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Riccardo Calcagno and Florian Heider

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Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

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Riccardo Calcagno - calcagno@em-lyon.com
EMLYON Business School

Florian Heider - florian.heider@ecb.int
European Central Bank and CEPR

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Liquidity, information aggregation, and market-based pay in an efficient market*

Riccardo Calcagno
EMLYON Business School

Florian Heider†
ECB and CEPR

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Abstract

This paper studies the usefulness of making the income of a CEO depend on the stock price of the firm he runs. We assume the stock market is efficient and find that other public information about CEO performance, e.g., accounting information, is not used to determine CEO pay. But because of the feedback loop between CEO actions and the stock price, the price does not fully reflect the consequences of CEO shirking for the value of the firm. The optimal incentive contract increases stock-based pay in order to increase the sensitivity of CEO income to shirking and thus deter it. This effect is stronger when traders have worse information, which can explain the prevalence of stock-based pay in hard-to-value firms. Our model derives a measure of the wedge between financial and economic efficiency, and generates new insights about the role of market conditions such as liquidity for optimal pay contracts.

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†Corresponding author. European Central Bank, Financial Research Division, 60314 Frankfurt, email: florian.heider@ecb.int

1 Introduction

This paper analyzes a new cost when using the stock price to incentivize managers. Stock-based pay is vulnerable to managerial shirking. Even in an efficient market, the stock price does not fully reflect the consequences of shirking for firm value. Yet, making the manager's incentive contract contingent on additional public information about the value of the firm (e.g., accounting information) does not help. The market is efficient and already incorporates this information in the stock price. Instead, the solution is to make the manager's pay more sensitive to movements in the stock price by giving him a lot of stock-based pay.

The issue of how much stock to give to top managers is highly controversial. There is considerable debate whether the level of Chief Executive Officer (CEO) pay is excessive, whether CEO pay is set and structured efficiently, and more generally, what kind of information should be used in CEO incentive contracts.[1] Moreover, the recent financial crisis highlighted weaknesses in the pay structures of bank CEOs, which in turn led to significant regulatory intervention in the United States and in Europe.[2]

Given the controversy and the possibility of having private contract setting curtailed by regulators, it is important to have a thorough understanding of the benefit and cost of stock-based CEO pay. We examine the benefit and cost in a parsimonious model in which the only extra assumption relative to a standard moral-hazard problem and a standard competitive noisy rational-expectations model of the stock market is that CEO pay cannot depend directly on the future value of the firm. CEOs take actions whose consequences can take a long time to realize, but not all CEO pay can be postponed until the distant future.[3]

The benefit of using the stock price as a measure of managerial performance therefore is the aggregation of traders' diverse information about the future value of the firm. The stock

[1] The large literature on these debates is reviewed in Murphy (2013) who focuses on historical and institutional factors, Frydman and Jenter (2010) who focus on the empirical evidence, and Edmans and Gabaix (2015), who focus on the various theories.

[2] See for example, Fahlenbrach and Stulz (2011). For a review of governance issues for banks including compensation, see Mehran and Mollineaux (2012).

[3] Gopalan, Milbourn and Song (2010) view top managers as "visionaries whose main role is to set strategy for the firm". Yet, the weighted average of the vesting periods of the different components of executive pay is less than two years (Gopalan, Milbourn, Song and Thakor, 2014). Similarly, Xu (2013) documents a typical contract length of around three years with a significant portion of "at will" contracts, which can end at any moment in time. If future firm value were contractible in our model, the stock price would never be used in the CEO's pay contract.

price provides advance information about the consequences of CEO actions. One can use the stock price as a communication device to access otherwise unavailable information.[4]

But there is a cost even in an efficient market in which the stock price accurately reflects the future value of the firm (conditional on all publicly available information) because the stock price does not fully reflect the consequences of CEO shirking. The reason is the feedback loop between CEO actions and the stock price. The stock price reflects fundamental factors as well as the consequences of CEO actions. Conversely, the manager understands the consequences of his actions for his pay via the stock price. In equilibrium, stock price formation and CEO actions must be consistent.

Stock price formation undermines the consistency between the stock price and the CEO's equilibrium action by giving him an incentive to shirk. The CEO's equilibrium action determines informed traders' prior belief about the future value of the firm. Trade in the company's stock occurs when traders, who monitor the firm, observe different, noisy signals about the future value of the firm and update their prior beliefs. The company's stock price is set by market markers who react to the order flow from informed (and uninformed) traders.

The CEO can always hide some of the value consequences of shirking behind trader's imprecise information about future firm value. When updating their prior beliefs about firm value, traders think the CEO takes the equilibrium action (even though he may be shirking) and interpret any new information as coming from fundamental factors or from noise in monitoring. As they have no ability to detect shirking directly—a low signal about future firm value can always come from low fundamentals or from noise in monitoring—, this is the only consistent behavior.[5]

The optimal pay contract ensures the absence of CEO shirking in equilibrium and hence establishes the consistency between stock price formation and CEO actions. The owners of the firm hire the CEO and design his pay contract with a full understanding of the feedback between CEO actions and the stock price. Importantly, they understand that the stock price

[4]This benefit is well understood, see for example the survey by Bond, Edmans, and Goldstein (2012). An early exposition of this benefit in the context of CEO pay is Diamond and Verrechia (1982) although there is no explicit modeling of information aggregation and pay depends negatively on the stock price because traders' common signal is independent of CEO actions.

[5]The CEO cannot fully hide behind fundamental factors because shirking makes a low signal about future firm value more likely, holding fundamental factors and noise in monitoring constant.

cannot fully reveal the consequences of CEO shirking for the future value of the firm.

Making CEO pay depend on other public sources of information about firm value (e.g., accounting information) cannot address the CEO's incentive to shirk. Because the information is public, it is available to market participants. In an efficient market, the stock price already incorporates all public information about the value of the firm. Hence, it should also include all public information about the consequences of CEO actions. Indeed, we show that the stock price in an efficient market is a sufficient statistic for future firm value. In line with the Holmström (1979) informativeness principle, the optimal pay contract will not be contingent on other public information.

The only way to address the CEO's incentive to shirk is to give him more stock-based pay. This is our main result. As the problem is the limited sensitivity of the stock price to deviations from the equilibrium action, and because using other public information does not help, the optimal contract must put more weight on the stock price to undo the limited sensitivity.

This insight offers a new take on some empirical regularities. For example, there is a lot of stock-based pay and thus a high sensitivity of pay to stock price movements in hard-to-value firms (i.e., young, high growth firms with a lot of intangible assets). The literature explains this with inefficient stock markets (Bolton, Scheinkman, and Xiong, 2006), stock price manipulation (Peng and Röell, 2014), and inefficient contracting (e.g., CEOs being able to design their own pay contracts as in Bebchuk and Fried, 2004).[6] Even though none of these frictions are present in our model, the CEO's pay contract puts more weight on the stock price if traders have worse information about the value of the firm.

When traders have worse private information, they update their prior beliefs about future firm value less, and consequently trade less aggressively. The price reacts less to trader's information, which gives the CEO more scope to hide the value consequences of shirking. To counter the reduced sensitivity of the stock price to shirking, he receives more stock based pay.

Similarly, the CEO receives more stock-based pay when there is worse public information

[6]In Goldman and Slezak (2006) the possibility to manipulate, however, reduces the extent to which CEO pay depends on the stock price.

about the value of the firm (even though his pay contract does not depend directly on public information) or when there is more uninformed trading of the company stock.

Our parsimonious model can also resolve some empirical puzzles about how market liquidity affects stock-based pay and firm performance. Some find that a more liquid market leads to more stock-based pay and enhances firm performance because of a higher information content of the stock price (Fang, Noe, and Tice, 2009). Others find the same positive link between liquidity and stock-based pay when there is a lower information content in the stock price (Jayaraman and Milbourn, 2011). And yet others find a negative link between liquidity and stock based pay (Kang and Liu, 2010).

Market liquidity, stock price informativeness, and CEO pay are all joint outcome (endogenous) variables. We can replicate the various empirical findings in our model by changing different underlying (exogenous) parameters.

For example, a liquid market is one where trading is possible without much price impact. When trading has little price impact, then the CEO has a strong incentive to shirk (because the stock price is not sensitive to the consequences of shirking on future firm value). Moreover, the market is liquid if there is a lot of uninformed trading because it allows informed traders to hide their trades better. Hence, higher market liquidity can go hand-in-hand with more stock-based pay and less informative stock prices. This is the opposite of Holmström and Tirole (1993) where more liquidity leads to more stock-based pay because the stock price is more informative.[7]

Similar endogeneity concerns may be the reason why it has been difficult to assess the value of disclosing information about the value of the firm to the market (Leuz and Wysocki, 2008). In our model, it does not matter whether the additional signal about firm value is public (e.g., accounting information) or private (e.g., internal performance measures) as long as its information content stays the same. The contract adjusts to make optimal use of either public or private information but overall incentives and hence economic efficiency do not change.

Like us, a number of papers explore the issue of stock-based pay theoretically in models in which the stock price is determined endogenously in the market place (Paul, 1992, Kim

[7]We comment on the differences between their model and ours below, and again in Section 5.

and Suh, 1993, Bushman and Indjejikian, 1993, Holmström and Tirole, 1993, Kang and Liu, 2010). But all these papers face the following puzzle:

> [I]t would seem that estimating the value of the firm and estimating the manager's contribution to firm value are essentially the same thing. Assuming the market is "efficient", one might expect stock price to "correctly" aggregate all of the information signals, *including accounting earnings*. This suggests that the optimal compensation contract should depend solely on the stock price. Since [...] the optimal contract places positive weight on earnings, I found this to be a puzzling result. (Lambert, 1993)

The puzzle is not present in our model. In line with intuition, an efficient stock market aggregates traders' private information as well as public information. The CEO's compensation contract does not depend on the public signal about the value of the firm. The contract only depends on the stock price.[8]

A distinguishing feature of our paper is the explicit modeling of how an efficient stock market aggregates traders' private information as well as public information. For this we add the presence of a public signal to Vives' (1995) competitive noisy rational expectations model with a risk-neutral market making sector. A competitive rational expectations model is well suited to take into account the public-good character of information in the stock price. A risk-neutral market making sector renders the stock market informationally efficient, which is a useful benchmark to study the informational role of stock prices in incentive contracts.[9]

Our model offers one explanation why CEO pay contracts appear simple relative to Holmström's (1979) informativeness principle according to which any informative signal, even if it is very noisy, should be included in the optimal incentive contract (see Edmans and Gabaix, 2015). If a signal is observed by stock market participants and the market is

[8]Taking it as given that the stock price is not a sufficient statistic for firm value, there is a large accounting literature that examines the role of public information (such as accounting information) in incentive contracts (for a survey, see for example Bushman and Smith, 2001).

[9]In earlier noisy rational expectations models such as Hellwig (1980), Grossman and Stiglitz (1980) or Kyle (1989), the stock price is not semi-strong efficient. There, informed traders also engage in market-making, and the stock price contains a risk-premium. For more information see Chapter 4 of Vives (2008). Kovalenko and Vives (2015) show that competitive rational expectations equilibria are a useful approximation of strategic behavior in moderately sized markets precisely when prices are semi-strong efficient.

efficient, then it is sufficient to include only the stock price in the optimal incentive contract (and disregard the signal itself).[10]

Our model also contributes to the question of what is the difference between the financial efficiency of stock markets and the real efficiency of economic decisions based on stock prices because of their informational role. Bond et al. (2012) distinguish between forecasting price efficiency (FPE) and revelatory price efficiency (RPE). In our model, forecasting price efficiency does not imply revelatory price efficiency. Even though the price is efficient and aggregates all relevant information (FPE), it does not fully reveal the consequences of CEO shirking for the future value of the firm (RPE). Yet, the CEO's incentive to shirk determines the design of the optimal incentive contract and hence economic efficiency.

