DISCUSSION PAPER SERIES

DP11242

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FINANCIAL ECONOMICS and MONETARY ECONOMICS AND FLUCTUATIONS

ISSN 0265-8003

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Discussion Paper 11242
Published 25 April 2016
Submitted 25 April 2016

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Acknowledgements
We thank José Antonio de Aguirre and Lorenzo Infantino for helpful conversations and suggestions. Kyle Dempsey and Natasha Rovo have provided excellent research assistantship. Financial support from the ERC Consolidator Grant No. 614879 (MONPMOD) is gratefully acknowledged.

Private Money Creation and Equilibrium Liquidity*

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April 24, 2016

Abstract

Can creation of private money fulfill the liquidity needs of the economy? The answer is no if the market of private money is run only by forces of perfect competition. Multiple equilibria are possible: there exist good equilibria with complete satiation of liquidity and absence of default on private money, and bad equilibria characterized by shortage of liquidity and partial default. In this framework, capital requirements, distortions to the demand or supply of private money, and the role of public liquidity in substituting private liquidity or in offsetting liquidity crises are investigated.

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1 Introduction

This paper considers an economy that lacks liquidity and studies the conditions under which creation of private money can provide it. The recent financial crisis has unveiled the existence of a shadow banking sector that for years has been able to provide some form of "quasi money." Suddenly, and this is the very origin of the crisis, transacting parties realized that what was believed to be a safe security – and therefore liquid – did not have appropriate backing in the quality of intermediaries' assets. What had been acceptable to satisfy liquidity needs became inadequate. The subsequent shortage of liquid assets produced a disruption in the real economy and a deep recession.[1]

Swings in creation and destruction of private money are not just a recent phenomenon. They have characterized almost every deep financial crisis throughout much of monetary history, with different names given to the intermediaries and their assets and liabilities. Economists have not abstained from the debate, offering opposing opinions on whether private liquidity should be issued and on the restrictions that should be imposed on the financial sector in doing so.[2]

Hundreds of years ago John Law, a Scottish financier, was one of the first proponents of the so called "real bills" doctrine. Under this view, liabilities of intermediaries acting without barriers to competition can satisfy the liquidity needs of the economy, as long as the assets of these intermediaries are free of risk ("real bills"), guaranteeing the safety of liabilities themselves.[3] Hayek (1976) took an extreme position, claiming that the creation of money should be completely privatized and run only by forces of competition. In his view, free competition was sufficient to enable the economy to achieve the efficient

[1] Brunnermeier (2009) and Stein (2010) provides an interesting account of the 2007-2008 credit and liquidity crunch.

[2] For a comprehensive perspective on the debate, see Aguirre (1985), Aguirre and Infantino (2013). Moreover, Sargent (2011) offers an interesting historical view on the tension between economic efficiency and financial stability.

[3] The theory has often been attributed to Adam Smith who instead fought John Law's ideas. Indeed, Smith (1976, p. 337) considered Law's idea of multiplying paper money – the so called Mississippi scheme – as the most extravagant project of banking. However, in Smith (1976, p. 323) there is a description of what has been identified as the origin of the "real bills" theory. The key aspect emphasized by Smith as opposed to Law is the ultimate convertibility of "real bills" into gold which avoids any excess money creation. Otherwise Smith (1976, p. 328) recognized the impossibility of distinguishing "real bills" from fictitious ones.

provision of liquidity. This perspective is also in line with a strong version of the "free-banking" theory, while weaker forms still envision the backing of liabilities with some sort of fractional reserve holdings.

More recently, monetary injections by the Federal Reserve during the 2008 financial crisis and the fact that many central banks are currently remunerating reserves have reopened this debate. Despite these very important developments, few papers provide frameworks in which private money created by financial intermediaries coexists with public liquidity, which allows policies related to the latter to be analyzed correctly.

The model presented in this paper addresses these issues. We begin with an economy with liquidity needs that are only partially fulfilled by publicly-issued securities. There is room for private money creation which is accepted in goods transactions as long as the issuer of private money remains solvent. In the baseline model, intermediaries freely compete to supply deposits – the quasi money – investing the borrowed resources in risky assets.

Our first result is that perfect competition is not enough to achieve efficiency. Multiple equilibria with different characteristics naturally arise. There is a good equilibrium in which efficiency is achieved through the supply of default-free private money. But there are also bad equilibria in which the liquidity needs are only satiated in good states of nature, while they are not in bad states because of intermediaries' default. There are even equilibria in which deposits are always partially or entirely seized in any contingency and therefore only held for their pecuniary return. This multiplicity shows the inherent fragility in the private supply of liquidity.

Our second main result is that the level of intermediaries' net worth is the critical determinant of the characteristics of the different equilibria. Therefore, a sufficiently high level of capitalization can enforce the efficient good equilibrium.

This result shares some similarities with the "real bills" doctrine, with some important caveats. According to this doctrine, assets of financial intermediaries should be completely safe. Intermediaries in our model, however, can invest in risky securities. What makes private money safe in our context is a sufficient level of capital. Regulation on holding "real bills" is substituted by regulation on holding capital. These different requirements have an important side effect. Under the "real bills" doctrine intermediaries transform risk-free private money into risk-free assets. In our framework, instead, they can invest in risky securities.

The backing of private money through capital requirements, however, is still prone to some fragility. First, it requires knowledge of the worst-case scenario: if some tail events are not accounted their realization can suddenly produce a shortage of liquidity and drop in consumption. Second, when we extend the model to add a real cost of default, we obtain richer results about the multiplicity of equilibria. For some levels of intermediary capitalization, there exist multiple equilibria associated with the same level of net worth. That is, for a given capitalization, the economy can experience both the good equilibrium with no default and the bad equilibrium in which private securities are partially seized in bad states of nature with the subsequent shortage of liquidity and drop in consumption. This suggests that net worth of intermediaries should be raised even more to be robust to appropriate stress-test exercises. This is another key result of our paper.

We then turn to the analysis of public liquidity and of alternative monetary/fiscal policy rules. Friedman (1960) advocated the separation between money and credit, putting barriers in place for intermediaries involved in credit creation to prevent their liabilities from playing a liquidity role. Related to this argument, we show that if the government can issue a sufficiently large level of interest-bearing public money (i.e. reserves or government bonds), then there is no role for privately-supplied money. However, interest-bearing public money requires adequate backing in terms of sufficiently high taxes that the government must levy.

Another important result of our analysis is that there are two arguments in favor of private money over public money. First, if the variability of the returns on the assets backing private money is not high then the amount of resources needed to adequately capitalize intermediaries is lower than the amount of real taxes needed to back public money. Second, if collecting large taxes is costly or unfeasible (due to considerations that are not captured by the model), then private liquidity creation is essential.

We also discuss the policy response to a liquidity crunch. If prices are flexible, the government can increase the real value of existing liquidity, offsetting the shortage by raising real taxes in order to reduce nominal prices, an argument related to the fiscal theory of the price level. Otherwise, the government could keep prices constant and inject more public money in the economy, as the Federal Reserve did in 2008. In the end, the way out of the crisis is to substitute the insufficient backing of private money with more backing of public money. The efficacy of this strategy relies on the ability of the government to raise real taxes. However, if increasing real taxes is not feasible, the ability of the government to stave off a liquidity crisis might be limited.

Our baseline analysis is then extended to address elements that distort demand or supply in the market for private money. On the demand side we allow households to pay a premium to get private insurance against the liquidity shock. Insurance does not solve the problem of multiplicity, but works to eliminate the sudden drop in consumption in default states. Moreover, private insurance is optimal whenever public insurance through public money is minimal. On the supply side, some barriers to competition (modelled in a market with monopolistic competition) raise the price of private money while also reducing demand and creating shortages even in no default equilibria.

Other papers have analyzed the interaction of private money issued by financial intermediaries and public money, mostly using overlapping generations models. Sargent and Wallace (1982) study the "real bills" doctrine in comparison with the quantity theory in a context in which private credit instruments and public money are always perfectly substitutes. We instead link their substitutability to the balance sheet conditions of intermediaries and of the government/central bank. We share with them a finance theory outlook to the problem in which the government/central bank can be interpreted as an intermediary. Another closely related paper is Bullard and Smith (2003). However, some of the results in that paper are related to the ability of money to eliminate dynamic inefficiencies in standard overlapping generations models. Our framework, instead, abstracts from the inefficiencies of OLG models by using an infinite-horizon formulation.

Kiyotaki and Moore (2002) present a model in which limited commitment gives rise to roles for money and for financial intermediaries. While there is a distinction between our framework and theirs in terms of the underlying assumption (risk vs. lack of commitment), financial intermediaries in both frameworks operate in a way that allows them to overcome the friction, by using either net worth (in our model) or by investing in a commitment technology (in their model).

In Brunnermeier and Sannikov (2016), private and public money coexist, but money has the function of store of value. More importantly, the focus of their paper is on the effects of nominal contracts and debt-deflation, while a central theme of our paper is the link between the value of money and the balance sheet of the issuer, including the possibility of default.

This paper is organized as follows. Section 2 presents a brief overview of the framework and discusses the main mechanism and results. It is followed in Section 3 by a thorough presentation of the model. Section 4 discusses the equilibrium under perfect competition, while Section 5 adds default costs. Section 6 studies the role of alternative specifications of the monetary/fiscal policy rules. Section 7 investigates the role of private insurance and the model with monopolistically competitive financial intermediaries. Section 8 concludes.

2 Equilibrium Liquidity

We use this section to expound some of the concepts we are going to develop in the model of this paper and discuss some of the key mechanisms at work. We first introduce the concept of liquidity and then illustrate how to value liquidity before briefly overviewing the equilibrium determination.

First, we assume that only debt can provide liquidity services, while other securities such as equity cannot. Second, in each state of nature, a debt security is liquid only if it is not defaulted in that state. In other words, a debt security does not provide liquidity services when it is defaulted. Our assumptions are in the same spirit as Gorton and Pennacchi (1990) and Stein (2012), although more general since they do not allow risky securities to provide liquidity services.[4] We instead link directly the liquidity value of a security with its safety in a particular contingency. The same security can be liquid and therefore accepted in trading goods in a favorable state, but unacceptable in a bad state if the promised payoff is even partially seized. This assumption can be simply justified by the existence of some time requirement to complete the default procedure. These delays are enough to prevent the use of the security in trading goods.

To price securities that have liquidity value, it is necessary to depart from standard asset pricing theory, for which assets are valued only by their pecuniary return.[5] Consider first a security with a promised payoff of one unit. In our model, it can be issued by the government or by private financial intermediaries. However, for reasons that will be explained below, the publicly-issued security is always free of credit risk and, therefore, always liquid. We can think of it as short-term government bonds or interest-bearing reserves with

[4]Stein (2012) excludes this possibility on the ground that information-sensitive securities can carry some problems of adverse selection between transacting parties; these informational problems are not modeled in Stein (2012).

[5]See Lagos (2010, 2011).

the central bank.[6] The analysis below will show that the household's demand of the publicly-issued security is flat at a price Q_t such that

$$Q_t = \beta E_t \left\{ \frac{P_t}{P_{t+1}} (1 + \mu_{t+1}) \right\} \tag{1}$$

where β is the consumer's intertemporal discount factor and P_t is the general price index. Accordingly $\beta P_t / P_{t+1}$ is the stochastic discount factor used to evaluate nominal payoffs between time t and any contingency at time $t+1$. The non-negative term μ_{t+1} captures the non-pecuniary return of the security. Households are willing to pay more for the publicly-issued security because they can exploit its liquidity services in purchasing goods. The liquidity premium can be just captured by the non-negative difference $Q_t - Q_t^f$ where $Q_t^f = \beta E_t \{P_t / P_{t+1}\}$ is the notional price of a security with similar credit-risk characteristic but no liquidity value.

