DISCUSSION PAPER SERIES

No. 1110

UNEMPLOYMENT, INVESTMENT AND SECTORAL REALLOCATION

Bankim Chadha and Fabrizio Coricelli

INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research
25–28 Old Burlington Street, London W1X 1LB

Centre for Economic Policy Research

DISCUSSION PAPER SUBSCRIPTION FORM

Subscriptions may be placed for all CEPR Discussion Papers or for those appearing under one or more of the Centre's five research programme areas: International Macroeconomics, International Trade, Industrial Organization, Financial Economics and Human Resources

Subscription orders will be invoiced quarterly in arrears The charge will be determined by the number of papers sent during the preceeding three months Invoices will be sent on 31 March, 30 June, 30 September and 31 December New subscriptions must start from one of these dates If no starting date is specified, the subscription will be started from the beginning of the next period Papers are charged at the rate of £3 ($5) Individual academics may obtain papers at the concessionary rate of £2 ($3) To qualify for this concession, the declaration below (*) must be signed

Back copies of papers from number 600 are available For more details, and information on out of print papers contact the Centre

I wish to place a subscription for:

- ☐ International Macroeconomics (IM) Discussion Papers
- ☐ International Trade (IT) Discussion Papers
- ☐ Industrial Organization (IO) Discussion Papers
- ☐ Financial Economics (FE) Discussion Papers
- ☐ Human Resources (HR) Discussion Papers

☐ *I wish to take advantage of the concessionary rate for individual academics I am affiliated to an academic institution and will finance my subscription personally

Name _____

Affiliation (if applicable) _____

Address _____

_____

_____

_____

Vat No. (if any) _____

Telephone _____ Fax _____

Signature _____ Date _____

Invoice Address if different from delivery address:

_____

_____

_____

Upon receipt of an invoice payment must be made by the date on the invoice in one of the following methods: i) Sterling cheque drawn on a UK bank; ii) Sterling Eurocheque endorsed with your card number; iii) US dollar cheque drawn on a US bank; iv) Credit card (VISA/Access/Barclaycard/Eurocard/Mastercard) – please quote card type, number and expiry date; v) Bank transfer in Sterling to our bank – please contact CEPR for details of our bank account

I want my subscription to start: ☐ 1 January ☐ 1 April ☐ 1 July ☐ 1 October

Return this form to 'The Subscription Officer', at the address below.

✂ _____

CEPR, 25–28 Old Burlington Street, London W1X 1LB. Tel: 44 71 734 9110. Fax: 44 71 734 8760.

UNEMPLOYMENT, INVESTMENT AND SECTORAL REALLOCATION

Bankim Chadha and Fabrizio Coricelli

Discussion Paper No. 1110
January 1995

Centre for Economic Policy Research
25–28 Old Burlington Street
London W1X 1LB
Tel: (44 71) 734 9110

ISSN 0265-8003

CEPR Discussion Paper No. 1110

January 1995

ABSTRACT

Unemployment, Investment and Sectoral Reallocation*

This paper presents a model of development of an economy comprised of a rural-agricultural sector and an urban-industrial sector. The interaction of investment with unemployment creates a channel for potentially divergent long-run outcomes. If the urban-industrial capital stock falls short of a threshold level, the urban-industrial sector will not develop. If the capital stock is high enough, there is a unique path by which it will develop. Between these two extremes is a region of indeterminacy where expectations can play a pivotal role in determining the long-run outcome.

JEL Classification: E2, O1, P5
Keywords: unemployment, development, restructuring, investment

Bankim Chadha
International Monetary Fund
700 H Street NW
Washington DC 20431
USA
Tel: (1 202) 623 6669

Fabrizio Coricelli
Department of Economics
University of Siena
Piazza San Francesco 7
53100 Siena
ITALY
Tel: (39) 577 298 799

*We would like to thank Andres Velasco for useful discussions. Any errors and all views expressed here are our own.

Submitted 7 October 1994

NON-TECHNICAL SUMMARY

The recent literature on endogenous growth and development has sought to explain sustained disparities of income levels and growth rates across countries. The principal mechanism whereby these models generate large and sustained divergences across countries is the existence of externalities. In this paper we identify an alternative mechanism based on the interaction of investment with unemployment, whereby sustained divergences in outcomes can occur.

The paper presents a model of economic development of an economy comprised of an agricultural-rural sector and an urban-industrial sector. Economic development is identified with a successful and sustainable accumulation of physical capital in the urban-industrial sector. There are two elements of the mechanism that create a channel for potentially divergent long-run outcomes. The first element of the mechanism is that unemployment evolves non-monotonically during the sectoral reallocation process. Starting from an initially underdeveloped urban-industrial sector, therefore, unemployment tends to display a hump-shaped pattern as the urban-industrial sector capital stock expands.

The second element of the mechanism is that wages in the urban-industrial sector are a decreasing function of unemployment. Such a notion, of course, corresponds to standard models of the Phillips curve. In our model, such a dependence arises from an explicit efficiency-wage mechanism in the urban-industrial sector. At any given capital stock, unemployment, by depressing wages, provides an impetus for employment expansion, which in turn raises the marginal productivity of capital.

Since unemployment is a hump-shaped function of the urban-industrial capital stock and unemployment, by depressing wages, raises the return to capital, in equilibrium the return to capital will be increasing in the initial stages of urban-industrial sector development and subsequently will be diminishing. We show that when firms in the urban-industrial sector behave myopically, and investment is determined entirely by current rates of return, there exists, in general, a critical threshold level of the urban capital stock. As long as the capital stock falls short of this level, the economy will be trapped in an equilibrium where the urban-industrial sector does not develop. Only if the initial capital stock somehow – through policy actions, external grants, exogenous shocks, or historical accident – exceeds this critical threshold level will the urban-industrial sector develop. When firms are forward looking, and

investment is determined by the present value of the stream of future returns on capital, the set of potential dynamic paths of the economy are richer and more complex. In particular, there are four regions of initial conditions that imply alternative dynamic paths. The two extreme cases encountered with myopic investment continue to represent possible outcomes. There is a threshold level of the capital stock below which there is no viable path of urban-industrial sector development. Thus the presence of forward-looking firms does not eliminate the 'development trap'. Similarly, there is a region defined by a sufficiently high level of capital, such that there is a unique, convergent, equilibrium path along which the urban-industrial sector will develop. Between these two extreme regions, there are two regions of indeterminacy. In these regions, the same initial condition on the capital stock is consistent simultaneously with a multiplicity of equilibrium paths by which the urban-industrial sector can develop *and* with a multiplicity of equilibrium paths by which the nascent urban-industrial sector can disappear. Thus, expectations can play a critical role in determining the long-run outcome.

The specific model presented was a stylized model of development of an economy dominated initially by a rural-agricultural sector to one dominated eventually by an urban-industrial sector. The mechanism identified is not, however, limited to the development context. The basic forces of the mechanism appear relevant in any sectoral reallocation process and could give rise to divergent outcomes in a wide variety of other contexts. These include: sectoral reallocation of factors due to shifts in demand or changes in technology; the massive reallocation of labour from state-owned to newly created private firms that is taking place in the countries of Eastern Europe and the former Soviet Union today; and sectoral reallocation of factors induced by trade reforms in developing countries.

Figure 4. Effect of an Increase in Internationally Given Relative Price of Urban-Industrial Sector Good.

