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DATA ABUNDANCE AND ASSET PRICE INFORMATIVENESS

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DATA ABUNDANCE AND ASSET PRICE INFORMATIVENESS

Abstract

Investors can acquire either raw or processed information about the payoff of risky assets. Information processing filters out the noise in raw information but it takes time. Hence, investors buying processed information trade with a lag relative to investors buying raw information. As the cost of raw information declines, more investors trade on it, which reduces the value of processed information, unless raw information is very unreliable. Thus, a decline in the cost of raw information can reduce the demand for processed information and, for this reason, the informativeness of asset prices in the long run.

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Data Abundance and Asset Price Informativeness*

Jérôme Dugast[†] Thierry Foucault[‡]

March 18, 2016

Abstract

Investors can acquire either raw or processed information about the payoff of a risky asset. Information processing filters out the noise in raw information but it takes time. Hence, investors buying processed information trade with a lag relative to investors buying raw information. As the cost of raw information declines, more investors trade on it, which reduces the value of processed information, unless raw information is very unreliable. Thus, a decline in the cost of raw information can reduce the demand for processed information and, for this reason, the informativeness of the asset price in the long run.

KEYWORDS: Price Informativeness, Information Processing, Markets for Information, Contrarian and momentum trading.

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“Increasingly, there is a new technological race in which hedge funds and other well-heeled investors armed with big data analytics analyze millions of twitter messages and other non-traditional information sources to buy and sell stocks faster than smaller investors can hit “retweet.”

in “How investors are using social media to make money,” Fortune, December 7, 2015.

1 Introduction

Improvements in information technologies change how information is produced and disseminated in financial markets. In particular, they enable investors to obtain huge amount of data at lower cost.¹ For instance, investors can now easily get on-line access to companies reports, economic reports, or other investors’ opinions (expressed on social medias) to assess the value of a stock.² Similarly, traditional data vendors like Reuters, Bloomberg, or new entrants like Dataminr and Eagle Alpha use so-called news analytics softwares to extract signals from the huge flow of unstructured data (news reports, press releases, stock market announcements, tweets etc.) available on the internet and then sell these signals to investors who feed them into their trading algorithms.³

How does this evolution affect the informativeness of asset prices? This question is important because ultimately price informativeness affects firms’ real decisions (see Bond, Edmans, and Goldstein (2014) for a survey). Models with costly information acquisition predicts that asset price informativeness should increase when the cost of information declines, either because more investors buy information (Grossman and Stiglitz (1980)) or because investors acquire more precise signals (Verrechia (1982)). As these models are static, they commingle the moments at which investors access to information and process it. Yet, in reality, filtering out noise from a signal takes time. In this paper, we show that, due to this delay, a decline in the cost of accessing information can reduce the long run informativeness of asset prices about fundamentals. Indeed, cheaper access to raw data increases the number of investors trading on very imprecise information. This effect makes prices more informative in the short run but it undermines traders’ incentive to

¹For instance, the cost of sending one trillion bits has declined from \$150,000 to \$0.17 from 1970 to 1999 (see “The new paradigm”, Federal Reserve Bank of Dallas, 1999.).

²For instance, websites such as StockTwits or Seeking Alphas allow investors to comment on stocks, share investment ideas, and provide, in real time, raw financial information pulled off from other social medias. For evidence that information exchanged on social medias contains value relevant information, see Chen et al.(2014).

³See “Rise of the news-reading machines,” Financial Times, January 26, 2010 and “How investors are using social medias to make money,” Fortune, December 7, 2015.

process information further. As a result, asset prices are less informative in the long run.

In our model, speculators can acquire either “raw” information or “processed” information about the payoff of a risky asset. The signal generated by raw information is correct (reveals the asset payoff) with probability θ or is just noise with probability $(1 - \theta)$. Thus, θ characterizes the reliability of raw information.⁴ The true nature of the signal (information/noise) can only be discovered after processing raw information further, which requires some time.

To account for this delay, we assume that speculators who buy processed information (“deep information” speculators) receive their signal with a lag relative to speculators who buy raw information (“shallow information” speculators). Specifically, raw information is available in period 1 while processed information is available in period 2, only. When they receive their signal, speculators can trade on it with risk neutral market makers and liquidity traders (as in Kyle (1985)).

Following Veldkamp (2006a,b), we assume that the cost of producing a signal is fixed but, once produced, the signal can be replicated for free so the marginal cost of providing information to an extra user is zero.⁵ We allow the cost of producing raw and processed information to be different, so that we can study the effects of decreasing the cost of raw information, holding constant the cost of processed information. Furthermore, markets for information are competitive: (i) raw and processed information are sold at competitive prices (i.e., information sellers make zero profits) and (ii) speculators’ profits from trading on each type of information net of the price paid to information sellers are zero. In this setup, we analyze how a decline in the cost of raw information affects equilibrium outcomes, in particular the number of investors buying each type of information in equilibrium (information demand) and the informational content of the asset price in the short run (period 1) and the long run (period 2).

We first show that a decrease in the *cost of raw information* can reduce or increase the demand for *processed information* in equilibrium. Indeed, a decrease in this cost raises the number of speculators trading on raw information and therefore the likelihood that the price of the asset, at the end of period 1, reflects the signal conveyed by raw

⁴Raw information does not need to be literally interpreted as completely unprocessed information. For instance, a buy or sell recommendation of a stock based on linguistic analysis of discussions on social medias about this stock relies on some automated information processing. However, such information processing is not as deep as that performed by financial analysts or investment advisors who spend time in evaluating firms business models, analyzing accounting statements, and building up evaluation models to determine the fundamental value of an asset (see, for instance, Chapters 7 and 8 in Pedersen (2015) for a description of the process for discovering information about stocks in some hedge funds).

⁵Shapiro and Varian (1999) argue that an important feature of information is that its fixed cost of production is large but its variable costs of reproduction are small.

information. If this signal is sufficiently reliable (specifically: $\theta \geq 1 - (\sqrt{2})^{-1}$), this effect reduces the profit from trading on processed information and therefore the demand for this type of information in equilibrium. When the cost of raw information becomes small enough, this crowding out effect can be so strong that it leads to a discontinuous drop in the equilibrium demand for processed information, from a strictly positive value to zero. At this point, the market for processed information ceases to exist.

If instead, raw information is very unreliable (i.e., $\theta < 1 - (\sqrt{2})^{-1}$), speculators trading on raw information frequently inject noise in prices, which creates profit opportunities for speculators trading on processed information. Indeed, they can exploit mispricings due to speculators trading on raw information when the latter, mistakenly, react to noise. For this reason, there exist parameter values for which a decrease in the cost of raw information indirectly raises the equilibrium demand for processed information.

We then study the implications of these effects for the informativeness of the equilibrium price at each date about the asset payoff. Other things equal, a reduction in the cost of raw information improves price informativeness in the short run and a reduction in the cost of processed information improves price informativeness in the long run. The reason is that the demand for a given type of information increases when its cost is reduced, as usual in models of trading with endogenous information acquisition.

More surprisingly, a reduction in the cost of raw information can lead to less informative prices in the long run.⁶ This happens when a reduction in this cost leads to a decline in the demand for processed information. Indeed, this decline increases the risk of persistent mispricing when shallow information speculators trade on noise while reducing the likelihood that the second period price fully reveals the asset payoff when shallow information speculators' signal is valid. In particular, when the cost of raw information is nil, long run price informativeness is always smaller than if speculators could only buy processed information, even if the cost of processed information gets arbitrarily small (but remains strictly positive).⁷ Thus, a decline in the cost of raw information can make

⁶There is at least as much information available in period 2 than in period 1 and strictly more if, in equilibrium, some speculators buy processed information. Thus, the informativeness of the price in period 2 is higher than in period 1. Yet, when the cost of raw information declines, the former can decrease, even though the latter increases.

⁷Brunnermeier (2005) considers a model of trading in a stock with two trading rounds. In the first trading round, there is information leakage: one investor receives a noisy signal about information that is made public just before the second trading round. Brunnermeier (2005) shows that the informativeness of the stock price in the second trading round is smaller than in the absence of information leakage. At first glance, this result looks similar to ours. However, the mechanisms in Brunnermeier (2005) and in our model are very different. In particular, there is no public information arrival in our model (so no trader speculates on this arrival) and our findings rely on endogenous information acquisition (the number of informed speculators is exogenous in Brunnermeier (2005)).

prices more informative in the short run and yet less informative in the long run.

This implication of the model is consistent with Weller (2016) who finds empirically a negative association between the activity of algorithmic traders (a class of traders who is likely to trade on relatively raw signals) and the informativeness of prices about future earnings. It also offers a possible interpretation of the empirical findings in Bai, Phillipon, and Savov (2015). For the entire universe of U.S. stocks, they find (see their Figure A.3) that stock price informativeness has been declining over time (they obtain the opposite conclusion for stocks in the S&P500 index). They attribute this evolution to change in the characteristics of firms that do not belong to the S&P500 index. Our model suggests that the reduction in the cost of raw information might be another explanatory factor for this evolution.

Our model has additional testable implications for the trade patterns of various types of investors. First the model predicts that the correlation between shallow and deep information speculators' order flows (the difference between their sales and buys) declines (and can even become negative) when the cost of raw information decreases. Indeed, deep information speculators trade in a direction opposite to that of shallow information speculators when they correct mispricings due to shallow information speculators' trades. Now, shallow information speculators are more likely to generate mispricings when they are more numerous, i.e., when the cost of raw information is small. For this reason, sales (resp., buys) by shallow information speculators are more likely to be followed by buys (resp. sales) from deep information speculators when the cost of raw information declines.

Second, deep information speculators' order flows and past returns are correlated. This correlation is negative when raw information is unreliable ($\theta \leq \frac{1}{2}$) and positive otherwise.⁸ Thus, in equilibrium, deep information speculators behave either like *contrarian* traders (they trade against past returns) or *momentum* traders (they trade in the same direction as past returns). Intuitively, when raw information is unreliable, price moves due to shallow information speculators' orders are more likely to be due to noise and therefore to be subsequently corrected by deep information speculators. The model also implies that, in absolute value, the covariance between past returns and deep information speculators' order flow should be inversely related to the cost of raw information.

Last, the direction of shallow information speculators' trade is positively correlated

⁸Standard models of informed trading (e.g., Kyle (1985)) predicts a zero correlation between the trades of informed investors at a given date and lagged returns (see Boulatov, Livdan and Hendershott (2012), Proposition 1, for instance.)

with future returns (from period 1 to period 2 in the model).⁹ However, this correlation becomes weaker when the cost of raw information declines. Indeed, this decline increases the mass of speculators trading on shallow information and therefore the likelihood that the first period price fully reflects their signal. When this happens, deep information speculators can only profitably trade on information that is orthogonal to shallow information speculators' signal. As a result, shallow information speculators' signal (or equivalently trade) has less predictive power for deep information speculators' trade and therefore the second period return.

All our predictions are about the effects of a decrease in the cost of raw information. Empiricists could test them by using shocks to the cost of accessing raw financial data. For instance, in 2009, the SEC mandated that financial statements be filed with a new language (the so called EXtensible Business Reporting Language) on the ground that it would lower the cost of accessing data for smaller investors.¹⁰ The implementation of this new rule or other shocks of the same nature could therefore be used to test some of our predictions.¹¹

Our paper contributes to the literature on costly information acquisition in financial markets and information markets (e.g., Grossman and Stiglitz (1980), Verrechia (1982), Admati and Pfleiderer (1986), Veldkamp (2006a,b), Cespa (2008), or Lee (2013)). Some models (e.g., Verrechia (1982) or Peress (2010)) have considered the possibility for informed investors to pay a cost to obtain more precise information. This cost can be interpreted as a cost of processing information. However, in these models, all investors trade simultaneously. In this case, a reduction in the cost of information leads investors to buy more precise information *and* make stock prices more informative (see Verrechia (1982), Corollary 4). In contrast, in our model, time is required to obtain information of greater precision. In this case, our analysis shows that a decline in the cost of raw

⁹This is not due to serial correlation in returns. In our model, the price of the asset at each date is equal to its expected value conditional on all available public information, i.e., the history of trades as in Kyle (1985). Hence, returns are serially uncorrelated in our model.

¹⁰See SEC (2009). In particular on page 129, the SEC notes that: “*If [XBRL] serves to lower the data aggregation costs as expected, then it is further expected that smaller investors will have greater access to financial data than before. In particular, many investors that had neither the time nor financial resources to procure broadly aggregated financial data prior to interactive data will have lower cost access than before interactive data. Lower data aggregation costs will allow investors to either aggregate the data on their own, or purchase it at a lower cost than what would be required prior to interactive data. Hence, smaller investors will have fewer informational barriers that separate them from larger investors with greater financial resources.*”

¹¹Interestingly, data vendors such as Dow Jones screen SEC filings by firms and release information contained in these filings through specialized services (e.g., Dow Jones Corporate Filing Alert). Thus, reduction in the cost of accessing these filings for data vendors are like reduction in the cost of producing raw information in our model.

information can reduce the value of processed information and therefore the demand for this type of information. As a result, price informativeness is reduced in the long run.

As in Froot, Scharfstein and Stein (1992) and Hirshleifer et al. (1994), our model features “early” (shallow information speculators) and “late” (deep information speculators) informed investors. In contrast to these models, however, investors can endogenously choose to trade early or late in our model and this choice determines the reliability of their information (late investors receive more reliable information).¹² For this reason, the implications of our model for price informativeness and trade patterns are *not* observationally equivalent.¹³ For instance, in Hirshleifer et al.(1994), the trades of early and late informed investors are always positively correlated (see their Proposition 2) while, instead, they can be negatively correlated in our model. Moreover, in Hirshleifer et al.(1994), the trades of late informed investors are not correlated with past returns (see their Proposition 3) while they are in our model.

The next section describes the model. Section 3 derives equilibrium prices at dates 1 and 2, taking the demands for raw and processed information as given. Section 4 endogenizes these demands . Section 5 derives the implications of the model for (a) asset price informativeness and (ii) price and trade patterns. Section 6 concludes. Proofs of the main results are in the appendix. Additional material is available in the on-line appendix on the authors’ website.

2 Model

We consider the market for a risky asset. Figure 1 describes the timing of actions and events in our model. There are four periods ($t \in \{0, 1, 2, 3\}$). The payoff of the asset, V , is realized at date $t = 3$ and is equal to $V = 1$ or $V = 0$ with equal probabilities. Trades take place at dates $t = 1$ and $t = 2$ among three types of market participants: (i) liquidity traders, (ii) a continuum of risk neutral speculators, and (iii) a competitive and risk neutral market-maker.

¹²In Holden and Subrahmanyam (2002), risk averse investors can choose to receive information at dates 1 or 2. However, the precision of investors’ signals is the same at both dates. In contrast, in our model, investors who process information are better informed than investors who just trade on raw information.

¹³In Froot et al.(1992), there exist equilibria in which a fraction of speculators trade on noise. However, there is no possibility for traders to correct price changes due to such trades. In contrast, in our model, deep information speculators correct price changes due to noise, after processing signals.

exploited only with a delay relative to raw (“shallow”) information.

We refer to speculators who buy raw information as *shallow information speculators* and to those who buy processed information as *deep information speculators*. We denote the mass of shallow and deep information speculators by α_1 and α_2 . In Section 4, we introduce a market for information and we endogenize the prices (F_r and F_p) and the demand for of each type of information, i.e., α_1 and α_2 .

As in Glosten and Milgrom (1985), we assume that each speculator can only buy or sell a fixed number of shares—normalized to one share—using market orders (i.e., orders that are non contingent on the contemporaneous execution price). If he decides to trade a speculator will optimally submit an order of the maximum size (one share) because he is risk neutral and too small to individually affect the equilibrium price. We denote by $x_{it} \in \{-1, 0, 1\}$, the market order submitted by speculator i trading at date t , with $x_{it} = 0$ if speculator i chooses not to trade and $x_{it} = -1$ (resp., $+1$) if he sells (resp., buys) the asset. We focus on equilibria in pure strategies in which all speculators of a given type play the same strategy (symmetric equilibria).¹⁵ Moreover, to facilitate the exposition, we assume that speculators who buy raw information trade at date $t = 1$. We show in the on-line appendix that this strategy is indeed optimal relative to the alternative, which is to wait until date 2 to exploit signal s .¹⁶ Thus, index t unambiguously identifies the type of a speculator. Therefore, henceforth, we drop index i when referring to the strategy of a speculator.

Liquidity Traders. At each date t , liquidity traders buy or sell the asset for exogenous reasons. Their aggregate demand at date t , denoted \tilde{l}_t , has a uniform distribution (denoted $\phi(\cdot)$) on $[-1, 1]$ and \tilde{l}_1 is independent from \tilde{l}_2 . As usual, liquidity traders ensure that the order flow at date t is not necessarily fully revealing and thereby allows speculators to make trading profits in equilibrium. This is necessary for information to have value in equilibrium (Grossman and Stiglitz (1980)).

