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## UNION DEBT MANAGEMENT

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# UNION DEBT MANAGEMENT


## Abstract

We study the role of government debt maturity in currency unions to identify whether debt management can help governments hedge their budgets against spending shocks. We first use a detailed dataset of debt portfolios of five Euro Area countries to run a battery of VARs, estimat- ing the responses of holding period returns to fiscal shocks. We find that government portfolios, which in our sample comprise mainly of nominal assets, have not been effective in absorbing idiosyncratic fiscal risks, whereas they have been very effective in absorbing aggregate risks. We then setup a formal model of optimal debt management with two countries, a benevolent planner, distortionary taxes and agggregate and idiosyncratic shocks. The theoretical model validates our empirical findings: nominal bonds are not optimal to insure against idiosyncratic shocks. Our key finding is that governments should introduce in their portfolios also inflation indexed long term debt since this allows them to take full advantage of fiscal hedging. Looking at the data, we find a sharp rise in issuances of inflation index bonds in France and in Italy since the beginning of the Euro. We show that bonds linked to French inflation were able to absorb both aggregate and idiosyncratic fiscal risks.



JEL Classification: E43, E62, H63

Keywords: Debt Management, Fiscal policy, Government Debt, Maturity Structure, Tax Smoothing



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# Union Debt Management *

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August 2019

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# 1 Introduction

A growing literature on optimal debt management (DM) in macroeconomic models with distortionary taxes advocates that governments should focus on issuing *long term nominal bonds*. In doing so, first of all, they would benefit from using inflation to stabilize debt in periods of high fiscal deficits (Chari and Kehoe (1999), Lustig et al (2008) and Faraglia et al (2013)). Moreover, they would be able to exploit the negative covariance between long bond prices and government deficits, the so called *fiscal hedging* channel of debt management (e.g. Angeletos (2002) and Buera and Nicolini (2004) (hereafter ABN) and Faraglia et al (2010)).

All these papers consider debt management in a closed economy setting. It is, however, not clear that the same conclusions apply when we consider the optimal structure of government debt in an open economy and especially in a currency union. There are, in fact, two main issues.

Firstly, the role of inflation in stabilizing macroeconomic variables, such as public debt, is limited in currency areas and in the presence of idiosyncratic shocks. This is a well understood problem for monetary policy in a currency area. If country members experience changes in spending levels that leave average spending roughly constant across the union, a change in inflation would make public debt more stable in some countries, but destabilize debt in others.

Secondly, the covariance between long bond prices and government deficits may be close to zero, when shocks are asymmetric and governments will not be able to benefit from fiscal insurance. This relates to the endogenous behavior of long bond prices and bond returns in a currency area. When nominal government bonds of creditworthy countries are held by domestic and foreign citizens in almost frictionless financial markets, or when long term interest rates follow closely future short term rates due to the common monetary policy, bond yields are highly correlated across countries. Nominal bond prices will, then, not respond negatively to idiosyncratic spending shocks, they may only respond to aggregate shocks that affect the overall spending levels in the currency area.

For these reasons, this paper studies DM in a currency union, both empirically and theoretically, to explore whether debt portfolios can help governments hedge against fiscal shocks.

In Section 2 we lay out our empirical analysis which tests formally for the presence of fiscal hedging in Euro Area government bond markets. We use a detailed dataset, also documented in Equiza-Goñi (2016), which contains information on bond prices and quantities of all types of debt in the portfolios of five Euro Area governments since 1999, the year the common currency was formally introduced.

Between 1999 and 2008, the period considered in our analysis, governments in the Euro Area have issued debt in a wide range of maturities including large amounts of long term bonds, and have mainly focused on issuing nominal debt. Nominal bond yields were highly correlated across countries, reflecting the markets' perception of stability of the Euro and the creditworthiness of its member countries. To test for the presence of fiscal hedging in this data, we estimate a series of panel VARs which enables us to identify the effects of fiscal shocks on the holding returns of government portfolios. We propose a novel identification strategy which separates idiosyncratic and aggregate spending shocks, and study their impact effects on the holding period returns. If these impacts are negative, then a rise in spending levels leads to a drop in the market value of government liabilities, and governments experience a capital gain. This is essentially the fiscal hedging channel of DM.

Our findings suggest that bond returns responded strongly to spending shocks over the sample period considered. However and most importantly, this only holds for shocks which affect the average level of spending in the Euro Area, which we identify as aggregate shocks. Whereas we find that shocks that do not affect average spending (s.a. shocks increasing spending levels in some countries but compensated by a drop in spending elsewhere), which we identify as idiosyncratic, did not have any effect on bond returns. Hedging against the latter types of shocks through DM therefore seems to have been nearly impossible.

In Section 3 we turn to a formal model of optimal DM in a currency union. We setup a model in which two countries, members of a currency area, face fiscal shocks and finance them through distortionary taxes and through debt portfolios. Key features of our model are monopolistic competition and sticky prices (e.g. Siu (2004) and Lustig et al (2008)), private sector preferences which exhibit a *mild bias* towards home goods, and government consumption that is allocated in home goods (e.g. Gali and Monacelli (2008)). Using this model we study the optimal policy under complete markets, assuming that a benevolent planner with full commitment sets taxes, prices and the debt portfolio of state contingent assets in both countries to maximise the joint welfare of their citizens. The assumption of a benevolent planner that maximises joint welfare, essentially amounts to assuming coordinated optimal monetary and fiscal policies, as in Siu (2004) and Lustig et al (2008), Faraglia et al (2013) and others in the DM literature and also Gali and Monacelli (2008) and Fahri and Werning (2017) in the international macro literature.

We first use the theoretical model to interpret our empirical findings and understand their implications. Our main quantitative experiment, presented in Section 4, attempts to decentralize the optimal complete market allocation using non-state contingent bonds of various maturities, as seen in the data. We consider the case of nominal bonds, and find that though these bonds are useful to hedge against aggregate shocks, in the presence of idiosyncratic shocks the complete market outcome generally cannot be attained. Nominal bond prices do not covary negatively with idiosyncratic fiscal shocks, or they vary too little to enable a government to use DM in order to fully insure against these types of shocks, exactly as in our empirical findings.

We then ask which kind of bonds can help the government to implement the complete market outcome regardless the type of shocks. We find that issuing inflation indexed debt can achieve this goal because in our model the covariance between real bond prices and government deficits is negative irrespective of the shock and in particular long term bonds can be used to fully exploit the fiscal hedging channel (similar to Angeletos (2002), Buera and Nicolini (2004) and Faraglia et al (2010)).

As in the data, in our model nominal yield curves are very strongly correlated across countries in the currency union. Therefore, nominal yields cannot absorb shocks that redistribute the fiscal burden across countries (our definition of idiosyncratic shocks). However, real yields are not strongly correlated, and governments can exploit these differences and complete the markets.

To complete the empirical analysis, in the last subsection of the paper we turn again our attention to our dataset to document the response of returns of inflation indexed debt to the fiscal shocks we identified in Section 2. Though three out of five countries in our sample did not issue (virtually) any inflation indexed bonds during the period considered, France has been using this instrument since 1999 and increasing significantly its share over time. This enables us to test for the presence of fiscal hedging in real yields. Our empirical estimates reveal a strong negative response of returns to both aggregate and idiosyncratic spending shocks, which implies that inflation indexed bonds can fully absorb fiscal risks as predicted by our theoretical findings.

This paper brings several new insights to the literature and relates to several strands. First, our empirical exercise in Section 2 is complementary to related empirical papers on fiscal hedging, for example Berndt et al (2012) and Faraglia et al (2008). Berndt et al (2012) use post World War II US data, identifying fiscal shocks as innovations to defense spending, and find a significant impact on holding period returns. Faraglia et al (2008) provide evidence of the presence of fiscal insurance in a panel of OECD countries studying the covariance between deficits and bond returns over the period 1970-2000. Our empirical analysis validates the fiscal hedging channel of DM focusing on the Euro Area.

Second, the post 2008-9 turmoil in the Euro Area bond markets and the sovereign debt crisis that followed, raised numerous concerns regarding the ability of governments to repay their debts, and the future of the Euro as a common currency. A substantial effort has been devoted in academic and policy circles to find ways to stabilize governments' budgets and promote risk sharing, in particular of fiscal risks, across countries. A recent stream of papers (see for example Fahri and Werning (2017), Dmitriev and Hoddenbagh (2015), Auclert and Ronglie (2014)) studies optimal fiscal transfers in models of currency unions (essentially fiscal unions). Other papers (e.g. Blanchard et al (2016), Acalin (2018), Shiller et al (2018)) have investigated whether introducing new types of debt (primarily GDP growth linked debt) can help resolve the debt instability problem. Our paper is complementary to this growing literature. In particular, we offer an alternative way for governments to share fiscal risks efficiently with other governments, i.e. across countries, and stabilize their budgets. Under complete markets, using transfers to distribute the fiscal burden across countries or using debt management, achieves the same effect. Equivalently, GDP denominated debt can, in our model, complete the market, as inflation indexed debt can.

Admittedly, at the center of today's policy debate in the Euro Area, is also the notion that various frictions hamper the implementation of optimal policies. Moral hazard concerns make difficult to reach consensus over fiscal transfers and, for the same reason, it is questionable whether marketable GDP-linked bonds can actually resolve governments' problems. Various financial market frictions, which our paper abstracts from, may also limit the scope of DM. In our view the potential presence of frictions, is a further reason to consider the DM as an additional policy margin. We leave, however, to future work to study the interplay between DM and fiscal transfers in a model with realistic frictions.

Finally, our findings are relevant for the literature on the so called 'equity home bias puzzle' in DSGE models[1], especially the recent strand in this literature which considers bonds and equities together (see for example Coeurdacier and Gourinchas (2017) and Coeurdacier and Rey (2013) and references therein). In these models households choose portfolios to hedge consumption against real exchange rate and non tradable income risks. However, the hedging motives of governments have not been yet considered in this context. Since governments are an important supplier of debt, accounting for their behavior explicitly seems a meaningful next step in the agenda.

# 2 Fiscal Insurance in the Euro Area

In this section we provide a formal test for the presence fiscal insurance in the Euro Area. Our sample covers five Euro Area countries: Belgium, France, Germany, Italy and Spain which account for roughly 80 percent of aggregate output and spending in the Euro Area. We employ panel SVARs estimating the responses of bond returns to government spending shocks. In Section 2.1 we briefly describe the idea behind fiscal insurance which motivates the empirical exercise. In Section 2.2 we describe the data used in our empirical analysis and the variables that enter in the SVARs. In section 2.3 we present our identification strategy to separately identify the effects of aggregate spending shocks and of idiosyncratic shocks. Our empirical results are described in subsections 2.4 and 2.5.

We find evidence of fiscal insurance against aggregate shocks. These shocks impinge large and significant changes in bond returns. In contrast, idiosyncratic shocks do not lead to significant changes in bond returns.

## 2.1 Definition of Fiscal Insurance

Consider the intertemporal budget constraint of a government that issues $N$ zero coupon bonds, $B^j$, of maturities $j = 1, 2, ...N$:

$$\underbrace{\frac{B^1_{t-1}}{\pi_t} + \sum_{j\geq 2}^{N} q_t^{j-1}\frac{B^j_{t-1}}{\pi_t}}_{\text{Market Value of Debt}} = \underbrace{E_t \sum_{k=0}^{\infty}\prod_{i=1}^{k}\frac{1}{r_{t+i}}S_{t+k}}_{\text{p.d.v of surpluses in } t} \equiv \widetilde{S}_t. \tag{1}$$

The left hand side is the (real) market value of outstanding debt and $q_t^j$ is the price of a nominal bond of maturity $j$. $\pi_t$ is the gross inflation rate between $t-1$ and $t$. The right hand side represents the (real) present value of fiscal surpluses, $S_t$, discounted by the real discount rate, $r$.

According to (1) a rise in spending levels in $t$, not fully compensated by future higher taxes, lowers the government's surplus, $\widetilde{S}_t$. The government can finance this shortage either through an increase in inflation or through a decrease in bond prices, $q^j$, experiencing a capital gain in her portfolio. This negative comovement between spending levels and (long) bond prices has been called "fiscal insurance" in the optimal DM literature.

[1]See for example Baxter and Jermann (1997), Coeurdacier et al (2010), Coeurdacier and Gourinchas (2017), Devereux and Sutherland (2007, 2008), Engel and Matsumoto (2009) and Coeurdacier and Rey (2013) amongst numerous others.

Using equation (1) and letting $D_{t-1} = \sum_{j\geq1}^{N} q_{t-1}^{j} B_{t-1}^{j}$ be the market value of debt issued in period $t-1$ , we can rewrite the intertemporal constraint of the government as:

$$\sum_{j\geq1}^{N} \frac{q_t^{j-1}}{\pi_t q_{t-1}^{j}} \frac{q_{t-1}^{j} B_{t-1}^{j}}{D_{t-1}} = \sum_{j\geq1}^{N} \underbrace{R_{t-1,t}^{j}}_{return} \underbrace{w_{t-1}^{j}}_{share} = \frac{\widetilde{S}_t}{D_{t-1}}, \tag{2}$$

where $R_{t-1,t}^{j} \equiv \frac{q_t^{j-1}}{\pi_t q_{t-1}^{j}}$ is the real holding period return of buying a bond of maturity $j$ in $t-1$ and selling it in period $t$ at price $q_t^{j-1}$, adjusted by the realized inflation rate and $w_{t-1}^{j} \equiv \frac{q_{t-1}^{j} B_{t-1}^{j}}{D_{t-1}}$ is the share of bonds of maturity $j$ in the government's portfolio.

In the next sections we will test the presence of fiscal insurance for a set of Euro Area countries using panel SVARs to identify exogenous innovations in government spending and looking at the response of the real payout of debt, $\sum_j R_{t-1,t}^{j} w_{t-1}^{j}$, to these shocks. A negative response of holding period returns to an increase in government spending will be interpreted as evidence of fiscal insurance.

## 2.2 Data and Variables

### 2.2.1 Description of the Data

**Bond Quantities and Prices.** Our dataset contains information on every outstanding security issued by the central governments of the countries considered (Belgium, France, Germany, Italy and Spain) over the period 1998 to 2013. For each asset we can identify the type, nominal or real, the maturity, from 1 quarter to 30 years, as well as the principal and the coupon payments. These observations have been gathered separately for each country. The uniqueness of this dataset lies on the fact that it determines the maturity structure of each country's government debt taking into account each bond issued not relying on any of the aggregated official time series publicly available. Equiza-Goñi (2016) documents the data sources and for brevity we refer the reader to this paper for details.

All countries issued short term zero coupon bonds maturing in less than a year after their issuance. These are called Treasury Certificates or Bills in Belgium, Bubills in Germany, Letras in Spain, BOTs in Italy, or BTFs in France. In general, their share fluctuates around 10 percent of all outstanding debt for each country, except for Germany where Bubills always represent less than 5 percent of total sovereign debt.

The rest of debt is long and several types of assets have been used by different countries. There are medium-term coupon bonds typically maturing 5 years after issuance: Bobls in Germany, BTAN in France, Bonos in Spain, etc.. There are also many types of long-term bonds: OLOs in Belgium, OATs in France, Bunds in Germany, etc. In general, these instruments pay fixed coupons. An exception is Italy that issues CTZs that do not pay coupons, and CCTs which are floating rate bonds.

In order to construct the shares of debt for different maturities, $w^j$ in equation (2), we convert all non-zero coupon bonds in our data set into zero-coupon bonds taking into account both principal repayment and coupons of each bond and their residual maturity; this procedure gives us face values of debt for every maturity $j$. We price the zero coupon bond quantities with the zero-coupon yields, to obtain market values of debt. Nominal bond yield estimates for zero coupon bonds from the BIS database are used to price all payment obligations.

Figure 1 plots the obtained shares of outstanding payment obligations of maturities up to a year (solid lines), between 1 and 4 years (dashed lines) between 4 and 7 years (dotted lines) and above 7 years (dashed-dotted lines).

[Figure 1 About Here]

The behavior of the shares reveals several noteworthy features of DM policy in the Euro Area. The shares of short and long term bonds are positive for all countries and are generally stable over time. Governments tend to increase slightly the share of longer maturities in periods where debt accumulates faster (e.g. towards the end of our sample).

Issuing more long term debt when overall debt rises is a common feature of DM in Europe and in the US.[2] When overall debt rises the refinancing risk increases and debt managers face a trade off between issuing more expensive and less risky debt, long term, or cheaper and riskier debt, short term. In general they prefer to issue long term debt to reduce overall refinancing risks of government portfolios.

It is worth noting that Italy increased the share of long term debt before entering the Euro. Italy faced a more steeply upward sloping yield curve than Germany, France and Belgium before the introduction of the Euro.[3] In relative terms, issuing long bonds was therefore more expensive. When long term yields fell, the Italian government begun substituting some of its short and medium term maturity debt with long bonds.

Though most of the debt issued by Euro area governments is nominal, some medium- and long-term bonds are indexed to inflation. Figure 2 plots the shares of inflation indexed debt in the five countries in our sample, over the period 1998-2013.