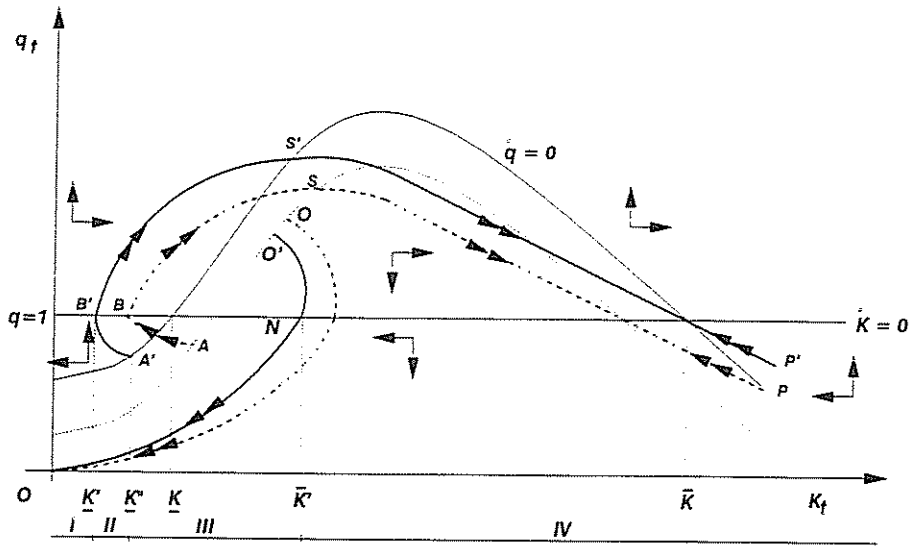
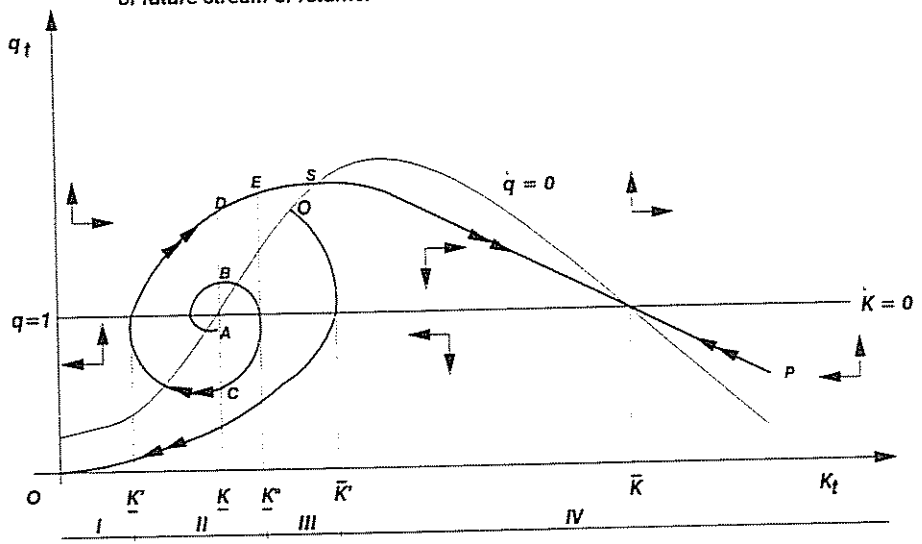
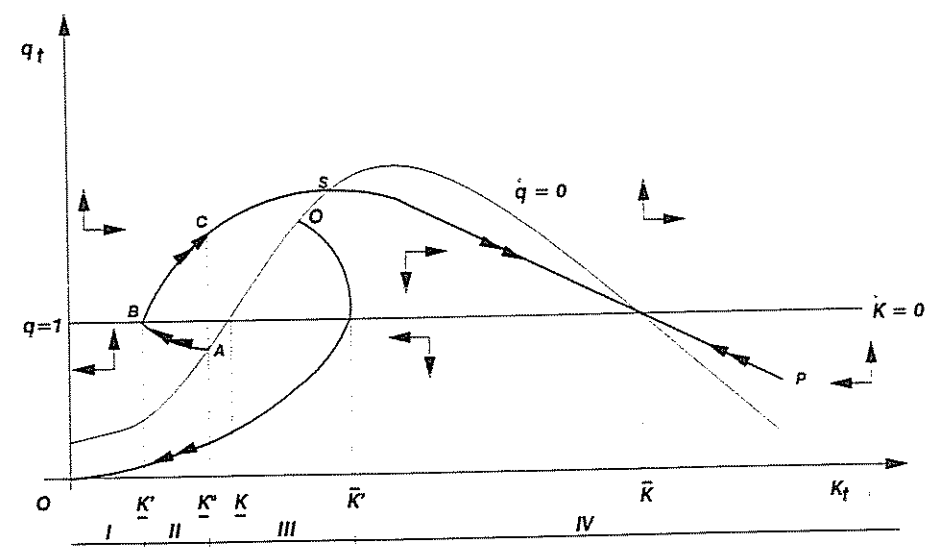


Figure 3. Dynamics of Capital Stock when investment is based on present value of future stream of returns.



3a. Case of Imaginary roots around $\underline{K}$



3b. Case of real roots around $\underline{K}$

1. Introduction

The recent literature on endogenous growth and development has sought to explain sustained disparities of income levels and growth rates across countries. The principal mechanism whereby these models generate large and sustained divergences across countries is the existence of externalities. Several types of externalities have been posited. Romer (1986) emphasizes spillovers across firms of knowledge acquired in the process of investment in physical capital that lead, in the aggregate, to increasing returns to capital accumulation. Lucas (1988) discusses external effects of skill levels on the production process in an industry. Azariadis and Drazen (1990) stress aggregate increasing returns to scale in the accumulation of human capital. Murphy, Shleifer and Vishny (1989) highlight the role of aggregate demand spillovers in affecting the development of an industrial sector. In this paper we identify an alternative mechanism based on the interaction of investment with unemployment, whereby sustained divergences in outcomes can occur.

We present a model of economic development of an economy comprised of an agricultural-rural sector and an urban-industrial sector. Economic development is identified with a successful and sustainable accumulation of physical capital in the urban-industrial sector. There are two elements of the mechanism that create a channel for potentially divergent long-run outcomes. The first element of the mechanism is that unemployment evolves nonmonotonically during the sectoral reallocation process. In our model, in the initial stages of urban-industrial sector expansion unemployment rises. After a critical stage of urban-industrial sector development, further expansion is accompanied by declines in unemployment. An increase in the urban-industrial capital stock leads to an expansion of employment, decreasing unemployment. However, as in the model of Harris and Todaro (1970), expanded job opportunities in the urban-industrial sector, by raising expected incomes, encourage migration from the rural-agricultural sector, increasing unemployment. Unemployment could, therefore, rise or fall with urban-industrial sector expansion depending on the relative strengths of the two effects. We find that at low levels of urban-industrial

sector development, the latter effect is strong, so that unemployment tends to rise. At higher levels of urban-industrial sector development, the former effect is strong, so unemployment tends to decline. Starting from an initially underdeveloped urban-industrial sector, therefore, unemployment tends to display a hump-shaped pattern as the urban-industrial sector capital stock expands. While the hump-shaped relationship between unemployment and the urban-industrial capital stock in our model results explicitly from the Harris-Todaro effect, it is worth emphasizing that there are several other reasons why unemployment may be expected to behave nonmonotonically during any sectoral reallocation process. In fact, when the change in unemployment is viewed as the difference between the rate at which one sector sheds labor and the rate at which the other absorbs labor, it should be apparent that only under special assumptions would the evolution of unemployment be expected to be monotonic or constant. It is easy to construct simple arithmetic examples where employment in one sector expands at a constant rate and contracts in the other at a constant rate such that unemployment evolves nonmonotonically.

The second element of the mechanism is that wages in the urban-industrial sector are a decreasing function of unemployment. Such a notion, of course, corresponds to standard models of the Phillips curve. In our model, such a dependence arises from an explicit efficiency-wage mechanism in the urban-industrial sector.1/ At any given capital stock, unemployment, by depressing wages, provides an impetus for employment expansion, which in turn raises the marginal productivity of capital.

Since unemployment is a hump-shaped function of the urban-industrial capital stock and unemployment, by depressing wages, raises the return to capital, in equilibrium the return to capital will be increasing in the initial stages of urban-industrial sector development and subsequently will

1/ Formally, when there are a large number of firms in the urban-industrial sector, who treat aggregate unemployment as exogenous to their actions, the dependence of firm-level wages on aggregate unemployment can be classified as an externality.

Figure 2. Dynamics of Capital Stock when investment is based on current returns.

