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EARLY OPTION EXERCISE: NEVER SAY NEVER†

Abstract

A classic result by Merton (1973) is that, except just before expiration or dividend payments, one should never exercise a call option and never convert a convertible bond. We show theoretically that this result is overturned when investors face frictions. Early option exercise can be optimal when it reduces short-sale costs, transaction costs, or funding costs. We provide consistent empirical evidence, documenting billions of dollars of early exercise for options and convertible bonds using unique data on actual exercise decisions and frictions. Our model can explain as much as 98% of early exercises by market makers and 67% by customers.

JEL Classification: G11, G12, G13 and G14
Keywords: convertible bonds, derivatives pricing, frictions, option exercise, short-sale costs and transaction costs

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I. Introduction: Never Exercise a Call and Never Convert a Convertible?

One of the classic laws of financial economics is that equity call options should never be exercised, except at expiration or just before dividend payments (Merton, 1973) and, similarly, convertible bonds should not be converted early (Brennan and Schwartz (1977), Ingersoll (1977a)). Stock lending fees are similar to dividends and can therefore give rise to early exercise as is well understood.[1] The fact that the financial friction of lending fees can lead to early exercise raises several broader questions: Which financial frictions lead to early exercise? When should we expect to observe friction-driven early exercise? Do customers of brokers, market makers, and other investors actually exercise early? Are actual lending fees and other financial frictions large enough to drive significant early exercise decisions? If so, to what extent are actual early exercise decisions driven by actual financial frictions?

We seek to address these questions theoretically and empirically. First, we show that early exercise can be optimal when agents face short-sale costs, transaction costs, or funding costs and we characterize both a lower and upper bound for the optimal exercise policy under such financial frictions. Second, we show empirically that investors indeed exercise equity options early and convert convertibles when facing these frictions, using unique data on actual exercise and conversion decisions.

To understand our result, first recall the famous arbitrage argument of Merton (1973): Rather than exercising a call option and receive the stock price S less the strike price X, an investor is better off shorting the stock, putting the discounted value of X in the money market, and possibly exercising the option at expiration — or selling the option to another agent who can do so. However, this arbitrage argument can break down when shorting is costly or agents face transaction costs or funding costs.

We introduce these financial frictions in a model. We first show that Merton's no-exercise

[1] Avellaneda and Lipkin (2009) mention that lending fees can in principle lead to early exercise and, more broadly, the point may be common knowledge among option traders and researchers even if we did not find other references.

rule holds even with "mild" frictions, meaning either (i) when short-sale costs and funding costs are small (even if transaction costs are large), or (ii) when transaction costs are small and the option price is above the intrinsic value (which can be driven by other agents facing low shorting and funding costs). However, we show that early exercise in fact *is* optimal when frictions are more severe such that the option price net of transaction costs is below intrinsic value and the option owner faces sufficiently high shorting and/or funding costs.

Finally, we show how the effects of financial frictions can be quantified in a continuous-time model in which the parameters can be directly calibrated to match the data. We show that exercise is justified when the stock price is above a lower exercise boundary, which we derive. The exercise boundary is decreasing in short-sale costs, margin requirement, and funding costs. Said differently, exercise happens earlier (i.e., for lower stock prices) with larger short-sale costs, larger margin requirements, and larger funding costs.

To intuitively understand our model and to illustrate its clear quantitative implications, consider the example of options written on the Ishares Silver Trust stock (the largest early exercise day in our sample of options on non-dividend paying stocks). Figure 1 shows the stock price of Ishares trust and the lower exercise boundary that we derive based on the short-sale cost (or "stock lending fee") and funding costs that we observe in our data. While exercise is never optimal before expiration without frictions, we see that the exercise boundary is finite due to the observed financial frictions. Furthermore, we see investors actually exercise shortly after the stock price crosses our model-implied lower exercise boundary.

This illustrative example provides evidence consistent with our model, but does it reflect broader empirical phenomena? To address this question, we collect and combine several large datasets. For equity options, we merge databases on option prices and transaction costs (OptionMetrics), stock prices and corporate events (CRSP), short-sale costs (Data Explorers), proxies for funding costs, and actual option exercises (from the Options Clearing Corporation). Focusing only on options on non-dividend-paying stocks, we find that 1.8 billion option contracts are exercised early (i.e., before Merton's rule) in the time period from 2003 to 2010, representing a total exercise value of $36.3 billion. (Of course, the amount of exercises before Merton's rule would be larger if we included dividend paying stocks, but for clarity we restrict attention to the most obvious violations.)

Consistent with our theory's qualitative implications, we find that early exercise is more likely when (i) the short-sale costs for the underlying stock are higher, (ii) the option's transaction costs are higher, (iii) the option is more in-the-money, and (iv) the option has shorter time to expiration. These results are highly statistically significant (due to the large amounts of data). Moreover, our data allows us to identify exercises for each of three types of agents, customers, market makers, and proprietary traders. We find that each type of agent exercises options early, including the professional market makers and proprietary traders, and that each type is more likely to do so when frictions are severe, consistent with our theory of rational exercises.

We also test the quantitative implications of the model more directly. For each option that is exercised early, we estimate the lower exercise boundary by solving our model-implied partial differential equation (PDE) based on the observed frictions. We find that 66–84% of all early exercise decisions in our data happen when the stock price is above the model-implied exercise boundary, depending on how input variables are estimated (and even higher if we exclude corporate events). The behavior of market makers is most consistent with our model (their exercise decisions coincide with the model-implied prediction in 86-98% of the cases), while customers of brokers make the most exercise decisions that we cannot explain and proprietary traders are in between, consistent with the idea that market makers are the most sophisticated and face the lowest frictions while customers of brokers face the highest frictions.

Furthermore, using logit and probit regressions, we find that real-world investors are more likely to exercise early when the stock price is above the model-implied exercise boundary. This results consistent with the model is both statistically and economically significant: The estimated probability that an option contract is exercised early, cumulated over a 20-trading-day period where the stock price is above the boundary, is 20.7% (21.8%) based on logit (probit) regressions. The corresponding probability when the stock price is below the boundary is 0.4% (0.4%). The large difference in exercise behavior across these two cases is a success for the model. The numbers also indicate that far from all options are exercised immediately when the stock price goes above our model-implied lower boundary (which is not surprising given that it is a lower bound).

We also entertain alternative potential reasons for early exercise and examine the issue of causality. Indeed, we find some early exercises that are not explained by our estimated model, and, some of the largest of those, are related to corporate events. Therefore, we repeat our analysis in the sub-sample where corporate events are excluded and find similar results. To test for causality, we consider the natural experiment of the short-sale ban of certain stocks in 2008 and conduct a difference-in-differences analysis that supports the idea that short-sale frictions leads to early exercise. Indeed, early exercise rose for options on affected stocks during the ban period relative to unaffected options.

For convertible bonds, we combine data on equities and short-sale costs with the Mergent FISD database on convertible bond features and actual conversions. We find 25.4 million early conversions, representing an equity value of $7.7 billion at conversion. The early conversion rates for convertible bonds is increasing in the short-sale cost of the stock and in the moneyness of the convertible bond, again consistent with our theory, but we note that this data set is smaller and subject to potential errors and inaccuracies.

Our paper complements the large literature following Black and Scholes (1973) and Merton (1973). Option prices have been found to be puzzlingly expensive (Longstaff (1995), Bates (2000), Bates (2003), Jackwerth (2000), Ni (2009), Constantinides, Jackwerth, and Perrakis (2009)) and several papers explain this based on frictions: Option prices are driven by demand pressure (Bollen and Whaley (2004), Garleanu, Pedersen, and Poteshman (2009)), are affected by transaction costs (Brenner, Eldor, and Hauser (2001) and Christoffersen, Goyenko, Jacobs, and Karoui (2011)), short-sale costs (Ofek, Richardson, and Whitelaw (2003), Avellaneda and Lipkin (2009)), funding constraints (Bergman (1995), Santa-Clara and Saretto (2009), Leipold and Su (2012)), embedded leverage (Frazzini and Pedersen (2011)), and interest-rate spreads and other portfolio constraints (Karatzas and

Kou (1998), Piterbarg (2010)). We complement the literature on how frictions affect option prices by showing that frictions also affect option exercises.

There also exist papers on option exercises, which document irrational early exercise decisions (Gay, Kolb, and Yung (1989),[2] Poteshman and Serbin (2003)), irrational failures of exercise of call options (Pool, Stoll, and Whaley (2008)) and put options (Barraclough and Whaley (2012)), and irrational delivery decisions (Gay and Manaster (1986)). We complement these findings by linking early exercise decisions to financial frictions, both theoretically and empirically, and by drawing a parallel to convertible bonds. Early exercise therefore exist both for rational and irrational reasons; while Poteshman and Serbin (2003) find that customers sometimes irrationally exercise early, we find that market makers and firm proprietary traders also frequently exercise early and that most exercises appear to be linked to financial frictions. Battalio, Figlewski, and Neal (2014) also find that option bid prices can be below intrinsic value, which is necessary condition for optimal early exercise, and our model helps explain why the option price can be this low.

Regarding convertible bonds, the literature has linked their prices to financial frictions (Mitchell, Pedersen, and Pulvino (2007), Agarwal, Naik, and Loon (2011)) and examined whether the companies call these bonds too late (often convertible bonds are also callable, see the literature following Ingersoll (1977b)), while we study early conversions by the owners of the convertible bonds due to financial frictions.

In summary, we characterize how frictions can lead to optimal early exercise of call options and conversion of convertibles, and we provide extensive empirical evidence consistent with our predictions. These findings overturn one of the fundamental laws of finance, providing another example that the basic workings of financial markets are affected by financial frictions with broader implications for economics.

[2] Gay, Kolb, and Yung (1989) study futures options, where early exercise can be optimal even without frictions. They also discuss transaction costs and after-hours exercise (where transaction costs in the underlying can be viewed as infinite), but, as, we show in Proposition 1, transaction costs are not sufficient to justify early exercise for equity call options.

II. Theory

We are interested in studying when it is optimal to exercise an American call option early, that is, during times other than expiration and days before ex-dividend days of the underlying stock. Such rational early exercises must be driven by frictions since they violate Merton's rule. We first consider a simple model to illustrate how early exercise can be optimal for an investor who is long an option (Section II.A) and next present a continuous-time model with testable quantitative predictions for early exercise (Section II.B).

A. When Is Early Exercise Optimal?

Consider an economy with three securities that all are traded at times 0 and 1: a risk-free security with interest rate $r^f > 0$, a non-dividend-paying stock, and an American call option with strike price $X > 0$ that expires at time $t = 1$. The stock price at time t is denoted S_t and the option price C_t. The stock price S_1 at time 1 can take values in $[0, \infty)$ and is naturally unknown at time 0 . The final payoff of the option is $C_1 = \max(S_1 - X, 0)$.

All agents are rational, wealth-maximizing price takers, subject to financial frictions. Agent i faces a proportional stock transaction cost of $\lambda^{i,S} \in [0, 1]$ per dollar stock sold. Furthermore, agent i faces a proportional transaction cost of $\lambda^{i,C} \in [0, 1]$ per dollar option sold. If agent i sells the stock short at time $t = 0$, agent i incurs a proportional securities-lending fee of $S_0 L^i$ at time 1, $L^i \geq 0$. If i is long the stock, agent i can lend out the stock and receive a proportional securities-lending fee of $S_0 l^i$ at time 1, where $l^i \in [0, L^i]$. Agent i also faces a funding cost of $F^i(x, y)$ at time 0 if the agent chooses to hold a value of $x \in \mathbb{R}$ of the stock and $y \in \mathbb{R}$ of the option. This funding cost could be due to an opportunity cost associated with binding capital requirement. Naturally, the funding cost is zero if the agent takes a zero position, $F^i(0, 0) = 0$, and increasing in the absolute sizes of x and y.[3]

We are interested in whether early exercise can be optimal. We therefore analyze whether a strategy is "dominated." We say that a strategy is dominated if there exists another

[3] Stated mathematically, the funding cost function has the property that for $x_2 \geq x_1 \geq 0$ then $F^i(x_2, y) \geq F^i(x_1, y) \geq 0$ for all i and $y \in \mathbb{R}$. Similarly, if $x_2 \leq x_1 \leq 0$ then $F^i(x_2, y) \geq F^i(x_1, y) \geq 0$ for all i and $y \in \mathbb{R}$, and similarly for the dependence on y.

strategy that generates at least as high cash flows in each time period and in every state of nature, and a strictly higher cash flow in some possible state. Further, early exercise is defined as being dominated if any possible strategy that includes early exercise is dominated. We assume that there exists no pure arbitrage net of transaction costs because such a strategy would trivially dominate all other strategies (or, said differently, all strategies are either non-dominated or dominated by a non-dominated strategy).

We first show that, under certain "mild" frictions, early exercise is always dominated. This result extends Merton's classic no-early-exercise rule and shows that the rule is robust to certain frictions. All proofs are in Appendix A

Proposition 1 (No Exercise with "Mild" Frictions)

Early exercise is dominated for an agent i that has:

i. zero short-sale and funding costs, i.e., $L^i = F^i = 0$ (regardless of all transaction costs); or

ii. a sale revenue of the option above the intrinsic value, $C_0(1-\lambda^{i,C}) > S_0 - X$. A sufficient condition for this high sale revenue is that agent i has zero option transaction costs, $\lambda^{i,C} = 0$, and the existence of another type of agents j with zero short-sale costs, funding costs, and stock transaction costs, $L^j = F^j = \lambda^{j,S} = 0$.

The first part of this proposition states that transaction costs alone cannot justify rational early exercise. The reasoning behind this is as follows: When the option is exercised it is either to get the underlying stock or to get cash. In the case where the option holder wants cash, exercising early and immediately selling the stock is dominated by hedging the option position through short-selling of the underlying stock and investing in the risk-free security. The transaction cost from selling the stock after early exercise and from selling the stock short are the same so positive transaction costs of the stock cannot in themselves make early exercise optimal.

In the case where the option holder wants stock, early exercise is dominated by holding on to the option, exercising later, and investing the strike price discounted back one period,

$\frac{X}{1+r^f}$, in the risk-free asset. Thereby the investor will still get the stock, but on top of that earn interest from the risk-free asset. This strategy does not involve any direct trading with the stock and, hence, is not affected by stock transaction costs. (Note that these two alternative strategies do not involve option transactions and hence dominate early exercise even with high option transaction costs.)

The second part of the proposition states that early exercise is also dominated if the option owner's net proceeds from selling the option exceeds intrinsic value. In this case, the owner is better off by selling the option than by exercising early. If there is a type of agents, j, who faces no short-sale costs, no funding costs, and no stock transaction costs then these agents value the option at strictly more than its intrinsic value (as explained above). Therefore, the option holder i prefers selling to j over exercising early if no option transaction costs apply.

While it is important to recognize that frictions need not break Merton's rule, we next show that Merton's rule indeed break down when frictions are severe enough. Specifically, a combination of short-sale costs and transaction costs can make early exercise optimal.

Proposition 2 (Rational Early Exercise with "Severe" Frictions)
Consider an agent i who is long a call option which is in-the-money taking stock transaction costs into account, $S_0(1-\lambda^{i,S}) > X$. Early exercise is not dominated for i if the revenue of selling the option is low, $C_0(1-\lambda^{i,C}) \leq S_0(1-\lambda^{i,S}) - X$ and one of the following holds:

a. the short-sale costs, L^i, is large enough or

b. the funding costs, F^i, is large enough.

The condition $C_0(1-\lambda^{i,C}) \leq S_0(1-\lambda^{i,S}) - X$ is satisfied if the option transaction cost $\lambda^{i,C}$ is large enough and/or the option price is low enough.

To understand the intuition behind who early exercise can be optimal, consider an option owner who wants cash now (with no risk of negative cash flows at time 1). Such an agent can either (i) sell the option, (ii) hedge it, or (iii) exercise early. Option (i) is not attractive (relative to early exercise) if the sale revenue after transaction costs is low. Further, option (ii) is also not attractive if the funding costs or short-sale costs (or those in combination) make hedging very costly. Therefore, option (iii), early exercise, can be optimal.

Note that a low option price can itself be a result of frictions. For instance, the option price is expected to be low if all agents face high short-sale costs and can earn lending fees from being long stocks as we explore further in the next section.

B. Quantifying Early Exercise: Exercise Boundaries and Comparative Statics

We next consider a model that is realistic and tractable enough that we can use its *quantitative* implications in our empirical analysis. We solve for a lower bound of the optimal exercise boundary in a continuous-time model in which all parameters have clear empirical counterparts. The exercise boundary is the critical value of the stock price above which exercise is optimal — so we can examine empirically whether people actually exercise when the stock price is above the lower boundary.

The model solution also allows us to derive interesting comparative statics, showing how the exercise decision depends on short-sale costs, funding costs, and margin requirements. To accomplish these quantitative results, we must assume that the stock has no transaction costs. Clearly, stocks have much lower transaction costs than options in the real world and we primarily included stock transaction costs in the previous sections to show that they are *not* the driver of early exercise (Proposition 1).

