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FINANCIAL INTERMEDIATION, CAPITAL ACCUMULATION, AND RECOVERY

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Abstract

This paper integrates a simple model of banks into a two-sector neoclassical growth model. The integration yields an analytically tractable framework with two coupled accumulation rules for household capital and bank equity. We analyze steady state properties, transition and recovery patterns, as well as policies to accelerate recoveries. After establishing existence, uniqueness and global stability of the steady state, we identify in particular five key results and predictions, and we provide a quantitative assessment. First, larger financial frictions in financial intermediation may increase banker wealth although total capital is depressed. Second, negative shocks to bank equity cause considerably larger downturns than comparable shocks to household wealth, but their persistence is similar. Third, temporary worsening of shocks to financial frictions (called "trust shocks") induces divergent reactions of household wealth and bank equity, causes a boom in the banking sector, and possibly in the economy – after an initial bust. Fourth, the model replicates typical patterns of financing over the business cycle: procyclical bank leverage, procyclical bank lending, and counter-cyclical bond financing. Finally, a combination of bailouts and dividend-payout-restrictions ensures a rapid build-up of bank equity after a slump in the banking sector and increases total production.

JEL Classification: E21, E32, F44, G21 and G28
Keywords: banking crises, business cycles, bust-boom cycles, capital accumulation, financial intermediation, macroeconomic shocks and recovery policies

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1 Introduction

Capital accumulation is a key determinant of medium and long-run economic performance and the dynamic behavior of an economy. Economic dynamics and steady states may be heavily affected by the way financial intermediaries alleviate financing frictions for firms and how financial intermediaries themselves are subject to other financial frictions. In this paper, we integrate a simple model of banks into a standard two-sector neoclassical growth model to address how financial intermediaries affect the dynamic responses of key macroeconomic variables to adverse shocks on capital, productivity, and the underlying financial friction. In addition, we discuss suitable policies to manage financial and banking crises.

Since the seminal contributions of Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke et al. (1996) it is well-understood that in an economy with financial frictions, even small temporary shocks can have persistent effects on economic activity through their impact on the net worth of levered agents. Moreover, financial frictions often cause amplifications of shocks because leverage and prices are affected.[1]

The contribution of our paper to the literature discussed in the next subsection is threefold. First, at the conceptual level we develop a parsimonious and analytically tractable model that combines a two-sector neoclassical growth model with a micro-founded banking model. Because of the tractability, our approach is amenable to analyze the stability and the transition dynamics of the system without the need to linearize around its steady state – a property that also simplifies the quantitative analysis tremendously and makes the model a convenient macro-banking module that can easily be integrated into more complex economic environments. Second, the model provides analytical and quantitative insights into macroeconomic dynamics in response to adverse shocks, explains typical business cycle patterns of key macro-financial variables, and characterizes the impact of financial frictions on long-run capital accumulation and production. Third, our analysis sheds light on the design of welfare-improving public policies in response to a financial crisis when banks take center stage in the propagation of shocks.

Financial frictions constitute the centerpiece of our macro-banking model, as they micro-found the existence of banks and determine the propagation of adverse shocks.[2] Specifically, we assume that a fraction of firms depends on bank lending as banks alleviate

[1]There have been important assessments of the quantitative magnitudes of such effects (see Carlstrom and Fuerst (1997)) and in recent work the interplay of monetary policy and financial frictions has been included (see Christiano et al. (2005), Christiano et al. (2007), Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Angeloni and Faia (2013))

[2]Gersbach and Rochet (2012) studies a static version of the same banking model in which bank equity capital cannot be accumulated. Gersbach et al. (2015) study banks in the two-sector Solow growth model.

the moral hazard problems of the entrepreneurs running the technology. Bank lending itself is in each period limited as bankers can only pledge a fraction of their revenues to depositors. As a consequence, banks face an endogenous leverage constraint which depends on capital returns in both types of firms. Households can also invest in the bonds issued by the other firms (the other sector) which have access to direct financing.

The need for bank lending (which we also call informed lending) coupled with the lack of full revenue pledgeability are the only financial frictions in our model.[3] In the baseline model, there are three types of agents: investors, bankers and workers. The latter do not save and consume their labor income. Investors and bankers have standard intertemporal utility functions and decide in each period on how much to save and to consume.[4] Their utility maximization problems yield two accumulation rules in investor wealth and bank equity, respectively. These rules are coupled in the sense that the investors' saving and investment policies depend on how bankers fare and vice versa. Both types of lending – informed lending by banks and uninformed lending through capital markets – enable capital accumulation in the respective sectors. Our modelling strategy yields closed-form accumulation rules for investor wealth and bank equity and thus, explicit economic dynamics. Furthermore, when bankers are more impatient than investors, the economy converges to a unique globally stable steady state in which financial frictions are binding.

We establish five key results and predictions of the model, and we provide a quantitative assessment using a calibrated version of the model. First, larger financial frictions and thus reduced pledgeability of bankers depresses total capital in the steady state but increases wealth of bankers. Second, temporary negative shocks to bank equity cause larger downturns than comparable shocks to investor wealth. Furthermore, both types of downturns exhibit considerable persistence. These results are consistent with empirical studies, indicating that banking crises typically lead to more pronounced downturns than standard recessions (see Bordo et al. (2001), Allen and Gale (2009), Schularick and Taylor (2012)). Moreover, Jorda et al. (2013) have shown that in general, downturns associated with financial crises involve higher accumulated output losses. Third, temporary negative shocks to financial frictions, e.g., increasing mistrust into the financial sector, cause diverging reactions of investor wealth and bank equity. Specifically, bank equity increases and induces a temporary boom in banking and eventually in the entire economy. Fourth, the model replicates typical patterns of financing over the business

[3] As we discuss in Subsection 3.2, the foundation of these frictions can be moral hazard problems à la Holmstrom and Tirole (1997), asset diversion (like Gertler and Karadi (2011) and Gertler and Kiyotaki (2010)) or non-alienability of human capital (like Hart and Moore (1994) and Diamond and Rajan (2000)).

[4] In the extensions we consider a version of the model in which there are only two types of agents: households, acting as investors and workers, and banks. To preserve clarity in exposition, we reserve the terminus household to that case only.

cycle: procyclical leverage of banks, procyclical bank lending and countercyclical bond financing (see Adrian and Shin (2014), Adrian and Boyarchenko (2012), Adrian and Boyarchenko (2013) and Nuño and Thomas (2012) for evidence). This holds if downturns are associated with negative productivity, bank equity or trust shocks – or any combination thereof. Fifth, when the economy is suffering from a banking crisis associated with a significant decline of bank equity and bank lending, a mix of investor financed bank recapitalization (bank bailout) and dividend payout restrictions can ensure a rapid build-up of bank equity and a more rapid recovery. A judicious mix of these policies can improve welfare of workers – and under certain conditions also for investors and workers together – while leaving intertemporal utility of bankers unchanged. These properties add to our understanding about the role of banking for economic activities and complement the flourishing literature.

The paper is organized as follows. In the next section, we relate the paper to the existing literature. Section 3 introduces the model. Section 4 defines and characterizes sequential market equilibria. Section 5 analyzes the steady state allocation and Section 6 establishes global stability of the system. Section 7 examines temporary shocks to productivity, investor wealth, bank equity and financial frictions and illustrates the quantitative behavior of the economy using a calibrated version of the model. Section 8 explores the scope of public policies and regulation to speed up recoveries when the economy is hit by a negative bank equity shock. Section 9 explores several extensions of the model and, finally, Section 10 concludes.

2 Relation to the Literature

The present paper addresses the impact of banks on macroeconomic dynamics and in particular how financial intermediation affects capital accumulation and output dynamics when the economy is hit by adverse shocks. This paper closely relates to three recent strands of the literature that integrate financial intermediation into macroeconomic models.

First, in the context of DSGE models, Meh and Moran (2010) and Angeloni and Faia (2013) have provided valuable insights regarding the bank capital transmission channel. Meh and Moran (2010) finds that this channel significantly amplifies and propagates the impact of technology shocks on inflation and output. Angeloni and Faia (2013) introduces banks subject to runs in a DSGE model. They show that combining countercyclical capital requirements with monetary polices that are conditional on the financial market conditions may constitute an optimal policy mix.

In this paper we abstract from price rigidities and outline a parsimonious model on how to integrate models of banking into neoclassical growth models. The ensuing analytically tractable coupled accumulation equations of investors' wealth and bank equity allow to study how macroeconomic dynamics are affected by financial intermediation and which policy mix can optimally respond to crises.

Second, our paper is complementary to recent papers that develop macroeconomic models with financial intermediation. Brunnermeier and Sannikov (2014) has stressed that in models similar to Bernanke et al. (1996) and Kiyotaki and Moore (1997) the economy's reaction to shocks is highly non-linear. In particular, if the economy is sufficiently far away from its steady state, even small shocks can lead to a substantial amplification and can generate a significant degree of endogenous business cycle risk. In contrast, near the steady state, the economy is robust to most shocks. Such non-linear effects of shocks have been shown in other models by He and Krishnamurthy (2013) as risk premia on equity increase sharply when financial constraints become binding. Rampini and Viswanathan (2014) examines the coupled dynamics of net worth of the corporate sector and financial intermediaries when the latter act as collateralization specialists. They show that a credit crunch can depress economic activity for a long time and recovery is delayed or stalled.

Compared to Brunnermeier and Sannikov (2014) and Rampini and Viswanathan (2014) we focus on a model in which saving and investment as well as the interest rate on bonds respond smoothly to shocks. Moreover, the relative productivity of financially constrained and unconstrained firms is endogenous and determined by the accumulation process. In principle, such a set-up suggests smoother reactions of the economy to shocks as for example in the work of Brunnermeier and Sannikov (2014) and Rampini and Viswanathan (2014) and thus our approach is taking a medium and long-term perspective. However, the special role of banks in the capital accumulation process with binding leverage constraints during an entire recovery process and potentially divergent reactions of investor wealth and bank capital can produce considerable output reactions which can substantially be amplified by negative shocks to productivity, bank equity and trust in a downturn. Moreover, particular shocks such as negative shocks to financial frictions may induce a reversal of fortunes: the (surprising) initial boom in banking can even reverse the economy from an initial bust into a boom.

Martinez-Miera and Suarez (2012) studies a dynamic general equilibrium model in which banks decide on their exposure to systemic shocks. Capital requirements reduce the impact of negative systemic shocks but they also lower the amount of credit and output in normal times. Optimal capital requirements balance these costs and benefits. Our model is complementary and focuses on the simultaneous build-up of bank equity and investor wealth and how those coupled accumulation processes are affected by unanticipated and anticipated shocks to productivity, wealth or financial frictions.

Third, regarding the policy results, our paper is closely related to Acharya et al. (2011) which studies dividend payments of banks in the financial crises of 2007–2009 and Acharya et al. (2013) which suggests that banks may pay excessively high dividends as they do not take into account the negative externalities these dividends exert on other banks, e.g., via interbank relationships (see also Onali (2014)). In our model, investor financed bank recapitalization and publicly enforced dividend payout restrictions speed up the recovery after a banking crisis and, in addition, can make workers – and under certain conditions even investors – better off while leaving the welfare of bankers unaffected.[5]

3 Capital Accumulation cum Banks

We integrate a simple model of banks into a two-sector Ramsey growth model. Time is discrete and denoted by $t \in \{0, 1, 2, \dots\}$. There are two production sectors with constant returns to scale production technologies in capital and labor that produce a homogenous good that serves as an *all-purpose* good. Sectors differ with respect to total factor productivity and access to capital markets. The good can be either consumed in the same period or invested as capital for production in future periods. The consumption good is the numéraire in the economy and its price is normalized to 1. There are three types of agents: workers, investors, and bankers.[6] While workers are hand-to-mouth consumers, investors and bankers accumulate capital governed by the respective intertemporal utility maximization problem. The details of the model are set out in this section.