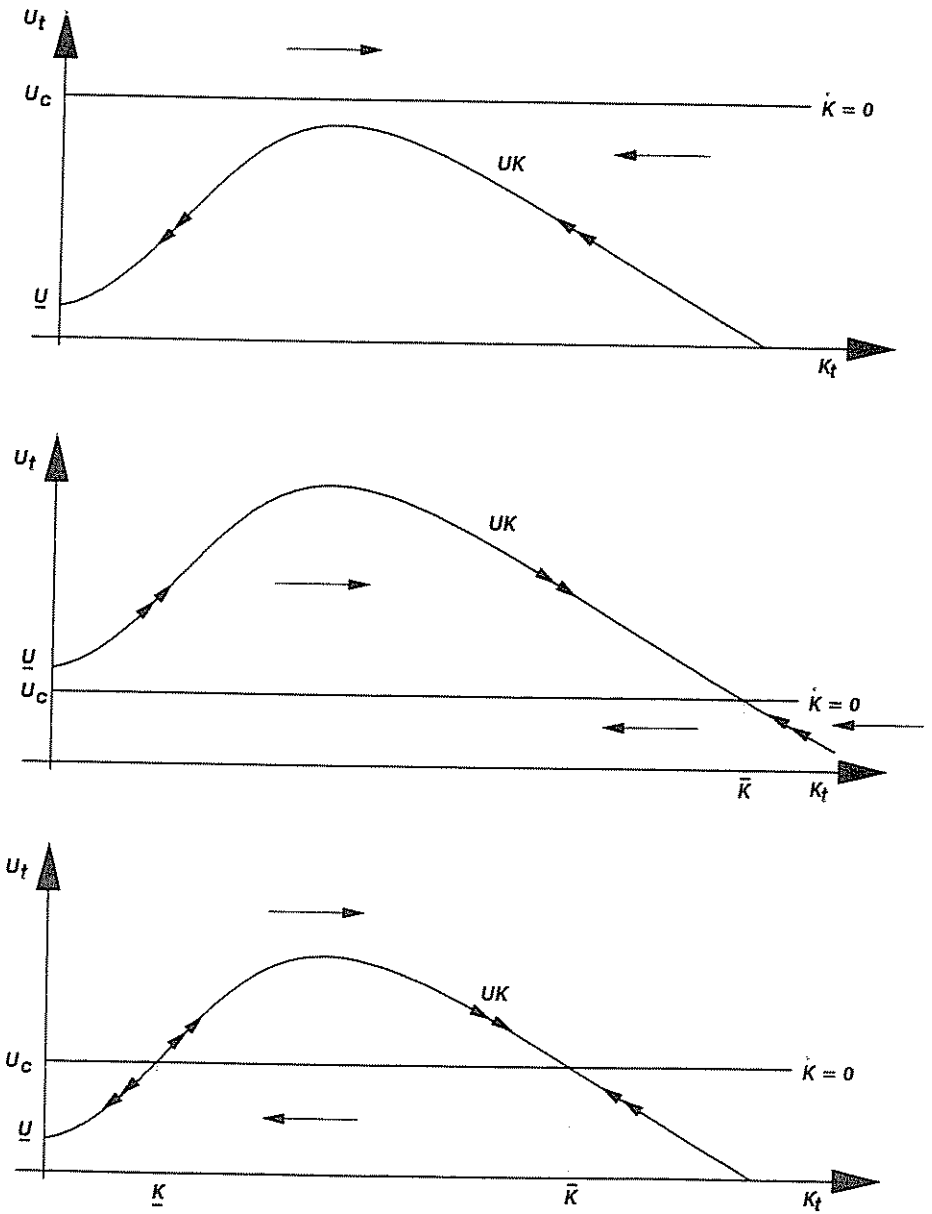
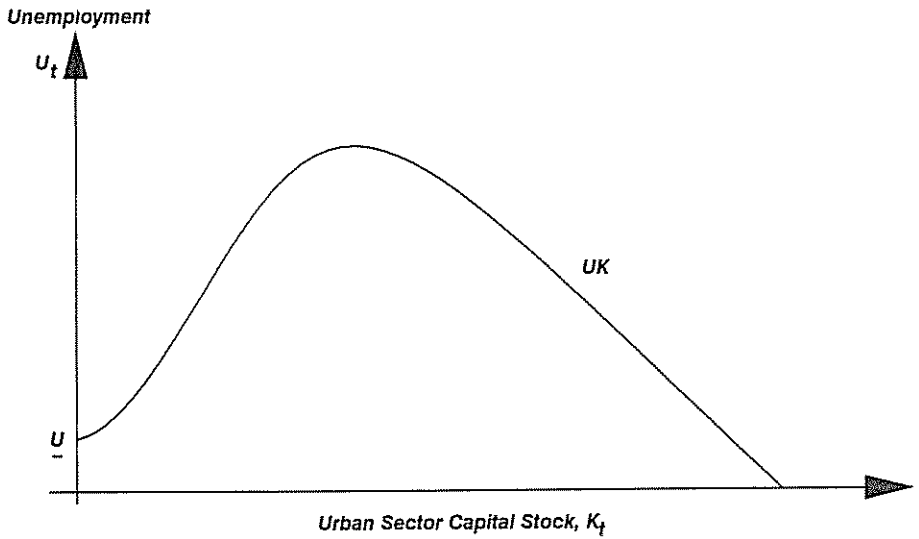


Figure 1. Unemployment and the Urban Capital Stock.



be diminishing. We show that when firms in the urban-industrial sector behave myopically, and investment is determined entirely by current rates of return, there exists, in general, a critical threshold level of the urban capital stock. As long as the capital stock falls short of this level, the economy will be trapped in an equilibrium where the urban-industrial sector does not develop. Only if the initial capital stock somehow--through policy actions, external grants, exogenous shocks, or historical accident--exceeds this critical threshold level will the urban-industrial sector develop. When firms are forward looking, and investment is determined by the present value of the stream of future returns on capital, the set of potential dynamic paths of the economy are richer and more complex. In particular, there are four regions of initial conditions that imply alternative dynamic paths. The two extreme cases encountered with myopic investment continue to represent possible outcomes. There is a threshold level of the capital stock below which there is no viable path of urban-industrial sector development. Thus the presence of forward looking firms does not eliminate the "development trap". Similarly, there is a region defined by a sufficiently high level of capital, such that there is a unique, convergent, equilibrium path along which the urban-industrial sector will develop. Between these two extreme regions, there are two regions of indeterminacy. In these regions, the same initial condition on the capital stock is consistent simultaneously with a multiplicity of equilibrium paths by which the urban-industrial sector can develop and with a multiplicity of equilibrium paths by which the nascent urban-industrial sector can disappear. As in the models of Krugman (1991) and Matsuyama (1991), expectations can play a critical role in determining the long-run outcome.

The paper is structured as follows. Section II presents a two sector model of sectoral reallocation and examines the interaction of investment with unemployment. Section III discusses the potential role of policies in affecting the development of the urban-industrial sector. Section IV contains some concluding remarks.

II. A Model of Sectoral Reallocation and Investment

Consider a small open economy comprised of a rural (agricultural) sector and an urban (manufacturing) sector. Each sector produces a (basket of) good(s), the relative prices of which are determined in the rest of the world. In the rural or agricultural sector, output is produced using labor and a fixed factor, land. The rural sector is assumed to be competitive in that labor earns its marginal product. In the urban or manufacturing sector, output is produced using labor and physical capital. Wages in the urban sector are determined by an efficiency wage mechanism that creates an incentive for employers to offer wages that exceed market clearing levels, resulting in unemployment. The population or labor supply is assumed to be constant and normalized to equal unity. Labor is assumed to be risk neutral.

The capital stock in the urban-industrial sector is treated as a predetermined variable in that it can only be adjusted gradually while employment can be adjusted immediately. While firms in the urban-industrial sector face adjustment costs associated with investment in physical capital, labor can be adjusted costlessly. This asymmetry has the advantage of keeping the analysis tractable and formally reflects the assumption that either the adjustment costs associated with physical capital are quantitatively more important than those associated with adjusting employment, or that the frequency at which decisions to invest in physical capital are made, exceeds that at which employment decisions are made. Since labor can move costlessly between sectors, it will maximize income at each point in time.

We first discuss the labor market and characterize equilibrium unemployment for a given level of the capital stock in the urban sector. We then examine the interaction of investment in the urban sector with aggregate unemployment.

1. The Labor Market

Rural (agricultural) sector

Output in the rural sector is produced according to

Since (effective) labor is paid its marginal product, and in the steady state the marginal product of capital equals the world real interest rate, it follows that the flow return to the firm is $r^*\tilde{K}$. The present value of the firm is, therefore, exactly $\tilde{K}$. Similarly, as K goes to zero, employment goes to zero and output goes to zero. The present value of the firm when it no longer exists is, therefore, exactly zero.