The optimal exercise decision is closely connected to the rational valuation of American options in the context of financial frictions. Hence, we seek to joint solve for the value of the option and the optimal exercise decision. We start in the classic Black-Scholes-Merton framework, where agents can invest in a risk-free money-market rate of $r^f > 0$ and a stock with price process S given by:

$$\mathrm{d}S(t) = S(t)\mu\mathrm{d}t + S(t)\sigma\mathrm{d}W(t) \tag{1}$$

where μ is the drift, σ is the volatility, and W is a Brownian motion. The stock can be traded without cost, but we consider the following financial frictions.

First, agents face short-sale costs, modeled based on standard market practices: To sell the stock short, an agent must borrow the share and leave the short-sale proceeds as collateral. Agent i's short-sale account must have an amount of cash equal to $S(t)$, which earns the interest rate $r^f - L^i$ (called the "rebate rate"). The fact that the rebate rate is below the money-market rate reflects an (implicit) continuous short-sale cost of L^i (called the "rebate rate specialness"). The securities lender — the owner of the share — holds the cash and must pay a continuous interest of $r^f - l^i$. Since he can invest the cash in the money market, this corresponds to a continuous securities-lending income of $l^i \in [0, L^i]$. We allow that the securities-lending fees depend on the agent i, and that lender earns less than the short-seller pays ($l^i < L^i$) since the difference is lost to intermediaries (custodians and brokers) and search costs and delays.[4]

The second friction that we consider is funding costs. In particular, there exists a wedge $\psi^i \geq 0$ between the agent's cost of capital and the risk free rate. The agent's margin account earns the risk-free money-market rate, $r^f > 0$, while the cost of capital is $r^f + \psi^i$ (in the sense that using his own equity for a risk free investment is associated with an opportunity cost of $r^f + \psi^i$). Such a capital cost can arise from costly equity financing and from a binding capital constraint.[5] The cash in the agent's margin account must be at least $K^i(x, y)$, depending on the number of stocks $x \in \mathbb{R}$ and options $y \in \mathbb{R}_+$.

Based on these assumptions about the stock dynamics and agent frictions, we seek to determine an option owner's optimal exercise policy. Consider an owner i of an American call option with expiration T and strike price X and his option valuation C^i. The option value is assumed to be a $\mathcal{C}^{1,2}$ function of time t and the stock price S so we apply Itô's lemma

[4] The institutional details of short selling and the over-the-counter securities-lending market are described in Duffie, Gârleanu, and Pedersen (2002) who also discuss why not all investors can immediately lend their shares in equilibrium.

[5] See Garleanu and Pedersen (2011) for an equilibrium model with binding margin requirements where such implicit capitals costs arise endogenously as ψ^i is the Lagrange multiplier of the margin requirement.

to write the option price dynamics as:

$$\mathrm{d}C^i(t) = \left(C_t^i + \frac{1}{2}\sigma^2 S^2 C_{SS}^i \right) \mathrm{d}t + C_S^i \mathrm{d}S(t) \tag{2}$$

where subscripts denote derivatives (e.g., C_{SS} is the second order derivative of the option value w.r.t. the stock price S), and we assume the natural condition that $C_S^i \geq 0$ for all i. To derive some bounds on the optimal exercise policy for any agent, we consider the strategies of two hypothetical "extreme" agents, $\underline{i}$ and $\bar{i}^{\psi}$. First, hypothetical agent $\underline{i}$ has the most strict frictions, leading to the lowest exercise boundary, $\underline{B}$, and the lowest option valuation, $\underline{C}$. To accomplish this lower bound, agent $\underline{i}$ is always short the stock, has the highest funding cost ($\psi^{\underline{i}} := \max_i \psi^i$), the highest short-sale cost ($L^{\underline{i}} := \max_i L^i$), and must have cash in his margin account equal to

$$K^{\underline{i}}(x, y) = m^{\underline{i},S} S|x| + (m^{\underline{i},C} - 1)\underline{C}y \tag{3}$$

where $m^{\underline{i},S}, m^{\underline{i},C} \in [0, 1]$. Given that the agent also owns options worth $\underline{C}y$, this expression corresponds to a margin equity of $m^{\underline{i},S} S|x| + m^{\underline{i},C}\underline{C}y$. Hence, $m^{\underline{i},S}$ is the margin requirement for the stock and $m^{\underline{i},C}$ is the margin requirement for the option. The required amount on the margin account approximates the real-world margin requirements in a way that is tractable enough for our analytical results. The real-world margin requirements differ across exchanges and market participants and are very complex (see, e.g., CBOE (2000)). All other agents i have looser margin requirements in the sense that

$$\begin{aligned} K^i(x_2, y) - K^i(x_1, y) &\leq m^{\underline{i},S}(x_1 - x_2)S \quad \text{for } x_2 \leq x_1 \text{ and } \forall y \in \mathbb{R} \\ K^i(x, y_2) - K^i(x, y_1) &\leq (m^{\underline{i},C} - 1)(y_2 - y_1)\underline{C} \quad \text{for } y_2 \geq y_1 \geq 0 \text{ and } \forall x \in \mathbb{R} \end{aligned} \tag{4}$$

The first condition says that a decrease in the number of stocks held increases the required margin cash at least as much for agent $\underline{i}$ as for i. Likewise, the second condition says that an increase in the number of options increases the required margin cash at least as much for agent $\underline{i}$ as for i.

To focus on the exercise strategy, we assume that agents cannot sell the option at or above $\underline{C}$. This assumption can be viewed as a large option transaction cost or as a result of low equilibrium option prices arising from other agents facing the same frictions.

Consider the portfolio dynamics of buying one (additional) option at price $\underline{C}$, hedging by selling (additional) $\underline{C}_S$ shares of the stock, and fully financing the strategy based on margin loans and the use of equity capital. The value of this fully-financed strategy evolves as according to:

$$\begin{aligned}&\left(\underline{C}_t + \frac{1}{2}\sigma^2 S^2 \underline{C}_{SS}\right) \mathrm{d}t + \underline{C}_S \mathrm{d}S(t) - (1 - m^{\underline{i},C})\underline{C} r^f \mathrm{d}t - m^{\underline{i},C}\underline{C}(r^f + \psi^{\underline{i}})\mathrm{d}t \\ &- \underline{C}_S \mathrm{d}S(t) + \underline{C}_S S \left(m^{\underline{i},S} r^f - m^{\underline{i},S}(r^f + \psi^{\underline{i}}) + (r^f - L^{\underline{i}})\right) \mathrm{d}t \end{aligned} \tag{5}$$

Let us carefully explain each of the terms in this central expression. The first two terms simply represent the dynamics of the option (as seen in Eqn. (2)). The next two terms represent the funding of the option. Specifically, $(1 - m^{\underline{i},C})\underline{C}$ can be borrowed against the option at the money-market funding cost r^f. The remaining option value, the margin requirement $m^{\underline{i},C}\underline{C}$, must be financed as equity at a rate of $r^f + \psi^{\underline{i}}$. The second line of (5) represents the terms stemming from the stock position and its financing. The first term is the stock dynamics, given the $\underline{C}_S$ number of shares sold. The last three terms capture the various financing costs. The stock sold short to hedge the option increases the required amount in the margin account by $\underline{C}_S S m^{\underline{i},S}$ which earns the interest r^f. This amount must be financed as equity at the rate $r^f + \psi^{\underline{i}}$. Agent $\underline{i}$ must deposit the cash from the stock sold short, $\underline{C}_S S$, on a short-sales account earning interest $r^f - L^{\underline{i}}$.

We are ready to state the free boundary problem for the option value and the exercise boundary $\underline{B}(T-t)$, which depends on the time to expiration $T-t$. First, the stock position is chosen to offset the risk of the option, so the stochastic terms involving $\mathrm{d}W(t)$ cancel out in (5). Second, as the portfolio is fully financed and the change in value is deterministic, the drift must also be zero. Setting the drift equal to zero yields the following PDE:

$$\underline{C}_t + \frac{1}{2}\sigma^2 S^2 \underline{C}_{SS} - (r^f + m^{\underline{i},C}\psi^{\underline{i}})\underline{C} + \underline{C}_S S(-m^{\underline{i},S}\psi^{\underline{i}} + r^f - L^{\underline{i}}) = 0 \ , \tag{6}$$

for all stock prices $S < \underline{B}(T-t)$. Whenever $S(t) \geq \underline{B}(T-t)$, the option is exercised and the following boundary conditions ensure that the problem is well-posed (Merton (1973)):

$$\begin{aligned}
\underline{C}(T,S) &= \max(S-X,0) && \\
\underline{C}(t,0) &= 0 && t<T \\
\underline{C}(t,S) &= S-X \qquad S \geq \underline{B}(T-t), && t<T \\
\underline{C}_S(t,\underline{B}(T-t)) &= 1 && t<T
\end{aligned} \tag{7}$$

The first condition is the standard boundary condition for the value of the option at the expiration date T. The second condition expresses that, if the stock is worthless, so is the option. The third condition imposes that the value of the option is equal to its intrinsic value at and above the exercise boundary. The fourth condition is the high-contact boundary condition (smooth-pasting condition), stating that the delta of the option goes to 1 as the stock price goes to the exercise boundary (both from above and below), which ensures the optimality of the exercise boundary in maximizing the option value (Kim (1990)). We note that the exercise boundary can take the value ∞ if early exercise is not optimal for any stock price at time t.

Likewise, to derive an upper exercise bound $\bar{B}^\psi$, we define a hypothetical agent $\bar{i}^\psi$ in a slightly more complex way: This agent is long the stock, has the lowest lending fee, and has extreme margin requirement among agents with a given level of funding cost ψ as specified in Appendix A. We derive the PDE for this agent's valuation $\bar{C}^\psi$ and exercise strategy:

$$\bar{C}^\psi_t + \frac{1}{2}\sigma^2 S^2 \bar{C}^\psi_{SS} - (r^f + m^{\bar{i}^\psi,C}\psi)\bar{C}^\psi + \bar{C}^\psi_S S(m^{\bar{i}^\psi,S}\psi + r^f - l^{\bar{i}^\psi}) = 0 \ , \quad S < \bar{B}(T-t) \tag{8}$$

subject to the same boundary conditions as (7) for $\bar{B}$ and $\bar{C}$. We see that the free boundary problems are mathematically equivalent to that arising from the pricing of an American call option in a Black-Scholes-Merton model with a modified interest rate and a continuous dividend yield. Specifically, in (6), the role of the interest rate is played by $r^f + m^{\underline{i},C}\psi^{\underline{i}}$ and the role of the dividend yield is played by $L^{\underline{i}} + \psi^{\underline{i}}(m^{\underline{i},C} + m^{\underline{i},S})$ and similarly for (8). This means that both problems can be solved by traditional numerical methods as we do in our empirical analysis. The solutions include the option values and optimal exercise boundaries, relying partly on results from Dewynne, Howison, Rupf, and Wilmott (1993).

Proposition 3 (Lower and Upper Exercise Boundaries)
(i) Any option owner i has a value of the option in $[\underline{C}, \bar{C}^{\psi^i}]$; exercise is dominated if $S(t) < \underline{B}(T-t)$ and failing to exercise is dominated when $S(t) > \bar{B}^{\psi^i}(T-t)$.
(ii) If $L^{\underline{i}} + \psi^{\underline{i}}(m^{\underline{i},C} + m^{\underline{i},S}) > 0$, then the lower exercise boundary, $\underline{B}$, is finite for all $t \leq T$ (i.e. early exercise for agent $\underline{i}$); otherwise, $\underline{B}$ is infinite for $t < T$.
(iii) Similarly, if $l^{\bar{i}} + \psi(m^{\bar{i},C} - m^{\bar{i},S}) > 0$, then the upper exercise boundary $\bar{B}^{\psi}(T-t)$ is finite for all $t \leq T$ (i.e. early exercise for agent $\bar{i}^{\psi}$); otherwise $\bar{B}^{\psi}$ is infinite for $t < T$.

This proposition provides several results that are useful for our empirical analysis. Indeed, while we assumed that the hypothetical agent $\underline{i}$ is short stock for $t < T$ and $\bar{i}^{\psi}$ is long stock for $t < T$, Proposition 3 provides more general results that apply to *any* agent, regardless of his stock position, about when early exercise is dominated and when it is dominated not to exercise early. In the empirical section, we derive the exercise bounds by numerically solving the PDEs, and we focus on the lower bound, $\underline{B}$, for several reasons: First, and most importantly, we are interested in whether the observed exercise decisions can be rationally justified and this is the case when the stock price is above the lower boundary. Second, the lower boundary applies to all agents, independent of ψ. Third, the lower boundary depends on the lending fee L which we observe (while we don't have data on the part l that accrues to the owner). The next proposition analyzes the nature of optimal early exercise decisions through a number of intuitive comparative statics.

Proposition 4 (Comparative Statics of the Exercise Boundaries)
The following comparative statics hold for the lower and upper exercise boundaries: $\underline{B}$ is

weakly decreasing in short-sale cost, funding costs, option margin requirements, and stock margin requirements; $\bar{B}^{\psi}$ *is weakly decreasing in lending fee and option margin requirement and increasing in stock margin requirement.*

To get a feel for the optimal early exercise behavior, we illustrate the comparative statics results of Proposition 4 with a numerical example as seen in Figure 3. To do this, we solve the PDE using the Crank-Nicolson finite difference method. In each of the four panels, the time to expiration $T-t$ is on the x-axis and the scaled exercise boundary is on the y-axis. Specifically, we scale the exercise boundary $\underline{B}(T-t)$ by the strike price X, which yields it a more intuitive number. For example, a value of $\underline{B}(T-t)/X = 1.6$ means that early exercise is optimal when the stock price is at least 60% in the money. Said differently, early exercise happens when $S(t) \geq \underline{B}(T-t)$ or, equivalently, when $S(t)/X \geq \underline{B}(T-t)/X$, and the latter scaled measure is more intuitive. Clearly, a lower exercise boundary corresponds to an earlier optimal exercise decision (i.e., for lower moneyness of the stock price).

The plots in Figure 3 vary the model parameters around the following base-case parameters for an agent who is long a call option and shorts the stock: the risk-free rate is $r^f = 2\%$, the volatility is $\sigma = 40\%$, the funding cost is $\psi^{\underline{i}} = 1\%$, the short-sale cost is $L^{\underline{i}} = 1\%$, the margin requirement for the stock is $m^{\underline{i},S} = 50\%$, and margin requirement for the option is $m^{\underline{i},C} = 100\%$.

The top left panel illustrates how higher short-sale costs correspond to a lower boundary, implying earlier optimal exercise, consistent with Proposition 4. This result is natural as short-sale costs make it costly to hedge the option. The top right panel shows that higher funding costs also make it optimal to exercise earlier, namely to free up capital. The bottom left panel shows that a higher option margin requirement encourages an earlier exercise. Finally, the bottom right panel shows that early exercise is delayed with higher volatility. To see why, recall that a higher volatility increases the value of optionality, therefore making it less attractive to exercise early.

In all the graphs we see that the optimal exercise boundary decreases in the time to expiration. In fact, it is a general result that the exercise boundary must be weakly decreasing in time to expiration. To understand this result, note that, if it is optimal to exercise a longer-dated option, then it must also be optimal to exercise a shorter-dated one (since you give up less optionality).

Finally, the figures provide *quantitative* insights into when we should expect early exercise due to frictions. In the base-case, early exercise is optimal when the stock price is 1.67 times the strike price a quarter before expiration and 1.27 times the strike price 10 days before expiration. We next turn to the empirical analysis, where we also implement our model for each option in a large data set and analyze whether the real-world exercise decisions occur when the stock price is above the exercise boundary that we calculate.

III. Data and Preliminary Analysis

This section describes our data sources, provides summary statistics, and outlines our empirical methodology. We start with the data and then turn to the summary statistics, which already shows large amounts of early exercises and early conversions both in terms of number of contracts and in terms of dollar value.

A. Data

Our study combines a number of very large data sets as described in Table I. For equity options, we combine the OptionMetrics database on U.S. option prices and option bid-ask spreads with the CRSP tape of U.S. equity prices and corporate events. We use data on the cost of short-selling stocks from Data Explorers, focusing on their Daily Cost of Borrow Score (DCBS), which is an integer from 1 to 10 with 1 indicating a low cost of shorting and 10 indicating a high one.

We analyze actual exercise behavior using data originally from the Options Clearing Corporation (OCC).[6] This data contains the number of contract exercises for each option series each day. The daily exercises can be separated into three groups of market participants,

[6]We are very grateful to Robert Whaley for providing this data.

namely exercises done by customers of brokers (retail customers and hedge funds), market makers, and firm proprietary traders. The option exercise data runs from July 2001 to and including August 2010. The data is missing in the months of November 2001, January and July 2002, and January 2006.

We use the Mergent FISD database on convertible bonds. This database provides time and amount of conversions together with total outstanding amount for convertible bonds. Finally, we use data on equity issues and mergers and acquisitions from Thomson One. This data set includes issue dates and settlement dates for equity issues and announcement dates, effective dates, and withdrawals dates for both acquiror and target in mergers and acquisitions.

B. Sample Selection

To identify option exercises that clearly violate Merton's rule, we focus on early exercise of options on stocks that do not pay dividends. In particular, we exclude any option series if OptionMetrics reports a non-zero forecast of future dividends during any day of the life time of the option. Further, we exclude observations of option series on the day of expiry to focus on early exercise. Lastly, we exclude options that do not follow the standard practice of the having an expiration date on the day after the third Friday in a given month (this excludes only a tiny fraction).

