DISCUSSION PAPER SERIES

DP11350

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INDUSTRIAL ORGANIZATION

ISSN 0265-8003

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Discussion Paper 11350
Published 24 June 2016
Submitted 24 June 2016

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When to Learn What in Bilateral Trade

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June 23, 2016

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1 Introduction

Gathering information, identifying the relevant parts and internalizing their implication is a process that takes time. Even when information is readily accessible, decision-makers typically allocate their attention to a variety of tasks, and hence cannot devote their full resources to information acquisition. Consequently, information gathering and processing is often gradual. For example, the owner of an asset may only gradually learn all the various attributes of his asset; it takes time and experience for a worker to realize his ability in the various dimensions of his job; and a forecaster or analyst only gradually learns and processes the relevant characteristics of a state of nature.

The gradual nature of information acquisition is a form of friction that introduces new strategic considerations to economic environments where the participating agents may be asymmetrically informed. In particular, time, even if not costly per-se, may signal how much - and possibly what type of - information has been acquired and processed. Consequently, the *amount* of information that is learned (the degree of asymmetric information between agents), and the *scheduling* of information acquisition (which piece of information is learned each period) are determined *strategically*. Our goal is to understand how these strategic considerations affect trade.

We take a small step towards achieving this goal by focusing on bilateral trade, the basic building block of any trading environment. To isolate the pure effect of strategic gradual learning, we analyze the case in which only *one party* can acquire private information, and where information acquisition has *no direct cost*. We assume that all the bargaining power is in the hands of the party that can acquire information, who makes a take-it-or-leave-it offer to the uninformed party. The side that can access information may be either the seller or the buyer - the exact identity does not affect our analysis. In some contexts it may be more natural for the buyer to be acquiring information.[1] In other contexts, only the party with the initial property rights over the asset can learn more about that asset. To fix ideas, we assume that it is the *seller* who acquires information.

The literature on bilateral trade with private information either begins its analysis when parties have *already acquired* their private information, or the parties decide in advance how much information to acquire and this information is acquired *instantaneously*.[2] We begin our analysis at a stage when both parties are symmetrically

[1] For example, start-ups that come up with technological innovations may lack information on the potential applications and marketability of that innovation. This information, however, may be more easily accessible to an experienced firm that is interesting in buying the start-up.

[2] This literature analyzes the implication of *costly* information acquisition. See e.g., Persico (2000)

uninformed and then explicitly model the *gradual* nature in which private information is strategically acquired. Consequently, both the *extent* of asymmetry in information, and the *nature* of the asymmetry, are determined *endogenously*. How does this affect the parties' ability to realize potential gain from trade? How does it affect the frequency and timing of trade?

To address these questions we propose a simple stylized model of bilateral trade over a multi-attribute asset. Each attribute is a binary random variable (an attribute is either available or not) and the attributes are i.i.d.. The value of the asset to the buyer and seller is a weighted sum of attributes, where the weights of the buyer may differ from those of the seller. The different attributes may be interpreted as representing different potential benefits that can be reaped from the asset, and the two parties have different opinions about their contribution to the overall value of the asset. Both the buyer and seller are initially uninformed about the values of the attributes, but the seller can learn the realizations of the attributes gradually over time. In each period, the seller decides whether to make a take-it-or-leave-it offer to the buyer or to inspect another attribute - in which case he chooses an attribute and observes its realization at no cost. We first assume that the buyer observes neither the attributes that were inspected nor their realizations. The solution concept we employ is a refinement of perfect Bayesian equilibrium that imposes some "natural" restrictions on out-of-equilibrium beliefs.

Our model is not meant to be a faithful description of a particular market or industry, but rather a stylized representation of trading situations that have the following key features. First, the buyer and seller disagree on the relative contribution of different attributes of the asset to its overall value. It is therefore unknown ex-ante which party can benefit most from the asset but it depends on the realized profile of attributes (in contrast to a standard lemons problem). Second, the time the asset has been with the seller signals to the buyer the extent to which the seller has already explored potential applications of the asset and learned whether they are worthwhile. If the seller is trying to sell the asset after holding on to it for a relatively long period, it is more likely that he has learned a lot about the asset and its potential.

To see what type of situations have the above features, think of an entrepreneur who develops a new technology or product, which may be used domestically or abroad. The entrepreneur can investigate the domestic demand for his invention, as well as the demand in various foreign countries. Such an investigation would take time and most likely would not yield results on all countries at the same time. Rather, the

and Bergmann and Välimäki (2002). Our analysis differs from these studies in that information is *costless* to acquire and information is gathered gradually over time.

entrepreneur would only gradually learn about the potential demand in different countries. In addition, the logistics, regulations, costs and cultural differences involved in conducting business in different countries may lead the entrepreneur to value the demand of one country over another. However, a foreign entity interested in buying the entrepreneur's invention is likely to put different values than the entrepreneur on the potential demand in different countries. This may be due to greater expertise of the potential buyer in these markets, or because the potential buyer faces less regulations, or because his corporate culture is closer to the corporate culture in these countries. If the potential buyer were to be approached by the entrepreneur with a sales offer, it would try to infer from the timing of the offer, how much the entrepreneur has learned about the global potential of its invention. The longer the entrepreneur held on to its invention, the more likely it is that he researched the potential demand for it.

This paper studies how the scheduling of information acquisition by a seller affects the extent to which gains from trade can be realized. We first show that the equilibrium sequencing of inspection depends *only* on the seller's weights: attributes are inspected in *decreasing* order of importance to him. Intuitively, the more important an attribute is, the more informative it is to learn its value. This allows us to establish necessary and sufficient conditions for the existence of ex-post efficient equilibria where *all* gains from trade are realized. Roughly speaking, these conditions state that there is some integer $k \leq n$ such that for each of the k most valuable attributes to the seller, the seller's value is sufficiently above that of the buyer's. Interestingly, when an ex-post efficient equilibrium exists it is the unique equilibrium. When it does not exist, then some gains from trade are realized in equilibrium only if the total weight that the seller gives to his k' least important attributes is lower than that of the buyer's for some k'. Conversely, if this condition is met, then some gains from trade are realized in any equilibrium.

When the buyer cannot observe which inspections the seller conducts the seller uses the timing of sale offers to signal this information. Even though in equilibrium the buyer infers what attributes the seller inspects, the fact that this information is not directly observed induces the seller to consider only his own valuation when deciding which attributes to inspect. We also study the case where the buyer observes which inspections are performed by the seller but does not observe the outcome of these inspections. In this case, the seller's incentive to get the most informative signal about his own value is mitigated by the incentive to affect the buyer's beliefs. Hence, in equilibrium the seller strategically schedules his inspections by considering the difference between his and the buyer's valuation of each attribute. Consequently, *some* gains from trade are always realized in equilibrium. *All* gains from trade are realized if and only if the seller's value is sufficiently above the buyer's for each attribute that is more valuable to the seller (specifically, the seller's excess valuation must exceed the aggregate excess valuation of the buyer over all attributes that the buyer values more). Observable inspection therefore enlarge the set of parameters for which ex-post efficiency can be attained. The rough intuition for this is that the seller schedules his inspections in a way that mitigates the adverse selection problem: When the buyer is offered to trade, he infers that this offer signals relatively more bad news to the seller than to the buyer.

The majority of the literature on bilateral trade considers buyers, who when offered a price, ask themselves "why did the seller make this particular price offer?". Our contribution is to add a time dimension, which leads the buyer to also ask, "why is the buyer making me an offer *now*?". If the buyer takes into account how long a good has been with the seller before it was put on sale, then the timing of a price offer may convey information on the amount and type of private information that the seller has. Hence, the seller can try and affect the information that the buyer gleans from the timing of offers by strategically choosing *what* to learn about its good and *when* to learn it. This can help mitigate the adverse selection problem that is inherent in trade under asymmetric information.

The paper is organized as follows. The next section discusses the related literature. Section 3 presents the basic model when inspections are unobservable, and Section 4 presents the analysis of this model. Section 5 studies the case of observable inspections. Concluding remarks are presented in Section 6. All proofs are in the text.

2 Related literature

The idea of strategic scheduling of learning was studied in Frug (2016) in the context of cheap talk. In that paper a sender, who is initially uninformed about the state of nature, has to decide each period which potential state to inspect and what cheap talk message to send. The author shows that full communication can be attained in equilibrium if the sender inspects states in opposite direction of his bias (e.g., if the sender's bias is towards high states, he inspects states from high to low).

The decision problem of which attributes to inspect when goods have several attributes was also analyzed in Klabjan, Olszewski and Wolinsky (2014). They consider the following single decision-maker problem. There is an object with several attributes, where each attribute's value is drawn from some distribution, and the decision-maker's payoff is a weighted average of the attributes' values. The decision-maker has to choose between the object and some fixed outside option. The decision-maker is initially uninformed about the attributes' values, but he can inspect each attribute at some cost. The authors consider the case in which the decision-maker can commit to inspect some set of attributes, and the no-commitment case. They show that when attributes have the same costs and are ranked by second-order stochastic dominance, no uninspected attribute will be dominated by an inspected attribute. The no-commitment case is analyzed for two special cases: binary distributions and equal costs and two independent and symmetric distributions with reservation utility equal to the expected value of the object. In the former case, the optimal order of inspection is decreasing in order of dominance and the latter case is in decreasing order of indeces that depend on the distribution and the cost.

