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Abstract

We show that any well-behaved demand function can be represented by its demand manifold, a smooth curve which relates the elasticity and convexity of demand. This manifold is a sufficient statistic for many comparative statics questions; leads naturally to characterizations of new families of demand functions which nest most of those used in applied economics; and connects assumptions about demand structure with firm behavior and economic performance. In particular, we show that the demand manifold leads to new insights about industry adjustment with heterogeneous firms, and provides a quantitative framework for measuring the effects of globalization.

JEL Classification: F23, F15, F12

Keywords: Heterogeneous Firms, Pass-Through, Quantifying Responses to Trade Liberalization, Super- and Sub-Convexity, Supermodularity

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1 Introduction

Assumptions about the structure of preferences and demand matter enormously for comparative statics in trade, industrial organization, and many other applied fields. Examples from international trade include competition effects (such as whether globalization reduces firms' markups), which depend on whether the elasticity of demand falls with sales;[1] and selection effects (such as whether more productive firms select into FDI rather than exports), which depend on whether the elasticity and convexity of demand sum to more than three.[2] Examples from industrial organization include pass-through (do firms pass on cost increases by more than dollar-for-dollar?), which depends on whether the demand function is log-convex,[3] and the welfare effects of third-degree price discrimination, which depend on how demand convexity varies with price.[4] In all these cases, the answer to an important real-world question hinges on a feature of demand which seems at best arbitrary and in some cases esoteric. All bar specialists may have difficulty remembering these results, far less explicating them and relating them to each other.

There is an apparent paradox here. These applied questions are all supply-side puzzles: they concern the behavior of firms or the performance of industries. Why then should the answers to them hinge on the shape of demand functions, and in many cases on their second or even third derivatives? The paradox is only apparent, however. In perfectly competitive models, shifts in supply curves lead to movements along the demand curve, and so their effects hinge on the slope or elasticity of demand. When firms are monopolists or monopolistic competitors, as in this paper, they do not have a supply function as such; instead, exogenous supply-side shocks or differences between firms lead to more subtle differences in behavior, whose implications depend on the curvature as well as the slope of the demand function.

Different authors and even different sub-fields have adopted a variety of approaches to

[1] See Krugman (1979) and Zhelobodko, Kokovin, Parenti, and Thisse (2012).
[2] See Helpman, Melitz, and Yeaple (2004) and Mrázová and Neary (2011).
[3] See Bulow and Pfleiderer (1983) and Weyl and Fabinger (2013).
[4] See Schmalensee (1981) and Aguirre, Cowan, and Vickers (2010).

these issues. Weyl and Fabinger (2013) show that many results can be understood by taking the degree of pass-through of costs to prices as a unifying principle. Macroeconomists frequently work with the "superelasticity" of demand, due to Kimball (1995), to model more realistic patterns of price adjustment than allowed by CES preferences. In our previous work (Mrázová and Neary (2011)), we showed that, since monopoly firms adjust along their marginal revenue curve rather than the demand curve, the elasticity of marginal revenue itself pins down some results. Each of these approaches focuses on a single demand measure which is a sufficient statistic for particular results. This paper goes much further than these, by developing a general framework that provides a new perspective on how assumptions about the functional form of demand determine conclusions about comparative statics.

The key idea we explore is the value of taking a "firm's eye view" of demand functions. To understand a monopoly firm's responses to infinitesimal shocks it is enough to focus on the local properties of the demand function it faces, since these determine its choice of output: the slope of demand determines the firm's level of marginal revenue, which it wishes to equate to marginal cost, while the curvature of demand determines the slope of marginal revenue, which must be decreasing if the second-order condition for profit maximization is to be met. Measuring slope and curvature in unit-free ways leads us to focus on the elasticity and convexity of demand, following Seade (1980), and we show that for any well-behaved demand function these two parameters are related to each other. We call the implied relationship the "demand manifold", and show that it is a sufficient statistic linking the functional form of demand to many comparative statics properties. It thus allows us to develop new comparative statics results and illustrate existing ones in a simple and compact way; and it leads naturally to characterizations of new families of demand functions which provide a parsimonious way of nesting existing ones, including most of those used in applied economics.[5]

[5] Demand functions used in recent work that fit into our framework include the linear (Melitz and Ottaviano (2008)), LES (Simonovska (2015)), CARA (Behrens and Murata (2007)), translog (Feenstra (2003)), QMOR (Feenstra (2014)), and Bulow-Pfleiderer (Atkin and Donaldson (2012)). See Section 3.3 and Appendices I and J.

A "firm's-eye view" is partial-equilibrium by construction, of course. Nevertheless, it can provide the basis for understanding general-equilibrium behavior. To demonstrate this, we show how our approach allows us to characterize the responses of outputs, prices and product variety in the canonical model of international trade under monopolistic competition due to Krugman (1979). We show how the quantitative magnitude of the model's properties can be related to the assumed demand function through the lens of the implied demand manifold. Furthermore, we use our approach to derive new results for the case of heterogeneous firms, as in Melitz (2003), but general demands, as in Zhelobodko, Kokovin, Parenti, and Thisse (2012) and Bertoletti and Epifani (2014).

The plan of the paper follows this route map. Section 2 introduces our new perspective on demand, and shows how the elasticity and convexity of demand condition comparative statics results. Section 3 shows how the demand manifold can be located in the space of elasticity and convexity, and explores how a wide range of demand functions, both old and new, can be represented by their manifold in a parsimonious way. Section 4 illustrates the usefulness of our approach by applying it to a canonical general-equilibrium model of international trade under monopolistic competition, and characterizing the implications of assumptions about functional form for the quantitative effects of exogenous shocks. Section 5 concludes, while the Appendix gives proofs of all propositions, discusses some extensions, and provides a glossary of terms used.

2 Demand Functions and Comparative Statics

2.1 A Firm's-Eye View of Demand

A perfectly competitive firm takes the price it faces as given. Our starting point is the fact that a monopolistic or monopolistically competitive firm takes the demand function it faces as given. Observing economists will often wish to solve for the full general equilibrium of the economy, or to consider the implications of alternative assumptions about the structure of preferences (such as discrete choice, representative agent, homotheticity, separability, etc.). By contrast, the firm takes all these as given and is concerned only with maximizing profits subject to the partial-equilibrium demand function it perceives. For the most part we write this demand function in inverse form, $p = p(x)$, with the only restrictions that consumers' willingness to pay is continuous, three-times differentiable, and strictly decreasing in sales: $p'(x) < 0$.[6] It is sometimes convenient to switch to the corresponding direct demand function, $x = x(p)$, with $x'(p) < 0$, the inverse of $p(x)$.

As explained in the introduction, we express all our results in terms of the slope and curvature of demand, measured by two unit-free parameters, the elasticity ε and convexity ρ of the demand function:

$$\varepsilon(x) \equiv -\frac{p(x)}{xp'(x)} > 0 \quad \text{and} \quad \rho(x) \equiv -\frac{xp''(x)}{p'(x)} \tag{1}$$

These are not unique measures of slope and curvature, and our results could alternatively be presented in terms of other parameters, such as the convexity of the direct demand function, or the Kimball (1995) superelasticity of demand. Appendix A gives more details of these alternatives, and explains our preference for focusing on ε and ρ.

Because we want to highlight the implications of alternative assumptions about demand, we assume throughout that marginal cost is constant.[7] Maximizing profits therefore requires that marginal revenue should equal marginal cost and should be decreasing with output. This imposes restrictions on the values of ε and ρ that must hold at a profit-maximizing equilibrium. From the first-order condition, a non-negative price-cost margin implies that the elasticity must not be less than one:

$$p + xp' = c \geq 0 \quad \Rightarrow \quad \varepsilon \geq 1 \tag{2}$$

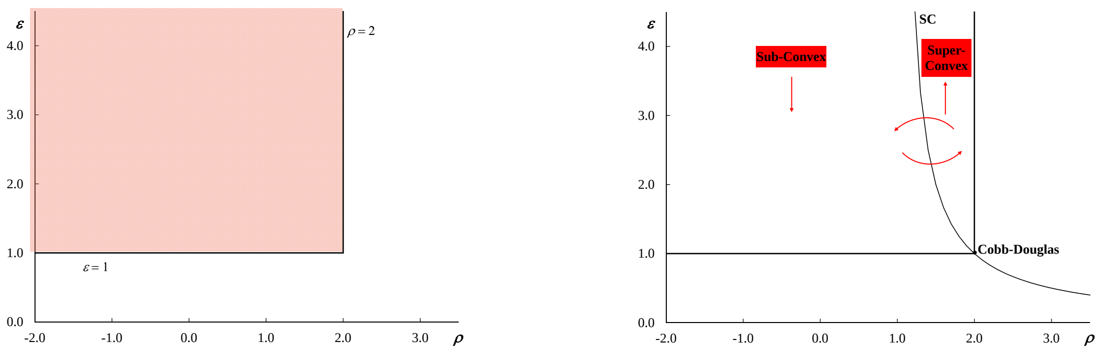
[6]We use "sales' throughout to denote consumption x, which in equilibrium equals the firm's output.

[7]Zhelobodko, Kokovin, Parenti, and Thisse (2012) show that variable marginal costs make little difference to the properties of models with homogeneous firms. In models of heterogeneous firms it is standard to assume that marginal costs are constant.

As for the second-order condition, if marginal revenue $p + xp'$ decreases with output, then our measure of convexity must be less then two:

$$2p' + xp'' < 0 \quad \Rightarrow \quad \rho < 2 \tag{3}$$

These restrictions can be visualized in terms of an admissible region in $\{\varepsilon, \rho\}$ space, as shown by the shaded region in Figure 1(a).[8]



(a) The Admissible Region

(b) The Super- and Sub-Convex Regions

Figure 1: The Space of Elasticity and Convexity

[8]The admissible region is $\{\varepsilon, \rho\} \in \{1 \leq \varepsilon \leq \infty, -\infty \leq \rho < 2\}$. In the figures that follow, we illustrate the subset of the admissible region where $\varepsilon \leq 4.5$ and $\rho \geq -2.0$, since this is where most interesting issues arise and it is also consistent with the available empirical evidence. (Broda and Weinstein (2006), Soderbery (2015) and Benkovskis and Wörz (2014) estimate median elasticities of demand for imports of 3.7 or lower.) Note that the admissible region is larger in oligopolistic markets, since both boundary conditions are less stringent than (2) and (3). See Appendix B for details.

2.2 The CES Benchmark

In general, both ε and ρ vary with sales. The only exception is the case of CES preferences or iso-elastic demands:[9]

$$p(x) = \beta x^{-1/\sigma} \quad \Rightarrow \quad \varepsilon = \sigma, \quad \rho = \rho^{CES} \equiv \frac{\sigma + 1}{\sigma} > 1 \tag{4}$$

Clearly this case is very special: both elasticity and convexity are determined by a single parameter. The curve labeled "SC" in Figure 1(b) illustrates the implied relationship between ε and ρ for all members of the CES family: $\varepsilon = \frac{1}{\rho - 1}$, or $\rho = \frac{\varepsilon+1}{\varepsilon}$. Every point on this curve corresponds to a different CES demand function: firms always operate at that point irrespective of the values of exogenous variables. In this respect too the CES is very special, as we will see. The Cobb-Douglas special case corresponds to the point $\{\varepsilon, \rho\} = \{1, 2\}$, and so has the dubious distinction of being just on the boundary of both the first- and second-order conditions.

The CES case is important in itself but also because it is an important boundary for comparative statics results. Following Mrázová and Neary (2011), we say that a demand function is "superconvex" at an arbitrary point if it is more convex at that point than a CES demand function with the same elasticity. Hence the eponymous SC curve in Figure 1(b) divides the admissible region in two: points to the right of the curve are strictly superconvex, points to the left are strictly subconvex, while all CES demand functions are both weakly superconvex and weakly subconvex. As we show in Appendix C, superconvexity also determines the relationship between demand elasticity and sales: the elasticity of demand increases in sales (or, equivalently, decreases in price), $\varepsilon_x \geq 0$, if and only if the demand function $p(x)$ is superconvex. So, ε is independent of sales only along the SC locus, it increases

[9] It is convenient to follow the widespread practice of applying the "CES" label to the demand function in (4), though this only follows from CES preferences in the case of monopolistic competition, when firms assume they cannot affect the aggregate price index. The fact that CES demands are sufficient for constant elasticity is obvious. The fact that they are necessary follows from setting $-\frac{p(x)}{xp'(x)}$ equal to a constant σ and integrating.

with sales in the superconvex region to the right, and decreases with sales in the subconvex region to the left.[10] These properties imply something like the comparative-statics analogue of a phase diagram: the arrows in Figure 1(b) indicate the direction of movement as sales rise.

2.3 Illustrating Comparative Statics Results

We can use our diagram to illustrate some of the comparative statics results discussed in the introduction. The results themselves are not new, but illustrating them in a common framework provides new insights and sets the scene for our discussion of the implications of particular demand functions in Section 3.

2.3.1 Competition Effects and Relative Pass-Through: Superconvexity

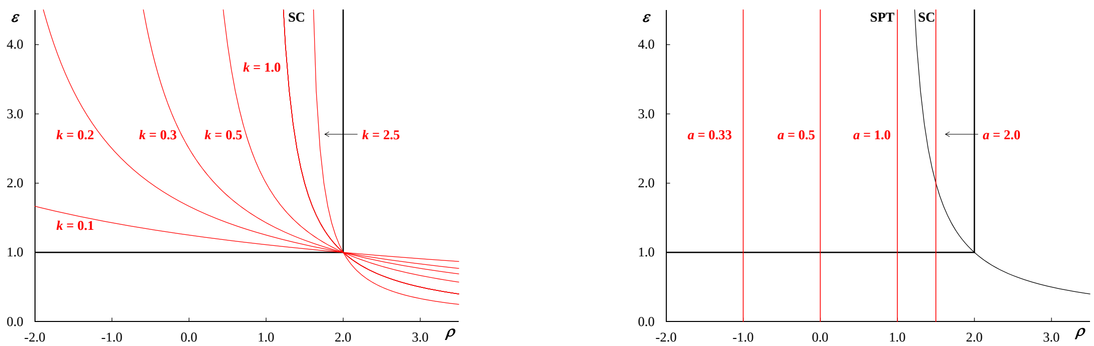
Superconvexity itself determines both competition effects and relative pass-through: the effects of globalization and of cost changes respectively on firms' proportional profit margins.[11] From the first-order condition, the relative markup $\frac{p-c}{p}$ equals $-\frac{xp'}{p}$, which is just the inverse of the elasticity ε. Hence, if globalization reduces incumbent firms' sales in their home markets, it is associated with a higher elasticity and so a lower markup if and only if demand is subconvex. Similarly, an increase in marginal cost c, which other things equal must lower sales, is associated with a higher elasticity and so a lower proportional profit margin, implying less than 100% pass-through, if and only if demands are subconvex:

$$\frac{d\log p}{d\log c} = \frac{\varepsilon-1}{\varepsilon}\frac{1}{2-\rho} > 0 \quad \Rightarrow \quad \frac{d\log p}{d\log c} - 1 = -\frac{\varepsilon+1-\varepsilon\rho}{\varepsilon(2-\rho)} > 0 \tag{5}$$

[10] Many authors, from Marshall onwards, have argued that subconvexity is intuitively more plausible. Moreover, it is consistent with much of the available empirical evidence. However, superconvexity cannot be ruled out either theoretically or empirically.