The key observation is that the equilibrium price Q_t cannot be understood simply from equation (1), which reflects only the demand side of public security. One must also look at the supply side of the security, as well as both the demand and supply of competing private sources of liquidity. Turning first to the central bank's supply of reserves, solvency of the government implies

$$\frac{B_{t-1}}{P_t} = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left(T_{t+j} + (Q_{t+j} - Q_{t+j}^f) \frac{B_{t+j}}{P_{t+j}} \right) \right\} \tag{2}$$

which is indeed critical to understand why the publicly-issued security is free of risk in our model. Given a supply B_t of securities of unitary promised face value, full backing is always attainable by drawing on three sources. The first is the present discounted value of real taxes, denoted by T, and corresponds to the first term on the right-hand side of the above solvency condition. The second source results from the liquidity properties of reserves which produce real rents that can back the value of outstanding obligations.[7] But again this source might be out of control of the government since it depends on the equilibrium between competing sources of liquidity and therefore on the equilibrium value of liquidity. Third, absent a complete backing derived from the first two sources, the price level can move to adjust the real value

[6] In our model, government includes both the treasury and the central bank.

[7] This source is often called seigniorage.

of promised obligations to meet available resources, as shown on the left-hand side of the solvency condition.[8] This is why solvency is not an issue for publicly-issued securities in our model.

The above setting is also useful to understand demand and supply of private liquidity and overall equilibrium liquidity. In our baseline model, we will show that household's demand for private liquidity, which takes the form of deposits issued by intermediaries, is flat at the price Q_t^D

$$Q_t^D = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left[(1 - I_{t+1})(1 - \chi_{t+1}) + (1 + \mu_{t+1}) I_{t+1} \right] \right\} \tag{3}$$

where I_{t+1} is an indicator function equal to one when the privately-issued security is not defaulted. In the case of default, χ_{t+1} is the seized fraction. Therefore, $Q_t^D = Q_t$ whenever financial intermediaries are solvent in all possible states.

It is critical to explain why the privately-issued security can be seized partially in some or all contingencies, unlike the public security. A solvency condition similar to that of the government applies:

$$(1-\chi_t)\frac{D_{t-1}}{P_t} - (1+r_t)K_{t-1}^I = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left(T_{t+j}^I + (Q_{t+j}^D - Q_{t+j}^f)\frac{D_{t+j}}{P_{t+j}} \right) \right\} \tag{4}$$

although there are some notable differences.[9] Physical capital, K_t^I, shows up as an asset with a return r_t – the exogenous force of our model – that can trigger default on deposits D_t. However, this is not the critical difference with respect to the solvency condition of the government. The key point rests on the limited backing of intermediaries' net liabilities which can only be resolved by (partial) default. First, adjusting the price level is not an option for a single intermediary. What remains are transfers from households, T_t^I, and resources obtained by issuing liabilities. However, the first channel can be limited by the fact that positive transfers from the private sector T_t^I are nothing more than negative dividends or injection of further net worth – and it is reasonable to assume that there is a bound on these resources as we

[8] A recent literature stemming from the work of Sims (2005) has studied the inflationary consequences of paying interest on central-bank reserves when the central bank faces losses because its portfolio of assets includes also risky securities.

[9] In the next section, we present a simpler model of intermediaries that live for only two periods in an overlapping way. This is to keep tractability and without losing generality.

will do. The liability channel is similar to the government's, but with two limitations. On the one hand, $Q_t^D \leq Q_t$, with strict inequality if there is a positive probability of default of the private intermediary. On the other hand, and more importantly, the amount of deposits is determined in equilibrium as a function of households' demand.

Finally, we draw the important distinction between the issuance of public and private liquidity. In the baseline model, we take as given a certain level of public liquidity that is created by the government. But private liquidity creation arises from the interaction between households and financial intermediaries. Therefore, while Equations (1) and (2) are sufficient to describe public liquidity, Equations (3) and (4) are not enough to completely characterize private liquidity creation. Assuming a market characterized by perfect competition, we show that the supply of private liquidity is infinitely elastic at the price

$$Q_t^D = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left[(1 - I_{t+1})(1 - \chi_{t+1}) + I_{t+1} \right] \right\}. \tag{5}$$

Demand (3) and supply (5) of private liquidity meet at

$$E_t \left\{ \frac{P_t}{P_{t+1}} \left[(1 + \mu_{t+1}) I_{t+1} \right] \right\} = E_t \left\{ \frac{P_t}{P_{t+1}} I_{t+1} \right\}.$$

The above equation delivers on this paper's main result. Private money creation under perfect competition implies multiplicity of equilibria. There can be equilibria in which deposits are never defaulted ($I_{t+1} = 1$ in all contingencies) and correspondingly the equilibrium level of deposits is enough to satiate all needs ($\mu_{t+1} = 0$ in all contingencies). There are also equilibria with partial default in some states or in all states. As consequence, a shortage of liquidity arises in these states, causing an inefficient level of consumption.

Key in our analysis is that the different equilibria stem from different levels of intermediaries' net worth, and therefore by the backing provided to private money. All these results will be derived and discussed in Section 4. We now present the details of the model.

3 Model

The model features four sets of actors: households, firms, financial intermediaries and a government (including treasury and central bank). We begin by describing each of these groups and then discuss equilibrium.

3.1 Households

Households are infinitely lived and have the following intertemporal preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + X_t\right], \tag{6}$$

where E_0 is the expectation operator at time 0 and β is the intertemporal discount factor with $0 < \beta < 1$. C_t and X_t denote consumption of the same good but done in different subperiods within period t: C_t is consumed in the first subperiod, X_t in the second.[10] There is, however, a financial friction putting barriers to which securities can be used to purchase consumption goods in the first subperiod. The limitation says that liquidity services can only be provided by any publicly and privately-created debt security so long as the security is not defaulted, even partially, in that particular state of nature in which it is exchanged for goods. Liquidity can then be supplied also by securities which have credit risk, but not in the state of nature where risk materializes. As discussed in the previous section, the liquidity friction can be simply justified by delays, due to the default procedure, in receiving the seized payoff in time to make purchases during the first subperiod.

We assume that only two securities can potentially provide liquidity services: a publicly-issued security (B), which has the interpretation of government debt or interest-bearing central-bank reserves; and deposits (D), which are privately created by financial intermediaries. At the beginning of a generic period t, households are subject to the liquidity constraint:

$$B_{t-1} + I_t D_{t-1} \geq P_t C_t, \tag{7}$$

where I_t is an indicator function taking value of one in the contingency where the payoff of deposits is not seized; P_t is the price level of the consumption good.[11] Later, we discuss assumptions under which government bonds are riskless and therefore always provide liquidity services.

[10] This assumption is reminiscent of the structure used in new monetarist models such as Lagos and Wright (2005), in which money is required for transactions in one of the subperiods. We share with those models a key assumption, namely the inability of the buyer to commit to settle payments after a transaction has taken place; that is, to buy using credit.

[11] Our result of perfect substitution between the liquidity provided by B_{t-1} and D_{t-1} is motivated by the results of Nagel (2014), who estimates a high elasticity of substitution between public and private liquidity.

According to (6), households display some aversion to risk, given by logarithmic utility in consuming goods during the first subperiod. At the same time, we make the simplifying assumption that utility is linear in the second subperiod. After purchasing X_t consumption goods, they make portfolio decisions regarding how many deposits, bonds, and capital to hold until next period. Their end-of-period budget constraint is

$$\begin{aligned} & P_t X_t + Q_t B_t + Q_t^D D_t + P_t K_t^H \leq (1 - I_t)(1 - \chi_t) D_{t-1} \\ & + [B_{t-1} + I_t D_{t-1} - P_t C_t] + P_{t-1} K_{t-1}^H \left(1 + i_t^K\right) + \Pi_t + \Pi_t^I - P_t T_t - N_t \end{aligned} \quad (8)$$

where Q_t and Q_t^D are the nominal prices of government bonds and deposits, respectively; χ_t is the rate of default on deposits; K_t^H is the holding of capital; and i_t^K is its nominal return. At the end of period, households receive profits from firms, Π_t, and from financial intermediaries, Π_t^I, pay lump-sum real taxes to the government, T_t, and finance net worth of financial intermediaries in the amount N_t.

Consumption and portfolio choices are implied by the maximization of (6) under the constraints (7) and (8). The assumption of linear utility with respect to second-subperiod consumption greatly simplifies the analysis to the extent that the Lagrange multiplier λ_t associated with the constraint (8) is just equal to the inverse of the price level, $\lambda_t = 1/P_t$. A further implication is that demand of goods in the first subperiod is:

$$C_t = \frac{1}{1 + \mu_t} \quad (9)$$

where μ_t/P_t is the Lagrange multiplier associated with the constraint (7). Since $\mu_t \geq 0$, $C_t \leq 1$ and at the first best $C_t = 1$. The first-best allocation follows from the fact that the relative price between the consumption of goods at the beginning and at the end of period is one. Absent the liquidity constraint (7), households would equate the marginal rate of substitution between the two consumptions, $1/C_t$ to their relative price, one. This implies the optimal level $C_t = 1$. Clearly, in the constrained economy, they will not exceed the first-best level and hoard any excess financial assets until the second subperiod.

To conclude the characterization of the household's problem, demand for reserves, deposits and capital are optimally allocated when their respective prices satisfy the following conditions:

$$Q_t = \beta E_t \left\{ \frac{P_t}{P_{t+1}} (1 + \mu_{t+1}) \right\}, \quad (10)$$

$$Q_t^D = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left[(1 - I_{t+1})(1 - \chi_{t+1}) + (1 + \mu_{t+1}) I_{t+1} \right] \right\}, \tag{11}$$

$$1 = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left(1 + i_{t+1}^K \right) \right\}. \tag{12}$$

An implication of (10) and (11) is that $Q_t \geq Q_t^D$, with strict inequality when deposits are seized in some contingency. Liquidity services provide benefits to the issuer by lowering borrowing costs.

Finally, a transversality condition applies, imposing an appropriate limit on the rate of growth of assets held by households

$$\lim_{j \to \infty} \beta^j \left(\frac{Q_{t+j} B_{t+j} + Q_{t+j}^D D_{t+j}}{P_{t+j}} + K_{t+j}^H \right) = 0. \tag{13}$$

Equation (13) holds almost surely looking forward from each time t and each contingency at time t.

3.2 Firms

Firms produce consumption goods using borrowed capital supplied by households and financial intermediaries. The production technology is $Y_t = A_t K_{t-1}^{\alpha}$, with $0 < \alpha < 1$, where Y_t is output, K_{t-1} is capital, and A_t is a technology parameter which is the only stochastic disturbance of the model. We simply assume that A_t takes two values A_{high} and A_{low} with probabilities $1 - \pi$ and π, such that $A_{high} > A_{low} > 0$. We denote $A \equiv (1 - \pi) A_{high} + \pi A_{low}$. Capital is borrowed at time $t - 1$ in the amount $P_{t-1} K_{t-1}$ and repaid at time t through the contingent nominal return $(1 + i_t^K)$ which is determined by the proceedings from selling goods. The simplifying assumption we make is that nothing is left to firms once borrowed capital is fully paid back.[12] This defines the nominal contingent return on capital at the rate

$$1 + i_t^K = \frac{P_t Y_t}{P_{t-1} K_{t-1}} = \frac{P_t}{P_{t-1}} A_t K_{t-1}^{\alpha - 1}. \tag{14}$$

The above equation can be used to determine the total demand of capital by firms, once we take expectations at time $t - 1$ and use (12):

$$K_{t-1} = [\beta A]^{\frac{1}{1-\alpha}}. \tag{15}$$

[12] Our modelling of firms' behavior is just meant to simplify the analysis and could be interpreted as if households/intermediaries were directly running production themselves.

