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JEL Classification: D82, D83, E44, G12, G14

Keywords: CDO, RMBS, disagreement, default correlation, credit risk, great recession, housing bubble

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Securitisation Bubbles: Structured finance with disagreement about default correlations

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February 2016

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The early 2000s have seen an enormous boom and bust in structured financial products, such as residential mortgage-backed securities (RMBSs) or collateralised debt obligations (CDOs). The standard 'Gaussian Copula' model used to quantify their credit risk was highly dependent on the choice of a single default correlation parameter that often required subjective judgement, as underlying assets were not standardised or only had a short history. This paper shows how moderate disagreement about default correlation increases the market value of the structured collateral considerably above that of its total cash-flow, as investors self-select into buying tranches they value more highly than others. The implied 'return to tranching' is sizeable for a typical RMBS, and an order of magnitude larger for CDOs backed by RMBS-tranches, whose cash-flow distribution is not bounded by a minimum recovery value and thus more sensitive to heterogeneous default correlations. In contrast, disagreement about average default probabilities, or recovery values, does not imply a large return to tranching.

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1 Introduction

"Structured finance' products, which allocate the cash-flow from a pool of collateral assets to different 'tranches' in order of pre-specified seniority, have been blamed both for their contribution to the US housing boom of the early 2000s, and for the large losses of financial institutions during the crisis that followed. In fact, structured assets, such as residential mortgage-backed securities (RMBSs) or collateralised debt obligations (CDOs), experienced an enormous rise in popularity between 2001 and 2007, when issuance of private-label mortgage-backed securities alone amounted to 5.8 trillion USD (Cordell et al., 2011), before issuance all but stopped by 2008. This boom has been attributed both to effiency-enhancing risk-management benefits, such as diversification and credit enhancement[1], and to a number of inefficient distortions, such as issuer moral hazard, regulatory arbitrage, or rating bias[2].

This paper points out an additional factor that may have fuelled the boom in structured finance and in collateral assets such as mortgage loans: disagreement about default correlations. The results of the highly stylised 'industry standard' Gaussian copula model of credit risk, used by both investors and rating agencies, depended in fact strongly on how correlated the defaults of collateral assets were (Coval et al., 2009). At the same time many collateral assets had no meaningful default histories, implying large uncertainty about their default correlation. This paper shows how disagreement about default correlation can strongly inflate the risk-neutral price of structured finance products as investors self-select into buying their preferred tranches. This is because, for investors who believe in low default correlation, diversification is powerful. Default rates thus vary little over time, and losses are concentrated in junior tranches, while senior tranches are essentially risk-free. For investors who believe defaults to be more strongly correlated, in contrast, risk is more reflective of aggregate conditions. They thus self-select into buying risky tranches, which they believe will pay whenever 'times are good', while senior tranches retain some risk of not paying fully in bad times.

The contribution of this paper is, first, to point out how disagreement about default correla-

[1] Structured finance assets share the diversification of credit risk through pooling of collateral assets with other forms of securitisation. Their particularity is the credit enhancement of more senior claims that are paid first and thus more appealing to investors with lower risk appetite, strict investment mandates, or concerns about their informational disadvantage relative to the issuer.

[2] Issuers had incentives to sell on credit-risk to investors to reduce regulatory capital requirements (see, e.g., Brunnermeier (2009)) and to exploit informational advantages about the true credit quality (Levitin and Wachter, 2012; Beltran et al., 2013). The complex nature and short history of structured finance assets opened room for subjectivity, and indeed positive bias, in the ratings by agencies for whom their active role in the structuring of assets was highly profitable (Griffin and Tang, 2011; 2012). Moreover, the additional risk implied by uncertain parameters such as probabilities and correlations of default, to which the valuation of structured finance assets were very sensitive, seems to have been under-appreciated by both investors and rating agencies, making structured finance assets again seem unduely attractive (Coval et al., 2009; Morini, 2011).

tions can increase the price of structured securitisations above the common expectation of their total cash-flow, and thus above the price of a simple, non-tranched ‘pass-through’ securitisation, as investors view different tranches as most profitable and thus self-select into paying high prices for them. The paper also shows how disagreement about mean payoffs, where optimists value all tranches higher than pessimists and are indifferent between buying collateral in tranches or not, does not imply such a ‘return to tranching’. The second contribution is to show how, when calibrating the industry standard quantitative credit risk model based on the Gaussian copula to capture the main features of CDOs backed by tranches of RMBS, this price increase can be up to several hundred basis points. This is because, contrary to the original RMBS whose price increase from disagreement is smaller if still substantial, CDO cash-flow is not protected by a minimum recovery value of the underlying collateral, amplifying the effect of default correlation on the distribution of payoffs.

A simple two-investor, two-loan example shows how any difference in perceived default correlations moves expected payoffs from a junior and a senior tranche in opposite directions. When asset supply is small relative to investor resources and tranches are priced at their highest valuation, this raises the price of the securitisation, equal to the average maximum tranche-valuation, above the maximum valuation of total collateral cash-flow, which is independent of default correlation. I call this price increase from structuring collateral cash-flow the ‘return to tranching’, and show how it is absent under disagreement about mean payoffs, where optimists have the highest valuation of all tranches, whose sum equals their valuation of total cash-flow. When supply is a substantial fraction of investor resources, and no single investor-type can afford to buy the loan pool, tranches still trade at a price equal to their maximum valuation with disagreement about default correlations, as opposing perceptions of tranche profitability make a large number of investors happy to pay high prices for different tranches. As a result, a rise in disagreement about default correlation always raises prices. Again, this is in contrast to disagreement about mean payoffs, where a large price discount is necessary for pessimists to participate. In the example, this implies that prices fall as disagreement about mean payoffs rises.

In a quantitative evaluation of this effect, the return to tranching is between 35 and 110 basis points for RMBS, and up to several hundred basis points for CDOs, which are more sensitive to disagreement about default correlations. A robustness analysis shows that the return to tranching is likely to be large when there is a large number of assets in the pool, average default probability is high, and there is strong expected comovement of loss severity with default rates. The fact that these characteristics all hold true for subprime RMBS suggests that investor disagreement about default correlation may have contributed to their boom in particular, and thus potentially to US house price inflation prior to the great recession. Importantly, the substantial ‘return to tranching’ provides an additional explanation for the popular ‘originate to distribute’ model of US mortgage finance prior to the crisis.

This paper studies the effect of heterogenous default correlations on the attractiveness of structured finance assets and the equilibrium price of collateral. To make this link most transparent, it focuses on a two-period environment where valuations simply equal discounted expected payoffs (by assuming risk-neutral investors). The analysis thus abstracts from other factors that contributed to the boom, such as heterogeneity in risk-aversion or investment mandates, information and incentive problems, regulatory arbitrage, or overoptimistic expectations about the riskiness of structured finance assets or the value of their underlying collateral (see, e.g., Fender and Mitchell (2005) for a discussion).

The paper assumes that investors agree on most features of the economy but agree to disagree about one or two model parameters. While this assumption is a simplification, it seems appealing in the case of structured finance assets for a number of reasons: first, contrary to most assets that may be valued using different models with potentially many dimensions of disagreement, structured finance products came with a unique "market standard" (Morini (2011), p. 127) model - the Gaussian copula with homogeneous correlations - with only three main parameters: the average default probability, the homogeneous correlation of defaults, and the value of loss given default. Like Griffin and Nickerson (2015), the quantitative analysis in this paper concentrates on default correlations, partly because, as the simpler example shows, disagreement about loss-given-default and average default probabilities do not lead to the self-selection at the heart of this paper, since the value of all tranches is monotonically declining in both parameters. Second, disagreement seems important in a situation where a short history provides little information about the stochastic properties of collateral assets such as subprime RMBS tranches, or, as a matter of fact, their constituent mortgages. Moreover, the parameter uncertainty that this implies seems to have been largely neglected by investors (Coval et al., 2009; Morini, 2011), making heterogeneous 'point'-beliefs an appealing, if simplifying, assumption. Finally, while investors should in principle use the information contained in market prices to adjust their beliefs, this effect will be muted in a market dominated by over-the-counter trades and with several unobserved dimensions of heterogeneity (most of which the analysis abstracts from) including a suspected lack of understanding of the complexity of the assets traded on the part of some participants.

