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JEL Classification: D52, E31, E43, E52, E62, E63

Keywords: incomplete markets, inflation, monetary policy, Fiscal policy, zero lower bound, Fiscal Multiplier, forward guidance

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Keywords: Price Level Determinacy, Ricardian Equivalence, Incomplete Markets, Inflation, Monetary Policy, Fiscal Policy, Policy Coordination

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1 Introduction

The prevailing view on monetary policy (Woodford, 2003; Galí, 2015) is that it works through setting nominal interest rates, that it controls the inflation rate, and that prices are determinate if monetary policy responds sufficiently strongly to inflation - the Taylor principle. Fiscal policy is largely irrelevant in this view.

In this paper, I propose that heterogeneous agent incomplete markets models offer a new and different perspective on these topics. While monetary policy continues to control the nominal interest rate, fiscal policy is now assigned a significant role. Two results stand out.

First, in contrast to the conventional view, in the absence of output growth and even when monetary policy operates an interest rate rule with a different inflation target, the long-run inflation rate is equal to the growth rate of nominal fiscal variables, which are controlled by fiscal policy. A tough, independent central bank is not only insufficient to ensure price stability in the long-run, it also has no direct control over long-run inflation, even if it follows an interest rate rule which satisfies the Taylor principle.[1] The fiscal determination of long-run inflation leads to a reinterpretation of one of Friedman's (1963, 1968, 1970) key propositions that "inflation is always and everywhere a monetary phenomenon in the sense that it is and can be produced only by a more rapid increase in the quantity of money than in output." Friedman assumed that the money supply, purposefully and astutely controlled by central banks, serves as a nominal anchor to control inflation. 50 years after Friedman (1963, 1968, 1970) however, central bank practice and academic research has shifted to using a short-term nominal interest rate as the policy instrument. The money supply is no longer controlled by monetary policy, but determined endogenously as clearing the money market. I incorporate these features into the model, which strips money of its anchoring role envisaged by Friedman. However, the nominal fiscal variables, controlled by fiscal policy, such as nominal government debt and spending, replace money in this role.

Second, incomplete markets models provide a new theory of price-level determination. Monetary policy works through setting an arbitrary sequence of nominal interest rates, for example through an interest rate rule. Fiscal policy sets sequences of nominal government spending, taxes, and government debt, for example through a fiscal rule, and these sequences satisfy the present value government budget constraint at all times and for all prices, that is, the fiscal policy is passive (Leeper, 1991). In this environment, I show that the steady-state price level is determinate, even if nominal interest rates are constant, and I derive conditions for policy rules that ensure local determinacy. It is this determinacy result which rules out sunspot-driven inflation movements and ensures that the price

[1] Central bank independence ensures, however, that the treasury cannot impose fiscal policies on the central bank, e.g. monetizing its debt to finance the government.

level and the inflation rate are uniquely determined by policy. As I will show, the price level is determined jointly by monetary and fiscal policy, and the long-run inflation rate by fiscal policy only.

To understand these results, it is sufficient to combine a few simple insights and it is instructive to start with a steady state. First, market incompleteness generates a precautionary savings demand such that the household sector effectively values real government bonds, i.e. their real value exceeds the value of tax liabilities. When, in addition, government bonds are nominal, shifts in the price level shift the real value of debt. The price level is then determined such that the demand for real bonds equals its supply. In contrast, if markets were complete, Ricardian equivalence would hold and the household sector would be indifferent to the amount of government bonds available, leaving the price level indeterminate.

Second, the steady-state real interest rate is not determined uniquely by a household discount factor, but instead depends on the amount of real bonds available. In incomplete markets models, households are willing to accept a lower return than suggested by their discount factor, as bonds allow households to smooth consumption more effectively in response to uninsurable idiosyncratic income shocks (Aiyagari, 1995). Bonds have a "liquidity premium", which under standard assumptions is decreasing in the amount of bonds households own. To absorb more government bonds, households thus require a higher real interest rate. This indicates, that depending on the amount of bonds available, a continuum of steady-state real interest rates is feasible. Monetary and fiscal policy now jointly choose one out of this continuum of potential steady-state real interest rates. Monetary policy sets the steady-state nominal interest rate, whereas fiscal policy sets the growth rate of nominal government debt. In a steady state, the value of real government debt is constant, such that the steady-state condition for fiscal policy is that the growth rate of nominal debt equals the inflation rate (in the absence of economic growth). The real interest rate is then determined by the Fisher equation, as the ratio of the nominal interest rate to the inflation rate. Clearly, this logic for determining the long-run inflation rate does not apply if the steady-state real interest rate is pinned down by the discount factor, as is the case in standard New Keynesian complete markets models.

Section 3 builds on these simple insights and shows that the steady-state price level is determinate in a large class of heterogenous agent incomplete markets models. I also explain the mechanism behind the determinacy result; why incomplete market models deliver determinacy and why complete markets models lead to indeterminacy when nominal interest rates are constant; why fiscal policy has to be partially nominal; why the Demand Theory of the Price Level proposed here is not the Fiscal Theory of the Price Level (FTPL); and why adding capital or money to the model does not alter these conclusions.

Outside of steady states, I demonstrate local determinacy for all monetary policy rules, that is, those not responding, responding weakly or responding strongly to prices in Section 4. Interestingly, by responding too strongly to price increases, fiscal policy may induce rather than remove indeterminacy. In such an expansionary fiscal policy scenario, reestablishing determinacy requires monetary policy to increase nominal interest rates in response to price increases. I provide a characterization of these determinacy conditions, including how responsive monetary policy has to be if fiscal policy is excessively expansionary.

It is important to emphasize that these results do not hold in all models in which Ricardian equivalence fails. For example, the price level is not determinate in an economy where a fraction of households simply consume their current income "hand-to-mouth", while the remaining households act according to the permanent income hypothesis (PIH).[2] The reason for the indeterminacy is that only permanent-income households hold bonds, and thus shifts in the value of public debt have no aggregate demand effects, but only shift consumption from one group to the other. Similar arguments apply to the perpetual youth model and its variants (Yaari, 1965; Blanchard, 1985; Bénassy, 2005, 2008), in which Ricardian equivalence does not hold, but the price level is indeterminate, as explained in the Appendix.

This paper shows that using the workhorse incomplete markets model not only provides an empirically superior model of consumption (Kaplan and Violante, 2014) - a key part of the monetary transmission mechanism - but also entails a windfall gain: the price level is determinate. This seems important, since a growing body of the literature has recently emerged which incorporates price rigidities into heterogeneous agent incomplete markets models (HANK).[3] One motivation to do so is that, while able to generate a realistic distribution of marginal propensities to consume, the textbook incomplete markets model does not allow output to be demand-determined, as prices are fully flexible, potentially limiting its applicability to many questions raised by the Great Recession. Adding a nominal side to the model and allowing for price rigidities however, forces us to address the same questions we confront in complete markets models: How is the price level determined? What type

[2]Two Agent New Keynesian (TANK) models are a popular model class, due to their theoretical elegance and tractability (Bilbiie, 2008, 2017, 2018).

[3]See Kaplan and Violante (2018) for a recent review of this emerging Heterogeneous Agents New Keynesian (HANK) literature. Additional references include Gornemann et al. (2012), Kaplan et al. (2018), Auclert (2016) and Lütticke (2015) who study monetary policy in a model with incomplete markets and pricing frictions, but with a different focus, emphasizing and quantifying several redistributive channels of the transmission mechanism of monetary policy, which are absent in standard complete markets models. Earlier contributions are Oh and Reis (2012) and Guerrieri and Lorenzoni (2017), who were among the first to add nominal rigidities to a Bewley-Imrohoroglu-Huggett-Aiyagari model. More recent contributions include McKay and Reis (2016) (Impact of automatic stabilizers), McKay et al. (2016) (Forward guidance), Bayer et al. (2019) (Impact of time-varying income risk), Ravn and Sterk (2017) (Increase in uncertainty causes a recession), Den Haan et al. (2017) (Increase in precautionary savings magnifies deflationary recessions), Auclert and Rognlie (2017) (Inequality and aggregate demand) and Auclert et al. (2018) (Intertemporal Keynesian Cross). While all these papers address issues that are complimentary to my paper, the price level is shown to be determined endogenously in equilibrium only in my paper. In terms of assumptions, the main reason for this difference is that government bonds are nominal here, whereas they are assumed to be real in the cited work. Only Hagedorn et al. (2017a) (Fiscal multiplier), Hagedorn et al. (2019) (Forward guidance) and Hagedorn et al. (2017b) (Model estimation) assume nominal bonds.

of monetary and fiscal policies ensure determinacy? How does taking the zero lower bound (ZLB) into account affect the conclusions derived from the model? The determinacy results in this paper provide these answers and show that they are quite different from the standard analysis based on complete markets. The results are derived within the standard framework in macroeconomics and dynamic public finance - heterogeneous agent incomplete markets models - and are obtained without invoking any assumptions on policy except that it is partially nominal. Nevertheless, the literature has overlooked this model property. Section 5 concludes and discusses how the novel theory proposed in this paper offers new answers to various issues in monetary economics.

2 A Heterogeneous Agent Incomplete Markets Model

In this Section, I describe a heterogeneous agents endowment economy with uninsurable idiosyncratic labor income risk in which markets are incomplete, based on Huggett (1993), where only a government bond can be traded, subject to exogenously imposed borrowing limits. I consider a cashless economy as in Woodford (2003) where monetary policy sets the nominal interest rate. The household sector is fully described in real terms, as this allows for a clean definition of aggregate savings in real terms in Section 2.4.

2.1 Households

The economy is populated by a continuum of households of measure one with preferences over consumption c_t

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where u is increasing and strictly concave, β is the subjective discount factor and the expectation is taken over realizations of idiosyncratic endowments. Agent endowment $\{e_t\}_{t=0}^{\infty}$ is stochastic and characterized by an N-state Markov chain that can take on values $e_t \in \mathcal{E} = \{e_1, \cdots, e_N\}$ with the transition probability characterized by $\vartheta(e'|e)$ and $\int e = 1$. A household saves real bonds a_{t+1} in period t subject to the credit constraint

$$a_{t+1} \geq -\bar{a} \tag{2}$$

for a credit limit $\bar{a}$.[4] The real real return is $1 + r_{t+1}$ and thus, at time t, the household faces the budget constraint

$$c_t + a_{t+1} \quad = \quad (1 + r_t)a_t + e_t - \tau_t, \tag{3}$$

where τ_t is a lump sum tax. The household choice problem can be written recursively, given initial conditions (a_0, e_0),

$$\begin{aligned} V_t(a, e; \Omega) = \max_{c \geq 0, a' \geq -\bar{a}} u(c) + \beta \sum_{e' \in \mathcal{E}} \vartheta(e'|e) V_{t+1}(a', e'; \Omega') \qquad & (4) \\ \text{subject to} \qquad c + a' = (1 + r_t)a + e - \tau_t & \\ \Omega' = \mathcal{T}(\Omega), & \end{aligned}$$

where a prime denotes the next period's value, $\Omega(a, e) \in \mathcal{M}$ is the distribution on the space $X = \mathcal{A} \times \mathcal{E}$, agent asset holdings $a \in \mathcal{A}$ and endowment $e \in \mathcal{E}$, across the population, which, together with the policy variables, determine the equilibrium prices. $\mathcal{T}$ is an equilibrium object that specifies the evolution of the distribution Ω in the set of probability measures $\mathcal{M}$ over X with σ-algebra $\mathbb{B}(X)$. The transversality condition is

$$\lim_{T \to \infty} E_0[\beta^T u'(c_T) a_{T+1}] \quad = \quad 0. \tag{5}$$

2.2 Government

Monetary policy sets the nominal interest i_t, for which I allow the sequence of nominal interest rates to be exogenous or to follow from some feedback rule. The aggregate Period t price level is P_t, the real interest rate $1 + r_{t+1} = (1 + i_{t+1}) \frac{P_t}{P_{t+1}}$ and the inflation rate is $1 + \pi_{t+1} = \frac{P_{t+1}}{P_t}$.[5] Since the steady state real interest rate can be negative and there is no money in this model, negative nominal interest rates are possible. Fiscal policy sets the nominal level of bonds B_t and adjusts nominal lump-sum

[4] An alternative assumption to an exogenous credit constraint $\bar{a}$ is to impose natural debt limits. The appendix considers this alternative assumption.

[5] It is important to bear in mind that here, as in all the recent literature on monetary economics, the central bank sets the nominal interest rate on short-term bonds and not the money supply, which gives rise to price level indeterminacy in the first place (Sargent and Wallace, 1975).

taxes T_t to satisfy the government budget constraint at all times[6]

$$T_t := (1 + i_t)B_t - B_{t+1}, \tag{6}$$

so that households pay taxes $\tau_t = T_t/P_t$ in equilibrium.

2.3 Equilibrium

The asset-market clearing condition is

$$\frac{B_{t+1}}{P_t} = \int a_{t+1} d\Omega_t \tag{7}$$

and the resource constraint reads

$$\int c_t d\Omega_t = \int e_t d\Omega_t = 1. \tag{8}$$

A *competitive equilibrium* is then a sequence of prices P_t, i_t, r_t, taxes T_t, bonds B_t, value function V_t with policy functions c, a' and a law of motion $\mathcal{T} : \mathcal{M} \to \mathcal{M}$ such that:

1. Households maximize utility, taking prices and government policies as given.
2. The government budget constraint is satisfied.
3. The resource constraint is satisfied.
4. The transversality condition (5) holds.
5. The asset market clears.
6. The aggregate law of motion $\mathcal{T}$ is generated by a' and p.

A *stationary competitive equilibrium* is a competitive equilibrium where $\Omega_t, 1+r_t$, $1+\pi_t$ and $1+i_t$ are constant and

$$\frac{B_t}{B_{t-1}} = \frac{T_t}{T_{t-1}} = \frac{P_t}{P_{t-1}} = 1 + \pi_{ss}. \tag{9}$$

[6]The government budget constraint is specified here in nominal terms. The only purpose of this specification is that it enables to easily distinguishing this paper's theory from the Fiscal Theory of the Price Level (FTPL) as I discuss below. As I show in the appendix, my theory allows for adding real and nominal government expenditures and more complicated real or nominal tax functions. The theory only requires that fiscal policy be partially nominal, i.e. not fully indexed to the price level.

2.4 The Savings Function

It is standard to characterize the stationary equilibrium through the asset market clearing condition and I follow this approach here. Given initial period t assets a_t, sequences of real interest rates $(r_t, r_{t+1}, r_{t+2}, \ldots)$ and taxes $(\tau_t, \tau_{t+1}, \ldots)$, each household maximizes utility yielding a sequence of savings $(a_{t+1}, a_{t+2}, \ldots)$. Aggregate savings at time t,

$$S_{t+1} = \int a_{t+1} d\Omega_t, \tag{10}$$

is then a function of the initial distribution of assets Ω_t, real interest rates and taxes,

$$S_{t+1} = \mathcal{S}(\Omega_t; 1 + r_t, 1 + r_{t+1}, 1 + r_{t+2}, \ldots; \tau_t, \tau_{t+1}, \ldots). \tag{11}$$

In a stationary equilibrium, the real interest rate r_{ss}, aggregate savings S_{ss} and the distribution of assets Ω_{ss} are invariant, and taxes $\tau_{ss} = r_{ss}S_{ss}$. Steady-state savings is the fixed point of

$$S_{ss} = \mathcal{S}(\Omega_{ss}; 1 + r_{ss}, 1 + r_{ss}, 1 + r_{ss}, \ldots; r_{ss}S_{ss}, r_{ss}S_{ss}, \ldots), \tag{12}$$

such that steady-state savings is a function $S(1 + r)$ of the real interest rate only.[7] The equilibrium is characterized through the asset market clearing condition,

$$S(1 + r) \quad = \quad \frac{B}{P}, \tag{13}$$

since equilibrium conditions 1, 2 and 4 are satisfied by construction of the S function, and the asset market clearing (condition 5) implies that the resource constraint (condition 3) is satisfied.[8] If the inflation rate π is not zero, bonds in a steady state issued at time t equal $B(1 + \pi)^t$ and the price level equals $P(1 + \pi)^t$ for initial values B and P, so that the term $(1 + \pi)^t$ term cancels itself out when computing the real value of bonds B/P.

3 Steady-state Price Level Determinacy

In this Section, I establish that the steady-state price level is determinate. This is particularly difficult, since the nominal interest rate is constant in a steady state and therefore, the Sargent and

[7] More precisely, the fixed point problem is to find the fixed point Ω_{ss} of the law of motion $\mathcal{T} : \mathcal{M} \to \mathcal{M}$ which then satisfies $S_{ss} = \int a d\Omega_{ss} = \mathcal{S}(\Omega_{ss}; 1 + r_{ss}, 1 + r_{ss}, 1 + r_{ss}, \ldots; r_{ss} \int a d\Omega_{ss}, r_{ss} \int a d\Omega_{ss}, \ldots)$. For existence and uniqueness under standard assumptions see for example Acemoglu and Jensen (2015) and Açikgöz (2018).

[8] Note that that this steady-state saving function is different from the savings function which just aggregates individual household decisions taking prices as given. This is on purpose. Substituting taxes τ_{ss}, using the government budget constraint, allows me to characterize the equilibrium through one equation (13) as a function of one unknown, the real interest rate.

Wallace (1975) critique applies fully. One way to understand why the price level in complete markets models is indeterminate is to note that the number of endogenous variables exceeds the number of equilibrium conditions by one. I show that the asset-market clearing condition in incomplete markets models provides an additional equation that is needed to determine the price level. The formal analysis is complemented with a graphical one, which together show that this result holds not only in a large class of incomplete market models, but in all models with a steady-state savings curve, which describes a trade-off between the real return and the quantity of the asset. Prices are assumed to be flexible throughout the steady-state analysis, since price stickiness is irrelevant for steady-state determinacy, simply because a steady state requires no price-adjustment other than an increase at the steady-state inflation rate (Nakajima and Polemarchakis, 2005). Price rigidities will be added in the local determinacy analysis in Section 4.

3.1 Asset-Market Clearing and Price-Level Determinacy

As is well known, the incomplete markets economy is in equilibrium if aggregate asset supply (households' savings) equals real aggregate asset demand (government bonds), which can be represented by the well-known Figure 1 (left panel).[9] Household asset demand $S(1+r)$ is an upward sloping function of the real interest rate $1+r$ which is smaller than $1/\beta$. Real steady-state asset supply from the government equals $\frac{B}{P} = \frac{B(1+\pi)^t}{P(1+\pi)^t} = \frac{B_{t+1}}{P_t}$, such that the equilibrium condition is

$$S(1+r) = \frac{B}{P}. \tag{14}$$

This is one equation with two unknowns, the real interest rate $1+r$ and the price level P. This suggests that a continuum of price levels (associated with a continuum of real interest rates), e.g. P_1, P_2, P_3, clears the asset market as illustrated in the right panel of Figure 1. I will now argue that equation (14) nevertheless determines the price level, since the real interest rate is determined by monetary and fiscal policy.

The reader familiar with the Bewley (1980) basic model of fiat money will notice the similarities but also the differences of the two approaches. Following the exposition of the Bewley model in Ljungqvist and Sargent (2012) but using my notation, the central bank provides a fixed supply of money M, the only asset in this economy. The derivation of the aggregate money demand function in Bewley, $L(\pi_{ss})$, is basically identical to the derivation of my S function with the important difference that L depends on the real return on money, $\frac{1}{1+\pi_{ss}}$. The steady-state inflation rate is zero so that the real return on money is 0. The steady-state price level P^* is then determined as a solution to

[9] For a textbook treatment of incomplete markets models and their steady states, see Ljungqvist and Sargent (2012).

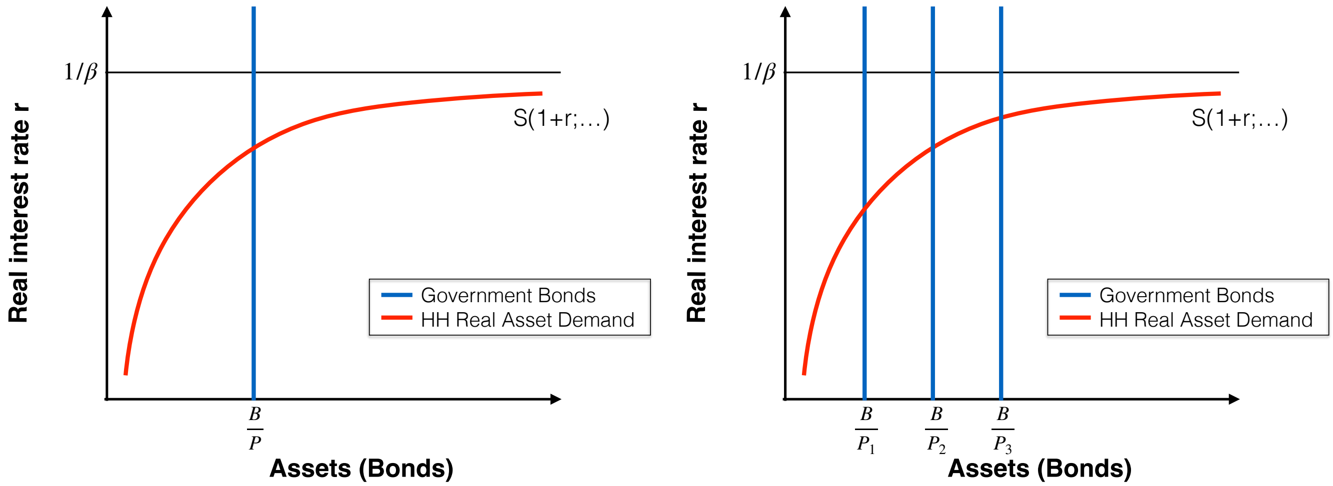

Figure 1: Asset market in incomplete markets economy

the money-market clearing condition

$$\frac{M}{P^*} = L(0), \tag{15}$$

which at first glance looks like replacing M with B and L with S in (14). One important difference is that (14) does not determine the price level yet since the real interest rate is an unknown whereas the money-market equation (15) does determine the price level. From a conceptual level, the approaches are quite different, mainly because of the Bewley model's different objective. The Bewley model provides a very elegant way to derive a demand for money from first principles, which leads to the quantity equation (15) known from textbook models. Unsurprisingly, a different objective means that the Bewley model misses elements that are crucial here. There are no bonds in the Bewley model, limiting the role of fiscal policy, the central bank sets money supply and does not and cannot freely set the nominal return on bonds.[10] I will explain these differences and the implications for price and inflation determination and fiscal and monetary policy in more detail in the next Sections.

3.2 How Monetary and Fiscal Policy Determine the Steady-State Real Interest Rate

A key step in the argument is to show how monetary and fiscal policy determine the steady-state real interest rate. In both complete and incomplete markets models, a Fisher relationship between

[10] Coexistence of bonds and money in the Bewley model would require that both assets have the same return, that is the return i_{ss} on bonds has to equal the return on currency i^m and thus i_{ss} cannot be set freely and different from i^m. Note that setting the nominal interest rate in the Sargent and Wallace (1975) critique means the nominal return on bonds and not an interest rate on some other asset such as currency.

the steady-state nominal interest i_{ss}, real interest rate r_{ss} and inflation π_{ss} holds:

$$1 + r_{ss} = \frac{1 + i_{ss}}{1 + \pi_{ss}}. \tag{16}$$

Monetary policy sets the steady-state nominal interest rate i_{ss}. Fiscal policy sets the growth rate of nominal debt (B) and adjusts nominal tax revenue (T) to balance the government budget. In a steady state, real tax revenue and real government debt are constant, such that the steady-state condition for fiscal policy is that the growth rates of nominal tax revenue and nominal debt all equal the inflation rate in the absence of economic growth (a prime denotes the next period's value),[11]

$$1 + \pi_{ss} = \frac{B' - B}{B} = \frac{T' - T}{T}. \tag{17}$$

Non-constant growth rate policies would be inconsistent with a steady state and are considered in Section 4. Note that a specific interpretation is assigned to these steady-state conditions: If fiscal policy decides on a 2% nominal growth rate in nominal debt, $\frac{B'-B}{B}$, then the steady-state condition for steady-state real government debt to be constant requires the steady-state inflation rate to equal 2% as well. The steady-state further requires nominal tax revenue T also to grow at 2%. It is important to note that these considerations do not determine the levels of real taxes and real debt, except in the sense that these are unchanging over time in a steady state. In particular, the price level has not yet been determined.

Equation (17) means that the inflation rate is equal to the growth rate of nominal government debt, implying that the equilibrium real interest rate is determined jointly by monetary and fiscal policy.[12] These conclusions about the steady-state inflation rate are valid, even if monetary policy implements an interest rate rule such as

$$i_{t+1} = \max(\bar{i} + \phi(\pi_t - \pi^*), 0), \tag{18}$$

for an inflation target π^*, an intercept $\bar{i}$ and $\phi > 0$. In this case, inflation is still determined by equation (17) and the steady-state nominal interest rate equals[13]

$$i^{ss} = \max(\bar{i} + \phi(\frac{B' - B}{B} - \pi^*), 0). \tag{19}$$

[11]With real economic growth of rate γ, $(1 + \pi_{ss})(1 + \gamma) = \frac{B'-B}{B} = \frac{T'-T}{T}$.

[12]Monetary and fiscal policy cannot simply implement an arbitrary steady-state real interest rate, but only one that is consistent with a steady state. In particular $\beta(1 + r_{ss}) < 1$, since otherwise, asset demand would become infinite, also a well-known result in incomplete markets models.

[13]For example if $\bar{i} = 0.02$, $\phi = 1.5$, debt grows at $\frac{B'-B}{B} = 0.02$ and the inflation target $\pi^* = 0$, steady-state inflation is then 2% and the nominal interest rate equals $i^{ss} = 0.02 + 1.5 * 0.02 = 0.05$. In the (less realistic) case that the inflation target of monetary policy $\pi^* = 0.04$ exceeds the 2% that follows from fiscal policy, the steady-state nominal interest rate equals $i^{ss} = \max(0.02 + 1.5(0.02 - 0.04), 0) = 0$ and inflation is still 2%.

Note that this line of reasoning requires a continuum of potential steady-state real interest rates, and not just one equal to $1/\beta$, as in complete markets models. Therefore, this logic for determining the long-run inflation rate does not apply if markets are complete.

3.3 Price Level Determinacy

I can now use equation (14) to determine the price level. Using the result that $(1 + r_{ss}) = \frac{1+i_{ss}}{1+\pi_{ss}}$ is set by policy to eliminate the real interest rate from the list of unknowns, equation (14) now has just one unknown, the price level P^*:

$$S(\frac{1 + i_{ss}}{1 + \pi_{ss}}) = \frac{B}{P^*}, \tag{20}$$

which serves to determine the unique price level, as illustrated in the left panel of Figure 2. This reasoning is based on an upward sloping savings curve, which delivers a unique asset market clearing price P^*. If $\overline{a} = 0$ (no borrowing), a sufficient condition for an upward sloping savings curve and thus uniqueness in real incomplete markets models is that the intertemporal elasticity of substitution is weakly greater than one (Achdou et al., 2017).[14] If this sufficient condition is not met, the underlying real model could have a backward bending savings curve, such that multiple real stationary equilibria exist. In this case, a unique price level is associated with each of the real equilibria. Unsurprisingly, this paper's theory overcomes nominal but not real indeterminacies.

There are two key assumptions to obtain price level determinacy. First, fiscal policy is nominal. Without this assumption, fiscal policy would be specified fully in real terms, and the equilibrium in the asset market would not depend on the price level, such that the equilibrium condition cannot be used to determine the price level.[15] Second, there is a steady-state aggregate asset demand function, which depends on the real interest rate. This is a standard result in models with heterogeneous agents and market incompleteness. I explain below why the arguments supporting the incomplete markets economy do not apply in complete markets environments.

The infinite horizon assumption ensures a non-degenerated savings curve in each period and in particular in a steady state that can be used to determine the price level. To understand this better, assume a finite horizon such that in the last period, the demand for bonds is zero. Asset market

[14]Household consumption $c(a, e; r, \tau) = c(a, e; r, rS)$ is decreasing in τ, so that an intertemporal elasticity of substitution weakly greater than one implies that $\frac{\partial c(a)}{\partial r} < 0$. Since household savings $s(a, e; r, \tau) = (1+r)a + e - \tau - c(a, e; r, \tau)$ aggregate savings

$$S = \int s(a, e; r, \tau) d\Omega_r = \int (1 + r)a + e - rS - c(a, e; r, rS) d\Omega_r = \underbrace{\int r(a - S) d\Omega_r}_{=0} + \int a + e - c(a, e; r, rS) d\Omega_r \tag{21}$$

is increasing in r, since $\int a + e - c(a, e; r, rS) d\Omega_r$ is increasing using the same arguments as in (Achdou et al., 2017) to show that the monotonicity of c carries over to the stationary distribution.

[15]I consider nominal government expenditure in Appendix A.I.2.

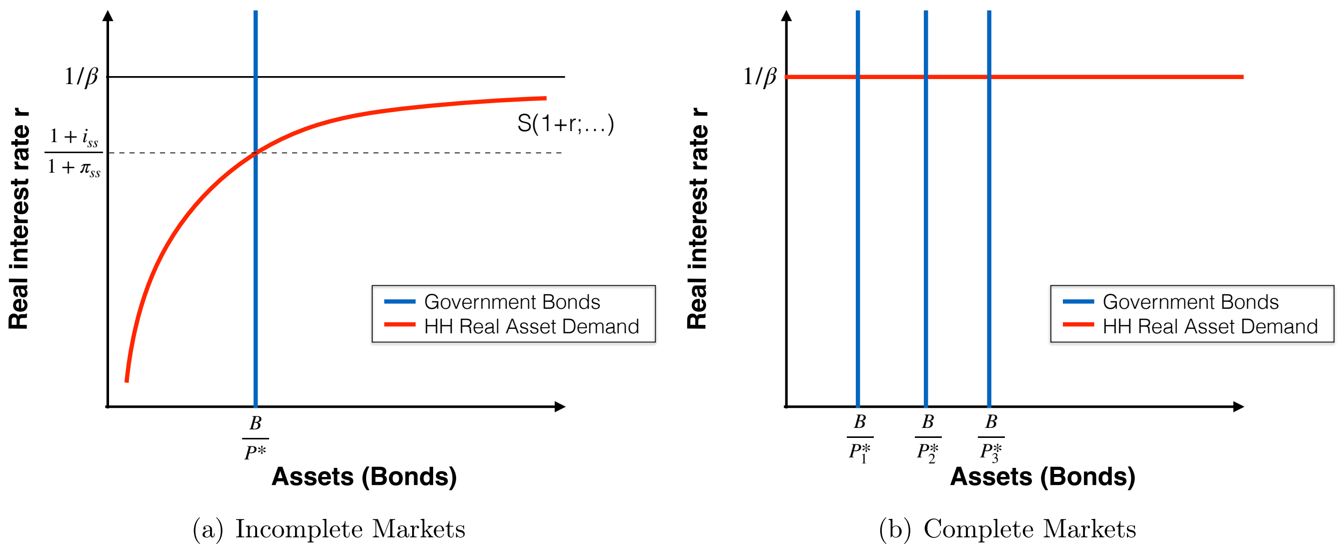

Figure 2: Asset Market Equilibrium: a) Determinacy b) Indeterminacy.

clearing then cannot be used to determine the price level, rendering the price level indeterminate. This last-period indeterminacy carries over to previous periods, such that the price level would be indeterminate in all periods, as for example in Geanakoplos and Mas-Colell (1989) and Balasko and Cass (1989).