3.1 Production

Production takes place in two different sectors labeled as sector M (market-financed) and sector I (intermediary-financed). Both sectors consist of a continuum of identical firms. The production technologies exhibit constant returns to scale in the production

[5] How taxes or subsidies may favorably impact the transition dynamics in a standard growth model with financial frictions has been studied by Itskhoki and Moll (2014). Our policy analysis is complementary as we focus on two policies that are typically applied in banking: bailout and recapitalization of banks coupled with dividend payment restrictions.

[6] Splitting the household sector into workers and investors guarantees the analytical tractability of the model. In Section 9 we consider the variant in which there is only one type of households that supplies labor and acts as investor. As will be shown, the steady state allocation is unaffected by this variant.

factors capital and labor, have positive and diminishing marginal returns regarding a single production factor and satisfy the Inada conditions. The assumption of constant returns to scale allows to represent each sector by an aggregate representative production technology. Specifically, technologies are of the Cobb-Douglas type

$$Y_t^j = Az^j \left(K_t^j\right)^{\alpha} \left(L_t^j\right)^{1-\alpha}, \quad j \in \{M, I\}$$

where A is an index of economy-wide total factor productivity, z^j is an index of sectoral total factor productivity and K_t^j and L_t^j denote capital and labor input in sector $j \in \{M, I\}$, respectively. In the basic version we set $A = 1$ without loss of generality. In the extension we also consider a stochastic version in which A varies over time.

The sectors differ with respect to their access to financial markets. Firms in sector M have access to financial markets and obtain capital directly from investors. In contrast, firms in sector I are excluded from financial markets and obtain capital exclusively from financial intermediaries, the banking sector (see De Fiore and Uhlig (2011)).[7]

Taking interest and wage rates as given, the representative firms in each sector $j = M, I$ choose capital and labor to maximize their period profits. Specifically, the optimization problems in period t read

$$\max_{\{K_t^j, L_t^j\}} \left\{ z^j \left(K_t^j\right)^{\alpha} \left(L_t^j\right)^{1-\alpha} - r_t^j K_t^j - w_t^j L_t^j \right\}, \quad j \in \{M, I\} \tag{1}$$

where w_t^j is the prevailing wage and r_t^j is the rental rate of capital in sector j and period t, respectively. The total amount of capital invested in both sectors is denoted by K_t and given by $K_t = K_t^M + K_t^I$. Total labor used in production is $L_t = L_t^M + L_t^I$. We assume that capital depreciates at rate δ $(0 < \delta < 1)$.

3.2 Capital Markets and Financial Frictions

There are no financial frictions in sector M whose firms directly borrow K_t^M units of capital from the investors on competitive capital markets. In contrast, firms in sector I cannot raise funds from investors as they cannot pledge their output. Thus, moral hazard is so severe that it prevents direct financing in sector I. There is a continuum of bankers and each banker owns and runs a financial intermediary – a bank. Bankers can alleviate the moral hazard problem of the entrepreneurs. Specifically, bankers evaluate and monitor entrepreneurs and enforce contractual obligations. For simplicity, we assume

[7] Typically, sector I consists of newer or smaller firms that cannot pledge repayment to investors in the capital market because of moral hazard.

that the direct costs of these activities are negligible.[8]

Bankers themselves raise funds from investors at the deposit rate r_t^D. A banker, however, cannot pledge the entire amount of repayments from entrepreneurs to investors. More specifically, the non-pledgeable part of repayments is θk_t^I if the banker has granted a loan of size k_t^I to entrepreneurs in period t. Let r_t^I denote the lending rate to sector I. Thus, a banker can only pledge $(1 + r_t^I - \theta)k_t^I$ to investors. The parameter θ ($\theta > 0$) measures the magnitude of the financial friction.

Moral hazard of entrepreneurs in sector I – which prevents them from a direct access to financial markets – coupled with the partial non-pledgeability of the financial intermediaries' revenues are the only financial frictions in the model. These frictions boil down to the presence of the parameter $\theta > 0$.[9] In the basic version of our model we abstract from explicitly incorporating aggregate risk, i.e. negative shocks are not anticipated.[10] In the extensions, we allow for aggregate risk regarding loan repayments and productivity.[11]

3.3 Workers and Investors

There is a continuum of workers with mass L ($L > 0$). Labor is sector-specific and can only be employed in one sector for productive use. Without loss of generality, we normalize each worker's labor endowment to unity and assume that he is endowed with l^M and $l^I = 1 - l^M$ units of labor that are inelastically supplied to production sectors M and I, respectively. Workers are essentially hand-to-mouth consumers: they earn labor income which is consumed entirely and immediately.[12] Aggregation is straightforward and without loss of generality. The representative worker earns $w_t^M L^M + w_t^I L^I$ where

[8] In Subsection 9.3 we discuss the impact of intermediation costs.

[9] The paramter θ can also be interpreted as the degree of trust in repayments by bankers.

[10] Individual repayment risk is possible as it can be fully diversified by investors.

[11] The partial non-pledgeability of revenues leads to moral hazard between bankers and investors as in Holmstrom and Tirole (1997) and can alternatively be traced back to the possibility of asset diversion (like Gertler and Karadi (2011) and Gertler and Kiyotaki (2010)) or non-alienability of human capital (like Hart and Moore (1994) and Diamond and Rajan (2000)). See Gersbach and Rochet (2013) for an in-depth discussion. Furthermore, assume that bankers that shirk in the current period cannot be excluded from seeking new funds from investors in the next period. This rules out that bankers can pledge revenues from future periods in order to attract more funds today. For example, consider the case of asset diversion. Suppose that a banker attempts to pledge $(1 - \theta')k_t^B$ in the current period with $\theta' < \theta$ in a long-term contract with more than one period, in which he invests k_t^B more than once. Then, he can divert θk_t^B in period t and seeks new funds in period $t + 1$. This is profitable and thus $(1 - \theta')k_t^B$ cannot be pledged.

[12] There are several reasons for which workers do not want to save and essentially behave like hand-to-mouth consumers, e.g. lower discount factors or borrowing constraints. We are agnostic about the specific reason. As reported in Challe and Ragot (2015), estimates of the share of hand-to-mouth households in the United States vary a lot and range from 15% to 60%. A recent study by Kaplan et al. (2014) finds that more than one-third of the population in the Unites States voluntarily saves little or nothing.

$L^M = l^M L$ and $L^I = l^I L = L - L^M$.

There is a continuum of investors with unit mass. Each investor is endowed with some units of the capital good which can be used for investment in bonds and deposits or for consumption. As disposable income is linear homogenous in wealth and the period utility function is homothetic, consumption and saving decisions are linear homogenous in wealth, too. This implies that the distribution of capital among investors has no impact on aggregate consumption and saving and we can restrict the analysis to a representative investor who takes sequences of rental rates of capital as given. Specifically, the representative investor has logarithmic period utility and solves the following problem

$$\max_{\{C_t^H, \Omega_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta_H^t \ln(C_t^H) \right\} \tag{2}$$

subject to

$$\begin{aligned} &C_t^H + \Omega_{t+1} = r_t^M K_t^M + r_t^D D_t + (1-\delta)\Omega_t \\ &K_t^M + D_t = \Omega_t \\ &\Omega_0 \text{ given} \end{aligned}$$

where D_t denotes the (aggregate) amount of deposits and Ω_{t+1} the (aggregate) amount of investor wealth at the beginning of period $t+1$. Moreover, $\beta_H = \frac{1}{1+\rho_H}$ $(0 < \beta_H < 1)$ denotes the discount factor and ρ_H the discount rate.

In any equilibrium, capital is employed in both sectors because Cobb-Douglas production functions satisfy the Inada conditions and labor allocations are fixed. Since investors are indifferent between bonds and deposits, competition implies that r_t^D is equal to r_t^M. Hence, the budget constraint of the investors in period t simplifies to

$$C_t^H + \Omega_{t+1} = \Omega_t(r_t^M + 1 - \delta). \tag{3}$$

3.4 Bankers

There is a continuum of banks with unit mass. Each banker is endowed with the amount e_0 of the all-purpose good. Total endowment of bankers is E_0. At the beginning of period t, a typical banker owns e_t which she uses as equity funding for her bank. She attracts additional funds $(k_t^I - e_t)$ from investors and lends k_t^I to entrepreneurs in sector I.[13]

[13]In principle, bankers could also invest their resources in sector M. However, this will not occur when the leverage constraint binds as bank equity is scarce in such circumstances and sector I pays higher returns on investments to bankers.

As investors can invest into sector M, a banker needs to be able to pledge (at least) $(1+r_t^M)(k_t^I - e_t)$ to investors in order to attract $(k_t^I - e_t)$. Hence, the banker faces the market-imposed leverage constraint

$$(1+r_t^M)(k_t^I - e_t) \leq k_t^I(1+r_t^I - \theta)$$

which can be rewritten as

$$k_t^I \leq \frac{1+r_t^M}{r_t^M - r_t^I + \theta} e_t. \tag{4}$$

Suppose financial frictions are binding.[14] In equilibrium we have $r_t^M - r_t^I + \theta > 0$. In this case, a banker is always better off attracting loanable funds and investing k_t^I in sector I thereby earning θk_t^I, than either investing only e_t in sector I thereby earning $r_t^I e_t$ or investing in sector M.[15] As bankers are price takers, profit maximization implies that condition (4) holds with equality. Since the binding leverage constraint is linear in equity, the behavior of the banking system can be represented by the behavior of a representative banker facing the aggregate leverage constraint

$$K_t^I = \frac{1+r_t^M}{r_t^M - r_t^I + \theta} E_t = \lambda_t E_t \tag{5}$$

where $\lambda_t = \frac{1+r_t^M}{r_t^M - r_t^I + \theta}$ denotes bank leverage.

We focus mostly on circumstances in which (5) holds in all periods. This requires that bank equity is and remains sufficiently scarce relative to the wealth of households in all periods, i.e. $r_t^I > r_t^M$. Note that disposable income of bankers in t is given by $\theta K_t^I - \delta E_t = (\theta\lambda_t - \delta)E_t$. The formal condition on scarcity of bank equity will be established in Section 4. For convenience, define $\Gamma \subseteq \mathbb{R}_+^2$ as the subset on the state space for which financial frictions are binding.

Alternatively, suppose that financial frictions are non binding, that is equity is relatively abundant. Thus, $(E, \Omega) \in \mathbb{R}_+^2 \setminus \Gamma$. Clearly, $r_t^I = r_t^M$ and disposable income of bankers in period t is $(1+r_t^M - \delta)E_t$.

The problem of the representative banker can be written as

$$\max_{\{C_t^B, E_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta_B^t \ln(C_t^B) \right\} \tag{6}$$

[14] In Section 5 we show that as long as bankers are sufficiently impatient, there exists a unique globally steady state in which financial frictions are binding.

[15] The leverage constraint implies $\theta k_t^I \leq (1+r_t^I)e_t + (k_t^I - e_t)(r_t^I - r_t^M)$.

subject to

$$C_t^B + E_{t+1} = \begin{cases} (\theta\lambda_t - \delta)E_t & \text{if } (E_t, \Omega_t) \in \Gamma \\ (1 + r_t^M - \delta)E_t & \text{if } (E_t, \Omega_t) \in \mathbb{R}_+^2 \setminus \Gamma \end{cases}$$

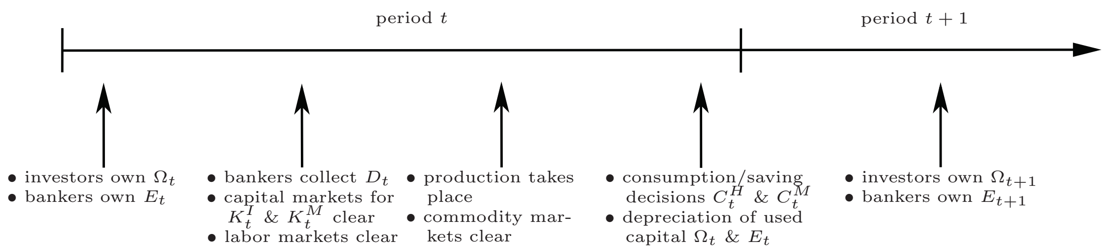
$$E_0 \text{ given}$$

where $\beta_B = \frac{1}{1+\rho_B}$ $(0 < \beta_B < 1)$ denotes the discount factor of the banker (ρ_B being the discount rate) and C_t^B denotes the consumption of bankers.