[11] This is the sense in which the term "pass-through" is used in international macroeconomics. See for example Gopinath and Itskhoki (2010).

More generally, loci corresponding to $100k\%$ pass-through are defined by $\varepsilon = \frac{1}{1-(2-\rho)k}$.[12] Figure 2(a) illustrates some of these loci for different values of k.



(a) Constant Proportional Pass-Through

(b) Constant Absolute Pass-Through

Figure 2: Loci of Constant Pass-Through

2.3.2 Absolute Pass-Through: Log-Convexity

The criterion for absolute or dollar-for-dollar pass-through from cost to price has been known since Bulow and Pfleiderer (1983). Differentiating the first-order condition $p + xp' = c$, we see that an increase in cost must raise price provided only that the second-order condition holds, which implies a different expression for the effect of an increase in marginal cost on the absolute profit margin:

$$\frac{dp}{dc} = \frac{1}{2-\rho} > 0 \quad \Rightarrow \quad \frac{dp}{dc} - 1 = \frac{\rho - 1}{2-\rho} \gtreqless 0 \tag{6}$$

Hence we have what we call "Super-Pass-Through", whereby the equilibrium price rises by more than the increase in marginal cost, if and only if ρ is greater than one. More generally, loci corresponding to a pass-through coefficient of a are defined by convexity values of $\rho = 2 - \frac{1}{a}$. Figure 2(b) illustrates some of these loci for different values of a. The one corresponding to $a = 1$, labeled "SPT", divides the admissible region into sub-regions of

[12] This is a family of rectangular hyperbolas, all asymptotic to $\{\varepsilon, \rho\} = \{\infty, \frac{2k-1}{k}\}$ and $\{0, \infty\}$, and all passing through the Cobb-Douglas point $\{\varepsilon, \rho\} = \{1, 2\}$.

sub- and super-pass-through. It corresponds to a log-linear direct demand function, which is less convex than the CES.[13] Hence superconvexity implies super-pass-through, but not the converse: in the region between the SPT and SC loci, pass-through is more than dollar-for-dollar but less than 100%. More generally, comparing Figures 2(a) and 2(b) shows that at any point the degree of absolute pass-through is greater than that of relative pass-through, and by more so the lower the elasticity; the implied relationship is: $a = \frac{\varepsilon}{\varepsilon-1}k$.

2.3.3 Selection Effects: Supermodularity

The third criterion for comparative statics responses that we can locate in our diagram arises in models with heterogeneous firms, where firms choose between two alternative ways of serving a market, such as the choice between exports and foreign direct investment (FDI) as in Helpman, Melitz, and Yeaple (2004).[14] Mrázová and Neary (2011) show that more efficient firms are sure to select into FDI only if their *ex post* profit function is *supermodular* in their own marginal cost c and the iceberg transport cost they face t. Supermodularity holds if and only if the elasticity of marginal revenue with respect to sales is less than one, which in turn implies that the elasticity and convexity of demand sum to more than three.[15] When this condition holds, a 10% reduction in the marginal cost of serving a market raises sales by more than 10%, so more productive firms have a greater incentive to engage in FDI than in exports. This criterion defines a third locus in $\{\varepsilon, \rho\}$ space, as shown in Figure 3. Once again it divides the admissible region into two sub-regions, one where either the elasticity or convexity or both are high, so supermodularity prevails, and the other where the

[13]Setting $\rho = 1$ implies a second-order ordinary differential equation $xp''(x) + p'(x) = 0$. Integrating this yields $p(x) = c_1 + c_2 \log x$, where c_1 and c_2 are constants of integration, which is equivalent to a log-linear direct demand function, $\log x(p) = \gamma + \delta p$.

[14]Mrázová and Neary (2011) show that the same criterion determines selection effects in a number of other cases, including the choice between producing in the high-wage "North" or building a new export plant in the low-wage "South" as in Antràs and Helpman (2004), and the choice of technique as in Bustos (2011).

[15]Let $\pi(c, t) \equiv \max_x [p(x) - tc]\, x$ denote the maximum operating profits which a firm with marginal production cost c can earn facing an iceberg transport cost of accessing the market equal to t. By the envelope theorem, $\pi_c = -tx$. Hence, $\pi_{ct} = -x - t\frac{dx}{dt} = -x - \frac{tc}{2p' + xp''} = -x + \frac{\varepsilon-1}{2-\rho}x$. Writing revenue as $R(x) = xp(x)$, so marginal revenue is $R' = p + xp'$, the elasticity of marginal revenue (in absolute value) is seen to be: $-\frac{xR''}{R'} = \frac{2-\rho}{\varepsilon-1}$. The results in the text follow by inspection.

profit function is submodular. The locus lies everywhere below the superconvex locus, and is tangential to it at the Cobb-Douglas point. Hence, supermodularity always holds with CES demands. However, when demands are subconvex and firms are large (operating at a point on their demand curve with relatively low elasticity), submodularity prevails, and so the standard selection effects may be reversed.

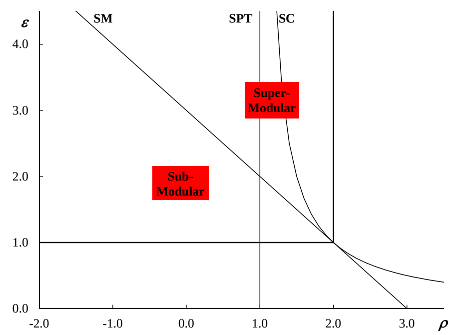


Figure 3: The Super- and Sub-Modular Regions

2.4 Summary

Figure 4 summarizes the results illustrated in this section. The three loci, corresponding to constant elasticity (SC), unit convexity (SPT), and unit elasticity of marginal revenue (SM), place bounds on the combinations of elasticity and convexity consistent with particular comparative statics outcomes. Of eight logically possible sub-regions within the admissible region, three can be ruled out because superconvexity implies both super-pass-through and supermodularity. From the figure it is clear that knowing the values of the elasticity and convexity of demand which a firm faces is sufficient to predict its responses to a very wide range of exogenous shocks, including some of the classic questions posed in the introduction.

Region	$\varepsilon_x > 0$	$\frac{dp}{dc} > 0$	$\varepsilon + \rho > 3$
1			
2			✓
3		✓	
4		✓	✓
5	✓	✓	✓

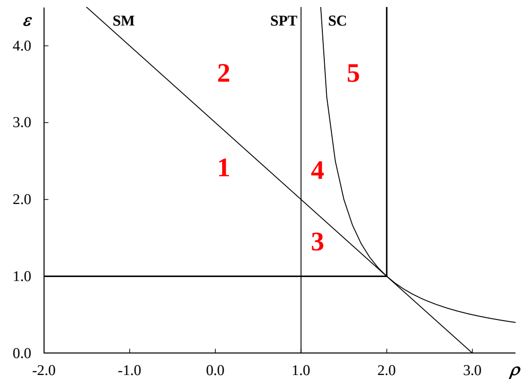


Figure 4: Regions of Comparative Statics

3 The Demand Manifold

3.1 Introduction

So far, we have shown how a very wide range of comparative statics responses can be signed just by knowing the values of ε and ρ which a firm faces. Next we want to see how different assumptions about the form of demand determine these responses. Formally, we seek to characterize the set Ω_{p_0}, which is the set of ε and ρ consistent with a given demand function $p = p_0(x)$, whose domain is $X_{p_0} \equiv \{x : x \in \mathbb{R}^+ \cup \{0\} \text{ and } p_0(x) \geq 0\}$:

$$\Omega_{p_0} \equiv \left\{ (\varepsilon, \rho) : \varepsilon = -\frac{p_0(x)}{xp_0'(x)}, \rho = -\frac{xp_0''(x)}{p_0'(x)}, \forall x \in X_{p_0} \right\} \tag{7}$$

We have already seen that this set and hence the comparative statics responses implied by particular demand functions are pinned down in one special case: with CES demands the firm is always at a single point in $\{\varepsilon, \rho\}$ space. Can anything be said more generally? The answer is “yes”, as the following result shows:

Proposition 1. *For every continuous, three-times differentiable, strictly-decreasing demand function, $p_0(x)$, other than the CES, the set Ω_{p_0} corresponds to a smooth curve in $\{\varepsilon, \rho\}$ space.*

The proof is in Appendix D. It proceeds by showing that, at any point on every demand function other than the CES, at least one of the functions $\varepsilon = \varepsilon(x)$ and $\rho = \rho(x)$ can be inverted to solve for x, and the resulting expression, denoted $x^{\varepsilon}(\varepsilon)$ and $x^{\rho}(\rho)$ respectively, substituted into the other function to give a relationship between ε and ρ:

$$\varepsilon = \bar{\varepsilon}(\rho) \equiv \varepsilon\left[x^{\rho}(\rho)\right] \quad \text{or} \quad \rho = \bar{\rho}(\varepsilon) \equiv \rho\left[x^{\varepsilon}(\varepsilon)\right] \tag{8}$$

We write this in two alternative ways, since at any given point only one may be well-defined, and, even when both are well-defined, one or the other may be more convenient depending on the context. The relationship between ε and ρ defined implicitly by (7) is not in general a function, since it need not be globally single-valued; but neither is it a correspondence, since it is locally single-valued. So we call it the "demand manifold" corresponding to the demand function $p_0(x)$. In the CES case, not covered by Proposition 1, we follow the convention that, corresponding to each value of the elasticity of substitution σ, the set Ω_{p_0} is represented by a point-manifold lying on the SC locus.

The first advantage of working with the demand manifold rather than the demand function itself is that it is located in $\{\varepsilon, \rho\}$ space, and so it immediately reveals the implications of assumptions made about demand for comparative statics. A second advantage, departing from the "firm's-eye-view" that we have adopted so far, is that the manifold is often independent of exogenous parameters even though the demand function itself is typically not. Expressing this in the language of Chamberlin (1933), exogenous shocks typically shift the perceived demand curve, but they need not shift the corresponding demand manifold. We call this property "manifold invariance". When it holds, exogenous shocks lead only to movements along the manifold, not to shifts in it. As a result, it is particularly easy to make comparative statics predictions. Clearly, the manifold cannot in most cases be invariant to changes in all parameters: even in the CES case, the point-manifold is not independent of the value of σ.[16] However, it is invariant to changes in any parameter ϕ which affects the

[16]The manifold corresponding to the linear demand function is an example which is invariant with respect

level term only; for ease of comparison with later functions, we write this in terms of both the direct and inverse CES demand functions:

$$x\,(p,\phi) = \delta\,(\phi)\,p^{-\sigma} \quad \Leftrightarrow \quad p\,(x,\phi) = \beta(\phi)x^{-1/\sigma} \tag{9}$$

It is particularly convenient that the CES point-manifold is invariant with respect to variables (such as income or the prices of other goods) that are endogenous in general equilibrium and affect only the level term, whereas the parameter σ with respect to which it is not invariant is a structural preference parameter. In the same way, for many demand functions, including some of the most widely-used, the manifold turns out to be invariant with respect to their level parameters, so it provides a parsimonious summary of their implications for comparative statics.[17] In the next two sub-sections, we show that manifold invariance provides a fruitful organizing principle for a very wide range of demand functions. Section 3.2 extends the CES demand functions from (9) in a non-parametric way, whereas Section 3.3 extends them parametrically by adding an additional power-law term. Section 3.4 then shows how the demand manifold can be used to infer the comparative statics implications of a particular demand function, while Section 3.5 notes some demand functions whose manifolds are not invariant with respect to any of their parameters.

3.2 Multiplicatively Separable Demand Functions

Our first result is that manifold invariance holds when the demand function is multiplicatively separable in ϕ:

Proposition 2. *The demand manifold is invariant to shocks in a parameter ϕ if either the direct or inverse demand function is multiplicatively separable in ϕ:*

---

to all demand parameters. Other examples are the Stone-Geary, CARA, and translog demand manifolds: see Section 3.3 below.

[17] The general form of the manifold can be written as either $\bar{\varepsilon}(\rho,\phi) = \varepsilon\left[x^{\rho}(\rho,\phi),\phi\right]$ or $\bar{\rho}(\varepsilon,\phi) \equiv \rho\left[x^{\varepsilon}(\varepsilon,\phi),\phi\right]$. Manifold invariance requires that either $\bar{\varepsilon}_{\phi} = \varepsilon_x x^{\rho}_{\phi} + \varepsilon_{\phi} = -\varepsilon_x \frac{\rho_{\phi}}{\rho_x} + \varepsilon_{\phi} = 0$ or $\bar{\rho}_{\phi} = \rho_x x^{\varepsilon}_{\phi} + \rho_{\phi} = -\rho_x \frac{\varepsilon_{\phi}}{\varepsilon_x} + \rho_{\phi} = 0$.

$$\text{(a)} \quad x(p,\phi) = \delta(\phi)\,\tilde{x}(p) \qquad \text{or} \qquad \text{(b)} \quad p(x,\phi) = \beta(\phi)\tilde{p}(x) \tag{10}$$

The proof is in Appendix E, and relies on the convenient property that, with separability of this kind, both the elasticity and convexity are themselves invariant with respect to ϕ. This result has some important corollaries. First, when utility is additively separable, the inverse demand function for any good equals the marginal utility of that good times the inverse of the marginal utility of income. The latter is a sufficient statistic for all economy-wide variables which affect the demand in an individual market, such as aggregate income or the price index. A similar property holds for the direct demand function if the indirect utility function is additively separable (as in Bertoletti and Etro (2013)), with the qualification that the indirect sub-utility functions depend on prices relative to income. (See Appendix E for details.) Summarizing:[18]

Corollary 1. *If preferences are additively separable, whether directly or indirectly, the demand manifold for any good is invariant to changes in aggregate variables (except for income, in the case of indirect additivity).*

Given the pervasiveness of additive separability in theoretical models of monopolistic competition, this is an important result, which implies that in many models the manifold is invariant to economy-wide shocks. We will see a specific application in Section 4, where we apply our approach to the Krugman (1979) model of international trade with monopolistic competition.

