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MECHANISM DESIGN APPROACH WITH  
DEMAND UNCERTAINTY AND MORAL  
HAZARD**

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***INDUSTRIAL ORGANIZATION***



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# A THEORY OF CROWDFUNDING - A MECHANISM DESIGN APPROACH WITH DEMAND UNCERTAINTY AND MORAL HAZARD

## Abstract

Crowdfunding provides innovation in that it enables entrepreneurs to contract with consumers before investment. Under aggregate demand uncertainty, this improves screening for valuable projects. Entrepreneurial moral hazard threatens this benefit. Studying the subsequent trade-off between screening and moral hazard, the paper characterizes optimal mechanisms. Popular all-or-nothing reward-crowdfunding schemes reflect their salient features. Efficiency is sustainable only if returns exceed investment costs by a margin reflecting the degree of moral hazard. Constrained efficient mechanisms exhibit underinvestment. As a screening tool for valuable projects, crowdfunding promotes social welfare. Crowdfunding complements rather than substitutes traditional entrepreneurial financing.

JEL Classification: D82, G32, L11, M31

Keywords: Crowdfunding, entrepreneurship, moral hazard, aggregate demand uncertainty, venture capital

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# 1 Introduction

Crowdfunding has, in recent years, attracted much attention as a new mode of entrepreneurial financing: through the internet many individuals — the crowd — provide funds directly to the entrepreneur rather than through a financial intermediary to whom is delegated the task to manage the investment.<sup>1</sup> Given the typical agency problems associated with entrepreneurial financing, the popularity of crowdfunding is surprising.<sup>2</sup> In particular, the seminal paper of Diamond (1985) suggests that crowdfunding cannot handle agency problems well, because, due to the large number of investors, the free-riding problem in monitoring the project is especially severe.

However, popular crowdfunding platforms like Kickstarter and Indiegogo do not only dispense with the financial intermediary, they also change the returns to investment. Instead of promising a monetary return, they promise investors only the good which the entrepreneur intends to develop. Hence, these so-called *reward-based* crowdfunding schemes have the feature that the entrepreneur's consumers become her investors. Next to eliminating the financial intermediary, a reward-crowdfunding platform therefore provides the innovation that it allows an entrepreneur to contract with her future consumers *before* the investment decision is taken.

The primary objective of this paper is to show that this latter innovation has important efficiency effects that offset threats of moral hazard. The basic intuition behind these efficiency gains is relatively straightforward. By directly contracting with consumers, the contract can elicit their demand and, thereby, obtain information about whether aggregate demand is large enough to cover the project's investment costs. Hence, by conditioning the investment decision on this information, crowdfunding has the potential to yield more efficient investment decisions.

In the presence of private demand information and entrepreneurial moral hazard, it is however not clear whether the contracting parties can actually realize these potential efficiency gains. Due to private information, consumers have to be given incentives to reveal their demand truthfully and, due to moral hazard, the entrepreneur

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<sup>1</sup>Mollick (2014) defines crowdfunding as ventures “without standard financial intermediaries”.

<sup>2</sup>E.g. The Economist (2012) reports that the “talk of crowdfunding as a short-lived fad has largely ceased” and regulatory reforms such as the JOBS Act from 2012 and SEC (2015) demonstrate that regulators also expect crowdfunding to persist.

has to be given incentives to properly invest. These incentive problems may thwart the intuitive efficiency effects of crowdfunding as pointed out above.

This leads us to the central research questions of this paper. Defining crowdfunding contracts as contracts between an entrepreneur and her final consumers before the entrepreneur's investment, we investigate the potential of such contracts to implement efficient allocations. In particular, we characterize (constrained) efficient contracts in the presence of entrepreneurial moral hazard and private information about demand, and compare these contracts to crowdfunding schemes in practise.

Modelling the entrepreneurial moral hazard as the entrepreneur's ability to embezzle investment funds without properly investing, we show that (constrained) efficient contracts use deferred payments to the entrepreneur to prevent moral hazard and minimize agency costs. These agency costs are strictly positive and proportional to both the project's investment size and the degree of moral hazard. They effectively augment the project's investment costs.

We show however that if the ex ante expected profitability of the project exceeds these augmented investment costs, then the additional agency costs do not prevent efficient outcomes. Hence, optimal crowdfunding contracts are constrained efficient only if the project's ex ante expected return is low. In this case, the second best exhibits the downward distortion that investment does not take place for projects with a small, but strictly positive social value. The distortion is needed to convince the entrepreneur that her deferred payments are large enough so that she refrains from embezzling the funds that are provided to her upfront. The downward distortion therefore ensures incentive compatibility of the entrepreneur's decision to invest.

We, moreover, show that agency costs are minimized by not fully revealing aggregate demand completely. At first sight this is a surprising result, because in the basic intuition provided above the potential efficiency effect of reward-crowdfunding is exactly the fact that it enables the entrepreneur to learn aggregate demand. Our formal analysis therefore refines this basic intuition and clarifies that the optimal degree of information revelation is in fact a subtle one: the entrepreneur should neither learn too little nor too much about aggregate demand. The reason is again related to the use of deferred payments for controlling the entrepreneur's moral hazard problem. In particular, if the entrepreneur learns that demand is small, she also learns that

her deferred payments are low, which exacerbates the moral hazard problem. Hence, by not revealing aggregated demand information perfectly, it is less costly to ensure incentive compatibility of the entrepreneur's decision to invest.

In order to discuss the extent to which our optimal contracts reflect real-life crowdfunding schemes, it is helpful to describe first how these schemes are used in practise. Kickstarter, the most successful crowdfunding platform to date, implements crowdfunding as follows.<sup>3</sup> First, the entrepreneur describes her project, consisting of the following three elements: 1) a description of the reward to the consumer, which is typically the entrepreneur's final product; 2) a "pledge level"  $p$ ; and 3) a "target level"  $T$ . After describing these elements, a number, say  $\tilde{n}$ , of consumers pledge contributions. If the sum of pledges exceed the target level, i.e. if  $\tilde{n} \cdot p \geq T$ , the entrepreneur receives the contribution  $p$  from each of the  $\tilde{n}$  pledging consumers and in return delivers to each of them the promised reward. If the pledged contributions lie below the target level,  $\tilde{n} \cdot p < T$ , then the project is cancelled; consumers withdraw their pledges and the entrepreneur has no obligations. Hence, given a specified reward, a pair  $(p, T)$  defines the crowdfunding scheme.

As we formally show, we can indeed implement both efficient and constrained-efficient contracts of our framework by a crowdfunding scheme  $(p, T)$ , where  $p$  represents the pledge level and  $T$  a target level that has to be reached in order to trigger the investment. Hence, the pledge  $p$  together with a target level  $T$  presents a simple but optimal way to elicit the consumer's private demand information and on which to condition the investment decision – even in the presence of moral hazard.<sup>4</sup> In addition, the hands-off approach of current crowdfunding platforms is in line with their role in our formal analysis: they are mediators in the sense of Myerson (1982).

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<sup>3</sup>For more details see <https://www.kickstarter.com/help/faq/kickstarter+basics?ref=footer>, last retrieved 14 December 2015.

<sup>4</sup>Crowdfunding schemes that trigger funding only if the target level is reached are called "all-or-nothing" schemes. They are commonly used by platforms that focus on for-profit projects (e.g. Kickstarter, Sellaband, and PledgeMusic). Platforms that focus on non-profit projects (e.g. GoFundMe) often use the alternative "keep-what-you-raise" system, where pledges are triggered even if the target level is not reached. The for-profit and non-profit platform Indiegogo actually gives the entrepreneur the choice between using the all-or-nothing (fixed funding) or keep-what-you-raise (flexible funding) model.

An at first sight important difference seems that Kickstarter does not use deferred payments to control entrepreneurial moral hazard. A somewhat broader view however clarifies that such deferred payments are used implicitly in the form of revenues from selling goods to non-crowdfunders in the after-market. In the case of Kickstarter these after-market revenues are substantial. In line with this view, crowdfunding platforms like PledgeMusic, which focus on products with an after-crowdfunding market that is arguably small, do use deferred payments explicitly.<sup>5</sup>

Finally, we argue that because crowdfunding schemes themselves are, in the presence of moral hazard, unable to attain full efficiency in general, they complement rather than substitute traditional forms of venture capital. Whereas the strength of crowdfunding lies in learning about aggregate demand, the advantage of venture capitalists (or banks) lies in controlling better entrepreneurial moral hazard. The two forms are therefore complementary and we expect them to converge in the future.

The rest of the paper is organized as follows. The next section discusses the related literature. Section 3 introduces the setup and takes an intuitive approach that identifies the main trade-offs. Section 4 sets up the problem as one of mechanism design. Section 5 characterizes (constrained) efficient mechanisms. Section 6 relates optimal mechanisms to real-life crowdfunding mechanisms and examines extensions. Section 7 concludes. All formal proofs are collected in the appendix.

## 2 Related literature

Being a relatively new phenomenon, the economic literature on crowdfunding is small but growing. Concerning crowdfunding's economic underpinnings, Agrawal et al. (2014) highlight the main issues. They emphasize entrepreneurial moral hazard and mention that crowdfunding can reduce demand uncertainty. Belleflamme et al. (2015) survey current crowdfunding platforms and also point out the use of crowdfunding "for market testing under uncertain aggregate demand."

Focusing on revenue-maximization rather than efficiency and fully abstracting from moral hazard, Cornelli (1996) considers a monopolistic firm that, after a fixed

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<sup>5</sup>PledgeMusic explicitly mentions that it uses deferred payments to prevent fraud: <http://www.pledgemic.com/blog/220-preventing-fraud>, last retrieved 2 November 2015.

initial investment, faces privately informed consumers. She fully characterizes the profit maximizing selling mechanism. Importantly, this mechanism engages in price discrimination and conditions the investment decision on the actual *composition* of aggregate contributions rather than the *sum* of aggregate contributions (see also Barbieri and Malueg, 2010). As a result, crowdfunding schemes that condition on the sum of pledges cannot price-discriminate optimally. Restricting attention to different ad hoc types of crowdfunding schemes, Belleflamme et. al (2014), Ellman and Hurkens (2014), and Chang (2015), however, point out that, even though suboptimal, crowdfunding still helps firms to price-discriminate consumers to some degree.<sup>6</sup>

With respect to this strand of the literature, the current paper emphasizes that crowdfunding schemes exhibits important efficiency effects unrelated to price discrimination. Emphasizing these differences also clarifies that crowdfunding may serve two distinctive role. First, as analyzed in this paper, crowdfunding schemes are a tool for screening for valuable projects, which requires identifying differences in the *aggregate* valuation of consumers. Second, crowdfunding schemes are a tool for price discrimination, which requires identifying and exploiting differences between the *individual* valuations of consumers.

Identifying these two distinctive roles of crowdfunding is also crucial for evaluating its welfare properties and addressing regulatory questions. Using crowdfunding as a screening tool for project value univocally benefits both welfare and consumers, whereas the use of crowdfunding as a tool for price discrimination has, at best, ambiguous effects on welfare and consumers (e.g. Bergemann et. al. 2015).

While there is little work in economics and finance that focuses on the firm's ability to screen for valuable projects by addressing consumers directly, the marketing literature explicitly addresses this issue in its subfield of market research, focusing on consumer surveys and product testing (e.g. Lauga and Ofek 2009). Ding (2007) however points out that marketing research mainly relies on voluntary, non-incentivized reporting by consumers. He emphasizes that consumers need to be given explicit incentives for revealing their information truthfully. In line with this view, we point out that crowdfunding schemes provide explicit incentives for truthtelling naturally.

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<sup>6</sup>Also Hakenes and Schlegel (2015) restrict attention to specific crowdfunding schemes although they do not focus on price discrimination but focus on costly information acquisition.

Empirical studies of crowdfunding aim at identifying the crucial features of crowdfunding projects. Studies such as Agrawal et al. (2011) and Mollick (2014) focus on the geographic origin of consumers relative to the entrepreneur, while Kuppuswamy and Bayus (2013) examine the role of social information for the project’s success. Focusing on investment-based crowdfunding, Hildebrand, et al. (2013) identifies an increased problem of moral hazard. Ordanini et al. (2011) present a marketing-based case study on crowdfunding and also note that crowdfunding blurs the boundaries between marketing and finance.

### 3 A Model of Crowdfunding

This section introduces the framework. It considers an entrepreneur, who, prior to her investment decision, directly interacts with privately informed consumers about whether they value the product. In order to clearly demonstrate the potential of crowdfunding, we first model and discuss the role of demand uncertainty, and introduce the problem of moral hazard only in a second step.

**The entrepreneur.** We consider a penniless entrepreneur, who needs an upfront investment of  $I > 0$  from investors to develop her product. After developing it, the entrepreneur can produce the good at some marginal cost  $c \in [0, 1)$ . The entrepreneur is crucial for realizing the project and cannot sell her idea to outsiders. We normalize interest rates to zero.

**The crowd.** We consider a total of  $n$  consumers and denote a specific consumer by the index  $i = 1, \dots, n$ . A consumer  $i$  either values the good,  $v_i = 1$ , or not,  $v_i = 0$ .<sup>7</sup> Hence, the  $n$ -dimensional vector  $v = (v_1, \dots, v_n) \in V \equiv \{0, 1\}^n$  represents the valuation profile of the consumers. We let  $\pi(v)$  denote its corresponding probability. As a result, the number of consumers with value  $v = 1$ , which we express by  $n_1$ , is:

$$\Pr\{n_1\} \equiv \sum_{\{v: \sum_i v_i = n_1\}} \pi(v).$$

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<sup>7</sup>The binary structure ensures that demand uncertainty expresses itself only concerning the question whether the entrepreneur should invest without affecting actual pricing decisions. It clarifies that the model’s driving force is *not* price discrimination. Section 6.4 discusses this in more detail.

Since the marginal costs  $c$  are smaller than 1, we can take  $n_1$  as the potential demand of the entrepreneur's good. Its randomness expresses the demand uncertainty.