Terminal Conditions

The characteristic equation of the matrix [J] around $\bar{K}$ is

$$\lambda^2 - r^*\lambda + \left(\frac{\partial \psi}{\partial q}\right)\theta\left(\frac{\partial G}{\partial U}\right)\left(\frac{\partial U}{\partial K}\right) = 0 \quad , \tag{A7}$$

and so the unstable root, say λ_1, is

$$\lambda_1 = \frac{1}{2}\left[r^* + \left((r^*)^2 + 4\left(\frac{\partial \psi}{\partial q}\right)\theta\left(\frac{\partial G}{\partial U}\right)\left(-\frac{\partial U}{\partial K}\right)\right]^{1/2}\right) \tag{A8}$$

It is straightforward to note that λ_1 is greater than r^*. On any explosive path, that is when K_t and q_t tend to infinity, the rate at which they eventually do so equals the value of the unstable root. Since the unstable root exceeds the discount rate it is straightforward to note that an explosive path violates the transversality condition.

Consider now an implosive path in figure 3. For concreteness, consider a path that lies a little below OO in figure 3a such that the trajectory hits the horizontal axis and q equals zero for a finite capital stock. At this point equation (19) is violated. Since the capital stock is positive, the present value of the stream of future marginal products must be positive. Similarly, consider a path that hits the vertical axis below the $K = 0$ locus. At this point the capital stock is zero while q is positive. Since q is less than 1 there are no incentives to accumulate capital. The capital stock will, therefore, remain at zero. It follows that the present value of the future stream of marginal products cannot be positive without violating equation (19).

Present value of the firm

When $K = \bar{K}$, investment equals zero, and the present value of the firm is

$$V_0 = \int_0^{\infty}\left[Q_{2t}(\bar{K}, L_{2t}) - W_{2t}\cdot L_{2t}\right]\exp(-r^*t)dt \quad , \tag{A9}$$

$$Q_{1t} = H_1\cdot(L_{1t})^{\beta} \quad , \qquad 0 < \beta < 1 \quad , \tag{1}$$

where Q_{1t} denotes output in the rural sector--sector 1; L_1 denotes the (share of the) labor force employed in the rural sector; H_1 is a constant; and the fixed factor, land, has been suppressed from the notation.

Labor in the rural sector is free to seek employment in the urban sector. In equilibrium, therefore, the wage in the rural sector, which equals the marginal product of labor in the sector, must equal the expected income from leaving the sector, that is

$$\beta(L_{1t})^{\beta-1} = \frac{1}{P}[\delta_t B + (1-\delta_t)W_{2t}] \quad , \tag{2}$$

where δ_t denotes the probability that a worker leaving the rural sector becomes unemployed; $(1-\delta_t)$ represents the probability that he obtains employment in the urban sector; B denotes the exogenous level of unemployment benefits paid by the government to the urban unemployed; W_2 denotes the real wage in the urban sector; and P denotes the internationally given relative price of the rural sector's good to the urban sector's good, which is assumed to be the numeraire.

Equation (2) implies that for any given value of δ_t, an increase in the level of unemployment benefits or wages in the urban sector will, by increasing the expected income of labor leaving the rural sector, lower employment in the rural sector. Similarly, a decline in δ_t, will create an outflow from the rural sector and lower employment.2/

Urban (manufacturing) sector

Worker effort in the urban sector is endogenously determined by an efficiency-wage mechanism. Following Phelps (1992) and Chadha et al. (1993), we assume that worker effort is an

2/ Assuming--as will be evident below is always the case--that wages in the urban sector always exceed the level of unemployment benefits.

increasing concave function of the differential of the real wage in the sector over the level of unemployment benefits, and of the aggregate level of unemployment.3/ Output is produced by a Cobb-Douglas production function with diminishing returns to labor measured in efficiency units

$$Q_{2t} = H_2[E(W_{2t} - B, U_t)\cdot L_{2t}]^{\alpha}K_t^{1-\alpha} \quad , \quad \text{where } 0 < \alpha < 1, \quad E(0, U_t) < 0 \text{ for all } U_t \quad , \tag{3}$$

where H_2 is a constant. Labor measured in efficiency units, or the effective labor input, $[E(W_{2t}-B,U_t)\cdot L_{2t}]$ is defined as the product of the effort of an individual worker, $E(W_{2t}-B,U_t)$, and the number of workers employed, L_{2t}. For simplicity we assume that the effort function is separable in the two arguments.

The firm's first order conditions for profit maximization can be solved for wages and employment as functions of the level of unemployment in the economy, which the representative firm treats as exogenous to its actions, and the stock of capital, which is predetermined at a point in time. Wages are determined by the well known condition in efficiency-wage models that the elasticity of effort with respect to the real wage is unity

$$\frac{\partial E(W_{2t} - B, U_t)}{\partial W_{2t}} \cdot \frac{W_{2t}}{E(W_{2t} - B, U_t)} = 1, \quad \text{so that} \quad W_{2t} = W_2(U) \quad , \tag{4}$$

where the sign underneath the argument in the W_2 function indicates the sign of the partial derivative.4/ Equation (4) implies that for any given level of unemployment benefits, an increase in the aggregate unemployment rate leads the firm to offer a lower wage to workers. This effect

3/ For a recent survey and overview of efficiency-wage models see Weiss (1990). Among the references cited there, for motivations of our specification see, in particular, Shapiro and Stiglitz (1984) and Calvo (1979).

4/ Since effort is an increasing function of the differential of the wage paid over unemployment benefits and effort is assumed to be negative if wages were to approach zero, it follows that the wage offered will always be greater than the level of unemployment benefits.

Since the urban wage is decreasing in unemployment, the first term, i.e. the term in round brackets, will be largest when employment in the rural sector is high, and when unemployment is low, so that the level of unemployment does not exert much downward pressure on the urban wage. The expression is likely to be positive, therefore, and the UK curve upward sloping, when the initial level of employment in the rural sector is high and the level of unemployment low.

Appendix 2

Local Stability

Linearizing equations (18) and (19) around any steady state, the Jacobian is

$$[J] = \begin{bmatrix} \partial\dot{q}/\partial q & \partial\dot{q}/\partial K \\ \partial\dot{K}/\partial q & \partial\dot{K}/\partial K \end{bmatrix} = \begin{bmatrix} r^* & -\theta(\partial G/\partial U)(\partial U/\partial K) \\ \partial\psi/\partial q & 0 \end{bmatrix} \tag{A4}$$

Denoting the roots of [J] by λ_i it follows that

$$\lambda_1 + \lambda_2 = r^* > 0 \quad , \tag{A5}$$

$$\lambda_1\cdot\lambda_2 = \left(\frac{\partial\psi}{\partial q}\right)\theta\left(\frac{\partial G}{\partial U}\right)\left(\frac{\partial U}{\partial K}\right) \gtrless 0 \quad \text{depending on} \quad \frac{\partial U}{\partial K} \gtrless 0 \tag{A6}$$

It follows immediately that when the linearization is taken around $\bar{K}$, where unemployment is a decreasing function of the urban capital stock, the sum of the roots is positive while the product is negative. Therefore, one root is positive while the other is negative so the system is saddlepath stable around $\bar{K}$. Around $\underline{K}$ on the other hand, when unemployment is an increasing function of the urban capital stock, the sum and product of the roots are both positive. It follows that both roots are either real and positive, or are imaginary with a positive real part. In either case, $\underline{K}$ is locally unstable.

Appendix 1

This appendix establishes the shape of the UK curve defined by equation (9) in the text.