The demand for capital is independent of the realization of the technology parameter and independent of conditions in the money market. Key in our analysis is that this amount of capital will always be available in supply through households. As a consequence, any disruption in financial intermediation will not affect equilibrium capital and output. This abstraction allows us to focus on the consequences that a financial crisis has for the supply of liquidity rather than on supply of goods.

A further implication of (14) and (15) is that the real return on capital is given by

$$1 + r_t^K \equiv (1 + i_t^K)\frac{P_{t-1}}{P_t} = \frac{1}{\beta}\frac{A_t}{A}$$

which defines a high and low level of the real interest rate, respectively r_{high}^K and r_{low}^K, associated with the two states of the exogenous disturbance.

3.3 Financial Intermediaries

For expositional purposes, we make the simplifying assumption that financial intermediaries live only for two periods in an overlapping way. Consider intermediaries which start to operate at time t and end their activity at time $t+1$. In the first period they collect funds by issuing deposits and raising net worth. Both sources come from households' investment. They have the technology to transform these funds into capital at the price P_t:

$$P_t K_t^I = N_t + Q_t^D D_t. \tag{16}$$

In the following period $t+1$ nominal profits of intermediation are given by

$$\Pi_{t+1}^I = (1 + i_{t+1}^K) P_t K_t^H - (1 - \chi_{t+1}) D_t, \tag{17}$$

reflecting the return on capital and the cost of repaying deposits. The rate of default, χ, is endogenous in our framework and depends on a simple and key assumption that financial intermediaries cannot deliver negative profits in the last period of their life. This assumption can be justified by the limited liability of intermediaries, which is consistent with the general view that private financial intermediaries have a limited backing from their shareholders.[13] Using (17), non-negative profits imply that the default rate is simply

[13] In the discussion of Section 2 the non-negative profit requirement corresponds to non-positive transfers T^I.

given by

$$\begin{aligned}\chi_{t+1} &= \max\left(0, 1-(1+i^K_{t+1})\frac{P_t K_t}{D_t}\right)\\ &= \max\left(0, 1-(1+i^K_{t+1})\frac{N_t + Q^D_t D_t}{D_t}\right), \quad (18)\end{aligned}$$

where the second line follows from the intermediaries' balance sheet (16). Default is more likely when the return on capital, the intermediaries' net worth and price of deposits are low enough. Everything else being equal, a higher level of deposits raises the default rate.

We now analyze the market structure in which intermediaries operate. In the baseline model we assume that the market is perfectly competitive. There is an infinite number of small financial intermediaries supplying a homogenous product in the form of a deposit security. All these intermediaries are marginal with respect to the size of the overall market in which there is free entry and exit. As a consequence they take prices as given. All these forces eliminate any rents from financial intermediation. To understand the source of rents, consider that households inject capital N_t into intermediaries and receive profits in the next period as the payoff of their investment. Rents from financial intermediation are completely abated if initial net worth is equal to the expected discounted value of profits through the discount factor used by households[14]

$$N_t = E_t\left\{\beta\frac{P_t}{P_{t+1}}\Pi^I_{t+1}\right\}. \quad (19)$$

We can now evaluate the right-hand side of (19) using (16) and (17) to obtain

$$\begin{aligned}E_t\left\{\beta\frac{P_t}{P_{t+1}}\Pi^I_{t+1}\right\} &= \beta E_t\left\{(1+i^K_{t+1})\frac{P_t}{P_{t+1}}(N_t + Q^D_t D_t) - \frac{P_t}{P_{t+1}}(1-\chi_{t+1})D_t\right\}\\ &= N_t + Q^D_t D_t - \beta E_t\left\{\frac{P_t}{P_{t+1}}(1-\chi_{t+1})\right\}D_t.\end{aligned}$$

Zero rents imply a flat supply of deposits at the price

$$Q^D_t = \beta E_t\left\{\frac{P_t}{P_{t+1}}(1-\chi_{t+1})\right\}, \quad (20)$$

[14] Households do not internalize the non-pecuniary benefits of operating intermediaries coming from the provision of liquidity services.

which reflects the expected discounted value of the payoff of the security. Financial intermediaries maximize profits, subject to the zero-rent condition.

3.4 Government

Government includes together the treasury and the central bank. For expositional simplicity, the only liability is B_t, central-bank reserves. At time $t-1$ government has to pay back B_{t-1} using newly issued securities B_t at price Q_t and collecting real lump-sum taxes T_t at the price P_t. Therefore its flow budget constraint is:

$$B_{t-1} = Q_t B_t + P_t T_t \,.$$

Iterating forward the last expression and combining it with (10), we get:

$$\frac{B_{t-1}}{P_t} = E_t\left\{\sum_{j=0}^{\infty}\beta^j\left(T_{t+j} + \beta\frac{\mu_{t+1+j}}{P_{t+1+j}}B_{t+j}\right)\right\} + \lim_{j\to\infty}\beta^j E_t\left\{\frac{Q_{t+j}B_{t+j}}{P_{t+j}}\right\}. \quad (21)$$

Let us first focus on the second term on the right-hand side. Households' transversality condition (13) together with the balance sheet of intermediaries (16) imply

$$\lim_{j\to\infty}\beta^j E_t\left\{\frac{Q_{t+j}B_{t+j}}{P_{t+j}}\right\} = \lim_{j\to\infty}\beta^j E_t^j\left\{K_{t+j} - \frac{N_{t+j}}{P_{t+j}}\right\} = -\lim_{j\to\infty}\beta^j E_t\left\{\frac{N_{t+j}}{P_{t+j}}\right\},$$

where the second equality follows from the observation that total capital is constant in equilibrium as shown under the firms' problem. If we just focus on equilibria in which real net worth of intermediaries does not grow at a rate higher than or equal to $1/\beta^j$ as j goes to infinity, then the second term on the right-hand side of (21) is zero and the intertemporal budget constraint of the government simplifies to

$$\frac{B_{t-1}}{P_t} = E_t\left\{\sum_{j=0}^{\infty}\beta^j\left(T_{t+j} + \beta\frac{\mu_{t+1+j}}{P_{t+1+j}}B_{t+j}\right)\right\}.$$

Another way to write it is to use (10):

$$\frac{B_{t-1}}{P_t} = E_t\left\{\sum_{j=0}^{\infty}\beta^j\left(T_{t+j} + (Q_{t+j} - Q^f_{t+j})\frac{B_{t+j}}{P_{t+j}}\right)\right\}, \quad (22)$$

where we have further defined Q_t^f as the notional price of a risk-free bond that does not provide liquidity services.

The government can always fully repay its outstanding liabilities – the reason for why central-bank reserves are always liquid in our model – by drawing on three main sources. First, the government can levy real taxes on households. Second, as reflected in the second term on the right-hand side of (22), liquidity premia lower the cost of borrowing and enhance the ability to repay debt, captured by a positive difference between the price of reserves and that of similar risk-less but not liquid security. Finally, the price level can adjust, reducing if necessary the amount of real resources to pay.

The government chooses two policy instruments in our model. In the baseline case, we assume that government sets the path of reserves and taxes $\{B_t, T_t\}$ given an initial condition on B. To simplify our analysis, we find it convenient to assume that the tax rule is of the form

$$T_t = (1-\beta)T - (Q_t - Q_t^f)\frac{B_t}{P_t}. \tag{23}$$

In each period, real taxes are proportional to a constant, T, and fall proportionally to the real value of outstanding reserves. The proportionality factor is captured by the liquidity premium $Q_t - Q_t^f$. The tax rule greatly simplifies our analysis since once it is substituted into (22) it yields

$$\frac{B_{t-1}}{P_t} = T. \tag{24}$$

A further simplification is to assume that reserves are in constant supply, $B_t = B$. It then follows that the specification of the monetary/fiscal policy determines, uniquely, a constant price level P. Later, in Section 6, we discuss the implications of alternative monetary-fiscal policy rules.

3.5 Equilibrium

We use a standard concept of equilibrium, in which households maximize utility, financial intermediaries operate under perfect competition, goods and asset markets clear, and the real value of government debt equals the present-discounted value of taxes and seigniorage, as shown in (22), given a monetary and fiscal policy rule. In particular, intermediaries take as given the price of deposits, Q_t^D. This means that free entry at a price Q_t^D occurs by intermediaries supplying deposits that are homogenous to those already in the market.

As will be clear later, this implies entering with the same level of net worth as other incumbents.[15]

We have already characterized some equilibrium results, namely that the level of capital is constant and that output just varies with the realization of technology. The price level is also constant given the specification of the monetary/fiscal policy rule.

The following set of equations is what is left to determine the remaining variables. The liquidity constraint (7) now simplifies to

$$B + I_t D_{t-1} \geq PC_t, \tag{25}$$

while first-subperiod consumption and the Lagrange multiplier μ_t are related through

$$C_t = \frac{1}{1+\mu_t}. \tag{26}$$

Demand for government bonds implies the following relationship between their price Q_t and the Lagrange multiplier

$$Q_t = \beta E_t(1 + \mu_{t+1}). \tag{27}$$

Demand and supply for deposits respectively yield the following price schedules

$$Q_t^D = \beta E_t \left\{(1 - I_{t+1})(1 - \chi_{t+1}) + (1 + \mu_{t+1})I_{t+1}\right\}, \tag{28}$$

$$Q_t^D = \beta E_t \left\{(1 - I_{t+1})(1 - \chi_{t+1}) + I_{t+1}\right\}. \tag{29}$$

Were χ_t exogenous and therefore also I_t, the above set of five equations would suffice to determine D_t, C_t, μ_t, Q_t and Q_t^D. However, key in our model is that χ_t is endogenous and depends on the limited backing of intermediaries

[15] An alternative route is to use a mechanism design approach in which intermediaries compete by offering the best contract to depositors, and thus can possibly post intermediary-specific prices of deposits. Under this approach, the equilibrium must be robust to a set of deviations that we currently do not allow; namely, the possibility that some intermediary deviates and offers deposits at a price different from the one used by all other intermediaries. This is because under perfect competition all intermediaries are small with respect to the market, sell a homogenous deposit security and take prices as given. Section (7) discusses the extension to a market of monopolistic competition in which the fact that deposits are non-homogeneous allows intermediaries to differentiate their choices.

captured by the non-negative constraint on profits. With constant prices, (18) simplifies to

$$\chi_{t+1} = \max\left(0, 1 - (1 + r^K_{t+1})\frac{N_t + Q^D_t D_t}{D_t}\right) \tag{30}$$

where the exogenous force triggering default is now the realized real return on capital, $1+r^K_{t+1}$. Recall that given the two state process for the exogenous disturbance A_t, it can take the two values $1+r^K_{high}$ and $1+r^K_{low}$. Equation (30) further shows that there are important endogenous feedback effects between the price of deposits, Q^D, and the default rate and between the level of deposits, D_t, and the default rate. What is not determined is the level of intermediaries' initial net worth, N_t. The next section will show that this level will be key to determine the characteristics of the equilibrium.