The market-maker. At date t , the market-maker absorbs the net demand from liquidity traders and speculators at this date (the “order flow”) at a price, p_t , equal to the expected payoff of the asset conditional on his information. As the market-maker does not observe \tilde{s} and \tilde{u} until $t = 3$, the price at date t only depends on the order flow

¹⁵This restriction is innocuous because there are no other equilibria than symmetric equilibria in pure strategies when speculators’ gross expected profits in equilibrium are strictly positive. This condition is necessarily satisfied in equilibrium when the price of a signal is strictly positive because no speculator would buy a signal if his expected trading profit, gross of the price paid for the signal, is zero.

¹⁶This is intuitive. Indeed, a shallow information speculator who waits until date 2 to exploit his signal s faces competition from better informed investors and takes the risk that the price at the end of date 1 reveals his signal, due to trades from other shallow information speculators.

history until this date (as in Kyle (1985)). Formally, let f_t be the order flow at date t :

$$f_t = \tilde{l}_t + \int_0^{\alpha_t} x_{it} di. \quad (2)$$

The stock price at date t is:

$$p_t = E[\tilde{V}|\Omega_t] = Pr[\tilde{V} = 1|\Omega_t], \quad (3)$$

where Ω_t is the market-maker's information set at date t ($\Omega_1 = \{f_1\}$ and $\Omega_2 = \{f_2, f_1\}$). At date 0, the price of the asset is $p_0 = E(V) = 1/2$. Henceforth, we denote the highest and smallest possible realizations of the order flow at date t by f_t^{max} and f_t^{min} . These are, respectively, $1 + \alpha_t$ (all investors present at date t buy) and $-(1 + \alpha_t)$ (all speculators present at date t sell).

3 Equilibrium Trading Strategies and Prices

In this section, we derive speculators' optimal trading strategies and equilibrium prices, for given values of α_1 and α_2 . This is a required step to analyze the equilibrium of the markets for raw and shallow information at date 0 because the decision to buy a specific type of information depends on the expected profits that a speculator can obtain with this information.

Let $\mu(s)$ be shallow information speculators' valuation for the asset at date 1 given their signal $s \in \{0, 1\}$. We have:

$$\mu(s) = Pr[V = 1|\tilde{s} = s].$$

Hence:

$$\mu(1) = \frac{1 + \theta}{2} > \frac{1}{2} \quad \text{and} \quad \mu(0) = \frac{1 - \theta}{2} < \frac{1}{2}.$$

A shallow speculator's market order only depends on s because this is the only source of information available to him. Hence, we denote the trading strategy of a shallow information speculator by $x_1(s)$. His expected profit when he receives signal s is:

$$\pi_1(\alpha_1, s) = x_1(s)(\mu(s) - E[p_1|\tilde{s}]).$$

Proposition 1. Let $\omega(x, \alpha_1) = \frac{\phi(x - \alpha_1)}{\phi(x - \alpha_1) + \phi(x + \alpha_1)}$. The equilibrium at date 1 is as follows:

1. Shallow information speculators buy the asset if $s = 1$ and sell it if $s = 0$ ($x_1(0) = -1$ and $x_1(1) = 1$).
2. The equilibrium stock price at date 1 is:

$$p_1^*(f_1) = Pr[\tilde{V} = 1 | \tilde{f}_1 = f_1] = \omega(f_1, \alpha_1)\mu(1) + (1 - \omega(f_1, \alpha_1))\mu(0), \quad (4)$$

for $f_1 \in [f_1^{min}, f_1^{max}]$.

3. Shallow information speculators' ex-ante (date 0) expected profit is:

$$\bar{\pi}_1(\alpha_1) = E(\pi_1(\alpha_1, s)) = \frac{\theta}{2} Max\{1 - \alpha_1, 0\}. \quad (5)$$

As $\phi(\cdot)$ is the density of the uniform distribution on $[-1, 1]$, we deduce from eq. (4) that:

$$p_1^*(f_1) = \begin{cases} \mu(0) & \text{if } f_1 \in [f_1^{min}, -1 + \alpha_1], \\ \frac{1}{2} & \text{if } f_1 \in [-1 + \alpha_1, 1 - \alpha_1], \\ \mu(1) & \text{if } f_1 \in [1 - \alpha_1, f_1^{max}]. \end{cases}$$

A realization of the order flow at date 1 in $[-1 + \alpha_1, 1 - \alpha_1]$ does not affect the market-maker's belief about the payoff of the asset because the likelihood of each realization in this interval is the same whether $V = 1$ or $V = 0$. Thus, the market-maker sets a price equal to the ex-ante expected value of the asset, $1/2$. The threshold $1 - \alpha_1$ is the largest possible realization of the order flow when shallow information speculators observe $s = 0$ because they all sell the asset in this case. Thus, if the order flow at date 1 is above this threshold, the market-maker infers that $s = 1$ and sets a price equal to $p_1 = \mu(1)$. Symmetrically, if the order flow at date 1 is smaller than $-1 + \alpha_1$, the market-maker infers that $s = 0$ and sets a price equal to $p_1 = \mu(0)$.

In sum, the order flow at date 1, \tilde{f}_1 , is completely uninformative about shallow information speculators' signal, s , when its realization is inside the interval $[-(1 - \alpha_1), (1 - \alpha_1)]$. Otherwise, it fully reveals this signal. Full revelation of shallow speculators' signal occurs therefore with a probability equal to $Min\{\alpha_1, 1\}$. This probability increases with the mass of shallow information speculators, α_1 , because, as their mass increases, their aggregate order size becomes larger relative to that of liquidity traders. Thus, they account for a larger fraction of the total order flow, which therefore becomes more informative. For this reason, shallow speculators' expected profit in the first period declines with α_1 . When $\alpha_1 \geq 1$, the mass of shallow information speculators is so large relative to the mass

of liquidity traders that the order flow at date 1 is always fully revealing (the interval $[-1 + \alpha_1, 1 - \alpha_1]$ is empty). In this case, speculators trading on raw information earn zero expected profits.

At $t = 2$, deep information speculators receive the signal (s, u) , and observe p_1 . Hence, we denote a deep information speculator's trading strategy by $x_2(s, u, p_1)$. His expected trading profit is:

$$\pi_2(\alpha_1, \alpha_2, s, u) = x_2(s, u, p_1)(E[V|u, s] - E[p_2|u, s, p_1]).$$

Proposition 2. *The equilibrium at date $t = 2$ is as follows:*

1. *If deep information speculators receive a signal $(s, 0)$, they buy one share if the price in the first period is smaller than $\frac{1}{2}$ (i.e., $x_2(s, 0, p_1) = 1$ if $p_1 < 1/2$), sell one share if the price in the first period is greater than $\frac{1}{2}$ (i.e., $x_2(s, 0, p_1) = -1$ if $p_1 > 1/2$), and do not trade otherwise (i.e., $x_2(s, 0, 1/2) = 0$). If instead deep information speculators receive the signal $(s, 1)$, they buy one share if $s = 1$ (i.e., $x_2(1, 1, p_1) = 1$) and sell one share if $s = 0$ (i.e., $x_2(0, 1, p_1) = -1$).*
2. *If $p_1 = \mu(1) = \frac{1+\theta}{2}$ then the stock price at date 2 is:*

$$p_2^*(f_2) = \begin{cases} \frac{1}{2} & \text{if } f_2 \in [f_2^{min}, -1 + \alpha_2], \\ \frac{1+\theta}{2} & \text{if } f_2 \in [-1 + \alpha_2, 1 - \alpha_2], \\ 1 & \text{if } f_2 \in [1 - \alpha_2, f_2^{max}]. \end{cases}$$

3. *If $p_1 = \mu(0) = \frac{1-\theta}{2}$ then the stock price at date 2 is:*

$$p_2^*(f_2) = \begin{cases} 0 & \text{if } f_2 \in [f_2^{min}, -1], \\ \frac{1-\theta}{2} & \text{if } f_2 \in [-1 + \alpha_2, 1 - \alpha_2], \\ \frac{1}{2} & \text{if } f_2 \in [1 - \alpha_2, f_2^{max}]. \end{cases}$$

4. If $p_1 = \frac{1}{2}$ then the stock price at date 2 is:

$$p_2^*(f_2) = \begin{cases} 0 & \text{if } f_2 \in [f_2^{min}, -1], \\ \frac{1-\theta}{2-\theta} & \text{if } f_2 \in [-1, \text{Min}\{-1 + \alpha_2, 1 - \alpha_2\}], \\ \frac{1}{2} & \text{if } f_2 \in [\text{Min}\{-1 + \alpha_2, 1 - \alpha_2\}, \text{Max}\{-1 + \alpha_2, 1 - \alpha_2\}] \\ \frac{1}{2-\theta} & \text{if } f_2 \in [\text{Max}\{-1 + \alpha_2, 1 - \alpha_2\}, 1] \\ 1 & \text{if } f_2 \in [1, f_2^{max}]. \end{cases}$$

5. Deep information speculators' ex-ante expected profit, $\bar{\pi}_2(\alpha_1, \alpha_2) \equiv E[\pi_2(\alpha_1, \alpha_2, s, u)]$, is:

$$\bar{\pi}_2(\alpha_1, \alpha_2) = \begin{cases} \frac{\theta}{2} [2\alpha_1(1-\theta)(1-\alpha_2) + (1-\alpha_1)(1-\alpha_2(2-\theta)^{-1})] & \text{if } \alpha_2 \leq 1, \\ \frac{\theta}{2} \left(\frac{1-\theta}{2-\theta}\right) (1-\alpha_1)(2-\alpha_2) & \text{if } 1 \leq \alpha_2 \leq 2, \\ 0 & \text{if } \alpha_2 > 2. \end{cases} \quad (6)$$

The nature of the deep information speculator's strategy at date 2 depends on whether $u = 1$ or $u = 0$. When $u = 1$, deep information speculators learn that the signal s is valid. Their strategy is then identical to that of shallow information speculators: they buy the asset if $s = 1$ (the asset payoff is high) and sell it if $s = 0$ (the asset payoff is zero). Thus, *conditional* on $u = 1$, their strategy at date 2 is independent from the price of the asset at the end of the first period.

In contrast, when $u = 0$, deep information speculators learn that s contains no information. Thus, their estimate of the asset payoff is equal to its unconditional expected value, $1/2$. Their strategy is then determined by the price of the asset at date 1. If $p_1 > 1/2$, deep information speculators optimally sell the asset because they expect that, on average, their sell orders will execute at a price greater than their valuation for the asset ($1/2$). Symmetrically, if $p_1 < 1/2$, deep information speculators optimally buy the asset. Finally, if $p_1 = 1/2$, not trading is weakly dominant for shallow information speculators because they expect their order to execute at a price equal to their valuation for the asset.¹⁷

¹⁷The reason is that, in this case, a deep information speculator expects (i) liquidity traders' aggregate demand to be zero on average and (ii) other deep information speculators' demand to be zero as well. Hence, a speculator expects the price at date 2 to be identical to the price at date 1 because his demand is negligible compared to speculators' aggregate demand.

Importantly, the likelihoods of up or down price movements at dates 1 and 2 are endogenous and depend on the demands for shallow and deep information, i.e., α_1 and α_2 . As an illustration, consider Figure 2. It shows possible realizations for equilibrium prices at each date conditional on $s = 1$ and $s = 0$ and, in each case, the transition probabilities from one price to another between two consecutive dates, assuming that $\alpha_1 \leq 1$ and $\alpha_2 \leq 1$.¹⁸ The unconditional probability of a given price path in equilibrium is obtained by multiplying the conditional likelihood of this path by $1/2$ because $s = 1$ or $s = 0$ with equal probabilities.

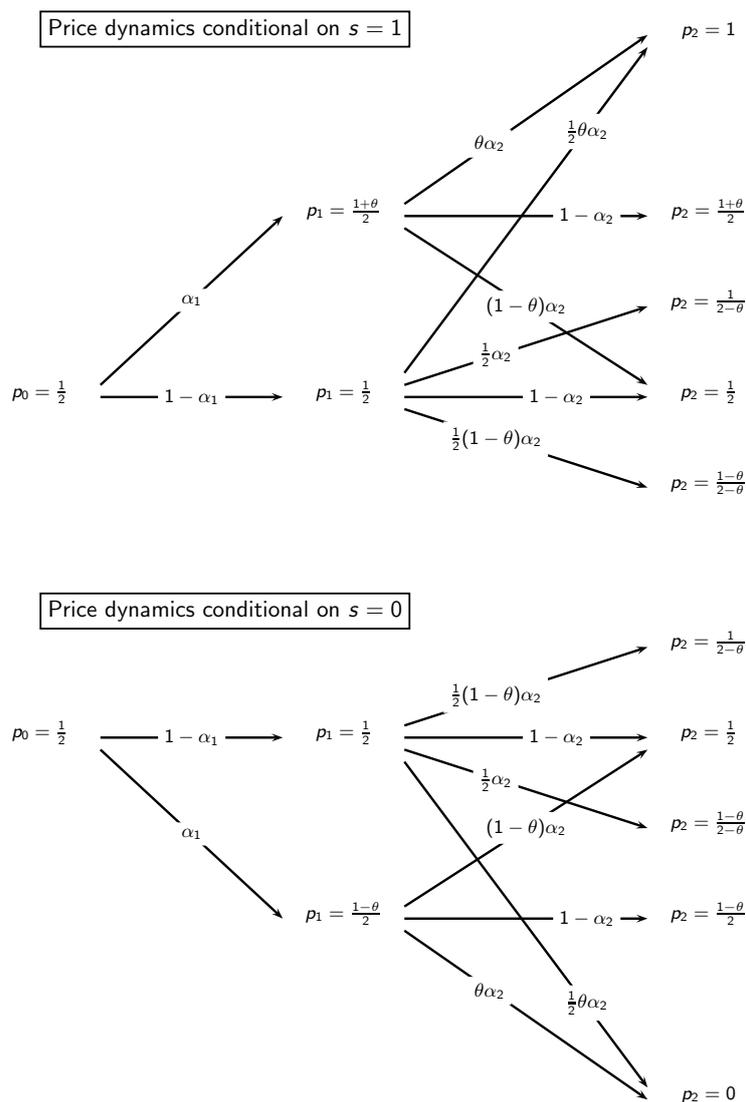


Figure 2: Price dynamics in equilibrium for $\alpha_1 \leq 1$ and $\alpha_2 \leq 1$. On each arrow, we indicate the probability of transition from the price at the start of the arrow to the price at which the arrow points.

¹⁸Transition probabilities are different when $\alpha_2 > 1$, which may happen in equilibrium; see next section.

For instance, suppose that $s = 1$. In this case, shallow information speculators buy the asset at date 1 and, with probability α_1 , the market maker infers from the order flow that $s = 1$. In this case, the price of the asset at date 1 increases relative to date 0. Otherwise (with probability $(1 - \alpha_1)$), it is unchanged.

Now, suppose the price increases at date 1, i.e., consider the case $p_1 = \frac{1+\theta}{2}$. Then, there are two possibilities. First, with probability θ , deep information speculators learn that the signal in period 1 was indeed informative. Hence, they also buy the asset and, with probability α_2 , their demand is so strong that the market maker infers that $V = 1$. Hence, the price goes up at date 2 relative to the price at date 1. The overall unconditional probability of two consecutive up movements in the price is therefore $\frac{\theta\alpha_1\alpha_2}{2}$. Alternatively, with probability $(1 - \theta)$, deep information speculators discover, in period 2, that s is just noise. Hence, deep information speculators sell the asset in period 2 because, given their information, it is overpriced. In this case, with probability α_2 , their supply is strong enough to push the price back to its initial level and they in fact correct the noise injected by shallow information speculators into prices. Thus, the unconditional probability of an up price movement followed by a down movement is $\frac{(1-\theta)\alpha_1\alpha_2}{2}$. Finally, in either case, there is a probability $(1 - \alpha_2)$ that the order flow at date 2 is uninformative. In this case, the price at date 2 is equal to the price at date 1.

It is immediate that a speculator's expected trading profit decreases when the number of speculators with the *same* type increases. This effect is standard in models of informed trading (e.g., Grossman and Stiglitz (1980)). Indeed, as more speculators trade on the same signal, the order flow is more informative about this signal, which erodes speculators' ability to make profits.

In contrast, the effect of an increase in the mass of shallow information speculators on deep information speculators' expected profit is ambiguous, as the next corollary shows. Let denote $\hat{\alpha}_2(\theta) = \frac{(1-2\theta)(2-\theta)}{2(2-3\theta+\theta^2)-1}$. Observe that $\hat{\alpha}_2(\theta) > 0$ iff $\theta < 1/2$ and that $\hat{\alpha}_2(\theta)$ goes to $2/3$ as θ goes to zero.

Corollary 1. *Deep information speculators' expected profit, $\bar{\pi}_2(\alpha_1, \alpha_2)$, increases with the mass of shallow information speculators, α_1 , if and only if $\alpha_2 < \hat{\alpha}_2(\theta)$ and $\theta \leq 1/2$. Otherwise, deep information speculators' expected profit decreases with the the mass of shallow information speculators.*

Shallow information speculators' orders tilt the direction of the order flow at date 1 in the direction of their signal s . Thus, if the price changes at date 1, this is always in the direction of their signal. A change in the price at date 1 has therefore two opposite effects

on deep information speculators' expected profit. If shallow information speculators' signal is valid ($u = 1$), the adjustment of the price at date 1 to signal s reduces the expected profit of trading on deep information. If instead the signal s is noise, the change in price at date 1 is a source of mispricing. Deep information speculators can then make profit on exploiting this mispricing.