[Figure 2 About Here]

At the beginning of the sample the shares of four out of five countries were equal to zero. France (dashed line) started issuing the first inflation indexed bonds in 1998 followed by Italy (line with triangles) in 2002. Between 1998 and 2016 the shares of these two countries have increased steadily and stabilized at around 12 percent of overall debt. In Germany the shares turned positive in 2007 and 2013 respectively, but remained below 5 percent of over all debt throughout the period documented. Belgium and Spain did not use this instrument throughout our sample period. The bonds issued have maturity ranging from 5 years to 32 years. Italy and France issued some bonds indexed to the Euro Area HICP as well as bonds indexed to the national CPI.

Due to the fact that issuances of inflation-indexed bonds were virtually zero for most of the sample period for many countries, our main empirical analysis will focus on nominal debt. In Section 4, however, we will expand our analysis to explore to which extent inflation-indexed bonds have helped the French government to hedge against fiscal shocks.

**GDP, Prices and Spending.** Our SVARs will include government spending, output and price variables. We retrieve real GDP, price indices (GDP deflator) and government spending for all countries considered from the OECD database. Government spending corresponds to consumption of final goods and services by the general government and thus excludes transfers and public investment. Over the period considered mean spending levels range between 21 percent of GDP, in the case of France, to 17 percent, in the case of Spain. Spending varies considerably over time; for example, in Belgium expenditures account for 17 percent of the variability of GDP (i.e. in terms of the ratio of standard deviations), and in Germany for 10 percent.

**Sample Selection.** Our sample covers the period 1998 (Q2) to 2008 (Q2). We exclude the 'turbulent years' of the financial and sovereign debt crises since we wish to focus on years when the markets perceived government default as extremely unlikely for all countries in our sample as assumed in the optimal DM literature. We thus truncate our sample in the second quarter of 2008 just before Lehman-Brothers filed for bankruptcy in September 2008. Moreover, we start the sample 3 quarters before January 1st 1999, the date of the official introduction of the Euro, to allow our VARs to have the sufficiently rich lag structure suggested by the usual criteria.

[2] See Greenwood et al. (2015) for US evidence.

[3] A plausible explanation for higher yields is inflation risk premia. In the early 90s markets feared that the Italian government could resort to inflation to finance its high debt. In the late 90s Italian monetary policy gained credibility because the exchange rate was locked to the ECU.

High long term yields were also the case in Spain, however, this country begun increasing its long term debt issuances even before 1999. By the start of the sample the overall the duration of Spanish debt is similar to the duration in Germany and Belgium. See online appendix.

### 2.2.2 Bond Return Variables

The previous subsection has explained how we have created the shares of outstanding obligations using bond quantities and prices. We now turn to the construction of the holding period returns used in the SVARs.

For maturity $j = 1, 2, ..., 120$, where $j$ denotes quarters, we have calculated the shares of debt for every maturity, $w^{i,j}_{t-1}$, and the holding period returns, $R^{i,j}_{t-1,t}$ using the zero coupon prices, $q^{i,j}_t$, and the GDP deflator to measure inflation, for every country $i$ in each period $t$.

We then construct the following return variables:

$$\mathscr{R}^{i,\underline{j},\overline{j}}_{t-1,t} \equiv \frac{\sum_{j=\underline{j}}^{j=\overline{j}}(R^{i,j}_{t-1,t} - 1)w^{i,j}_{t-1}}{\sum_{j=\underline{j}}^{j=\overline{j}} w^{i,j}_{t-1}}$$

where $w^{i,j}_{t-1}$ is the share of the market value of debt of maturity $j$ over the total market value of debt issued by government $i$ in $t-1$. $\underline{j}$ and $\overline{j}$ are the lowest and upper bounds of maturities included in the composite return $\mathscr{R}^{i,\underline{j},\overline{j}}_{t-1,t}$.

In our empirical analysis we estimate separate VARs assuming different values for $\underline{j}$ and $\overline{j}$. Our baseline model sets $\underline{j} = 1$ and $\overline{j} = 120$ and $\mathscr{R}^{i,1,120}_{t-1,t}$ denotes the return on the overall portfolio of government $i$ (as in the LHS of equation (2)). In order to identify the effects of spending shocks on different maturity segments we also use values for $\{\underline{j}, \overline{j}\} : \{1, 4\}, \{5, 16\}, \{17, 28\}$, and $\{29, 120\}$ corresponding to short, medium, long and very long maturities respectively.

Figures 3 and 4 plot the resulting holding period return series. Figure 3 shows the series of the aggregate portfolio ($\underline{j} = 1, \overline{j} = 120$) for each of the five countries in our sample. The bottom panel displays the inflation adjusted returns and the top panel the unadjusted nominal returns. For completeness, we extend the sample period to include observations up to 2013 as in the previous section.

[Figure 3 About Here]

Figure 4 tracks the behavior of the inflation adjusted returns for each of the different maturity segments considered. The top left panel shows the returns of short term bonds, the top right medium term, the bottom left long term (maturity 4-7 years) and the bottom right very long maturity debt (>7 years).

[Figure 4 About Here]

From the figures it is evident that prior to the financial crisis, the sample used in our VARs, holding period returns were very strongly correlated across countries. The lowest correlation coefficient of the inflation adjusted returns on the aggregate portfolio is 0.85 (between Italy and Belgium). The unadjusted return series are slightly more correlated. However, after 2008 Q3, the cross-sectional correlations became considerably weaker; most notably, the returns on Italian and Spanish debt followed a different pattern than in France and Germany: the correlation coefficient between Italy's return series and the returns on German debt is -0.06; the analogous correlation between Spain and Germany is 0.22.

As it is well known, during the crisis, Italy and Spain faced high credit spreads as financial markets revised upwards their expectations of the two countries to withdraw from the Euro and default on their debt. In contrast, before the crisis (1999-2008), the Euro was considered a stable arrangement and returns were basically aligned across countries.

Figure 4 shows that these patterns are driven mainly by the behavior of the returns on longer maturity bonds. It is evident from the plots that returns of long term bonds exhibited higher volatility and weaker correlations during the crisis period. Before the crisis returns on long term bonds, were more volatile than returns on short maturities, but they were strongly correlated across countries.

Finally, comparing the behavior of the shares shown in Figure 1 and the pattern followed by the return series in Figures 3 and 4, it is evident that changes in the real payout of debt $\mathscr{R}^{i,1,120}_{t-1,t}$ do not reflect changes in the shares $w$.

## 2.3 Identification of Aggregate and Idiosyncratic Spending Shocks

Our goal is to quantify the response of holding returns to government spending shocks and simultaneously to separately identify the impact of aggregate and idiosyncratic shocks. We thus make two identification assumptions. The first assumption serves to identify spending shocks in general and the second to separate the effects of aggregate shocks from the effects of idiosyncratic shocks. To identify spending shocks, we follow Blanchard and Perotti (2002), Burriel et al (2010), Beetsma and Giuliodori (2011) (among others) and assume that government consumption reacts with lags to innovations in output.[4] To separately estimate the impact of aggregate and idiosyncratic shocks, we assume that country specific spending does not impact instantaneously the average spending level of the five countries in our sample.

Formally, let $\mathscr{X}_t^i \equiv \left(\widetilde{g}_t^a, \widetilde{g}_t^i, \widetilde{Y}_t^i, \widetilde{P}_t^i, \mathscr{R}_{t-1,t}^{i,\underline{j},\bar{j}}\right)$ denote the vector of variables that enter in the VAR, where $\widetilde{g}_t^a$ denotes the log of the weighted average of real government spending in $t$ across the five countries in our sample (with weights corresponding to relative GDPs), $\widetilde{g}_t^i$ is the log of real government spending in country $i$[5], $\widetilde{Y}_t^i$ the log of real GDP, $\widetilde{P}_t^i$ the log of the GDP deflator. We detrended each of these variables using a one-sided Hodrick-Prescott filter with a smoothing parameter of 8330, following Berndt et al. (2012) that shows evidence of fiscal insurance in the U.S. also in a VAR setting.[6]

Our baseline specification is the following pooled panel VAR:

$$\mathscr{X}_t = \mathscr{B}(L)\mathscr{X}_{t-1} + \nu_t, \quad \nu_t = \mathscr{M}\epsilon_t \tag{3}$$

where $\mathscr{X}$ stacks the $\mathscr{X}^i$ matrices, $\nu_t$ represents the reduced form innovations, $\epsilon_t$ are the structural shocks which have diagonal variance-covariance matrix $\mathscr{D}$. $\mathscr{B}(L)$ is the lag polynomial in the reduced form VAR. Identification of the structural shocks is achieved through the choice of the square matrix $\mathscr{M}$ that maps the errors $\epsilon_t$ to the reduced form errors $\nu_t$. We include country fixed effects in the estimation. To condense notation we omit them from (3).

We use the Choleski decomposition and assume that $\mathscr{M} = \mathscr{L} \cdot \mathscr{D}^{1/2}$ where $\mathscr{L} \cdot \mathscr{D}^{1/2}$ is a lower diagonal matrix. Given the ordering of the variables in the VAR, we identify the aggregate spending shock as the shock that impacts all variables instantaneously and the idiosyncratic spending shock as the shock that does not impact (within the same period) average spending. Both spending shocks are identified as in Blanchard and Perotti (2002) assuming that spending responds with lags to innovations in prices and output. The return series enters last in $\mathscr{X}$ since both output shocks and price shocks impact returns instantaneously.

Our identification strategy is similar to Bernardini et al (2018) who estimate spending multipliers in US states using Blanchard and Perotti's scheme. The first variable in their VAR is aggregate spending, the second variable is a redistribution index which captures state specific shocks that leave total spending levels unchanged. Employing this strategy in our setting, idiosyncratic shocks capture precisely the component of spending that redistributes the fiscal burden across countries: a positive shock in one country is compensated by the opposite shock in another country. Note that this does not require shocks to occur simultaneously and cancel each other out. Suppose for example that there is a rise in spending in Germany, but no change in spending elsewhere. Then, since Germany is a large country the shock increases the average spending level in the Euro area, hence there is a component which represents an aggregate shock. On top of this, there are also several idiosyncratic

[4] Because our data is quarterly assuming even two lags is reasonable. More lags would probably be less sensible as the use of Blanchard and Perotti's scheme in low frequency (annual) data is seen as problematic (e.g. Ravn et al (2012)). Born and Mueller (2011) have however shown that the scheme works well even with annual data.

[5] Note that as in Perotti (2004) and Burriel et al (2010) we adjusted government spending by adding deviations of the deflator of spending from its trend and assuming an elasticity of 0.5. The trend is obtained through a standard HP filter. In this way we cleaned the government consumption series from cyclical variations in prices.

[6] Notice that the value 8330 is equivalent to removing frequencies of around 15 year. Also, the fact that the HP filter only uses past values assures that the temporal ordering of the data is preserved. Given our short time span, we use a pre-sample from 1981(Q1) to detrend the series. In the online appendix we show that our results are robust towards alternative values of the smoothing parameter, towards not using the pre-sample and/or removing instead simple linear or quadratic trends. Finally, the series of debt returns is not detrended.

shocks. Germany experiences a positive idiosyncratic shock, all of the other countries in our sample experience negative idiosyncratic shocks, keeping their spending levels constant.

This obviously is a simplistic example, since in practice all countries simultaneously experience changes in their spending levels. Changes in the average spending level thus do not originate in one country. However, this example helps to clarify that the opposite is also not true: it is not necessary that fiscal policies in all countries coordinate and increase spending levels, in order to produce an aggregate shock. Idiosyncratic and aggregate shocks will occur simultaneously in our empirical model and they will not load in different periods (see online appendix for evidence on this)[7].

Finally, a few recent papers have drawn caution on the ability of the Blanchard and Perotti identification scheme to isolate exogenous shocks spending in structural VAR models. Ramey (2011) and Leeper, Walker, and Yang (2013) have highlighted the limitations of these models in accounting for 'fiscal foresight', i.e. when fiscal measures not observable to the econometrician are known in advance to private agents. One approach to deal with this problem is to augment the VAR with forecasts errors of spending (e.g. Auerbach and Gorodnichenko (2012)). Another is to augment the VAR with variables that may react to news about spending, for example, bond returns, stock prices, spreads etc (Sims (2012)). The only database which contains forecasts of spending for the countries in our sample is the OECD database, however these are not available at quarterly frequency. On the other hand in our model we use returns on government debt that obviously react to news about spending. For this reason we are confident that our VAR can provide reliable estimates of the structural shocks to spending.[8]

## 2.4 Baseline Estimates

### 2.4.1 Estimates from a SVAR: All Shocks Together

Before considering separately idiosyncratic and aggregate shocks, it is instructive to consider in a first stage the effects of total spending on holding period returns. For this exercise we run model (3) removing average spending from the state vector and letting $\mathscr{X}_t^i \equiv \left(\widetilde{g}_t^i, \widetilde{Y}_t^i, \widetilde{P}_t^i, \mathscr{R}_{t-1,t}^{i,\underline{j},\overline{j}}\right)$.[9] Our estimates then pull together both the aggregate and idiosyncratic shock components, since each of the series $\widetilde{g}_t^i$ is influenced by both types of shocks. Given the ordering, we identify the shocks to spending through the Blanchard and Perotti (2002) scheme. Applying the usual information criteria, our reduced form estimates indicate that including two lags in the polynomial $\mathscr{B}(L)$ is appropriate.

Figure 5 reports the impulse responses of each variable to a one standard deviation shock in 'total' spending where the composite return is $\mathscr{R}_{t-1,t}^{i,1,120}$, the return on the entire portfolio. The top left panel plots the response of spending, the top right output, the bottom left the price level and on the bottom right we plot the response of returns. The dashed lines in each plot indicate the one standard error confidence intervals (equivalent to the 16th and 84th percentiles as is typical in the

[7]See online appendix for graphs showing average and country series of real government consumption that are included in our benchmark VAR (in natural logarithms, detrended, etc), as well as our estimated aggregate and idiosyncratic shocks in 1998-2008. We did the forecast error variance decomposition. We find that 27% of country-specific government spending forecast errors are caused on impact by aggregate shocks, whereas the other 73% are caused by unforeseen idiosyncratic shocks. Obviously, 100% of the variability of average government spending forecasts is produced, on impact, by aggregate shocks.

[8]Following Auerbach and Gorodnichenko (2012), we tested for the significance of the correlation between the components of actual and forecasted growth rates of real government consumption that are orthogonal to the information in our panel VAR. We used in each quarter the closest preceding forecast provided by the OECD in June and December (thus, biannually). The correlation coefficient is 0.1 (with p-value equal to 0.18) for 1998-2008. Similarly, the correlation between our panel VAR innovations to government spending and the residuals obtained from regressing the spending growth forecasts on our VAR lagged values is 0.08 (with p-value of 0.24). Thus, since there is no significant correlation between the additional information of OECD forecasts and the residuals resulting from our VAR, we conclude that fiscal foresight is not an issue in our analysis.

[9]A similar exercise has been performed by Beetsma and Giuliodori (2011) with Euro Area data to identify the effects of spending on output. Their state vector includes spending, output, long term rates and real exchange rates. Their identification assumption is the same as the one employed here.

literature).[10]

Following a shock to government spending, prices increase and output increases albeit not substantially. Notice, however, that the estimated response of real output falls within the range of previous estimates in the literature, which typically give a multiplier close to one for the Euro Area.[11]

[Figure 5 About Here]

Focusing on the bottom right panel of Figure 5, we notice that the return variable drops significantly. Table 1, first column, reports the point estimate: the total return on the portfolio drops by 33.8 basis points (bps) on impact, and this drop is statistically significant. This constitutes a large impact effect of spending on returns.[12] The effect can be interpreted as follows: when spending levels rise, the payout of government debt is lowered and governments enjoy capital gains (equivalently investors suffer capital losses). This provides *strong evidence of fiscal insurance* in the Euro Area.

[Table 1 About Here]

In rows 2 to 5 of the first column of Table 1 we record the effect of varying the maturity of debt on our estimates. The second row reports the magnitude of the response of returns of maturity less than or equal to one year. The remaining rows report the impact effects of the shock on maturities between 1 and 4 years, 4 and 7 and above 7 years respectively. All responses are large and statistically significant. Moreover, the impact effect is larger the longer the maturity of the bonds included in $\mathscr{R}$. This finding is in line with the predictions of theoretical models of optimal DM theory which suggests that governments should use long maturities to exploit better the fiscal insurance channel.[13]

### 2.4.2 Fiscal Hedging against Aggregate and Idiosyncratic Shocks

We now turn our attention to the main focus of our exercise: to identify separately the impacts of aggregate and idiosyncratic shocks on holding period returns. The second and third columns of Table 1 summarize the results of the estimation of the VAR in (3) with $\mathscr{X}_t^i \equiv \left(\widetilde{g}_t^a, \widetilde{g}_t^i, \widetilde{Y}_t^i, \widetilde{P}_t^i, \mathscr{R}_{t-1,t}^{i,\underline{j},\overline{j}}\right)$, as defined in Section 2.3. The second column of the table reports the impact effect on the returns of an aggregate spending shock, the third column the effect of an idiosyncratic shock. The results are as follows: first, in response to an aggregate shock holding period returns drop significantly. The estimated impact is roughly equal to 86 bps, more than twice the impact estimated in the previous subsection. Second, the estimated impact in the case of idiosyncratic shocks is roughly 13 bps, however, it is not statistically significant. This suggests that fiscal hedging is powerful, but only in the case of aggregate shocks.

