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Designing Central Banks for Inflation Stability*

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July 18, 2016

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Well-designed central banks can uniquely determine a stable inflation rate following either active Taylor's rules or interest-rate pegs. They should receive an initial transfer of capital, hold only risk-free assets, rebate their income to the treasury. This system prevents permanent liquidity traps and inflationary spirals without further need of treasury's support beyond the initial capitalization. Instead, if the central bank engages in purchases of risky securities, fiscal support is required to uniquely back the value of money. Absent treasury's support and with a risky composition of assets, inflationary spirals and deflationary traps can develop due to self-fulfilling expectations or credit events.

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1 Introduction

The determination of prices has been at the center of the economic debate since the existence of monetary systems with discussions ranging from the policies that monetary institutions should follow to the assets that they should hold to back the value of money.

The instability and volatility associated with environments pervaded by multiple equilibria can undermine the role of the central bank in achieving inflation goals. The literature has mainly fixed this problem by imposing some degree of "activism" on the actions of the fiscal authority to back the price level (see among others Cochrane, 2011).

This paper contributes to the literature by proposing a monetary/fiscal policy regime that can uniquely implement a stable inflation target where the "activism" is left to the central bank. There are four key ingredients of the proposed regime: 1) an active Taylor's rule; 2) a passive fiscal policy; 3) a transfer policy between central bank and treasury characterized by three elements: 3a) an injection of real capital at the inception of the central bank and, thereafter, 3b) a central-bank remittances' policy that transfers all central bank's profits to the treasury, 3c) a modified remittances' policy which acts off equilibrium to trim inflationary spirals; finally 4) a riskless composition of the assets of the central bank.

It is surely hard to disentangle in the proposed specification whether a feature is more pertinent to the monetary or the fiscal authority. However –under assumptions 1 and 2– the interesting implication is that once the central bank receives initial capital, holds only risk-free securities, and commits to rebate positive profits to the treasury then deflationary solutions or permanent liquidity trap cannot arise in equilibrium. Under the same conditions, the central bank is able to provide internal resources –without any need of fiscal support– to appropriately back the value of money and avoid that inflationary spirals develop.

The main conclusion of this work is that a central bank – appropriately designed to be financially independent – has all the power and ability to determine the value of money at the desired level without the need of further treasury's support beyond the initial capitalization.

A theory of price determination through the balance sheet of the central bank emerges when an interest-rate peg replaces the active Taylor's rule, item 1, further reinforcing the main result. In this case, conditions 2, 3a and 4 must hold while the commitment to the remittances' policy 3c must also be taken in equilibrium replacing 3b. Interestingly, this remittances' policy implies that all central bank's income, which is never negative, is rebated in each period to the treasury and therefore that the value of money is backed relying only on internal resources, provided again that the central bank is appropriately capitalized at the inception.

The riskless composition of the central bank's assets and the initial injection of capital are key to leave complete autonomy to the monetary authority in determining the price level. However, global uniqueness can be ensured even if the central bank holds risky securities, therefore relaxing assumption 4, while maintaining unchanged all other features of the monetary/fiscal policy regime specified above. In this case, though, the central bank is no longer financially independent since the treasury is expected to eventually back the losses of the central bank when they occur. Accordingly price determination involves a direct support from the fiscal authority.

What will happen instead if the central bank engages in purchases of risky securities without any fiscal support? Uniqueness is no longer guaranteed. Inflationary spirals can be equilibria triggered either by shifts in inflation expectations which are self-fulfilling or by credit events lowering the value of long-term assets. A deflationary trap may also emerge in the latter case.

This paper is related to an important literature that has discussed the issue of price determination in general equilibrium monetary economies ranging from the fiscal theory of the price level as in Cochrane (2001), Leeper (1991), Sims (1994, 2000, 2013), Woodford (1995, 2001) to theories of price determination through active interest-rate rules supported by fiscal backing as in Benhabib et al. (2001, 2002), Schmitt-Grohé and Uribe (2000), Sims (2013), Woodford (2003). Cochrane (2011) provides an extensive and critical discussion of results of determinacy achieved through Taylor's rules.

With respect to all this literature, the contribution of this work is to emphasize that the determination of the price level can be left to the central bank without any fiscal backing or support resting on an appropriate design of how central banks should operate starting from their capital, composition of assets, remittances' policy and policy rule. One of the main insights of this work stands on the separation between the budget constraint of the treasury and that of the central bank as suggested by a recent literature following Bassetto and Messer (2013), Benigno and Nisticò (2015), Del Negro and Sims (2015), Hall and Reis (2015), Reis (2015), Sims (2000, 2005).

Bassetto (2004) and Obstfeld and Rogoff (1983) are closely related papers since they also rely only on central bank's actions or internal resources to defeat unpleasant or explosive solutions. Bassetto (2004) considers the possibility that the central bank can threat the private sector to set negative nominal interest rates to counteract the occurrence of a deflationary solution. Obstfeld and Rogoff (1983) argues that the central can stop hyperinflations by guaranteeing a minimal real redemption value for money. A similar mechanism applies in our model against inflationary solutions relying on the backing given by the initial level of capital together with the threat of transferring the real value of the central bank to the treasury at the desired price level. In contrast with Obstfeld and Rogoff (1983), the solution of this paper should disallow inflationary solutions as equilibria rather than just stop them in line with the discussion of Cochrane (2011).

The structure of the paper is the following. Section 2 presents a simple monetary model. Section 3 discusses the monetary/fiscal policy regime that supports a unique stable inflation rate. Section 4 extends the model to include long-term securities and a non-pecuniary value of money balances. It also discusses the implication of departing from the proposed regime analyzing unconventional balance-sheet policies without any treasury's support. Section 5 concludes.

2 Model

I present a simple endowment monetary economy in which central bank and financial markets start their operations at time t_0. There are three agents in the economy: households, treasury and central bank. I will now move to describe their behavior.

2.1 Household

Households have an intertemporal utility of the form:

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} U(C_t) \tag{1}$$

where β is the intertemporal discount factor with $0 < \beta < 1$, C is a consumption good and $U(\cdot)$ is a concave function, twice continuously differentiable, increasing in C.