The data from OptionMetrics is merged with the exercise data from OCC on date, ticker, option ticker, strike price, and expiry date. For each option series, we further merge the data with that of the corresponding stock from CRSP and Data Explorers based on CUSIP.

We further clean the data in a number of ways. We exclude any option series (i) where the underlying according to CRSP has ex-date for a distribution event (split, dividend payment, exchanges, reorganizations etc.) within the observed life time of the option series; (ii) where the underlying at some point according to OptionMetrics was expected to have an ex-dividend date within the observed life time of the option series; (iii) which has records with different strike prices for the same series, different underlying identifiers (secid or CUSIP), or different expiry dates (indicating data errors or changes in the contract); (iv) which has no data is available on the last trading day before expiry (indicating some possible outside event); (v) which has several records for the same day in OptionMetrics; (vi) for which another option series with same underlying stock ticker, option ticker (root of option symbol for old option symbol and first part of symbol for new OSI symbol), strike price, and expiry date observed on the same day exists in the data (vii) which has settlement special, e.g. AM-settlement; or (viii) which for some day during its observed life time has no matching observation in the CRSP data for the underlying stock.

We note that the OCC data only has records of exercises, meaning that option series that never experienced an exercise (before or at expiry) are not part of that sample. Hence, by requiring a match between Option Metrics and the OCC data, our sample only includes options that were exercised at some point. An alternative approach is to include our entire OptionMetrics sample and assume that option series missing in the OCC data were never exercised. Since we do not know whether the OCC data is complete, neither approach is perfect. To resolve this, we focus on whether the observed early option exercises can be explained by financial frictions, not whether people fail to exercise when they should (failure to exercise has been studied by Pool, Stoll, and Whaley (2008)).

Convertible bond data is acquired through Mergent FISD. Our sample only includes convertible bonds that at some point in time were converted (including at a dividend, at maturity, or as a response to a call). If Mergent FISD has not been able to identify the exact day of a conversion they set the date to the end of the quarter or even fiscal year (the latter seems to be the case only rarely). This makes it difficult to identify whether the conversion happened on the day before ex-dividend or not in these cases. To avoid problems related to these issues, we only include bonds where the underlying did not have any distribution events (including dividend payments) during the sample period, using data of distribution events from CRSP. We also exclude bonds where the underlying is first observed in CRSP more than one day later than the offering date of the bond. Furthermore, we exclude bonds that at some point had an exchange offer or a tender offer, or where the underlying is not either a common stock or an American depository stock. If Mergent FISD data has no maturity date or conversion price of the bond, then it is also excluded. The original conversion prices of the bonds are recorded in FISD and through the cumulative adjustment factors provided by CRSP they can be updated to reflect any changes e.g. due to stock splits.[7] We only include observations from days where bond holders had the right to convert early. If CRSP data for the underlying is missing starting at some point in time, we exclude the observations from five days before this happened and onwards, to avoid inclusion of conversions related to some kind of exogenous corporate event. If the day of the initial observation of the bond in Mergent FISD is after the offering day of the bond, we exclude observations before this initial observation. If the bond has been partly or fully called at some point in time, then we exclude all observations which are less than three months earlier or three months later to this event. This measure is taken in order to avoid inclusion of conversions that are a response to a call from the issuer (and hence not early conversion initiated by the bond holder), though it is not guaranteed that we catch all such events. Likewise, if the record shows any reorganization or exchange of the bond, observations from three months before this event and onwards are excluded.

C. Summary Statistics

Table II provides summary statistics for our final sample. We see a substantial amount of early exercises and conversions. Panel A reports the total number of early exercises of equity options, that is, exercises that violate Merton's rule. Our data contain 1,806 million early exercises, representing a total exercise value of $36.3 billion or a total intrinsic value of $22.8 billion. Naturally, the exercises are concentrated among in-the-money options, especially deep-in-the-money options. Our data does contain a small fraction of exercised out-of-the-money options, which could be due to measurement error or investors exercising to save transaction costs when they want the actual stock. Measurement error may occur for instance when options are exercised during the day and we measure the moneyness based

[7]If e.g. a stock is split in two, a convertible bond with this stock as underlying will have its conversion price halved at the same time.

on the end-of-day price.

Table II also shows that early exercises are concentrated among shorter-term options. This finding is consistent with our theory, since short-term options have less optionality, but it could also be driven by the simple fact that there is a larger open interest of such options. Our formal empirical tests therefore consider the number of exercises as a fraction of the open interest.

The final part of Panel A in Table II shows that all types of agents exercise early for billions of dollars. Customers of brokers exercise early with a total strike price of \$16.0 billion, firm proprietary traders for a total of about \$2.0 billion, and market makers for a total of \$18.3 billion.

Panel B in Table II reports the total number of early conversions of convertible bonds. In our data, 25 million bonds are converted early, corresponding to about \$5.7 billion worth of principal or \$7.7 billion of equity value. A few conversions of out-of-the-money bonds are seen, which relates to the definition of moneyness applied. A convertible bond is considered out-of-the-money if the price of the underlying stock is less than conversion price, in-the-money if stock price is up to 25% above conversion price, and deep-in-the-money if stock price is more than 25% above conversion price. Conversion price is defined by the principal amount of bond that must be converted to get one stock. Our definition of moneyness is not perfect: Indeed, the market value of a non-convertible bond can deviate some from the face value, so it might actually be attractive to convert even when it is out-of-the-money according to the above definition. While this adds noise to our analysis, we see no reason that it would drive our conclusions.

Figure 2 shows the evolution of relative share of early option exercise over time for the three different agent types observed. The picture is dominated by Customers (which includes retail customers and hedge funds) and Market Makers. Interestingly, the share of early exercise for the professional Market Makers has increased over time. We next discuss how we use this exercise and conversion data to test our theoretical predictions.

D. Variables of Interest and Methodology

We are interested in the fraction of options that are exercised and how this relates to our theoretical predictions. For each day t and each option series i in our sample, we define the options that are exercised EX as a fraction of the open interest OI on the close of the day before.

$$EX_t^i = \frac{\#\text{exercised options}_t^i}{\max\{OI_{t-1}^i, \#\text{exercised options}_t^i\}} \tag{9}$$

We take the maximum in the denominator to ensure that EX is between zero and one, including the rare instances when the number of exercises is greater than the open interest the day before (which must be due to options that are bought and exercised on the same day or data errors). Similarly, in our logit and probit regressions, we compare the number of exercises to the number of "trials" given by $\max\{OI_{t-1}^i, \#\text{exercised options}_t^i\}$.

For each daily observation of an option series, we measure the option transaction costs as the relative bid-ask spread constructed in the following way:

$$TCOST_t^i = \frac{\text{ask price}^{s(i)} - \text{bid price}^{s(i)}}{(\text{ask price}^{s(i)} + \text{bid price}^{s(i)})/2} \tag{10}$$

Here, the superscript i denotes the option series and $s(i)$ is the corresponding at-the-money option series with the same underlying stock, defined as the series with the smallest absolute difference between stock price and strike price (where $s(i)$ may be equal to i itself). We use the at-the-money option instead of the option itself to avoid endogeneity issues. The possibility of exercising the option early will itself affect bid and ask prices, especially for deep-in-the-money options. The bid price will in such cases often go below, but not much below, the intrinsic value. As a result the intrinsic value will somewhat be a floor for how much the bid price can go below the ask price. Our focus is to test how the general level of transaction costs of an option series affects early exercise in the first place.

We measure the short-sale fee, L, as follows. For each stock and each date, we observe its Daily Cost of Borrow Score (DCBS), which is an integer score from from 1 to 10. We map this DCBS score to a short-sale fee level by using the median among all stocks with this DCBS score that have data on both their DCBS score and their fee level. In the analysis based on short-sale fees, we only include stock-dates with non-missing DCBS scores.

The model-implied optimal exercise boundary $\underline{B}^j(T-t)$ for any option j on day t is computed as follows. We numerically solve the PDE problem (6)–(7) that takes frictions into account using the observed characteristics on any date t using the Crank-Nicolson finite-difference method. The stock volatility σ is set as the 60-day average historical volatility. We use the historical volatility as an objective measure of risk and note that we cannot use the standard Black-Scholes implied volatility for two reasons: (i) in many of the interesting cases when options are exercised early, option prices don't satisfy Merton's lower bound and, therefore, the Black-Scholes implied volatility cannot be computed; and, more broadly, (ii) the Black-Scholes implied volatility does not take frictions into account, so it should be a biased estimate of volatility when frictions are severe. For the risk-free rate r^f, we use the Fed Funds rate, the margin requirement of the stock is set to 50%, the margin requirement of the option i set to 100%, the funding cost ψ is set as the LIBOR-OIS interest-rate spread (based on Garleanu and Pedersen (2011)), and the short-sale and funding costs are as defined above. We measure the stock price S as the closing price on the given day.

Similarly to the exercise measure EX for equity options, we are interested in the daily converted bonds as a fraction of the total outstanding amount. We define $CONV_t$ as amount converted on day t divided by the sum of amount converted and outstanding amount after the conversion. (Equivalently, the denominator is the amount outstanding before the conversion.)

$$CONV_t^i = \frac{\text{Amount converted}_t^i}{\text{Amount converted}_t^i + \text{Amount outstanding after conversion}_t^i} \tag{11}$$

IV. Empirical Results: Never Exercise a Call Option?

We turn to formally testing the link between early options exercises and financial frictions. We first sort the exercises by short-sale costs and transaction costs to analyze the connection between early exercises and financial frictions in a simple way. Next, we test the model more directly by considering whether the stock price is above or below the model-implied lower exercise boundary at the time of exercise. Finally, we use multivariate logit and probit regressions to analyze how the propensity to exercise can be explained by the model and how it depends on the joint effects of a number of option characteristics.

Table III and Figure 4 show how the fraction of early option exercises EX (defined in (9) above) varies with short-sale costs. We see that the fraction of early exercise decisions increases monotonically in the short-sale cost, consistent with our model. Among options with minimal short-sale costs, the fraction of options exercised early is only 0.17%, while among options with the highest short-sale costs, the fraction exercised is above 4%. As seen in the table, the difference in these two extreme groups is highly statistically significant.

Table III and Figure 4 further consider the exercises broken down by moneyness. Naturally, there are virtually no out-of-the-money options that are exercised (which would clearly be irrational or due to measurement error), and the exercises are concentrated among deep-in-the-money options, as we would expect. Splitting the data by expiration, the table and figure show that the exercises are more frequent for shorter-term options. This is consistent with our theory as the benefits of postponing exercising is smaller (smaller optionality) for shorter-term options. Again we see that the option exercises increase in short-sale costs within expiration group.

Lastly, we split the data by agent types, that is, across customers of brokers (retail customers and hedge funds), market makers, and firm proprietary traders. We see that all types of agents exercise early and more so when the short-sale costs are higher. We note that the absolute magnitude of the numbers should not be compared across groups for the following reason: Our data do not contain open interest by agent type, so we measure the number of exercises by each agent type as a fraction of the total open interest. Hence, the fraction of exercises by firm proprietary traders may be low simply because this agent type trades few options relative to the total open interest. In any case, the pattern of an increasing propensity to exercise as short-sale costs increase is consistent with our theory.

Table IV and Figure 5 show how the fraction of early exercises varies with the transaction costs. We measure transaction costs as the relative bid-ask spread of the at-the-money with the same expiration ($TCOST$) as defined in (10) above. We assign all observations an integer score from 1 to 5 (low to high transaction costs) based on this measure, using the full sample quintile breakpoints. Further, observations are classified in three groups: out-of-the-money (with stock price below strike price), in-the-money options (with stock price up to 25% above strike price), and deep-in-the-money options (with stock price more than 25% above strike price).

We see that the fraction of options exercised increases monotonically with the transaction costs. This pattern holds overall, for each moneyness group, and for each type of agents. The absolute of numbers in Table IV are smaller on average than the numbers in Table III. This is because 81% of the data have a short-sale cost code (DCBS) equal to 1 (as classified by Data Explorers) and, as expected, the fraction of exercises is low in this group as seen in Table III. In Table IV, our groups by transaction costs are more balanced.

In summary, consistent with our model's qualitative predictions, we have seen that option exercises increase in short-sale costs and transaction costs and that these patterns tend to hold within groups sorted by moneyness, expiration, and agent types. Next, we seek to test our model's quantitative predictions. In particular, for each exercised option, we compare the model-implied lower exercise boundary to the high of the stock price at the day of the exercise as seen in Table V. The table reports the fraction of exercises consistent with our model for each agent type and for all agents (in each row of the table). We look at three different samples: full sample (Panel A), a sample excluding corporate events (Panel B), and a sample including only options with more than 9 months to expiration (Panel C). Each column of the table corresponds to a specific set of assumptions underlying the model, with increasingly conservative assumptions going from left to right. In the left-most column, we estimate each stock's volatility based on the 60-day realized volatility. Given that volatility is mean-reverting, the next column uses a lower estimate of future volatility, namely the minimum of the current 60-day volatility and its median in the OptionMetrics sample (June 1 2001–January 31 2012). The third column uses both the conservative volatility estimate and a conservative estimate of short-sale costs, namely the 90th percentile of short-sale costs within each group (rather than the median observed cost).

For all cases of model input assumptions, we see that the majority of option exercises happen when the stock price is above the model-implied lower exercise boundary. The fraction of exercises consistent with our model is highest for market makers, which could be because these agents face the lowest financial frictions and are the most active market participants. Naturally, the fraction of exercises consistent with the model increases in the columns with more conservative assumptions (by construction). The model can explain 98% of the market makers' exercises with the most conservative assumptions, a very large fraction in light of the remaining noise. Table V Panel B consistently reports higher numbers than Panel A, which shows that excluding corporate events disproportionately excludes early exercises not explained by the model. Section IV.B elaborates this result. Table V.C focuses on the exercises that happen with more than 9 months to expiration. This sample has a larger fraction of exercises relative to full sample that are in line with the estimated model. For market makers, the fraction of exercises consistent with the model are as high as 98.3% to 99.7%, depending on the model inputs.

Lastly, we study the propensity to exercise in a logit and probit regression setting. To do so, we need in principle to compute the model-implied lower exercise boundary for each type of option and each date, including days when no exercises are observed. The very large amount of data combined by the numerical complexity in solving our model's PDE makes such a complete analysis unfeasible. To address this issue, we look at a sub-sample only consisting of one day per month, namely 17 days before option expiry (which is the day after the third Friday in every month). The sub-sample analysis is sufficient to obtain statistically significant results and we have confirmed that our model independent results hold up in the full sample (i.e., by regressing the propensity to exercise on characteristics such as short-sale costs). For each of the selected 80 dates, we compute the optimal exercise boundary for each option by solving the PDE problem (6)–(7) that takes frictions into account.

If we run a pooled logit (or probit) regression using all options on all of the selected dates, then we get highly significant results consistent with our model (not reported). However, such standard errors would be heavily downward biased since investors usually exercise many options simultaneously, generating a strong correlation across options on a given date. To address this correlation issue, we proceed as follows. First, we run a logit (or probit) regression for each sub-sample of three dates (the last sub-sample has only two dates). This generates an estimated vector of parameters, $\hat{\theta}_s$, for each sub-sample s. Second, we estimate the full-sample parameters and their standard errors based on the insight of Fama and MacBeth (1973) that each parameter can be viewed as sampled from parameters' distribution. In particular, we estimate the full-sample parameters as the sample average, $\hat{\theta} = 1/27 \sum_s \hat{\theta}_s$, and the standard errors based on the sample standard deviation corrected for possible auto-correlation (Newey-West correction with automatic lag selection using a Bartlett-kernel, Newey and West (1987, 1994)). This estimation method is relatively immune to cross-sectional correlation in option exercises and assumes that any time-series correlation is captured by the Newey-West correction.

Table VI reports the results, where Panel A is the logit regressions and Panel B is probit. The first regression specification simply considers how the propensity to exercise depends on the indicator that the stock price is above the lower exercise boundary ($S > \underline{B}$). We see that the estimated coefficient for $S > \underline{B}$ is positive, consistent with our model, and the effect is highly statistically significant. The estimated probability of exercise on a day with $S > \underline{B}$ is $1/(1 + \exp(8.54 - 4.09)) = 1.2\%$, while estimated probability on other days is $1/(1 + \exp(8.54 - 0)) = 0.02\%$. Cumulated over 20 trading days in a simple way, these probabilities correspond to aggregate exercise probabilities of $1 - (1 - 1.2\%)^{20} = 20.7\%$ when the stock price is above the exercise boundary and $1 - (1 - 0.02\%)^{20} = 0.4\%$ otherwise. We see that the exercise behavior is very different depending on whether the stock price is above or below the exercise boundary.

The second specification shows a similar result, but where we allow the exercise probability to increase in the distance between the stock price and the boundary using the variable $(S - \underline{B})^+/\underline{B}$. The third specification shows that having a bid price for the option below the intrinsic value is also a significant predictor of early option exercise, which is also consistent with our model (and a basic trade off between selling the option vs. exercising it). Of course, the option bid price is an endogenous variable, so this regression does not address the deeper question of *why* option prices can be so low that early exercise can make sense. The first specification based on our model is not subject to the same issue since the exercise boundary is computed based on the observed financial frictions.