Differentiating equation (9)

$$\frac{dU_t}{dK_t} = \frac{-\left[\frac{\partial L_{1t}}{\partial K_t} + \frac{\partial L_{2t}}{\partial K_t}\right]}{\left[\frac{\partial L_{1t}}{\partial U_t} + \frac{\partial L_{2t}}{\partial U_t} + 1\right]} \tag{A1}$$

The denominator is always positive. Differentiating equation (4), the numerator in (A1) is

$$\left[\delta_t(W_{2t} - B) - P(1 - L_{1t})(-F''(L_{1t}))\right] \frac{i}{P} \quad (1 - L_{1t}) \frac{1}{(-F''(L_{1t}))} (\partial L_{2t}/\partial K_t) \tag{A2}$$

where $F''(L_{1t})$ denotes the second derivative of the rural sector production function. Since the terms outside the square brackets are all positive, the sign of this expression will be determined by the sign of the expression in square brackets, so that

$$\text{Sign of } \left(\frac{dU_t}{dK_t}\right) = \text{Sign of } \left[\left(W_2(U_t) - \frac{P\beta}{(L_{1t})^{1-\beta}}\right) - \frac{P(1 - L_{1t})\ \beta(1 - \beta)}{(L_{1t})^{2-\beta}}\right] \tag{A3}$$

the difference of two positive terms.

Note several features of the expression in (A3). First, as L_{1t} becomes small, the absolute value of the second term increases and the expression must, at some value of L_{1t}, become negative. In fact, as L_{1t} approaches zero the expression approaches negative infinity. Since L_{1t} is a decreasing function of K_t, it follows that as K increases, the slope of the UK curve eventually becomes negative. Second, for small values of K the larger the size of the rural sector, and the smaller the unemployment rate, the more likely it is that the expression in (A3) will be positive. Note that when L_{1t} is large the second term is small. In fact as L_{1t} approaches unity the second term approaches zero. The magnitude of the first term depends also on the level of unemployment.

corresponds to standard models of the Phillips curve. Here, however, the effect results from the assumption that an increase in the aggregate unemployment rate raises each worker's effort at the existing wage. This allows the firm to lower the wage offered and still obtain the same effort level from workers.

The profit-maximizing level of employment in the urban sector can be expressed as a function of the wage, and substituting in equation (4), is

$$L_{2t} = \frac{\left[\alpha\ H_2\right]^{\frac{1}{1-\alpha}} \left[\ E(W_{2t} - B,\ U_t\ )\right]^{\frac{\alpha}{1-\alpha}} K_t}{\left[W_{2t}\right]^{\frac{1}{1-\alpha}}} = L_2[W_2(U_t),\ U_t\ ,\ K_t\ ] = L_2(U_t\ ,\ K_t\ ) \tag{5}$$

Urban employment is, therefore, an increasing function of unemployment and the urban capital stock. The former effect results from the fact that an increase in unemployment depresses wages, leading to a higher profit-maximizing level of employment at any given capital stock.

Equilibrium Unemployment

To keep the analysis tractable we assume, in the tradition of Harris and Todaro (1970) that the urban sector randomly selects the desired number of workers from the pool of workers who are not employed in the rural sector. 5/ The probability that a worker who leaves the rural sector is employed in the urban sector is defined by

$$1 - \delta_t = \frac{L_{2t}}{U_t + L_{2t}} \tag{6}$$

5/ An alternative approach is to allow for job tenure and define the probability of an unemployed worker obtaining a job in the urban sector as developed in the "flows" approach to labor markets as, for example, presented in Blanchard and Diamond (1992). Unlike the flows approach where employment is a predetermined variable, under our assumptions, employment can be adjusted discretely. However, it can be shown that along an equilibrium path, i.e. where the urban capital stock adjusts smoothly, our approach is consistent with the flows approach. See Chadha et al (1993).

We impose that in general equilibrium the perceived probabilities are equal to their actual values. Substituting the definition in equation (6) into the equilibrium condition in equation (2), equilibrium employment in the rural sector can be expressed as

$$L_{1t} = L_{1t}\,[U_t\,,\ W_2\,(U_t)\,;\ L_2\,(U_t\,,\ K_t\,)\,;\ P] \qquad (7)$$

Rural sector employment is thus a decreasing function of the urban sector capital stock, but appears to be an ambiguous function of the level of unemployment. Unemployment has three effects on rural sector employment. Unemployment directly increases rural sector employment by increasing δ_t which deters migration from the sector. Similarly, increased unemployment depresses urban wages, deterring migration from the rural sector. As urban employment expands with lower wages, however, job opportunities in the urban sector expand, which lowers employment in the rural sector. Under certain conditions6/--which we assume to hold here--the first two effects always dominate the third and employment in the rural sector is an increasing function of the unemployment rate. Rural sector employment can, therefore, be expressed as

$$L_{1t} = L_1(U_t\,,\ K_t\,;\ P\,) \qquad (8)$$

General equilibrium in the economy at each point in time is defined by the identity

6/ The condition is that the elasticity of worker effort in the urban sector with respect to unemployment be less than a positive constant, that is

$$\frac{\partial E_t}{\partial U_t}\,\frac{U_t}{E_t} < \frac{1-\alpha}{\alpha}$$

Rural sector employment is then an unambiguous increasing function of the unemployment rate. This condition is not necessary. It is, however, sufficient. For a detailed discussion see Chadha et al. (1993).

Romer, Paul M., (1986), "Increasing Returns and Long-Run Growth", Journal of Political Economy, October, 94, 1002-38.

Shapiro, C. and J.E. Stiglitz (1984), "Equilibrium Unemployment as a Worker Discipline Device", American Economic Review, Vol. 74, no. 3, pp. 433-444.

Velasco, Andres, (1992), "Animal Spirits, Capital Repatriation, and Investment", C.V. Starr Center for Applied Economics, Research Report No. 92-43, Department of Economics, New York University, New York.

Weiss, Andrew (1990), Efficiency Wages: Models of Unemployment, Layoffs, and Wage Dispersion, Princeton University Press, Princeton, New Jersey.

References

Aghion, Philippe and Olivier Blanchard, (1993), "On the Speed of Transition in Eastern Europe", mimmeo., March.

Azariadis, Costas and Allan Drazen, (1990), "Threshold Externalities in Economic Development", Quarterly Journal of Economics, 105, May, pp. 501-526.

Blanchard, Olivier J. and Peter Diamond (1992), "The Flow Approach to Labor Markets", American Economic Association Papers and Proceedings, Volume 82, No. 2, pp. 354-359, May.

Blanchard, Olivier J. and Stanley Fischer (1989), Lectures on Macroeconomics, MIT press, Massachusetts.

Calvo, Guillermo A. (1978), "Urban Unemployment and Wage Determination in LDCs: Trade Unions in the Harris-Todaro Model", International Economic Review, Vol. 19, no. 1, pp. 65-81.

Calvo, Guillermo A. (1979), "Quasi-Walrasian Theories of Unemployment", American Economic Review: Papers and Proceedings, Vol. 69, no. 2, pp. 102-107.

Chadha, Bankim, Fabrizio Coricelli and Kornélia Krajnyák (1993), "Economic Restructuring, Unemployment, and Growth In a Transition Economy", International Monetary Fund Staff Papers, Vol. 40, no. 4, pp. 744-780.

Gavin, Michael, (1993), "Unemployment and the Economics of Gradualist Policy Reform", mimeo., Columbia University, January.

Harris, J.R. and M.P. Todaro, (1970), "Migration, Unemployment and Development: A Two Sector Analysis", American Economic Review, 60, March, pp. 126-142.

Krugman, Paul, (1991), "History Versus Expectations", Quarterly Journal of Economics, 106, May, pp. 651-667.

Lucas, Robert E., Jr. (1988), "On the Mechanics of Economic Development", Journal of Monetary Economics, Vol. 22, pp. 3-42.

Matsuyama, Kiminori, (1991), "Increasing Returns, Industrialization, and Indeterminacy of Equilibrium", Quarterly Journal of Economics, 106, May, pp. 617-650.

Murphy, Kevin M., Andrei Shleifer and Robert Vishny, (1989), "Industrialization and the Big Push", Journal of Political Economy, 97, October, 1003-26.

Phelps, Edmund, (1992), "Consumer Demand and Equilibrium Unemployment in a Working Model of the Customer-Market Incentive Wage Model", Quarterly Journal of Economics, 107, August, 1003-1032.

$$L_1(U_t, K_t, P) + L_2(U_t, K_t) + U_t = 1 \qquad (9)$$

which yields a solution for the equilibrium level of unemployment as a function of the urban sector capital stock.

