog property together imply that $\Delta_+ \psi(a,b) < \Delta_+ \psi(b,a)$ if and only if $a > b$. This, together with the fact that $n_+ > n_-$, implies that the above expression is strictly positive, and therefore that $\bar{q}_M^{br}$ is positive. That the best-response threshold is strictly positive implies that any equilibrium threshold is positive as well, and a positive fraction of the electorate therefore prefer to abstain in equilibrium. ■

Proof of Theorem 5. This theorem is proven in the text. ■

Appendix B: Regression Tables

Variable	Coef.	Std. Err.	z	P>z	95% C.I.
$High \times P25$	0.021	0.012	1.81	0.071	$[-0.002, 0.045]$
$High \times P50$	0.019	0.006	3.26	0.001	$[0.008, 0.031]$
$High \times M0$	0.008	0.006	1.47	0.142	$[-0.003, 0.020]$
$High \times M25$	0.011	0.011	1.04	0.298	$[-0.010, 0.032]$
$High \times M50$	0.032	0.022	1.45	0.147	$[-0.011, 0.075]$
$Low \times P0$	0.420	0.062	6.77	0.000	$[0.299, 0.542]$
$Low \times P25$	0.293	0.038	7.63	0.000	$[0.218, 0.368]$
$Low \times P50$	0.284	0.050	5.69	0.000	$[0.186, 0.382]$
$Low \times M0$	0.361	0.093	3.89	0.000	$[0.179, 0.543]$
$Low \times M25$	0.359	0.040	9.08	0.000	$[0.281, 0.436]$
$Low \times M50$	0.272	0.061	4.5	0.000	$[0.154, 0.391]$
Constant	0.003	0.006	0.54	0.588	$[-0.009, 0.015]$
Observations	14,400		σ_u	.209	ρ .313
Subjects	360		σ_e	.309	

Table 1: Random effects GLS regression of the probability of abstention on a constant and a number of dummies indicating the interaction between voter type, voting rule, and level of partisanship.

Appendix C: Instructions for the Experiment

Welcome and thank you for taking part in this experiment. Please remain quiet and switch off your mobile phone. It is important that you do not talk to other participants during the entire experiment. Please read these instructions very carefully; the better you understand the instructions the more money you will be able to earn. If you have further questions after reading the instructions, please give us a sign by raising your hand out of your cubicle. We will then approach you in order to answer your questions personally. Please do not ask aloud.

During the experiment all sums of money are listed in ECU (for Experimental Currency Unit). Your earnings during the experiment will be converted to euros at the end and paid to you in cash. The exchange rate is 40 ECU = 1€. The earnings will be added to a participation payment of 4€.

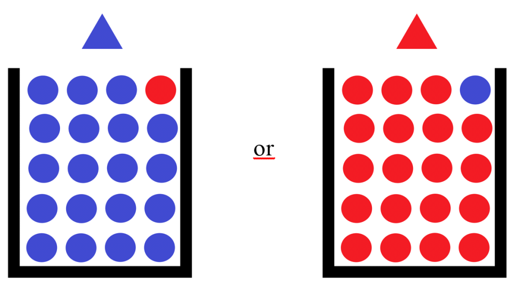
At the beginning of this experiment, participants will be randomly and anonymously divided into sets of 12 participants. These sets remain unaltered for the entire experiment, but you will never be told who is in your set. The experiment is divided into 40 rounds.

The rules are the same for all participants and for all rounds. In each round, participants in each set are divided into two groups of 6 participants. In a given round you will only interact with the participants in your group for that round. The earnings in each round will depend partly on your own decision, partly on the decisions of the other participants in your group, and partly on chance.

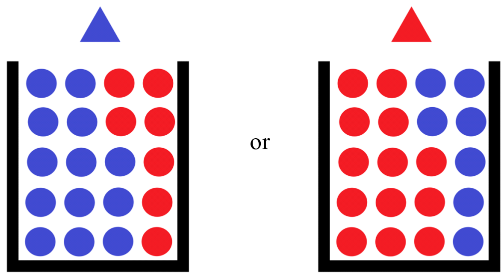
The Triangle Color. There is a triangle, and at the beginning of each round, the color of the triangle will be chosen randomly. With **50%** probability it will be blue ▲, and with **50%** probability it will be red ▲. You will not know the color of the triangle, but each member of your group will receive a hint. Your objective as a group will be to guess the color of the triangle.

Types. As a hint of the color of the triangle, each group member will observe the color of one ball, drawn from an urn filled with 20 red and blue balls. First, however, each group member will be assigned a type: with **40% probability** you will be designated as **Type B** and will receive a big hint; with **60% probability**, you will be designated as **Type S** and will receive a small hint. Types will be assigned independently for each member of the group, so you and the other members of your group might have different types. You will learn your own type, but will not know the types of the other members of your group.