**Investing without demand uncertainty.** Consider as a benchmark the case of perfect information, where the realized demand  $n_1$  is observable so that the investment decision can directly condition on it. It is socially optimal that the entrepreneur invests if the project's revenue,  $n_1$ , covers the costs of production  $I + n_1c$ , i.e. if

$$n_1 \geq \bar{n} \equiv \frac{I}{1-c}.$$

In this case, the project generates an ex ante expected aggregate surplus of

$$S^* = \sum_{n_1=\bar{n}}^n \Pr\{n_1\}[(1-c)n_1 - I],$$

which is assumed to be strictly positive. Note that by investing whenever  $n_1 \geq \bar{n}$  and, subsequently, selling the good at a price  $p = 1$ , the entrepreneur can appropriate the full surplus. Given that the entrepreneur obtains the funds, this behavior represents her optimal strategy. Anticipating the entrepreneur's optimal behavior, a competitive credit market is willing to lend the amount  $I$  at the normalized interest rate of zero. Hence, perfect information yields an efficient outcome.

**Investing with demand uncertainty.** Next consider the setup with demand uncertainty, i.e. the entrepreneur must decide to invest  $I$  without knowing  $n_1$  if she wants to sell the good at some price  $p$ . If she does invest, it clearly remains optimal to set a price  $p = 1$ . Hence, expected profits from investing are

$$\bar{\Pi} = \left( \sum_{n_1=0}^n \Pr\{n_1\}(1-c)n_1 \right) - I.$$

It is therefore profitable to invest only if  $\bar{\Pi} \geq 0$ . Even though the price  $p = 1$  does not leave any consumer rents, the entrepreneur's decision to invest leads either to under- or over-investment. For parameter constellations such that  $\bar{\Pi} < 0$ , the entrepreneur will not invest and, hence, under-investment results (because the good is not produced for any  $n_1 > \bar{n}$ , where it would be efficient to produce). For the parameter constellation  $\bar{\Pi} \geq 0$ , the entrepreneur does invest  $I$ , but this implies over-investment (because she produces the good also when it turns out that  $n_1 < \bar{n}$ ).

**Crowdfunding without moral hazard.** We next consider the case of demand uncertainty but with an "all-or-nothing reward-based crowdfunding scheme" ( $p, T$ )

as introduced in the introduction. That is, the investment is now governed by a contract pair  $(p, T)$  with the interpretation that if the total amount of pledged funds,  $P$ , exceeds  $T$ , then the entrepreneur obtains it “all”: she receives the pledges  $P$ , invests, and produces a good for each consumer who pledged. If the total amount of pledges  $P$  falls short of  $T$ , then the entrepreneur obtains “nothing”: the pledges are not triggered, the entrepreneur receives no funding, and she does not invest.

It is straightforward to see that this crowdfunding scheme enables the entrepreneur to extract the maximum aggregate surplus  $S^*$  and, thereby, achieve an efficient outcome. Indeed, for any  $p \in (0, 1]$ , it is optimal for the consumer to pledge  $p$  if and only if  $v = 1$ . As a result, exactly  $n_1$  consumers sign up so that the sum of pledges equals  $P = n_1 p$ . Hence, the project is triggered whenever  $T \leq n_1 p$ . Hence, an all-or-nothing crowdfunding scheme  $(p, T)$  with  $p \in (0, 1]$  yields the entrepreneur the expected profit

$$\Pi^c(p, T) = \sum_{n_1=T/p}^n \Pr\{n_1\}[(1-c)p - I].$$

Clearly, a pledge level  $p = 1$  and target level  $T = \bar{n}$  maximize profits, enabling the entrepreneur to extract the surplus  $S^*$  and yielding an efficient outcome.

Apart from stressing the surprisingly simple way by which the crowdfunding pair  $(p, T)$  resolves the problem of demand uncertainty, it is worthwhile to point out two additional features. First, even without any active coordination between consumers, it circumvents any potential coordination problems. This is because of the schemes second feature: it eliminates any strategic uncertainty concerning both the behavior and the private information of other consumers. In other words, the all-or-nothing pledge system induces a game between the consumers, in which it is a (weakly) dominant strategy for each individual consumer  $i$  to pledge if and only if he values the product. In the jargon of mechanism design, the all-or-nothing crowdfunding scheme  $(p, T) = (1, \bar{n})$  implements the first best in dominant strategies.

**Moral hazard.** The setup until now abstracted from problems of moral hazard. Consumers are sure to obtain the promised good if their pledge is triggered. In practice, consumers may however worry about whether the entrepreneur will deliver a good that meets the initial specifications, or deliver the good at all.

We capture the problem of moral hazard by assuming that, after the entrepreneur obtains the money from the crowdfunding platform, she can “make a run” for it with

a share  $\alpha \in [0, 1]$ . When the entrepreneur “runs”, she does not incur any investment or production costs and consumers do not obtain their goods. The parameter  $\alpha$  measures the weakness of the institutional environment to prevent moral hazard. For the extreme  $\alpha = 0$ , there is effectively no moral hazard, whereas for the extreme,  $\alpha = 1$ , the entrepreneur can keep all the pledges without incurring any costs.

It is important to stress that this modeling approach captures several types of moral hazard problems. First, we can take the running literally: The entrepreneur is able to flee with the share  $\alpha P$  without being caught. Or, alternatively, run with the amount  $P$  but with an expected fine of  $(1 - \alpha)P$ .<sup>8</sup> Second, at a reduced cost of  $(1 - \alpha)P < I - \tilde{n}c$  the entrepreneur can provide the consumer a product that matches the formal description but is still worthless to the consumer.<sup>9</sup> Third, by a (possibly expected) cost  $(1 - \alpha)P$ , the entrepreneur can convincingly claim that the project failed so that, without fearing any legal repercussions, she need not deliver the product and keep the pledges.<sup>10</sup>

In order to see that moral hazard undermines the simple crowdfunding scheme  $(p, T)$ , note that, facing aggregated pledges  $P$ , the entrepreneur obtains a payoff  $\alpha P$  from running and a profit  $P - I - cP/p$  from investing. Hence, she runs if

$$\alpha P > P - I - cP/p. \tag{1}$$

The inequality not only holds for the extreme  $\alpha = 1$  but also for any  $\alpha \geq 1 - c/p$ . For all these cases, consumers rationally expect that the entrepreneur will not deliver the

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<sup>8</sup>E.g. the project “Code hero” raised \$170,954 but never delivered its rewards, Polygon.com states “His critics believe he has run off with the money raised from the kickstarter campaign” (<http://www.polygon.com/2012/12/18/3781782/code-hero-kickstarter-interview>, last retrieved 3 November 2015), whereas the Kickstarter campaign “Asylum Playing Cards” resulted in legal fines “against a crowdfunded project that didn’t follow through on its promise to backers” (<http://www.atg.wa.gov/news/news-releases/ag-makes-crowdfunded-company-pay-shady-deal>, last retrieved 8 November 2015).

<sup>9</sup>E.g. the crowdfunding project “Healbe GoBe” raised much controversy about whether the delivered product actually works (see for instance <http://blog.belgoat.com/24-hours-with-my-healbe-gobe/>, last retrieved 8 November 2015).

<sup>10</sup>E.g. Kickstarter refers to this possibility explicitly: “If a creator is making a good faith effort to complete their project and is transparent about it, backers should do their best to be patient and understanding while demanding continued accountability from the creator.” (<https://www.kickstarter.com/help/faq/kickstarter%20basics#Acco>, last retrieved 8 November 2015).

product so that they will not be willing to participate in the crowdfunding scheme.

In the remainder of this section, we introduce two intuitive but ad hoc changes to the crowdfunding scheme  $(p, T)$  that reduces entrepreneurial moral hazard. Using a mechanism design approach, the next section proves that the two changes lead to mechanisms that are indeed optimal in the class of all possible mechanisms.

**Deferred payments.** An intuitive way to mitigate the moral hazard problem is to transfer the consumer's pledges to the entrepreneur only *after* having produced the good. Because the penniless entrepreneur needs at least the amount  $I$  to develop the product, such a delay in payments is possible only up to the amount  $I$ .

Hence, a first, ad hoc step towards mitigating the moral hazard problem is to adjust the crowdfunding scheme  $(p, T)$  and introduce deferred payments as follows. As before, the variable  $p$  represents the pledge level of an individual consumer and  $T$  the target level which the sum of pledges,  $P$ , has to meet before the investment is triggered. Different from before however, the entrepreneur, after learning  $P$ , first obtains only the required amount  $I$  for developing the product and receives the remaining part  $P - I$  only after delivering the good to consumers.

In order to characterize crowdfunding schemes with deferred payments that prevent moral hazard, note that the entrepreneur now obtains only the payoff  $\alpha I$  from a run and the payoff  $P - I - cP/p$  from realizing the project. Hence, she has no incentive to run if

$$\alpha I \leq P - I - cP/p \Rightarrow P \geq \bar{P} \equiv \frac{(1 + \alpha)pI}{p - c}. \quad (2)$$

In particular, the deferred crowdfunding scheme with a pledge level of  $p = 1$  and a target  $T = (1 + \alpha)I/(1 - c)$  does not induce any running by the entrepreneur. Given this scheme, a consumer with value  $v = 1$  is willing to pledge  $p = 1$  and the scheme leads to an equilibrium outcome in which all consumers with  $v = 1$  pledge and the project is triggered when at least  $T$  consumers have the willingness to pay of 1, i.e. if  $n_1 > (1 + \alpha)I/(1 - c)$ . Although the scheme prevents moral hazard, it, for any  $\alpha > 0$ , does not attain the efficient outcome, because its target level is larger than the socially efficient one; the scheme exhibits under-investment.

**The information trade-off.** We argued that a crowdfunding scheme with deferred payments can circumvent the moral hazard problem. Since this deferred crowdfund-

ing scheme does not yield an efficient outcome, the question arises whether there are more sophisticated schemes that do better. To show that this is indeed the case, note first that with respect to choosing the efficient investment decision, the entrepreneur only needs to learn whether  $n_1$  is above or below  $\bar{n}$ . The value of  $n_1$  itself is immaterial.

Yet, as inequality (2) reveals, the moral hazard problem intensifies if the entrepreneur obtains full information about  $P$ . As discussed, this inequality has to hold for any possible realization of  $P \geq T$  in order to prevent the entrepreneur from running. Because the constraint is most stringent for  $P = T$ , a crowdfunding scheme  $(p, T)$  prevents moral hazard if and only if the target  $T$  exceeds the threshold  $\bar{P}$ .

In contrast, if the entrepreneur would only learn that  $P$  exceeds  $T$ , but not the exact value of  $P$  itself, then she rationally anticipates an expected payoff

$$E[P|P \geq T] - I - cE[P|P \geq T]/p$$

from not running. Hence, this adapted scheme prevents moral hazard if  $E[P|P \geq T]$  rather than  $T$  exceeds  $\bar{P}$ . Since the conditional expectation  $E[P|P \geq T]$  is at least  $T$ , a crowdfunding scheme that reveals only whether  $P$  exceeds  $T$  can deal with the moral hazard problem more efficiently.

Hence, in the presence of both demand uncertainty and moral hazard, the information extraction problem becomes a sophisticated one, because the extraction of demand information interacts with the moral hazard problem. As a result, the entrepreneur should learn neither too much nor too little.

The analysis up to now has been ad hoc. By starting with a simple crowdfunding scheme as used in practise and adapting it in two way — introducing deferred payments and reducing its informativeness — we improved its efficiency properties. The ad hocness of these two changes raises, however, the question whether even more efficient crowdfunding mechanisms exist.

In order to address this question, the next two sections study the crowdfunding problem as one of optimal mechanism design. It formally proves that the *payout-deferred, information-restricted, all-or-nothing reward-crowdfunding scheme*  $(p, T)$  towards which we argued intuitively is indeed optimal in the class of all mechanisms. In other words, such crowdfunding schemes are an indirect implementation of the optimal direct mechanism that we introduce and study next.

## 4 Crowdfunding and Mechanism Design

In this section we cast the entrepreneur's economic problem into a problem of mechanism design and characterize optimal mechanisms. In order to treat the entrepreneur's moral hazard, we use the framework of Myerson (1982), which can handle both ex ante private information and moral hazard. This generalized framework introduces a mediator, who coordinates the communication between economic agents and gives incentive compatible recommendations concerning the unobservable actions that underlie the moral hazard problem. One of the insights from this analysis is that crowdfunding platforms play exactly the role of a mediator in the sense of Myerson (1982). The section's main result is to confirm formally that the payout-deferred, information-restricted, all-or-nothing reward-crowdfunding scheme as identified in the previous section is a (possibly constrained) efficient mechanism.

**Economic Allocations.** In order to cast the entrepreneur's investment problem in a framework of mechanism design, we first make precise the feasible economic allocations: Crowdfunding seeks to implement an allocation between one cash-constrained entrepreneur, player 0, and  $n$  consumers, players 1 to  $n$ . It involves monetary transfers and production decisions. Concerning monetary transfers, consumers can make transfers to the entrepreneur both before and after the entrepreneur's investment decision. We denote the ex ante transfer from consumer  $i$  to the entrepreneur by  $t_i^a$  and the ex post transfer by  $t_i^p$ . Concerning the production decisions, the allocation describes whether the entrepreneur invests,  $x_0 = 1$ , or not,  $x_0 = 0$ , and whether the entrepreneur produces a good for consumer  $i$ ,  $x_i = 1$ , or not,  $x_i = 0$ . Consequently, an economic allocation is a collection  $a = (t, x)$  of transfers  $t = (t_1^a, \dots, t_n^a, t_1^p, \dots, t_n^p) \in \mathbb{R}^{2n}$  and outputs  $x = (x_0, \dots, x_n) \in X \equiv \{0, 1\}^{n+1}$ .