From rows 2-5 in Table 1 we see that the above findings hold also for each of the maturity segments considered. Across all segments aggregate spending shocks have strong negative effects on returns, and the effects are statistically significant. For idiosyncratic shocks, the impact effects are small and are significant only in the case of short and medium term bonds.

[10] See for example Blanchard and Perotti (2002) or Beetsma and Giuliodori (2011). The confidence intervals are computed assuming normality of errors and using Monte Carlo simulations (1000 replications). We apply the same procedure to all estimates reported in this section.

[11] See for example Burriel et al (2010)). Since spending is about 20 percent output in our sample, a spending shock equivalent to 1 percent of GDP increases output by 0.75 percentage points. Thus, despite our short time span (only 10 years compared to other studies), we obtain a multiplier not far from 1.

[12] Though $\mathscr{R}$ continues to respond to the shock even after the initial period, our focus here is on the initial impact. Given that we have constructed the returns $\mathscr{R}$ using time varying portfolio weights,$w_t^{i,j}$, at impact the weights are predetermined (see previous derivations) but in the subsequent periods the weights may adjust to the shock. This could be due for example to the fact that debt management offices adjust the maturity of new issuances in response to the slope of the yield curve, i.e. issue more of a cheaper security. Focusing on initial period responses we avoid this problem.

[13] It is not surprising that even assets of maturity less than or equal to 1 year give a strong and statistically significant impact. Since our horizon is quarterly the prices of six month and one year nominal debt respond to the shock. Changes in $P$ also exert an influence since we adjusted returns by inflation.

## 2.5 Robustness

The evidence presented so far has two important implications for debt management in the Euro Area. First, fiscal hedging exerts a powerful influence on the behavior of returns on government debt and second, this influence is mainly accounted for by shifts in aggregate spending. We now turn to alternative specifications of our empirical model to test the robustness of our findings. We consider here three different versions of the model: first, we replace the pooled VAR in (3) with separate VARs for each country employing the same identification strategy of Section 2.3 but now averaging the estimates over the five countries in our sample. Second, we replace the average spending level (the first variable in the pooled VAR in (3)) with the first principal component of the spending series of the countries and run another pooled panel. Third, we run again model (3) replacing returns $\mathscr{R}$ with holding period returns not adjusted by inflation.

### 2.5.1 Evidence from Separate VARs

Since our sample contains 41 observations for each country, the advantage of a pooled VAR is that it limits the number of parameters that needs to be estimated. On the other hand, a pooled VAR can be criticized for imposing too much structure and restrictions on the estimated coefficients. In this section we put aside the efficiency gains of the pooled panel and allow for heterogeneity in the estimated coefficients. We run the following model:

$$\mathscr{X}_t^i = \mathscr{B}^i(L)\mathscr{X}_{t-1}^i + \nu_t^i, \quad \nu_t^i = \mathscr{M}^i \epsilon_t^i \tag{4}$$

where superscript $i$ denotes a country. The state vector $\mathscr{X}^i$ is as defined before. The lag polynomials $\mathscr{B}^i(L)$ are again of second order. After estimating model (4) we again apply the Cholesky decomposition to identify the structural innovations to spending. Table 2 reports the results. The impact effects now correspond to the average responses from the five VARs.[14]

[Table 2 About Here]

In column 2 of the table we report the impacts of aggregate shocks.[15] Notice that aggregate shocks continue to yield considerable impacts on returns. This confirms our previous findings that movements in aggregate spending in the Euro Area lead to debt devaluations.

Column 3 shows the analogous impacts of idiosyncratic shocks. Now all coefficients are insignificant and moreover have the 'wrong sign': returns increase in response to spending shocks so that governments experience capital losses. This holds for the overall portfolio as well as for the different maturity segments considered in the second to fifth rows of the table.

Our previous findings are therefore confirmed when we allow for heterogeneity in the estimated parameters. Aggregate shocks are the key driving force behind fiscal hedging in the Euro Area.

### 2.5.2 Using First Principal Component for Aggregate Shocks

Our identification of aggregate and idiosyncratic shocks relied on the assumption that idiosyncratic disturbances do not influence the average spending series. In Section 3 we will use a theoretical model to show that shocks which do not influence average spending are not insurable when governments issue nominal bonds. The identifying assumption we have employed so far will thus match the model's definition of aggregate and idiosyncratic shocks. Nevertheless, in an empirical section devoted to fiscal hedging, it is important to establish robustness towards using alternative series as first variables in the VAR. We experiment with using the first principal component ($\mathscr{P}$) of the spending series of the five countries in our sample. We run a panel VAR as in (3) with $\mathscr{X}_t^i \equiv \left(\mathscr{P}_t, \widetilde{g}_t^i, \widetilde{Y}_t^i, \widetilde{P}_t^i, \mathscr{R}_{t-1,t}^{i,j,\bar{j}}\right)$.

[Table 3 About Here]

[14] The averages are weighted by relative GDP as we did to construct $\widetilde{g}^a$.

[15] For completeness column 1 reports the estimated responses for total spending as in Section 2.4.1, now allowing for heterogeneous coefficients across countries.

The results are shown in the first two columns of Table 3. For brevity we only report here the impacts of aggregate and idiosyncratic shocks separately. The estimated responses are similar to those reported in Table 2. Although they are quantitatively smaller in the case of aggregate shocks, they are significant, whereas responses to idiosyncratic shocks are not significant. Therefore, using the first principal component, as opposed to the weighted average of spending as first variable in the VAR, makes very little difference for our findings.

For completeness, columns 3-4 of Table 3 report the estimates of the model of subsection 2.3.1 allowing for heterogeneity in coefficients. The results are similar to the ones documented in Table 2. We continue to find very strong evidence of fiscal hedging against aggregate shocks and little evidence against country specific shocks.

### 2.5.3 Returns not Adjusted by Inflation

As explained in section 2.2.2 to construct $\mathscr{R}$ we divided the return from buying a nominal asset in $t-1$ and selling it in $t$ by the rate of inflation. In order to isolate the impact of spending shocks on nominal bond prices we now run VAR (3) replacing $\mathscr{R}$ with holding period returns not adjusted by inflation. The results are shown in Table 4. Notice that, not surprisingly, now the estimated coefficients are smaller in absolute value, as these returns do not reflect unexpected higher (lower) current inflation due to positive (negative) spending shocks. However, they remain strongly significant in the case of aggregate shocks (except for maturities shorter than one year) and not significant in the case of idiosyncratic shocks.

Therefore, part of the response of bond returns $\mathscr{R}$ to spending shocks is accounted for by inflation but aggregate shocks have substantial impacts on nominal bond returns.

[Table 4 About Here]

### 2.5.4 Further Robustness Checks

Our main empirical analysis focuses on the subperiod 1999-2008. As discussed previously, the fiscal hedging argument presupposes that countries are creditworthy and for this reason we have dropped the years of the financial crisis. During the crisis Italy and Spain experienced high spreads because investors considered more likely that these countries would default and exit the Euro. Therefore if we add post 2008 observations to our VARs we could obtain negative effects of spending shocks on returns because higher spending increases debt and makes default more likely.[16] We would thus confound the standard fiscal hedging impact of shocks on returns with the default impact.

Nevertheless it is interesting to extend our estimates to the crisis period and for this reason we now run VARs including observations up to 2013. The results are shown in Table 5 , where the top panel reports the pooled panel estimates and the bottom panel the estimates from running separate VARs (as in Section 2.5.1). The results don't change when we use the longer sample. Aggregate shocks continue to affect returns, whereas idiosyncratic shocks do not.

[Table 5 About Here]

The online appendix presents additional robustness checks of the main results reported in this section. In particular, we have used alternative detrending methods (including linear and quadratic trends), a richer lag structure or different inflation measures. We have found essentially the same results: fiscal hedging is driven by aggregate shocks in the Euro Area.

[16]Note that this could also mean that idiosyncratic shocks would have a significant impact. If a rise in spending increases the spread of one country, then since part of the shock is idiosyncratic, returns would comove negatively with the idiosyncratic shock.

# 3 Model and Ramsey Policy

## 3.1 Model

The empirical findings of the previous section indicated that debt management in the Euro Area benefited from fiscal insurance against spending shocks. However, this only held for aggregate shocks. Governments could not insure against idiosyncratic shocks through nominal assets.

We now turn to the theory and use a theoretical model in the spirit of the optimal DM literature to interpret our empirical results but also to investigate what types of debt are optimal in a currency union.

Our baseline model is a Ramsey policy equilibrium in a currency area which consists of two countries, $A$ and $B$. The model can be seen as a variant of (adding another country to) the models of Angeletos (2002) and Buera and Nicolini (2004). We assume that a benevolent planner under full commitment sets taxes and issues debt to finance exogenous shocks to government spending. Markets are complete. In addition, the model features monopolistic competition and sticky prices as in Siu (2004) and Lustig et al (2008) and it is close to Faia and Monacelli (2004). In this section we first derive the equilibrium and setup the planning problem under the assumption governments can issue debt in state contingent securities. Then, we ask if the optimal allocation can be decentralized through DM.

### 3.1.1 Uncertainty

Uncertainty in our model derives from fluctuations in the spending levels of the two countries. Let $G_t^i$ denote the level of exogenous government spending in country $i = A, B$. We assume that spending has two components: a common component denoted by $g_t^c$, which influences the spending levels of the two countries symmetrically, and a country specific component $g_t^i$, which is i.i.d across countries. We assume $G_t^i = g_t^c + g_t^i$.

$G_t^i$, $g_t^c$ and $g_t^i$ are first order Markov processes. Let $N$ be the total number of possible realizations of the vector $G_t = [G_t^A, G_t^B]$ of joint spending. This joint process evolves according to the transition matrix $\overline{\mu}$. Finally, let $s_t \in \{s_1, ..., s_N\}$ denote the state in $t$ and $s^t$ represent the history of shocks from dates 0 to $t$.

The above parameterization of uncertainty is typical in multi-country business cycle models[17] and it is useful to make our theoretical analysis easily comparable to the empirical analysis of the previous section. The spending process assumed has both an idiosyncratic and an aggregate component; as in the data, we will identify idiosyncratic shocks taking deviations of $G_t^i$ from the average spending level and aggregate shocks as shocks to average spending.

To clarify this, consider the following example. Suppose we have $N = 4$, $g_t^c$ constant and $g_t^i \in \{\underline{g}, \overline{g}\}$. The vector of possible outcomes is:

$$\left(G_t^A - g^c, G_t^B - g^c\right) \in \left\{ \left(\underline{g}, \underline{g}\right), \left(\overline{g}, \underline{g}\right), \left(\underline{g}, \overline{g}\right), \left(\overline{g}, \overline{g}\right) \right\}$$

and average spending is

$$\frac{\sum_i G_t^i}{2} \in \left\{ g^c + \underline{g}, g^c + \frac{\overline{g} + \underline{g}}{2}, g^c + \frac{\overline{g} + \underline{g}}{2}, g^c + \overline{g} \right\}.$$

In this example, a shock which shifts $G_t$ from state 1 to state 4 is an aggregate shock; the difference between the spending in country $i$ and the average spending, $G_t^i - \frac{\sum_i G_t^i}{2}$, equals 0. A shock which shifts spending from state 2 to state 3 is an idiosyncratic shock, since there is no shift in the average.

[17] See for example, Krueger et al (2011) for an analogous parameterization of TFP shocks in a multicountry setup.

### 3.1.2 Preferences, Aggregators and Price Vectors

Each country is populated by a representative household, and therefore countries are of equal size. Households have identical preferences and for $i = A, B$ the discounted utilities are:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^i, N_t^i), \tag{5}$$

where $C_t^i$ is consumption in period $t$ in country $i$ and $N_t^i$ denotes aggregate hours. We define $C_t^i$ as

$$C_t^i(s^t) = \left[ (1-\alpha)^{\frac{1}{\eta}} \left( C_{i,t}^i(s^t) \right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left( C_{-i,t}^i(s^t) \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{6}$$

where $C_{i,t}^i$ is the consumption in country $i$ of a composite bundle of goods produced in $i$ and $C_{-i,t}^i$ is the composite good produced abroad and consumed in $i$. $\alpha$ governs the degree of home bias in preferences. If $\alpha < \frac{1}{2}$, country $i'$s consumption is biased towards the domestic good. $\eta$ determines the elasticity of substitution between home and foreign goods.

We assume that $C_{j,t}^i$, $i, j = A, B$, are aggregates over infinitely many varieties of differentiated products with the aggregates defined as:

$$C_{A,t}^i(s^t) = \left[ \int_0^1 \left( C_{A,t}^i(s^t, a) \right)^{\frac{\epsilon-1}{\epsilon}} da \right]^{\frac{\epsilon}{\epsilon-1}} \quad \text{and} \quad C_{B,t}^i(s^t) = \left[ \int_0^1 \left( C_{B,t}^i(s^t, b) \right)^{\frac{\epsilon-1}{\epsilon}} db \right]^{\frac{\epsilon}{\epsilon-1}},$$

where $\epsilon$ governs the elasticity of substitution across differentiated varieties.

Goods $a$ and $b$ have the same price in both countries and thus the law of one price holds. The price indices for the goods produced in countries $A$ and $B$ are:

$$P_t^A(s^t) = \left[ \int_0^1 \left( P_t^A(s^t, a) \right)^{1-\epsilon} da \right]^{\frac{1}{1-\epsilon}} \quad \text{and} \quad P_t^B(s^t) = \left[ \int_0^1 \left( P_t^B(s^t, b) \right)^{1-\epsilon} db \right]^{\frac{1}{1-\epsilon}}.$$

Standard results give us the following demand functions:

$$C_{A,t}^i(s^t, a) = \left( \frac{P_t^A(s^t, a)}{P_t^A(s^t)} \right)^{-\epsilon} C_{A,t}^i(s^t) \quad \text{and} \quad C_{B,t}^i(s^t, b) = \left( \frac{P_t^B(s^t, b)}{P_t^B(s^t)} \right)^{-\epsilon} C_{B,t}^i(s^t)$$

for generic products $a$ (produced in $A$) and $b$ (produced in $B$) consumed in $i$.

From (6) the optimal choices of the bundles $C_{j,t}^i$ are given by:

$$C_{i,t}^i(s^t) = (1-\alpha) \left( \frac{P_t^i(s^t)}{P_{C,t}^i(s^t)} \right)^{-\eta} C_t^i(s^t) \quad \text{and} \quad C_{-i,t}^i(s^t) = \alpha \left( \frac{P_t^{-i}(s^t)}{P_{C,t}^i(s^t)} \right)^{-\eta} C_t^i(s^t),$$

where $P_{C,t}^i(s^t) = \left[ (1-\alpha) P_t^i(s^t)^{1-\eta} + \alpha P_t^{-i}(s^t)^{1-\eta} \right]^{\frac{1}{1-\eta}}$ is the consumer price index in country $i$.

### 3.1.3 Asset Markets

Our aim is to characterize policy in an equilibrium with complete markets. As discussed previously, we will first solve the households' and the governments' problems under the assumption that agents trade intertemporally with state contingent assets. Let $D_t(s^{t+1})$ denote a state contingent security bought in $t$, which pays one unit of income in $t+1$ if history $s^{t+1}$ is realized. $Q_t(s^{t+1})$ is the price of this asset. Moreover, assume that $D_t^{i,G}(s^{t+1})$ denotes the quantity of this state contingent asset purchased by the government in $i$ in period $t$. Analogously, $D_t^{i,H}(s^{t+1})$ is the quantity bought by the household in $i$.

We write the budget constraint of the government in country $i$ as:

$$D_{t-1}^{i,G}(s^t) + P_t^i(s^t)G_t^i(s^t) = \sum_{s^{t+1}} D_t^{i,G}(s^{t+1})Q_t(s^{t+1}) + \tau_t^i(s^t)W_t^i(s^t)N_t^i(s^t), \tag{7}$$

where $\tau_t^i(s^t)$ denotes the tax rate levied on the labor income of country $i$s household, $W_t^i(s^t)N_t^i(s^t)$. $P_t^i(s^t)G_t^i(s^t)$ is the nominal value of spending of the government.