3.5 Sequence of Events

It is useful to outline the sequence of events within a particular period.

Figure 1: Sequence of Events



4 Sequential Markets Equilibrium

In this section, we analyze the sequential markets equilibrium which is defined as follows

Definition 1. *A sequential markets equilibrium is a sequence of factor prices and allocations* $\{w_t^M, w_t^I, r_t^M, r_t^I, K_t^M, K_t^I, C_t^H, C_t^B, \Omega_{t+1}, E_{t+1}\}_{t=0}^{\infty}$ *such that*

1. *Given* Ω_0 *and* $\{r_t^M\}_{t=0}^{\infty}$, *the allocation* $\{C_t^H, \Omega_{t+1}\}_{t=0}^{\infty}$ *solves the representative investor's problem* (2).

2. *Given* E_0 *and* $\{r_t^M,\, r_t^I\}_{t=0}^{\infty}$, *the allocation* $\{C_t^B, E_{t+1}\}_{t=0}^{\infty}$ *solves the representative banker's problem* (6).

3. *For each* $t \geq 0$, *given* $(w_t^M, w_t^I, r_t^M, r_t^I)$, *the allocation* $(K_t^M, K_t^I, L_t^M, L_t^I)$ *solves the representative firms' maximization problems* (1).

4. Factor and output markets clear.

Our analysis proceeds in two steps. The first step characterizes the intraperiod factor allocation, i.e. the investment policies K_t^M and K_t^I. The second step confines attention to the intertemporal consumption-saving decision which determines capital accumulation Ω_{t+1} and E_{t+1}.

4.1 Intraperiod Equilibrium

We consider the equilibrium in a typical period t with given initial capital stock (E_t, Ω_t). The firms' first-order conditions in both sectors and market clearing in the labor market ($L_t^M = L^M$ and $L_t^I = L^I$) yield the usual conditions for (gross) interest rates and wages

$$r_t^j = \alpha z^j \left(\frac{K_t^j}{L^j}\right)^{\alpha-1}, \quad j \in \{M, I\} \tag{7}$$

$$w_t^j = (1-\alpha) z^j \left(\frac{K_t^j}{L^j}\right)^{\alpha}, \quad j \in \{M, I\}. \tag{8}$$

We distinguish two different cases: when financial frictions are non-binding and thus irrelevant and when financial frictions are binding.

4.1.1 Irrelevant Financial Frictions

When financial frictions do not matter for aggregate capital investments, the (gross) interest rates r_t^I and r_t^M have to be equal such that

$$\frac{K_t^I}{L^I} = \left(\frac{z^I}{z^M}\right)^{\frac{1}{1-\alpha}} \frac{K_t^M}{L^M}.$$

Defining $z \doteq \left(\frac{z^I}{z^M}\right)^{\frac{1}{1-\alpha}}$ and $\ell = \frac{L^I}{L^M}$, the aggregate resource constraint $K_t^M + K_t^I = \Omega_t + E_t$ yields

$$K_t^{M*} = \frac{\Omega_t + E_t}{1 + z\ell} = \frac{K_t}{1 + z\ell}$$

$$K_t^{I*} = z\ell \frac{\Omega_t + E_t}{1 + z\ell} = \frac{z\ell K_t}{1 + z\ell}.$$

Net earnings of bankers are $(1+r_t^M)E_t$. Incentive compatibility requires that net earnings are at least as large as the non-pledgeable part θK_t^I. Thus,

$$E_t \geq \theta z\ell \frac{\Omega_t + E_t}{(1 + r_t^M)(1 + z\ell)} = \frac{\theta z\ell}{(1 + r_t^M)(1 + z\ell)} K_t.$$

Using the equilibrium interest rate condition (7) and K_t^{M*} we get

$$E_t \geq \frac{\theta z\ell}{\left(1+\alpha z^M\left(\frac{K_t}{L^M(1+z\ell)}\right)^{\alpha-1}\right)(1+z\ell)} K_t \doteq \bar{E}(K_t) \tag{9}$$

where $\bar{E}(K_t)$ denotes the lower bound of bank equity to make financial frictions irrelevant in a particular period t given some overall capital K_t in the economy.

4.1.2 Relevant Financial Frictions

We next turn to the case in which financial frictions matter in period t and thus the leverage constraint is binding for given initial capital stocks (E_t, Ω_t). Financial frictions matter if $r_t^I > r_t^M$ which implies

$$\frac{K_t^I}{K_t - K_t^I} < z\ell \quad \Leftrightarrow \quad K_t^I < \frac{z\ell K_t}{1+z\ell}.$$

As the leverage constraint is binding, $K_t^I = \lambda_t E_t$. Thus, for any given (E_t, Ω_t), the aggregate leverage constraint (5) yields

$$\lambda_t = \frac{K_t^I}{E_t} = \frac{1+\alpha z^M\left(\frac{K_t-\lambda_t E_t}{L^M}\right)^{\alpha-1}}{\theta+\alpha z^M\left(\frac{K_t-\lambda_t E_t}{L^M}\right)^{\alpha-1} - \alpha z^I\left(\frac{\lambda_t E_t}{L^I}\right)^{\alpha-1}}.$$

Rewriting this condition and defining the auxiliary function $\varphi(\lambda_t)$, equilibrium leverage λ_t satisfies

$$\varphi(\lambda_t) \doteq \alpha z^M\left(\frac{\Omega_t + E_t - \lambda_t E_t}{L^M}\right)^{\alpha-1}\left(1-\frac{1}{\lambda_t}\right) - \frac{1}{\lambda_t} + \theta - \alpha z^I\left(\frac{\lambda_t E_t}{L^I}\right)^{\alpha-1} = 0. \tag{10}$$

Note that $\varphi(\lambda_t)$ is increasing for $\lambda_t \in \left(1, \frac{z\ell}{1+z\ell}\frac{K_t}{E_t}\right)$.[16] Clearly, evaluated at the lower bound of the interval, $\varphi(1) < 0$. In contrast, evaluating $\varphi\left(\frac{z\ell}{1+z\ell}\frac{K_t}{E_t}\right)$ delivers

$$\varphi\left(\frac{z\ell}{1+z\ell}\frac{K_t}{E_t}\right) = \alpha z^M\left(\frac{K_t}{L^M(1+z\ell)}\right)^{\alpha-1}\left(1-\frac{(1+z\ell)}{z\ell}\frac{E_t}{K_t}\right) - \frac{(1+z\ell)}{z\ell}\frac{E_t}{K_t} + \theta - \alpha z^I\left(\frac{z\ell K_t}{L^I(1+z\ell)}\right)^{\alpha-1}.$$

At this point, interest rates in both sectors are equal: $\alpha z^M\left(\frac{K_t}{L^M(1+z\ell)}\right)^{\alpha-1} = \alpha z^I\left(\frac{z\ell K_t}{L^I(1+z\ell)}\right)^{\alpha-1}$.

[16] A priori, it is not excluded that the interval is degenerated. For E_t arbitrary small, we observe from (10) that leverage can be arbitrarily large.

Given the definitions of z and ℓ, $\varphi\left(\frac{z\ell}{1+z\ell}\frac{K_t}{E_t}\right)$ simplifies to

$$\varphi\left(\frac{z\ell}{1+z\ell}\frac{K_t}{E_t}\right) = \theta - \frac{(1+z\ell)}{z\ell}\frac{E_t}{K_t}\left(1+\alpha z^M\left(\frac{K_t}{L^M(1+z\ell)}\right)^{\alpha-1}\right).$$

Observe that $\varphi\left(\frac{z\ell}{1+z\ell}\frac{K_t}{E_t}\right)$ is decreasing in E_t and attains zero when $E_t = \bar{E}(K_t)$. However, as financial frictions matter, $E_t < \bar{E}(K_t)$, we get $\varphi\left(\frac{z\ell}{1+z\ell}\frac{K_t}{E_t}\right) > 0$.

Because $\varphi(\lambda_t)$ is continuous and strictly monotone for $E_t < \bar{E}(K_t)$, the intermediate value theorem delivers existence and uniqueness of a λ_t^* satisfying $\varphi(\lambda_t^*) = 0$. The equilibrium factor allocations are then straightforwardly given by

$$\begin{aligned} K_t^{M*} &= K - \lambda_t^* E_t = \Omega_t - (\lambda_t^* - 1)E \\ K_t^{I*} &= \lambda_t^* E_t. \end{aligned}$$

4.1.3 Existence and Uniqueness of Intraperiod Equilibrium

The preceding considerations culminate in the following proposition:

Proposition 1 (Intraperiod Equilibrium: Factor Allocation)**.**
For all pairs (E_t, K_t) with $0 < E_t < K_t$, there exists a unique equilibrium.

(i) If $E_t \geq \bar{E}(K_t)$, financial frictions do not matter. The capital allocation is given by $K_t^{M*} = \frac{K_t}{1+z\ell}$ and $K_t^{I*} = z\ell\frac{K_t}{1+z\ell}$.

(ii) If $E_t < \bar{E}(K_t)$, financial frictions matter. The bank leverage λ_t^* is the solution to $\varphi(\lambda^*) = 0$ and the capital allocation is given by $K^{M*} = \Omega - (\lambda^* - 1)E$ and $K^{I*} = \lambda^* E$.

4.1.4 Comparative Statics when Financial Frictions Matter

In the case when financial frictions matter, the intraperiod equilibrium allows for straightforward and useful comparative statics. Because $\frac{\partial\varphi}{\partial z^M} > 0$, $\frac{\partial\varphi}{\partial z^I} < 0$, $\frac{\partial\varphi}{\partial\Omega_t} < 0$, $\frac{\partial\varphi}{\partial E_t} > 0$, and $\frac{\partial\varphi}{\partial\theta} > 0$, total differentiation of the equilibrium condition $\varphi(\lambda_t) = 0$ yields

Corollary 1. *Suppose that financial frictions matter, i.e. $E_t < \bar{E}(K_t)$. Then,*

(i) λ_t increases in z^I and Ω_t.

(ii) λ_t decreases in z^M, E_t and θ.

Thus, when financial frictions matter, equilibrium bank leverage is an increasing function of the productivity of small and medium sized firms (z^I) and of investors' wealth (Ω_t). It is a decreasing function of the productivity of large firms (z^M), of aggregate bank equity (E_t), and of the magnitude of financial frictions (θ).