A second corollary of Proposition 2 comes by noting that, setting $\delta(\phi)$ in (10)(a) equal to market size s, yields the following:

Corollary 2. *The demand manifold is invariant to neutral changes in market size:* $x(p,s) = s\tilde{x}(p)$.

[18] Another class of demand functions nested by (10) is that recently introduced by Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012).

This corollary is particularly useful since it does not depend on the functional form of the individual demand function. An example which illustrates this is the logistic direct demand function: see Appendix F for details.

Finally, a third corollary of Proposition 2 is the dual of Corollary 2, and comes from setting $\beta(\phi)$ in (10)(b) equal to quality q:

Corollary 3. *The demand manifold is invariant to neutral changes in quality:* $p(x, q) = q\tilde{p}(x)$.

Baldwin and Harrigan (2011) call this assumption "box-size quality": the consumer's willingness to pay for a single box of a good with quality level q is the same as their willingness to pay for q boxes of the same good with unit quality. Though special, it is a very convenient assumption, widely used in international trade, so it is useful that the comparative statics predictions of any such demand function are independent of the level of quality.

3.3 Bipower Demand Functions

The second class of demand functions which exhibit manifold invariance comes from adding a second power-law term to the CES case (9), giving a "Bipower" or "Double CES" form:

Proposition 3. *The demand manifold is invariant to shocks in a parameter ϕ if either the direct or inverse demand function takes a bipower form:*

$$\text{(a)} \quad x(p,\phi) = \gamma(\phi)p^{-\nu} + \delta(\phi)p^{-\sigma} \quad \Leftrightarrow \quad \bar{\rho}(\varepsilon) = \frac{\nu+\sigma+1}{\varepsilon} - \frac{\nu\sigma}{\varepsilon^2} \tag{11}$$

$$\text{(b)} \quad p(x,\phi) = \alpha(\phi)x^{-\eta} + \beta(\phi)x^{-\theta} \quad \Leftrightarrow \quad \bar{\rho}\,(\varepsilon) = \eta + \theta + 1 - \eta\theta\varepsilon \tag{12}$$

The proof is in Appendix G. Note that these direct and inverse demand functions have very different implications for behavior. However, they have the same functional form except that the roles of x and p are reversed, so results proved for one can be applied immediately to the other. Clearly, the manifolds in (11) and (12) are invariant with respect to the level parameters $\{\gamma, \delta\}$ and $\{\alpha, \beta\}$, so changes in exogenous variables such as income or market size which only affect these parameters do not shift the manifold. (Hence we can suppress ϕ from here on.) Putting this differently, we need four parameters to characterize each demand function, but only two to characterize the corresponding manifold, which allows us to place bounds on the comparative statics responses reviewed in Section 2.

However, the level parameters in (11) and (12) are also qualitatively important, as the following proposition shows:

Proposition 4. *The bipower direct demand functions in* (11) *are superconvex if and only if both γ and δ are positive. Similarly, the bipower inverse demand functions in* (12) *are superconvex if and only if both α and β are positive.*

The proof is in Appendix H.[19] The two sets of parameters thus play very different roles. The power-law exponents $\{\nu, \sigma\}$ and $\{\eta, \theta\}$ determine the location of the manifold, whereas the level parameters $\{\gamma, \delta\}$ and $\{\alpha, \beta\}$ determine which "branch" of a particular manifold is relevant: the superconvex branch if they are both positive, the subconvex one if either of them is negative. (They cannot both be negative since both sales and price are nonnegative.) How this works is best understood by considering some special cases, which, as we will see, include some of the most widely-used demand functions in applied economics.

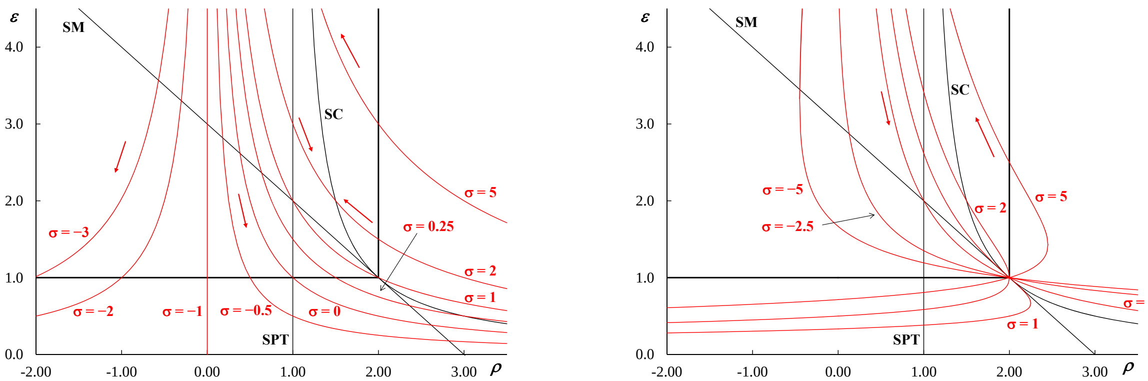
Two special cases of the bipower direct class (11) are of particular interest.[20] The first, where $\nu = 0$, is the family of demand functions due to Pollak (1971).[21] The direct demand function is now a "translated" CES function of price: $x(p) = \gamma + \delta p^{-\sigma}$; while the demand manifold is a rectangular hyperbola: $\bar{\rho}(\varepsilon) = \frac{\sigma+1}{\varepsilon}$. Figure 5(a) illustrates some members of the Pollak family. They include many widely-used demand functions, including the CES ($\gamma = 0$), linear ($\sigma = -1$), Stone-Geary (or "LES" for "Linear Expenditure System": $\sigma = 1$),

[19] An implication of this result is that, if the market demand function arises from aggregating across two groups with different CES preferences, then it must be superconvex.

[20] For further details on these demand functions, see Appendices I.1 and I.2. A third special case is the family of demand functions implied by the quadratic mean of order r expenditure function introduced by Diewert (1976) and extended to monopolistic competition by Feenstra (2014). See Appendix I.4 for details.

[21] Because ν and σ enter symmetrically into (11), it is arbitrary which we set equal to zero. For concreteness and without loss of generality we assume $\delta \neq 0$ and $\sigma \neq 0$ throughout.

and “CARA” (“constant absolute risk aversion”: the limiting case as σ approaches zero).[22] Manifolds with σ greater than one have two branches, one each in the sub- and superconvex regions, implying different directions of adjustment as sales increase, as indicated by the arrows.[23]



(a) The Pollak Family

(b) The PIGL Family

Figure 5: Demand Manifolds for Some Bipower Direct Demand Functions

A second important special case of (11) comes from setting $\nu = 1$. This gives the “PIGL” (“price-independent generalized linear”) system of Muellbauer (1975): $x(p) = \frac{1}{p}\left(\gamma + \delta p^{1-\sigma}\right)$, which implies that expenditure $px(p)$ is a translated-CES function of price. From (11), the manifold is given by: $\bar{\rho}(\varepsilon) = \frac{(\sigma+2)\varepsilon-\sigma}{\varepsilon^2}$. Figure 5(b) illustrates some PIGL demand manifolds. Among these is the LES, the only case which is a member of both the Pollak and PIGL families (since $\nu = 0$ and $\sigma = 1$ are equivalent to $\nu = 1$ and $\sigma = 0$ in (11)). Another member of the PIGL family is the translog, $x(p) = \frac{1}{p}\left(\gamma' + \delta' \log p\right)$, which is the limiting case as σ approaches one so $\bar{\rho}(\varepsilon) = \frac{3\varepsilon-1}{\varepsilon^2}$.[24] From the firm’s perspective, this is consistent with both

[22] To show that CARA demands are a special case, rewrite the constants as $\gamma = \gamma' + \frac{\delta'}{\sigma}$ and $\delta = -\frac{\delta'}{\sigma}$, and apply l’Hôpital’s Rule, which yields the CARA demand function $x = \gamma' + \delta' \log p$, $\delta' < 0$.

[23] These branches correspond to the same value of σ but to different values of γ and/or δ, and so to different demand functions. No bipower demand function as defined in Proposition 3 can be subconvex for some values of output and superconvex for others. Recalling Figure 1(b), this is only possible if the manifold is horizontal where it crosses the superconvexity locus. Appendix K gives an example of a demand function with this property.

[24] To show this, rewrite the constants as $\gamma = \gamma' - \frac{\delta'}{1-\sigma}$ and $\delta = \frac{\delta'}{1-\sigma}$, and apply l’Hôpital’s Rule, which yields the translog demand function.

the Almost Ideal or "AIDS" model of Deaton and Muellbauer (1980), and the homothetic translog of Feenstra (2003).

Just as the general bipower direct and bipower inverse demand functions in (11) and (12) are dual to each other, so also two special cases of (12), dual to the special cases of (11) just considered, are of particular interest. The first of these comes from setting η in (12) equal to zero, giving the inverse demand function $p(x) = \alpha+\beta x^{-\theta}$. This is the iso-convex or "constant pass-through" family of Bulow and Pfleiderer (1983), recently empirically implemented by Atkin and Donaldson (2012). The second important special case of (12) comes from setting η equal to one. This gives the "inverse PIGL" ("price-independent generalized linear") system, which is dual to the direct PIGL system considered earlier: expenditure $xp(x)$ is now a "translated-CES" function of sales: $p(x) = \frac{1}{x}(\alpha + \beta x^{1-\theta})$. Further details about these demand functions and their manifolds are given in Appendix J.

3.4 Demand Manifolds and Comparative Statics

It should be clear how a given demand manifold helps to understand the comparative statics implications of the corresponding demand function. To take a specific example, consider the Stone-Geary demand function (represented by the curves labeled $\sigma = 1$ in Figure 5(a) and $\sigma = 0$ in Figure 5(b)). Referring back to Figures 2 and 4 in Section 2, we can conclude without the need for any calculations that Stone-Geary demands are always subconvex, and that they imply less than absolute pass-through and submodular profits for small firms but the opposite for large ones. Inspecting the figures shows that, qualitatively, these properties are similar to those of the CARA and translog demand functions (though see the next paragraph), but quite different from those of the CES on the one hand or the linear demand function on the other.

Comparing demand functions in terms of their manifolds can also draw attention to hitherto unsuspected results. An example, which is suggested by Figure 5, is that the translog is the only bipower demand function that is both subconcave and supermodular throughout the admissible region: see the curve labeled $\sigma = 1$ in Figure 5(b). it turns out that we can go further and show that the translog is the only member of a broader class of demand functions, characterized in terms of their manifolds, that satisfies these conditions. We call the class in question a "contiguous bipower" manifold, since it expresses ρ as a bipower function of ε, where the exponents of ε are contiguous integers, k and $k+1$; this includes both bipower direct and bipower inverse demands, for which k equals -2 and zero, respectively:[25]

Lemma 1. *The translog is the only demand function with a contiguous bipower manifold, $\rho = a_1\varepsilon^k + a_2\varepsilon^{k+1}$, where k is an integer, that is always both strictly subconvex and strictly supermodular in the interior of the admissible region.*

This is an attractive feature: the translog is the only demand function from a very broad family that allows for competition effects (so mark-ups fall with globalization) but also implies that larger firms always serve foreign markets via FDI rather than exports.

3.5 Demand Functions that are Not Manifold-Invariant

In the rest of the paper we concentrate on the demand functions introduced here which have manifolds that are invariant with respect to at least some parameters. However, even in more complex cases when the demand manifold has the same number of parameters as the demand function, it typically provides an economy of information by highlighting which parameters matter for comparative statics. Appendix L presents two examples of this kind that nest some important cases, such as the "Adjustable Pass-Through" (APT) demand function of Fabinger and Weyl (2012).

[25] The proof is in Appendix I.3.

4 Monopolistic Competition in General Equilibrium

4.1 Globalization with Homogeneous Firms

To illustrate the power of the approach we have developed in previous sections, we turn next to apply it to a canonical model of international trade, a one-sector, one-factor, multi-country, general-equilibrium model of monopolistic competition. To highlight the new features of our approach, we focus on the case considered by Krugman (1979), where countries are symmetric and trade is unrestricted. Following Krugman (1979) and a large subsequent literature, we model globalization as an increase in the number of countries in the world economy.[26] We first consider in this sub-section the case where firms are homogeneous, and then extend to heterogeneous firms in Section 4.2. On the consumer side, we assume that preferences are symmetric, and that the elasticity of demand depends only on consumption levels, which in symmetric equilibrium means on the amount consumed of a typical variety, denoted by x. From Goldman and Uzawa (1964), this is equivalent to assuming additively separable preferences as in Dixit and Stiglitz (1977) and Krugman (1979).

Symmetric demands and homogeneous firms imply that we can dispense with firm subscripts from the outset. Industry equilibrium requires that firms maximize profits by choosing output y to set marginal revenue MR equal to marginal cost MC, and that profits are driven to zero by free entry (so average revenue AR equals average cost AC):

$$\text{Profit Maximization (MR=MC):} \quad p = \frac{\varepsilon(x)}{\varepsilon(x) - 1} c \tag{13}$$

$$\text{Free Entry (AR=AC):} \quad p = \frac{f}{y} + c \tag{14}$$

[26]The effects of changes in trade costs are considered in Mrázová and Neary (2014).

The model is completed by market-clearing conditions for the goods and labor markets:

$$\text{Goods-Market Equilibrium (GME):} \quad y = kLx \tag{15}$$

$$\text{Labor-Market Equilibrium (LME):} \quad L = n\,(f + cy) \tag{16}$$

Here L is the number of worker/consumers in each country, each of whom supplies one unit of labor and consumes an amount x of every variety; k is the number of identical countries; and n is the number of identical firms in each, all with total output y, so $N = kn$ is the total number of firms in the world. Since all firms are single-product by assumption, N is also the total number of varieties available to all consumers.