4 Perfect competition

Our model is one of coexistence between public and private liquidity. Given the government's backing privileges, B is always available for liquidity purposes. For private liquidity to play a role, we need to make assumptions that limit the availability of public liquidity. As already discussed, first-best consumption requires $C_t = 1$, above which any excess liquidity will be hoarded. As shown in (25), if B/P is greater than one, there is no need to have private liquidity. Therefore, we set an upper bound on taxes, $T < 1$, implying $B/P < 1$. At this point, we can simply justify this limit with political constraints that preclude very high level of taxation. In Section 6 we will further explain the interaction between the specification of the monetary/fiscal policy rule and equilibrium liquidity.

The key question addressed in this section is whether private liquidity can become a perfect substitute for scarce public liquidity.[16] The market

[16] We can also inquire whether the economy can just live with private liquidity, relinquishing publicly-issued money of any such role as proponents of "free banking" or of "real bills" doctrine have been arguing for a long time. Opponents to such theories stress that the disappearance of public money would create problems for the determinacy of price level. This objection in not valid in our model, provided that some government-issued securities remain always in positive supply and are held by households just for their pecuniary return. Even if B has no liquidity value, the intertemporal budget constraint (22) is still an equilibrium condition with $Q_t = Q^f_t$, rendering it possible to determine the price

structure under analysis in this baseline model is of the same form as that advocated by extreme theories of "free banking" discussed by Hayek (1976), for example. Indeed, it is a completely unfettered system characterized by perfect competition that eliminate any operational rents from financial intermediation. Demand for deposits implies a flat schedule at the price given by (28). Also, supply of deposits implies a flat supply at the price (29). At their meeting point,

$$E_t\left\{(1+\mu_{t+1})I_{t+1}\right\} = E_t\left\{I_{t+1}\right\}. \tag{31}$$

The reading of equation (31) yields one of the key results of this paper. Perfect competition is not enough to avoid instability, in the sense of multiple equilibria. There are indeed equilibria with complete satiation of liquidity in all contingencies – the above equation is satisfied by $I_{t+1} = 1$, $\mu_{t+1} = 0$ and $\chi_{t+1} = 0$ in all states of nature. But, there are other equilibria in which there is satiation and no default in some contingencies ($I_{t+1} = 1$, $\mu_{t+1} = 0$ and $\chi_{t+1} = 0$ in these contingencies) and default with shortage of liquidity in the remaining contingencies ($I_{t+1} = 0$, $\mu_{t+1} > 0$ and $\chi_{t+1} > 0$). Finally, there are equilibria with default and liquidity shortages in all states of nature. We now characterize all these equilibria.

4.1 Good equilibrium

In the good equilibrium, intermediaries are always solvent and there is complete satiation of liquidity ($\mu_t = 0$) in all states of nature. It follows that consumption in the first subperiod is at the efficient level $C_t = 1$. Prices of government bonds and deposits are equated at $Q_t = Q_t^D = \beta$, since both provide maximal liquidity services. Using (25), the level of deposits is given by $D_t = 1 - B/P = 1 - T > 0$, complementing the supply of public money.[17]

What makes this equilibrium feasible is the complete solvency of intermediaries. Profits should be non-negative in any state, which needs to be

level with appropriate policies of the kind already discussed. Woodford (1995) has already made this point in a different environment.

[17] In the model, any level of deposits greater than or equal to $1 - T$ can arise in equilibrium. That is because households and intermediaries have access to the same technology. Thus, households can either invest directly in capital or hold excess deposits and have banks investing in capital on their behalf. However, if there are intermediation costs, households are better off by holding only the minimum amount of deposits required for liquidity purposes. Our result $D_t = 1 - T$ can thus be viewed as arising from the limiting case in which the cost of intermediation goes to zero.

checked only for the lowest realization of the return on capital:

$$(1 + r^K_{low})P_t K^I_t - D_t \geq 0.$$

By using the previous result $D = 1 - T$ and the balance-sheet constraint (16) to substitute for $P_t K^I_t$ in the above inequality, we obtain a lower bound on net worth, $N_t \geq \bar{N}$, defined by

$$\bar{N} = (1 - T)\left[(1 + r_{low})^{-1} - \beta\right], \tag{32}$$

which is positive since $(1 + r_{low}) < 1/\beta$.

Net worth should be sufficiently high for the no default equilibrium to exist. What happens when net worth falls below the threshold? This opens the possibility of equilibria with default, explored in the next Section.

4.2 Default equilibria

We now analyze default equilibria characterized by insolvency (even partially) of financial intermediaries. There are several types of these equilibria depending on (i) the rate of default and (ii) whether default occurs only in the low state or in all contingencies. We first investigate equilibria in which there is default only in the low state, and then consider equilibria with the possibility of default in all contingencies.

The feedback loop in the default equilibrium is related to the lower price of deposits Q^D_t. Due to the lower price, banks must pay a higher return on deposits. In the low state, though, banks do not have enough resources due to the realized low productivity; therefore, they default on their promises. Anticipating default, households are willing to hold deposits only if their return includes a premium for the possibility of default, lowering Q^D_t:

$$Q^D = \beta[(1 - \pi) + \pi(1 - \chi_{low})].$$

Therefore, this is an equilibrium.

What is interesting is that default in the low state does not prevent the economy from being fully satiated of liquidity in the high state. Indeed, equation (31) implies that $\mu_t = 0$ in the high state and therefore that the amount of deposits held by households is the same as in the good equilibrium, $D = 1 - B/P = 1 - T$. Denoting with *high* and *low* the state of nature characterized respectively by high and low productivity, consumption C_t is now state contingent and given by $C_{high} = 1$ and $C_{low} = B/P = T < 1$. A liquidity shortage arises in the low state because transacting parties do not accept defaulted securities for liquidity purposes.

The default equilibrium shares some features in common with a liquidity crisis. In the good state, consumption allocation is the same as in the good equilibrium. However, deposits bear some credit risk. When that risk materializes in the bad state, private liquidity is defaulted and the economy experiences a liquidity crunch with a sudden fall in consumption.

To have default in the low state, the zero-profit constraint should bind and therefore

$$(1 + r^K_{low})(N + Q^D D) - (1 - \chi_{low})D = 0. \tag{33}$$

Solving for the level of net worth consistent with a rate of default χ_{low} in the low state, we obtain:

$$N = \frac{1 - T}{1 + r^K_{low}} \left[(1 - \chi_{low}) - \beta(1 + r^K_{low})(1 - \pi\chi_{low})\right] \tag{34}$$

which is decreasing in χ_{low} and is equal to $\bar{N}$ when $\chi_{low} = 0$. As net worth decreases, the rate of default rises. This weakly negative relationship between net worth and the rate of default is a key result of the baseline model with interesting implications for market regulation that we will discuss in the next section. Here, we simply characterize all the possible default equilibria. To this end, consider that as χ_{low} increases, Q^D further falls which in turn depresses profits also in the high state:

$$\Pi^I_{high} = (1 + r^K_{high})(N + Q^D D) - D, \tag{35}$$

up to the point in which there is also default in this state. By combining (33) and (35) under $\Pi^I_{high} = 0$, we find that $\bar{\chi}_{low} = 1 - (1 + r_{low})/(1 + r_{high})$ is the default rate in the low state at which default is also triggered in the high state. Above $\bar{\chi}_{low}$, the relationship between the two rates of default is given by

$$\chi_{high} = 1 - \left(\frac{1 + r_{high}}{1 + r_{low}}\right)(1 - \chi_{low}). \tag{36}$$

As default happens also in the high state, shortages of liquidity are widespread in the economy with a drop in first-subperiod consumption $C_{low} = C_{high} = B/P$ in any contingency. This does not prevent households from holding deposits only for their pecuniary return. The key question now is to understand at which level of net worth default is also triggered in the high state. Net worth should fall to zero for this to happen. This is not surprising (as we have discussed in Section 3.3), since expected profits are always equal to net worth under perfect competition. Considering that endogenous default is triggered by zero profits and that there is default in both states, net worth should be necessarily zero. Interestingly now at zero net worth, there are multiple equilibria with different rate of defaults where χ_{low} is in the range $[\bar{\chi}_{low}, 1]$ and χ_{high} determined by (36).

4.3 Regulation

The model is consistent with efficiency of competition only if intermediaries are subject to capital regulation. A key result of previous discussion is that there is a monotone non-increasing relationship between intermediaries' net worth and the default rate. As N rises above zero, χ_{low} falls. Sufficiently high net worth, above $\bar{N}$, is enough to enforce the good equilibrium with liquidity efficiently supplied in all possible contingencies.

According to our results, extreme forms of "free banking" (defined as completely unregulated forces of competition) are not desirable. Perfect competition in our model abates to zero all rents from financial intermediation, until net worth is always equal to expected profits. But nothing pins down the equilibrium level of net worth.[18] Therefore, to enforce the good equilibrium, regulation is needed.

However, there is no contradiction between our results and what is proposed by "real bills" supporters, with some important wrinkles. Indeed, under "real bills", "regulation" is directed toward the quality of intermediaries' assets by requiring them to hold only safe security. In our model, "real bills" would be represented by a risk-free security with a real return always equal to $1/\beta$.[19] Repeating the above analysis while restricting intermediaries to hold "real bills", the threshold defining the existence of the good equilibrium becomes zero, i.e. $\bar{N} = 0$. This implies no regulation whatsoever on capital

[18] It is now clear that intermediaries entering the market and taking the equilibrium price Q_t^D as given have to choose the same level of net worth as incumbents to supply a homogenous deposit security with respect to the market. This is why any level of non-negative net worth is consistent with an equilibrium.

[19] Central-bank reserves could serve as "real bills" although in the original theory proponents had in mind sources of private indebtedness without liquidity value.

but hides two important requirements: first, that "real bills" are available; second, that intermediaries involved in the creation of private money should not supply risky loans. Therefore, there is no integration between money and credit market. At most it is limited to the credit market for safe securities. This is what provides the appropriate backing that makes private-money creation safe. Instead, intermediaries in our model do take risky position in lending which can have unmodelled benefits for capital accumulation and growth.[20] Capital requirements are key to make deposits safe in our model.

The level of net worth is one way to identify the type of equilibrium in our model. In particular, to select the good equilibrium, the macro-prudential threshold above which net worth of intermediaries should stay can be written as

$$\bar{N} = D\left(\min_s \frac{1}{1+r_s} - \frac{1}{1+r}\right).$$

For a given level of deposits, one needs to compute the worst realization of the distribution of real returns on capital across states s and compare it, in a discounted way, with the real risk-free return $1 + r$. Intermediaries' net worth has to be enough to cover the worst-case scenario. Though useful, this prescription reveals itself to be fragile in practice. Regulators need to understand the full distribution of real returns and evaluate the worst case, which is a difficult task. Mistakes in this computation could lead to lower capital requirements and open the possibility to default equilibria, characterized by full liquidity in the high state and a liquidity crunch in the low state.

The analysis of the previous section shows another possible way to characterize equilibria simply by looking at the ratio between net worth and deposits. Denote the variable $\Gamma \equiv N/D$, which is a measure of leverage. When net worth is high, above $\bar{N}$, or $\Gamma \geq [\min_s(1 + r_s)^{-1} - (1 + r)^{-1}]$, the good equilibrium exists with liquidity available in all states of nature. As N falls below $\bar{N}$ and correspondingly Γ falls, shortages of liquidity happen in the low state and default monotonically rises from 0 to $\bar{\chi}_{low}$. At the latter level, $N = \Gamma = 0$, and default occurs in the high state as well, while liquidity collapses in all contingencies.