In sum, deep information speculators can benefit from a price movement at date 1 if $\alpha_2 \leq \hat{\alpha}_2(\theta)$ and $\theta \leq 1/2$, i.e., when (i) shallow information speculators frequently trade on noise (θ is small) and (ii) deep information speculators' trades do not reveal their information too frequently (α_2 is small), as Figure 3 shows. In this case, an increase in the mass of shallow information speculators, α_1 , increases deep information speculators' expected profit because it increases the likelihood of a price movement at date 1, which is a pre-requisite for deep information speculators to benefit from mispricings due to shallow information speculators. Otherwise, the reverse is true: an increase in the mass of shallow information speculators erodes deep information speculators' ability to profit from deep information because it increases the likelihood that the price at date 1 reflects some of their information. Thus, an increase in the mass of shallow information speculators can be a positive or a negative externality for deep information speculators, depending on parameter values.

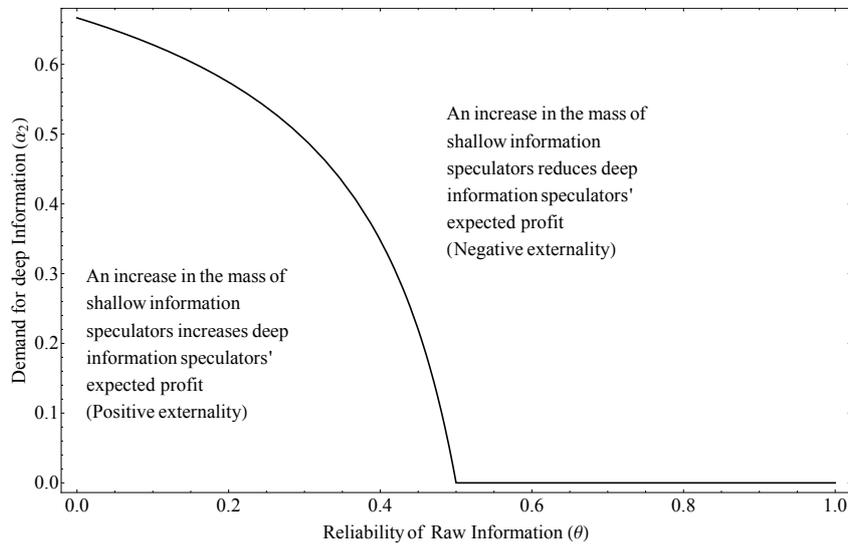


Figure 3: This figure plots the curve that represents the function $\hat{\alpha}_2(\theta)$ and shows the sets of values of θ and α_2 for which a marginal increase in the demand for shallow information (α_1) exerts a negative externality or a positive externality on deep information speculators (Corollary 1).

4 Equilibrium in the markets for deep and shallow information

In this section, we derive the equilibrium demands (α_2^* and α_1^*) and prices in the markets for deep and shallow information. The fixed cost of producing information is high but reproduction costs are negligible (see Shapiro and Varian (1999)). Thus, as in Veldkamp (2006a,b), we assume that the costs of producing deep information (the signal (s, u)) and shallow information (the signal s) are fixed and equal to C_p and C_r , respectively.

In practice, suppliers of raw and shallow information are different. For instance, Reuters, Bloomberg, or firms like Dataminr specialize in the distribution of relatively raw information while financial intermediaries (e.g., securities analysts) sell processed information. Thus, we treat the markets for shallow and deep information as two distinct markets.

As in Veldkamp (2006a,b), we also assume that markets for information are competitive. This means that if some speculators buy information of a given type (e.g., deep information) then buyers and sellers of this type of information just break even. In particular, if the equilibrium mass of speculators buying deep information is $\alpha_2^e > 0$ then the equilibrium price of deep information, F_p^e , is:

$$F_p^e = \frac{C_p}{\alpha_2^e}, \quad (7)$$

so that sellers of deep information just make zero expected profits. Similarly, if the mass of speculators buying shallow information is $\alpha_1^e > 0$ in equilibrium, the price of raw information is:

$$F_r^e = \frac{C_r}{\alpha_1^e}. \quad (8)$$

We first analyze the determination of the equilibrium in the market for deep information, holding the mass of shallow information speculators fixed. This is without loss of generality because the equilibrium value of α_1 is independent of the equilibrium value of α_2 (while the reverse is not true; see below). Thus, α_1 can be treated as a parameter in the analysis of the equilibrium of the market for deep information.

Let $\bar{\pi}_2^{net}(\alpha_1, \alpha_2^e, F_p^e) = \bar{\pi}_2(\alpha_1, \alpha_2^e) - F_p^e$ be the expected profit of deep information speculators net of the price of deep information in equilibrium. In an interior equilibrium of the market for deep information (i.e., an equilibrium in which $\alpha_2^e > 0$), the net expected profit of deep information speculators is just zero because the market for deep information

is competitive, i.e.,

$$\bar{\pi}_2^{net}(\alpha_1, \alpha_2^e, F_p^e) = 0. \quad (9)$$

This is a natural condition if there is free entry of speculators. Indeed, if $\bar{\pi}_2^{net}(\alpha_1, \alpha_2^e, F_p^e) > 0$, additional speculators would buy information at price F_p^e (so that α_2^e is not the equilibrium demand) while if $\bar{\pi}_2^{net}(\alpha_1, \alpha_2^e, F_p^e) < 0$, speculators would be better off not buying information (which contradicts the fact that $\alpha_2^e > 0$).

The zero profit condition (9) implies that in an interior equilibrium, deep information speculators' *aggregate* net expected profit (denoted $\pi_2^{net,a}(\alpha_1, \alpha_2^e) = \alpha_2^e \bar{\pi}_2^{net}(\alpha_1, \alpha_2^e, F_p^e)$) is zero as well. Thus, using eq.(7), we deduce that α_2^e is an interior equilibrium if and only if it solves:

$$\pi_2^{net,a}(\alpha_1, \alpha_2^e) = \pi_2^{gross,a}(\alpha_1, \alpha_2^e) - C_p = 0, \quad (10)$$

where $\pi_2^{gross,a}(\alpha_1, \alpha_2) = \alpha_2 \bar{\pi}_2(\alpha_1, \alpha_2)$ denotes deep information speculators' *gross* expected profit in equilibrium, for a given value of α_1 . Condition (10) is equivalent to:

$$\pi_2^{gross,a}(\alpha_1, \alpha_2^e) = C_p. \quad (11)$$

Thus, in an interior equilibrium, the equilibrium demand for deep information is such that the aggregate *gross* expected profit of deep information speculators is equal to the cost of producing deep information. In sum, a competitive equilibrium is a pair (F_2^e, α_2^e) that solves equations (8) and (11).

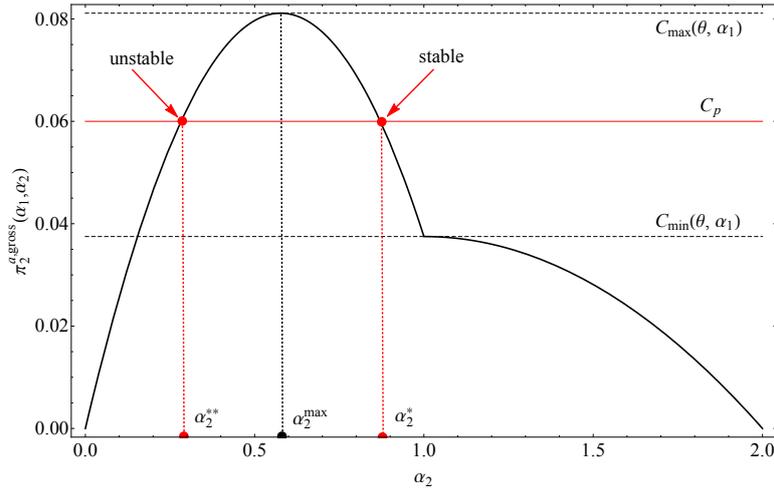


Figure 4: This figure represents deep information speculators' aggregate gross expected profit as a function of the demand for deep information, α_2 .

Deep information speculators' aggregate gross expected profit, $\pi_2^{gross,a}(\alpha_1, \alpha_2)$, is hump-shaped in the demand for deep information, α_2 , holding α_1 fixed (see Figure 4). We denote

by $\alpha_2^{max}(\alpha_1, \theta)$ the mass of deep information speculators that maximizes their aggregate gross expected trading profit. Using eq.(6), we obtain:

$$\alpha_2^{max}(\alpha_1, \theta) = \frac{(2 - \theta)(1 - (2\theta - 1)\alpha_1)}{2(1 + (2(2 - \theta)(1 - \theta) - 1)\alpha_1)}, \quad (12)$$

which is always less than 1. We deduce from eq.(6) that the maximum aggregate gross expected trading profits for deep information speculators, denoted $C_{max}(\theta, \alpha_1)$, is:

$$C_{max}(\theta, \alpha_1) \equiv \pi_2^{gross,a}(\alpha_1, \alpha_2^{max}) = \frac{\theta(1 - (2\theta - 1)\alpha_1)\alpha_2^{max}}{4}. \quad (13)$$

First, consider the case in which $C_p < C_{max}(\theta, \alpha_1)$, as assumed in Figure 4. For $\alpha_2 \in [\alpha_2^{max}, 2]$, deep information speculators' aggregate gross expected profit decreases in α_2 from $C_{max}(\theta, \alpha_1)$ to zero. Thus, there is a unique $\alpha_2^* \in (\alpha_2^{max}, 2)$ solving eq.(11) for $0 < C_p < C_{max}$. It follows that, $\alpha_2^e = \alpha_2^*$ and $F_2^e = C_p/\alpha_2^*$ is the unique interior equilibrium of the market for deep information for which α_2^e is on the decreasing portion of deep information speculators' aggregate gross expected profit. We call this equilibrium E^* .

In general, as Figure 4 shows, there is another value of α_2 , denoted α_2^{**} , solving eq.(11). This value is necessarily on the increasing segment of deep information speculators' aggregate gross expected profit (see Figure 4) since α_2^* is the unique solution on the decreasing segment, as we just explained. Thus, $\alpha_2^{**} < \alpha_2^{max} < \alpha_2^*$. The situation in which, $\alpha_2^e = \alpha_2^{**}$ and $F_2^e = C_p/\alpha_2^{**}$ is another possible interior equilibrium of the market for deep information. We call this equilibrium E^{**} .

Equilibrium E^{**} is unlikely to be obtained for two reasons. First, as $\alpha_2^{**} < \alpha_2^*$, the price of deep information is higher in this equilibrium. Thus, if information sellers could post their price before demand for information is realized, one information seller would have an incentive to undercut the price of information obtained in equilibrium E^{**} .¹⁹ Second, this equilibrium is unstable because α_2^{**} is on the increasing segment of the deep information speculators' aggregate gross expected profit.²⁰ Thus, when $C_p \leq C_{max}(\theta, \alpha_1)$,

¹⁹In our model, this deviation is not profitable because buyers and sellers of information make their decisions *simultaneously*. Thus, an information seller who undercuts the competitive price in equilibrium E^{**} still expects demand for information to be α_2^{**} and therefore undercutting is not optimal. In contrast, if a seller information can post a price before buyers make their decision then there are prices for deep information above the price in equilibrium E^* but below the price in equilibrium E^{**} that make both the deviating seller better off (because it attracts more demand and therefore better covers the fixed cost of producing information) and each individual buyer better off taking as given other buyers' actions (because the price is below the price obtained in equilibrium E^*). Thus, equilibrium E^{**} does not survive a slight change in the order of moves in the information market.

²⁰Following Manzano and Vives (2012), we say that an equilibrium of the market for shallow in-

the unique stable interior equilibrium of the market for deep information is equilibrium E^* , in which $\alpha_2^e = \alpha_2^*$ and $F_2^e = C_p/\alpha_2^*$. We therefore focus on this equilibrium when an interior equilibrium exists.

Now consider the case in which $C_p \geq C_{max}(\theta, \alpha_1)$. In this case, eq.(11) has no solution because, for any α_2 , deep information speculators' gross aggregate profit is smaller than C_p ((see Figure 4)). Thus, there is no price for deep information at which transactions between buyers and sellers of deep information are mutually profitable. Consequently, when $C_p \geq C_{max}(\theta, \alpha_1)$, the unique equilibrium of the market for deep information is a corner equilibrium in which $\alpha_2^e = 0$.²¹

The next proposition provides the closed form solution for the equilibrium demand for deep information, α_2^* , when $C_p \leq C_{max}(\theta, \alpha_1)$.

Lemma 1. Let $C_{min}(\theta, \alpha_1) = \frac{\theta(1-\theta)(1-\alpha_1)}{2(2-\theta)}$.

1. If $C_p < C_{max}(\theta, \alpha_1)$, the unique stable interior equilibrium of the market for deep information is as follows. The demand for deep information is:

$$\alpha_2^*(\theta, \alpha_1, C_p) = \begin{cases} \alpha_2^{max}(\theta, \alpha_1) \left(1 + \sqrt{1 - \frac{C_p}{C_{max}(\theta, \alpha_1)}}\right) & \text{if } C_{min}(\theta, \alpha_1) \leq C_p \leq C_{max}(\theta, \alpha_1), \\ 1 + \sqrt{1 - \frac{C_p}{C_{min}(\theta, \alpha_1)}} & \text{if } 0 \leq C_p < C_{min}(\theta, \alpha_1), \end{cases}$$

and the equilibrium price of deep information is $F_p^* = \frac{C_p}{\alpha_2^*}$.

2. If $C_p > C_{max}(\theta, \alpha_1)$, there is no equilibrium in which buyers and sellers of deep information can trade deep information at a mutually profitable price. Thus, the market for deep information does not exist in equilibrium ($\alpha_2^* = 0$).

Not surprisingly, as the cost of producing deep information declines (starting from C_{max}), the mass of traders buying this information increases (see Figure 4). When this cost falls below C_{min} , the mass of traders buying deep information becomes larger than 1, so that the expression for their expected profit changes (see eq.(6)). This explains why the expression for α_2^* varies depending on whether C_p is above or below the threshold C_{min} .

formation is stable if when one slightly perturbrates α_2 around α_2^e and, at this point, (i) reduces α_2 if $\pi_2^{gross,a}(\alpha_1, \alpha_2^e) < C_p$ (i.e., if buying deep information is strictly unprofitable in the aggregate) or (ii) increases α_2 if $\pi_2^{gross,a}(\alpha_1, \alpha_2^e) > C_p$ (i.e., if buying deep information is strictly profitable in the aggregate) then one is brought back to α_2^e . If this is not the case, the equilibrium is unstable.

²¹This corner equilibrium is stable and can also be an equilibrium when $C_p < C_{max}(\theta, \alpha_1)$. Indeed, if speculators expect the price of deep information to be infinite then no speculators buy deep information, which means that there is no finite price at which deep information sellers can break even. In this case, however, the market for deep information is viable in principle since there is a stable interior equilibrium. It is therefore natural to focus on the interior equilibrium.

The next proposition provides the equilibrium of the market for shallow information. As the derivation of this equilibrium is similar and simpler than that of the equilibrium of the market for deep information, we relegate the proof of Lemma 2 to the on-line appendix for brevity.

Lemma 2. *1. If $C_r < \frac{\theta}{8}$, the unique stable equilibrium of the market for shallow information and this equilibrium is as follows. The equilibrium mass of shallow information speculators is:*

$$\alpha_1^*(\theta, C_r) = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2C_r}{\theta}} \quad (14)$$

and the equilibrium price of deep information is $F_r^ = \frac{C_r}{\alpha_1^*}$.*

2. If $C_r \geq \frac{\theta}{8}$, there is no equilibrium in which buyers and sellers of shallow information can trade raw information at a mutually profitable price. Thus, the market for shallow information does not exist in equilibrium ($\alpha_1^ = 0$).*

Thus, in equilibrium, the mass of shallow information speculators, α_1^* , increases when the cost of producing shallow information decreases. This observation, combined with Corollary 3, implies that a decrease in the cost of producing shallow information, C_r , has also an effect on the equilibrium demand for deep information. The next corollary analyzes this effect. Let $\bar{C}_r(\theta) = \frac{\theta}{2} \left(\frac{1}{4} - \max \left(\frac{(1-\theta)^2 + \theta^2}{(1-2\theta)[2(1-\theta)(2-\theta)-1]} - \frac{1}{2}, 0 \right)^2 \right)$ and $\bar{C}_p(\theta) = \frac{\theta(1-\theta)^2(2-\theta)(1-2\theta)}{(2(1-\theta)(2-\theta)-1)^2}$.