Equations (13) to (16) comprise a system of four equations in four endogenous variables, p, x, y and n, with the wage rate set equal to one by choice of numéraire. To solve for the effects of globalization, an increase in the number of countries k, we totally differentiate the equations, using "hats" to denote logarithmic derivatives, so $\hat{x} \equiv d\log x$, $x \neq 0$:

$$\text{MR=MC:} \quad \hat{p} = \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon\,(\varepsilon - 1)}\hat{x} \tag{17}$$

$$\text{AR=AC:} \quad \hat{p} = -(1 - \omega)\hat{y} \tag{18}$$

$$\text{GME:} \quad \hat{y} = \hat{k} + \hat{x} \tag{19}$$

$$\text{LME:} \quad 0 = \hat{n} + \omega\hat{y} \tag{20}$$

Consider first the MR=MC equilibrium condition, equation (17). Clearly p and x move together if and only if $\varepsilon + 1 - \varepsilon\rho > 0$, i.e., if and only if demand is subconvex. This reflects the property noted in Section 2.2: higher sales are associated with a higher proportional mark-up, $\frac{p}{c}$, if and only if they imply a lower elasticity of demand. As for the free-entry condition, equation (18), it shows that the fall in price required to maintain zero profits following an increase in firm output is greater the smaller is $\omega \equiv \frac{cy}{f+cy}$, the share of variable in total costs, which is an inverse measure of returns to scale. This looks like a new parameter but in equilibrium it is not. It equals the ratio of marginal cost to price, $\frac{c}{p}$, which because of profit maximization equals the ratio of marginal revenue to price $\frac{p+xp'}{p}$, which in turn is a monotonically increasing transformation of the elasticity of demand ε: $\omega = \frac{c}{p} = \frac{p+xp'}{p} = \frac{\varepsilon-1}{\varepsilon}$. Similarly, equation (20) shows that the fall in the number of firms required to maintain full employment following an increase in firm output is greater the larger is ω. It follows by inspection that all four equations depend only on two parameters, which implies:

Lemma 2. *The local comparative statics properties of the symmetric monopolistic competition model with respect to a globalization shock depend only on ε and ρ.*

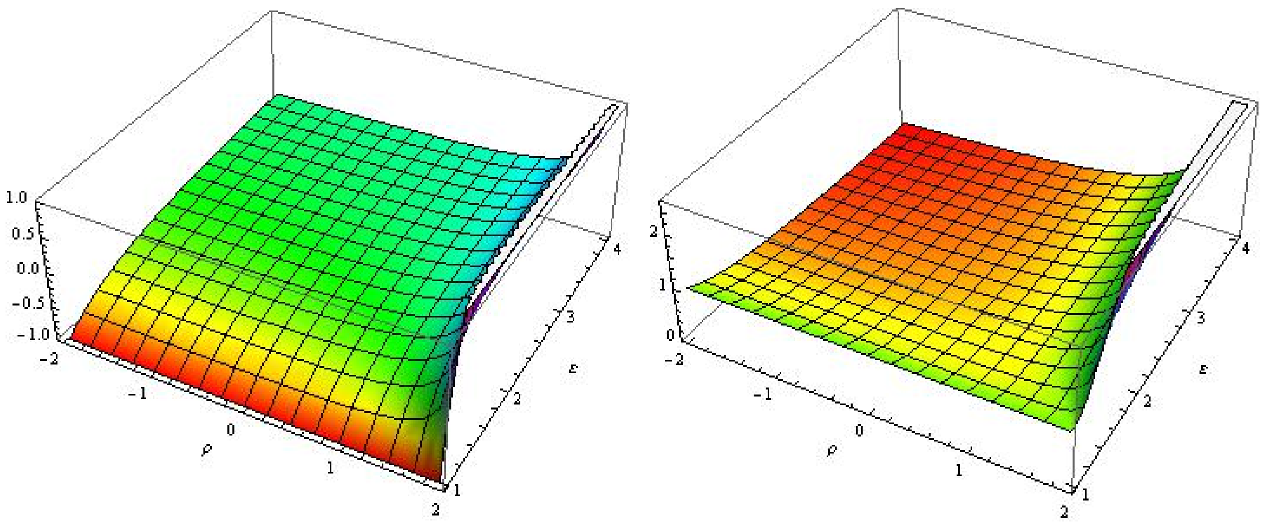
To see the implications of this in detail, solve for the effects of globalization on outputs, prices and the number of firms in each country:

$$\hat{y} = \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon\,(2-\rho)}\hat{k}, \qquad \hat{p} = -\frac{1}{\varepsilon}\hat{y}, \qquad \hat{n} = -\frac{\varepsilon - 1}{\varepsilon}\hat{y} \tag{21}$$

(Details of the solution are given in Appendix M.) The signs of these depend solely on whether demands are sub- or superconvex, i.e., whether $\varepsilon + 1 - \varepsilon\rho$ is positive or negative. With subconvexity we get what Krugman (1979) called "sensible" results: globalization prompts a shift from the extensive to the intensive margin, with fewer but larger firms in each country, as firms move down their average cost curves and prices of all varieties fall. With superconvexity, as noted by Zhelobodko, Kokovin, Parenti, and Thisse (2012), all these results are reversed. (See also Neary (2009).) The CES case, where $\varepsilon + 1 - \varepsilon\rho = 0$, is the boundary one, with firm outputs, prices, and the number of firms per country unchanged. The only effects that hold irrespective of the form of demand are that consumption per head of each variety falls and the total number of varieties produced in the world and consumed in each country rises:

$$\hat{x} = -\frac{1}{2-\rho}\frac{\varepsilon - 1}{\varepsilon}\hat{k} \;<\; 0, \qquad \hat{N} = \frac{(\varepsilon-1)^2 + (2-\rho)\,\varepsilon}{\varepsilon^2\,(2-\rho)}\hat{k} \;>\; 0 \tag{22}$$

Qualitatively these results are not new. The new feature that our approach highlights is that their quantitative magnitudes depend only on two parameters, ε and ρ, the same ones on which we have focused throughout. Hence we can invoke the demand manifold apparatus developed in Section 3.



(a) Changes in Prices

(b) Changes in the Number of Varieties

Figure 6: The Effects of Globalization

To illustrate how this works, Figure 6 gives the quantitative magnitudes of changes in the two variables that matter most for welfare: prices and the number of varieties. In each panel, the vertical axis measures the proportional change in either p or N following a unit increase in k as a function of the elasticity and convexity of demand. The two-dimensional surfaces shown are independent of the functional form of demand, so we can combine them with the results on demand manifolds from Section 3 to read off the quantitative effects of globalization implied by different assumptions about demand. We know already from equations (21) and (22) that prices fall if and only if demand is subconvex and that product variety always rises. The figures show in addition that lower values of the elasticity of demand are usually associated with greater falls in prices and larger increases in variety;[27] while more

[27]Though these properties are reversed if demands are highly convex: $\hat{p}/\hat{k}$ is increasing in ε if and only if $\rho < 1 + \frac{2}{\varepsilon}$, and $\hat{N}/\hat{k}$ is decreasing in ε if and only if $\rho < \frac{2}{\varepsilon}$.

convex demand is always associated with greater increases (in absolute value) in both prices and the number of varieties.

To summarize this sub-section, Lemma 2 implies that the demand manifold is a sufficient statistic for the positive effects of globalization in the Krugman (1979) model, just as it is for the comparative statics results discussed in Section 2.

4.2 Heterogeneous Firms

The case of homogeneous firms is of independent interest, and also provides a useful reference point for understanding the comparative statics of a model with heterogeneous firms and general demands. Consider the same model as before, except that now firms differ in their marginal costs c, which, as in Melitz (2003), are given by an exogenous distribution $g(c)$, with support on $[\underline{c}, \overline{c}]$. The maximum operating profit that a firm can earn varies inversely with its own marginal cost c. Through the inverse demand function $p(y, \lambda, k)$, it also depends positively on the size of the global economy k, and negatively on an aggregate index of all prices and income, λ:

$$\pi(\underset{-}{c}, \underset{-}{\lambda}, \underset{+}{k}) \equiv \max_{y} \left[p(y, \lambda, k) - c\right] y \tag{23}$$

In much of what follows, we assume that preferences are additively separable, so λ equals the marginal utility of income. However, this specification is also consistent with a much broader class of demands which Pollak (1972) calls "generalized additive separability".[28] A key implication of this class is that, in monopolistic competition, where individual firms are infinitesimal relative to the industry, λ is endogenous to the industry, but exogenous to firms, and so can be interpreted as a measure of the degree of competition each firm faces.

With homogeneous firms, equation (14) in Section 4.1 gave a free-entry condition which was common to all firms. With heterogeneous firms, this must be replaced by two conditions. First is the zero-profit condition for marginal firms, which requires that their operating profits

[28] The class includes indirectly additive utility functions, and quasi-linear quadratic preferences as in Melitz and Ottaviano (2008).

equal the common fixed cost f:

$$\pi(c_0, \lambda, k) = f \tag{24}$$

This determines the cutoff cost c_0 as a function of λ and k. Second is the zero-expected-profit condition for all firms, which requires that their expected operating profits equal the common fixed cost f plus the sunk cost of entering the industry f_e:

$$\bar{\pi}(\lambda, k) \equiv \int_{\underline{c}}^{\bar{c}} \pi(c, \lambda, k) g(c)\, \mathrm{d}c = f + f_e \tag{25}$$

Expected operating profits are conditional on incurring the sunk cost of entry, not conditional on actually entering, and so they are independent of the cutoff c_0. Equation (25) thus determines the level of competition as a function of the size of the world economy k.

We can now derive the effects of globalization on the profile of profits across firms. Combining the profit function and equation (25) gives:

$$\hat{\pi} = \frac{\lambda \pi_\lambda}{\pi} \hat{\lambda} + \frac{k \pi_k}{\pi} \hat{k} = \Bigg(\underbrace{\frac{k \pi_k}{\pi}}_{(M)} - \underbrace{\frac{\lambda \pi_\lambda}{\pi} \frac{\bar{\pi}}{\lambda \bar{\pi}_\lambda} \frac{k \bar{\pi}_k}{\bar{\pi}}}_{(C)} \Bigg) \hat{k} \tag{26}$$

This shows that globalization has a market-size effect, given by (M), which tends to *raise* each firm's profits. In addition it has a competition effect, given by (C): because *all* firms' profits rise at the initial level of competition, the latter must increase to ensure that expected profits remain equal to the fixed cost of entry. This in turn tends to *reduce* each firm's profits. The net outcome is indeterminate in general. However, with additive separability, equation (26) takes a particularly simple form (see Appendix N for details):

$$\hat{\pi} = \left(1 - \frac{\varepsilon}{\bar{\varepsilon}} \right) \hat{k} \tag{27}$$

where $\bar{\varepsilon} \equiv \int_{\underline{c}}^{\bar{c}} \frac{\pi(c)}{\bar{\pi}} \varepsilon(c) g(c)\, \mathrm{d}c$ is the elasticity faced by the average firm. Thus the market-size effect is one-for-one (given λ, all firms' profits increase proportionally with k), while the competition effect is greater than one if and only the elasticity a firm faces is greater than the average elasticity. The implications for the response of profits across firms are immediate, recalling that firms face an elasticity of demand that falls with their sales if and only if demands are subconvex:

Proposition 5. *With additive separability, globalization pivots the profile of profits across firms around the average firm; if and only if demands are subconvex, profits rise for firms above the average, and by more the larger a firm's initial sales.*

A corollary of this result is the effect of globalization on the extensive margin. Using (27) to evaluate the change in the marginal cost of the cutoff firm defined by (24) gives:

$$\hat{c}_0 = \frac{1}{\varepsilon_0 - 1}\left(1 - \frac{\varepsilon_0}{\bar{\varepsilon}}\right)\hat{k} \tag{28}$$

where $\varepsilon_0 \equiv \varepsilon(c_0)$ is the elasticity faced by the marginal firm. The marginal firm has the lowest sales and so, when demands are subconvex, the highest elasticity: $\varepsilon_0 > \bar{\varepsilon}$. Hence the competition effect dominates and the least efficient firms exit. By contrast, when demands are superconvex, ε_0 is less than $\bar{\varepsilon}$, so the threshold firm becomes *more* profitable, and globalization encourages entry of less efficient firms.

In the same way we can solve for the effects of globalization at the intensive margin. As shown in Appendix N, the changes in the profiles of firm outputs and prices are given by:

$$\hat{y} = \left[\underbrace{\frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon\,(2-\rho)}}_{*} + \frac{\bar{\varepsilon} - \varepsilon}{\bar{\varepsilon}}\,\frac{\varepsilon - 1}{\varepsilon\,(2-\rho)}\right]\hat{k}, \qquad \hat{p} = -\underbrace{\frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon^2\,(2-\rho)}}_{*}\,\frac{\varepsilon}{\bar{\varepsilon}}\,\hat{k} \tag{29}$$

These changes in output and price for each firm equal the corresponding changes in the homogeneous-firms case (given by (21) and denoted here by $*$), adjusted for the difference between that firm's elasticity ε and the average elasticity $\bar{\varepsilon}$. Hence, the changes in output and price of the average firm are identical to those in the homogeneous-firms case; for example, that for the average price is the same as that shown in Figure 6(a). For firms other than the average, the output and price changes have two components. The first is the same as in the homogeneous-firms case, for each individual firm; so, for example, as in Section 4.1, we can evaluate this for price changes by combining Figure 6(a) with the appropriate demand manifold. The second is a correction factor reflecting the difference between the elasticity ε faced by the firm in question and the average elasticity $\bar{\varepsilon}$. As with firm profits in (27), the sign of this correction factor depends on whether demand is super- or subconvex. In particular, equation (29) shows that, if demand is subconvex, then, relative to the homogeneous-firms benchmark, outputs increase by more, and prices fall by less in larger firms.

5 Conclusion

In this paper we have presented a new way of relating the structure of demand functions to the positive and normative properties of monopolistic and monopolistically competitive markets. By adopting a "firms'-eye view" of demand, we have shown how the elasticity and convexity of demand determine many comparative statics responses. In turn, we have shown how the relationship between these two parameters, which we call the "demand manifold," provides a parsimonious representation of an arbitrary demand function, and a sufficient statistic for many comparative statics results. The manifold is particularly useful when it is unaffected by changes in exogenous variables, a property which we call "manifold invariance." We have introduced some new families of demand systems which exhibit manifold invariance, and have shown that they nest many of the most widely used functions in applied theory. For example, our "bipower direct" family provides a natural way of nesting translog, CES and linear demand functions.[29]

To illustrate the usefulness of our approach, we have shown that it allows a parsimonious way of understanding how monopolistically competitive economies adjust to external shocks, as well as a framework for quantifying the effects of globalization. The demand manifold

[29] Alternative ways of nesting translog and CES demands, though with considerably more complicated demand manifolds, appear in Novy (2013) and in Pollak, Sickles, and Wales (1984).

turns out to be a sufficient statistic for the positive implications of globalization in general equilibrium.

Many extensions of our approach naturally suggest themselves. There are many other topics where functional form plays a key role in determining the implications of a given set of assumptions, and where our approach of focusing on the elasticity and convexity of a pivotal function yields important insights. For example, in ongoing work we show that the same approach can be applied to the utility function, to derive results on globalization and welfare, and to the slope of the demand function, viewed as a function in itself, to derive results on variable pass-through and departures from Gibrat's Law. Further applications to choice under uncertainty and to oligopoly immediately come to mind. As for our application to the effects of globalization in monopolistic competition, the framework we have presented can be extended to allow for trade costs.[30] Finally, the families of demand functions we have introduced provide a natural setting for estimating relatively flexible functional forms, and direct attention towards the parameters that matter for comparative statics predictions.