Next, we discuss how the analysis of the baseline model changes when we consider default costs. As we will show, this additional feature exacerbates the problems of multiplicity of equilibria along an interesting dimension.

[20]If we were assuming that households do not supply capital, disruption in financial intermediation would create a collapse in output.

5 Default costs

In the baseline model, intermediaries issue non-contingent deposit liabilities that can be used for transactions in the first subperiod. In the event of default on such securities, the only consequence is that deposits lose their transaction value. In this section, we extend the model by introducing a fixed real cost of default c, independent of the size of the intermediary's balance sheet, that must be paid using the value of assets before repaying depositors. This cost captures the expenses associated with the bankruptcy process, or a more general unmodelled loss of value associated with default.[21]

A key implication of this extension is that we obtain a multiplicity of equilibria for a given level of net worth N, for some level of N.

The cost of default is often used to model the optimality of debt following the costly state verification approach of Townsend (1979). However, in our framework, we simply assume that intermediaries can issue both equity and debt, without formalizing the optimality of such contracts.[22] Given our assumption that intermediaries can also issue state contingent equity (net worth), our results are richer than those in standard models with costly state verification. In particular, we obtain a good equilibrium in which net worth acts as a buffer to protect depositors against default, as in Section 4.1. In this good equilibrium, the contractual outcome between intermediaries and depositors resembles the risk-free, non-contingent deposit contract derived by Gorton and Pennacchi (1990).

We now characterize all equilibria with default costs. Consider a financial intermediary born at time t. Profits at time $t+1$ are now

$$\Pi_{t+1}^{I} = (1 + i_{t+1}^{K}) P K_t^H - (1 - \chi_{t+1}) D_t - (1 - I_{t+1}) P c,$$

in which the cost c is incurred only when default happens. Throughout the analysis, we are still making the assumptions that imply a constant price level P.

The cost of default has important consequences for equilibrium. First consider the supply decisions of intermediaries. Expected profits are

$$E_t \left\{ \beta \Pi_{t+1}^{I} \right\} = N_t + Q_t^D D_t - \beta E_t \left\{ (1 - \chi_{t+1}) \right\} D_t - \beta E_t \left\{ (1 - I_{t+1}) P c \right\}.$$

[21] For instance, Veronesi and Zingales (2010) use data from the 2008 financial crisis and estimate that bankruptcy of a financial intermediary would have destroyed about 22% of enterprise value.

[22] There are extensions of the basic Townsend (1979) model that justifies the coexistence of debt and equity, see e.g. Boyd and Smith (1999).

which are lowered by default costs. Perfect competition is still assumed, reducing to zero the rents of financial intermediation. The implied supply schedule is of the form:

$$Q_t^D = \beta E_t \left\{(1 - \chi_{t+1})\right\} + \beta \frac{Pc}{D_t} E_t \left\{(1 - I_{t+1})\right\},$$

showing a negative relationship between the price and the level of deposits. The higher the supply of deposits, the lower is the impact of the fixed bankruptcy cost on intermediaries' balance sheets, and the lower is the deposit price.

Demand for deposits is unchanged. Supply and demand now meet at

$$E_t \left\{(1 + \mu_{t+1}) I_{t+1}\right\} = E_t \left\{I_{t+1}\right\} + \frac{Pc}{D_t} E_t \left\{(1 - I_{t+1})\right\}. \tag{37}$$

By inspecting the above equation, it is easy to see that again in the good equilibrium with no default ($I_{t+1} = 1$ in all states) liquidity is supplied as needed to satiate the consumer, $\mu_{t+1} = 0$ in all states. It follows that the conditions found before for the lower bound on net worth do not change. The intuition is simple: in the good equilibrium the cost of default is never suffered and this is why it does not change the conditions for its existence.

Default equilibria are now different since intermediaries will supply deposits at a higher price to compensate for the fall in expected profits. In our simple two-state model, equation (37) simplifies to

$$(1 - \pi)(1 + \mu_{high}) = (1 - \pi) + \frac{Pc}{D}\pi \tag{38}$$

when we consider default only in the low state. Since c is positive, μ_{high} is also positive. This is the first important difference with respect to the baseline model. There is some shortage of liquidity even in the high state of the default equilibrium as opposed to what happens in the baseline model where instead full liquidity was available in that state. Therefore, moving from a good equilibrium to a default equilibrium now creates a drop in the level of deposits.

To evaluate the equilibrium level of deposits, note again that $C_{high} = d + b = 1/(1 + \mu_{high})$ having defined real variables with lowercases, d and b for deposit and central bank reserves, respectively. We can substitute C_{high} into the above equation to find that d is the non-negative root of a second-order polynomial $P(d)$ of the form

$$P(d) = d^2 + (b + c\pi_R - 1)d + bc\pi_R$$

in which we have defined π_R as the ratio of the probability of the two states, $\pi_R \equiv \pi/(1-\pi)$. Moreover, d should be in the interval $[0, 1-b]$. The study of the roots can be greatly simplified by looking at how they vary with the level of public liquidity available. When b goes to zero, there are two solutions $d = 0$ and $d = 1 - c\pi_R$. As b rises, the smaller root increases while the larger decreases.

This represents another important difference with respect to the baseline model. In the default equilibria, there can be multiple equilibrium levels of deposits all implying some shortage in the high state. However, this multiplicity can be easily reduced by making some innocuous assumptions. Note first that our economy does not have a store of value, like currency, which can also provide liquidity services. Assume now that the government can still issue interest-bearing reserves and at the same time can also supply currency. Following previous discussion, both securities always provide liquidity service since they are fully backed. However, demand for currency is zero as long as Q is less than one since currency is dominated in return by reserves, while demand for reserves is zero if Q exceeds one. Therefore the value of one is an upper bound for Q implying using (27) that μ_{high} is also appropriately bounded.[23] This allows us to disregard the lower root of the polynomial $P(d)$ because it implies too high a value of μ_{high} in equation (38).

In what follows, we restrict attention to the higher root of the polynomial. Still, we find interesting departures from the baseline model when we look at the relationship between net worth and the default rate. Consider the zero-profit condition in the low state of the default equilibria:

$$\begin{aligned} N &= \frac{D}{1+r_{low}^K}\left[(1-\chi_{low}) - \beta(1+r_{low}^K)(1-\pi\chi_{low})\right] \\ &\quad + \frac{Pc}{1+r_{low}^K}\left[(1-\beta(1+r_{low}^K)\pi\right]. \end{aligned}$$

There is now an additional term (on the second line) due to the positive default cost c. We must examine whether this level of net worth can be higher than the threshold required to enforce the good equilibria, breaking then the monotone relationship between net worth and default rate found in the baseline model. Consider first the case in which b is close to zero and

[23] Using (27) and noting that $C_{low} = b = 1/(1+\mu_{low})$, the upper bound on $1+\mu_{high}$ can be expressed as $1+\mu_{high} \leq 1/[\beta(1-\pi)] - \pi_R/b$. Finally using (38) a lower bound on d can be found.

equilibrium deposits are equal to $d \simeq 1 - c\pi_R$. In the limit $\chi_{low} \to 0$, the equation above implies that

$$\tilde{N} = \bar{N} + \frac{Pc}{1+r^K_{low}}\left[\frac{1-2\pi}{1-\pi} + \frac{\pi^2}{1-\pi}\beta(1+r^K_{low})\right].$$

The threshold $\tilde{N}$ now exceeds than $\bar{N}$ for several parametrization (for example, just set $\pi = 1/2$). Critically, default costs can now produce multiple equilibria for the same level of net worth. Indeed, in the example above, the good equilibrium with no default coexists with equilibria characterized by partial default when net worth is in the range $[\bar{N}, \tilde{N}]$. Shifts in confidence driving expectations to include the possibility of default can be self-fulfilling. The mechanism can be understood as follows. Consider for simplicity a very small probability of realizing the low state. Let households expect default in the low state. This expectation feeds into higher borrowing costs to the point in which the current level of net worth is not enough to ensure solvency. Indeed, if the intermediary defaults, it has to pay the cost c which is the reason why current net worth is insufficient to cover full reimbursement of deposits. This explains also why partial default is also an equilibrium for the same level of net worth as in the good equilibrium.

Ultimately, capital requirements should be higher than the threshold $\tilde{N}$ to insure that intermediaries are always solvent. This supports a stricter macro-prudential requirement than that of the baseline model.

To complete the characterization of all the default equilibria, consider that as net worth further decreases the default rate χ_{low} rises. Zero net worth again triggers default also in the high state but now combined with a higher level of default in the low state, $\tilde{\chi}_{low} = 1-(1+r_{low})/(1+r_{high})+c/d$. Moreover, at zero net worth the default rates in the two states are linked by

$$\chi_{high} = 1 - \left(\frac{1+r_{\max}}{1+r_{\min}}\right)\left[(1-\chi_{low}) + \frac{c}{d}\right],$$

with $\chi_{low} \in [\tilde{\chi}_{low}, 1]$.

As in the baseline model, when net worth is zero, there is a shortage of private liquidity and deposits can be held only for their pecuniary return. Figure 1 shows the relationship between net worth and default rate for different levels of public liquidity, B/P.[24] The continuous green line identifies

[24] We use the following calibration: $\pi = 0.1$, $c = 0.1$, $\beta = 0.99$, $1 + r_{low} = 1$.

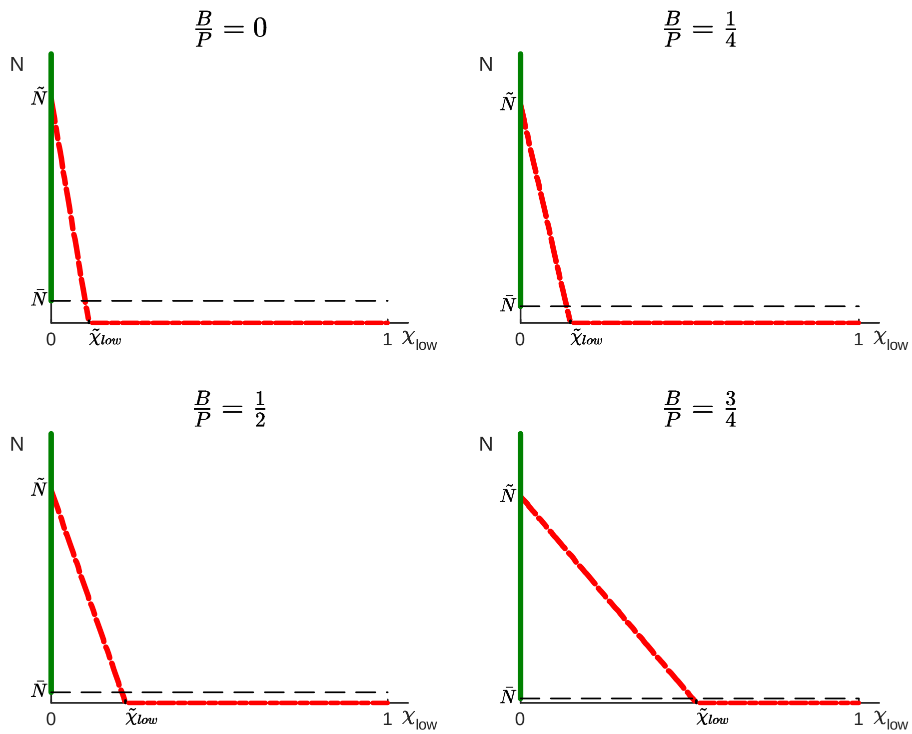

Figure 1: Relationship between level of net worth and type of equilbria for different levels of public liquidity B/P. N is net worth, χ_{low} is the default rate in the low state, while $\tilde{\chi}_{low}$ is defined in the text. The continuous green line shows the level of net worth for which the good equilibrium arises. The dashed red line shows the relationship between net worth and default rate in the default equilibria. In the region $[\bar{N}, \tilde{N}]$ there are multiple equilibria.

the levels of net worth consistent with the good equilibrium, while the dashed red line draws the relationship between net worth and default rate in the low state, χ_{low}, of the default equilibria. Consistent with the discussion above, multiple equilibria arise in the region $[\bar{N}, \tilde{N}]$. It is interesting to note that in the numerical example, and for all the chosen levels of B/P, the multiplicity region can be quite large relative to the level of net worth $\bar{N}$ above which the good equilibrium exists. This implies that capital requirements should be quite tight to enforce uniquely the good equilibrium. Moreover, as B/P rises, $\bar{N}$ falls because the equilibrium level of deposits is lower while the threshold $\tilde{\chi}_{low}$ increases.