3.4 Representative Agent and Hand-to-Mouth Consumers

The above reasoning does not extend to representative agent environments, so that the price level is indeterminate if markets are complete. The key implication of complete markets is that the steady-state real interest rate is determined by the discount factor only, $(1 + r_{ss})\beta = 1$, whereas in incomplete market models, the real interest rate depends on virtually all model primitives, so that the equilibrium in the asset market can be represented equivalently as

$$\frac{1 + i_{ss}}{1 + \pi_{ss}} = 1 + r_{ss}(B/P^*), \tag{22}$$

which again determines the steady-state price level P^* and where $r_{ss}(B/P)$ is the real interest rate which makes households willing to hold B/P real assets in steady state. The counterpart in a representative agent model is

$$\frac{1 + i_{ss}}{1 + \pi_{ss}} = 1 + r_{ss} = 1/\beta, \tag{23}$$

which no longer depends on the price level, which is therefore indeterminate. The right panel of Figure 2 illustrates the indeterminacy, depicting supply and demand in the asset market as before, with incomplete markets. The difference is that the steady-state savings curve is a vertical line at the steady-state interest rate $1/\beta$, whereas it is an upward sloping curve in a model with incomplete markets. An equivalent interpretation of the reason for indeterminacy is that the representative household is willing to absorb any amount of real bonds in a steady state with real interest rate $1/\beta$. The vertical asset demand curve with complete markets then reflects the result that the real interest rate is independent of the quantity of real bonds, such that a continuum of price levels, e.g. P_1^*, P_2^*, P_3^*, satisfies all equilibrium conditions.

The same arguments apply to models in which a fraction of households simply consume their current income, "hand-to-mouth", while the remaining households act according to the permanent income hypothesis (PIH). Since hand-to-mouth consumers do not participate in the asset market, the real interest rate is determined by the discount factor of PIH households only, $(1 + r_{ss})\beta = 1$, and equilibrium in the asset market is again characterized through

$$\frac{1 + i_{ss}}{1 + \pi_{ss}} = 1 + r_{ss} = 1/\beta, \tag{24}$$

which does not depend on the price level, implying that it is indeterminate. This model shows that it is not heterogeneity by itself that leads to the result. Rather it is the combination of heterogeneity and market incompleteness that leads to precautionary savings and a non-degenerated aggregate savings function, implying price level determinacy. By the same argument, permanent heterogeneity in productivity, but otherwise complete markets, will not lead to price level determinacy either, since again $(1 + r_{ss})\beta = 1$ in a steady state.

The graphical representation also suggests another simple way to understand why the price level is indeterminate in complete markets, but determinate in incomplete markets models. The number of endogenous variables exceeds the number of equilibrium conditions by one if markets are complete, rendering the price level indeterminate. Market incompleteness provides an extra non-redundant equation, namely the asset-market clearing condition (20). At first glance, the argument might seem wrong, since the asset market has to clear in both models. That is correct, but in the first model, this equation is redundant, as it merely ensures that the steady-state real interest rate is equal to $1/\beta$, with no role whatsoever for bonds or the price level. In contrast, the savings curve in incomplete markets models defines a trade-off between the real interest rate and the value of bonds. This extra and non-redundant clearing condition, together with $1+r_{ss} = \frac{1+i_{ss}}{1+\pi_{ss}}$, then determines the price level as illustrated analytically and graphically above. Appendix A.I shows that the reasoning for incomplete markets does not extend to perpetual youth models or to representative agent models with aggregate risk, so that the price level is also indeterminate in these models.

3.5 Adding Capital in Production

The same determinacy result holds in a model with investment I_t and capital K_t. To see this, assume a production function $Y_t = F(K_t, h_t) = F(K_t, 1)$ where labor is inelastically supplied, $h_t = 1$, and capital accumulates as

$$K_{t+1} = F(K_t, h_t) + (1-\delta)K_t - \int c_t d\Omega_t, \tag{25}$$

for a depreciation rate δ. Households rent their labor services $h_t e_t = e_t$ to firms for a real wage $w_t = F_h(K_t, 1)$ such that the budget constraint changes to

$$c_t + a_{t+1} \quad = \quad (1+r_t)a_t + w_t e_t - \tau_t. \tag{26}$$

Bonds and capital are perfect substitutes, so that asset market clearing requires

$$K_{t+1} + \frac{B_{t+1}}{P_t} \quad = \quad \int a_{t+1} d\Omega_t = S_{t+1} \tag{27}$$

and that both assets have the same return,

$$F_K(K_t, 1) + (1-\delta) \quad = \quad 1 + r_{t+1} = (1+i_{t+1})\frac{P_t}{P_{t+1}}. \tag{28}$$

The definition of the steady-state savings function S is as in Section 2.4, with the only difference being that lump-sum taxes are levied only to cover the interest rate payments on bonds which in equilibrium are equal to $\frac{B_t}{P_{t-1}} = S_t - K_t$, so that now

$$\tau_{ss} = r_{ss}(S_{ss} - K_{ss}). \tag{29}$$

In a steady state the two conditions (27) and (28) are

$$K^* + \frac{B}{P^*} \quad = \quad S(1+r_{ss}) = S(\frac{1+i_{ss}}{1+\pi_{ss}}), \tag{30}$$

$$F_K(K^*, 1) + (1-\delta) \quad = \quad 1 + r_{ss} = \frac{1+i_{ss}}{1+\pi_{ss}}, \tag{31}$$

which together determine the steady-state values of the two endogenous variables K^* and P^*. In particular, the price level is determinate using again the asset market clearing condition, but now taking into account that there are two assets, bonds and capital. The asset market clearing condition again provides the additional equation needed for price level determinacy, while the second equation is used only to determine the capital stock.

3.6 Money Demand, Endogenous Money and Open-Market Operations

I now show that the determinacy result derived so far in a cashless economy extends to models where households have a non-trivial demand for money. To generate a demand for money, I assume preferences over consumption c_t and real money balances m_t

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(c_t) + \mu(m_t)), \tag{32}$$

where μ is increasing and concave. A household carries nominal money $P_{t-1}m_t$ into period t from the previous period and acquires money $P_t m_{t+1}$ in that period. Households then maximize utility for a budget constraint

$$c_t + a_{t+1} + m_{t+1} = (1 + r_t)a_t + m_t \frac{P_{t-1}}{P_t} + e_t - \tau_t, \tag{33}$$

subject to the credit constraint (2) and $m_{t+1} \geq 0$. Aggregate steady-state assets and real money are

$$S = \int a d\Omega \qquad \text{and} \qquad L = \int m d\Omega, \tag{34}$$

where Ω is now the distribution on agents' assets a, real balances m and endowment e across the population. Both S and L are functions of the real interest rate $1 + r$, the inflation rate $1 + \pi$ and taxes τ,

$$S = S(1 + r, 1 + \pi, \tau) \qquad \text{and} \qquad L = L(1 + r, 1 + \pi, \tau). \tag{35}$$

Using that steady-state seigniorage revenue is $L\frac{\pi}{1+\pi}$ and thus $\tau = rS - L\frac{\pi}{1+\pi}$ determines savings and money as fixed points to

$$S = S(1 + r, 1 + \pi, rS - L\frac{\pi}{1 + \pi}) \qquad \text{and} \qquad L = L(1 + r, 1 + \pi, rS - L\frac{\pi}{1 + \pi}) \tag{36}$$

and yields steady-state savings and money as a function of the real interest rate and the inflation rate

$$S = S(1 + r, 1 + \pi) \qquad \text{and} \qquad L = L(1 + r, 1 + \pi). \tag{37}$$

The central bank adjusts money supply M to satisfy whatever money demand households have given the nominal interest rate $i_{ss} > 0$ set by the central bank,[16] rendering M an endogenous variable.[17]

[16]While the model without money allows for $i_{ss} < 0$, adding money precludes negative nominal interest rates but allows for $i_{ss} = 0$ if there is a finite satiation level of money.

[17]The same argument holds when the central bank pays an interest rate i^m on money holdings. Again, the central bank has to satisfy household money demand. Paying interest rates on reserves also does not overcome the indeterminacy issue and only changes the opportunity costs of holding money. I thus omit this complication. Diba and Loisel (2017) allow for $i^m > 0$ and in contrast to this paper, assume that the central bank sets money supply M and not the nominal interest rate on bonds i. It is this latter (and not the first) assumption on setting M that delivers determinacy in their model.

The steady-state price level P^* and money M^* are determined as solutions to

$$\frac{M^*}{P^*} = L(\frac{1+i_{ss}}{1+\pi_{ss}}, 1+\pi_{ss}), \tag{38}$$

$$\frac{B}{P^*} = S(\frac{1+i_{ss}}{1+\pi_{ss}}, 1+\pi_{ss}). \tag{39}$$

Now there are two equations in two unknowns M^* and P^*, where B and π_{ss} are set by fiscal policy and i is set by monetary policy. The central bank has to provide nominal money

$$M^* = P^* \int m d\Omega = P^* L(\frac{1+i_{ss}}{1+\pi_{ss}}, 1+\pi_{ss}) \tag{40}$$

to implement the nominal interest rate $1+i_{ss}$. Here the assumption is that households exchange consumption goods for money. If one assumes instead that households obtain money through open market operations, then P^* and M^* solve

$$\frac{M^*}{P^*} = L(\frac{1+i_{ss}}{1+\pi_{ss}}, 1+\pi_{ss}), \tag{41}$$

$$\frac{B-M^*}{P^*} = S(\frac{1+i_{ss}}{1+\pi_{ss}}, 1+\pi_{ss}), \tag{42}$$

again with two equations in two unknowns, or equivalently P^* solves

$$\frac{B}{P^*} = S(\frac{1+i_{ss}}{1+\pi_{ss}}, 1+\pi_{ss}) + L(\frac{1+i_{ss}}{1+\pi_{ss}}, 1+\pi_{ss}). \tag{43}$$

Clearly, equation (41) alone does not determine the price level, since the central bank sets i and not M, which adjusts endogenously to satisfy the quantity equation. Instead it is the asset market clearing condition that determines the price level, taking fiscal and monetary policy variables as given.

Both the real value of money M and of bonds B are constant in a steady state,

$$1+\pi_{ss} = \frac{B'-B}{B} = \frac{M'-M}{M}. \tag{44}$$

With open-market operations, households hold $B-M$ nominal bonds and money M in a steady state, which both grow at the same rate as the supply of nominal bonds B, which again equals the inflation rate:

$$1+\pi_{ss} = \frac{B'-B}{B} = \frac{(B-M)'-(B-M)}{B-M} = \frac{M'-M}{M}. \tag{45}$$

This is a steady-state condition which will hold in any model with government bonds and money.[18]

[18]If the central bank sets $i_{ss}=0$ and the utility function is such that money demand is finite at $i_{ss}=0$, households

In simple textbook models, the central bank sets the money supply and according to the standard interpretation, determines the steady-state inflation rate equal to the growth rate of money. If the central bank sets M'/M to 2 percent then the inflation rate equals 2 percent in a steady state. If, in another steady state, M'/M equals 4 percent, then the inflation rate equals 4 percent. Fiscal policy then has no choice but to adjust the growth rate of bond supply to be equal to the inflation rate determined by monetary policy. Causality runs from M'/M to π_{ss} to B'/B. What is different here is that money is not a policy instrument, but adjusts to money demand and price movements, so that the causality runs from π_{ss} to M'/M. Fiscal policy sets B'/B which is then equal to the steady-state inflation rate, so that the causality runs from fiscal policy to inflation. Again, this is a comparison of steady states, which first requires fiscal policy to be consistent with steady-state conditions, since otherwise the economy is not in steady state. And secondly, this adds a causal interpretation, as fiscal policy follows a simple constant debt-growth-rate policy. If this simple rule sets B'/B equal to 2 percent, then the inflation rate equals 2 percent, and setting B'/B equal to 4 percent implies an inflation rate of 4 percent. In contrast to the FTPL, no game between the fiscal and the monetary authority needs to be specified. The result here is just a combination of a steady-state condition and the fact that fiscal policy implements a constant debt growth-rate policy, and monetary policy a constant nominal interest rate. These two policies are consistent, as all equilibrium conditions are satisfied, so that no game needs to be specified to resolve any inconsistencies.

In this sense, fiscal policy can determine the long-run inflation rate (and the price level) through commitment to such a simple rule. Below, I will consider endogenous fiscal rules which respond to movements in prices and output and characterize which rules imply determinacy. Determinacy means that causality runs from policy to prices and inflation and that reverse causality is ruled out. An example of indeterminacy and thus reverse causality would be a policy which keeps the nominal interest rate and real bonds constant. The steady-state equation $1 + \pi_{ss} = \frac{B'-B}{B}$ would still hold, but would be driven by inflation sunspots fully accommodated by fiscal policy through adjustments in nominal bonds.

are indifferent between holding bonds and money, leaving the ratio of money to bond holdings indeterminate. This is however irrelevant to the arguments, since $S() + L()$ is well defined, the total supply of nominal assets still grows at rate $\frac{B'-B}{B}$ equal to the inflation rate, and the price level is determinate using (43).

3.7 Fiscal Theory of the Price Level (FTPL)

The FTPL[19] takes a different route to determine the steady-state price level. The starting point is the government budget constraint in a complete markets model, which in steady state is

$$\frac{B}{P} = \sum_{t=0}^{\infty} \beta^t s = \frac{s}{1-\beta}, \tag{46}$$

for a real primary surplus s and using the real interest rate as $1 + r = 1/\beta$. In the model in Section 2, the real primary surplus $s = \tau$ for real lump-sum taxes τ.[20] The solution for the price level is then

$$P = \frac{B(1-\beta)}{s}. \tag{47}$$

One way to understand why the price level in complete markets models is indeterminate is to note that the number of endogenous variables exceeds the number of equilibrium conditions by one. The FTPL provides an additional equation, as it assumes that the government budget constraint is satisfied by only one price level. While this is not the approach pursued in this paper, there is a similarity. The FTPL uses an equation which states that nominal debt divided by the price level is equal to "something real", the discounted present value of primary surpluses. This paper uses a different equation - the asset market clearing condition - which also states that nominal debt divided by the price level is equal to "something real", namely real aggregate savings.

There are, however, two reasons why the two theories are different and why the FTPL does not operate here. First, the government budget is specified in nominal terms, so that the nominal primary surplus equals $\Sigma = T$ for nominal lump sum taxes T ($\Sigma = T - G$ if nominal government expenditures $G > 0$). The government budget constraint in real terms is

$$\frac{B}{P} = \frac{\Sigma}{P(1-\beta)}. \tag{48}$$

Since the surplus is in nominal and not in real terms, the price level shows up on both sides of the equation, preventing us from solving the equation for P. The government budget constraint now has to be satisfied in nominal terms,

$$B = \frac{\Sigma}{1-\beta}, \tag{49}$$

and if it is, the corresponding real constraint is then satisfied for all price levels P, rendering P

[19]Developed by Leeper (1991), Sims (1994, 1997), Woodford (1995, 1997, 1998a,b), Dupor (2000) and Cochrane (1999, 2001, 2005) building on Sargent and Wallace (1981).

[20]In models where real government spending $g > 0$ (as in the model in Appendix A.I.2), the real primary surplus $s = \tau - g$.

indeterminate.

A second difference is that here, taxes T are adjusted to balance the government budget at all times, taking prices as given. In contrast, the FTPL assumes the government does not take prices as given when choosing B and s, so that nominal bonds and the real surplus are chosen first and then prices adjust. More generally, the FTPL requires fiscal policy to be active (Leeper, 1991), that is, the fiscal authority does not adjust taxes to balance the budget at all times. Without this assumption, fiscal policy is passive, taxes are adjusted to balance the budget and the FTPL is not operating and the equation stating that nominal debt divided by the price level is equal to “something real” is not well-defined. Since this paper assumes a passive fiscal policy, the price level is determinate but not according to the FTPL. A further discussion of the differences between my theory and the FTPL using Ljungqvist and Sargent’s (2012) “Ten Monetary Doctrines” environment is provided in Appendix A.II.[21]

4 Local Determinacy

To obtain a characterization of local determinacy, I build on the key insight in Werning (2015), that using an as-if representative agent economy as a reference model enables deriving theoretical results in incomplete markets models. The first step therefore extends the analysis in Werning (2015) to models with a positive number of assets - government bonds - and a balanced government budget. The second step defines the incomplete markets model as a departure from this reference model, which renders the theoretical analysis tractable. While household decisions are nontrivial, the behavior of aggregate variables can then be characterized.

Both steps merely describe the consumption/savings decisions of households - the consumption block of the model - since all other parts of the model are identical. In all models, households take the same sequences of real interest rates $(\{1+r_t\}_{t=0}^{\infty}$, tax rates $\{\tau_t\}_{t=0}^{\infty}$ and output $\{Y_t\}_{t=0}^{\infty}$ as given, and initial aggregate assets are A_0.

[21]Kocherlakota and Phelan (1999) (KP) impose a limiting condition, $\lim_{t\to\infty}(M_t+(1+i_{t+1})B_t)\prod_{s=1}^{t+1}\frac{1}{1+i_s}=0$, on the government budget (equation (25) in KP) to rule out the FTPL (M is money). In their complete markets model, this condition is equivalent to the household transversality condition which has to be satisfied in any equilibrium. But the KP condition is not equivalent to the transversality condition when markets are incomplete, which, for example, and in contrast to KP, is consistent with a real interest rate below the growth rate of the economy. For example, the KP limiting condition would not be satisfied in a steady state where $i\equiv 0$ (ZLB), constant $M,B>0$ and nominal taxes equal to nominal government spending, $T=G$, although the government budget constraint and the TVC (5) are clearly satisfied. Therefore (25) in KP does not apply in incomplete markets models and does not indicate whether or not the FTPL is operating.

4.1 The reference model

I first define and characterize the as-if representative agent consumption/saving problem and then construct a transfer scheme in the incomplete markets model, such that both models deliver identical paths of aggregate consumption. I follow Werning (2015) and assume that $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ and $\bar{a} = 0$. To obtain the same real interest rate in the incomplete and complete markets model, I set the discount factor in the latter model to $\beta^{CM} = 1/(1 + r_{ss}) < 1$.[22] Consumption and assets in the complete markets allocation are denoted with a CM superscript.

4.1.1 The representative agent economy

The representative household maximizes

$$\sum_{t=0}^{\infty} (\beta^{CM})^t \frac{(C_t^{CM})^{1-\sigma}}{1-\sigma} \tag{50}$$

subject to the budget constraint

$$C_t^{CM} + A_{t+1}^{CM} = (1 + r_t)A_t^{CM} + Y_t - \tau_t, \tag{51}$$

with initial assets A_0. All variables - consumption C_t^{CM}, assets A_t^{CM}, output Y_t and taxes τ_t - are in real terms. Consumption in Period 0 is a function of current and future interest rates, tax rates and output,

$$C_0^{CM} = \mathcal{C}^{CM}(\{1 + r_s\}_{s=0}^{\infty}, \{\tau_s\}_{s=0}^{\infty}, \{Y_s\}_{s=0}^{\infty}, A_0) \tag{52}$$

with elasticities evaluated at steady-state values r_{ss}, τ_{ss}, Y_{ss} and A_{ss}:[23]

$$\epsilon_{C_0,A_0}^{CM} = \frac{\partial C_0}{\partial A_0}\frac{A_{ss}}{C_{ss}} = \frac{1-\beta^{CM}}{\beta^{CM}}\frac{A_{ss}}{C_{ss}}; \tag{53}$$

$$\epsilon_{C_0,1+r_0}^{CM} = \frac{\partial C_0}{\partial 1 + r_0}\frac{1 + r_{ss}}{C_{ss}} = \frac{1-\beta^{CM}}{\beta^{CM}}\frac{A_{ss}}{C_{ss}}; \tag{54}$$

$$\forall k \geq 0: \quad \epsilon_{C_0,Y_k}^{CM} = \frac{\partial C_0}{\partial Y_k}\frac{Y_{ss}}{C_{ss}} = (1-\beta^{CM})(\beta^{CM})^k \frac{Y_{ss}}{C_{ss}}; \tag{55}$$

$$\forall k \geq 0: \quad \epsilon_{C_0,\tau_k}^{CM} = \frac{\partial C_0}{\partial \tau_k}\frac{\tau_{ss}}{C_{ss}} = -(1-\beta^{CM})(\beta^{CM})^k \frac{\tau_{ss}}{C_{ss}}; \tag{56}$$

$$\forall k \geq 1: \quad \epsilon_{C_0,1+r_k}^{CM} = \frac{\partial C_0}{\partial 1 + r_k}\frac{1 + r_{ss}}{C_{ss}} = (\frac{\tau_{ss}}{C_{ss}} - \frac{1}{\sigma})(\beta^{CM})^k. \tag{57}$$

[22] The appendix describes the necessary adjustments if $1 + r_{ss} \leq 0$ or $\bar{a} > 0$.

[23] A standard result in the New Keynesian literature is "horizon invariance", that is, a change in the interest rate at time t_1 or time t_2 has the identical effect on current consumption. As explained in McKay et al. (2016), horizon invariance is an equilibrium result and the effect of interest rate changes decays in the horizon in a household problem.

The presence of taxes $\tau_{ss} > 0$ implies that an interest rate increase has a wealth effect which leads to higher period 0 consumption. I assume this wealth effect to be dominated by the substitution effect, $\frac{\tau_{ss}}{C_{ss}} - \frac{1}{\sigma} = \frac{(1-\beta^{CM})A_{ss}}{\beta^{CM}C_{ss}} - \frac{1}{\sigma} < 0$, such that the interest rate elasticity $\epsilon^{CM}_{C_0,1+r_k} < 0$ for all $k \geq 1$. Time consistency implies that the same function $\mathcal{C}^{CM}$ with the same elasticities describes future period $t \geq 0$ consumption

$$C_t^{CM} = \mathcal{C}^{CM}(\{1+r_s\}_{s=t}^{\infty}, \{\tau_s\}_{s=t}^{\infty}, \{Y_s\}_{s=t}^{\infty}, A_t^{CM}). \tag{58}$$

Aggregate savings are

$$A_{t+1}^{CM} = Y_t^{CM} - C_t^{CM} - \tau_t + (1+r_t)A_t^{CM} \tag{59}$$

with initial condition $A_0^{CM} = A_0 = A_{ss}$.

4.1.2 The as-if economy

I now construct an individual- and time-dependent transfer scheme $\Delta_{i,t}$ in the incomplete markets economy, which yields an as-if economy with aggregate consumption and assets as in the complete markets model. The consumption and asset choices of individual i at time t in the steady state, where both the real interest rates and output are at their steady-state levels, are denoted $c_{i,t}^{ss}$ and $a_{i,t}^{ss}$ respectively. These choices depend on the full history of states $(e_{i,0}, e_{i,1}, \ldots, e_{i,t})$ and the initial asset level $a_{i,0}$ of individual i, so that $c_{i,t}^{ss} = c(e_{i,0}, e_{i,1}, \ldots, e_{i,t}; a_{i,0})$ and $a_{i,t}^{ss} = a(e_{i,0}, e_{i,1}, \ldots, e_{i,t}; a_{i,0})$, but this dependence is dropped for notational convenience. Households' period t income is $e_{i,t}Y_t$. For a sequence of aggregate consumption, assets, output, interest rates and taxes$\{C_t^{CM}, A_t^{CM}, Y_t, r_t, \tau_t\}_{t=0}^{\infty}$ define transfers

$$\begin{aligned} \Delta_{i,t} \;:=\; & \frac{A_{t+1}^{CM} - A_{ss}}{A_{ss}} a_{i,t+1}^{ss} + a_{i,t}^{ss}\{(1+r_{ss}) - \frac{A_t^{CM}}{A_{ss}}(1+r_t)\} \\ + \; & \frac{C_t^{CM} - C_{ss}}{C_{ss}} c_{i,t}^{ss} + (Y_{ss} - Y_t)e_{i,t} - r_{ss}A_{ss} + \tau_t, \end{aligned} \tag{60}$$

where C_{ss} and A_{ss} are aggregate steady-state consumption and savings in both the as-if and the complete markets economies. The as-if (AI) economy and the complete markets economy have the same interest rates, wages, tax rates, and the same output sequence, and households face the budget constraint

$$a_{i,t+1}^{AI} = (1+r_t)a_{i,t}^{AI} - c_{i,t}^{AI} + e_{i,t}Y_t - \tau_t + \Delta_{i,t} \tag{61}$$

in the as-if economy, where $\Delta_{i,t}$ is exogenous to the household. The appendix shows that these transfers are purely redistributive in the cross-section, $\int \Delta_{i,t} d\Omega_t = 0$ and household i's optimal choices at time t in the as-if economy are

$$c_{i,t}^{AI} = \frac{C_t^{CM}}{C_{ss}} c_{i,t}^{ss}; \qquad a_{i,t+1}^{AI} = \frac{A_{t+1}^{CM}}{A_{ss}} a_{i,t+1}^{ss}, \tag{62}$$

implying that aggregate consumption and assets are identical in the two economies,

$$C_t^{AI} = \int c_{i,t}^{AI} d\Omega_t = \int \frac{C_t^{CM}}{C_{ss}} c_{i,t}^{ss} d\Omega_t = C_t^{CM} \frac{C_{ss}}{C_{ss}} = C_t^{CM}, \tag{63}$$

$$A_{t+1}^{AI} = \int a_{i,t+1}^{AI} d\Omega_t = \int \frac{A_{t+1}^{CM}}{A_{ss}} a_{i,t+1}^{ss} d\Omega_t = A_{t+1}^{CM} \frac{A_{ss}}{A_{ss}} = A_{t+1}^{CM}. \tag{64}$$

We thus obtain

$$C_t^{AI} = C_t^{CM} = \mathcal{C}^{CM}(\{1+r_s\}_{s=t}^{\infty}, \{\tau_s\}_{s=t}^{\infty}, \{Y_s\}_{s=t}^{\infty}, A_t^{CM}), \tag{65}$$

implying that the consumption elasticities (53) - (57) coincide in the two economies. Note that the consumption and savings responses in the AI economy are the combination of the changes in r, τ, A, Y and the associated changes in the transfers $\Delta_{i,t}$. This is on purpose since the changes in $\Delta_{i,t}$ ensure that the elasticities are identical. Without these changes in $\Delta_{i,t}$, the aggregate consumption and savings responses in the as-if and the representative agent economies would differ. The transfers $\Delta_{i,t}$ also ensure that the elasticities are time-invariant, $\epsilon_{C_t,A_t}^{AI} = \epsilon_{C_0,A_0}^{AI}, \epsilon_{C_t,1+r_t}^{AI} = \epsilon_{C_0,1+r_0}^{AI}, \epsilon_{C_t,Y_{t+k}}^{AI} = \epsilon_{C_0,Y_k}^{AI}, \epsilon_{C_t,\tau_{t+k}}^{AI} = \epsilon_{C_0,\tau_k}^{AI}, \epsilon_{C_t,1+r_{t+k}}^{AI} = \epsilon_{C_0,1+r_k}^{AI}$, since (65) shows that aggregate consumption can be written as a time-independent function and the elasticities are always evaluated at the same steady-state values r_{ss}, τ_{ss}, Y_{ss} and A_{ss}.

4.2 The incomplete markets economy

Werning (2015) allows for cyclical income risk in modeling the incomplete markets economy as a departure from the as-if complete markets economy. I follow his approach, but use a different departure. In the as-if economy, the $\Delta_{i,t}$ transfers redistribute the tax burden, labor income and asset income such that all households choices are linearly homogeneous in the aggregate variables. I now disable the redistribution of the tax burden outside steady states through adjusting the construction of the transfers Δ_{it}.

I choose this approach since output is constant with flexible prices and thus, cyclical risk would have no impact in the benchmark analysis. The logic underlying my and Werning's approaches is however the same. The starting point is the household budget constraint which reads

$$\tilde{c}^{IM}_{i,t} + \tilde{a}^{IM}_{i,t+1} = (1 + r_t)\tilde{a}^{IM}_{i,t} + e_{i,t} - \tau_t + \Delta_{i,t}, \tag{66}$$

for consumption choices $\tilde{c}^{IM}_{i,t}$ and asset choices $\tilde{a}^{IM}_{i,t+1}$. The transfers $\Delta_{i,t}$ are as defined in (60) with the important modification that taxes are now set to their steady-state value τ_{ss} when constructing $\Delta_{i,t}, C^{CM}_t$ and A^{CM}_t,

$$\Delta_{i,t} := \frac{A^{CM}_{t+1} - A_{ss}}{A_{ss}} a^{ss}_{i,t+1} + a^{ss}_{i,t}\{(1 + r_{ss}) - \frac{A^{CM}_t}{A_{ss}}(1 + r_t)\} + \frac{C^{CM}_t - C_{ss}}{C_{ss}} c^{ss}_{i,t} + (Y_{ss} - Y_t)e_{i,t}, \tag{67}$$

where the complete markets aggregates A^{CM}_{t+1} and C^{CM}_t are derived from the representative agent economy with budget constraints

$$C^{CM}_t + A^{CM}_{t+1} = (1 + r_t)A^{CM}_t + Y_t - \tau_{ss}. \tag{68}$$

Solving this model yields a consumption function $\mathcal{C}^{IM}()$ describing Period 0 consumption as a function of initial assets, interest rates and taxes

$$C_0 = \mathcal{C}^{IM}(\{1 + r_s\}^{\infty}_{s=0}, \{\tau_s\}^{\infty}_{s=0}, \{Y_s\}^{\infty}_{s=0}, A^{IM}_0),$$

where the argument A^{IM}_0 means that the initial asset distribution is the steady-state asset distribution shifted by the same factor A^{IM}_0/A_{ss} for every household.

If $\tau_t = \tau_{ss}\ \forall t$, the incomplete markets economy coincides with the as-if economy in Section 4.1.2. If taxes are not at their steady-state level τ_{ss}, the economy deviates from complete markets, since each household has to pay the tax τ_t and there is no insurance or redistribution to replicate the complete markets response. Instead, households have different marginal propensities to consume (MPCs) in response to a tax change, so that the economy can generate the salient features of incomplete markets models documented in Hagedorn et al. (2017a) and Auclert et al. (2018). As a result, aggregating the individual responses results in an aggregate consumption response - the aggregate MPC - larger than $1 - \beta^{CM}$, the complete markets aggregate MPC.