The household's budget constraint is:

$$\frac{B_t + X_t}{1 + i_t} + M_t \leq B_{t-1} + X_{t-1} + M_{t-1} + P_t(Y - C_t) - T_t^F. \tag{2}$$

Households can invest their financial wealth in money M_t which is a non-interest bearing security issued by the central bank with the role of store of value. They can also invest in interest-bearing reserves, X_t, issued as well by the central bank at the risk-free nominal interest rate i_t and can lend or borrow using short-term securities, B_t, at the same interest rate i_t. Y is a constant endowment of the only good traded; T_t^F are lump-sum taxes levied by the treasury. There are no financial markets before time t_0, therefore B_{t_0-1}, X_{t_0-1}, M_{t_0-1} are all equal to zero. The household's problem is subject to a borrowing-limit condition

$$\lim_{T\to\infty}\left\{R_{t_0,T}\left(\frac{B_T + X_T}{1 + i_T} + M_T\right)\right\} \geq 0, \tag{3}$$

looking forward from time t_0 where $R_{t_0,T}$ is the nominal stochastic discount factor that is used to evaluate nominal wealth at time T with respect to nominal wealth at time t_0, with $T > t_0$. It is also required for the existence of an intertemporal budget constraint that the nominal interest rate is non-negative $i_t \geq 0$ since a store of value is available, and that

$$\sum_{T=t_0}^{\infty} R_{t_0,T}\left\{P_T C_T + \frac{i_T}{1 + i_T} M_T\right\} < \infty \tag{4}$$

since there is no limit to the ability of households to borrow against future income.

Households choose consumption, and asset allocations to maximize utility (1) under constraints (2), (3) and (4), given the initial conditions. The set of first-order conditions imply the Euler equation

$$\frac{U_c(C_t)}{P_t} = \beta(1 + i_t)\frac{U_c(C_{t+1})}{P_{t+1}} \tag{5}$$

at each time $t \geq t_0$ assuming interior solutions.

Money, as an asset, is dominated in return by reserves and moreover does not provide non-pecuniary benefits. Its demand is zero whenever $i_t > 0$. However, when $i_t = 0$, money becomes a perfect substitute of reserves.

To conclude the characterization of the household's problem, a transversality condition applies and therefore (3) holds with equality, given the equilibrium nominal stochastic discount factor

$$R_{t_0,T} = \beta^{T-t_0} \frac{U_c(C_T)}{U_c(C_{t_0})} \frac{P_{t_0}}{P_T}.$$

2.2 Treasury

The treasury raises lump-sum taxes T_t^F (net of transfers) from the private sector and receives remittances T^C (when T^C is positive) or makes transfers to the central bank (when T^C is negative). The treasury can finance its deficit through short-term debt (B^F) at the price $1/(1+i_t)$, facing the following flow budget constraint

$$\frac{B_t^F}{1+i_t} = B_{t-1}^F - T_t^F - T_t^C$$

given initial condition $B_{t_0-1}^F = 0$.

2.3 Central bank

The central bank issues non-interest bearing liabilities, money M_t^C, and interest-bearing liability, reserves X_t^C, to hold short-term securities, B_t^C. Central-bank net worth, N_t^C – the difference between the market value of assets and liabilities – is given by

$$N_t^C \equiv \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t}, \tag{6}$$

while its law of motion depends on the profits that are not distributed to the treasury:

$$N_t^C = N_{t-1}^C + \Psi_t^C - T_t^C \tag{7}$$

where Ψ_t^C are central bank's profits:[1]

$$\Psi_t^C = \frac{i_{t-1}}{1+i_{t-1}} (B_{t-1}^C - X_{t-1}^C). \tag{8}$$

[1] Profits of central bank are defined as the net income derived from the portfolio of assets and liabilities which once fully distributed is such to keep the central bank's nominal net worth constant. This is in line with similar definitions given by Bassetto and Messer (2013), Del Negro and Sims (2015) and Hall and Reis (2015).

Considering the definition (6), profits can also be written as

$$\Psi_t^C = i_{t-1}(N_{t-1}^C + M_{t-1}^C). \tag{9}$$

Combining (6), (7) and (8), the central bank's flow budget constraint follows:

$$\frac{B_t^C - X_t^C}{1+i_t} - M_t^C = B_{t-1}^C - X_{t-1}^C - M_{t-1}^C - T_t^C,$$

given initial conditions $B_{t_0-1}^C, X_{t_0-1}^C, M_{t_0-1}^C$ all equal to zero.

2.4 Equilibrium

In equilibrium consumption is equal to the endowment at all times and therefore constant, $C_t = Y$. The Fisher's equation holds

$$1 + i_t = \frac{1}{\beta}\frac{P_{t+1}}{P_t}. \tag{10}$$

Given the existence of a storage technology, the economy is subject to the zero-lower bound on nominal interest rate

$$i_t \geq 0. \tag{11}$$

The household's transversality condition can be simplified to

$$\lim_{T\to\infty}\left\{\beta^{T-t_0}\left(\frac{P_{t_0}}{P_T}\right)\left(M_T + \frac{B_T + X_T}{1+i_T}\right)\right\} = 0, \tag{12}$$

while the bound (4) can be written as

$$\sum_{T=t_0}^{\infty}\beta^{T-t_0}\left\{Y + \frac{i_T}{1+i_T}\frac{M_T}{P_T}\right\} < \infty$$

which is naturally satisfied given the properties of money demand.

The flow budget constraints of treasury and central bank are respectively

$$\frac{B_t^F}{1+i_t} = B_{t-1}^F - T_t^F - T_t^C, \tag{13}$$

$$\frac{B_t^C - X_t^C}{1+i_t} - M_t^C = B_{t-1}^C - X_{t-1}^C - M_{t-1}^C - T_t^C, \tag{14}$$

while equilibrium in the securities market closes the model

$$B_t + B_t^C = B_t^F, \tag{15}$$

$$M_t = M_t^C, \tag{16}$$

$$X_t = X_t^C. \tag{17}$$

Recall that the demand of money is zero when the nominal interest is positive and is infinitely elastic when the interest rate is zero – at which point money becomes a perfect substitute of reserves. Without losing generality, I can set $M_t^C = M_t = 0$. Therefore a rational-expectations equilibrium is a collection of processes $\{P_t,\ i_t,\ T_t^F,\ T_t^C,\ B_t,\ B_t^C,\ B_t^F,\ X_t\}_{t=t_0}^{\infty}$ that satisfy (10)-(15) at each date $t \geq t_0$ given (17), $M_t = M_t^C = 0$ at all times and initial conditions M_{t_0-1}, $B_{t_0-1}^C$, $B_{t_0-1}^F$,X_{t_0-1} all equal to zero. Since (12) is a bound, there are four degrees of freedom to specify the monetary/fiscal policy regime.

3 Designing central banks for inflation stability

In this section, I show how to design the monetary/fiscal policy regime in order to uniquely implement inflation at the target Π^*. The four degrees of freedom left for the specification of the monetary/fiscal policy regime are taken in turn by: 1) an active Taylor's rule that sets a path for $\{i_t\}_{t=t_0}^{\infty}$ as a function of the inflation rate; 2) a passive fiscal policy setting the sequence of lump-sum taxes $\{T_t^F\}_{t=t_0}^{\infty}$; 3) a central-bank transfer policy specifying $\{T_t^C\}_{t=t_0}^{\infty}$ and characterized by three elements: 3a) an injection of real capital at the birth of the central bank and, thereafter, 3b) a central-bank remittances' policy that transfers all central bank's profits to the treasury, 3c) a modified remittances' policy which acts off equilibrium to trim inflationary spirals; finally 4) a central bank's balance-sheet policy that specifies in an appropriate way holdings of short-term assets $\{B_t^C\}_{t=t_0}^{\infty}$.