The major difference between the above paper and our own is that we consider a *two-player game*, where the seller's inspection decisions affect the buyer's beliefs, and hence the equilibrium price. This is precisely our innovation, to introduce gradual attainment of information into a strategic setting, where the sequencing of learning has strategic implications. In particular, a key feature of our model (which is absent in Klabjan et. al. (2014)) is the potential presence of adverse selection which the seller tries to influence through his inspection strategy. Thus, the efficient composition of the inspection set will be determined so as to mitigate the adverse selection problem, and will depend on both the seller's and buyer's valuations. While there are no direct costs of inspection in our model there are indirect costs through the effect of inspection on the continuation payoff, which is determined endogenously in equilibrium.

In our paper we show that in the no-commitment case, any strategy that does not inspect the most important attributes to the seller before a price offer is dominated by a strategy that inspects attributes in decreasing order of important to the seller. To show this, we fix the strategy of the buyer and consider the single decision-maker problem in which at every period t the seller decides between choosing the object or accepting period t's outside option. This is reminiscent of the result on decreasing order of dominance in Klabjan et. al. (2014), though the frameworks and methods of proof are very different.

Our model has the features that a buyer tries to infer the seller's private information from the timing of a price offer, and the seller attempts to manipulate these inferences by deciding what to learn and when to make an offer. Similar features appear in Taylor (1999) and Kaya and Kim (2015), who study a lemons market where a seller sequentially meets buyers, each of whom observes some noisy signal about the quality of the good. The authors are interested in the question, how are the buyers' beliefs affected by the length of time in which a good was on the market but not sold? Taylor (1999) studies a two-period model of two-sided asymmetric information and compares the trading outcomes under different assumptions regarding the observability of past events. In general, buyers become more pessimistic about the quality of the asset over time. In Kaya and Kim (2015), however, the buyers may become more pessimistic or more optimistic over time, depending on the prior beliefs. On the one hand, a longer duration may signal poor quality since no one was willing to buy the good. On the other hand, a longer duration may actually signal high quality since the seller was unwilling to compromise on the price.[3]

In contrast to these papers, the seller in our model strategically decides when to learn (if at all) each piece of information. Since he meets only one buyer, the buyer's beliefs about the quality of the asset are based only on the information that the seller may have gathered during that period, and which persuaded him to put the asset for sale. Similar to Kaya and Kim (2015) a longer duration can in principle mean both positive and negative news. On the one hand, if the seller waited a long time, then he has accumulated more private information - including information which is more valuable to the buyer. If this information persuaded him to sell, then it should mean bad news for the buyer. On the other hand, if the seller did not sell at an earlier period it means that the information he gathered early on was probably good. In equilibrium the price asked at a late period must be lower than the price asked at an earlier period since otherwise, the seller would wait with the offer. This means that in equilibria where trade can occur in multiple periods, the buyers' beliefs become more pessimistic as time goes by (since equilibrium prices are less than or equal to the buyer's willingness-to-pay). Furthermore, when the buyer observes the seller's inspections (the case analyzed in Section 5), his beliefs about the quality of the asset is affected by both the price offer and by the actual learning schedule.

Our paper touches on the question of whether less private information can allow parties to realize more gains from trade. In the context of a lemons market Levin (2001) showed that more asymmetric information can either increase or decrease the amount of trade. We do not model a lemons market since there are no strict gains from trade ex-ante. Nevertheless, as in Levin (2001) more asymmetry in information does not necessarily hinder trade. In particular, while the buyer and seller are symmetri-

[3]Both Kaya and Kim (2015) and our own paper deal with dynamic adverse selection. By now there is a huge literature that studies different aspects of this topic, many of which are not directly related to our model. We therefore discuss only those papers that are closest to our own.

cally uninformed, all gains from trade can be realized when the seller gathers private information. In our model, in contrast to Levin (2001), both the nature and the extent of informational asymmetry develops gradually and endogenously.

A central theme of our work is that it is important not only what the seller *learns* but also when he learns each piece of information. A related theme, studied in Guttman, Kremer and Skrzypacz (2014), is that it is important not only what an agent *discloses* but also when he discloses it. They study a two period model where each period an agent may observe up to two signals of the quality of his asset and has to decide whether to reveal any of the signals he has observed so far and not disclosed yet. His decision affects each period's market price, which equals the expected value of the asset, conditional on the history. The authors show that the price at the end of the second period given disclosure of one signal is higher if the signal is disclosed later in the game. As their paper and ours analyze completely different frameworks, the only commonality is that both papers highlight how an informed agent can affect the inferences of an uninformed agent by strategically choosing the timing of an action that conveys information (in our case the action is a price offer, while in their case it is disclosure of hard information).

3 A model

A given asset is characterized by n attributes, $X_1, ..., X_n$. An attribute X_i is a binary random variable that gets the value $x_i = 1$ (the asset *has* the attribute X_i) with probability θ and value $x_i = 0$ (the asset does *not* have the attribute X_i) with probability $1-\theta$ (a capital letter denotes a random variable while a small case letter denotes a realized value). Let $\Omega = \{0,1\}^n$ be the set of all possible profiles of realizations of the n attributes. The random variables $X_1, ..., X_n$ are *i.i.d*, and a profile of realizations $x = (x_1, \ldots, x_n)$ is referred to as a state of nature. We denote by x_{-k} the profile of realizations of all attributes other than X_k.

There is a seller and a buyer whose valuations for the asset are given by the weighted sums $V_S(x) = \sum_{i=1}^n \alpha_i x_i$ and $V_B(x) = \sum_{i=1}^n \beta_i x_i$, respectively, where $\alpha_i, \beta_i > 0$ for each i. We denote the players' preferences by $\alpha \equiv (\alpha_1, \ldots, \alpha_n)$ and $\beta \equiv (\beta_1, \ldots, \beta_n)$ and normalize $\sum_{i=1}^n \alpha_i = 1$. Hence, there are ex-ante strict gains from trade if and only if $\sum_{i=1}^n \beta_i > 1$. We assume that there exist a pair of states, x and x' such that $V_S(x) > V_B(x)$ and $V_B(x') > V_S(x')$. Thus, the ex-post efficient allocation of the asset is state-dependent.[4] We further assume that the players have clear rankings over the

[4] In contrast to a pure lemons problem where the buyer values the asset more than the seller in

importance of the different attributes, and $\alpha_1 > \cdots > \alpha_n$. We will therefore refer to the i-th most valuable attribute, from the seller's perspective, as either attribute i or X_i.

The buyer does not observe the realized values of the attributes before purchasing the asset. The seller is initially uninformed about these values but can learn them over time in the following manner. At the end of each period $t \leq n$ the seller decides whether to make a take-it-or-leave-it price offer to the buyer, or to inspect one of the attributes that have not been previously inspected. Thus, if each period the seller inspects an attribute, then at the end of period t he would make a decision knowing the realizations of t attributes (at the end of period 0 the seller makes a decision without knowing any realization). If a price offer is made, then the game ends with trade if the buyer accepts the offer, and it ends with no trade if the buyer rejects the offer. If the seller inspects an attribute, then he observes the realized value of that attribute. The buyer observes neither the attributes that were inspected nor their realizations. If no price offer is made at the end of n periods, then the game ends with no trade.

There is no cost to inspecting attributes, hence the players' payoffs depend only on the realized state of nature and the terms of trade (if any). If the game ends with trade, then the seller receives the price, while the buyer receives his value of the asset minus the price paid. If the game ends with no trade, then the seller receives his value of the asset while the buyer receives a payoff of zero. Both players do not discount future payoffs. The distribution over Ω, the preferences α and β, and the seller's learning technology are commonly known.

A strategy for the buyer specifies for each pair consisting of a time period $t \leq n$ and a price p_t, a decision of whether to accept or reject the offer. In order for the seller to have a best response to the buyer's strategy we restrict attention to buyer's strategies with the property that for every period there is a maximal price that the buyer is willing to accept. We identify any buyer's strategy with the vector of maximal prices accepted by the buyer, $\overline{p} = (\bar{p}_0, \bar{p}_1, ..., \bar{p}_n)$.

A nonterminal (seller's private) history h_t is given by a sequence of attributes inspected before period t, and the values of these attributes $(j_1, ..., j_{t-1}; x_{j_1}, ..., x_{j_{t-1}})$. Let H be the set of all nonterminal histories. A strategy for the seller is a pair of functions $(o(h), f(h))$, where $o(h)$ is an *inspection function* that assigns to every history $h \in H$ an attribute that was not inspected in h and $f(h)$ is a *trading function* that assigns to every history h either a price offer, which is a non-negative number, or no price offer, which is denoted $\varnothing$. Note that inspection and trading functions are defined

every state of nature.

independently of each other.