6 Interaction with monetary/fiscal policy

In this section, we explore more deeply the role of public liquidity and therefore of alternative monetary/fiscal policy rules. We start with an analysis of the mix of policies that implement the Friedman rule, and then study whether the government can react to a liquidity crunch.

6.1 Friedman rule

Consider again the equilibrium of the baseline model. Under scarcity of public money, the key result was that competition among intermediaries regulated only by a minimum capital requirement can succeed to fulfill all the liquidity needs of the economy. Implicitly, we had the following assignment of tasks: monetary policy to target the price level, and competition with regulation to fix the shortage of liquidity.

Often, in similar contexts, the Friedman rule – i.e. reduce to zero the return differential between assets with different liquidity properties but same credit-risk characteristics – has been advocated as a solution to shortages of liquidity. In the good equilibrium of our baseline model, Friedman rule is clearly not needed since private money creation is able to satisfy all liquidity needs.

Let us instead address the reverse problem: namely, what will happen if monetary policy follows the Friedman rule? To this end, consider a monetary/fiscal rule that implements the Friedman rule. The rule eliminates any difference between the price of central bank reserves:

$$Q_t = \beta E_t \left\{ P_t/P_{t+1}(1 + \mu_{t+1}) \right\}$$

and the price of a risk-free nominal bond with no liquidity value:

$$Q_t^f = \beta E_t \left\{ P_t / P_{t+1} \right\}.$$

The Friedman rule requires to set monetary/fiscal policy in a way that $\mu_{t+1} = 0$ in all states of nature. This is possible if public liquidity is supplied in enough quantity to always enable the economy reaching the efficient level of consumption. To this end, considering the first-subperiod budget constraint of households and abstracting from deposits, it should be the case that $B_{t-1}/P_t = C_t = 1$ in all contingencies. Assume again that reserves are in constant supply at B while the tax rule is specified as in (23) where now $T = 1$. This mix of policies can implement the Friedman rule with a constant price level $P_t = B$ as it can be easily seen from the intertemporal budget constraint of the government which now collapses to

$$\frac{B}{P_t} = T = 1.$$

It is worth noting that in implementing the Friedman rule we have not found any requirement pointing to say that the economy should be deflated at the rate given by the intertemporal discount factor of the consumer. Actually, full satiation with public liquidity can be consistent with a constant price level. Moreover, the same result can be obtained through any other rate of inflation or deflation. This stems from the fact that central bank reserves pay an interest rate which provides the additional monetary-policy instrument that frees monetary policy from any constraint in achieving a certain rate of inflation or deflation.[25]

Once the government follows the Friedman rule, private-liquidity creation becomes irrelevant.[26] We have therefore two contrasting cases, one in which private financial intermediation is not needed to provide liquidity services because of the Friedman rule and another in which the Friedman rule is not needed because of private liquidity creation. The right choice rests on the evaluation of the costs under the two options. To this end, our analysis has shown that satisfying the liquidity needs of the economy with only public

[25] Canzoneri, Cumby and Diba (2015) and Kayshap and Stein (2016) discuss the role of paying interest rate on reserves for the Friedman rule when intermediaries do not internalize the consequences of making excess use of deposits.

[26] Financial intermediation would be critical if we assumed an essential role for intermediaries in supplying capital, setting to zero then the supply of capital of households.

liquidity may require a high level of lump-sum taxes. Indeed, in the baseline model, we added an upper bound on real taxes that can be thought as a reduced-form approach to capture constraints coming from distortionary taxation, the need to limit government debt in order to avoid sovereign crises, or other political-economy constraints. Taking into account all these considerations may limit the ability of the government to supply enough liquidity.

In our model, there are no welfare differences between the equilibrium implemented by the Friedman rule with constant prices and that with private money creation appropriately regulated. However, if (i) the securities in which intermediaries invest are not too risky, and (ii) if there were costs associated with issuing net worth and with taxation, and (iii) if such costs were comparable, then privately-created liquidity would save resources and would be preferred to the Friedman rule. This is because supplying an efficient amount of public liquidity requires the government to levy a constant lump-sum real tax equal to 1. On the other hand, achieving efficiency through only private-money creation requires an injection of real capital equal to $\bar{N}/P$ each period, where $\bar{N}$ is given by (32) and $D/P = 1$. The latter resources, $\bar{N}/P$, are lower than the real taxes $T = 1$ if $(1 + r_{low})^{-1} - \beta < 1$, a condition that is verified whenever the variability of the real rate is not that high.[27] Therefore, the costs associated with net worth and private-money creation would be lower than the costs associated with taxation and with the Friedman rule.

All in all, our analysis suggests that a well-regulated competitive market of financial intermediation can indeed satiate the liquidity needs of the economy (with the caveats we discussed earlier). One of these caveats is the possibility of a liquidity crunch if the capitalization of intermediaries turns out to be insufficient. Our next objective is to examine whether it is possible to design government intervention in the market of liquidity to react to a liquidity crisis while leaving room to private-money creation in good times.

6.2 Policy response to a liquidity crunch

Consider again the baseline model and assume that intermediaries' net worth is not enough to enforce the good equilibrium but allows a default equilibrium. As previously discussed, private money can satiate the needs of the economy in the high state but a liquidity shortage happens in the low state.

[27] It is always true when $r_{low} \geq 0$. In terms of primitive parameters the requirement $(1 + r_{low})^{-1} - \beta < 1$ can be written as $(1 - \pi)(A_{high}/A_{low} - 1) < 1$.

Does there exist a monetary/fiscal rule that can avoid the low-state liquidity shortage without making private-money creation irrelevant in the high state?

Under the specification of the monetary/fiscal rule of the baseline model, consumption in the high state is equal to the efficient level, $C_{high} = 1$, while consumption in the low state depends on the real value of public liquidity, $C_{low} = B_{t-1}/P_t$. To counteract the liquidity crisis, we propose two alternative policies. In the first, the price level adjusts, and this adjustment can exactly achieve the objective proposed, with however some drawbacks. The other alternative – to adjust reserves – does not completely achieve the objective, but presents less drawbacks.

Consider first the adjustment of the price level. To reach efficiency in the liquidity crisis, the price level should move to $P_{t,low} = B_{t-1}$. This adjustment in prices can be achieved by an appropriate state-contingent specification of the monetary/fiscal policy rule. Consider the simple case in which the path of reserves is kept constant, $B_t = B$ and real taxes in the high state are such that $B/P_{high} < 1$, consistent with the objective that private money creation remains essential in the high state. Given these assumptions, the objective is to find the level of taxes in the low state that implements the desired equilibrium. Since there is full satiation of liquidity in both states in the desired equilibrium, it follows that $\mu_{high} = \mu_{low} = 0$ and $Q_t = Q_t^f$ at all times. Using these results, we can write the intertemporal budget constraint of the government (21) under the two states as

$$\frac{B}{P_{high}} = T_{high} + \frac{\beta}{1-\beta}\left[(1-\pi)T_{high} + \pi T_{low}\right]$$

$$1 = \frac{B}{P_{low}} = T_{low} + \frac{\beta}{1-\beta}\left[(1-\pi)T_{high} + \pi T_{low}\right].$$

It is clear by comparing the two equations that $T_{low} > T_{high}$ since $B/P_{high} < 1$. But why should taxes increase during the liquidity crisis? The reason is that there is a shortage of demand for goods in the first subperiod because fewer liquid assets are available. To increase demand, the purchasing power of the remaining assets should increase, lowering the price level. Higher lump-sum taxes reduce the overall wealth of households and decrease the overall demand for consumption goods (first and second subperiod). For a given supply of goods the price level should fall to equilibrate the goods market. The drop of the price level increases the real value of liquid assets, allowing to achieve efficiency. Therefore, the liquidity crisis can be exactly offset by the fall in prices. Note, though, that the only friction in our model is the liquidity constraint. If we posit other frictions like price rigidity, this will make the adjustment of prices sluggish and unable to completely counteract the liquidity crisis. If instead there are wage rigidities and labor-market frictions, the decline in prices and the subsequent rise in real wages can depress employment. Considering all these arguments, a fall in the price level is not at all desirable during a liquidity crisis.

The other option available to the government is to counteract, at least to some extent, the liquidity crunch through a temporary rise in reserves while keeping constant the price level.[28] In the model, if a crisis hits unexpectedly in the first subperiod, the central bank has no time to increase reserves and thus liquidity is not sufficient. Even relaxing the timing and allowing the central bank to partially increase public liquidity on impact might not be enough to satiate all the liquidity needs. This view is consistent with what happened in 2008: the collapse of Lehman Brothers was followed by an immediate crisis and a strong response by the Federal Reserve, but the liquidity problem persisted for some time.[29]

If the crisis is expected to persist, the central bank can raise first-subperiod consumption and reach the efficient level by increasing reserves in such a way that $B_t/P = 1$, keeping constant the price level at P. As soon as the economy exits the low productivity state, the excess liquidity can be withdrawn leaving room for private-money creation.[30]

The most interesting result, however, is that in either way – temporarily lowering the price level or raising reserves – the government must raise lump-sum real taxes to meet the increase in real public liabilities. The reason is in the very origin of the liquidity crisis that starts from an insufficient backing of private intermediaries and therefore of private money. To counteract it, the government should supply more of its safe securities. A higher supply of safe government debt requires stronger backing through higher taxes. At the end,

[28] Benigno and Nisticò (2013) reaches a similar conclusion, in a different model, when evaluating optimal monetary policy following an exogenous liquidity shock. Reis (2015) also emphasizes the importance of central-bank reserves given their safe-asset quality during periods of financial disruption.

[29] For instance, insufficient liquidity seems to be the primary cause for the deviation from arbitrage observed during the 2008 crisis; see Mancini-Griffoli and Ranaldo (2011).

[30] The other drawback of this policy is that if the return to normal conditions is unexpected, public liquidity remains too high level even in the high state, temporarily making irrelevant private-money supply.

the way out of the crisis is to substitute the insufficient backing of private money with more backing of public money.[31] If government intervention is not immediately available or infeasible, there is not much hope to avert the liquidity crisis.

7 Extensions

In this section we return to the baseline model with a constant price level and discuss two simple deviations. In the first, we allow households to insure the value of deposits through brokers by paying a fee. This, however, distorts the demand for deposit. The second deviation considers a departure from perfect competition. In this extension, we analyze the implications of a model with monopolistically-competitive financial intermediaries.