The separation between the budget constraints of treasury and central bank is the novelty of this work and the critical innovation that allows the central bank to uniquely determine the price level relying mainly on its own actions and resources. Without this distinction, the specification of the monetary/fiscal policy regime collapses to two elements: the interest-rate policy and the tax policy. In this case, as shown by the literature, some degree of "activism" is requested to the fiscal authority to determine the price level. In this work, instead, this "activism" is left to the central bank.

I now discuss in details the specification of the monetary/fiscal policy regime with respect to the sequences $\{i_t, T_t^F, T_t^C, B_t^C\}_{t=t_0}^{\infty}$.

3.1 An active Taylor's rule

To characterize the first element of the monetary/fiscal policy regime I assume that monetary policy follows an interest-rate rule of the form

$$1 + i_t = \max\left(\frac{\Pi^*}{\beta}\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}, 1\right) \tag{18}$$

where Π^* is the inflation target with $\Pi^* > \beta$ and ϕ_π is a policy parameter, with $\phi_\pi > 1$, capturing the reaction of the interest rate to inflation in reference to its target.

The interest-rate rule (18) has interesting implications once combined with the Fisher's equation. After substituting (18) into (10), I obtain a non-linear difference equation

$$\frac{\Pi_{t+1}}{\Pi^*} = \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}. \tag{19}$$

This difference equation has a stationary solution with inflation equal to the target $\Pi_t = \Pi^*$ at all times $t \geq t_0$. Since the nominal interest rate is bounded below by zero, there is also another stationary solution in which $\Pi_t = \beta$ for each $t \geq t_0$. There are as well solutions in which inflation takes diverging paths if $\Pi_{t_0} > \Pi^*$ or solutions with deflationary spirals if $\Pi_{t_0} < \Pi^*$. In the latter case, all solutions converge to the deflationary equilibrium in a finite period of time.[2]

3.2 A passive fiscal policy

The second ingredient of the monetary/fiscal policy regime is a passive fiscal policy, which is in the same spirit of that of Leeper (1991) with an important qualification since it now strictly refers to the rule followed by the treasury

[2]Note that P_{t_0-1} is not defined. Therefore it can be set freely at the level $P_{t_0-1} = P^*(\Pi^*)^{-1}$ to imply $P_{t_0} = P^*$ in the stationary solution in which $\Pi_t = \Pi^*$.

and not by the whole government, which instead includes also the central bank. This distinction follows the specification of Benigno and Nisticò (2015). Real taxes are set following the rule

$$\frac{T_t^F}{P_t} = \bar{T}^F - \phi_c \frac{T_t^C}{P_t} + \phi_f \frac{B_{t-1}^F}{P_t}, \tag{20}$$

with $\phi_c = 1$ and $\phi_f > 0$. They depend negatively on the remittances that the treasury receives from the central bank and positively on the past level of short-term real debt, given an initial condition $B_{t_0-1}^F = 0$; $\bar{T}^F$ is a constant. The fiscal rule is called passive since it ensures that the treasury's real liabilities are stationary for any equilibrium path of prices and nominal interest rates. In particular the following limiting condition

$$\lim_{T\to\infty} \left\{ \beta^{T-t_0} \left(\frac{P_{t_0}}{P_T} \right) \frac{B_T^F}{1+i_T} \right\} = 0 \tag{21}$$

holds. Note that $\phi_c = 1$ and $\phi_f > 0$ are necessary and sufficient conditions for the rule (20) to imply (21) given (13).[3]

3.3 Central-bank remittances' policy and balance-sheet policy

The monetary system starts at time t_0 with the creation of the central bank. At its inception the central bank receives a real transfer $\tilde{N}_{t_0}^C$ from the treasury

$$\frac{T_{t_0}^C}{P_{t_0}} = -\tilde{N}_{t_0}^C$$

which is item 3a in the specification of the monetary/fiscal policy regime. This initial capitalization is covered by the treasury through lump-sum real taxes levied on the household according to (20). Given this real transfer, the central bank begins its operations with the following budget constraint

$$\frac{B_{t_0}^C - X_{t_0}}{P_{t_0}(1+i_{t_0})} = \tilde{N}_{t_0}^C,$$

consistently with equation (14) and initial conditions.

[3] See the discussion of Benigno and Nisticò (2015).

After time t_0 the central bank follows a transfer policy of the form

$$\frac{T_t^C}{P_t} = \frac{\Psi_t^C}{P_t} \tag{22}$$

for each $t > t_0$ specifying that all central bank's profits are transferred to the treasury. This corresponds to item 3b in the specification of the monetary/fiscal policy regime. The rule is symmetric and therefore also requires that losses are promptly covered by the treasury. But, as it will be clear in a moment, this never happens given the riskless composition of the central bank's assets.

An implication of the assumed remittances' policy is that central bank's nominal net worth is constant and positive at any time, as it is shown by (7), i.e. $N_t^C = N_{t-1}^C = ... = P_{t_0}\tilde{N}_{t_0}^C$ and therefore

$$\frac{B_t^C - X_t}{(1+i_t)} = P_{t_0}\tilde{N}_{t_0}^C,$$

at each time $t \geq t_0$.

The remittances' rule, the initial capitalization of the central bank, a safe-asset balance-sheet composition together imply that central bank's income is non-negative and always positive whenever the nominal interest rate is above zero, as shown in (9). This means that the central bank is financially independent from the treasury, i.e. it does not need treasury's support except for the initial capitalization. Moreover the central bank makes positive transfers to the treasury when the nominal interest rate is positive.

Item 3c of the monetary/fiscal policy regimes specifies an off-equilibrium change of the remittances's policy from (22) to

$$\frac{T_t^C}{P_t} = i^* \frac{P_{t_0}\tilde{N}_{t_0}^C}{P^*(\Pi^*)^{t-t_o}}, \tag{23}$$

starting from a generic time $t > t_0$ onward if unstable inflationary paths develop, where $i^* = \beta^{-1}\Pi^* - 1$. The effectiveness of this threat will be clear in the next subsection.