Proposition 3. *1. For $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$, a decrease in the cost of shallow information reduces the equilibrium demand for deep information ($\frac{\partial \alpha_2^*}{\partial C_r} > 0$).*

2. For $\theta \leq \frac{\sqrt{2}-1}{\sqrt{2}}$, a decrease in the cost of shallow information increases the equilibrium demand for deep information (i.e., $\frac{\partial \alpha_2^}{\partial C_r} < 0$) if $C_r < \bar{C}_r(\theta)$ and $C_p > \bar{C}_p(\theta)$. Otherwise, a decrease in the cost of shallow information reduces the equilibrium demand for deep information.*

As shown in Corollary 1, an increase in the mass of shallow information speculators can exert a negative or a positive externality on deep information speculators. The externality is positive if and only if $\alpha_2^* < \hat{\alpha}_2(\theta)$ (see Corollary 1). In the proof of Proposition 3, we show that this condition is equivalent to $\theta \leq \frac{\sqrt{2}-1}{\sqrt{2}}$, $C_r < \bar{C}_r(\theta)$, and $C_p > \bar{C}_p(\theta)$. In this case, a decrease in the cost of shallow information triggers, directly, an increase in the

equilibrium demand for shallow information and thereby, *indirectly*, an increase in the value of deep information and therefore the demand for this type of information.

Otherwise (e.g., when $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$), an increase in the mass of shallow information speculators exerts a negative externality on deep information speculators. In this case, the value of deep information and therefore the demand for this information decline when the cost of shallow of information decreases.²² Figure 5 illustrates this point by showing the equilibrium demand for deep information for two different levels (high and low) of the cost of raw information.

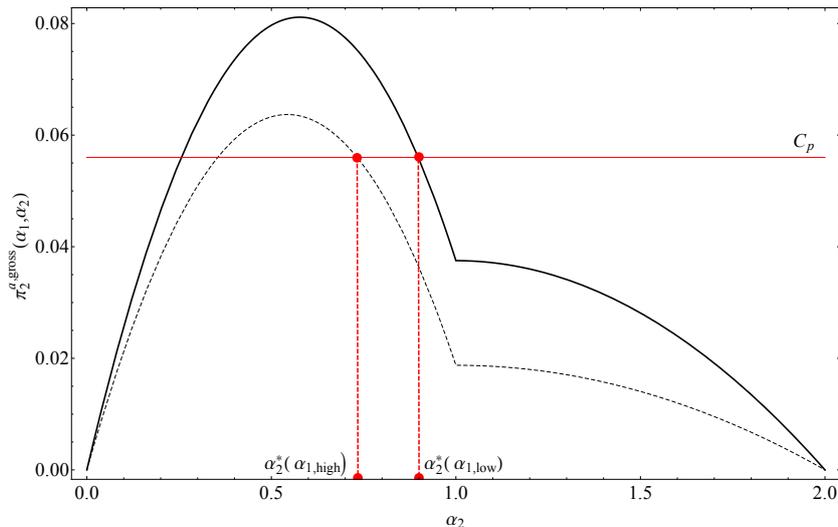


Figure 5: This figure represents deep information speculators' aggregate profit as a function of the demand for deep information for two different values of cost of raw information. The equilibrium demand for shallow information when this cost is high (resp. low) is denoted by $\alpha_{1,high}$ (resp., $\alpha_{1,low}$). The corresponding stable equilibria of the market for deep information in each case are, respectively, $\alpha_2^*(\alpha_{1,high})$ and $\alpha_2^*(\alpha_{1,low})$.

As the next corollary shows, this effect can lead to a complete breakdown in the market for deep information (a discontinuous drop to zero of the demand for shallow information), even though this market would exist in the absence of a market for shallow information.

Proposition 4. Suppose $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$ and $\frac{\theta(1-\theta)}{4} < C_p < \frac{\theta(2-\theta)}{8}$. There exists a threshold

²²Lee (2013) considers a static model of trading in which investors can buy information on one of two independent fundamentals (factors) for an asset. In his model, a decrease in the cost of acquiring information on one fundamental can increase or reduce the number of investors buying information on the other fundamental. Our Proposition 3 has a similar flavor. However, the mechanisms and information structures in Lee (2013) and our model are very different. In Lee (2013)'s model, traders informed on different fundamentals trade simultaneously and externalities arise from the fact that trades by one type of informed trader affects (negatively) liquidity for the other type and (positively) the ability of the other type to conceal his trades. In our model, these effects cannot play out because shallow and deep information speculators trade at different dates.

$\hat{C}_r(\theta, C_p)$ (defined in the proof of the proposition) such that if $C_r \geq \hat{C}_r$, $\alpha_2^* \geq \bar{\alpha}_2^{max} > 0$ while if $C_r < \hat{C}_r$, $\alpha_2^* = 0$.

Thus, the demand for deep information discontinuously drops to zero when C_r passes below $\hat{C}_r(\theta, C_p)$. Indeed, as the cost of shallow information declines, more speculators choose to trade on shallow information, which erodes deep information speculators' aggregate expected profits. When the cost of shallow information is just equal to the threshold \hat{C}_r , the largest possible value for deep information speculators' gross expected trading profit is just equal to the cost of producing deep information, C_p . At this point, any further decrease in the cost of shallow information implies that deep information speculators' aggregate gross expected trading profit is smaller than the cost of producing deep information. Thus, there is no price at which the sellers and buyers of deep information can find profitable to trade together. Hence, the market for deep information is not viable when $C_r < \hat{C}_r(\theta, C_p)$ and therefore ceases to exist.

Figure 6 illustrates this result. As the cost of shallow information C_r declines, the demand for shallow information increases (dotted line) while the demand for deep information declines (plain line). At $C_r = \hat{C}_r \approx 0.06$, the demand for shallow information discontinuously drops from $\alpha_2^* \approx 0.6$ to zero.

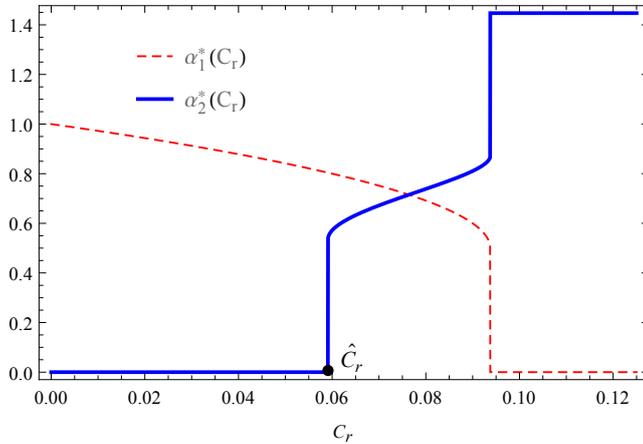


Figure 6: Demands for shallow information (red dotted line) and deep information (blue thick line) as a function of the cost of raw information C_r (X-axis), with $\theta = 0.75$ and $C_p = 0.06$

5 Implications

5.1 Price Informativeness

We now study how a change in the cost of shallow information affects price informativeness. In the absence of informed trading at dates 1 and 2 ($\alpha_1 = \alpha_2 = 0$), the asset price at each date is constant ($p_0 = p_1 = p_2 = 1/2$) and is therefore completely uninformative about the asset payoff. In this benchmark case, the average squared pricing error (the difference between the asset payoff and its price) is therefore $E[(\tilde{V} - p_0)^2] = 1/4$. We measure price informativeness at date t , denoted $\mathcal{E}_t(C_r, C_p)$ by the *difference* between the average pricing error in the benchmark case (completely uninformative prices) and the average pricing error at date t in equilibrium, i.e., by:

$$\mathcal{E}_t(C_r, C_p) = \frac{1}{4} - E[(\tilde{V} - p_t^*)^2] \quad (15)$$

The more informative is the price at date t in equilibrium, the larger is $\mathcal{E}_t(C_r, C_p)$. The *largest* possible value for $\mathcal{E}_t(C_r, C_p)$ is obtained if the price at date t is fully informative ($p_t = V$) and is therefore equal to $1/4$. The smallest possible value is equal to zero and is obtained when the price at date t is uninformative.²³ Thus, $\mathcal{E}_t(C_r, C_p)$ belongs to $[0, 1/4]$.

It is natural to interpret $\mathcal{E}_1(C_r, C_p)$ as a measure of price informativeness in the short run and $\mathcal{E}_2(C_r, C_p)$ as a measure of price informativeness in the long run since the latter measures the informational content of the stock market *after* information processing. Thus, it corresponds to a low frequency measure of price informativeness. This is the frequency that matters for decision makers (e.g., managers) when they rely on stock prices for their decisions.²⁴

Intuitively, long run price informativeness is at least equal to short run price informativeness because the market maker has at least as much information at date 1 than he has at date 2 ($\Omega_1 \subset \Omega_2$). It is strictly higher (i.e., $\mathcal{E}_2(C_r, C_p) > \mathcal{E}_1(C_r, C_p)$) when $\alpha_2^* > 0$ because trades at date 2 contain new information if some speculators trade on deep information. Otherwise, if $\alpha_2^* = 0$, long run price informativeness is equal to short run price informativeness because $p_2^* = p_1^*$ with certainty.

We now study in detail the effects of a change in the costs of producing information

²³As $p_t = E[V|\Omega_t]$, we have $E[(\tilde{V} - p_t^*)^2] = E[Var[V|\Omega_t]]$. Thus, $\mathcal{E}_t(C_r, C_p) = Var(V) - E[Var[V|\Omega_t]]$. Hence, price informativeness at date t is higher when, on average, the price at this date provides a more accurate signal of the asset payoff.

²⁴See Bond, Edmans, and Goldstein (2012) for a review of the literature linking real decisions by firms (e.g., investment) to the informativeness of stock prices.

on price informativeness. The following corollary studies how a change in the cost of deep information (C_p) affects price informativeness in equilibrium (i.e., accounting for the effects of a change in this cost on the prices and demands of deep and shallow information).

Corollary 2. *A reduction in the cost of deep information has no effect on short run price informativeness ($\frac{\partial \mathcal{E}_1(C_r, C_p)}{\partial C_p} = 0$) while it (weakly) increases long run price informativeness ($\frac{\partial \mathcal{E}_2(C_r, C_p)}{\partial C_p} \leq 0$).*

A decrease in the cost of deep information raises the demand for deep information in equilibrium and therefore leads to more informative prices at date 2. This effect is standard in models with endogenous information acquisition (e.g., Grossman and Stiglitz (1980)): when the cost of information production falls, the demand for information increases and prices become more informative.

More surprisingly and novel, our next proposition shows that this logic does not necessarily apply when one considers a decline in the cost of shallow information (C_r). Indeed, even though a decline in this cost improves price informativeness in the short run, it can *impair* long run price informativeness.

Proposition 5. *A reduction in the cost of shallow information (weakly) increases short run price informativeness ($\frac{\partial \mathcal{E}_1(C_r, C_p)}{\partial C_r} \leq 0$). However, its effect on long run price informativeness is ambiguous. Specifically, suppose that (i) $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$ or (ii) $\theta \leq \frac{\sqrt{2}-1}{\sqrt{2}}$ and $C_r > \bar{C}_r(\theta)$, or (iii) $\theta \leq \frac{\sqrt{2}-1}{\sqrt{2}}$ and $C_p < \bar{C}_p(\theta)$ so that a reduction in C_r leads to a decrease in the demand for deep information. In this case, a reduction in the cost of deep information:*

1. *Reduces price informativeness when $C_p \leq C_{min}(\theta, \alpha_1^*)$.*
2. *Reduces price informativeness when $C_{min}(\theta, \alpha_1^*) \leq C_p \leq C_{max}(\theta, \alpha_1^*)$, if and only if $\Upsilon(\alpha_1^*, \alpha_2^*, \theta, C_p) > 0$.*
3. *Increases price informativeness when $C_p \geq C_{max}(\theta, \alpha_1^*)$,*

where $\Upsilon(\alpha_1^*, \alpha_2^*, \theta, C_p)$ is a function defined in the appendix.

Short run price informativeness increases when the cost of shallow information falls because it leads more speculators to buy shallow information. As shallow information is not always noise ($\theta > 0$, as otherwise no investor buys shallow information), the increase in the mass of shallow information speculators makes the asset price more informative

at date 1. However, when $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$ or (ii) $\theta \leq \frac{\sqrt{2}-1}{\sqrt{2}}$ and $C_r > \bar{C}_r(\theta)$, or (iii) $\theta \leq \frac{\sqrt{2}-1}{\sqrt{2}}$ and $C_p < \bar{C}_p(\theta)$, this effect triggers a drop in the demand for deep information because it reduces trading profits on this type of information (see Corollary 3 and Figure 6). This indirect effect of a reduction in the cost of shallow information tends to decrease long run price informativeness.

This second effect dominates for a large range of parameter values. That is, in many cases, a reduction in the cost of shallow information leads to a drop in long run price informativeness (a drop in $\mathcal{E}_2(C_r, C_p)$), despite its positive effect on short run price informativeness. Figure 7 illustrates this finding by showing the evolution of $\mathcal{E}_1(C_r, C_p)$ and $\mathcal{E}_2(C_r, C_p)$ as a function of the cost of raw information, C_r , for specific parameter values.

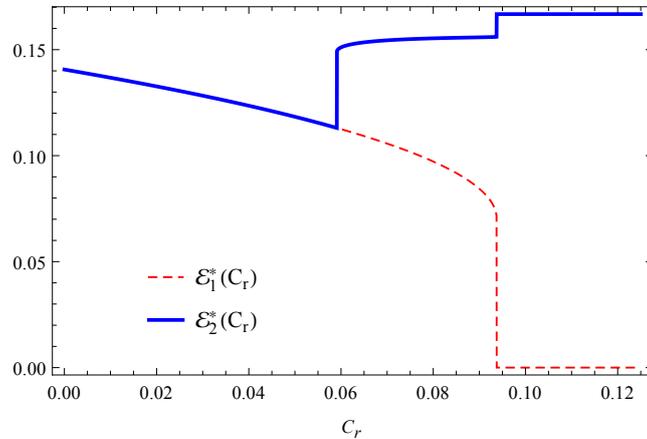


Figure 7: Price informativeness at date $t = 1$ (red dotted line) and date $t = 2$ (blue thick line) as a function of the cost of raw information C_r (X-axis), with $\theta = 0.75$ and $C_p = 0.06$

When the cost of raw information, C_r , is large, there is no trading on raw information ($\alpha_1^* = 0$) and the demand for deep information is strong ($\alpha_2^* \geq 1$ in our example because C_p is less than $C_{min}(0)$). As the cost of raw information declines, the demand for shallow information starts increasing and consequently the demand for deep information, α_2^* , drops (see Figure 6). Short run price informativeness increases but long run price informativeness drops (see Figure 7). When $C_r = \hat{C}_r \simeq 0.06$, deep information speculators' gross aggregate expected profit is just equal to the cost of producing information. At this point, if C_r decreases further, the demand for deep information discontinuously drops to zero (as implied by Corollary 4) and long run price informativeness drops discontinuously as well. Long run price informativeness is then just equal to short run price informativeness. As C_r keeps declining, the demand for shallow information increases. Hence, short run price informativeness improves and long run price informativeness does

as well because both short run and long run price informativeness are now equal. However, even if $C_r = 0$ (i.e., $\alpha_1^* = 1$), price informativeness at date 2 is smaller than when the cost of shallow information is so high ($C_r \geq \frac{\theta}{8}$) that no investor trades on it ($\alpha_1^* = 0$). Specifically, $\mathcal{E}_2(0, C_p) = 0.14$ while $\mathcal{E}_2(C_r, C_p) = 0.17$ for $C_r \geq \frac{\theta}{8}$ in the numerical example considered in Figure 7. The next proposition shows that this conclusion holds more generally.

Proposition 6. *When $0 < C_p \leq \frac{\theta(2-\theta)(1-(2\theta-1)^2)}{8}$, long run price informativeness is always smaller when shallow information is free ($C_r = 0$) than when it is so costly that no investor buys it in equilibrium ($C_r > \frac{\theta}{8}$).*

Arguably, progress in information technologies have reduced *both* the cost of access to information and the cost of processing information. However, as Proposition 6 shows, this evolution does not imply that long run price informativeness should improve. In fact, for any level of the cost of processing information, if $\theta < 1$, there is a level of the cost of raw information that is small enough such that long run price informativeness is lower than when there is no demand for shallow information.²⁵

Parts 1 and 3 of Proposition 5 hold for all parameter values. Part 2 requires the conditions on parameters that are stated in the proposition. When these conditions do not hold then a reduction in the cost of shallow information improves long run price informativeness because an increase in the mass of shallow information speculators exerts a positive externality on deep information speculators. That is, it raises the value of deep information and thereby leads to an increase in the demand for this information (see Corollary 3).

The arrival of public news in financial markets (e.g., earnings announcements) can present trading opportunities for speculators because news often require additional interpretations (see, for instance, Engelberg et al.(2012) for a discussion of the literature on this possibility and supporting evidence). One way to test our predictions is therefore to use news arrival in a stock as a proxy for date $t = 0$ in the model (p_0 is then the price of the stock just after news arrival). Subsequent to the news arrival, speculators get additional information privately that helps them to refine their interpretation of the news. Using this interpretation of the timing of events in our model, one could test its implications for price informativeness (and other implications developed in the next section) by considering the effect of change in the cost of raw information on the informativeness of

²⁵This follows from Proposition 6 and the continuity of $\mathcal{E}_2(C_r, C_p)$ in C_r for C_r sufficiently close to zero. The condition $\theta < 1$ is required because for $\theta = 1$, the condition on C_p in Proposition 6 can never be satisfied.

prices about, say, future earnings (a proxy for V) at various dates after news arrival. The model predicts that a decline in the cost of raw information should make prices shortly after news arrival (say, one day; $t = 1$ in the model) more informative and those at dates further away from the news (say, one week; $t = 2$ in the model) less informative.