Throughout the analysis, investors rely on their own beliefs, rather than those of rating agencies alone. While the market for structured finance products was indeed a 'rated market', evidence from surveys suggests that ratings were only one of many elements in the assessment of credit risks by investors (Fender and Mitchell, 2005). Moreover, the junior tranches where, as it turns out, disagreement leads to the strongest differences in value, were usually held by specialist investors who are likely to rely less heavily on ratings in their judgements (see Fender and

Mitchell (2005), p. 70). In fact, prices of non-AAA RMBS tranches seem to have incorporated information over and above credit ratings (Adelino, 2009). Note also that later in the boom, a substantial part of the collateral in CDOs was made up of credit default swaps (CDS) that referenced RMBS tranches, and thus effectively involved the short-sale of RMBS tranches to the CDO.[3] The analysis in this paper abstracts from short-sales. But the fact that hedge funds, which are usually taken to be less risk-averse than other investors, often were the counterparties to this trade, buying insurance against default of RMBS tranches via CDS, bears witness to the importance of disagreement among investors. Specifically, the popular 'long-short' strategy of buying the equity tranche of a CDO and CDS referencing its mezzanine tranches corresponds exactly to that of an investor who believes in higher-than-average correlation of the underlying collateral assets, which makes both the equity tranche more likely to pay, and the mezzanine tranche more likely to default.

2 A model of investors with heterogeneous beliefs

This section presents the general model environment and illustrates by use of a simple example how structuring collateral cash-flow in tranches can strongly boost the market value of a loan pool when investors disagree about the default correlation. This is because, by making extreme outcomes more likely, an increase in the perceived default correlation raises the expected payoffs from junior tranches that benefit from unusually low default rates, but lowers payoffs from senior tranches that suffer when default rates are high. Investors that perceive high correlation thus have a preference for the junior tranche in the example, while their low-correlation colleagues prefer the senior tranche, which they regard as relatively riskless. That investors view different tranches as most profitable has two effects: first, it raises the sum of maximum valuation of the tranches above the maximum valuation of the total collateral cash-flow (or of the non-tranched pass-through securitisation); and second, with different investors happy to pay high prices for different tranches, it increases the funds available for investment at these maximum valuations, and thus dampens the discount at which tranches trade in equilibrium relative to their highest valuation. Both effects are absent, or less strong, with disagreement about mean payoffs: all tranches are valued most by 'optimists', who value the collateral cash-flow with and without structuring equally; and, because optimists lack resources to buy all assets and pessimists thus need incentives to invest, equilibrium prices of all tranches include a discount on the highest valuation.

[3] The Financial Crisis Inquiry Commission estimates that the share of derivatives such as CDS in CDO collateral increased from 7 % in 2004 to 27 % in 2006 (Financial Crisis Inquiry Commission (2011), p. 191). Given the unregulated nature of most CDS trades, however, these figures should be taken with caution.

2.1 The General Environment

Consider an economy with $k = 1, .., K$ types of risk-neutral investors who live for two periods $t = 1, 2$, and whose preferences are $U_k = c_k + \frac{1}{R}E(c'_k)$, where c_k and c'_k are j's consumption in the first and second period respectively, $E(.)$ is the mathematical expectations operator and $\frac{1}{R}$ is the common discount factor. At the beginning of period 1, investors receive a non-storable consumption endowment a, which they can consume or invest in assets collateralised by a given pool of $n = 1, ..., N$ mortgages of face value and mass 1, which are sold by a single originator. In period 2 a stochastic fraction d of mortgages defaults. Defaulted mortgages pay recovery value $V_{rec} < 1$. Let $\overline{\pi}^k$ denote investor k's belief about the homogeneous default probability of mortgages.

The originator maximises current profits from selling the loan pool to investors. I compare two ways in which she sells the loan pool: as shares in a 'pass-through' securitisation that pays all investors their share in the total cash-flow that the collateral generates, equal to $1-d(1-V_{rec})\ \forall d$; or structured as an RMBS by splitting the cash-flow into 'tranches' that receive payments in strict order of their pre-specified seniority. Specifically, tranche 1 promises to make a total payment of $a_1 < 1$ to its holders in period 2, where a_1 is the 'detachment point' of tranche 1, and receives any cash-flow that defaulting and non-defaulting mortgages generate until a total of a_1 is reached. Tranche 2 promises to pay $a_2 - a_1$, where $a_1 < a_2 < 1$, but only receives cash-flow once a_1 has been paid to holders of the first tranche, etc.

Given one of the two securitisation possibilities - structured or pass-through - an equilibrium is defined as a vector of prices such that the originator maximises current profits, investors maximise utility, and demand for all assets equals supply. Investor optimality implies that the expected return from all positive investments in their portfolio must be the same, at least equal to R, and, in case it exceeds R, strictly higher than that from assets they do not hold.

2.2 Disagreement in a simplified model

The rest of this section looks at the special case of two mortgages $n = 1, 2$, for simplicity each of unit mass, and two investors $k = 1, 2$, who believe defaults to occur with probability $\overline{\pi}^k$ and whose different beliefs about default correlation are captured by π^k_{DD}, the conditional probability of one mortgage defaulting given default of the other.[4] Note that the conditional probability of the second loan being repaid, given payoff by the first, which equals $\pi^k_{NN} = 1 - \overline{\pi}(2 - \pi^k_{DD})$, is increasing in π^k_{DD}, as is the variance of default rates $Var^k_d = 2[\overline{\pi}(1-2\overline{\pi})+\pi\pi^k_{DD}]$. For simplicity,

[4]Note that this maps into the default correlation, calculated by assigning the numerical value 0 to the outcome of default and 1 to its complement, equal to $\rho^k_D = \frac{\overline{\pi}^k\pi^k_{DD}-\overline{\pi}^{k2}}{\overline{\pi}^k(1-\overline{\pi}^k)}$.

set $R = 1$ and $V_{rec} = 0$.

Investors value the cash-flow of the mortgage pool by its expected discounted payoff

$$V^k_{sec} = 2(1 - \overline{\pi}^k) \tag{1}$$

When the cash-flow is split into 2 tranches with face value 1 each, the value of the senior tranche is reduced by the probability of joint default $\overline{\pi}^k \pi^k_{DD}$

$$V^k_{sen} = 1 - \overline{\pi}^k \pi^k_{DD} \tag{2}$$

The junior tranche in turn only pays when both mortgages pay, so

$$V^k_{jun} = 1 - 2\overline{\pi}^k + \overline{\pi}^k \pi^k_{DD} \tag{3}$$

While all valuations are declining in the default probability $\overline{\pi}^k$, the default correlation parameter π^k_{DD} leaves that of the mortgage pool as a whole unaffected, but affects the valuation of tranches in opposite ways. Particularly, a rise in π^k_{DD} increases the value of the junior tranche, which gains from a rise in the probability of the extreme case of both mortgages paying off.

Disagreement about default correlation implies that different investors regard different tranches as particularly profitable. This affects equilibrium prices in two ways: first, it raises the sum of maximum tranche valuations above the maximum (in fact, in the simple example, common) valuation of total collateral cash-flow; and second, as different investors value different tranches most highly, it increases the funds available for investment at high prices. The first effect raises the market price of the structured securitisation above that of the non-tranched pass-through securitisation when investor demand strongly outweighs supply and all tranches are priced at their maximum valuation. The second effect dampens the equilibrium price discount that usually lowers prices of assets relative to their maximum valuation under investor disagreement.[5] To point out both effects, this section studies the market value of both the structured securitisation, and the non-tranched pass-through securitisation under two different assumptions about endowments. First, I study 'maximum prices', that arise in a situation where $a > 2$, so any

[5]The literature on heterogeneous investor beliefs has focused on disagreement about mean payoffs, which Miller (1977)'s seminal article shows to raise asset prices above the median valuation in the absence of short-selling, as the marginal investor becomes more optimistic as long as the asset is only bought by a minority of investors. Geanakoplos (2003) and Geanakoplos (2010) introduce leverage, which raises prices by increasing the resources of optimists. Simsek (2013) shows how leverage dampens the effect of belief disagreements on prices when optimists have relatively positive views on the distribution of relatively bad realisations of shocks. Broer and Kero (2014), in contrast, show how disagreement about the payoff variance increases prices with leverge and risk-neutrality, as high-variance investors buy the upside risk embodied in leveraged asset purchases.

single investor can purchase the mortgage pool, as well as its tranches, at their expected value for any $\overline{\pi}^k$ and π^k_{DD}. Prices of all assets thus simply equal their highest expected discounted cash-flow across investors. This case is similar to that analysed in the classical literature on disagreement about mean payoffs following Miller (1977), where a minority of optimist investors buys any particular asset.