The household's budget constraint can be written as

$$P_{C,t}^i(s^t)C_t^i(s^t) + \sum_{s^{t+1}} D_t^{i,H}(s^{t+1})Q_t(s^{t+1}) = D_{t-1}^{i,H}(s^t) + (1-\tau_t^i(s^t))W_t^i(s^t)N_t^i(s^t) + \Pi_t^i(s^t), \tag{8}$$

where $\Pi_t^i(s^t)$ denotes profits from monopolistically competitive firms in country $i$.[18]

### 3.1.4 Household Optimization: Choice of Hours and Assets

Households maximize (5) subject to the sequence of budget constraints (8). From the first order conditions of this program, which are standard and we omit for brevity, it is straightforward to show that

$$\frac{W_t^i(s^t)}{P_{C,t}^i(s^t)}(1-\tau_t^i(s^t)) = -\frac{U_{N,t}^i(s^t)}{U_{C,t}^i(s^t)} \tag{9}$$

characterizes the optimal choice of hours and moreover, state-contingent debt prices $Q_t(s^{t+1})$ satisfy

$$Q_t(s^{t+1}) = \beta \frac{P_{C,t}^i(s^t)}{P_{C,t+1}^i(s^{t+1})} \frac{U_{C,t+1}^i(s^{t+1})}{U_{C,t}^i(s^t)} \pi(s^{t+1}|s^t). \tag{10}$$

From (10) we can derive the following risk sharing condition which equates the ratio of marginal utilities to the consumption based real exchange rate under complete markets (see Backus and Smith (1993) and Kollmann (1995)): $\kappa \frac{U_{C,t}^B(s^t)}{U_{C,t}^A(s^t)} = \frac{P_{C,t}^B(s^t)}{P_{C,t}^A(s^t)}$. $\kappa$ is a constant which reflects the relative wealth endowments of the private sectors in the two countries. For the rest of our analysis we will assume that the two countries have the same initial wealth as we want to focus on symmetric equilibria. We thus set $\kappa = 1$ and

$$\frac{U_{C,t}^B(s^t)}{U_{C,t}^A(s^t)} = \frac{P_{C,t}^B(s^t)}{P_{C,t}^A(s^t)}. \tag{11}$$

### 3.1.5 Firms: Flexible and Sticky Prices

Aggregate output is composed by a continuum of differentiated intermediate products. Each product is produced by a monopolistically competitive firm operating a linear technology and labor is the sole input in production. We assume that a fraction $\nu$ of the intermediate goods firms, in both countries, have to set their prices one period in advance. The remaining firms are flexible price firms and their optimal price is set within the period. To simplify, we denote sticky price firms using a superscript $\mathscr{s}$; $\mathscr{f}$ denotes flexible price firms.

[18]Notice that we assume complete home bias in equities. This assumption is made for simplicity. It may seem strict for the Euro Area, however, with fiscal shocks being the only source of risk and under complete bond markets, the model does not give an incentive to households to hedge by buying foreign equity. Analogously, the complete market model will have little to say about whether government bonds should be held by domestic or foreign citizens.

The generic $\mathscr{f}$ type firm, $\imath$, in country $i$ solves a static optimization program, maximizing profits subject to the demand curve given by $Y_{t,\imath}^{i,\mathscr{f}}(s^t) = \left(\frac{P_{t,\imath}^{i,\mathscr{f}}(s^t)}{P_t^i(s^t)}\right)^{-\epsilon} Y_t^i(s^t)$ where $Y_t^i(s^t)$ denotes aggregate output produced in $i$. The optimal price level is given by

$$P_{t,\imath}^{i,\mathscr{f}}(s^t) = \frac{\epsilon}{\epsilon - 1} W_t^i(s^t). \tag{12}$$

$\mathscr{s}$ type firms set prices to maximize profits subject to demand, conditional on date $t-1$ information. For brevity we state this standard program in the appendix where we also show that the optimal price solves:

$$P_{t,\imath}^{i,\mathscr{s}}(s^{t-1}) = \frac{\epsilon}{\epsilon - 1} \frac{E_{t-1}\left(\frac{U_{C,t}^i(s^t)}{P_{C,t}^i(s^t)} W_t^i(s^t) Y_t^i(s^t) P_t^i(s^t)^{\epsilon}\right)}{E_{t-1}\left(\frac{U_{C,t}^i(s^t)}{P_{C,t}^i(s^t)} Y_t^i(s^t) P_t^i(s^t)^{\epsilon}\right)}, \tag{13}$$

where $U_{C,t}^i$ denotes the marginal utility of consumption used by the household to evaluate profits.

Finally, since all flexible (sticky) price firms in country $i$ set the same price level, the price index of goods produced in $i$, $P_t^i$ is written as:

$$P_t^i(s^t) = \left[\nu P_{t,\imath}^{i,\mathscr{s}}(s^t)^{1-\epsilon} + (1-\nu) P_{t,\imath}^{i,\mathscr{f}}(s^t)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}.$$

### 3.1.6 Resource constraints

Following Gali and Monacelli (2008) we assume that government spending is allocated exclusively to domestic goods. Aggregate output in country $i$ is

$$Y_t^i(s^t) = G_t^i(s^t) + C_{i,t}^A(s^t) + C_{i,t}^B(s^t). \tag{14}$$

In equilibrium, output equals total hours. Letting $N_t^i = \left[\int_0^1 \left(N_{t,\imath}^i\right)^{\frac{\epsilon-1}{\epsilon}} d\imath\right]^{\frac{\epsilon}{\epsilon-1}}$ and using the expressions for consumption demands derived previously, we can express (14) as

$$N_t^i(s^t) = G_t^i(s^t) + (1-\alpha)\Phi_t^i(s^t)^{\eta} C_t^i(s^t) + \alpha\left(\frac{P_{C,t}^{-i}(s^t)}{P_{C,t}^i(s^t)}\right)^{\eta} \Phi_t^i(s^t)^{\eta} C_t^{-i}(s^t), \tag{15}$$

where $\Phi_t^i(s^t) \equiv \frac{P_{C,t}^i(s^t)}{P_t^i(s^t)}$.

## 3.2 Equilibria with Complete Markets and Ramsey Policies

### 3.2.1 Implementability

A competitive equilibrium is a set of allocations, prices and tax policies such that households and firms optimize and markets clear. The Ramsey planner chooses from the set of competitive equilibria the one that maximizes welfare. As it is common in the literature (e.g. Lucas and Stockey (1983)), we adopt the primal approach and we simplify the planner's program by dispensing with some variables and constraints. For the sake of brevity, we relegate all derivations to the online appendix. We define here the set of sufficient implementability conditions for the Ramsey program.

**Proposition 1: Implementability, Complete Markets**

*An allocation* $\{C_t^i, C_{i,t}^i, C_{-i,t}^i\}$, $\{N_t^i, N_{t,\imath}^{i,\mathscr{s}}, N_{t,\imath}^{i,\mathscr{f}}\}$, $\{Y_t^i, Y_{t,\imath}^{i,\mathscr{s}}, Y_{t,\imath}^{i,\mathscr{f}}\}$ *with prices* $\{P_{C,t}^i, P_t^i, P_{t,\imath}^{i,\mathscr{s}}, P_{t,\imath}^{i,\mathscr{f}}, W_t^i\}$ *and policies* $\{\tau_t^i\}$ *is a competitive equilibrium if and only if the following equations hold:*

$$N_t^i(s^t) = \begin{cases} G_t^i(s^t) + (1-\alpha)(\Phi_t^i(s^t))^{\eta} C_t^i(s^t) + \alpha \omega_t(s^t)^{-\eta} (\Phi_t^i(s^t))^{\eta} C_t^{-i}(s^t) & \text{if } i = A \\ G_t^i(s^t) + (1-\alpha)(\Phi_t^i(s^t))^{\eta} C_t^i(s^t) + \alpha \omega_t(s^t)^{\eta} (\Phi_t^i(s^t))^{\eta} C_t^{-i}(s^t) & \text{if } i = B \end{cases} \tag{16}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t S_t^i(s^t) = D_{-1}^{i,G} \frac{U_{C,0}^i(s^0)}{P_{C,0}^i(s^0)} \tag{17}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t U_{C,t}^i(s^t) \left( C_t^i(s^t) + \frac{G_t^i(s^t)}{\Phi_t^i(s^t)} - \frac{N_t^i(s^t)}{\Phi_t^i(s^t)} \right) = (D_{-1}^{i,H} - D_{-1}^{i,G}) \frac{U_{C,0}^i(s^0)}{P_{C,0}^i(s^0)} \tag{18}$$

$$E_{t-1} \left( U_{C,t}^i(s^t) N_t^i(s^t) \frac{\left( \frac{N_{t,i}^{i,\mathscr{s}}(s^t)}{N_t^i(s^t)} \right)^{\frac{\epsilon-1}{\epsilon}}}{\Phi_t^i(s^t)} \right) = E_{t-1} \left( U_{C,t}^i(s^t) N_t^i(s^t) \frac{\left( \frac{N_{t,i}^{i,\mathscr{f}}(s^t)}{N_t^i(s^t)} \right)^{-\frac{1}{\epsilon}}}{\Phi_t^i(s^t)} \frac{N_{t,i}^{i,\mathscr{s}}(s^t)}{N_t^i(s^t)} \right) \tag{19}$$

$$\frac{\Phi_t^B(s^{t-1}, s_k)}{\Phi_t^B(s^{t-1}, s_1)} \left( \frac{N_{t,i}^{B,\mathscr{s}}(s^{t-1}, s_1) N_t^B(s^{t-1}, s_k)}{N_{t,i}^{B,\mathscr{s}}(s^{t-1}, s_k) N_t^B(s^{t-1}, s_1)} \right)^{-\frac{1}{\epsilon}} = \frac{\omega_t(s^{t-1}, s_1)}{\omega_t(s^{t-1}, s_k)} \frac{\Phi_t^A(s^{t-1}, s_k)}{\Phi_t^A(s^{t-1}, s_1)} \left( \frac{N_{t,i}^{A,\mathscr{s}}(s^{t-1}, s_1) N_t^A(s^{t-1}, s_k)}{N_{t,i}^{A,\mathscr{s}}(s^{t-1}, s_k) N_t^A(s^{t-1}, s_1)} \right)^{-\frac{1}{\epsilon}}, \tag{20}$$

*$k = 2, 3, ..N$ and where $\omega_t \equiv \frac{U_{C,t}^A}{U_{C,t}^B}$ and $\Phi_t^i$ in equilibrium is a function of $\omega_t$, $\Phi_t^i = \Phi^i(\omega_t)$.*

**Proof**: See Online Appendix

Let us briefly describe these objects. (16) is the resource constraint of the economy that we obtain from (15) after substituting out prices. (17) is the intertemporal budget constraint of the government, stating that at period 0 the outstanding value of government debt $D_{-1}^{i,G}$ in country $i$ must be equal to the present discounted value of current and future government surpluses, where $S_t^i \equiv (\tau_t^i W_t^i N_t^i - P_t^i G_t^i) \frac{U_{C,t}^i}{P_{C,t}^i}$. Under complete markets (17) is sufficient for the equilibrium, and it is not necessary to keep track of the intertemporal budget constraints after period 0 (see e.g. Buera and Nicolini (2004)). (18) is the 'intertemporal current account constraint' of the household in $i$.

Finally, (19) is the first order condition of the sticky price firm and (20) is the so called 'sticky price constraint' (see e.g. Siu (2004)) which states that $P_{t,i}^{i,\mathscr{s}}(s^{t-1})$ is constant across all $s^t$ given $s^{t-1}$ or that the price of an $\mathscr{s}$ type firm set in $t-1$ is fixed and independent of the state $s_t$.

Since prices are sticky, we cannot dispense with these equations; (19) and (20) are necessary implementability conditions because this is an open economy model. In the closed economy setup these conditions can be satisfied as residuals.[19] However, in the open economy constraints (19) and (20) will not generally be 'slack' and therefore need to be accounted for in Ramsey policy. In the online appendix we demonstrate this property with an analytical example.

### 3.2.2 Ramsey Policies

The Ramsey planner seeks to maximize:

$$\max_{\left\{C_t^i, C_{i,t}^i, N_{t,i}^{i,\mathscr{s}}, N_{t,i}^{i,\mathscr{f}}\right\}} \sum_{i=A,B} \sum_{t=0}^{\infty} \sum_{s^t} \mu(s^t) \beta^t \left\{ U(C_t^i(s^t), N_t^i(s^t)) \right\}$$

[19] The reader can verify this by noting that $D_{t-1}^{i,G}(s^t)$ and $D_{t-1}^{i,H}(s^t)$, which satisfy the intertemporal constraints of governments and households, are not uniquely determined, however the ratios $\frac{D_{t-1}^{i,G}(s^t)}{P_{C,t}^i}$ and $\frac{D_{t-1}^{i,H}(s^t)}{P_{C,t}^i}$ are determined. Thus given a price sequence which satisfies 19) and (20) we can find values for $D_{t-1}^{i,G}(s^t)$ and $D_{t-1}^{i,H}(s^t)$ to satisfy intertemporal solvency.

subject to the implementability conditions (16) to (20).

For brevity we leave to the appendix the derivation of the Lagrangean for the Ramsey program and the first order conditions, which together with constraints (16) to (20) give us the system of equations that we need to solve to characterize the Ramsey policy. We also discuss in detail the numerical algorithm we employ to approximate the solution to the Ramsey program. A noteworthy feature of the solution is that, since in (19) we have expectations of future variables, the optimal allocation is history dependent. As in Faraglia et al (2010) this makes ours a non-standard application of the complete market model since typically under complete markets allocations are only influenced by the current state $s_t$ (e.g. ABN). In this set up the only case where allocations are history independent is when we assume $corr(G_t^A, G_t^B) = 1$, i.e. when shocks are perfectly correlated across countries. Under this assumption our model essentially becomes a closed economy model and constraints (19) and (20) can be dropped.

# 4 Debt Management with Non-State Contingent Debt

We now investigate whether the complete market allocation can be decentralized through non-state contingent bonds. The arguments developed in this section follow closely ABN and Faraglia et al (2010). We attempt to recover optimal government portfolios solving a linear system of equations which represents the government's intertemporal budget constraints for each possible state of the economy. If we can find a solution to this system in every period $t$ then portfolios of non-state contingent assets can replicate the payoffs of Arrow Debreu securities and markets can be completed. If not, then we cannot decentralize the complete market allocation with non-state contingent debt and conclude that markets are effectively incomplete.

## 4.1 Introducing Non State Contingent Bonds

### 4.1.1 Nominal Bonds

Let us first assume that governments can issue nominal bonds in different maturities. Also assume that the private sector in each country can buy (or sell) any security. Let $\mathcal{J}$ denote the set of $N$ maturities issued. The number of assets in the market is then equal to the number of the possible realizations of $G_t$ and in theory enough to complete the markets. Let $\mathcal{J}_k$ denote the $k$-th element of $\mathcal{J}$.

All bonds are nominal. $q_{n,t}^{i,j}$ is the price of a nominal asset, $n$, which promises one unit of income in period $t+j$ and is issued by the government in country $i$.

A bond of maturity $j$ issued by a government in country $A$ is priced $q_{n,t}^{A,j} = \beta^j E_t\left(\frac{U_{C,t+j}^A}{U_{C,t}^A}\frac{P_{C,t}^A}{P_{C,t+j}^A}\right)$ by the home household according to his first order condition. The same asset is priced $q_{n,t}^{A,j} = \beta^j E_t\left(\frac{U_{C,t+j}^B}{U_{C,t}^B}\frac{P_{C,t}^B}{P_{C,t+j}^B}\right)$ by the household in country $B$. Then under no arbitrage this price needs to satisfy:

$$q_{n,t}^{A,j} = \beta^j E_t\left(\frac{U_{C,t+j}^A}{U_{C,t}^A}\frac{P_{C,t}^A}{P_{C,t+j}^A}\right) = \beta^j E_t\left(\frac{U_{C,t+j}^B}{U_{C,t}^B}\frac{P_{C,t}^B}{P_{C,t+j}^B}\right). \tag{21}$$

The same holds for a bond of the same maturity issued by the government in country $B$. Given that the payoff of a bond of maturity $j$ is the same, and equal to one, in both countries under no arbitrage the above conditions imply that the price of nominal debt of maturity $j$ issued by each country is equal and $q_{n,t}^j = q_{n,t}^{A,j} = q_{n,t}^{B,j}$.