How does unemployment evolve with increases in the urban capital stock? Note that the left hand side of equation (9) is an increasing function of unemployment. It is, however, an ambiguous function of the urban capital stock. On the one hand, an increase in the urban capital stock creates jobs in the urban sector, reducing unemployment. On the other hand expanded job opportunities in the urban sector create an outflow of labor from the rural sector, increasing unemployment. Unemployment is, in general, therefore, an ambiguous function of the urban capital stock. Appendix 1 shows that if the initial condition of the economy prior to the development of an urban sector is characterized by a large share of employment in the rural sector and low unemployment, then equation (9), referred to hereafter as the UK curve, will be hump-shaped. If the rural sector is viewed as primarily a subsistence sector which can, in principle, support the entire labor force, albeit at a relatively low wage, then this appears to be a reasonable initial condition and this is the case we focus on in the remainder of the paper. Figure 1 plots such a hump-shaped UK curve with unemployment rising as the urban sector capital stock increases and eventually declining as the capital stock continues to increase. It is straightforward to note from equation (9) that with increases in K along UK, while unemployment behaves nonmonotonically, the rural sector declines

monotonically and the urban sector expands monotonically.7/ It is possible to show that δ_t declines monotonically as the economy moves rightward along the UK curve.8/

2. Investment in the Urban (Industrial) Sector

The Marginal Product of Capital

It is useful to begin by considering the behavior of the marginal product of capital. Differentiating the urban production function in equation (3), the marginal product of capital is

$$\frac{\partial Q_{2t}}{\partial K_t} = (1-\alpha)H_2^{\alpha}[E(W_{2t} - B, U_t)\cdot L_{2t}]^{\alpha}K_t^{-\alpha} \qquad (10)$$

Substituting in the firm's employment rule from equation (5), the marginal product of capital at any point in time, that is for any level of the capital stock given by history, can be expressed simply as a function of the aggregate unemployment rate as

$$\left.\frac{\partial Q_{2t}}{\partial K_t}\right|_{L_{2t}=L_{2t}^*} = \left[(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}H_2^{\frac{1}{1-\alpha}}\right]\left|\frac{E(W_2(U_t)-B,U_t)}{W_2(U_t)}\right|^{\frac{\alpha}{1-\alpha}} = \theta\cdot G(U_t) \qquad (11)$$

where θ is a constant. It is straightforward to show that

$$\frac{\partial G}{\partial U} = \theta\frac{\alpha}{1-\alpha}\left|\frac{E(W_{2t}-B,U_t)}{W_2}\right|^{\frac{\alpha}{1-\alpha}}\frac{1}{E_t}\left(\frac{\partial E}{\partial U_t}\right) > 0, \quad \text{and so} \quad \frac{\partial Q_{2t}}{\partial K_t} = \theta G(U_t) \qquad (12)$$

7/ To the left of the peak unemployment rate, since the urban capital stock and unemployment are both rising, urban employment must be expanding. Since urban employment and unemployment are both increasing, rural employment must be declining. To the right of the peak unemployment rate, since the urban capital stock is rising and unemployment is falling, rural employment must be declining. Since rural employment and unemployment are both declining, urban employment must be rising.

8/ On the left hand side of the peak unemployment rate, note that

$$(dL_{1t}) = (\partial L_{1t}/\partial\delta)d\delta + (\partial L_{1t}/\partial W_{2t})(dW_2)$$

Therefore, $d\delta$ must be negative. On the right hand side of the peak unemployment rate, U_t is falling, while $(1-L_t^1)$ is rising, therefore δ_t which equals $U_t/(1-L_t^1)$ must continue to fall.

IV. Concluding Remarks

This paper has identified a mechanism based on the behavior of unemployment, which initially rises and subsequently falls, during the sectoral reallocation process, and the interaction of investment with unemployment that can lead to divergent long-run outcomes. In particular, there is a threshold initial capital stock in the emerging sector below which sectoral reallocation will not occur. If the initial capital stock is high enough, there is a unique, convergent path by which sectoral reallocation will occur. Between these two extremes there is a region of indeterminacy where expectations can play a pivotal role in determining the long run outcome. The specific model presented was a stylized model of development of an economy dominated initially by a rural-agricultural sector to one dominated eventually by an urban-industrial sector. The mechanism identified is not, however, limited to the development context. The basic forces of the mechanism appear relevant in any sectoral reallocation process and could give rise to divergent outcomes in a wide variety of other contexts. These include: sectoral reallocation of factors due to shifts in demand or changes in technology; the massive reallocation of labor from state-owned to newly created private firms that is taking place in the countries of Eastern Europe and the former Soviet Union today;21/ and sectoral reallocation of factors induced by trade reforms in developing countries.22/

21/ See Aghion and Blanchard (1993).

22/ Gavin (1993) examines the implications of trade reform for unemployment.

place the economy onto a path where urban-industrial development is self-sustaining. In the absence of external grants, the task for policy in placing it on a self-sustaining path of urban-industrial development is a formidable one. If policy succeeded in shrinking the development trap region such that the economy could break out of this region, it would find itself in a region of indeterminacy. While self-sustaining development would then become possible, it would not necessarily occur. Policy would then need to engineer a coordination of private sector expectations onto an optimistic path.

There are two sets of policy actions that can shrink the development trap region. The first set of policy actions operates on the labor market. By increasing unemployment, depressing wages and increasing the return to capital, policy can increase the set of initial conditions which lead to successful development of the urban-industrial sector. A fundamental dilemma of labor market policies in the present context is created by the fact that increases in unemployment, though costly to the economy, are necessary for successful development of the urban-industrial sector. The second set of policy actions are those that directly impact the returns from accumulating capital, such as a capital subsidy.

Both policies that create unemployment and those that subsidize capital in the urban-industrial sector entail budgetary costs. A balanced-budget policy intended to break the economy out of the development trap region would require taxing the rural-agricultural sector to pay for unemployment benefits and/or capital subsidies to the urban-industrial sector. Technical parameters will determine whether a balanced budget path exists such that the economy can break out of the development trap region. If a balanced-budget path does not exist, policies to break out of this region will require deficit financing. If there are constraints on the ability of the government to borrow, or other concerns which constrain the magnitude of the deficits it can run, then such constraints may preclude policies to break out of the development trap region.

The marginal product of capital is, in equilibrium, therefore, an increasing function of unemployment. Higher unemployment lowers wages in the urban sector. For a given capital stock, lower wages imply a higher level of employment. Increased employment, in turn, increases the marginal productivity of capital. Equation (12) implies that in equilibrium the marginal product of capital for any individual firm, and hence for the economy in the aggregate, will qualitatively mirror the UK curve in figure 1. At low levels of urban capital, as unemployment rises with increases in the capital stock, the marginal product of capital will rise. The economy, therefore, exhibits increasing returns to capital in this region. Once the urban capital stock exceeds the level corresponding to the peak unemployment rate in figure 1, increases in the urban capital stock lower unemployment, and the marginal product of capital will diminish. The relationship between the marginal product of capital, unemployment, and the level of capital can be summarized as

$$\frac{\partial Q_{2t}}{\partial K_t} = \theta \cdot G(U(K_t)) \qquad (13)$$
$$\cdot \ \cap$$

where $\cap$ is used to denote the hump-shaped behavior of unemployment as a function of the capital stock.

We turn now to an examination of the dynamics of the capital stock. We first discuss an ad-hoc model where investment is determined by the current rate of return on capital. We then examine the dynamic behavior of the economy when investment decisions are based on the entire future stream of returns to capital.

Investment based on current rates of return

It is assumed that for a small open economy net investment at home is an increasing function of the discrepancy between the actual rate of return to capital, r_t, and the exogenously given rate of return, r_t^*, in the international capital market

$$K_t = \gamma\left|r_t - r^*\right|, \qquad \text{for } K_t \geq 0 \tag{14}$$

Assuming a competitive market for capital, the rate of return (rental) on capital at home is given at any point in time by the marginal product of capital in the urban sector. Substituting equation (12) into equation (14), the dynamics of the capital stock can be represented as a function of unemployment as

$$K_t = \gamma\left|\theta \cdot G(U_t) - r^*\right| \tag{15}$$

Equation (15), along with the labor market equilibrium condition in equation (9), that is the UK curve which defines the equilibrium level of unemployment at each capital stock, then completely describe the dynamics of the economy. Note that equation (15) implies a critical unemployment level, say U_c, at which the capital stock is stationary. If unemployment exceeds U_c the return on capital exceeds the world interest rate and capital will be accumulated. When unemployment is below U_c, capital will be decumulated. Depending on the technological parameters determining the marginal productivity of capital and the level of the world real interest rate, there are then three possible configurations for the phase diagram. These three cases are depicted in figure 2. Single arrows are used to denote the forces of motion in the system when the economy is off the $\dot{K} = 0$ locus. Double arrows are used to denote the actual path of the economy.