[30] The implications of combining trade costs and general non-CES preferences have been considered by Bertoletti and Epifani (2014), Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012), and Mrázová and Neary (2014).

Appendices

A Alternative Measures of Slope and Curvature

As our measure of demand slope, we work throughout with the price elasticity of demand, which can be expressed in terms of the derivatives of either the inverse or the direct demand functions $p(x)$ and $x(p)$: $\varepsilon \equiv -\frac{p}{xp'} = -\frac{px'}{x}$. Many authors have used the inverse of this elasticity, $e \equiv -\frac{x}{px'} = \frac{1}{\varepsilon}$, under a variety of names: the elasticity of marginal utility: $e = -\frac{d\log u'(x)}{d\log x}$; the "relative love for variety" as in Zhelobodko, Kokovin, Parenti, and Thisse (2012); or (in monopoly equilibrium) the profit margin or Lerner Index of monopoly power: $e = \frac{p-c}{p}$. This has the advantage that its definition is symmetric with those of curvature ρ and of "temperance" χ, to be discussed below. Offsetting advantages of using ε include its greater intuitive appeal, and the fact that it focuses attention on the region of parameter space where comparative statics results are ambiguous.

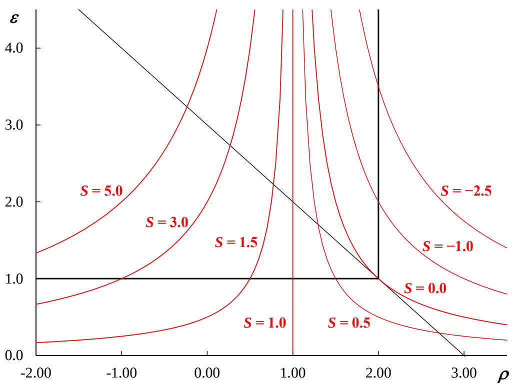
Turning to measures of curvature, the convexity of inverse demand which we use throughout equals the elasticity of the slope of inverse demand, $\rho \equiv -\frac{xp''}{p'} = -\frac{d\log p'(x)}{d\log x}$. Its importance for comparative statics results was highlighted by Seade (1980), and it is widely used in industrial organization, for example by Bulow, Geanakoplos, and Klemperer (1985) and Shapiro (1989). An alternative measure is the convexity of the direct demand function $x(p)$: $r(p) \equiv -\frac{px''(p)}{x'(p)}$. A convenient property is that e and r are dual to ε and ρ:

$$e \equiv -\frac{x}{px'} = \frac{1}{\varepsilon} \qquad r \equiv -\frac{px''}{x'} = \frac{pp''}{(p')^2} = \varepsilon\rho \tag{30}$$

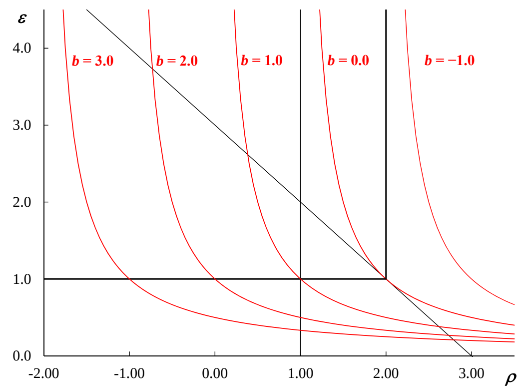
$$\varepsilon \equiv -\frac{p}{xp'} = \frac{1}{e} \qquad \rho \equiv -\frac{xp''}{p'} = \frac{xx''}{(x')^2} = er \tag{31}$$

We use these properties in the proof of Proposition 3 in Appendix G below.

Yet another measure of demand curvature, widely used in macroeconomics, is the superelasticity of Kimball (1995), defined as the elasticity with respect to price of the elasticity of demand, $S \equiv \frac{d \log \varepsilon}{d \log p}$. Positive values of S allow for asymmetric price setting in monopolistic competition. It is related to our measures as follows: $S = \frac{d \log \varepsilon}{d \log x} \frac{d \log x}{d \log p} = \left(\frac{x \varepsilon_x}{\varepsilon}\right)(-\varepsilon) = \varepsilon + 1 - \varepsilon\rho$ (using (33)), so it is positive if and only if demand is subconvex. Figure 7(a) illustrates loci of constant superelasticity, $\rho = \frac{\varepsilon + 1 - S}{\varepsilon}$. Formally, they correspond to the family of Pollak manifolds, $\bar{\rho}(\varepsilon) = \frac{\sigma + 1}{\varepsilon}$, displaced rightwards to be symmetric around the log-linear ($\rho = 1$) rather than the linear ($\rho = 0$) demand manifold.



(a) Iso-Superelasticity Loci



(b) Klenow-Willis Demand Manifolds

Figure 7: Kimball Superelasticity

Kimball himself did not present a parametric family of demand functions. Klenow and Willis (2006) introduce a parametric family which has the property that the superelasticity is a linear function of the elasticity: $S = b\varepsilon$. Substituting for S leads to the family of demand manifolds $\bar{\rho}(\varepsilon) = \frac{(1-b)\varepsilon + 1}{\varepsilon}$, which are lateral displacements of the CES locus. Figure 7(b) illustrates some members of this family.

We note in footnotes some implications of these alternative measures. The choice between them is largely a matter of convenience. We express all our results in terms of ε and ρ, partly because this is standard in industrial organization, partly because (unlike e and r) the inverse demand functions are easily integrated to obtain the direct utility function, and partly because (unlike ε and S) they lead to simple restrictions on the shape of the demand manifold as shown in Proposition 3. However, our results could just as well be expressed in terms of e and r or of ε and S. Details of these alternative ways of presenting them are available on request.

B Oligopoly

We consider only monopoly and monopolistic competition in the text, but our approach can also be applied to oligopolistic markets. Even in the simplest case of Cournot competition between n firms producing an identical good, this leads to extra complications. Now we need to distinguish market demand X from the sales of a typical firm i, x_i, with the elasticity and convexity of the demand function $p(X)$ defined in terms of the former: $\varepsilon \equiv -\frac{p}{Xp'}$ and $\rho \equiv -\frac{Xp''}{p'}$. The first-order condition is now $p + x_i p' = c_i \geq 0$. Since this differs between firms, the restriction it implies for the admissible region must be expressed in terms of market shares $\left(\omega_i \equiv \frac{x_i}{X}\right)$: $\varepsilon \geq \max_i(\omega_i)$, which attains its lower bound of $\frac{1}{n}$ when firms are identical.[31] As for the second-order condition, it becomes $2p' + x_i p'' < 0$, implying that $\rho < 2\min_i\left(\frac{1}{\omega_i}\right)$, which attains its upper bound of $2n$ when firms are identical. A different restriction on convexity comes from the stability condition: $\rho < n+1$. This imposes a tighter bound than the second-order condition provided the largest firm is not "too" large: $\max_i(\omega_i) < \frac{2}{n+1}$. Relative to the monopoly case, the admissible region expands unambiguously, except in the boundary case of a dominant firm, where $\max_i(\omega_i) = 1$. Equally important in oligopoly, as we know from Bulow, Geanakoplos, and Klemperer (1985), is that many comparative statics results hinge on strategic substitutability: the marginal revenue of firm i is decreasing in the output of every other firm. This is equivalent to $p' + x_i p'' < 0$, $\forall i$, which in our notation implies a restriction on convexity that lies within the admissible region: $\rho < \min_i\left(\frac{1}{\omega_i}\right) \geq 1$, which attains its upper bound of n when firms are identical.

[31] See Mathiesen (2014) for further discussion.

C Preliminaries: A Key Lemma

We make repeated use of the following result:

Lemma 3. *Consider a twice-differentiable function $g(x)$. Both the double-log convexity of $g(x)$ and the rate of change of its elasticity can be expressed in terms of its first and second derivatives as follows:*

$$\frac{d^2 \log g}{d(\log x)^2} = x\frac{d}{dx}\left(\frac{xg'}{g}\right) = \frac{xg'}{g}\left(1 - \frac{xg'}{g} + \frac{xg''}{g'}\right) \tag{32}$$

For most of the paper, $g(x)$ is the inverse demand function $p(x)$, and the result can be expressed in terms of the demand elasticity and convexity:

$$\frac{d^2 \log p}{d(\log x)^2} = \frac{x\varepsilon_x}{\varepsilon^2} = -\frac{1}{\varepsilon}\left(1 + \frac{1}{\varepsilon} - \rho\right) \tag{33}$$

Recalling equation (4), this gives the result in Section 2.2 that the elasticity of demand increases with sales if and only demand is superconvex. Qualitatively the same outcome comes from applying Lemma 3 to the direct demand function, replacing $g(x)$ by $x(p)$, and making use of (30) and (31):

$$\frac{d^2 \log x}{d(\log p)^2} = -p\frac{d\varepsilon}{dp} = -\varepsilon\left(1 + \varepsilon - \varepsilon\rho\right) \tag{34}$$

We use a different application of the Lemma to prove Proposition 1 in Section D below. Now, let $g(x)$ denote the absolute value of the demand slope $-p'(x)$, so (32) becomes:

$$\frac{d^2 \log(-p')}{d(\log x)^2} = -x\rho_x = -\rho\left(1 + \rho - \chi\right) \tag{35}$$

The parameter χ is a unit-free measure of the third derivative of the demand function, which, following Kimball (1992) and Eeckhoudt, Gollier, and Schneider (1995), we call the "Coefficient of Relative Temperance," or simply "temperance." The result in (35) that the change in convexity as sales rise depends only on temperance and convexity itself parallels that in (33) that the change in elasticity as sales rise depends only on convexity and elasticity itself.

All these expressions are zero in the CES case given by (4), when all three parameters depend only on the elasticity σ: $\{\varepsilon, \rho, \chi\}_{CES} = \left\{\sigma, 1 + \frac{1}{\sigma}, 2 + \frac{1}{\sigma}\right\}$.

D Proof of Proposition 1

We wish to prove that, except in the CES case, only one of ε_x and ρ_x can be zero at any x. Recall from equations (33) and (35) that $\varepsilon_x = \frac{\varepsilon}{x}\left[\rho - \frac{\varepsilon+1}{\varepsilon}\right]$ and $\rho_x = \frac{\rho}{x}(1 + \rho - \chi)$, where $\chi \equiv -\frac{xp'''}{p''}$. We have already seen that ε_x can be zero only along the CES locus. As for $\rho_x = 0$, there are two cases where it can equal zero. The first is where $\rho = 0$. From (35), this implies that ε_x equals $-\frac{\varepsilon+1}{x}$, which is non-zero. The second is where $1 + \rho - \chi = 0$. As we saw in Section 3.3, this implies that the demand function takes the iso-convex or Bulow-Pfleiderer form: $p(x) = \alpha + \beta x^{-\theta}$. The intersection of this with $\varepsilon_x = 0$ is the CES limiting case of Bulow-Pfleiderer as sales tend towards zero: see Figure 10(a) below. Hence we can conclude that the only cases where both ε_x and ρ_x equal zero at a given x lie on a CES demand function.

E Proof of Proposition 2

If demands are multiplicatively separable in ϕ, both the elasticity and convexity are independent of ϕ. In the case of inverse demands, $p(x, \phi) = \beta(\phi)\tilde{p}(x)$ implies:

$$\varepsilon = -\frac{p(x,\phi)}{xp_x(x,\phi)} = -\frac{\tilde{p}(x)}{x\tilde{p}'(x)} \quad \text{and} \quad \rho = -\frac{xp_{xx}(x,\phi)}{p_x(x,\phi)} = -\frac{x\tilde{p}''(x)}{\tilde{p}'(x)} \tag{36}$$

A special case of this is additive preferences: $\int_{\omega\in\Omega} u[x(\omega)]\,d\omega$. The first-order condition is $u'[x(\omega)] = \lambda^{-1}p(\omega)$, which implies that the perceived indirect demand function can be written in multiplicative form: $p(x,\phi) = \lambda(\phi)\tilde{p}(x)$.

Similar derivations hold for direct demands. If $x\left(p,\phi\right) = \delta\left(\phi\right)\tilde{x}\left(p\right)$ then:

$$\varepsilon = -\frac{p x_p(p,\phi)}{x(p,\phi)} = -\frac{p\tilde{x}'(p)}{\tilde{x}(p)} \quad \text{and} \quad \rho = \frac{x(p,\phi)x_{pp}(p,\phi)}{[x_p(p,\phi)]^2} = \frac{\tilde{x}(p)\tilde{x}''(p)}{[\tilde{x}'(p)]^2} \tag{37}$$

We also have a similar corollary, the case of indirect additivity, where the indirect utility function can be written as: $\int_{\omega\in\Omega} v\left[p(\omega)/I\right] d\omega$. Roy's Identity implies that: $v'\left[p(\omega)/I\right] = -\lambda x(\omega)$, where λ is the marginal utility of income, from which the direct demand function facing a firm can be written in multiplicative form: $x(p/I,\phi) = -\lambda^{-1}(\phi)\tilde{x}(p/I)$.

F Market Size and the Logistic Demand Function

To illustrate Proposition 2 that the demand manifold is independent of market size, consider the logistic direct demand function, equivalent to a logit inverse demand function (see Cowan (2012)):

$$x(p,s) = \left(1+e^{p-a}\right)^{-1} s \quad \Leftrightarrow \quad p(x,s) = a - \log\frac{x}{s-x} \tag{38}$$

Here x/s is the share of the market served: $x \in [0,s]$; and a is the price which induces a 50% market share: $p = a$ implies $x = \frac{s}{2}$. The elasticity equals $\varepsilon = p\frac{s-x}{s}$, while the convexity equals $\rho = \frac{s-2x}{s-x}$, which must be less than one. Eliminating x and p yields a closed-form expression for the manifold:

$$\bar{\varepsilon}(\rho) = \frac{a - \log(1-\rho)}{2-\rho} \tag{39}$$

which is invariant with respect to market size s though not with respect to a. Figure 8 illustrates this for values of a equal to 2 and 5.[32]

The logistic is just one example of a whole family of demand functions, many of which can be derived from log-concave distribution functions: Bergstrom and Bagnoli (2005) give

[32] The value of ρ determines market share and the level of price relative to a: $x = \frac{1-\rho}{2-\rho}s$ and $p = a - \log(1-\rho)$. In particular, when the function switches from convex to concave (i.e., ρ is zero), the elasticity equals $\frac{a}{2}$, market share is 50%, and $p = a$.