Denote by $B_{n,t}^{i,j}$ the face value of nominal debt, $n$, of maturity $j$ issued by the government in $i$ in period $t$. Following ABN, Faraglia et al (2010) and Lustig et al (2008) we assume that all debt is bought back one period after issuance. The period budget constraint of the government in country

$i$ is given by:

$$\sum_{j\in\mathscr{J}} q_{n,t}^{j-1} B_{n,t-1}^{i,j} = \sum_{j\in\mathscr{J}} q_{n,t}^{j} B_{n,t}^{i,j} + P_{C,t}^{i} \frac{S_t^i}{U_{C,t}^i}. \tag{22}$$

According to (22) the government finances its fiscal deficit, $-P_{C,t}^{i}\frac{S_t^i}{U_{C,t}^i}$, and the outstanding debt level, $\sum_{j\in\mathscr{J}} q_{n,t}^{j-1} B_{n,t-1}^{i,j}$, with a portfolio of newly issued bonds. Iterating forward we can retrieve the intertemporal budget constraint of the government:

$$\sum_{j\in\mathscr{J}} \frac{q_{n,t}^{j-1}}{(1+\pi_t^i)} \frac{B_{n,t-1}^{i,j}}{P_{C,t-1}^{i}} = E_t \sum_{k=0}^{\infty} \beta^k \frac{S_{t+k}^i}{U_{C,t}^i} \equiv \Upsilon_t^i. \tag{23}$$

Note that this constraint needs to be satisfied for every history $s^t$. As ABN and Faraglia et al (2010), we use (23) to characterise the optimal government portfolios. The bond quantities need to solve the following systems of equations:

$$\underbrace{\begin{bmatrix} \frac{q_{n,t}^{\mathscr{J}_1-1}(s^{t-1},s_1)}{1+\pi_t^i(s^{t-1},s_1)} & \frac{q_{n,t}^{\mathscr{J}_2-1}(s^{t-1},s_1)}{1+\pi_t^i(s^{t-1},s_1)} & \cdots & \frac{q_{n,t}^{\mathscr{J}_N-1}(s^{t-1},s_1)}{1+\pi_t^i(s^{t-1},s_1)} \\ \vdots & \cdots & & \\ \frac{q_{n,t}^{\mathscr{J}_1-1}(s^{t-1},s_N)}{1+\pi_t^i(s^{t-1},s_N)} & \frac{q_{n,t}^{\mathscr{J}_2-1}(s^{t-1},s_N)}{1+\pi_t^i(s^{t-1},s_N)} & \cdots & \frac{q_{n,t}^{\mathscr{J}_N-1}(s^{t-1},s_N)}{1+\pi_t^i(s^{t-1},s_N)} \end{bmatrix}}_{\mathscr{Q}_{n,t}} \cdot \underbrace{\begin{bmatrix} \frac{B_{n,t-1}^{i,\mathscr{J}_1}(s^{t-1})}{P_{C,t-1}^i(s^{t-1})} \\ \vdots \\ \frac{B_{n,t-1}^{i,\mathscr{J}_N}(s^{t-1})}{P_{C,t-1}^i(s^{t-1})} \end{bmatrix}}_{\mathscr{B}_{n,t}} = \underbrace{\begin{bmatrix} \Upsilon_t^i(s^{t-1},s_1) \\ \vdots \\ \Upsilon_t^i(s^{t-1},s_N) \end{bmatrix}}_{\Upsilon_t}$$

.The above system gives a unique solution $\mathscr{B}_{n,t} = (\mathscr{Q}_{n,t})^{-1}\Upsilon_t$ for optimal debt maturities if $\mathscr{Q}_{n,t}$ is invertible. If the solution exists then governments can generate the required state contingent payoffs in $t$ through issuing bonds quantities in $t-1$ and exploiting ex post variation in bond returns in $t$ to complete the markets.

In the next subsections we will provide several numerical examples which show that $\mathscr{Q}_{n,t}$ is an ill-conditioned/nearly non-invertible matrix. Before doing so we lay out a simple argument to explain why that is the case. Without loss of generality consider the case where $N = 4$ assuming also that spending follows the process:

$$(G_t^A, G_t^B) \in \Big\{ (\underline{g},\underline{g}), (\overline{g},\underline{g}), (\underline{g},\overline{g}), (\overline{g},\overline{g}) \Big\}. \tag{24}$$

for some transition probabilities $\overline{\mu}$. As explained in section (3.1.1), idiosyncratic shocks occur in states $s_2$ and $s_3$.

Assume that the government issues four different maturities $\mathscr{J} = \{\mathscr{J}_1, \mathscr{J}_2, \mathscr{J}_3, \mathscr{J}_4\},$. Then, the bond prices matrix $\mathscr{Q}_{n,t}$ of the outstanding debt is:

$$\mathscr{Q}_{n,t} = \begin{bmatrix} \frac{q_{n,t}^{\mathscr{J}_1-1}(s^{t-1},s_1)}{1+\pi_t^i(s^{t-1},s_1)} & \frac{q_{n,t}^{\mathscr{J}_2-1}(s^{t-1},s_1)}{1+\pi_t^i(s^{t-1},s_1)} & \frac{q_{n,t}^{\mathscr{J}_3-1}(s^{t-1},s_1)}{1+\pi_t^i(s^{t-1},s_1)} & \frac{q_{n,t}^{\mathscr{J}_4-1}(s^{t-1},s_1)}{1+\pi_t^i(s^{t-1},s_1)} \\ \frac{q_{n,t}^{\mathscr{J}_1-1}(s^{t-1},s_2)}{1+\pi_t^i(s^{t-1},s_2)} & \frac{q_{n,t}^{\mathscr{J}_2-1}(s^{t-1},s_2)}{1+\pi_t^i(s^{t-1},s_2)} & \frac{q_{n,t}^{\mathscr{J}_3-1}(s^{t-1},s_2)}{1+\pi_t^i(s^{t-1},s_2)} & \frac{q_{n,t}^{\mathscr{J}_4-1}(s^{t-1},s_2)}{1+\pi_t^i(s^{t-1},s_2)} \\ \frac{q_{n,t}^{\mathscr{J}_1-1}(s^{t-1},s_3)}{1+\pi_t^i(s^{t-1},s_3)} & \frac{q_{n,t}^{\mathscr{J}_2-1}(s^{t-1},s_3)}{1+\pi_t^i(s^{t-1},s_3)} & \frac{q_{n,t}^{\mathscr{J}_3-1}(s^{t-1},s_3)}{1+\pi_t^i(s^{t-1},s_3)} & \frac{q_{n,t}^{\mathscr{J}_4-1}(s^{t-1},s_3)}{1+\pi_t^i(s^{t-1},s_3)} \\ \frac{q_{n,t}^{\mathscr{J}_1-1}(s^{t-1},s_4)}{1+\pi_t^i(s^{t-1},s_4)} & \frac{q_{n,t}^{\mathscr{J}_2-1}(s^{t-1},s_4)}{1+\pi_t^i(s^{t-1},s_4)} & \frac{q_{n,t}^{\mathscr{J}_3-1}(s^{t-1},s_4)}{1+\pi_t^i(s^{t-1},s_4)} & \frac{q_{n,t}^{\mathscr{J}_4-1}(s^{t-1},s_4)}{1+\pi_t^i(s^{t-1},s_4)} \end{bmatrix}.$$

Due to the risk sharing condition (11) and the idiosyncratic nature of the shocks, in our numerical results we find that $\frac{U_{C,t}^i(s^t)}{P_{C,t}^i(s^t)}$ will not vary (or will not vary a lot) across $s_2$ and $s_3$. It will hold that

$$\frac{U_{C,t}^A(s^{t-1},s_2)}{P_{C,t}^A(s^{t-1},s_2)} = \frac{U_{C,t}^B(s^{t-1},s_2)}{P_{C,t}^B(s^{t-1},s_2)} \approx \frac{U_{C,t}^A(s^{t-1},s_3)}{P_{C,t}^A(s^{t-1},s_3)} = \frac{U_{C,t}^B(s^{t-1},s_3)}{P_{C,t}^B(s^{t-1},s_3)}$$

For the same reason, the expectations $E_t\left(\frac{U^i_{C,t+j}}{P^i_{C,t+j}}\right)$ for $j \geq 1$ will also not vary across these states. Therefore, nominal bond prices will satisfy:

$$q^j_{n,t}(s^{t-1}, s_2) \approx q^j_{n,t}(s^{t-1}, s_3) \quad \text{for} \quad j \in \mathscr{J}.$$

Given this property and given the equilibrium inflation rates, we can write row 3 of $\mathcal{Q}_{n,t}$ as $\frac{1+\pi^i_t(s^{t-1},s_2)}{1+\pi^i_t(s^{t-1},s_3)}$ times row 2. Matrix $\mathcal{Q}_{n,t}$ has two (nearly) linearly dependent rows and is therefore ill-conditioned.

Notice that these properties are in line with our empirical findings in Section 2 where we have established that the countries in our sample could not hedge against idiosyncratic shocks. As we showed, nominal returns do not respond at all to these shocks (Table 4). When we considered the real returns on nominal bonds, they could respond weakly (Table 1). This effect was due to the response of inflation to the spending shock (see subsection 2.5.3). Here also, inflation may change in response to an idiosyncratic shock, however, as the above derivations show, there is still not enough variability in real returns to complete the market. Issuing nominal bonds and relying on the endogenous response of inflation to create the required state contingent payoffs in $\mathcal{Q}_{n,t}$ will not work. There are two reasons why governments cannot complete the markets through inflation. The first is that prices are sticky; as in Siu (2004) using inflation to reduce debt when spending is high, distorts the allocation of labour across firms. Second, even in the case of fully flexible prices $\nu = 0$, because of (11) it is not possible to set inflation rates in both countries so that the intertemporal budget constraints of both governments are satisfied (which must hold under complete markets).[20]

Will the government be able to use nominal debt to hedge against aggregate shocks in the model? Yes. In our numerical experiments below we will demonstrate that in models with pure aggregate fiscal risks markets can be completed through nominal debt because

$$\frac{U^A_{C,t}(s^{t-1}, s_1)}{P^A_{C,t}(s^{t-1}, s_1)} = \frac{U^B_{C,t}(s^{t-1}, s_1)}{P^B_{C,t}(s^{t-1}, s_1)} \neq \frac{U^A_{C,t}(s^{t-1}, s_4)}{P^A_{C,t}(s^{t-1}, s_4)} = \frac{U^B_{C,t}(s^{t-1}, s_4)}{P^B_{C,t}(s^{t-1}, s_4)}$$

and

$$q^j_{n,t}(s^{t-1}, s_1) \neq q^j_{n,t}(s^{t-1}, s_4) \quad \text{for} \quad j \in \mathscr{J}.$$

### 4.1.2 Inflation Indexed Debt

If nominal bonds not always help to complete the markets, will issuing inflation indexed debt achieve this result? We will show that this is indeed the case as real bonds can replicate the payoffs of Arrow-Debreu securities.

A real bond, $r$, of residual maturity $j$, issued by country $A$ in period $t$, is a bond that pays out $(1+\pi^A_{t+1})(1+\pi^A_{t+2})...(1+\pi^A_{t+j})$ at $t+j$, where $\pi^A$ denotes CPI inflation in country A. From the household's optimization we can show that the price of this bond is $q^{A,j}_{r,t} = \beta^j E_t\left(\frac{U^A_{C,t+j}}{U^A_{C,t}}\right)$ when priced by the household in $A$. The same bond can be priced also by the household in $B$ and the household in $B$ sets $q^{A,j}_{r,t} = \beta^j E_t\left(\frac{U^B_{C,t+j}}{U^B_{C,t}}\frac{P^B_{C,t}}{P^B_{C,t+j}}\frac{P^A_{C,t+j}}{P^A_{C,t}}\right)$. No arbitrage imposes that prices of the same asset need to be equalised, therefore we have

$$q^{A,j}_{r,t} = \beta^j E_t\left(\frac{U^A_{C,t+j}}{U^A_{C,t}}\right) = \beta^j E_t\left(\frac{U^B_{C,t+j}}{U^B_{C,t}}\frac{P^B_{C,t}}{P^B_{C,t+j}}\frac{P^A_{C,t+j}}{P^A_{C,t}}\right) \tag{25}$$

[20] In the online appendix we consider the fully flexible price model and show that the well known result of Chari and Kehoe (1999) which states that governments can complete the markets with a single nominal bond by targeting a path of prices that satisfy government budget solvency cannot hold in the currency union. We discuss optimal DM under this scenario.

for $j \in \mathcal{J}$. It is easy to show that the risk sharing condition (11) is sufficient to satisfy the above condition. Moreover it is worth noting that, differently than the case of nominal bonds, $q_{r,t}^{A,j} \neq q_{r,t}^{B,j}$ as these bonds have different payoffs as they compensate investors for domestic inflation. In the case of inflation indexed bonds the government's intertemporal constraint of country $i$ is:

$$\sum_{j\in\mathcal{J}} q_{r,t}^{i,j-1} \frac{B_{r,t-1}^{i,j}}{P_{C,t-1}^{i}} = E_t \sum_{k=0}^{\infty} \beta^k \frac{S_{t+k}^{i}}{U_{C,t}^{i}}, \tag{26}$$

where $B_{r,t-1}^{i,j}$ is the quantity of real bonds, $r$, of maturity $j$ issued by the government in country $i$ in $t-1$. As in the case of nominal bonds we follow the complete market approach to DM and try to find the portfolio that can solve the following system of equations:

$$\underbrace{\begin{bmatrix} q_{r,t}^{i,\mathcal{J}_1-1}(s^{t-1},s_1) & q_{r,t}^{i,\mathcal{J}_2-1}(s^{t-1},s_2) & \dots & q_{r,t}^{i,\mathcal{J}_N-1}(s^{t-1},s_1) \\ \vdots & \cdots & & \\ & & & \\ q_{r,t}^{i,\mathcal{J}_1-1}(s^{t-1},s_N) & q_{r,t}^{i,\mathcal{J}_2-1}(s^{t-1},s_N) & \dots & q_{r,t}^{i,\mathcal{J}_N-1}(s^{t-1},s_N) \end{bmatrix}}_{\mathcal{Q}_{r,t}} \cdot \underbrace{\begin{bmatrix} \frac{B_{r,t-1}^{i,\mathcal{J}_1}(s^{t-1})}{P_{C,t-1}^{i}(s^{t-1})} \\ \vdots \\ \frac{B_{r,t-1}^{i,\mathcal{J}_N}(s^{t-1})}{P_{C,t-1}^{i}(s^{t-1})} \end{bmatrix}}_{\mathcal{B}_{r,t}} = \begin{bmatrix} \Upsilon_t^i(s^{t-1},s_1) \\ \vdots \\ \Upsilon_t^i(s^{t-1},s_N) \end{bmatrix}. \tag{27}$$

To gain intuition on why issuing inflation indexed debt may help governments complete the market recall that because of condition (11), the ratios $\frac{U_{C,t}^i(s^t)}{P_{C,t}^i(s^t)}$ will not vary across idiosyncratic shock states in our simulations; however, the quantities $U_{C,t}^i(s^t)$ and $E_t(U_{C,t+j}^i)$ will vary across these states and so will the expected growth of marginal utility. Therefore, $\mathcal{Q}_{r,t}$ will be invertible and a portfolio can be determined.

## 4.2 Calibration

We follow the related DM literature to calibrate the model's parameters (e.g. Buera and Nicolini (2004), Faraglia et al (2010), Siu (2004) and Lustig et al (2008)). To calibrate the open economy dimension of our model we borrow values from Faia and Monacelli (2004). Table 6 summarizes our choices.

[Table 6 About Here]

Each model period corresponds to one quarter. We set the discount rate $\beta = 0.98$ and we assume that household preferences are of the form:

$$\log(C_t^i) - \chi \frac{N_t^{i\,1+\gamma}}{1+\gamma}$$

and choose a value $\gamma = 2$ which gives a Frisch elasticity of 0.5.

We normalize hours to 1/3 of the unitary time endowment in the deterministic steady state. Moreover, we assume that the level of spending equals 20 percent of GDP in steady state. $\chi$ is calibrated so that tax revenues balance the governments' budgets in steady state.

We set the home bias parameter $\alpha = 0.4$, the demand elasticity parameter $\eta = 2$ and the elasticity of substitution across varieties parameter $\epsilon = 6$. These values are taken from Faia and Monacelli (2004) and we treat them as our benchmark.

Lustig et al (2008) and Siu (2004) assume fractions of sticky prices $\nu$ in their models equal to 0.05 and 0.08 respectively. These values are somewhat low, since the aim of these papers is to show that assuming even mild degrees of price stickiness can have dramatic effects on optimal policy. We consider a wider range of values for parameter $\nu$ letting $\nu = 0.05$ be the lowest value in the range considered. This enables us to illustrate the impact of varying the degree of price stickiness on optimal portfolios.

In order characterize the properties of optimal debt management transparently, we consider two cases: the case where governments can issue only two maturities, as in the benchmark of ABN, and the case where they can issue four maturities (as in Buera and Nicolini (2004), Faraglia et al (2010)). In the first case we choose a maturity structure $\mathcal{J} = \{1, 40\}$, with a short bond of one quarter and a long bond of forty quarters. In the second case we choose $\mathcal{J} = \{1, 4, 20, 40\}$, two short and two long bonds.[21]

The government can complete the markets with non state contingent assets only if the number of shock realizations is equal to the number of assets issued. If we use only two maturities then we must set $N = 2$ and we obviously have to consider separately the case where shocks are idiosyncratic and the case where they are aggregate. If shocks are idiosyncratic then the vector of possible outcomes is assumed to be:

$$(G_t^A, G_t^B) \in \Big\{ (\overline{g}, \underline{g}), (\underline{g}, \overline{g}) \Big\}$$

and if shocks are aggregate:

$$(G_t^A, G_t^B) \in \Big\{ (\underline{g}, \underline{g}), (\overline{g}, \overline{g}) \Big\}.$$

When we use four maturities then we set $N = 4$ and the vector of joint spending levels is

$$(G_t^A, G_t^B) \in \Big\{ (\underline{g}, \underline{g}), (\overline{g}, \underline{g}), (\underline{g}, \overline{g}), (\overline{g}, \overline{g}) \Big\}$$

as in (24), that is assuming an i.i.d. spending process for each country.[22] Finally, the Markov transition matrices across (joint) spending states are given by:

$$\Pi_\rho = \begin{bmatrix} \rho & 1-\rho \\ 1-\rho & \rho \end{bmatrix}$$

for the case where $N = 2$. For the case $N = 4$ we create the joint transition matrix $\Pi_\rho \otimes \Pi_\rho$. We set $\rho = 0.95$ as Faraglia et al (2010).

The results reported in the following pages are generated by a simulation of the model of 1000 periods assuming $D_{-1}^{i,G} = D_{-1}^{i,H} = 0$ for $i = A, B$. For more details on the numerical algorithm we refer the reader to the appendix.