Our model provides a (statistical) measure of the difference between FPE and RPE. The stock price is the best linear estimator of firm value with a prior based on the CEO's equilibrium action. But the optimal performance measure is the best linear estimator of firm value with a *diffuse* prior (i.e., one that does not focus on a particular CEO action). The estimation error of the optimal performance measure determines the wedge between FPE and RPE.

Finally, we examine the benefit and cost of stock-based pay in the context of the feedback loop between the stock price and the future value consequences of past CEO actions. But stock prices also contain valuable information for future CEO actions (e.g., information about the profitability of investment opportunities). Such prospective information also leads to feedback loops, whose implications are explored, for example, in Dow and Gorton (1997), Goldstein and Guembel (2008), Bond and Goldstein (2015), and Edmans, Goldstein, and Jiang (2015).

The paper is organized as follows. Section 2 presents the model. Section 3 analyses the case in which an unbiased signal about the future value of the firm (such as accounting information) is publicly available. Section 4 analyses the case when such a signal only is privately available. Section 5 considers the collection of better information by traders. Section 6 presents the empirical implications of our analysis. Section 7 concludes. All proofs are in Appendix A.

[10] Another avenue is explored in Chaigneau, Edmans, and Gottlieb (2016) who show that an informative signal may receive zero weight in the optimal incentive contract when the contract is constrained by limited liability.

2 The model

There are four dates, $t = 0, 1, 2, 3$ and five types of agents: a firm owner, a firm manager (CEO), informed traders, uninformed market-makers, and noise traders. At $t = 0$, the owner of a publicly traded firm hires the manager and gives him an incentive contract. The manager then chooses an unobservable level of effort that affects the value of the firm. At $t = 1$, the stock market opens where informed traders, uninformed market-makers and noise traders interact. At $t = 2$, the tenure of the manager comes to an end. He is paid and leaves the firm. At $t = 3$, the firm is liquidated and the proceeds are distributed among the shareholders of the firm.

2.1 Agents

The owner of the publicly traded firm is risk-neutral, value-oriented and does not trade in the stock market. She represents a collective of well-diversified, patient shareholders who own the firm until its liquidation. She cares about the value of the firm v at $t = 3$ net of any income I paid to her manager.

The manager is risk-averse with CARA utility U over net income and coefficient of risk-aversion r. Through his choice of effort e at $t = 0$ the manager affects the value of the firm at $t = 3$, $v = e + \theta$, where $\theta \sim N(0, \tau_\theta^{-1})$ represents random factors that are outside his control.[11] The manager's effort e is unobservable and costs him $\frac{k}{2}e^2$ (in money terms). If he does not join the firm at $t = 0$, his utility is zero. The manager is not allowed to participate in the stock market as this would be considered as insider trading.

A mass-one continuum of risk-averse, competitive informed traders buy and sell the firm's shares in the market at $t = 1$ after they observe some unbiased but (possibly very) imprecise information about the future value of the firm v. Specifically, each informed trader $i \in [0, 1]$ privately observes a signal $s_i = v + \varepsilon_i$, where ε_i are i.i.d. with $\varepsilon_i \sim N(0, \tau_\varepsilon^{-1})$. An informed trader i maximizes CARA utility with risk aversion ρ by submitting a demand schedule x_i at $t = 1$ and holding his position until $t = 3$. When an informed trader submits his demand schedule x_i, he uses his own private signal s_i as well as all other public infor-

[11] In the following we denote with $\tau_i = \frac{1}{\sigma_i^2}$ the precision of random variable i with variance σ_i^2.

mation available at $t = 1$. This includes the stock price p. Informed traders have rational expectations and condition their demand on the stock price because it contains information about v derived from the trading decisions of *all* informed traders.

In addition to informed traders, there are uninformed traders who trade for exogenous, random liquidity reasons, e.g., stochastic life cycle motives. We denote their demand by $u \sim N(0, \tau_u^{-1})$, which is independent of all other random variables. As usual, such noise trading is necessary to disguise the actions of the informed traders.

Finally, uninformed, risk-neutral and competitive market makers stand ready to buy (sell) the firm's shares at $t = 1$ when the price is low (high). They observe the aggregate limit order book L resulting from the joint demand schedules of informed traders and the demand of noise traders, $L = \int_0^1 x_i di + u$, as well as any public information available at $t = 1$.

2.2 Information and incentive contract

We assume the manager's tenure ends at $t = 2$ and he receives his income I at that moment. Hence, the manager's income cannot depend on the final value of the firm v, which becomes available only at $t = 3$. This captures the real life problem that the actions of a top manager, especially strategic decisions, affect the value of his firm well into the distant future and not all pay can be postponed until then (especially when the manager leaves the firm before).

There are two sources about firm value that can be used to incentivize the manager: the stock price p, which is the result of trading the firm's shares at $t = 1$, and additional unbiased but noisy information about the final value of the firm, $y = v + \eta$, where $\eta \sim N(0, \tau_\eta^{-1})$ is independent of all other random variables. This additional, exogenous information is a useful benchmark to evaluate the role of the endogenous stock price.

In our benchmark case the additional information y is publicly available when the firm's shares are traded at $t = 1$. In that case, we think of y as accounting information. Informed traders and market makers take publicly available accounting information into account when they make their buy and sell decisions. In Section 4 we study the case in which the additional signal y is not publicly available, but still can be used for contracting (e.g., internal performance measures that are not readily available to outsiders). To capture this,

y now becomes available only at $t = 2$ when the manager leaves and is paid. In that case, informed traders and market makers cannot condition their decisions on the signal y, yet the manager's income can still depend on it.

As usual, we consider incentive contracts that are linear in performance measures. The manager's income I contains a fixed wage W, market-based pay that depends on the stock price Mp and remuneration based on accounting information Ay:

$$I = W + Mp + Ay \tag{1}$$

2.3 Risk-neutral pricing

The presence of a competitive, risk-neutral market making sector ensures that the stock price is efficient:

$$p = E[v - W - Mp - Ay|L, y] \tag{2}$$

The share price at $t = 1$ equals the expectation of the liquidation value of the firm at $t = 3$ minus any payment to the manager when he leaves at $t = 2$, conditional on the order book L and publicly available accounting information y.[12,13]

It will be convenient to work with a version of the stock price that is gross of any payment to the manager:

$$\hat{p} = p + I = W + Ay + (1 + M)p \tag{3}$$

The price $\hat{p}$ is informationally equivalent to p because the contract terms (W, A, M) as well as p and y are known at $t = 1$. Now we can write efficient pricing (2) as

$$\hat{p} = E[v|L, y]. \tag{4}$$

In terms of $\hat{p}$, the manager's income is

[12] We normalize the number of shares in the company stock to one.

[13] In Section 4, the realization of y is not available when the price is formed at $t = 1$.

$$I = w + m\hat{p} + ay, \tag{5}$$

where $w = \frac{W}{1+M}$, $m = \frac{M}{1+M}$ and $a = \frac{A}{1+M}$.

Figure 1 summarizes the sequence of events.

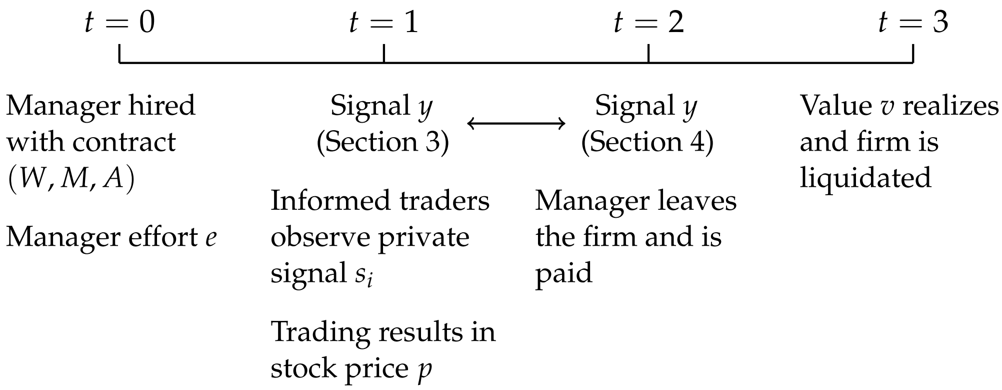

Figure 1: Sequence of events

3 Additional signal is publicly available

In this section, we solve the case in which the realization of the signal y occurs at $t = 1$, i.e., when trading takes place. In this case we can think of y as accounting information. We first state the well-known first best and second best benchmarks. Next, we solve for the stock price taking effort as given. Then we solve for the incentive contract given the stock price. An equilibrium requires consistency between the stock price and the incentive contract. We discuss our results and offer an interpretation in terms of the inference problem for market participants and for firm owners. We close the section with a presentation of comparative statics.

3.1 First best and second best benchmarks

The first best occurs when the manager's effort is observable and contractible. Then the manager receives a fixed wage, exerts effort $e^{FB} = \frac{1}{k}$, and the expected value of the firm at

$t = 0$ is $E[v - I] = \frac{1}{2k}$.

The second best occurs when the manager's effort is not observable but his income can depend on the final value of the firm v so that $I = W + Sv$. Because final value v is available for contracting, the signal y is superfluous. The second-best contract has a variable component S of[14]

$$S = \frac{1}{1 + rk\tau_\theta^{-1}}, \tag{6}$$

induces an effort of

$$e^{SB} = \frac{1}{k(1 + rk\tau_\theta^{-1})}, \tag{7}$$

and leads to an expected value of the firm at $t = 0$ of

$$E[v - I] = \frac{1}{2k(1 + rk\tau_\theta^{-1})}. \tag{8}$$

3.2 Pricing

Our standard CARA-normal model of the stock market admits equilibria that are linear (see Vives, 1995, 2008). Hence, we can write the demand schedule of an informed trader i at $t = 1$ as:

$$x_i(s_i, \hat{p}, y) = \beta s_i + f(\hat{p}, y), \tag{9}$$

where β measures how aggressively a trader uses his private information s_i and the function f is linear in $\hat{p}$ and y.

The joint demand of informed and noise traders gives the following aggregate limit order book:

$$\begin{aligned} L &= \int_0^1 (\beta s_i + f(\hat{p}, y))\, di + u = \beta v + f(\hat{p}, y) + u \\ &= z + f(\hat{p}, y), \end{aligned}$$

where $z = \beta(e + \theta) + u$ denotes the part of the order book that depends on traders' aggregate private information garbled by noise trading.

[14]See for example Prendergast (1999).

Market makers observe the order book L, the price $\hat{p}$, and the public signal y. This information is equivalent to observing the informed part of the order book z and the public signal y.[15] We can therefore write efficient pricing (4) as

$$\hat{p} = E[v|z, y],$$

which we solve in the following proposition:

Proposition 1 *If the signal y is available at $t = 1$, the stock price $\hat{p}$ is given by:*

$$\hat{p} = (1 - \lambda\beta - \gamma)e^* + (\lambda\beta + \gamma)(e + \theta) + \lambda u + \gamma\eta, \tag{10}$$

where e^* is the hypothesized equilibrium effort of the manager, e is the actual effort, $\beta = \frac{\tau_\varepsilon}{\rho}$, $\lambda = \frac{\beta\tau_u}{\beta^2\tau_u + \tau_\theta + \tau_\eta}$, and $\gamma = \frac{\tau_\eta}{\beta^2\tau_u + \tau_\theta + \tau_\eta}$.

The stock price is the weighted average of expected firm value (e^*) and actual firm value ($e + \theta$) plus two noise terms, one caused by noise trading u and the other one caused by noise in accounting information η.

The aggressiveness β with which informed traders base their demand on their private information s_i is the ratio of the precision of that information to their risk aversion.