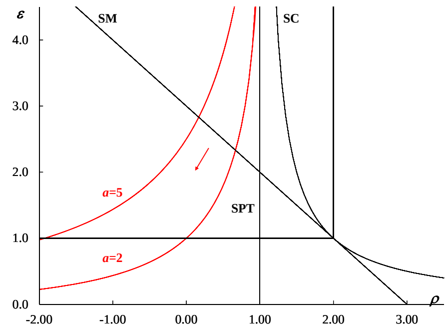


Figure 8: The Demand Manifold for the Logistic Demand Function

a comprehensive review of these. The power of the approach introduced in the last section is that we can immediately state the properties of all these functions: they imply sub-pass-through and, *a fortiori*, subconvexity, while they are typically supermodular for low values of output and submodular for high values. Any shock, such as a partial-equilibrium increase in market size, which raises the output of a monopoly firm, implies an adjustment as shown by the arrow in the figure.

G Proof of Proposition 3

We can exploit the duality between the direct and inverse demand functions in the proposition and concentrate on either one of them. It is most convenient to focus on the inverse demands in (12). Sufficiency follows by differentiating $p(x)$ and calculating the manifold directly. Necessity follows by setting $\rho(x) = a + b\varepsilon(x)$, where a and b are constants, and solving the resulting Euler-Cauchy differential equation. This proves the result in (12): a bipower inverse demand function is necessary and sufficient for a manifold such that ρ is affine in ε. With appropriate relabeling this in turn implies that a bipower direct demand function is necessary and sufficient for an affine *dual* manifold, that is to say, an equation linking the dual parameters r and e: $\bar{r}(e) = \nu + \sigma + 1 - \nu\sigma e$. Recalling from (30) that $e = \frac{1}{\varepsilon}$ and $r = \varepsilon\rho$ gives the result in (11).

To prove sufficiency, we first define $A \equiv \alpha x^{-\eta}$ and $B \equiv \beta x^{-\theta}$, so the demand function can be written as $p(x) = A + B$. Calculating the first and second derivatives yields: $xp' = -\eta A - \theta B$ and $x^2 p'' = \eta(\eta+1)A + \theta(\theta+1)B$. Adding x^2p'' to $\eta\theta p$ yields:

$$x^2 p'' + \eta\theta p = (\eta + \theta + 1)(\eta A + \theta B). \tag{40}$$

Using the expression for xp', this implies:

$$x^2 p'' + (\eta + \theta + 1)xp' + \eta\theta p = 0. \tag{41}$$

Dividing by xp' gives the desired result: $\bar{\rho}(\varepsilon) = \eta + \theta + 1 - \eta\theta\varepsilon$.

To prove necessity, assume the manifold is affine, so $\rho(x) = a + b\varepsilon(x)$ where a and b are constants. Substituting for $\rho(x)$ and $\varepsilon(x)$ and collecting terms yields:

$$x^2 p''(x) + axp'(x) - bp(x) = 0 \tag{42}$$

To solve this second-order Euler-Cauchy differential equation, we change variables as follows: $t = \log x$ and $p(x) = g(\log x) = g(t)$. Substituting for $p(x) = g(t)$, $p'(x) = \frac{1}{x}g'(t)$ and $p''(x) = \frac{1}{x^2}[g''(t) - g'(t)]$ into (42) gives a linear differential equation:

$$g''(t) + (a-1)g'(t) - bg(t) = 0 \tag{43}$$

Assuming a trial solution $g(t) = e^{\lambda t}$ gives the characteristic polynomial: $\lambda^2 + (a-1)\lambda - b = 0$, whose roots are $\lambda = \frac{1}{2}\left[-(a-1) \pm \sqrt{(a-1)^2 + 4b}\right]$. Only real roots make sense, so we assume $(a-1)^2 + 4b \geq 0$. If the inequality is strict, the roots are distinct and the general solution is given by $g(t) = \alpha e^{\lambda_1 t} + \beta e^{\lambda_2 t}$, where α and β are constants of integration. If $(a-1)^2 = -4b$, the roots are equal and the general solution is given by $g(t) = (\alpha + \beta t)e^{\lambda t}$. In both cases, the solution may be found by switching back from t and $g(t)$ to $\log x$ and

$p(x)$, recalling that $e^{\lambda \log x} = x^{\lambda}$. Hence, in the first case, $p(x) = \alpha x^{\lambda_1} + \beta x^{\lambda_2}$, and in the second case, $p(x) = (\alpha + \beta \log x) x^{\lambda}$.[33] The final step is to note that the sum of the roots is $\lambda_1 + \lambda_2 = 1 - a$ and their product is $\lambda_1 \lambda_2 = b$, which implies the relationship between the coefficients of the manifold and those of the implied demand function stated in the proposition. This completes the proof of (12), while that of (11) follows immediately by duality, as already noted.

H Proof of Proposition 4

Substituting from the bipower direct demand manifold in (11) into the condition for superconvexity, $\rho \geq \frac{\varepsilon+1}{\varepsilon}$, and using the fact that the elasticity of demand equals $\varepsilon = \frac{\nu\gamma p^{-\nu}+\sigma\delta p^{-\sigma}}{\gamma p^{-\nu}+\delta p^{-\sigma}}$, yields:

$$\rho - \frac{\varepsilon+1}{\varepsilon} = -\frac{1}{\varepsilon^2}\left(\varepsilon-\nu\right)\left(\varepsilon-\sigma\right) = \gamma\delta H \tag{44}$$

where $H \equiv \frac{1}{\varepsilon^2}\frac{(\sigma-\nu)^2 p^{-\sigma}}{(\gamma p^{-\nu}+\delta p^{-\sigma})^2}$, which is positive. Hence, superconvexity requires that γ and δ must have the same sign, which implies (since at least one of them must be positive) that they must both be positive, which proves the first part of Proposition 4.

Similarly, substituting from the bipower inverse demand manifold in (12) into the condition for superconvexity, and using the fact that the elasticity of demand equals $\varepsilon = \frac{\alpha x^{-\eta}+\beta x^{-\theta}}{\eta\alpha x^{-\eta}+\theta\beta x^{-\theta}}$, yields:

$$\rho - \frac{\varepsilon+1}{\varepsilon} = -\frac{1}{\varepsilon}\left(\eta\varepsilon-1\right)\left(\theta\varepsilon-1\right) = \alpha\beta H' \tag{45}$$

where $H' \equiv \frac{1}{\varepsilon}\frac{(\eta-\theta)^2 x^{-\eta-\theta}}{\left(\eta\alpha x^{-\eta}+\theta\beta x^{-\theta}\right)^2}$, which is positive. It follows that both α and β must be positive for superconvexity, which proves the second part of Proposition 4.

[33] We do not present the case of equal roots separately in the statement of Proposition 3 in the text: the economic interpretation is more convenient if we view it as the limiting case of the general expression as η approaches zero. See, for example, footnotes 22 and 24, which illustrate this for CARA and translog demands respectively.

I Examples of Bipower Direct Demands

I.1 Properties of Pollak Demands

With $\nu = 0$, the elasticity of demand becomes $\varepsilon = \frac{\sigma\delta p^{-\sigma}}{\gamma+\delta p^{-\sigma}} = \sigma\frac{\sigma\delta p^{-\sigma}}{x} = \sigma\frac{x-\gamma}{x}$. It follows that σ, δ and $x-\gamma$ must have the same sign. The sign of σ also determines whether the inverse demand function is logconvex or not. The CARA demand function is the limiting case when $\sigma \to 0$: the direct demand function becomes $x = \gamma' + \delta' \log p$, $\delta' < 0$, which implies that the inverse demand function is log-linear: $\log p = \alpha + \beta x$, $\beta < 0$.[34] The CARA manifold is $\bar{\rho}(\varepsilon) = \frac{1}{\varepsilon}$, which is a rectangular hyperbola through the point $\{1.0, 1.0\}$. Hence the CARA function is the dividing line between two sub-groups of demand functions and their corresponding manifolds, with σ either negative or positive. For negative values of σ, γ is an upper bound to consumption: the best-known example of this class is the linear demand function, corresponding to $\sigma = -1$. By contrast, for strictly positive values of σ, γ is the lower bound to consumption and there is no upper bound. Especially in the LES case, it is common to interpret γ as a "subsistence" level of consumption, but this requires that it be positive, which (when σ and δ are positive) only holds if demand is superconvex. All members of the Pollak family with positive σ are translated-CES functions, and, as the arrows in Figure 5(a) indicate, they asymptote towards the corresponding "untranslated-CES" function as sales rise without bound; for example, the LES demand function, with σ equal to one, asymptotes towards the Cobb-Douglas. Table 1 summarizes the three possible cases of this family of demand functions.

Pollak showed that these are the only demand functions that are consistent with both additive separability and quasi-homotheticity (so the expenditure function exhibits the "Gorman Polar Form"). Just as (11) is dual to (12), so the Pollak family of direct demand

[34] As noted by Pollak, this demand function was first proposed by Chipman (1965), who showed that it is implied by an additive exponential utility function. Later independent developments include Bertoletti (2006) and Behrens and Murata (2007). Differentiating the Arrow-Pratt coefficient of absolute risk aversion defined in footnote 36 gives $\frac{\partial A(x)}{\partial x} = -\frac{u'u'''-(u'')^2}{(u'')^2} = -\frac{pp''-(p')^2}{(p')^2} = 1-\varepsilon\rho$, so absolute risk aversion is constant if and only if $\varepsilon = \frac{1}{\rho}$.

	$\gamma > 0$	$\gamma < 0$
$\sigma > 0$, $\delta > 0$	1. Superconvex; logconvex: $x > \gamma > 0$	2. Subconvex; logconvex
$\sigma < 0$, $\delta < 0$	3. Subconvex; logconcave: $\gamma > x > 0$	n/a

Table 1: Properties of Pollak Demand Functions

functions is dual to the Bulow-Pfleiderer family of inverse demand functions. An implication of this is that, corresponding to the property of Bulow-Pfleiderer demands that marginal revenue is linear in price, Pollak demands exhibit the property that the marginal *loss* in revenue from a small increase in price is linear in sales.[35] This implies that the coefficient of absolute risk aversion for these demands is hyperbolic in sales, which is why, in the theory of choice under uncertainty, they are known as "HARA" ("hyperbolic absolute risk aversion") demands following Merton (1971).[36]

I.2 Properties of PIGL Demands

With $\nu = 1$, the elasticity of demand becomes $\varepsilon = \frac{\gamma p^{-1}+\sigma\delta p^{-\sigma}}{\gamma p^{-1}+\delta p^{-\sigma}}$. Subtracting one gives: $\varepsilon - 1 = \frac{(\sigma-1)\delta p^{-\sigma}}{\gamma p^{-1}+\delta p^{-\sigma}} = (\sigma - 1)\,\frac{px-\gamma}{px}$. It follows that $\sigma - 1$, δ and $px - \gamma$ must have the same sign. In addition, the demand manifold is $\rho = \frac{(\sigma+2)\varepsilon-\sigma}{\varepsilon^2}$, so convexity is increasing in σ. Combining these results with Proposition 4, there are three possible cases of this demand function. For σ less than one, the demand function is less convex than the translog (i.e., PIGLOG) case, δ is negative and γ is positive. For σ greater than one, δ is positive, the demand function is more convex than the translog case, and it is subconvex if γ is negative, otherwise it is superconvex. These properties are dual to those of the inverse PIGL demand functions in Appendix J.3, and, like the latter, they can be related to whether the elasticity of marginal revenue with respect to *price* is greater or less than one (the value of one corresponding to

[35] Recall from footnote 37 that Bulow-Pfleiderer demands $p(x) = \alpha + \beta x^{-\theta}$ satisfy the property: $p + xp' = \theta\alpha + (1-\theta)p$. Switching variables, we can conclude that Pollak demands $x(p) = \gamma + \delta p^{-\sigma}$ satisfy the property: $x + px' = \sigma\gamma + (1-\sigma)x$.

[36] The Arrow-Pratt coefficient of absolute risk aversion is $A(x) \equiv -\frac{u''(x)}{u'(x)}$. With additive separability this becomes $A(x) = -\frac{p'(x)}{p(x)} = -\frac{1}{px'(p)}$. Using the result from footnote 35, this implies: $A(x) = \frac{1}{\sigma(x-\gamma)}$, which is hyperbolic in x.

the PIGLOG case). Note finally that the limiting case of PIGL demand function when σ approaches zero is the LES, the only demand function which is a subset of both PIGL and Pollak. The LES case is special in another respect: as can be seen in Figure 5(a), it is the only member of the PIGL family for which ε is monotonic in ρ along the manifold. In all other cases the manifold is vertical at $\{\varepsilon, \rho\} = \{\frac{2\sigma}{\sigma+2}, \frac{(\sigma+2)^2}{4\sigma}\}$. For $\sigma < 0$ it is not defined for $\rho < \frac{(\sigma+2)^2}{4\sigma}$, while for $\sigma > 0$ it is not defined for $\rho > \frac{(\sigma+2)^2}{4\sigma}$.

I.3 Proof of Lemma 1: Uniqueness of the Translog

We wish to show that the translog is the only demand function with a manifold of the "contiguous bipower" form $\rho = a_1\varepsilon^k + a_2\varepsilon^{k+1}$ that is always both strictly subconvex and strictly supermodular in the interior of the admissible region. The proof proceeds by showing that these conditions require that the demand manifold satisfy three distinct restrictions. These enable us to isolate the translog demand function as the only candidate.

First, it is clear by inspection that, if a demand function is always both subconvex and supermodular, then its manifold must pass through the Cobb-Douglas point, $\{\varepsilon, \rho\} = \{1, 2\}$. Hence the parameters must satisfy $a_1 + a_2 = 2$.

Second, the slope of the manifold, $\frac{d\rho}{d\epsilon} = a_1 k\varepsilon^{k-1} + a_2(k+1)\varepsilon^k$, must be greater than that of the SM locus and less than that of the SC locus at $\{1, 2\}$. Both of these slopes equal -1: $\frac{d\rho}{d\epsilon}\Big|_{SM} = -1$ everywhere, and $\frac{d\rho}{d\epsilon}\Big|_{SC} = -\frac{1}{\varepsilon^2} = -1$ at $\{1, 2\}$. Hence the parameters must satisfy $a_1 k + a_2(k+1) = -1$. This and the previous restriction can be solved for a_1 and a_2 in terms of k: $a_1 = 3 + 2k$ and $a_2 = -(1 + 2k)$.