7.1 Insurance through brokers

A key assumption of our baseline model is that deposits provide liquidity benefits only in states of nature in which they are not defaulted (even partially). We now relax this restriction by assuming the existence of other financial intermediaries – brokers – who can exchange deposits at their fair value, before default happens. The security issued by brokers is free of risk and therefore can be used by the bearer to purchase consumption goods in the first subperiod. This additional layer of financial intermediation is inspired by the banking history of the nineteenth century (see Gorton and Mullineaux, 1985). At that time, banks freely issued their notes which were made liquid in a secondary market by brokers trading them in exchange for specie. Brokers had incentives to monitor the quality of the assets backing bank notes, and the quote in the secondary market revealed that information. The exchange of notes into specie made them indirectly liquid since specie were accepted as a medium of exchange. Brokers were then able to redeem the notes at the issuing bank, making profits or losses.[32]

[31] If the government is unable to increase public liquidity permanently, but is able to increase it during a crisis, then the liquidity injection is only temporary. Robatto (2016) presents a model in which a liquidity injection that is temporary worsens the crisis in some circumstances.

[32] There is an important distinction between bank notes and demand deposits since the latter, unlike the former, is both a claim on a bank and on an agent's account at that bank. This distinction is not captured in our model and therefore bank notes correspond

The timing is as follows. At the beginning of the first subperiod of period t, after deposits have been issued and before default is realized, brokers propose an insurance contract to households under which they supply a security of value $1 - E_{t-1}\chi_t$ (the fair value of deposit at that point) in return for a premium f_t. The security issued by the broker is free of risk and can be used by households to purchase goods in the first subperiod. Once default happens brokers are able to recover the value of deposits from the financial intermediary.[33] Brokers' profits are given by

$$\Pi_t^B = (1 - \chi_t)D_{t-1} - (1 - E_{t-1}\chi_t - f_t)D_{t-1},$$

where both the proportional fee f_t and the brokers' profits are rebated to households each period. Brokers can make positive expected profits by charging a positive fee f_t. However, the key assumption is that the security they issue is free of any risk and therefore liquid. This is possible only if they are always solvent. We still assume limited backing of any financial intermediary –and therefore of brokers– which translates into a non-negative profit requirement on Π_t^B. Perfect competition in supplying risk-less securities assures that profits of brokers in the low state are zero.[34] It follows that the fee is determined by $f_t = \chi_{low,t} - E_{t-1}\chi_t$.

Depositors should find convenient to exchange deposits with the security supplied by brokers. For this to happen, the gain in expected utility in the first subperiod should be enough to compensate for the fee. This requirement can be written formally as

$$E_{t-1} \ln\left(\frac{B}{P} + I_t\frac{\bar{D}}{P}\right) \leq \ln\left(\frac{B}{P} + (1 - E_{t-1}\chi_t - f)\frac{D_{t-1}}{P}\right), \tag{39}$$

where the left-hand side captures expected utility from first subperiod consumption, when no insurance is available, evaluated at the optimal level of

to our definition of deposits. Moreover, historically, bank notes are barely liquid without a secondary market. Instead, in the model that follows, we will anyway assume that deposits (or bank notes) have the same liquidity properties as in the baseline model which, we will show, can be enhanced by the action of brokers in default states.

[33] Recall that a key assumption made in our baseline model was that depositors could recover the seized value of deposits only in the second subperiod. Here brokers are able to circumvent this restriction and recover the realized value in the first subperiod.

[34] Perfect competition cannot reduce to zero all profits. Otherwise, in the absence of backing, securities issued by brokers will not be free of risk.

deposits, $\tilde{D}/P = 1 - B/P$, derived in Section 4.[35] The right-hand side measures the utility under insurance taking into consideration the possibility of a different optimal level of deposits. Next, we analyze the optimal level of deposits, then come back to evaluate the above inequality.

Consider first how insurance changes the liquidity constraint of households since all deposits are now exchanged at their fair value through brokers after paying the fee f_t

$$B + D_{t-1}(1 - E_{t-1}\chi_t - f_t) \geq PC_t. \tag{40}$$

Given the above liquidity constraint, the household's optimization problem implies a flat demand for deposits at the price

$$Q_t^J = \beta\left(1 - E_t\chi_{t+1} - f_t\right)(1 + E_t\mu_{t+1})$$

while supply remains unchanged at (29). They now meet at

$$\left(1 - E_t\chi_{t+1} - f_t\right)(1 + E_t\mu_{t+1}) = 1 - E_t\chi_{t+1} \tag{41}$$

implying that the Lagrange multiplier μ_{t+1} is no longer state contingent and now given by:[36]

$$\mu = E_t\mu_{t+1} = \frac{f}{(1 - E\chi - f)}.$$

The Lagrange multiplier μ is zero only if $f = 0$, but since $f = \chi_{low} - E\chi$ this is only possible when there is no default as in the good equilibrium. This result is not surprising. When deposits are free of risk, there is always consumption insurance and therefore no role for brokers unless they operate at a zero fee. The analysis of the good equilibrium would follow exactly the no-insurance case. However, when f is positive, which happens in default equilibria, the multiplier μ is an increasing function of the expected rate of default. With this result in hand, we can derive the optimal level of consumption, which is also not state contingent, given by

$$C = 1 - \frac{f}{(1 - E\chi)} = \frac{1 - \chi_{low}}{(1 - E\chi)}. \tag{42}$$

Insurance clearly works since consumption will be perfectly equalized across states even in the case of default on deposits. However the somewhat interesting result is that insurance does not reach efficiency unless $\chi_{low} = 0$, which

[35] We are still assuming constant B and P.

[36] In what follows we consider stationary equilibria and drop the time index.

only happens in the no-default equilibria. Otherwise, consumption falls as the default rate rises. This is a new result with respect to the case of no insurance in which consumption dropped only in the low state.

Although the brokers' securities are liquid, the inherent risky characteristics of the original deposit securities are transferred into a positive fee – which is the reason why brokers' securities are liquid – implying an inefficiently low demand of private liquidity given by

$$\frac{D}{P} = \frac{1}{1 - E\chi} - \frac{B/P}{(1 - \chi_{low})}. \tag{43}$$

Equation (43) is obtained using (40) and (42) noting that $(1 - E\chi - f) = 1 - \chi_{low}$. Equilibrium deposits can even be an increasing function of the default rate, at least for small χ and low values of B/P.[37] As default rises, households are willing to hold more deposits since higher holdings will partially offset the haircut of brokers and provide a buffer of liquidity. However, the increase in deposits does not prevent consumption to fall with the default rate, as discussed before.

We now turn to analyze the solvency of financial intermediaries. As already noted, the conditions for the existence of the good equilibrium are the same as in the baseline model: net worth should be greater than $\bar{N}$, with $\bar{N}$ given by (32). Consider now equilibria with default only in the low state. Again the critical condition on net worth as a function of other variables is still as in the baseline model:

$$N_t = \frac{D}{1 + r_{low}^K}\left[(1 - \chi_{low}) - \beta(1 + r_{low}^K)(1 - \pi\chi_{low})\right],$$

where now deposits vary with the default rate as shown in (43) setting $\chi_{high} = 0$. There are two contrasting channels influencing the relationship between net worth and the default rate. On the one hand, D can rise with the default rate; on the other, the term in the square brackets decreases. This second channel dominates.[38] Therefore, there is a monotone non-increasing relationship between the level of net worth and the default rate. Further increases in χ_{low} trigger default in the high state as well once χ_{low} reaches the level $\bar{\chi}_{low} = 1 - (1 + r_{low})/(1 + r_{high})$. Above $\bar{\chi}_{low}$ the relationship between

[37] For deposits to be positive, it is required that $(1 - \chi_{low}) - (1 - E\chi)B/P$ is positive.

[38] This can be easily seen by assuming B=0 which is the case in which deposits always increase with the rate of default and at the highest speed.

the two default rates is again given by (36). As in the baseline model, net worth is zero when default happens also in the high state. At this level of net worth there are multiple default equilibria. However, the important difference with respect to the baseline case is that these equilibria are no longer associated with drop of consumption in the bad state, compared to the good state. It is true, however, that consumption drops with higher default rates, but in a smooth way.[39]

We emphasize that insurance through brokers does not reduce the multiplicity of equilibria. Having characterized all the possible equilibria, we can now study the conditions under which insurance is optimal from the point of view of the consumer. As already discussed, insurance is irrelevant in the good equilibria. In the default equilibria in which $\chi_{low} \in (0, \bar{\chi}_{low}]$, the inequality (39) can be written using (43) and the equilibrium premium f as

$$\left(\frac{B}{P}\right)^{\pi} \leq \frac{1-\chi_{low}}{1-\pi\chi_{low}} = 1 - \frac{f}{1-\pi\chi_{low}}$$

which is in general true for low enough supply of public liquidity and relatively high probability of realization of the low state.[40] The key result is that whenever public liquidity does not provide much insurance, then it is optimal to have private insurance. This is in line with the historical evidence of the early nineteenth century in which public money was not available, and a secondary market for private bank notes developed.

When χ_{low} exceeds $\bar{\chi}_{low}$ and there is also default in the high state, the inequality (39) can be written using (43) and (36), as

$$\left(\frac{B}{P}\right) \leq \frac{1-\chi_{low}}{1-E\chi} = 1 - \frac{f}{1-E\chi} = \beta(1+r_{low})$$

which is exactly the condition required for deposits to be positive. It is satisfied again when public liquidity is low and the variability of the real rate is not that high.[41] In the latter case, the required premium to make brokers' securities free of risk can be small since χ_{low} and χ_{high} are closer.

[39] When $(1-\chi_{low}) - (1-E\chi)B/P \leq 0$ equilibrium deposits fall to zero and only public liquidity will be used for first-subperiod consumption.

[40] Note also the if the above inequality is true the condition for a positive level of deposit is also satisfied, since $B/P \leq (B/P)^{\pi}$.

[41] Recall that $1/\beta$ is the expected real interest rate.

7.2 Monopolistic competition

We now discuss how equilibrium changes when the supply side of the deposit market becomes distorted by departures from perfect competition. We amend the model to analyze a market in which financial intermediaries are monopolistically competitive. Let us assume that there are K wholesale financial intermediaries. A generic type j supplies $D_t(j)$ at the price $Q_t^D(j)$. The model is also enriched by a retail financial intermediary that invests in a portfolio of all deposit securities financing it with a structured security D_t which is the following combination of the simple deposit securities

$$D_t = \left[\left(\frac{1}{K} \right)^{\frac{1}{\theta}} \sum_{j=1}^{K} (D_t(j))^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

where θ, with $\theta > 1$, captures the degree of substitution of the securities $D_t(j)$ in the structured product. At time t the balance sheet of the retail intermediary is

$$Q_t^D D_t = \sum_{j=1}^{K} Q_t(j) D_t(j)$$

where Q_t^D is the price of the structured product given by

$$Q_t^D = \left[\frac{1}{K} \sum_{j=1}^{K} \left(Q_t^D(j) \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

We assume that households do not have access to the wholesale market and can only invest in the structured security D_t issued by the retail intermediary at the price Q_t^D. At time $t+1$, the payoff of the retail intermediary's portfolio is fully transferred to the household according to an aggregate default rate χ_t defined by

$$(1-\chi_t) D_t = \sum_{j=1}^{K} (1-\chi_t(j)) D_t(j).$$

Given the framework outlined above nothing changes in the optimization problem of the household where now Q_t^D and D_t have to be understood as the price and quantity of the structured product. As before, if D is partially seized– and this is the case if at least one deposit $D_t(j)$ is not fully reimbursed– the security D is not accepted in goods transaction. This is reminiscent of the 2008 financial crisis where structured securities lost their liquidity entirely even if only few of the embedded primitive securities were defaulted.