The fourth specification includes all these variables jointly. We see that all three variables remain positive and statistically significant, both with logit and probit. Lastly, the fifth specification also includes a number of control variables. Both of the model-implied variables continue to be positive, and $1_{(S>\underline{B})}$ remains highly significant in all specifications. Several of the other variables are also significant, suggesting that the precise probability of exercise is a complicated function of the observable data. We expect differences across investors in terms of the frictions that they face, the frequency with which they observe the markets, and their estimate of future volatility. While some investors' exercise decision may be captured by the distance between the stock price S and the boundary $\underline{B}$ that we calculate, other investors may exercise based on their individual-specific frictions, and this idiosyncratic variation may be partly picked up up by the control variables. Of course, economic models of individual behavior are always less precise than those of prices (which aggregate individual noise) and, thus, these imperfections echo those in the literature on individual investors' refinancing of mortgage bonds. Still, our results support that early exercise decisions are driven by frictions in most cases, especially for market makers.

Since using the LIBOR-OIS spread is an imperfect measure of funding costs (and does not vary in the cross-section), we have also repeated the analysis with the exercise boundary computed for a zero funding cost (not show, but available upon request from the authors). Our results continue to hold qualitatively in this specification, indicating the robustness of our results and that short-sale frictions are important drivers. However, a larger number of exercises naturally happen below this alternative boundary, which could indicate that the funding costs may also play a role.

A. A Natural Experiment: The Short-Sale Ban of 2008

To further test the theory that early option exercises are caused by, and not only correlated with, frictions we consider exercise behavior during the short-sale ban in 2008. After the market closed on September 18, 2008, the U.S. Securities and Exchange Commission (SEC) introduced an emergency order that baned short-sale of certain stocks (SEC release no. 34-58592). Boehmer, Jones, and Zhang (2013) provide a useful summary of the time line of the events. Initially, the ban applied to a list of 797 tickers for financial stocks (c.f. SEC's emergency order). On Sunday September 21, SEC send out an amendment to the emergency order (SEC release no. 34-58611) where the exchanges became responsible for adding and deleting stocks to the list of banned stocks such that the ban would include all financial firms, leading to around 200 additions and a few deletions.

The short sale ban serves as a shock to the short-sale frictions. Of course, the fall of 2008 which a volatile period with many events, but the fact that the ban only applied to certain firms allows us to control general events in this time period. Specifically, we compare the exercise behavior before and after the ban for, respectively, affected and unaffected firms (a so-called difference-in-differences approach often used to study causality). Our theory predicts that options on affected stocks should experience an increase in early exercise relative to unaffected stocks.

For this natural experiment, we obtain data from Nasdaq allowing us to identify every stock that was affected by the ban. We create an indicator variable for each option series taking the value 1 for options written on *Affected stocks*. Further, we create a *Ban period* indicator variable for the period where the ban was in effect (September 19 to October 8, both included).

To consider the affects of the ban on exercise behavior, we include these indicators and, importantly, their interaction in the logit and probit regressions. To estimate standard errors, modify the Fama-MacBeth procedure used in Table VI as follows. Focusing on September to October, 2008, we randomly divide the sample into 10 equal-sized sub-samples of option-series/days, each large enough to include several observations for all four combinations of stocks being financial/non-financial and during/outside the ban period. We compute the model-implied exercise boundary based on the ex ante short-sale costs measured at the end of August, 2008 (and we also use this ex ante short-sale cost as a control variable). We use this ex ante short-sale cost to avoid having our results be driven by confounding effects due to the ban's effect on the securities-lending market.

Table VII reports the results of the regressions. The key coefficient is the coefficient on *Affected stock* $\times$ *Ban period.* The fact that this coefficient is positive supports that early exercise of options on banned stocks increased during the ban period, relative to early exercise of options on non-banned stocks. This serves as evidence that increased short-sale frictions causes an increase in early exercises. We find this results both with probit and logit regressions and both with and without date fixed effects.

B. Alternative Reasons for Early Exercise

Lastly, we consider alternative potential drivers of early exercise such as corporate events. To begin the search for alternative explanations, we first consider the top five observations where market makers exercise options when the stock price is below the most conservative boundary, ranked on the number of contracts exercised on a given day for a given series (recall that market makers exercise is consistent with the model in 98.4% of the cases as seen in Table V.A, so we study the top 5 cases among the remaining 1.6%).

Interestingly, three of the five cases are associated with corporate events.[8] Corporate

[8] The observation with most such unexplained early exercise is from January 7, 2004 where market makers exercised 5957 option contracts written on Univision Communications Inc. with expiration January 17, 2004 and strike price 35.00 USD. The Thomson One database for corporate events shows that, on the same day as the exercises, Univision announced a Follow-On issue of the Univision stock of 600 million USD and that "the underwriter may engage in activities that stabilize, maintain or otherwise affect the price" of the stock. Such stabilizing activities can limit the expected downside of the stock and reduce the optionality value of in-the-money options, making early exercise induced by frictions more attractive. Also, hedging activities from lead underwriters could lead to a temporary increase in short-sale costs, enough to induce early exercise, but for a short enough period to not be observed in the short-sale cost data. The second-largest of the the unexplained early exercise by market makers happened Monday July 26, 2010 for options written on Americredit with expiration August 21, 2010 and strike price 22.50 USD. Four days earlier, on July 22, an acquisition of AmeriCredit by General Motors was announced to take effect October 1, 2010 at the price of 24.50 USD per share. If the market expects the the deal to go through for certain, the stock will act as a risk-free asset after the initial price adjustment. In fact, the volatility of the stock did become tiny until the

events could be important for two reasons: first, they could simply affect the parameters of the model, e.g., lower the volatility, which would lead us to mis-estimate the exercise boundary. More worryingly, corporate events could drive early exercise for reasons unrelated to the model. To address these concerns. we have repeated our analysis in the sub-sample that excludes all option-days when the underlying stock experiences a corporate event according to the Thomson One database. In particular, we exclude data around equity issues (from 30 days before to 50 days after or settlement date, whichever comes first) and around mergers and acquisitions for both the target stocks and the acquiror (from 5 days before the announcement until the offer is effective or withdrawn, or, if no end date is observed, until 180 days after the announcement).

Table V.B, shows that our model explains the early exercise behavior better in the sub-sample that excludes corporate events. Also, we have repeated our logit/probit regressions in the sub-sample that excludes corporate events, and the results do not change qualitatively (available upon request from the authors). Hence. it appears that corporate events could create noise as an alternative driver of early exercise, but it does not appear to be driving our results (as an omitted variable).

In addition to equity issuance and mergers and acquisitions, using shares for voting could be a consideration. However, rather than exercising an option, shares can in principle be obtained in the securities lending market, and, therefore, the model should capture this effect via the lending fees and short-sales costs. Further, Christoffersen, Geczy, Musto, and Reed (2007) find that "the average vote sells for zero," suggesting that this is not a major driver of our results. Further, we expect that a number of the important voting events are excluded by the filter applied above.

A final reason for early exercise could be that investors are not fully rational as shown by Gay and Manaster (1986) and Poteshman and Serbin (2003). This could help explain why some options are exercised even when the stock price is below the model-implied boundary.

successful acquisition. Given a vanishing volatility the observed frictions are large enough to justify early exercise by our model. Number five of the list is 2190 option contracts exercised with Autonation Inc. as underlying stock on April 11, 2006. Autonation had an outstanding tender offer to buy back some of its own shares that expired April 12, 2006. The exercise might be induced by this tender offer expiration.

However, irrationality does not appear to be whole picture as seen from the model's explanatory power and the natural experiment of Section IV.A. Further, the fact that early exercise is also observed by market makers who are professional investors likely to make rational decisions also suggest that a number of early exercise decisions are driven by frictions.

Gay, Kolb, and Yung (1989) address transaction costs and the opportunity to exercise after the closing of the futures market. They study options where exercise can be optimal even without frictions so, in their case, transactions cost and market closure can change the optimal exercise time. We show that transaction costs and closed stock exchanges alone are not sufficient to rationalize early exercise of call options although transaction costs do play a role in combination with other frictions (cf. Proposition 1 and Proposition 2).[9]

In summary, there could be alternative reasons for early exercise such as corporate events, but, within the limits of any empirical study, our results suggest that frictions constitute a separate driver of early exercise.

V. Empirical Results: Never Convert a Convertible?

We next consider how early conversions of convertible bonds are related to financial frictions. Table VIII reports our results. We see that early conversions are more frequent among companies with large short-sale costs for the equity, consistent with the theory.

The table breaks down the conversions by the moneyness of the convertible bonds. We consider a convertible bond to be out-of-the-money if the price of the underlying stock is less than conversion price, in-the-money if stock price is up to 25% above conversion price, and deep-in-the-money if stock price is more than 25% above conversion price. As expected, we see that conversions are concentrated among in-the-money and deep-in-the-money convertibles. We see a few conversions of out-of-the-money bonds, which relates to the definition of moneyness discussed in Section III.B.

More importantly, we see that conversion rates increase monotonically in short-sale costs

[9]Closed stock exchange corresponds to extreme stock transaction costs so that no positive value can be gained from selling the stock. Proposition 1 shows that stock transaction costs are not sufficient to drive early exercise.

both for in-the-money and deep-in-the-money convertibles, providing further evidence consistent with the theory. However, we note that the difference across groups is not statistically significant due to the small and noisy dataset.

VI. Conclusion: Never Say Never Again

A classic rule in financial economics states that, except just before expiration or dividend payments, one should never exercise a call option and never convert a convertible bond. This rule is ubiquitous in option theory and taught in most introductory finance classes. We show that this rule breaks down — theoretically and empirically — when financial frictions are introduced, just as frictions break the Modigliani-Miller Theorem, the Law of One Price, and other classic rules in financial economics.

Our theory shows that early exercise of options can be rational in light of financial frictions and, indeed, we would expect early exercise to occur. Consistent with our theory, the empirical propensity to exercise equity options is increasing in the short-sale costs, transaction costs, and moneyness, and decreasing in the time to expiration. We find that options are exercised by customers of brokers, market makers, and firm proprietary traders and, for each group, exercises are more prevalent when the financial frictions are more severe. Our model further implies that it can be optimal to convert a convertible bond. We document a number of early conversions of convertible bonds, especially among stocks with high short-sale costs.

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A. Appendix: Derivations and Proofs

Derivation of PDE for agent $\bar{i}^{\psi}$

For any $\psi \in \mathbb{R}_+$ we consider agent $\bar{i}^{\psi}$ with $\psi^{\bar{i}^{\psi}} = \psi$, who we show have the highest exercise boundary and option valuation among all agents with $\psi^i = \psi$. Agent $\bar{i}^{\psi}$ is always long stock. Let the function for the required amount on the margin account have the form

$$K^{\bar{i}^{\psi}}(x, y) = m^{\bar{i}^{\psi},S} S|x| + (m^{\bar{i}^{\psi},C} - 1)\bar{C}^{\psi} y \tag{12}$$

where $x \in \mathbb{R}$ is the number of stocks held, $y \in \mathbb{R}_+$ is the number of options held, and $\bar{C}^{\psi}$ is $\bar{i}^{\psi}$'s valuation of the option. Also, $m^{\bar{i}^{\psi},S} \geq 0$ and $m^{\bar{i}^{\psi},C} \geq 0$.

Let $l^{\bar{i}^{\psi}} = \min_{i \in \{j:\psi^j = \psi\}} l^i$ and for $i \in \{j : \psi^j = \psi\}$ let

$$\begin{aligned} &K^i(x_2, y) - K^i(x_1, y) \leq m^{\bar{i}^{\psi},S}(x_2 - x_1)S \quad \text{for } x_1 \leq x_2 \text{ and } \forall y \in \mathbb{R} \\ &K^i(x, y_2) - K^i(x, y_1) \geq (m^{\bar{i}^{\psi},C} - 1)(y_2 - y_1)\bar{C}^{\psi} \quad \text{for } y_2 \geq y_1 \geq 0 \text{ and } \forall x \in \mathbb{R} \end{aligned} \tag{13}$$

The first line expresses that an increase in the number of stocks does not increase the required amount on the margin account for agent i by more than it increases the required amount on the margin account for agent $\bar{i}^{\psi}$. The second line expresses that an increase in the number of options increases the amount $\bar{i}^{\psi}$ can borrow through the margin account by at least as much as it increases the amount agent i can borrow.

Next, we want to derive the PDE for agent $\bar{i}^{\psi}$ exercise boundary, $\bar{B}^{\psi}$, and option valuation, $\bar{C}^{\psi}$. Consider the portfolio dynamics for agent $\bar{i}^{\psi}$ of buying one (additional) option at price $\bar{C}^{\psi}$, hedging by selling (additional) $\bar{C}^{\psi}_S$ shares of the stock, and fully financing the strategy based on margin loans and the use of equity capital. The value of this fully-financed strategy evolves as according to:

$$\begin{aligned} &\left(\bar{C}^{\psi}_t + \frac{1}{2}\sigma^2 S^2 \bar{C}^{\psi}_{SS}\right) \mathrm{d}t + \bar{C}^{\psi}_S \mathrm{d}S(t) - (1 - m^{\bar{i}^{\psi},C})\bar{C}^{\psi} r^f \mathrm{d}t - m^{\bar{i}^{\psi},C}\bar{C}^{\psi}(r^f + \psi)\mathrm{d}t \\ &- \bar{C}^{\psi}_S \mathrm{d}S(t) + \bar{C}^{\psi}_S S\left(-m^{\bar{i}^{\psi},S} r^f + m^{\bar{i}^{\psi},S}(r^f + \psi) + (r^f - l^{\bar{i}^{\psi}})\right) \mathrm{d}t \end{aligned} \tag{14}$$

The first two terms simply represent the dynamics of the option (as seen in Eqn. (2)). The next two terms represent the funding of the option. Specifically, $(1 - m^{\bar{i}^{\psi},C})\bar{C}^{\psi}$ can be borrowed against the option at the money-market funding cost r^f. The remaining option value, the margin requirement $m^{\bar{i}^{\psi},C}\bar{C}^{\psi}$, must be financed as equity at a rate of $r^f + \psi^{\underline{i}}$.

The second line of (14) represents the terms stemming from the stock position and its financing. The first term is the stock dynamics, given the $\bar{C}^{\psi}_S$ number of shares sold. The last three terms capture the various financing costs. The stock sold to hedge the option decreases the long stock position thereby decreasing the required cash on the margin account by $\bar{C}^{\psi}_S S m^{\bar{i}^{\psi},S}$ which earns the interest r^f. This cash can instead be invested at the rate $r^f + \psi$. Agent $\bar{i}^{\psi}$ reduces the cash that it borrows on the stock lending account by $\bar{C}^{\psi}_S S$ thereby saving the interest paid on this amount. The interest rate on the stock lending account is $r^f - l^{\bar{i}^{\psi}}$. The stochastic terms cancel out. The fully financed strategy with deterministic drift must have drift equal zero for the option valuation to be correct. This leads to the PDE stated in (8). The conditions follow from the fact that the option is an American call option and ensure that the solution will entail the exercise boundary that optimizes the option value, c.f. Merton (1973) and Kim (1990).

Proof of Proposition 1

If the agent wants cash and exercises to immediately sell the stock in the market, the net proceeds would be $S_0(1 - \lambda^{i,S}) - X$. If the investor instead places $\frac{X}{1+r^f}$ in the risk-free asset and sells short the stock, the cash flow would now be $S_0(1 - \lambda^{i,S}) - \frac{X}{1+r^f}$ (as there are no funding or short-sale costs). In the next period the option can be exercised paid by the money from the risk-free asset and the received stock can be used to close the short position. This has a net payment of zero. Thereby the cash flow of the strategy dominates the one created by exercising and selling the stock (since $r^f > 0$).

If the investor wants the exposure to the stock and exercises to get the stock, the cash flow is $-X$. The alternative dominating strategy is to place $\frac{X}{1+r^f}$ in the risk-free asset and wait and exercise later. This costs less today (as $r^f > 0$), and the stock purchase can be made cash flow neutral at time 1 by the money from the risk-free asset. No value is forgone by not owning the stock in the meantime as there is no lending fee, $l^i = 0$, and no funding costs are incurred.

Consider *(ii)* where the sale revenue of the option is above the intrinsic value for agent i, $C_0(1 - \lambda^{i,C}) > S_0 - X$. If agent i wants cash, exercising the option and selling the stock obviously gives less than selling the option. If agent i wants stock, selling the option and buying the stock has a net cost of $S_0 - C_0(1 - \lambda^{i,C})$ which is less than what it costs to gain the stock through exercise, X. Hence, no matter if agent i wants stock or cash, early exercise is dominated.

A sufficient condition for $C_0(1-\lambda^{i,C}) > S_0 - X$ is that agent i faces no option transaction costs, $\lambda^{i,C} = 0$, and the existence of another type of agents j with zero short-sale costs, funding costs, and stock transaction costs. We have just shown above that for an investor without short-sale costs, funding costs or stock transaction costs the value of the option will be at least $S_0 - \frac{X}{1+r^f}$. If such unconstrained agents j exist, the option price must be at least $S_0 - \frac{X}{1+r^f}$ to avoid arbitrage. If $\lambda^{i,C} = 0$ we get: $C_0(1 - \lambda^{i,C}) = C_0 \geq S_0 - \frac{X}{1+r^f} > S_0 - X$.