Partly because of the sheer size of the supply of some collateral assets, such as US residential mortgages, this section also looks at 'equilibrium prices', where endowments equal $a = 2(1 - max\{\overline{\pi}^k\}) - \epsilon$ for a small positive number ϵ, such that no single investor can buy the whole securitisation, or all of its tranches, at their highest expected value across investors. As we will see, this leaves prices unaffected with disagreement about default correlations, as both investors are happy to invest at a price equal to the maximum valuaton of 'their' tranche and have enough funds to afford its total supply. With disagreement about mean payoffs, in contrast, in order to attract pessimists, equilibrium prices are lower than the valuation by optimists.

2.2.1 Disagreement about default correlation π^k_{DD}

Suppose agents agree on the probability of default $\overline{\pi}^1 = \overline{\pi}^2 = \overline{\pi}$, but disagree on their correlation, with $\pi^1_{DD} > \pi^2_{DD}$. Denote maximum prices as P^{max}, equilibrium prices as P^{equ}, but suppress their superscripts whenever the two are equal.

Result 1 ***The market price of the pass-through securitisation*** P_{sec}
The maximum and equilibrium prices of the non-tranched pass-through securitisation are the same, and equal the common expectation of collateral cashflow so $P_{sec} = (1 - \overline{\pi})$.

Result 2 ***The market prices of the tranched securitisation*** $P_{sec|tr}$ ***exceeds*** P_{sec}.
The equilibrium market price of the structured securitisation is the same as its maximum price, equal to the sum of the highest expected tranche payoffs across investors, so $P_{sec|tr} = 2(1 - \overline{\pi}^k) + \overline{\pi}\Delta\pi_{DD} > P_{sec}$, *for the disagreement measure* $\Delta\pi_{DD} = \pi^1_{DD} - \pi^2_{DD}$.
Proof: *Under both assumptions about endowments, investors can afford the tranche they value most highly at a price equal to their own, maximum valuation, so* $P_{jun} = V^1_{jun}$ *and* $P_{sen} = V^2_{sen}$, *for* P_{jun} *and* P_{sen} *the prices of the junior and senior tranche, respectively. Both investors strictly prefer to buy their tranche over buying the other and are indifferent between buying and consuming. The prices thus also clear the market for all assets. By arbitrage, the price of the structured securitisation is equal to the sum of the two tranches.*

Corollary 1 ***An Increase in disagreement about default correlation*** $\Delta\pi_{DD} > 0$ ***increases*** $P_{sec|tr}$.

2.2.2 Disagreement about default probability $\overline{\pi}^k$

Suppose instead agents disagree about the probability of default, with $\overline{\pi}^1 > \overline{\pi}^2$, but share the same belief about their correlation $\pi^1_{DD} = \pi^2_{DD} = \pi_{DD}$. Call investor 2 the 'optimist', and investor 1 the 'pessimist'.

Result 3 ***Maximum prices: no 'return to tranching'***
The maximum prices of the structured and pass-through securitisation $P^{max}_{sec|tr}$ and P^{max}_{sec} are the same, equal to the cash-flow expected by the optimist $2(1-\overline{\pi}^2)$.

Result 4 ***Equilibrium market prices are below maximum prices***
The equilibrium price of the pass-through securitisation is equal to the valuation by the pessimist $P^{equ}_{sec} = 2(1-\overline{\pi}^1)$. That of the structured securitisation lies between the expected values of pool cash-flow perceived by the optimist and the pessimist. In the special case $\pi_{DD} = 1$ (perfectly correlated defaults) $P^{equ}_{sec|tr} = P^{equ}_{sec} = 2(1-\overline{\pi}^1)$.

Result 5 ***An increase in disagreement decreases equilibrium prices***
When disagreement about the default probability increases by $d\overline{\pi} = d\overline{\pi}^1 = -d\overline{\pi}^2 > 0$, the equilibrium price of the tranched securitisation $P^{equ}_{sec|tr}$ falls.
Proof: *Since $\frac{V^1_{sen}}{V^1_{jun}} > \frac{V^2_{sen}}{V^2_{jun}}$, the pessimist has a comparative advantage in the senior tranche: she would never buy the junior tranche at a relative price $\frac{P_{sen}}{P_{jun}}$ that makes the senior tranche attractive to the optimist. This implies $P^{equ}_{sen} = V^1_{sen}$, and $P^{equ}_{jun} = \frac{P^{equ2}_{sen}}{V^2_{sen}} V^2_{jun}$ by arbitrage of the optimist. This yields*

$$P^{equ}_{sec|tr} = P^{equ}_{sen} + P^{equ}_{jun} = V^1_{sen}[1 + \frac{V^2_{jun}}{V^2_{sen}}] \quad (4)$$

$$= (1-\overline{\pi}^1)[1 + \frac{1 - 2\overline{\pi}^2 + \overline{\pi}^2 \pi_{DD}}{1-\overline{\pi}^2}] \quad (5)$$

It is easy to show that $P^{equ}_{sec|tr}$ is declining in $d\overline{\pi}$.

Disagreement about default correlation thus raises the market price of the structured securitisation above the common expected value of its cash-flow, as originators can exploit investor disagreement about which tranche is the most profitable. An increase in disagreement therefore raises originator profits. Disagreement about default probabilities, in contrast, makes all tranches most profitable for 'optimists', who may, however, lack the resources to buy them. To lure pessimists into the market, prices have to be lower, and fall as disagreement increases.

This strong result, of equilibrium prices that fall with disagreement about mean payoffs, depends on the stylised wealth distribution across two investors in the simple example, where effectively, pessimists price the asset. With a more general distribution prices may rise or fall depending on the relative asset supply. But whenever optimists cannot buy the whole collateral pool, price rises are contained by the need to incentivise agents with lower valuations to participate.[6] In contrast, the result that tranche prices equal their maximum valuations is true even with a more general wealth distribution, as the next section will show.

3 The quantitative implications of investor disagreement

This section concentrates on disagreement about default correlation and tries to quantify its effect on market prices. This task is made easier by the fact that, before the crisis, there was a clear "market standard" (Morini (2011), p. 127) for pricing and rating structured finance assets: the Gaussian copula model with homogeneous correlation among assets. This section shows how, in a calibration to a typical subprime RMBS, moderate disagreement about its correlation parameter increases the market value of collateral by about between 50 and 110 basis points. Prices for CDOs backed by (homogeneous) RMBS tranches, in contrast, can experience a price increase of several hundred basis points.

3.1 A Gaussian copula model

Consider a loan pool consisting of N mortgages. Investor k believes mortgage n to default whenever the following condition is met

$$x_n = \rho_k \cdot M + \sqrt{1-\rho_k^2} \cdot M_n < \overline{x} = \mathbb{N}^{-1}(\overline{\pi}),\ M, M_n \propto N(0,1) \tag{6}$$

Here, x_n is an index variable that can be interpreted as the value of creditor n's assets. It equals the weighted average of an aggregate factor M, capturing economy-wide conditions, and a loan- or borrower-specific factor M_n, which are both distributed according to the standard normal distribution. Investors agree that loan n defaults whenever the index x_n falls below a threshold $\overline{x}$ equal to the inverse normal distribution evaluated at the default probability $\overline{\pi}$, which this section assumes is shared by all investors. Investors disagree, however, about

[6] The literature on heterogeneous investor beliefs about mean payoffs assumes that the asset supply of any individual asset is small relative to available investment funds, such that a minority of optimists buys them (see e.g. Miller (1977), p.1153). The sheer size of the market for certain kinds of securitised collateral, such as US residential mortgages, makes this assumption less appealing in the context of structured finance.

the importance of aggregate conditions in determining loan defaults, as summarised by the parameter ρ_k. Specifically, ρ_k^2 equals the correlation between two individual creditors' asset values perceived by investor k. To understand the effect of ρ_k on default rates, note that for a large enough number of loans, when $\rho_k = 0$, the default rate of the pool d, equal to the share of loans whose asset values are less then the threshold, equals $\overline{\pi}$ with certainty. Investors with higher perceived ρ_k believe individual defaults to comove more strongly, and thus expect d to be less tightly distributed around $\overline{\pi}$, with a maximum variance of $\overline{\pi}(1-\overline{\pi})$ for $\rho_k = 1$. Together with the recovery value in case of default V_{rec}, ρ_k and $\overline{\pi}$ completely determine the distribution of the cash-flow from the mortgage pool equal to $\mathbb{C} = 1 - d(1 - V_{rec})\ \forall d$. This cash-flow is distributed either proportionally to investors' shares in a pass-through securitisation, or to tranches of an RMBS in order of their seniority. Its parsimony is one reason why this model, first introduced by Li (2000) and refined by Laurent and Gregory (2005), had become the standard for quantifying portfolio credit risk before the crisis.