Big Hints. If your type is Type B, you will receive a big hint. First, an urn will be filled with **19 balls** that are the same color as the triangle, and **1 ball** of the opposite color (a total of 20 balls). If the triangle is blue ▲, for example, then the urn will be filled with 19 blue balls and 1 red ball. If the triangle is red ▲, the urn will be filled with 1 blue ball and 19 red balls. As a Type B individual, you will observe the color of one ball, drawn randomly from this urn. If other members of your group are designated as Type B, they will also observe one ball from this same urn. They might observe the same ball you observed, or a different ball.



Small Hints. If your type is Type S, you will receive a small hint. First, an urn will be filled with **13 balls** that are the same color as the triangle, and **7 balls** of the opposite color (a total of 20 balls). If the triangle is blue ▲, for example, then the urn will be filled with 13 blue balls and 7 red balls. If the triangle is red ▲, the urn will be filled with 7 blue ball and 13 red balls. As a Type S individual, you will observe the color of one ball, drawn randomly from this urn. If other members of your group are designated as Type S, they will also observe one ball from this same urn. They might observe the same ball you observed, or a different ball.



Your Voting Decision. Your voting decision is one of three options: (1) vote Blue, (2) vote Red, or (3) Abstain from voting.

Regardless of your decision (vote Blue, vote Red, or Abstain), your choice might be changed with some probability:

• With a probability of **65%** (or 13 out of 20) your voting decision choice will be maintained.

• With a probability of **10%** (or 1 out of 10) your voting decision will be replaced by a computer who will **Abstain**.

• With a probability of 12.5% (or 1 out of 8) your voting decision will be replaced by a computer who will vote **Blue**.

• With a probability of 12.5% (or 1 out of 8) your voting decision will be replaced by a computer who will vote **Red**.

At the end of each round you will be told whether your voting decision was maintained or replaced. If your vote is replaced, you will also be told how a computer voted in your place.

The other members of your group will cast votes in the same fashion, and like you, their votes might randomly be replaced by computers. At the end of each round, you will see the final vote cast by each of your group members, but you will not be told whether their original vote choices were replaced by computers or not.

Your Payoff. Your payoff in a given round will be the same for all members in your group. Your payoff will depend **only** on the numbers of Blue and Red votes in your group (and **not** on the number of abstentions).

[P]

• If the color of the triangle receives **more** votes than the other color, your payoff will be **100**.

• If the color of the triangle receives **fewer** votes than the other color, your payoff will be **0**.

• If the color of the triangle and the other color receive **equal** numbers of votes, your payoff will be **50**.

Example 1: Suppose that the triangle is red ▲ and that there are 3 Blue votesand 2 Red votes. Since there are fewer votes for the color of the triangle than for the other color, your payoff is 0 ECUs.

Example 2: Suppose that the triangle is red ▲ and that there are 0 Blue votes and 2 Red votes. Since there are more votes for the color of the triangle than for the other color, your payoff is 100 ECUs.

The following table lists your payoff, for any possible combination of Blue and Red votes.

Number of votes for the other color

Number of votes for the color of the triangle	0	1	2	3	4	5	6
0	50	0	0	0	0	0	0
1	100	50	0	0	0	0	
2	100	100	50	0	0		
3	100	100	100	50			
4	100	100	100				
5	100	100					
6	100						

[M]

Your payoff in will be the **percentage** of votes that have the **same** color as the triangle (if this percentage is not an entire number, the payment will be rounded to the closest entire number). If there are no votes (because everyone abstains) then your payoff is 50.

Example 1: Suppose that the triangle is red ▲ and that there are 3 Blue votes and 2 Red votes. Since 40% (i.e. two out of five) of the votes match the color of the Triangle, your payoff is 40.

Example 2: Suppose that the triangle is red ▲ and that there are 0 Blue votes and 2 Red votes. Since 100% (i.e. two out of two) of the votes match the color of the Triangle, your payoff is 100.

The following table lists your payoff, for any possible combination of Blue and Red votes.

Number of votes for the other color

Number of votes for the color of the triangle	0	1	2	3	4	5	6
0	50	0	0	0	0	0	0
1	100	50	33	25	20	17	
2	100	67	50	40	33		
3	100	75	60	50			
4	100	80	67				
5	100	83					
6	100						

Information at the end of each Round. Once you and all the other participants have made your choices and these choices have been randomly replaced (or not), the round will be over. At the end of each round, you will receive the following information about the round: the color of the triangle, your vote, and the total numbers of Blue votes, Red votes, and abstentions in your group. You will also observe [M: the percentage of votes that match the color of the Triangle, and] [P: whether the color of the Triangle received more, equal or fewer votes than the other color, and] the payoff for your group.

Final Earnings. After the 40 rounds are over, the computer will randomly select 5 of the 40 rounds and you will receive the rewards that you had earned in each of those rounds. Each of the 40 rounds has the same chance of being selected.

Control Questions. Before starting the experiment, you will have to answer some control questions in the computer terminal. Once you and all the other participants have answered all the control questions, Round 1 will begin.

Questionnaire. After the experiment, we will ask you to complete a short questionnaire, which we need for the statistical analysis of the experimental data. The data of the questionnaire, as well as all your decisions during the experiments will be anonymous.