Third, the curvature of the manifold, $\frac{d^2\rho}{d\epsilon^2}\Big|_M = a_1(k-1)k\varepsilon^{k-2} + a_2 k(k+1)\varepsilon^{k-1}$, must be greater than that of the SM locus and less than that of the SC locus at $\{1, 2\}$. These curvatures are: $\frac{d^2\rho}{d\epsilon^2}\Big|_{SM} = 0$ everywhere, and $\frac{d^2\rho}{d\epsilon^2}\Big|_{SC} = \frac{1}{\varepsilon^3} = 1$ at $\{1, 2\}$. Hence the parameters must satisfy $0 \leq a_1(k-1)k + a_2 k(k+1) \leq 1$. Substituting for a_1 and a_2 and simplifying gives: $0 \leq -2k(k+2) \leq 1$. Only two integer values of k satisfy these inequalities: $k = 0$ is the SM locus itself, which is not in the interior of the admissible region; that leaves $k = -2$, implying $\rho = -\varepsilon^{-2} + 3\varepsilon^{-1} = \frac{3\varepsilon-1}{\varepsilon^2}$, the translog demand manifold, as was to be proved.

I.4 QMOR Demand Functions

Diewert (1976) introduced the quadratic mean of order r expenditure function, which implies a general functional form for homothetic demand functions. Feenstra (2014) considers a symmetric special case and shows how it can be adapted to allow for entry and exit of goods, so making it applicable to models of monopolistic competition. In our notation, the resulting family of demand functions, taking a "firm's eye view", is:

$$x(p) = \gamma p^{-(1-r)} + \delta p^{-\frac{2-r}{2}} \tag{46}$$

This is clearly a member of the bipower direct family, with $\nu = 1 - r$ and $\sigma = \frac{2-r}{2}$. Hence, from Proposition 3, its demand manifold is:

$$\bar{\rho}(\varepsilon) = \frac{(2-r)(3\varepsilon - 1 + r)}{2\varepsilon^2} \tag{47}$$

In the limit as $r \to 0$, this becomes $\bar{\rho}(\varepsilon) = \frac{3\varepsilon-1}{\varepsilon^2}$, which is the translog manifold discussed in the text. Figure 9 illustrates this demand manifold for a range of values of r. For $r = 2$ it coincides with the $\rho = 0$ vertical line: i.e., a linear demand function from the firm's perspective. For negative values of r (i.e., more convex than the translog), the manifolds extend into the superconvex region. However, this is for arbitrary values of γ and δ. Feenstra (2014) shows that these parameters, which depend on real income and on prices of other goods, must be of opposite sign when the demand function (46) is derived from expenditure minimization. Hence, from Proposition 4, QMOR demands are not consistent with superconvexity, though in other respects they allow for considerable flexibility in modeling homothetic demands.

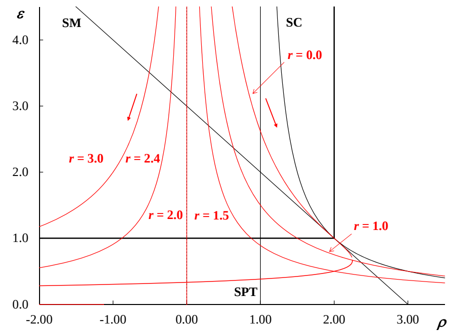


Figure 9: Demand Manifolds for QMOR Demand Functions

J Examples of Bipower Inverse Demands

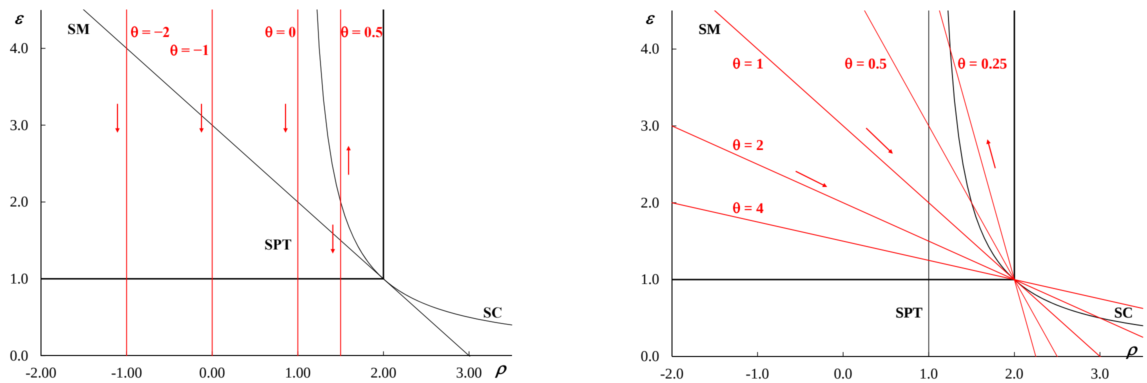
J.1 Bulow-Pfleiderer and Inverse PIGL Demands

As noted in the text, the first sub-case of the bipower inverse demand functions in (12) we consider comes from setting η equal to zero, giving the iso-convex or "constant pass-through" family of Bulow and Pfleiderer (1983): $p(x) = \alpha + \beta x^{-\theta}$. Convexity ρ equals a constant $\theta + 1$, so from (6) $\frac{1}{1-\theta}$ measures the degree of absolute pass-through for this system. Pass-through can be more than 100%, as in the CES case ($\alpha = 0$, $\theta = \frac{1}{\sigma} > 0$); equal to 100%, as in the log-linear direct demand case ($\theta \to 0$, so $p(x) = \alpha' + \beta' \log x$, implying that $\log x(p) = \gamma + \delta p$); or less than 100%, as in the case of linear demand ($\theta = -1$ so pass-through is 50%).

This family has many other attractive properties. It is necessary and sufficient for marginal revenue to be affine in price.[37] It can be given a discrete choice interpretation: it equals the cumulative demand that would be generated by a population of consumers if their preferences followed a Generalized Pareto Distribution.[38] Finally, as shown by Weyl and Fabinger (2013) and empirically implemented by Atkin and Donaldson (2012), it allows the division of surplus between consumers and producers to be calculated without knowledge of quantities. Figure 10(a) shows the demand manifolds for some members of this family;

[37] See Appendix J.2 below for more details.
[38] See Bulow and Klemperer (2012).

see Appendix J.2 below for more details.[39]



(a) The Bulow-Pfleiderer Family

(b) The Inverse PIGL Family

Figure 10: Demand Manifolds for Some Bipower Inverse Demand Functions

The second case of (12) considered in the text comes from setting η equal to one, yields the "inverse PIGL" ("price-independent generalized linear") system: $p(x) = \frac{1}{x}(\alpha + \beta x^{1-\theta})$. This system implies that the elasticity of marginal revenue defined in footnote 15 is constant and equal to θ: $\eta = 1$ implies from (12) that $-\frac{xR''}{R'} = \frac{2-\rho}{\varepsilon-1} = \theta$. The limiting case as $\theta \to 1$ is the inverse "PIGLOG" ("price-independent generalized logarithmic") or inverse translog, $p(x) = \frac{1}{x}(\alpha' + \beta' \log x)$.[40] This implies that the elasticity of marginal revenue is unity, and so, as noted in Mrázová and Neary (2011), it coincides with the supermodularity locus: $\eta = \theta = 1$ implies from (12) that $\bar{\rho}\,(\varepsilon) = 3 - \varepsilon$. Figure 10(b) shows the demand manifolds for some members of this family. Details are given in Appendix J.3 below.

[39] Note how the behavior implied by these manifolds differs from the Pollak case in Figure 5(a), especially in the super-pass-through region. With Bulow-Pfleiderer demands, firms diverge from the CES benchmark along the SC locus as sales increase, whereas with Pollak demands they converge towards it; both these statements hold whether demands are super- or subconvex. This allows a simple visualization of the limiting behavior of a monopolistically competitive sector as market size increases without bound. See Dhingra and Morrow (2011) for a related discussion.

[40] To show this, take the limit as in footnote 24.

J.2 Properties of Bulow-Pfleiderer Demands

With $\eta = 0$, the elasticity of demand becomes: $\varepsilon = \frac{\alpha+\beta x^{-\theta}}{\theta\beta x^{-\theta}} = \frac{p}{\theta\beta x^{-\theta}} = \frac{p}{\theta(p-\alpha)}$. It follows that θ, β and $p-\alpha$ must have the same sign. The sign of θ also determines whether the direct demand function is logconvex (i.e., whether it exhibits super-pass-through) or not: recall that $\rho - 1 = \theta$. There are therefore three possible cases of this demand function: see Table 2. As shown by Bulow and Pfleiderer (1983), these demands are necessary and sufficient for marginal revenue to be affine in price. Sufficiency is immediate: marginal revenue is $p + xp' = \theta\alpha + (1-\theta)p$. Necessity follows by solving the differential equation $p(x) + xp'(x) = a + bp(x)$, which yields $p(x) = \frac{a}{1-b} + c_1 x^{b-1}$, where c_1 is a constant of integration.

	$\alpha > 0$	$\alpha < 0$
$\theta > 0$, $\beta > 0$	1. Superconvex; logconvex: $p > \alpha > 0$	2. Subconvex; logconvex
$\theta < 0$, $\beta < 0$	3. Subconvex; logconcave: $\alpha > p > 0$	n/a

Table 2: Properties of Bulow-Pfleiderer Demand Functions

J.3 Properties of Inverse PIGL Demands

With $\eta = 1$, so the elasticity of demand becomes $\varepsilon = \frac{\alpha x^{-1}+\beta x^{-\theta}}{\alpha x^{-1}+\theta\beta x^{-\theta}}$, its value less one can be written in two alternative ways: $\varepsilon - 1 = \frac{(1-\theta)\beta x^{1-\theta}}{\alpha+\theta\beta x^{1-\theta}} = (1-\theta)\frac{px-\alpha}{\theta px+(1-\theta)\alpha}$. It follows that $1-\theta$, β and $px - \alpha$ must have the same sign. (Recall that θ itself equals $\frac{2-\rho}{\varepsilon-1}$ and so must be positive in the admissible region.) The value of $1-\theta$ also determines whether the demand function is supermodular or not: substituting from the demand manifold $\bar{\rho}(\varepsilon) = 2 + (1-\varepsilon)\theta$ into the condition for supermodularity gives $\varepsilon + \rho > 3 \;\Leftrightarrow\; (\varepsilon - 1)(1-\theta) > 0 \;\Leftrightarrow\; \theta < 1$. Combining these results with Proposition 4 shows that there are three possible cases of this demand function, as shown in Table 3.

	$\alpha > 0$	$\alpha < 0$
$\theta < 1,\ \beta > 0$	1. Superconvex; supermodular: $px > \alpha > 0$	2. Subconvex; supermodular
$\theta > 1,\ \beta < 0$	3. Subconvex; submodular: $\alpha > px > 0$	n/a

Table 3: Properties of Inverse PIGL Demand Functions

K Exponential Inverse Demand

In this section we introduce a demand function, the inverse exponential, which is an example of one that, for the same parameter values, is sometimes sub- and sometimes superconvex:[41]

$$p(x) = \alpha + \beta \exp(-\gamma x^{\delta}) \tag{48}$$

where $\gamma > 0$ and $\delta > 0$. The elasticity and convexity of demand are found to be:

$$\varepsilon(x) = \frac{p}{p-\alpha}\frac{1}{\delta\gamma x^{\delta}} \qquad \text{and} \qquad \rho(x) = \delta\gamma x^{\delta} - \delta + 1 \tag{49}$$

Solving the latter for γx^{δ} as a function of ρ and substituting into the former yields a closed-form expression for the demand manifold:

$$\bar{\varepsilon}(\rho) = \frac{1 + \frac{\alpha}{\beta}\exp\left(\frac{\rho+\delta-1}{\delta}\right)}{\rho+\delta-1} \tag{50}$$

This is invariant with respect to γ and also depends only on the ratio of α and β, not on their levels. Differentiating with respect to ρ shows that, provided $\frac{\alpha}{\beta}$ is strictly positive, the demand function is subconcave for low values of ρ, which from (49) implies low values of x, but superconcave for high ρ and x:

$$\bar{\varepsilon}_{\rho} = \frac{-\delta + \frac{\alpha}{\beta}(\rho-1)\exp\left(\frac{\rho+\delta-1}{\delta}\right)}{\delta(\rho+\delta-1)^2} \tag{51}$$

[41] Mrázová and Neary (2011) consider the properties of R&D cost functions of this form.

Figure 11 illustrates some demand functions and the corresponding manifolds from this class for a range of values of α, assuming $\beta = \gamma = 1$ and $\delta = 2$. A superconvex range in the admissible region is possible only for parameter values such that the minimum point of the manifold lies above the Cobb-Douglas point, $\{\varepsilon, \rho\} = \{1, 2\}$, i.e., only for $\alpha > \beta\delta \exp\left(-\frac{\delta+1}{\delta}\right)$, which for the assumed values of β and δ is approximately $\alpha > 0.446$.

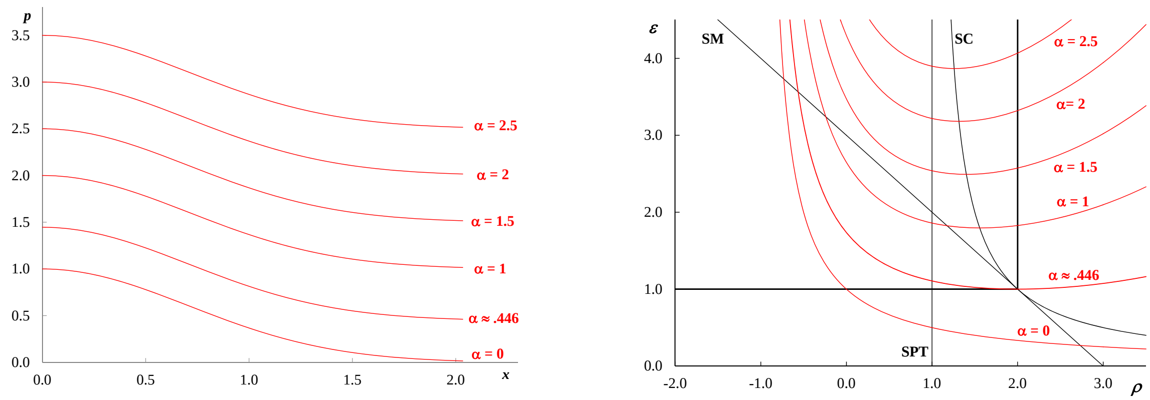


Figure 11: Inverse Exponential Demand Functions and Manifolds

L Demand Functions that are not Manifold-Invariant

In this section we introduce two new demand systems whose demand manifolds can be written in closed form, though they depend on all the parameters, and so are not manifold invariant. We consider in turn: the "Doubly-Translated CES" super-family, which nests both the Pollak and Bulow-Pfleiderer families; and the "Translated Bipower Inverse" super-family, which nests both the "APT" (Adjustable pass-through) system of Fabinger and Weyl (2012) and a new family which we call the inverse "iso-temperance" system.[42]

[42] A third super-family is the dual of the second, the "Translated Bipower Direct" super-family. Reversing the roles of p and x in equation (54) below leads to a "dual" manifold giving the inverse elasticity e as a function of the direct convexity r with the same form as (56). Special cases of this include the dual of the APT system and the direct "iso-temperance" system (i.e., the demand system necessary and sufficient for $-px'''/x''$ to be constant). It does not seem possible to express the manifold $\bar{\varepsilon}(\rho)$ in closed form for this family.