To conclude the specification of the monetary/fiscal policy regime, the last degree of freedom is taken by the specification of the balance-sheet policy that sets $\left\{B_t^C\right\}_{t=t_0}^{\infty}$ as an appropriate positive sequence.[4] Given the balance-

[4]Appropriate means consistent with a non-negative equilibrium level of reserves at all times.

sheet policy and the remittances' policy, and considering the Taylor's rule, equation (14) can be used to determine the path of reserves.[5]

3.4 Implementation of equilibrium

Recall that (19) has multiple solutions. The above-specified monetary/fiscal policy regime carries important implications. First, it can eliminate deflationary solutions. Combine the transversality condition (3) with (21) to obtain that in equilibrium real central bank's net worth should be appropriately bounded

$$\lim_{T\to\infty}\left\{\beta^{T-t_0}\frac{P_{t_0}}{P_T}N_T^C\right\}=0. \tag{24}$$

Use the result that nominal net worth is constant and positive into (24) to get

$$\lim_{T\to\infty}\left\{\beta^{T-t_0}\frac{P_{t_0}}{P_T}\right\}=0. \tag{25}$$

In equilibrium prices cannot fall at a rate equal or higher than β. This excludes the stationary solution $\Pi_t=\beta$ and all other solutions in which Π_{t_0} starts below Π^* and converges to the deflationary trap in a finite period of time. Permanent liquidity traps in which the nominal interest rate is at its bound are not equilibria given the specification of the monetary/fiscal policy regime.

Key to eliminate deflationary solutions is that the central bank is committed to rebate all nominal profits to the treasury and keep nominal net worth constant. However, these commitments may be questioned – an issue that needs further discussion.

Suppose that a deflationary spiral develops with $\Pi_t=\beta$ then (24) and the monetary/fiscal policy regime imply that

$$\lim_{T\to\infty}\left\{N_T^C\right\}=P_{t_0}\tilde{N}_{t_0}>0.$$

The transversality condition is violated: the long-run net debt position of the household is positive corresponding specular to a positive net asset position of the central bank. The reason why the deflationary solution cannot be an equilibrium is that households should borrow more than what is allowed by

[5] In the case in which the nominal interest rate is zero, equation (14) can jointly determine reserves and money.

the natural borrowing limit in order for consumption to be equal to output in each period. In particular, borrowing should be above what they can repay with certainty using the presented-discounted value of their net income. The need of an excess borrowing rests on the fact that the central bank is retaining, in the long run, a positive value of resources in its balance sheet, as shown by the inequality above. In particular given the deflationary path and the commitment to maintain nominal net worth constant, the central bank is also implicitly taking the commitment to let its real net worth explode.

The critical question to ask is how credible this retention of resources is, given that the missing resources are exactly what prevents the private sector from repaying the amount of debt needed for the deflationary solution to develop as an equilibrium. The treasury could tax the central bank and expropriate entirely its net worth to rebate it to the private sector. This extraordinary measure can be even more justified by noting that during the liquidity trap profits are zero and therefore remittances to the treasury are zero. The treasury could question why the central bank is allowing its net worth to increase in real terms without rebating any resource to the treasury.

There are two arguments that can be used to counteract these criticisms. The first is related to the independence of the central bank, an institution that once designed should not be questioned in its operations and remittances' policy. The second reason is more model dependent. The deflationary equilibrium fails to arise simply because at time t_0 the private sector needs to borrow more than what it can afford taking into account that the central bank is retaining resources. Therefore, it is already at time t_0 that counterparties in the credit market have to be sure about the solvency of the private sector and therefore be sure of what happens to the net worth of the central bank – whether at the end is entirely expropriated and rebated to the private sector. If there is even a small probability that this does not happen or even a small amount of capital remains at the central bank then the equilibrium will not form because the transversality condition is violated.

Going back to the proposed solution, the founding elements of the specified monetary/fiscal policy regime are key to get rid of the deflationary solutions. First note that if the central bank does not receive capital at time t_0 (item 3a) – while I maintain the other elements of the monetary-fiscal policy regime unchanged– then deflationary solutions are not excluded as equilibria, as shown in (24) since $N_T = N_{T-1} = ... = 0$. Therefore the initial capitalization of the central bank is critical to obtain the result. Moreover, the assumption that the central bank holds only short-term risk-free securities (item 4) ensures that profits of the central bank are positive when the nominal interest rate is above zero, in which case they are fully rebated to the treasury according to the remittances' policy (22) (item 3b). Except for the inception period, the central bank does not need treasury's support. Having received initial capital, the central bank can defeat deflationary solutions by relying only on internal resources.

So far, I have eliminated deflationary solutions, but still inflationary spirals could develop. If inflation starts higher than Π^*, it could continually increase. The threat to switch the remittances' policy from (22) to (23), which is item 3c in the specification of the monetary/fiscal policy regime, can work to trim these unstable solutions. To this end, consider the intertemporal budget constraint of the central bank which is the mirror image of the transversality condition of the household having assumed a passive fiscal policy

$$\frac{N_t^C}{P_t} = \sum_{T=t+1}^{\infty} \beta^{T-t} \frac{T_T^C}{P_T}. \tag{26}$$

The left-hand side captures the real value of the central bank given by its net worth. The value of the central bank should be equal to the present-discounted value of the remittances transferred to the treasury.

Let us suppose that an inflationary solution develops that pushes prices up to an explosive path. To eliminate this solution as an equilibrium, the central bank should switch to a remittances' policy that backs appropriately the value of money. In particular the change in remittances' policy should bring prices back to the trajectory they would have followed under the equilibrium $\Pi_t = \Pi^*$ at all times. A commitment to change the remittances' policy to (23) from a generic time $t > t_0$ onward would work.

By substituting (23) into (26) and considering that the level of net worth reached at time t, i.e. $N_t^C = P_{t_0} \tilde{N}_{t_0}^C$, is the one implied by the remittances' policy (22), it follows that the threat to switch to the remittances' policy (23) determines the price level at time t consistently with the path that prices would have followed given the equilibrium with stable inflation Π^*, i.e. $P_t = P^*(\Pi^*)^{t-t_o}$. This threat can only succeed when $\tilde{N}_{t_0}^C \neq 0$. It is therefore again critical that the initial real level of capital of the central bank is positive, $\tilde{N}_{t_0}^C > 0$, on top of the assumed remittances' policy and balance-sheet policy. Given (23), forces that eliminate arbitrage opportunities operate to make bond prices at time t_0 aligned with those arising in the equilibrium with $\Pi_t = \Pi^*$ at all times.

The other important feature of the above proposal rests on the ability of the central bank to back the value of currency using again its own resources and without treasury's support. Indeed, the remittances implied by the policy (23) are always positive provided again that $\tilde{N}_{t_0}^C > 0$.

Finally, it is important to note that the above threat can also be used to defeat deflationary solutions. However, being a threat, it seems less appealing than the proposal of eliminating the deflationary spirals through a monetary/fiscal policy regime which acts both on and off the equilibrium path.