The coefficient λ is the sensitivity of the price to the order flow $\beta(e + \theta) + u$. Its inverse λ^{-1} is a standard measure of liquidity (see Kyle, 1985). The market is more liquid if market markers worry less about trading against informed traders. Liquidity increases (lower λ) when there is more noise trading (lower τ_u), more precise accounting information (higher τ_η) and firm value is less volatile (higher τ_θ).[16]

The quality of the price $\hat{p}$ as predictor of firm value is given by

$$\tau \equiv Var[v|\hat{p}]^{-1} = \beta^2\tau_u + \tau_\theta + \tau_\eta. \tag{11}$$

[15]The price will be a linear combination of z and y. The order book is a linear combination of z, $\hat{p}$ and y where z can be inferred because the function f is known. Hence, observing L, $\hat{p}$ and y is equivalent to observe z and y.

[16] The comparative statics of liquidity with respect to trading aggressiveness β are ambiguous. A higher trading aggressiveness leads to more liquidity iff $\frac{\tau_\varepsilon}{\rho} > \sqrt{\frac{\tau_\eta + \tau_\theta}{\tau_u}}$.

Stock price informativeness increases when there is more aggressive trading on private information (higher β), less noise trading (higher τ_u), firm value is less volatile (higher τ_θ) and accounting information is more precise (higher τ_η).

The pricing function (10) shows it is important to distinguish between equilibrium effort e^* and actual effort e. The equilibrium effort determines the market's prior expectation of firm value whereas actual effort (including deviations from equilibrium effort, i.e., shirking) drives firm value v and thus determines traders' signals s_i.

3.3 Incentive contracting

The optimal incentive contract for the manager (w, m, a) at $t = 0$ maximizes the expected value of the firm net of the manager's income subject to his incentive compatibility and participation constraints,

$$\max_{w,m,a} E[v - I] \tag{12}$$

$$\text{subject to} \qquad e = \arg\max_{\hat{e}} \; E[I] - \tfrac{r}{2} Var[I] - \tfrac{k}{2}\hat{e}^2 \tag{13}$$

$$\text{and} \qquad E[I] - \tfrac{r}{2} Var[I] - \tfrac{k}{2} e^2 \geq 0, \tag{14}$$

where we have used the certainty equivalent for the CARA utility of the manager.

The next proposition states the solution to the contracting problem (12)-(14):

Proposition 2 *The optimal managerial contract places no weight on publicly available accounting information, $a^* = 0$. The weight on the stock price is*

$$m^* = \frac{1 + \tau_\theta \Sigma}{1 + kr\left(\tau_\theta^{-1} + \Sigma\right)}, \tag{15}$$

where $\Sigma = \left(\beta^2 \tau_u + \tau_\eta\right)^{-1}$.

The optimal contract does not depend directly on accounting information because this information is publicly available at $t = 1$ when trading occurs. As risk-neutral pricing renders the market semi-strong efficient, the accounting information is already incorporated in the stock price. The manager's income therefore depends indirectly on accounting information via the stock price. Making his income also depend directly on accounting information adds no new information but increases the variance of income, which is costly given the risk-aversion of the manager.

We discuss the weights on the stock price and the interpretation of Σ in more detail in the next section.[17] But note that the weight on the stock price m^* reverts to the weight S in the second best (6) when $\Sigma = 0$.

The next proposition derives the implications of the optimal contract for managerial equilibrium effort and expected firm value.

Proposition 3 *The equilibrium level of manager effort is*

$$e^* = \frac{1}{k\left(1 + kr\left(\tau_\theta^{-1} + \Sigma\right)\right)}, \tag{16}$$

and the expected net value of the firm is

$$E[v - I] = \frac{1}{2k\left(1 + kr\left(\tau_\theta^{-1} + \Sigma\right)\right)}. \tag{17}$$

As in the case of the contract weight m^*, the expressions for equilibrium effort and expected net firm value are equal to the corresponding expressions in the second best, (7)-(8), when $\Sigma = 0$.

For completeness, we also state the weight on the stock price prior to normalization, p (the weight on accounting information remains zero, $A^* = 0$):

$$M^* = \frac{m^*}{1 - m^*} = \frac{1 + \tau_\theta \Sigma}{-\tau_\theta \Sigma + kr\left(\tau_\theta^{-1} + \Sigma\right)} \tag{18}$$

3.4 Discussion

As seen in Propositions 2 and 3, Σ plays a key role in our analysis of optimal market-based pay. When $\Sigma = 0$, we are back to the second best (as if final firm value was contractible).

[17] The expression for the fixed wage w is not interesting. It is given by the binding participation constraint.

Hence, Σ is a measure of the inefficiency of market-based pay. To understand Σ, it is useful to write the manager's income under the optimal contract as

$$I = w + \frac{1}{1 + rk(\tau_\theta^{-1} + \Sigma)}\hat{v} \tag{19}$$

where

$$\hat{v} = (1 + \tau_\theta \Sigma)\hat{p}. \tag{20}$$

Written this way, the contracting outcome is equivalent to a situation in which there is only one performance measure $\hat{v}$ (instead of $\hat{p}$ and potentially y). And this performance measure has additional noise Σ relative the second best (compare the denominator in (19) with the one in (6)).[18]

The next proposition offers an interpretation of the performance measure $\hat{v}$ in terms of the inference problem for those who design the manager's incentive contract:

Proposition 4 *The measure $\hat{v}$ (as defined in (20)) is the best linear estimator of v given the stock price $\hat{p}$, the publicly available signal y, and a diffuse prior, $Var[v] = \infty$. The estimator is unbiased in the sense that $E[\hat{v}\,|v] = v$ and its estimation error $Var[\hat{v} - v]$ is Σ.*

Finding the optimal incentive contract for the manager is equivalent to finding an optimal estimator of firm value without any prior knowledge about managerial effort (the diffuse prior). By contracting on this estimator, the risk-averse manager is then exposed to the volatility of firm value τ_θ^{-1} and the estimation error Σ. Appendix B gives a graphical illustration of this result.

The optimal estimator $\hat{v}$ has two properties: it does not depend on the public signal y and it is different from the stock price $\hat{p}$.

Not including the public signal y is fully in line with the Holmström (1979) informativeness principle. It follows from risk-neutral pricing, which renders the stock price semi-strong efficient and a sufficient statistic for v with respect to public information (e.g., accounting information).

[18] See Banker and Datar (1989) as well as Pendergast (1999) for such a decomposition of the contract into "provision of incentives" (19) and "information aggregation" (20).

The estimator $\hat{v}$ is a linear transformation of the stock price $\hat{p}$. The difference between the estimator $\hat{v}$ and the stock price $\hat{p}$ reflects the difference between real efficiency and price efficiency. And Σ is a measure of this difference.

The price $\hat{p}$ is the best linear estimator of v given the available information at $t = 1$ when trading occurs. At $t = 1$, informed traders and market makers know the manager's incentive contract and thus have a prior about firm value based on the equilibrium effort e^*. With this prior, the variance of firm value is $Var[v] = \tau_\theta^{-1}$.

In contrast, the owner who designs the contract has no specific prior about the manager's effort. When designing the optimal contract at $t = 0$ she has to consider all possible effort levels e and therefore has to use a diffuse prior, $Var[v] = \infty$. Of course in equilibrium $e = e^*$, but this cannot be taken for granted when deriving the optimal contract.

More specifically, the estimator $\hat{v}$ scales the stock price $\hat{p}$ up by a factor larger than one, $1 + \tau_\theta \Sigma > 1$. The scaling up counters the limited sensitivity of the stock price to managerial shirking. To see this, we rewrite the scaling factor as

$$1 + \tau_\theta \Sigma = 1 + \frac{\tau_\theta}{\beta^2 \tau_u + \tau_\eta} = \frac{\beta^2 \tau_u + \tau_\eta + \tau_\theta}{\beta^2 \tau_u + \tau_\eta} = \frac{1}{\lambda\beta + \gamma} > 1,$$

and rewrite the pricing equation (10) as:

$$\hat{p} = e^* + (\lambda\beta + \gamma)\left[(e - e^*) + \theta\right] + \lambda u + \gamma\eta.$$

The scaling factor in the optimal contract is the inverse of the sensitivity of the stock price to deviations from equilibrium effort, $(\lambda\beta + \gamma)$.

The reason for the inverse link between the scaling factor in the optimal contract and the sensitivity of the stock price to shirking is the incentive constraint for the manager. Replacing the incentive constraint (13) with its first-order condition, we have:[19]

$$(\lambda\beta + \gamma)m + a = ke \tag{21}$$

The right-hand side of the incentive constraint is the marginal cost for the manager when he

[19]This is possible because the manager's problem is concave.

choses an effort level e. The left-hand side is his marginal benefit. The benefit depends on the contract terms m and a but also on the sensitivity of the performance measures to effort. The sensitivity of accounting information to effort is one. The sensitivity of the stock price to effort is less than one, $(\lambda\beta + \gamma) < 1$. If the incentive contract did not increase the sensitivity of the manager's income to the stock price by factor $(\lambda\beta + \gamma)^{-1} > 1$, and undo the limited sensitivity of the stock price to effort, then the contract would violate the incentive constraint and induce the manager to deviate from the equilibrium effort.

Statistically, the stock price is an unbiased estimator of firm value, $E[\hat{p}|v] = v$, only on the equilibrium path, i.e., when $e = e^*$. Off the equilibrium path, when $e \neq e^*$, the stock price is biased, $E[\hat{p}|v] \neq v$. This gives the manager an incentive to deviate from e^* that is countered by using the measure $\hat{v}$, which is unbiased both on and off the equilibrium path, $E[\hat{v}|v] = v$.

The limited sensitivity of the stock price to deviations from equilibrium effort, $e - e^*$, is the disadvantage of the stock price as a measure of managerial performance. The information content of the stock price depends on informed trading. Informed traders observe the signal (and the stock price) and use them to update their prior belief about the value of the firm. But when they update their prior belief, informed traders think the CEO takes the equilibrium action e^* (which they infer from the CEO's incentive contract). Thus, they attribute all variation in the signal to variation in the noise terms θ and ε. Hence, the stock price does not fully reflect the value consequences of possible deviations from equilibrium effort.

The stock price would only fully reveal deviations from equilibrium effort ($\lambda\beta = 1$) if informed traders observed perfectly future firm value ($\tau_\varepsilon = \infty$), if there was no uninformed trading ($\tau_u = \infty$), or if the public signal revealed perfectly future firm value ($\tau_\eta = \infty$). But each of these situations is incompatible with trade.

3.5 Comparative Statics

The next proposition shows how the sensitivity of the manager's income I to the stock price $\hat{p}$ changes as parameters governing trading behavior change.[20]

Proposition 5 *Market-based pay m^* increases when*

i) firm value is less volatile (higher τ_θ)

ii) there is more noise trading (lower τ_u)

iii) informed traders trade less aggressively (lower β), that is, when they are more risk-averse (higher ρ) or they have less precise information (lower τ_ε)

iv) the public signal (accounting information) is less precise (lower τ_η)

Scaling up the stock price by $1 + \tau_\theta \Sigma$ to counter the limited sensitivity of the stock price to the manager's deviation from equilibrium effort (equation (20)) drives the comparative statics. Whenever the stock price is *not* sensitive to the manager's shirking, his income becomes *more* sensitive to the stock price.

The stock price is not sensitive to the manager's shirking when the volatility of firm value is low (high τ_θ). When the value of the firm does not vary much, the adverse selection problem between market makers and informed traders is low, and the market is liquid (low λ). In a liquid market, the stock price is not sensitive to shirking because it does not react much to variations in the order flow.

The stock price also is not sensitive to shirking when there is a lot of noise trading (low τ_u). With a lot of noise trading, the adverse selection problem in the market is low because informed traders can hide their orders better. Again, the market is liquid and shirking does not move the stock price much.

The same intuition holds when informed traders behave less aggressively (low β), either because their are more risk-averse or because they have less precise information about the value of the firm. When informed traders behave less aggressively, then there is less adverse selection, more liquidity and a lower sensitivity of the stock price to managerial shirking.