A meaningful local determinacy analysis requires a model that combines elements of the complete and incomplete markets models. If taxes are at their steady state values, $\tau_t = \tau_{ss} \forall t$, then the economy should behave like a complete markets economy. If taxes are not at their steady state values, the economy should look like an incomplete markets model. The challenge is to combine these elements within one model in a mutually consistent way. The appendix describes the necessary adjustments of the transfers and of discount factors such that aggregate consumption and assets satisfy ($\forall t \geq 0$)

$$C_t^{IM} = \mathcal{C}^{IM}(\{1+r_s\}_{s=t}^{\infty}, \{\tau_s\}_{s=t}^{\infty}, \{Y_s\}_{s=t}^{\infty}, A_t^{IM}), \tag{69}$$

$$A_{t+1}^{IM} = A_t^{IM}(1+r_t) - C_t^{IM} + Y_t - \tau_t, \tag{70}$$

and household i's optimal choices at time t in this economy are again linearly homogenous in aggregates,

$$c_{i,t}^{IM} = \frac{C_t^{IM}}{C_{ss}} c_{i,t}^{ss}; \qquad a_{i,t+1}^{IM} = \frac{A_{t+1}^{IM}}{A_{ss}} a_{i,t+1}^{ss}. \tag{71}$$

If taxes are at their steady-state level in all periods, $\tau_t = \tau_{ss}\ \forall t$, aggregate consumption is then identical to as-if complete markets aggregate consumption,

$$\begin{aligned} C_t^{IM} &= \mathcal{C}^{IM}(\{1+r_s\}_{s=t}^{\infty}, \{\tau_s = \tau_{ss}\}_{s=t}^{\infty}, \{Y_s\}_{s=t}^{\infty}, A_t^{IM}) \\ &= \mathcal{C}^{CM}(\{1+r_s\}_{s=t}^{\infty}, \{\tau_s = \tau_{ss}\}_{s=t}^{\infty}, \{Y_s\}_{s=t}^{\infty}, A_t^{IM}). \end{aligned} \tag{72}$$

At the same time, for tax changes, this economy features the aggregate MPCs of an incomplete markets model. Using the as-if economy consumption elasticities in (53) - (57), I obtain the Period 0 saving elasticities for

$$\begin{aligned} A_1^{IM} &= Y_0 - \tau_t + (1+r_0)A_0 - C_0^{IM} \\ &= Y_0 - \tau_t + (1+r_0)A_0 - \mathcal{C}^{IM}(\{1+r_s\}_{s=0}^{\infty}, \{\tau_s\}_{s=0}^{\infty}, \{Y_s\}_{s=0}^{\infty}, A_0), \end{aligned} \tag{73}$$

Result 1. *Elasticities in the incomplete markets model:*

$$\begin{aligned} \epsilon_0^A := \epsilon_{A_1^{IM},A_0} &= (1 + r_{ss} - \frac{\partial C_0^{IM}}{\partial A_0})\frac{A_{ss}}{A_{ss}} = (\frac{1}{\beta^{CM}} - \frac{1-\beta^{CM}}{\beta^{CM}})\frac{A_{ss}}{A_{ss}} = 1; \\ \epsilon_0^r := \epsilon_{A_1^{IM},1+r_0} &= (A_{ss} - \frac{\partial C_0^{IM}}{\partial 1+r_0})\frac{1+r_{ss}}{A_{ss}} = (A_{ss} - (1-\beta^{CM})A_{ss})\frac{1+r_{ss}}{A_{ss}} = 1; \\ \epsilon_0^Y := \epsilon_{A_1^{IM},Y_0} &= (1 - \frac{\partial C_0^{IM}}{\partial Y_0})\frac{Y_{ss}}{A_{ss}} = (1-(1-\beta^{CM}))\frac{Y_{ss}}{A_{ss}} = \beta^{CM}\frac{Y_{ss}}{A_{ss}}; \\ \forall k \geq 1: \quad \epsilon_k^Y := \epsilon_{A_1^{IM},Y_k} &= -\frac{\partial C_0^{IM}}{\partial Y_t}\frac{Y_{ss}}{A_{ss}} = -(1-\beta^{CM})(\beta^{CM})^k\frac{Y_{ss}}{A_{ss}}; \\ \forall k \geq 1: \epsilon_k^r := \epsilon_{A_1^{IM},1+r_k} &= -\frac{\partial C_0^{IM}}{\partial 1+r_k}\frac{1+r_{ss}}{A_{ss}} = (\frac{1}{\sigma} - \frac{\tau_{ss}}{C_{ss}})(\beta^{CM})^k\frac{C_{ss}}{A_{ss}} \end{aligned}$$

Since the consumption response to taxes is not the same as in the as-if economy, the τ-elasticities

$$\epsilon_0^\tau := \epsilon_{A_1^{IM},\tau_0} = (-1 - \frac{\partial C_0^{IM}}{\partial \tau_0})\frac{\tau_{ss}}{A_{ss}}; \qquad \epsilon_k^\tau := \epsilon_{A_1^{IM},\tau_k} = -\frac{\partial C_0^{IM}}{\partial \tau_k}\frac{\tau_{ss}}{A_{ss}} \qquad \forall k \geq 1 \tag{74}$$

are not either, which requires deriving additional properties that hold in all incomplete markets models. Note that these elasticities are time-invariant, this means $\epsilon_{A_{t+1}^{IM},\tau_{t+k}} = \epsilon_{A_{1,\tau_k}^{IM}}$, since (69) shows that aggregate consumption can be written as a time-invariant function of aggregate assets and all elasticities are evaluated at the steady-state values $\Omega_{ss}, r_{ss}, \tau_{ss}, Y_{ss}$. Similarly, (72) implies that the elasticities in Result 1 are time-invariant.

The first result characterizes the permanent MPC, the consumption response to a permanent increase in taxes. Define the savings response $\forall k \geq 0, \tilde{\epsilon}_k^\tau = \epsilon_k^\tau \frac{A_{ss}}{\tau_{ss}}$, then

Result 2.

$$-1 < \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau < 0. \tag{75}$$

The result states that in incomplete markets models, households respond to a permanent increase in transfers by increasing consumption and also (precautionary) savings. Result 2 can be restated in terms of marginal propensities to consume. Define $mpc_0^k = -\frac{\partial C_0^{IM}}{\partial \tau_k}$, the period 0 aggregate consumption response to a \$1 transfer in period k, so that $\forall k \geq 1, \tilde{\epsilon}_k^\tau = mpc_0^k$ and $\tilde{\epsilon}_0^\tau = -1 + mpc_0^0$. Therefore

$$\sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau = -1 + \sum_{k=0}^{\infty} mpc_0^k,$$

so that Result 2 is equivalent to a condition on the permanent MPC,

$$0 < \sum_{k=0}^{\infty} mpc_0^k < 1.$$

The first step of the proof is to recognize that Acemoglu and Jensen's 2015 comparative static framework applies. The second step is then an application of their Lemma 1, which builds on Topkis' monotonicity theorem, or equivalently of their Theorem 6. Note that if markets were complete, a permanent increase in transfers increases household consumption one-for-one and leaves savings unaffected. Indeed, for $k \geq 1$, $\tilde{\epsilon}_k^\tau = (1 - \beta^{CM})(\beta^{CM})^k$ and $\tilde{\epsilon}_0^\tau = -\beta^{CM}$ in the complete markets economy, implying $\sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau = 0$.

Results 1 and 2 describe households' partial equilibrium responses to more initial assets and to higher taxes. In general equilibrium, these responses are linked, as a higher level of government debt means more assets for households, but at the same time higher taxes. I therefore define the savings response of an increase in initial assets, taking into account that this leads to higher current and future interest rate payments,

$$\tilde{\epsilon}_0^A := \epsilon_0^A + \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau \frac{i_{ss} - \pi_{ss}}{1 + \pi_{ss}}, \tag{76}$$

which, by the above results satisfies,

$$1 > \tilde{\epsilon}_0^A > 0. \tag{77}$$

Similarly, define the savings response of an increase in the (initial) interest rate, taking into account that this leads to higher current and future interest rate payments,

$$\tilde{\epsilon}_0^r := \epsilon_0^r + \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau \frac{i_{ss} - \pi_{ss}}{1 + \pi_{ss}}, \tag{78}$$

which by the above results also satisfies,

$$1 > \tilde{\epsilon}_0^r > 0. \tag{79}$$

The next result states properties of the marginal consumption responses to a one-time transfer in Period k.

Result 3. *For $k = 0$:*

$$0 \geq \tilde{\epsilon}_0^\tau \frac{1 + i_{ss}}{1 + \pi_{ss}} \geq -1$$

For $k \geq 1$:

$$\xi_k^{(0)} = \tilde{\epsilon}_k^\tau \geq 0 \tag{80}$$

$$\xi_k^{(1)} = \xi_k^{(0)} - \frac{1 + i_{ss}}{1 + \pi_{ss}} \xi_{k+1}^{(0)} = \tilde{\epsilon}_k^\tau - \frac{1 + i_{ss}}{1 + \pi_{ss}} \tilde{\epsilon}_{k+1}^\tau > 0 \tag{81}$$

If prudence is not too strong:[24]

$$\xi_k^{(2)} = \xi_k^{(1)} - \frac{1 + i_{ss}}{1 + \pi_{ss}} \xi_{k+1}^{(1)} = \tilde{\epsilon}_k^\tau - 2(\frac{1 + i_{ss}}{1 + \pi_{ss}}) \tilde{\epsilon}_{k+1}^\tau + (\frac{1 + i_{ss}}{1 + \pi_{ss}})^2 \tilde{\epsilon}_{k+2}^\tau \geq 0 \tag{82}$$

$$\xi_k^{(3)} = \xi_k^{(2)} - \frac{1 + i_{ss}}{1 + \pi_{ss}} \xi_{k+1}^{(2)} = \tilde{\epsilon}_k^\tau - 3(\frac{1 + i_{ss}}{1 + \pi_{ss}}) \tilde{\epsilon}_{k+1}^\tau + 3(\frac{1 + i_{ss}}{1 + \pi_{ss}})^2 \tilde{\epsilon}_{k+2}^\tau - (\frac{1 + i_{ss}}{1 + \pi_{ss}})^3 \tilde{\epsilon}_{k+3}^\tau \geq 0 \tag{83}$$

Again, these results are implicit statements about marginal propensities to consume, formalizing the quantitative results in Hagedorn et al. (2017a) and Auclert et al. (2018). The Period 0 marginal propensity to consume is larger than $(1 - \beta^{CM})$, the complete markets MPC. Lump-sum transfers

[24] I verify in the appendix that both properties hold in the calibrated model in Hagedorn et al. (2017a), and that Result 3 (and 2) hold for asymptotically time-invariant MPCs (Auclert et al., 2019a).

have a smaller effect on current consumption the further into the future they are paid. However this argument does not extend to the transfer schemes $\xi_k^{(1)}$ and $\xi_k^{(2)}$ if prudence is too strong.[25] I therefore impose a bound on prudence which ensures that any transfer scheme ($\xi_k^{(1)}$ or $\xi_k^{(2)}$) has a larger effect on current consumption than the same scheme shifted into the future by one period ($\frac{1+i_{ss}}{1+\pi_{ss}}\xi_{k+1}^{(1)}$ or $\frac{1+i_{ss}}{1+\pi_{ss}}\xi_{k+1}^{(2)}$). Note that without credit constraints, no transfer scheme $\xi_k^{(1)}, \xi_k^{(2)}, \xi_k^{(3)}$ would affect household intertemporal budget constraints, so that $\xi_k^{(1)} = \xi_k^{(2)} = \xi_k^{(3)} = 0 \ \forall k \geq 1$ if markets were complete.[26]

4.3 Local Determinacy Criterion

The steady state is locally unique, if there is no other equilibrium in which all variables are within a neighborhood of their steady-state values. To check local determinacy of the steady state, it is sufficient to check for the log-linearized economy (Woodford, 2003).[27] As I will show below, the linearized model is of the form

$$\sum_{k=-j}^{\infty} \Theta_k p_{t+k} = 0, \tag{84}$$

where j is the number of predetermined variables and $\Theta_k \in \mathbb{R}$. The standard approach, following Blanchard and Kahn (1980), of computing eigenvalues does not apply, due to the infinite number of leads. I therefore resort to the determinacy criterion developed in Onatski (2006), which not only allows for an infinite lead, but is also theoretically more tractable than Blanchard and Kahn (1980) in the presence of predetermined variables.[28] Define

$$\Theta(\lambda) = \sum_{k=-j}^{\infty} \Theta_k e^{-ik\lambda} \tag{85}$$

and the winding number as the number of times the graph of $\Theta(\lambda)$ rotates around zero counter-clockwise when λ goes form 0 to 2π.

Result 4. [Onatski (2006)] *A generic model as in (84) has a unique bounded solution if the winding number of $\Theta(\lambda)$ is equal to zero.*

[25]If prudence is strong, paying $\frac{1+i_{ss}}{1+\pi_{ss}}$ in Period 1 and taxing $(\frac{1+i_{ss}}{1+\pi_{ss}})^2$ in Period 2 can lead to a larger aggregate period 0 consumption response than paying one unit in Period 0 and taxing $\frac{1+i_{ss}}{1+\pi_{ss}}$ in Period 1, $\frac{1+i_{ss}}{1+\pi_{ss}}(mpc_0^1 - \frac{1+i_{ss}}{1+\pi_{ss}}mpc_0^2) > mpc_0^0 - \frac{1+i_{ss}}{1+\pi_{ss}}mpc_0^1$. The reason is that, if prudence is strong, the first scheme has a large effect on the precautionary savings motives of Period 0 unconstrained households and thus on their consumption.

[26]The properties $\xi_k^{(2)} \geq 0$ and $\xi_k^{(3)} \geq 0$ are sufficient, but not necessary for the local determinacy results.

[27]Note that this always involves - independently of the number of leads and lags - an application of the infinite-dimensional inverse function theorem due to the infinite time horizon.

[28]Auclert et al. (2019a) also use the Onatski criterion for a quantitative assessment.

A univariate model (84) is always generic.[29] Onatski (2006) also assumes that the model (84) has no unit roots so that $\Theta(\lambda) \neq 0$ for all $\lambda \in \mathbb{R}$.

The idea behind the determinacy proofs is to show that the real part of the complex number $\Theta(\lambda)$ is always positive, $\text{Re}\{\Theta(\lambda)\} > 0$, which has two implications. First, the model has no unit roots. Second, the winding number is zero, since the graph of $\Theta(\lambda)$ is within the plane of positive real numbers and a circle around zero necessarily requires that $\text{Re}(\Theta(\lambda))$ can be positive and negative.

Onatski (2006) shows that for models with a finite lead, his criterion coincides with that derived in Blanchard and Kahn (1980). This is easy to see for a model of first-order,

$$p_t + \Theta_1 p_{t+1} = 0, \tag{86}$$

where the Onatski criterion requires that

$$1 + \Theta_1 e^{-i\lambda} \tag{87}$$

does not circle around 0. The Blanchard&Kahn determinacy criterion requires that the eigenvalue $1/\Theta_1$ of $p_{t+1} = \frac{1}{\Theta_1} p_t$ be larger than 1, implying that $1 + \Theta_1 e^{-i\lambda}$ circles around 1 with radius $|\Theta_1| < 1$. Thus, the circle does not contain zero, and the winding number is zero, implying determinacy using Onatski (2006).

To establish $\text{Re}\{\Theta(\lambda)\} > 0$, I use $Re(e^{-ik\lambda}) = \cos(-k\lambda) = \cos(k\lambda) = Re(e^{ik\lambda})$ and define $\tilde{\Theta}_k = \Theta_k + \Theta_{-k}$ ($\tilde{\Theta}_0 = \Theta_0$, and $\tilde{\Theta}_{-k} = 0$ for $k \geq j+1$) to first show that

$$\text{Re}\{\Theta(\lambda)\} = \sum_{k=0}^{\infty} \tilde{\Theta}_k \cos(k\lambda) \tag{88}$$

and then apply a result from Fejér (1928, 1936):[30]

Proposition 1. [Fejér (1928, 1936)] *If* $\forall k \geq 0 : \alpha_k \geq 0, \alpha_k - \alpha_{k+1} \geq 0, \alpha_k - 2\alpha_{k+1} + \alpha_{k+2} \geq 0,$ *then*

$$\frac{\alpha_0}{2} + \sum_{k=1}^{\infty} \alpha_k \cos(k\lambda) \geq 0 \quad \forall \lambda \in (0, 2\pi).$$

If in addition $\alpha_0 - 2\alpha_1 + \alpha_2 > 0$*, then*

$$\frac{\alpha_0}{2} + \sum_{k=1}^{\infty} \alpha_k \cos(k\lambda) \geq \frac{\alpha_0 + \alpha_2}{2} - \alpha_1 > 0 \quad \forall \lambda \in (0, 2\pi).$$

[29] Sims (2007) points out that genericity cannot be taken for granted in multivariate models and needs to be checked for each model before applying Onatski's winding number criterion. The transformation to a univariate model thus delivers the additional benefit that genericity "comes for free".

[30] The Proposition excludes $\lambda = 0$, since certain analytical properties cannot be guaranteed. However, it will be clear from the application of this proposition in the determinacy proofs that the claims extend to $\lambda = 0$ here.

4.4 Local Determinacy: Flexible Prices

I now use the Onatski (2006) criterion to assess the local determinacy of the incomplete markets model defined as a departure from a complete markets model in Section 4.2. I first assume that prices are flexible, that fiscal policy follows a stationary exogenous policy and sets a fixed amount of nominal government debt, and that monetary policy sets a constant nominal interest rate, and I allow for feedback policy rules and price rigidities below. The period t savings function is equal to

$$\begin{aligned} S_{t+1} = A_{t+1}^{IM} &= Y_t - \tau_t + (1+r_t)A_t^{IM} - C_t^{IM} \\ &= Y_t - \tau_t + (1+r_t)A_t^{IM} - \mathcal{C}^{IM}(\{1+r_s\}_{s=t}^{\infty}, \{\tau_s\}_{s=t}^{\infty}, \{Y_s\}_{s=t}^{\infty}, A_t^{IM}), \end{aligned} \tag{89}$$

and is thus a function of current assets and future real interest rates, tax rates and output levels with, by construction, the same elasticities derived in Section 4.2 for A_1^{IM}. The starting point is the Period t asset-market-clearing condition

$$S_{t+1}(\{1+r_s\}_{s=t}^{\infty}, \{\tau_s\}_{s=t}^{\infty}, \{Y_s\}_{s=t}^{\infty}, A_t^{IM}) = \frac{B_{t+1}}{P_t}, \tag{90}$$

which equates households' aggregate asset demand S_{t+1} to the real supply of government bonds $\frac{B_{t+1}}{P_t}$. Linearizing the asset-market-clearing condition around the steady state yields

$$\sum_{k=0}^{\infty} \left\{ \epsilon_{k+1}^r \hat{r}_{t+k+1} + \epsilon_k^\tau \hat{\tau}_{t+k} + \epsilon_k^Y \hat{Y}_{t+k} \right\} + \epsilon_0^r \hat{r}_0 + \epsilon_0^A \hat{A}_t = \hat{B}_{t+1} - \hat{p}_t, \tag{91}$$

where $\hat{p}_t = \log(P_t/P_t^*)$ is the log deviation of the price level from steady state for the steady-state price $P_t^* = P^*(1+\pi_{ss})^t$, $\hat{r}_{t+1} = \log\left(\frac{1+r_{t+1}}{1+r_{ss}}\right)$ is the log real interest rate, $\hat{B}_{t+1}$ the log deviation of nominal bonds, $\hat{\tau}_t$ the log deviation of taxes and $\hat{Y}_t = \log(Y_t/Y^*)$ the log deviation of output from steady state (also equal to the output gap, since the natural rate of output is constant in the absence of real disturbances). The elasticities ϵ_k^r, ϵ_k^Y, ϵ_0^A and ϵ_k^τ are defined in Result 1 and in (74).

The aim of the determinacy proof is to use equilibrium conditions to substitute for $\hat{r}$, $\hat{\tau}$, $\hat{Y}$ and $\hat{B}$ in the asset market clearing condition, which is then the only remaining equilibrium condition. First, note that output is constant, with flexible prices and without exogenous disturbances, $\hat{Y}_t = 0$. In the presence of price rigidities, output is not necessarily constant and affects savings, as I will discuss in Section 4.6. Second, the real interest rate satisfies the Fisher equation,

$$\hat{r}_{t+1} = \hat{i}_{t+1} + \hat{p}_t - \hat{p}_{t+1},$$

where $\hat{i}_{t+1} = \log\left(\frac{1+i_{t+1}}{1+i_{ss}}\right)$ is the log nominal interest rate. Third, the tax rate is endogenous and satisfies the government budget constraint

$$\frac{B_{t+1}}{P_t} = (1+i_t)\frac{B_t}{P_t} - \tau_t. \tag{92}$$

Linearizing around the steady state yields, using in a steady state $A_{ss} = B_{t+1}/P_t$ and $P_{t+1}/P_t = B_{t+1}/B_t = 1 + \pi_{ss}$,

$$\hat{B}_{t+1} - \hat{p}_t = \hat{i}_t \frac{1+i_{ss}}{1+\pi_{ss}} + \frac{1+i_{ss}}{1+\pi_{ss}}(\hat{B}_t - \hat{p}_t) - \frac{\tau_{ss}}{A_{ss}}\hat{\tau}_t. \tag{93}$$

Since both nominal interest rates and government bonds are constant, $\hat{i}_{t+1} = 0$ and $\hat{B}_t = \hat{B}_{t+1} = 0$, this simplifies to

$$\frac{\tau_{ss}}{A_{ss}}\hat{\tau}_t = -\frac{1+i_{ss}}{1+\pi_{ss}}\hat{p}_t + \hat{p}_t = \frac{\pi_{ss} - i_{ss}}{1+\pi_{ss}}\hat{p}_t. \tag{94}$$

Using these three steps in (91), $\hat{A}_t = \hat{B}_t - \hat{p}_{t-1} = -\hat{p}_{t-1}$ and $\tilde{\epsilon}_k^\tau = \frac{A_{ss}}{\tau_{ss}}\epsilon_k^\tau$ renders the asset market clearing condition a univariate equation in prices,

$$\sum_{k=0}^{\infty}\left\{\epsilon_{k+1}^r(\hat{p}_{t+k} - \hat{p}_{t+k+1}) + \tilde{\epsilon}_k^\tau \frac{\pi_{ss} - i_{ss}}{1+\pi_{ss}}\hat{p}_{t+k}\right\} + \epsilon_0^r(\hat{p}_{t-1} - \hat{p}_t) - \epsilon_0^A \hat{p}_{t-1} = -\hat{p}_t, \tag{95}$$

It follows that a price sequence $\{\hat{p}_t\}_{t=-1}^{\infty}$ is an equilibrium if and only if the asset market clears.

To prove determinacy, one has to show that the steady state price, that is $\hat{p}_t = 0$, is the only equilibrium. In particular one has to rule out a price sequence in which all prices shift up by p percent, $\hat{p}_t = p$ for all t. If markets were complete, this price sequence satisfies (95) since $\tilde{\epsilon}_0^r = 1$,

$$\sum_{k=0}^{\infty}\left\{\epsilon_{k+1}^r(p - p)\right\} = p(\tilde{\epsilon}_0^r - 1) = 0, \tag{96}$$

and would constitute an equilibrium, rendering the complete markets model indeterminate. Indeed, the real value of bonds, household wealth and savings would decrease by p percent, implying asset market clearing. And lower real debt would mean lower interest rate income and lower taxes of the same magnitude, leaving household consumption choices unaffected by Ricardian equivalence.

With incomplete markets, however, this logic breaks down. While the real value of bonds and household wealth again decrease by p percent, the lower wealth now affects consumption and thus savings decisions, reflected in $\tilde{\epsilon}_0^r < 1$, so that (96) no longer holds. Instead, demand exceeds supply by $p(1 - \tilde{\epsilon}_0^r) > 0$. Since $\epsilon_{k+1}^r > 0$, asset market clearing requires a decrease in real interest rates (on average), so as to lower demand, implying that future prices increase and move away from the steady state. In the Blanchard&Kahn framework, this would mean an eigenvalue larger than one and prices diverging to infinity. I obtain, applying the Onatski (2006) criterion to (95):

Result 5. *The model with a constant level of nominal interest rates and bonds has a unique bounded solution, that is, the economy is locally determinate.*

To provide further intuition, I now consider a limited information, special case of the incomplete markets model. The type of informational friction is deliberately kept simple here.[31] Households' period t information set only contains $1+r_t, 1+r_{t+1}, \tau_t$ so that the asset market clearing condition now reads

$$S_{t+1}(1+r_t, 1+r_{t+1}, \tau_t, A_t^{IM}) = \frac{B_{t+1}}{P_t}. \tag{97}$$

Other parts of the model work as before, for example future taxes might change, but households' information set does not reflect this. Linearizing (97) around the steady-state yields

$$\epsilon_1^r(\hat{p}_t - \hat{p}_{t+1}) + \tilde{\epsilon}_0^\tau \frac{\pi_{ss} - i_{ss}}{1+\pi_{ss}} \hat{p}_t + \epsilon_0^r(\hat{p}_{t-1} - \hat{p}_t) - \epsilon_0^A \hat{p}_{t-1} = -\hat{p}_t, \tag{98}$$

which simplifies to, defining consistently with (78), $\tilde{\epsilon}_0^r = \epsilon_0^r + \tilde{\epsilon}_0^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}$,

$$\hat{p}_{t+1} = (1 + \frac{1-\tilde{\epsilon}_0^r}{\epsilon_1^r})\hat{p}_t, \tag{99}$$

with eigenvalue $1 + \frac{1-\tilde{\epsilon}_0^r}{\epsilon_1^r}$. This one-dimensional model is determinate, since $1 - \tilde{\epsilon}_0^r > 0$ and $\epsilon_1^r > 0$ and thus the eigenvalue is larger than one. To grasp the intuition for this determinacy result, suppose that $P_t > P^*$ is an equilibrium price which implies a fall in the real value of debt, since, for the time being nominal debt is constant. The drop in real debt exceeds the fall in savings as $\tilde{\epsilon}_0^r < 1$. Since $\epsilon_1^r > 0$, asset market clearing then requires a decrease in the real interest rate, which is equivalent to $P_{t+1} > P_t$ because the nominal interest rate is constant. Iterating this argument shows that the price level diverges monotonically to infinity, or equivalently, that the eigenvalue is larger than one, meaning that the economy is locally determinate.

4.5 Local Determinacy: Policy Rules

The previous section establishes price-level determinacy when monetary and fiscal policy are constant. I now extend the analysis and allow for policies responding to price deviations from their

[31] An emerging literature, for example Paciello and Wiederholt (2014), Maćkowiak and Wiederholt (2015), Angeletos and Lian (2018), Farhi and Werning (2019) and Auclert et al. (2019b), features some form of informational frictions with or without limited processing capacity. While the informational frictions are much less involved here than in these papers, the simple model here suggests a wider applicability of my results.

respective steady-state values, and establish conditions for policy rules which deliver local determinacy. Interestingly, policy rules do not overcome indeterminacy, and may even induce it. Prices are again assumed to be flexible such that output is always at its natural level. Below I add sticky prices, implying output deviations from the natural level, and policy responding to these deviations.

I assume an interest rate rule,

$$\hat{i}_{t+1} = \varphi^i \hat{p}_t, \tag{100}$$

where φ^i is the response to price deviations. Since prices are the state-variables, it is convenient to specify the rule in terms of prices and not of inflation. Similarly, I assume a rule for nominal debt

$$\hat{B}_{t+1} = \varphi^B \hat{p}_t \tag{101}$$

with a price response φ^B. Taxes are still set to balance the government budget constraint,

$$\begin{aligned} \frac{\tau_{ss}}{A_{ss}}\hat{\tau}_t &= \frac{1+i_{ss}}{1+\pi_{ss}}\hat{i}_t + \frac{1+i_{ss}}{1+\pi_{ss}}(\hat{B}_t - \hat{p}_t) - (\hat{B}_{t+1} - \hat{p}_t) \\ &= \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}}\hat{p}_{t-1} + \frac{1+i_{ss}}{1+\pi_{ss}}(\varphi^B \hat{p}_{t-1} - \hat{p}_t) - (\varphi^B - 1)\hat{p}_t. \end{aligned} \tag{102}$$

The asset market clearing condition, again after substituting using equilibrium conditions, is

$$\begin{aligned} &\sum_{k=0}^{\infty} \Big\{ \epsilon^r_{k+1}(\varphi^i \hat{p}_{t+k} + \hat{p}_{t+k} - \hat{p}_{t+k+1}) + \tilde{\epsilon}^\tau_k [\varphi^i \hat{p}_{t+k-1}\frac{1+i_{ss}}{1+\pi_{ss}} + \frac{1+i_{ss}}{1+\pi_{ss}}(\varphi^B \hat{p}_{t+k-1} - \hat{p}_{t+k}) - (\varphi^B - 1)\hat{p}_{t+k}] \Big\} \\ + \;& \epsilon^r_0(\varphi^i \hat{p}_{t-1} + \hat{p}_{t-1} - \hat{p}_t) + \epsilon^A_0(\varphi^B - 1)\hat{p}_{t-1} = (\varphi^B - 1)\hat{p}_t, \end{aligned}$$

Determinacy requires showing that $\hat{p}_t = 0$ is the only equilibrium. It is again instructive to first consider the price sequence $\hat{p}_t = p > 0 \, (\forall t)$, and to show that demand exceeds supply, leading to the condition

$$\varphi^i \left(\epsilon^r_0 + \frac{\epsilon^r_1}{1-\beta^{CM}} + \frac{1+i_{ss}}{1+\pi_{ss}} \sum_{k=0}^{\infty} \tilde{\epsilon}^\tau_k \right) > (1-\varphi^B)(\tilde{\epsilon}^r_0 - 1). \tag{103}$$

Note that this condition nests Result 5, as it indicates determinacy for constant policy ($\varphi^i = \varphi^B = 0$) since $\tilde{\epsilon}^r_0 - 1 < 0$.[32] It is easy to establish the conditions for local determinacy for two special cases.

[32]Note that steady-state prices are not necessarily constant, and inflation can be negative or positive in this case. In particular, no price target for the central bank needs to be specified, since the economy is already determinate even if the nominal interest-rate is kept constant (Result 5). In contrast to complete markets models (Woodford, 2003; Giannoni, 2014) and similarly for the analytical HANK model with fiscal policy in real terms (Bilbiie, 2019), the role of monetary policy is not to render an indeterminate economy determinate here, but instead, monetary policy plays a role only when fiscal policy is too aggressive.

When monetary policy is constant ($\varphi^i = 0$) and only fiscal policy responds, or when fiscal policy is constant ($\varphi^B = 0$) and only monetary policy responds. If monetary policy is constant ($\varphi^i = 0$), the economy is locally determinate if $\varphi^B < 1$. The intuition is straightforward as to why determinacy can be ensured only if parameter restrictions on fiscal policy are imposed. Consider debt policy first and assume that $P_t > P^*$. If debt policy does not respond to prices, $\varphi^B = 0$, $P_t > P^*$ implies a fall in the real value of debt, so that households require lower real interest rates or equivalently, higher inflation rates, so as to absorb less real debt. That is, prices move further away from the steady state and thus cannot constitute an equilibrium. By contrast, if debt policy is aggressive, $\varphi^B > 1$, this reasoning does not work, since in this case, $P_t > P^*$ implies a policy-induced increase in the real value of debt. Households then require higher real interest rates, if they are willing to absorb more real debt. If the nominal interest rate does not respond to prices, this requires a fall in prices, implying indeterminacy. In the Blanchard&Kahn framework, this case would correspond to an eigenvalue less than one.

Monetary policy works through three channels. Tighter monetary policy increases the initial interest rate $1 + r_0$, rendering households richer, and aggregate savings increase by ϵ_0^r. Second, higher interest rates also induce an intertemporal substitution of consumption, implying higher savings as reflected by $\frac{\epsilon_1^r}{1-\beta^{CM}}$. Third, higher interest rates increase the tax burden and thus reduce savings, $\frac{1+i_{ss}}{1+\pi_{ss}} \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau < 0$. While condition (103) always ensures determinacy, I only consider the empirically relevant case that a tightening of monetary policy leads to a drop in consumption and an increase in savings.[33] Given this realistic restriction, there are no restrictions on monetary policy, so as to respond to prices, and the economy is determinate for all $\varphi^i \geq 0$, even including $\varphi^i = 0$.

Condition (103) combines monetary and fiscal policy. An expansionary fiscal policy ($\varphi^B > 1$) now induces determinacy, but monetary policy has to be sufficiently contractionary, that is, φ^i has to be sufficiently high. The intuition behind this result builds on the above explanations of the determinacy of monetary and fiscal policy. Suppose again $P_t > P^*$ and $\varphi^B > 1$. Again, real debt increases, so that households require higher real interest rates to be willing to absorb more real debt. If monetary policy were passive, this would require a fall in prices. But if the nominal interest rate increases more than the required real interest rate, $\hat{p}_t = p$ would then imply that real interest rates are too high. As a consequence, future prices must again increase to bring the real interest rate down. That is prices move away from the steady state, implying determinacy.[34]

In contrast to Result 5, it is now not sufficient to rule out the price sequence $\hat{p}_t = p$ to ensure determinacy and the condition needs to be modified. For natural sign restrictions, $\varphi^i \geq 0, \varphi^B \geq 0$, I

[33] The condition, $\epsilon_0^r + \frac{\epsilon_1^r}{1-\beta^{CM}} + \frac{1+i_{ss}}{1+\pi_{ss}} \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau > 0$, is quite weak, since $\frac{1+i_{ss}}{1+\pi_{ss}} \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau \geq -1$ and $\frac{\epsilon_1^r}{1-\beta^{CM}} \approx 70$ for standard calibration choices (Kaplan et al., 2018).