The Gaussian copula model (6) conveniently allows investors to value not just RMBS, backed by mortgages directly, but also CDOs backed by a pool of J RMBS tranches whose cash-flow simply equals the sum of cash-flows from the individual tranches. The crucial additional parameter that determines the CDO's payoff distribution relative to that of any individual $RMBS$ tranche is the additional diversification gain from pooling. To introduce disagreement about the correlation of tranches in the CDO pool, I assume that investor k perceives the aggregate factor M in (6) to be the sum of a global factor $\mathbb{M}$ and an RMBS-specific factor M_j

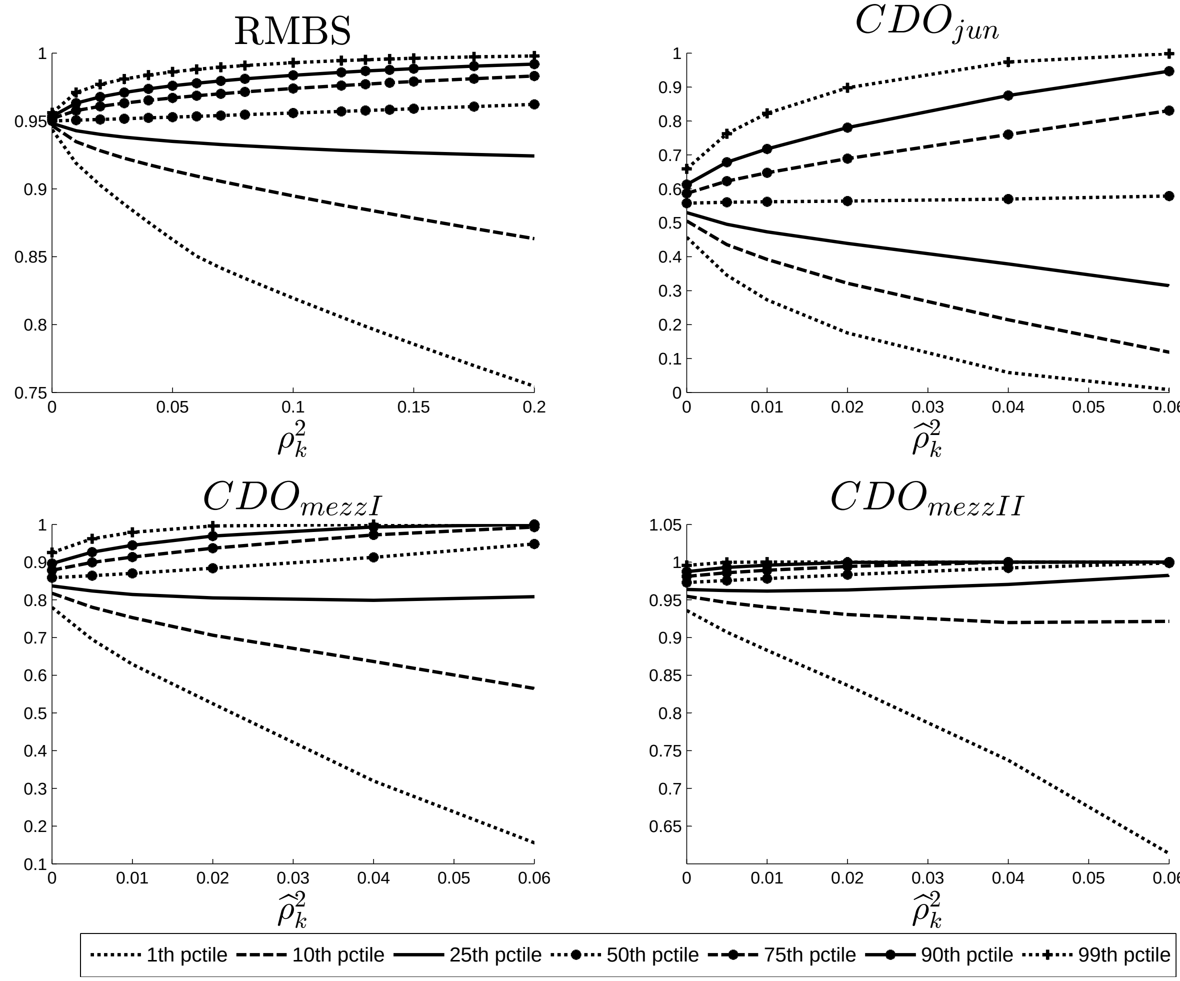
$$M = \rho'_k \cdot \mathbb{M} + \sqrt{1 - \rho'^2_k} \cdot M_j,\ \ \mathbb{M}, M_j \propto N(0,1) \tag{7}$$

The CDO's payoff distribution is thus determined by a 3-factor Copula, with an asset correlation that investor k perceives to equal ρ_k^2 for mortgages in the same RMBS, and $\widehat{\rho}_k^2 = \rho_k^2 \rho'^2_k \leq \rho_k^2$ for mortgages in different RMBS pools.

3.2 Payoff distributions and valuation of tranches

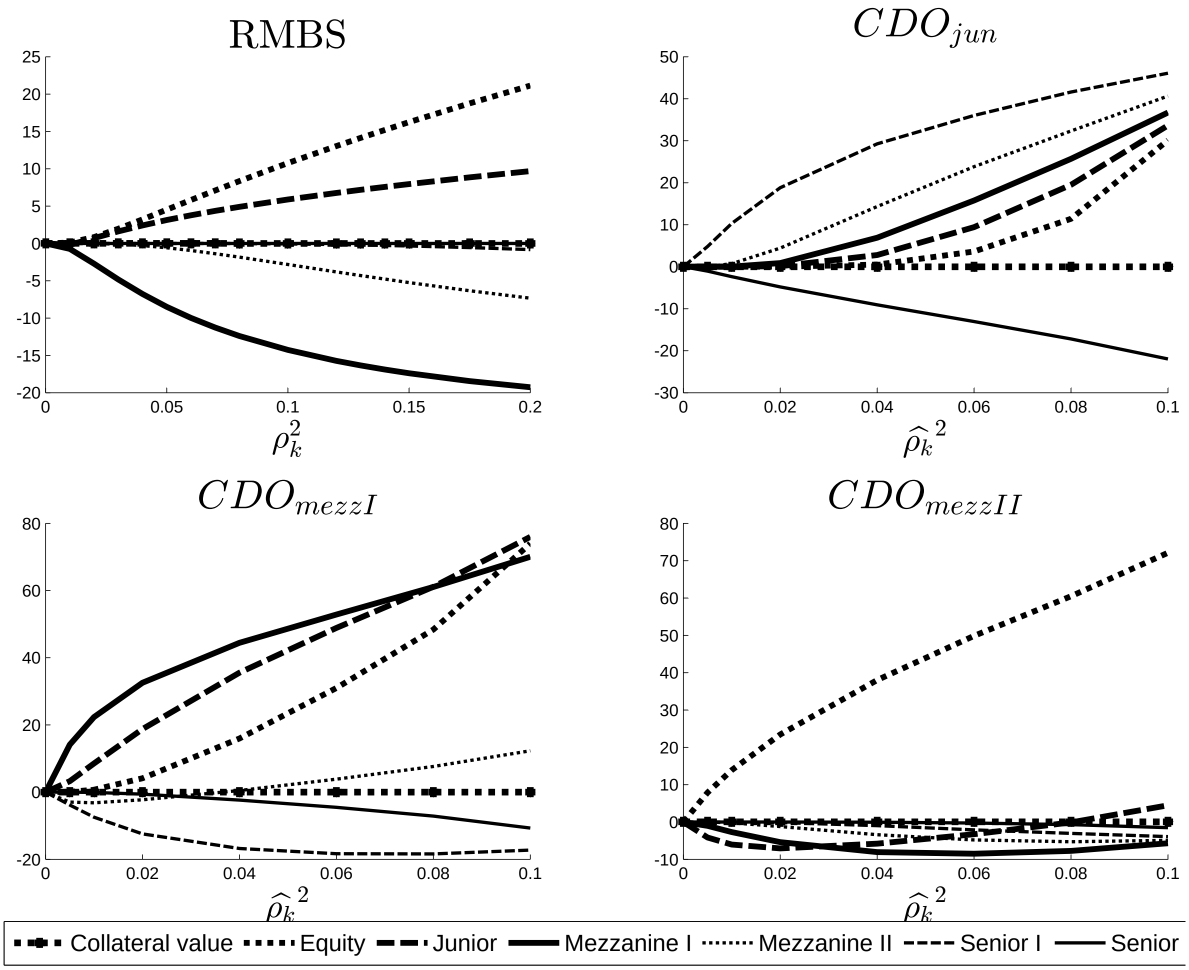
This section illustrates the effect of disagreement about the asset correlations ρ_k^2 and $\widehat{\rho}_k^2$ on the perceived distributions of collateral cash-flow and the expected payoffs of RMBS and CDO tranches. We consider structures, recovery values and average default probabilities, as well as a number of loans in RMBSs (5000) and of RMBS tranches in a CDO (100) roughly typical for securities backed by US subprime mortgages prior to the crisis. Since both RMBS and CDOs usually had similarly granular structures, the analysis uses the same 6 tranche structure as Coval et al. (2009) for both, consisting of an equity tranche (100-97 percent), a junior tranche (97-93 percent), mezzanine tranches I and II (93-88 and 88-80 percent respectively) and senior tranches

Figure 1: Distribution of collateral payoffs



The figure shows the distribution of collateral payoffs in an RMBS (upper left panel) and CDOs with homogenous collateral consisisting of junior (upper right panel), mezzanine I (lower left panel), and mezzanine II (lower right panel) RMBS tranches. The picture is based on a pool of 5000 mortgages with a common default probability $\overline{\pi}$ equal to 12.5 percent, and a recovery value V_{rec} that equals 60 *percent* on average, but comoves weakly with the default rate as explained in the main text.