I have therefore found a specification of the monetary/fiscal policy regime which ensures uniqueness of equilibrium under an active interest-rate rule without relying on treasury's support, except for the initial provision of capital.

3.5 Interest-rate pegs

I now modify the interest-rate policy, item 1 of the monetary/fiscal policy regime, by assuming an interest-rate peg of the form $i_t = \bar{\imath}$ for each t in which $\bar{\imath}$ is a non-negative constant. The analysis can be easily generalized to include time-varying pegs. Inserting the policy rule into the Fisher's equation, I can obtain

$$\frac{P_{t+1}}{P_t} = \beta(1 + \bar{\imath}) \equiv \bar{\Pi},$$

at each time $t \geq t_0$. The above equilibrium condition determines the inflation rate at $\bar{\Pi}$, but it does not uniquely pin down the price level at time t_0.

Consider now the remaining specification of the monetary/fiscal policy regime. I still maintain assumptions 2 and 4. It remains to specify the remittances' policy, which is item 3 of the monetary/fiscal policy regime. First, as done earlier, I assume that at time t_0 the treasury transfers real resources to the central bank in the amount $\tilde{N}_{t_0}^C$ (item 3a). Given the assumption of passive fiscal policy and the initial injection of real capital, the following intertemporal budget constraint of the central bank holds at time t_0

$$\tilde{N}_{t_0}^C = \sum_{T=t_0+1}^{\infty} \beta^{T-t_0} \frac{T_T^C}{P_T} \tag{27}$$

while for each $t > t_0$ the constraint can be written as

$$\frac{N_t^C}{P_t} = \sum_{T=t+1}^{\infty} \beta^{T-t} \frac{T_T^C}{P_T}. \tag{28}$$

The objective here is to find a non-negative remittances' policy from period $t_0 + 1$ onward that can determine uniquely the initial price level at a desired level $P_{t_0} = \bar{P}$ and moreover in such a way that $P_t = \bar{\Pi}^{t-t_0}\bar{P}$ for each $t \geq t_0$.

Let us assume a remittances' policy of the form

$$\frac{T_t^C}{P_t} = \bar{\imath} \frac{P_{t_0} \tilde{N}_{t_0}^C}{\bar{P}(\bar{\Pi})^{t-t_o}} \tag{29}$$

for each $t \geq t_0 + 1$ which is of the same form as (23) where the targets i^*, P^*, Π^* are substituted by $\bar{\imath}$, $\bar{P}$ and $\bar{\Pi}$, respectively.[6]

Using the remittances' rule into (27), it follows that $P_{t_0} = \bar{P}$ provided $\bar{\Pi} > \beta$ and $\tilde{N}_{t_0}^C \neq 0$. The Fisher's equation implies that $P_t = \bar{\Pi}^{t-t_0}\bar{P}$ for each $t \geq t_0$ and therefore that the desired path of prices is implemented. It is also easy to see that the remittances' policy (29) satisfies (28) at the equilibrium prices for each $t > t_0$, provided again $\bar{\Pi} > \beta$.

A theory of price determination through the balance-sheet constraint of the central bank emerges akin to the fiscal theory of the price level but with the important difference that it is now the central bank that backs the value of money. A critical element for the validity of the theory is the initial injection of real capital into the central bank (item 3a). Without this capitalization, it will not be possible to determine the initial price level. Moreover, the initial capital enables the central bank to transfer positive remittances to the treasury and to back the value of money in all periods without the need of additional treasury's support: the remittances in (29) are positive if and only if $\tilde{N}_{t_0}^C > 0$. Another implication of rule (29) is that in equilibrium nominal net worth is constant at the level $N_t = P_{t_0}\tilde{N}_{t_0}^C$ for each $t \geq t_0$.

An important assumption behind the simple framework of this section is the riskless composition of central bank's assets. This is critical to generate positive central bank's income. In the next section, I generalize the model along two dimensions: 1) to include long-term securities and therefore allow an unconventional composition of the assets of the central bank; 2) to

[6] Note that item 3c, which under an active interest-rate policy was used only off the equilibrium path, is now used on the equilibrium path and replaces item 3b.

characterize a non-pecuniary role for money balances and therefore include seigniorage revenues in the analysis.

4 Unconventional balance-sheet policies and price determination

I consider now a simple extension of the above model in which preferences of the representative household change to

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[U(C_t) + V\left(\frac{M_t}{P_t}\right)\right] \tag{30}$$

where now $V(\cdot)$ is a non-decreasing twice-continuously differentiable function of real money balances with $V_m(\cdot) = 0$ for $M_t/P_t \geq \bar{m}$ where $\bar{m}$ indicates a finite level of money balances at which there is satiation.

The household's budget constraint is modified to:

$$M_t + \frac{B_t + X_t}{1+i_t} + Q_t D_t \leq M_{t-1} + B_{t-1} + X_{t-1} + (1-\varkappa_t)(1+\delta Q_t)D_{t-1} + P_t(Y - C_t) - T_t^F. \tag{31}$$

where D_t indicates long-term securities issued at a price Q_t. The security available has decaying coupons: by lending Q_t units of currency at time t, geometrically decaying coupons are delivered equal to $1, \delta,\ \delta^2, \delta^3...$ in the following periods and in the case of no default.[7] The variable $\varkappa_t$ on the right-hand side of (31) captures the possibility that long-term securities can be partially seized by exogenous default. The household's borrowing limit is

$$\lim_{T\to\infty} \left\{ R_{t_0,T} \left(M_T + \frac{B_T + X_T}{1+i_T} + Q_T D_T \right) \right\} \geq 0.$$

The set of first-order conditions is now enriched by a demand of real money balance of the form

$$\frac{M_t}{P_t} \geq L\left(C_t, i_t\right)$$

with

$$i_t \geq 0$$

[7] The stock of long-term asset follows the law of motion $D_t = Z_t + (1-\delta)D_{t-1}$, where Z_t is the amount of new long-term lending, if positive, supplied at time t. See among others Woodford (2001).

in which at least one of the two inequalities above must hold with equality at any time. The function $L(\cdot,\cdot)$ is defined by $L(\cdot,\cdot) \equiv V_m^{-1}(U_c(C)i_t/(1+i_t))$ which is non decreasing in C and non-increasing in i with $L(C_t, 0) = \bar{m}$.

Absence of arbitrage opportunities implies that

$$Q_t = \beta \frac{U_c(C_{t+1})}{U_c(C_t)} \frac{P_t}{P_{t+1}} (1 - \varkappa_{t+1})(1 + \delta Q_{t+1}) \tag{32}$$

from which a "fundamental" solution for long-term bond prices follows:

$$Q_t = \sum_{T=t}^{\infty} \delta^{T-t} \beta^{T+1-t} \frac{U_c(C_{T+1})}{U_c(C_t)} \left(\frac{P_t}{P_{T+1}} \right) \prod_{j=t+1}^{T+1} (1 - \varkappa_j).$$

In a perfect-foresight equilibrium the return on long-term bonds is also equal to the short-term interest rate as shown by combining (5) and (32)

$$r_{t+1} = i_t \tag{33}$$

with the return on long-term bonds defined by $r_{t+1} \equiv (1-\varkappa_{t+1})(1+\delta Q_{t+1})/Q_t - 1$.