A history $h = (j_1, ..., j_i; x_{j_1}, ..., x_{j_i})$ is *consistent* with o if $o(j_1, ..., j_t; x_{j_1}, ..., x_{j_t}) = j_{t+1}$ for all $t < i$. Note that given any inspection function $\hat{o}$, and an index $t \in \{0, 1, .., n\}$, every sequence of t realizations $y \in \{0,1\}^t$, uniquely determines an order of inspection $\hat{o}_y$ until time t, and thus, determines a unique history $h = (\hat{o}_y; y)$ that is consistent with $\hat{o}$. We call the restriction of the inspection function o to histories that are consistent with o, an *inspection plan.*

Fix an inspection function o and a strategy for the buyer $\overline{p}$. Let $f(o, \overline{p})$ be an *optimal trading function* given o and $\overline{p}$.

Definition 1 *An inspection function o is said to be weakly dominated if there exists an inspection function o' such that for every strategy of the buyer $\overline{p}$, the seller's expected payoff from $(o', f(o', \overline{p}))$ is at least as high as the expected payoff from $(o, f(o, \overline{p}))$, and for at least one buyer's strategy it is strictly higher.*

The solution concept we employ is pure-strategy perfect Bayesian equilibrium (PBE) in which the seller uses an undominated inspection function, and at every history the buyer believes that seller is using an undominated inspection function. Henceforth, whenever we write "equilibrium" we mean this refinement of pure-strategy PBE.

The buyer's and seller's strategies induce an assignment of trade outcomes ("trade" or "no trade") to each possible state $x \in \Omega$. We refer to this assignment as a trade-outcome of the game. An equilibrium is *efficient* if it maximizes the ex-ante expected surplus among all equilibria. An equilibrium is *ex-post efficient* if at every state of nature the trade-outcome of the equilibrium allocates the asset to the player with the highest valuation.

Discussion. The focus of this paper is on the *strategic* effect of gradual information acquisition. Hence, to isolate this effect we assume that inspection is costless. Adding costs of information would only confound the strategic motivation of acquiring information with its direct effect on payoffs.

Our assumption that at most one price offer is made keeps the analysis tractable by reducing the possible histories on which a player can make his action contingent. Without this assumption, a player's action could depend on previous price offers, and in constructing equilibrium strategies one would need to specify what each player would do after every possible history of rejected price offers. This complicated the analysis without adding substance.

Our decision to focus on PBE rather than on sequential equilibrium simplifies the construction of equilibria since there are many off equilibrium histories (which include both deviations from the equilibrium order of inspection and from the histories where price offers are supposed to be made) for which one has to specify a perturbation in order to satisfy the consistency requirement. More importantly, the notion of sequential equilibrium does not impose more discipline on out-of-equilibrium beliefs relative to the structural restrictions that we impose on beliefs.

4 Analysis

This section explores the implications of strategic gradual learning by the seller. To get some intuition for the model suppose there are only two attributes and that $\beta_1+\beta_2=1$. Using the notation defined above, X_1 is the most important attribute to the seller (thus, $\alpha_1>\frac{1}{2}$). If $\alpha_1\leq\beta_1$ gains from trade cannot be realized in equilibrium (as is the case when the seller is fully informed). On the other hand, if $\alpha_1>\beta_1$, there exists an ex-post efficient equilibrium and, moreover, this is the unique equilibrium outcome of the game. In equilibrium, the seller inspects X_1 at $t=1$, and makes a price offer at the end of period 1 if and only if $x_1=0$. No price offer is made at the end of period 2.

The key feature of this equilibrium is the seller's strategic use of time. If a price offer is made, it takes place at the end of period 1. By the time the good is offered for sale, the seller could not have inspected both attributes, and from the ex-ante perspective, it is in his best interest to inspect X_1 first. Thus, the early timing of the price offer enables the seller to "commit" not to condition his trade decision on the realization of X_2. Notice that ex-post efficiency is reached in equilibrium independently of the value of θ. In contrast, if θ is sufficiently low (a high likelihood that the asset is "a lemon"), it would be impossible to realize any gains from trade if the seller is fully informed at the outset. Our aim in this section is to understand how these insights extend to the multiple attribute case.

4.1 The seller's equilibrium strategy

In this subsection we identify key properties of the seller's equilibrium strategy and explore their implications. We begin by characterizing the unique undominated inspection function.

Let o^α be an inspection function that assigns each non-terminal history h the attribute with the highest α_i among those attributes that were not inspected in h, i.e.,

$$o^\alpha(j_1,...,j_i;x_{j_1},...,x_{j_i})=\min\{k:k\in\{1,...,n\}\backslash\{j_i,...,j_i\}\},$$

Thus, o^α induces an inspection plan according to which, at period t the seller inspects X_t, independently of the realizations along the inspection process.

Lemma 1 *For any strategy of the buyer $\overline{p}$, the seller's strategy $(o^\alpha, f(o^\alpha, \overline{p}))$ is a best response.*

Proof. Let h^k be a history in which there are k attributes left to be inspected. Let $G(h^k)$ denote the continuation game that follows h^k. Given an inspection function o and a buyer's strategy $\overline{p}$, we denote by $o|_{h^k}$ and $(\bar{p}_{n-k}, ..., \bar{p}_n)$ the inspection function and buyer's strategy restricted to $G(h^k)$.[5] We denote by $f(o^\alpha|_{h^k}, (\bar{p}_{n-k}, ..., \bar{p}_n))$ an optimal trading function in $G(h^k)$, given $(o^\alpha|_{h^k}, (\bar{p}_{n-k}, ..., \bar{p}_n))$. We claim that for any non-terminal history h^k and for any buyer's strategy $(\bar{p}_{n-k}, ..., \bar{p}_n)$, the seller's strategy $(o^\alpha|_{h^k}, f(o^\alpha|_{h^k}, (\bar{p}_{n-k}, ..., \bar{p}_n)))$ is a best response in $G(h^k)$. This will prove the lemma since $(o^\alpha|_{h^n}, f(o^\alpha|_{h^n}, (\bar{p}_0, \bar{p}_1 ..., \bar{p}_n))) = (o^\alpha, f(o^\alpha, \overline{p}))$.

The proof proceeds by induction on k, the number of remaining attributes. If only one attribute is left, the claim is trivially true. Assume that the claim is proven for some k. Consider some history h^{k+1} where there are $k+1$ attributes left to be inspected. Denote these attributes by $x^1, ..., x^{k+1}$ and assume that they are ordered according the the seller's order of importance (i.e., in a decreasing order of α_i). Let $(\bar{p}_{n-(k+1)}, ..., \bar{p}_n)$ be some sequence of maximal prices that the buyer is willing to accept from now onwards. By the induction hypothesis, after h^{k+1} the seller has a best response to $(\bar{p}_{n-(k+1)}, ..., \bar{p}_n)$ with the property that when there are only k remaining attributes, these are inspected in decreasing order of α_i independently of the realizations of the inspected attributes. Denote this best response by q_1 and let x^j be the first inspected attribute in q_1. If $j = 1$, then the proof is complete. Assume that $j > 1$. Then q_1 induces the following order of inspection:

$$x^j, x^1, x^2, ..., x^{j-1}, x^{j+1}, ..., x^k, x^{k+1}, \tag{1}$$

We now show that there is a best response to $(\bar{p}_{n-(k+1)}, ..., \bar{p}_n)$, which induces an order of inspection that swaps the first two attributes in the inspection order of q_1 :

$$x^1, x^j, x^2, ..., x^{j-1}, x^{j+1}, ..., x^k, x^{k+1}, \tag{2}$$

This will conclude the proof because after x^1 is inspected, only k attributes are left and we can apply the induction hypothesis again.

[5] Note that $\bar{p}_{n-k}$ is the maximal price the buyer is willing to pay if the seller makes a sales offer at the beginning of $G(h^k)$, prior to any inspection.

Denote the trading assignment under q_1 by $g : S \to \{hold, sell\}$, where S is the set of all sequences of $\{0,1\}$ with length of at most $k+1$. If $g(\{0\}) = g(\{1\})$, that is, the (immediate) trading decision after the inspection of x^j is independent of the value of x^j, the seller can sell exactly the same profiles of qualities under both inspection orders, at every period.