We next investigate how an aggregate productivity shock affects leverage in the intraperiod equilibrium. Suppose that both parameters of total factor productivity, z^M and z^I, are hit by the same relative shock.

$$\epsilon \doteq \frac{\Delta z^M}{z^M} = \frac{\Delta z^I}{z^I}.$$

Then, the change of $\varphi(\lambda_t)$ at a particular value λ_t, denoted by $\Delta\varphi$, is given by

$$\Delta\varphi = \left(r_t^M(\lambda_t - 1) - \lambda_t r_t^I\right)\frac{\epsilon}{\lambda_t}.$$

As $\theta\lambda_t = \lambda_t r_t^I - (\lambda_t - 1)r_t^M + 1$ from condition (10), we finally arrive at

$$\frac{\Delta\varphi}{\epsilon} = \frac{1}{\lambda_t} - \theta = \left(\frac{r_t^M(1-\theta) - r_t^I}{r_t^M - r_t^I + \theta}\right)\frac{1}{\lambda_t} < 0$$

where the inequality follows from $r_t^I > r_t^M$. Since $\varphi(\lambda_t)$ is strictly increasing in λ_t, we get

Corollary 2. *Suppose that financial frictions matter. Then $\frac{\partial\lambda_t}{\partial\epsilon} > 0$, where ϵ is a proportional change of z^M and z^I.*

Corollary 2 has important implications: If the economy is hit by a positive (negative) aggregate productivity shock, leverage increases (decreases). Hence, book leverage in the banking industry is procyclical – a pattern that is well documented (see Adrian and Shin (2014), Adrian and Boyarchenko (2012), Adrian and Boyarchenko (2013), Nuño and Thomas (2012)).

We now consider the impact of wealth shocks to the factor allocation. In particular, suppose E_t increases and thus the overall amount of capital K_t in the economy increases by the same amount. We first note that a positive equity shock (and thus a simultaneous increase of E_t and K_t) has two opposing effects on K_t^I. On the one hand, a higher amount of equity *ceteris paribus* increases K_t^I. On the other hand, from Corollary 1 we know that leverage declines which *ceteris paribus* lowers K_t^I. However, as the next result demonstrates, the first effect always dominates.

Corollary 3. *Suppose that financial frictions matter. Then, an increase in bank equity E_t (and a corresponding increase of K_t) raises K_t^I.*

Corollary 3 is obtained from the following contradiction. Suppose K_t^I does not increase and thus r_t^I does not fall. As more capital will be employed in sector M, r_t^M declines. However, as $\frac{\partial \lambda_t}{\partial r_t^M} < 0$, leverage increases. Since bank equity is also higher, K_t^I increases, which is a contradiction.

In a similar vein, we establish the corresponding impact of an increase in Ω_t on K_t^M.

Corollary 4. *Suppose that financial frictions matter. Then, an increase in household wealth Ω_t (and a corresponding increase of K_t) raises K_t^M.*

Key comparative statics are summarized in Table 1.

Table 1: Comparative Statics

	Leverage λ	Bonds K^M	Loans K^I	Output Y
Productivity ($\epsilon > 0$)	$+$	$-$	$+$	$+$
Investor Wealth ($\Delta\Omega > 0$)	$+$	$+$	$+$	$+$
Bank Equity ($\Delta E > 0$)	$-$	$\pm$	$+$	$+$

First, productivity shocks $\epsilon > 0$: As of Corollary 2, leverage and loan finance are increasing in the joint productivity shock ϵ. Because of $K_t^{M*} = \Omega_t - (\lambda_t^* - 1)E$, bond finance decreases. Second, investor wealth shocks $\Delta\Omega > 0$: The positive response of leverage and loan finance in response to a shock to investor wealth directly follows from Corollary 1, whereas the positive reaction of bond finance has been established in Corollary 4. Third, bank equity shocks $\Delta E_t > 0$: According to Corollaries 1 and 3, leverage is decreasing and loan finance is increasing in bank equity. For the bond finance response, there are basically two opposing effects. On the one hand, if equity increases, banks can attract more loanable funds which increases credit supply pushing loan finance up. On the other hand, leverage is negatively affected by the increase in bank equity, which reduces credit supply pushing loan finance down. Although the overall effect on loan finance is positive, it is not clear whether the increase is sufficient to dominate the implemented equity increase. Thus, the impact on bond finance $K_t^{M*} = \Omega_t - (\lambda_t^* - 1)E$ is ambiguous. Finally, productivity shocks, investor wealth shocks and bank equity shocks are positively correlated to output as either productivity or total capital increase.

4.2 Intertemporal Consumption-Saving Decision

As both bankers and investors have homothetic utility and disposable income is linear homogenous in wealth, the investors' and bankers' consumption-saving policies are linear homogenous in wealth. The policy functions depend on the current state (E_t, Ω_t), and in particular whether financial frictions are binding and how equilibrium returns and leverage are determined at state (E_t, Ω_t). In effect, both types of agents have constant saving rates out of end-of-period net-worth. Suppressing the dependency of r_t^M and r_t^I from the state variables (E_t, Ω_t) for convenience, this leads to the following proposition:

Proposition 2 (Intertemporal Equilibrium: Consumption and Saving)**.**

(i) The necessary conditions for the solution of the banker's problem imply

$$\begin{aligned} C_t^B &= (1-\beta_B)(1+r_t^B)E_t \\ E_{t+1} &= \beta_B(1+r_t^B)E_t \end{aligned}$$

where r_t^B is the (net) return on equity in period t given by

$$r_t^B \doteq \begin{cases} \theta\lambda(r_t^M, r_t^I) - \delta - 1 & \textit{if} \quad E_t < \bar{E}(K_t) \\ r_t^M - \delta & \textit{if} \quad E_t \geq \bar{E}(K_t). \end{cases}$$

(ii) The necessary conditions for the solution of the investor's problem imply

$$\begin{aligned} C_t^H &= (1-\beta_H)(1+r_t^M-\delta)\Omega_t \\ \Omega_{t+1} &= \beta_H(1+r_t^M-\delta)\Omega_t. \end{aligned}$$

We note that from condition (10) we can represent the (net) return on equity in the friction case in a convenient way:

$$\begin{aligned} r_t^B &= \theta\lambda(r_t^M, r_t^I) - \delta = \lambda(r_t^M, r_t^I)(1+r_t^I) - \big(\lambda(r_t^M, r_t^I) - 1\big)(1+r_t^M) - \delta - 1 \\ &= r_t^M + \lambda(r_t^M, r_t^I)(r_t^I - r_t^M) - \delta. \end{aligned}$$

This formula reveals that the bankers benefit from higher capital return in sector I and from leverage. For the reminder of this paper we make the assumption that bankers are more impatient than investors, i.e. $\beta_B < \beta_H$ or $\rho_B > \rho_H$.

5 Steady State

In this section, we first prove the existence and uniqueness of the steady state and second analyze the impact of financial frictions and technological progress on the steady state allocation.

5.1 Existence and Uniqueness of Steady State

In a steady state, allocations and prices are constant across time. For now suppose that the economy is at a steady state in which financial frictions are relevant. As $E_{t+1} = E_t$ and $\Omega_{t+1} = \Omega_t$, the saving policies provided in Proposition 2 yield

$$\hat{r}^M = \delta + \rho_H \tag{11}$$

$$\theta\hat{\lambda} = \delta + \rho_B + 1. \tag{12}$$

where $\hat{\lambda}$, $\hat{r}^M$, $\hat{r}^I$, etc. denote steady state values. Moreover, combining the definition of λ with the steady state condition for bank equity leads to

$$\hat{r}^I = \hat{r}^M + \frac{\theta(\rho_B - \rho_H)}{1 + \delta + \rho_B} = \delta + \rho_H + \frac{\theta(\rho_B - \rho_H)}{1 + \delta + \rho_B}.$$

Clearly, $\hat{r}^I > \hat{r}^M$ which is consistent with our presupposition that financial frictions matter.

Using the equilibrium interest rates $\hat{r}^M = \alpha z^M \left(\frac{\hat{K}^M}{L^M}\right)^{\alpha-1}$ and $\hat{r}^I = \alpha z^I \left(\frac{\hat{K}^I}{L^I}\right)^{\alpha-1}$ and the bankers' investment policy $\hat{K}^I = \hat{\lambda}\hat{E}$, the steady-state allocation computes as

$$\hat{K}^M = \left(\frac{\alpha z^M}{\hat{r}^M}\right)^{\frac{1}{1-\alpha}} L^M \tag{13}$$

$$\hat{K}^I = \left(\frac{\alpha z^I}{\hat{r}^I}\right)^{\frac{1}{1-\alpha}} L^I \tag{14}$$

$$\hat{E} = \left(\frac{\alpha z^I}{\hat{r}^I}\right)^{\frac{1}{1-\alpha}} \frac{\theta}{1 + \delta + \rho_B} L^I \tag{15}$$

$$\hat{\Omega} = \hat{K} - \hat{E}. \tag{16}$$

So far we have assumed that financial frictions matter at the steady state. We next show that there does not exist a steady state in which financial frictions do not matter. Suppose that a steady state without frictions exists. In such a steady state $E_t \geq \bar{E}(K_t)$.

Thus,

$$
\begin{aligned}
r_t^M = r_t^I &= \alpha z^M \left(\frac{K_t}{L^M(1+z\ell)} \right)^{\alpha-1} = \alpha z^I \left(\frac{z\ell K_t}{L^I(1+z\ell)} \right)^{\alpha-1} \\
E_{t+1} &= \beta_B \left(1 + \alpha z^M \left(\frac{K_t}{L^M(1+z\ell)} \right)^{\alpha-1} - \delta \right) E_t \\
\Omega_{t+1} &= \beta_H \left(1 + \alpha z^M \left(\frac{K_t}{L^M(1+z\ell)} \right)^{\alpha-1} - \delta \right) \Omega_t.
\end{aligned}
$$

Since $\Omega_{t+1} = \Omega_t$ and thus $r_t^M - \delta = \rho_H$, we get $E_{t+1} < E_t$: bank equity declines which is a contradiction. Therefore, there does not exist any steady state in which financial frictions do not matter. Indeed, from these observations we are able to derive a stronger result for transitional dynamics and global stability which is the topic of Section 6. However, let us first summarize our finding in the following proposition.

Proposition 3 (Existence and Uniqueness of Steady State)**.**
The laws of motion described by Proposition 2 yield a unique steady state $(\hat{E}, \hat{\Omega})$. Specifically, the steady state capital allocation is given by equations (11) *to* (16)*.*

The allocation described in the Proposition is second-best in the following sense. Consider a social planner who can control the allocation of investors' capital across sectors and time, but is subject to the same financial friction as in the model involving markets, i.e. the bankers retain the non-pledgeable part of investments in sector I.[17] Then, starting from the steady state allocation, there is no feasible reallocation of capital of investors across sectors and time which would constitute a Pareto improvement for investors and workers.[18]

5.2 Financial Frictions, Technological Progress and the Steady State

As a simple consequence of Proposition 3, we obtain

Corollary 5. *An increase of the intensity of financial frictions, i.e. an increase of θ,*

(i) lowers the steady-state value $\hat{K}$,

(ii) increases bank equity $\hat{E}$ if bankers are not too impatient.

[17] We exclude taxation of capital endowments.

[18] This is a consequence of the first welfare theorem, applied to an economy with investors and workers only, and in which capital returns in sector I are reduced by the amount θ.

The statement for $\hat{K}$ follows immediately from the fact that a higher value of θ increases $\hat{r}^I$, and thus lowers $\hat{K}^I$. The impact on $\hat{E}$ is more involved. Differentiation yields

$$\frac{\partial \hat{E}}{\partial \theta} = \frac{1}{1+\delta+\rho_B}\left\{\frac{1}{\hat{r}^I}\left(\frac{\alpha z^I}{\hat{r}^I}\right)^{\frac{1}{1-\alpha}}\left(\hat{r}^M - \frac{\alpha}{1-\alpha}\frac{\theta(\rho_B-\rho_H)}{1+\delta+\rho_B}\right)\right\}L^I. \qquad (17)$$

When ρ_B is sufficiently close to ρ_H, we get $\frac{\partial \hat{E}}{\partial \theta} > 0$.