**Feasible Allocations.** By the very nature of the crowdfunding problem, the entrepreneur does not have the resources to finance the required investment  $I > 0$ . The entrepreneur's financial constraints imply the following restrictions on feasible allocations. First, if the entrepreneur invests ( $x_0 = 1$ ), the transfers of the consumers must be enough to cover the investment costs  $I$ . Moreover, the entrepreneur cannot make any net positive ex ante transfers to consumers if she does not invest ( $x_0 = 0$ ). Second, aggregate payments must be enough to cover the entrepreneur's investment

and production costs. To express these two feasibility requirements, we say that an allocation  $a = (t, x)$  is *budget-feasible* if

$$\sum_{i=1}^n t_i^a \geq Ix_0 \text{ and } \sum_{i=1}^n t_i^a + t_i^p \geq Ix_0 + c \sum_i x_i. \quad (3)$$

In addition, an entrepreneur can only produce a good to a consumer if she developed it. To express this feasibility requirement, we say that an allocation  $a = (t, x)$  is *development feasible* if, whenever the good is produced for at least one consumer, the entrepreneur invested in its development:

$$\exists i : x_i = 1 \Rightarrow x_0 = 1. \quad (4)$$

This condition logically implies that if  $x_0 = 0$  then  $x_i = 0$  for all  $i$ .

Let the set  $A \subset \mathbb{R}^{2n} \times \{0, 1\}^{n+1}$  denote the set of budget- and development-feasible allocations, i.e. allocations that satisfy (3) and (4).

**Payoffs.** Let the  $n$ -dimensional vector  $v = (v_1, \dots, v_n) \in V \equiv \{0, 1\}^n$  represent the valuation profile of the consumers. We denote the probability of  $v \in V$  by  $\pi(v)$  and the conditional probability of  $v_{-i} \in V_{-1} \equiv \{0, 1\}^{n-1}$  given  $v_i$  as  $\pi_i(v_{-i}|v_i)$ . Assuming that individual types are drawn independently, it holds  $\pi_i(v_{-i}|0) = \pi_i(v_{-i}|1)$  so that we can express the conditional probability simply as  $\pi_i(v_{-i})$ .<sup>11</sup> Moreover, we assume that consumers are identical:  $\pi_i(v_{-k}) = \pi_j(v_{-k})$  for any  $v_{-k} \in V_{-1}$  and  $i, j$ .

A feasible allocation  $a \in A$  yields a consumer  $i$  with value  $v_i$  the payoff

$$U_i(a|v_i) = v_i x_i - t_i^a - t_i^p;$$

and the entrepreneur the payoff

$$\Pi(a) = \sum_{i=1}^n (t_i^a + t_i^p) - c \cdot \sum_{i=1}^n x_i - Ix_0 \geq 0,$$

where the inequality follows directly from the second inequality in (3), implying that any feasible allocation yields the entrepreneur a non-negative payoff.

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<sup>11</sup>Although we introduce an independence assumption here to avoid possible complications due to correlated private information, we stress that all our results hold also with correlated values. This is so because the efficient scheme does not leave any information rents even with independence.

**Efficiency.** An output schedule  $x \in X$  is *Pareto efficient* in state  $v$  if and only if it maximizes the *aggregate net surplus*

$$S(x|v) \equiv \Pi(a) + \sum_{i=1}^n U_i(a|v_i) = \sum_{i=1}^n (v_i - c)x_i - Ix_0.$$

With respect to efficiency, two different types of production decisions matter: the overall investment decision  $x_0$  and the individual production decisions  $x_i$ . Given  $v_l = 0 < c < v_h = 1$ , efficiency with respect to the individual allocations requires  $x_i = v_i$ . This yields a surplus of  $\sum_i v_i(1 - c) - I$ .

Defining

$$\bar{n} \equiv \frac{I}{1 - c}; V^0 \equiv \{v : \sum_i v < \bar{n}\}; V^1 \equiv \{v : \sum_i v \geq \bar{n}\}; \text{ and } \pi^* \equiv \sum_{v \in V^1} \pi(v),$$

we can fully characterize the *Pareto efficient output schedule*  $x^*(v)$  as follows. For  $v \in V^0$ , it exhibits  $x_0^* = x_i^* = 0$  for all  $i$ . For  $v \in V^1$ , it exhibits  $x_0^* = 1$  and  $x_i^* = v_i$  for all  $i$ .<sup>12</sup> Under an efficient output schedule, the entrepreneur invests only if  $v \in V^1$ , implying that  $\pi^*$  expresses the ex ante probability that the project is executed.

Although transfers are immaterial for Pareto efficiency, we must nevertheless ensure that the efficient output schedule  $x^*(v)$  can indeed be made part of some feasible allocation  $a \in A$ . In order to specify one such feasible allocation, we define the *first best* allocation  $a^*(v)$  as follows. For  $v \in V^1$ , it exhibits  $x_i = t_i^a = v_i = 1$  and  $t_i^p = 0$ . For  $v \in V^0$ ,  $a^*(v)$  is defined by  $x_i = t_i^a = x_i = t_i^a = t_i^p = 0$ . By construction  $a^*(v)$  is feasible and yields an ex ante expected gross surplus (gross of investment costs) of  $W^*$ , where

$$W^* \equiv \sum_{v \in V^1} \sum_i \pi(v) v_i (1 - c) \quad (5)$$

We further say that an output schedule  $x : V \rightarrow X$  is *development efficient* if

$$x_0(v) = 1 \Rightarrow \exists i : x_i(v) = 1. \quad (6)$$

This condition is the converse of development feasibility (4). If it does not hold, it implies the inefficiency that there is a state  $v$  in which the entrepreneur invests  $I$  but no consumer consumes the good. Although technically feasible, a schedule that is not development efficient is not Pareto efficient, since it wastes the investment  $I > 0$ .

<sup>12</sup>For  $\sum_i v = \bar{n}$ , the output schedule  $x_0^* = x_i^* = 0$  is also efficient, but this is immaterial (and can only arise for the non-generic case that  $I$  is a multiple of  $1 - c$ ).

For future reference, the following lemma summarizes these considerations.

**Lemma 1** *The first best allocation  $a^*(v)$  is feasible and exhibits an output schedule that is development efficient. It yields an expected net surplus of  $W^* - \pi^*I$ .*

**Mechanisms.** We next turn to mechanisms. A *mechanism*  $\Gamma$  is a set of rules between the entrepreneur and the  $n$  consumers that induces a game. Its outcome is an allocation  $a \in A$  with payoffs  $\Pi(a)$  and  $U_i(a|v_i)$ . In line with Myerson (1982), we interpret the crowdfunding platform as the mediator, who runs the mechanism; it coordinates the communication between participants and enforces the rules of the game which the mechanism specifies.

At the end of Section 3, we considered one such mechanism: the *payout-deferred, information-restricted reward-crowdfunding scheme*  $\Gamma^{CF} = (p, T)$ . This mechanism induces the following game between the entrepreneur and consumers: Each consumer  $i$  sends to the platform a confidential binary message  $m_i \in \{0, 1\}$ . If  $\sum_i m_i p < T$ , the platform enforces the allocation  $t_i^a = t_i^p = x_i = x_0 = 0$ . If  $\sum_i m_i p \geq T$ , the platform enforces the ex ante payments  $t_i^a = I / \sum_i m_i$ . In case the entrepreneur does not run, it subsequently enforces outputs  $x_i = m_i$  and transfers  $t_i^p = p - t_i^a$ . In case the entrepreneur runs, the outputs  $x_i = 0$  and transfers  $t_i^p = 0$  result.

The main task of this section is to demonstrate that this type of mechanism is indeed optimal. In order to do so, we first introduce a different class of mechanisms: direct mechanisms.

A *direct mechanism* is a function  $\gamma : V \rightarrow A$ , which induces the following game.<sup>13</sup> First, consumers simultaneously and independently send a (confidential) report  $v_i^r$  about their values to the platform. Based on the collected reports  $v^r$  and in line with the rules  $\gamma$ , the platform collects the funds  $T^a = \sum_i t_i^a(v^r)$  from the consumers and transfers them to the entrepreneur together with the recommendation  $x_0(v^r)$  about whether to invest  $I$ . To capture the moral hazard problem, we explicitly assume that the platform cannot coerce the entrepreneur into following the recommendation  $x_0 = 1$ . That is, the entrepreneur is free to follow or reject it. If, however, the entrepreneur follows the recommendation, the platform enforces the production schedule  $x(v^r) =$

<sup>13</sup>Hence, we study the extreme where the entrepreneur contracts with *all*  $n$  consumers. In section 6.2 we address extensions where the mechanism can reach only a subset of all consumers.

$(x_1(v^r), \dots, x_n(v^r))$  and the transfers  $t_i^p(v^r)$ . If the entrepreneur does not follow the recommendation to invest, but runs, then individual production schedules are 0, and no ex post transfers flow, i.e.  $x_i = t_i^p = 0$ . Moreover, consumers forfeit their ex ante transfers  $t_i^a$ .

A direct mechanism  $\gamma$  is *incentive compatible* if its induced game as described above has a perfect Bayesian equilibrium in which 1) consumers are *truthful* in that they reveal their values honestly, i.e.  $v_i^r = v_i$ , and 2) the entrepreneur is *obedient* in that she follows the recommendation, i.e.  $x_0 = x_0(v^r)$ .

To formalize the notion of truthful revelation, we define

$$X_i(v_i) \equiv \sum_{v_{-i} \in V_{-i}} x_i(v_i, v_{-i}) \pi_i(v_{-i});$$

and

$$T_i(v_i) \equiv \sum_{v_{-i} \in V_{-i}} (t_i^a(v_i, v_{-i}) + t_i^p(v_i, v_{-i})) \pi_i(v_{-i}).$$

Consequently, we say that a direct mechanism  $\gamma$  is *truthful* if

$$v_i X_i(v_i) - T_i(v_i) \geq v_i X_i(v'_i) - T_i(v'_i) \text{ for all } i \in I \text{ and } v_i, v'_i \in V_i. \quad (7)$$

To formalize the notion of obedience, we define for a direct mechanism  $\gamma$  the set  $\mathcal{T}^a$  as the set of possible aggregate ex ante transfers which the mechanism can induce conditional on recommending investment:

$$\mathcal{T}^a \equiv \{T^a | \exists v \in V : \sum_{i=1}^n t_i^a(v) = T^a \wedge x_0(v) = 1\}.$$

Given this set we define for any  $T^a \in \mathcal{T}^a$  the set  $V(T^a)$  which comprises all states that induce a recommendation to invest together with a total transfer  $T^a$ :

$$V(T^a) \equiv \{v \in V | x_0(v) = 1 \wedge \sum_i t_i^a(v) = T^a\}.$$

Upon receiving a recommendation to invest, the entrepreneur has received some transfer  $T^a \in \mathcal{T}^a$  and has a belief  $\pi(v|T^a)$  that the state is  $v$ . These beliefs are Bayes' consistent whenever

$$\pi(v|T^a) \equiv \begin{cases} \frac{\pi(v)}{\sum_{v' \in V(T^a)} \pi(v')} & \text{if } v \in V(T^a); \\ 0 & \text{otherwise.} \end{cases}$$

With this notation, we say that a direct mechanism  $\gamma$  is *obedient* if for any  $T^a \in \mathcal{T}^a$  and after obtaining the recommendation to invest,  $x_0 = 1$ , the entrepreneur is better off investing than taking the money and run, given her updated belief  $\pi(v|T^a)$ :

$$\sum_{v \in V} \sum_{i=1}^n \pi(v|T^a)(t_i^p(v) - cx_i(v)) + T^a - I \geq \alpha T^a, \text{ for all } T^a \in \mathcal{T}^a. \quad (8)$$

We say that a direct mechanism is incentive compatible if and only if it is truthful and obedient.

By its nature, participation in the crowdfunding mechanism is voluntary so that it must yield the consumers and the entrepreneur at least their outside option. Taking these outside options as 0, the entrepreneur's participation is not an issue, because, as argued, any feasible allocation yields the entrepreneur a non-negative payoff. In contrast, a consumer's participation in an incentive compatible direct mechanism is *individual rational* only if

$$v_i X_i(v_i) - T_i(v_i) \geq 0 \text{ for all } i \in I \text{ and } v_i \in V_i. \quad (9)$$

To summarize, we say that a direct mechanism  $\gamma$  is *feasible*, if it is incentive compatible and individual rational for each consumer.<sup>14</sup> A feasible direct mechanism yields consumer  $i$  with valuation  $v_i$  the utility

$$U_i(v_i) \equiv v_i X_i(v_i) - T_i(v_i); \quad (10)$$

and the entrepreneur an expected payoff

$$\Pi = \sum_{v \in V} \pi(v) \Pi(\gamma(v)). \quad (11)$$

Finally, we say that two feasible direct mechanisms  $\gamma = (t, x)$  and  $\gamma' = (t', x')$  are *payoff-equivalent* if they lead to identical payoffs to each consumer type  $v_i$ :

$$\sum_{v_{-i} \in V_{-i}} \pi(v_{-i}) U_i(\gamma(v), v_i) = \sum_{v_{-i} \in V_{-i}} \pi(v_{-i}) U_i(\gamma'(v), v_i), \quad \forall i, v_i;$$

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<sup>14</sup>This implicitly assumes that the mechanism has “perfect consumer reach” in that every consumer is aware and can participate in the mechanism. As an extension that yields important additional insights, Subsection 6.2 studies the effect of imperfect consumer reach.

and the entrepreneur:

$$\sum_{v \in V} \pi(v) \Pi(\gamma(v)) = \sum_{v \in V} \pi(v) \Pi(\gamma'(v)).$$

**Implementability.** An *allocation function*  $f : V \rightarrow A$  specifies for any value profile  $v$  an allocation  $a \in A$ . It is *implementable* if there exists a mechanism  $\Gamma$  such that the induced game has a perfect Bayesian equilibrium outcome in which the induced allocation coincides with  $f(v)$  for every  $v \in V$ . In this case, we say  $\Gamma$  *implements*  $f$ .

Likewise, an *output schedule*  $x : V \rightarrow X$  specifies for any value profile  $v$  an output schedule  $x \in X$ . It is *implementable* if there exists a mechanism  $\Gamma$  such that the induced game has a perfect Bayesian equilibrium outcome in which the induced output coincides with  $x(v)$  for every  $v \in V$ . In this case, we say  $\Gamma$  implements output schedule  $x(\cdot)$ .