Assume that $g(\{0\}) = sell$ and $g(\{1\}) = hold$. Consider the seller's strategy under which he inspects the attributes according to the order (**??**), and his trade decisions are given by g. Denote this strategy by q_2. The probability of trade at each period is the same under q_1 and q_2. Thus, to compare between q_1 and q_2 we need to compare the (expected) value of the good, if it is not traded. Let $Hold(q)$ denote the set of states of nature in which the good is not traded under the strategy q. Note that we can obtain $Hold(q_2)$ from $Hold(q_1)$ by replacing every $x \in Hold(q_1)$ such that $x^j = 1$ and $x^1 = 0$ with $y \notin Hold(q_1)$ such that $y^j = 0$, $y^1 = 1$, and $y^i = x^i$ for all $i \notin \{1, j\}$. Since $\alpha^1 > \alpha^j$, the seller values y more than x. Therefore, $\mathbb{E}[V_S(x|x \in Hold(q_1))] \leq \mathbb{E}[V_S(x|x \in Hold(q_2))]$. ■

Lemma 2 *Let $o \neq o^\alpha$. Then, there exist $\bar{p}$ and h such that the seller's expected payoff after h, given o^α is strictly higher than his payoff given o.*

Proof. Let h be a shortest history where $o^\alpha(h) \neq o(h)$ with the property that for each same-length history h' such that $o^\alpha(h') \neq o(h')$ the seller's expected valuation of the good after h is weakly lower than it is after h'. Let $|h|$ denote the length of h, and let $j = o(h)$ and $k = o^\alpha(h)$. Note that $k < j$. Let $V_S(h, x^k = 0)$ and $V_S(h, x^j = 0)$ be the seller's expected valuations of the good after h, given that $x^k = 0$ or $x^j = 0$, respectively. Note that $V_S(h, x^k = 0) < V_S(h, x^j = 0)$ because $\alpha_k > \alpha_j$. Let $\bar{p}$, be the buyer's strategy where $\bar{p}_{|h|+1} \in (V_S(h, x^k = 0), V_S(h, x^j = 0))$ and $\bar{p}_t = 0$ for all $t \neq |h| + 1$. Note that the seller's expected payoff after h under o^α is strictly higher than his payoff under o as under o the seller would never sell the good while under o^α he sells with positive probability (if after h the seller learns that $x_k = 0$), for a price that is above his expected valuation of the good. ■

Taken together, Lemmas 1 and 2 have the following implication.

Corollary 1 *o^α is the unique undominated inspection function.*

This means that in equilibrium, the seller's inspection function satisfies that after any history, and *regardless of the realizations along that history*, the attribute with the highest α_i among the uninspected attributes is inspected. It follows that along the equilibrium path the attributes are inspected in a decreasing order of α_i.

Definition 2 *A strategy for the seller is said to be* ***closed from below*** *if for every j and for every x_{-j}, if the seller makes a price offer at t when the state is $(x_{-j}, x_j = 1)$, then he makes a price offer at some $t' \leq t$ when the state is $(x_{-j}, x_j = 0)$.*

This property is the dynamic counterpart of adverse-selection type of considerations on the part of the seller in an environment where his information evolves such that new pieces of information arrive *independently* of the information acquired in the past. In the present model, where learning is endogenous, this corresponds to cases where the inspection *plan* can be represented by one sequence of inspections that does not depend on past realizations (for example the inspection plan induced by o^{α}). To illustrate this definition consider the following simple example.

Example 1. Suppose there are only three attributes. Let (o^{α}, f) be a strategy where f is given in the table below.

h	f
$(1;0)$	$\theta\beta_2 + \theta\beta_3$
$(1,2;1,0)$	$\beta_1 + \theta\beta_3$
other	$\varnothing$

The inspection function o^{α} induces the order of inspection X_1, X_2, X_3, regardless of the realizations observed. To see why this strategy is closed from below notice first that for all states (x_1, x_2, x_3) such that $x_1 = 0$, a price offer is made at the same period $(t = 1)$. Now, consider the state $(x_1 = 1, x_2 = 0, x_3 = 1)$ in which there is a price offer at $t = 2$. Note that in the state $(x_1 = 1, x_2 = 0, x_3 = 0)$, the seller also offers the good for sale at $t = 2$, and if the state is $(x_1 = 0, x_2 = 0, x_3 = 1)$, the good has been offered for sale already at $t = 1$. $\square$

Lemma 3 *WLOG the seller's strategy in equilibrium is closed from below.*

Proof. Let e be an equilibrium such that there are t, j and x_{-j} for which $(x_{-j}, x_j = 1)$ is offered for sale at $p_t = \overline{p}_t$ and $(x_{-j}, x_j = 0)$ is not offered for sale at any period $\tau \leq t$. Since the seller's strategy is consistent with an equilibrium, the inspection function must be o^{α}. The seller's trade function must be measurable with respect to the seller's information at each period, therefore, our negation assumption implies that $j \leq t$. It follows that, at period t, the seller's expected value of the good at state $(x_{-j}, x_j = 1)$ is strictly higher than his expected value of the good at state $(x_{-j}, x_j = 0)$. Therefore, if it is optimal for the seller to offer the good for sale given his information at period

t for the price p_t in state $(x_{-j}, x_j = 1)$, he can be better off if he deviates and offers the good for sale at period t for the price p_t given his information at time t in state $(x_{-j}, x_j = 0)$. ■

Note that in an equilibrium where trade occurs with positive probability in multiple periods the prices cannot increase over time. Otherwise, the seller would simply wait to the period where the price is highest. Taken together with Lemma 3, this property of prices allows us to place an upper bound on accepted price offers in equilibrium. This observation will prove useful in the continuation.

Lemma 4 *In any equilibrium with trade, prices are equal to at most* $\theta \sum_{i=1}^{n} \beta_i$.

Proof. Let e be an equilibrium with trade. Let t be the earliest period in which there is trade at a price $p_t > \theta \sum_{i=1}^{n} \beta_i$ under e. Note that there cannot be any trade prior to t since, by the definition of t all earlier prices would be lower and, instead of selling before t, the seller would benefit from waiting until time t.

Denote by S_t the set of states in which the good is traded at time t under e. Because the seller's equilibrium strategy is closed from below, for any $j \in \{1, \ldots, n\}$ and any x_{-j}, if $(x_{-j}, x_j = 1) \in S_t$, then $(x_{-j}, x_j = 0) \in S_t$. This holds for any x_{-j}, and thus, conditional on S_t, the probability that $x_j = 1$ is bounded above by θ. Therefore,

$$\bar{p}_t \leq \mathbb{E}[V_B(x|x \in S_t)] \leq \theta \sum_{i=1}^{n} \beta_i$$

a contradiction. ■

4.2 Realizing gains from trade in equilibrium

Now that we understand how the seller schedules his inspections in equilibrium, we turn to identify conditions under which the buyer and seller realize gains from trade in equilibrium. We first ask whether there are equilibria in which the buyer and seller realize *all* the gains from trade, and if so, under what conditions.

We begin by introducing some useful notations. Let $I^S = \{i : \alpha_i > \beta_i\}$ denote the set of attributes that the seller values strictly more than the buyer. If the seller inspects the attributes according to o^α, his information after t periods is represented by the vector $(x_1, ..., x_t, X_{t+1}, ..., X_n)$. The interpretation is that the seller knows the realizations of $\{X_1, ..., X_t\}$ (hence the lower case letters), but does not know the realizations of $\{X_{t+1}, ..., X_n\}$. We will sometimes summarize the seller's information at the end of time t by the t-prefix of the state of nature, $x(t) = (x_1, ..., x_t)$. To save on notation, we suppress the conditional expectation symbol when there is no risk of confusion (for example, we write $V_S(x(t))$ instead of $\mathbb{E}[V_S(x|x(t))]$). Let ε_i denote the realization of x such that $x_i = 1$ and $x_k = 0$ for all $k \neq i$. We denote by $\mathbf{1}$ and $\mathbf{0}$ the vectors $(1, 1, \ldots, 1)$, and $(0, 0, ..., 0)$ respectively.

Our first main result identifies the necessary and sufficient conditions for an ex-post efficient equilibrium in our environment. Interestingly, it involves only conditions on α and β, so the following result is independent of θ. Intuitively, there is an ex-post efficient equilibrium if and only if the attributes that are *relatively* more important to the seller (than to the buyer) are also the most important attributes to the seller in *absolute* terms, and in addition, the difference between the players' valuations of each such attribute is large enough.

Proposition 1 *There exists an ex-post efficient equilibrium if and only if there exists k such that $I^S = \{1, 2, ..., k\}$ and $\alpha_j - \beta_j \geq \sum_{i>k}(\beta_i - \alpha_i)$ for all $j \leq k$.*

Proof. (*Necessity*). Assume by contradiction that there are indices $i < j$ such that $\alpha_i \leq \beta_i$ and $\alpha_j > \beta_j$. By ex-post efficiency, in state ε_j the good has to be allocated to the seller. Since $j > i$, for every $t \geq i$, $V_S(\varepsilon_j(t)) < V_S((\sum_{i \notin I^S} \varepsilon_i)(t))$. Therefore, if, in an equilibrium, the seller chooses to keep the good at state ε_j, he would not sell the good in state $(\sum_{i \notin I^S} \varepsilon_i)$ in that equilibrium, in contradiction to ex-post efficiency. Thus, a necessary condition for an ex-post efficient equilibrium is that $I^S = \{1, 2, ..., k\}$ for some k. Assume now that $\alpha_j - \beta_j < \sum_{i \notin I^S}(\beta_i - \alpha_i)$ for some $j \in I^S$. In this case, $V_B(\varepsilon_j + \sum_{i \notin I^S} \varepsilon_i) > V_S(\varepsilon_j + \sum_{i \notin I^S} \varepsilon_i)$, and so, by ex-post efficiency, the good has to be allocated to the buyer. On the other hand, ex-post efficiency implies that in state ε_j, the good has to stay with the seller. Since $V_S((\varepsilon_j + \sum_{i \notin I^S} \varepsilon_i)(t)) \geq V_S(\varepsilon_j(t))$ for all t, and the inequality is strict whenever $(\varepsilon_j + \sum_{i \notin I^S} \varepsilon_i)(t) \neq \varepsilon_j(t)$, these two trading decisions are not consistent with an equilibrium. Necessity follows.