Proof of Proposition 2

This proof shows that there exists a strategy involving early exercise that is not dominated by any strategy not involving early exercise if frictions are large enough.

We introduce the following notation for the portfolio held by some agent: The number of stocks, $\alpha_0 \in \mathbb{R}$, the amount invested in the risk-free asset, $\beta_0 \in \mathbb{R}$, and the number of options, $\gamma_0 \in \mathbb{R}$.

Let an agent i who is long option, $\gamma_0 > 0$, want as much cash as possible at time 0 without any negative cash flow in any possible state at time 1. One way to do this is to exercise the option position early and close the positions in stock and risk-free asset. If stocks needs to be sold to close the position, stock transaction costs apply. However, in the event that more can be earned through lending out the stock and borrow the present value of the future earned lending fee than through selling the stock, this is done instead. If stock must be acquired this can be done either by buying stock or buying options and exercise immediately. The cheapest way is chosen. The cash-flow at time 0 of this strategy can be described by K:

$$K := \begin{cases} (\alpha_0+\gamma_0)S_0(1-\lambda^S)+\beta_0-\gamma_0 X & \text{if } \alpha_0+\gamma_0 \geq 0 \text{ and } 1-\lambda^S \geq \frac{l^i}{1+r^f} \\ (\alpha_0+\gamma_0)\frac{S_0 l^i}{1+r^f}+\beta_0-\gamma_0 X & \text{if } \alpha_0+\gamma_0 \geq 0 \text{ and } 1-\lambda^S < \frac{l^i}{1+r^f} \\ (\alpha_0+\gamma_0)S_0+\beta_0-\gamma_0 X & \text{if } \alpha_0+\gamma_0 < 0 \text{ and } S_0 \leq C_0+X \\ (\alpha_0+\gamma_0)(C_0+X)+\beta_0-\gamma_0 X & \text{if } \alpha_0+\gamma_0 < 0 \text{ and } S_0 > C_0+X \end{cases} \tag{15}$$

We refer to this strategy as *the liquidating early exercise strategy.*

Any strategy without early exercise is identified by the allocation of wealth between the three assets.[10] This allocation can be denoted by $(\alpha_1, \beta_1, \gamma_1)$ for the number of stocks, amount invested in risk-free asset, and number of options respectively. The cash-flow at time 0 for an arbitrary strategy not involving early exercise can be described by the function J_1 where:

$$\begin{aligned} J_1(\alpha_1, \beta_1, \gamma_1) :=& (\alpha_0-\alpha_1)(S_0-\mathbf{1}_{(\alpha_0-\alpha_1>0)}S_0\lambda^S)+(\beta_0-\beta_1) \\ &+(\gamma_0-\gamma_1)(C_0-\mathbf{1}_{(\gamma_0-\gamma_1>0)}\lambda^{i,C}C_0)-F^i(\alpha_1 S_0, \gamma_1 C_0) \end{aligned}$$

We want to show that there for agent i exists a strategy involving early exercise that is not dominated by any strategy not involving early exercise if frictions are severe enough. Our candidate for such a non-dominated strategy involving early exercise is the liquidating early exercise strategy.

In order for a strategy to dominate the liquidating early exercise strategy the strategy must have non-negative net-liabilities at time 1 in all possible states. Specifically this means that for every stock agent i is short agent i must also be long at least 1 option and have no less than $\frac{X+S_0 L^i}{1+r^f}$ in the risk-free asset. As the stock has unlimited upside the option is needed

[10] Admittedly, the same portfolio can be obtained through several ways of trading. E.g. the number of stocks held can be reduced by 1 either by selling one stock or selling two stocks and buying one back immediately, corresponding to money burning when stock transaction costs are positive. For our purpose it is sufficient to consider strategies with the cheapest way to obtain the portfolio. If such strategies cannot dominate early exercise neither can strategies where money is given up for nothing.

to secure no negative payoff, and the cash is needed to pay the possible future exercise of the option and the short-sale fee of the stock. Moreover, for every option agent i is short, i must be at least long 1 stock, and not use the risk-free asset to borrow more than $\frac{S_0 l^i}{1+r^f}$. The stock is needed to be able to honor a possible future exercise of the option in all states. Since the stock could also turn out to be worthless at time 1 there cannot be borrowed more than $\frac{S_0 l^i}{1+r^f}$, the present value of the only secure income raised from lending out the stock in the period. Only strategies where this is fulfilled can possible dominate the liquidating early exercise strategy. The maximal cash-flow that can be obtained at time $t = 0$ without early exercise and without net-liabilities at time $t = 1$ is then the solution to this problem:

$$\max_{\alpha_1,\beta_1,\gamma_1} J_1(\alpha_1, \beta_1, \gamma_1) \tag{16}$$

$$\text{s.t. } \alpha_1 + \gamma_1 \geq 0 \tag{17}$$

$$\beta_1 + \mathbf{1}_{(\alpha_1<0)}\alpha_1 \frac{X + S_0 L^i}{1 + r^f} + \mathbf{1}_{(\alpha_1>0)}\alpha_1 \frac{S_0 l^i}{1 + r^f} \geq 0 \tag{18}$$

All strategies are either non-dominated or dominated by a non-dominated strategy. Since domination is a transitive property this implies that it is sufficient to show that there exists no non-dominated strategy not involving early exercise that dominates the liquidating early exercise strategy.

If there exists a non-dominated strategy involving selling the option (i.e. $\gamma_1 < \gamma_0$) and $C_0(1-\lambda^{i,C}) \leq S(1-\lambda^{i,S})-X$ then there is a corresponding non-dominated strategy involving early exercise; namely the one with the same trades except that instead of selling the options the same number of options are exercised early and the obtained stock sold immediately. And hence the statement in the proposition is fulfilled.

This means that it is sufficient to show that there are no strategies that does not involve selling the option or exercising it early that dominate the liquidating early exercise strategy. Not selling the option corresponds to $\gamma_1 \geq \gamma_0$. Combining this with (17) we get the inequality $\gamma_1 \geq \max(-\alpha_1, \gamma_0)$. Note that J_1 is decreasing in γ_1 for $\gamma_1 \geq 0$ meaning that this inequality must bind in optimum. Likewise, J_1 is decreasing in β_1 which means that (18) must bind in optimum. We can now substitute both β_1 and γ_1 and reformulate the problem as:

$$\max_{\alpha_1} J_2(\alpha_1) \tag{19}$$

where we define the function J_2 by:

$$\begin{aligned} J_2(\alpha_1) :=& J_1(\alpha_1, -\mathbf{1}_{(\alpha_1<0)}\alpha_1\frac{X+S_0L^i}{1+r^f} - \mathbf{1}_{(\alpha_1>0)}\alpha_1\frac{S_0l^i}{1+r^f}, \max(-\alpha_1,\gamma_0)) \\ =&(\alpha_0-\alpha_1)(S_0-\mathbf{1}_{(\alpha_0-\alpha_1>0)}S_0\lambda^S)+\beta_0+\mathbf{1}_{(\alpha_1<0)}\alpha_1\frac{X+S_0L^i}{1+r^f} \\ &+\mathbf{1}_{(\alpha_1>0)}\alpha_1\frac{S_0l^i}{1+r^f}+(\gamma_0-\max(-\alpha_1,\gamma_0))C_0-F^i(\alpha_1S_0,-\max(-\alpha_1,\gamma_0)C_0) \end{aligned} \tag{20}$$

First, we want to show part *a.* in the proposition. The proposition claims that when $S_0(1-\lambda^{i,S}) > X$ and $C_0(1-\lambda^{i,C}) \leq S_0(1-\lambda^{i,S})-X$ then early exercise is not dominated for i if the short-sale cost, L^i, is large enough. Specifically we show that $L^i > \frac{Xr^f}{S_0}$ is sufficient.

Given this we show that the optimal cash-flow solving (19) is smaller than the cash flow from the liquidating early exercise strategy (15), regardless of funding costs. Since funding costs, F^i, are non-negative it is sufficient to show that the maximum of J_2 is smaller than (15) for $F^i = 0$. So in the following we consider the case where $F^i = 0$. Then J_2 is piece-wise linear in α_1 with a kinks at $\{-\gamma_0, \alpha_0, 0\}$. So a global maximum for the J_2 will either be at $-\gamma_0$, α_0, 0 or be non-existing. The maximum is non-existing if J_2 goes to infinity for α_1 going to either ∞ or $-\infty$. We check the last first:

$$\begin{aligned} \lim_{\alpha_1\to-\infty} J_2(\alpha_1) =& \lim_{\alpha_1\to-\infty}\left[(\alpha_0-\alpha_1)S_0(1-\lambda^{i,S})+\beta_0+\alpha_1\frac{X+S_0L^i}{1+r^f}+(\gamma_0+\alpha_1)C_0\right] \\ =&\alpha_0S_0(1-\lambda^{i,S})+\beta_0+\gamma_0C_0+\lim_{\alpha_1\to-\infty}\left[\alpha_1[-S_0(1-\lambda^{i,S})+\frac{X+S_0L^i}{1+r^f}+C_0]\right] \end{aligned} \tag{21}$$

It must hold that $C_0 + X \geq S_0(1-\lambda^{i,S})$, otherwise there would be an arbitrage strategy by buying options, exercise immediately and sell the stock. Given this and that $L^i > \frac{Xr^f}{S_0}$ the entire expression (21) is diverging to $-\infty$. We next check $\alpha_1 \to \infty$:

$$\lim_{\alpha_1 \to \infty} J_2(\alpha_1) = \lim_{\alpha_1 \to \infty} \left[(\alpha_0 - \alpha_1) S_0 + \beta_0 + \alpha_1 \frac{S_0 l^i}{1 + r^f} \right]$$
$$= \alpha_0 S_0 + \beta_0 + \lim_{\alpha_1 \to \infty} \left[\alpha_1 (-S_0 + \frac{S_0 l^i}{1 + r^f}) \right]$$

No-arbitrage implies that $S_0 > \frac{S_0 l^i}{1+r^f}$. Otherwise an arbitrage gain could be made through buying stocks and lending them out. This in turn implies that J_2 does not go to infinity as α_1 goes to infinity.

We now evaluate the expression in $\alpha_1 = \alpha_0 \leq -\gamma_0$:

$$J_2(\alpha_0) = \beta_0 + \alpha_0 \frac{X + S_0 L^i}{1 + r^f} + (\gamma_0 + \alpha_0) C_0 \tag{22}$$

This is strictly smaller than the cash flow from early exercise in (15) (using that $L^i > \bar{L} = \frac{X r^f}{S_0}$ and $\alpha_0 + \gamma_0 \leq 0$). Next, evaluate where $\alpha_1 = \alpha_0 \in (-\gamma_0, 0]$:

$$J_2(\alpha_0) = \beta_0 + \alpha_0 \frac{X + S_0 L^i}{1 + r^f} \tag{23}$$

This is strictly smaller than (15) (using that $\alpha_0 + \gamma_0 > 0$ and $L^i > \bar{L} = \frac{X r^f}{S_0}$). Evaluating J_2 where $\alpha_1 = \alpha_0 > 0$:

$$J_2(\alpha_0) = \beta_0 + \alpha_0 \frac{S_0 l^i}{1 + r^f}$$

which is strictly smaller than (15).

Next, we evaluate where $\alpha_1 = 0$:

$$J_2(0) = \alpha_0 (S_0 - \mathbf{1}_{(\alpha_0 > 0)} S_0 \lambda^S) + \beta_0 \tag{24}$$

This is also smaller than the cash flow at time $t = 0$ from the liquidating early exercise strategy described by (15).

The final evaluation of the expression is where $\alpha_1 = -\gamma_0$:

$$J_2(-\gamma_0) = (\alpha_0 + \gamma_0)(S_0 - \mathbf{1}_{(\alpha_0+\gamma_0>0)} S_0 \lambda^{i,S}) + \beta_0 - \gamma_0 \frac{X + S_0 L^i}{1 + r^f} \tag{25}$$

Again, this is strictly smaller than (15) since $L^i > \bar{L} = \frac{X r^f}{S_0}$. Hence it has been proved that for sufficiently high short-sale costs early exercise is not dominated regardless of funding costs if $C_0(1 - \lambda^{i,C}) \leq S_0(1 - \lambda^{i,S}) - X$ and $S_0(1 - \lambda^{i,S}) - X > 0$. Now, we want to show that a similar result holds for sufficiently high funding costs instead of short-sale costs.

The proposition states that if $S_0(1 - \lambda^{i,S}) > X$ and $C_0(1 - \lambda^{i,C}) \leq S_0(1 - \lambda^{i,S}) - X$ then early exercise is not dominated when the funding costs, F^i, is large enough. By F^i being large enough we specifically mean that $F^i(x, y) \geq \bar{F}(|x| + |y|)$ where $\bar{F} > \frac{r^f X}{(1+r^f)(S_0+C_0)}$. When this is the case J_3 will be smaller than J_2 where:

$$\begin{aligned} J_3(\alpha_1) :=& (\alpha_0 - \alpha_1)(S_0 - \mathbf{1}_{(\alpha_0-\alpha_1>0)} S_0 \lambda^S) + \beta_0 + \mathbf{1}_{(\alpha_1<0)} \alpha_1 \frac{X + S_0 L^i}{1 + r^f} + \mathbf{1}_{(\alpha_1>0)} \alpha_1 \frac{S_0 l^i}{1 + r^f} \\ &+ (\gamma_0 - \max(-\alpha_1, \gamma_0)) C_0 - \bar{F}(|\alpha_1 S_0| + \max(-\alpha_1, \gamma_0) C_0) \end{aligned} \tag{26}$$

So it is sufficient to show that J_3 is smaller than (15) given that $\bar{F} > \frac{r^f X}{(1+r^f)(S_0+C_0)}$. In the following we assume that this condition is fulfilled. J_3 is piecewise linear with kinks in $\{-\gamma_0, \alpha_0, 0\}$. In order for a global maximum to exist for J_3 it must not go to ∞ as α_1 goes to either ∞ or $-\infty$. We check this:

$$\begin{aligned} &\lim_{\alpha_1 \to -\infty} J_3(\alpha_1) \\ =& \lim_{\alpha_1 \to -\infty} (\alpha_0 - \alpha_1) S_0 (1 - \lambda^S) + \beta_0 + \alpha_1 \frac{X + S_0 L^i}{1 + r^f} + (\gamma_0 + \alpha_1) C_0 + \alpha_1 \bar{F}(S_0 + C_0) \\ =& \alpha_0 + \beta_0 + \gamma_0 C_0 + \lim_{\alpha_1 \to -\infty} \left(\alpha_1 [-S_0(1 - \lambda^S) + \frac{X + S_0 L^i}{1 + r^f} + C_0 + \bar{F}(S_0 + C_0)] \right) \end{aligned}$$

The expression in the squared brackets is positive since no-arbitrage implies that $S(1-\lambda^S) \leq C + X$. If this inequality was not fulfilled an arbitrage strategy would be to buy options, exercise them immediately and sell the obtained stocks. We conclude that J_3 does not go to

∞ for α_1 going to $-\infty$.

$$\begin{aligned}
&\lim_{\alpha_1 \to \infty} J_3(\alpha_1) \\
=&\lim_{\alpha_1 \to \infty} (\alpha_0 - \alpha_1)S_0 + \beta_0 + \alpha_1 \frac{S_0 l^i}{1+r^f} - \bar{F}(\alpha_1 S_0 + \gamma_0 C_0) \\
=&\alpha_0 + \beta_0 - \bar{F}\gamma_0 C_0 + \lim_{\alpha_1 \to \infty} \left[\alpha_1 [-S_0 + \frac{S_0 l^i}{1+r^f} - \bar{F} S_0] \right]
\end{aligned}$$

Since $S_0 > \frac{S_0 l^i}{1+r^f}$ it follows that this does not go to infinity. Next, we evaluate J_3 where $\alpha_1 = -\gamma_0$:

$$J_3(-\gamma_0) = (\alpha_0 + \gamma_0)(S_0 - \mathbf{1}_{(\alpha_0+\gamma_0>0)} S_0 \lambda^S) + \beta_0 - \gamma_0 \frac{X + S_0 L^i}{1+r^f} - \bar{F}(\gamma_0 S_0 + \gamma_0 C_0)$$

This is strictly smaller than the payoff from early exercise in (15) since $\bar{F} > \frac{r^f X}{(1+r^f)(S_0+C_0)}$. For $\alpha_1 = 0$ we get:

$$J_3(0) = \alpha_0(S_0 - \mathbf{1}_{(\alpha_0>0)} S_0 \lambda^S) + \beta_0 - \bar{F}\gamma_0 C_0$$

which is strictly smaller than (15) since $S(1 - \lambda^{i,S}) - X > 0$. Next, evaluate where $\alpha_1 = \alpha_0 \leq -\gamma_0$:

$$J_3(\alpha_0) = \beta_0 + \alpha_0 \frac{X + S_0 L^i}{1+r^f} + (\gamma_0 + \alpha_0)C_0 + \alpha_0 \bar{F}(S_0 + C_0)$$

which is strictly smaller than (15). Evaluate where $\alpha_1 = \alpha_0 \in (-\gamma_0, 0]$:

$$\begin{aligned}
J_3(\alpha_0) &= \beta_0 + \alpha_0 \frac{X + S_0 L^i}{1+r^f} - \bar{F}(-\alpha_0 S_0 + \gamma_0 C_0) \\
&= \beta_0 + \alpha_0 \frac{X + S_0 L^i}{1+r^f} + \alpha_0 \bar{F}(S_0 + C_0) - \bar{F} C_0(\alpha_0 + \gamma_0)
\end{aligned}$$

which is strictly smaller than (15). Finally, we evaluate where $\alpha_1 = \alpha_0 > 0$:

$$J_3(\alpha_0) = \beta_0 + \alpha_0 \frac{S_0 l^i}{1+r^f} - \bar{F}(\alpha_0 S_0 + \gamma_0 C_0)$$

which is also strictly smaller than (15). Hence, for funding cost large enough, i.e. $F^i(x,y) \geq \bar{F}(|x|+|y|)$ where $\bar{F} > \frac{r^f X}{(1+r^f)(S_0+C_0)}$, early exercise is not dominated (regardless of short-sale costs).