By the revelation principle, an allocation function  $f(\cdot)$  is implementable if and only if there exists a feasible direct mechanism  $\gamma$  with  $\gamma(v) = f(v)$  for any  $v \in V$ . Likewise, an output schedule  $x(\cdot)$  is implementable if and only if there exists a direct mechanism  $\gamma = (x_\gamma, t_\gamma)$  such that  $x_\gamma(v) = x(v)$  for any  $v \in V$ . Hence, as usual, the revelation principle motivates incentive compatibility as one of the defining requirements of feasibility. A first question that arises is whether an efficient output schedule is always implementable. Considering a specific version of the model, the next proposition demonstrates that this is not the case:

**Proposition 1** *For  $I = n - 1/2$ ,  $\alpha = 1$ , and  $c = 0$ , the efficient output schedule  $x^*(v)$  is not implementable.*

The proposition implies that, in general, the efficient output is not implementable. The main driver behind this inefficiency result is a tension between the entrepreneur's budget constraint and the moral hazard problem. For consumers to make sure that the entrepreneur realizes her project, it does not suffice to give her simply the required amount  $I$  to invest. Due to the moral hazard problem, she must also be given an incentive to actually invest this money. The proposition shows that for the efficient output schedule  $x^*$  this is, in general, not possible.

The proposition raises questions about which output schedules are generally implementable and about the conditions under which the efficient schedule is imple-

mentable. To answer these questions we investigate the mechanism design problem further. The following lemma shows that with respect to development-efficient allocations, we may reduce the class of feasible direct mechanisms further.

**Lemma 2** *If  $\gamma = (t, x)$  is feasible and  $x$  is development-efficient then there is a feasible and payoff equivalent direct mechanism  $\hat{\gamma} = (\hat{t}, x)$  with*

$$\sum_i \hat{t}_i^a(v) = Ix_0(v), \forall v \in V. \quad (12)$$

The lemma implies that with respect to development-efficient mechanisms there is no loss of generality in restricting attention to feasible direct mechanisms that give the entrepreneur exactly the amount  $I$  if the entrepreneur is to develop the product. The lemma therefore makes precise the suggestion of the previous section that a mechanism should provide the entrepreneur with the minimal amount of information for reducing demand uncertainty; effectively, she should only be told that the demand of consumers ensures that the project has a positive NPV, but not more. The main step in proving this result is to show that obedience remains satisfied when we replace different aggregate levels of ex ante payments by a single one.<sup>15</sup>

The lemma simplifies the mechanism design problem in two respects. First, under condition (12), condition (3) reduces to

$$\sum_{i=1}^n t_i^p(v) \geq c \sum_i x_i(v). \quad (13)$$

Second, under condition (12), we have  $\mathcal{T}^a = \{I\}$  so that the obedience constraint (8) must only be respected with regard to  $I$ :

$$\sum_{v \in V} \sum_{i=1}^n \pi(v|I)(t_i^p(v) - cx_i(v)) \geq \alpha I. \quad (14)$$

## 5 (Constrained) efficient mechanisms

In this section we characterize second best mechanisms  $\gamma^{sb} = (x^{sb}, t^{sb})$  that maximize aggregate surplus in the presence of demand uncertainty and moral hazard. We are

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<sup>15</sup>The lemma fails for development-inefficient mechanisms so that we cannot dispense with the restriction to development-efficient mechanisms.

especially interested in determining the circumstances under which these second best mechanisms cannot implement the efficient output schedule  $x^*$ .

Recall that a feasible direct mechanism  $\gamma$  yields a surplus of

$$\sum_{v \in V} \pi(v) S(x(v)|v) = \sum_{v \in V} \pi(v) \left[ \sum_i^n (v_i - c)x_i(v) - Ix_0(v) \right]. \quad (15)$$

Clearly  $\gamma^{sb}$  cannot yield more than  $W^* - \pi^*I$ , which is generated under the efficient output schedule  $x^*$ . Indeed, Proposition 1 showed that, in general,  $\gamma^{sb}$  achieves strictly less.

As  $\gamma^{sb}$  is necessarily development-efficient, we can find it by maximizing (15) subject to the constraints (7), (9), (12), (13), and (14), because these constraints characterize the set of implementable allocation functions that are development-efficient. The maximization problem yields the following partial characterization of  $\gamma^{sb}$ :

**Lemma 3** *The individual rationality constraint of consumers with the high value  $v_i = 1$  does not restrict the second best mechanism  $\gamma^{sb}$ . The second best mechanism exhibits  $x_i(0, v_{-i}) = X_i(0) = T_i(0) = 0$ , and  $T_i(1) = X_i(1)$  for all  $i = 1, \dots, n$ .*

It follows from the previous lemma that the second best mechanism  $\gamma^{sb}$  is a solution to the problem

$$\begin{aligned} \mathcal{P} : \max_{x(\cdot), t(\cdot)} \quad & \sum_{v \in V} \pi(v) \left[ \sum_i^n (v_i - c)x_i(v) - Ix_0(v) \right] \\ \text{s.t.} \quad & T_i(1) = X_i(1) \text{ for all } i; \end{aligned} \quad (16)$$

$$\sum_{v \in V} \sum_{i=1}^n \pi(v|I)(t_i^p(v) - cx_i(v)) \geq \alpha I; \quad (17)$$

$$T_i(0) = 0 \text{ for all } i; \quad (18)$$

$$\sum_{i=1}^n t_i^a(v) = Ix_0(v); \quad (19)$$

$$\sum_{i=1}^n t_i^p(v) \geq \sum_i cx_i(v); \quad (20)$$

$$x_i(v) = 1 \Rightarrow x_0(v) = 1; \quad (21)$$

$$x_i(0, v_{-i}) = 0, \forall v_{-i} \in V_{-i}. \quad (22)$$

Recalling that  $\pi^*$  represents the ex ante probability that the project is executed under the efficient schedule  $x^*$ , we obtain the following result.

**Proposition 2** *The efficient output schedule  $x^*$  is implementable if and only if  $W^* \geq \bar{W}^\alpha \equiv (1 + \alpha)\pi^*I$ . If implementable, the indirect payout-deferred, information-restricted, all-or-nothing reward-crowdfunding scheme  $\bar{\Gamma}^{CF} = (\bar{p}, \bar{T})$  with  $\bar{p} \equiv 1$  and  $\bar{T} \equiv I/(1 - c)$  implements  $x^*$ .*

Proposition 2 makes precise the parameter constellation under which the first best  $x^*$  is implementable: only if the efficient production schedule  $x^*$  generates a surplus that exceeds the ex ante expected investment costs  $(1 + \alpha)$  times. For values of  $W^*$  in between  $\pi^*I$  and  $(1 + \alpha)\pi^*I$ , the optimal mechanism exhibit inefficiencies.

Intuitively, the driver behind this inefficiency is a tension between the entrepreneur's budget constraint and her moral hazard problem. For consumers to make sure that the entrepreneur realizes her project, it does not suffice to give her simply the required amount  $I$  to invest. Due to the moral hazard problem, she must also be given an incentive to actually invest this money. As the proposition shows, this effectively requires consumers to pay the entrepreneur the run-away payoff  $\alpha I$  as a deferred payment. This amount represents the agency costs associated with the entrepreneur's moral hazard problem. They augment investment costs by a factor  $\alpha$ .

Whenever the ex ante gross surplus does not exceed the expected investment costs by the factor  $\alpha$ , the efficient output schedule,  $x^*$ , is not implementable so that the second best output schedule  $x^{sb}$  does not coincide with  $x^*$ . We next characterize both the second best and the type of inefficiencies it exhibits.

**Proposition 3** *For  $W^* < \bar{W}^\alpha$ , the constrained efficient output schedule  $x^{sb}$  exhibits i)  $x_i^{sb}(v) = v_i$  whenever  $x_0^{sb}(v) = 1$ ; ii)  $x_0^{sb}(v) = 0$  whenever  $x_0^*(v) = 0$ ; and iii)  $x_0^{sb}(v) = 1$  whenever  $\sum v_i > (1 + \alpha)I/(1 - c)$ . Moreover, an indirect payout-deferred, information restricted, all-or-nothing reward-crowdfunding scheme  $\Gamma^{CF} = (p, T)$  implements  $x^{sb}$ .*

The first part of the proposition shows that the constrained efficient output schedules are only distorted with respect to the investment decision but not to the individual assignments. The second part of the proposition shows that the second best output schedule is distorted downwards rather than upwards. The third part shows that at most the allocations for which aggregate valuations lie in the range between

$W^*$  and  $\bar{W}^\alpha$  are downward distorted. This final statement also implies that for the constrained efficient output schedule it matters only whether the sum of valuations exceed a target level  $T$ . As a result, the second best scheme can be implemented indirectly by a payout-deferred and information-restricted reward-crowdfunding scheme  $(1, T)$ .

## 6 Interpretation and Discussion

This section interprets the optimal direct mechanism as derived in the previous section and relates it to crowdfunding platforms in practise. It further discusses extensions and robustness of the results.

### 6.1 Comparison to current crowdfunding platforms

Relating our theoretical results to current crowdfunding platforms, our first observation concerns the role of the crowdfunding platform itself. In our formal analysis the platforms structures the communication between entrepreneur and consumers, and executes the mechanism. We note that this is consistent with the role that crowdfunding platforms play in practise. Platforms such as Kickstarter emphasize that they themselves are not involved in the development of the projects and take no responsibility for the entrepreneur’s project. Wikipedia therefore calls the platform “a moderating organization”.<sup>16</sup> Tellingly, the technical term of the platform’s role in our mechanism design is “mediator” (e.g. Myerson 1982). Although the platform’s role seems only minor, it is nevertheless crucial. Due to commitment problems, it cannot be performed by either the entrepreneur or the consumers.

A second notable feature of optimal direct mechanisms is that they do not exhibit negative transfers. Hence, the entrepreneur does not share her revenue after the investment:  $t_i^p(v) \geq 0$  for all  $i$ . Optimal mechanisms do therefore not turn consumers into investors; the optimal crowdfunding scheme is reward-based instead of investment-based. This feature is consistent with popular reward-crowdfunding

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<sup>16</sup>See <https://en.wikipedia.org/wiki/Crowdfunding> and <https://www.kickstarter.com/help/faq/kickstarter%20basics#Acco> for explaining the limited involvement of Kickstarter, last retrieved 8 November 2015.

platforms such as Kickstarter, which explicitly prohibit any monetary transfers to crowdfunders.<sup>17</sup>

A third feature of optimal direct mechanisms is that they condition the investment decision on the sum of reported valuations rather than each consumer's report individually. This is consistent with the many all-or-nothing pledge schemes such as Kickstarter. In other words, these schemes are indirect mechanisms that implement this type of conditional investment.

A fourth feature of optimal direct mechanisms is a deferred payout to prevent moral hazard. Some but definitely not all crowdfunding platforms do so. For instance, PledgeMusic, a crowdfunding platform specialized in raising money for music recordings, uses deferred payouts to prevent fraud.<sup>18</sup> In the next section, we however point out that crowdfunding platforms such as Kickstarter use deferred payments implicitly in the form of sales to consumers who did not participate in the crowdfunding campaign.

A final notable feature of optimal direct mechanisms is that they provide only information about whether the sum of pledges exceeds the target and not the total sum of pledges itself. In line with Lemma 2 any additional information is not needed to implement (constrained) efficient outcomes, and schemes that provide more information may exacerbate the moral hazard problem. Current crowdfunding platforms do not reflect this feature. Currently all crowdfunding platforms are fully transparent and announce publicly the total amount of pledges rather than just whether the target level was reached.

## 6.2 Limited Consumer Reach

The previous section suggests that current crowdfunding schemes deal with moral hazard suboptimally. In particular, they rarely use deferred payments, which, as shown by PledgeMusic, seems relatively straightforward to implement. On the other hand, there seems nevertheless little indication that in practise fraud is a prevalent

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<sup>17</sup>See <https://www.kickstarter.com/rules?ref=footer>, last retrieved 22 July 2015. The next section argues however that a limited reach of the platform results in crowd-investment.

<sup>18</sup>See <http://www.pledgemusic.com/blog/220-preventing-fraud>, last retrieved 20 July 2015.

problem in crowdfunding. Mollick (2014), for instance, reports that fraudulent failures for Kickstarter projects are very rare (well below 5%).

Since crowdfunding platforms currently reach only a very small part of potential demand, this section argues that even though crowdfunding schemes do not use deferred payouts explicitly, they do so implicitly. The argument is that the entrepreneur's prospect to sell her products to consumers who did not participate in crowdfunding acts as a direct substitute for deferred payments.<sup>19</sup>

Motivated by the observation that crowdfunding allows entrepreneurs to contract with consumers before their investment, our formal analysis took this idea to the extreme and implicitly assumed that the entrepreneur could contract with *every* potential consumer. Given this extreme position, the revelation principle implies that there is indeed no loss of generality in assuming that mechanisms allow consumers to acquire the product only through the mechanism. This changes when, for some exogenous reason, not all consumers can participate in the mechanism. In practise this is a highly relevant concern, because a share of consumers may fail to notice the crowdfunding scheme, not have access to the internet, or only arrive in the market after the product has been developed. Hence, a relevant extension of our framework is to consider mechanisms, which, for some exogenous reason, reach consumers imperfectly.

In order to make this more concrete, consider an extension of the model in which only a share of  $\beta \in (0, 1)$  can partake in the mechanism. Already the pure proportional case that a consumer's ability to participate is independent of his valuation, yields new important insights.

Note first that for this pure proportional case, the crowdfunding scheme is still able to elicit the project's value: a pledge by  $\tilde{n}$  consumers means that the project is worth  $n_1 = \tilde{n}/\beta$ . Consequently, investment is efficient if and only if

$$\tilde{n}/\beta \geq I/(1 - c) \Rightarrow \tilde{n} \geq \bar{n}(\beta) \equiv \beta I/(1 - c).$$

It is straightforward to see that the previous analysis still applies when we factor in  $\beta$ .

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<sup>19</sup>Interestingly, this may not be the case for very specialized crowdfunding platforms such as PledgeMusic, who explicitly use deferred payout systems.