(*Sufficiency*). We begin by specifying the buyer's strategy and beliefs. The buyer believes that the seller follows the unique undominated inspection function, and in case of a price offer at time t, he believes that $x_j = 0$ for all $j \leq t$. Hence, the buyer accepts any price offer up to $\theta \sum_{i>t} \beta_i$.

The seller's strategy is as follows. He inspects the attributes according to o^α. After histories consistent with o^α, the seller makes a price offer only at $t = k$ if and only if $x_j = 0$ for all $j \leq k$. Conditional on making a price offer at $t = k$, he asks for $\theta \sum_{i>k} \beta_i$. The remaining off-equilibrium components of the seller's strategy can be completed by backwards induction.

Given the buyer's beliefs, at each t he accepts price offers up to his expected value of the good. Hence, his strategy is sequentially rational.

Consider a seller who learns that $x_j = 0$ for all $j \leq k$ and sells at the end of that period. The seller's expected payoff is the same as if he would inspect all the attributes and sell at a price equal to the buyer's value of the asset - assuming the buyer knows the realized values of all the attributes. By our assumption on α and β, the seller would extract the maximal surplus, conditional on $x_j = 0$ for all $j \leq k$, since the value of the buyer is (weakly) higher than the seller's for every realization of $(X_{k+1}, ..., X_n)$. Suppose instead that, he does not sell given $x_j = 0$ for all $j \leq k$. Then, in every realization of the attributes, either the seller sells at some $t > k$ for a price that is weakly lower than the buyer's expected value at time t (because the buyer believes that all realizations up to and including t are zero), or the seller keeps the good and receives a value, which is no greater than the buyer's value (since $\alpha_j \leq \beta_j$ for $j > k$). It follows that the seller cannot gain by not selling at $t = k$ when he learns that $x_j = 0$ for all $j \leq k$ in the first k periods.

We now show that if the seller follows o^α, he does not want to deviate and offer the good for sale after observing a prefix of zeros of length $l < k$. After such history, he can offer the good for sale and get $\theta \sum_{i>l} \beta_i$. Alternatively, he can wait another period, inspect X_{l+1} and sell the good at the end of next period (or never) if and only if $x_{l+1} = 0$. The seller's payoff from this behavior, given his information is

$$\theta(\alpha_{l+1} + \theta \sum_{i>l+1} \alpha_i) + (1-\theta)\theta \sum_{i>l+1} \beta_i > \theta(\beta_{l+1} + \theta \sum_{i>l+1} \beta_i) + (1-\theta)\theta \sum_{i>l+1} \beta_i = \theta \sum_{i>l} \beta_i$$

where the inequality follows from our assumption that $l < k$, which implies that $\alpha_l > \beta_l$ and $\sum_{i>l+1} \alpha_i > \sum_{i>l+1} \beta_i$.

To conclude, we show that if $x_j = 1$ for some $j \leq k$, the seller does not want to deviate and make a price offer at any $t \geq j$. The seller's expected utility from keeping the good, given the information at time t is at least

$$\alpha_j + \theta \sum_{i>t} \alpha_i > \theta(\alpha_j + \sum_{i>t} \alpha_i) > \theta(\beta_j + \sum_{i>t} \beta_i) > \theta \sum_{i>t} \beta_i$$

where the first inequality follows from $\theta < 1$, the second inequality follows from our assumption that for all $i < k$, $\alpha_i > \beta_i$, and $\alpha_j + \sum_{i>k} \alpha_i > \beta_j + \sum_{i>k} \beta_i$. The expression in RHS is the price the seller can get at period t if he offers the good for sale. Sufficiency follows. ■

Interestingly, when the conditions of Proposition 1 are met, ex-post efficiency is not only consistent with equilibrium but it is also the *unique* equilibrium outcome. To see this, notice that the seller can guarantee himself the payoff he obtained in the above ex-post efficient equilibrium by deviating to the strategy he plays in that equilibrium. Recall that under our equilibrium refinement, the buyer's most pessimistic beliefs given a (on or off-equilibrium) price offer at the end of period t are that $x_i = 0$ for all $i \leq t$. Thus at $t = k$, the buyer will accept any price offer up to $\theta \sum_{i>k} \beta_i$. Since in the equilibrium presented in the proof of Proposition 1 the whole expected surplus goes to the seller, any profile of strategies that generates a lower total surplus in expectation cannot be consistent with an equilibrium.

Another notable implication of Proposition 1 is that when ex-post efficiency is attainable in equilibrium, there is a unique period in which trade can take place. In other words, whenever an equilibrium involves multiple periods with trade on the equilibrium path, the trade-outcome of the equilibrium is not ex-post efficient. The timing of trade, however, is endogenous and it is determined so that the seller has just enough time to inspect all of the attributes he values more than the buyer, and only those attributes.

Since the result given in proposition 1 holds for any value of θ, this stands in sharp contrast to a benchmark where the seller is fully informed. It is easy to see that if θ is sufficiently low, no gains from trade can be realized in the latter case.

While ex-post efficiency can be achieved in equilibrium, it is feasible only under somewhat narrow circumstances. A natural question that arises is, under what conditions does there exist an equilibrium with *non-trivial trade* in the sense that the buyer and seller realize *some* positive gains? The answer to this question turns out to depend on whether there is a "tail" in the seller's ranking of attributes, which is more valuable to the buyer. Fix some $j < n$ and consider the $n - j$ least important attributes to the seller (the "$n - j$ tail"). Define $R_j \equiv \sum_{i>j}(\beta_i - \alpha_i)$ as the buyer's excess valuation of $n - j$ least important attributes to the seller.

Proposition 2 *There exists an equilibrium with non-trivial trade only if $R_j > 0$ for some j.*

Proof. Let e be an equilibrium with non-trivial trade and assume by contradiction that for all $j \in \{0, ..., n-1\}$, $R_j \leq 0$. Let S the set of profiles that are sold in e and denote by $\eta_i = \frac{1}{|S|} \cdot |\{x \in S : x_i = 1\}|$ the proportion of elements $x \in S$ with $x_i = 1$. We now show that if $i < j$, then $\eta_i \leq \eta_j$. Assume by contradiction that there exist $i < j$, such that $\eta_i > \eta_j$. Then, there is $x_{-i,-j}$ such that (1) $(x_{-i,-j}, x_i = 1, x_j = 0) \in S$ and (2) $(x_{-i,-j}, x_i = 0, x_j = 1) \notin S$. Since the seller's strategy is closed from below, from (1) we know that $(x_{-i,-j}, x_i = 0, x_j = 0) \in S$. Moreover, since $(x_{-i,-j}, x_i = 0, x_j = 1) \notin S$, it must be the case that the profile $(x_{-i,-j}, x_i = 0, x_j = 0)$ is offered for sale at the end of period $t \geq j$ (as the realization of X_j is known to the seller when $(x_{-i,-j}, x_i = 0, x_j = 0)$ is offered for sale). Applying closure from below again, we conclude that $(x_{-i,-j}, x_i = 1, x_j = 0)$ is offered for sale at $t \geq j$. Since $\alpha_i > \alpha_j$, trading decisions (1) and (2) cannot be optimal at the same time.

Therefore,

$$\begin{array}{rcc} \mathbb{E}[V_S(x|x \in S) - \mathbb{E}[V_B(x|x \in S)] & = & \sum_{i=1}^{n} (\alpha_i - \beta_i)\eta_i \\ & = & (\alpha_n - \beta_n)\eta_n + \sum_{i=1}^{n-1} (\alpha_i - \beta_i)\eta_i \\ & \geq & \sum_{i=n-1}^{n} (\alpha_i - \beta_i)\eta_{n-1} + \sum_{i=1}^{n-1} (\alpha_i - \beta_i)\eta_i \\ & \vdots & \vdots \\ & \geq & \sum_{i=1}^{n} (\alpha_i - \beta_i)\eta_1 \\ & \geq & 0 \end{array}$$

Where the inequalities follow from our assumption that $R_j \leq 0$ for every $j \leq n - 1$ and from the fact that $\eta_1 \leq \ldots \leq \eta_n$. It follows that $\mathbb{E}[V_S(x|x \in S)] \geq \mathbb{E}[V_B(x|x \in S)]$ implying that the trade under e does not realize any gains from trade, a contradiction. ■

The next proposition shows that if the necessary condition for an equilibrium with non-trivial trade holds, then in any equilibrium the players realize some gains from trade.