## 4.3 Baseline Results

**Nominal Bonds and Idiosyncratic Shocks.** Consider the case $N = 2$. The top panel of Table 7 reports the optimal portfolios of country $A$ and the long bond price in the two spending states when shocks are assumed idiosyncratic. The portfolios for country $B$ are symmetric and we omit them. The numbers reported are averages of face values and prices over the simulation. The columns report the optimal portfolios when we increase the degree of price stickiness.

As the third and fourth rows of the table show, bond prices are virtually identical across the high and low spending states. This implies that matrix $Q_{n,t}$ is nearly non-invertible. The optimal portfolios which result from inverting the ill-conditioned matrix feature large investments in the long term asset and short term debt issuance. For example for a low level of price stickiness, $\nu = 0.05$, we obtain $B_n^{A,1} = 3.8 * 10^4$ and $B_n^{A,40} = -8.5 * 10^4$. When $\nu = 0.75$ we have $B_n^{A,1} = 0.5 * 10^4$ and $B_n^{A,40} = -1.1 * 10^4$.

[Table 7 About Here]

[21] The results we report below are robust towards assuming other maturity structures.

[22] We assume an i.i.d. process since if we included $g^c$ we would need at least $N = 8$. This would complicate unnecessarily the analysis.

The numbers reported in the table correspond to the average portfolios. However, since the linear problem we solve is badly scaled, many times we observe that the optimal positions are reversed and governments issue long term instead of short term debt. When this happens portfolios are an order of magnitude larger than the averages reported in Table 7. For example, for $\nu = 0.05$ and some $t_k$, the portfolios becomes $B^{A,1}_{n,t_k} = -1.2 * 10^5$ and $B^{A,40}_{n,t_k} = 2.8 * 10^5$.

The results suggest that markets are virtually incomplete. Even though in our simulations we can invert the matrix $\mathcal{Q}_{n,t}$, this is probably due to the numerical approximation of the model equilibrium rather than a feature of the equilibrium itself.

In the light of the above findings we can conclude that nominal bonds do not enable governments to hedge against idiosyncratic spending shocks and complete the markets. As in our empirical model of Section 2, bond prices (virtually) do not comove with spending shocks. The little variability in prices that the model produces, can only be exploited if governments take incredibly large and erratic positions in the bond market. These recommendations do not seem of any policy relevance.

**Inflation Indexed Bonds and Idiosyncratic Shocks.** We now consider the case where the government issues only inflation indexed bonds assuming again $N = 2$. The results are shown in Table 8.

[Table 8 About Here]

Regardless the degree of price stickiness the optimal DM strategy features issuances of long term debt financed through savings in the short term asset. Our model's predictions are similar to those of Angeletos (2002); bond prices covary negatively with spending in our model (see rows 3-4 of the table) and the government finds optimal to issue long term debt in order to take advantage of fiscal insurance. Because the variability of prices is not large, however, the optimal positions are several times GDP which is on average equal to 1/3 in the model. This is a standard prediction of the theory (see Buera and Nicolini (2004) and Faraglia et al (2010)).

Assuming a higher fraction of sticky prices leads to a fanning out of the positions. When $\nu = 5\%$, the government issues roughly 253 times GDP in the long term bond, however, when $\nu = 75\%$ we obtain long bond issuances equal to 421 times GDP. This is due to the fact that a higher degree of price stickiness leads to a smaller impact of spending shocks on consumption (this property follows clearly from (11)). Since after 39 quarters consumption in expectation reverts back to its mean value, most of the variability of real long bond prices is explained by current consumption. This makes the price of long term inflation indexed debt less responsive to spending shocks, when the degree of price stickiness increases. A government which seeks to take full advantage of fiscal hedging must then fan out its position.

In the case of inflation indexed bonds the optimal portfolios are stable through time. We do not observe reversals of the portfolios in any period in our simulations, as we did previously with nominal debt. Portfolios are not constant as in ABN, but as we illustrate in the online appendix, they only change slightly through time and the qualitative patterns documented in Table 8 holds at all simulated model periods.

We can then conclude that inflation indexed bonds do enable governments to hedge against idiosyncratic spending shocks and complete the markets. Bond prices do comove negatively with spending shocks and governments can exploit fiscal insurance.

**Aggregate Shocks.** So far we have concluded that in an equilibrium with pure idiosyncratic shocks nominal bonds cannot decentralize the complete market allocation whereas inflation indexed bonds can. We now turn to the case of aggregate shocks only, assuming again $N = 2$.

Since now shocks are perfectly correlated across countries, the model becomes essentially isomorphic to a closed economy model. The optimal allocation features zero inflation and the degree of price stickiness exerts no influence. Under aggregate shocks the planner does not find optimal to use surprise inflation to ward off fiscal shocks, since inflation distorts the allocation of labor across flexible and sticky price goods and since it is possible to exploit variations in the price of real debt to smooth taxes across time.

When shocks are aggregate the government can complete the markets even when it issues nominal bonds. Precisely because inflation is always zero, nominal bonds and inflation indexed bonds are equivalent and the optimal bond portfolio is the same across the two model versions. For our parametrization the optimal portfolio is $(B_r^{A,1}, B_r^{A,40}) = (B_n^{A,1}, B_n^{A,40}) = (-13.6, 30.1)$ and it is constant over time due to the fact that prices are constant. Also in this case governments desire to issue long bonds and finance this position through short term savings. Trivially the optimal portfolio does not change as we vary the degree of price stickiness $\nu$ since as noted previously under complete markets and aggregate shocks prices remain constant through time.

**Varying the Number of States.** Until now we have considered the case of idiosyncratic and aggregate shocks separately. In this section we consider the case when our economy features both aggregate and idiosyncratic shocks, setting $N = 4$.

Our previous finding that markets are effectively incomplete under nominal debt continues to hold. When the government has access to 4 nominal bonds of different maturities, two of the rows of $Q_{n,t}$ continue to be linearly dependent and $Q_{n,t}$ is not invertible. For brevity we leave this case outside the tables.

It is more interesting to consider two alternative cases: the case where governments can issue four inflation indexed bonds of different maturities and the case where they issue two nominal bonds and two real instruments.

The top panel of Table 9 reports the results of the case with only real bonds. The model continues to suggest that the government should issue long term debt in order to exploit fiscal insurance. The position of short term assets, $B_r^{A,1} + B_r^{A,4}$, is negative and the position on long term bonds, $B_r^{A,20} + B_r^{A,40}$, remains positive across all values of $\nu$.[23] Governments desire to issue long bonds, with most of the debt being concentrated in either the 20 quarter maturity (when $\nu$ is low) or the 40 quarter bond (for high values of $\nu$).[24].

[Table 9 About Here]

The bottom panel of Table 9 considers the case where governments issue nominal bonds in two maturities (one quarter and 4 quarters) and issue real long term bonds.[25]

Issuing a mixture of nominal and real assets, allows the government to complete the markets, since nominal debt is effective in dealing with aggregate shocks whereas real debt can be used to absorb idiosyncratic shocks. As the table shows, government debt is again concentrated in the long term assets, $B_r^{A,20} + B_r^{A,40} > 0$. The optimal portfolio features larger positions (in absolute terms) than those reported in the top panel of Table 9. Unsurprisingly, nominal bond prices do not covary as strongly with deficits as real prices do, and fanning out the positions is necessary to complete the markets.

## 4.4 Robustness and Extensions

**Varying the Degree of Home Bias.** We now study how the optimal portfolio varies when we assume a stronger home bias, decreasing $\alpha$ from 0.4 to 0.2. We assume again that $N = 2$ and that shocks can be only aggregate or only idiosyncratic, as in our benchmark case. Table 10 reports the results. For brevity we focus here only on inflation indexed bonds.

[Table 10 About Here]

[23] This is also true when we calculate the market value of the bonds.

[24] It shouldn't be surprising that governments issue small amounts of very short bonds and, in some calibrations, buy the longest maturity available. Faraglia et al (2010) find a similar property in some of the 4 state models they consider, the exact values of the bond quantities that emerge from these models are somewhat sensitive to the calibration. However, the principle that optimal debt is of long maturity, continues to hold

[25] As we have seen most of the inflation indexed debt issue in Europe is long term. For this reason we let the shortest maturities to be nominal debt, though we could complete the market with two nominal bonds of any maturity.

Comparing these results with the benchmark in Table 8, in the case of idiosyncratic shocks, a stronger home bias leads to a slight fanning in of the bond positions. A lower $\alpha$ leads to a larger impact of an idiosyncratic shock to consumption. Therefore, it leads to larger variability in real long bond prices and governments can issue fewer long bonds to hedge against the shock. Notice also that this effect weakens as we increase the degree of price stickiness in the economy, since, as discussed previously, assuming a larger fraction of sticky price firms reduces the responsiveness of consumption to fiscal shocks.

The bottom panel of Table 10 considers the case of aggregate spending shocks. Clearly, in this case $\alpha$ exerts no influence on portfolios since the model is essentially equivalent to the closed economy model.

Our finding that markets can be completed through real debt issuances holds under any $\alpha < \frac{1}{2}$. When $\alpha$ equals a half, the CPIs are equal across countries and an efficient allocation sets the ratio of marginal utilities constant. In this case, consumption does not respond to idiosyncratic shocks and so real debt prices do not vary across idiosyncratic spending states. Without a mild degree of home bias governments cannot complete markets even when they issue inflation indexed debt.

**Varying the Elasticity of Home and Foreign Goods.** Our baseline calibration has assumed $\eta = 2$ and this assumption follows Faia and Monacelli (2004). Most papers in the open economy DSGE literature assume values in the range $[1, 2]$ and a recent stream of studies[26], which estimate structural DSGEs with Bayesian methods, narrow this range further to $[1.5, 2]$. We choose the lower bound of these estimates and set $\eta = 1.5$ to check the robustness of findings.

The results displayed in Table 11 suggest that assuming a lower $\eta$ increases the variability of bond prices and again governments slightly reduce the size of bond positions. However, the qualitative prediction that governments desire to issue long term debt remains.

[Table 11 About Here]

**Union Wide Inflation Indexed Debt.** As a final numerical experiment we study optimal debt management in the case where governments issue debt indexed to Union wide inflation. As was discussed in Section 2 two countries in our sample, France and Italy, have issued bonds linked to Euro Area inflation. We therefore find interesting to extend our analysis to consider the effect of issuing these types of bonds in the model. For brevity all details of this version of the model are provided in the online appendix.

Our findings suggest that governments continue to desire to issue long term debt, however the optimal portfolio positions are now even larger than before. For example, in our benchmark calibration, where $N = 2$, $\nu = 0.05$ and shocks are idiosyncratic, long debt is 1065 times larger than GDP. When the prices are stickier and $\nu = 0.75$, the long bond position increases to 2085 times of GDP. These positions are obviously extremely large.

This is because while governments can complete the markets through issuing debt indexed to union-wide inflation, these bonds, in terms of their price responses to spending shocks, are essentially somewhere between nominal debt and debt indexed to country inflation. Bond prices respond less to the shocks which in turn implies a weaker comovement with idiosyncratic shocks. Governments thus have to leverage more on the size of the positions to achieve an optimal fiscal insurance and complete the markets.

When shocks are purely aggregate, bond positions do not change relative to the benchmark as expected. For brevity we leave the above findings outside the tables.

To summarise, the optimality of issuing long term debt is a robust prediction of our model. We have shown this under a wide range of plausible calibrations of the model, considering alternative values for the degree of price stickiness, the degree of home bias and the elasticity parameter $\eta$, as

[26] See for example De Walque, Smets, and Wouters (2005), Rabanal and Tuesta (2005), Justiniano and Preston (2006).

well as alternative inflation indexing schemes. Under all cases real bond prices comove negatively with government deficits and this implies that the rows of $\mathcal{Q}_{r,t}$ are linearly independent, so that a solution could be found to system (27) and the optimal portfolios recovered.

Finally, we have seen that the optimal portfolios that emerge from our complete market model are slightly sensitive towards varying the model's parameters and indexing assumptions and bond positions are large multiples of GDP. These findings are in line with the analogous findings in Buera and Nicolini and Faraglia et al (2010).

### 4.5 Inflation Indexed Debt in the Euro Area

The main takeaway from the theoretical analysis of this section is that inflation indexed debt provides a better hedge to governments against fiscal shocks. Before concluding our paper in the next section, we turn again our attention to our dataset to provide evidence that this prediction is in line with the Euro Area bond market data.

Since France is the only country that has a positive share of inflation indexed bonds throughout the entire sample period, we use this country to test the prediction that these bonds can absorb both aggregate and idiosyncratic spending shocks. We apply the methodology developed in Section 2 and run a VAR letting the weighted average spending be the first variable and French spending the second, followed by output, prices and returns. We run two separate VARs, one with observations up until 2008 and another extending our dataset to include observations until the third quarter of 2018. We run the second VAR because focusing on one country we only have 41 observations and thus our estimates on the shorter sample are likely to be noisy.

[Table 12 About Here]

For the sake of comparison the first two columns of Table 12 show the impact effects of the spending shock on the holding period returns of nominal bonds. In both samples considered, the results highlighted in Section 2 continue to hold. Spending shocks lower returns, however, this is only the case for aggregate shocks. Idiosyncratic shocks have a statistically insignificant impact on returns.

Columns 3-4 show the analogous responses in the case of inflation indexed bonds. Now the results are different: for both aggregate and idiosyncratic shocks the coefficients are negative and significant. Idiosyncratic shocks have an even larger impact on returns than aggregate shocks. This suggests that real bonds are particularly effective in absorbing these types of shocks confirming our theoretical analysis.

As discussed in section 2.2, Figure 2 shows that France has steadily increased the share of inflation indexed debt in its portfolio throughout the 2000s. The literature provides mainly two reasons to justify this choice: first real bonds are a commitment device against inflating away public debt (see Garcia and van Rixtel (2007) for a review) and second, real bonds are demanded by long horizon investors (e.g. pension funds) because they provide insurance against inflation risks. French inflation rates, however, have been low and stable; the commitment story does not seem plausible. While the second argument could have played a role, this paper provides an additional explanation: long term inflation indexed bonds have a hedging value for the French government after entering in the currency union.

## 5 Conclusions

The complete market model of debt management advises governments in a currency union to issue inflation indexed long term debt. In this way, budgets are insulated from the effects of spending shocks and taxes smoothed, since portfolios absorb a large part of the fiscal shocks. Our paper builds on the insights of the recent macro debt management literature, bringing together its positive and normative elements. Our analysis aims to shed light on the management of government debt in currency areas, which, since recently, has received considerable attention in policy circles.

A number of fruitful extensions of our framework are worth discussing briefly here. First, our model builds on the presumption that markets can be completed, and thus abstracts from possibly important financial market frictions which governments may find difficult to circumvent when they design policies. A recent stream of papers in the closed economy DM literature has explored the interplay between frictions and optimal maturity (e.g. Faraglia et al (2018), Debortoli et al (2017), Hansen et al (2015)). Useful insights can be drawn from these papers and applied to the Union DM model that we propose. However, in these recent papers the distinction between real and nominal bonds is less crucial than it is in our framework, and therefore it is important to explore whether frictions impact differently nominal and real debt markets. This can be argued for example for the US bond markets, where it has been documented that inflation indexed bonds are less liquid.[27] It would be interesting to know whether higher yields associated with lower liquidity, reduce the attractiveness of these types of assets for debt managers in a currency area.

Second, introducing more shocks to the model is required to study jointly the optimal portfolios of private agents along with the optimal DM by governments, and thus bring together the hedging motives of governments and individuals. This will enable to allow trade in equity into our model and thus draw valuable insights from the vast literature on cross country equity portfolios.

Finally, the interplay between fiscal transfers and DM is also worth exploring.

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Table 1: **Responses of Real Returns to Spending Shocks (Pooled-Panel VAR)**

| | **All shocks** | | **Aggregate** | | **Idiosyncratic** | |
|---|---|---|---|---|---|---|
| All maturities | $-33.8^{\star}$ | (0.01) | $-86.3^{\star}$ | (0.00) | $-13.4$ | (0.40) |
| $\leq$ 1 year | $-15.2^{\star}$ | (0.00) | $-9.3^{\star}$ | (0.18) | $-17.3^{\star}$ | (0.00) |
| 1-4 years | $-28.6^{\star}$ | (0.00) | $-57.9^{\star}$ | (0.00) | $-17.0^{\star}$ | (0.08) |
| 4-7 years | $-49.0^{\star}$ | (0.01) | $-127.2^{\star}$ | (0.00) | $-18.1$ | (0.38) |
| $>$ 7 years | $-50.2^{\star}$ | (0.10) | $-156.9^{\star}$ | (0.01) | $-8.4$ | (0.81) |

Notes: The table reports initial period responses (in terms of basis points) of bond returns to a government spending shock (1% increase in government consumption) in pooled panel VARs (see section 2.3 for details). Our data is quarterly and covers the period 1998Q2-2008Q2. All variables (spending, output, prices and returns) are detrended using a one-side HP filter with smoothing parameter of 8330. P-values are reported in parentheses while * denotes that zero is outside the one-standard-deviation confidence interval obtained with Monte Carlo simulations (1000 replications).