Proof of Proposition 3

Both the problem for the lower boundary, (6)–(7), and the problem for the upper boundary, (8) with its boundary conditions, are mathematical equivalent to problem of pricing American call options in the Black-Scholes-Merton model with continuous dividend yield which is the problem Dewynne, Howison, Rupf, and Wilmott (1993) study. *(ii)+(iii)* follow from their results. If what corresponds to the dividend yield is zero or negative (in *(ii)* $L^{\underline{i}} + \psi^{\underline{i}}(m^{\underline{i},C} + m^{\underline{i},S})$, in *(iii)* $l^{\bar{i}} + \psi(m^{\bar{i},C} - m^{\bar{i},S})$) Merton's lower bound still holds and early exercise is always dominated implying infinite exercise boundaries for $t < T$. If what corresponds to the dividend yield is positive the exercise boundary is finite.

Next, we will prove *(i)*. If $S(t) < \underline{B}(T-t)$ then $\underline{C} > S(t) - X$, it is dominated for agent $\underline{i}$ to exercise early, and (6) must hold. The equation is based on the fact that a fully financed position with deterministic drift must have drift zero. Now, what is the right valuation and exercise strategy for agent i who owns one option? What if i assigned the same value to the option as $\underline{i}$ and also hedged the option by selling off $\underline{C}_S$ stocks and financed the equity amount at its equity rate, $r^f + \psi^i$? The risk of the strategy would be zero because of the perfect hedge leaving only a deterministic dt-term times this value:

$$\begin{aligned} &\underline{C}_t + \frac{1}{2}\sigma^2 S^2 \underline{C}_{SS} - [K^i(x - \underline{C}_S, 1) - K^i(x,0) + \underline{C}](r^f + \psi^i) \\ &+ \underline{C}_S S(r^f - \tilde{l}^i) + [K^i(x - \underline{C}_S, 1) - K^i(x,0)]r^f \end{aligned} \tag{27}$$

where $x \in \mathbb{R}$ is an arbitrary number of stocks held by i. $\tilde{l}^i$ is equal to L^i if $x \leq 0$, l^i if $x \geq \underline{C}_S$, and $\frac{x}{\underline{C}_S} l^i + \frac{\underline{C}_S - x}{\underline{C}_S} L^i$ if $0 < x < \underline{C}_S$. The last case reflects the case where some but not all of the stock sold to hedge the option must be shorted. $[K^i(x - \underline{C}_S, 1) - K^i(x,0)]$ represents the increase in required amount on the margin account relative to not holding and hedging one option. The change in required equity is the change in margin requirement plus $\underline{C}$, as the hedge must have the same cash flow as if the option was sold for $\underline{C}$. We can now show that (27) is greater or equal to zero:

$$\begin{aligned}
(27) \geq & \underline{C}_t + \frac{1}{2}\sigma^2 S^2 \underline{C}_{SS} - [m^{\underline{i},S} S \underline{C}_S + (m^{\underline{i},C} - 1)\underline{C}]\psi^i - \underline{C}(r^f + \psi^i) + \underline{C}_S S(r^f - \tilde{l}^i) \\
= & \underline{C}_t + \frac{1}{2}\sigma^2 S^2 \underline{C}_{SS} - [m^{\underline{i},S} S \underline{C}_S + m^{\underline{i},C}\underline{C}]\psi^i - \underline{C} r^f + \underline{C}_S S(r^f - \tilde{l}^i) \\
\geq & \underline{C}_t + \frac{1}{2}\sigma^2 S^2 \underline{C}_{SS} - [m^{\underline{i},S} S \underline{C}_S + m^{\underline{i},C}\underline{C}]\psi^{\underline{i}} - \underline{C} r^f + \underline{C}_S S(r^f - L^{\underline{i}}) \\
= & 0
\end{aligned}$$

The first inequality follows from (4), the second inequality follows from $\psi^i \leq \psi^{\underline{i}}$ and $L^i \leq L^{\underline{i}}$, besides the assumptions in the model for all agents i: $L^i \geq l^i \geq 0$, $\psi^i \geq 0$, and $r^f > 0$. The last equality follows from (6). This non-negative drift means that i must assign a value to the option no smaller than agent $\underline{i}$ does, because the option could be hedged and generate a risk-free payment of at least $\underline{C}$ pr. option while for sure not give any negative cash flow in the future; i.e. $C^i \geq \underline{C}$. Since $S(t) < \underline{B}(T-t)$ then $\underline{C} > S(t) - X$ and thus $C^i > S(t) - X$. This means that it is dominated for i to exercise the option early.[11]

Turn to the upper bound part of *(i)*. We will show that if it is not dominated for i not to exercise early neither is it for $\bar{i} := \bar{i}^{\psi^i}$, implying that it is dominated for i not to exercise whenever $S(t) > \bar{B}^{\psi^i}(T-t)$ (proof by contrapositive). At any point in time where it is not dominated for i not to exercise early, i can hedge the option position by selling C^i_S stocks and financing it at equity rate $r^f + \psi^i$. The perfect hedge leaves a deterministic $\mathrm{d}t$ term that must be equal zero in order for the option to be correctly valued. This equality that must hold is

$$\begin{aligned}
& C^i_t + \frac{1}{2}\sigma^2 S^2 C^i_{SS} - [K^i(x - C^i_S, 1) - K^i(x, 0) + C^i](r^f + \psi^i) \\
& + C^i_S S(r^f - \tilde{l}^i) + [K^i(x - C^i_S, 1) - K^i(x, 0)]r^f = 0
\end{aligned} \tag{28}$$

Combining (28) with (13) and $l^i \geq l^{\bar{i}}$ we get that

[11]The proposition and proof could easily be extended from showing that it is dominated to exercise the option if i holds one option to show that it is also dominated to exercise the whole or a fraction of any positive option position. This would require heavier notation without changing the method of the proof.

$$C^i_t + \frac{1}{2}\sigma^2 S^2 C^i_{SS} - [(m^{\bar{i},C} - 1)\bar{C}^{\psi^i} - m^{\bar{i},S} C^i_S S]\psi^i + C^i_S S(r^f - l^{bari}) - C^i(r^f + \psi^i) \geq 0 \quad (29)$$

What if $\bar{i}$ like i assigned the value C^i to the option and also hedged an option by selling off C^i_S stocks and finance the required equity amount at its equity rate, $r^f + \psi^i$? The risk of the strategy would be zero because of the perfect hedge leaving only a deterministic dt-term times this value:

$$\begin{aligned}
&C^i_t + \frac{1}{2}\sigma^2 S^2 C^i_{SS} - [K^{\bar{i}}(x - C^i_S, 1) - K^{\bar{i}}(x, 0) + C^i](r^f + \psi^{\bar{i}}) \\
&+ C^i_S S(r^f - l^{\bar{i}}) + [K^{\bar{i}}(x - C^i_S, 1) - K^{\bar{i}}(x, 0)]r^f \qquad (30)\\
=&C^i_t + \frac{1}{2}\sigma^2 S^2 C^i_{SS} - [(m^{\bar{i},C} - 1)\bar{C}^{\psi^i} - m^{\bar{i},S} C^i_S S]\psi^i + C^i_S S(r^f - l^{\bar{i}}) - C^i(r^f + \psi^i)
\end{aligned}$$

This equals the left-hand side of (29) and hence must also be at least zero. This non-negative drift implies that $\bar{i}$ must assign a value to the option that is not immediately exercised no smaller than the value assigned by i, $\bar{C}^{\psi^i} \geq C^i$. C^i is no smaller than $S - X$ since it is not dominated for i not to exercise early. And hence it is not dominated for $\bar{i}$ not to exercise the option early when it is not dominated for i not to exercise the option early. This means that it is dominated for i not to exercise the option early whenever it is dominated for $\bar{i} = \bar{i}^{\psi^i}$ not to exercise the option early, which is the case whenever $S(t) > \bar{B}(T - t)$.

Proof of Proposition 4

This result follows from Proposition 3. $\underline{B}$ is the exercise boundary for agent $\underline{i}$. Consider the hypothetical agent j who, like $\underline{i}$, is short stock for $t < T$ and has the same funding cost as $\underline{i}$: $\psi^j = \psi^{\underline{i}}$. Also, let $L^j < L^{\underline{i}}$ and

$$K^j(x, y) = m^{\underline{i},S} S|x| + (m^{\underline{i},C} - 1)\hat{C} y \qquad (31)$$

for some $\hat{C} \geq \underline{C}$. Adding such agent j to the pool of agents will not change the characteristics of the "extreme" hypothetical agent $\underline{i}$, since $L^{\underline{i}} \geq L^j$, $\psi^{\underline{i}} \geq \psi^j$, and (4) is satisfied for $i = j$. Proposition 3 then gives that it is dominated for agent j to exercise whenever $S(t) < \underline{B}(T - t)$

and that $C^j \geq \underline{C}$ (remember that $m^{\underline{i},C} \in [0,1]$). Specifically this is true when $\hat{C} = C^j$, and when this is true j is identical to $\underline{i}$ except that j has smaller short-sale costs. The PDE with boundary constraints that j's option valuation must satisfy is identical to the problem for $\underline{i}$ stated in (6)–(7) except that L^j replaces L^i. The solution likewise gives an exercise boundary. The comparative statics for $\underline{B}$ when changing $L^{\underline{i}}$ thus can be examined by looking at the difference for the exercise boundaries for agent $\underline{i}$ and agent j. Since it is dominated for j to exercise whenever $S(t) < \underline{B}(T-t)$ the exercise boundary for j is at least as large as for $\underline{i}$. The implication is that $\underline{B}$ is weakly decreasing in short-sale cost $L^{\underline{i}}$. This argument can be repeated in an equivalent way for both funding costs, $\psi^{\underline{i}}$, option margin requirements, $m^{\underline{i},C}$, and stock margin requirements $m^{\underline{i},S}$.

Correspondingly, the argument can be replicated for the comparative statics for $\bar{B}^\psi$ for lending fee, $l^{\bar{i}^\psi}$, option margin requirement $m^{\bar{i}^\psi,C}$, and stock margin requirement, $m^{\bar{i}^\psi,S}$. We choose to show it for option margin requirement. Consider an agent k who is long stock for $t < T$ and has funding cost $\psi^k = \psi$, lending fee $l^k = l^{\bar{i}^\psi}$ and

$$K^k(x,y) = m^{\bar{i}^\psi,S} S|x| + (m^{k,C} - 1)\hat{C}y \tag{32}$$

for $m^{k,C} > m^{\bar{i}^\psi,C}$ and some $\hat{C} \leq \bar{C}^\psi$. The requirements that agent $\bar{i}^\psi$ must satisfy for all agents i then in particular are satisfied for agent $i = k$. Proposition 3 then implies that the option value C^k is no larger than C^ψ. This then also holds for $\hat{C} = C^k$. When this is the case k is identical to $\bar{i}^\psi$ except for the increased option margin requirement. The exercise boundary for k will be the solution to a problem equal to (8) with its boundary conditions except that $m^{k,C}$ replaces $m^{\bar{i}^\psi,C}$. Thus we can study how an increase in the option margin requirements for agent $\bar{i}^\psi$ affects $\bar{B}^\psi$ by comparing the exercise boundary for agent $\bar{i}^\psi$ and agent k. Proposition 3 gives that it is dominated for agent k not to exercise whenever $S(t) > \bar{B}^\psi(T-t)$ implying that k's exercise boundary is no larger than $\bar{i}^\psi$'s exercise boundary. In conclusion agent $\bar{i}^\psi$'s exercise boundary decreases weakly when $m^{\bar{i}^\psi,C}$ increases.

Table I
Data Sources.

This table shows the data sources used in our study, the variables that we use, the start and end date of each data source, the number of securities, and the number of observations (which is the number of rows in the data).

Data set	Data	Start date	End date	Number of call options/ convertible bond series	Number of underlying securities	Number of observations
CRSP[1]	Dividends, prices, corporate events	30-08-1985	31-12-2011		23,597	18,314,652
OCC Exercises[2]	Exercises of equity options	01-07-2001	31-08-2010	821,052	5,727	7,852,739
OptionMetrics	Option prices, open interests, volatilities, expected future dividends	01-01-1996	31-01-2012	3,949,199	7,509	355,259,334
Data Explorers[3]	Short-sale costs	19-06-2002	03-12-2012		41,188	55,139,348
Mergent FISD[4]	Convertible bond features and conversions	30-08-1985	05-06-2012	4,539	1,731	14,194
Bloomberg	LIBOR-OIS spreads	02-01-1990	22-01-2013			8,620
Thomson One[5]	Equity Issues, mergers and acquisitions	12-01-1963	17-09-2015		402,101	439,456

[1] CRSP data start in 1926, but we only use it when we have option and convertible bond data.

[2] Data for the months November 2001, January and July 2002, and January 2006 are missing.

[3] We focus on the Daily Cost of Borrow Score (DCBS), which is first observed from October 22, 2003.

[4] Mergent FISD has earlier bond observations, but this is the first date a convertible bond can be observed

[5] The number of underlyings and observations for Thomson One are based on the sub-sample from 2003-01-01 to 2010-12-31

Table II

Summary Statistics.

This table summarizes the number of early exercises and early conversions in the sample used in our study. It reports the number of contracts exercised and bonds converted. The total numbers are broken into categories of Moneyness, Expiration, and Agent Type performing the exercise. Options are defined as "out of the money" if the closing stock price is below the strike price, "in the money" if the stock price is 0–25% above the strike price, and "deep in the money" if the stock price is more than 25% higher than the strike price. For convertible bonds the definition is parallel with conversion price used instead of strike price.

Panel A: Early Exercises of Equity Call Options in sample

	Exercises (number of contracts, millions)	Exercises (value of strike, USD millions)	Exercises (intrinsic value, USD millions)
All	1,806	36,250	22,811
By moneyness			
Out of the money	2	38	-1
In the money	480	14,170	1,942
Deep in the money	1,324	22,042	20,870
By time to expiration			
Less than 3 months	1,720	35,422	21,199
Between 3 and 9 months	72	690	1,006
More than 9 months	13	137	605
By agent type			
Customer	808	16,007	6,338
Firm	81	1,955	998
Market maker	916	18,288	15,475

Panel B: Early Conversions of Convertible Bonds in sample

	Conversions (number of bonds, millions)	Conversions (principal amount, USD millions)	Conversions (value of stock, USD millions)
All	25.4	5,655	7,732
By moneyness			
Out of the money	3.8	2,063	1,255
In the money	15.4	1,001	1,127
Deep in the money	6.3	2,591	5,350

Table III
Early Exercise of Equity Options by Short-Sale Costs.

This table shows the average number of early option exercises as a fraction of open interest on the previous day for options sorted on the short-sale cost of the underlying equity. The table further classifies options by their moneyness, expiration, and agent type. Options are defined as "out of the money" if the closing stock price is below the strike price, "in the money" if stock price is 0–25% above strike price, and "deep in the money" if stock price is more than 25% higher than strike price. We note that the number of exercises for each agent type is reported as a fraction of the total open interest since our data do not include open interest by agent type. Standard errors are reported in parenthesis.