Figure 2: Expected value of tranches



For the case of tranches of RMBS (upper left panel) and of CDOs with homogenous collateral consisisting of 100 junior (upper right panel), mezzanine I (lower left panel), and mezzanine II (lower right panel) RMBS tranches, the figure shows the difference between their payoffs expected by an investor with perceived asset correlation ρ_k^2 or $\widehat{\rho}_k^2$ (depicted along the bottom axis) and that expected by a 'zero correlation' investor (whose ρ_k^2 or $\widehat{\rho}_k^2$ equals 0), as a percentage of the underlying collateral's face value (the 'width' of the tranche).

I and II (80 to 65 and 65 to 0 percent).

The upper left panel of Figure 1 shows how the perceived distribution of cash-flows from a pool of 5000 mortgages changes as a function of the perceived asset correlations ρ_k^2 in the copula model (6).[7] To draw the figure, the remaining two parameters are chosen to capture features of the US subprime mortgage market and assumed to be common to all investors for simplicity. Specifically, I choose a default probability $\overline{\pi}$ equal to 12.5 percent, and a recovery value V_{rec} that comoves inversely with the default rate d in a range of $+/-15$ percentage points around its average of $\overline{V}_{rec} = 60$ percent, in order to account for longer time-until-foreclosure and lower resale values when default rates are high.[8] For $\rho_k^2 = 0$, the distribution in Figure 1 collapses around the expected payoff equal to $1 - d(1 - \overline{V}_{rec}) = 95$ percent. As ρ_k^2 rises, the distribution fans out at a decreasing rate. Even for high correlations, however, the lowest percentile remains above 75 percent, as payoffs are protected by the recovery value.

The remaining panels of figure 1 depict the perceived payoff distributions from pools of 100 junior RMBS tranches of equal seniority, each generated by the copula model (6) and (7). This illustration is based on a value of ρ_k^2 equal to 0.1 and considers RMBSs backed by the cash-flow distribution depicted in the upper left panel, structured in 6 tranches as described above. The upper right (lower left, lower right) panel of figure 1 depicts the resulting distribution of cash-flows from a pool of 100 junior (mezzanine I, mezzanine II) RMBS tranches as a function of the perceived 'remaining' asset correlation $\widehat{\rho}_k^2$. The payoff distributions of the CDO collateral differ in several important aspects from that of the RMBS. Apart from the difference in mean payoffs (increasing in the seniority of the tranche), the lower number of collateral assets implies that their payoff remains uncertain even when the diversification gain is perceived to be perfect (equivalent to a weight on the global factor of $\widehat{\rho}_k = 0$). More importantly, junior tranches of RMBS are not protected by the recovery value of the underlying mortgages from the bottom, and, just as the underlying tranche payoffs, are not bounded away from a full payoff at the top. The distributions thus fan out more strongly, implying higher payoff variances. For example, probabilities of a zero payoff are greater than 1 percent for junior RMBS tranches when $\widehat{\rho}_k^2$ exceeds 6 percent, while for mezzanine tranches, the upper percentiles bunch increasingly at 100 percent as $\widehat{\rho}_k^2$ increases above 2 percent.

To illustrate how the heterogeneous perceived payoff distributions depicted in Figure 1 affect

[7] To interpret the magnitudes, note that ρ_k^2 and $\widehat{\rho}_k^2$ do not equal default correlations. In fact, as Figure 3 in the appendix shows, the correlation between default events of any two mortgages in the RMBS is about half as large as the correlation of the underlying asset value x_n.

[8] The default rates for subprime mortgages differed strongly over time, fluctuating around 10 percent during the years of strong house price growth up to 2006 and increasing to 40 percent thereafter (see, e.g., Beltran et al. (2013)). The recovery value equals $V_{rec} = 0.6 + (d - \overline{d})$, where $\overline{d}$ is the benchmark default rate of 60 percent, but is bounded by a minimum of 45 percent.

the expected payoffs of RMBS and CDO tranches, Figure 2 shows the difference between their payoffs expected by an investor with perceived asset correlation ρ_k^2 or $\widehat{\rho}_k^2$ and that expected by a 'zero correlation' investor (whose ρ_k^2 or $\widehat{\rho}_k^2$ equals 0), as a percentage of the underlying collateral's face value (the 'width' of the tranche).

As expected, in the case of RMBS depicted in the upper left panel, as the perceived correlation ρ_k^2 increases and the variance of payoffs rises, the collateral value, or total expected payoff from the mortgage pool (the starred dashed line flat at 0), is unchanged as all investors share the same average default probability. Because the 'zero correlation' investor expects the payoff from the mortgage pool to equal 95 percent with certainty, she deems the junior and equity tranches, with attachment points close to or above 95 percent, to be worth nothing or little. High ρ_k^2 investors, in contrast, who perceive both a larger downside and upside risk, think that junior tranches are more likely to pay off, while they expect the mezzanine tranches to default with positive probability. Interestingly, the downside risk is never strong enough for investors to considerably disagree about the valuation of the senior tranches.

In line with the stronger rise in payoff variation of CDO collateral in figure 1, the disagreement about tranche valuations is more widely spread there, and differences in valuation an order of magnitude larger as a fraction of collateral face value. Thus, an investor who perceives no diversification gain from pooling RMBS tranches ($\rho_k^2 = \widehat{\rho}_k^2 = 0.1$) expects payoffs as a fraction of the face value from the three most junior CDO tranches backed by mezzanine I collateral to be between 70 and 80 percentage points higher than his counterpart who expects diversification gains to be perfect ($\rho_k^2 = 0.1$, $\widehat{\rho}_k^2 = 0$).[9]

4 The return to tranching

From figure 2, it is evident how disagreement about default correlations may raise securitisation profits if originators can sell tranches backed by collateral cash-flow to investors who value them particularly highly, rather than selling the cash-flow as a pass-through securitisation at its common, lower valuation. The extent to which this is possible, however, depends on the distribution of resources across investors. Like the simple example in Section 2.2, this section looks at two ways of determining prices: 'maximum' prices that simply equal the maximum valuation of investors (and are infact equilibrium prices if the endowment of any single investor type was large enough relative to the loan supply); and 'equilibrium' prices, where the endowments of several investors are needed to buy the loan pool. This is, again, in order to show how disagreement about default correlations has two effects, raising the sum of maximum tranche

[9]A final difference between RMBS and CDO tranche valuations is that the latter do not necessarily rise or fall monotonically as $\widehat{\rho}_k^2$ rises and the middle of the distribution 'thins out".

values above the maximum valuation of total collateral cash-flow; and requiring only a small, if any, discount in equilibrium prices as many investors are happy to invest at maximum prices of different tranches.