The supply side changes because wholesale intermediaries face a demand for their deposit of the form

$$D_t(j) = \frac{1}{K} \left(\frac{Q_t^D(j)}{Q_t^D} \right)^{-\theta} D_t, \tag{44}$$

as a result of how different deposit securities are packaged into the structured product. In a market of monopolistically-competitive suppliers, wholesale intermediaries can choose the price of their security $Q_t^D(j)$ internalizing demand (44) and taking aggregate variables as given. The optimal choice implies the following flat supply schedule at the price

$$Q_t^D(j) = \beta(1+\tau)E_t \left\{ (1 - I_{t+1}(j))(1 - \chi_{t+1}(j)) + I_{t+1}(j) \right\}, \tag{45}$$

where we have defined $\tau \equiv \theta/(\theta - 1) - 1 \geq 0$.[42] Under monopolistic competition, intermediaries supply deposits at a higher price where τ captures the market power. In what follows we first look at a symmetric equilibrium in which all intermediaries make the same choices, in particular assuming that they are all capitalized with the same level of net worth. Later, we depart from this assumption and consider the case in which intermediaries enter the market by choosing the level of net worth that maximizes their profits. This is now possible, unlike in the perfect competition case, since deposit securities are no longer homogenous.

Let us first consider equilibria under the assumption that intermediaries are all capitalized at the same level. Denote in particular by D_t^s the supply of deposits of each single intermediary and note that the aggregate supply of the structured product is given by $D_t = K \cdot D_t^s$.

Expected profits of a generic wholesale financial intermediary are now

$$E_t \left\{ \beta \Pi_{t+1}^I \right\} = N_t + \tau \beta E_t \left\{ (1 - I_{t+1})(1 - \chi_{t+1}) + I_{t+1} \right\} D_t^s \tag{46}$$

which clearly shows the rents of monopoly power captured by the second addendum on the right-hand side. The existence of these rents determines the entry of a finite number K of wholesale intermediaries into the market once a fixed cost Φ is paid. We turn to this analysis later.

[42] We are still analyzing an equilibrium with constant price level.

Demand of households is still (28), which now refers to the price of the structured product. However, in equilibrium $Q_t^D(j) = Q_t$, and we can equate demand and supply implying:

$$E_t\left\{(1+\mu_{t+1})I_{t+1}\right\} = E_t\left\{I_{t+1}\right\} + \tau E_t\left\{(1 - I_{t+1}(j))(1 - \chi_{t+1}(j)) + I_{t+1}(j)\right\},$$

which shows that even in the case of no default ($I_{t+1} = 1$ in all states of nature), there is shortage of liquidity ($\mu_{t+1} > 0$) in all contingencies. It is easy to see that the equilibrium level of the structured product is

$$\frac{D}{P} = \frac{1}{1+\tau} - \frac{B}{P} < 1 - \frac{B}{P}, \tag{47}$$

which is lower than the efficient level because of the monopoly rents in supplying wholesale deposits. Rents are reflected in higher security prices which dampen the equilibrium level of liquidity below the efficient level even in the no-default equilibrium.[43]

To support the good equilibrium, intermediaries' net worth should be above the threshold $\bar{N}^m$:

$$N \geq \bar{N}^m = \frac{D^s}{1 + r_{low}^K}\left[1 - \beta(1+\tau)(1 + r_{low}^K)\right],$$

which, once expressed in terms of the variable Γ, is now a lower requirement with respect to the perfect competition case because the higher price of deposits reduces the borrowing costs of intermediary.[44] Lastly, we must determine D^s. Knowing D, we need to derive the number of wholesale intermediaries by equalizing expected profits (46) and the fixed entry cost Φ plus initial net worth N_t.[45] This condition determines the supply of deposits of each intermediary at $D^s = P\Phi/(\beta\tau)$, implying together with (47) that the equilibrium number of intermediaries is:

$$K = \frac{\beta\tau}{\Phi}\left(\frac{1}{1+\tau} - \frac{B}{P}\right).$$

In the good equilibrium K is decreasing in the fixed cost and in the level of public liquidity available. The number of intermediaries is also increasing in the market power τ for reasonable parametrization (e.g. small B/P).

[43]To have a positive level of equilibrium deposits, we assume that $B/P < 1/(1+\tau)$.

[44]In the analysis we further make the assumption that the monopolistic power is not that high so that $\beta(1+\tau)(1+r_{low}^K) \leq 1$.

[45]Recall the no-rent condition (19).

We now move to analyze equilibria with default, first in just the low state. Equilibrium between supply and demand implies:

$$\frac{D}{P} = \frac{1}{1+\tau\frac{1-\pi\chi_{low}}{1-\pi}} - \frac{B}{P}.$$

When χ_{low} tends to zero aggregate deposits are lower than in the good equilibrium by a discrete amount. The reason for this fall in aggregate deposits is that the price charged by wholesale intermediaries (which is fully passed through into the price of the structured product) is inefficiently high. As soon as there is a small probability of default, there is a discontinuity in the liquidity properties of deposits between the high and low state, as opposed to the good equilibrium where deposits are always liquid. This discontinuity and the inefficiently high price of deposits explain the fall in the demand for the structured product. As the rate of default, χ_{low}, rises, the price of deposits falls and pushes a bit up the demand of the structured product.

This non-monotone relationship between aggregate deposits and the default rate, however, bears no consequences for the relationship between net worth and default rate at the level of the wholesale intermediaries. The latter two variables are linked through the following equation:

$$N = \frac{D^s}{1+r^K_{low}}\left[(1-\chi_{low}) - \beta(1+r^K_{low})(1+\tau)(1-\pi\chi_{low})\right]. \tag{48}$$

As before, D^s is determined by the equalization of expected profits to the fixed entry cost paid by each intermediary now at $D^s = P\Phi/(\beta\tau(1-\pi\chi_{low}))$, which is increasing in the rate of default. However, note that the level of net worth N defined in (48) reaches its maximum $\bar{N}^m$ when $\chi_{low} \to 0$ and otherwise is decreasing in χ_{low}. Consistent with the baseline model, there is a monotone non-increasing relationship between net worth and the default rate. Given D^s and D, we can determine the equilibrium number of intermediaries in the default equilibrium

$$K = \beta\tau\frac{(1-\pi\chi_{low})}{\Phi}\left(\frac{1}{1+\tau\frac{1-\pi\chi_{low}}{1-\pi}} - \frac{B}{P}\right).$$

At low levels of χ_{low}, the aggregate level of deposits falls along with the number of intermediaries. Then, as χ_{low} rises, aggregate deposits increase and K might also increase to meet the higher level. Finally, as net worth falls further, χ_{low} increases to the point where default occurs also in the high state. The threshold for χ_{low} is the same as in the baseline model $\bar{\chi}_{low} = 1-(1+r_{low})/(1+r_{high})$ but the level of net worth at which this happens is now negative, because of monopoly rents. Actually when $N = -\Phi$ default happens also in the high state. At this level, the relationship between defaults in the two states is again given by (36). This is interesting because if we assume the reasonable restriction that the initial net worth of intermediary is non-negative, then the good equilibrium coexists with equilibria with default but only in the low state for χ_{low} in the range $[0,\tilde{\chi}^{m}_{low}]$ where $\tilde{\chi}^{m}_{low} \equiv (1 - \beta(1+r^{K}_{low})(1+\tau))/(1-\beta(1+r^{K}_{low})(1+\tau)\pi)$. Monopoly rents make some equilibria infeasible at a non-negative level of net worth.

We now discuss how the analysis changes when intermediaries can also choose the level of net worth at which they enter the market. Given previous results, consider the expected profits of a generic intermediary j choosing net worth $N_t(j)$ and setting the optimal price $Q^D_t(j)$:

$$E_t\left\{\beta\Pi^{I}_{t+1}(j)\right\} = N_t(j) + \frac{\tau}{1+\tau}\frac{1}{K}Q^D_t(j)^{1-\theta}(Q^D_t)^{\theta}D_t$$

where we have used equation (46) and demand (44). Note first that expected profits are decreasing with the optimal price $Q^D_t(j)$ given by (45), which in turn is decreasing with the default rate. Since the zero-profit condition requires $E_t\left\{\beta\Pi^{I}_{t+1}(j)\right\} = N_t(j)+\Phi$, an intermediary j would like to set $Q^D_t(j)$ to the lowest feasible value. This is possible if net worth is reduced the most and therefore set to zero which implies a positive default rate $\tilde{\chi}^{m}_{low}$ in the low state. It turns out that this is the optimal strategy for each intermediary. Therefore there is only one equilibrium in which every intermediary sets $N_t = 0$ and deposits will be safe in the high state while seized in the low state at the rate $\tilde{\chi}^{m}_{low}$. This case shows a race to the bottom since all intermediaries enter the market with the lowest possible level of capital.

Two main lessons come from this extension. First, when we consider that initial net worth is out of the control of financial intermediaries, the inefficiencies of monopolistic competition result in a shortage of liquidity even in the good equilibrium, to the point that the economy is not fully satiated. As a positive side effect, the monopoly rents and therefore the lower borrowing costs reduce the level of net worth required to enforce the good equilibrium with the consequence that at non-negative levels of net worth some bad equilibria are now ruled out. Second, if intermediaries also choose their own level of net worth, there is a race to the bottom following the attempt of maximizing individual expected profits. This yields to a sub-optimal equilibrium in which every intermediary enters with zero net worth. Default and shortage of liquidity arise in the low state.

8 Conclusion

We have presented a framework to study equilibria with private money creation in a model in which both public and private liquidity play a role for transactions.

If the availability of public liquidity is limited because of upper bounds on taxes, there is room for private money creation in normal times. Competition should be supplemented by regulation on intermediaries' capital to enable the economy to reach efficiency in all contingencies. Private-money creation can even be a resource saving option when the variability of the return of the asset backing private money is not very high. However, under insufficient backing of private money, liquidity crises can occur, featuring a sudden drop in consumption. In this case we also argue that the same limit on real taxes might give rise to limitations to policy action.

We are aware that we have omitted some important real-world features, such as the fact that some of the public liquidity takes the form of central bank reserves that can only be held by banks. But we consider our model as a first step in addressing the important topic of private and public liquidity determination. A debate which has been at the center of economists' thoughts for hundreds years but that has received not much attention in modern economic analysis

We see at least two possible extensions of our framework. First, we have limited the focus of our analysis only on the consequences that financial disruption has on the liquidity market. There can be, however, important effects on the supply of credit with interesting spillovers between credit and money markets that could be explored in more complicated frameworks. Second, we have analyzed stylized models of market interaction like perfect competition and monopolistic competition. An interesting result under monopolistic competition is that intermediaries might have an incentive to increase their profits relative to other competitors by lowering the price of deposits and therefore increasing the rate of default. But this incentive goes against that of the buyer of deposits that would prefer instead to have safe assets to satisfy their liquidity benefits. The analysis could be extended to more sophisticated markets eventually characterized by informational asymmetries between buyers and sellers or different forms of competition such that the buyers' liquidity benefits could be internalized and partly ripped by sellers. We leave these extensions to future work.

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