[20]As M^* is monotone increasing in m^* (see equation (18)), Proposition 5 also describes the comparative statics of the sensitivity of income to the stock price p.

Finally, the stock price is not sensitive to the manager's shirking when the public signal y is less precise (low τ_η). Now the intuition is slightly different because less precise public information increases adverse selection in the market and decreases liquidity (high λ). But less precise public information also decreases the sensitivity of stock price to the public signal, γ. The direct effect via γ outweighs the indirect effect via λ and overall makes the stock price less sensitive to shirking.

4 Additional signal is privately available

Suppose the signal y realizes only at $t = 2$ so that it is not available when trading takes place. The signal y now represents internal performance measures that are contractible but not readily observable by outsiders (e.g., whether projects have been implemented within agreed time or cost frames).

The manager's income still is given by (1), but now we assume the contract is implemented by a combination of a cash payment and shares transferred from the owner to the manager at $t = 2$. Specifically, the fixed wage W and the variable income based on the stock price Mp are paid in cash, while income based on the internal performance measure Ay is paid in shares.[21]

With this implementation the per-share liquidation value of the firm at $t = 3$ is $v - W - Mp$. The fraction of shares α transferred from the owner to the manager is such that $Ay = \alpha E[v - W - Mp\,|p]$, where the expectation is the share value of the firm at $t = 2$ (when the manager leaves the firm).[22]

As before, competitive, risk-neutral market makers ensure efficient pricing

$$p = E[v - W - Mp|L],$$

we use an informationally-equivalent stock price that is gross of any cash payment to the

[21] This assumption is the same as in Holmström and Tirole (1993). It is necessary to obtain a closed-form solution. Otherwise, the trading aggressiveness β will depend on the terms of the incentive contract because informed traders will have to forecast the impact of pay that depends on the future realization of y on the net value of the shares they trade.

[22] If the stock market was open at $t = 2$, informed traders received no new information and noise trading at $t = 2$ was iid to noise trading at $t = 1$, then the stock price at $t = 2$ would be equal to $E[v - W - Mp|p]$.

manager

$$\hat{p} = p + W + Mp = E[v|L],$$

and the manager's income in terms of $\hat{p}$ is

$$I = w + m\hat{p} + ay,$$

where $w = \frac{W}{1+M}$, $m = \frac{M}{1+M}$ and $a = A$.

The next proposition describes the pricing function. It is the analogue of Proposition 1 when the realization of the signal y is not seen by the market.

Proposition 6 *If the signal y is not available at $t = 1$, the stock price $\hat{p}$ is given by:*

$$\hat{p} = (1 - \lambda\beta)e^* + \lambda\beta(e + \theta) + \lambda u, \tag{22}$$

where e^* is the hypothesized equilibrium effort of the manager, e is the actual effort, $\beta = \frac{\tau_\varepsilon}{\rho}$ and $\lambda = \frac{\beta\tau_u}{\beta^2\tau_u+\tau_\theta}$. The informativeness of the stock price is $\tau \equiv Var[v|\hat{p}]^{-1} = \tau_\theta + \beta^2\tau_u$.

The stock price when the signal y is not available at $t = 1$ (22) is the same as in the case in which the signal y is publicly available at $t = 1$ but has zero information content, $\tau_\eta = 0$.

The following proposition characterizes the optimal contract when the signal y is not available at $t = 1$ when trading takes place.

Proposition 7 *When the signal y is not available at $t = 1$, the optimal contract weights on the stock price and the signal are:*

$$\begin{aligned} m^* &= \frac{1 + (\tau_\theta - \tau_\eta)\Sigma}{1 + kr\left(\tau_\theta^{-1} + \Sigma\right)} \\ a^* &= \frac{\tau_\eta\Sigma}{1 + kr\left(\tau_\theta^{-1} + \Sigma\right)}, \end{aligned}$$

where as before $\Sigma = (\beta^2\tau_u + \tau_\eta)^{-1}$. The equilibrium effort and the expected value of the firm are the same as in the case in which y is available at $t = 1$.

The unavailability of the signal y at $t = 1$ changes the optimal contract for the manager. The manager's income now depends directly on the signal y, $a^* > 0$, because the market can no longer incorporate this information into the stock price.

To interpret the contract weights, it is useful to consider again an aggregate performance measure

$$\hat{v} = (1 + \tau_\theta \Sigma)\hat{p} + \tau_\eta \Sigma (v - \hat{p}) \tag{23}$$

such that (19) holds. The estimation error Σ of the aggregate performance measure $\hat{v}$ is the same as in the case in which y is available at $t = 1$. As before, the aggregate measure scales up the stock price by $1 + \tau_\theta \Sigma > 1$ to counter the limited sensitivity of the stock price to shirking by the manager, $\lambda\beta < 1$.

The aggregate measure now uses the information in y, but only the information that is not already contained in the stock price, $v - \hat{p}$. This way, the total pay-for-performance sensitivity ($m^* + a^*$) is the same as in the case in which y is available at $t = 1$ (where m^* is given by (15) and $a^* = 0$):

$$m^* + a^* = \frac{1 + \tau_\theta \Sigma}{1 + kr\left(\tau_\theta^{-1} + \Sigma\right)}$$

For economic efficiency, it does not matter whether the signal y is disclosed to the market at $t = 1$ or not. If it is disclosed, the stock price incorporates it and the incentive contract does not depend directly on y. If it is not disclosed, the stock price cannot incorporate this information but the optimal incentive contract's direct use of y leads to the same estimation error, the same total pay-for-performance sensitivity, the same manager effort and the same expected net firm value as in Proposition 3.

The next proposition collects the comparative statics of the contract terms m^* and a^*. The comparative statics of the total pay-for-performance sensitivity $m^* + a^*$ are the same as in Proposition 5 (where $a^* = 0$).

Proposition 8 *Market-based pay m^* increases when firm value is less volatile (higher τ_θ) and the signal y is less precise (lower τ_η). Iff $r < \frac{\tau_\theta}{k}\left(\frac{\tau_\theta}{\tau_\eta} - 1\right)$, then market-based pay increases when there is more noise trading (lower τ_u) and informed traders trade less aggressively (lower β), either because they are more risk-averse (higher ρ) or they have less precise information (lower τ_ε). Pay based on the signal y, a^*, increases when firm value is less volatile (higher τ_θ), there is more noise trading* *(lower τ_u), informed traders trade less aggressively (lower β, i.e., higher risk-aversion ρ or less precise information, lower τ_ε), and the signal y is more precise (higher τ_η).*

As long as the risk aversion of the manager is not too large, the comparative statics of market-based pay are as in the case in which y is public information at $t = 1$ (see Proposition 5).

The comparative statics of the weight on the signal y (when it is not available to the market at $t = 1$), a^*, are intuitive. There is more pay based on the signal y when it is more precise or the stock price is less informative. This is best seen in the relative contract weight:

$$\frac{m^*}{a^*} = \frac{\tau}{\tau_\eta}. \tag{24}$$

The relative weight of market- to non-market-based pay is given by the ratio of stock price informativeness τ (=$Var[v|\hat{p}]^{-1}$) and the precision of the signal y, τ_η (=$Var[y|v]^{-1}$).[23]

5 Endogenous information

In this section we examine the case of endogenous information, i.e., traders can invest in more precise information. So far traders' information was exogenous in our model. We found that better information for traders leads to less market-based pay because it increases the sensitivity of the stock price to shirking by the manager. Yet, more noise trading leads to more market-based pay. These two effects work against each other when information is endogenous. With more noise trading, traders can hide their trades better, make larger expected profits, and therefore have an incentive to invest in more precise information.

To endogenize traders' information, we now allow them to increase the precision of their signals, τ_ε, at a cost $c(\tau_\varepsilon)$ at $t = 0$ before they observe their signals s_i and before trading occurs. For tractability, we specify a linear cost $c(\tau_\varepsilon) = c\tau_\varepsilon$ and consider the case in which the signal y is revealed only after trading takes place. Hence, the stock price and the contract

[23]Banker and Datar (1989) propose to examine the relative contract weights. The relative contact weights do not depend on the characteristics of the manager (risk aversion, cost of effort), which makes the ratio of contract weights particularly useful for empirical work. When two performance measures are uncorrelated, the ratio is given by the measures' signal-to-noise ratios. This is not the case here as both the signal y and the stock price $\hat{p}$ depend on the value of the firm v and thus are correlated.

are given by Propositions 6 and 7, respectively, except that τ_ε now is a choice variable.

The next proposition solves for a trader's optimal choice of signal precision and computes the comparative statics of the optimal contract with respect to noise trading.

Proposition 9 *If* $\sqrt{8c\rho\tau_\theta + 1} > 2c\rho\tau_\theta + 1$ *the optimal signal precision is*

$$\tau_\varepsilon^* = \sqrt{\frac{\rho}{2c\tau_u}\left(\sqrt{8c\rho\tau_\theta + 1} - (2c\rho\tau_\theta + 1)\right)}.$$

With a linear cost of acquiring more precise information, market-based pay m^* *and total pay-for-performance sensitivity* $m^* + a^*$ *are independent of noise trading.*

The optimal signal precision increases when there is more noise trading. So while more noise trading has a direct positive effect on the extent of market-based pay (or on the total pay-for-performance sensitivity), it has an indirect negative effect via the improvement of the quality of traders' information. With a linear cost of acquiring better information, the indirect negative effect is so strong that it exactly off-sets the direct positive one. If the cost was slightly convex, then the indirect effect would be less strong and more noise trading again leads to more market-based pay.

It is worth contrasting our result to Holmström and Tirole (1993) where more noise trading leads to more information acquisition by a single informed trader and more market-based pay. Unlike in our model, noise trading in their model has *no* direct effect on market-based pay and it has an indirect effect because, like in our model, the trader invests in better information. But unlike in our model, such better information leads to *more* market-based pay. Hence, the channel through which the optimal contract of the manager reacts to market conditions is different in their analysis. The difference comes from using a Kyle (1985) market model in which the informativeness of the stock price is constant and the higher precision of the single trader's signal directly translates into lower stock price volatility and hence more market-based pay. This is not the case in our large competitive market with (infinitely) many informed traders.[24]

[24]See also Verrecchia (1982) who examines information acquisition in a competitive noisy rational expectations model like ours. Like us, he finds more noise trading leads to a less informative stock price despite the acquisition of better information.

6 Empirical implications

In our model, the manager's incentive contract relies more on the stock price when the stock price is less sensitive to managerial shirking. The sensitivity to shirking in turn depends on market conditions. Because both the stock price and the optimal contract are equilibrium outcomes, our model can shed new light on evidence that links liquidity, price informativeness, pay-for-performance sensitivity (PPS) and firm value, all of which are potentially endogenous variables in empirical studies.

There is conflicting evidence on the issue whether and how liquidity is good for governance via market-based CEO pay and for firm performance. Fang at al. (2009) show that more liquidity (a lower effective spread when executing a trade) leads to better firm performance, and this effect operates via an increase of the information content of stock prices and a larger PPS of CEO contracts. Jayaraman and Milbourn (2011) also find a positive effect of liquidity on PPS, but in a setting where more liquidity reduces the information content of stock prices (they examine the addition of a firm to the S&P 500 index). In Kang and Lui (2010), a higher probability of informed trading (PIN) leads to higher PPS (holding constant firm risk, which itself has a negative effect on PPS). As the PIN is a measure of adverse selection (see Easley et al., 1996), this is evidence of a negative effect of liquidity on PPS.

In our model, the relationship between stock market liquidity (λ^{-1}), price informativeness (τ) and PPS ($m^* + a^*$) pay is multi-facetted, as these are outcome variables. We can replicate the various empirical findings in our model in terms of variation of the underlying parameters. For example, a more liquid market occurs together with a higher PPS (as in Fang et al., 2009, and Jayaraman and Milbourn, 2011) when there is more noise trading (lower τ_u), firm value is less volatile (higher τ_θ), and is possible when informed traders act less aggressively (lower β).[25] Such a positive relationship between liquidity and PPS can go hand-in-hand with a less informative stock price (as in Jayaraman and Milbourn, 2011) when more noise trading or less aggressive informed trading is the underlying cause.