One problem with this approach is that one must take a stand on what are short and long run prices. One way to circumvent this empirical issue is to measure the effect of a reduction in the cost of raw information on the *average* price observed over some period of time, after the arrival of news (date 0). For instance, consider the average price over periods 1 and 2 in our model: $\bar{p}^* = \frac{p_1^* + p_2^*}{2}$. The informativeness of the average price is measured by:

$$\mathcal{E}^{average}(C_r, C_p) = \frac{1}{4} - E[(\tilde{V} - \bar{p}^*)^2] \quad (16)$$

Using the fact that $(V - p_2)$ is orthogonal to $p_2 - p_1$, we obtain after some algebra that:²⁶

$$\mathcal{E}^{average}(C_r, C_p) = 0.75 \times \mathcal{E}_2(C_r, C_p) + 0.25 \times \mathcal{E}_1(C_r, C_p). \quad (17)$$

Thus, as one could expect, the informativeness of the average price over periods 1 and 2 is a weighted average of long run price informativeness and short run price informativeness. Long run price informativeness counts relatively more than short run price informativeness since it receives a weight of 75%.

A decline in the cost of raw information improves short run price informativeness but it can reduce long run price informativeness. Hence, eq.(17) implies that this decline should have a non monotonic effect on the informativeness of the average price over a given period of time. Specifically, numerical simulations show that as C_r declines, the informativeness of the average price first increases (the positive effect on short run price informativeness dominates the negative effect on long run price informativeness) and then decreases (the negative effect on long run price informativeness dominates). Figure 8 illustrates this pattern for specific parameter values (the informativeness of the average price is given by the green dotted line). It also shows that the informativeness of the average price when $C_r = 0$ is strictly smaller than the informativeness of the average price when C_r is so large that no speculator chooses to buy raw information.

²⁶Indeed, we have $E[(\tilde{V} - \bar{p}^*)^2] = E[(\tilde{V} - p_2^*)^2] + 0.25E[(p_2^* - p_1^*)^2]$ because $(V - p_2^*)$ is orthogonal to $p_2^* - p_1^*$. Moreover, for the same reason, we also have $E[(\tilde{V} - p_1^*)^2] = E[(V - p_2^*)^2] + E[(p_2^* - p_1^*)^2]$. Hence, $E[(p_2^* - p_1^*)^2] = E[(\tilde{V} - p_1^*)^2] - E[(V - p_2^*)^2]$. We deduce that $E[(\tilde{V} - \bar{p}^*)^2] = 0.75E[(\tilde{V} - p_2^*)^2] + 0.25E[(V - p_1^*)^2]$, which implies that eq.(17). holds true.

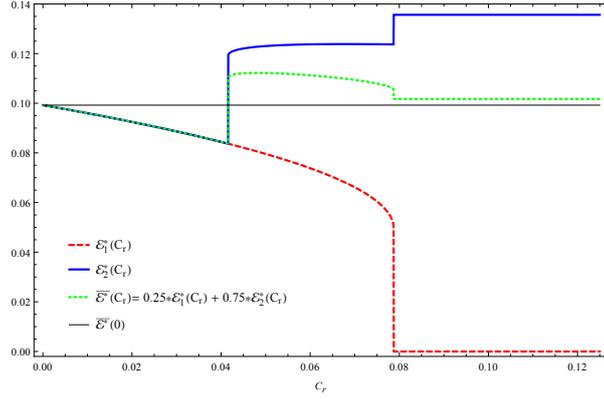


Figure 8: Price informativeness at date $t = 1$ (red dotted line), at date $t = 2$ (blue thick line), and Informativeness of the average price (green dotted line) as a function of the cost of raw information C_r (X-axis), with $\theta = 0.63$ and $C_p = 0.065$

5.2 Price and Trade Patterns

In this section, we analyze in more detail the return and trade patterns induced by speculators' equilibrium behavior. Our goal is to derive the predictions of our model for the effects of a decrease in the cost of shallow information on the relationships between (i) the trades of shallow and deep information speculators, (ii) past returns and the trades of deep information speculators, (iii) future returns and the trades of shallow information speculators. These predictions could be tested with data on trades by each type of speculators. For instance, discretionary long-short equity hedge funds rely on fundamental analysis of stocks to decide whether to buy or sell them while other hedge funds (or trading desks within these funds) specialize in trading on very high frequency signals (see Pedersen (2015), Chapters 7 and 9). The former are deep information speculators while the latter are shallow information speculators according to our terminology.

To focus on the interesting case, we assume that $C_r < \frac{\theta}{8}$ so that some speculators buy shallow information in equilibrium ($\alpha_1^* > 0$). Moreover, we assume that $C_p < C_{max}(\theta, \alpha_1^*(\theta, C_r))$ so that some speculators buy deep information ($\alpha_2^* > 0$). If one of these two conditions are not satisfied then one type of speculators does not trade ($x_1 = 0$ or $x_2 = 0$) and all covariances below are just equal to zero.

Corollary 3. *In equilibrium, the covariance between the trades of deep and shallow information speculators is:*

$$Cov(x_1, x_2) = \theta - (1 - \theta)\alpha_1^*(\theta, C_r),$$

This covariance declines when the cost of shallow information declines and becomes negative if $\theta < \frac{1}{2}$ and $C_r < \frac{\theta^2(2\theta-1)}{2(1-\theta)}$.

Figure 9, Panel A, illustrates Corollary 3. It shows the covariance between trades of shallow and deep information speculators against the reliability of shallow information, θ , for various values of the cost of producing shallow information. As explained previously, this covariance is zero when this cost is so large relative to the reliability of information, θ , that no speculator buys shallow information ($C_r > \frac{\theta}{8}$) or so small that no speculator buys deep information ($C_p > C_{max}(\theta, \alpha_1^*(\theta, C_r))$), which happens for θ large enough, holding C_r constant. For intermediate values of θ , the covariance increases with θ and can be positive or negative. Moreover, holding θ fixed, it decreases as the cost of raw information declines.

The intuition for Corollary 3 is as follows. Deep information speculators trade in the same direction as shallow information speculators when $u = 1$, i.e., when shallow information speculators do receive valid signals. They trade in opposite direction if $u = 0$ (shallow information speculators' signal is just noise) and the price at date 1 moves in reaction to the trades at this date. Holding θ constant, the probability of the latter event is low when the cost of shallow information is large because shallow information speculators are too few to move prices. Hence, for sufficiently high values of C_r , shallow and deep information speculators often trade in the same direction and therefore $Cov(x_1, x_2) > 0$. As the cost of shallow information declines, the likelihood that shallow information speculators move prices is higher because more speculators trade on shallow information. This effect raises the likelihood that deep information speculators trade in a direction opposite to that of shallow information speculators. For this reason, as the cost of shallow information declines, the covariance between the trades of shallow and deep information speculators becomes weaker and can even become negative if shallow information speculators' signal is sufficiently unreliable (i.e., if $\theta < 1/2$).

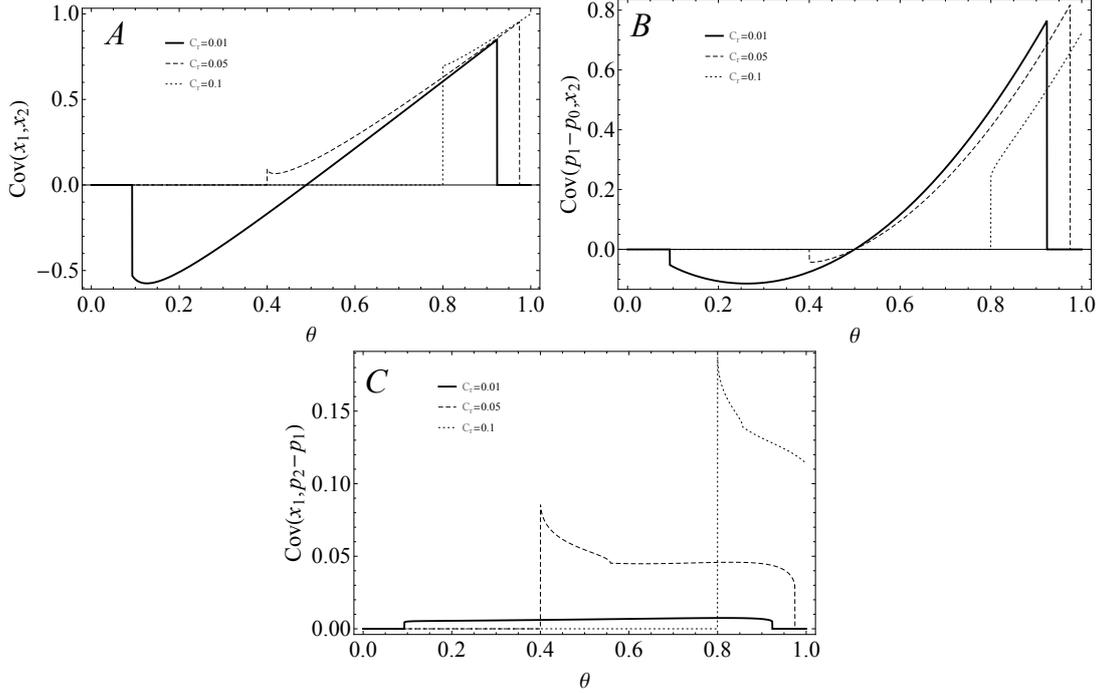


Figure 9: Panel A shows the covariance between shallow and deep information speculators' trades ($\text{Cov}(x_1, x_2)$) as a function of θ . Panel B shows the covariance between the return at date 1 and speculators' trades at date 2 as a function of θ . Panel C shows the covariance between speculators' trades at date 1 and the return at date 2 as a function of θ . In each case, various values of C_r are considered: $C_r = 0.1$ (dotted lines), $C_r = 0.05$ (dashed lines), $C_r = 0.01$ (thick lines). In all cases $C_p = 0.02$.

We define the return of the asset from date t to date $t + 1$ as $r_{t+1} = p_{t+1} - p_t$.

Corollary 4. *In equilibrium, the covariance between the first period return ($r_1 = p_1 - p_0$) and the trade of a deep information speculator is:*

$$\text{Cov}(r_1, x_2) = \theta(2\theta - 1)\alpha_1^*.$$

Hence, deep information speculators' orders are negatively correlated with the first period return if and only if $\theta < \frac{1}{2}$. Furthermore, a decline in the cost of shallow information, C_r , raises the absolute value of the covariance between deep information speculators' trade and the first period return.

Figure 9 (Panel B) illustrates this result. Conditional on a price change at date 1, the likelihood that deep information speculators trade against this change increases with the likelihood, $(1 - \theta)$, that shallow information speculators trade on noise. This explains why, for $\theta < \frac{1}{2}$, $\text{Cov}(r_1, x_2) < 0$. Thus, deep information speculators behave like *momentum traders* when $\theta > \frac{1}{2}$ (the direction of their trades is positively related to the

lagged return) and *contrarian traders* (the direction of their trades is negatively related to lagged return) when $\theta < \frac{1}{2}$. Moreover, holding θ constant, the relationship between past returns and deep information speculators' trades becomes stronger when the cost of shallow information declines. The reason is that this decline triggers an increase in the demand for shallow information and therefore the likelihood that shallow information speculators' trades will affect prices at date 1.

Corollary 5. *In equilibrium, the covariance between shallow information speculators' trades and the second period return, $r_2 = p_2 - p_1$, is positive and equal to:*

$$Cov(x_1, r_2) = \begin{cases} \frac{\theta(1-\alpha_1^*)\alpha_2^*}{2(2-\theta)}, & \text{when } C_{min}(\theta, \alpha_1^*) \leq C_p \leq C_{max}(\theta, \alpha_1^*), \\ \frac{\theta(1-\alpha_1^*)(1-(1-\theta)(1-\alpha_2^*))}{2(2-\theta)}, & \text{when } C_p \leq C_{min}(\theta, \alpha_1^*). \end{cases} \quad (18)$$

This covariance decreases when the cost of shallow information declines if (i) $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$ or (ii) $\theta \leq \frac{\sqrt{2}-1}{\sqrt{2}}$ and $C_r \geq \bar{C}_r(\theta)$, or (iii) $\theta \leq \frac{\sqrt{2}-1}{\sqrt{2}}$ and $C_p \leq \bar{C}_p(\theta)$. In contrast, it always increases when the cost of deep information declines.

When the cost of raw information declines, the mass of shallow information speculators increases and it becomes increasingly likely that the price at the end of the first period reveals s . This implies that in the second period, deep information speculators can only trade on the component of their signal that is uncorrelated with shallow information speculators (i.e., the innovation in the expectation of the asset payoff due to the observation of u). This effect lowers the covariance between shallow information speculators' trade and the second period return (see Figure 9, Panel C) because the latter is increasingly determined by factors independent from shallow information speculators' signal (the realization of u and liquidity traders' orders in the second period).

In contrast, a decline in the cost of processing information has the opposite effect: it strengthens the covariance between shallow information speculators' trade and the second period return. The reason is that this decrease raises the mass of speculators trading on deep information and thereby the likelihood that the return in the second period reveals their information. As this information is correlated with shallow information speculators' signal s , the predictive power of shallow information speculators' trade for future returns increases.

6 Conclusion

In this paper, we show that the reduction in the cost of accessing data for investors can impair asset price informativeness. In our theory, investors can acquire either raw information or processed information about the payoff of a risky asset. Information processing filters out the noise in raw information but it takes time. Hence, investors buying processed information trade with a lag relative to investors buying raw information. As the cost of raw information declines, more investors trade on it, which reduces the value of processed information, unless raw information is very unreliable. Thus, a decline in the cost of raw information can reduce the demand for deep information and for this reason the informativeness of the asset price in the long run (i.e., after information is processed). The model also predicts that this decline should affect correlations between (i) trades of speculators buying processed and raw information, (ii) trades of speculators buying processed information and past returns, and (iii) trades of speculators buying raw information and future returns.

Future research could test these implications by considering technological changes that reduce the cost of access to raw information. We believe that recent improvements in technologies to disseminate information in digital form offer many opportunities in this respect. Another interesting issue is to analyze how reductions in the cost of raw information affect firms' real decisions. If firms rely on stock prices for their real decisions (e.g., investment), a drop in long run price informativeness might ultimately make their investment decisions less sensitive to stock prices and possibly less efficient. Last, firms themselves are often the source of raw information on which investors trade (e.g., by disclosing more or less information on their websites). According to our theory, the amount of raw information provided by firms is not innocuous as it ultimately affects incentives for the production of processed information. Studying optimal release of raw information by firms in this context is another interesting venue for future research.

Appendix A

Proof of Proposition 1.

Step 1: Stock Price at date 1. The equilibrium price at date 1 satisfies (see eq.(3)):

$$p_1^*(f_1) = Pr[V = 1 | \tilde{f}_1 = f_1] = \frac{Pr[\tilde{f}_1 = f_1 | V = 1]Pr[V = 1]}{Pr[\tilde{f}_1 = f_1]}. \quad (19)$$

Shallow information speculators buy the asset at date 1 when they observe $s = 1$. Hence, conditional on $V = 1$, aggregate speculators' demand is α_1 with probability $(1 + \theta)/2$ and $-\alpha_1$ with probability $(1 - \theta)/2$. Thus:

$$Pr[\tilde{f}_1 = f_1 | V = 1] = \left(\frac{1 + \theta}{2}\right)\phi(f_1 - \alpha_1) + \frac{1 - \theta}{2}\phi(f_1 + \alpha_1). \quad (20)$$

Furthermore, by symmetry:

$$Pr[\tilde{f}_1 = f_1] = \frac{1}{2}\phi(f_1 - \alpha_1) + \frac{1}{2}\phi(f_1 + \alpha_1). \quad (21)$$

Substituting (20) and (21) in (19) and using the fact that $Pr[V = 1] = 1/2$, we obtain (4).

Step 2: Shallow information speculators' strategies. For a given trade x_1 , a shallow information speculator's expected profit when he observes signal s is:

$$\pi_1(\alpha, s) = x_1(\mu(s) - E[p_1 | s]).$$

As $p_1^*(f_1) = E[V | \tilde{f}_1]$ and the market-maker's information set at date 1 is coarser than speculators' information set, we have:

$$\mu(0) \leq p_1^* \leq \mu(1),$$

with a strict inequality when $f_1 \in [-1 + \alpha_1, 1 - \alpha_1]$ because in this case the order flow at date 1 contains no information (all realizations of the order flow in this interval are equally likely conditional on $V = 0$ or $V = 1$). Therefore:

$$\mu(0) < E[p_1^* | s] < \mu(1),$$

when $\alpha_1 < 1$. Thus, in this case, it is a strictly dominant strategy for a speculator to buy the asset when $s = 1$ and sell the asset when $s = 0$. It follows that the equilibrium at date 1 is unique when $\alpha_1 < 1$. When $\alpha_1 \geq 1$, $[-1 + \alpha_1, 1 - \alpha_1]$ is an empty set and $p_1^*(f_1) = \mu(s)$ for all values of f_1 . Hence, a speculator obtains a zero expected profit for all x_1 whether $s = 1$ or $s = 0$. Buying the asset when $s = 1$ and selling the asset when $s = 0$ is then weakly dominant.