The flow budget constraint of the treasury is unchanged assuming that it can only issue short-term securities. This assumption is made to simplify the analysis and without losing generality.

The central bank can instead invest also in long-term securities, D_t^C. Net worth, N_t^C, and profits, Ψ_t^C, are therefore given by

$$N_t^C \equiv Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t}, \tag{34}$$

$$\Psi_t^C = i_{t-1}(N_{t-1}^C + M_{t-1}^C) + (r_t - i_{t-1}) Q_{t-1} D_{t-1}^C. \tag{35}$$

Profits show now an additional component that represents the excess gains or losses of holding long-term securities with respect to a riskless portfolio. Since the excess return on these securities can be negative due to unexpected shocks, the latter component may as well be negative – the more so the larger are the holdings of long-term securities – producing income losses for the central bank.

Combining (7), (34) and (35), the central bank's flow budget constraint follows:

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t) D_{t-1}^C + B_{t-1}^C - X_{t-1}^C - M_{t-1}^C - T_t^C,$$

given initial conditions $D^C_{t_0-1}, B^C_{t_0-1}, X^C_{t_0-1}, M^C_{t_0-1}$ all equal to zero.

I now characterize in a compact way the equilibrium conditions of this more general model.

4.1 Equilibrium

The Fisher's equation still holds

$$1 + i_t = \frac{1}{\beta}\frac{P_{t+1}}{P_t}, \tag{36}$$

while the equilibrium price of long-term securities is:

$$Q_t = \sum_{T=t}^{\infty} \delta^{T-t}\beta^{T+1-t}\left(\frac{P_t}{P_{T+1}}\right)\prod_{j=t+1}^{T+1}(1-\varkappa_j). \tag{37}$$

Demand of real money balances is given by

$$\frac{M_t}{P_t} \geq L(Y, i_t) \tag{38}$$

with

$$i_t \geq 0 \tag{39}$$

and the complementary slackness condition

$$i_t\left[\frac{M_t}{P_t} - L(Y, i_t)\right] = 0. \tag{40}$$

The household's transversality condition can be simplified to

$$\lim_{T\to\infty}\left\{\beta^{T-t_0}\left(\frac{P_{t_0}}{P_T}\right)\left(M_T + \frac{B_T + X_T}{1+i_T} + Q_T D_T\right)\right\} = 0, \tag{41}$$

while the bound (4) can be written as

$$\sum_{T=t_0}^{\infty}\beta^{T-t_0}\left[Y + \frac{i_T}{1+i_T}L(Y, i_T)\right] < \infty$$

which is naturally satisfied.

The flow budget constraints of treasury and central bank are respectively

$$\frac{B_t^F}{1+i_t} = B_{t-1}^F - T_t^F - T_t^C, \tag{42}$$

$$Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t^C - \frac{X_t^C}{1+i_t} = (1-\varkappa_t)(1+\delta Q_t) D_{t-1}^C + B_{t-1}^C - X_{t-1}^C - M_{t-1}^C - T_t^C, \tag{43}$$

while equilibrium in the securities market closes the model

$$B_t + B_t^C = B_t^F, \tag{44}$$

$$M_t = M_t^C, \tag{45}$$

$$X_t = X_t^C, \tag{46}$$

$$D_t + D_t^C = 0. \tag{47}$$

A rational-expectations equilibrium is a collection of processes $\{P_t,\ i_t,\ M_t,\ Q_t,\ T_t^F,\ T_t^C,\ B_t,\ B_t^C,\ B_t^F,\ D_t^C,\ X_t\}_{t=t_0}^{\infty}$ that satisfy (36)-(44) at each date $t \geq t_0$ given (45)-(47) and initial conditions M_{t_0-1}, $B_{t_0-1}^C$, $B_{t_0-1}^F$,D_{t_0-1},X_{t_0-1} all equal to zero. Since (41) is a bound and considering the complementary slackness condition (40), there are now five degrees of freedom to specify the monetary/fiscal policy regime.

4.2 Inflation stability

There is an additional degree of freedom to consider in the specification of the monetary/fiscal policy regime with respect to the simple model of Section 3. If I assume $D_t^C = 0$, the results of Section 3 can be restated both under an active Taylor's rule and an interest-rate peg. However, there is one important difference to point out since now the central bank can get revenues from seigniorage because money balances provide non-pecuniary benefits. The value of the central bank, indeed, changes to

$$\frac{N_t^C}{P_t} + \sum_{T=t}^{\infty} \beta^{T-t} \frac{i_T}{1+i_T} \frac{M_T}{P_T} = \sum_{T=t+1}^{\infty} \beta^{T-t} \frac{T_T^C}{P_T}, \tag{48}$$

and is now given by the sum of net worth and seigniorage revenues. This additional source of revenues brings implications when I build the remittances' policy that trims off-equilibrium inflationary paths under an active Taylor's rule. A commitment to change the remittances policy to

$$\frac{T_t^C}{P_t} = \frac{1}{\beta}\frac{i_{t-1}}{1+i_{t-1}}\frac{M_{t-1}}{P_{t-1}} + i^* \frac{P_{t_0}\tilde{N}_{t_0}^C}{P^*(\Pi^*)^{t-t_o}}, \tag{49}$$

would work in this case. In particular, the off-equilibrium remittances' policy (49) replaces (23) under item 3c. Similarly, one should appropriately adjust the remittances' policy under an interest-rate peg to get determinacy. In any case, the general conclusion of the analyses of Section 3 remains unchanged since the central bank can still rely on internal resources – and now to a larger extent given the seigniorage revenues – to back the value of money.

The significant extension to consider is the one in which the central bank holds also long-term securities. Consider an active Taylor's rule and maintain all other elements of the fiscal/monetary policy regime unchanged, like in the simple model, on top of including non-zero holding of long-term assets. The steps of Section 3 would follow exactly along the same lines, and the equilibrium $\Pi_t = \Pi^*$ at all times will be uniquely implemented using (49) in place of (23) as an off-equilibrium threat. There is however one key difference. By holding long-term securities, central bank's profits can turn negative if there are unexpected movements in the prices of long-term bonds due to credit or interest-rate shocks. If this happens, the treasury should promptly support the central bank to cover income losses according to the remittances' policy (22).