The dynamic path of the capital stock is trivial to characterize in the cases represented in panels 2a and 2b. In panel 2a where the $K = 0$ locus always exceeds the UK curve, the world real interest rate always exceeds the rate of return on capital at home. For any initial capital stock given by history, the urban (industrial) sector will decumulate capital until it disappears and the economy is specialized in the production of the rural sector good. In panel 2b the $K = 0$ locus intersects the UK curve uniquely. $\bar{K}$ then represents the unique stable long-run equilibrium capital stock.

An increase in the internationally given relative price of the urban-industrial sector's good, by increasing unemployment, and raising the return to capital can thus shrink the development trap region, and increase the set of initial conditions for which the urban-industrial sector can develop. In addition, it lowers the initial capital stocks comprising the region of indeterminacy, thus increasing the set of initial conditions for which self-sustaining urban-industrial sector development will occur.

Analogous to changes in the external terms of trade, P, changes in the internal terms of trade brought about by taxing rural-agricultural output, for example, would shift the UK curve up. So would wage or employment taxes and declines in multifactor productivity in the rural-agricultural sector.

An urban output or employment subsidy has slightly different effects. It can be shown that such subsidies will shift the UK curve to the left in figure 1, maintaining the vertical intercept. While the subsidy leads to an expansion of employment in the urban sector, lowering unemployment, the expanded job opportunities encourage migration from the rural sector, raising unemployment. To the left of the peak unemployment rate, unemployment rises and to the right it falls. Output or wage subsidies to the urban sector will, therefore, lead to a decline in $\underline{K}$ and $\bar{K}$.

Consider a subsidy to holding capital. The flow return to the firm from each unit of capital it owns is then the marginal product of capital plus the flow subsidy from the government. It should be evident from equation (19) that the effects of an increase in the subsidy to capital would be analogous to that of a decline in P discussed above which raised the return on capital at each capital stock. The development trap region will, therefore, shrink and the region of indeterminacy will shift to the left.

For a poor economy with a low initial capital stock that falls in the development trap region, the analysis presents a strong case for external assistance tied to developing the urban-industrial capital stock. Direct grants of physical capital from abroad, by changing the initial condition, can

migration to the urban sector, 18/ and increases unemployment. In terms of figure 3, an increase in unemployment raises the return to capital at each capital stock and implies from equation (19) that the $\dot{q} = 0$ locus will shift up. The $\dot{K} = 0$ locus is unchanged and $\underline{K}$ falls while $\bar{K}$ rises. These changes are depicted in figure 4 which is drawn for an initial configuration of the economy given by figure 3b.

What are the implications for the four regions? Consider the original path ABSP prior to the decline in P. For the range of capital stocks spanned by the segment BSP, the marginal product of capital is higher as a consequence of the decline in P. At the new steady state the marginal product is unchanged. Therefore, q, the present value of the future marginal products of capital must be higher for each capital stock spanned by the segment BSP, and the segment BSP must shift up. The forces of motion of the system then imply that the new path must intersect the unchanged $\dot{K} = 0$ locus at a point to the left of B such as B'(and the new $\dot{q} = 0$ locus at a point such as A'19/). The development trap region, therefore, shrinks.

Consider now the original ray OO. Following the increase in the relative price of the urban sector's good, the marginal product of capital is higher at each capital stock in the region 0 to $\bar{K}$'. q must, therefore, be higher. It follows that the point of intersection of the new locus, OO', with the unchanged $\dot{K} = 0$ locus, at a point such as N, must occur at a lower capital stock than previously. The region of indeterminacy thus shifts to the left.20/

18/ See equation (2).

19/ The fact that even though the marginal product of capital is higher at each capital stock but a portion of the segment A'B' lies below the original path results from the fact that on the new path the capital stock declines by more. In this region, that is to the left of the peak in the $\dot{q} = 0$ locus, the marginal product of capital declines with decreases in the capital stock. Therefore, q can be lower.

20/ We are not asserting that the entire ray OO shifts to the left. Only the portion below the $\dot{K} = 0$ locus. What happens above the $\dot{K} = 0$ locus depends on whether the OO ray ever crosses over the capital stock corresponding to the peak unemployment rate.

Starting from any initial capital stock, the economy will converge to $\bar{K}$ at which point the return on capital equals the world real interest rate.

In panel 3c of figure 3 the $\dot{K} = 0$ locus intersects the UK curve twice. $\bar{K}$ again represents a stable long run equilibrium. However, the economy will only naturally attain this long-run equilibrium if the initial capital stock exceeds $\underline{K}$. If the initial capital stock lies below $\underline{K}$ the economy will decumulate capital until the urban capital stock falls to zero. Note further that as capital is decumulated in this region, the return on capital falls, so that capital decumulation reinforces itself. The capital stock $\underline{K}$ corresponds to popular notions of "bottlenecks" in that it represents a threshold level of capital. As long as the level of capital falls short of this level the urban (industrial) sector will not develop. Were the initial endowment of capital somehow raised above $\underline{K}$ the economy would naturally traverse along the UK curve in a self-sustaining manner and converge to the high capital equilibrium. The role of policy in engineering such an outcome is discussed in the next section.

Among the three cases depicted in figure 2, figures 2a and 2b should really be viewed as special cases of figure 2c. Were the economy characterized by figures 2a or 2b, a shock such as a change in the world real interest rate or a change in the terms of trade could easily result in a re-characterization of the economy as in figure 2c. This is, therefore, the case we focus on in the next subsection where investment is determined by the present value of the future stream of returns on capital.

Investment based on present value of future returns

To focus on the question of immediate interest we assume that firms in the urban sector own their capital stock and finance investment through retained earnings. It is further assumed that

firms discount the future at the world real interest rate.9/ The representative firm in the urban sector maximizes at each point in time the present discounted value of profits (cash flows), that is

$$V_0 = \int_0^{\infty} [Q_{2t} - I_t - C(I_t) - W_{2t}L_{2t}]\exp(-r^*t)dt \quad , \tag{16}$$

subject to $\dot{K}_t = I_t$, (17)

and an initial capital stock given by history. Q_{2t} is defined in equation (3), I_t denotes investment and $C(I_t)$ is a convex function of investment that represents the installation costs of capital. It follows immediately that as long as there are no adjustment costs associated with labor, the static first order conditions for profit maximization with respect to wages and employment presented in section II.1 above continue to apply. As is well known10/, the dynamic conditions for a maximum are

$$q_t = 1 + C'(I_t) \quad , \quad \text{or} \quad \dot{K}_t = \psi(q_t - 1) \quad \text{where } \psi' > 0 \quad , \tag{18}$$

$$\text{and} \quad q_t = r^* q_t - \frac{\partial Q_{2t}}{\partial K_t} = r^* q_t - \theta \cdot G(U(K_t)) \tag{19}$$

q_t is the shadow price of a unit of installed capital.11/ Equation (19) can be integrated forward to show that q_t represents the present value of the future stream of marginal products of a unit of

9/ Explicitly allowing firms to borrow and lend on the international capital market to finance investment would not alter the basic predictions of the model derived below.

10/ See, for example, Blanchard and Fischer (1989).

11/ In deriving the last part of equation (19), equation (13) was used to substitute for the marginal product of capital.

Figure 3b presents the case of real roots around $\underline{K}$. Since the roots are real, the system will not exhibit cycles around $\underline{K}$ as in figure 3a. Figure 3b shows, however, that the nonuniqueness of paths in region II can still occur. Except for the absence of oscillations, therefore, the classification of regions is exactly as in figure 3a.

While we have characterized the existence of four regions, in general, we note that in special cases regions I or II may not exist. In figure 3a, for example, it is possible that the left outermost spiral passes through the vertical axis at $q=1$, so that $\underline{K}'$ is zero. Region I will, therefore, not exist. Similarly, as another example consider the possibility in figure 3b that there is a unique trajectory that emanates from a point where $K=\underline{K}$ and q slightly greater than 1 that places the economy eventually on SP. In this case region II will not exist.