But we can also rationalize a negative relationship between liquidity and PPS as in Kang and Liu (2010). This occurs when public information becomes less precise (lower τ_η) and it

[25] Provided β is below the threshold in footnote 16.

also is possible when informed traders act less aggressively.[26]

Patterns in longer, aggregate time series support the evidence in Jayaraman and Milbourn (2011) of a positive relationship between stock market liquidity and the extent to which CEO pay depends on the stock price. In their survey, Frydman and Jenter (2010) document a strong increase in stock-based pay. While in 1970s, such pay accounts for 16% of overall CEO pay, it accounts for 60% in the early 2000s. Bai et al. (2015) document a strong increase in liquidity as measured by monthly share turnover, from 1.6% in 1960 to 20% in 2014. They note "the opportunity to trade in a liquid market also increases the incentive to produce information in the first place". In terms of our model, this again points to an increase in noise trading (lower τ_u) because more noise trading increases liquidity, increases informed trading and traders' incentive to invest in better information (as in Section 5), and leads to a higher PPS.[27]

Our model also sheds light on the open question whether stronger CEO incentives (more PPS) improve efficiency, e.g., lead to higher firm value (see for example the survey by Edmans and Gabaix, 2015). Again, this is a difficult question because the manager's incentive contract and his effort are endogenous. In our model, the relationship between between PPS and expected net firm value ($E[v - I]$) can be positive or negative. More PPS leads to less firm value when there is more noise trading, worse public information, or when traders act less aggressively. More PPS leads to higher firm value when firm volatility decreases.

There is also a debate whether more disclosure of financial information leads to better economic outcomes. The survey by Leuz and Wysocki (2008) finds little consensus in the literature and "few insights into the [...] economic efficiency of reporting and disclosure regulation". Better information in our model always increases efficiency. For example, a more precise signal y (higher τ_η) or more precise information for traders (higher τ_ε) lead higher expected net firm value $E[v - I]$). This is not surprising, especially as we do not specify the cost of such better information.

[26]Provided β is above the threshold in footnote 16.

[27]Bai et al. (2015) find a positive link between liquidity and stock price informativeness, which appears at odds with a decrease in τ_u in our model. But their empirical measure of price informativeness is the variance of future earnings predicted by the stock price. Price informativeness in our model, τ, is the inverse of the variance of firm value (possibly proxied by future earnings) conditional on the stock price, which is a different concept (it is the inverse of the regression error in Bai et al.).

But our model also has a novel implication for the disclosure of information holding the quality of information constant. Revealing the signal y to the public does not affect economic efficiency. It does not affect the manager's effort nor the expected net value of the firm. When the signal y is public, the market is more liquid and more informative than when the signal y is private (compare Proposition 1 and (11) to Proposition 6). But the PPS of the manager's contract stays the same because the contract adjusts. It puts more weight on the stock price and less (i.e., no) weight on the signal y when the signal is public. An empirical investigation of the effect of disclosure on economic efficiency must therefore distinguish between having better information and having public information (holding constant the quality of information). And it must hold constant endogenous variables such as CEO pay.

Next, our paper generates empirical implications from our comparative statics exercises (Propositions 5 and 8). First, we already discussed how more noise trading leads to more liquidity and more PPS (as evidenced in Jayaraman and Milbourn, 2011). As more noise trading also reduces prices informativeness—and this is robust to endogenous information collection—the effect is different from the usual intuition in Holmström and Tirole (1993).

Second, the PPS should be higher in less volatile firms (higher τ_θ). The substantial literature on the empirical relationship between firm volatility and PPS finds mixed results (see Edmans and Gabaix, 2015). Normally, the intuition for a negative relationship between firm volatility and PPS relies on the standard risk-incentive trade-off. Our analysis with endogenous trading strengthens the negative relationship because the market is more liquid and the stock price is less sensitive to managerial shirking when firm volatility is low.

Third, the PPS increases when public information about the future value of the firm y is less precise (lower τ_η). We obtain this result even though the optimal contract does not depend directly on public information. We also obtain the result when the signal y is private, i.e., when there is worse internal information. Moreover, in that case the relative weight on market-based pay should increase (see equation (24)).[28]

Finally, the PPS increases when trader's information about the future value of the firm is less precise (lower τ_ε). That less information in the market leads to more market-based

[28] An accounting literature started by Lambert and Larcker (1987) and Sloan (1993) examines relative contract weights and generally finds evidence in support of efficient contracting models like ours. However, they do not treat the stock price as an endogenous variable whose information content depends on trading conditions.

pay is perhaps the most novel empirical implication of our model. It is in line with the positive relationship between PPS and more uninformed (noise) trading and the negative relationship between PPS and the quality of public information. We are not aware of any formal test, but firms with growth options or intangible assets, which are harder to value, tend to put more weight on the stock price in their pay structures. According to our model, they do this to counter the limited sensitivity of the stock price to managerial shirking.[29]

7 Conclusions

In this paper we examine the design of optimal market-based pay for a CEO when the future value of the firm is not contractible. In accordance with intuition, the stock price in an informationally (semi-strong) efficient market is a sufficient statistic for public information with respect to CEO effort. Hence, the optimal pay contract only includes the stock price and disregards other public information about the value consequences of CEO effort.

But because of the feedback loop between CEO effort and the stock price, the price does not fully reflect a CEO's deviation from equilibrium. We show that the stock price is not an unbiased estimator of future firm value off the equilibrium path. When traders observe their noisy information about the future value of the firm, they think the CEO takes the equilibrium effort. They cannot act as if they expect a deviation because then there can never be consistency between traders' beliefs and the design of the contract, which implements the equilibrium effort. A CEO can therefore always hide partially behind other fundamental factors that drive firm value.

The optimal contract counters a CEO's incentive to deviate from equilibrium effort by giving him more stock-based pay than in the second-best, i.e., the hypothetical case in which future firm value was contractible.

Our parsimonious rational competitive model yields a number of new testable implications (e.g., when traders have worse information a CEO should receive more stock-based pay) and offers a reinterpretation of existing, sometimes contradictory, empirical evidence

[29] Of course, there are other explanations. For example, such firms can use the high value of their stock as "currency" to attract and incentivize talent, they use market-based pay to induce risk-taking, or it is the optimal response to the greater uncertainty about the ability to manipulate the stock price for those firms (as in Peng and Röell, 2014).

(e.g., about the role of market liquidity).

Generally, market conditions and the optimal pay contract are joint outcomes that depend on underlying parameters in subtle ways. Any regulation that intervenes with private contracting must therefore proceed cautiously. For example, we show that it is optimal in a liquid market for the CEO to receive a lot of stock-based pay. This is necessary so that one can profit from the stock market's power to aggregate information while, at the same time, limit a CEO's incentive to shirk. Imposing a cap on CEO pay therefore can be counterproductive. It is more efficient to tackle the underlying friction—CEO actions may take a very long time to shape firm value—directly, e.g., by adding clawback provisions to CEO pay or by improving the information about a firm's long-term value that is available to the market.

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Appendix A: Proofs

Throughout the proofs, we use the following standard result:

Result 1 (Conditional expectation) *Let Y_i be a $(n_i \times 1)$ a multi-variate normally distributed vector with mean μ_i, i=1,2, and variance-covariance matrices Σ_{ij}, then*

$$Y_2|Y_1 = y_1 \sim N([\mu_2 + \Sigma_{21}\Sigma_{11}^{-1}(y_1 - \mu_1)], [\Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}])$$

Proof of Proposition 1

Applying Result 1, we obtain

$$\begin{aligned}
\hat{p} &= E[v\,|z,y] = e^* \left(1 - \frac{\beta^2\tau_u + \tau_\eta}{\beta^2\tau_u + \tau_\theta + \tau_\eta}\right) + \frac{\beta\tau_u}{\beta^2\tau_u + \tau_\theta + \tau_\eta} z + \frac{\tau_\eta}{\beta^2\tau_u + \tau_\theta + \tau_\eta} y \\
&= e^* (1 - \beta\lambda - \gamma) + \lambda z + \gamma y \\
&= e^* (1 - \beta\lambda - \gamma) + (\beta\lambda + \gamma)(e + \theta) + \lambda u + \gamma\eta
\end{aligned}$$

with $\lambda = \frac{\beta\tau_u}{\beta^2\tau_u+\tau_\theta+\tau_\eta}$ and $\gamma = \frac{\tau_\eta}{\beta^2\tau_u+\tau_\theta+\tau_\eta}$.

From the standard maximization of traders' CARA utility using the certainty equivalent, we know that

$$x_i = \frac{E[v - \hat{p}\,|s_i,z,y]}{\rho Var[v\,|s_i,z,y]} = \frac{E[v\,|s_i,z,y] - \hat{p}}{\rho Var[v\,|s_i,z,y]} = \frac{e^*\tau_\theta + z\beta\tau_u + s_i\tau_\varepsilon + y\tau_\eta - \hat{p}(\beta^2\tau_u+\tau_\theta+\tau_\varepsilon+\tau_\eta)}{\rho}$$

and substituting for $\hat{p} = E[v\,|z,y] = \frac{e^*\tau_\theta+\beta\tau_u z+y\tau_\eta}{\beta^2\tau_u+\tau_\theta+\tau_\eta}$ one obtains

$$x_i(s_i,\hat{p}) = \frac{e^*\tau_\theta+\beta\tau_u z+s_i\tau_\varepsilon+y\tau_\eta - \frac{e^*\tau_\theta+\beta\tau_u z+y\tau_\eta}{\beta^2\tau_u+\tau_\theta+\tau_\eta}\left(\beta^2\tau_u+\tau_\theta+\tau_\varepsilon+\tau_\eta\right)}{\rho} = \frac{\tau_\varepsilon}{\rho}\left(s_i - \hat{p}\right),$$

from which we can identify $\beta = \frac{\tau_\varepsilon}{\rho}$.

Proof of Proposition 2

We first verify that the participation constraint (14) binds at the optimum. Suppose this was not the case. Then $E[I] = \frac{r}{2}Var[I] + \frac{k}{2}e^2 + \varepsilon$, $\varepsilon > 0$. Notice that neither the variance $Var[I]$ nor the incentive constraint depend on the fixed wage W. The owner-principal could improve on her payoff $E[v] - E[I]$ by reducing W and hence, $E[I]$, by ε, which is a contradiction to the contract being optimal.