[34] With complete markets, $\tilde{\epsilon}_0^r = 1$, and condition (103) reduces to $\varphi^i > 0$, the determinacy condition for price level targeting (Woodford, 2003; Giannoni, 2014; Bilbiie, 2019).

obtain[35,36]

Result 6. *The model with a policy rule (100) for nominal interest rates and (101) for bonds with natural sign restrictions, $\varphi^i \geq 0, \varphi^B \geq 0$, has a unique bounded solution, that is, the economy is locally determinate if*

$$\varphi^i[\frac{\epsilon_1^r}{1+\beta^{CM}} - (1+\tilde{\epsilon}_0^\tau \frac{1+i_{ss}}{1+\pi_{ss}})] > (1-\varphi^B)(\tilde{\epsilon}_0^r - 1) + 2\varphi^B(1+\tilde{\epsilon}_0^\tau \frac{1+i_{ss}}{1+\pi_{ss}}). \tag{104}$$

The need to rule out negative or alternating price sequences requires modifying the term for the intertemporal substitution channel of monetary policy in (103), $\frac{\epsilon_1^r}{1+\beta^{CM}}$. This includes, but is more involved than merely ruling out negative eigenvalues larger than -1, and leads to a weaker lower bound on the magnitude of this channel. Similarly, ruling out price sequences other than $\hat{p}_t = p$ also requires replacing the remaining terms describing monetary policy for $\hat{p}_t = p$ in (103), $\varphi^i \epsilon_0^r$ and $\varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau$. If the previous period's price is low, $P_{t-1} < P^*$, households then start Period t with fewer assets if $\varphi^B > 0$ and obtain lower interest rate income if $\varphi^i > 0$, rendering them poorer. As a result, households lower savings, but at the same time Period t taxes are lowered which increases savings. Combining both effects leads to a savings decrease, which requires replacing the term $\varphi^i(\epsilon_0^r + \frac{1+i_{ss}}{1+\pi_{ss}} \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau)$ in (103) by the term $-(\varphi^i + 2\varphi^B)(1+\tilde{\epsilon}_0^\tau \frac{1+i_{ss}}{1+\pi_{ss}})$ in condition (104).

4.6 Local Determinacy: Sticky Prices

The appendix adds price-adjustment costs to the model as in Rotemberg (1982), and shows that this leads to the standard New Keynesian linearized Phillips curve[37]

$$\hat{\pi}_t = \kappa \hat{Y}_t + \beta^{CM} \hat{\pi}_{t+1}, \tag{105}$$

which allows expressing $\hat{Y}_t$ in terms of prices

$$\hat{Y}_t = \frac{\hat{p}_t - \hat{p}_{t-1} - \beta^{CM}(\hat{p}_{t+1} - \hat{p}_t)}{\kappa}. \tag{106}$$

Using this to substitute for $\hat{Y}_t$ in the asset market clearing condition (91) renders Onatski's criterion applicable and I obtain:

[35] While a negative $\varphi^B < 0$ pushes towards determinacy, and if negative enough, determinacy is obtained without imposing any condition, monetary policy with $\varphi^i < 0$ now pushes towards indeterminacy, rendering the determinacy condition tighter. The appendix provides the details for these empirically less relevant scenarios.

[36] The proof in the appendix shows that the condition can be slightly strengthened.

[37] Note that profits in the linearized model are discounted using the steady-state real interest rate and that the inverse of the steady-state real interest rate, $\beta^{CM} = \frac{1+\pi_{ss}}{1+i_{ss}}$.

Result 7. *Allowing for price rigidities does not change the determinacy criterion. For natural sign restrictions ($\varphi^i, \varphi^B \geq 0$), the economy is locally determinate if*

$$\varphi^i[\frac{\epsilon_1^r}{1+\beta^{CM}} - (1+\tilde{\epsilon}_0^r \frac{1+i_{ss}}{1+\pi_{ss}})] > (1-\varphi^B)(\tilde{\epsilon}_0^r - 1) + 2\varphi^B(1+\tilde{\epsilon}_0^r \frac{1+i_{ss}}{1+\pi_{ss}}). \tag{107}$$

Considering the price sequence $\hat{p}_t = p\,\forall t$ indicates why allowing for sticky prices does not alter the criterion. For these prices, (106) implies that $\hat{Y}_t = 0\,\forall t$, so that the asset market clearing equation, and thus the determinacy criterion, are unchanged. The proof builds on this insight and then rules out all other price sequences such that $\hat{p}_t = 0$ is the only remaining equilibrium price, implying determinacy. The underlying reason for an unchanged determinacy criterion is that adding price rigidities only affects the magnitude of price changes, but not the condition for determinacy, echoing results in Nakajima and Polemarchakis (2005). (In)determinacy is about whether an equilibrium price $\hat{p}_t \neq 0$ exists. The magnitude of the price deviation from 0 is irrelevant. The economy would be indeterminate if $\hat{p}_t = 2\%$ constitutes an equilibrium for flexible prices, or if sticky prices constrain the price deviation to 1%. The finding on the irrelevance of price stickiness is also consistent with the determinacy criterion in New Keynesian representative agent economies, which requires the coefficient for inflation in the interest rate rule to exceed 1, independently of the degree of price stickiness.[38]

Price rigidities alter the determinacy analysis, both in the representative agent and the incomplete markets economies if the policy rules also respond to output deviations. I thus now allow policy rules to respond not only to price but also to output deviations,

$$\hat{i}_{t+1} = \varphi^i \hat{p}_t + \varphi_Y^i \hat{Y}_t, \tag{108}$$

$$\hat{B}_{t+1} = \varphi^B \hat{p}_t + \varphi_Y^B \hat{Y}_t. \tag{109}$$

Imposing natural sign restrictions - $\varphi_Y^i > 0$ (higher interest rate in a boom) and $\varphi_Y^B < 0$ (expansionary fiscal policy in a recession) shows that the local determinacy condition is unaffected by φ_Y^i and φ_Y^B under two assumptions (precisely stated in the appendix). Firstly, monetary policy operates through an intertemporal substitution effect and through its fiscal effects. I assume that the first effect outweighs the second such that an increase in nominal interest rates increases savings and lowers consumption as observed in the data. Secondly, I impose an upper bound on φ_Y^B which approximately states $\beta^{CM}\epsilon_0^Y \geq -\varphi_Y^B(mpc_0^0 + \beta^{CM} - 1)$. If markets are complete, the two conditions

[38] Another property of the basic New Keynesian is that the norm of the complex eigenvalues falls, if price rigidities increase, what can be interpreted as rigid prices slowing down the adjustment of inflation. The norm of the eigenvalue of the transition matrix equals the determinant of the transition matrix which is increasing in κ (see Woodford, 2003, Appendix C.2). The eigenvalues of the simple New Keynesian representative agent model are complex, if the inflation coefficient in the interest rate rule satisfies the mild condition of being larger than $1 + \frac{(\kappa\sigma+(1-\beta))^2}{4\beta\kappa\sigma}$, which, for standard parameter choices, equals about 1.01.

are always satisfied since $mpc_0^0 = 1 - \beta^{CM}$, implying that both conditions are also satisfied if the incomplete markets allocation is sufficiently close to the complete markets allocation. The reason why price rigidities require additional assumptions is that the policy responses to output deviations have fiscal consequences, which affect consumption and savings in incomplete markets models. The two assumptions impose bounds on the magnitude of these fiscal effects.

5 Conclusion

This paper shows that the price level is determinate in Bewley-Imrohoroglu-Huggett-Aiyagari heterogenous agents incomplete markets models. A key finding is that the price level is determined jointly by monetary and fiscal policy, with long-run inflation determined by the growth rate of nominal government debt, even if monetary policy is operating an interest rate rule with a different inflation target. The nominal anchor - nominal fiscal variables - is controlled by fiscal policy, which therefore has the power to set the long-run inflation rate.

Price level determinacy is derived within the standard incomplete markets framework, with a partially nominal fiscal policy and without any additional new assumptions. The determinacy result is then merely a consequence of well-known properties of aggregate consumption, savings and real interest rates in incomplete markets models. The literature has however, not recognized how combining these properties yields determinacy.

This novel theory of price and inflation determination calls for a rethinking of various issues in monetary economics, which should be addressed in future research. Applied to recent attempts by the ECB to increase inflation in the Euro area, the findings in this paper suggest that these efforts are unlikely to succeed. Instead, higher inflation would require an expansion of nominal fiscal spending by Euro area member states, in order to stimulate nominal demand, assigning an important role to large countries such as Germany. A fiscal stimulus by a small country within the Euro area would have very little impact on inflation, as it has only a negligible effect on area-wide demand, but would lead to a real exchange rate appreciation (with probable adverse economic consequences) for this small country.

If the US or the world economy in the future finds itself stuck in a liquidity trap with zero nominal and real interest rates for an extended period, the results in this paper suggest an easy solution. Although the ZLB prevents further cuts of the nominal interest rate, fiscal policy can increase the growth rate of nominal spending and therefore the inflation rate, leading to lower real interest rates, provided that this policy is sufficiently persistent and credible to households and firms. If instead, fiscal policy were to implement an austerity plan of bringing low inflation rates to around zero, then the real interest rate would also hover around zero, even in the long run.

The theory set out in this paper also offers a different perspective on US inflation history. After experiencing high rates in the 1970s, the 1980s witnessed success in keeping inflation low. The standard interpretation is that central banks eventually recognized that keeping inflation low was their primary objective and as a consequence, were successful in doing so. The framework proposed in this paper suggests that it was not in fact the change in monetary policy that kept inflation in check, but rather a shift to a less expansionary fiscal policy during the Reagan administration, perhaps imposed by the prolonged high nominal interest rates set by central banks under chairman Paul Volcker and the resulting high deficits. Having established a framework with a determinate price level allows for a rigorous study that sheds new light on these and many more important policy and empirical questions such as forward guidance (Hagedorn et al., 2019) and fiscal policy in a liquidity trap (Hagedorn et al., 2017a).

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APPENDIX

A.I Appendix to Section 3

In this Section, I first explain price level determinacy in incomplete markets models with a natural borrowing constraint (Section A.I.1). In the remainder of this Section, I discuss additional aspects of steady-state price level determinacy, supplementing the arguments in Section 3, with:

- Fully Indexed Bonds and Nominal Government Expenditure (Section A.I.2).
- Explaining price level indeterminacy in perpetual youth models (Section A.I.3).
- Explaining price level indeterminacy in representative agent models with aggregate risk (Section A.I.4).

A.I.1 Price Level Determinacy with Natural Debt Limits (NDL)

In the main text, debt limits are exogenous. With such debt limits, the intuition (already in Bewley, 1980) for the determinacy result is that with incomplete markets, the household sector effectively values real bonds, and fiscal policy determines the level of nominal government debt.

The NDL assumption differs from exogenous credit constraints, since it merely requires that the household can definitely repay the loan. In a steady state for an interest rate $r > 0$ and $e_1 = \min\{e \mid e \in \mathcal{E}\}$ this yields

$$a_{t+1} \geq -\frac{e_1}{r}. \tag{A1}$$

If the real interest rate $r < 0$, which is possible in incomplete markets models, an exogenous constraint as (2) needs to be imposed, and we are back to the exogenous constraint case. I therefore focus on the $r > 0$ case.

The NDL can give rise to a Modigliani-Miller theorem of government finance, where there is a continuum of government policies that leads to the same equilibrium consumption choices as in (Wallace, 1981; Chamley and Polemarchakis, 1984; Peled, 1985; Sargent and Smith, 1987). An incomplete markets model with NDL allows for such a theorem, where the level of real government debt is irrelevant and thus does not affect household consumption choices (Bhandari et al., 2017). This type of real indeterminacy translates into steady-state price level indeterminacy, if there are two stationary equilibria with the same real interest rate $1 + r_{ss}$ and the same household consumption choices but with different price levels P_1 and P_2.

To characterize the type of fiscal policies which give rise to such an indeterminacy, we define the nominal taxes of an agent with endowment e as $T(e)$. Without government expenditure, taxes are only raised to pay for interest payments on bonds, so that the steady-state government budget constraint is satisfied,

$$\int T(e)d\Omega = iB. \tag{A2}$$

Consider a household in the equilibrium with price level P_1, endowment e and beginning-of-period real assets $a_1(e)$. Let $a_2(e)$ be real assets of this household in the other price P_2 equilibrium. For the consumption choice of this household to be the same across the two equilibria, the budget constraint has to be the same, implying[39]

$$r_{ss}(a_1(e) - a_2(e)) = \frac{T(e)}{P_1} - \frac{T(e)}{P_2}. \tag{A3}$$

In the model with exogenous credit constraints, this condition would not leave household consumption choices unchanged. For example, consider a credit-constrained household in the P_1 equilibrium. Then $a_2(e) < a_1(e)$ is not feasible, as it violates the credit constraint and if $a_2(e) > a_1(e)$, some of the additional wealth is consumed. With NDL, this argument could break down, as some Ricardian equivalence-type argument kicks in. With NDL, the credit constraint depends on future after-tax income and thus the credit constraint is relaxed if future tax obligations fall. If tax obligations fall enough, this could render $a_2(e) < a_1(e)$ feasible. I will show and verify below what "falling enough" means, but I proceed now just using (A3) and assume that the consumption choice is unaffected. This implies that the difference in assets across the two equilibria carried to the next period is $a_1(e) - a_2(e)$. For the consumption choice in the next period to be unaffected requires again that the budget constraints be the same, implying

$$r_{ss}(a_1(e) - a_2(e)) = \frac{T(e')}{P_1} - \frac{T(e')}{P_2} \tag{A4}$$

with the important difference that taxes depend on next period's endowment e'. Iterating this

[39] The Period t intertemporal budget constraint for $e_t = e$ implies

$$(1 + r_{ss})(a_{1,t}(e) - a_{2,t}(e)) = E_t \sum_{s=t}^{\infty} \Big(\frac{1}{1 + r_{ss}}\Big)^{s-t} \Big(\frac{T(e_s)}{P_1} - \frac{T(e_s)}{P_2}\Big)$$

and the Period $t+1$ intertemporal budget constraint for $e_{t+1} = e$ implies

$$(1 + r_{ss})(a_{1,t+1}(e) - a_{2,t}(e_t)) = E_{t+1} \sum_{s=t+1}^{\infty} \Big(\frac{1}{1 + r_{ss}}\Big)^{s-t-1} \Big(\frac{T(e_s)}{P_1} - \frac{T(e_s)}{P_2}\Big)$$

so that the differences in beginning of Period t assets , $a_{1,t}(e) - a_{2,t}(e)$, and of Period $t+1$ assets, $a_{1,t+1}(e) - a_{2,t+1}(e)$, are identical,

$$a_{1,t}(e) - a_{2,t}(e) = a_{1,t+1}(e) - a_{2,t+1}(e),$$

so that these terms cancel in (A3).

argument shows that

$$r_{ss}(a_1(e) - a_2(e)) = \frac{T(\hat{e})}{P_1} - \frac{T(\hat{e})}{P_2} \tag{A5}$$

for all $\hat{e} \in \mathcal{E}$, that is the right side is constant. Therefore,

$$\forall \hat{e} \in \mathcal{E} : \quad \frac{T(e) - T(\hat{e})}{P_1} = \frac{T(e) - T(\hat{e})}{P_2}, \tag{A6}$$

implying, since $P_1 \neq P_2$, that taxes do not depend on e and are lump-sum,

$$T(e) = T. \tag{A7}$$

In line with the arguments in the literature cited above, lump-sum taxation implies that constructing a continuum of equilibria is straightforward, but more subtle than merely multiplying all prices by some number $\lambda > 0$. Start with one price level P and an associated stationary asset distribution described through Ω. We now consider a different price $\hat{P} \neq P$, and construct a different stationary equilibrium.

Define $\delta = (\frac{T}{\hat{P}} - \frac{T}{P})/r$ and shift the distribution of assets by δ, that is, if a household holds a in the P stationary equilibrium, then it holds $\hat{a} = a + \delta$ in the new $\hat{P}$ stationary equilibrium. Household budget constraints change to

$$(1 + r)\hat{a} - c + e - \frac{T}{\hat{P}} = (1 + r)(a + \delta) - c + e - \frac{T}{P} - r\delta = (1 + r)a - c + e + \delta = a' + \delta = \hat{a}'. \tag{A8}$$

That is, for the same consumption choice c, the household now carries δ more assets, $a' + \delta$, to the next period, consistent with the idea that the asset distribution is shifted for each household in all states by δ. Following (Bhandari et al., 2017) it is now straightforward to show that the same consumption choices are optimal in both equilibria. The main idea is the same as in Ricardian equivalence proofs. The difference in lump-sum taxes is equal to the difference in interest payments for each household, such that a higher level of assets is merely used to cover higher tax payments without affecting consumption choices. The government budget constraint is also satisfied,

$$\frac{T}{\hat{P}} = \frac{T}{P} + r\delta = r(\frac{B}{P} + \delta) = r \int (a_i + \delta) di = r \int \hat{a}_i di. \tag{A9}$$

Note that in none of these equilibria is the real value of bonds owned by a household equal to the present value of its tax obligations. The reason is simple: households hold different amounts of bonds, but all pay the same amount of taxes. Instead, the difference in the real value of a household's bonds across equilibria is equal to the difference in its present value of taxes. This also explains why it is not sufficient simply to multiply the price P with λ. Such a multiplication would change the tax obligation for everyone by the same amount, x, but the real value of household bonds is not changed by the same absolute amount, but only by the same percentage, $1/\lambda$. MPC heterogeneity implies that this redistribution of wealth does not leave consumption choices unaffected.

These arguments show that the only scenario in which the price level is indeterminate, is with NDL and lump-sum taxes as the only tax instrument. In this case, constructing aggregate steady-state savings as in the main text as a fixed point of

$$S = \mathcal{S}(1 + r, \tau = rS) \tag{A10}$$

is not well-defined. If S is a fixed point, then $S + \delta$ is a fixed point as well, as I have just shown.

In all other cases - exogenous borrowing constraints or non-lump sum taxes - the price level is again determinate and the fixed point is well-defined, since no Modigliani-Miller theorem of government finance holds. Shifts in taxes affect consumption choices and thus savings decisions in such a way that no other fixed point is obtained. This latter scenario is the empirically relevant one. The NDL allocation is too close to the complete markets outcome and thus cannot match the empirical facts that motivate the heterogenous agents model. Income heterogeneity also restricts the size of lump-sum taxes to be less than the lowest income level, requiring a richer tax code to cover government expenditures (Werning, 2007). This renders the indeterminacy case an interesting but purely theoretical possibility. The NDL also does not lead to indeterminacy when government expenditure is (partially) nominal, as I show in the next Section.

A.I.2 Price-Level Determinacy with Fully Indexed Bonds, Nominal Government Expenditures and more on NDL

I now generalize the discussion of price-level determinacy to a scenario in which government bonds are real, and show that the price level is nevertheless determinate if spending and taxes are (partially) nominal. For illustrative purposes, I consider the extreme case of the real value of government bonds fixed at B^{real} and government spending and taxes entirely nominal.[40] One example is without government bonds, $B^{real} = 0$, and where nominal taxes are equal to nominal government spending in each period, $T = G$.

The inflation rate is again determined by fiscal policy

$$1 + \pi_{ss} = \frac{T' - T}{T} = \frac{G' - G}{G}. \tag{A11}$$

[40] This assumption also makes clear that the theory in my paper is different from the Fiscal Theory of the Price Level (FTPL), where the price level is determined such that the real value of bonds clears the government present-value budget constraint. Obviously the FTPL has no bite, if the real value of bonds is fixed and nominal taxes are set to balance the budget each period.

As in the main text, steady-state savings depend on the real interest rate, $1+r$, and the real value of taxes, T/P. The difference from the main text is that now, the real value of taxes and thus savings depend on the price level, such that the asset market clearing condition is

$$S = \mathcal{S}(1+r, T/P) = B^{real}, \tag{A12}$$

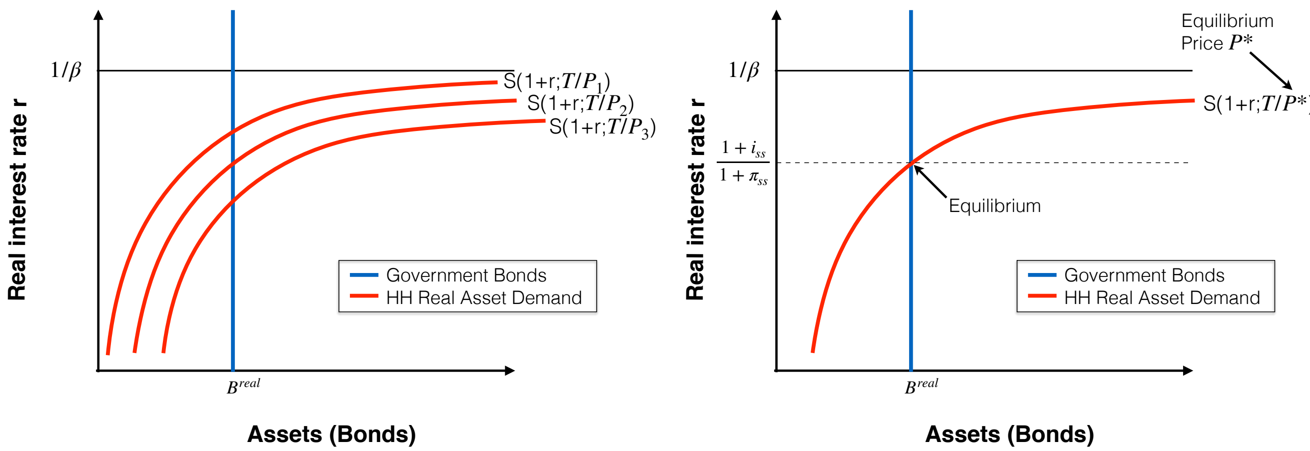
where $\frac{T}{P} = \frac{G}{P} - \omega + B^{real} i$ for a linear endowment tax ω and $i = r$ assuming $\pi = 0$. Note that this is no longer a fixed point in S, since real interest rate payments are fixed at $B^{real} r$, implying that even with NDL, the price level is determinate. The left panel of Figure 3 shows that different price levels shift the savings curve, while the real value of bonds is unchanged at B^{real}. The reason why household real asset demand depends on the level of real taxes, T/P, for a fixed real interest rate, is explained by heterogeneity and incomplete markets, and is formally established in Result 2. These features imply failure of the permanent income hypothesis and that agents, as a result of this failure, engage in precautionary savings. A lower steady-state level of real taxes increases both demand and (precautionary) savings. This reasoning extends to changes in the price level, which translate one-for-one into changes in real taxes, since the nominal level of taxes is fixed.

The intuition is straightforward. A higher steady-state price level lowers real government consumption, since fiscal policy is nominal, and at the same time, lowers the tax burden for the private sector by the same amount. Households, however, do not spend all of the tax rebate on consumption, but instead, use some of the tax rebate to increase their precautionary savings. This less than one-for-one substitution of private sector demand for government consumption implies a drop in aggregate demand (households plus government demand) and an increase in household asset demand. This would require an adjustment of the real interest rate so as to stimulate demand and lower savings, such that both the goods and the asset markets clear. The steady-state real interest rate cannot adjust to equate supply and demand, because it is pinned down by the nominal interest rate set by monetary policy, and by the inflation rate, which is equal to the growth rate of nominal government spending. Therefore, the equilibrium price level P^* must adjust such that demand equals supply when the real interest rate equals $1 + r_{ss} = \frac{1+i_{ss}}{1+\pi_{ss}}$ and solves

$$S(\frac{1+i_{ss}}{1+\pi_{ss}}, T/P^*) = B^{real}, \tag{A13}$$

as the right panel of Figure 3 illustrates. These arguments extend to cases in which government bonds are nominal. In particular, the price level is determinate even with NDL, and for all tax policies, as long as government expenditure is (partially) nominal. If bonds B are nominal, taxes now satisfy

$$\frac{T}{P} = \frac{G}{P} - \omega + i\frac{B}{P}, \tag{A14}$$



(a) Savings Curve

(b) Equilibrium

Figure 3: Asset Market Equilibrium with Price-Indexed Government Debt B^{real}.

such that steady-state savings are a fixed point of

$$S = \mathcal{S}(1+r, \frac{G}{P} - \omega + rS). \tag{A15}$$

Although this is again a fixed-point problem, the above arguments for showing price-level indeterminacy with NDL do not apply here. To observe this, suppose to the contrary that there are two different equilibrium prices P_1 and P_2. The same arguments as above show that a necessary condition is again that taxes are lump-sum, so that $\omega = 0$. Shifting the assets of each household by δ, where

$$\frac{B}{P_2} = \frac{B}{P_1} + \delta, \tag{A16}$$

then requires

$$\frac{T}{P_2} = \frac{T}{P_1} + \delta r \tag{A17}$$

to keep household budget constraints unaffected. But then, the steady-state government budget constraint for P_2 is not satisfied, if it is satisfied for P_1

$$\begin{aligned}
& \frac{G}{P_2} + r\frac{B}{P_2} - \frac{T}{P_2} && \text{(A18)} \\
= \; & \frac{G}{P_2} + r(\frac{B}{P_1} + \delta) - (\frac{T}{P_1} + \delta r) && \text{(A19)} \\
= \; & \frac{G}{P_2} + r\frac{B}{P_1} - \frac{T}{P_1} && \text{(A20)} \\
= \; & \frac{G}{P_2} \underbrace{- \frac{G}{P_1} + \frac{G}{P_1}}_{=0} + r\frac{B}{P_1} - \frac{T}{P_1} && \text{(A21)} \\
= \; & \frac{G}{P_2} - \frac{G}{P_1} + \underbrace{\frac{G}{P_1} + r\frac{B}{P_1} - \frac{T}{P_1}}_{=0} && \text{(A22)} \\
= \; & \frac{G}{P_2} - \frac{G}{P_1} \neq 0. && \text{(A23)}
\end{aligned}$$

The intuition is straightforward. The shift in prices not only shifts taxes one-for-one with interest rate payments on debt, but also shifts real government spending if it is partially nominal. This leaves the household budget constraint unaffected, but not the government budget constraint, as government expenditure shows up in the latter, but not in the former constraint.

A.I.3 Price Level Indeterminacy: Perpetual Youth Model

Similar arguments for showing indeterminacy in complete markets models apply in "perpetual youth" models (Yaari, 1965; Blanchard, 1985) since the steady-state interest rate is again equal to the discount rate, but now adjusted for the probability of death or retirement, so that $(1 + r_{ss})\tilde{\beta} = 1$ in a steady state for the adjusted discount rate $\tilde{\beta}$. Again, the steady-state real interest rate is independent of the price level and only the change in prices, $1 + \pi_{ss}$, but not the level itself is determined.

In this class of models, however, this is not the only equilibrium, if the Samuelson dynamic inefficiency condition is satisfied. In this case, both a bubbleless as well as a continuum of bubbly equilibria exist, a scenario explored in recent work by Galí (2017). Whereas most papers assume that the bubble is a real asset affecting the stock market or housing market, a monetary bubble may coexist, so that money has value as in Samuelson's work. As a result, there is a continuum of equilibria, each associated with a different value of money (= different size of monetary bubble) and each associated with a different price level. As an example, suppose that a bubble exists that has a real value of one. In one equilibrium, nominal money has a value of one, the price level is one and there are thus no real bubbles. In another equilibrium, the price level is two and there is a real bubble with a value one half. Alternatively, the price level is three and the real bubble has a value of two thirds. Or the real bubble has a value of one and money has no value.

Bénassy (2005, 2008) make a particular choice on the size of the monetary bubble through ruling out real bubbles (the first case in the previous example) and conditional on this choice, find a unique bubbly price level. This approach however, does not overcome the indeterminacy problem in the "bubbleless" equilibrium and it rules out other bubbly equilibria with different price levels by assumption.[41] Bénassy (2005, 2008) need to make this equilibrium selection in order to obtain a well-defined demand for money (or more generally for nominal government liabilities), since the Samuelson logic only delivers the existence of a monetary equilibrium, but not uniqueness. This shows again, as in the Hand-to-Mouth economy, that the failure of Ricardian equivalence is a necessary but not sufficient condition for price-level determinacy.

A.I.4 Price Level Indeterminacy: Complete Markets and Aggregate Risk

Price level indeterminacy also arises in representative agent economies with aggregate risk. Assume there are n aggregate shocks $s_1, \ldots, s_n$ with associated consumption levels of the representative household $c_1, \ldots, c_n$ and marginal utilities of consumption $u_1, \ldots, u_n$. The FOCs for nominal bonds are therefore

$$\frac{u_i}{\tilde{P}_i} = \beta \frac{1 + i_{ss}}{1 + \pi} \sum_{j=1}^{n} q_{ij} \frac{u_j}{\tilde{P}_j},$$

where $q_{ij} = Prob(s_j \mid s_i)$, $\tilde{P}_i$ is the price level in state s_i (normalized by the price trend) and the inflation rate π, which is equal to the constant growth rate of nominal debt. Since consumption c_i is equal to endowment in this state s_i, marginal utilities u_i do not depend on prices. Therefore, for each $\kappa > 0$ multiplying all prices in all states by κ is also an equilibrium, establishing indeterminacy. Adding aggregate risk to an economy with PIH-households and hand-to-mouth households also does not overcome the indeterminacy problem. The same arguments for the representative agent now apply to the PIH households.

A.II Monetary and Fiscal Policy and Ljungqvist and Sargent's (2012) Ten Monetary Doctrines

In this Section, I explain the differences between the FTPL and my theory in more detail. The starting point is Chapter 26 on "Fiscal-Monetary Theories of Inflation" in Ljungqvist and Sargent (2012), which considers a policy designed to differentiate between the initial Period $t = 0$ ("short run") and the remaining Periods $t \geq 1$ ("long run"). I adopt this setting in my model and thus also

[41] Bénassy (2005, 2008) implicitly assumes a particularly strong dynamic inefficiency condition - the population growth rate exceeds the real interest rate (which exceeds $1/\beta$) - since consumption by the initial generation would otherwise eventually exceed GDP.

allow for real government expenditures g_t, such that the Period t government budget constraint is

$$T_t := (1 + i_t)B_t + P_t g_t - B_{t+1} \tag{A24}$$

and assume that

$$\begin{aligned} g_t &= g \quad \forall t \geq 0 \\ \tau_t &= \tau \quad \forall t \geq 0 \\ B_t &= B \quad \forall t \geq 1 \\ T_t &= T \quad \forall t \geq 1, \\ i_t &= i \quad \forall t \geq 0, \end{aligned} \tag{A25}$$

where I permit initial bonds $B_0 \neq B$ and initial lump-sum taxes $T_0 \neq T$. The steady-state inflation rate is $\pi_{ss} = 0$.