4.1 Equilibrium prices

When otherwise identical, risk-neutral investors disagree about the value of a single asset and short-selling is ruled out, the equilibrium price is typically determined by a marginal investor who is indifferent between buying or not. Investors with higher valuations invest all their resources such that total demand equals the supply at the equilibrium price. With several assets and general disagreement, however, it is not clear that an equilibrium price vector exists, or that it is unique. In the context of this paper, with I tranches, the equilibrium price vector p_i of tranches $i = 1, ...I$ ordered by seniority has to be such that supply equals demand for every tranche, and that any investor k expects to earn an equal return R^k not smaller than R, and greater than on any assets she does not hold, on all positive investments in her portfolio. Since this implies $(K+1) \cdot I$ equilibrium conditions, solving for any such equilibrium is often complex. In the case of RMBS, however, as Figure 2 suggests, the valuation of tranches is typically monotone in ρ_k^2: high ρ_k^2 types value junior tranches more than low ρ_k^2 types, while the reverse is true for senior tranches. The analysis exploits this feature to calculate the equilibrium price of all tranches in the example of a subprime RMBS with a small number of ρ_k^2 types. As in Section 2, I compare these equilibrium prices to 'maximum' prices that pertain in a version of the economy where a is large relative to the supply of mortgages, such that any of the finite ρ_k types can potentially purchase the whole securitisation. Thus, assets are valued at their maximum valuation across investors. Given the monotonicity of valuations in ρ_k, this economy is equivalent to an economy with only two types whose ρ_ks equal the bounds of the support for ρ_k.

4.2 Calibrating disagreement

As argued above, the distribution of cash endowments across ρ_k-types is crucial for equilibrium prices. This section does not aim at an exact calibration of this distribution. Rather, I first choose a support for $\{\rho_k\}_{k=1}^K \in [\rho_{min}, \rho_{max}]$ that is meant to be conservative, and is informed by three facts: first, the large uncertainty around estimates of the parameter ρ^2 and $\widehat{\rho}^2$ on short histories of data; second, the ratings of RMBS and CDO tranches by the main ratings agencies; and finally, the default experience in the US subprime mortgage market.[10] I then show how

[10] I do not consider information from RMBS and CDO tranche prices. This is, first, because their prices are often determined over the counter and thus unobserved. Moreover, the aim of the exercise

the distribution of cash endowments on that support is only of limited importance even for equilibrium prices.

In order to illustrate the uncertainty surrounding estimates of asset correlation ρ^2 in the copula model (6) on short histories of data, Figure 4 in the appendix shows the 10th and 90th percentile of estimates $\widetilde{\rho}^2$ for three different numbers of observations on default rates d. Even with a number of observations equal to 50, which far exceeds those on subprime mortgage defaults even at the yearly frequency, the uncertainty around $\widetilde{\rho}^2$ is huge.

I use two pairs of values $\{\rho^2_{min}, \rho^2_{max}\}$ which I label 'weak' and 'strong' disagreement. In the case of weak disagreement, to capture the intuition that structures were designed to give senior tranches top credit ratings, and that rating agencies often had optimistic assessments of default probabilities (Griffin and Tang, 2011), I choose a ρ^2_{min} equal to the maximum correlation compatible with an AAA rating of both senior RMBS tranches.[11] To calibrate ρ^2_{max}, I assume that a small number of investors perceived a small but significant probability of default rates rising to levels experienced during the crisis. In the case of 'weak' disagreement I interpret this to be a 0.5 percent probability of default rates in the RMBS pool reaching 40 percent or more, as observed in 2007 for US subprime mortgages (see, e.g., Beltran et al. (2013), especially figure 4). This calibration yields values of ρ^2_{min} and ρ^2_{max} equal to 7 and 12 percent, respectively (equivalent to default correlations of roughly 3 and 5 percent). In a 'strong' disagreement specification, I extend the range of values such that the ρ^2_{min}-investor would also just give the mezzanine II tranche an AAA rating, and such that the ρ^2_{max} investor perceives a probability of 1 percent of default rates rising to 40 percent or above. This yields values of ρ^2_{min} and ρ^2_{max} equal to 2 and 16 percent, respectively (corresponding to default correlations of 1 and 6.5 percent).

To calibrate the two extremes of the $\widehat{\rho}^2_k$ distribution, capturing disagreement about the additional diversification gain from pooling RMBS tranches, is more difficult. I choose $\widehat{\rho}^2_{min}$ to yield a default correlation of mezzanine RMBS tranches approximately equal to that calculated by Griffin and Nickerson (2015) for the main rating agencies, when fixing ρ^2 at a mean value of 10 percent.[12] This yields a diversification gain $1-\rho'^2_k$ in equation (7) equal to 88 percent, or ${\rho'_k}^2 = 0.12$. This diversification gain has been criticised as too optimistic, partly since most RMBS were already geographically diversified (see, e.g., Cordell et al. (2011)). I thus use this value as the lower bound of the ρ'_k-distribution ${\rho'_{min}}^2$, and set ${\rho'_{max}}^2$ to yield a lower additional

is to isolate the effect of disagreement about default correlations on equilibrium prices via expected payoffs. Observed prices, on the other hand, were affected by many other factors, such as risk aversion or investment mandates of some investors.

[11] In line with this, Ashcraft et al. (2010), p.13 find that the average fraction of subprime RMBS that received a AAA rating was 82 percent.

[12] Griffin and Nickerson (2015) back out an average default correlation of 0.03 for Moody's, and 0.042 for S&P. We choose $\widehat{\rho}^2_{min}$ to yield a default correlation of mezzanine II tranches equal to 3.8 percent.

diversification of 70 and 50 percent in the weak and strong disagreement cases respectively.

Contrary to 'maximum' prices, where tranches are priced by investors on the bound of the support of ρ^2 and $\widehat{\rho}^2$, the distribution on that support matters for equilibrium prices. I choose a particularly simple, uniform distribution on $[\rho^2_{min}, \rho^2_{max}]$ with 5 support points. I then calibrate the supply of mortgages and investors' consumption endowment such that at least three ρ^2_k-types are needed to buy the mortgage-pool. Specifically, I normalise the mass at each support point to 1 and set the endowment a equal to $\frac{2}{7}$ for all investors.[13] This implies that 20 percent of the consumption endowment is located at the extremes. As it turns out, this usually suffices for all junior tranches, whose valuation increases with ρ^2_k, to be bought by the ρ^2_{max} investor.

4.3 Disagreement, investor selection, and the return to tranching subprime mortgage pools

This section concentrates on the 'return to tranching' subprime mortgage pools into RMBS, defined as the difference in market values between the RMBS and that of the collateral when sold as a non-tranched, pass-through securitisation. Table 1 shows how, when all prices are at their maximum, this return equals 35 (85) basis points in the weak (strong) disagreement case. Interestingly, just like in Section 2, the equilibrium prices of all RMBS tranches are approximately equal to maximum prices. The return to tranching in Table 1, however, is <u>higher</u> with equilibrium prices. This is because the equilibrium price of the non-tranched, pass-through securitisation equals its (less than maximal) valuation by the median investor, as opposed to the maximum valuation of the ρ^2_{min} investor. Contrary to the simple example of Section 2.2, a higher perceived ρ^2_k lowers the valuation of the pass-through securitisation because loss-given default is assumed to rise with default rate d, implying a concave relationship between payoffs and default rates. So expected payoffs decline with the variance of defaults, or with the correlation parameter ρ^2_k, due to a Jensen's inequality effect. Section 5 evaluates the importance of this effect for the results.

Equilibrium tranche prices equal their maximum valuation for two reasons. First, like in the simple example in Section 2.2, ρ^2_{max} investors can afford to buy the two most junior tranches – which they value most highly – because their small attachment range and low payoff probability

[13] These values ensure that, should the originator decide to sell the loan-pool as a non-tranched pass-through securitisation, it is the median-ρ^2_k type who is the marginal investor that determines its price, equal to her expectation of collateral cash-flow. This is possible because, contrary to the simple example in Section 2.2, disagreement about ρ^2 implies disagreement about expected collateral cash-flow whenever the recovery value V_{rec} comoves negatively with the default rate d. A higher perceived ρ^2_k (and thus higher perceived default rate variance) reduces the expected collateral cash-flow, as defaults are expected to be numerous when loss-given-default is large.

Table 1: Return to tranching

	Max	**Equ**
Weak disagreement about ρ^2	34	44
Strong disagreement about ρ^2	84	111

The table presents the return to the originator of selling the mortgage pool in tranches rather than as a non-tranched pass-through securitisation, measured in basis points (100th of a percent) of the latter's market price.

make them cheap. Similarly, the two mezzanine tranches, whose valuation is declining with ρ^2 in the upper left panel of Figure 2, are affordable to the ρ^2_{min} investors. Second, the valuation of the remaining 2 senior tranches, which are not affordable to any single investor since their attachment range is large and expected loss negligible, is approximately the same across the three investors with the lowest ρ^2_k, who all view them as basically riskless. Thus, their price is, again, equal to their maximum valuation. Moreover, none of the investors has an arbitrage opportunity at these prices, as they are indifferent between their investments and consuming their endowment. In fact, this is a general pattern which in this setting of structured assets dampens the equilibrium effect that reduces asset prices under disagreement in other contexts: tranching makes the valuation of the bulk of senior tranches insensitive to disagreement. Rather, disagreement about valuations is concentrated in the junior tranches, which are cheap and can thus be bought by a small number of specialised investors. This increases the equlibrium return to tranching, as the collateral pool as a whole, or the pass-through securitisation, is typically priced at less than its highest valuation.