Step 3: Shallow information speculators' expected profit. Suppose that $s = 1$, so that shallow information speculators' valuation for the asset is $\mu(1)$. Given their equilibrium strategy, shallow information speculators' aggregate demand is then α_1 . Thus, the aggregate demand for the asset at date 1 is above the threshold $-1 + \alpha_1$. Accordingly, the price at date 1 is either $1/2$ if $f_1 \in [-1 + \alpha_1, 1 - \alpha_1]$ or $\mu(1)$ if $f_1 \geq 1 - \alpha_1$. In the former case, shallow information speculators earn a zero expected profit while in the later case, their expected profit is $\mu(1) - 1/2 = \theta/2$. Now we have:

$$Prob(f_1 \in [-1 + \alpha_1, 1 - \alpha_1] | s = 1) = Prob(l_1 \in [-1, 1 - 2\alpha_1]) = Max\{1 - \alpha_1, 0\}.$$

Thus, conditional on $s = 1$, shallow information speculators' expected profit is $\frac{\theta}{2}Max\{1 - \alpha_1, 0\}$. By symmetry, this is also the case when $s = -1$. Thus, $\bar{\pi}_1(\alpha_1) = \frac{\theta}{2}Max\{1 - \alpha_1, 0\}$.

Proof of Proposition 2.

Step 1. Stock Price at date 2. We first derive the equilibrium stock price when speculators behave as described in part 1 of Proposition 2.

Case 1. Suppose first that $p_1 = \mu(1)$. In this case, the market maker knows that $s = 1$. Hence, the remaining uncertainty is about u . If $u = 1$, deep information speculators buy the asset at date 2 and, therefore, the total demand for the asset belongs to $[-1 + \alpha_2, f_2^{max}]$. If $u = 0$, deep information speculators sell the asset since $p_1 > 1/2$ and therefore the total demand for the asset belongs to $[f_2^{min}, 1 - \alpha_2]$. For $\alpha_2 \leq 1$, we have $1 - \alpha_2 > -1 + \alpha_2$. Thus, if $f_2 \in [f_2^{min}, -1 + \alpha_2]$, market makers infer that $u = 0$ and set $p_2^* = E(V | s = 1, u = 0) = 1/2$. Symmetrically if $f_2 \in [1 - \alpha_2, f_2^{max}]$, they infer that $u = 1$ and they set $p_2^* = E(V | s = 1, u = 1) = 1$. Intermediate realizations of f_2 (those in $[-1 + \alpha_2, 1 - \alpha_2]$) are equally likely when $u = 1$ or when $u = 0$. Thus, they convey no information on u . Hence, for these realizations: $p_2^* = E(V | s = 1) = \mu(1)$. For $\alpha_2 > 1$, the reasoning is unchanged but the intermediate case never occurs. This yields Part 2

of the proposition.

Case 2. When $p_1 = \mu(0)$, the reasoning is symmetric to that followed when $p_1 = \mu(0)$ (Case 1). Part 3 of the proposition follows.

Case 3. Now consider the case in which $p_1 = 1/2$. In this case, the market outcome at date 1 conveys no information to the market maker. Thus, from his viewpoint, there are three possible states at date 2: $\{u = 1, s = 1\}$, $\{u = 0\}$, and $\{u = 1, s = -1\}$. Given deep information speculators' trading strategy, the corresponding total demand for the asset at date 2 has the following support: $[-1 + \alpha_2, f_2^{max}]$ if $\{u = 1, s = 1\}$, $[-1, 1]$ if $\{u = 0\}$, and $[f_2^{min}, 1 - \alpha_2]$ if $\{u = 1, s = -1\}$.

Thus, if $f_2 > 1$, the market maker infers that $\{u = 1, s = 1\}$ and if $f_2 < -1$, he infers that $\{u = 1, s = -1\}$. Hence, in the first case $p_2^* = 1$ and in the second case $p_2^* = 0$. Now, consider intermediate realizations for f_2 , i.e., $f_2 \in [-1, 1]$. First, suppose $f_2 \in [-1, \text{Min}\{-1 + \alpha_2, 1 - \alpha_2\}]$. Such a realization is possible only if $u = 0$ or if $\{u = 1, s = 0\}$. Thus, in this case:

$$p_2^* = \text{Pr}[u = 0 | f_2 \in [-1, \text{Min}\{-1 + \alpha_2, 1 - \alpha_2\}]] \times \frac{1}{2}.$$

Now,

$$\text{Pr}[u = 0 | f_2 \in [-1, \text{Min}\{-1 + \alpha_2, 1 - \alpha_2\}]] = \frac{\text{Pr}[f_2 \in [-1, \text{Min}\{-1 + \alpha_2, 1 - \alpha_2\}] | u = 0](1 - \theta)}{\text{Pr}[f_2 \in [-1, \text{Min}\{-1 + \alpha_2, 1 - \alpha_2\}]]},$$

that is

$$\text{Pr}[u = 0 | f_2 \in [-1, \text{Min}\{-1 + \alpha_2, 1 - \alpha_2\}]] = \frac{2(1 - \theta)}{2 - \theta}.$$

Thus, for $f_2 \in [-1, \text{Min}\{-1 + \alpha_2, 1 - \alpha_2\}]$, $p_2^* = \frac{(1 - \theta)}{2 - \theta}$. The case, in which $f_2 \in [\text{Max}\{-1 + \alpha_2, 1 - \alpha_2\}, 1]$ is symmetric: such a realization of the order flow is possible only if $u = 0$ or if $\{u = 1, s = 1\}$. Thus, in this case,

$$p_2^* = \text{Pr}[u = 1, s = 1 | f_2 \in [\text{Max}\{-1 + \alpha_2, 1 - \alpha_2\}, 1]] + \text{Pr}[u = 0 | f_2 \in [\text{Max}\{-1 + \alpha_2, 1 - \alpha_2\}, 1]] \frac{1}{2}. \quad (22)$$

Using the fact that deep information speculators buy if $\{u = 1, s = 1\}$ and stay put if $u = 0$ (since we are in the case in which $p_1^* = 1/2$), we deduce from eq.(22):

$$p_2^* = \frac{1}{2 - \theta}.$$

Finally, realizations of $f_2 \in [Min\{-1 + \alpha_2, 1 - \alpha_2\}, Max\{-1 + \alpha_2, 1 - \alpha_2\}]$ are equally likely in each possible state when $p_1 = 1/2$. Thus, observations of f_2 in this range are uninformative and the equilibrium price in this case is $p_2^* = 1/2$. This achieves the proof of Part 4 of the proposition.

Step 2. Deep information speculators' Trading strategies. Let $\mu(u, s)$ be the expected payoff of the asset conditional on u and s . This is deep information speculators' valuation of the asset at date 2. Suppose $p_1^* = \mu(1)$ first. In this case $s = 1$ and deep information speculators' valuation for the asset is either $\mu(1, 1) = 1$ or $\mu(0, 1) = 1/2$. Moreover, in this case, the equilibrium price of the asset at date 2 is such that:

$$\mu(0, 1) \leq p_2^* \leq \mu(1, 1),$$

with a strict inequality when $f_2 \in [-1 + \alpha_2, 1]$. This interval is never empty for $\alpha_2 \leq 2$. Thus, we can proceed exactly as in the proof of Proposition 1 to show that it is a dominant strategy for shallow information speculators to (i) buy the asset if their expectation of the value of the asset is $\mu(1, 1)$ and $p_1 = \mu(1)$ and (ii) sell the asset if their expectation of the value of the asset is $\mu(0, 1)$ and $p_1 = \mu(1)$. A similar reasoning implies that it is a dominant strategy for or shallow information speculators to (i) sell the asset if their expectation of the value of the asset is $\mu(1, 0)$ and $p_1 = \mu(0)$ and (ii) buy the asset if their expectation of the value of the asset is $\mu(0, 1)$ and $p_1 = \mu(0)$.

Now consider the case in which $p_1 = 1/2$ and $u = 1$. In this case, we have:

$$\mu(1, 0) \leq p_2^* \leq \mu(1, 1),$$

with a strict inequality for some realizations of f_2 . Thus, again, we conclude that it is a dominant strategy for shallow information speculators to (i) sell the asset if their expectation of the value of the asset is $\mu(1, 0)$ and $p_1 = 1/2$ and (ii) buy the asset if their expectation of the value of the asset is $\mu(1, 1)$ and $p_1 = 1/2$.

The remaining case is the case in which $p_1 = 1/2$ and $u = 0$. In this case, a deep information speculator expects other deep information speculators to stay put in equilibrium. Suppose that one deep information speculator deviates from this strategy by trading x_2 shares in $[-1, 1]$. His

effect on aggregate demand is infinitesimal. Hence, he expects $f_2 = l_2$ and therefore he expect f_2 to be uniformly distributed on $[-1, 1]$. Therefore, using the expression for p_2^* when $p_1 = 1/2$, the speculator expects to trade at:

$$E(p_2^* | p_1 = 1/2, f_2 \in [-1, 1]) = 1/2 - \frac{\theta(\text{Min}\{-1 + \alpha_2, 1 - \alpha_2\} + \text{Max}\{-1 + \alpha_2, 1 - \alpha_2\})}{4(2 - \theta)} = 1/2.$$

As the speculator expects the payoff to be $\mu(0, 0) = 1/2$, his expected profit is therefore $x_2(\mu(0, 0) - E(p_2^* | p_1 = 1/2, f_2 \in [-1, 1])) = 0$. Thus, the deviation yields a zero expected profit and therefore not trading is weakly dominant for the speculator when $p_1^* = 1/2$ and $u = 0$.

In sum we have shown that the trading strategy described in Part 1 of Proposition 2 is optimal for a deep information speculator, if he expects other traders to follow this strategy and if prices at date 2 are given as in Parts 2, 3, and 4 of Proposition 2.

Step 3. Deep information speculators' expected profit. Case 1: $p_1 = \mu(1)$. In this case a deep information speculator buys the asset if $u = 1$ and sells the asset if $u = 0$. He then makes a profit if and only if $p_2 = p_1 = \mu(1)$, i.e., if $f_2 \in [-1 + \alpha_2, 1 - \alpha_2]$. The likelihood of this event is $\text{Max}\{1 - \alpha_2, 0\}$ whether $u = 1$ or $u = 0$. Thus, the expected profit of a deep information speculator if $p_1 = \mu(1)$ is $\text{Max}\{1 - \alpha_2, 0\}(\theta \times (1 - \mu(1)) + (1 - \theta) \times (\mu(1) - 1/2)) = \text{Max}\{1 - \alpha_2, 0\}\theta(1 - \theta)$.

Case 2: $p_1 = \mu(0)$. The case is symmetric to Case 1 and therefore the expected profit of a deep information speculator conditional on $p_1 = \mu(0)$ is also $\text{Max}\{1 - \alpha_2, 0\}\theta(1 - \theta)$ in this case.

Case 3: $p_1 = 1/2$. In this case a speculator trades the asset only if $u = 1$. Suppose first that $s = 1$. Using Parts 2, 3, and 4 of Proposition 2, the table below gives the probability of each possible realization for the equilibrium price at date 2 conditional on $\{u, s, p_1\} = \{1, 1, 1/2\}$ and the associated profit for a shallow information speculator (taking into account that shallow information speculators buy the asset at date 2 if $u = s = 1$).

Equilibrium price at date 2: p_2^*		Prob if $\alpha_2 \leq 1$	Prob if $1 < \alpha_2 \leq 2$	Speculator's profit
0		0	0	1
$\frac{1-\theta}{2-\theta}$		0	0	$\frac{1}{2-\theta}$
$\frac{1}{2}$		$1 - \alpha_2$	0	$\frac{1}{2}$
$\frac{1}{2-\theta}$		$\frac{\alpha_2}{2}$	$\frac{(2-\alpha_2)}{2}$	$\frac{1-\theta}{2-\theta}$
1		$\frac{\alpha_2}{2}$	$\frac{\alpha_2}{2}$	0

We deduce that if $\{u, s, p_1\} = \{1, 1, 1/2\}$, a deep information speculator obtains an expected profit of $(1 - \alpha_2(2 - \theta)^{-1})/2$ if $\alpha_2 \leq 1$ and $\frac{(2-\alpha_2)(1-\theta)}{2(2-\theta)}$ if $1 < \alpha_2 \leq 2$. The case in which $u = 1$, $s = 0$, and $p_1 = 1/2$ is symmetric and therefore yields the same expected profit for a deep information speculator. Thus, when $p_1 = 1/2$, a deep information speculator's expected profit is $\theta(1 - \alpha_2(2 - \theta)^{-1})/2$ if $\alpha_2 \leq 1$ and $\frac{\theta}{2} \frac{(2-\alpha_2)(1-\theta)}{(2-\theta)}$ if $1 < \alpha_2 \leq 2$.

Cases 1 and 2 happen with probability $\alpha_1/2$ each while case 3 happens with probability $(1 - \alpha_1)$. We deduce that if $\alpha_2 \leq 1$, the expected profit of a deep information speculator is:

$$\bar{\pi}(\alpha_1, \alpha_2) = \frac{\theta}{2} (2\alpha_1(1 - \alpha_2)(1 - \theta) + (1 - \alpha_1)((1 - \alpha_2(2 - \theta)^{-1})).$$

while if $1 < \alpha_2 \leq 2$, the expected profit of a shallow speculator is:

$$\bar{\pi}(\alpha_1, \alpha_2) = \frac{\theta}{2} \frac{(2 - \alpha_2)(1 - \alpha_1)(1 - \theta)}{(2 - \theta)}.$$

Proof of Corollary 1. When $1 \leq \alpha_2 \leq 2$, it immediately follows from eq.(6) that $\frac{\partial \bar{\pi}_2}{\partial \alpha_1} < 0$ for all values of θ . When $\alpha_2 \leq 1$, using eq.(6), we obtain:

$$\frac{\partial \bar{\pi}_2}{\partial \alpha_1} = \frac{\theta}{2} \left[1 - 2\theta - \left(2(1 - \theta) - \frac{1}{2 - \theta} \right) \alpha_2 \right]. \quad (23)$$

If $\theta \leq 1/2$, it follows that:

$$\frac{\partial \bar{\pi}_2}{\partial \alpha_1} > 0 \quad \text{iff} \quad \alpha_2 \leq \hat{\alpha}_2(\theta),$$

when $\alpha_2 \leq 1$. if instead, $\theta > 1/2$, there exists a threshold $1/2 < \hat{\theta} < 1$ such that $2(1 - \theta) - \frac{1}{2 - \theta} < 0$ iff $\theta > \hat{\theta}$. Thus, if $1/2 < \theta < \hat{\theta}$ then $\frac{\partial \bar{\pi}_2}{\partial \alpha_1} < 0$. Finally, if $\theta > \hat{\theta}$, $\frac{\partial \bar{\pi}_2}{\partial \alpha_1} > 0$ iff $\alpha_2 > \frac{1-2\theta}{2(1-\theta)-(2-\theta)^{-1}}$, which is impossible since the R.H.S. of this inequality is strictly larger than one.

Proof of Lemma 1. As explained in the text, $\alpha_2^* = 0$ when $C_p \geq C_{max}$ and $\alpha_2^* \in (\alpha_2^{max}, 2)$ when $0 < C_p < C_{max}$. Let $C_{min}(\theta, \alpha_1)$ be the value of C_p such that $\alpha_2^* = 1$. Thus, C_{min} solves $\pi_2^{gross,a}(\alpha_1, 1) = C_{min}$. Using eq.(6) and the definition of $\pi_2^{gross,a}(\alpha_1, \alpha_2)$, we deduce that $C_{min}(\theta, \alpha_1) = \frac{\theta}{2} \frac{\theta(1-\theta)(1-\alpha_1)}{2(2-\theta)}$. As $\pi_2^{gross,a}(\alpha_1, \alpha_2)$ decreases continuously in both α_2 for $\alpha_2 \in (\alpha_2^{max}, 2)$ and C_p , we deduce that $\alpha_2^* \leq 1$ for $C_p > C_{min}$ (case 1) and $\alpha_2^* \geq 1$ for $C_p \leq C_{min}$ (case 2).

In case 1, using eq.(6) and eq.(11), we deduce that α_2^* solves:

$$\alpha_2^* \bar{\pi}_2(\alpha_1, \alpha_2^*) - C_p = \frac{\theta}{2} \alpha_2 \left[1 - (2\theta - 1)\alpha_1 - \left(\frac{1}{2-\theta} + \left(2(1-\theta) - \frac{1}{2-\theta} \right) \alpha_1 \right) \alpha_2 \right] - C_p = 0. \quad (24)$$

This equation has two roots in α_2 but only one is larger than α_2^{max} , as required in equilibrium.