III. The Role of Policies in Affecting Development

We first examine the effects of once-and-for-all changes in exogenous parameters and policy variables and then discuss the general role of policies and external assistance in bringing about the development of the urban-industrial sector. There are, in principle, a large number of experiments that are relevant: changes in the terms of trade, output tax-cum-subsidies, employment and wage taxes and subsides, changes in unemployment benefits, productivity or technology shocks, investment and capital subsidies, changes in the world real interest rate, etc. For reasons of space we shall describe only one of these in detail and simply mention the effects of some others.17/

Consider first an increase in the internationally given relative price of the urban sector's good. From equation (9) note that such an increase, which corresponds to a decline in P, would raise unemployment at each capital stock, that is shift the UK curve up in figure 1. An increase in the relative price of the urban sector's good lowers the real wage in the rural sector encouraging

17/ A fuller description of some of these experiments can be found in Chadha et al. (1993).

The simultaneous existence of equilibrium paths that converge to a zero urban capital stock with paths that converge to a positive capital stock, $\bar{K}$, imply that there is an indeterminacy as to which long-run equilibrium the economy will converge to. If expectations were somehow coordinated along an optimistic path such as the spiral emanating at A, capital would be accumulated, the urban sector would develop and the economy would converge in the long run to $\bar{K}$. If expectations were uniformly pessimistic as on the ray OO, the economy would decumulate capital and the urban sector would eventually disappear. Appendix 2 shows that the present value of the firm at a steady state level of $\bar{K}$, is just $\bar{K}$, and exceeds the present value of a firm at the origin, which is zero. Firms are, therefore, better off at $\bar{K}$ than at the origin. Even though firms are better off at $\bar{K}$, there is nothing in the system to ensure that the economy will pick an initial condition for q such that $\bar{K}$ is attained. In this region expectations are the key to determining the long-run equilibrium.16/

Region III in figure 3a can be considered a special case of region II. Rather than a multiplicity of paths that place the economy on SP, however, there is now a unique path that would place the economy eventually on SP. A notable feature of this equilibrium trajectory is that in the early stages of this region q is rising--along ES in figure 3a--so that investment must be accelerating. The ray OO continues to represent an equilibrium path that cannot be ruled out.

Finally, if the initial capital stock lies in region IV, then the saddle path, SP, represents the unique convergent equilibrium path. All other paths can be ruled out in that they will eventually violate either the transversality condition or the arbitrage relation for q. The optimal path is, therefore, the saddle path and for any capital stock the economy will traverse the saddlepath converging to $\bar{K}$ in the long run.

16/ The models developed by Matsuyama (1991), Krugman (1991) and Velasco (1992) display similar regions where the long-run outcome is, in principle, indeterminate.

capital. In addition to equations (18) and (19), for a maximum, the transversality condition must hold, that is

$$\lim_{t\to\infty} K_t q_t \exp(-r^* t) = 0 \qquad (20)$$

To characterize the phase diagram in the (K, q) plane, note from equation (18) that the $\dot{K} = 0$ locus is a horizontal line at $q = 1$. For values of q above 1, the capital stock is increasing and vice versa for values of q below 1. Equation (19) shows that the $\dot{q} = 0$ locus will be hump-shaped because the marginal product of capital is an increasing function of unemployment and unemployment is a hump-shaped function of the capital stock. For values of q above the $\dot{q} = 0$ locus, q is increasing and vice versa for values of q below the locus.

There are again three configurations of the phase diagram that correspond closely to the three cases presented in figure 2. First, the $\dot{K} = 0$ locus could lie above the $\dot{q} = 0$ locus for all values of K. Second, the two loci could intersect uniquely. Finally, the loci could intersect twice. For reasons noted earlier, the first two cases can be thought of as special cases of the third configuration. We, therefore, focus on the third case which is depicted in figure 3. The capital stocks corresponding to the two steady states defined by the intersections of the $\dot{q} = 0$ and $\dot{K} = 0$ loci are denoted $\underline{K}$ and $\bar{K}$.

Appendix 2 establishes that around the steady state corresponding to $\underline{K}$ the dynamical system has either two positive roots or two imaginary roots with positive real parts, so that the system is locally unstable. Around the steady state corresponding to $\bar{K}$, the dynamical system has one stable root and one unstable root, so that in this vicinity the system is saddlepath stable.12/ We now

12/ Given the existence of multiple steady states it would, of course, be ideal to carry out a global analysis of the dynamical system. As is well known, however, global analysis is not a straightforward matter. See Matsuyama (1991).

proceed to provide a taxonomy of possible equilibrium trajectories for ranges of initial capital stocks based on the forces of motion of the system when the economy is off the $q = 0$ and $K = 0$ loci. These forces can be qualitatively identified without requiring linear approximations. We argue that there exist, in general, four regions into which initial capital stocks can be classified.

To establish the taxonomy of regions some observations on terminal conditions are useful. Appendix 2 shows that any explosive trajectory on which q and K rise without bound can be ruled out as an equilibrium path as it would violate the transversality condition. Similarly, any implosive path that leads eventually to either one of q or K equalling zero while the other is positive13/, would violate the arbitrage relation in equation (19) that q equal the present value of the stream of future marginal products of a unit of capital. Such implosive paths can, therefore, also be ruled out.14/ An implosive path where q and K monotonically converge to zero, however, does not violate the transversality condition or the arbitrage relation for q, and is a potential equilibrium path.

Since the steady state defined by $K=\underline{K}$ and $q=1$ would be attained only by fluke and is unstable in that and any small perturbation would result in the economy permanently moving away from it, we do not dwell on its existence as a potential long-run equilibrium. Since it is an unstable equilibrium, it is natural to consider an unstable path that emanates in the vicinity of $K=\underline{K}$ and $q=1$ such that once sufficient capital is accumulated, the economy can attain exactly the saddlepath passing through $\bar{K}$. Figure 3a depicts such a path for the case where the roots of the system around $\underline{K}$ are imaginary. The path originates at point A. Because the roots are imaginary with positive real parts, the solution can then display explosive oscillations, spiralling outward and hitting exactly the saddlepath, hereafter denoted as the curve SP, which passes through $K=\bar{K}$ and $q=1$, such that

13/ That is a path in the (q, K) plane that hits either the horizontal or the vertical axis.
14/ As can any path that entails an anticipated jump in q.

long-run equilibrium occurs at $\bar{K}$. This path then suggests a classification of initial capital stocks into four regions as is done in figure 3a: I, II, III, and IV.

In region I, for initial capital stocks between 0 and $\underline{K}'$, which is the capital stock corresponding to the left intersection of the outermost spiral of the trajectory emanating at A with the $K = 0$ locus, there are no continuous paths by which the economy can attain $\bar{K}$ in the long run. In this region the unique equilibrium trajectory is given by the portion of the ray OO in the region. The economy will thus decumulate capital until the urban sector disappears. $\underline{K}'$ thus represents a threshhold capital stock. An economy with a historically poorly developed urban (industrial) sector will be trapped in this region.

Region II in figure 3a is comprised of initial capital stocks demarcated by the left and right intersections of the outermost spiral emanating from point A with the $\dot{K} = 0$ locus. To determine possible paths for initial capital stocks in this region, several observations are in order. First, note that for any initial capital stock in this region there exists a path that can lead the economy to $\bar{K}$. Second, while such a path exists for each capital stock, there is a multiplicity of such paths. In figure 3a, for an initial capital stock of $\underline{K}$, any of the values of q corresponding to points A, B, C or D would place the economy eventually on SP. There is thus a multiplicity of equilibrium paths consistent with eventual convergence to $\bar{K}$. It is worth emphasizing that each of the values of q corresponding to A, B, C or D is perfectly consistent with the fundamentals and is self-fulfilling. Clearly the initial condition of D, which represents the most optimistic expectation of the present value of future marginal products of capital, implies fastest convergence to $\bar{K}$. Third, note that the ray OO, along which q and K both monotonically converge to zero, and the economy eventually specializes in the rural (agricultural) sector good, also represents a possible equilibrium path in this region.15/

15/ The ray OO, will also likely spiral around $q=1$ and $K=\underline{K}$, so there are in fact also multiple paths leading to the origin.