We do not model the mortgage market in this simple exercise, assuming for simplicity that a single originator reaps all the surplus from a return to tranching of between 45 and 110 basis points in equilibrium. An alternative, more complex environment where the surplus accrues to mortgage borrowers, would predict a fall in mortgage rates from selling the loan pool in tranches that equals the return to tranching calculated above. We think that the magnitude of this fall is sizeable when compared to the low real interest rates of the early 2000s and the expected loss rates on mortgage pools of only 5 percent. As it turns out, the return from tranching pools of RMBS tranches into CDOs, however, is an order of magnitude larger. Unfortunately, it is prohibitively complex to compute the equilibrium prices of tranches and collateral pools in the case of CDOs. The following analysis therefore concentrates on returns to (re-)tranching RMBS tranches into CDOs based on their maximum prices, noting that the previous results and intuition suggest these to be a lower bound for the equilibrium estimates.

Table 2: Return to re-tranching

	$Junior$	$Mezz_I$	$Mezz_{II}$
Weak disagreement about $\widehat{\rho}^2$	496	207	44
Strong disagreement about $\widehat{\rho}^2$	960	400	85

The table presents the return from selling a pool of junior / mezzanine I / mezzanine II RMBS tranches in the form of a CDO, measured in basis points (100th of a percent) of the price of the non-structured pool.

Table 3: Return to structuring the mortgage pool

	Weak disagreement about ρ^2	**Strong disagreement about** ρ^2
Weak disagreement about $\widehat{\rho}^2$	55	98
Strong disagreement about $\widehat{\rho}_2$	75	111

The table presents the difference in market value of a pool of 100 mortgage pools that are structured as an RMBS, whose tranches are then pooled with those of equal seniority from other RMBS and structured as CDOs, rather than sold as separate non-tranched pass-through securitisations, measured in basis points (100th of a percent) of the latters' market price.

4.4 CDOs and the 'returns to re-tranching"

Table 2 shows the returns to (re-)tranching RMBS tranches of three different levels of seniority into CDOs when investors disagree about $\widehat{\rho}_k^2$ but agree on ρ^2, the asset correlation within any given RMBS (set equal to 10 percent). The returns for both weak (top row) and strong disagreement (bottom row) are an order of magnitude higher than for RMBS, explained by the greater sensitivity of the CDOs' cash-flow distributions (in figure 1) and tranche valuations (in figure 2) to disagreement about $\widehat{\rho}_k^2$. Particularly, to an originator, selling junior RMBS tranches as a CDO yields a return of between 500 and 1000 basis points, as disagreement increases tranche values strongly and the price of the non-structured RMBS tranches is low due to a high average expected loss (somewhat below 50 percent, corresponding to an average RMBS payoff in the middle of the junior tranche's attachment range).

Table 3 presents the return to tranching any given mortgage pool as an RMBS and repackaging its tranches as CDOs, equal to the difference in price relative to that of the non-structured pass-through securitisation. The extra return on the RMBS from 're-tranching' its tranches as CDOs (i.e. the difference between the values in Table 3 and those in the first column of Table 1) is sizeable (with values between 10 and 40 basis points), but small relative to the large returns from packaging junior RMBS tranches in CDOs in Table 2. This is because the value of RMBS is dominated by the senior tranches whose expected payoff is perceived to be, essentially, riskless due to the substantial recovery value of 60 percent on average, and thus

Table 4: Return to structuring the mortgage pool for different values of $\overline{\pi}$

	$\overline{\pi} = 7.5\%$	$\overline{\pi} = 20\%$
Weak disagreement about ρ^2	21	61
Strong disagreemen about ρ^2	57	161

The table presents the return to the originator of selling the mortgage pool in tranches rather than as a non-tranched pass-through securitisation, measured in basis points (100th of a percent) of the latter's market price, for different values of the default probability $\overline{\pi}$.

unaffected by disagreement.

5 Robustness

This section shows how the effect of disagreement on the price of the structured collateral cash-flow increases with the default probability $\overline{\pi}$, and when recovery values V_{rec} depend negatively on default rates, as in the benchmark analysis, as opposed to a constant V_{rec}.[14] The fact that high default rates and default-rate-sensitive recovery values are characteristics of US subprime mortgage markets suggests that investor disagreement about default correlation may have been of particular importance there.

5.1 Alternative assumptions about the default probability

Tables 4 and 5 show the returns from structuring the mortgage pool at values of the default probability $\overline{\pi}$ higher and lower than the benchmark of 12.5 percent.[15] Note that asset correlations between 0 and 1 map into default variances between 0 and $\overline{\pi}(1-\overline{\pi})$. At default probabilities below 50 percent, any given disagreement about ρ thus implies a larger disagreement about the distribution of default rates as $\overline{\pi}$ increases. For example, when the probability of default equals 20 percent, the maximum return to tranching is between 35 and 75 basis points higher than in the benchmark case. Similarly, the maximum return when also selling RMBS tranches to investors who disagree about $\widehat{\rho}_k^2$, or the diversification gain of pooling different RMBS tranches, rises to about 210 basis points in this case, as can be seen in Table 5.

[14] A previous version of this paper also shows how the effect is higher with a larger number of assets in the pool, reducing stochastic 'noise' in default rates, and thus increasing the role of the default parameter ρ in determining default distributions.

[15] The effect of alternative assumptions about the recovery value on the payoff distribution, and thus on the prices of CDO and RMBS tranches, is very similar to that of alternative default probabilities, and thus omitted here.

Table 5: Return to re-tranching for different values of $\overline{\pi}$

	$\overline{\pi} = 7.5\%$	$\overline{\pi} = 20\%$
Weak (weak) disagreement about ρ^2 ($\widehat{\rho}^2$)	28	91
Weak (strong) disagreement about ρ^2 ($\widehat{\rho}^2$)	37	123
Strong (weak) disagreement about ρ^2 ($\widehat{\rho}^2$)	61	182
Strong (strong) disagreement about ρ^2 ($\widehat{\rho}^2$)	66	207

The table presents the return to the originator of selling the mortgage pool in tranches rather than as a non-tranched pass-through securitisation, measured in basis points (100th of a percent) of the latter's market price, for different values of the default probability $\overline{\pi}$.

Table 6: Return to structuring the mortgage pool with constant recovery value V_{rec}

	Max	**Equ**
Weak disagreement about ρ^2	30	30
Strong disagreement about ρ^2	71	71

The table presents the return to the originator of selling the mortgage pool in tranches rather than as a non-tranched pass-through securitisation, measured in basis points (100th of a percent) of the latter's market price, when the recovery value V_{rec} is constant at 60 percent.

5.2 Constant recovery value V_{rec}

The benchmark quantitative results of this paper were derived under the reasonable assumption that recovery values of mortgages are negatively affected by realised default rates in order to account for longer time-until-foreclosure and lower resale values when default rates are high. Since losses on individual mortgages are then large (small) when many (few) loans default, this increases the payoff variance from the mortgage portfolio for any given $\rho > 0$, and thus amplifies the effect of disagreement about ρ. Indeed tables 6 and 7 show that returns from structuring the mortgage pool are between 5 and 10 basis points lower when the recovery value V_{rec} is constant at 60 percent. Interestingly, both returns in Table 6 are now equal. To understand this, remember that the higher equilibrium return in the benchmark results was entirely due to a lower equilibrium price of the pass-through securitisation. This was because, when the recovery value declines with default rate d, payoffs fall faster as d rises, implying a concave relationship between payoffs and default rates. Expected payoffs thus decline with the variance of defaults, or with the correlation parameter ρ, due to a Jensen's inequality effect. When the recovery value is constant, in contrast, this effect of ρ on the price of the pass-through securitisation is absent. So investors agree about its expected value and there is no difference between maximum valuation and equilibrium price.

Table 7: Return to re-tranching with constant recovery value V_{rec}

	Weak disagreement about ρ^2	**Strong disagreement about ρ^2**
Weak disagreement about $\widehat{\rho}^2$	45	83
Strong disagreement about $\widehat{\rho}^2$	60	95

The table presents the partial equilibrium return to the originator of selling the mortgage pool in tranches rather than as a non-tranched pass-through securitisation, measured in basis points (100th of a percent) of the latter's market price, when the recovery value V_{rec} is constant at 60 percent.