The mean and variance of the manager's income are:

$$\begin{aligned}
E[I] &= w + m[(1 - \lambda\beta - \gamma)e^* + (\lambda\beta + \gamma)e] + ae \\
Var[I] &= m^2[(\lambda\beta+\gamma)^2\tau_\theta^{-1} + \lambda^2\tau_u^{-1} + \gamma^2\tau_\eta^{-1}] + a^2(\tau_\theta^{-1} + \tau_\eta^{-1}) + 2ma((\lambda\beta+\gamma)\tau_\theta^{-1} + \gamma\tau_\eta^{-1})
\end{aligned}$$

We replace the incentive constraint (13) with its first-order condition (which is possible because the agent's problem is concave):

$$(\lambda\beta + \gamma)m + a = ke \tag{A.1}$$

After substituting the binding participation constraint into the objective function and using (A.1) to substitute for e, problem (12) becomes:

$$\begin{aligned}&\max_{m,a} \frac{1}{k}(m(\lambda\beta+\gamma)+a)\\ &-\frac{r}{2}\left[m^2[(\lambda\beta+\gamma)^2\tau_\theta^{-1}+\lambda^2\tau_u^{-1}+\gamma^2\tau_\eta^{-1}]+a^2(\tau_\theta^{-1}+\tau_\eta^{-1})+2ma((\lambda\beta+\gamma)\tau_\theta^{-1}+\gamma\tau_\eta^{-1})\right]\\ &-\frac{1}{2k}(m(\lambda\beta+\gamma)+a)^2\end{aligned}$$

The first order conditions with respect to m and a are:

$$\begin{aligned}\lambda\beta+\gamma &= m\left[kr\left((\lambda\beta+\gamma)^2\tau_\theta^{-1}+\lambda^2\tau_u^{-1}+\gamma^2\tau_\eta^{-1}\right)+(\lambda\beta+\gamma)^2\right]\\ &\quad +a\left[kr((\lambda\beta+\gamma)\tau_\theta^{-1}+\gamma\tau_\eta^{-1})+(\lambda\beta+\gamma)\right]\\ 1 &= m\left[kr\left((\lambda\beta+\gamma)\tau_\theta^{-1}+\gamma\tau_\eta^{-1}\right)+(\lambda\beta+\gamma)\right]+a\left[kr\left(\tau_\theta^{-1}+\tau_\eta^{-1}\right)+1\right]\end{aligned}$$

Solving for m and a yields

$$\begin{aligned}m &= \tau_\theta\frac{\frac{\beta}{\lambda}\tau_u}{\tau_\theta\tau_\eta+\beta^2\tau_u\tau_\theta+kr\tau_\theta+kr\tau_\eta+kr\beta^2\tau_u}\\ a &= \tau_\theta\frac{\tau_\eta-\frac{\beta}{\lambda}\tau_u\gamma}{\tau_\theta\tau_\eta+\beta^2\tau_u\tau_\theta+kr\tau_\theta+kr\tau_\eta+kr\beta^2\tau_u}\end{aligned}$$

We rearrange using $\Sigma=\left(\beta^2\tau_u+\tau_\eta\right)^{-1}$ and $\tau=\beta^2\tau_u+\tau_\theta+\tau_\eta$ to obtain:

$$\begin{aligned}m &= \frac{1}{1+kr\left(\tau_\theta^{-1}+\Sigma\right)}\Sigma\tau\\ a &= \frac{1}{1+kr\left(\tau_\theta^{-1}+\Sigma\right)}\Sigma\left(\tau_\eta-\gamma\tau\right)\end{aligned}$$

Substituting for $\gamma=\frac{\tau_\eta}{\tau}$ into the second expression one finds $a=0$. Also, $\Sigma\tau=\frac{\beta^2\tau_u+\tau_\theta+\tau_\eta}{\beta^2\tau_u+\tau_\eta}=1+\tau_\theta\Sigma$ which provides expression (15).

Proof of Proposition 3

The equilibrium ($e=e^*$) effort is recovered from the incentive constraint (A.1):

$$\begin{aligned}e^* &= \frac{1}{k}\left(\frac{1}{1+kr\left(\tau_\theta^{-1}+\Sigma\right)}\Sigma\tau\left(\frac{\beta^2\tau_u}{\tau}+\frac{\tau_\eta}{\tau}\right)\right)\\ &= \frac{1}{k}\left(\frac{1}{1+kr\left(\tau_\theta^{-1}+\Sigma\right)}\Sigma\left(\beta^2\tau_u+\tau_\eta\right)\right)\\ e^* &= \frac{1}{k}\left(\frac{1}{1+kr\left(\tau_\theta^{-1}+\Sigma\right)}\right),\end{aligned}$$

which is the expression in the proposition.

From the binding participation constraint, we have

$$E[I] = \frac{k}{2}(e^*)^2 + \frac{r}{2}Var[I]$$

where e^* is the equilibrium effort. Hence,

$$E[v - I] = e^* - \frac{k}{2}(e^*)^2 - \frac{r}{2}Var[I]$$

Substituting for e^* in (16) we have:

$$e^* - \frac{k}{2}(e^*)^2 = \frac{1}{2k}\left(\frac{1 + 2kr(\sigma_\theta^2 + \Sigma)}{[1 + kr(\sigma_\theta^2 + \Sigma)]^2}\right) \tag{A.2}$$

The variance of the CEO's income $I = w + m\hat{p}$ is

$$Var[I] = \left[\frac{\Sigma\tau}{1 + kr(\sigma_\theta^2 + \Sigma)}\right]^2 \left[(\lambda\beta + \gamma)^2\sigma_\theta^2 + \lambda^2\sigma_u^2 + \gamma^2\sigma_\eta^2\right]$$

Because $\Sigma\tau = \frac{1}{\lambda\beta+\gamma}$, the expression for the variance becomes

$$Var[I] = \left[\frac{1}{1 + kr(\sigma_\theta^2 + \Sigma)}\right]^2 \left[\sigma_\theta^2 + \frac{\lambda^2}{(\lambda\beta + \gamma)^2}\sigma_u^2 + \frac{\gamma^2}{(\lambda\beta + \gamma)^2}\sigma_\eta^2\right]$$

Next, we have

$$\begin{aligned}
\frac{\lambda^2}{(\lambda\beta + \gamma)^2} &= \lambda^2(\Sigma\tau)^2 = \left(\frac{\beta\tau_u}{\beta^2\tau_u + \tau_\eta}\right)^2 \\
\frac{\gamma^2}{(\lambda\beta + \gamma)^2} &= \gamma^2(\Sigma\tau)^2 = \left(\frac{\tau_\eta}{\beta^2\tau_u + \tau_\eta}\right)^2
\end{aligned}$$

With this, we can further simplify the expression for the variance

$$\begin{aligned}
Var[I] &= \left[\frac{1}{1 + kr(\sigma_\theta^2 + \Sigma)}\right]^2 \left[\sigma_\theta^2 + \frac{\beta^2\tau_u^2}{(\beta^2\tau_u + \tau_\eta)^2}\frac{1}{\tau_u} + \frac{\tau_\eta^2}{(\beta^2\tau_u + \tau_\eta)^2}\frac{1}{\tau_\eta}\right] \\
&= \left[\frac{1}{1 + kr(\sigma_\theta^2 + \Sigma)}\right]^2 [\sigma_\theta^2 + \Sigma] \qquad \text{(A.3)}
\end{aligned}$$

Combining (A.2) and (A.3) yields

$$\begin{aligned}
E[v - I] &= \frac{1}{2k}\left(\frac{1 + 2kr(\sigma_\theta^2 + \Sigma)}{[1 + kr(\sigma_\theta^2 + \Sigma)]^2}\right) - \frac{r}{2}\left[\frac{1}{1 + kr(\sigma_\theta^2 + \Sigma)}\right]^2 [\sigma_\theta^2 + \Sigma] \\
&= \frac{1}{2k}\left(\frac{1 + 2kr(\sigma_\theta^2 + \Sigma) - kr(\sigma_\theta^2 + \Sigma)}{[1 + kr(\sigma_\theta^2 + \Sigma)]^2}\right) \\
&= \frac{1}{2k}\left(\frac{1 + kr(\sigma_\theta^2 + \Sigma)}{[1 + kr(\sigma_\theta^2 + \Sigma)]^2}\right) \\
&= \frac{1}{2k}\left(\frac{1}{1 + kr(\sigma_\theta^2 + \Sigma)}\right)
\end{aligned}$$

which is the desired result.

Proof of Proposition 4

We start showing that $\hat{v} = \Sigma\tau\hat{p}$ is the efficient linear estimator of v with a diffuse prior given the observation of $\hat{p}$ and y. Notice that substituting $\Sigma = \left(\beta^2\tau_u + \tau_\eta\right)^{-1}$, $\tau = \beta^2\tau_u + \tau_\theta + \tau_\eta$, and using the definitions of λ and γ (Proposition 1), one obtains

$$\hat{v} = \frac{1}{\lambda\beta + \gamma}\hat{p}, \tag{A.4}$$

an expression that will be used in the following.

Linearity requires

$$\hat{v} = b\hat{p} + cy \tag{A.5}$$

while efficiency requires

$$\min_{b,c} Var[\hat{v} - v] \tag{A.6}$$

Writing $\hat{v}$ as in (A.5) and using the expression for $\hat{p}$ as in (10), we have

$$\hat{v} - v = b(1 - \lambda\beta - \gamma)e^* + [b(\lambda\beta + \gamma) + c - 1]v + b\lambda u + (b\gamma + c)\eta$$

so that

$$Var[\hat{v} - v] = [b(\lambda\beta + \gamma) + c - 1]^2\sigma_\theta^2 + (b\lambda)^2\sigma_u^2 + (b\gamma + c)^2\sigma_\eta^2. \tag{A.7}$$

Using (A.7), the first-order conditions for (A.6) are:

$$\begin{aligned} b &= \frac{\beta\sigma_\eta^2\sigma_\theta^2}{\left(\sigma_\eta^2\sigma_u^2 + \sigma_\theta^2\sigma_u^2 + \beta^2\sigma_\eta^2\sigma_\theta^2\right)\lambda} = \frac{\beta\sigma_\eta^2}{\left(\frac{\sigma_\eta^2\sigma_u^2}{\sigma_\theta^2} + \sigma_u^2 + \beta^2\sigma_\eta^2\right)\lambda} \\ c &= \frac{\sigma_\theta^2\left(\lambda\sigma_u^2 - \beta\gamma\sigma_\eta^2\right)}{\left(\sigma_\eta^2\sigma_u^2 + \sigma_\theta^2\sigma_u^2 + \beta^2\sigma_\eta^2\sigma_\theta^2\right)\lambda} = \frac{\left(\lambda\sigma_u^2 - \beta\gamma\sigma_\eta^2\right)}{\left(\frac{\sigma_\eta^2\sigma_u^2}{\sigma_\theta^2} + \sigma_u^2 + \beta^2\sigma_\eta^2\right)\lambda} \end{aligned}$$

As $\sigma_\theta^2 \to \infty$ (diffuse priors), the expressions become

$$\begin{aligned} b &= \frac{\beta\sigma_\eta^2}{\lambda\sigma_u^2 + \lambda\beta^2\sigma_\eta^2} \\ c &= \frac{\lambda\sigma_u^2 - \beta\gamma\sigma_\eta^2}{\lambda\sigma_u^2 + \lambda\beta^2\sigma_\eta^2} \end{aligned}$$

Divide both the numerator and denominator of the expression for b by $\beta\sigma_\eta^2$ to get

$$b = \frac{1}{\frac{\lambda\sigma_u^2}{\beta\sigma_\eta^2} + \lambda\beta}$$

We obtain (A.4) if $\frac{\lambda\sigma_u^2}{\beta\sigma_\eta^2} = \gamma$ and $c = 0$. The first condition holds because

$$\frac{\lambda\sigma_u^2}{\beta\sigma_\eta^2} = \tau_\eta\frac{\lambda}{\beta\tau_u} = \tau_\eta\frac{\frac{\beta\tau_u}{\beta^2\tau_u + \tau_\theta + \tau_\eta}}{\beta\tau_u} = \frac{\tau_\eta}{\beta^2\tau_u + \tau_\theta + \tau_\eta} = \gamma.$$

Finally, $c = 0$ because $\lambda\sigma_u^2 = \beta\gamma\sigma_\eta^2$, or equivalently $\frac{\lambda\sigma_u^2}{\beta\sigma_\eta^2} = \gamma$ (as just shown).

The estimator $\hat{v}$ is unbiased because

$$E[\hat{v}\,|v] = E[b\hat{p}\,|v] = \tfrac{1}{\lambda\beta+\gamma}E[\hat{p}\,|v] = \tfrac{1}{\lambda\beta+\gamma}\,(\lambda\beta+\gamma)\,v = v,$$

where we used (10) to substitute for $\hat{p}$.