In order to see that, with limited consumer reach, a classic reward crowdfunding scheme  $(p, T)$  without deferred payments can withstand moral hazard even for the extreme case  $\alpha = 1$ , note that when only a share of  $\beta$  potential consumers can be reached, inequality (1), which describes the condition under which the entrepreneur has a strict incentive to run, changes to

$$\alpha P > P/\beta - I - cP/(p\beta) \Rightarrow \beta > \bar{\beta} \equiv \frac{1 - c/p}{\alpha + I/P}. \quad (23)$$

Hence, whereas, under full consumer reach ( $\beta = 1$ ), a reward crowdfunding scheme  $(p, T)$  without deferred payments is unable to withstand moral hazard whenever  $\alpha > 1 - c/p - I/P$ , it does withstand moral hazard when its consumer reach is limited to  $\beta < \bar{\beta}$ . The reason for this follows the logic behind deferred payments: the limited consumer reach effectively implies that a pledge level  $P$  constitutes a deferred payment of  $P/\beta - P > 0$ .

Apart from reducing the threat of moral hazard, the extension reveals an important additional effect: consumers may become actual investors when the share of crowdfunding consumers  $\beta$  is small. To see this, note that, because the entrepreneur needs the amount  $I$  to develop the product, the (average) ex ante transfer of a pledging consumer needs to be at least  $I/\tilde{n}$ . When  $\beta$  is small in the sense that  $\bar{n}(\beta)$  is smaller than 1, it follows that for  $\tilde{n}$  close to  $\bar{n}(\beta)$ , the consumer's ex ante transfer *exceeds* his willingness to pay. Individual rationality then implies that the ex post transfer to the consumer is *negative*. Hence, the optimal mechanism turns consumers into investors; they finance the entrepreneur's investment *and* share in her revenues.

As noted, reward crowdfunding schemes such as Kickstarter explicitly prohibit monetary transfers to crowdfunders. Our formal analysis confirms that this is indeed not needed if the investment  $I$  is small compared to the number of crowdfunding consumers, but for large investments such restrictions may matter.<sup>20</sup>

Finally note that all-or-nothing crowdfunding schemes also give consumers a strict incentive to participate in the crowdfunding scheme, even if they have the option to

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<sup>20</sup>Ordanini et al. (2011) report the case of Cameesa, a Chicago based clothing company which in 2008 introduced an all-or-nothing crowdfunding model, but also shared revenue with its crowdfunders. Supporters of a successful project not only obtained the shirt, but also shared in some of the revenue of its future sales. (see <http://www.cnet.com/news/cameesa-a-threadless-where-customers-are-also-investors/>, last retrieved 22 July 2015).

wait and buy the product later in the after market. This is so, because a consumer may be pivotal for the entrepreneur's decision whether to produce the good. Hence, facing a crowdfunding scheme  $(p, T)$  a consumer is strictly better off participating (provided that he expects that the after market price is not lowered, which in our setup would indeed not be the case). Hence, next to eliciting the consumer's valuation in an incentive compatible manner, crowdfunding schemes also exhibit features, which make participation incentive compatibility.<sup>21</sup>

### 6.3 Entrepreneur's private cost information

Both the first best and the optimal (constrained) efficient mechanism condition on the entrepreneur's investment cost  $I$  and marginal cost  $c$ . The optimal mechanism, therefore, exploits the assumption that costs are public information. In practise, one may worry that the entrepreneur is better informed about these costs than consumers. In this case, costs are the entrepreneur's private information so that if the mechanism is to condition on this information, it must incentivize the entrepreneur to reveal them. This may potentially lead to additional agency costs and new distortions.

Note that because the optimal mechanism implements an allocation in which the entrepreneur obtains all the rents, one might be tempted to argue that the entrepreneur's incentives are fully aligned with social welfare so that the mechanism automatically gives her the proper incentives to reveal her private information truthfully. This reasoning is however incomplete, because it only considers a unilateral deviation. Hence, even though the reasoning implies that a single deviation to misreport costs is not profitable, the *combined* deviation of misreporting costs and, subsequently, running away with the money may be profitable. Therefore, the crowdfunding scheme  $\bar{\Gamma}^{CF}$  implements the first best with private information about costs only if  $W^* \geq \bar{W}^\alpha$  and a combined deviation is not profitable.

To examine the implications of this in closer detail, suppose that  $W^* \geq \bar{W}^\alpha$  so that, under the assumption that  $I$  and  $c$  are public information, the efficient output

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<sup>21</sup>Next to the probability to be pivotal and the consumer's expectation of the price in the after market, a consumer's specific incentives to participate will also depend on other factors from which our model abstracts: time-preferences, the probability that the project will succeed, and the possibility that the consumer can better judge the product after it has been successfully produced.

$x^*$  is implementable by the scheme  $\bar{\Gamma}^{CF}$ . Now first suppose that only  $c$  is private information. It is then straightforward to show that, the scheme  $\bar{\Gamma}^{CF}$  automatically induces the entrepreneur to report  $c$  truthfully, provided that she does not run away. In order to see that  $\bar{\Gamma}^{CF}$  does not imply an incentive compatibility with respect to the *combined* deviation of misreporting and running away, define  $\bar{\pi}(T)$  as the probability that the sum of consumer valuations exceeds  $T$ , i.e.

$$\bar{\pi}(T) \equiv \sum_{\{v: \sum_i v_i \geq T\}} \pi(v).$$

Now suppose the platform offers the scheme  $\bar{\Gamma}^{CF}$  with the target  $\bar{T} = I/(1-c)$ , but, not observing  $c$ , asks the entrepreneur to report it before implementing the scheme. Instead of reporting truthfully and obtain the gross revenue  $W^*$ , the strategy to report  $c^r$  and subsequently run away yields the entrepreneur  $\alpha\bar{\pi}(I/(1-c^r))I$ . Hence, the best deviation for the entrepreneur is to report  $c^r = 0$  and subsequently run away with the payment  $I$ , yielding the deviation payoff  $\alpha\bar{\pi}(I)I$ . With private information about the cost  $c$ , the scheme  $\bar{\Gamma}^{CF}$  therefore withstands moral hazard only if  $W^* - \pi^*I \geq \alpha\bar{\pi}(I)I$ , whereas, with public information about the cost  $c$ , it withstands moral hazard for  $W^* - \pi^*I \geq \alpha\pi^*I$ . Since  $\pi^* = \bar{\pi}(I/(1-c)) \leq \bar{\pi}(I)$ , the moral hazard constraint when  $c$  is private information is stricter. This reasoning leads to the following lemma.

**Lemma 4** *Suppose  $I$  is public information, whereas  $c$  is the entrepreneur's private information. Then the crowdfunding contract  $\bar{\Gamma}^{CF}$  implements the efficient output schedule  $x^*$  if and only if  $W^* \geq (\pi^* + \alpha\bar{\pi}(I))I$ .*

A comparison of the lemma to Proposition 2 reveals that with private information about  $c$ , the efficient output is more difficult to implement. The intuition for this result is that by reporting a lower cost  $c$ , the entrepreneur can raise the probability that the project is financed and this increases the threat of moral hazard.

It is straightforward to extend the previous reasoning to the case where also  $I$  is private information. In this case, an implementation of  $\bar{\Gamma}^{CF}$  requires the entrepreneur to report both the investment cost  $I$  and the marginal costs  $c$ . Again, misreporting  $(I, c)$  does not lead to a higher payoff if the entrepreneur does not run, while reporting  $(I^r, c^r)$  and subsequently running away yields the pay  $\bar{\pi}(I^r)I^r$ . Defining

$$\hat{I} = \arg \max_{I^r} \bar{\pi}(I^r)I^r,$$

the maximum deviation payoff is  $\alpha\bar{\pi}(\hat{I})\hat{I}$ . Hence, the scheme  $\bar{\Gamma}^{CF}$  withstands moral hazard if  $W^* - \pi^*I \geq \alpha\bar{\pi}(\hat{I})\hat{I}$ . The reasoning leads to the following proposition.

**Proposition 4** *Suppose  $(I, c)$  are the entrepreneur's private information. Then the crowdfunding contract  $\bar{\Gamma}^{CF}$  implements the efficient output schedule  $x^*$  if and only if  $W^* \geq \pi^*I + \alpha\bar{\pi}(\hat{I})\hat{I}$ .*

The proposition shows that even though the additional private information increases agency costs, the efficient outcome remains implementable if the ex ante gross surplus  $W^*$  is large enough. Moreover, agency costs remain zero if there is no moral hazard ( $\alpha = 0$ ) to begin with. Hence, private information about costs only affect outcomes to the extent that it intensifies the moral hazard problem proportionally.

## 6.4 Crowdfunding and price discrimination

In our formal analysis, we assumed that consumers either do not value the good or value it at the same positive amount. This allows us to focus on the problem of aggregate demand uncertainty and side-step issues of price discrimination.

An explicit comparison of our results to papers that focus on price discrimination, however, enables to identify two distinct economic roles of crowdfunding: screening for overall project value vs. screening for consumer-specific value.

To make this more precise, note that a defining feature of an all-or-nothing crowdfunding scheme  $(p, T)$  is that it conditions the investment decision on the *sum* of pledges. Cornelli (1996, p.18), however, shows that, for achieving optimal price discrimination, the actual *composition* of this sum rather than the sum itself matters. As a result, crowdfunding schemes cannot deal with the price-discrimination problem optimally (see also Barbieri and Malueg 2010). In contrast, our results show that, even in the presence of entrepreneurial moral hazard, conditioning on the sum of pledges is optimal when screening for overall project value.

Distinguishing between the two roles of crowdfunding is also crucial for evaluating its welfare effects and implications for regulation. Using crowdfunding as a screening device for project value unambiguously benefits consumers and aggregate welfare, whereas the welfare properties of crowdfunding as a tool for price discrimination are, at best, ambiguous. Hence, from a regulatory perspective one may want to

encourage the use of crowdfunding for value screening, while dissuade its use for price discrimination. One way of doing so is to allow crowdfunding schemes to condition on the sum of valuations but not on the exact composition of this sum.

## 6.5 Crowdfunding vs. Venture Capitalists

By enabling direct interactions with consumers before the investment, crowdfunding leads to a transformation of the entrepreneurial business model. Ordanini et al. (2011) emphasize that this transformation takes place at a fundamental level, blurring the traditional separation of finance and marketing.<sup>22</sup> In the traditional model, venture capitalists (or banks) attract capital from consumers to finance entrepreneurs, who subsequently use this capital to produce goods and market them to consumers. In this traditional model, finance and marketing are naturally separated and run along different channels. In contrast under reward crowdfunding, finance and marketing run along the same channel: the crowdfunding platform.

Although this fundamental perspective is correct if one views reward crowdfunding as an exclusive alternative to specialized venture capitalists, we emphasize that crowdfunding and venture capital financing are not mutually exclusive. On the contrary, we view the two forms as highly complementary. In line with Diamond (1984), we see the advantage of venture capitalists (or banks) in reducing the moral hazard problem, which in terms of the paper’s model implies a reduction in  $\alpha$ . In contrast, the strength of crowdfunding lies in learning about the economic value of the project.

Because the analysis of a fully-fledged model which combines venture capitalists and crowdfunding lies outside the scope of the current paper, we just mention that we see no reason why a venture capitalist may not use crowdfunding to learn about demand or why after a successful crowdfunding campaign an entrepreneur may not approach a venture capitalist. Indeed, Dingman (2013) reports that exactly this occurred in the case of the Pebble Smart Watch. Venture capitalist decided to support the entrepreneur’s project only after a successful crowdfunding campaign on Kickstarter. Quoting a managing partner of a venture capitalist firm: “What venture capital always wants is to get validation, and with Kickstarter, he [i.e. the

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<sup>22</sup>In contrast, “investment-based crowdfunding” upholds the traditional separation between finance and marketing, because the consumers and the crowd-investors do typically not coincide.

entrepreneur] could prove there was a market.”

## 7 Conclusion

Crowdfunding provides the innovation that, already before the product is developed, an entrepreneur can contract with her potential consumers. In the presence of aggregate demand uncertainty, this enables entrepreneurs to use crowdfunding as a tool to screen for valuable projects and thereby improve investment decisions. Our formal analysis confirms that optimal mechanisms do indeed take on this role of screening, even in the presence of moral hazard. Current all-or-nothing reward crowdfunding schemes such as used by Kickstarter reflect the main features of these optimal mechanisms. In particular, they are consistent with the idea that these schemes are used to improve the selection of entrepreneurial projects. This promotes social welfare.

Our analysis further shows that, despite the effectiveness of reward crowdfunding schemes in screening for project value, their susceptibility to entrepreneurial moral hazard may prevent the implementation of fully efficient outcomes. In particular, crowdfunding can attain fully efficient outcomes only if the project’s ex ante expected gross return exceeds its ex ante expected investment costs by a markup whose size reflects the severeness of the moral hazard problem. This markup reflect the agency costs due to entrepreneurial moral hazard. Constrained efficient mechanisms exhibit underinvestment, but still reflect crucial features of current all-or-nothing reward crowdfunding schemes.

Because crowdfunding schemes by themselves are, in the presence of moral hazard, unable to attain efficiency in general, we see them as complements rather than substitutes for traditional venture capital. We therefore expect a convergence of the two financing forms so that venture capitalists can provide their expertise in reducing moral hazard, while crowdfunding platforms enable a better screening for project value. Current policy measures such as the US JOBS Act and its implementation in SEC (2015) will make such mixed forms easier to develop and to take advantage of their respective strengths. The website of the crowdfunding platform Rockethub already explicitly mentions this possible effect of the JOBS Act.<sup>23</sup>

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<sup>23</sup>See <http://www.rockethub.com/education/faq#jobs-act-index>, last retrieved 22 July 2015.