6 Conclusion

This paper proposes a new, additional reason for the popularity of structured assets prior to the financial crisis: disagreement about default correlations. Investors who believe in high default correlation like junior tranches, who they think pay whenever aggregate times are good while they expect senior assets to incur losses in bad times. Low-correlation investors think the risk to senior assets is low, while junior assets are, essentially, junk. We showed how this preference for different assets makes investors pay high prices for tranches of different seniority, and thus raises the price of the structured securitisation above that of collateral cash-flow sold in simple shares, as a pass-through securitisation. With disagreement about mean payoffs, where optimists value the securitisation highest whether tranched or not, there is no additional price rise from structuring collateral cash-flow. Our quantitative analysis showed how this 'return to tranching' cash-flow over selling it as a pass-through securitisation may have been substantial for RMBSs. It may have been even more important, however, for CDOs, whose cash-flow distribution is not bounded below by a positive minimum recovery value of the collateral and thus more affected by default correlation. We think that this large 'return to tranching' thus provides an additional explanation for the boom in structured finance and, particularly, for the popular 'originate to distribute' model of US mortgage finance prior to the crisis.

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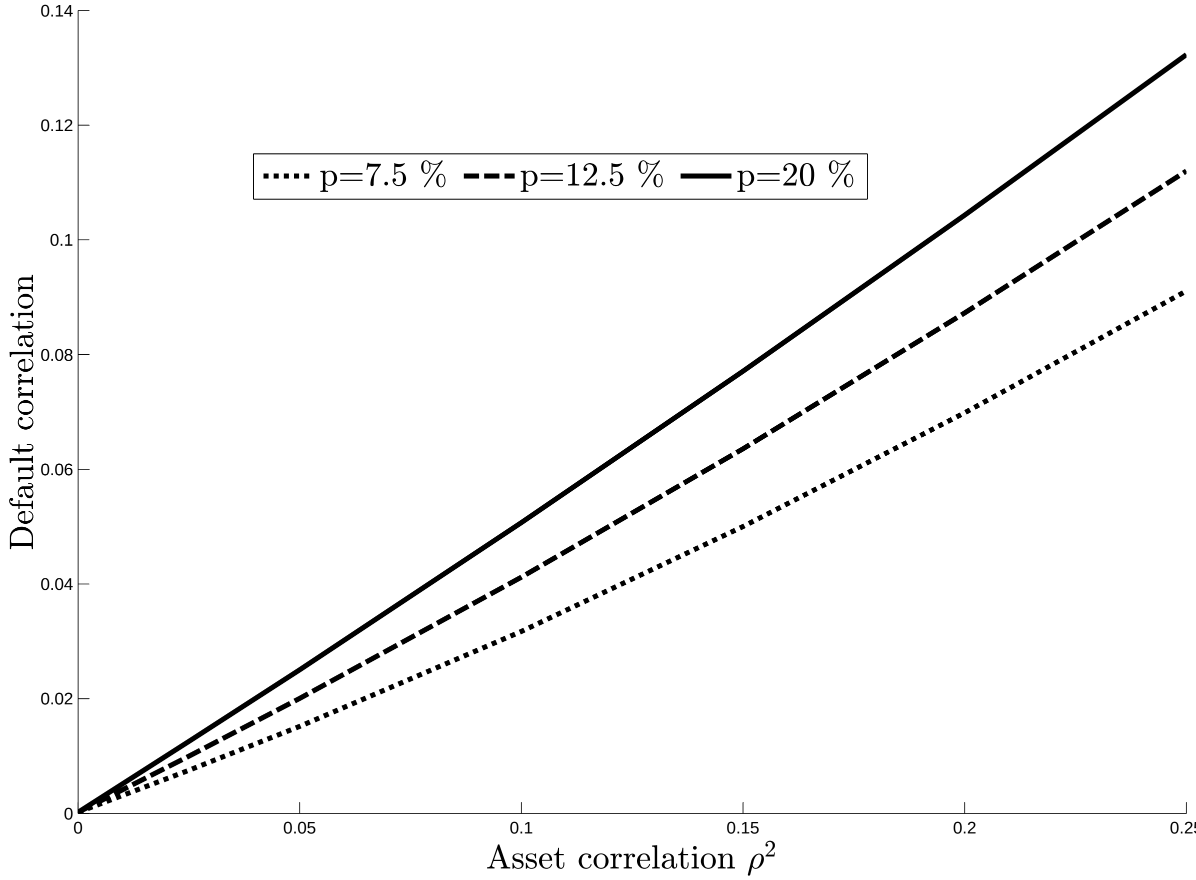
Figure 3: Default and asset correlation

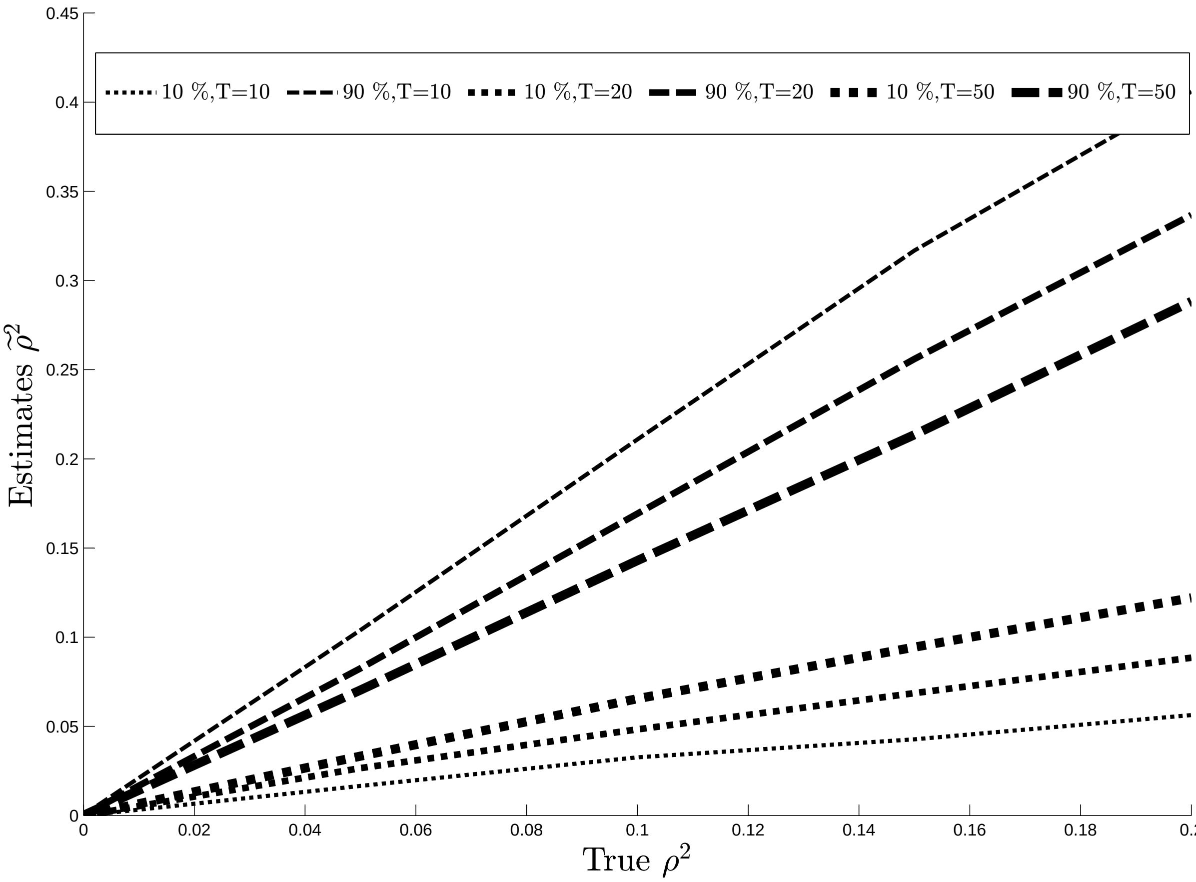


The figure shows the default correlation as a function of the asset correlation ρ^2 and the probability of default $\overline{\pi}$.

7 Appendix

7.1 Additional figures

Figure 4: Percentiles of $\widetilde{\rho}^2$ distribution



The figure shows the 10th and 90th percentiles of the distribution of estimates for the asset correlation in the copula model (6). Estimates are based on the simulation of 5000 times series of default rates of length T, for a mortgage pool of 5000 loans with default probability of 12.5 percent. We use a method of moments estimator that chooses, for every simulation, the asset correlation that yields the same variance of default rates in a long simulation of 5000 periods as in the short time series.