	1 Low cost of shorting	2	3	4	5	6	7	8	9	10 High cost of shorting
All	0.17%	0.31%	0.44%	0.58%	0.85%	1.21%	1.74%	2.57%	3.05%	4.28%
	(0.00%)	(0.00%)	(0.01%)	(0.01%)	(0.02%)	(0.02%)	(0.03%)	(0.04%)	(0.05%)	(0.07%)
By moneyness										
Out of the money	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%
	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.01%)
In the money	0.10%	0.18%	0.24%	0.28%	0.31%	0.32%	0.38%	0.41%	0.50%	0.63%
	(0.00%)	(0.00%)	(0.01%)	(0.01%)	(0.02%)	(0.02%)	(0.02%)	(0.03%)	(0.03%)	(0.05%)
Deep in the money	0.31%	0.55%	0.78%	1.06%	1.51%	2.23%	3.13%	4.52%	5.75%	8.44%
	(0.00%)	(0.01%)	(0.01%)	(0.02%)	(0.03%)	(0.04%)	(0.05%)	(0.08%)	(0.09%)	(0.14%)
By time to expiration										
Less than 3 months	0.28%	0.51%	0.71%	0.99%	1.49%	2.10%	2.97%	4.51%	5.32%	7.58%
	(0.00%)	(0.01%)	(0.01%)	(0.02%)	(0.03%)	(0.04%)	(0.05%)	(0.08%)	(0.09%)	(0.14%)
Between 3 and 9 months	0.02%	0.05%	0.09%	0.10%	0.21%	0.29%	0.48%	0.61%	0.88%	1.15%
	(0.00%)	(0.00%)	(0.01%)	(0.01%)	(0.01%)	(0.02%)	(0.02%)	(0.03%)	(0.04%)	(0.07%)
More than 9 months	0.01%	0.03%	0.07%	0.08%	0.10%	0.14%	0.23%	0.53%	0.40%	0.29%
	(0.00%)	(0.00%)	(0.01%)	(0.01%)	(0.02%)	(0.03%)	(0.03%)	(0.05%)	(0.05%)	(0.05%)
By agent type										
Customer	0.11%	0.19%	0.24%	0.32%	0.43%	0.56%	0.76%	0.96%	1.24%	1.76%
	(0.00%)	(0.00%)	(0.00%)	(0.01%)	(0.01%)	(0.02%)	(0.02%)	(0.02%)	(0.03%)	(0.04%)
Firm	0.01%	0.02%	0.02%	0.03%	0.06%	0.07%	0.11%	0.13%	0.17%	0.17%
	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.01%)	(0.01%)	(0.01%)	(0.01%)
Market maker	0.05%	0.11%	0.18%	0.23%	0.37%	0.58%	0.86%	1.48%	1.64%	2.36%
	(0.00%)	(0.00%)	(0.00%)	(0.01%)	(0.01%)	(0.02%)	(0.02%)	(0.03%)	(0.03%)	(0.05%)

Table IV

Early Exercise of Equity Options by Transaction Costs.

This table shows the average number of early option exercises as a fraction of open interest on the previous day for options sorted on the transaction costs. Transaction costs for options are measured daily as the bid-ask spread divided by the mid price for the at-the-money option with same underlying, expiration, and observation day. Observations are grouped into quintiles by transaction costs. The table further classifies options by their moneyness, expiration, and agent type. Options are defined as "out of the money" if the closing stock price is below the strike price, "in the money" if stock price is 0–25% above strike price, and "deep in the money" if stock price is more than 25% larger than strike price. We note that the number of exercises for each agent type is reported as a fraction of the total open interest since our data do not include open interest by agent type. Standard errors are reported in parenthesis.

	1 Low T-cost	**2**	**3**	**4**	**5 High T-cost**	**5-1**
All	0.13%	0.16%	0.21%	0.27%	0.50%	**0.37%**
	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	**(0.00%)**
By moneyness						
Out of the money	0.00%	0.00%	0.00%	0.00%	0.00%	**0.00%**
	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	**(0.00%)**
In the money	0.02%	0.03%	0.05%	0.11%	0.35%	**0.33%**
	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	**(0.00%)**
Deep in the money	0.25%	0.34%	0.47%	0.62%	0.98%	**0.73%**
	(0.00%)	(0.00%)	(0.00%)	(0.01%)	(0.01%)	**(0.01%)**
By time to expiration						
Less than 3 months	0.29%	0.31%	0.36%	0.42%	0.62%	**0.33%**
	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	**(0.00%)**
Between 3 and 9 months	0.04%	0.04%	0.04%	0.05%	0.07%	**0.03%**
	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	**(0.00%)**
More than 9 months	0.02%	0.02%	0.04%	0.04%	0.03%	**0.00%**
	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	**(0.00%)**
By agent type						
Customer	0.06%	0.08%	0.11%	0.16%	0.35%	**0.30%**
	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	**(0.00%)**
Firm	0.01%	0.01%	0.01%	0.01%	0.02%	**0.02%**
	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	**(0.00%)**
Market maker	0.07%	0.07%	0.08%	0.10%	0.12%	**0.05%**
	(0.00%)	(0.00%)	(0.00%)	(0.00%)	(0.00%)	**(0.00%)**

Table V

Early Exercise of Equity Options: Model-Implied Lower Exercise Boundary.

This table shows the fraction of the observed early option exercise decisions that can be rationalized by our model for our full sample (Panel A), a sample excluding corporate events (Panel B), and a sample of options with more than 9 months to expiration (Panel C). An early exercise can be rationalized by our model when the daily high of the stock price, S, is above the model-implied estimated lower exercise boundary, $\underline{B}$. Here, $\underline{B}$ is computed as the solution to the PDE (6)–(7) based on the estimated volatility, short-sale cost, and funding cost. The table reports this fraction for all agents and by agent type (across the different rows) and for four different implementations of the model (across the columns). The first column estimates the volatility as the 60-day historical volatility and the short-sale cost as the median short-sale cost among stock loans of this type (DCBS score). The second column uses a more conservative (i.e., lower) estimate of expected future volatility. The third column also uses a conservative (i.e., higher) estimate of short-sale costs, namely the 90% percentile among stock loans of this type. The fourth column has the most conservative boundary, which is 90% of the estimated boundary from column three.

	Exercises w/ S>B	**Exercises w/ S>B, conservative volatility estimate**	**Exercises w/ S>B, conservative volatility and short-sale cost estimates**	**Exercises w/ S>B, conservative boundary**
		Panel A. Early exercises: full sample		
All	65.8%	75.1%	79.4%	84.2%
By agent type				
Customer	41.3%	51.3%	58.2%	66.8%
Firm	64.2%	76.6%	79.7%	84.2%
Market maker	86.0%	94.4%	96.7%	98.4%
		Panel B. Early exercises: excluding corporate events		
All	69.4%	77.3%	81.8%	86.4%
By agent type				
Customer	45.0%	54.0%	61.3%	70.0%
Firm	70.4%	80.6%	83.9%	87.8%
Market maker	88.1%	94.9%	97.3%	98.9%
		Panel C. Early exercises: more than 9 months to expiration		
All	82.9%	88.4%	89.8%	90.1%
By agent type				
Customer	53.0%	65.3%	69.2%	70.3%
Firm	72.9%	90.7%	91.1%	91.3%
Market maker	98.3%	99.5%	99.7%	99.7%

Table VI

Early Exercise of Equity Options: Regression Analysis.

This table shows logit (Panel A) and probit (Panel B) regressions for the determinants of early option exercises. The dependent variable is 1 for every option contract that is exercised early and 0 for every contract outstanding at end of the previous day that is not exercised early on a given day. The independent variables are as follows. The first variable is the indicator that the stock closing price S is above the model-implied lower exercise boundary, $\underline{B}$, and the second variable is $(S-\underline{B})^{+}/\underline{B}$. The third variable is the indicator that the option's closing bid price, C_{bid}, is below the intrinsic value given the strike price, X. Moneyness is S/X. Short-sale cost score is the observed DCBS score, where a higher score indicates a higher cost. Bid-ask spread is the relative bid-ask spread for the option closest to at-the-money with same underlying and expiry date. Time to expiration is in 100 days. Historical volatility is estimated based on the previous 60 days stock returns. The LIBOR-OIS spread is in basis points. We report estimated t-statistics in parenthesis based on standard errors that account for cross-sectional and time-series correlations using the method of Fama and MacBeth (1973). First, the regressions are estimated in each 3-month sub-sample. Second, the full sample parameters are estimated as the sample means of the estimates and the standard errors are estimated based on the sample standard deviation of parameter estimates across sub-samples, using the Newey-West correction.

Panel A: Logit

Independent variables	(1)	(2)	(3)	(4)	(5)
$1_{(S>\underline{B})}$	4.09			1.84	1.19
	(13.42)			(9.84)	(6.06)
$[(S-\underline{B})/\underline{B}]^{+}$		2.13		0.45	0.39
		(5.98)		(2.72)	(1.30)
$1_{(C\_bid<S-X)}$			5.23	4.46	3.92
			(20.74)	(17.93)	(23.47)
Moneyness					0.03
					(0.32)
Short-sale cost score					0.33
					(10.25)
Bid-ask spread					0.26
					(3.19)
Time to expiration					-1.61
					-(6.20)
Historical volatility					0.73
					(5.50)
LIBOR-OIS spread					0.25
					(0.99)
Intercept	-8.54	-7.75	-10.63	-10.70	-12.89
	(38.83)	-(36.70)	-(53.54)	-(50.09)	-(6.25)
Method	Logit	Logit	Logit	Logit	Logit

Panel B: Probit

Independent variables	(1)	(2)	(3)	(4)	(5)
$1_{(S>\underline{B})}$	1.29			0.62	0.42
	(11.71)			(9.54)	(5.91)
$[(S-\underline{B})/\underline{B}]^{+}$		0.91		0.24	0.20
		(5.81)		(2.89)	(1.78)
$1_{(C\_bid<S-K)}$			1.45	1.22	1.15
			(17.04)	(17.85)	(18.13)
Moneyness					0.03
					(0.82)
Short-sale cost score					0.13
					(9.88)
Bid-ask spread					0.12
					(6.44)
Time to expiration					-0.57
					-(6.00)
Historical volatility					0.24
					(6.71)
LIBOR-OIS spread					0.06
					(0.79)
Intercept	-3.54	-3.35	-4.06	-4.10	-4.73
	(46.99)	-(58.35)	-(85.96)	-(79.33)	-(7.19)
Method	Probit	Probit	Probit	Probit	Probit

Table VII

Early Exercise of Equity Options: Short-Sale Ban Difference-in-Differences.

This table shows logit and probit regressions that study whether the short-sale ban caused a rise in early option exercises for affected stocks. The short-sale ban applied to US financial stocks and serves as a natural experiment. The dependent variable is 1 for every option contract exercised early and 0 for every contract outstanding at end of the previous day not exercised early. The "Ban period" and "Affected stock" are dummy variables for the ban period (Sep. 19 to Oct. 8) and the affected stocks, respectively. Using a difference-in-differences analysis, the key variable in the interaction: The positive, statistically significant "Affected stock × Ban period" supports that the short-sale ban increased early exercise for options written on affected stocks relative to options on unaffected stocks. T-statistics are in parenthesis.

Independent variables	(1)	(2)	(3)	(4)
$1_{(S>\underline{B})}$	1.43	1.49	0.54	0.58
	(11.21)	(8.88)	(14.09)	(11.79)
$[(S-\underline{B})/\underline{B}]^{+}$	1.09	1.32	0.58	0.67
	(4.66)	(4.98)	(5.83)	(5.81)
$1_{(C\_bid<S-X)}$	3.96	4.24	1.38	1.52
	(9.64)	(9.18)	(10.46)	(9.62)
Moneyness	-0.16	-0.32	-0.11	-0.17
	-(0.96)	-(1.63)	-(1.75)	-(2.19)
Aug 08 short-sale cost score	0.31	0.31	0.12	0.12
	(13.41)	(13.22)	(11.90)	(11.64)
Bid-ask spread	0.22	0.15	0.13	0.09
	(1.05)	(0.92)	(1.46)	(1.26)
Time to expiration	-1.58	-1.77	-0.59	-0.66
	-(9.95)	-(8.30)	-(9.75)	-(8.06)
Historical volatility	-0.34	-0.28	-0.08	-0.06
	-(2.30)	-(1.90)	-(1.51)	-(1.21)
Affected stock	0.60	0.74	0.26	0.31
	(2.04)	(2.58)	(1.88)	(2.40)
Ban period	0.49		0.17	
	(3.12)		(2.87)	
Affected stock × Ban period	**1.19**	**1.21**	**0.54**	**0.51**
	(2.95)	**(3.16)**	**(3.34)**	**(3.44)**
LIBOR-OIS spread	0.00		0.00	
	-(1.21)		-(1.39)	
Intercept	Yes	No	Yes	No
Date fixed effect	No	Yes	No	Yes
Method	Logit	Logit	Probit	Probit

Table VIII

Early Conversion of Convertible Bonds.

This table shows the average amount of early conversion as a fraction of the outstanding amount for convertible bonds sorted on the short-sale costs of the underlying equity. The observations are grouped into Low, Medium, and High cost of shorting based on the DCBS score from Data Explorers. "Low" cost reflects a DCBS score of 1, "Medium" a score of 2–5, and "High" 6–10. The table further classifies convertible bonds by their moneyness. Convertible bonds are defined as "out of the money" when stock price is below conversion price, "in the money" when stock price is at or up to 25% above the conversion price and "deep in the money" when stock price is more than 25% above conversion price. The conversion price defines the amount of face value of bond that must be converted to obtain one share of the underlying stock. Standard errors are reported in parenthesis.

	1 Low cost of shorting	2	3 High cost of shorting	3-1
Panel A: All	0.05%	0.05%	0.12%	**0.07%**
	(0.01%)	(0.02%)	(0.07%)	**(0.09%)**
Panel B: By Moneyness				
Out of the money	0.02%	0.00%	0.00%	**-0.03%**
	(0.01%)	(0.00%)	(0.00%)	**(0.02%)**
In the money	0.06%	0.07%	0.16%	**0.07%**
	(0.03%)	(0.07%)	(0.13%)	**(0.13%)**
Deep in the money	0.07%	0.12%	0.29%	**0.19%**
	(0.02%)	(0.06%)	(0.18%)	**(0.23%)**

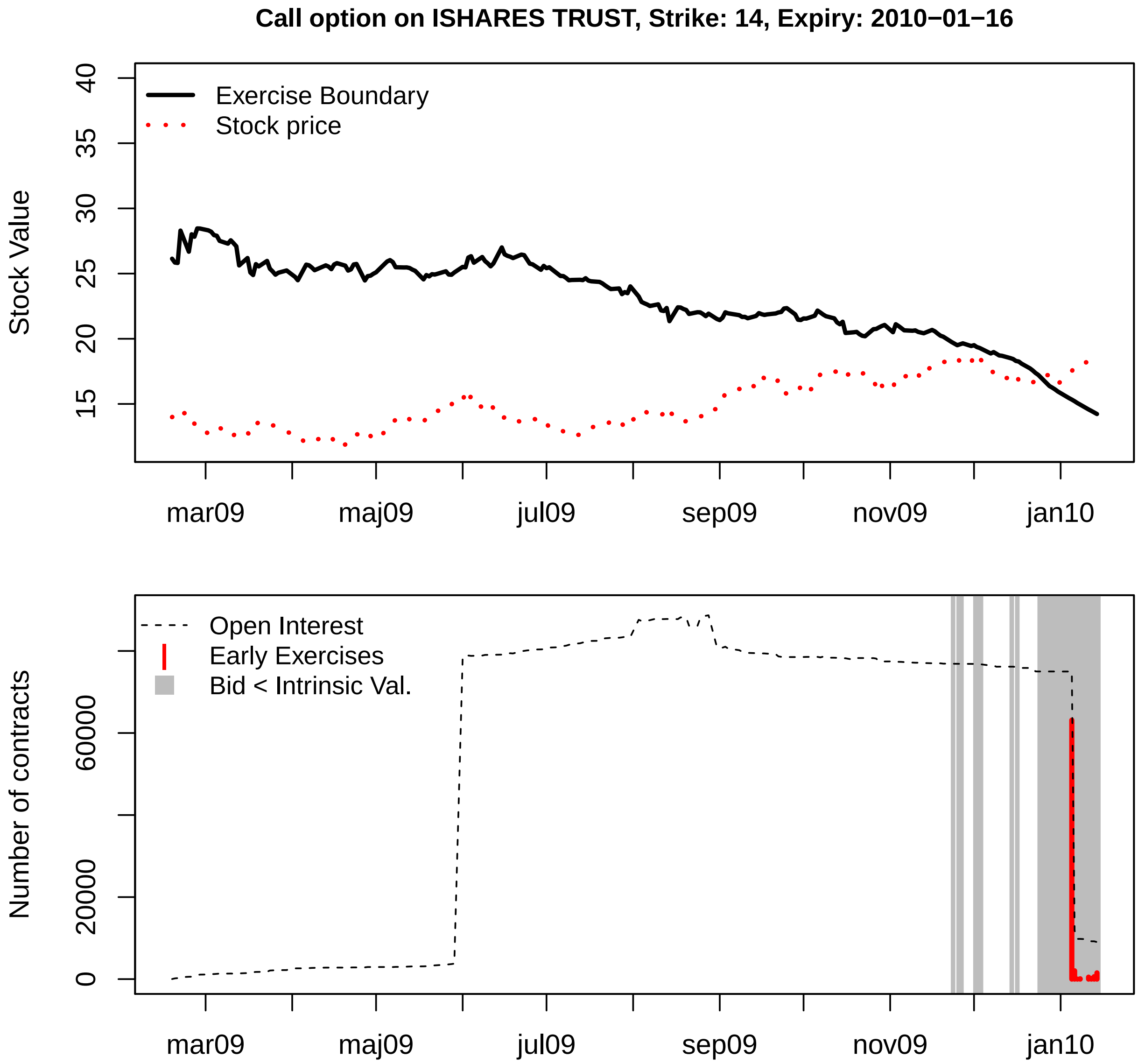

Figure 1: **Early Exercise of Call Options before Expiration: IShares Trust.** The upper panel shows the daily closing price of Ishares Trust stock (silver) and the model-implied lower exercise boundary based on the following parameters: The risk-free rate is the Fed funds rate, the volatility is estimated as the 60-day historical volatility, the short-sale fee is from Data Explorers, the funding cost is the LIBOR-OIS spread, and the assumed margin requirements are $m^{i,C} = 100\%$ for the option and $m^{i,S} = 50\%$ for the stock. The lower panel shows the open interest and early exercise of the option. Shortly after the stock price is above the exercise boundary, 84% of the open interest is exercised in one day. Early exercises are also observed the following days. The closing bid-price of the option is below closing bid-price of the stock minus strike price in periods with gray background.