Proposition 3 *If $R_j > 0$ for some j, then any equilibrium involves non-trivial trade.*

Proof. Suppose $R_j > 0$ for some j. Assume, by contradiction, that there exists an equilibrium in which no gains from trade are realized. Let j^* be the smallest index such that $R_{j^*} > 0$. Suppose the seller inspects his j^* most important attributes in the first j^* periods and if $x_i = 0$ for all $i \leq j^*$, he offers the asset for sale for a price $\theta \sum_{i>j^*} \beta_i$ (otherwise, he keeps the asset). By our refinement, the buyer would accept a price offer of $\theta \sum_{i>j^*} \beta_i$ at the end of period j^*. By the definition of j^*, the seller would make a positive profit from this sale. Since in the original equilibrium no gains from trade are realized, we have found a profitable deviation, a contradiction. ■

The following example illustrates an equilibrium with non-trivial trade. It has the feature that trade may occur in multiple periods.

Example 2. Let $n = 4$, $\theta = 0.8$ and

i	1	2	3	4
α_i	0.4	0.3	0.2	0.1
β_i	0.1	0.2	0.3	0.4

Note that $\sum_{i=1}^{4} \beta_i = 1$ so that there are no gains from trade ex-ante. The sufficient condition for non-trivial trade is satisfied because $R_1 = 0.3$. There exists an equilibrium with the following properties. The seller inspects the attributes in a decreasing order of α_i. If $x_1 = 0$, then the asset is traded in period 1 for a price of 0.72. If $x_1 = 1$ and $x_2 = 0$, then the asset is traded in period 2 for a price of 0.66.

Notice the strategic use of time in this equilibrium: The same trade-outcome can be achieved by inspecting the first two attributes during the first two periods, and offering the asset for sale only in the second period, if at least one of the inspected attributes equals zero. However, this one-sale strategy is inconsistent with equilibrium. If such an equilibrium existed, the good would be traded for $p_2 = \mathbb{E}[V_B(x|x_1 = 0 \textit{ or } x_2 = 0)] \approx 0.69$. After observing $x_1 = 0$ in period 1, if the seller plays according to the suggested equilibrium, his payoff is p_2 (he will sell for sure at $t = 2$). However, by making an off-equilibrium price offer at the end of period 1, the seller can guarantee himself a payoff of $\mathbb{E}[V_B(x|x_1 = 0)] = 0.72$ (which corresponds to the highest price offer the buyer would accept under most pessimistic beliefs, consistent with our equilibrium refinement). $\square$

After identifying the conditions for an ex-post efficient equilibrium and the conditions under which some gains from trade can be realized in equilibrium, our third objective is to understand the structure of the most efficient equilibria for cases in which ex-post efficiency is not attainable. At present, we are unable to provide a full characterization of such equilibria for an arbitrary number of attributes and all α, β, and θ. As illustrated in Example 2, an equilibrium can have complicated structures even for few attributes. In general, efficient equilibria heavily depend on the parameters of the environment and in particular, they can exhibit many periods with trade or strategic withholding of a (tradable) good. However, we are able to characterize the efficient equilibrium of our model for the case where the difference between the fully informed seller benchmark and our gradual learning environment is probably the most prominent.

We say that an equilibrium exhibits "severe adverse selection" if the seller never sells the good if at least one of the inspected attributes has the high realization. That is, a sales offer in equilibrium means that the seller has the worst signal about the asset's quality. Notice that if the seller is fully informed, then there cannot be an equilibrium with severe adverse selection *and* non-trivial trade. Hence, this type of equilibria is unique to an environment in which the seller can be partially informed.

Lemma 5 *In an equilibrium with severe adverse selection the seller extracts the entire expected surplus, conditional on the seller's information at the time of sale.*

Proof. Suppose there exists an equilibrium with severe adverse selection. By Corollary 1, if the seller offers the asset for sale in period t, then until that period he inspected his t most important attributes. It follows that there exists a unique history $h_t = (1, \ldots, t; 0, \ldots, 0)$ after which trade occurs. Since the buyer's beliefs are consistent, he believes that when a price offer is made at time t, the history h_t occurred with certainty. By sequential rationality, the buyer would accept any price offer up to $\mathbb{E}[V_B(x|h_t)] = \theta \sum_{i>t} \beta_i$. Since, by assumption, trade occurs at t it must be that $R_t \geq 0$. Hence, the best response of the seller is to ask for $\theta \sum_{i>t} \beta_i$ ■

Do equilibria with severe adverse selection exist in our model? If so, are there environments in which these are the only equilibria?

Proposition 4 *Let* $m = \arg\max_i R_i$ *and assume that* $R_m > 0$.

(1) If $R_i \leq 0$ *for all* $i \neq m$*, then for any* $\theta \in (0, 1)$ *there exists an equilibrium with severe adverse selection in which trade can occur only in period* m*.*

(2) There exists $\hat{\theta} > 0$ *such that for all* $\theta < \hat{\theta}$ *there exist only equilibria with adverse selection, and in these equilibria trade can occur only in period* m*.*

Proof. *Proof of (1).* By assumption, $R_i \leq 0$ for all $i \neq m$. Consider the following profile of strategies and beliefs. The seller inspects the attributes according to o^α and makes a price offer of $\theta \sum_{i>m} \beta_i$ at time m if and only if $x_i = 0$ for all $i \leq m$. No price offer is made at any $t \neq m$. At every period t the buyer accepts any price below or equal to $\theta \sum_{i>t} \beta_i$. This is rationalized by the buyer's most pessimistic beliefs.

We now show that the suggested profile of strategies and beliefs constitute an equilibrium. The buyer's beliefs are consistent with our refinement and his actions are optimal given those beliefs. To verify whether the seller has a profitable deviation, suppose he sells the asset at some period $t \neq m$. By assumption, $R_t \leq 0$. It follows that

$$V_S(x(t)) \geq V_S(\mathbf{0}(t)) = \theta \sum_{i>t} \alpha_i \geq \theta \sum_{i>t} \beta_i,$$

where the RHS is the highest price the seller can get by offering the good for sale at period t, implying that the seller is weakly better off not selling the asset at t.

Assume that the seller's information in period m is given by $(0, ..., 0, X_{m+1}, ..., X_n)$. The seller then prefers to sell the good and get $\theta \sum_{i>m} \beta_i$ rather than keep the good and get $\theta \sum_{i>m} \alpha_i$. To see that no other seller type at time m wants to sell note that it suffices to show that a seller of type $\tilde{x}(m) = (0, ..., 0, 1, X_{m+1}, ..., X_n)$ prefers to keep the good. This follows from

$$V_S(\tilde{x}(m)) > V_S(\mathbf{0}(m-1)) = \theta \sum_{i>m-1} \alpha_i \geq \theta \sum_{i>m-1} \beta_i \geq \theta \sum_{i>m} \beta_i.$$

Proof of (2). We first find a threshold $\theta^* > 0$ such that for all $\theta < \theta^*$ any equilibrium satisfies that a price offer is made at time t if and only if $t \leq m$ and $x_i = 0$ for all $i \leq m$. By Lemma 4, if $\theta < \theta^* = \alpha_n / \sum_{i=1}^{n} \beta_i$ then, in equilibrium, the seller is strictly better off keeping the asset whenever one or more attributes have a realization of one. Therefore, it must be a severe adverse-selection equilibrium.

From the definition of m it follows that if $\theta < \theta^*$, then there cannot be an equilibrium in which the asset is sold in period $t > m$. To see why, suppose there is an equilibrium with trade in period $t > m$. Consider the seller in period m after observing that $x_i = 0$ for all $i \leq m$. If he offers the asset for sale now, the buyer would accept a price of $\theta \sum_{i>m} \beta_i$, while keeping the good and following the equilibrium strategy yields an expected payoff of at most $\max_t [\theta \sum_{i>m} \alpha_i + (1-\theta)^{t-m} \theta R_t]$. Since $R_m > R_t$, the seller strictly prefers to deviate and sell at time m for any $\theta > 0$.

We now show that for sufficiently low values of θ there does not exist an equilibrium in which the asset is sold at $t < m$. Since $R_m > R_t$ for all $t < m$ there exists $\theta^{**} \in (0, \theta^*)$ such that, for any $\theta < \theta^{**}$, and for all $t < m$, $(1-\theta)^{m-t} R_m > R_t$. Assume $\theta < \theta^{**}$. Then, in any equilibrium with non-trivial trade before period m, trade occurs after a unique history in which $x_i = 0$ for all $i \leq t$ for some $t < m$. Suppose the seller deviates from the proposed equilibrium by keeping the asset in period t, continuing to inspect attributes in decreasing order of α_i and offering the asset for sale in period m if and only if $x_i = 0$ for all $i \leq m$. Our equilibrium refinement implies that the buyer would accept an off-equilibrium price offer of $\theta \sum_{i>m} \beta_i$ in period m. Since $\theta < \theta^{**}$ the seller would strictly gain from this deviation, a contradiction.