L.1 The "Doubly-Translated CES" Super-Family

We can nest the Pollak and Bulow-Pfleiderer families as follows: $p(x) = \alpha + \beta (x - \gamma)^{-\theta}$.[43] The elasticity and convexity of this function are:

$$\varepsilon(x) = \frac{1}{\theta}\frac{p}{p-\alpha}\frac{x-\gamma}{x} \qquad \rho(x) = (\theta+1)\frac{x}{x-\gamma} \tag{52}$$

When γ is zero this reduces to the Bulow-Pfleiderer case. Assuming $\gamma \neq 0$, we have $\rho \neq \theta+1$, and so the expression for ρ in (52) can be solved for x: $x = \frac{\rho}{\rho-(\theta+1)}\gamma$. Substituting into the expression for ε yields:[44]

$$\bar{\varepsilon}(\rho) = \left[1 + a_1\left(\frac{1}{\rho - a_2}\right)^{a_3}\right]\frac{a_4}{\rho} \tag{53}$$

where: $a_1 = \frac{\alpha}{\beta}\left\{(\theta+1)\gamma\right\}^{\theta}$, $a_2 = \theta+1$, $a_3 = \theta$, and $a_4 = \frac{\theta+1}{\theta}$. This is a closed-form expression for the manifold but it depends on all four parameters, except in special cases such as the Pollak family, when, with $\alpha = 0$, it reduces to $\bar{\varepsilon}(\rho) = \frac{\theta+1}{\theta}\frac{1}{\rho}$. Nevertheless, the general demand manifold (53) allows for considerable economy of information: three of its four parameters depend only on the exponent θ in the demand function, and the fourth parameter, a_1, is invariant to rescalings of the demand function parameters which keep $\frac{\alpha}{\beta}\gamma^{\theta}$ constant.

L.2 The "Translated Bipower Inverse" Super-Family

This demand function adds an intercept α_0 to the bipower inverse family given by (12):

$$p(x) = \alpha_0 + \alpha x^{-\eta} + \beta x^{-\theta} \tag{54}$$

[43] After we developed this family, we realized that it had already been considered in the working paper version of Zhelobodko, Kokovin, Parenti, and Thisse (2012), who call it the "Augmented-HARA" system.

[44] Here and elsewhere, the parameters must be such that, when the exponent (here θ) is not an integer, the expression which is raised to the power of that exponent is positive.

Differentiating gives the elasticity and convexity:

$$\varepsilon(x) = \frac{\alpha_0 x^{\eta} + \alpha + \beta x^{\eta-\theta}}{\eta\alpha + \theta\beta x^{\eta-\theta}} \qquad \rho(x) = \frac{\eta\,(\eta+1)\,\alpha + \theta\,(\theta+1)\,\beta x^{\eta-\theta}}{\eta\alpha + \theta\beta x^{\eta-\theta}} \tag{55}$$

Assuming as before that $\rho \neq \theta + 1$, and also that $\eta \neq \theta$, we can invert $\rho(x)$ to solve for x: $x(\rho) = \left[\frac{\eta\alpha}{\theta\beta}\frac{(\eta+1)-\rho}{\rho-(\theta+1)}\right]^{\frac{1}{\eta-\theta}}$. Substituting into $\varepsilon(x)$ gives a closed-form expression for the manifold:

$$\bar{\varepsilon}(\rho) = \frac{\rho - a_1}{a_2} + (a_3 - \rho)^{a_4}(\rho - a_5)^{a_6} a_7 \tag{56}$$

where: $a_1 = \eta + \theta + 1$, $a_2 = -\eta\theta$, $a_3 = \eta + 1$, $a_4 = \frac{\eta}{\eta-\theta}$, $a_5 = \theta + 1$, $a_6 = -\frac{\theta}{\eta-\theta}$, and $a_7 = \left(\frac{\eta}{\beta}\right)^{\frac{\eta}{\eta-\theta}}\left(\frac{\theta}{\alpha}\right)^{-\frac{\theta}{\eta-\theta}}\frac{\alpha_0}{\eta\theta(\eta-\theta)}$. In general, this depends on the same five parameters as the demand function (54), though once again it allows for considerable economy of information: all but a_7 depend only on the two exponents η and θ, and a_7 itself is unaffected by changes in the other three demand-function parameters that keep $\alpha^{\frac{\theta}{\eta-\theta}}\beta^{\frac{-\eta}{\eta-\theta}}\alpha_0$ constant. Equation (56) is best understood by considering some special cases:

(1) Bipower Inverse: The cost in additional complexity of the "translation" parameter α_0 is apparent. Setting this equal to zero, the expression simplifies to give the bipower inverse manifold as in Proposition 3: $\bar{\rho}(\varepsilon) = 1 + \eta + \theta - \eta\theta\varepsilon$.

(2) APT Demands: Fabinger and Weyl (2012) show that the pass-through rate (in our notation, $\frac{dp}{dc} = \frac{1}{2-\rho}$) is quadratic in the square root of price if and only if the inverse demand function has the form of (54) with $\eta = 2\theta$. This reduces the number of parameters by one, so the demand manifold simplifies to: $\bar{\varepsilon}(\rho) = \frac{1+3\theta-\rho}{2\theta^2} - \frac{[(2\theta+1)-\rho]^2}{\rho-(\theta+1)}\frac{2\alpha}{\beta^2\theta^2}\alpha_0$.

(3) Iso-Temperance Demands: Setting $\eta = -1$ is sufficient to ensure that temperance, $\chi \equiv -\frac{xp'''}{p''}$, is constant, equal to $\theta + 2$. It is also necessary. To see this, write $xp''' = -\chi p''$, where χ is a constant, and integrate three times, which yields $p(x) = c_0 + c_1 x + \frac{c_2}{(1-\chi)(2-\chi)}x^{2-\chi}$, where c_0, c_1 and c_2 are constants of integration. This is identical to (54) with $\eta = -1$ and $\theta = \chi - 2$. Note that iso-convexity implies iso-temperance, but the converse does not hold; just as CES implies iso-convexity, but the converse does not hold.

These special cases and the general demand manifold in (56) allow us to infer the comparative statics implications of this family of demand functions. Moreover, if we are mainly interested in pass-through, we do not need to work with the demand manifold at all, since the key conditions in (6) and (35) do not depend on the elasticity of demand (a point stressed by Weyl and Fabinger (2013)). In such cases, our approach can be applied to the *slope* rather than the *level* of demand. By relating the elasticity and convexity of this slope to each other, we can construct a "demand-slope manifold" corresponding to any given demand function, and the properties of this manifold are very informative about when pass-through is increasing or decreasing with sales. In ongoing work, we show that the demand-slope manifolds of the APT and iso-temperance demand functions are particularly convenient in this respect.

M Calculating the Effects of Globalization

To solve for the results in (21), use (19) to eliminate $\hat{x}$ from (17) and then solve (17) and (18) for $\hat{p}$ and $\hat{y}$, with $\hat{n}$ determined residually by (20). The results in (22) are obtained by using $\hat{x} = \hat{y} - \hat{k}$ and $\hat{N} = \hat{k} + \hat{n}$.

N Heterogeneous Firms with Additive Separability

The first step is to calculate the elasticities of the maximum profit function:

$$\pi(c, \lambda, k) \equiv \max_y \Big[p(y, \lambda, k) - c\Big] y \quad \text{where:} \quad p(y, \lambda, k) = \lambda^{-1} u'(y/kL) \tag{57}$$

For later use, the first and second derivatives of the inverse demand function, expressed in terms of elasticities, are:

$$\frac{yp_y}{p} = -\frac{1}{\varepsilon}, \qquad \frac{\lambda p_\lambda}{p} = -1 \quad \text{and} \quad \frac{kp_k}{p} = \frac{1}{\varepsilon} \tag{58}$$

$$\frac{yp_{yy}}{p_y} = -\rho, \qquad \frac{yp_{y\lambda}}{p_\lambda} = -\frac{1}{\varepsilon} \quad \text{and} \quad \frac{yp_{yk}}{p_k} = \frac{1}{\varepsilon} \tag{59}$$

Using the envelope theorem, the derivatives of the profit function are:

$$\pi_c = -y, \quad \pi_\lambda = -\lambda^{-2}u'y = -\lambda^{-1}py, \quad \pi_k = -\frac{y^2 u''}{\lambda k^2 L} = -\frac{y^2}{k}p_y \tag{60}$$

These in turn can be expressed in terms of elasticities, making use of the first-order condition $p + yp_y = c$:

$$\frac{c\pi_c}{\pi} = -\frac{cy}{\pi} = -\frac{c}{p-c} = -(\varepsilon - 1), \quad \frac{\lambda\pi_\lambda}{\pi} = -\frac{py}{\pi} = -\frac{p}{p-c} = -\varepsilon, \tag{61}$$

$$\frac{k\pi_k}{\pi} = -\frac{y^2 p_y}{(p-c)y} = \frac{yp_y}{yp_y} = 1 \tag{62}$$

Aggregating these gives the elasticities of aggregate profits:

$$\frac{\lambda\bar{\pi}_\lambda}{\bar{\pi}} = \int_{\underline{c}}^{\bar{c}} \frac{\pi(c)}{\bar{\pi}} \frac{\lambda\pi_\lambda(c)}{\pi(c)} g(c)\,\mathrm{d}c = -\int_{\underline{c}}^{\bar{c}} \frac{\pi(c)}{\bar{\pi}} \varepsilon(c) g(c)\,\mathrm{d}c = -\bar{\varepsilon}, \tag{63}$$

$$\frac{k\bar{\pi}_k}{\bar{\pi}} = \int_{\underline{c}}^{\bar{c}} \frac{\pi(c)}{\bar{\pi}} \frac{k\pi_k(c)}{\pi(c)} g(c)\mathrm{d}c = \int_{\underline{c}}^{\bar{c}} \frac{\pi(c)}{\bar{\pi}} g(c)\mathrm{d}c = 1 \tag{64}$$

Using these results, we can solve for the effect of globalization on the degree of competition λ by totally differentiating the zero-expected-profit condition (25):

$$\hat{\lambda} = -\left(\frac{\lambda\bar{\pi}_\lambda}{\bar{\pi}}\right)^{-1} \frac{k\bar{\pi}_k}{\bar{\pi}} = \frac{1}{\bar{\varepsilon}}\,\hat{k} \tag{65}$$

The next step is to solve for the effects of globalization at the intensive margin. Totally differentiating the first-order condition, and making use of (58) and (59), the partial elasticities of output with respect to λ and k are given by:

$$\frac{\lambda y_\lambda}{y} = -\frac{\varepsilon - 1}{2 - \rho} \quad \text{and} \quad \frac{ky_k}{y} = 1 \tag{66}$$

Hence, the total derivative of output with respect to k, allowing for the indirect effect via the level of competition, is:

$$\hat{y} = \left(\frac{k y_k}{y} + \frac{\lambda y_\lambda}{y} \frac{k}{\lambda} \frac{d\lambda}{dk} \right) \hat{k} = \left(1 - \frac{\varepsilon - 1}{2 - \rho} \frac{1}{\bar{\varepsilon}} \right) \hat{k} \tag{67}$$

where we use (65) and (66) to simplify. Adding and subtracting $\frac{\varepsilon-1}{2-\rho}\frac{1}{\varepsilon}\hat{k}$ gives the decomposition in (29) in the text.

As for prices, equation (17) in Section 4.1, which relates price changes to changes in per capita consumption x, continues to hold for each individual firm. The change in x in turn can be derived from the goods-market equilibrium condition (19):

$$\hat{x} = \hat{y} - \hat{k} = \left(1 - \frac{\varepsilon - 1}{2 - \rho} \frac{1}{\bar{\varepsilon}} - 1 \right) \hat{k} = - \frac{\varepsilon - 1}{2 - \rho} \frac{1}{\bar{\varepsilon}} \hat{k} \tag{68}$$

Substituting into (17) gives the change in prices:

$$\hat{p} = - \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon (\varepsilon - 1)} \frac{\varepsilon - 1}{2 - \rho} \frac{1}{\bar{\varepsilon}} \hat{k} = - \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon \bar{\varepsilon} (2 - \rho)} \hat{k} = - \frac{\varepsilon}{\bar{\varepsilon}} \frac{\varepsilon + 1 - \varepsilon\rho}{\varepsilon^2 (2 - \rho)} \hat{k} \tag{69}$$

O Glossary of Terms (Web Appendix)

In this appendix we note some alternative definitions of terms that we use in the text. The text can be read independently of the glossary.

CES: Strictly speaking, the label "CES" refers to the utility function from which the demand functions (9) are derived, and not to the demand functions themselves. We follow standard usage and refer to "CES" demands throughout.

Log-Convexity: We follow standard practice and describe a function $f(x)$ as log-convex at a point $(x_0, f(x_0))$ if and only if $\log f(x)$ is convex in x at $(x_0, f(x_0))$.

Manifold: Each of the demand manifolds we present is a one-dimensional smooth manifold, or a smooth plane curve in the Euclidean plane R^2. Each is defined by an equation $f(\varepsilon, \rho) =$

0, where $f : R^2 \to R$ is a smooth function, and the partial derivatives $\frac{\partial f}{\partial \varepsilon}$ and $\frac{\partial f}{\partial \rho}$ are never both zero. Strictly speaking, a manifold cannot have a self-intersection point, whereas the relationship between ε and ρ could exhibit such a feature.

Pollak or HARA Demands: The demand functions due to Pollak (1971) which we consider in Section 3.3 are sometimes called "HARA" ("hyperbolic absolute risk aversion") demands following Merton (1971). In the present context the former label seems more appropriate. Pollak characterized the preferences that are consistent with these demands in a non-stochastic multi-good setting, showing that they are the only ones that are consistent with both additive separability and quasi-homotheticity; whereas Merton focused on portfolio allocation in a stochastic one-good setting.

Superconvexity: Following Mrázová and Neary (2011), we describe a function $f(x)$ as superconvex at a point $(x_0, f(x_0))$ if and only if $\log f(x)$ is convex in $\log x$ at $(x_0, f(x_0))$. Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012) use the term "log-convex" for such a function, whereas Kingman (1961) uses the term "superconvex" as a synonym for the more widely-used sense of log-convexity, i.e., $\log f(x)$ convex in x.

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