Finally, in order to focus on the trade-off between demand uncertainty and entrepreneurial moral hazard — which we view as two fundamental first order effects in crowdfunding — our analysis necessarily abstracts from many other relevant aspects and makes many simplifying assumptions. For instance, we do not address the role of crowdfunders in promoting the product or the dynamics in pledging behavior in actual crowdfunding schemes. We further model the entrepreneur’s investment technology as a deterministic one, leading to a well-defined private good with known quality and costs, no network effects, or any other form of externalities. Apart from pointing out that crowdfunding and external capital provision in the form of venture capital are complements, we also do not provide a formal analysis of the interaction between external financing and reward crowdfunding. We moreover do not address possible issues concerning the platform’s commitment to enforce the mechanism honestly. Since the platform is a long-term player we conjecture that it can uphold its honesty by well-known reputational arguments in repeated games (see Strausz, 2005). This however requires payments to the platform, another aspect we do not touch upon. Although we consider all these issues important and relevant, they lie outside the scope of the current investigation, but point to a fruitful line of future research.

## Appendix

This appendix collects the formal proofs.

**Proof of Lemma 1:** Follows directly from the text Q.E.D.

**Proof of Proposition 1:** Let  $\mathbf{1}^n$  denote the vector  $(1, \dots, 1) \in \mathbb{R}^n$ . Since  $\bar{n} = I/(1-c) = n - 1/2$ , it follows  $V^1 = \{\mathbf{1}^n\}$  and  $V^0 = V \setminus V^1$  so that the efficient output schedule  $x^*(v)$  exhibits  $x_0^*(v) = x_i^*(v) = 0$  for  $v \neq \mathbf{1}^n$ , and  $x_0^*(v) = x_i^*(v) = 1$  for  $v = \mathbf{1}^n$ . We show, by contradiction, that a feasible direct mechanism  $\gamma^*$  that implements  $x^*(v)$  does not exist.

For suppose to the contrary that such a direct mechanism does exist, then there exists a transfer schedule  $t$  so that the direct mechanism  $\gamma^* = (x^*, t)$  is feasible. Since  $x_0^*(v) = 1$  implies  $v = \mathbf{1}^n$ , it follows that  $\mathcal{T}^a$  is a singleton and for all  $T^a \in \mathcal{T}^a$ , it holds  $V(T^a) = \{\mathbf{1}^n\}$ . Consequently,  $p(\mathbf{1}^n|T^a) = 1$  and  $\pi(v|T^a) = 0$  for all  $v \neq \mathbf{1}^n$ . Since  $\alpha = 1$  and  $c = 0$ , (8) rewrites after multiplying by  $p(\mathbf{1}^n)$  therefore as

$$\sum_{i=1}^n t_i^p(\mathbf{1}^n)p(\mathbf{1}^n) \geq Ip(\mathbf{1}^n). \quad (24)$$

Since  $x_0(\mathbf{1}^n) = 1$  the first inequality in (3) implies after multiplying with  $p(\mathbf{1}^n)$

$$\sum_{i=1}^n t_i^a(\mathbf{1}^n)p(\mathbf{1}^n) \geq Ip(\mathbf{1}^n). \quad (25)$$

Note further that the second inequality in (3) for each  $v \neq \mathbf{1}^n$  implies

$$\sum_{i=1}^n t_i^a(v) + t_i^p(v) \geq 0 \quad (26)$$

Multiplying with  $\pi(v)$  and adding over all  $v \neq \mathbf{1}^n$  yields

$$\sum_{v \neq \mathbf{1}^n} \sum_{i=1}^n (t_i^a(v) + t_i^p(v))\pi(v) \geq 0 \quad (27)$$

Combining (24), (25), and (27) yields

$$\sum_i \sum_{v \in V} (t_i^a(v) + t_i^p(v))\pi(v) \geq 2Ip(\mathbf{1}^n) = (2n-1)p(\mathbf{1}^n), \quad (28)$$

where the equality uses  $I = n - 1/2$ .

We now show that (28) contradicts (9). First note that (9) for  $v_i = 0$  implies after a multiplication by  $p_i(0)$  for each  $i$

$$\sum_{v_{-i} \in V_{-i}} (t_i^a(0, v_{-i}) + t_i^p(0, v_{-i}))p(0, v_{-i}) \leq 0. \quad (29)$$

Summing over  $i$  it follows

$$\sum_i \sum_{v_{-i} \in V_{-i}} (t_i^a(0, v_{-i}) + t_i^p(0, v_{-i}))p(0, v_{-i}) \leq 0. \quad (30)$$

Likewise, since  $X_i(1) = p_i(\mathbf{1}^{n-1})$ , (9) for  $v_i = 1$  implies after a multiplication with  $p_i(1)$  and using  $p_i(1) = p(1, v_{-i})$  that for each  $i$

$$\sum_{v_{-i} \in V_{-i}} (t_i^a(1, v_{-i}) + t_i^p(1, v_{-i}))p(1, v_{-i}) \leq p(\mathbf{1}^n). \quad (31)$$

Summing over  $i$  yields

$$\sum_i \sum_{v_{-i} \in V_{-i}} (t_i^a(1, v_{-i}) + t_i^p(1, v_{-i}))p(1, v_{-i}) \leq p(\mathbf{1}^n)n. \quad (32)$$

Combining (30) and (32) yields

$$\sum_i \sum_{v \in V} (t_i^a(v) + t_i^p(v))\pi(v) \leq p(\mathbf{1}^n)n. \quad (33)$$

But since  $2n - 1 > n$ , this contradicts (28).

Q.E.D.

**Proof of Lemma 2:** Fix a feasible  $\bar{\gamma} = (\bar{t}, \bar{x})$  with  $\bar{x}$  development efficient and define for each  $v$ ,

$$\bar{K}(v) \equiv \sum_i \bar{t}_i^a(v) - I\bar{x}_0(v).$$

Feasibility of  $\bar{\gamma}$  means  $\bar{x}(v)$  satisfies (3) for all  $v \in V$ , and therefore  $\bar{K}(v) \geq 0$  for all  $v \in V$ . For any state  $v$ , let  $\bar{n}(v) \equiv \sum_i \bar{x}_i(v)$  represent, for a given  $v$ , the total number of consumers with  $x_i = 1$ . For any state  $v$  with  $\bar{x}_0(v) = 0$ , define  $\hat{t}_i^a(v) \equiv 0$  and  $\hat{t}_i^p(v) \equiv \bar{t}_i^a(v) + \bar{t}_i^p(v)$ . For  $\bar{x}_0(v) = 1$  define  $\hat{t}_i^a(v) \equiv \bar{t}_i^a(v) - \bar{x}_i(v)\bar{K}(v)/\bar{n}(v)$  and  $\hat{t}_i^p(v) \equiv \bar{t}_i^p(v) + \bar{x}_i(v)\bar{K}(v)/\bar{n}(v)$ . Since  $\bar{x}$  is feasible and development efficient, it holds  $\bar{n}(v) > 0$  if and only if  $\bar{x}_0(v) = 1$ . Hence, the transformed transfer schedule  $\hat{t}$  is well-defined. By construction, we have  $\hat{t}_i^a(v) + \hat{t}_i^p(v) = \bar{t}_i^a(v) + \bar{t}_i^p(v)$  for all  $v$ , and  $\sum_i \hat{t}_i^a(v) = 0$  for any  $v$  with  $\bar{x}_0(v) = 0$ , and  $\sum_i \hat{t}_i^a(v) = \sum_i \bar{t}_i^a(v) - \bar{x}_i(v)\bar{K}(v)/\bar{n}(v) = \sum_i \bar{t}_i^a(v) - \bar{K}(v) = I$  for any  $v$  with  $\bar{x}_0(v) = 1$ . Hence, the allocation  $(\hat{t}(v), \bar{x}(v))$  satisfies (12). Because the allocation  $(\bar{t}(v), \bar{x}(v))$  is development feasible, also the allocation  $(\hat{t}(v), \bar{x}(v))$  is development feasible. Moreover, from  $\hat{t}_i^a(v) + \hat{t}_i^p(v) = \bar{t}_i^a(v) + \bar{t}_i^p(v)$  it follows that  $(\hat{t}, \bar{x})$  is also budget-feasible, truthful, and individual rational, given that  $(\bar{t}, \bar{x})$  is so by assumption.

In order to show that  $(\hat{t}, \bar{x})$  is feasible, it only remains to show that it is obedient, i.e., satisfies (8). To show this, define for  $\bar{T}^a \in \bar{\mathcal{T}}^a$

$$\bar{P}(\bar{T}^a) = \sum_{v \in \bar{V}(\bar{T}^a)} \pi(v),$$

Now since,  $\bar{\gamma} = (\bar{t}, \bar{x})$  is obedient by assumption, (8) holds for any  $\bar{T}^a \in \bar{\mathcal{T}}^a$ . Given that  $\bar{T}^a = \sum_i \bar{t}_i^a(v)$  for any  $v$  such that  $\bar{\pi}(v|\bar{T}^a) > 0$ , we can rewrite (8) as

$$\sum_{v \in V} \sum_{i=1}^n \bar{\pi}(v|\bar{T}^a) (\bar{t}_i^p(v) - c\bar{x}_i(v) + \bar{t}_i^a(v)) - I \geq \alpha \bar{T}^a, \text{ for all } \bar{T}^a \in \bar{\mathcal{T}}^a. \quad (34)$$

From  $\hat{t}_i^a(v) + \hat{t}_i^p(v) = \bar{t}_i^a(v) + \bar{t}_i^p(v)$ , this rewrites as

$$\sum_{v \in V} \sum_{i=1}^n \bar{\pi}(v|\bar{T}^a) (\hat{t}_i^p(v) - c\bar{x}_i(v) + \hat{t}_i^a(v)) - I \geq \alpha \bar{T}^a, \text{ for all } \bar{T}^a \in \bar{\mathcal{T}}^a. \quad (35)$$

Because, by construction  $\sum_i \hat{t}_i^a(v) = I$  for  $v$  such that  $\bar{\pi}(v|\bar{T}^a) > 0$ , this rewrites as

$$\sum_{v \in V} \sum_{i=1}^n \bar{\pi}(v|\bar{T}^a) (\hat{t}_i^p(v) - c\bar{x}_i(v)) \geq \alpha \bar{T}^a, \text{ for all } \bar{T}^a \in \bar{\mathcal{T}}^a. \quad (36)$$

Moreover, since feasibility implies that  $\bar{T}^a \geq I$ , the previous inequality implies that

$$\sum_{v \in V} \sum_{i=1}^n \bar{\pi}(v|\bar{T}^a) (\hat{t}_i^p(v) - c\bar{x}_i(v)) \geq \alpha I, \text{ for all } \bar{T}^a \in \bar{\mathcal{T}}^a. \quad (37)$$

It follows after a further multiplication by  $\bar{P}(\bar{T}^a)$  that

$$\sum_{v \in V} \sum_{i=1}^n \bar{\pi}(v|\bar{T}^a) \bar{P}(\bar{T}^a) (\hat{t}_i^p(v) - c\bar{x}_i(v)) \geq \alpha I \cdot \bar{P}(\bar{T}^a), \text{ for all } \bar{T}^a \in \bar{\mathcal{T}}^a. \quad (38)$$

By definition of  $\bar{\pi}(v|\bar{T}^a)$ , we have  $\bar{\pi}(v|\bar{T}^a) \bar{P}(\bar{T}^a) = \pi(v) \mathbf{1}_{v \in \bar{V}(\bar{T}^a)}$ , where  $\mathbf{1}_A$  is the indicator function which is 1 if the statement  $A$  is true and 0 otherwise. Thus we may rewrite the former inequality as

$$\sum_{v \in V} \sum_{i=1}^n \pi(v) \mathbf{1}_{v \in \bar{V}(\bar{T}^a)} (\hat{t}_i^p(v) - c\bar{x}_i(v)) \geq \alpha I \cdot \bar{P}(\bar{T}^a), \text{ for all } \bar{T}^a \in \bar{\mathcal{T}}^a. \quad (39)$$

Summing over all  $\bar{T}^a \in \bar{\mathcal{T}}^a$ , we obtain

$$\sum_{\bar{T}^a \in \bar{\mathcal{T}}^a} \sum_{v \in V} \sum_{i=1}^n \pi(v) \mathbf{1}_{v \in \bar{V}(\bar{T}^a)} (\hat{t}_i^p(v) - c\bar{x}_i(v)) \geq \sum_{\bar{T}^a \in \bar{\mathcal{T}}^a} \alpha I \cdot \bar{P}(\bar{T}^a). \quad (40)$$

Denoting by  $\hat{V}(\cdot)$  and  $\hat{P}(\cdot)$  under the mechanism  $\hat{\gamma}$  the corresponding sets  $\bar{V}(\cdot)$  and probabilities  $\bar{P}(\cdot)$  under the mechanism  $\bar{\gamma}$ , we can, after noting that  $\hat{\mathcal{T}}^a = \{I\}$  and  $\hat{V}(I) = \{v | \bar{x}_0(v) = 1\} = \cup_{\bar{T}^a \in \bar{\mathcal{T}}^a} \bar{V}(\bar{T}^a)$ , rewrite the previous inequality as

$$\sum_{v \in V} \sum_{i=1}^n \pi(v) \mathbf{1}_{v \in \hat{V}(I)} (\hat{t}_i^p(v) - c\bar{x}_i(v)) \geq \alpha I \cdot \bar{P}(\cup_{\bar{T}^a \in \bar{\mathcal{T}}^a} \bar{T}^a), \quad (41)$$

which we can further rewrite as

$$\sum_{v \in V} \sum_{i=1}^n \pi(v) \mathbf{1}_{\{\bar{x}_0(v)=1 \wedge \sum_i \hat{t}_i^p(v)=I\}} (\hat{t}_i^p(v) - c\bar{x}_i(v)) \geq \alpha I \cdot \hat{P}(I), \quad (42)$$

but, since for  $\hat{\gamma}$  we have  $\hat{\mathcal{T}}^a = \{I\}$ , this is equivalent to

$$\sum_{v \in V} \sum_{i=1}^n \hat{\pi}(v | \hat{T}^a) (\hat{t}_i^p(v) - c\bar{x}_i(v)) - \hat{T}^a - I \geq \alpha I \cdot \hat{P}(I), \text{ for all } \hat{T}^a \in \hat{\mathcal{T}}^a. \quad (43)$$

Hence,  $\hat{\gamma}$  satisfies (8) so that  $\hat{\gamma} = (\hat{t}, \bar{x})$  is obedient. To complete the proof note that since  $\hat{t}_i^a(v) + \hat{t}_i^p(v) = \bar{t}_i^a(v) + \bar{t}_i^p(v)$ , the feasible direct mechanism  $\hat{\gamma} = (\hat{t}, \bar{x})$  is payoff equivalent to original mechanism  $\bar{\gamma} = (\bar{t}, \bar{x})$ . Q.E.D.