This root is:

$$\alpha_2^* = \alpha_2^{max}(\theta, \alpha_1) \left(1 + \sqrt{1 - \frac{C_p}{C_{max}(\theta, \alpha_1)}} \right).$$

In case 2 ($C_p \leq C_{min}$), $\alpha_2^* \geq 1$. Thus, using eq.(6) and eq.(11), we deduce that α_2^* solves:

$$\alpha_2^* \bar{\pi}_2(\alpha_1, \alpha_2^*) - C_p = \frac{\theta}{2} \frac{1-\theta}{2-\theta} (1-\alpha_1) \alpha_2 (2-\alpha_2) - C_p = 0. \quad (25)$$

This equation again has two roots in α_2 but only one is larger than 1 (as required). This root is:

$$\alpha_2^* = 1 + \sqrt{1 - \frac{C_p}{C_{min}(\theta, \alpha_1)}}.$$

Proof of Proposition 3. As C_r affects α_2^* only through its effect on α_1^* , we have:

$$\frac{\partial \alpha_2^*}{\partial C_r} = \left(\frac{\partial \alpha_2^*}{\partial \alpha_1} \right) \left(\frac{\partial \alpha_1^*}{\partial C_r} \right). \quad (26)$$

It is immediate from Lemma 2 that $\frac{\partial \alpha_1^*}{\partial C_r} \leq 0$. Thus, eq.(26) implies that $\frac{\partial \alpha_2^*}{\partial C_r} \geq 0$ iff $\frac{\partial \alpha_2^*}{\partial \alpha_1} < 0$.

Thus, in the rest of this proof, we sign $\frac{\partial \alpha_2^*}{\partial \alpha_1}$.

Remember that for $C_p < C_{max}$, $\alpha_2^* > \alpha_2^{max}$ and α_2^* solves:

$$\pi_2^{gross,a}(\alpha_1, \alpha_2^*) = C_p.$$

Thus, using the implicit function theorem and the definition of $\pi_2^{net,a}(\alpha_1, \alpha_2, C_p)$, we have

$$\frac{\partial \alpha_2^*}{\partial \alpha_1} = - \frac{\frac{\partial}{\partial \alpha_1} [\alpha_2 \bar{\pi}_2(\alpha_1, \alpha_2)]_{\alpha_2=\alpha_2^*}}{\frac{\partial}{\partial \alpha_2} [\alpha_2 \bar{\pi}_2(\alpha_1, \alpha_2)]_{\alpha_2=\alpha_2^*}}. \quad (27)$$

As $\alpha_2^* > \alpha_2^{max}$, we have $\frac{\partial}{\partial \alpha_2} [\alpha_2 \bar{\pi}_2(\alpha_1, \alpha_2)]_{\alpha_2=\alpha_2^*} < 0$. We deduce from eq.(27) that $\frac{\partial \alpha_2^*}{\partial \alpha_1} > 0$ iff $\frac{\partial}{\partial \alpha_1} [\alpha_2 \bar{\pi}_2(\alpha_1, \alpha_2)]_{\alpha_2=\alpha_2^*} < 0$.

Case 1: $1/2 < \theta$ or $C_p < C_{min}(\theta, \alpha_1)$. If $\theta > 1/2$, we deduce from Corollary 1 that deep information speculator's expected profit, $\bar{\pi}_2$, decreases with α_1 . If $C_p < C_{min}(\theta, \alpha_1)$, we deduce from Proposition 1 that $\alpha_2^* > 1$. Therefore, using Corollary 1 again, deep information speculator's net expected profit, $\bar{\pi}_2$, decreases with α_1 . Hence, for $\theta > 1/2$ or $C_p < C_{min}(\theta, \alpha_1)$, we have:

$$\frac{\partial [\alpha_2 \bar{\pi}_2(\alpha_1, \alpha_2^*)]}{\partial \alpha_1} < 0.$$

We deduce that if $\theta > 1/2$ or $C_p < C_{min}(\theta, \alpha_1)$ then $\frac{\partial \alpha_2^*}{\partial \alpha_1} < 0$ and therefore $\frac{\partial \alpha_2^*}{\partial C_r} > 0$.

Case 2: $\theta < 1/2$ and $C_{min}(\theta, \alpha_1) < C_p < C_{max}(\theta, \alpha_1)$. Using Corollary 1, we deduce that deep information speculator's expected profit, $\bar{\pi}_2$, increases with α_1 iff $\alpha_2^*(\alpha_1) < \hat{\alpha}_2(\theta)$. Thus, if this condition is satisfied then $\frac{\partial}{\partial \alpha_1} [\alpha_2 \bar{\pi}_2(\alpha_1, \alpha_2)]_{\alpha_2=\alpha_2^*} < 0$ and therefore $\frac{\partial \alpha_2^*}{\partial \alpha_1} > 0$. Thus, in this case, $\frac{\partial \alpha_2^*}{\partial C_r} > 0$. The rest of the proof consists in showing that the conditions (i) $\theta < \frac{\sqrt{2}-1}{\sqrt{2}}$, (ii) $C_r < \bar{C}_r(\theta)$, and (iii) $C_p > \bar{C}_p(\theta)$ are necessary and sufficient for $\alpha_2^*(\alpha_1) < \hat{\alpha}_2(\theta)$. For brevity, we provide the proof of this result in the on-line appendix. As $\frac{\sqrt{2}-1}{\sqrt{2}} < 1/2$, the proposition follows.

Proof of Proposition 4. When $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$, we show in the on-line appendix that $C_{max}(\theta, \alpha_1)$ decreases with α_1 . Moreover, using eq.(13), we obtain $C_{max}(\theta, 1) = \frac{\theta(1-\theta)}{4}$ and $C_{max}(\theta, 0) = \frac{\theta(2-\theta)}{8}$. Thus, for each $C_p \in [\frac{\theta(1-\theta)}{4}, \frac{\theta(2-\theta)}{8}]$, there exists a unique $\alpha_1^c(\theta, C_p)$ such that:

$$C_p = C_{max}(\theta, \alpha_1^c).$$

Moreover, for $\alpha_1^* > \alpha_1^c$, $C_p < C_{max}(\theta, \alpha_1^*)$ while for $\alpha_1^* < \alpha_1^c$, $C_p > C_{max}(\theta, \alpha_1^*)$. We deduce from Lemma 1, that for $\alpha_1^* > \alpha_1^c$, $\alpha_2^*(\theta, \alpha_1^*) > \alpha_2^{max}$ while for $\alpha_1^* < \alpha_1^c$, $\alpha_2^*(\theta, \alpha_1^*) = 0$. The corollary follows by defining \hat{C}_r as the value of C_r such that $\alpha_1^*(\theta, \hat{C}_r) = \alpha_1^c(\theta, C_p)$.

Proof of Corollary 2.

Using Proposition 1 (or Figure 2), we obtain that:

$$\mathcal{E}_1(C_r, C_p) = \begin{cases} 0 & \text{if } C_r \geq \frac{\theta}{8}, \\ \frac{\alpha_1^*(\theta, C_r)\theta^2}{4} & \text{if } C_r \leq \frac{\theta}{8}, \end{cases} \quad (28)$$

and

$$\mathcal{E}_2(C_r, C_p) = \begin{cases} \mathcal{E}_1(C_r, C_p) & \text{if } C_p \geq C_{max}(\theta, \alpha_1^*), \\ \frac{\theta}{4} \left[1 - (1 - \alpha_1^* \left(1 - \frac{\alpha_2^*}{2-\theta} \right) - (1 - \theta)\alpha_1^*(1 - \alpha_2^*)) \right] & \text{if } C_{min}(\theta, \alpha_1^*) \leq C_p \leq C_{max}(\theta, \alpha_1^*), \\ \frac{\theta}{4} \left[1 - (1 - \frac{\theta(1-\theta)}{2-\theta}\alpha_1^*)(2 - \alpha_2^*) \right] & \text{if } C_p \leq C_{min}(\theta, \alpha_1^*), \end{cases} \quad (29)$$

where to simplify notations we have omitted the arguments of functions α_2^* and α_1^* . As α_1^* does not depend on the cost of deep information, we deduce from eq.(28) that price informativeness at date 1 is not affected by a change in C_p .

As explained in the text, $\mathcal{E}_2(C_r, C_p) \leq \mathcal{E}_1(C_r, C_p)$ and this inequality is strict if $\alpha_2^* > 0$, i.e., if $C_p < C_{max}(\theta, \alpha_1^*)$. In this range of value for C_p , it is immediate from eq.(29) that price informativeness at date 2 increases with α_2^* . As α_2^* declines when C_p decreases, we deduce that price informativeness at date 2 increases when C_p declines for $C_p < C_{max}(\theta, \alpha_1^*)$. For $C_p > C_{max}(\theta, \alpha_1^*)$, price informativeness at date 2 is equal to price informativeness at date 1 and therefore independent of C_p .

Proof of Proposition 5.

Part 1: Effect of C_r on short run price informativeness. We know from Proposition 2 that α_1^* weakly increases when C_r increases. Hence, we deduce from eq.(28) that $\mathcal{E}_1^*(C_r, C_p)$ weakly decreases when C_r decreases.

Part 2: Effect of C_r on long run price informativeness. We consider three different cases depending on the value of C_p .

Case 1. Consider first the case in which $C_p < C_{min}(\theta, \alpha_1^*)$. In this case, $\alpha_2^* \geq 1$ (Proposition 1). Using eq.(6) and eq.(29), we have:

$$\mathcal{E}_2^*(C_r, C_p) = \frac{\theta}{4} - \frac{1}{2}\bar{\pi}_2(\alpha_1^*, \alpha_2^*), \quad (30)$$

where we omit the arguments of functions α_1^* and α_2^* to simplify notations. Now, as $\alpha_2^* > 0$, in equilibrium, $\alpha_2^* \bar{\pi}_2^* = C_p$ (see eq.(10)). Thus, we deduce from (30) that:

$$\mathcal{E}_2^* = \frac{\theta}{4} - \frac{1}{2} \frac{C_p}{\alpha_2^*}. \quad (31)$$

As $C_p < C_{min}(\theta, \alpha_1^*)$, we deduce from the analysis of Case 1 in the proof of Proposition 3 that α_2^* decreases when C_r decreases. Hence, from eq.(31), we deduce that if $C_p < C_{min}(\theta, \alpha_1^*)$ then $\mathcal{E}_2^*(C_r, C_p)$ decreases when C_r decreases.

Case 2. Now consider the case in which $C_{min}(\theta, \alpha_1^*) < C_p < C_{max}(\theta, \alpha_1^*)$. In this case, $0 < \alpha_2^* \leq 1$ (Proposition 1). Using eq.(6) and eq.(29), we have:

$$\mathcal{E}_2^*(C_r, C_p) = \frac{\theta}{4} - \frac{1}{2} \left(\bar{\pi}_2(\alpha_1^*, \alpha_2^*) - \frac{\theta(1-\theta)}{2} \alpha_1^*(1-\alpha_2^*) \right), \quad (32)$$

where we again omit the arguments of functions α_1^* and α_2^* to simplify notations. As $\alpha_2^* > 0$, in equilibrium, $\alpha_2^* \bar{\pi}_2^* = C_p$ (see eq.(10)). Thus, we can rewrite eq.(32) as:

$$\mathcal{E}_2^*(C_r, C_p) = \frac{\theta}{4} - \frac{1}{2} \left(\frac{C_p}{\alpha_2^*} - \frac{\theta(1-\theta)}{2} \alpha_1^*(1-\alpha_2^*) \right), \quad (33)$$

Using the fact that C_r affects α_2^* only through its effect on α_1^* , we deduce from eq.(32) that:

$$\frac{\partial \mathcal{E}_2^*(C_r, C_p)}{\partial C_r} = \left(\frac{1}{2} \frac{\partial \alpha_1^*}{\partial C_r} \right) \left(\frac{\partial \alpha_2^*}{\partial \alpha_1^*} \left(\frac{C_p}{\alpha_2^{*2}} - \frac{\theta(1-\theta)}{2} \alpha_1^* \right) + \frac{\theta(1-\theta)}{2} (1-\alpha_2^*) \right), \quad (34)$$

As $\frac{\partial \alpha_1^*}{\partial C_r} \leq 0$, we deduce that the sign of $\frac{\partial \mathcal{E}_2^*(C_r, C_p)}{\partial C_r}$ is opposite to the sign of the following function:

$$G(\alpha_1^*, \alpha_2^*) \equiv \frac{\partial \alpha_2^*}{\partial \alpha_1^*} \left(\frac{C_p}{\alpha_2^{*2}} - \frac{\theta(1-\theta)}{2} \alpha_1^* \right) + \frac{\theta(1-\theta)}{2} (1-\alpha_2^*) \quad (35)$$

To determine the sign of $G(\alpha_1^*, \alpha_2^*)$, we first compute $\frac{\partial \alpha_2^*}{\partial \alpha_1^*}$. Using eq.(27), we obtain:

$$-\frac{\partial \alpha_2^*}{\partial \alpha_1^*} = \frac{\frac{\partial [\alpha_2^* \bar{\pi}_2(\alpha_1^*, \alpha_2^*)]}{\partial \alpha_1^*}}{\frac{\partial [\alpha_2^* \bar{\pi}_2(\alpha_1^*, \alpha_2^*)]}{\partial \alpha_2^*}} = \frac{\alpha_2^* \frac{\partial \bar{\pi}_2(\alpha_1^*, \alpha_2^*)}{\partial \alpha_1^*}}{\alpha_2^* \frac{\partial \bar{\pi}_2(\alpha_1^*, \alpha_2^*)}{\partial \alpha_2^*} + \bar{\pi}_2(\alpha_1^*, \alpha_2^*)}.$$

Moreover, as $0 < \alpha_2^* \leq 1$, we deduce from Proposition 2 that:

$$\bar{\pi}_2(\alpha_1^*, \alpha_2^*) = \frac{\theta}{2} \left\{ 1 - (2\theta - 1)\alpha_1^* - \left[\frac{1}{2-\theta} + \left(2(1-\theta) - \frac{1}{2-\theta} \right) \alpha_1^* \right] \alpha_2^* \right\}.$$

This implies that

$$\frac{\partial \bar{\pi}_2(\alpha_1^*, \alpha_2^*)}{\partial \alpha_1^*} = -\frac{\theta}{2} \left[2\theta - 1 + \left(2(1 - \theta) - \frac{1}{2 - \theta} \right) \alpha_2^* \right],$$

$$\frac{\partial \bar{\pi}_2(\alpha_1^*, \alpha_2^*)}{\partial \alpha_2^*} = -\frac{\theta}{2} \left[\frac{1}{2 - \theta} + \left(2(1 - \theta) - \frac{1}{2 - \theta} \right) \alpha_1^* \right].$$

Therefore,

$$-\frac{\partial \alpha_2^*}{\partial \alpha_1^*} = \frac{\alpha_2^* \left[2\theta - 1 + \left(2(1 - \theta) - \frac{1}{2 - \theta} \right) \alpha_2^* \right]}{\alpha_2^* \left[\frac{1}{2 - \theta} + \left(2(1 - \theta) - \frac{1}{2 - \theta} \right) \alpha_1^* \right] - \frac{2C_p}{\theta} \frac{1}{\alpha_2^*}}$$

The denominator of this expression is equal to $-\frac{\partial[\alpha_2^* \bar{\pi}_2(\alpha_1^*, \alpha_2^*)]}{\partial \alpha_2^*}$. This derivative is strictly positive in equilibrium (see the discussion that precedes Proposition 1). Hence, we deduce that $G(\alpha_1^*, \alpha_2^*) < 0$ iff:

$$\begin{aligned} & \alpha_2^* \left[2\theta - 1 + \left(2(1 - \theta) - \frac{1}{2 - \theta} \right) \alpha_2^* \right] \left(\frac{2C_p}{\theta} \frac{1}{\alpha_2^{*2}} - (1 - \theta) \alpha_1^* \right), \\ & - (1 - \theta)(1 - \alpha_2^*) \left[\alpha_2^* \left(\frac{1}{2 - \theta} + \left(2(1 - \theta) - \frac{1}{2 - \theta} \right) \alpha_1^* \right) - \frac{2C_p}{\theta} \frac{1}{\alpha_2^*} \right] > 0 \end{aligned}$$

After some algebra, one can show that this condition is equivalent to:

$$\Upsilon(\alpha_1^*, \alpha_2^*(\alpha_1^*), \theta, C_p) > 0,$$

where

$$\Upsilon(\alpha_1^*, \alpha_2^*(\alpha_1^*), \theta, C_p) \equiv \frac{1 - \theta}{2 - \theta} \left(\frac{2C_p}{\theta} - \alpha_2^*(1 - \alpha_2^*) \right) + 2C_p \left(\frac{1}{\alpha_2^*} - 1 \right) - \alpha_1^* \alpha_2^* \frac{(1 - \theta)^2}{2 - \theta}. \quad (36)$$

In sum, $G(\alpha_1^*, \alpha_2^*) < 0$ iff $\Upsilon(\alpha_1^*, \alpha_2^*(\alpha_1^*), \theta, C_p) > 0$. Thus, when $C_{min}(\theta, \alpha_1^*) < C_p < C_{max}(\theta, \alpha_1^*)$, $\frac{\partial \mathcal{E}_2^*(C_r, C_p)}{\partial C_r} > 0$ iff $\Upsilon(\alpha_1^*, \alpha_2^*(\alpha_1^*), \theta, C_p) > 0$.