I use the savings function $S_{t+1}(\Omega_t, \frac{1+i}{P_t}, 1 + r_{t+1}, 1 + r_{t+2}, \ldots; \tau_t, \tau_{t+1}, \ldots)$ but with some modifications to simplify solving the model backwards. Ω_t is now the joint distribution of nominal assets and productivity at the beginning of Period t and savings S_{t+1} depend on $\frac{1+i}{P_t}$. Savings S_{t+1} still depend on the sequence of future real interest rates and taxes.[42]

Using $1 + r_{t+k+1} = (1 + i)\frac{P_{t+k}}{P_{t+k+1}} \forall k \geq 0$ and the government budget constraint (A24), the equilibrium asset market clearing condition in Period t is then

$$S_{t+1}(\Omega_t, \frac{1+i}{P_t}, (1+i)\frac{P_t}{P_{t+1}}, (1+i)\frac{P_{t+1}}{P_{t+2}}, \ldots; g_t + \frac{(1+i)B_t - B_{t+1}}{P_t}, g_{t+1} + \frac{(1+i)B_{t+1} - B_{t+2}}{P_{t+1}}, \ldots)$$
$$-\frac{B_{t+1}}{P_t} = 0. \tag{A26}$$

Assume now that such an equilibrium sequence of savings, distributions and prices exists and that a steady state is reached after N periods. Given this sequence of equilibrium distributions, the sequence of prices can easily be determined backwards.[43] Condition (A26) is upward sloping in the price P_t, since $\epsilon_1^r + 1 - \epsilon_0^r - i\tilde{\epsilon}_0^\tau > 0$ as shown in Section 4, implying a unique solution P_t, taking all other future prices as given. Starting with the steady-state price level $P_N = P^*$, this argument allows computing the full sequence of prices backwards. First, the price P_{N-1} is calculated, taking P_N as given as the

[42]Note that the Period $t-1$ real value of nominal assets b_t acquired in Period $t-1$ is b_t/P_{t-1} and that the real Period t payoff is $b_t/P_{t-1}(1+i)\frac{P_{t-1}}{P_t} = b_t\frac{1+i}{P_t}$, explaining why Period t savings depend on $\frac{1+i}{P_t}$. Household Period t real asset income thus equals $b_t\frac{1+i}{P_t}$ and the household problem is fully real.

[43]Computing an equilibrium requires an iteration of two steps. Given a sequence of distributions, we compute the sequence of prices backwards. Given a sequence of prices and thus real interest rates, we compute the household problems which yield a new sequence of distributions.

solution to

$$S_N(\Omega_{N-1}, \frac{1+i}{P_{N-1}}, (1+i)\frac{P_{N-1}}{P_N}, 1+r_{ss}, \ldots; g_t + i\frac{B}{P_{N-1}}, g + i\frac{B}{P_N}, \ldots) - \frac{B}{P_{N-1}} = 0.$$

In the next step, P_{N-2} is the solution to

$$S_{N-1}(\Omega_{N-2}, \frac{1+i}{P_{N-2}}, (1+i)\frac{P_{N-2}}{P_{N-1}}, , (1+i)\frac{P_{N-1}}{P_N}, 1+r_{ss}, \ldots; g_t + i\frac{B}{P_{N-2}}, g + i\frac{B}{P_{N-1}}, \ldots) - \frac{B}{P_{N-2}} = 0.$$

This procedure is iterated until the initial price level P_0 is computed. If $B_{-1} = B$, then

$$P_t \quad = \quad P^* \quad \forall t \geq 0. \tag{A27}$$

The Fiscal Theory of the Price Level (FTPL) takes a different approach. To clarify the difference, now I assume that markets are complete and that lump-sum taxes are in real terms and fixed at τ.

The Period t government budget constraint in real term reads

$$\frac{B_{t+1}}{P_t} = (1+r_t)\frac{B_t}{P_{t-1}} + g - \tau. \tag{A28}$$

Since aggregate endowment and thus consumption are constant, the real interest rate $1 + r_t = 1 + r$ is also constant. The intertemporal government budget constraint states that

$$\frac{B_1}{P_0} = \sum_{t=1}^{\infty} \frac{1}{(1+r)^t}(\tau - g) \tag{A29}$$

and thus, we obtain for the initial debt level B_0

$$\frac{B_0}{P_0} \quad = \quad \frac{1}{1+i}[\frac{B_1}{P_0} + (\tau - g)] = \frac{1}{1+i}\sum_{t=0}^{\infty} \frac{1}{(1+r)^t}(\tau - g) = \frac{(\tau - g)(1+r)}{r(1+i)}, \tag{A30}$$

which using the complete markets property $1 + r = 1/\beta$, yields

$$\frac{B_0}{P_0} \quad = \quad \frac{\tau - g}{(1+i)(1-\beta)}. \tag{A31}$$

The FTPL assumes that g and τ are exogenous, so that the initial price level P_0^{FTPL} is determined as the ratio of outstanding nominal debt B_0 to the present value of the primary surplus.

Apparently, the determination of the price level when markets are incomplete, or when the FTPL is operating, are different. In the first case, the asset market clearing condition is used, so that the price level depends on savings demand. In the second case, the government budget constraint is used, so that the price level depends on the initial outstanding government debt level B_0 and the fiscal variables g and τ.

If $B_0 = B$ the difference becomes even clearer. Suppose that both theories delivered the same price level, $P^* = P^{FTPL}$. Then

$$S(1+r) = \frac{B}{P^*} = \frac{B}{P^{FTPL}} = \frac{\tau - g}{r}, \tag{A32}$$

implying that steady-state savings, using $1+r = 1+i$, equals

$$S(1+r) = S(1+i) = \frac{\tau - g}{i}, \tag{A33}$$

which is generically not true. For example, it is well known that savings converge to infinity if $1+r \to 1/\beta$, whereas the right side does not,

$$\lim_{1+i \to 1/\beta} S(1+i) = \infty > \lim_{1+i \to 1/\beta} \frac{\tau - g}{i}. \tag{A34}$$

The reason for the difference is that P^* is determined as clearing the asset market and that the price P_0^{FTPL} is determined as satisfying the government budget constraint. This constraint also has to be satisfied in the first case, although it is not the price, but the lump-sum tax T which ensures this, such that both the asset market clears and the budget constraint is satisfied when markets are incomplete. Also, fiscal policy is passive in the first case, whereas it is active in the FTPL (Leeper, 1991).

Both constraints are also satisfied when the FTPL is operating. The government budget constraint is satisfied by construction. The asset market clears as well, but in the context of the FTPL markets being complete and asset market clearing only pinning down the real interest rate $1 + r = 1/\beta$. This is a consequence of Ricardian equivalence, since the private sector is willing to absorb any equilibrium amount of real bonds. Using the FTPL in different environments in which Ricardian equivalence does not hold, for example if markets are incomplete, requires the private sector to be willing to absorb the real value of debt, which satisfies the government budget constraint.

A.III Proofs and Derivations

A.III.1 Derivations of Section 4 [Local Determinacy]

Derivations of Section 4.1.2 [Properties of $\Delta_{i,t}$]

The transfer scheme is

$$
\begin{aligned}
\Delta_{i,t} &= \frac{A^{CM}_{t+1} - A_{ss}}{A_{ss}} a^{ss}_{i,t+1} + a^{ss}_{i,t}\{(1+r_{ss}) - \frac{A^{CM}_{t}}{A_{ss}}(1+r_t)\} \\
&+ \frac{C^{CM}_{t} - C_{ss}}{C_{ss}} c^{ss}_{i,t} + (Y_{ss} - Y_t)e_{i,t} - r_{ss}A_{ss} + \tau_t, \\
&= \frac{A^{CM}_{t+1} - A_{ss}}{A_{ss}} a^{ss}_{i,t+1} - a^{ss}_{i,t}(\frac{A^{CM}_{t}}{A_{ss}} - 1) - r_t \frac{A^{CM}_{t}}{A_{ss}}(a^{ss}_{i,t} - A_{ss}) + r_{ss}(a^{ss}_{i,t} - A_{ss}) \\
&+ \frac{C^{CM}_{t} - C_{ss}}{C_{ss}} c^{ss}_{i,t} + (Y_{ss} - Y_t)e_{i,t} - r_t A^{CM}_{t} + \tau_t,
\end{aligned}
$$

The definition (59) of A^{CM}_{t+1} implies that these transfers are cross-sectionally purely redistributive

$$
\int \Delta_{i,t} d\Omega_t = (A^{CM}_{t+1} - A_{ss}) - (A^{CM}_{t} - A_{ss}) + (C^{CM}_{t} - C_{ss}) + (Y_{ss} - Y_t) - r_t A^{CM}_{t} + \tau_t = 0.
$$

For individual choices

$$
c^{AI}_{i,t} = \frac{C^{CM}_{t}}{C_{ss}} c^{ss}_{i,t}; \qquad a^{AI}_{i,t+1} = \frac{A^{CM}_{t+1}}{A_{ss}} a^{ss}_{i,t+1}, \tag{A35}
$$

the household budget constraints are satisfied:

$$
\begin{aligned}
& (1+r_t)a^{AI}_{i,t} - c^{AI}_{i,t} + e_{i,t}Y_t - \tau_t + \Delta_{i,t} \\
=& \frac{A^{CM}_{t+1} - A_{ss}}{A_{ss}} a^{ss}_{i,t+1} + (1+r_{ss})a^{ss}_{i,t} - c^{ss}_{i,t} + e_{i,t}Y_{ss} - \tau_{ss} \\
=& \frac{A^{CM}_{t+1} - A_{ss}}{A_{ss}} a^{ss}_{i,t+1} + a^{ss}_{i,t+1} \\
=& a^{AI}_{i,t+1}.
\end{aligned}
$$

The Consumption Euler-equations are satisfied. If the credit constraint is not binding, then

$$
(c^{ss}_{i,t})^{-\sigma} = \beta(1+r_{ss})E_t(c^{ss}_{i,t+1})^{-\sigma}. \tag{A36}
$$

holds in the steady state. The Euler equation in the complete markets model holds

$$
(C^{CM}_{i,t})^{-\sigma} = \beta^{CM}(1+r_{t+1})E_t(C^{CM}_{i,t+1})^{-\sigma}. \tag{A37}
$$

Combining these two equations implies that the consumption Euler equation holds for the new allocation

$$(c_{i,t}^{AI})^{-\sigma} = (c_{i,t}^{ss})^{-\sigma}(\frac{C_t^{CM}}{C_{ss}})^{-\sigma} = \beta(1+r_{ss})E_t(c_{i,t+1}^{ss})^{-\sigma}\beta^{CM}(1+r_{t+1})E_t(\frac{C_{t+1}^{CM}}{C_{ss}})^{-\sigma}. \quad \text{(A38)}$$

$$= \beta(1+r_{t+1})E_t(c_{i,t+1}^{AI})^{-\sigma} \quad \text{(A39)}$$

If the credit constraint is binding in the steady state for individual i at time t, $a_{it+1}^{ss} = 0$, then the same is true in the new allocation, since the same arguments show that

$$(c_{i,t}^{ss})^{-\sigma} > \beta(1+r_{ss})E_t(c_{i,t+1}^{ss})^{-\sigma}. \quad \text{(A40)}$$

implies

$$(c_{i,t}^{AI})^{-\sigma} > \beta(1+r_{t+1})E_t(c_{i,t+1}^{AI})^{-\sigma}. \quad \text{(A41)}$$

Aggregate consumption and savings in the incomplete markets economy are now

$$C_t^{AI} = \int c_{i,t}^{AI} d\Omega_t = \int \frac{C_t^{CM}}{C_{ss}} c_{i,t}^{ss} d\Omega_t = C_t^{CM}\frac{C_{ss}}{C_{ss}} = C_t^{CM}, \quad \text{(A42)}$$

$$A_t^{AI} = \int a_{i,t}^{AI} d\Omega_t = \int \frac{A_t^{CM}}{A_{ss}} a_{i,t}^{ss} d\Omega_t = A_t^{CM}\frac{A_{ss}}{A_{ss}} = A_t^{CM}, \quad \text{(A43)}$$

that is, the paths of aggregate consumption and savings coincide in and away from steady state. We thus have for $t = 0$,

$$C_0^{AI} = C_0^{CM} = \mathcal{C}(\{1+r_t\}_{t=0}^{\infty}, \{\tau_t\}_{t=0}^{\infty}, \{Y_t\}_{t=0}^{\infty}, A_0) \quad \text{(A44)}$$

and for $t > 0$,

$$C_t^{AI} = C_t^{CM} = \mathcal{C}^{CM}(\{1+r_s\}_{s=t}^{\infty}, \{\tau_s\}_{s=t}^{\infty}, \{Y_s\}_{s=t}^{\infty}, A_t^{CM}). \quad \text{(A45)}$$

Modifications if $\bar{a} > 0$

Note that some simple modifications reveal that the result extends to the case $\bar{a} > 0$. The set of households is split into two groups, one with non-negative assets and the other with negative assets. Let $\tilde{A}_{ss}$ be the aggregated steady-state assets of the first group with nonnegative assets, such that $A_{ss} - \tilde{A}_{ss}$ are aggregated assets of the second group with negative assets. Again define $c_{i,t}^{AI} = \frac{C_t^{CM}}{C_{ss}} c_{i,t}^{ss}$ but

$$a_{i,t}^{AI} = \begin{cases} a_{i,t}^{ss} & \text{if } a_{i,t}^{ss} < 0 \\ \frac{\tilde{A}_t^{CM}}{\tilde{A}_{ss}} a_{i,t}^{ss} & \text{if } a_{i,t}^{ss} \geq 0 \end{cases}$$

for $\tilde{A}_t^{CM} := A_t^{CM} + \tilde{A}_{ss} - A_{ss}$. The transfer is then defined as

$$\Delta_{i,t} = a_{i,t+1}^{AI} - a_{i,t+1}^{ss} + a_{i,t}^{ss}(1+r_{ss}) - a_{i,t}^{AI}(1+r_t) + c_{i,t}^{AI} - c_{i,t}^{ss} + (Y_{ss} - Y_t)e_{i,t} - r_{ss}A_{ss} + \tau_t.$$

Since

$$\int a_{i,t}^{AI} d\Omega_t = \int_{a_{i,t}^{ss} \geq 0} \frac{\tilde{A}_t^{CM}}{\tilde{A}_{ss}} a_{i,t}^{ss} d\Omega_t + \int_{a_{i,t}^{ss} < 0} a_{i,t}^{ss} d\Omega_t = \tilde{A}_t^{CM} + A_{ss} - \tilde{A}_{ss} = A_t^{CM},$$

the transfers are again cross-sectionally purely redistributive

$$\int \Delta_{i,t} d\Omega_t \quad = \quad (A_{t+1}^{CM} - A_{ss}) - (A_t^{CM} - A_{ss}) + (C_t^{CM} - C_{ss}) + (Y_{ss} - Y_t) - r_t A_t^{CM} + \tau_t = 0.$$

The household budget constraints are satisfied:

$$\begin{aligned} & (1+r_t)a_{i,t}^{AI} - c_{i,t}^{AI} + e_{i,t}Y_t - \tau_t + \Delta_{i,t} \\ = \; & a_{i,t+1}^{AI} - a_{i,t+1}^{ss} + (1+r_{ss})a_{i,t}^{ss} - c_{i,t}^{ss} + e_{i,t}Y_{ss} - \tau_{ss} \\ = \; & a_{i,t+1}^{AI}. \end{aligned}$$

The Consumption Euler-equations are satisfied by the same arguments used for $\bar{a} = 0$ since consumption changes by the same factor for everyone. Aggregate consumption and savings are then

$$C_t^{AI} = \int c_{i,t}^{AI} d\Omega_t \quad = C_t^{CM} \quad \text{and} \quad A_t^{AI} = \int a_{i,t}^{AI} d\Omega_t \quad = A_t^{CM}, \tag{A46}$$

that is, the paths of aggregate consumption and savings coincide in and away from steady state.

Derivations of Section 4.2 [Construction of Transfers]

The idea is again to construct transfers such that household consumption and asset choices are linear homogeneous in aggregate variables. The approach starts with the economy described in the main text where the household budget constraint (66) reads

$$\tilde{c}_{i,t}^{IM} + \tilde{a}_{i,t+1}^{IM} \quad = \quad (1+r_t)\tilde{a}_{i,t}^{IM} + e_{i,t} - \tau_t + \Delta_{i,t}, \tag{A47}$$

and $\Delta_{i,t}$ is defined in (67). Solving this model yields Period 0 consumption as a function of initial assets, interest rates and taxes

$$C_0 = \mathcal{C}^{IM}(\{1+r_s\}_{s=0}^{\infty}, \{\tau_s\}_{s=0}^{\infty}, \{Y_s\}_{s=0}^{\infty}, A_0^{IM}),$$

where the argument A_0^{IM} means that the initial asset distribution is the steady-state asset distribution shifted by the same factor A_0^{IM}/A_{ss} for every household.

Now define a sequence of aggregate consumption and assets iteratively for $t = 0, 1, 2, \ldots$,

$$\begin{aligned}\tilde{C}_0^{IM} &= C_0,\\ \tilde{A}_{t+1}^{IM} &= \tilde{A}_t^{IM}(1+r_t) - \tilde{C}_t^{IM} + Y_t - \tau_t,\\ \tilde{C}_{t+1}^{IM} &= \mathcal{C}^{IM}(\{1+r_s\}_{s=t+1}^{\infty}, \{\tau_s\}_{s=t+1}^{\infty}, \{Y_s\}_{s=t+1}^{\infty}, \tilde{A}_{t+1}^{IM}).\end{aligned}$$

Note that $\tilde{A}_{t+1}^{IM} \geq 0$ since $\tilde{C}_t^{IM}$ aggregates optimal consumption choices given aggregate assets $\tilde{A}_t^{IM}$. The individual choices underlying these aggregates do not constitute an equilibrium. A Period 0 household forms expectation about its Period t choices which are not necessarily identical with the actual Period t choices underlying the construction of $\tilde{C}_t^{IM}$. I therefore construct a new economy with different individual choices but with the same aggregate outcomes. I thus use these aggregate series to define a transfer scheme $\tilde{\Delta}_{i,t}$,

$$\begin{aligned}\tilde{\Delta}_{i,t} &:= \frac{\tilde{A}_{t+1}^{IM} - A_{ss}}{A_{ss}} a_{i,t+1}^{ss} + a_{i,t}^{ss}\{(1+r_{ss}) - \frac{\tilde{A}_t^{IM}}{A_{ss}}(1+r_t)\}\\ &+ \frac{\tilde{C}_t^{IM} - C_{ss}}{C_{ss}} c_{i,t}^{ss} + (Y_{ss} - Y_t)e_{i,t} - \tau_{ss} + \tau_t,\end{aligned} \tag{A48}$$

so that the household budget constraint reads

$$c_{i,t}^{IM} + a_{i,t+1}^{IM} = (1+r_t)a_{i,t}^{IM} + e_{i,t}Y_t - \tau_t + \tilde{\Delta}_{i,t}. \tag{A49}$$

The construction of the aggregate consumption and asset sequence in (A48) implies that the transfer scheme is again cross-sectionally purely redistributive

$$\int \tilde{\Delta}_{i,t} d\Omega_t = (\tilde{A}_{t+1}^{IM} - A_{ss}) - (\tilde{A}_t^{IM} - A_{ss}) + (\tilde{C}_t^{IM} - C_{ss}) + (Y_{ss} - Y_t) - r_t\tilde{A}_t^{IM} + \tau_t = 0.$$

I also adjust the sequence of discount factors to

$$\hat{\beta}_t = \beta \frac{(1+r_{ss})(\tilde{C}_t^{IM})^{-\sigma}}{(1+r_{t+1})(\tilde{C}_{t+1}^{IM})^{-\sigma}}. \tag{A50}$$

Household i's optimal choices at time t in this economy are

$$c_{i,t}^{IM} = \frac{\tilde{C}_t^{IM}}{C_{ss}} c_{i,t}^{ss}; \qquad a_{i,t+1}^{IM} = \frac{\tilde{A}_{t+1}^{IM}}{A_{ss}} a_{i,t+1}^{ss}. \tag{A51}$$

To see why, note first that the household budget constraints are satisfied:

$$\begin{aligned}
& (1+r_t)a_{i,t}^{IM} - c_{i,t}^{IM} + e_{i,t}Y_t - \tau_t + \tilde{\Delta}_{i,t} \\
= \quad & \frac{\tilde{A}_{t+1}^{IM} - A_{ss}}{A_{ss}} a_{i,t+1}^{ss} + (1+r_{ss})a_{i,t}^{ss} - c_{i,t}^{ss} + e_{i,t}Y_{ss} - \tau_{ss} \\
= \quad & \frac{\tilde{A}_{t+1}^{IM} - A_{ss}}{A_{ss}} a_{i,t+1}^{ss} + a_{i,t+1}^{ss} \\
= \quad & a_{i,t+1}^{IM}.
\end{aligned}$$

The consumption Euler-equations are also satisfied since the construction of $\hat{\beta}_t$ implies that

$$\hat{\beta}_t(1+r_{t+1})(\tilde{C}_{t+1}^{IM})^{-\sigma} = \beta \frac{(1+r_{ss})(\tilde{C}_t^{IM})^{-\sigma}}{(1+r_{t+1})(\tilde{C}_{t+1}^{IM})^{-\sigma}}(1+r_{t+1})(\tilde{C}_{t+1}^{IM})^{-\sigma} = \beta(1+r_{ss})(\tilde{C}_t^{IM})^{-\sigma} \tag{A52}$$

and thus

$$(c_{i,t}^{IM})^{-\sigma} = (c_{i,t}^{ss})^{-\sigma}(\frac{\tilde{C}_t^{IM}}{C_{ss}})^{-\sigma} \quad = \quad \beta(1+r_{ss})E_t(c_{i,t+1}^{ss})^{-\sigma}\frac{\hat{\beta}_t(1+r_{t+1})}{\beta(1+r_{ss})}(\frac{\tilde{C}_{t+1}^{IM}}{C_{ss}})^{-\sigma}. \tag{A53}$$

$$= \quad \hat{\beta}_t(1+r_{t+1})E_t(c_{i,t+1}^{IM})^{-\sigma} \tag{A54}$$

Aggregate consumption and savings are then

$$C_t^{IM} = \int c_{i,t}^{IM} d\Omega_t \quad = \quad \tilde{C}_t^{IM} = \mathcal{C}^{IM}(\{1+r_s\}_{s=t}^{\infty}, \{\tau_s\}_{s=t}^{\infty}, \{Y_s\}_{s=t}^{\infty}, A_t^{IM}), \tag{A55}$$

$$A_t^{IM} = \int a_{i,t}^{IM} d\Omega_t \quad = \quad \tilde{A}_t^{IM}, \tag{A56}$$

that is, the paths of aggregate consumption and savings coincide in the two economies. The construction also ensures that the elasticities are by construction time-invariant. For tax changes,

$$\frac{\partial C_t^{IM}}{\partial \tau_{t+k}} \quad = \quad \frac{\partial \mathcal{C}^{IM}(\{1+r_{ss}\}_{s=t}^{\infty}, \{\tau_{ss}\}_{s=t}^{\infty}, \{Y_{ss}\}_{s=t}^{\infty}, A_{ss}^{IM})}{\partial \tau_{t+k}} \tag{A57}$$

$$= \quad \frac{\partial \mathcal{C}^{IM}(\{1+r_{ss}\}_{s=0}^{\infty}, \{\tau_{ss}\}_{s=0}^{\infty}, \{Y_{ss}\}_{s=0}^{\infty}, A_{ss}^{IM})}{\partial \tau_k} = \frac{\partial C_0^{IM}}{\partial \tau_k} \tag{A58}$$

This economy features the same aggregate MPC as the previous one, since by construction $C_t^{IM} = \tilde{C}_t^{IM}$, but the two economies differ at the individual level. Instead of a potentially large heterogeneity of individual MPCs, all households now adjust their consumption proportionally. However, since only the aggregate consumption response matters for local determinacy which coincides in the two economies, this is irrelevant.

All other elasticities are also time-invariant and coincide with the complete markets elasticities.

For interest rates,

$$\frac{\partial C_t^{IM}}{\partial 1+r_{t+k}} = \frac{\partial C_0^{IM}}{\partial 1+r_k} = \frac{\partial \mathcal{C}^{IM}(\{1+r_{ss}\}_{s=0}^{\infty},\{\tau_{ss}\}_{s=0}^{\infty},\{Y_{ss}\}_{s=0}^{\infty},A_{ss}^{IM})}{\partial 1+r_k} \tag{A59}$$

$$= \frac{\partial \mathcal{C}^{CM}(\{1+r_{ss}\}_{s=0}^{\infty},\{\tau_{ss}\}_{s=0}^{\infty},\{Y_{ss}\}_{s=0}^{\infty},A_{ss}^{IM})}{\partial 1+r_k}, \tag{A60}$$

output,

$$\frac{\partial C_t^{IM}}{\partial Y_{t+k}} = \frac{\partial C_0^{IM}}{\partial Y_k} = \frac{\partial \mathcal{C}^{IM}(\{1+r_{ss}\}_{s=0}^{\infty},\{\tau_{ss}\}_{s=0}^{\infty},\{Y_{ss}\}_{s=0}^{\infty},A_{ss}^{IM})}{\partial Y_k} \tag{A61}$$

$$= \frac{\partial \mathcal{C}^{CM}(\{1+r_{ss}\}_{s=0}^{\infty},\{\tau_{ss}\}_{s=0}^{\infty},\{Y_{ss}\}_{s=0}^{\infty},A_{ss}^{IM})}{\partial Y_k}, \tag{A62}$$

and initial assets

$$\frac{\partial C_t^{IM}}{\partial A_t^{IM}} = \frac{\partial C_0^{IM}}{\partial A_0^{IM}} = \frac{\partial \mathcal{C}^{IM}(\{1+r_{ss}\}_{s=0}^{\infty},\{\tau_{ss}\}_{s=0}^{\infty},\{Y_{ss}\}_{s=0}^{\infty},A_{ss}^{IM})}{\partial A_0^{IM}} \tag{A63}$$

$$= \frac{\partial \mathcal{C}^{CM}(\{1+r_{ss}\}_{s=0}^{\infty},\{\tau_{ss}\}_{s=0}^{\infty},\{Y_{ss}\}_{s=0}^{\infty},A_{ss}^{IM})}{\partial A_0^{IM}}. \tag{A64}$$

The "IM" incomplete markets economy satisfies the requirements for the local determinacy analysis. It is a consistent model that describes an incomplete markets model as a departure from a complete markets model. And the elasticities evaluated at the steady-state are time invariant with properties derived in the main text.

Derivations of Section 4.2 [Properties MPCs]

Result 2: MPC, permanent transfer

The proof is largely an application of the results in Acemoglu and Jensen (2015). Acemoglu and Jensen (2015) define a positive shock as a change in an exogenous parameter which leads to an increase in a household's decision variable. I therefore have to show that a permanent increase in transfers is a positive shock, since this implies that a permanent increase in transfers leads to an increase in individual savings in an incomplete markets model.

Lemma 1 in Acemoglu and Jensen (2015) provides sufficient conditions for an exogenous parameter change to be a positive shock. I therefore need to check the assumptions of Lemma 1 in Acemoglu and Jensen (2015), which builds on Topkis' monotonicity theorem.[44] Acemoglu and Jensen (2015) show that an incomplete markets model satisfies their assumptions 1 and 3.

What remains to be shown (to apply Lemma 1) is that a household's budget constraint has strict

[44] One could also apply Theorem 6 to a single individual, noting that market aggregates (real interest rates, aggregate output,...) are constant in a household problem.

complementarities, that is, if for two asset levels $a_t^2 \geq a_t^1$ and two lump-sum transfers $\tau^2 \geq \tau^1$, if two assets choices satisfy a_{t+1} and $\tilde{a}_{t+1}$ satisfy

$$a_{t+1} \leq (1+r)a_t^1 + e_t + \tau^2; \qquad \tilde{a}_{t+1} \leq (1+r)a_t^2 + e_t + \tau^1 \tag{A65}$$

then it holds that

$$\min(a_{t+1}, \tilde{a}_{t+1}) \leq (1+r)a_t^1 + e_t + \tau^1 \tag{A66}$$

$$\max(a_{t+1}, \tilde{a}_{t+1}) \leq (1+r)a_t^2 + e_t + \tau^2, \tag{A67}$$

which is obviously correct. Furthermore recognizing that u has strictly increasing differences,

$$\frac{\partial^2 u\big((1+r)a_t + e_t + \tau - a_{t+1}\big)}{\partial a_{t+1}\partial \tau} > 0, \tag{A68}$$

renders Lemma 1 applicable, implying that a permanent increase in transfers increases savings.

This result ensures that each household's savings are non-decreasing. To establish that aggregate savings strictly increase, it is sufficient to show this for one individual. Using the arguments in Edlin and Shannon (1998) establishes this for all unconstrained households. Alternatively, consider the first-order conditions of household i with the highest consumption level $c_{i,t}$ in Period t, implying that this household is unconstrained and that the consumption Euler equation holds with equality:

$$u'(c_{i,t}) = \beta(1+r_{ss})E_{i,t}u'(c_{i,t+1}).$$

If savings were unchanged for all households, then a necessary condition for unchanged savings to be optimal for household i is,

$$u''(c_{i,t}) = \beta(1+r_{ss})E_{i,t}u''(c_{i,t+1}),$$

the derivative of the consumption Euler equation with respect to a permanent transfer increase (which is consumed in each period). Using CRRA utility and $\frac{c_{i,t}}{c_{i,t+1}} \geq 1$ with strict inequality for at least one $t+1$ state (follows from $\beta(1+r_{ss}) < 1$), then implies

$$u'(c_{i,t}) = \beta(1+r_{ss})E_{i,t}u'(c_{i,t+1})\frac{c_{i,t}}{c_{i,t+1}} > \beta(1+r_{ss})E_{i,t}u'(c_{i,t+1}),$$

a contradiction, establishing that savings strictly increase for this individual and thus aggregate savings strictly increase. Similar arguments establish that consumption is non-decreasing for unconstrained households, so that aggregate consumption strictly increases, since constrained households strictly increase their consumption.

Result 3: MPCs, transitory transfer

First, for $k = 0$, the result that

$$0 \geq \tilde{\epsilon}^{\tau}_0 \frac{1 + i_{ss}}{1 + \pi_{ss}} \geq -1$$

is equivalent to

$$1 - \beta^{CM} \leq mpc^0_0 \leq 1,$$

and follows, for example, from Carroll and Kimball (1996, 2005), Huggett (2004), Holm (2018), who establish that the consumption function is concave in wealth, and that it approaches the complete markets consumption function when wealth converges to infinity.

Second, for $k \geq 1$:

$$\tilde{\epsilon}^{\tau}_k \geq 0$$

is equivalent to

$$mpc^k_0 \geq 0.$$

This follows, since the Euler equation implies that unconstrained households increase consumption in Periods k and $k - 1$. They increase consumption in Period k, the time of the transfer, since the consumption function is strictly increasing in wealth. The consumption Euler equation then implies that unconstrained households which increase consumption in some state in Period k also increase consumption in Period $k - 1$ through saving less. Households constrained in Period $k - 1$ do not respond. The same arguments apply to Periods $k-1$ and $k-2$, $k-2$ and $k-3$ and so on, establishing that consumption increases in all these periods including Period 0.

Third, for $k \geq 1$:

$$\tilde{\epsilon}^{\tau}_k - \frac{1 + i_{ss}}{1 + \pi_{ss}} \tilde{\epsilon}^{\tau}_{k+1} \geq 0 \tag{A69}$$

is equivalent to

$$mpc^k_0 \geq \frac{1 + i_{ss}}{1 + \pi_{ss}} mpc^{k+1}_0. \tag{A70}$$

In words, a transfer of one unit paid in Period $k \geq 1$ has a larger effect on Period 0 consumption than a transfer $\frac{1+i_{ss}}{1+\pi_{ss}}$ paid in Period $k + 1$. This is a result of credit constraints which prevent constrained households from increasing current consumption when future income increases.

For credit-unconstrained households in Period k, i.e. those with positive savings from Period k to $k + 1$, the two numbers are the same, $mpc^k_0 = \frac{1+i_{ss}}{1+\pi_{ss}} mpc^{k+1}_0$. This follows from the observation that any consumption plan that is feasible if a transfer is paid in Period k is also feasible if a transfer $\frac{1+i_{ss}}{1+\pi_{ss}}$ is paid in Period $k + 1$ and vice versa.