**Proof of lemma 3:** The first statement follows because the incentive constraint (7) for  $v_i = 1$ , and the individual rationality (9) of a consumer with value  $v = 0$  imply the individual rationality (9) for  $v_i = 1$ . That is,  $1 \cdot X_i(1) - T_i(1) \geq 1 \cdot X_i(0) - T_i(0) \geq 0 \cdot X_i(0) - T_i(0) \geq 0$ .

To see  $x_i(0, v_{-i}) = 0$ , note that if not, then  $x_i(0, v_{-i}) = 1$ . But then lowering it to 0 raises the objective (15) by  $p(0, v_{-i})c$ . This change is feasible, because it keeps constraints (7) for  $v_i = 0$ , (9), and (12) unaffected, while relaxing the constraints (7) for  $v_i = 1$ , (13), and (14). The statement  $X_i(0) = 0$  then follows as a corollary.

To see  $T_i(0) = 0$ , note that (9) implies  $T_i(0) \leq 0$ . But if  $T_i(0) < 0$ , then raising each  $t_i^p(0, v_{-i})$  and  $t_i^p(1, v_{-i})$  by  $T_i(0)/\pi_i(v_{-i})$  for each  $v_{-i} \in V_{-i}$  leads to a feasible mechanism with  $T_i(0) = 0$  and the same value for the objective (15). The adapted mechanism is feasible since the change does not affect (7) and (12), and, by construction, satisfies (9) for  $v_i = 0$  so that, by the first argument of this lemma, it also satisfies (9) for  $v_i = 1$ . The raises in  $t_i^p(v)$  further relaxes (13) and (14). Consequently, there is no loss of generality in assuming that, at the optimum,  $T_i(0) = 0$ .

To see  $T_i(1) = X_i(1)$ , note that (7) for  $v_i = 1$  together with  $X_i(0) = T_i(0) = 0$  imply  $T_i(1) \leq X_i(1)$ . But if  $T_i(1) < X_i(1)$ , then we can raise all  $t_i^p(1, v_{-i})$  by  $\varepsilon > 0$

such that  $T_i(1) = X_i(1)$ . The increase is feasible and does not affect the objective (15). To see that the change is feasible, note that it relaxes constraint (7) for  $v_i = 0$  and, by construction, satisfies (7) for  $v_i = 1$ . It further does not affect (9) for  $v_i = 0$  and, by the first part of the lemma, the constraint (9) for  $v_i = 1$  is redundant. It also does not affect (12), while relaxing (13) and (14). Consequently, there is no loss of generality in assuming that, at the optimum,  $T_i(1) = X_i(1)$ . Q.E.D.

**Proof of Proposition 2:** Recalling that  $\pi^* = \sum_{v \in V^1} \pi(v)$ , define

$$\pi^*(v) \equiv \begin{cases} \pi(v)/\pi^* & \text{if } x_0^*(v) = 1; \\ 0 & \text{otherwise,} \end{cases}$$

The proposition's condition  $W^* \geq (1 + \alpha)\pi^*I$  is then equivalent to

$$\sum_{v \in V} \sum_i \pi^*(v) v_i (1 - c) \geq (1 + \alpha)I. \quad (44)$$

We first prove that under condition (44) the first best is implementable by constructing a transfer schedule  $\hat{t}$  such that the direct mechanism  $\gamma^* = (\hat{t}, x^*)$  is feasible and therefore implements  $x^*$ . For any  $v$  such that  $x_0^*(v) = 0$ , set  $\hat{t}_i^a(v) = \hat{t}_i^p(v) = 0$ . For any  $v$  such that  $x_0^*(v) = 1$ , let  $\bar{x}^*(v) \equiv \sum_i x_i^*(v) > 0$  represents the efficient number of goods to be produced in state  $v$ .<sup>24</sup> Set  $\hat{t}_i^a(v) = x_i^*(v)I/\bar{x}^*(v)$  and  $\hat{t}_i^p(v) = x_i^*(v)(1 - I/\bar{x}^*(v))$ .

We show that the resulting mechanism  $\gamma^* = (\hat{t}, x^*)$  is direct and feasible. More specifically, for each  $v \in V$  the allocation  $\gamma^*(v)$  satisfies (3) so that  $\gamma^*$  is direct (it trivially satisfies (4), since  $x^*$  does so by construction). Moreover, the direct mechanism  $\gamma^*$  satisfies (7), (8), and (9) for each  $v \in V$ .

To show (3) for  $v$  such that  $x_0^*(v) = 0$ , note that  $\sum_i \hat{t}_i^a(v) = 0 = Ix_0^*(v)$ , and that  $\sum_i \hat{t}_i^a(v) + \hat{t}_i^p(v) = 0 = Ix_0^*(v) + c \sum_i x_i^*(v)$ , since  $x_i^*(v) = 0$  for all  $i$  whenever  $x_0^*(v) = 0$ . To show (3) for  $v$  such that  $x_0^*(v) = 1$ , note that  $\sum_i \hat{t}_i^a(v) = \sum_i x_i^*(v)I/\bar{x}^*(v) = I = Ix_0^*(v)$  and  $\sum_i \hat{t}_i^a(v) + \hat{t}_i^p(v) = \sum_{\{i: x_i^*(v)=1\}} (\hat{t}_i^a(v) + \hat{t}_i^p(v)) + \sum_{\{i: x_i^*(v)=0\}} (\hat{t}_i^a(v) + \hat{t}_i^p(v)) = \sum_{\{i: x_i^*(v)=1\}} 1 + \sum_{\{i: x_i^*(v)=0\}} 0 = \sum_i v_i \geq I + \sum_i cv_i = Ix_0^*(v) + c \sum_i x_i^*(v)$ , where the inequality holds because  $x^*(0) = 1$  is efficient by assumption so that  $\sum_i v_i \geq I + \sum_i cv_i$ . Hence,  $\gamma(v) \in A$  for all  $v$  so that the mechanism  $\gamma^*$  is direct.

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<sup>24</sup> $\bar{x}^*(v)$  is greater than 0, since  $x_0^*(v) = 1$  and  $x^*$  is development-efficient.

To show (7) and (9) note that  $x_i^*(0) = 0$  implies  $X_i^*(0) = 0$  and, by construction of  $\hat{t}$ , also  $T_i^*(0) = 0$ . Moreover,  $X_i^*(1) \geq 0$  and  $T_i^*(1) \geq 0$ . For  $v_i = 0$ , it therefore follows  $v_i X_i^*(v_i) - T_i^*(v_i) = 0 \cdot X_i^*(0) - T_i^*(0) = 0 \leq -T_i^*(1) = 0 \cdot X_i^*(1) - T_i^*(1)$  so that (7) and (9) are satisfied for  $v_i = 0$ . To see that they are also satisfied for  $v_i = 1$ , note that  $1 \cdot X_i^*(1) - T_i^*(1) = \sum_{v_{-i}} \pi_i(v_{-i}) [x_i^*(1, v_{-i}) - \hat{t}_i^a(1, v_{-i}) - \hat{t}_i^p(1, v_{-i})] = 0 = 1 \cdot X_i^*(0) - T_i^*(0)$ .

Finally, to show (8), first note that for  $\gamma^*$  we have  $\mathcal{T}^* = \{I\}$  and  $\pi(v|I) = \pi^*(v)$  so that we only need to show  $\sum_{v \in V} \sum_{i=1}^n \pi^*(v) [\hat{t}_i^p(v) - c x_i^*(v)] \geq \alpha I$ , which follows from

$$\sum_{v \in V} \sum_{i=1}^n \pi^*(v) [\hat{t}_i^p(v) - c x_i^*(v)] = \sum_{i=1}^n \sum_{v: x_i^*(v)=1} \pi^*(v) [1 - I/\bar{x}^*(v) - c] = \quad (45)$$

$$= \sum_{i=1}^n \sum_{v: x_i^*(v)=1} \pi^*(v) (1 - c) - I = \sum_{i=1}^n \sum_{v \in V} \pi^*(v) v_i (1 - c) - I \geq \alpha I, \quad (46)$$

where the inequality uses (44).

We next show that if condition (44) is violated so that

$$\sum_{v \in V} \sum_i \pi^*(v) v_i (1 - c) < (1 + \alpha) I, \quad (47)$$

then there does not exist a transfer schedule  $\hat{t}$  such that the direct mechanism  $\gamma = (\hat{t}, x^*)$  is feasible. In particular, we show there does not exist a transfer schedule  $\hat{t}$  such that  $(\hat{t}, x^*)$  satisfies (16)-(22).

For the efficient output schedule  $x^*$  it holds  $V^1 = \{v | x_0^*(v) = 1\}$  and  $V^0 = \{v | x_0^*(v) = 0\}$  and  $V = V^1 \cup V^0$ .

For  $v \in V^0$ , it therefore holds  $x_i(v) = 0$  so that conditions (19) and (20) taken together imply  $\sum_i t_i^a(v) + t_i^p(v) \geq 0$ . Multiplying by  $\pi(v)$  and summing up over all  $v$  in  $V^0$  yields

$$\sum_{v \in V^0} \sum_i \pi(v) [t_i^a(v) + t_i^p(v)] \geq 0 \quad (48)$$

For  $v \in V^1$ , (19) implies  $\sum_i t_i^a(v) = I$ . Multiplying by  $\pi(v)$  and summing up over all  $v$  in  $V^1$  yields

$$\sum_{v \in V^1} \sum_i \pi(v) t_i^a(v) = \sum_{v \in V^1} \pi(v) I = \pi^* I \quad (49)$$

Since (19) implies  $\mathcal{T}^a = \{I\}$ , it follows that  $\pi^* \cdot \pi(v|I) = \pi(v)$  for  $v \in V^1$  and  $\pi(v|I) = 0$  for all  $v \in V^0$ . Hence, after a multiplication by  $\pi^*$  we can rewrite (17) as

$$\sum_{v \in V^1} \sum_i \pi(v) t_i^p(v) \geq \alpha \pi^* I + \sum_{v \in V^1} \sum_i \pi(v) c x_i^*(v) \quad (50)$$

Combining (49) and (50) yields

$$\sum_{v \in V^1} \sum_i \pi(v) [t_i^a(v) + t_i^p(v)] \geq (1 + \alpha) \pi^* I + \sum_{v \in V^1} \sum_i \pi(v) c x_i^*(v) \quad (51)$$

Since  $x_i^*(v) = 0$  for  $v \in V^0$ , (51) together with (48) imply

$$\sum_{v \in V} \sum_i \pi(v) [t_i^a(v) + t_i^p(v)] \geq (1 + \alpha) \pi^* I + \sum_{v \in V} \sum_i \pi(v) c x_i^*(v) \quad (52)$$

Since  $x_i^*(v) = v_i$  for  $v \in V^1$  and  $x_i^*(v) = 0$  for  $v \in V^0$ , multiplying (47) by  $\pi^*$  and rearranging terms yields

$$(1 + \alpha) \pi^* I + \sum_{v \in V} \sum_i \pi(v) c x_i^*(v) > \sum_{v \in V} \sum_i \pi(v) x_i^*(v). \quad (53)$$

Combining this latter inequality with inequality (52) yields

$$\sum_{v \in V} \sum_i \pi(v) [t_i^a(v) + t_i^p(v)] > \sum_{v \in V} \sum_i \pi(v) x_i^*(v). \quad (54)$$

Condition (16) implies after multiplying by  $p_i(1)$  and summing over all  $i$

$$\sum_i \sum_{v_{-i}} p(1, v_{-i}) [t_i^a(1, v_{-i}) + t_i^p(1, v_{-i})] = \sum_i \sum_{v_{-i}} p(1, v_{-i}) x_i^*(1, v_{-i}). \quad (55)$$

Similarly, (18) implies after multiplying by  $p_i(0)$  and summing over all  $i$

$$\sum_i \sum_{v_{-i}} p(0, v_{-i}) [t_i^a(0, v_{-i}) + t_i^p(0, v_{-i})] = 0 = \sum_i \sum_{v_{-i}} p(0, v_{-i}) x_i^*(0, v_{-i}), \quad (56)$$

because  $x_i^*(0, v_{-i}) = 0$ .

Combining the latter two inequalities yields

$$\sum_i \sum_{v \in V} \pi(v) [t_i^a(v) + t_i^p(v)] = \sum_i \sum_{v \in V} \pi(v) x_i^*(v), \quad (57)$$

but this contradicts (54). Hence, under (47) there does not exist a direct mechanism  $\gamma = (t, x^*)$  that satisfies (16)-(22) and, hence,  $x^*$  is not implementable. Q.E.D.

**Proof of Proposition 3:** Consider a maximizer  $\bar{\gamma} = (\bar{t}, \bar{x})$  of problem  $\mathcal{P}$ .

To show that it satisfies the first statement, note that (22) directly implies that for  $v_i = 0$  it holds  $x_i(v_i, v_{-i}) = v_i$ . So it is left to prove  $x_0^{sb}(1, v_{-i}) = 1 \Rightarrow \bar{x}_i(1, v_{-i}) = 1$ . Suppose to the contrary that there exists some  $\bar{v} \in V$  with some  $\bar{v}_i = 1$  so that  $\bar{x}_0(\bar{v}) = 1$  and  $\bar{x}_i(1, \bar{v}_{-i}) = 0$ . Then by raising both  $\bar{x}_i(1, \bar{v}_{-i})$  and the corresponding  $\bar{t}_i^p(1, \bar{v}_{-i})$  by 1, the objective (15) is raised by  $p(1, \bar{v}_{-i})(1-c) > 0$ , while the constraints (16), (18),(19),(21), and (22) are unaffected, and (17) and (20) are relaxed.