An important consequence of Corollary 5 is that more severe financial frictions depress the total amount of capital and the part owned by investors, but not the wealth of bankers in the steady state if bankers are not too impatient. The reason is subtle. A higher value of θ lowers leverage. However, when ρ_B is close to ρ_H, a steady state requires that $\hat{r}^I$ is close to $\hat{r}^M$ and thus $\hat{K}^I$ is close to $\frac{\hat{K}^M}{z\ell}$. As the latter is independent of θ, variations of θ have little effect on $\hat{K}^I$ for ρ_B close to ρ_H. Because $\hat{K}^I = \hat{\lambda}\hat{E}$, a higher value of θ is associated with a higher value of $\hat{E}$.

The consequences of an proportional increase of z^M and z^I by the same factor $1+\epsilon$ in each period are straightforward. Leverage and interest rates at the steady state are independent of the technology level. The capital allocation $\hat{K}^I$ and $\hat{K}^M$ and bank equity $\hat{E}$ are proportional to $(1+\epsilon)^{\frac{1}{1-\alpha}}$. As a result, the share of capital intermediated by banks in the economy is independent of the level of total productivity.

6 Stability and Permanent Shocks

In this section we focus on global dynamics and establish the global stability of the economic system. Moreover, we explore the entire dynamics when the system is affected by permanent shocks to financial frictions.

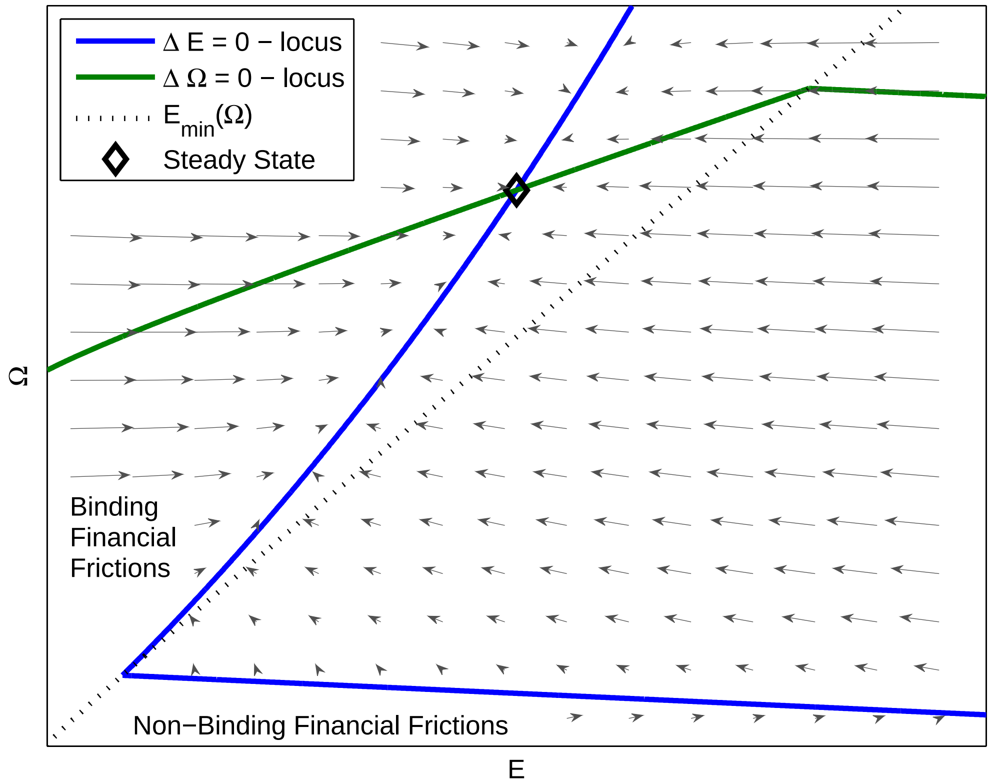
6.1 Global Stability

Global stability of the economy is established in two steps. We provide a brief informal outline of the steps and defer the formal argument to appendix A.

In the first step, one can show that for any initial (E_0, Ω_0) with $E_0 \geq \bar{E}(K_0)$, i.e. for any initial endowment for which financial frictions do not matter, the endogenous capital accumulation always reaches an equity and wealth allocation (E_τ, Ω_τ) in finite time τ such that financial frictions matter. In other words, bank equity and investors wealth always converge the the partition on (Ω, E)-space in which financial frictions matter.

The second step then shows that for any (E_τ, Ω_τ) with $E_\tau \leq \bar{E}(K_\tau)$, i.e. for any equity and wealth allocation for which financial frictions matter, the economic system converges

Figure 2: Phase Diagram



towards its unique steady state.

Taken together, these two observations culminate into the following proposition:

Proposition 4 (Global Stability of Steady State with Financial Frictions)**.**
For any initial $(E, \Omega) \in \mathbb{R}^2_+$*, the economy converges to the unique steady state in which financial fractions matter.*

For illustration, we depict the resulting global dynamics in the phase diagram, Figure 2.

6.2 Permanent Shocks to Financial Frictions

There are several examples for permanent shocks to the financial friction between depositors and bankers that materialize in an increase in θ. For instance, it can become more difficult to enforce contractual obligations which worsens the moral hazard problem. Another example is decreasing trust in the banking sector as a result of shifted beliefs about repayment behavior of bankers.

Suppose that the economy is at the steady state $(\hat{E}(\theta), \hat{\Omega}(\theta))$ associated with some level

θ of the financial friction. Suppose that in period $t = 1$, the economy is hit by a negative permanent shock that affects the financial friction. Specifically, assume that θ increases to θ' ($\theta' > \theta$). We first establish an analytical result regarding the consequences of such shocks for bankers.

Proposition 5. *Suppose that ρ_B is sufficiently close to ρ_H and the economy is hit by a negative permanent shock to financial frictions ($\theta \rightarrow \theta' > \theta$). Then, the intertemporal utility of bankers after the shock is higher than in the steady state associated with θ.*

Proposition 5 follows from the following considerations. Bank equity increases from period $t = 1$ onwards from $\hat{E}(\theta)$ to $\hat{E}(\theta')$ as direct consequence of Corollary 5. This means that during the transition phase $\theta'\lambda_t$ has to be larger than $\delta + \rho_B + 1$ and thus consumption of bankers during the transition phase is higher than in the steady state associated with $(\hat{E}(\theta), \hat{\Omega}(\theta))$. As the (net) steady state return on equity is independent of financial frictions, the bankers will have higher utility in each period when the economy is hit by an adverse shock to financial frictions.[19]

In contrast to bankers, investors and workers are hurt by higher financial frictions. Workers are also hurt in the long-run as aggregate wages decline towards the new steady state associated with $\theta' > \theta$. For investors, however, the intraperiod utility losses vanish over time as the interest rate r_t^M converges to $\hat{r}^M = \delta + \rho_H$, which is independent of θ.

7 Temporary Shocks and Quantitative Analysis

This section analyzes the behavior of the economy when it is hit by temporary wealth, productivity or trust shocks. To highlight the forces at work, we examine the impact of each type of shock separately. In this section, we assume that shocks are unanticipated. In Section 9, we discuss how the findings are affected when such shocks are anticipated. We provide an in-depth discussion of the main mechanisms and supplement it by a quantitative analysis. For this reason, we first outline the calibration strategy before discussing each type of shock separately.

7.1 Calibration

There are 10 model parameters: the production parameters α, z^M, and z^M, the depreciation rates δ^H and δ^B, the time preference factors β_H and β_B, the financial friction θ,

[19]We note that we do not have shown that the movement from $\hat{E}(\theta)$ to $\hat{E}(\theta')$ is monotonic. However, as initially $\theta'\lambda_t$ is larger than $\delta + \rho_B + 1$, a potential overshooting of bank equity above $\hat{E}(\theta')$ later on would not invalidate the conclusion.

and labor endowment L^M and L^I. The calibration is based on annual US data from 1998 to 2004 taken – with one exception – from the Federal Reserve Economic Data (FRED) and the Federal Deposit Insurance Corporation (FDIC) Call Report Data and proceeds as follows. As only the relative productivity affects the steady state allocation, we normalize $z^M = 1$ without loss of generality. In a similar vein, we normalize $L^M = 1$. The output elasticity of capital is set to $\alpha = 0.36$, which is in the range of values suggested in the literature.

We equalize depreciation rates on household wealth and bank equity, i.e. $\delta^B = \delta^H = \delta$. Let $\overline{s}$ denote the aggregate saving rate. The capital-to-output ratio simplifies to $K/Y = s/\delta$. Choosing the saving rate to match the gross-saving-to-GNP ratio taken from the FRED NIPA accounts, i.e. $\overline{s} = 0.1872$, and setting the capital-to-output ratio the implied series from the Penn World Table, i.e. $\overline{K/Y} = 3$, delivers $\delta = 0.0624$.

According to FRED, we set the average return on bank equity to 14.26 percent: $\overline{r}^B = 0.1426$. Moreover, taking the calibration target for bank leverage from the aggregated Call Report Data provided by the FDIC, $\overline{\lambda} = 10.7449$, the definition for return on equity $\overline{r}^B = \theta\overline{\lambda} - \delta - 1$ yields $\theta = 0.1121$ and the steady state condition for bank equity delivers $\beta_B = 1/(\theta\overline{\lambda} - \delta) = 0.8752$.

According to De Fiore and Uhlig (2011), the average bond-to-loan finance ratio in the US is $\overline{K^M/K^I} = 1.5000$. In order to attribute return differences solely to relative differences in productivity z, capital-to-labor ratios in sector M and I have to be equal. Thus, we set $L^I = 2/3$. Furthermore, combining the target for the bond-to-loan finance ratio with the aggregate resource constraint delivers $K^I = K/(1 + \overline{K^M/K^I})$. The steady state condition for household wealth and the leverage condition read

$$\frac{K^M}{L^M} = \left(\frac{\alpha z^M}{\frac{1}{\beta_H} + \delta - 1}\right)^{1/(1-\alpha)}$$
$$\frac{K^I}{L^I} = \left(\frac{\alpha\overline{\lambda}z^I}{(\overline{\lambda} - 1)\left(\frac{1}{\beta_H} + \delta - 1\right) + \theta\overline{\lambda} - 1}\right)^{1/(1-\alpha)}.$$

Using $K^M = \overline{K^M/K^I}K^I$ we combine both conditions to substitute for K^I and obtain

$$z^I(L^I)^{1-\alpha} = \frac{(\overline{\lambda} - 1)\left(\frac{1}{\beta_H} + \delta - 1\right) + \theta\overline{\lambda} - 1}{\frac{1}{\beta_H} + \delta - 1}\frac{z^M}{\overline{\lambda}}\left(\frac{L^M}{\overline{K^M/K^I}}\right)^{1-\alpha}. \tag{18}$$

Moreover, the capital-to-output ratio reads

$$\begin{aligned}\overline{K/Y} &= \frac{(1+\overline{K^M/K^I})K^I}{z^I(K^I)^\alpha(L^I)^{1-\alpha} + z^M(\overline{K^M/K^I})^\alpha(K^I)^\alpha(L^M)^{1-\alpha}} \\ &= \frac{1+\overline{K^M/K^I}}{z^I(L^I)^{1-\alpha} + z^M(\overline{K^M/K^I})^\alpha(L^M)^{1-\alpha}}(\overline{K^M/K^I})^{\alpha-1}\frac{\alpha z^M}{\frac{1}{\beta_H}+\delta-1}(L^M)^{1-\alpha}, \quad (19)\end{aligned}$$

where the second line uses the steady state condition for investor wealth and the bond-to-loan finance ratio to substitute for K^I. Using (18) in (19) and solving for β_H, we get

$$\beta_H = \left(1-\delta+\frac{\alpha\overline{\lambda}\frac{1+\overline{K^M/K^I}}{\overline{K/Y}} - (\theta\overline{\lambda}-1)}{\overline{\lambda}(1+\overline{K^M/K^I})-1}\right)^{-1} = 0.9485$$

which is within the range of values used in the literature for annual calibration. Finally, equation (18) yields $z^I = 1.0704$. The calibrated parameter values and the calibration targets are summarized in Table 2.