Table 2: **Responses of Real Returns to Spending Shocks (Average of Country Responses)**

| | **All shocks** | | **Aggregate** | | **Idiosyncratic** | |
|---|---|---|---|---|---|---|
| All maturities | $-16.5$ | (0.76) | $-84.8^{\star}$ | (0.12) | 19.1 | (0.79) |
| $\leq$ 1 year | $-1.0$ | (0.92) | $-11.3$ | (0.42) | 2.6 | (0.85) |
| 1-4 years | $-7.6$ | (0.83) | $-61.7^{\star}$ | (0.09) | 15.6 | (0.72) |
| 4-7 years | $-29.5$ | (0.67) | $-128.6^{\star}$ | (0.08) | 19.9 | (0.82) |
| $>$ 7 year | $-25.3$ | (0.82) | $-147.9^{\star}$ | (0.21) | 43.4 | (0.77) |

Notes: The table reports initial period *average responses* of bond returns to government spending shocks. We run a separate VAR for each country, then identified the response of return variables to the shocks and averaged using as weights the relative mean GDP over the sample period. The sample and the identification strategy (ordering of the variables) used are the same as in Table 1. P-values are reported in parentheses while * denotes that zero is outside the one-standard-deviation confidence interval obtained with Monte Carlo simulations (1000 replications).

Table 3: **Responses of Real Returns to Spending Shocks (Principal Components)**

| | **Pooled Panel** | | | | **Average** | | | |
|---|---|---|---|---|---|---|---|---|
| | **Aggregate** | | **Idiosyncratic** | | **Aggregate** | | **Idiosyncratic** | |
| All maturities | $-34.6^{\star}$ | (0.00) | $-12.9$ | (0.38) | $-36.1^{\star}$ | (0.08) | $-27.6$ | (0.67) |
| $\leq$ 1 year | $-7.9^{\star}$ | (0.05) | $-15.4^{\star}$ | (0.00) | $-7.2^{\star}$ | (0.15) | $-7.4$ | (0.57) |
| 1-4 years | $-26.2^{\star}$ | (0.00) | $-13.6^{\star}$ | (0.13) | $-30.0^{\star}$ | (0.02) | $-22.1$ | (0.60) |
| 4-7 years | $-51.2^{\star}$ | (0.00) | $-17.4$ | (0.36) | $-54.9^{\star}$ | (0.05) | $-38.2$ | (0.65) |
| $>$ 7 year | $-56.6^{\star}$ | (0.01) | $-14.2$ | (0.66) | $-54.7^{\star}$ | (0.22) | $-32.5$ | (0.82) |

Notes: We used the same sample and identification strategy (ordering of the variables) as in Table 1. However, the first variable in the panel VAR is now the first principal component of (the log of) the government spending series of the countries in the sample. P-values are reported in parentheses while * denotes that zero is outside the one-standard-deviation confidence interval obtained through Monte Carlo simulations (1000 replications).

Table 4: **Responses of Nominal Returns to Spending Shocks (Pooled-Panel VAR)**

| | **All shocks** | | **Aggregate** | | **Idiosyncratic** | |
|---|---|---|---|---|---|---|
| All maturities | $-20.8^{\star}$ | (0.13) | $-74.0^{\star}$ | (0.00) | 1.0 | (0.95) |
| $\leq$ 1 year | $-3.8^{\star}$ | (0.05) | $-51.7$ | (0.13) | 13.5 | (0.76) |
| 1-4 years | $-16.1^{\star}$ | (0.05) | $-47.3^{\star}$ | (0.01) | $-2.8$ | (0.77) |
| 4-7 years | $-35.9^{\star}$ | (0.05) | $-115.2^{\star}$ | (0.00) | $-3.7$ | (0.86) |
| $>$ 7 years | $-37.1^{\star}$ | (0.30) | $-144.3^{\star}$ | (0.03) | 6.0 | (0.86) |

Notes: The table reports reponses of returns to spending shocks identified in a pooled panel VAR as in Table 1, however, the return variables here are not adjusted for inflation. P-values are reported in parentheses while * denotes that zero is outside the one-standard-deviation confidence interval obtained

Table 5: **Responses of Real Returns to Spending Shocks (1998-2013)**

| | **All shocks** | | **Aggregate** | | **Idiosyncratic** | |
|---|---|---|---|---|---|---|
| **Pooled-Panel VAR** | | | | | | |
| All maturities | $-36.1^{\star}$ | (0.02) | $-90.9^{\star}$ | (0.00) | 10.8 | (0.53) |
| $\leq$ 1 year | $-18.5^{\star}$ | (0.00) | $-15.1$ | (0.02) | $18.5^{\star}$ | (0.00) |
| 1-4 years | $-30.0^{\star}$ | (0.00) | $-46.2^{\star}$ | (0.01) | $-19.6^{\star}$ | (0.06) |
| 4-7 years | $-51.8^{\star}$ | (0.01) | $-126.8^{\star}$ | (0.00) | $-17.7$ | (0.41) |
| $>$ 7 years | $-66.3^{\star}$ | (0.04) | $-195.3^{\star}$ | (0.00) | $-5.9$ | (0.88) |
| **Average of Country Responses** | | | | | | |
| All maturities | $-41.6$ | (0.47) | $-99.1^{\star}$ | (0.10) | 3.8 | (0.96) |
| $\leq$ 1 year | $-7.9$ | (0.42) | $-12.2$ | (0.37) | $-3.6$ | (0.78) |
| 1-4 years | $-33.6^{\star}$ | (0.32) | $-60.5^{\star}$ | (0.09) | $-9.1$ | (0.82) |
| 4-7 years | $-67.4$ | (0.34) | $-139.9^{\star}$ | (0.07) | $-10.4$ | (0.90) |
| $>$ 7 years | $-62.1$ | (0.61) | $-191.8^{\star}$ | (0.16) | 36.5 | (0.81) |

Notes: The table reports reponses of returns to spending shocks identified in a pooled panel VAR as in Table 1, however, the return variables here are not adjusted for inflation. P-values are reported in parentheses while * denotes that zero is outside the one-standard-deviation confidence interval obtained

Table 6: **The Model Parameters**

| Parameter | Symbol | Value | Target/Reference |
|---|---|---|---|
| | | *Preferences and Technology* | |
| Discount Rate | $\beta$ | 0.98 | Faraglia et al (2010) |
| Disutility of Labor (curvature) | $\gamma$ | 2 | Elasticity of 0.5 |
| Disutility of Labor (level) | $\chi$ | | Steady state tax |
| Time Working | $\overline{h}$ | $\frac{1}{3}$ | Normalization |
| Home Bias Parameter | $\alpha$ | 0.4 | |
| Substitution / Varieties | $\epsilon$ | 6 | Faia and Monacelli (2004) |
| Substitution / Domestic vs. Foreign goods | $\eta$ | 2 | |
| Fraction of Sticky Prices | $\nu$ | 0.05 | Siu (2004) |
| | | *Spending* | |
| High Spending State | $\overline{g}$ | $(1+0.07)g_s$ | |
| Low Spending State | $\underline{g}$ | $(1-0.07)g_s$ | Faraglia et al (2010) |
| Persistence | $\rho$ | 0.95 | |

Note: The table summarizes the values of the model parameters under the baseline calibration. The second column gives the symbol used in text and the last column reports the targets or refers to papers followed to set the values. $g_s$ is used to denote the steady state level of spending (20 percent of GDP). See text for further details.

Table 7: **Optimal Portfolios and Prices: Nominal Bonds and Idiosyncratic Shocks**

| % Sticky Prices | $\nu = 5\%$ | $\nu = 10\%$ | $\nu = 25\%$ | $\nu = 50\%$ | $\nu = 75\%$ |
|---|---|---|---|---|---|
| | | | *Portfolios* | | |
| $B_n^{A,1}$ | 38917 | 19721 | 7269 | 9369 | 5227 |
| $B_n^{A,40}$ | -85569 | -43363 | -15984 | -20618 | -11494 |
| | | | *Bond Prices* | | |
| $q_n^{A,39}(\overline{g})$ | 0.4548 | 0.4548 | 0.4548 | 0.4548 | 0.4548 |
| $q_n^{A,39}(\underline{g})$ | 0.4548 | 0.4548 | 0.4548 | 0.4548 | 0.4547 |

Notes: The table reports optimal portfolios and long bond prices under $N = 2$ (two spending states) and under idiosyncratic shocks. The numbers reported in the top panel are face values of short- 1 period- debt ($B_n^{A,1}$) and long - 40 quarters- debt ($B_n^{A,40}$). The bottom panel reports repurchase prices (averages over simulations) for the long bond in the case where spending is high (low) $q_n^{A,39}(\overline{g})$ ($q_n^{A,39}(\underline{g})$).

Table 8: **Optimal Portfolios: Inflation Indexed Bonds and Idiosyncratic Shocks**

| % Sticky Prices | $\nu = 5\%$ | $\nu = 10\%$ | $\nu = 25\%$ | $\nu = 50\%$ | $\nu = 75\%$ |
|---|---|---|---|---|---|
| | | *Portfolios* | | | |
| $B_r^{A,1}$ | -30.7 | -41.2 | -42.9 | -48.2 | -64.1 |
| $B_r^{A,40}$ | 89.6 | 90.6 | 94.5 | 106.1 | 140.9 |
| | | *Bond Prices* | | | |
| $q_r^{A,39}(\overline{g})$ | 0.453 | 0.453 | 0.453 | 0.453 | 0.453 |
| $q_r^{A,39}(\underline{g})$ | 0.457 | 0.457 | 0.457 | 0.457 | 0.456 |

Notes: The table reports optimal portfolios under $N = 2$ (two spending states). The top panel shows the case of idiosyncratic shocks and the bottom panel the case of aggregate shocks. The numbers reported are face values of short- 1 period- debt ($B_r^{A,1}$) and long - 40 quarters- debt ($B_r^{A,40}$).

Table 9: **Optimal Portfolios: Varying the Number of States**

| % Sticky Prices | $\nu = 5\%$ | $\nu = 10\%$ | $\nu = 25\%$ | $\nu = 50\%$ | $\nu = 75\%$ |
|---|---|---|---|---|---|
| | | *Inflation Indexed Only* | | | |
| $B_r^{A,1}$ | 0.5 | 0.5 | 0.5 | 0.5 | 0.6 |
| $B_r^{A,4}$ | -30.4 | -30.4 | -30.3 | -30.1 | -29.1 |
| $B_r^{A,20}$ | 55.1 | 53.4 | 47.1 | 28.5 | -21.2 |
| $B_r^{A,40}$ | -20.6 | -18.2 | -8.9 | 18.2 | 90.6 |
| | | *Inflation Indexed +Nominal* | | | |
| $B_n^{A,1}$ | -0.2 | -0.0 | 0.6 | 2.4 | 7.7 |
| $B_n^{A,4}$ | -38.3 | -38.5 | -39.6 | -42.7 | -53.3 |
| $B_r^{A,20}$ | -70.6 | -71.6 | -75.3 | -84.9 | -99.4 |
| $B_r^{A,40}$ | 185.5 | 187.2 | 193.5 | 210.2 | 242.4 |

Notes: The table reports optimal portfolios under $N = 4$ (four spending states). The top panel shows the case where the government issues only inflation indexed debt. The bottom panel considers the case where short and medium term debt (maturities 1 and 8 quarters respectively) is nominal. The numbers reported are face values ($B_r^{A,j}$ and $B_n^{A,j}$) of maturities $j = 1, 4, 20, 40$ where $j$ denotes quarters.

Table 10: **Optimal Portfolios: Varying the Degree of Home Bias**

| % Sticky Prices | $\nu = 5\%$ | $\nu = 10\%$ | $\nu = 25\%$ | $\nu = 50\%$ | $\nu = 75\%$ |
|---|---|---|---|---|---|
| | | | *Idiosyncratic Shocks* | | |
| $B_r^{A,1}$ | -32.3 | -32.9 | -35.2 | -42.3 | -63.2 |
| $B_r^{A,40}$ | 71.0 | 72.4 | 77.5 | 93.0 | 139.1 |
| | | | *Aggregate Shocks* | | |
| $B_r^{A,1}$ | -13.6 | -13.6 | -13.6 | -13.6 | -13.6 |
| $B_r^{A,40}$ | 30.1 | 30.1 | 30.1 | 30.1 | 30.1 |

Notes: The table reports optimal portfolios under $N = 2$ (two spending states). The calibration of the model sets $\alpha = 0.2$ (higher degree of home bias). All other parameters are unchanged relative to Table 8.

Table 11: **Optimal Portfolios: Varying the Elasticity of Home and Foreign Goods**

| % Sticky Prices | $\nu = 5\%$ | $\nu = 10\%$ | $\nu = 25\%$ | $\nu = 50\%$ | $\nu = 75\%$ |
|---|---|---|---|---|---|
| | | | *Idiosyncratic Shocks* | | |
| $B_r^{A,1}$ | -28.5 | -29.1 | -31.5 | -38.6 | -59.8 |
| $B_r^{A,40}$ | 62.6 | 64.0 | 69.2 | 84.9 | 131.6 |
| | | | *Aggregate Shocks* | | |
| $B_r^{A,1}$ | -13.6 | -13.6 | -13.6 | -13.6 | -13.6 |
| $B_r^{A,40}$ | 30.1 | 30.1 | 30.1 | 30.1 | 30.1 |

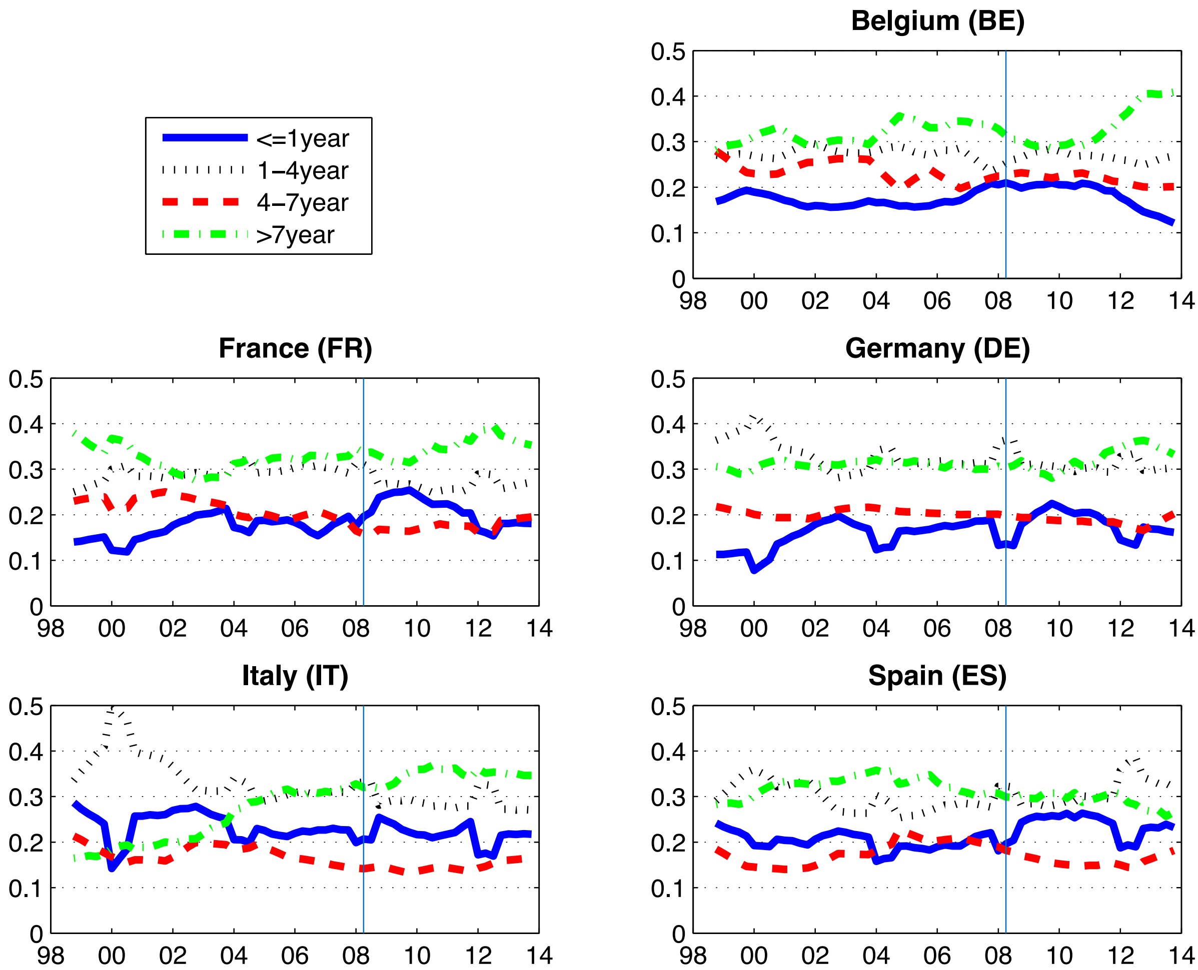
Notes: The table reports optimal portfolios under $N = 2$ (two spending states). The calibration of the model sets $\eta = 1.5$. All other parameters are unchanged relative to Table 8.