To show the second statement, suppose to the contrary that  $\bar{\gamma} = (\bar{t}, \bar{x})$  exhibits  $\bar{x}_0(\bar{v}) = 1$ , while  $x_0^*(\bar{v}) = 0$  for some  $\bar{v} = (\bar{v}_1, \dots, \bar{v}_n)$ . Define  $I^1 = \{i | \bar{x}_i(\bar{v}) = 1\}$  as the set of consumers who receive the good under  $\bar{\gamma}$  and the value realization  $\bar{v}$ . Since  $\bar{\gamma}$  is, by assumption, a maximizer of  $\mathcal{P}$ , it must hold that  $I^1$  is non-empty and, due to (22), for all  $i \in I^1$  it holds  $v_i = 1$ . But since  $x_0^*(v) = 0$ , it follows  $\sum_{i \in I^1} \bar{v}_i(1-c) \leq \sum_i \bar{v}_i(1-c) < I$ . Now consider an alternative mechanism  $\hat{\gamma} = (\hat{t}, \hat{x})$  that is identical to  $\bar{\gamma}$  except that  $\hat{x}_0(\bar{v}) = \hat{x}_i(\bar{v}) = 0$  and for all  $i \in I^1$  it exhibits  $\hat{x}_i(\bar{v}) = 0$ ,  $\hat{t}_i^p(\bar{v}) = \bar{t}_i^p(\bar{v}) - c$ , and  $\hat{t}_i^a(\bar{v}) = \bar{t}_i^a(\bar{v}) - 1 + c$ . First note that a comparison of the objective (15) evaluated at  $\hat{\gamma}$  and  $\bar{\gamma}$  yields a difference of  $p(\bar{v})[I - \sum_{i \in I^1} (1-c)]$ , which is positive. Hence,  $\bar{\gamma}$  is not a solution to  $\mathcal{P}$  if  $\hat{\gamma}$  is feasible. In order to see that  $\hat{\gamma}$  is feasible, we verify that it satisfies (3), (4), (7), (8), and (9) using that  $\bar{\gamma}$  satisfies these constraints by assumption.

To verify the first inequality in (3), note  $\sum_i \hat{t}_i^a(\bar{v}) = \sum_{i \in I^1} (\bar{t}_i^a(\bar{v}) - 1 + c) + \sum_{i \notin I^1} \bar{t}_i^a(\bar{v}) \geq I - \sum_{i \in I^1} (1-c) \geq 0 = I \hat{x}_0(\bar{v})$ , where the first inequality follows because  $\bar{\gamma}$  satisfies (3) and the second inequality was already established above.

To verify the second inequality in (3) note  $\sum_i (\hat{t}_i^a(\bar{v}) + \hat{t}_i^p(\bar{v})) = \sum_{i \in I^1} (\bar{t}_i^a(\bar{v}) + \bar{t}_i^p(\bar{v}) - 1) + \sum_{i \notin I^1} (\bar{t}_i^a(\bar{v}) + \bar{t}_i^p(\bar{v})) \geq I + c \sum_{i \in I^1} \bar{x}_i(\bar{v}) - \sum_{i \in I^1} 1 = I - \sum_{i \in I^1} (1-c) \geq 0 = I \hat{x}_0(\bar{v}) + c \sum_i \hat{x}_i(\bar{v})$ , where the first inequality follows because  $\bar{\gamma}$  satisfies (3).

Noting that, because  $\bar{\gamma}$  satisfies (4), it trivially follows that also  $\hat{\gamma}$  satisfies (4), we continue to verify (7) and (9). Note that, by construction,  $\hat{x}_i(\bar{v}) - \hat{t}_i^a(\bar{v}) - \hat{t}_i^p(\bar{v}) = \bar{x}_i(\bar{v}) - \bar{t}_i^a(\bar{v}) - \bar{t}_i^p(\bar{v})$  so that  $\hat{X}_i(v_i) - \hat{T}_i(v_i) = \bar{X}_i(v_i) - \bar{T}_i(v_i)$ . Because  $\bar{\gamma}$  satisfies (7) and (9), therefore, also  $\hat{\gamma}$ .

Finally, to verify (8) note that for  $\bar{\gamma}$  we have  $\mathcal{T}^a = \{I\}$  so that this is also the case for  $\hat{\gamma}$ . Hence, (8) reduces to (18). To see that  $\hat{\gamma}$  satisfies this constraint, note that  $\sum_{v \in V} \sum_{i=1}^n \pi(v|I) (\hat{t}_i^p(v) - c \hat{x}_i(v)) = \sum_{v \neq \bar{v}} \sum_{i=1}^n \pi(v|I) (\hat{t}_i^p(v) - c \hat{x}_i(v)) +$

$$\begin{aligned}
& \sum_i p(\bar{v}|I)(\hat{t}_i^p(\bar{v}) - c\hat{x}_i(\bar{v})) = \sum_{v \neq \bar{v}} \sum_i \pi(v|I)(\bar{t}_i^p(v) - c\bar{x}_i(v)) + \sum_i p(\bar{v}|I)\hat{t}_i^p(\bar{v}) \\
& = \sum_{v \neq \bar{v}} \sum_i \pi(v|I)(\bar{t}_i^p(v) - c\bar{x}_i(v)) + \sum_{i \notin I^1} p(\bar{v}|I)\hat{t}_i^p(\bar{v}) + \sum_{i \in I^1} p(\bar{v}|I)\hat{t}_i^p(\bar{v}) \\
& = \sum_{v \neq \bar{v}} \sum_i \pi(v|I)(\bar{t}_i^p(v) - c\bar{x}_i(v)) + \sum_{i \notin I^1}^n p(\bar{v}|I)\bar{t}_i^p(\bar{v}) + \sum_{i \in I^1}^n p(\bar{v}|I)(\bar{t}_i^p(\bar{v}) - c) = \\
& \sum_{v \neq \bar{v}} \sum_i \pi(v|I)(\bar{t}_i^p(v) - c\bar{x}_i(v)) + \sum_{i \notin I^1}^n p(\bar{v}|I)(\bar{t}_i^p(\bar{v}) - c\bar{x}_i(\bar{v})) + \sum_{i \in I^1}^n p(\bar{v}|I)(\bar{t}_i^p(\bar{v}) - \\
& c\bar{x}_i(\bar{v})) = \sum_{v \in V} \sum_i \pi(v|I)(\bar{t}_i^p(v) - c\bar{x}_i(v)) \geq \alpha I.
\end{aligned}$$

To show the proposition's third statement, consider a mechanism  $\bar{\gamma} = (\bar{x}, \bar{t})$  which satisfies (16)-(22) and there is a  $\bar{v}$  such that  $\bar{x}_0(\bar{v}) = 0$ , while  $\sum_i \bar{v}_i > (1 + \alpha)I/(1 - c)$ . We show that  $\bar{\gamma}$  is not a solution to  $\mathcal{P}$ , because there exists a  $(\hat{x}, \hat{t})$  that also satisfies (16)-(22) but yields a strictly higher surplus than  $\bar{\gamma}$ . More specifically, let  $(\hat{x}, \hat{t})$  be identical to  $(\bar{x}, \bar{t})$  except that  $\hat{x}_i(\bar{v}) = \bar{v}_i$ ,  $\hat{t}_i^a(\bar{v}) = \bar{t}_i^a(\bar{v}) + \bar{v}_i \cdot I / \sum_j \bar{v}_j$ , and  $\hat{t}_i^p(\hat{v}) = \bar{t}_i^p(\hat{v}) + \bar{v}_i(1 - I / \sum_j \bar{v}_j)$ .

Note first that the difference in surplus between  $(\hat{x}, \hat{t})$  and  $(\bar{x}, \bar{t})$  is  $p(\bar{v})[(1 - c) \sum_j \bar{v}_j - I] > 0$ . It remains to be checked that  $(\hat{x}, \hat{t})$  satisfies (16)-(22). That it satisfies (16), (18), (21), and (22) follows directly, because  $(\bar{x}, \bar{t})$  satisfies these constraints by assumption and  $(\hat{x}, \hat{t})$  is a transformation of  $(\bar{x}, \bar{t})$  which preserves them.

Since (19) holds for  $(\bar{x}, \bar{t})$ ,  $(\hat{x}, \hat{t})$  trivially satisfies it for all  $v \neq \hat{v}$ . It, however, also holds for  $\hat{v}$ , since  $\sum_i \hat{t}_i^a(\bar{v}) = \sum_i \bar{t}_i^a(\bar{v}) + \sum_i \bar{v}_i \cdot I / \sum_j \bar{v}_j = I\bar{x}_0(\bar{v}) + I = I = I\hat{x}_0(\bar{v})$ . Similarly (20), holds for all  $v \neq \bar{v}$ , while for  $\bar{v}$  it follows  $\sum_i \hat{t}_i^p(\bar{v}) = \sum_i \bar{t}_i^p(\bar{v}) + \sum_i \bar{v}_i(1 - I / \sum_j \bar{v}_j) \geq \sum_i \hat{x}_i(\bar{v})(1 - I / \sum_j \bar{v}_j) > \sum_i \hat{x}_i(\bar{v})c$ , where the first inequality uses that  $(\bar{x}, \bar{t})$  satisfies (20), and the second inequality follows from the proposition's presumption that  $\sum_j \bar{v}_j > (1 + \alpha)I/(1 - c)$ , as this implies  $c < 1 - I / \sum_j \bar{v}_j$ .

Finally, to see that  $(\hat{x}, \hat{t})$  satisfies (17) because  $(\bar{x}, \bar{t})$  does so, first define

$$\bar{R}(v) = \sum_i [\bar{t}_i^p(v) - c\bar{x}_i(v)] \text{ and } \hat{R}(v) = \sum_i [\hat{t}_i^p(v) - c\hat{x}_i(v)].$$

It holds  $\bar{R}(v) = \hat{R}(v)$  for all  $v \neq \bar{v}$ , while for  $\bar{v}$  it follows  $\hat{R}(\bar{v}) = \sum_i [\hat{t}_i^p(\bar{v}) - c\hat{x}_i(\bar{v})] = \sum_i [\bar{t}_i^p(\bar{v}) + \bar{v}_i(1 - I / \sum_j \bar{v}_j) - c\bar{v}_i] \geq \sum_i \bar{v}_i(1 - I / \sum_j \bar{v}_j - c) = \sum_i \bar{v}_i(1 - c) - I > \alpha I$ , where the first inequality uses that  $(\bar{x}, \bar{t})$  satisfies (20), and the final inequality uses the proposition's presumption that  $\sum_i \bar{v}_i > (1 + \alpha)I/(1 - c)$ .

Since  $(\bar{x}, \bar{t})$  satisfies (17), the definition of  $\pi(v|I)$  implies that it holds

$$\sum_{\{v: \bar{x}_0(v)=1\}} \pi(v)\bar{R}(v) \geq \alpha I \cdot \sum_{\{v: \bar{x}_0(v)=1\}} \pi(v).$$

Combining this with the previously established inequality  $\hat{R}(\bar{v}) > \alpha I$ , it follows

$$p(\bar{v})\hat{R}(\bar{v}) + \sum_{\{v:\bar{x}_0(v)=1\}} \pi(v)\bar{R}(v) \geq \alpha I [p(\bar{v}) + \sum_{\{v:\bar{x}_0(v)=1\}} \pi(v)].$$

But since  $\{v : \hat{x}_0(v) = 1\} = \{v : \bar{x}_0(v) = 1\} \cup \{\bar{v}\}$ , this is equivalent to

$$\sum_{\{v:\hat{x}_0(v)=1\}} \pi(v)\hat{R}(v) \geq \alpha I \cdot \sum_{\{v:\bar{x}_0(v)=1\}} \pi(v),$$

which is equivalent to saying that  $(\hat{x}, \hat{t})$  satisfies (17).

To show the proposition's last statement, suppose that, to the contrary, there exists a solution  $\bar{\gamma} = (\bar{x}, \bar{t})$  of problem  $\mathcal{P}$  for which such a  $T$  does not exist. In this case, there exist a valuation profile  $\bar{v}$  and  $\hat{v}$  with  $\sum_j \bar{v}_j > \sum_j \hat{v}_j$  such that  $\bar{x}_0(\bar{v}) = 0$  and  $\bar{x}_0(\hat{v}) = 1$ . Since  $\sum_j \bar{v}_j > \sum_j \hat{v}_j$ , we can find a bijective correspondence  $k : \{1, \dots, n\} \rightarrow \{1, \dots, n\}$  such that  $\hat{v}_j = 1$  implies  $\bar{v}_{k(j)} = 1$ . Fix the correspondence  $k$  and its inverse  $k^{-1}$  and define the mechanism  $\hat{\gamma} = (\hat{x}, \hat{t})$  by  $\hat{x}_0(v) = \bar{x}_0(v)$  for all  $v \neq \hat{v}, \bar{v}$ ,  $\hat{x}_0(\bar{v}) = 1$  and  $\hat{x}_0(\hat{v}) = 0$ , and  $\hat{x}_i(v) = \bar{x}_{k^{-1}(i)}(v)$ ,  $\hat{t}_i^a(v) = \bar{t}_{k^{-1}(i)}^a(v)$ ,  $\hat{t}_i^p(v) = \bar{t}_{k^{-1}(i)}^p(v)$ . Since  $\bar{\gamma}$  satisfies by assumption all constraints (16)-(22) of problem  $\mathcal{P}$ , so does  $\hat{\gamma}$ . They also yield the same objective (15). But since  $\sum_j \bar{v}_j > \sum_j \hat{v}_j$ , mechanism  $\hat{\gamma}$  exhibits at least one  $i$  such that  $\bar{v}_i = 1$  and  $x_i(\bar{v}) = 0$ . By the first statement of this proposition,  $\hat{\gamma}$  is not optimal, since there exists a feasible  $\tilde{\gamma}$  which yields a strictly larger surplus. Consequently, we obtain the contradiction that  $\bar{\gamma}$  is not optimal. Q.E.D.

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