Table 2: Parameters and Calibration Targets

Parameter	α	z^M	z^I	δ^H	δ^B
Value	0.3600	1.0000	1.0704	0.0624	0.0624
Parameter	β^H	β^B	θ	L^M	L^I
Value	0.9485	0.8752	0.1121	1.0000	0.6667
Calibration Target	s	$\overline{K/Y}$	λ	r^B	$\overline{K^M/K^I}$
Value	0.1872	3.0000	10.7449	0.1426	1.5000

In order to assess the calibration strategy, we compute the implied size of the banking sector relative to GDP. Specifically, as K^I is equivalent to the asset side of the bank balance sheet, the relative size of the banking sector to GDP amounts to $\hat{K}^I/\hat{Y} = (1+\overline{K^M/K^I})^{-1}\overline{K/Y} = 1.2000$. Because we calibrate to the steady state we have to compare the model outcome with the pre-crisis average. Specifically, choosing 1998-2008 as reference period, we get a relative bank sector size of 0.7839. One reason for the higher relative bank sector size in our model is that we abstract from retained earnings in the production sector as an additional source to finance investment. The steady state allocation and further non-targeted equilibrium statistics are provided in Table 3.

Table 3: Steady State Allocation and Non-Targeted Statistics

Steady State Allocation	$\hat{E}$	$\hat{K}$	$\hat{K}^M$	$\hat{K}^I$	$\hat{C}^H$	$\hat{C}^B$
Value	0.3606	9.6871	5.8123	3.8749	0.5066	0.0514
Non-Targeted Statistic	$\hat{r}^M$	$\hat{r}^I$	$\hat{w}^M$	$\hat{w}^I$	$\hat{K}^I/\hat{Y}$	
Value	0.1167	0.1249	1.2060	1.2909	1.2000	

7.2 Wealth Shocks

We consider two different wealth shocks, distinguished whether the shocks fall primarily on bankers or investors. The main insights are: First, when levered banks perform financial intermediation to a sector that would otherwise not have access to capital markets, wealth shocks get amplified and are more persistent. Second, shocks hitting the banking sector directly are initially more severe than shocks hitting investors first, even when shocks are similar in magnitude. The persistence of these shocks, however, is similar.

We are agnostic about the sources of wealth shocks in this exercise. However, in the context of banks, it is useful to provide two clarifying examples. Suppose, first, that bankers can engage in additional risky activities[20] in a scale that is proportional to their equity.[21] Specifically, these activities yield κ^h ($\kappa^h > 0$) per unit of investments with probability ω and κ^l ($\kappa^l < 0$) with probability $1 - \omega$. Suppose that such activities yield a positive expected payoff such that they might be beneficial for bankers. In the case when financial frictions matter, these investment possibilities change the net return on bank equity in period t as follows

$$\tilde{r}_t^B = \begin{cases} \theta\lambda(r_t^M, r_t^I) + \kappa^h T - \delta - 1 & \text{with probability } \omega \\ \theta\lambda(r_t^M, r_t^I) + \kappa^l T - \delta - 1 & \text{with probability } 1 - \omega \end{cases} \tag{20}$$

where T is the scaling factor with regard to bank equity when bankers invest in risky activities. In this section we assume that bankers attach a zero probability to negative

[20] In an international context such assets may be financial claims in other countries. Examples of such type of behaviour in the recent financial crisis of 2008/2009 can be found in Hellwig (2009).

[21] This can occur when such investments require bankers' wealth as a backup.

events. Then, if we define $\delta' = \kappa^h T - \delta$, the extended version is isomorphic to the baseline version of the model as long as bad events are unanticipated. Suppose, second, as in Holmstrom and Tirole (1997) that motivating bankers to monitor entrepreneurs requires a higher payment to bankers when they are successful than when they fail. If the events success/failure are perfectly positively correlated across banks, aggregate bank equity declines with positive probability.

We illustrate the model dynamics using the following parameterization for the wealth shocks and depict the respective impulse response functions in Figure 3. Shocks either reduce bank equity or investor wealth by 1 percent of the total steady state capital stock. As a result, bank equity and investor wealth shocks amount to 25.27 and 1.04 percent of equity and investor wealth, respectively, and could be associated with a banking crisis and a (moderate) financial crisis, respectively. The size of the bank equity shock is large but not unreasonable for periods like the Great Recession in which inside and outside equity measured in book or market values have dropped substantially.

First, consider a shock to bank equity in $t = 0$. According to Corollaries 1 and 3, bank leverage increases and investment in sector I decreases. However, the effect on investment in sector M is ambiguous. Suppose that bank leverage is initially unaffected by the bank equity shock. Investors reduce their deposits by a multiple λ_0 of the bank equity shock in order to keep the leverage constant and use their excess funds to invest into bonds. As a result, K_0^M increases. However, when bank leverage adjusts, the increase in bank leverage dampens the withdrawal of deposits and thus dampens the increase in bond finance. In particular, when bank leverage is sufficiently sensitive, the increase in leverage can even revert the investors' portfolio choice such that deposits ultimately increase and investment into bonds decrease. For the remainder of the analysis, we confine attention to the case in which bank leverage is sufficiently insensitive to a bank equity shocks, i.e.

$$\frac{\mathrm{d}K_0^M}{\mathrm{d}E_0} = \frac{\partial K_0}{\partial E_0} - \frac{\partial K_0^I}{\partial E_0} = 1 - \frac{\partial \lambda_0}{\partial E_0} E_0 - \lambda_0 < 0,$$

which is also the predominant pattern for a wide range of parameters and calibration targets. Actually, this is also consistent with empirical evidence documenting a shift from loan to bond finance in recessions when equity shocks and productivity shocks are positively correlated. As K_0^I decreases and K_0^M increases, there is a shift of resources from the more productive sector I to the less productive sector M, which finally leads to a drop in total output. Clearly, shocks are more amplified the more levered banks are. Because low investment induces low consumption and saving in the banking sector, next period equity remains low, inducing similar dynamics in period $t+1$ and the subsequent periods.

Note that the more sensitive the bank leverage to changes in the capital-to-labor ratios is, the more dampened is the decline of deposits, and the higher are bank profits during the crisis. As a result, banks rebuild their equity stock more easily and shocks are less persistent. Evidently, the sensitivity of bank leverage plays an important role in the propagation of adverse shocks.[22]

Second, consider a shock to investor wealth in $t = 0$. As Ω_0 declines, λ_0 declines according to Corollary 3 and K_0^M declines according to Corollary 4. Clearly, when λ_0 declines, $K_0^I = \lambda_0 E_0$ declines as well. As before this reduces bank equity in the next period which leads to amplification and persistence of shocks similar to the ones discussed previously. However, as shocks are only indirect, the decline in bank equity is not as emphasized as before. Thus, shocks to investor wealth get relatively less amplified but their persistence is comparable to bank equity shocks.

In order to measure dynamic output losses, we define two measures to assess the consequences when either bankers or investors experience negative shocks to their wealth. Without loss of generality, we assume that the economy is initially at its steady state and is hit by either a bank equity shock ΔE or an investor wealth shock $\Delta\Omega$. For comparison, we assume that the overall capital stock declines by the same percentage points, i.e. we set $\frac{\Delta E}{\hat{K}} = \frac{\Delta\Omega}{\hat{K}} = \mu$ where μ is the relative decline of the capital stock.

Definition 2. *The bank equity multiplier and household wealth multiplier are defined as*

1. *Bank Equity Multiplier (BEM_μ)*

$$BEM_\mu \doteq \frac{\frac{\sum_{t=0}^{\infty}(Y_t - \hat{Y})(\beta_H)^t}{\hat{Y}}}{\frac{\Delta E}{\hat{K}}}.$$

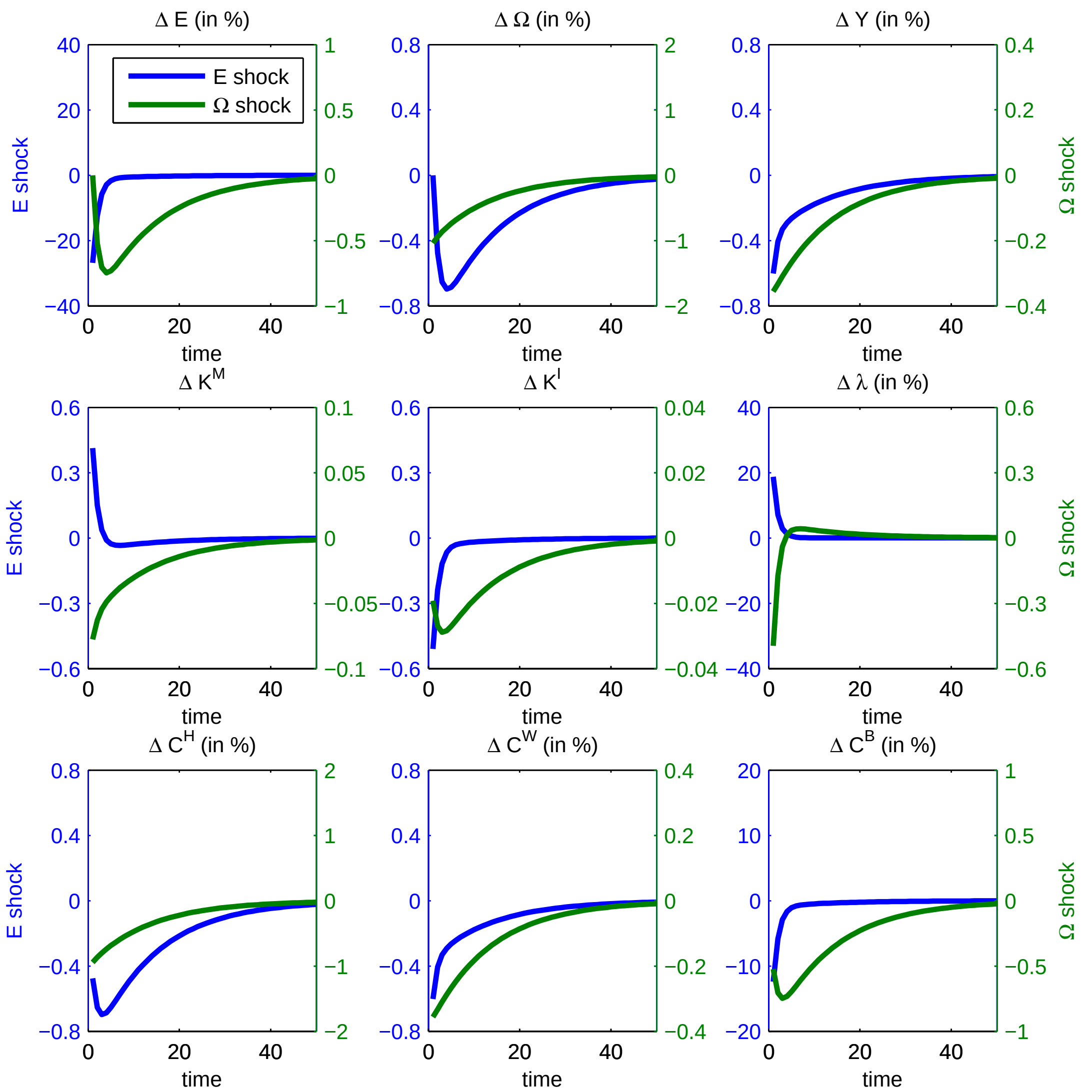
2. *Household Wealth Multiplier (HWM_μ)*

$$HWM_\mu \doteq \frac{\frac{\sum_{t=0}^{\infty}(Y_t - \hat{Y})(\beta_H)^t}{\hat{Y}}}{\frac{\Delta\Omega}{\hat{K}}}.$$

Table 4 reports the bank equity and investor wealth multipliers, respectively. The impact of the bank equity shock is larger, although both values are of the same order of

[22]Labor mobility is an important determinant for the sensitivity of bank leverage: Since labor is immobile the reactions of capital returns and equilibrium bank leverage are stronger than in the case of complete labor mobility. As a consequence, the amplification and persistence of wealth shocks is smaller when labor is immobile (see Gersbach et al. (2015) for an extreme amplification and persistence of wealth shocks in the two-sector Solow model with banks). A deeper discussion of this link between labor markets and financial intermediation is beyond the scope of the paper.