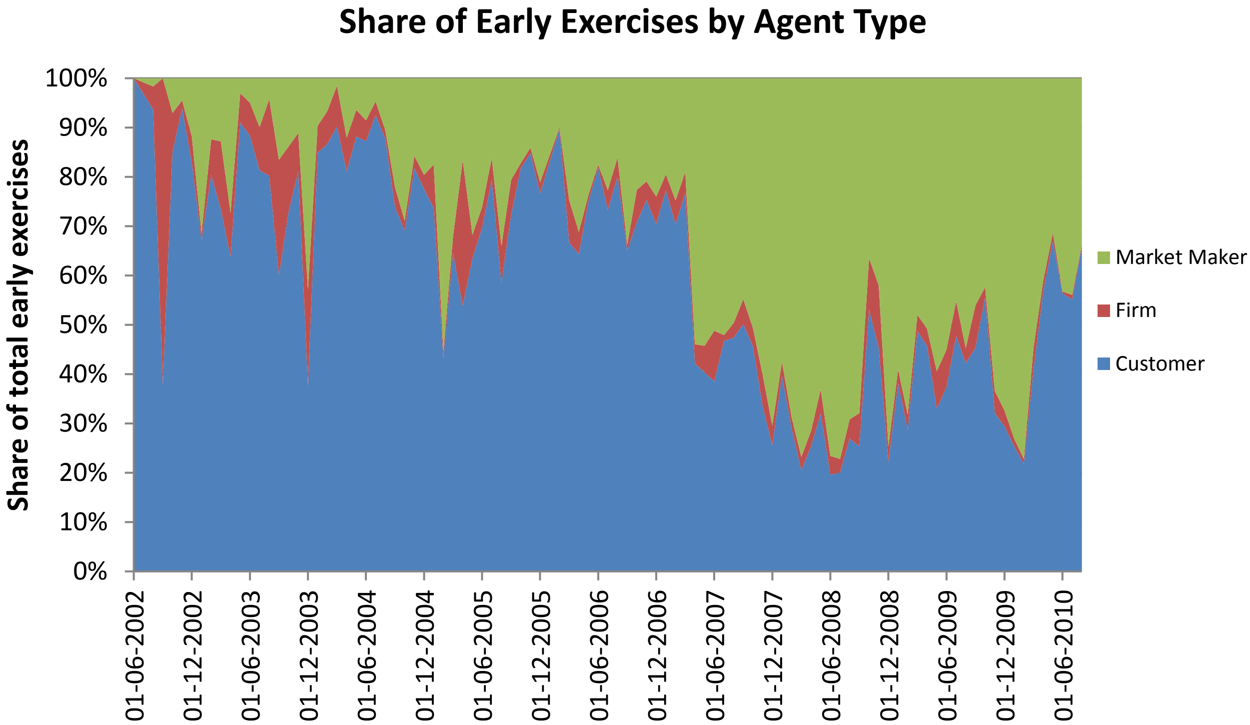

Figure 2: **Share of Early Exercises by Agent Type Over Time.** This figure shows how the monthly relative share of the number of total early exercises are distributed among the three agent types: Customers (retail costumers and hedge funds), Firms (proprietary traders), and Market Makers. We see early exercises for all three groups and an increasing share from Market Makers over time.

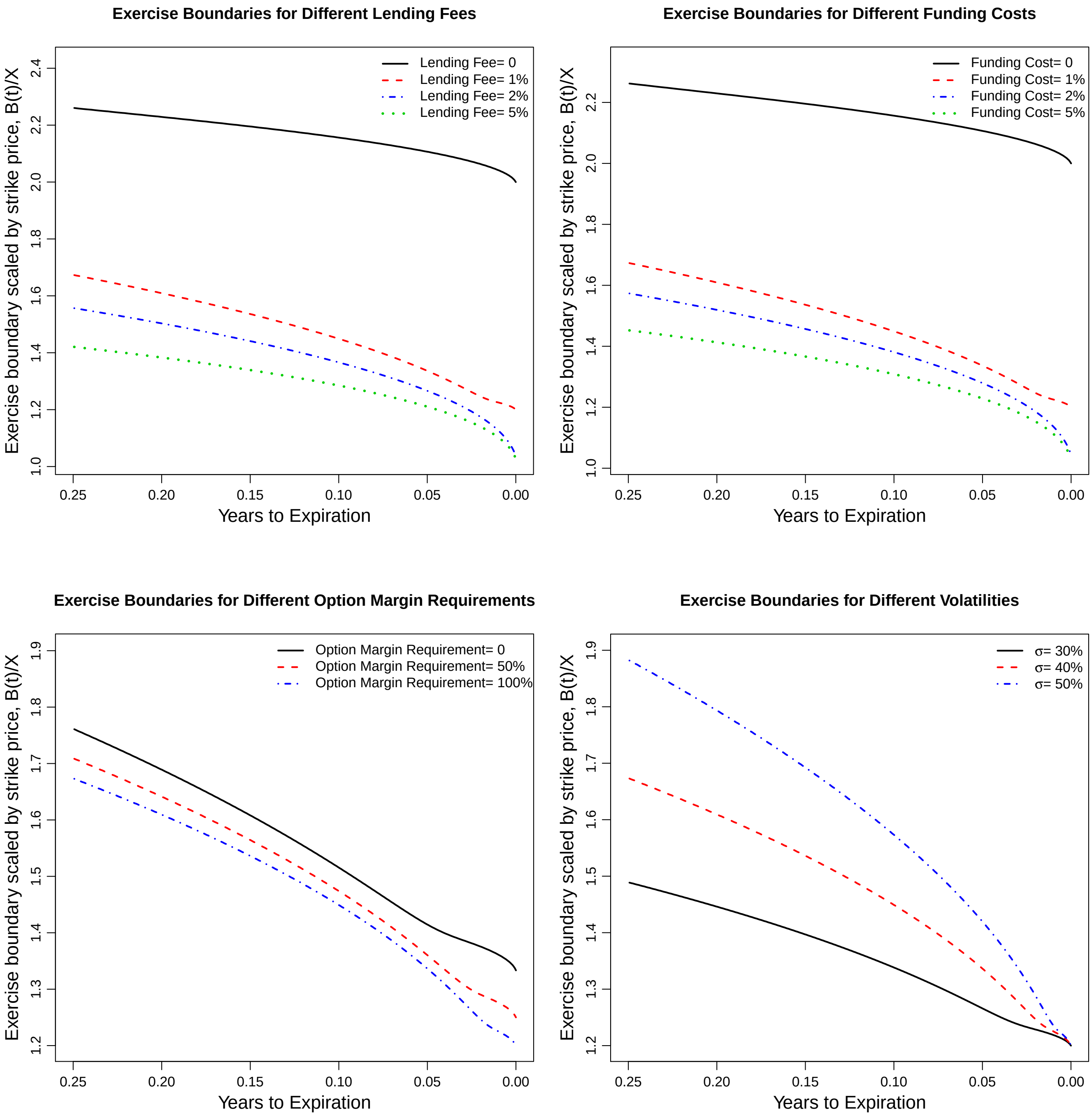

Figure 3: **The Lower Exercise Boundary with Frictions: Comparative Statics.** This figure shows theoretical lower exercise boundaries for equity options with frictions for an agent who is short stock. The boundary is a solution to the PDE (6)–(7) for a stock that pays no dividend. Each graph varies the parameters around a base-case cost where the risk-free rate is $r^f = 2\%$, the lending fee is $L^{\underline{i}} = 1\%$, the funding cost is $\psi^{\underline{i}} = 1\%$, the volatility is $\sigma = 40\%$, and margin requirements are $m^{\underline{i},S} = 50\%$ for the stock and $m^{\underline{i},C} = 100\%$ for the option. Early exercise is seen to be increasing in lending fees, funding costs, option margin requirements, and decreasing in volatility and time to expiration.

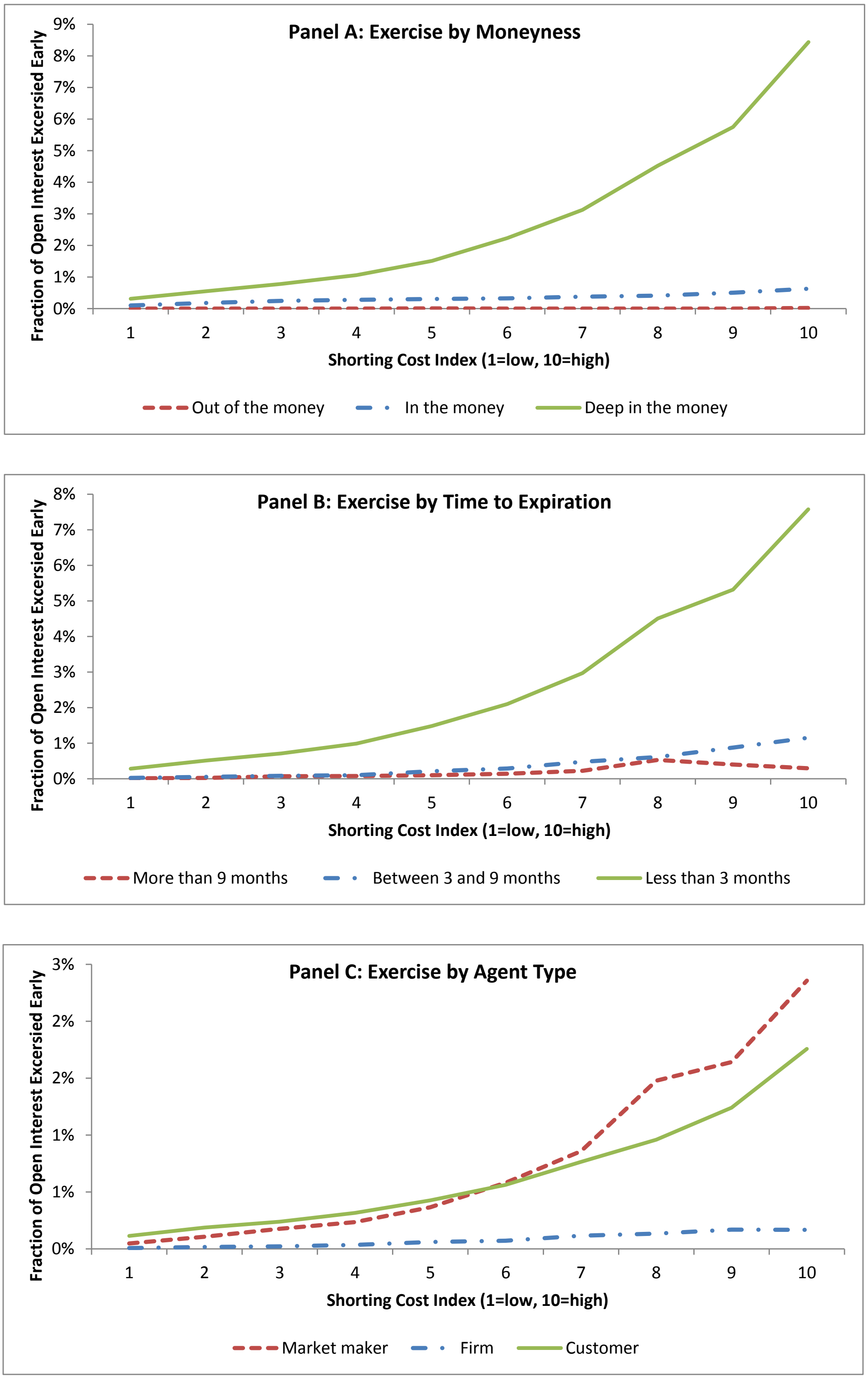

Figure 4: **Actual Early Exercise of Equity Options for Varying Short-Sale Costs.** This figure shows the empirical fraction of equity options exercised early as a function of the short-sale costs. Panel A groups the data by the moneyness of the option, Panel B by expiration, and Panel C by agent type. Consistent with our theory, early exercise is increasing in short-sale costs, increasing in moneyness, decreasing in time to expiration, and the exercise pattern is prevalent for all agent types including professional market makers and firm proprietary traders.

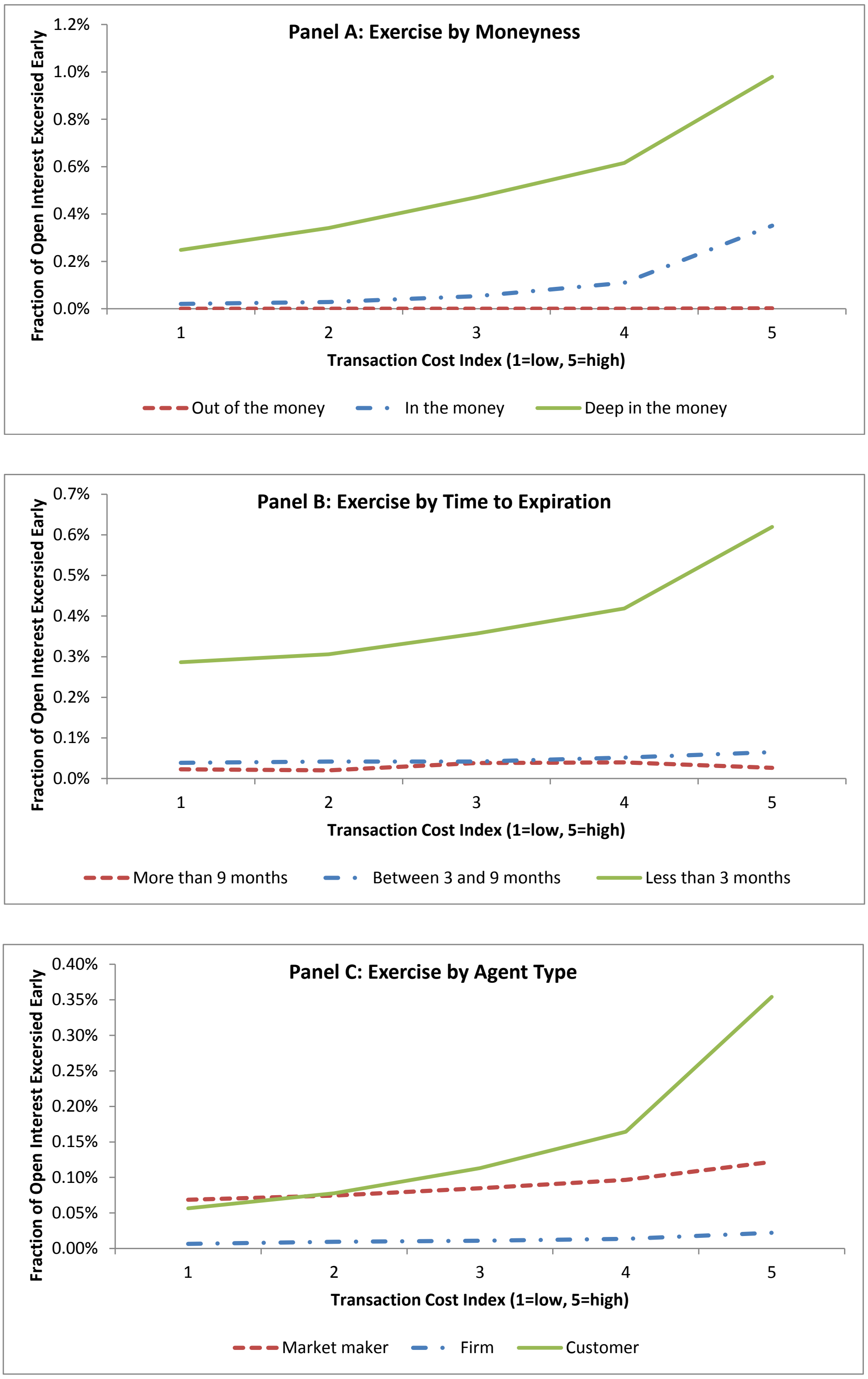

Figure 5: **Actual Early Exercise of Equity Options for Varying Transaction Costs.** This figure shows the empirical fraction of equity options exercised early as a function of transaction costs. The daily data on option series are divided in quintiles based on their transaction costs, measured each day as the bid-ask spread divided by the mid price for the corresponding at-the-money option series with the same expiration and underlying stock. Panel A groups the data by the moneyness of the option, Panel B by expiration, and Panel C by agent type. Consistent with our theory, early exercise is increasing in transaction costs, increasing in moneyness, and decreasing in time to expiration.