Given this backing, I cannot anymore claim that the central bank can achieve a stable inflation rate by relying only on internal resources. Treasury's support is critical to achieve the desired inflation target on top of the initial injection of capital.

But, what will happen when the treasury does not support the central bank except for the initial capitalization? This is the question I am addressing in the next section.

4.3 Lack of treasury's support

I focus in this section on the case in which the interest-rate policy is set as an active Taylor's rule while fiscal policy is still passive as in (20). Therefore items 1 and 2 in the specification of the monetary/fiscal policy regime still apply in this subsection. The balance-sheet policy specifies appropriate positive sequences $\{B_t^C,\, D_t^C\}_{t=t_0}^{\infty}$ and therefore also allows for positive holdings of long-term debt. Concerning item 3, I still maintain the assumption that at time t_0 the treasury provides the initial capital through which the central bank starts its operations (item 3a). However, after time t_0, remittances are assumed to be non-negative, $T_t^C \geq 0$, excluding any possible support from the treasury. In particular, I assume a remittances' policy in which the central bank transfers all its income to the treasury provided nominal net worth is not below the initial level $\bar{N}$.[8] Therefore for each $t > t_0$ $T_t^C = \max(\Psi_t^C, 0)$ whenever $N_t \geq \bar{N}$ and $T_t^C = 0$ if $N_t < \bar{N}$. This remittances' policy has a real-world counterpart in the deferred-asset regime currently used by the Federal Reserve System for which, whenever capital falls, the central bank stops making remittances and accounts for a deferred asset in its balance sheet paid later by retained earnings. Only once the deferred asset is paid in full, the central bank returns to rebate profits to the treasury.

At this point I am leaving open the possibility that the central bank can also switch, off equilibrium, to another remittances' policy to see whether this threat really exists based only on internal resources, since treasury's support is now excluded.

4.3.1 Interest-rate risk

I study whether $\Pi_t = \Pi^*$ for each $t \geq t_0$ remains the unique global equilibrium. Consider the case in which the private sector expects instead $\Pi_{t+1} < \Pi^*$ at a certain time t and therefore a deflationary path implied by the Taylor's rule combined with the Fisher's equation. Given this switch in expectations, the price of long-term bonds rises and the central bank benefits of a capital gain on the holdings of long-term securities. Under the assumed remittances' policy the capital gain is immediately rebated to the treasury. Net worth remains constant and (25) ensures that solutions with deflationary paths are not equilibria. Therefore sudden shifts in expectations towards a deflationary path are not self-fulfilling equilibria given the monetary-fiscal policy regime assumed.

What will instead happen if the private sector expects $\Pi_{t+1} > \Pi^*$ at a certain time t and thereafter an increasing path of inflation consistent with the Taylor's rule and the Fisher's equation? The price of long-term bonds at time t would unexpectedly fall and this would cause losses for the central bank on the holdings of long-term securities. Note that using equations

[8] I define $\bar{N} \equiv P_{t_0}\tilde{N}_{t_0}$.

(19) and (37), Q_t can be expressed as a function of only Π_{t+1} and moreover $Q(\Pi_{t+1})$ is decreasing in Π_{t+1}.[9]

Since there is no treasury's support, the income loss translates into a fall in central bank's net worth. In the previous section, I showed that these inflationary spirals are not equilibria since the central bank is able to back equilibrium prices by using the threat (49). Here this backing may be at risk since central bank's net worth can instead fall. Given its dependence on r_t and therefore on Q_t, the level reached by central bank's net worth at time t is also a function of Π_{t+1}

$$N^C(\Pi_{t+1}) = i_{t-1}(N_{t-1}^C + M_{t-1}^C) + (r(\Pi_{t+1}) - i_{t-1})Q_{t-1}D_{t-1}^C, \tag{50}$$

in which I implicitly assume that central bank's income is negative and therefore that remittances are zero consistently with what prescribed by the deferred-asset regime.

Define now the seigniorage flow

$$s(\Pi_t) = \frac{\max\left[i(\Pi_t), 0\right]}{1 + \max\left[i(\Pi_t), 0\right]} L(i(\Pi_t), Y),$$

and let $\mathcal{S}_t$ the expected discounted value of the seigniorage

$$\mathcal{S}_t \equiv \sum_{T=t}^{\infty} \beta^{T-t} s(\Pi_T).$$

Note again that given the Taylor's rule and the Fisher's equation I can also write $\mathcal{S}_t$ as function of only Π_t, i.e. $\mathcal{S}(\Pi_t)$.

The central bank has insufficient backing to defeat an adverse shift in expectations when the following inequality holds

$$\frac{N^C(\Pi_{t+1})}{P^*(\Pi^*)^{t-t_0}} + \mathcal{S}(\Pi^*) < 0. \tag{51}$$

The left-hand side captures the real value of the central bank, again given by the two components: the real value of net worth and the present-discounted value of seigniorage. Nominal net worth is evaluated at the level it would reach if expectations at time t were to shift to an inflationary path as implied by (50). Instead, seigniorage and prices are evaluated at the stable inflation

[9] I am implicitly assuming that $\varkappa_t = 0$ at all times.

equilibrium, $\Pi_t = \Pi^*$ at all times. When (51) holds, the intertemporal constraint (48) shows that the presented discounted value of remittances to the treasury is negative and therefore that the central bank has insufficient "internal resources" to back the price level consistently with the inflation target Π^*. Following an upward shift in inflation expectations, it is not possible to find a positive remittances' policy that backs the value of money at the desired target. Inflationary equilibria are not defeated.[10]

The multiplicity of equilibria disappears if the treasury supports the central bank by covering income losses as I discussed in Section 4.2. But here treasury's support is excluded showing then its relevance for inflation stability when central banks hold also long-term assets.

Finally note that the results of this section share similarities with those discussed in Del Negro and Sims (2015) but with an important difference. In their analysis, multiplicity appears as a shift to a different interest-rate rule, since it is Π^* that changes across equilibria. In my analysis, the Taylor's rule remains unchanged and the multiplicity arises along the multiple solutions that the Taylor's rule and the Fisher's equation inherently imply as shown in (19) given the inability of the central bank alone to trim some of these paths using internal resources.[11]

4.3.2 Credit risk

I consider now the consequences of an unexpected realization of a credit event. Starting from a perfect foresight equilibrium in which $\varkappa_t = 0$ at all times, assume that at time t there is default $\varkappa_t > 0$ or alternatively that private agents expect at time t that $\varkappa_T > 0$ at some future date $T > t$. In both cases, the time-t return on long-term bonds unexpectedly falls which could lead to negative profits and to a fall in net worth. I will now evaluate whether this unexpected shock, given a deferred asset regime, creates problems to the uniqueness of the solution I found in Section 3.