Figure 3: Response to Wealth Shock



magnitude since the persistence of shocks is similar. The reason is that shocks to investor wealth trigger a subsequent decline of bank equity that delays the recovery similar to direct shocks to bank equity.

Focusing on consumption, we find that both shocks affect consumption of investors, workers, and bankers in a similar way, albeit bank equity shocks induce stronger reactions of the consumption paths. In terms of consumption equivalent units, the welfare costs of a bank equity shock and a investor wealth shock are of the same order of magnitude. Hence, the bank capital transmission channel has important welfare implications even for agents that are not directly affected by the shock in the initial period. In contrast, bank equity shocks induce an order of magnitude larger welfare costs for bankers than a comparable one on investor wealth (see Table 4). In essence, the quantitative results highlight the role banks in transmitting adverse shocks even when banks are not directly affected by shocks in first place.

Table 4: Welfare Costs and Output Multipliers of Negative Capital Shocks

	Output Multipliers	Welfare Costs		
		Investor	Worker	Banker
Bank Equity Shock	5.2073	0.3504	0.1696	3.0894
Household Wealth Shock	4.9553	0.4002	0.1543	0.5522

7.3 Productivity Shocks

This section considers isolated productivity shocks in sectors I and M as well as a joint productivity shock that leaves the relative productivity in both sectors unaffected. The main results are as follows: First, sector specific shocks lead to substantially different dynamics. While negative shocks in sector I induce a drop in bank equity and investor wealth, shocks in sector M lead to a temporary boom in the banking sector. Second, joint productivity shocks induce procyclical bank leverage and countercyclical bond-to-loan finance ratios.

Shocks are parameterized as a two percent decrease in productivity $z^j, j = \{M, I\}$ for one period and the impulse response functions for the distinct shocks are depicted in Figure 4.

Consider a productivity shock in sector I in $t = 0$. According to Corollary 1, a negative productivity shock in sector I immediately reduces bank leverage. Because initial bank equity is unaffected, this implies that investors withdraw deposits and use these funds to invest in bonds: K_0^M increases whereas $K_0^I = \lambda_0^I E_0$ decreases. An immediate output drop ensues and, as a result, next period equity is below its steady state level. Moreover, because r_0^M decreases, next period investor wealth falls short of its steady state value as well. With productivity returning to its pre-shock value in the subsequent period, the situation is similar to the negative wealth shocks analyzed in the previous section and the dynamics unfold in a similar vein with an overshooting of bank leverage.

The situation is different when the economy is hit by a negative (one-period) productivity shock in sector M only. According to Corollary 1, leverage increases. Because of $K_0^I =$

$\lambda_0 E_0$, loan finance decreases and consequently, loan finance increases. As the bankers' profits increase, next period bank equity holdings exceeds its steady state value. On the contrary, investor wealth remains below its steady state value. Hence, bankers benefit from the bust in period 0 while investors are hurt. The recovery towards the steady states starts with a bank equity allocation above and an investor wealth allocation below its steady state value, respectively.

Joint productivity shocks in sector M and I are dominated by the dynamics of the shock in sector I. If the economy is hit by a negative aggregate productivity shock, Corollary 2 states that leverage in period 0 declines which also causes a decline of bank equity in period 1. As more capital will be employed in sector M in period 0 (and productivity declines), the return r_0^M falls such that investor wealth in period 1 is lower than without shocks. From period 1 onwards, the economy starts to recover: investor wealth and bank equity start to build up again. Hence, a (one-period) negative aggregate productivity shock depresses all accumulation processes and thus output.

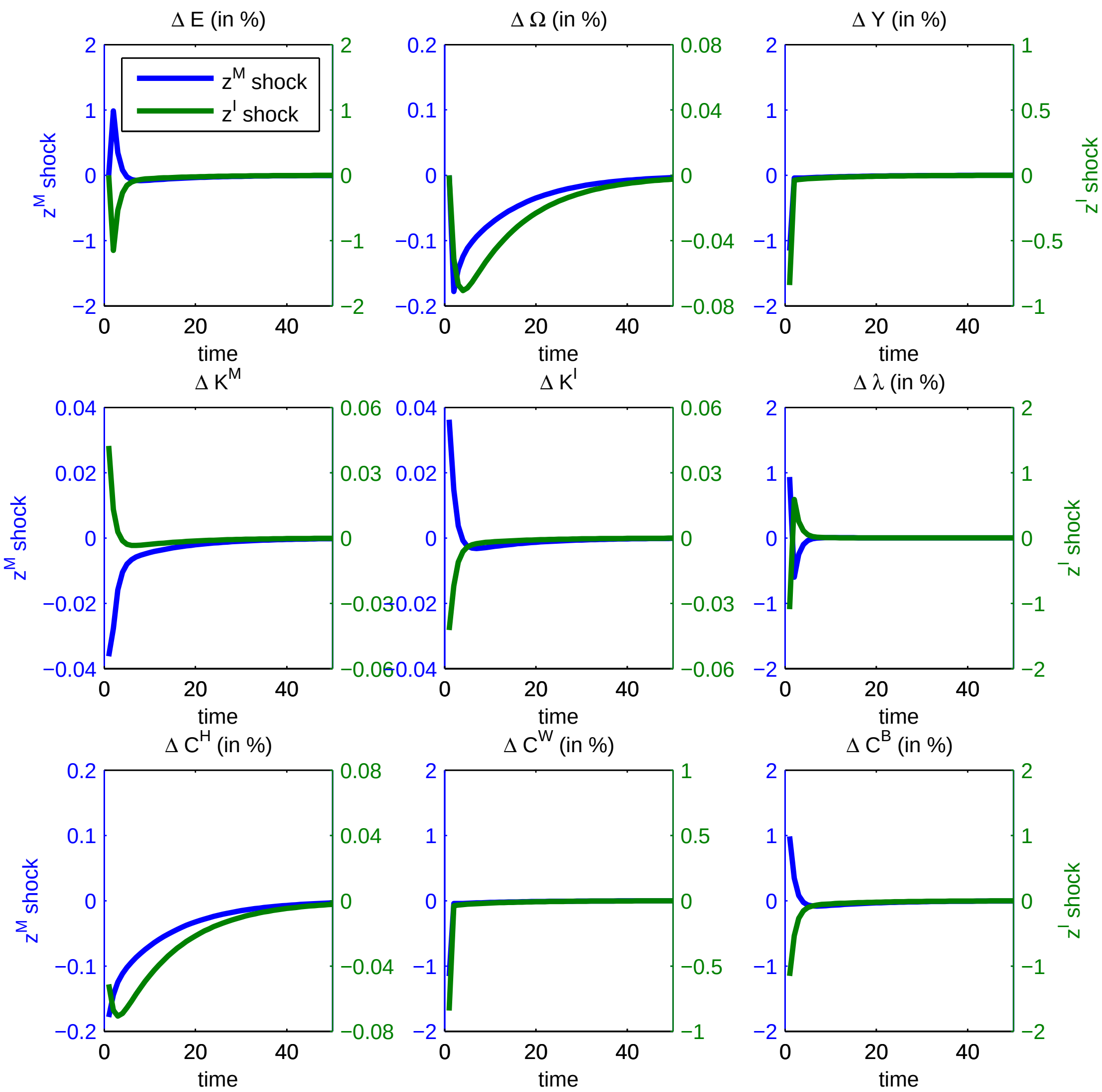
Procyclicality of bank leverage and countercyclicality of the bond-to-loan finance ratio are well-documented patterns in the literature. Adrian and Shin (2014) and Adrian and Boyarchenko (2012) establish that such properties arise when banks are confronted with a VaR constraint and, thus, the larger the risk the lower is leverage. This implies that leverage is procyclical as the level of risk is countercyclical. In our context, leverage is determined by the *skin in the game* constraint bankers face and this constraint is lessened if aggregate productivity increases. In addition, the bond-to-loan finance ratio is countercyclical, as documented e.g. in De Fiore and Uhlig (2011).

Welfare losses are moderate for investors and workers. In contrast, bankers even realize welfare gains when an isolated productivity shock hits sector M. The reason is that the increase in leverage generates higher profits and therefore rises the bankers' consumption path. When the shock hits sector I, however, the bankers' welfare loss is higher than the welfare loss of investors and workers. The quantitative results are summarized in Table 5.

7.4 Worsening of Financial Frictions (Trust Shocks)

This section discusses the impact of a temporary shock to the financial friction. The main result is as follows: A negative shock to financial frictions boosts accumulation of bank equity while it depresses output. When the shock ends, higher level of bank equity may allow temporary higher investment in sector I thereby boosting output. Trust shocks are calibrated as a 30 percent increase in the financial friction parameter θ for 2 consecutive periods, i.e. $\theta' = 1.30\ \theta$.

Figure 4: Response to Productivity Shock



According to Corollary 1, bank leverage decreases when the economy is hit by a trust shock in $t = 0$. Thus, investors shift resources from deposits to bonds which increases K_0^M and since bankers are more restricted in their investment choice, they reduce K_0^I. However, as θ increases, banks can divert a larger fraction of their investment. As long as $\frac{\partial \lambda_0}{\partial \theta} < 1$ (which is a predominant pattern for a large variety of parameters), bankers actually realize higher revenues and therefore build up equity above the steady state value. Because interest rate r_0^M declines, investor wealth is lower than in the steady state. For the consecutive periods with high θ', there can even be a boom in output as investment in the more efficient sector I increases. Clearly, as soon as the friction

Table 5: Welfare Costs of Negative Productivity Shocks

	Welfare Costs		
	Investor	Worker	Banker
Shock in Sector M only	0.0619	0.0794	-0.1293
Shock in Sector I only	0.0353	0.0572	0.2735
Shock in Sectors M and I	0.0974	0.1374	0.1444

returns to its normal value θ, the dynamics unfold according to a situation in which bank equity is relatively abundant and investor wealth is relatively scarce. Thus, the dynamics follow a similar pattern as under isolated negative productivity shocks in sector M. The respective impulse responses are depicted in Figure 5.

Our considerations also imply that a worsening of financial frictions leads to comparatively large welfare gains for the bankers through the induced boost in bank equity, whereas investors suffer a welfare loss (see Table 6).