Consider some $\theta < \theta^{**}$. From the previous paragraphs it follows that if there exists an equilibrium, then it must have the property that trade occurs after a unique history in which $x_i = 0$ for all $i \leq m$, and the price paid is $\theta \sum_{i>m} \beta_i$. To verify that such an equilibrium exists note that the pair of assesments from the proof of (1) constitutes an equilibrium. ■

To illustrate part (1) of Proposition 4 let $\alpha = (0.5, 0.3, 0.2)$ and $\beta = (0.3, 0.6, 0.1)$. Then $R_1 = 0.2$ while $R_i \leq 0$ for all $i \neq 1$. Hence, for any $\theta \in (0, 1)$ there is an equilibrium with the following properties: trade occurs only in period 1, it occurs if and only if $x_1 = 0$, and the price is $(0.7)\theta$. Note that if $\beta = (0.6, 0.3, 0.1)$, then by proposition 2, there is no trade in equilibrium.

To illustrate part (2) consider Example 2. There is an equilibrium in which trade can occur either in period 1 if $x_1 = 0$, or in period 2 if $x_1 = 1$ and $x_2 = 0$. Indeed, condition (1) of Proposition 4 is violated: $R_i > 0$ for all $i > 1$. However, if $\theta < 0.1$, then all the equilibria will have the following properties: trade occurs only in period 2, it occurs if and only if $x_1 = x_2 = 0$, and the sale price is $(0.7)\theta$.

5 Observable inspections

Up until now we assumed that the inspection process is unobservable. This meant that the seller can only use time to signal the amount and *type* of information acquired before a price offer. It turns out that the dominant inspection plan completely ignores the buyer's valuations and inspects attributes only according to the seller's order of importance.

There are many cases, however, where the process of information acquisition is observable. For instance, recall our entrepreneur example from the Introduction. Suppose that in order to estimate the demand in a given country, the firm has to send specialists to that country to conduct surveys. Even though the potential buyer will not observe the estimates collected by the firm, it may nevertheless be able to observe which markets were inspected.

In this section we explore the effects of strategic gradual learning when the buyer observes which attributes are inspected, but not their realizations. Intuitively, when the inspection is observable, the seller has an additional, more direct instrument to signal what information is available to him. The following example illustrates the seller's strategic gradual learning when it is observable.

Example 3. Let $n = 4$, $\theta = 0.9$ and

i	1	2	3	4
α_i	0.50	0.25	0.13	0.12
β_i	0.71	0.14	0.08	0.07
$\alpha_i - \beta_i$	−0.21	0.11	0.05	0.05

Assume first that the buyer does not observe the seller's inspections. Then by Proposition 2, there is no trade in equilibrium.

Assume next that the buyer does observe the seller's inspections. The buyer's beliefs about the inspected attributes are therefore pinned down, and hence, our equilibrium refinement has no bite. It can then be shown that there exists an equilibrium with the following properties. The seller begins by inspecting X_4, his *least important* attribute. If $x_4 = 0$, he offers the asset for sale at a price equal to $V_B(x|x_4 = 0) = \theta(\beta_1+\beta_2+\beta_3) = 0.837$. If $x_4 = 1$, he continues by inspecting X_3. If $x_3 = 0$, he offers the asset for sale at a price equal to $V_B(x|x_4 = 1, x_3 = 0) = \beta_4 + \theta(\beta_1 + \beta_2) = 0.835$. If $x_4 = x_3 = 1$, the seller then inspects X_2 and offers the asset for sale at a price $V_B(x|x_4 = x_3 = 1, x_2 = 0) = \beta_4 + \beta_1 + \theta\beta_1 = 0.789$ if $x_2 = 0$. If $x_4 = x_3 = x_2 = 1$ the seller keeps the asset.

If the seller follows the above inspection plan, then the buyer's beliefs about the seller's type are consistent with the seller's strategy and thus, the buyer accepts the above price offers. After any other history, the buyer believes that all inspected attributes had a realized value of zero. □

Example 3 illustrates that if the buyer observes the seller's inspections, then instead of inspecting his most important attributes, the seller strategically schedules his inspections to persuade the buyer that sale offers convey more bad news for the seller than to the buyer. This requires the seller to take both the buyer's and his own valuations into account when scheduling his inspections. We are interested in understanding the implications of observing which inspections are carried out (but not their results) on the players' ability to realize gains from trade.

Recall that when inspections are unobserable, ex-post efficient equilibrium exists only if the seller's most important attributes are the ones that he values more than the buyer, and the difference between the players' valuations of each such attribute is sufficiently large. The next result establishes that when inspections are observable, ex-post efficiency can be achieved in equilibrium for a larger set of parameters. Intuitively, now the seller can credibly focus his inspection on any subset of attributes. Therefore, it is no longer required that the most important attributes to the seller would be exactly the attributes he values more than the buyer.

Proposition 5 *There exists an ex-post efficient equilibrium if and only if* $\alpha_j - \beta_j \geq \sum_{i \notin I^S}(\beta_i - \alpha_i)$ *for all* $j \in I^S$.

Proof. *(Necessity).* Suppose there exists an ex-post efficient equilibrium. Assume, by contradiction, that there is a $j \in I^S$ such that $\alpha_j - \beta_j < \sum_{i \notin I^S}(\beta_i - \alpha_i)$. Ex-post efficiency implies that in state ε_j the good is allocated to the seller. Note that for any history of inspections h, $V_S(\varepsilon_j|h) \leq V_S((\varepsilon_j + \sum_{i \notin I^S} \varepsilon_i)|h)$. Moreover, if given h, the seller can distinguish between ε_j and $(\varepsilon_j + \sum_{i \notin I^S} \varepsilon_i)$, this inequality is strict. This means that regardless of the seller's inspection order, if he finds it optimal to keep the good at state ε_j, then he also prefers to keep the good at state $\varepsilon_j + \sum_{i \notin I^S} \varepsilon_i$. However, our negation assumption implies that $V_S(\varepsilon_j + \sum_{i \notin I^S} \varepsilon_i) < V_B(\varepsilon_j + \sum_{i \notin I^S} \varepsilon_i)$ in contradiction to the premise that an ex-post efficient equilibrium exists.

(Sufficiency). We construct an ex-post efficient equilibrium. We begin with the buyer's strategy and beliefs. Whenever a price offer is made, the buyer believes that all inspected attributes had a realized value of zero. Thus, whenever a price offer is made, the buyer is willing to accept any price offer up to $\theta \sum_{i \notin INS} \beta_i$, where INS denotes the set of inspected attributes.

The seller's plan of behavior is to inspect the attributes in a decreasing order of $(\alpha_i - \beta_i)$, make a price offer at the end of time $t = |I^S|$ if and only if all of the attributes inspected equal zero, and not to offer the good for sale at any $t \neq |I^S|$. To obtain a full description of the seller's strategy, the plan of behavior should be completed with a description of the seller's actions after off-equilibrium histories. This can be done by backwards induction, given the buyer's strategy.

Given the seller's strategy, the buyer's strategy is sequentially rational. We now show that the seller does not have a profitable deviation given the buyer's strategy.

First, we show that after observing a realization of zeros for all x_j, $j \in I^S$, the seller prefers to sell the good immediately (for a price equal to the buyer's expected valuation). By doing so, the seller obtains a payoff equal to what he would get if he would inspect all the attributes and sell at a price equal to the buyer's value of the asset - assuming the buyer knows the realized values of all the attributes. By our assumptions on α and β, the seller would extract the maximal surplus, conditional on $x_j = 0$ for all $j \in I^S$, since the value of the buyer is (weakly) higher than the seller's for every realization of attributes not in I^S. Suppose instead that the seller does not sell given $x_j = 0$ for all $j \in I^S$. Then in every realization of the attributes, he either sells at some $t > \left|I^S\right|$ for a price that is weakly lower than the buyer's expected value at time t (because the buyer believes that all realizations up to and including t are zero), or the seller keeps the good and receives a value, which is no greater than the buyer's value (since $\alpha_j \leq \beta_j$ *for* $j \notin I^S$). It follows that the seller cannot gain by not selling at $t = \left|I^S\right|$ when he learns that $x_j = 0$ for all $j \in I^S$.

We now show that for any $t < \left|I^S\right|$, the seller does not want to sell the good in case he observes only realizations of zeros up to t. Assume by contradiction that there exists a period $t < \left|I^S\right|$ such that after observing a sequence of zeros up to that period, the seller prefers to deviate and offer the good for sale. Denote by INS_t the set of inspected attributes at period t. The seller's payoff from keeping the good at the end of period t is at least as high as his payoff from the following plan: Inspect the attribute X_j, $j = \arg\max_{i \notin INS_t}(\alpha_i - \beta_i)$, offer the good for sale at $t+1$ if $x_j = 0$, and keep the good if $x_j = 1$. The seller's expected payoff from this plan, given his information at the end of time t, is

$$(1-\theta)(\theta \sum_{i \notin INS_t} \beta_i - \theta\beta_j) + \theta(\alpha_j + \theta \sum_{i \notin INS_t} \alpha_i - \theta\alpha_j) >$$

$$> (1-\theta)(\theta \sum_{i \notin INS_t} \beta_i - \theta\beta_j) + \theta(\beta_j + \theta \sum_{i \notin INS_t} \beta_i - \theta\beta_j) =$$

$$= \theta \sum_{i \notin INS_t} \beta_i,$$

where the inequality follows from our assumption that $t < \left|I^S\right|$, which, in turn, implies that $\alpha_j > \beta_j$, and $\alpha_j - \beta_j \geq \sum_{i \notin I^S}(\beta_i - \alpha_i)$. The expression on the RHS corresponds to the seller's payoff from selling the good at the end of period t, and the inequality shows that he prefers *not* to sell at the end of that period.