Finally, we show that Σ measures the estimation error $Var[\hat{v}-v]$. From the definition of $\hat{v}$ one obtains

$$\begin{aligned} Var(\hat{v}) &= Var(\Sigma\tau\hat{p}) \\ &= \Sigma^2\tau^2 Var(\hat{p}) \end{aligned}$$

Given risk-neutral pricing, we can decompose the variance as[30]

$$Var(\hat{p}) = Var(v) - Var(v\,|\hat{p}\,) = \tau_\theta^{-1} - \tau^{-1}$$

so that

$$Var(\hat{v}) = \Sigma^2\tau^2\left(\frac{1}{\tau_\theta} - \frac{1}{\tau}\right),$$

which after some manipulation and using $\lambda\beta = \frac{\beta^2\tau_u}{\tau} = 1 - \frac{\tau_\theta}{\tau}$ can be written as

$$Var(\hat{v}) = \frac{\Sigma\tau}{\tau_\theta}.$$

As $\hat{v}$ is the efficient, linear, unbiased estimator of v, we have $\hat{v} = v + \epsilon$, where the error term ϵ represents the estimation error and it is orthogonal to $\hat{v}$. Then

$$Var(\hat{v}) = Var(v) + Var(\epsilon).$$

We substitute for $Var(\hat{v})$:

$$\frac{\Sigma\tau}{\tau_\theta} = \frac{1}{\tau_\theta} + Var(\epsilon)$$

so that

$$Var(\epsilon) = \frac{\Sigma\tau}{\tau_\theta} - \frac{1}{\tau_\theta} = \frac{1}{\tau_\theta}\left(\Sigma\tau - 1\right)$$

Finally, we use $\Sigma = \left(\beta^2\tau_u + \tau_\eta\right)^{-1}$ and $\tau = \beta^2\tau_u + \tau_\theta + \tau_\eta$ to obtain:

$$Var(\epsilon) = \frac{1}{\tau_\theta}\left(\tfrac{\beta^2\tau_u+\tau_\theta+\tau_\eta}{\beta^2\tau_u+\tau_\eta} - 1\right) = \frac{1}{\tau_\theta}\left(\tfrac{\tau_\theta}{\beta^2\tau_u+\tau_\eta}\right) = \Sigma$$

Proof of Proposition 5

Substitute for Σ in m^* to obtain

$$m^* = \frac{\tau_u\beta^2\tau_\theta + \tau_\theta^2 + \tau_\eta\tau_\theta}{\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + kr\tau_\theta + kr\tau_\eta + kr\beta^2\tau_u}.$$

[30] $Var(v) = Var(E[v|\hat{p}]) + E[Var(v|\hat{p})]$, where $E[v\,|\hat{p}] = \hat{p}$ and $Var(v|\hat{p})$ is constant.

Then

$$
\begin{aligned}
\frac{dm^*}{d\tau_\theta} &= \frac{kr\beta^4\tau_u^2 + \beta^2\tau_u\tau_\theta^2 + 2kr\beta^2\tau_u\tau_\theta + 2kr\beta^2\tau_u\tau_\eta + \tau_\theta^2\tau_\eta + kr\tau_\theta^2 + 2kr\tau_\theta\tau_\eta + kr\tau_\eta^2}{\left(\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + kr\tau_\theta + kr\tau_\eta + kr\beta^2\tau_u\right)^2} > 0 \\
\frac{dm^*}{d\tau_u} &= -\beta^2\frac{\tau_\theta^3}{\left(\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + kr\tau_\theta + kr\tau_\eta + kr\beta^2\tau_u\right)^2} < 0 \\
\frac{dm^*}{d\beta} &= -2\beta\tau_u\frac{\tau_\theta^3}{\left(\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + kr\tau_\theta + kr\tau_\eta + kr\beta^2\tau_u\right)^2} < 0 \\
\frac{dm^*}{d\tau_\eta} &= -\frac{\tau_\theta^3}{\left(\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + kr\tau_\theta + kr\tau_\eta + kr\beta^2\tau_u\right)^2} < 0
\end{aligned}
$$

Proof of Proposition 6

Exactly as the proof of Proposition 1 with $\tau_\eta = 0$.

Proof of Proposition 7

The mean and variance of the manager's income are:

$$
\begin{aligned}
E[I] &= w + m[(1-\lambda\beta)e^* + \lambda\beta e] + ae \\
Var[I] &= m^2[(\lambda\beta)^2\tau_\theta^{-1} + \lambda^2\tau_u^{-1}] + a^2(\tau_\theta^{-1} + \tau_\eta^{-1}) + 2ma(\lambda\beta\tau_\theta^{-1})
\end{aligned}
$$

The first-order condition of the manager's problem (the incentive constraint) is

$$
m\lambda\beta + a - ke = 0 \tag{A.8}
$$

With this, the optimization problem (12) becomes:

$$
\max_{m,a}\frac{1}{k}(m\lambda\beta + a) - \frac{r}{2}\left[m^2[(\lambda\beta)^2\tau_\theta^{-1} + \lambda^2\tau_u^{-1}] + a^2(\tau_\theta^{-1} + \tau_\eta^{-1}) + 2ma(\lambda\beta\tau_\theta^{-1})\right] - \frac{1}{2k}(m\lambda\beta + a)^2
$$

The first order conditions with respect to m and a are:

$$
\begin{aligned}
\lambda\beta &= m\left[kr\left((\lambda\beta)^2\tau_\theta^{-1} + \lambda^2\tau_u^{-1}\right) + (\lambda\beta)^2\right] + a\left[kr\lambda\beta\tau_\theta^{-1} + \lambda\beta\right] \\
1 &= a\left[kr\left(\tau_\theta^{-1} + \tau_\eta^{-1}\right) + 1\right] + m\left[kr\lambda\beta\tau_\theta^{-1} + \lambda\beta\right]
\end{aligned}
$$

Solving for m and a yields

$$
\begin{aligned}
m &= \frac{\beta}{\lambda}\tau_u\frac{\tau_\theta}{\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + kr\tau_\theta + kr\tau_\eta + kr\beta^2\tau_u} \\
a &= \tau_\theta\frac{\tau_\eta}{\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + kr\tau_\theta + kr\tau_\eta + kr\beta^2\tau_u},
\end{aligned}
$$

which rearranged, substituting for λ from Proposition 6, and using $\Sigma^{-1} \equiv (\beta^2\tau_u + \tau_\eta)$ become the desired expressions.

We obtain the optimal effort from the incentive constraint (A.8):

$$
\begin{aligned}
e &= \frac{1}{k}\left[\frac{1}{1+kr\left(\tau_\theta^{-1}+\Sigma\right)}\left(\lambda\beta(1+(\tau_\theta-\tau_\eta)\Sigma)+\tau_\eta\Sigma\right)\right] \\
&= \frac{1}{k}\left[\frac{1}{1+kr\left(\tau_\theta^{-1}+\Sigma\right)}\Sigma\left(\lambda\beta\tau+\tau_\eta\right)\right] \\
&= \frac{1}{k}\left[\frac{1}{1+kr\left(\tau_\theta^{-1}+\Sigma\right)}\right],
\end{aligned}
$$

which is identical to (16).

The variance of the CEO's income $I = w + m^*\hat{p} + a^*y$ is

$$
\begin{aligned}
Var[I] = &\left[\frac{1+(\tau_\theta-\tau_\eta)\Sigma}{1+kr(\sigma_\theta^2+\Sigma)}\right]^2\left[(\lambda\beta)^2\sigma_\theta^2+\lambda^2\sigma_u^2\right]+\left[\frac{\tau_\eta\Sigma}{1+kr(\sigma_\theta^2+\Sigma)}\right]^2\left[\sigma_\theta^2+\sigma_\eta^2\right] \\
&+2\frac{1+(\tau_\theta-\tau_\eta)\Sigma}{1+kr(\sigma_\theta^2+\Sigma)}\lambda\beta\frac{\tau_\eta\Sigma}{1+kr(\sigma_\theta^2+\Sigma)}\sigma_\theta^2
\end{aligned}
$$

where the last term is the covariance between $m^*\hat{p}$ and a^*y. Because $1+(\tau_\theta-\tau_\eta)\Sigma = \Sigma\tau$, the expression for the variance becomes

$$
Var[I] = \left[\frac{\Sigma}{1+kr(\sigma_\theta^2+\Sigma)}\right]^2\left[(\tau\lambda\beta)^2\sigma_\theta^2+(\tau\lambda)^2\sigma_u^2+\tau_\eta^2\sigma_\theta^2+\tau_\eta^2\sigma_\eta^2+2\tau\lambda\beta\tau_\eta\sigma_\theta^2\right]
$$

Next, because $\tau\lambda = \beta\tau_u$ we have

$$
\begin{aligned}
Var[I] &= \left[\frac{\Sigma}{1+kr(\sigma_\theta^2+\Sigma)}\right]^2\left[(\beta^2\tau_u)^2\sigma_\theta^2+(\beta\tau_u)^2\sigma_u^2+\tau_\eta^2\sigma_\theta^2+\tau_\eta^2\sigma_\eta^2+2\beta^2\tau_u\tau_\eta\sigma_\theta^2\right] \\
&= \left[\frac{\Sigma}{1+kr(\sigma_\theta^2+\Sigma)}\right]^2\left[[(\beta^2\tau_u)^2+\tau_\eta^2+2\beta^2\tau_u\tau_\eta]\sigma_\theta^2+\beta^2\tau_u+\tau_\eta\right]
\end{aligned}
$$

Using $\Sigma = \frac{1}{\beta^2\tau_u+\tau_\eta}$, the expression for the variance becomes

$$
\begin{aligned}
Var[I] &= \left[\frac{1}{1+kr(\sigma_\theta^2+\Sigma)}\right]^2\left[\frac{(\beta^2\tau_u)^2+\tau_\eta^2+2\beta^2\tau_u\tau_\eta}{(\beta^2\tau_u+\tau_\eta)^2}\sigma_\theta^2+\Sigma\right] \\
&= \left[\frac{1}{1+kr(\sigma_\theta^2+\Sigma)}\right]^2\left[\sigma_\theta^2+\Sigma\right]
\end{aligned}
$$

which is identical to (A.3).

Proof of Proposition 8

Substituting $\Sigma = \left(\beta^2\tau_u+\tau_\eta\right)^{-1}$ in the expressions for m^* and a^* in Proposition 7 yields

$$
\begin{aligned}
m^* &= \frac{\tau_u\beta^2\tau_\theta+\tau_\theta^2}{\tau_\theta\tau_\eta+\beta^2\tau_u\tau_\theta+k\tau_\theta r+k\tau_\eta r+k\beta^2\tau_u r} \\
a &= \frac{\left(\beta^2\tau_u+\tau_\eta\right)^{-1}\tau_\eta}{1+kr\left(\tau_\theta^{-1}+\left(\beta^2\tau_u+\tau_\eta\right)^{-1}\right)}
\end{aligned}
$$

so that

$$\frac{\partial m}{\partial \tau_\theta} = \frac{\tau_\theta^2\tau_\eta + \beta^2\tau_u\tau_\theta^2 + k\tau_\theta^2 r + k\beta^4\tau_u^2 r + 2k\tau_\theta\tau_\eta r + 2k\beta^2\tau_u\tau_\theta r + k\beta^2\tau_u\tau_\eta r}{\left(\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + k\tau_\theta r + k\tau_\eta r + k\beta^2\tau_u r\right)^2} > 0$$

$$\frac{\partial m}{\partial \tau_u} = \beta^2\tau_\theta\frac{-\tau_\theta^2 + \tau_\eta\tau_\theta + k\tau_\eta r}{\left(\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + k\tau_\theta r + k\tau_\eta r + k\beta^2\tau_u r\right)^2} < 0 \quad \text{iff} \quad kr < \tau_\theta\left(\frac{\tau_\theta}{\tau_\eta} - 1\right)$$

$$\frac{\partial m}{\partial \beta} = 2\beta\tau_u\tau_\theta\frac{-\tau_\theta^2 + \tau_\eta\tau_\theta + k\tau_\eta r}{\left(\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + k\tau_\theta r + k\tau_\eta r + k\beta^2\tau_u r\right)^2} < 0 \quad \text{iff} \quad kr < \tau_\theta\left(\frac{\tau_\theta}{\tau_\eta} - 1\right)$$

$$\frac{\partial m}{\partial \tau_\eta} = -\left(\tau_u\beta^2\tau_\theta + \tau_\theta^2\right)\frac{\tau_\theta + kr}{\left(\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + k\tau_\theta r + k\tau_\eta r + k\beta^2\tau_u r\right)^2} < 0$$

$$\frac{\partial a}{\partial \tau_\theta} = k\tau_\eta r\frac{\tau_u\beta^2 + \tau_\eta}{\left(\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + k\tau_\theta r + k\tau_\eta r + k\beta^2\tau_u r\right)^2} > 0$$

$$\frac{\partial a}{\partial \tau_u} = -\beta^2\tau_\theta\tau_\eta\frac{\tau_\theta + kr}{\left(\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + k\tau_\theta r + k\tau_\eta r + k\beta^2\tau_u r\right)^2} < 0$$

$$\frac{\partial a}{\partial \beta} = -2\beta\tau_u\tau_\theta\tau_\eta\frac{\tau_\theta + kr}{\left(\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + k\tau_\theta r + k\tau_\eta r + k\beta^2\tau_u r\right)^2} < 0$$

$$\frac{\partial a}{\partial \tau_\eta} = \tau_\theta\frac{\beta^2\tau_u\tau_\theta + k\tau_\theta r_m + k\beta^2\tau_u r_m}{\left(\tau_\theta\tau_\eta + \beta^2\tau_u\tau_\theta + k\tau_\theta r + k\tau_\eta r + k\beta^2\tau_u r\right)^2} > 0$$