If net worth falls but not to the point that it jeopardizes the profitability of the central bank, the central bank itself maintains the capacity to recover

[10] Moreover, note that (51) with an equality sign defines a threshold $\tilde{\Pi}_{t+1}$ such that for all $\Pi_{t+1} \geq \tilde{\Pi}_{t+1}$, the inequality holds. It is easy to see that the threshold $\tilde{\Pi}_{t+1}$ is lower, the higher the holdings of long-term bonds are, the lower the seigniorage revenues are, the lower the initial level of capital is, the more distant time 't' is.

[11] There might be also self-fulfilling inflationary equilibria in which inflation is stable at Π^* and then switches to an inflationary path at some point in time.

previous losses using retained earnings and therefore can restore the initial level of net worth in a finite period of time $T > t$. The following condition should be satisfied at time t

$$\frac{N^C(\varkappa, \Pi_{t+1})}{P^*(\Pi^*)^{t-1-t_0}\Pi_t} + \mathcal{S}(\Pi_t) > 0 \tag{52}$$

evaluated when $\Pi_{t+1} = \Pi_t = \Pi^*$ in which $N^C(\varkappa, \Pi_{t+1})$ defines the level of net worth reached at time t given the unexpected credit event $\varkappa$. The function $N^C(\varkappa, \Pi_{t+1})$ shows also the dependence on Π_{t+1} through the price of long-term bonds Q_t.

If the above inequality holds at the equilibrium $\Pi_t = \Pi^*$, the intertemporal constraint (48) reveals that the presented discounted value of remittances to the treasury is positive notwithstanding the credit event. Under a deferred assets regime and in a perfect-foresight equilibrium, the latter result implies that central bank's net worth returns to its initial level $\bar{N}$ in a finite period of time. Therefore $\Pi_t = \Pi^*$ is still an equilibrium even though it is not unique. The conclusions of the previous subsection still hold showing that inflationary spirals are also equilibria. In fact, given credit losses, it is even easier that central-bank net worth turns out to be insufficient to provide the appropriate off-equilibrium threat that defeats inflationary solutions. Equation (52) can be indeed violated at $\Pi_t = \Pi^*$ for some $\Pi_{t+1} > \Pi^*$.

What is instead going to happen when the inequality sign of (52) evaluated at $\Pi_{t+1} = \Pi_t = \Pi^*$ is reversed perhaps following a stronger credit event? Using again (48), it follows that the present discounted value of remittances to the treasury is going to be negative in this case. This is not feasible under a deferred-asset regime since T_t^C should be non-negative. Therefore $\Pi_t = \Pi^*$ is no longer an equilibrium.

Maintaining the Taylor's rule unchanged, I now investigate if there are other possible equilibria. Consider the case in which, at the time of the credit event, there is also a shift in expectations towards a deflationary solution starting from Π_t in the range $\beta \leq \Pi_t < \Pi^*$. Observe that when $\beta \leq \Pi_t < \Pi^*$ at time t the deflationary trap $\Pi_{\tilde{T}} = \beta$ is reached in a finite period of time $\tilde{T} \geq t$ and remains thereafter for all the following periods. In order for the deflationary trap to be an equilibrium it must be that nominal net worth converges to zero

$$\lim_{T \to \infty} \left\{ N_T^C \right\} = 0,$$

as implied by (24). As a consequence, remittances should be always zero after (and including) period t since $N_T < \bar{N}$ for each $T \geq t$.[12] Using this observation into the equilibrium condition (48), it follows that a deflationary equilibrium exists if and only if

$$\frac{N_T^C}{P^*(\Pi^*)^{t-1-t_0} \prod_{j=t}^{T} \Pi_j} + \mathcal{S}(\Pi_T) = 0$$

for each $T \geq t$, where the path of inflation is given by (19) starting from an inflation rate Π_t in the range $\beta \leq \Pi_t < \Pi^*$ while net worth follows

$$N_T^C = N_{T-1}^C + i_{T-1}(N_{T-1}^C + M_{T-1}),$$

starting from the level reached at time t, $N^C(\varkappa, \Pi_{t+1})$, and considering the implied equilibrium interest rate and money holdings. Since $\Pi_{\tilde{T}} = \beta$ is reached in a finite period of time and at the same date seigniorage is zero, $\mathcal{S}(\beta) = 0$, it follows that net worth reaches zero in the same period, $N_{\tilde{T}}^C = 0$.[13] Note that time $\tilde{T}$ can even coincide with time t, meaning that a credit event combined with a shift toward deflationary expectations can suddenly bring the economy into a permanent liquidity trap. Deflationary equilibria may therefore occur following a sizeable credit event, though it is worth noting that this can happen depending on certain parameterization and surely not for all possible $\varkappa$.

Consider now inflationary solutions. They can be equilibria if and only if

$$\frac{N_t^C(\varkappa, \Pi_{t+1})}{P^*(\Pi^*)^{t-1-t_0}\Pi_t} + \mathcal{S}(\Pi_t) > 0 \tag{53}$$

at time t. Whereas (52) can be violated and therefore $\Pi_t = \Pi^*$ is not an equilibrium, paths instead in which Π_t starts above Π^* and increases thereafter, consistently with the Taylor's rule and the Fisher's equation, might be feasible provided seigniorage revenues are enough to make those paths an equilibrium. On the contrary, assume that (53) is violated for all $\Pi_t > \Pi^*$ then inflationary solutions are not equilibria.

A sizeable credit event implies therefore that the stable inflation target Π^* is no longer an equilibria. There could exist inflationary and deflationary

[12] Recall that under a deferred-asset regime $T_t^C = 0$ whenever $N_t < \bar{N}$.

[13] It should be that $N^C(\varkappa, \Pi_{t+1}) \leq 0$ at time t.

equilibria but it is also possible that no equilibrium exists. In this case, equilibria can form if the central bank changes the Taylor rule centering it around a different inflation target Π^* or if the treasury starts to support the central bank. The central bank has to give up either its inflation target or its financial independence.

5 Conclusion

I have described a monetary/fiscal policy regime that can uniquely implement a stable inflation rate in a simple endowment monetary economy. The important feature of the regime is that once the central bank is appropriately designed with an initial level of capital, a specified remittances' policy and the requirement of holding only riskless securities then it is equipped with all the relevant tools to defeat deflationary and inflationary spirals without the need of fiscal support. The elements underlined are not new compared with the evidence on how central banks are designed and some of them are consistent with what economists have been arguing for hundreds years.[14] What is new is that they can determine uniquely a stable inflation rate, once combined with a Taylor's rule or interest-rate pegs, something that the related literature has hardly ever managed to achieve without treasury's "activism".

The proposal of this work is based on commitments and threats which can be questioned, although one has to admit that those commitments and threats do not involve any external support of the central bank. However, they might be subject to public debate or renegotiations with the government along the way. I leave for future works these political-economy considerations.

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