For constrained households this argument is incorrect, as they cannot transfer resources from Period $k+1$ to Period k. A transfer paid in Period $k+1$ does not affect Period k consumption, or consumption in previous periods, so that $mpc_0^{k+1} = 0$ for these constrained households. They increase their consumption in Period k and in previous periods for a transfer paid in Period k, so that $mpc_0^k \geq 0$.

The response of Period 0 aggregate consumption - the sum of the responses of Period k constrained and unconstrained households - is therefore larger if one unit is paid in Period $k \geq 1$ than if a transfer $\frac{1+i_{ss}}{1+\pi_{ss}}$ is paid in Period $k+1$, $mpc_0^k \geq \frac{1+i_{ss}}{1+\pi_{ss}} mpc_0^{k+1}$.

<u>Fourth</u>, for $k \geq 1$:

$$(\tilde{\epsilon}_k^\tau - \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_{k+1}^\tau) - \frac{1+i_{ss}}{1+\pi_{ss}}(\tilde{\epsilon}_{k+1}^\tau - \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_{k+2}^\tau) \geq 0 \tag{A71}$$

is equivalent to

$$(mpc_0^k - \frac{1+i_{ss}}{1+\pi_{ss}} mpc_0^{k+1}) - \frac{1+i_{ss}}{1+\pi_{ss}}(mpc_0^{k+1} - \frac{1+i_{ss}}{1+\pi_{ss}} mpc_0^{k+2}) \geq 0. \tag{A72}$$

Expressed in words, scheme I, a transfer of one unit paid in Period $k \geq 1$ and a tax $\frac{1+i_{ss}}{1+\pi_{ss}}$ in Period $k+1$, has a larger effect on Period 0 consumption than the same scheme shifted by one period, namely scheme II, a transfer $\frac{1+i_{ss}}{1+\pi_{ss}}$ paid in Period $k+1$ and a tax $(\frac{1+i_{ss}}{1+\pi_{ss}})^2$ in Period $k+2$. Note that without credit-constraints both schemes would produce identical consumption responses.

To show this if prudence is sufficiently small, I first assume quadratic utility, i.e. no prudence, and show that the aggregate consumption response is strictly smaller in Period $m-1$ than in Period $m \leq k$. If utility is quadratic (no prudence) then marginal utility is linear, $u_0 - u_1 c$ with $u_0, u_1 > 0$. The consumption Euler equation for an unconstrained household i is

$$u_0 - u_1 c_{i,t} = \beta^{CM}(1+r_{ss})E_{i,t}(u_0 - u_1 c_{i,t+1}). \tag{A73}$$

Solving for $c_{i,t}$ yields

$$c_{i,t} = \frac{u_0}{u_1}(1 - \beta^{CM}(1+r_{ss})) + \beta^{CM}(1+r_{ss})E_{i,t}c_{i,t+1}. \tag{A74}$$

Credit constraints now imply that the aggregate Period $m-1$ consumption change, $\int \Delta c_{i,m-1} di$, is smaller than the Period m change, $\int \Delta c_{i,m} di$, for both schemes I and II:

$$\begin{aligned} \int \Delta c_{i,m-1} di = \int_{a_{i,m}>0} \Delta c_{i,m-1} di = \beta^{CM}(1+r_{ss}) \int_{a_{i,m}>0} E_{i,m-1}\Delta c_{i,m} di \\ \leq \quad \beta^{CM}(1+r_{ss}) \int \Delta c_{i,m} di < \int \Delta c_{i,m} di, \end{aligned}$$

where the first equality makes use of the result that only unconstrained agents adjust Period $m-1$ consumption, the second equality is implied by the consumption Euler equation for quadratic utility and the first inequality makes use of the result that $\Delta c_{i,m} \geq 0$ for all i, as shown in step 3 above.[45] This implies the desired result since the aggregate consumption response in scheme I, $\int \Delta c^I_{i,0} di$

$$\int \Delta c^I_{i,0} di \geq \int \Delta c^{II}_{i,1} di > \int \Delta c^{II}_{i,0} di, \tag{A75}$$

where the first inequality holds, since households in scheme I in Period $m-1$ can imitate the corresponding households in scheme II in Period m and thus can have higher consumption: It is feasible for household i to set $c^I_{i,m} = c^{II}_{i,m+1}$ for $m = 1, \ldots k$, meaning that budget and credit constraints are satisfied. This consumption path would leave household i in scheme I with $(1 + r_{ss})\Delta c^{II}_{i,0} \geq 0$ additional resources in Period 0, implying that consumption in Period 0 (and future periods) increases, that is $\Delta c^I_{i,0} \geq \Delta c^{II}_{i,1}$. The second inequality is the result of decreasing aggregate consumption responses shown before. Continuity implies that the results also hold if prudence is close enough to zero.

Fifth, analogous arguments show, for $k \geq 1$:

$$\tilde{\epsilon}^\tau_k - 3(\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^\tau_{k+1} + 3(\frac{1+i_{ss}}{1+\pi_{ss}})^2\tilde{\epsilon}^\tau_{k+2} - (\frac{1+i_{ss}}{1+\pi_{ss}})^3\tilde{\epsilon}^\tau_{k+3} \geq 0, \tag{A76}$$

which is equal to zero in the absence of credit constraints.

Finally, note that weaker inequalities than those shown here are used in the subsequent proofs: First,

$$\tilde{\epsilon}^\tau_k - (1 + \frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^\tau_{k+1} + \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^\tau_{k+2} \geq 0 \tag{A77}$$

is used, which follows from

$$\tilde{\epsilon}^\tau_k - 2(\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^\tau_{k+1} + (\frac{1+i_{ss}}{1+\pi_{ss}})^2\tilde{\epsilon}^\tau_{k+2} \geq 0, \tag{A78}$$

since

$$[\tilde{\epsilon}^\tau_k - (1 + \frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^\tau_{k+1} + \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^\tau_{k+2}] - [\tilde{\epsilon}^\tau_k - 2(\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^\tau_{k+1} + (\frac{1+i_{ss}}{1+\pi_{ss}})^2\tilde{\epsilon}^\tau_{k+2}] \tag{A79}$$

$$= (\frac{1+i_{ss}}{1+\pi_{ss}} - 1)(\tilde{\epsilon}^\tau_{k+1} - \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^\tau_{k+2}) \geq 0. \tag{A80}$$

[45] Note that the second equality could be a "$>$" if prudence is strong, which could imply that $\int \Delta c_{i,m-1} di > \int \Delta c_{i,m} di$.

Second

$$\beta^{CM}\tilde{\epsilon}^{\tau}_{k} - (2+\beta^{CM})\tilde{\epsilon}^{\tau}_{k+1} + (2+\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^{\tau}_{k+2} - \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^{\tau}_{k+3} \geq 0 \tag{A81}$$

is used, which follows from

$$\tilde{\epsilon}^{\tau}_{k} - 3(\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^{\tau}_{k+1} + 3(\frac{1+i_{ss}}{1+\pi_{ss}})^2\tilde{\epsilon}^{\tau}_{k+2} - (\frac{1+i_{ss}}{1+\pi_{ss}})^3\tilde{\epsilon}^{\tau}_{k+3} \geq 0, \tag{A82}$$

since

$$\begin{aligned}
& [\beta^{CM}\tilde{\epsilon}^{\tau}_{k} - (2+\beta^{CM})\tilde{\epsilon}^{\tau}_{k+1} + (2+\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^{\tau}_{k+2} - \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^{\tau}_{k+3}] && \text{(A83)}\\
- \;& [\beta^{CM}\tilde{\epsilon}^{\tau}_{k} - 3\tilde{\epsilon}^{\tau}_{k+1} + 3(\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^{\tau}_{k+2} - (\frac{1+i_{ss}}{1+\pi_{ss}})^2\tilde{\epsilon}^{\tau}_{k+3}] && \text{(A84)}\\
= \;& (1-\beta^{CM})(\tilde{\epsilon}^{\tau}_{k+1} - 2\frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^{\tau}_{k+2} + (\frac{1+i_{ss}}{1+\pi_{ss}})^2\tilde{\epsilon}^{\tau}_{k+3}) \geq 0, && \text{(A85)}
\end{aligned}$$

using that $(\beta^{CM}-1)\frac{1+i_{ss}}{1+\pi_{ss}} = 1 - \frac{1+i_{ss}}{1+\pi_{ss}}$.

<u>Verifying the properties of $\xi^{(2)}_k$ and $\xi^{(3)}_k$ in Result 3.</u>

I use the calibrated incomplete markets model in Hagedorn et al. (2017a) to calculate[46]

$$\begin{aligned}
\xi^{(2)}_k = \xi^{(1)}_k - \frac{1+i_{ss}}{1+\pi_{ss}}\xi^{(1)}_{k+1} &= \tilde{\epsilon}^{\tau}_{k} - 2(\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^{\tau}_{k+1} + (\frac{1+i_{ss}}{1+\pi_{ss}})^2\tilde{\epsilon}^{\tau}_{k+2} \geq 0\\
\xi^{(3)}_k = \xi^{(2)}_k - \frac{1+i_{ss}}{1+\pi_{ss}}\xi^{(2)}_{k+1} &= \tilde{\epsilon}^{\tau}_{k} - 3(\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^{\tau}_{k+1} + 3(\frac{1+i_{ss}}{1+\pi_{ss}})^2\tilde{\epsilon}^{\tau}_{k+2} - (\frac{1+i_{ss}}{1+\pi_{ss}})^3\tilde{\epsilon}^{\tau}_{k+3} \geq 0
\end{aligned}$$

Figure 4 shows the results, confirming that $\xi^{(2)}_k \geq 0$ and $\xi^{(3)}_k \geq 0$ for all $k \geq 1$.

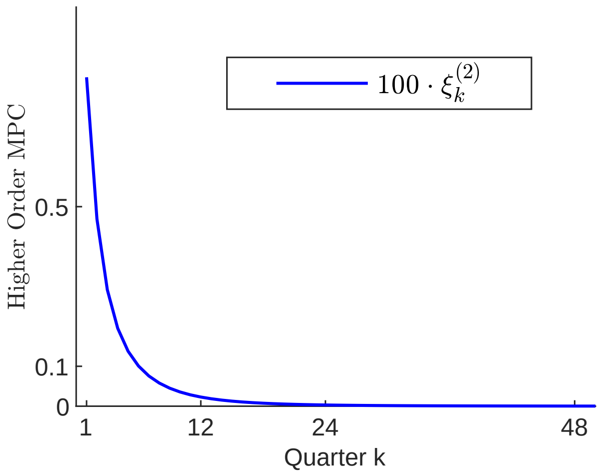
The finding that $\xi^{(2)}_k \geq 0$ implies the weaker inequality

$$\tilde{\epsilon}^{\tau}_{k} - (1+\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^{\tau}_{k+1} + \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^{\tau}_{k+2} = mpc^k_0 - (1+\frac{1+i_{ss}}{1+\pi_{ss}})mpc^{k+1}_0 + \frac{1+i_{ss}}{1+\pi_{ss}}mpc^{k+2}_0 \geq 0, \tag{A86}$$

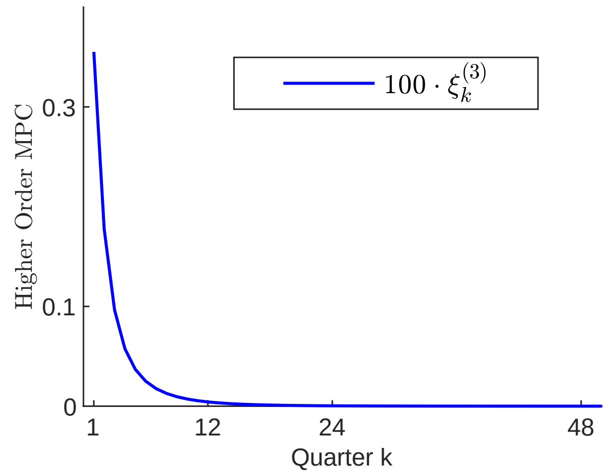
a property used in the subsequent proofs. Similarly, $\xi^{(3)}_k \geq 0$ implies the weaker inequality

$$\begin{aligned}
& \beta^{CM}\tilde{\epsilon}^{\tau}_{k} - (2+\beta^{CM})\tilde{\epsilon}^{\tau}_{k+1} + (2+\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}^{\tau}_{k+2} - \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^{\tau}_{k+3} && \text{(A87)}\\
= \;& \beta^{CM}mpc^k_0 - (2+\beta^{CM})mpc^{k+1}_0 + (2+\frac{1+i_{ss}}{1+\pi_{ss}})mpc^{k+2}_0 - \frac{1+i_{ss}}{1+\pi_{ss}}mpc^{k+3}_0 && \text{(A88)}\\
\geq \;& 0. && \text{(A89)}
\end{aligned}$$

[46] I thank Kurt Mitman for running these experiments.



(a) $\xi_k^{(2)} = \tilde{\epsilon}_k^\tau - 2(\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}_{k+1}^\tau + (\frac{1+i_{ss}}{1+\pi_{ss}})^2\tilde{\epsilon}_{k+2}^\tau$



(b) $\xi_k^{(3)} = \tilde{\epsilon}_k^\tau - 3(\frac{1+i_{ss}}{1+\pi_{ss}})\tilde{\epsilon}_{k+1}^\tau + 3(\frac{1+i_{ss}}{1+\pi_{ss}})^2\tilde{\epsilon}_{k+2}^\tau - (\frac{1+i_{ss}}{1+\pi_{ss}})^3\tilde{\epsilon}_{k+3}^\tau$

Figure 4: Transfer schemes $\xi_k^{(2)} \geq 0$ and $\xi_k^{(3)} \geq 0$

again the property used in the subsequent proofs.

Verifying Result 3 and 2 for asymptotically time-invariant MPCs (Auclert et al., 2019a)

Define $mpc_t^{t+k} = -\frac{\partial C_t^{IM}}{\partial \tau_{t+k}}$ as the Period t aggregate consumption response to a \$1 transfer in period $t+k$, where $k \in \mathbb{Z}$. For $k \geq 0$ and $t = 0$, the "left MPCs" mpc_t^{t+k} are equal to mpc_0^k in the main text. Auclert et al. (2019a) show that mpc_t^{t+k} is time-invariant for $t \to \infty$, i.e. $1 > \lambda_1 > \lambda_2$ exist such that the "right MPCs" decay at rate λ_1,

$$\lim_{t\to\infty} mpc_t^{t+k} = (\lambda_1)^{-k} \lim_{t\to\infty} mpc_t^t, \quad \text{for } k \leq 0,$$

the "left MPCs" decay at rate λ_2,

$$\lim_{t\to\infty} mpc_t^{t+k} = (\lambda_2)^{k} \lim_{t\to\infty} mpc_t^t, \quad \text{for } k \geq 0$$

and the left decay is faster than the right decay, $\lambda_1 > \frac{1+i_{ss}}{1+\pi_{ss}}\lambda_2$.

Then condition (82) holds at the limit $\forall k \geq 0$,

$$\begin{aligned}
\lim_{t\to\infty} \xi_{t,t+k}^{(2)} &= \lim_{t\to\infty} \left\{ mpc_t^{t+k} - 2(\frac{1+i_{ss}}{1+\pi_{ss}}) mpc_t^{t+k+1} + (\frac{1+i_{ss}}{1+\pi_{ss}})^2 mpc_t^{t+k+2} \right\} \\
&= \lim_{t\to\infty} mpc_t^{t+k} \left\{ 1 - 2(\frac{1+i_{ss}}{1+\pi_{ss}})\lambda_2 + (\frac{1+i_{ss}}{1+\pi_{ss}})^2 (\lambda_2)^2 \right\} \\
&= \lim_{t\to\infty} mpc_t^{t+k} \left\{ (1 - \frac{1+i_{ss}}{1+\pi_{ss}}\lambda_2)^2 \right\} \geq 0
\end{aligned}$$

and for condition (83) in the limit $\forall k \geq 0$,

$$\begin{aligned}
\lim_{t\to\infty} \xi_{t,t+k}^{(3)} &= \lim_{t\to\infty} \left\{ mpc_t^{t+k} - 3(\frac{1+i_{ss}}{1+\pi_{ss}}) mpc_t^{t+k+1} + 3(\frac{1+i_{ss}}{1+\pi_{ss}})^2 mpc_t^{t+k+2} - (\frac{1+i_{ss}}{1+\pi_{ss}})^3 mpc_t^{t+k+3} \right\} \\
&= \lim_{t\to\infty} mpc_t^{t+k} \left\{ 1 - 3(\frac{1+i_{ss}}{1+\pi_{ss}})\lambda_2 + 3(\frac{1+i_{ss}}{1+\pi_{ss}})^2 (\lambda_2)^2 - (\frac{1+i_{ss}}{1+\pi_{ss}})^3 (\lambda_2)^3 \right\} \\
&= \lim_{t\to\infty} mpc_t^{t+k} \left\{ (1 - \frac{1+i_{ss}}{1+\pi_{ss}}\lambda_2)^3 \right\} \geq 0.
\end{aligned}$$

For Result 2, Auclert et al. (2018) show that

$$\sum_{k=0}^{\infty} mpc_{t+k}^t (\frac{1+i_{ss}}{1+\pi_{ss}})^{-k} = 1$$

and (Auclert et al., 2019a) that $mpc_t^{t-k} = mpc_{t+k}^t$ holds in the limit. Together, these two results imply at the limit,

$$1 = \lim_{t\to\infty} \sum_{k=0}^{\infty} mpc_t^{t-k} (\frac{1+i_{ss}}{1+\pi_{ss}})^{-k} = \lim_{t\to\infty} \sum_{k=0}^{\infty} mpc_t^t (\frac{1+\pi_{ss}}{1+i_{ss}}\lambda_1)^k.$$

Therefore

$$\lim_{t\to\infty}\sum_{k=0}^{\infty} mpc_t^{t+k} = \lim_{t\to\infty}\sum_{k=0}^{\infty} mpc_t^t(\lambda_2)^k < \lim_{t\to\infty}\sum_{k=0}^{\infty} mpc_t^t(\frac{1+\pi_{ss}}{1+i_{ss}}\lambda_1)^k = 1,$$

meaning that the permanent MPC is less than one, which is equivalent to Result 2.

Derivations of Section 4.4 [Local Determinancy, Flexible Prices]

Result 5: Flexible Prices

As shown in the main text, the linearized asset market clearing condition after all substitutions is

$$\sum_{k=0}^{\infty}\left\{\epsilon_{k+1}^r(\hat{p}_{t+k}-\hat{p}_{t+k+1}) - \tilde{\epsilon}_k^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}\hat{p}_{t+k}\right\} + \epsilon_0^r(\hat{p}_{t-1}-\hat{p}_t) - \epsilon_0^A\hat{p}_{t-1} = -\hat{p}_t. \tag{A90}$$

Collecting $\hat{p}$ terms and re-arranging yields

$$(1+\epsilon_1^r-\epsilon_0^r-\tilde{\epsilon}_0^\tau\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}})\hat{p}_t = \sum_{k=1}^{\infty}(\epsilon_k^r-\epsilon_{k+1}^r+\tilde{\epsilon}_k^\tau\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}})\hat{p}_{t+k} \tag{A91}$$

so that the Onatski function

$$\Theta(\lambda) = (1+\epsilon_1^r-\epsilon_0^r-\tilde{\epsilon}_0^\tau\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}) - \sum_{k=1}^{\infty}(\epsilon_k^r-\epsilon_{k+1}^r+\tilde{\epsilon}_k^\tau\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}})e^{-ik\lambda}. \tag{A92}$$

I now show that $\text{Re}\{\Theta(\lambda)\} > 0$ for all $\lambda \in [0, 2\pi]$, implying that the winding number is zero and thus Onatski (2006) implies that there is a unique solution, that is, the economy is locally determinate. Since $\epsilon_k^r - \epsilon_{k+1}^r > 0, \lim_{k\to\infty}\epsilon_k^r = 0, \epsilon_0^r = 1$ (Result 1), $\tilde{\epsilon}_k^\tau \geq 0$ for $k \geq 1$ (Result 3) and $\cos(\cdot) \leq 1$

$$\text{Re}\{\Theta(\lambda)\} = (1+\epsilon_1^r-\epsilon_0^r-\tilde{\epsilon}_0^\tau\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}) - \sum_{k=1}^{\infty}(\epsilon_k^r-\epsilon_{k+1}^r+\tilde{\epsilon}_k^\tau\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}})\cos(-k\lambda) \tag{A93}$$

$$\geq (1+\epsilon_1^r-\epsilon_0^r-\tilde{\epsilon}_0^\tau\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}) - \sum_{k=1}^{\infty}(\epsilon_k^r-\epsilon_{k+1}^r+\tilde{\epsilon}_k^\tau\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}) \tag{A94}$$

$$= 1-\tilde{\epsilon}_0^r > 0, \tag{A95}$$

by Result 2. Thus the graph of $\Theta(\lambda)$ is within the plane of positive real numbers and does not encircle zero, implying a winding number of zero and local determinacy.

Derivations of Section 4.5 [Policy rules]

Condition (103): Flexible Prices, Policy Rules, Ruling out $\hat{p}_t = p$

Obviously, noting that

$$\sum_{k=0}^{\infty}\varphi^i\epsilon^r_{k+1}=\varphi^i\epsilon^r_1\sum_{k=0}^{\infty}(\beta^{CM})^k=\varphi^i\frac{\epsilon^r_1}{1-\beta^{CM}}.$$

Result 6: Flexible Prices, Policy Rules

As shown in the main text, the linearized asset market clearing condition after all substitutions is

$$\sum_{k=0}^{\infty}\Big\{\epsilon^r_{k+1}(\varphi^i\hat{p}_{t+k}+\hat{p}_{t+k}-\hat{p}_{t+k+1})+\tilde{\epsilon}^\tau_k[\frac{1+i_{ss}}{1+\pi_{ss}}\varphi^i\hat{p}_{t+k-1}+\frac{1+i_{ss}}{1+\pi_{ss}}(\varphi^B\hat{p}_{t+k-1}-\hat{p}_{t+k})-(\varphi^B-1)\hat{p}_{t+k}]\Big\}$$
$$+\epsilon^r_0(\varphi^i\hat{p}_{t-1}+\hat{p}_{t-1}-\hat{p}_t)+\epsilon^A_0(\varphi^B-1)\hat{p}_{t-1}=(\varphi^B-1)\hat{p}_t,$$

Collecting $\hat{p}$ terms and re-arranging yields

$$\begin{aligned}
&[\tilde{\epsilon}^\tau_0\varphi^B\frac{1+i_{ss}}{1+\pi_{ss}}+\tilde{\epsilon}^\tau_0\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}+\varphi^i\epsilon^r_0+\varphi^B\epsilon^A_0]\hat{p}_{t-1}\\
+\ &[(1-\varphi^B)-\epsilon^r_0+(1+\varphi^i)\epsilon^r_1-(\tilde{\epsilon}^\tau_0-\frac{1+i_{ss}}{1+\pi_{ss}}\epsilon^\tau_1)\varphi^B+\tilde{\epsilon}^\tau_1\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}-\tilde{\epsilon}^\tau_0\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}]\hat{p}_t\\
-\ &\sum_{k=1}^{\infty}[\epsilon^r_k-(1+\varphi^i)\epsilon^r_{k+1}+(\tilde{\epsilon}^\tau_k-\frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^\tau_{k+1})\varphi^B-\tilde{\epsilon}^\tau_{k+1}\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}+\tilde{\epsilon}^\tau_k\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}]\hat{p}_{t+k}\\
=\ &0.
\end{aligned}$$

and thus the Onatski function is

$$\begin{aligned}
&\Theta(\lambda)\\
=\ &[(1-\varphi^B)-\epsilon^r_0+(1+\varphi^i)\epsilon^r_1-(\tilde{\epsilon}^\tau_0-\frac{1+i_{ss}}{1+\pi_{ss}}\epsilon^\tau_1)\varphi^B+\tilde{\epsilon}^\tau_1\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}-\tilde{\epsilon}^\tau_0\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}]\\
+\ &[\tilde{\epsilon}^\tau_0\varphi^B\frac{1+i_{ss}}{1+\pi_{ss}}+\tilde{\epsilon}^\tau_0\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}+\varphi^i\epsilon^r_0+\varphi^B\epsilon^A_0]e^{i\lambda}\\
-\ &\sum_{k=1}^{\infty}[\epsilon^r_k-(1+\varphi^i)\epsilon^r_{k+1}+(\tilde{\epsilon}^\tau_k-\frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^\tau_{k+1})\varphi^B-\tilde{\epsilon}^\tau_{k+1}\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}+\tilde{\epsilon}^\tau_k\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}]e^{-ik\lambda}.
\end{aligned}$$

I show that $\text{Re}\{\Theta(\lambda)\}>0$, implying that the winding number is zero, since $\Theta(\lambda)$ is located in the plane of positive real numbers and thus does not circle around 0. Onatski's theorem then implies that there is a unique solution, that is, the economy is (locally) determinate.

The proof uses two auxiliary results. First,

$$\sum_{k=0}^{\infty}\varphi^i\epsilon^r_{k+1}e^{-ik\lambda}=\varphi^i\epsilon^r_1\sum_{k=0}^{\infty}(\beta^{CM}e^{-i\lambda})^k=\frac{\varphi^i\epsilon^r_1}{1-\beta^{CM}e^{-i\lambda}}.$$

Second,

$$\frac{1}{1+\beta^{CM}} \leq \mathrm{Re}\{\frac{1}{1-\beta^{CM}e^{-i\lambda}}\}.$$

This follows from

$$\mathrm{Re}\{\frac{1}{1-\beta^{CM}e^{-i\lambda}}\} = \frac{1-\beta^{CM}\cos(\lambda)}{1-2\beta^{CM}\cos(\lambda)+(\beta^{CM})^2} \tag{A96}$$

with λ derivative

$$\frac{[(\beta^{CM})^3-\beta^{CM}]\sin(\lambda)}{[1-2\beta^{CM}\cos(\lambda)+(\beta^{CM})^2]^2}, \tag{A97}$$

which is equal to zero within the interval $[0,2\pi)$ iff $\lambda = 0, \pi$. Since $\cos(0) = 1, \cos(\pi) = -1$, $\mathrm{Re}\,\frac{1}{1-\beta^{CM}e^{-i\lambda}}$ attains its maximum $\frac{1}{1-\beta^{CM}}$ at $\lambda = 0$ and its minimum $\frac{1}{1+\beta^{CM}}$ at $\lambda = \pi$.

Using $\epsilon^r_k - \epsilon^r_{k+1} \geq 0, \tilde{\epsilon}^\tau_k - \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^\tau_{k+1} \geq 0, \tilde{\epsilon}^\tau_k \geq 0$ and $\cos(-\lambda) = \cos(\lambda)$,

$$\begin{aligned}
& \mathrm{Re}\,\Theta(\lambda) \\
\geq\; & [(1-\varphi^B) - \epsilon^r_0 + \epsilon^r_1 - (\tilde{\epsilon}^\tau_0 - \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^\tau_1)\varphi^B + \tilde{\epsilon}^\tau_1\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}} - \tilde{\epsilon}^\tau_0\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}] \\
+\; & [\tilde{\epsilon}^\tau_0\varphi^B\frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}^\tau_0\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i\epsilon^r_0 + \varphi^B\epsilon^A_0]\cos(\lambda) \\
+\; & \sum_{k=1}^{\infty}[\tilde{\epsilon}^\tau_{k+1}\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}]\cos(-k\lambda) - \sum_{k=1}^{\infty}[(\tilde{\epsilon}^\tau_k - \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}^\tau_{k+1})\varphi^B + \tilde{\epsilon}^\tau_k\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}] \\
-\; & \sum_{k=1}^{\infty}[\epsilon^r_k - \epsilon^r_{k+1}] + \varphi^i\epsilon^r_1\,\mathrm{Re}\{\frac{1}{1-\beta^{CM}e^{-i\lambda}}\},
\end{aligned}$$

which is equal to

$$
\begin{aligned}
&= [(1-\varphi^B) - \epsilon_0^r - \tilde{\epsilon}_0^\tau \varphi^B + \tilde{\epsilon}_1^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}}] \\
&+ [\tilde{\epsilon}_0^\tau \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}_0^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon_0^r + \varphi^B \epsilon_0^A] \cos(\lambda) \\
&+ \sum_{k=1}^{\infty} [\tilde{\epsilon}_{k+1}^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}}] \cos(-k\lambda) - \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}} + \sum_{k=1}^{\infty} \tilde{\epsilon}_k^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}} \varphi^B \\
&+ \varphi^i \epsilon_1^r \operatorname{Re}\{\frac{1}{1-\beta^{CM} e^{-i\lambda}}\} \\
&\geq [(1-\varphi^B) - \epsilon_0^r - \tilde{\epsilon}_0^\tau \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}_1^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}}] \\
&+ [\tilde{\epsilon}_0^\tau \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}_0^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon_0^r + \varphi^B \epsilon_0^A + \tilde{\epsilon}_2^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}}] \cos(\lambda) \\
&+ \sum_{k=2}^{\infty} [\tilde{\epsilon}_{k+1}^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}}] \cos(k\lambda) \\
&+ (\varphi^B - 1) \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}} + \frac{\varphi^i \epsilon_1^r}{1+\beta^{CM}}.
\end{aligned}
$$

That is, we have

$$
\begin{aligned}
&\operatorname{Re}\Theta(\lambda) \\
\geq\ & \underbrace{[(1-\varphi^B) - \epsilon_0^r - \tilde{\epsilon}_0^\tau \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}_1^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + (\varphi^B - 1) \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}} + \frac{\varphi^i \epsilon_1^r}{1+\beta^{CM}}]}_{:=\alpha_0/2} \\
+\ & \underbrace{[\tilde{\epsilon}_0^\tau \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}_0^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon_0^r + \varphi^B \epsilon_0^A + \tilde{\epsilon}_2^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}}]}_{:=\alpha_1} \cos(\lambda) \\
+\ & \sum_{k=2}^{\infty} \underbrace{[\tilde{\epsilon}_{k+1}^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}}]}_{=:\alpha_k} \cos(k\lambda)
\end{aligned}
$$

I first consider the benchmark

- $\varphi^i \geq 0$ and $[\tilde{\epsilon}_0^\tau \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}_0^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon_0^r + \varphi^B \epsilon_0^A] \geq 0$.

The latter condition holds if $\varphi^B \geq 0$, since $-\tilde{\epsilon}_0^\tau \frac{1+i_{ss}}{1+\pi_{ss}} < 1 = \epsilon_0^A$.

Application of Proposition 1 requires showing that

$$\forall k \geq 0: \qquad \alpha_k \geq 0, \alpha_k - \alpha_{k+1} \geq 0, \alpha_k - 2\alpha_{k+1} + \alpha_{k+2} \geq 0$$

and

$$\alpha_0 - 2\alpha_1 + \alpha_2 > 0.$$

- For $k \geq 2$, Result 3 and (A77) impliy that

$$\begin{aligned}
&\tilde{\epsilon}^{\tau}_k - 2\tilde{\epsilon}^{\tau}_{k+1} + \tilde{\epsilon}^{\tau}_{k+2} = (\tilde{\epsilon}^{\tau}_k - \tilde{\epsilon}^{\tau}_{k+1}) - (\tilde{\epsilon}^{\tau}_{k+1} - \tilde{\epsilon}^{\tau}_{k+2}) \geq (\tilde{\epsilon}^{\tau}_k - \tilde{\epsilon}^{\tau}_{k+1}) - \frac{1+i_{ss}}{1+\pi_{ss}}(\tilde{\epsilon}^{\tau}_{k+1} - \tilde{\epsilon}^{\tau}_{k+2}) \geq 0,\\
&\tilde{\epsilon}^{\tau}_k - \tilde{\epsilon}^{\tau}_{k+1} \geq 0,\\
&\tilde{\epsilon}^{\tau}_k \geq 0,
\end{aligned}$$

and thus $\alpha_k - 2\alpha_{k+1} + \alpha_{k+2} \geq 0, \alpha_k - \alpha_{k+1} \geq 0, \alpha_k \geq 0$.