Case 3. Last consider the case in which $C_{max}(\theta, \alpha_1^*) < C_p$. In this case, we deduce from eq.(29) that $\mathcal{E}_2(C_r, C_p) = \mathcal{E}_1(C_r, C_p)$. As $\mathcal{E}_1(C_r, C_p)$ increases when C_r decreases, we obtain that this is also the case for $\mathcal{E}_2(C_r, C_p)$ when $C_{max}(\theta, \alpha_1^*) < C_p$.

Proof of Proposition 6.

Note that $\alpha_1^* = 0$ for all $C_r \geq \frac{\theta}{8}$. Thus, $\mathcal{E}_2(C_r, C_p) = \mathcal{E}_2(\frac{\theta}{8}, C_p)$ for $C_r \geq \frac{\theta}{8}$. We denote the

difference in price informativeness at date 2 when $C_r = 0$ and when $C_r \geq \frac{\theta}{8}$, for a given C_p , by $\Delta\mathcal{E}_2(C_p)$. That is:

$$\Delta\mathcal{E}_2(C_p) \equiv \mathcal{E}_2\left(\frac{\theta}{8}, C_p\right) - \mathcal{E}_2(0, C_p) \quad (37)$$

Observe that:

$$C_{max}(\theta, 0) = \frac{\theta(2-\theta)}{8} > C_{min}(\theta, 0) = \frac{\theta(1-\theta)}{2(2-\theta)} > C_{max}(\theta, 1) = \frac{\theta(1-\theta)}{4},$$

and that

$$\alpha_2^{max}(\theta, 0) = 1 - \frac{\theta}{2}, \text{ and } \alpha_2^{max}(\theta, 1) = \frac{1}{2}.$$

Case 1. First, consider the case in which $C_p \in [0, C_{max}(\theta, 1)]$. In this case, using Proposition 1 and the previous observations, we obtain that if $C_r = 0$ then

$$\alpha_2^* = \frac{1}{2} \left(1 + \sqrt{1 - \frac{4}{\theta(1-\theta)} C_p} \right) < 1$$

and that if $C_r \geq \frac{\theta}{8}$ then

$$\alpha_2^* = 1 + \sqrt{1 - \frac{2(2-\theta)}{\theta(1-\theta)} C_p} > 1$$

Hence, using eq.(29) and eq.(37), the previous observations, and the fact that $\alpha_1^* = 1$ if $C_r = 0$, we obtain

$$\Delta\mathcal{E}_2(C_p) = \frac{\theta}{4} \left[(1-\theta) \frac{1}{2} \left(1 - \sqrt{1 - \frac{4}{\theta(1-\theta)} C_p} \right) - \frac{1-\theta}{2-\theta} \left(1 - \sqrt{1 - \frac{2(2-\theta)}{\theta(1-\theta)} C_p} \right) \right].$$

We deduce that:

$$\frac{4}{\theta} \frac{\partial \Delta\mathcal{E}_2}{\partial C_p} = \frac{1}{\theta} \left(\frac{1}{\sqrt{1 - \frac{4}{\theta(1-\theta)} C_p}} - \frac{1}{\sqrt{1 - \frac{2(2-\theta)}{\theta(1-\theta)} C_p}} \right).$$

This is always positive iff $\frac{4}{\theta(1-\theta)} C_p > \frac{2(2-\theta)}{\theta(1-\theta)} C_p$, which is always true. Thus, for $C_p \in [0, C_{max}(\theta, 1)]$, $\Delta\mathcal{E}_2(C_p)$ increases with C_p . As $\Delta\mathcal{E}_2(0) = 0$ and $\Delta\mathcal{E}_2(C_{max}(\theta, 1)) > 0$, we obtain that $\Delta\mathcal{E}_2(C_p) > 0$ when $C_p \in [0, C_{max}(\theta, 1)]$.

Case 2. Now, consider the case in which $C_p \in [C_{max}(\theta, 1), C_{min}(\theta, 0)]$. In this case, using

Lemma 1 and the previous observations, we obtain that if $C_r = 0$ then

$$\alpha_2^* = 1 + \sqrt{1 - \frac{2(2-\theta)}{\theta(1-\theta)}C_p} > 1,$$

and if $C_r \geq \frac{\theta}{8}$ then

$$\alpha_2^* = 0.$$

Hence, using eq.(29) and eq.(37), and the fact that $\alpha_1^* = 1$ if $C_r = 0$, we obtain

$$\Delta\mathcal{E}_2(C_p) = \frac{\theta}{4} \left[1 - \theta - \frac{1-\theta}{2-\theta} \left(1 - \sqrt{1 - \frac{2(2-\theta)}{\theta(1-\theta)}C_p} \right) \right] = \frac{\theta}{4} \left[\frac{(1-\theta)^2}{2-\theta} \frac{1-\theta}{2-\theta} \sqrt{1 - \frac{2(2-\theta)}{\theta(1-\theta)}C_p} \right] > 0.$$

Case 3. Finally suppose that $C_p \in [C_{min}(\theta, 0), C_{max}(\theta, 1)]$. In this case, using Lemma 1 and the previous observations, we obtain that if $C_r = 0$ then

$$\alpha_2^* = \left(1 - \frac{\theta}{2}\right) \left(1 + \sqrt{1 - \frac{8}{\theta(2-\theta)}C_p}\right) < 1,$$

and if $C_r \geq \frac{\theta}{8}$ then

$$\alpha_2^* = 0.$$

Hence, using eq.(29) and eq.(37), and the fact that $\alpha_1^* = 1$ if $C_r = 0$, we obtain that

$$\Delta\mathcal{E}_2 = \frac{\theta}{4} \left[\frac{1}{2} - \theta + \frac{1}{2} \sqrt{1 - \frac{8}{\theta(2-\theta)}C_p} \right],$$

which is positive if $C_p \leq (1 - (2\theta - 1)^2) \frac{\theta(2-\theta)}{8}$.

Proof of Corollary 3. Using the first parts of Propositions 1 and 2, we deduce that:

$$x_1 = \mathbb{I}_{s=1} - \mathbb{I}_{s=0}, \text{ with } s = u \times V + (1-u) \times \epsilon, \quad (38)$$

$$x_2 = u \times [\mathbb{I}_{V=1} - \mathbb{I}_{V=0}] + (1-u) \times [\mathbb{I}_{p_1=(1-\theta)/2} - \mathbb{I}_{p_1=(1+\theta)/2}], \quad (39)$$

where \mathbb{I} denotes the indicator function, which is equal to one when the statement in brackets

holds. As $\mathbb{E}[x_1] = \mathbb{E}[x_2] = 0$, we deduce from eq. (38) and eq.(39) that:

$$\begin{aligned} Cov(x_1, x_2) &= \mathbb{E}[x_1 x_2] = \frac{1}{2} \mathbb{E}[x_2 | s = 1] - \frac{1}{2} \mathbb{E}[x_2 | s = 0], \\ &= \frac{\theta}{2} \mathbb{E}[x_2 | V = 1, u = 1] + \frac{1}{2} (1 - \theta) \frac{\alpha_1^*}{Q} \mathbb{E} \left[x_2 | \epsilon = 1, u = 0, p_1 = \frac{1 + \theta}{2} \right], \\ &\quad - \frac{\theta}{2} \mathbb{E}[x_2 | V = 0, u = 1] - \frac{1}{2} (1 - \theta) \frac{\alpha_1^*}{Q} \mathbb{E} \left[x_2 | \epsilon = 0, u = 0, p_1 = \frac{1 - \theta}{2} \right], \\ &= \theta - (1 - \theta) \alpha_1^*. \end{aligned}$$

As α_1^* increases when C_r declines, we deduce that $Cov(x_1, x_2)$ decreases when C_r decreases.

Moreover, $Cov(x_1, x_2) < 0$ iff:

$$\alpha_1^*(\theta, C_r) \geq \frac{\theta}{1 - \theta}.$$

Substituting $\alpha_1^*(\theta, C_r)$ by its expression in eq.(14), we deduce that $Cov(x_1, x_2) < 0$ iff $\theta < 1/2$ and $C_r < \frac{\theta^2(2\theta-1)}{2(1-\theta)}$.

Proof of Corollary 4. Using the second part of Proposition 1 and the first part of Proposition 2, we deduce that:

$$p_1 = \frac{1}{2} + \frac{\theta}{2} \mathbb{I}_{f_1 > 1 - \alpha_1^*} - \frac{\theta}{2} \mathbb{I}_{f_1 < -1 + \alpha_1^*} \quad (40)$$

$$x_2 = U \times [\mathbb{I}_{V=1} - \mathbb{I}_{V=0}] + (1 - u) \times [\mathbb{I}_{p_1=(1-\theta)/2} - \mathbb{I}_{p_1=(1+\theta)/2}]. \quad (41)$$

As $\mathbb{E}[x_2] = 0$ and $\mathbb{E}[p_1] = 1/2$, we deduce from (40) and (41) that:

$$\begin{aligned} Cov(p_1, x_2) &= \mathbb{E}[(p_1 - 1/2)x_2] = \frac{\theta \alpha_1^*}{4} \left\{ \mathbb{E} \left[x_2 | s = 1, p_1 = \frac{1 + \theta}{2} \right] - \mathbb{E} \left[x_2 | s = 0, p_1 = \frac{1 - \theta}{2} \right] \right\} \\ &= \frac{\theta^2}{4} \alpha_1^* \mathbb{E} \left[x_2 | V = 1, u = 1, p_1 = \frac{1 + \theta}{2} \right] + \frac{\theta(1 - \theta)}{4} \alpha_1^* \mathbb{E} \left[x_2 | \epsilon = 1, u = 0, p_1 = \frac{1 + \theta}{2} \right] \\ &\quad - \frac{\theta^2}{4} \alpha_1^* \mathbb{E} \left[x_2 | V = 0, u = 1, p_1 = \frac{1 - \theta}{2} \right] - \frac{\theta(1 - \theta)}{4} \alpha_1^* \mathbb{E} \left[x_2 | \epsilon = 0, u = 0, p_1 = \frac{1 - \theta}{2} \right] \\ &= \theta(2\theta - 1) \alpha_1^*. \end{aligned}$$

As α_1^* increases when C_r declines, we deduce that $|Cov(p_1, x_2)|$ increases when C_r decreases.

Proof of Corollary 5.

We first compute the expression for $Cov(x_1, r_2)$ given in eq.(18). As $E[x_1] = 0$,

$$Cov(x_1, r_2) = E[(p_2 - p_1)x_1] - E[p_2 - p_1]E[x_1] = E[(p_2 - p_1)x_1]. \quad (42)$$

Now:

$$\begin{aligned}
E[p_1 x_1] &= \frac{1}{2}(E[p_1 x_1 | s = 1] + E[p_1 x_1 | s = 0]) = \frac{1}{2}(E[p_1 | s = 1] - E[p_1 | s = 0]) \\
&= \frac{1}{2} \left((1 - \alpha_1^*) \frac{1}{2} + \alpha_1 \frac{1 + \theta}{2} \right) - \frac{1}{2} \left((1 - \alpha_1^*) \frac{1}{2} + \alpha_1^* \frac{1 - \theta}{2} \right) \\
&= \frac{\theta \alpha_1^*}{2}.
\end{aligned} \tag{43}$$

Similarly , we have that:

$$E[p_2 x_1] = \frac{1}{2}(E[p_2 | s = 1] - E[p_2 | s = 0]). \tag{44}$$

We first compute $E[p_2 | s = 1]$. We have:

$$E[p_2 | s = 1] = \alpha_1^* E \left[p_2 \mid s = 1, p_1 = \frac{1 + \theta}{2} \right] + (1 - \alpha_1^*) E \left[p_2 \mid s = 1, p_1 = \frac{1}{2} \right]. \tag{45}$$

The event $p_1 = \frac{1 + \theta}{2}$ implies that $s = 1$. Thus,

$$E \left[p_2 \mid s = 1, p_1 = \frac{1 + \theta}{2} \right] = E \left[p_2 \mid p_1 = \frac{1 + \theta}{2} \right] = p_1 = \frac{1 + \theta}{2},$$

where the third equality follows from the fact that the equilibrium price is a martingale. Moreover, using Proposition 2, we deduce that if $\alpha_2^* < 1$:

$$\begin{aligned}
E \left[p_2 \mid s = 1, p_1 = \frac{1}{2} \right] &= \frac{1}{2}(1 - \theta)\alpha_2^* \times \frac{1 - \theta}{2 - \theta} + (1 - \alpha_2^*) \times \frac{1}{2} + \frac{1}{2}\alpha_2^* \times \frac{1}{2 - \theta} + \frac{1}{2}\theta\alpha_2^* \times 1 \\
&= \frac{1}{2} + \alpha_2^* \left(-\frac{1}{2} + \frac{(1 - \theta)^2}{2(2 - \theta)} + \frac{1}{2(2 - \theta)} + \frac{\theta}{2} \right) \\
&= \frac{1}{2} + \frac{\theta\alpha_2^*}{2(2 - \theta)}.
\end{aligned}$$

and if $\alpha_2^* \geq 1$,

$$\begin{aligned}
E \left[p_2 \mid s = 1, p_1 = \frac{1}{2} \right] &= (1 - \theta) \frac{1}{2} + \theta \left[\frac{\alpha_2^*}{2} \times 1 + \left(1 - \frac{\alpha_2^*}{2} \right) \times \frac{1}{2 - \theta} \right] \\
&= \frac{1}{2} + \theta \left[\frac{\alpha_2^*}{2} \left(1 - \frac{1}{2 - \theta} \right) + \frac{1}{2 - \theta} - \frac{1}{2} \right] \\
&= \frac{1}{2} + \theta \left[\frac{\theta}{2(2 - \theta)} + \frac{1 - \theta}{2(2 - \theta)} \alpha_2^* \right] \\
&= \frac{1}{2} + \frac{\theta}{2(2 - \theta)} [1 + (1 - \theta)(\alpha_2^* - 1)].
\end{aligned}$$

We deduce from these expressions and eq.(45) that:

$$E[p_2|s = 1] = \begin{cases} \frac{1+\theta\alpha_1^*}{2} + \frac{\theta(1-\alpha_1^*)\alpha_2^*}{2(2-\theta)} & \text{if } \alpha_2^* \leq 1, \\ \frac{1+\theta\alpha_1^*}{2} + \frac{\theta(1-\alpha_1^*)}{2(2-\theta)}[1 - (1-\theta)(1-\alpha_2^*)] & \text{if } \alpha_2^* > 1. \end{cases} \quad (46)$$

Proceeding in a similar way, we obtain after some algebra that

$$E[p_2|s = 1] = \begin{cases} \frac{1-\theta\alpha_1^*}{2} - \frac{\theta\alpha_2^*}{2(2-\theta)} & \text{if } \alpha_2^* \leq 1, \\ \frac{1-\theta\alpha_1^*}{2} - \frac{\theta}{2(2-\theta)}[1 - (1-\theta)(1-\alpha_2^*)] & \text{if } \alpha_2^* > 1. \end{cases} \quad (47)$$

After some algebra, we deduce from equations (42), (43), (44), (46), and (47) that:

$$Cov(x_1, r_2) = \begin{cases} \frac{\theta(1-\alpha_1^*)\alpha_2^*}{2(2-\theta)} & \text{if } \alpha_2^* \leq 1, \\ \frac{(1-\alpha_1^*)\theta}{2(2-\theta)}[1 - (1-\theta)(1-\alpha_2^*)] & \text{if } \alpha_2^* > 1, \end{cases} \quad (48)$$

which is equivalent to eq.(18) because $\alpha_2^* > 1$ iff $C_p < C_{min}(\theta, \alpha_1^*)$ and $\alpha_2^* \leq 1$ iff $C_{min}(\theta, \alpha_1^*) \leq C_p \leq C_{max}(\theta, \alpha_1^*)$.

As α_2^* decreases with C_p and α_1^* does not depend on C_p , it is immediate from eq.(18) that $Cov(x_1, r_2)$ increases when C_p decreases. Moreover, if (i) $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$ or (ii) $\theta \leq \frac{\sqrt{2}-1}{\sqrt{2}}$ and $C_r \geq \bar{C}_r(\theta)$, or (iii) $\theta \leq \frac{\sqrt{2}-1}{\sqrt{2}}$ and $C_p \leq \bar{C}_p(\theta)$ then α_2^* decreases when C_r decreases. Thus, as α_1^* increases when C_r decreases, we deduce that $Cov(x_1, r_2)$ decreases when $C - r$ decreases if i) $\theta > \frac{\sqrt{2}-1}{\sqrt{2}}$ or (ii) $\theta \leq \frac{\sqrt{2}-1}{\sqrt{2}}$ and $C_r \geq \bar{C}_r(\theta)$, or (iii) $\theta \leq \frac{\sqrt{2}-1}{\sqrt{2}}$ and $C_p \leq \bar{C}_p(\theta)$.

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