To conclude, we note that the seller never wants to sell the good after observing $x_j = 1$ for some $j \in I^S$. To see this, let j be the first attribute such that $x_j = 1$, and consider the seller's payoff from deviating to a plan according to which he continues inspection (according to some inspection order) and offers the good for sale after some history of realizations. Denote by INS the set of attributes inspected when the seller makes a price offer. The seller's payoff from selling the good after this history is $\theta \sum_{i \notin INS} \beta_i$. However, his payoff from keeping the good after this history is at least $\alpha_j + \theta \sum_{i \notin INS} \alpha_i$, which, by our assumptions on α and β, is greater than $\sum_{i \notin INS} \beta_i$. Sufficiency follows. ■

When ex-post efficiency is unattainable in equilibrium, can parties realize some of the gains from trade?

Proposition 6 *Any equilibrium exhibits non-trivial trade.*

Proof. Assume, by contradiction, that there exists an equilibrium with no trade. By assumption, $I^S \neq \{1, 2, ..., n\}$. Consider the following deviation by the seller. He inspects all the attributes in I^S and offers the good for sale for the price of $\theta \sum_{j \notin I^S} \beta_j$ if and only if $\theta \sum_{j \notin I^S} \beta_j > V_S(x|h)$ where $h = (j_1, ..., j_i; x_{j_1}, ..., x_{j_i})$ such that $I^S =$

$\{j_1, ..., j_i\}$. Note that the seller strictly prefers to sell after the history $h = (j_1, ..., j_i : 0, ..., 0)$. Since the buyer observes the inspections, regardless of his beliefs, he would accept this price offer since his most pessimistic beliefs after observing this sequence of inspections is that all inspected attributes have a realization of zero. We therefore found a profitable deviation for the seller, a contradiction. ■

Proposition 6 stands in contrast with our finding in the previous section that when inspections are unobservable, there is no trade in equilibrium if $R_j \leq 0$ for all j.

We conclude this section by addressing equilibria in which trade can take place in multiple periods. Such equilibria emphasize the dynamic aspect of learning in contrast to static information acquisition. Example 3 that opened this section illustrated such an equilibrium, which had the following features: The seller first inspected the attributes that are more valuable to him than to the buyer, he conducted the inspection in a decreasing order of β_i, and trade could take place at every period until the seller reached an attribute more valuable to the buyer. Our next result shows that these features arise more generally when θ is sufficiently high and the buyer and seller agree ex-ante on the value of the asset.

Proposition 7 *Assume that* $\sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} \beta_i$. *There exists* $\hat{\theta} < 1$ *such that for all* $\theta > \hat{\theta}$, *there exists an equilibrium in which trade can take place at each of the first* $|I^S|$ *periods.*

Proof. Let $\hat{k} = |I^S|$ and denote by $X^1, ..., X^{\hat{k}}$ the β-order of I^S. That is, β^i is increasing in i for $i \leq \hat{k}$. Let $X^{\hat{k}+1}, ..., X^n$ be a fixed (continuation) ordering of attributes not in I^S. Consider the following seller's strategy. He inspects attributes in the order $X^1, ..., X^n$. If $x^1 = 0$, he offers the asset for sale for a price of $p_1(\theta) = \theta \sum_{i=2}^{n} \beta^i$. If $x^k = 0$ in period $k \in \{2, ..., \hat{k}\}$, then he offers the asset for sale at a price $p_k(\theta) = \sum_{i=1}^{k-1} \beta^j + \theta \sum_{i=k+1}^{n} \beta^j$. No price offer is made after period $\hat{k}$. The seller's strategy after histories where the order of inspection is inconsistent with $X^1, ..., X^n$ is set to be optimal given the buyer's beliefs (described below) and can be completed by backwards induction.

Since the inspection in the first $\hat{k}$ periods is in $\beta - \mathit{order}$ of I^S (and all β's are distinct), there exists $\hat{\theta}_1 < 1$ such that for all $\theta \in (\hat{\theta}_1, 1)$, $p_k(\theta)$ is decreasing in k. In what follows, assume that $\theta > \hat{\theta}_1$.

The buyer's beliefs after on-equilibrium inspection sequences are consistent with the seller's strategy (therefore, on-equilibrium price offers are accepted). If an off-equilibrium price offer is made, the buyer believes that all inspected attributes equal zero. This assumption has two implications. First, it ensures that the seller has no incentive to deviate from the inspection order prescribed above. Second, it guarantees that given the order of inspection $X^1, ..., X^n$, the seller would never find it optimal to make a price offer after period $\hat{k}$.

Assume that the seller has just learned that $x^k = 0$ for some $k \leq \hat{k}$. Offering the good for sale immediately yields a payoff of $p_k(\theta)$. We now show that there exists $\hat{\theta}_2 < 1$ such that for all $\theta \in (\hat{\theta}_2, 1)$, the seller is better off selling the good immediately. Note that selling the good immediately is strictly better than any strategy in which the seller will eventually sell the good with certainty since all future prices are below $p_k(\theta)$. Consider strategies in which there are future realizations of the attributes for which the seller will keep the good. In particular, if the seller learns that $x^j = 1$ for all $j > k$, such that $j \in I^S$ he keeps the good. The seller's expected utility at the end of period $\hat{k}$ conditional on $x^j = 1$ for all $j \in I^S$ and $j \neq k$ is $\sum_{i=1}^{k-1} \alpha^i + \sum_{i=k+1}^{\hat{k}} \alpha^i + \theta \sum_{i=\hat{k}+1}^{n} \alpha^i$, and it approaches $\sum_{i=1}^{n} \alpha^i - \alpha^k$ as θ approaches 1. However, $p_k(\theta) = \sum_{i=1}^{k-1} \beta^j + \theta \sum_{i=k+1}^{n} \beta^j \underset{\theta \to 1}{\longrightarrow} \sum_{i=1}^{n} \beta^i - \beta^k$. Since $\sum_{i=1}^{n} \alpha_i = \sum_{i=1}^{n} \beta_i$ and $\beta^k < \alpha^k$, there exist $\hat{\theta}_2 < 1$, such that for all $\theta \in (\hat{\theta}_2, 1)$, the price $p_k(\theta)$ is higher then the seller's expected utility, for any realization of $X^{k+1}, ...X^{\hat{k}}$. Hence, for $\theta > \max\{\hat{\theta}_1, \hat{\theta}_2\}$, upon finding that $x^k = 0$ for some $k \leq \hat{k}$, the seller prefers to sell the good immediately.

To complete the proof, note that after a history of realizations $x^1 = ... = x^k = 1$, for some $k \in \{1, ..., \hat{k}\}$, the seller cannot benefit from offering the good for sale instead of following the continuation of the above suggested strategy. To see this observe that $V_S(x|x^1 = ... = x^k = 1) > \theta$, and recall that $p_k(\theta) < \theta$ for all $k \in \{1, ..., \hat{k}\}$. ■

When there are ex-ante strict gains from trade, then for a sufficiently high θ the seller may sell the asset right away without inspecting any of the attributes. Hence, the insight of Proposition 7 is that even when the buyer and seller have the same ex-ante value for the asset, non-trivial trade can occur in multiple time periods.

6 Concluding remarks

Commitment. The seller in our model has the discretion to decide each period whether to continue inspecting and in which order. This raises the question of whether more gains from trade can be realized if the seller were able to commit to an inspection plan. For example, the seller could commit to an inspection order but keep the discretion of when to stop and make a price offer ("order commitment"). Alternatively, he could commit to inspect a specific set of attributes before making a price offer ("set commitment").

It turns out that these forms of commitment are sometimes equivalent to the case of observable inspections without commitment. For instance, both order and set-commitment induce the same necessary and sufficient condition for ex-post efficiency as in the case of observable inspection without commitment. In addition, under both forms of commitment the set of attributes that are inspected in equilibria with severe adverse selection is the same set that would be inspected under observable inspections without commitment.

Future directions. Although this paper focuses on a bilateral trade environment we think the question of strategic acquisition of information is relevant in many other environments where it has yet to be explored. For example, a job candidate's decision of how to sequence job interviews can have strategic implications. In each interview the candidate gets a signal about his ability and prospective employers may make inferences about a candidate's information about his quality from the timing of the interview (relative to the period in which the candidate was available). Another example is a mechanism-design environment in which agents are not endowed with private information at the outset, but rather collect this information gradually over time. The agents then decide what to learn each period and when to send their report to the planner, who in turn, has to decide how to respond to the content and timing of reports.

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