- For $k = 1$, $[\tilde{\epsilon}^{\tau}_0 \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}^{\tau}_0 \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon^r_0 + \varphi^B \epsilon^A_0] \geq 0$ and Result 3 imply

$$\begin{aligned}
&\alpha_1 \geq 0,\\
&\alpha_1 - \alpha_2 = [\tilde{\epsilon}^{\tau}_0 \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}^{\tau}_0 \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon^r_0 + \varphi^B \epsilon^A_0] + \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}^{\tau}_2 - \tilde{\epsilon}^{\tau}_3] \geq 0,\\
&\alpha_1 - 2\alpha_2 + \alpha_3 = [\tilde{\epsilon}^{\tau}_0 \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}^{\tau}_0 \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon^r_0 + \varphi^B \epsilon^A_0] + \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}^{\tau}_2 - 2\tilde{\epsilon}^{\tau}_3 + \tilde{\epsilon}^{\tau}_4] \geq 0.
\end{aligned}$$

- For $k = 0$, I now show that condition (104) implies that $\alpha_0 - 2\alpha_1 + \alpha_2 > 0$.

Using $\epsilon^r_0 = \epsilon^A_0 = 1$, it follows that

$$\begin{aligned}
& \alpha_0 - 2\alpha_1 + \alpha_2\\
= \;& 2[(1-\varphi^B) - \epsilon^r_0 - \tilde{\epsilon}^{\tau}_0 \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}^{\tau}_1 \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + (\varphi^B - 1)\sum_{k=0}^{\infty} \tilde{\epsilon}^{\tau}_k \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}} + \frac{\varphi^i \epsilon^r_1}{1+\beta^{CM}}]\\
- \;& 2[\tilde{\epsilon}^{\tau}_0 \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}^{\tau}_0 \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon^r_0 + \varphi^B \epsilon^A_0 + \tilde{\epsilon}^{\tau}_2 \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}}]\\
+ \;& [\tilde{\epsilon}^{\tau}_3 \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}}]\\
= \;& 2(\varphi^B - 1)\sum_{k=0}^{\infty} \tilde{\epsilon}^{\tau}_k \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}} - 4\varphi^B(1 + \tilde{\epsilon}^{\tau}_0 \frac{1+i_{ss}}{1+\pi_{ss}})\\
+ \;& 2\varphi^i[\frac{\epsilon^r_1}{1+\beta^{CM}} - 1 + (\tilde{\epsilon}^{\tau}_1 + \tilde{\epsilon}^{\tau}_3/2 - \tilde{\epsilon}^{\tau}_0 - \tilde{\epsilon}^{\tau}_2)\frac{1+i_{ss}}{1+\pi_{ss}}]\\
> \;& 0,
\end{aligned}$$

which is implied by condition (104) in Result 6, since Result 3 implies $\tilde{\epsilon}^{\tau}_1 + \tilde{\epsilon}^{\tau}_3/2 - \tilde{\epsilon}^{\tau}_2 \geq 0$, explaining footnote 36. This in turn implies that

$$\begin{aligned}
&\alpha_0 - \alpha_1 = (\alpha_0 - 2\alpha_1 + \alpha_2) + (\alpha_1 - \alpha_2) > 0,\\
&\alpha_0 > \alpha_1 > 0.
\end{aligned}$$

The assumptions of Proposition 1 are thus satisfied so that

$$\begin{aligned}\operatorname{Re}\Theta(\lambda) &\geq (\varphi^B-1)\sum_{k=0}^{\infty}\tilde{\epsilon}_k^{\tau}\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}} - 2\varphi^B(1+\tilde{\epsilon}_0^{\tau}\frac{1+i_{ss}}{1+\pi_{ss}}) \\ &\quad +\varphi^i[\frac{\epsilon_1^r}{1+\beta^{CM}} - 1 + (\tilde{\epsilon}_1^{\tau}+\tilde{\epsilon}_3^{\tau}/2-\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_2^{\tau})\frac{1+i_{ss}}{1+\pi_{ss}}] \\ &> 0,\end{aligned}$$

implying local determinacy. For completeness, note that

$$\operatorname{Re}\{\Theta(0)\} \geq \alpha_0/2+\alpha_1+\sum_{k=2}^{\infty}\alpha_k \geq \alpha_0/2+\alpha_1 > 0.$$

- If $\varphi^i \geq 0$, $[\tilde{\epsilon}_0^{\tau}\varphi^B\frac{1+i_{ss}}{1+\pi_{ss}}+\tilde{\epsilon}_0^{\tau}\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}+\varphi^i\epsilon_0^r+\varphi^B\epsilon_0^A] < 0$ (implying $\varphi^B \leq 0$) then

$$\begin{aligned}&\operatorname{Re}\{\Theta(\lambda)\} \\ \geq\ & [(1-\varphi^B)-\epsilon_0^r-\tilde{\epsilon}_0^{\tau}\varphi^B\frac{1+i_{ss}}{1+\pi_{ss}}+\tilde{\epsilon}_1^{\tau}\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}+(\varphi^B-1)\sum_{k=0}^{\infty}\tilde{\epsilon}_k^{\tau}\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}+\frac{\varphi^i\epsilon_1^r}{1+\beta^{CM}}] \\ +\ & [\tilde{\epsilon}_0^{\tau}\varphi^B\frac{1+i_{ss}}{1+\pi_{ss}}+\tilde{\epsilon}_0^{\tau}\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}+\varphi^i\epsilon_0^r+\varphi^B\epsilon_0^A] \\ +\ & \underbrace{\{[\tilde{\epsilon}_2^{\tau}\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}]\}}_{:=\alpha_1}\cos(\lambda) \\ +\ & \sum_{k=2}^{\infty}\underbrace{[\tilde{\epsilon}_{k+1}^{\tau}\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}]}_{=:\alpha_k}\cos(k\lambda)\end{aligned}$$

with

$$\begin{aligned}\alpha_0 &= [(1-\varphi^B)-\epsilon_0^r-\tilde{\epsilon}_0^{\tau}\varphi^B\frac{1+i_{ss}}{1+\pi_{ss}}+\tilde{\epsilon}_1^{\tau}\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}+(\varphi^B-1)\sum_{k=0}^{\infty}\tilde{\epsilon}_k^{\tau}\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}+\frac{\varphi^i\epsilon_1^r}{1+\beta^{CM}}] \\ &+ [\tilde{\epsilon}_0^{\tau}\varphi^B\frac{1+i_{ss}}{1+\pi_{ss}}+\tilde{\epsilon}_0^{\tau}\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}}+\varphi^i\epsilon_0^r+\varphi^B\epsilon_0^A] \\ &= \underbrace{[(\varphi^B-1)\sum_{k=0}^{\infty}\tilde{\epsilon}_k^{\tau}\frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}+\varphi^i(1+\tilde{\epsilon}_0^{\tau}\frac{1+i_{ss}}{1+\pi_{ss}})+\frac{\varphi^i\epsilon_1^r}{1+\beta^{CM}}]}_{=:\tilde{\alpha}_0>0}+\tilde{\epsilon}_1^{\tau}\varphi^i\frac{1+i_{ss}}{1+\pi_{ss}} \\ &> 0,\end{aligned}$$

without any assumptions, since $\varphi^B \leq 0$. Since $\tilde{\alpha}_0 > 0$ and $\varphi^i \geq 0$, Result 3 implies that the assumptions of Fejér (1928, 1936) are satisfied and thus $\operatorname{Re}\{\Theta(\lambda)\} > 0$.

- If $\varphi^i < 0$ then

$$\begin{aligned}
& \mathrm{Re}\,\Theta(\lambda) \\
\geq\ & [(1-\varphi^B) - \epsilon_0^r - \tilde{\epsilon}_0^\tau \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + (\varphi^B - 1) \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}} + \frac{\varphi^i \epsilon_1^r}{1+\beta^{CM}}] \\
-\ & \left| [\tilde{\epsilon}_0^\tau \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon_0^r + \varphi^B \epsilon_0^A] \right|,
\end{aligned}$$

which needs to be positive to ensure determinacy.

If $[\tilde{\epsilon}_0^\tau \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon_0^r + \varphi^B \epsilon_0^A] \leq 0$, the condition then simplifies to

$$\sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + (\varphi^B - 1) \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}} + \varphi^i (1 + \frac{\epsilon_1^r}{1+\beta^{CM}}) > 0.$$

If $[\tilde{\epsilon}_0^\tau \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon_0^r + \varphi^B \epsilon_0^A] > 0$ then the condition simplifies to

$$(\varphi^B - 1) \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}} - 2\varphi^B (1 + \tilde{\epsilon}_0^\tau \frac{1+i_{ss}}{1+\pi_{ss}}) + \sum_{k=0}^{\infty} \tilde{\epsilon}_k^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i (\frac{\epsilon_1^r}{1+\beta^{CM}} - 1) > 0.$$

Both conditions are strong, since $\varphi^i < 0$, so that monetary policy now pushes towards indeterminacy.

Derivations of Section 4.6 [Rigid Prices]

Result 7: Rigid Prices

Allowing for rigid prices adds $\sum_{k=0}^{\infty} \epsilon_k^Y \hat{Y}_{t+k}$ to asset demand, so that the asset market clearing condition now reads

$$\begin{aligned}
& \sum_{k=0}^{\infty} \Big\{ \epsilon_{k+1}^r (\varphi^i \hat{p}_{t+k} + \hat{p}_{t+k} - \hat{p}_{t+k+1}) + \tilde{\epsilon}_k^\tau [\varphi^i \hat{p}_{t+k-1} \frac{1+i_{ss}}{1+\pi_{ss}} + \frac{1+i_{ss}}{1+\pi_{ss}} (\varphi^B \hat{p}_{t+k-1} - \hat{p}_{t+k}) - (\varphi^B - 1)\hat{p}_{t+k}] \Big\} \\
& + \epsilon_0^r (\varphi^i \hat{p}_{t-1} + \hat{p}_{t-1} - \hat{p}_t) + \epsilon_0^A (\varphi^B - 1) \hat{p}_{t-1} + \sum_{k=0}^{\infty} \epsilon_k^Y \hat{Y}_{t+k} = (\varphi^B - 1)\hat{p}_t,
\end{aligned}$$

Using the Phillips curve

$$\hat{Y}_t = \frac{1}{\kappa}[(1+\beta^{CM})\hat{p}_t - \hat{p}_{t-1} - \beta^{CM} \hat{p}_{t+1}],$$

implies that

$$
\begin{aligned}
& \sum_{k=0}^{\infty} \epsilon_k^Y \hat{Y}_{t+k} \\
= \; & \frac{1}{\kappa} \sum_{k=0}^{\infty} \epsilon_k^Y [(1+\beta^{CM})\hat{p}_{t+k} - \hat{p}_{t+k-1} - \beta^{CM}\hat{p}_{t+k+1}] \\
= \; & \frac{1}{\kappa} \sum_{k=1}^{\infty} [\epsilon_k^Y (1+\beta^{CM}) - \epsilon_{k+1}^Y - \beta^{CM}\epsilon_{k-1}^Y]\hat{p}_{t+k} + \frac{1}{\kappa}[\epsilon_0^Y(1+\beta^{CM}) - \epsilon_1^Y]\hat{p}_t - \frac{1}{\kappa}\epsilon_0^Y \hat{p}_{t-1}
\end{aligned}
$$

Using this and collecting $\hat{p}$ terms and re-arranging the asset market clearing condition yields

$$
\begin{aligned}
& [\tilde{\epsilon}_0^\tau \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}_0^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon_0^r + \varphi^B \epsilon_0^A]\hat{p}_{t-1} \\
+ \; & [(1-\varphi^B) - \epsilon_0^r + (1+\varphi^i)\epsilon_1^r - (\tilde{\epsilon}_0^\tau - \frac{1+i_{ss}}{1+\pi_{ss}}\epsilon_1^\tau)\varphi^B + \tilde{\epsilon}_1^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} - \tilde{\epsilon}_0^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}]\hat{p}_t \\
- \; & \sum_{k=1}^{\infty} [\epsilon_k^r - (1+\varphi^i)\epsilon_{k+1}^r + (\tilde{\epsilon}_k^\tau - \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_{k+1}^\tau)\varphi^B - \tilde{\epsilon}_{k+1}^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}_k^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}]\hat{p}_{t+k} \\
+ \; & \frac{1}{\kappa} \sum_{k=1}^{\infty} [\epsilon_k^Y (1+\beta^{CM}) - \epsilon_{k+1}^Y - \beta^{CM}\epsilon_{k-1}^Y]\hat{p}_{t+k} + \frac{1}{\kappa}[\epsilon_0^Y(1+\beta^{CM}) - \epsilon_1^Y]\hat{p}_t - \frac{1}{\kappa}\epsilon_0^Y \hat{p}_{t-1} \\
= \; & 0
\end{aligned}
$$

and thus the Onatski function is

$$
\begin{aligned}
& \Theta(\lambda) \\
= \; & [(1-\varphi^B) - \epsilon_0^r + (1+\varphi^i)\epsilon_1^r - (\tilde{\epsilon}_0^\tau - \frac{1+i_{ss}}{1+\pi_{ss}}\epsilon_1^\tau)\varphi^B + \tilde{\epsilon}_1^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} - \tilde{\epsilon}_0^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}] \\
+ \; & [\tilde{\epsilon}_0^\tau \varphi^B \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}_0^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \varphi^i \epsilon_0^r + \varphi^B \epsilon_0^A]e^{i\lambda} \\
- \; & \sum_{k=1}^{\infty} [\epsilon_k^r - (1+\varphi^i)\epsilon_{k+1}^r + (\tilde{\epsilon}_k^\tau - \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_{k+1}^\tau)\varphi^B - \tilde{\epsilon}_{k+1}^\tau \varphi^i \frac{1+i_{ss}}{1+\pi_{ss}} + \tilde{\epsilon}_k^\tau \frac{i_{ss}-\pi_{ss}}{1+\pi_{ss}}]e^{-ik\lambda} \\
+ \; & \frac{1}{\kappa} \sum_{k=1}^{\infty} [\epsilon_k^Y (1+\beta^{CM}) - \epsilon_{k+1}^Y - \beta^{CM}\epsilon_{k-1}^Y]e^{-ik\lambda} + \frac{1}{\kappa}[\epsilon_0^Y(1+\beta^{CM}) - \epsilon_1^Y] - \frac{1}{\kappa}\epsilon_0^Y e^{i\lambda}.
\end{aligned}
$$

Allowing for rigid prices thus adds

$$
\frac{1}{\kappa} \sum_{k=1}^{\infty} [\epsilon_k^Y (1+\beta^{CM}) - \epsilon_{k+1}^Y - \beta^{CM}\epsilon_{k-1}^Y]e^{-ik\lambda} + \frac{1}{\kappa}[\epsilon_0^Y(1+\beta^{CM}) - \epsilon_1^Y] - \frac{1}{\kappa}\epsilon_0^Y e^{i\lambda}
$$

to the Onatski function, and I show that the real part of this additional term is nonnegative. The same condition (104) as with flexible prices then ensures $\text{Re}\{\Theta(\lambda)\} > 0$ and thus determinacy.

Result 1 implies that

– For $k \geq 2$:

$$[\epsilon_k^Y(1+\beta^{CM}) - \epsilon_{k+1}^Y - \beta^{CM}\epsilon_{k-1}^Y] = 0.$$

– For $k = 1$:

$$[\epsilon_1^Y(1+\beta^{CM}) - \epsilon_2^Y - \beta^{CM}\epsilon_0^Y] = \frac{Y_{ss}}{A_{ss}}\{-(1-\beta^{CM})(1+\beta^{CM})(\beta^{CM}) + (1-\beta^{CM})(\beta^{CM})^2 - (\beta^{CM})^2\}$$
$$= \frac{Y_{ss}}{A_{ss}}\{-(\beta^{CM} - (\beta^{CM})^3) + ((\beta^{CM})^2 - (\beta^{CM})^3) - (\beta^{CM})^2\} = -\frac{Y_{ss}}{A_{ss}}\beta^{CM}.$$

– For $k = 0$:

$$\epsilon_0^Y(1+\beta^{CM}) - \epsilon_1^Y = \frac{Y_{ss}}{A_{ss}}\{(1+\beta^{CM})\beta^{CM} + (1-\beta^{CM})\beta^{CM}\} = 2\beta^{CM}\frac{Y_{ss}}{A_{ss}}.$$

Therefore

$$\frac{1}{\kappa}\sum_{k=1}^{\infty}[\epsilon_k^Y(1+\beta^{CM}) - \epsilon_{k+1}^Y - \beta^{CM}\epsilon_{k-1}^Y]e^{-ik\lambda} + \frac{1}{\kappa}[\epsilon_0^Y(1+\beta^{CM}) - \epsilon_1^Y] - \frac{1}{\kappa}\epsilon_0^Y e^{i\lambda}$$
$$= \frac{Y_{ss}}{\kappa A_{ss}}(-\beta^{CM}e^{-i\lambda} + 2\beta^{CM} - \beta^{CM}e^{i\lambda}) \tag{A98}$$

with real part

$$\text{Re}\{\frac{Y_{ss}}{\kappa A_{ss}}(-\beta^{CM}e^{-i\lambda} + 2\beta^{CM} - \beta^{CM}e^{i\lambda})\} = \frac{2\beta^{CM}Y_{ss}}{\kappa A_{ss}}\text{Re}\{(1-e^{-i\lambda})\} \geq 0 \tag{A99}$$

since $\text{Re}\{e^{-i\lambda}\} \leq 1$.

<u>Results: Rigid prices, policy rules</u>

I make two assumptions. First, in an incomplete markets model, monetary policy affects savings through the intertemporal substitution channel and a fiscal channel. The intertemporal substitution channel operates as in complete markets models, so that higher interest rates lead to higher savings. The fiscal channel is a combination of two effects. An increase in the interest rate makes households richer and leads to higher savings. At the same time, higher interest rates lead to higher taxes which reduce savings. I assume that the intertemporal substitution channel outweighs the fiscal channel, such that an increase in nominal interest rates increases savings,

$$\epsilon_1^r > (1+\beta^{CM})(1+\tilde{\epsilon}_0^\tau\frac{1+i_{ss}}{1+\pi_{ss}}). \tag{A100}$$

Second, an increase in debt induced by an output change makes households richer and leads to higher savings and at the same time, to higher taxes which reduce savings. I assume an upper bound ϵ_0^Y on the sum of these two effects,

$$\epsilon_0^Y \geq -\frac{3-\beta^{CM}}{2}\varphi_Y^B(1+\frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_0^\tau),$$

which approximately means $\beta^{CM}\epsilon_0^Y \geq -\varphi_Y^B(mpc_0^0+\beta^{CM}-1)$. The proof requires an even weaker condition once monetary policy is taken into account:

$$\epsilon_0^Y\Big(1+\frac{\varphi_Y^i}{2\sigma}\Big) \geq -\frac{3-\beta^{CM}}{2}(\varphi_Y^i+\varphi_Y^B)\Big(1+\frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_0^\tau\Big). \tag{A101}$$

Note that if markets are complete, both conditions (A100) and (A101) are satisfied, since $1+\frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_0^\tau = 0$. The conditions are also satisfied if the incomplete markets economy is close enough to the complete markets economy.

Allowing for monetary and fiscal policy to respond to output changes, parametrized through φ_Y^B and φ_Y^i, adds

$$\varphi_Y^B\hat{Y}_t$$

to the supply of assets and

$$\sum_{k=0}^{\infty}\Big\{\epsilon_{k+1}^r\varphi_Y^i\hat{Y}_{t+k}+\tilde{\epsilon}_k^\tau\varphi_Y^i\frac{1+i_{ss}}{1+\pi_{ss}}\hat{Y}_{t+k-1}+\tilde{\epsilon}_k^\tau\varphi_Y^B(\frac{1+i_{ss}}{1+\pi_{ss}}\hat{Y}_{t+k-1}-\hat{Y}_{t+k})\Big\}+\epsilon_0^r\varphi_Y^i\hat{Y}_{t-1}+\epsilon_0^A\varphi_Y^B\hat{Y}_{t-1}$$

to the demand for assets.

The idea is again to show that the real part of the Onatski function is nonnegative. I therefore first consider each term separately, before summing them and showing that the real part is nonnegative. First,

$$\begin{aligned}
&\varphi_Y^B\sum_{k=0}^{\infty}\tilde{\epsilon}_k^\tau(\frac{1+i_{ss}}{1+\pi_{ss}}\hat{Y}_{t+k-1}-\hat{Y}_{t+k})\\
=\;&\frac{\varphi_Y^B}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\sum_{k=1}^{\infty}\{(1+2\beta^{CM})\tilde{\epsilon}_{k+1}^\tau-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_k^\tau+(\beta^{CM})^2\tilde{\epsilon}_{k-1}^\tau-\tilde{\epsilon}_{k+2}^\tau\}\hat{p}_{t+k}\\
+\;&\frac{\varphi_Y^B}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\Big\{\{(1+2\beta^{CM})\tilde{\epsilon}_1^\tau-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_0^\tau-\tilde{\epsilon}_2^\tau\}\hat{p}_t+\{(1+2\beta^{CM})\tilde{\epsilon}_0^\tau-\tilde{\epsilon}_1^\tau\}\hat{p}_{t-1}-\tilde{\epsilon}_0^\tau\hat{p}_{t-2}\Big\}.
\end{aligned}$$

The real part of the corresponding Onatski function equals

$$\begin{aligned}
& \frac{\varphi_Y^B}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\sum_{k=1}^{\infty}\{(1+2\beta^{CM})\tilde{\epsilon}_{k+1}^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_k^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_{k-1}^{\tau}-\tilde{\epsilon}_{k+2}^{\tau}\}\cos(-k\lambda) \\
+ & \frac{\varphi_Y^B}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\Big\{\{(1+2\beta^{CM})\tilde{\epsilon}_1^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_2^{\tau}\}+\{(1+2\beta^{CM})\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_1^{\tau}\}\cos(\lambda)-\tilde{\epsilon}_0^{\tau}\cos(2\lambda)\Big\} \\
\geq & \frac{\varphi_Y^B}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\sum_{k=3}^{\infty}\{(1+2\beta^{CM})\tilde{\epsilon}_{k+1}^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_k^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_{k-1}^{\tau}-\tilde{\epsilon}_{k+2}^{\tau}\} \\
+ & \frac{\varphi_Y^B}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\{(1+2\beta^{CM})\tilde{\epsilon}_3^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_2^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_1^{\tau}-\tilde{\epsilon}_4^{\tau}\}\cos(-2\lambda) \\
+ & \frac{\varphi_Y^B}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\{(1+2\beta^{CM})\tilde{\epsilon}_2^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_1^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_3^{\tau}\}\cos(-\lambda) \\
+ & \frac{\varphi_Y^B}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\Big\{\{(1+2\beta^{CM})\tilde{\epsilon}_1^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_2^{\tau}\}+\{(1+2\beta^{CM})\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_1^{\tau}\}\cos(\lambda)-\tilde{\epsilon}_0^{\tau}\cos(2\lambda)\Big\}
\end{aligned}$$

since the natural sign restriction $\varphi_Y^B \leq 0$ and Result 3 and (A81) show that for $k \geq 3$, $(1+2\beta^{CM})\tilde{\epsilon}_{k+1}^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_k^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_{k-1}^{\tau}-\tilde{\epsilon}_{k+2}^{\tau} > 0$.

For the next term,

$$\epsilon_0^A\varphi_Y^B\hat{Y}_{t-1}-\varphi_Y^B\hat{Y}_t = \frac{1}{\kappa}\Big\{\varphi_Y^B[(1+\beta^{CM})\hat{p}_{t-1}-\hat{p}_{t-2}-\beta^{CM}\hat{p}_t]-\varphi_Y^B[(1+\beta^{CM})\hat{p}_t-\hat{p}_{t-1}-\beta^{CM}\hat{p}_{t+1}]\Big\},$$

with Onatski function

$$\begin{aligned}
& \frac{\varphi_Y^B}{\kappa}\Big\{\beta^{CM}\cos(-\lambda)-(1+2\beta^{CM})+(2+\beta^{CM})\cos(\lambda)-\cos(2\lambda)\Big\} \\
= & \frac{\varphi_Y^B}{\kappa}\Big\{-(1+2\beta^{CM})+2(1+\beta^{CM})\cos(\lambda)-\cos(2\lambda)\Big\}.
\end{aligned}$$

For monetary policy parameterized through $\varphi_Y^i \geq 0$, the first term:

$$\begin{aligned}
& \sum_{k=0}^{\infty}\tilde{\epsilon}_k^{\tau}\varphi_Y^i\frac{1+i_{ss}}{1+\pi_{ss}}\hat{Y}_{t+k-1} = \frac{\varphi_Y^i}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\sum_{k=0}^{\infty}\tilde{\epsilon}_k^{\tau}[(1+\beta^{CM})\hat{p}_{t+k-1}-\hat{p}_{t+k-2}-\beta^{CM}\hat{p}_{t+k}] \\
= & \frac{\varphi_Y^i}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\{\sum_{k=0}^{\infty}[\tilde{\epsilon}_{k+1}^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_{k+2}^{\tau}-\beta^{CM}\tilde{\epsilon}_k^{\tau}]\hat{p}_{t+k}+[\tilde{\epsilon}_0^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_1^{\tau}]\hat{p}_{t-1}-\tilde{\epsilon}_0^{\tau}\hat{p}_{t-2}\}.
\end{aligned}$$

The real part of the corresponding Onatski function equals

$$
\begin{aligned}
& \frac{\varphi_Y^i}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\Big\{\sum_{k=0}^{\infty}[\tilde{\epsilon}_{k+1}^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_{k+2}^{\tau}-\beta^{CM}\tilde{\epsilon}_k^{\tau}]\cos(-k\lambda)+[\tilde{\epsilon}_0^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_1^{\tau}]\cos(\lambda)-\tilde{\epsilon}_0^{\tau}\cos(2\lambda)\Big\} \\
\geq\ & \frac{\varphi_Y^i}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\sum_{k=3}^{\infty}[\tilde{\epsilon}_{k+1}^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_{k+2}^{\tau}-\beta^{CM}\tilde{\epsilon}_k^{\tau}] \\
+\ & \frac{\varphi_Y^i}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}_3^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_4^{\tau}-\beta^{CM}\tilde{\epsilon}_2^{\tau}-\tilde{\epsilon}_0^{\tau}]\cos(2\lambda) \\
+\ & \frac{\varphi_Y^i}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}_2^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_3^{\tau}-\beta^{CM}\tilde{\epsilon}_1^{\tau}+\tilde{\epsilon}_0^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_1^{\tau}]\cos(\lambda) \\
+\ & \frac{\varphi_Y^i}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}_1^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_2^{\tau}-\beta^{CM}\tilde{\epsilon}_0^{\tau}],
\end{aligned}
$$

since $\varphi^i \geq 0$ and by Result 3 and (A77), $\forall k \geq 3 : \tilde{\epsilon}_{k+1}^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_{k+2}^{\tau}-\beta^{CM}\tilde{\epsilon}_k^{\tau} \leq 0$.