Table 12: **Responses of Real Returns to Spending Shocks of French Debt**

| | **Nominal bonds** | | | | **Real Bonds** | | | |
|---|---|---|---|---|---|---|---|---|
| | **Aggregate** | | **Idiosyncratic** | | **Aggregate** | | **Idiosyncratic** | |
| 1998-2008 | $-84.8^{\star}$ | (0.10) | $-12.1$ | (0.92) | $-94.7^{\star}$ | (0.14) | $-224.3^{\star}$ | (0.12) |
| 1998-2018 | $-91.7^{\star}$ | (0.09) | $-13.5$ | (0.91) | $-77.8^{\star}$ | (0.16) | $-170.8^{\star}$ | (0.15) |

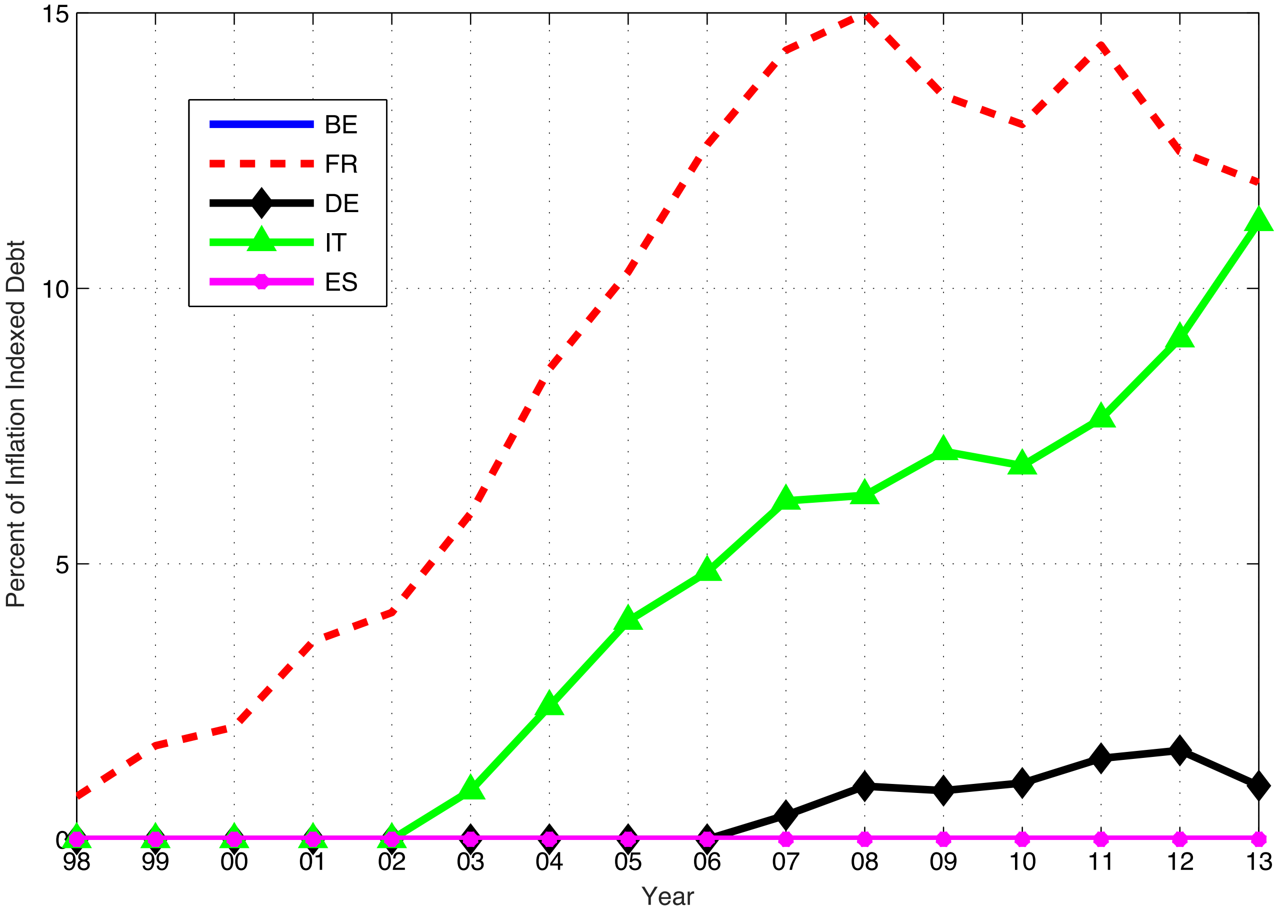
Notes: The table reports responses of returns to spending shocks. The left panel shows the impact effects of return variables of nominal bonds and the right panel shocks returns of bonds linked to French inflation. P-values are reported in parentheses while * denotes that zero is outside the one-standard-deviation confidence interval obtained through Monte Carlo simulations (1000 replications).

Figure 1: **Portfolio Shares: Various Maturity Segments**



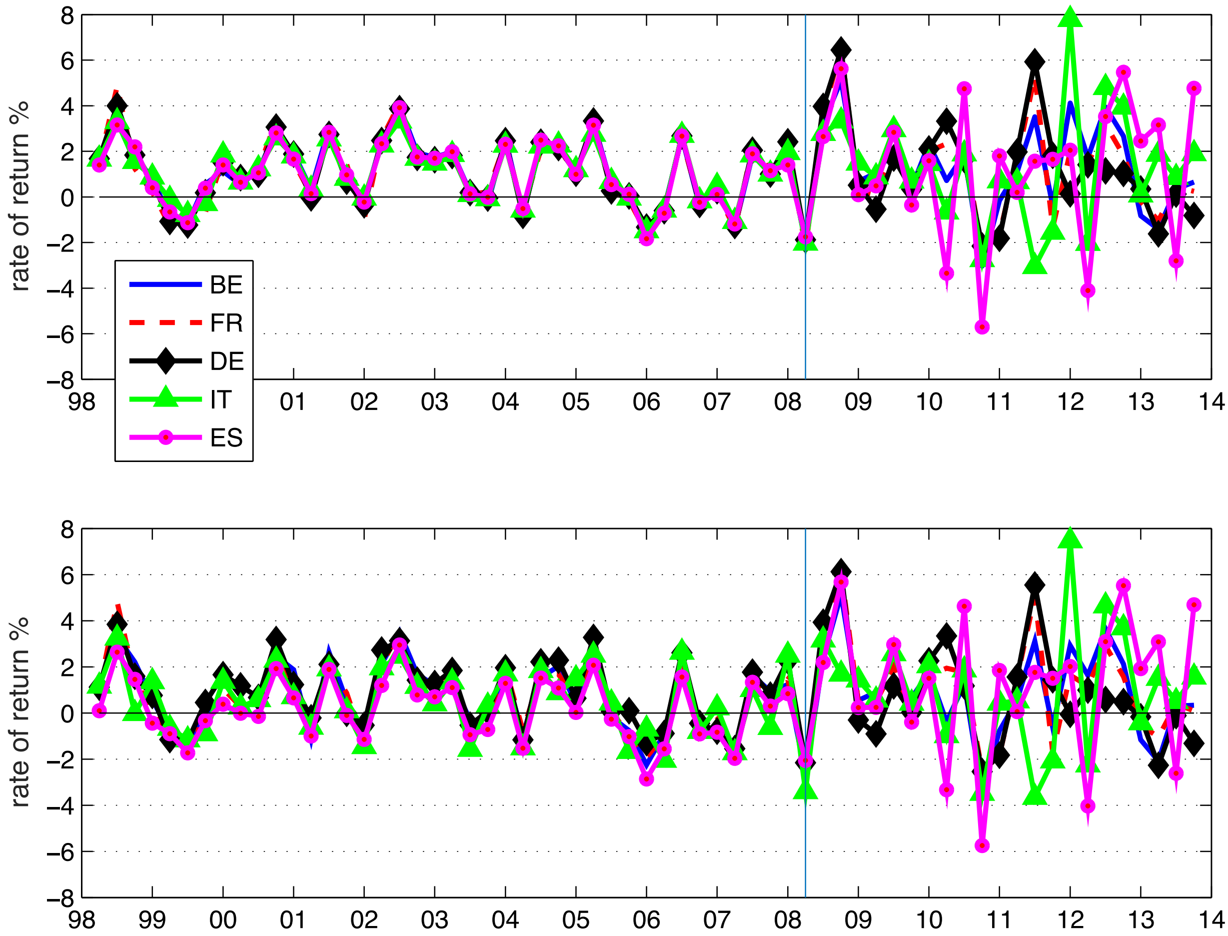

Notes: The figure shows the shares $w^j$ of short term (maturity up to one year), medium term (maturity between 1 and 4 years), long term (4-7 years) and very long term debt (≥7 years) debt in government portfolios. To compute these shares we count both principal repayments and coupons using the remaining life of each bond to assign maturity (see text for details). The vertical lines mark the beginning of the financial crisis at the second quarter of 2008.

Figure 2: **Shares of Inflation Indexed Debt in Government Portfolios**



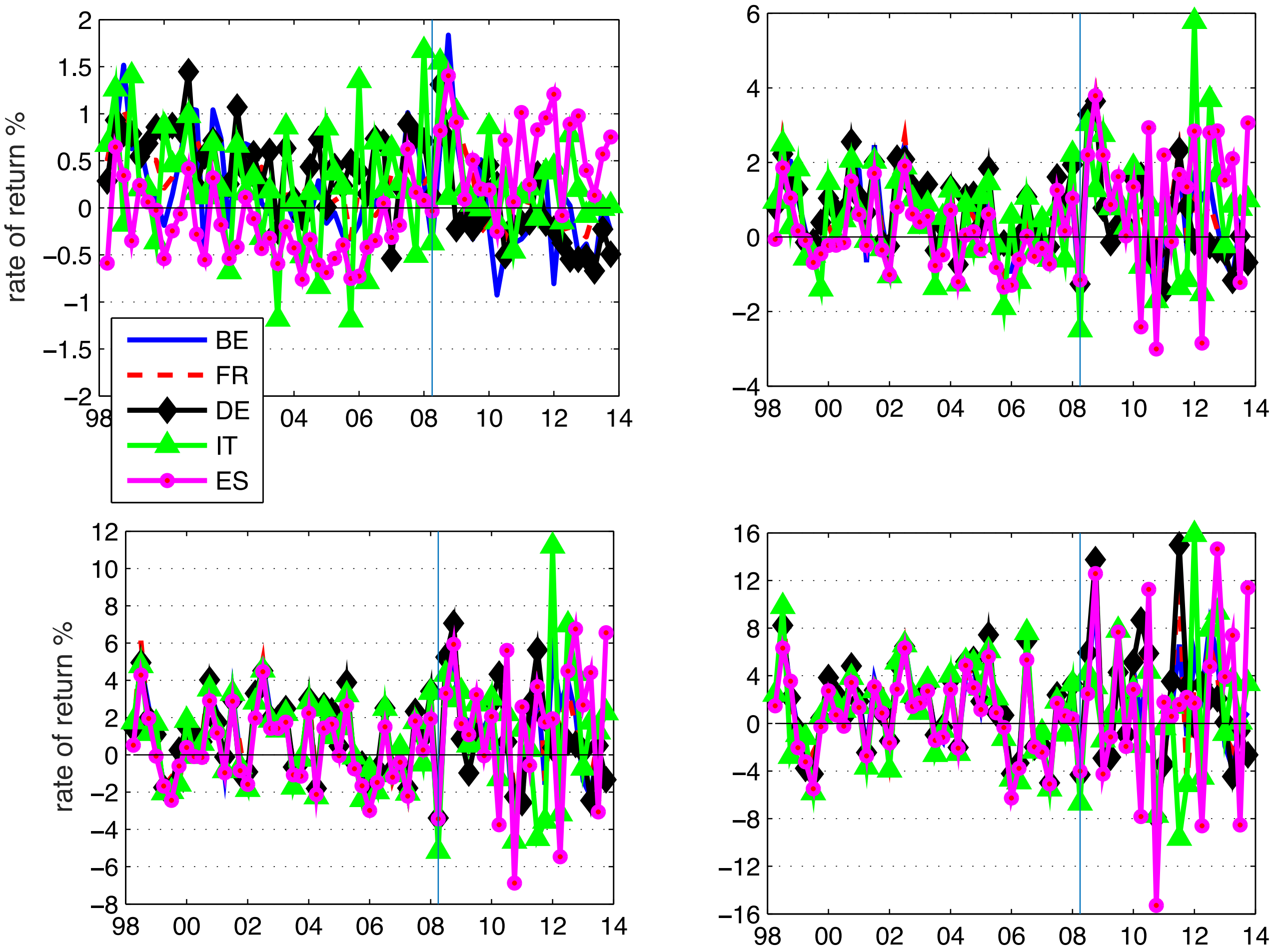

Notes: The figure shows the shares of inflation indexed long maturity debt over total government debt in each country.

Figure 3: **Holding Period Returns: Aggregate Portfolios**



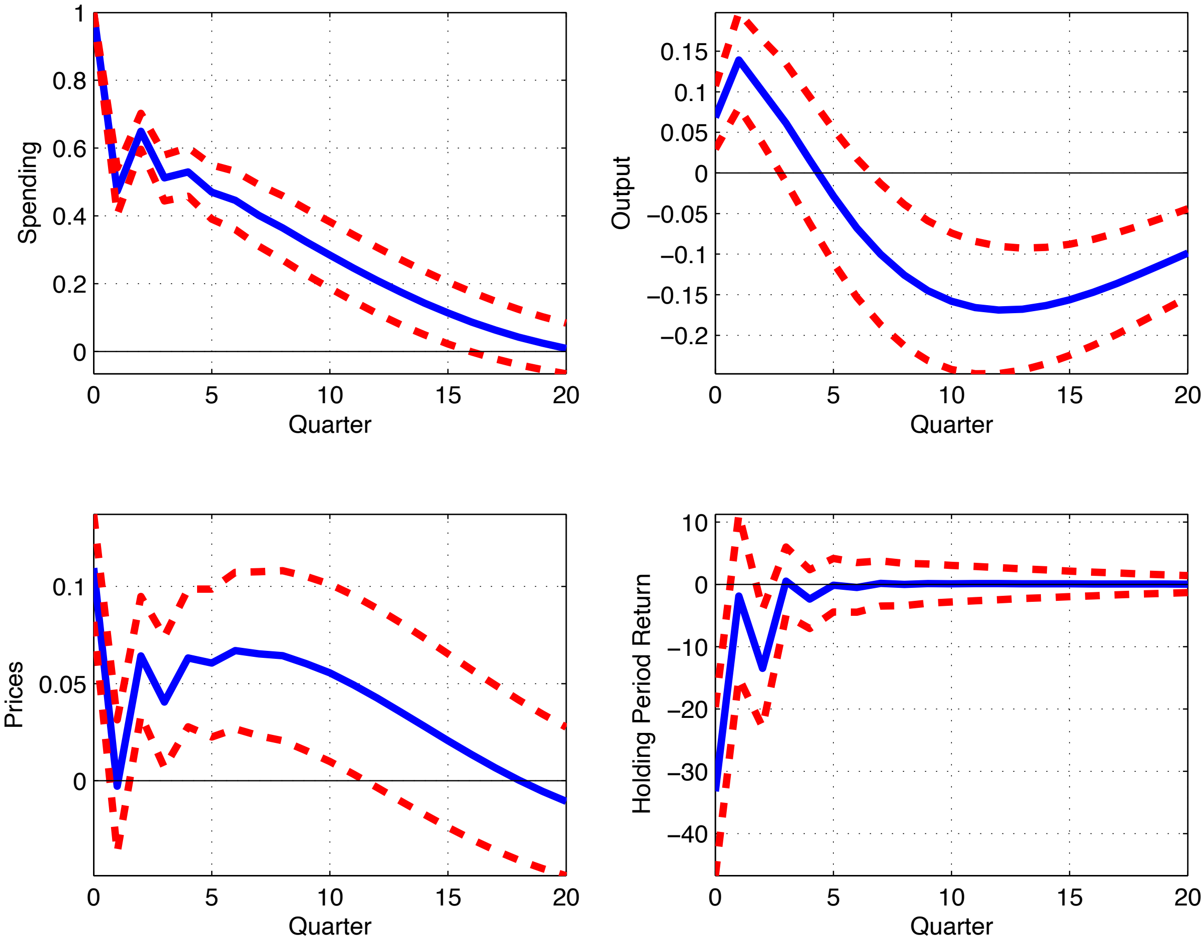

Notes: The figure shows the holding period return series. The top panel shows returns not adjusted by realized inflation. The bottom panel plots the return series $\mathscr{R}$ on the aggregate portfolio, setting $(\underline{j} = 1, \overline{j} = 120)$.

Figure 4: **Holding Period Returns: Various Maturity Segments**




Notes: The figure shows the holding period return series. The top left panel plots the return series for short term debt, ($\underline{j} = 1, \overline{j} = 4$). The top right panel sets ($\underline{j} = 5, \overline{j} = 16$) (medium term debt), the bottom left ($\underline{j} = 17, \overline{j} = 28$) (long term debt) and the bottom right sets ($\underline{j} = 29, \overline{j} = 120$) (very long term debt).

Figure 5: **Impulse Responses One Standard Deviation in Spending**




Notes: The figure plots the impulse responses of government spending (top left), output (top right), prices (bottom left) and holding returns (bottom right) to a one standard deviation shock in 'total' spending. The estimates derive from the panel S-VAR in Section 2.4.1. See text for further details.