Proof of Proposition 9

In order to compute the optimal choice of signal precision τ_ε, consider first the trading profit Π a trader i expects to earn once he has chosen τ_ε and conditional on observing the signal s_i and the stock price $\hat{p}$:

$$\begin{aligned}
E[\Pi\,|s_i,\hat{p}] &= x_i(s_i,\hat{p})\left(E[v\,|s_i,\hat{p}] - \hat{p}\right) \\
&= \frac{\tau_\varepsilon}{\rho}\left(s_i - \hat{p}\right)\left(E[v\,|s_i,\hat{p}] - \hat{p}\right) \\
&= \frac{\tau_\varepsilon}{\rho}\left(s_i - \hat{p}\right)\left(\frac{\tau_\varepsilon s_i + \left(\beta^2\tau_u + \tau_\theta\right)\hat{p}}{\tau_\varepsilon + \beta^2\tau_u + \tau_\theta} - \hat{p}\right) \\
&= \frac{\tau_\varepsilon}{\rho}\left(s_i - \hat{p}\right)\left(\frac{\tau_\varepsilon\left(s_i - \hat{p}\right)}{\tau_\varepsilon + \beta^2\tau_u + \tau_\theta}\right) \\
&= \frac{\tau_\varepsilon^2\left(s_i - \hat{p}\right)^2}{\rho\left(\tau_\varepsilon + \beta^2\tau_u + \tau_\theta\right)}
\end{aligned}$$

His expected trading profit at $t = 0$ (before knowing the realizations of s_i and $\hat{p}$) is:

$$E[\Pi] = E\left[\frac{\tau_\varepsilon^2\left(s_i - \hat{p}\right)^2}{\rho\left(\tau_\varepsilon + \beta^2\tau_u + \tau_\theta\right)}\right] = \frac{\tau_\varepsilon^2}{\rho\left(\tau_\varepsilon + \beta^2\tau_u + \tau_\theta\right)}E[(s_i - \hat{p})^2] \tag{A.9}$$

with

$$\begin{aligned}
E[(s_i-\hat{p})^2] &= E[(e^*+\theta+\varepsilon_i-E[v\,|z])^2] \\
&= E\left[\left(e^*+\theta+\varepsilon_i-\left(e^*+\frac{\beta\tau_u}{\beta^2\tau_u+\tau_\theta}(z-\beta e^*)\right)\right)^2\right] \\
&= E\left[\left(\theta+\varepsilon_i-\frac{\beta\tau_u}{\beta^2\tau_u+\tau_\theta}z+\frac{\beta^2\tau_u}{\beta^2\tau_u+\tau_\theta}e^*\right)^2\right] \\
&= E\left[\left(\theta+\varepsilon_i-\frac{\beta^2\tau_u}{\beta^2\tau_u+\tau_\theta}\theta-\frac{\beta\tau_u}{\beta^2\tau_u+\tau_\theta}u\right)^2\right] \\
&= E\left[\left(\left(1-\frac{\beta^2\tau_u}{\beta^2\tau_u+\tau_\theta}\right)\theta+\varepsilon_i-\frac{\beta\tau_u}{\beta^2\tau_u+\tau_\theta}u\right)^2\right] \\
&= \left(\frac{\tau_\theta}{\beta^2\tau_u+\tau_\theta}\right)^2\frac{1}{\tau_\theta}+\frac{1}{\tau_\varepsilon}+\frac{\beta^2\tau_u^2}{(\beta^2\tau_u+\tau_\theta)^2}\frac{1}{\tau_u} \\
&= \frac{1}{\beta^2\tau_u+\tau_\theta}+\frac{1}{\tau_\varepsilon}
\end{aligned}$$

where we have used $e=e^*$ (at $t=0$ a trader expects the manager to stick to his equilibrium strategy).

Substituting the last expression into (A.9), we have:

$$E[\Pi]=\frac{\tau_\varepsilon^2}{\rho\left(\tau_\varepsilon+\beta^2\tau_u+\tau_\theta\right)}\left(\frac{1}{\beta^2\tau_u+\tau_\theta}+\frac{1}{\tau_\varepsilon}\right)=\frac{\tau_\varepsilon}{\rho\left(\beta^2\tau_u+\tau_\theta\right)}=\frac{\rho\tau_\varepsilon}{\tau_u\tau_\varepsilon^2+\rho^2\tau_\theta}$$

The optimal precision τ_ε^* solves:

$$\max_{\tau_\varepsilon} E[\Pi]-c\tau_\varepsilon$$

The first order condition is

$$-c\rho^4\tau_\theta^2+\rho^3\tau_\theta-2c\rho^2\tau_u\tau_\theta\tau_\varepsilon-\rho\tau_u\tau_\varepsilon-c\tau_u^2\tau_\varepsilon^2=0$$

The solution to this quadratic equation is $\tau_\varepsilon^2=\frac{\rho}{2c\tau_u}\left(\sqrt{8c\rho\tau_\theta+1}-(2c\rho\tau_\theta+1)\right)$, whose unique positive root (if it exists) gives the expression for τ_ε^* in the proposition.

Substituting the expression for the optimal precision τ_ε^* into the optimal contract weights (from Proposition 7) yields

$$\begin{aligned}
m^* &= \tau_\theta\frac{\left(\sqrt{8c\rho\tau_\theta+1}-(2c\rho\tau_\theta+1)\right)+2c\tau_\theta}{\left(\sqrt{8c\rho\tau_\theta+1}-(2c\rho\tau_\theta+1)\right)(\tau_\theta+kr)+2c\left(\tau_\theta\tau_\eta+kr\tau_\theta+kr\tau_\eta\right)} \\
m^*+a^* &= \tau_\theta\frac{\left(\sqrt{8c\rho\tau_\theta+1}-(2c\rho\tau_\theta+1)\right)+2c\left(\tau_\eta+\tau_\theta\right)}{\left(\sqrt{8c\rho\tau_\theta+1}-(2c\rho\tau_\theta+1)\right)(\tau_\theta+kr)+2c\left(\tau_\theta\tau_\eta+kr\tau_\theta+kr\tau_\eta\right)},
\end{aligned}$$

which are independent of τ_u and hence no longer depend on noise trading.

Appendix B: A graphical illustration

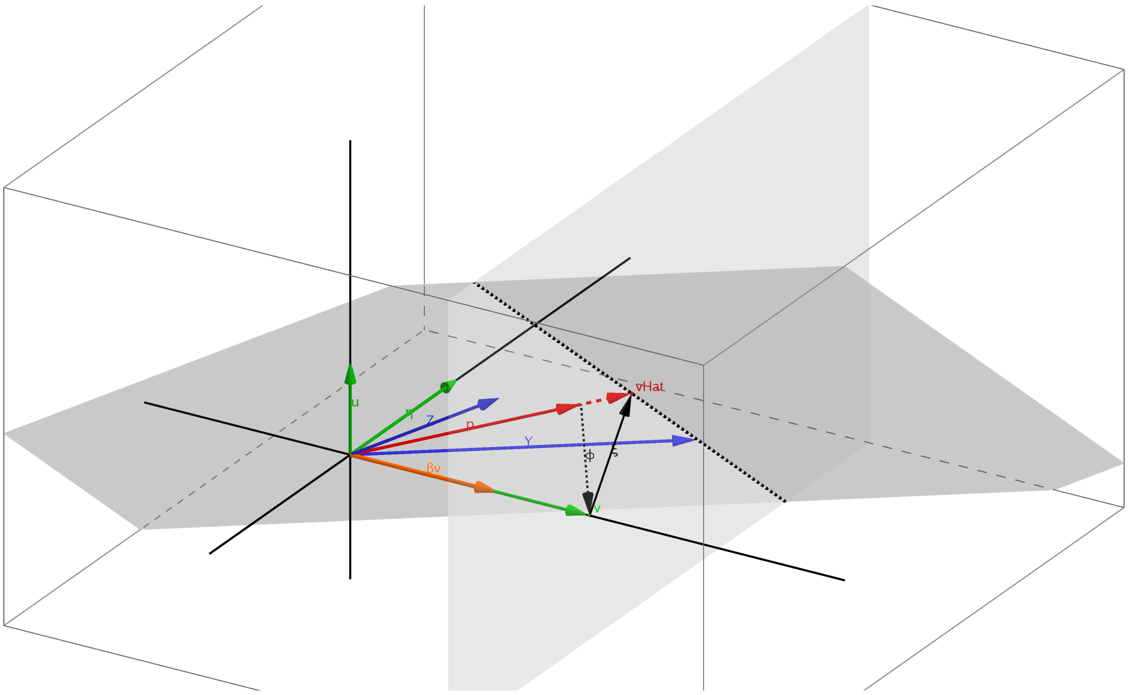

Figure 2: Graphical illustration of the information problem

Random variables can be represented by vectors whose length is given by their variance. Orthogonality means that two random variables are uncorrelated. The basis of the space we are working in is given by firm value (v), noise trading (u), and the noise in the signal y (η), which are all orthogonal to each other. These are depicted in green. The information in the order book is $z = \beta v + u$ (in blue). The extra signal is $y = v + \eta$ (also in blue). The space spanned by the blue vectors z and y, the slanted dark grey plane, represents public information about firm value at $t = 1$ (when the signal y is publicly available). An efficient stock price p (in red) is the projection of firm value v into that space. The error of that projection is ϕ. The inverse of the length of ϕ is τ, the informativeness of the stock price. Any measure of firm value m used in an incentive contract must be unbiased in the sense that $E[m|v] = v$ for all effort levels (i.e., on and off the equilibrium path), otherwise the incentive compatibility constraint is violated. Hence, the measure m has to be in the space orthogonal to v, represented by the light grey, vertical plane. The intersection of the two planes, the dotted line, represents unbiased performance measures based on publicly available information. For example, accounting information y represents a point in that space. The price p, however, does not reach that space, it is not an unbiased estimator of firm value for all effort levels. To make the information in the stock price unbiased for all effort levels (and thus usable for incentive contracting), one must use $\hat{v}$, the extension of p to the vertical plane. The error of using $\hat{v}$ as a performance measure is ξ, whose length is Σ. Note that $\hat{v}$ is the best performance measure. It is the closest point to v that lies on the dotted line. Hence, accounting information is not used in the incentive contract.