The next term,

$$
\sum_{k=0}^{\infty}\epsilon_{k+1}^{r}\varphi_Y^i\hat{Y}_{t+k} = \frac{\varphi_Y^i}{\kappa}\sum_{k=0}^{\infty}\epsilon_{k+1}^{r}[(1+\beta^{CM})\hat{p}_{t+k}-\hat{p}_{t+k-1}-\beta^{CM}\hat{p}_{t+k+1}] \quad \text{(A102)}
$$

$$
= \frac{\varphi_Y^i}{\kappa}\sum_{k=1}^{\infty}[\epsilon_{k+1}^{r}(1+\beta^{CM})-\epsilon_{k+2}^{r}-\beta^{CM}\epsilon_k^{r}]\hat{p}_{t+k}+\frac{\varphi_Y^i}{\kappa}[\epsilon_1^{r}(1+\beta^{CM})-\epsilon_2^{r}]\hat{p}_t-\frac{\varphi_Y^i}{\kappa}\epsilon_1^{r}\hat{p}_{t-1} \quad \text{(A103)}
$$

$$
= \frac{\varphi_Y^i}{\kappa}\epsilon_1^{r}(\hat{p}_t-\hat{p}_{t-1}) \quad \text{(A104)}
$$

since $\epsilon_{k+1}^{r}(1+\beta^{CM})-\epsilon_{k+2}^{r}-\beta^{CM}\epsilon_k^{r}=0$ for $k \geq 1$ and $\epsilon_2^{r}=\beta^{CM}\epsilon_1^{r}$, which leads to an Onatski function with real part

$$
\frac{\varphi_Y^i}{\kappa}\epsilon_1^{r}(1-\cos(\lambda)). \quad \text{(A105)}
$$

The last term

$$
\epsilon_0^{r}\varphi_Y^i\hat{Y}_{t-1} = \frac{\varphi_Y^i}{\kappa}\big\{-\beta^{CM}\hat{p}_t+(1+\beta^{CM})\hat{p}_{t-1}-\hat{p}_{t-2}\big\}
$$

leads to the real part of the Onatski function

$$
\frac{\varphi_Y^i}{\kappa}\big\{-\beta^{CM}+(1+\beta^{CM})\cos\lambda-\cos(2\lambda)\big\}.
$$

Summing all these terms and adding (A99),

$$
\mathrm{Re}\{\frac{Y_{ss}}{\kappa A_{ss}}\beta^{CM}2(1-e^{-i\lambda})\},
$$

yields

$$
\begin{aligned}
& \operatorname{Re}\{\Theta(\lambda)\} \\
\geq\ & \frac{\varphi_Y^i}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\sum_{k=3}^{\infty}[\tilde{\epsilon}_{k+1}^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_{k+2}^{\tau}-\beta^{CM}\tilde{\epsilon}_{k}^{\tau}] \\
+\ & \frac{\varphi_Y^B}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\sum_{k=3}^{\infty}\{(1+2\beta^{CM})\tilde{\epsilon}_{k+1}^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_{k}^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_{k-1}^{\tau}-\tilde{\epsilon}_{k+2}^{\tau}\} \\
+\ & \Big[\frac{\varphi_Y^i}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}_3^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_4^{\tau}-\beta^{CM}\tilde{\epsilon}_2^{\tau}-\tilde{\epsilon}_0^{\tau}]-1\Big\} \\
+\ & \frac{\varphi_Y^B}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}\{(1+2\beta^{CM})\tilde{\epsilon}_3^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_2^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_1^{\tau}-\tilde{\epsilon}_4^{\tau}-\tilde{\epsilon}_0^{\tau}\}-1\Big\}\Big]\cos(2\lambda) \\
+\ & \Big[\frac{\varphi_Y^i}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}_2^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_3^{\tau}-\beta^{CM}\tilde{\epsilon}_1^{\tau}+\tilde{\epsilon}_0^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_1^{\tau}]+1+\beta^{CM}-\epsilon_1^{r}\Big\} \\
+\ & \frac{\varphi_Y^B}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}\{(1+2\beta^{CM})\tilde{\epsilon}_2^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_1^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_3^{\tau}+(1+2\beta^{CM})\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_1^{\tau}\}\Big\} \\
+\ & \frac{\varphi_Y^B}{\kappa}2(1+\beta^{CM})-2\frac{Y_{ss}}{\kappa A_{ss}}\beta^{CM}\Big]\cos(\lambda) \\
+\ & \frac{\varphi_Y^i}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}_1^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_2^{\tau}-\beta^{CM}\tilde{\epsilon}_0^{\tau}]+\epsilon_1^{r}-\beta^{CM}\Big\} \\
+\ & \frac{\varphi_Y^B}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}\{(1+2\beta^{CM})\tilde{\epsilon}_1^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_2^{\tau}\}-(1+2\beta^{CM})\Big\}+2\frac{Y_{ss}}{\kappa A_{ss}}\beta^{CM}
\end{aligned}
$$

Define the $\cos(\lambda)$ coefficient

$$
\begin{aligned}
\alpha_1 \ :=\ & \Big[\frac{\varphi_Y^i}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}_2^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_3^{\tau}-\beta^{CM}\tilde{\epsilon}_1^{\tau}+\tilde{\epsilon}_0^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_1^{\tau}]+1+\beta^{CM}-\epsilon_1^{r}\Big\} \\
+\ & \frac{\varphi_Y^B}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}\{(1+2\beta^{CM})\tilde{\epsilon}_2^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_1^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_3^{\tau}+(1+2\beta^{CM})\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_1^{\tau}\}\Big\} \\
+\ & \frac{\varphi_Y^B}{\kappa}2(1+\beta^{CM})-2\frac{Y_{ss}}{\kappa A_{ss}}\beta^{CM}\Big]
\end{aligned}
$$

and the $\cos(2\lambda)$ coefficient,

$$
\begin{aligned}
\alpha_2 \ :=\ & \Big[\frac{\varphi_Y^i}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}_3^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_4^{\tau}-\beta^{CM}\tilde{\epsilon}_2^{\tau}-\tilde{\epsilon}_0^{\tau}]-1\Big\} \\
+\ & \frac{\varphi_Y^B}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}\{(1+2\beta^{CM})\tilde{\epsilon}_3^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_2^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_1^{\tau}-\tilde{\epsilon}_4^{\tau}-\tilde{\epsilon}_0^{\tau}\}-1\Big\}\Big].
\end{aligned}
$$

For α_1:

$$
\begin{aligned}
\alpha_1 &\leq \frac{\varphi_Y^i}{\kappa}\{(1+\frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_0^\tau)(1+\beta^{CM}) - \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_1^\tau - \epsilon_1^r\} \\
&+ \frac{\varphi_Y^B}{\kappa}\{(1+2\beta^{CM})(1+\tilde{\epsilon}_0^\tau\frac{1+i_{ss}}{1+\pi_{ss}}) - \tilde{\epsilon}_1^\tau\frac{1+i_{ss}}{1+\pi_{ss}} + (1-\beta^{CM})\} - 2\frac{Y_{ss}}{\kappa A_{ss}}\beta^{CM} \\
&= \frac{1}{\kappa}[\varphi_Y^B(1+2\beta^{CM}) + \varphi_Y^i(1+\beta^{CM})](1+\tilde{\epsilon}_0^\tau\frac{1+i_{ss}}{1+\pi_{ss}}) \\
&- \frac{1}{\kappa}\{\varphi_Y^i\epsilon_1^r + (\varphi_Y^i+\varphi_Y^B)\tilde{\epsilon}_1^\tau\frac{1+i_{ss}}{1+\pi_{ss}} - \varphi_Y^B(1-\beta^{CM}) + 2\frac{Y_{ss}}{A_{ss}}\beta^{CM}\} \\
&\leq \frac{\varphi_Y^B+\varphi_Y^i}{\kappa}(1+\beta^{CM})(1+\tilde{\epsilon}_0^\tau\frac{1+i_{ss}}{1+\pi_{ss}}) - \frac{\varphi_Y^i}{\kappa}\epsilon_1^r - \frac{\varphi_Y^i}{\kappa}\tilde{\epsilon}_1^\tau\frac{1+i_{ss}}{1+\pi_{ss}} - 2\frac{Y_{ss}}{\kappa A_{ss}}\beta^{CM},
\end{aligned}
$$

since Result 3, (A81) and $\epsilon_0^\tau = -1 + mpc_0^0$ imply

$$
(1+2\beta^{CM})\tilde{\epsilon}_2^\tau - (2\beta^{CM} + (\beta^{CM})^2)\tilde{\epsilon}_1^\tau + (\beta^{CM})^2\tilde{\epsilon}_0^\tau - \tilde{\epsilon}_3^\tau \geq -(\beta^{CM})^2
$$

and

$$
\begin{aligned}
& \varphi_Y^B\{(\beta^{CM} + \tilde{\epsilon}_0^\tau - \tilde{\epsilon}_1^\tau\frac{1+i_{ss}}{1+\pi_{ss}})\} + \varphi_Y^B(1-\beta^{CM}) \\
=\ & \varphi_Y^B\{(\beta^{CM} + (-1+mpc_0^0) - mpc_0^1\frac{1+i_{ss}}{1+\pi_{ss}}) + (1-\beta^{CM})\} \\
=\ & \varphi_Y^B\{mpc_0^0 - mpc_0^1\frac{1+i_{ss}}{1+\pi_{ss}})\} \\
\leq\ & 0.
\end{aligned}
$$

showing that $(1+\beta^{CM})(1+\tilde{\epsilon}_0^\tau\frac{1+i_{ss}}{1+\pi_{ss}}) - \epsilon_1^r < 0$ implies $\alpha_1 \leq 0$.

For α_2:

$$
\begin{aligned}
\alpha_2 &= \Big[\frac{\varphi_Y^i}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}_3^\tau(1+\beta^{CM}) - \tilde{\epsilon}_4^\tau - \beta^{CM}\tilde{\epsilon}_2^\tau - \tilde{\epsilon}_0^\tau] - 1\Big\} \\
&+ \frac{\varphi_Y^B}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}\big\{(1+2\beta^{CM})\tilde{\epsilon}_3^\tau - (2\beta^{CM} + (\beta^{CM})^2)\tilde{\epsilon}_2^\tau + (\beta^{CM})^2\tilde{\epsilon}_1^\tau - \tilde{\epsilon}_4^\tau - \tilde{\epsilon}_0^\tau\big\} - 1\Big\}\Big] \\
&\leq \Big[\frac{\varphi_Y^i}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}[-\tilde{\epsilon}_0^\tau] - 1\Big\} + \frac{\varphi_Y^B}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}[-\tilde{\epsilon}_0^\tau] - 1\Big\}\Big] \\
&= -\Big[\frac{\varphi_Y^i}{\kappa}\Big\{1 + \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_0^\tau\Big\} + \frac{\varphi_Y^B}{\kappa}\Big\{1 + \frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_0^\tau\Big\}\Big],
\end{aligned}
$$

since by Result 3, $(1+2\beta^{CM})\tilde{\epsilon}_3^\tau - (2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_2^\tau + (\beta^{CM})^2\tilde{\epsilon}_1^\tau - \tilde{\epsilon}_4^\tau \geq 0$. Therefore

$$\begin{aligned} -4\alpha_2 - \alpha_1 &\geq 4\Big[\frac{\varphi_Y^i}{\kappa}\Big\{1+\frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_0^\tau\Big\} + \frac{\varphi_Y^B}{\kappa}\Big\{1+\frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_0^\tau\Big\}\Big] \\ &- \frac{\varphi_Y^B+\varphi_Y^i}{\kappa}(1+\beta^{CM})(1+\tilde{\epsilon}_0^\tau\frac{1+i_{ss}}{1+\pi_{ss}}) \\ &+ \frac{\varphi_Y^i}{\kappa}\epsilon_1^r + \frac{\varphi_Y^i}{\kappa}\tilde{\epsilon}_1^\tau\frac{1+i_{ss}}{1+\pi_{ss}} + 2\frac{Y_{ss}}{\kappa A_{ss}}\beta^{CM} \\ &= \frac{3-\beta^{CM}}{\kappa}(\varphi_Y^i+\varphi_Y^B)\Big(1+\frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_0^\tau\Big) \\ &+ \frac{\varphi_Y^i}{\kappa}\epsilon_1^r + \frac{\varphi_Y^i}{\kappa}\tilde{\epsilon}_1^\tau\frac{1+i_{ss}}{1+\pi_{ss}} + 2\frac{Y_{ss}}{\kappa A_{ss}}\beta^{CM} > 0, \end{aligned}$$

which follows from

$$-\frac{3-\beta^{CM}}{2}\varphi_Y^B\Big(1+\frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_0^\tau\Big) \leq \beta^{CM}\frac{Y_{ss}}{A_{ss}} = \epsilon_0^Y,$$

or, taking monetary policy into account from

$$-\frac{3-\beta^{CM}}{2}(\varphi_Y^i+\varphi_Y^B)\Big(1+\frac{1+i_{ss}}{1+\pi_{ss}}\tilde{\epsilon}_0^\tau\Big) \leq \epsilon_0^Y\Big(1+\frac{\varphi_Y^i}{2\sigma}\Big).$$

Next, I show that $-4\alpha_2 - \alpha_1 > 0$ implies that

$$\Xi(\lambda) = \alpha_1\cos(\lambda) + \alpha_2\cos(2\lambda)$$

is minimized at $\lambda = 0$ with value $\alpha_1 + \alpha_2$. The derivative

$$\Xi'(\lambda) = -\alpha_1\sin(\lambda) - 2\alpha_2\sin(2\lambda)$$

has four zeros: $0, \pi, \arctan\Big(\pm\sqrt{\frac{16\alpha_2^2-\alpha_1^2}{4\alpha_2}}, \frac{-\alpha_1}{4\alpha_2}\Big)$ in $(-\pi,\pi]$. The second derivative evaluated at $\lambda = 0$

$$\Xi''(\lambda=0) = -\alpha_1\cos(0) - 4\alpha_2\cos(0) = -\alpha_1 - 4\alpha_2 > 0,$$

implying that $\lambda = 0$ is a minimum, since $\Xi''(\lambda=\pi) = -\alpha_1\cos(\pi) - 4\alpha_2\cos(2\pi) = \alpha_1 < 0$ and one of the two other zeros - $\arctan\left(\pm\sqrt{\frac{16\alpha_2^2-\alpha_1^2}{4\alpha_2}}, \frac{-\alpha_1}{4\alpha_2}\right)$ - is positive and the other is negative. Thus

$$
\begin{aligned}
& \mathrm{Re}\{\Theta(\lambda)\} \\
\geq\ & \frac{\varphi_Y^i}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\sum_{k=3}^{\infty}[\tilde{\epsilon}_{k+1}^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_{k+2}^{\tau}-\beta^{CM}\tilde{\epsilon}_k^{\tau}] \\
+\ & \frac{\varphi_Y^B}{\kappa}\frac{1+i_{ss}}{1+\pi_{ss}}\sum_{k=3}^{\infty}\{(1+2\beta^{CM})\tilde{\epsilon}_{k+1}^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_k^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_{k-1}^{\tau}-\tilde{\epsilon}_{k+2}^{\tau}\} \\
+\ & \Big[\frac{\varphi_Y^i}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}_3^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_4^{\tau}-\beta^{CM}\tilde{\epsilon}_2^{\tau}-\tilde{\epsilon}_0^{\tau}]-1\Big\} \\
+\ & \frac{\varphi_Y^B}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}\{(1+2\beta^{CM})\tilde{\epsilon}_3^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_2^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_1^{\tau}-\tilde{\epsilon}_4^{\tau}-\tilde{\epsilon}_0^{\tau}\}-1\Big\}\Big] \\
+\ & \Big[\frac{\varphi_Y^i}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}_2^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_3^{\tau}-\beta^{CM}\tilde{\epsilon}_1^{\tau}+\tilde{\epsilon}_0^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_1^{\tau}]+1+\beta^{CM}-\epsilon_1^{\tau}\Big\} \\
+\ & \frac{\varphi_Y^B}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}\{(1+2\beta^{CM})\tilde{\epsilon}_2^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_1^{\tau}+(\beta^{CM})^2\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_3^{\tau}+(1+2\beta^{CM})\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_1^{\tau}\}\Big\} \\
+\ & \frac{\varphi_Y^B}{\kappa}2(1+\beta^{CM})-2\frac{Y_{ss}}{\kappa A_{ss}}\beta^{CM}\Big] \\
+\ & \frac{\varphi_Y^i}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}[\tilde{\epsilon}_1^{\tau}(1+\beta^{CM})-\tilde{\epsilon}_2^{\tau}-\beta^{CM}\tilde{\epsilon}_0^{\tau}]+\epsilon_1^{r}-\beta^{CM}\Big\} \\
+\ & \frac{\varphi_Y^B}{\kappa}\Big\{\frac{1+i_{ss}}{1+\pi_{ss}}\{(1+2\beta^{CM})\tilde{\epsilon}_1^{\tau}-(2\beta^{CM}+(\beta^{CM})^2)\tilde{\epsilon}_0^{\tau}-\tilde{\epsilon}_2^{\tau}\}-(1+2\beta^{CM})\Big\}+2\frac{Y_{ss}}{\kappa A_{ss}}\beta^{CM} \\
=\ & 0,
\end{aligned}
$$

since all terms cancel.

Local Determinacy, $1+r_{ss}=\frac{1+i_{ss}}{1+\pi_{ss}}\leq 0$

Note that $1+r_{ss}\leq 0$ is not sufficient to rule out bubbles, implying that the real incomplete markets economy features multiple equilibria. The analysis in this paper is about nominal determinacy and thus cannot remove this real multiplicity. The following arguments therefore refer to the bubble-free equilibrium and cannot ensure a unique equilibrium more generally.

An economy with $1+r_{ss}\leq 0$ cannot be considered as a departure to a complete markets economy. I therefore make use of the general approach for incomplete markets models in Section 4.2, which I developed above in Section A.III.1.

I follow the same steps and first solve the model, which yields Period 0 consumption as a function of initial assets, interest rates, output, and taxes

$$C_0=\mathcal{C}^{IM}(\{1+r_s\}_{s=0}^{\infty},\{\tau_s\}_{s=0}^{\infty},\{Y_s\}_{s=0}^{\infty},\tilde{A}_0^{IM}),$$

where the argument $\tilde{A}_0^{IM}$ means that the initial asset distribution is the steady state asset distribution shifted by the same factor $\tilde{A}_0^{IM}/A_{ss}$ for every household. Consumption at later periods is defined as

$$\tilde{C}_{t+1}^{IM} = \mathcal{C}^{IM}(\{1+r_s\}_{s=t+1}^{\infty}, \{\tau_s\}_{s=t+1}^{\infty}, \{Y_s\}_{s=t+1}^{\infty}, \tilde{A}_{t+1}^{IM}), \tag{A106}$$

and aggregate assets satisfy

$$\tilde{A}_{t+1}^{IM} = \tilde{A}_t^{IM}(1+r_t) - \tilde{C}_t^{IM} - Y_t - \tau_t.$$

I then use the same Δ transfer scheme as in Section A.III.1 such that household i's optimal choices at time t in this economy are

$$c_{i,t}^{IM} = \frac{\tilde{C}_t^{IM}}{C_{ss}} c_{i,t}^{ss}; \qquad a_{i,t+1}^{IM} = \frac{\tilde{A}_{t+1}^{IM}}{A_{ss}} a_{i,t+1}^{ss} \tag{A107}$$

and aggregate consumption and savings are then

$$C_t^{IM} = \int c_{i,t}^{IM} d\Omega_t = \tilde{C}_t^{IM} = \mathcal{C}^{IM}(\{1+r_s\}_{s=t}^{\infty}, \{\tau_s\}_{s=t}^{\infty}, \{Y_s\}_{s=t}^{\infty}, A_t^{IM}), \tag{A108}$$

$$A_t^{IM} = \int a_{i,t}^{IM} d\Omega_t = \tilde{A}_t^{IM}. \tag{A109}$$

The linearized asset market clearing condition is the same as in the main text,

$$E_t \sum_{k=0}^{\infty} \left\{ \epsilon_{k+1}^r (\hat{p}_{t+k} - \hat{p}_{t+k+1}) - \tilde{\epsilon}_k^{\tau} \frac{i_{ss} - \pi_{ss}}{1+\pi_{ss}} \hat{p}_{t+k} \right\} + \epsilon_0^r (\hat{p}_{t-1} - \hat{p}_t) - \epsilon_0^A \hat{p}_{t-1} = -\hat{p}_t, \tag{A110}$$

but since $\tau_{ss} = 0$ if $r_{ss} = 0$, the $\tau-$elasticity is replaced with the derivative,

$$\epsilon_0^{\tau} := (-1 - \frac{\partial C_0^{IM}}{\partial \tau_0}) \frac{1}{A_{ss}}; \qquad \epsilon_k^{\tau} := -\frac{\partial C_0^{IM}}{\partial \tau_k} \frac{1}{A_{ss}} \qquad \forall k \geq 1, \tag{A111}$$

$\tilde{\epsilon}_k^{\tau} = \epsilon_k^{\tau} A_{ss}$ and $\hat{\tau}_t = \tau_t - \tau_{ss}$ is the difference, so that

$$\frac{\hat{\tau}_t}{A_{ss}} = -\frac{1+i_{ss}}{1+\pi_{ss}} \hat{p}_t + \hat{p}_t = \frac{\pi_{ss} - i_{ss}}{1+\pi_{ss}} \hat{p}_t. \tag{A112}$$

Using the same notation for the other elasticities as in the main text, collecting $\hat{p}$ terms and re-arranging yields

$$(1 + \epsilon_1^r - \epsilon_0^r - \tilde{\epsilon}_0^{\tau} \frac{i_{ss} - \pi_{ss}}{1+\pi_{ss}}) \hat{p}_t = \sum_{k=1}^{\infty} (\epsilon_k^r - \epsilon_{k+1}^r + \tilde{\epsilon}_k^{\tau} \frac{i_{ss} - \pi_{ss}}{1+\pi_{ss}}) \hat{p}_{t+k} \tag{A113}$$

so that

$$\Theta(\lambda) = (1 + \epsilon_1^r - \epsilon_0^r - \tilde{\epsilon}_0^\tau \frac{i_{ss} - \pi_{ss}}{1 + \pi_{ss}}) - \sum_{k=1}^{\infty} (\epsilon_k^r - \epsilon_{k+1}^r + \tilde{\epsilon}_k^\tau \frac{i_{ss} - \pi_{ss}}{1 + \pi_{ss}}) e^{-ik\lambda}. \tag{A114}$$

I again show that $\text{Re}\{\Theta(\lambda)\} > 0$ for all $\lambda \in [0, 2\pi]$, implying that the winding number is zero, and thus Onatski (2006) implies that there is a unique solution, that is, the economy is locally determinate. The findings in McKay et al. (2017) imply that, $\forall k \geq 0 : \epsilon_k^r \geq \epsilon_{k+1}^r \geq 0$ and $\lim_{k \to \infty} \epsilon_k^r = 0$.[47] Since Result 3 applies in any incomplete markets model $\tilde{\epsilon}_k^\tau \geq 0$ for $k \geq 1$. Therefore

$$\begin{aligned} \text{Re}\{\Theta(\lambda)\} &= (1 + \epsilon_1^r - \epsilon_0^r - \tilde{\epsilon}_0^\tau \frac{i_{ss} - \pi_{ss}}{1 + \pi_{ss}}) - \sum_{k=1}^{\infty} (\epsilon_k^r - \epsilon_{k+1}^r + \tilde{\epsilon}_k^\tau \frac{i_{ss} - \pi_{ss}}{1 + \pi_{ss}}) \cos(-k\lambda) && \text{(A115)} \\ &\geq (1 + \epsilon_1^r - \epsilon_0^r - \tilde{\epsilon}_0^\tau \frac{i_{ss} - \pi_{ss}}{1 + \pi_{ss}}) - \sum_{k=1}^{\infty} (\epsilon_k^r - \epsilon_{k+1}^r + \tilde{\epsilon}_k^\tau \frac{i_{ss} - \pi_{ss}}{1 + \pi_{ss}}) && \text{(A116)} \\ &= 1 - \tilde{\epsilon}_0^r > 0, && \text{(A117)} \end{aligned}$$

since Result 2 holds and implies that $\tilde{\epsilon}_0^r < 1$.[48] Thus the graph of $\Theta(\lambda)$ is within the plane of positive real numbers and does not encircle zero, implying a winding number of zero and local determinacy.

The model with price rigidities

I follow the standard approach in the New Keynesian literature so as to add price stickiness to the model.

Households First, labor h_t is elastically supplied and preferences are

$$\sum_{t=0}^{\infty} \beta^t (u(c_t) - v(h_t)), \tag{A118}$$

where v is increasing and strictly convex. Agents rent their labor services, $h_t e_t$, to firms for a real wage w_t and the budget constraint is

$$c_t + a_{t+1} = (1 + r_t) a_t + w_t h_t e_t - \tau_t. \tag{A119}$$

[47]Note that these are properties of the partial equilibrium household consumption/saving problem, where output and fiscal policy are unchanged. Note also that Werning (2015) and Hagedorn et al. (2019) present equilibrium results and thus do not apply here.

[48]Note that an increase in r_0 is equivalent to a one-time period zero transfer to households, implying that $\epsilon_0^r < 1$ since the marginal propensity to consume is larger than $1 - \beta^{CM}$.

Final Good Producer A competitive representative final-goods producer aggregates a continuum of intermediate goods y_{jt} indexed by $j \in [0,1]$ and with prices p_{jt}:

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $\epsilon > 1$. Given a level of aggregate demand Y_t, cost minimization for the final goods producer implies that the demand for the intermediate good j is given by

$$y_{jt} = y(p_{jt}; P_t, Y_t) = \left(\frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t, \tag{A120}$$

where P_t is the (equilibrium) price of the final good and can be expressed as

$$P_t = \left(\int_0^1 p_{jt}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}.$$

Intermediate good producer Each intermediate good j is produced by a monopolistically competitive producer using labor input n_{jt}. The production technology is linear,

$$y_{jt} = n_{jt}.$$

Intermediate producers hire labor at the nominal wage $P_t w_t$ in a competitive labor market. With this technology, the real marginal cost of a unit of the intermediate good is

$$mc_{jt} = w_t.$$

Each firm chooses its price, so as to maximize profits subject to real price-adjustment costs as in Rotemberg (1982),

$$\Phi\left(p_{jt}, p_{jt-1}\right) Y_t = \Phi\left(\frac{p_{jt}}{p_{jt-1}} - \pi^{ss}\right) Y_t \tag{A121}$$

which depend on the set price p_{jt} and on the previous period's price p_{jt-1}. Costs Φ are increasing and convex in its first argument and zero in a steady state with price level P_t^*, $\Phi\left(P_t^*, P_{t-1}^*\right) = 0$ and $\lim_{p_{jt} \to \infty} \Phi\left(p_{jt}; p_{jt-1}\right) = \infty$.

Given the previous period's individual price p_{jt-1} and the aggregate state $(P_t, Y_{t,}, w_t, r_t)$, the firm chooses this period's price p_{jt} to maximize the present discounted value of future profits. The firm satisfies all demand $y(p_{jt}; P_t, Y_t)$ by hiring the necessary amount of labor,

$$n_{jt} = y(p_{jt}; P_t, Y_t) = \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t. \tag{A122}$$

The firm's pricing problem is

$$V_t(p_{jt-1}) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} y(p_{jt}; P_t, Y_t) - w_t y(p_{jt}; P_t, Y_t) - \Phi(p_{jt}; p_{jt-1}) Y_t + \beta^{CM} V_{t+1}(p_{jt}).$$

In equilibrium, all firms choose the same price, and thus, aggregate consistency implies $p_{jt} = P_t$ for all j and t. Accordingly, $\frac{p_{jt}}{p_{jt-1}} = \frac{P_t}{P_{t-1}} = 1 + \pi_t$ and $\frac{p_{jt+1}}{p_{jt}} = \frac{P_{t+1}}{P_t} = 1 + \pi_{t+1}$. The equilibrium real profit of each intermediate goods firm is then

$$d_t = Y_t(1 - w_t).$$

This does not include price adjustment costs, because I follow the preferred interpretation of those costs in Rotemberg (1982) as being virtual—they affect optimal choices but do not cause real resources to be expended. Thus these costs affect firms' pricing decisions, but neither lower their profits nor enter the aggregate resource constraint. None of my conclusions are affected by this assumption, since steady-state adjustment costs are zero. Household i receives a share λ_i of real profits d_t at time t.

Wage setting The assumptions for wages and labor supply are made to replicate the textbook New Keynesian counterpart. I therefore assume that wages are flexible and that a middleman firm (e.g. a union) solves the aggregation problem such that all households provide the same amount of labor. As in the representative agent literature, each household i provides differentiated labor services $h_{it} e_{it}$ which are transformed by the union into an aggregate effective labor input, H_t, using the following technology:

$$H_t = \left(\int_0^1 s_{it} (h_{it})^{\frac{\epsilon_w - 1}{\epsilon_w}} di\right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{A123}$$

where ϵ_w is the elasticity of substitution across differentiated labor.

The union is assumed to maximize[49]

$$\max_{\{h_{it}\}} \int \left(w_t h_{it} - \frac{v(h_{it})}{u'(C_t)}\right) di$$

where C_t is aggregate consumption, and the competitive wage w_t is taken as given. The absence of

[49] Equivalently, one can think of a continuum of unions, each setting hours for a representative part of the population with $\int s = 1$ at all times.

wage adjustment costs implies that the problem is static.[50] For each household i, I obtain the same first-order condition

$$w_t = \frac{v'(h_{it})}{u'(C_t)}, \tag{A124}$$

which in equilibrium, where $Y_t = C_t = H_t = h_{it}$, reads

$$w_t = \frac{v'(Y_t)}{u'(Y_t)}. \tag{A125}$$

Adjustment of Δ_{it}

Section 4.1.2 in the main text constructs a transfer scheme Δ_{it}, such that the evolution of aggregates is identical in the incomplete and complete markets economies. I now show how to modify Δ_{it} in the model with endogenous labor and sticky prices to obtain the same result. I therefore replace $(Y_{ss} - Y_t)e_{i,t}$ in the definition (60) of Δ_{it} with $(w_{ss}h_{ss} - w_t h_t)e_{i,t} + \lambda_i(d_{ss} - d_t)$ so that now

$$\begin{aligned} \Delta_{i,t} \quad &:= \quad \frac{A_{t+1}^{CM} - A_{ss}}{A_{ss}} a_{i,t+1}^{ss} + a_{i,t}^{ss}\{(1 + r_{ss}) - \frac{A_t^{CM}}{A_{ss}}(1 + r_t)\} \\ &+ \quad \frac{C_t^{CM} - C_{ss}}{C_{ss}} c_{i,t}^{ss} + (w_{ss}h_{ss} - w_t h_t)e_{i,t} + \lambda_i(d_{ss} - d_t) - r_{ss}A_{ss} + \tau_t, \end{aligned} \tag{A126}$$

These transfers are again cross-sectionally purely redistributive

$$\begin{aligned} \int \Delta_{i,t} d\Omega_t &= (A_{t+1}^{CM} - A_{ss}) - (A_t^{CM} - A_{ss}) + (C_t^{CM} - C_{ss}) + (w_{ss}h_{ss} - w_t h_t) + (d_{ss} - d_t) - r_t A_t^{CM} + \tau_t \\ &= (A_{t+1}^{CM} - A_{ss}) - (A_t^{CM} - A_{ss}) + (C_t^{CM} - C_{ss}) + (Y_{ss} - Y_t) - r_t A_t^{CM} + \tau_t = 0. \end{aligned}$$

Household budget constraints are satisfied

$$\begin{aligned} & (1 + r_t)a_{i,t}^{AI} - c_{i,t}^{AI} + e_{i,t}w_t h_t + \lambda_i d_t - \tau_t + \Delta_{i,t} \\ = \; & \frac{A_{t+1}^{CM} - A_{ss}}{A_{ss}} a_{i,t+1}^{ss} + (1 + r_{ss})a_{i,t}^{ss} - c_{i,t}^{ss} + e_{i,t}w_{ss}h_{ss} + \lambda_i d_{ss} - \tau_{ss} \\ = \; & \frac{A_{t+1}^{CM} - A_{ss}}{A_{ss}} a_{i,t+1}^{ss} + a_{i,t+1}^{ss} \\ = \; & a_{i,t+1}^{AI}. \end{aligned}$$

The consumption Euler equation is unchanged and thus also satisfied. The same arguments apply to the transfers defined in Section 4.2 and are thus omitted here.

[50] Wage setting here can be thought of as the flexible wage competitive limit of the wage setting model in Hagedorn et al. (2017a).

Derivation of Phillips Curve

The firm's pricing problem is

$$V_t(p_{jt-1}) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} y(p_{jt}; P_t, Y_t) - w_t y(p_{jt}; P_t, Y_t) - \Phi\left(\frac{p_{jt}}{p_{jt-1}} - \pi_{ss}\right) Y_t + \beta^{CM} V_{t+1}(p_{jt}),$$

subject to the constraints $n_{jt} = y(p_{jt}; P_t, Y_t) = \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t$.
Equivalently

$$V_t(p_{jt-1}) \equiv \max_{p_{jt}} \frac{p_{jt}}{P_t} \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t - w_t \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} Y_t - \Phi\left(\frac{p_{jt}}{p_{jt-1}} - \pi_{ss}\right) Y_t + \beta^{CM} V_{t+1}(p_{jt})$$

The FOC w.r.t p_{jt}

$$(1-\epsilon)\left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} \frac{Y_t}{P_t} + \epsilon w_t \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon-1} \frac{Y_t}{P_t} - \Phi'\left(\frac{p_{jt}}{p_{jt-1}} - \pi_{ss}\right) \frac{Y_t}{p_{jt-1}} + \beta^{CM} V'_{t+1}(p_{jt}) = 0$$

and the envelope condition

$$V'_{t+1}(p_{jt}) = \Phi'\left(\frac{p_{jt+1}}{p_{jt}} - \pi_{ss}\right) \frac{p_{jt+1}}{p_{jt}} \frac{Y_{t+1}}{p_{jt}}.$$

Combining the FOC and the envelope condition

$$(1-\epsilon)\left(\frac{p_{jt}}{P_t}\right)^{-\epsilon} \frac{Y_t}{P_t} + \epsilon w_t \left(\frac{p_{jt}}{P_t}\right)^{-\epsilon-1} \frac{Y_t}{P_t} - \Phi'\left(\frac{p_{jt}}{p_{jt-1}} - \pi_{ss}\right) \frac{Y_t}{p_{jt-1}} + \beta^{CM} \Phi'\left(\frac{p_{jt+1}}{p_{jt}} - \pi_{ss}\right) \frac{p_{jt+1}}{p_{jt}} \frac{Y_{t+1}}{p_{jt}} = 0$$

Finally, on the basis that all firms choose the same price in equilibrium, that $\frac{p_{jt+1}}{p_{jt}} = \pi_{t+1}$, and dividing by $Y_t/P_t = Y_t/p_{jt}$ yields the non-linear Phillips curve:

$$(1-\epsilon) + \epsilon w_t - \Phi'(\pi_t - \pi_{ss}) \pi_t + \beta^{CM} \Phi'(\pi_{t+1} - \pi_{ss}) \pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0. \tag{A127}$$

Linearization of the Phillips Curve

A linearization of (A127) around the steady state yields

$$\epsilon \varphi \hat{Y}_t - \theta \hat{\pi}_t + \beta^{CM} \theta \hat{\pi}_{t+1} = 0 \tag{A128}$$

where $\theta = \Phi''(0), \Phi'(0) = 0$ and $\hat{w}_t = \varphi \hat{Y}_t$ (from linearizing (A125)) and $\hat{\pi}$ is the deviation of inflation from its steady-state value π_{ss}. Equivalently, for $\kappa = \frac{\epsilon\varphi}{\theta}$

$$\hat{\pi}_t = \kappa \hat{Y}_t + \beta^{CM} \hat{\pi}_{t+1} \tag{A129}$$

or in terms of prices

$$\hat{p}_t - \hat{p}_{t-1} = \kappa \hat{Y}_t + \beta^{CM}(\hat{p}_{t+1} - \hat{p}_t), \tag{A130}$$

where $\hat{p}_t$ is the deviation from the steady-state price $P_t^* = P^*(1 + \pi_{ss})^t$.