Table 6: Welfare Costs of Negative Trust Shocks

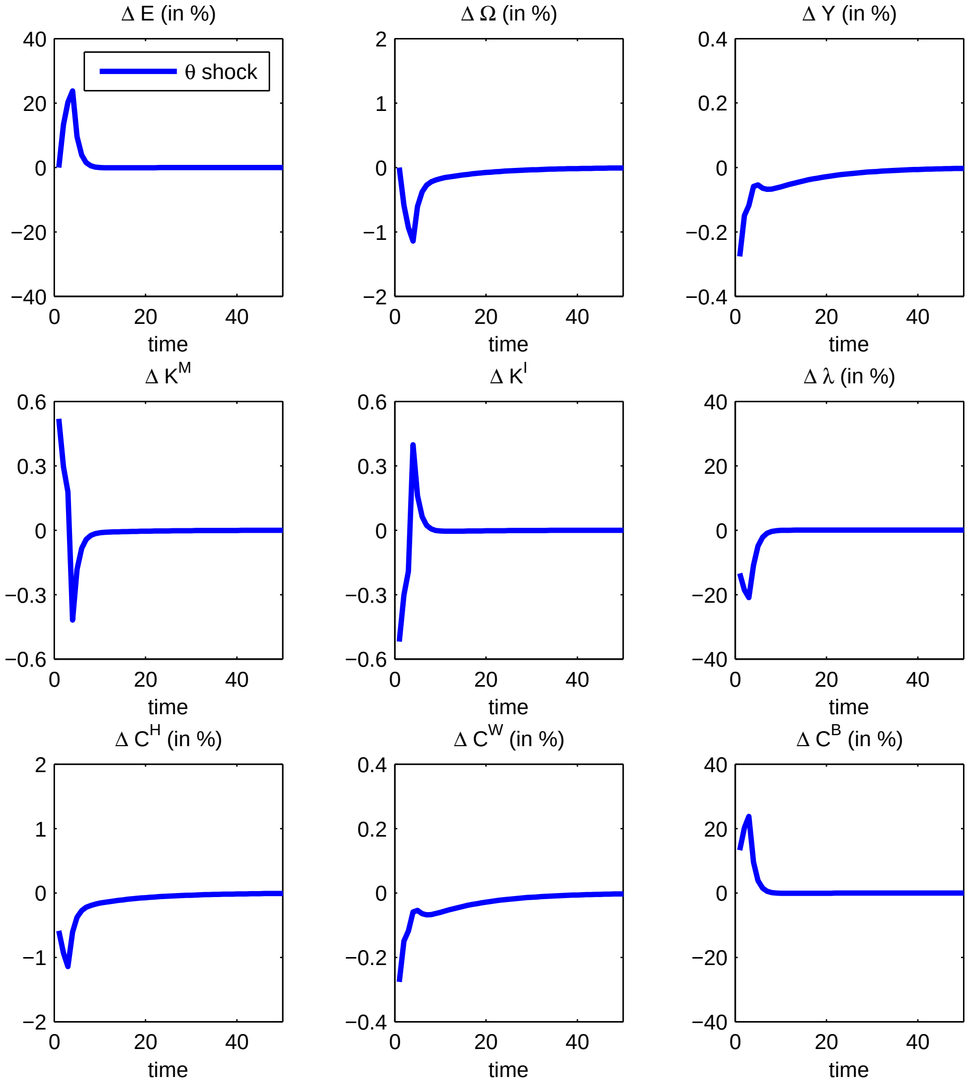
	Welfare Costs		
	Investor	Worker	Banker
Trust Shock	0.2460	0.0576	-6.5206

7.5 Bond and Loan Finance over the Business Cycle

The correlations of the two forms of financing – bank loans and bonds – with the business cycle are well-documented: Total bank loans are markedly procyclical (with some lag) while the level of bonds reacts little or is even countercyclical (De Fiore and Uhlig (2015) and Contessi et al. (2013)). We can account for this empirical observation in three steps.

Step 1: If normal recessions are associated with a negative productivity shock in both sectors, Corollary 2 delivers procyclical bank leverage and countercyclical bond-to-loan finance ratios.

Figure 5: Response to Trust Shock



Step 2: If the recession is accompanied by an initial sharp decline in bank equity, we refer to this situation as a banking crisis. Following our discussion of the bank equity shocks in Subsection 7.2, we find that the procyclicality of bank leverage and the countercyclicality of the bond-to-loan finance ratio is reinforced.

Step 3: If the banking crisis is accompanied by an increase of θ as trust in repayment pledges decline,[23] the cyclical pattern of bank loans and bonds in downturns of the types described in Step 1 or Step 2 is again reinforced as discussed in Subsection 7.4.

[23]According to Bloom et al. (2012) downturns are associated with a general increase of uncertainty which in our context could be interpreted as less trust in repayment pledges.

Table 7 reports welfare effects associated for the different steps when shock sizes are calibrated as previously described. The change of θ seems to be a strong mechanism that easily turns the bankers welfare losses into welfare gains.[24]

A more elaborated estimation and calibration of changes in the uncertainty is beyond the scope of the paper.

Table 7: Welfare Costs

	Welfare Costs		
	Investor	Worker	Banker
Productivity Shock	0.0974	0.1374	0.1444
Productivity and Equity Shock	0.4479	0.3077	3.2701
Productivity, Equity, and Trust Shock	0.6850	0.3863	-2.9661

8 Managing Recoveries

The previous section showed that negative shocks to bank equity can lead to persistent output losses and a comparatively slow recovery. In this section, we explore the scope of public policies and regulation when the economy is hit by negative shocks in order to speed up the economic recovery. We discuss two particular policies: dividend payout restrictions alone or a combination of dividend payout restrictions and investors financed capital injections. In order to simplify the exposition, we abstract from capital depreciation for the analytical results.

8.1 Dividend Payout Restrictions

Suppose the economy is at its steady state and at the end of period $\tau = 0$, i.e. after investment and production took place and before savings/investments decisions are made, it experiences a bank equity shock $\Delta E > 0$: End-of-period bank equity declines from $(1 + \hat{r}_0^B)\hat{E}$ to $(1 + \hat{r}_0^B)\hat{E} - \Delta E$. In order to speed up recovery, the regulator could implement limits on dividend payout ratios[25] which – in our model – translates into

[24]How changes of θ could be estimated from the increase of uncertainty in downturns (see Bloom et al. (2012)) is beyond the scope of this paper.

[25]Those regulations have been a prominent theme after the financial crises (e.g. Acharya et al. (2011)).

higher forced savings rates of bankers. Let $d_0 > \beta_B$ denote the forced higher saving rate the regulator prescribes. Bank equity at the beginning of period $\tau = 1$ satisfies $E_1^{lf} = \beta_B((1+\hat{r}_0^B)\hat{E} - \Delta E) < d_0((1+\hat{r}_0^B)\hat{E} - \Delta E) = E_1^{dr}$ with E_1^{lf} and E_1^{dr} denoting equity under laissez-faire and dividend payout restrictions, respectively. Applying the principle successively for a time period $[0, \tau']$, the regulator can engineer a more rapid build-up of bank equity when setting a sequence $\{d_\tau\}_{\tau=0}^{\tau'}$. We obtain

Proposition 6 (Dividend Payout Restrictions)**.**
Suppose that the economy is hit by a negative bank equity shock at $\tau = 0$. Then, appropriate restrictions of dividend payout ratios during a recovery time $[0, \tau']$, accelerate the convergence to the steady state, increase aggregate output and workers' consumption in all periods.[26]

8.2 Dividend Payout Restrictions and Capital Injections

Suppose again that bank equity declines from $\hat{R}_0^B \hat{E}$ by $\Delta E > 0$ at the end of period $\tau = 0$ and before savings-investment decisions are taken. Consider two policy instruments. First, limits on dividend payout ratios and second, capital transfer ΔT ($\Delta T > 0$) from investors to the banking sector. We refer to this policy package as *balanced bailout.* The idea is to simultaneously increase bank equity at the end of period $\tau = 0$ and to force a more rapid accumulation of bank equity.

We construct the policy scheme as follows. Suppose that the regulator makes a small one-time transfer ΔT from investors to bankers and imposes a payout restriction which forces bankers to retain a fraction $d_0 > \beta_B$ of their wealth. Hence, dividend payouts are restricted to $1 - d_0 < 1 - \beta_B$ at the end of period $\tau = 0$. Then, the regulator follows a scheme that produces the same consumption path for bankers as in the case when there are no policy interventions.[27] A formal description and analysis of the scheme is deferred to appendix B.

Because bankers obtain the same consumption path as under laissez faire, bankers are indifferent between regulation and laissez faire. With balanced bailout the total capital stock, K_t, and capital employed in sector I, K_t^I, exceed their laissez-faire values in all periods. As a result, total wage income of workers is always higher than under laissez such that workers unambiguously benefit from such a scheme. For investors, however,

[26] The impact on investors is more involved. Total consumption of workers and investors will increase.

[27] Policies that would make bankers better off in the accelerated recovery than without policy interventions may introduce moral hazard. Bankers may have an incentive to pay out more dividends, thereby consuming more, in order to cause a negative bank equity shock thereby inducing a bailout. However, there are no incentives for such a behavior at the individual level as banks can trigger a bailout only collectively.

the result is ambiguous: although benefiting from faster recoveries with higher returns, investors suffer from financing the initial capital injection to the banking sector. Which effect finally dominates depends on the specific calibration. The reason for this is as follows. If leverage is high, bank equity shocks also lead to high reductions in loan supply and thus to high output losses. Recessions are deep and persistent such that policies that avoid the bank capital transmission channel to unfold – like the investor financed capital injections – may be even welfare improving for investors.

We summarize the above results in the following proposition:

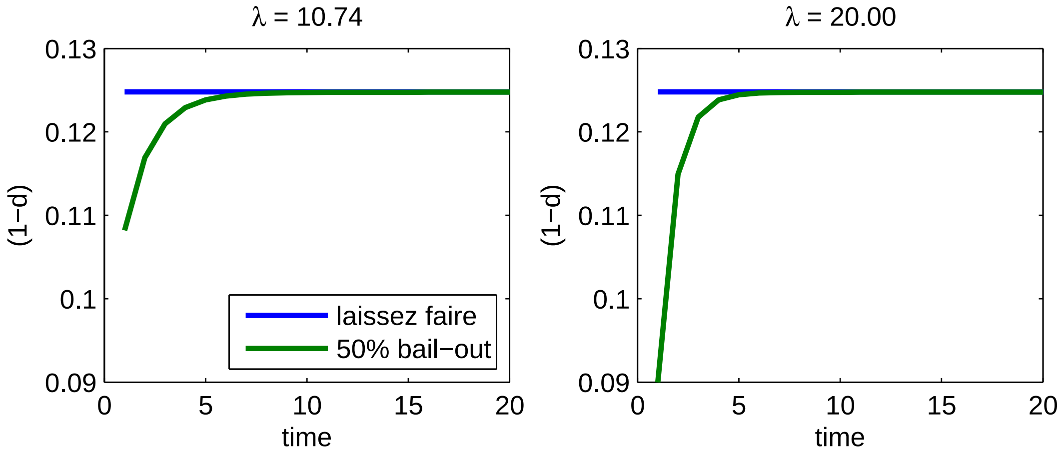
Proposition 7 (Dividend Payout Restrictions and Capital Injections)**.**
Suppose that a negative bank equity shock occurs and suppose that investors are sufficiently patient. Then, there exists an initial transfer ΔT from investors to bankers and a sequence of dividend payout restrictions on banks $\{1 - d_t\}_{t=0}^{\infty}$ such that life-time utility of bankers is the same as under laissez-faire and workers are better off.

Table 8 provides the welfare effects first, for the laissez-faire and second, for a 50 percent capital injection, i.e. $\Delta T = 0.5\Delta E$. We also consider an alternative calibration in which we change the calibration target for the steady state leverage to 20. Finally, Figure 6 informs about the respective time path of dividend payout restrictions.

Table 8: Welfare Costs of Negative Equity Shocks under Different Policy Regimes

Policy	Welfare Costs		
	Investor	Worker	Banker
$\lambda = 10.74$			
Laissez-faire	0.2901	0.1395	5.8112
50 Percent bailout	0.3092	0.1190	5.8112
$\lambda = 20.00$			
Laissez-faire	0.3193	0.1441	10.0426
50 Percent bailout	0.3318	0.1244	10.0426

Figure 6: Dividend Payout Restrictions



9 Extensions

This section we discuss several extensions and variations of the basic version. We restrict attention to four extensions that are particularly important for the conceptual understanding of the forces at work and the interpretation of the model. Throughout this section, we assume that bankers are less patient than investors and thus financial frictions always matter at the steady state.

9.1 Anticipated Bank Equity Shocks

This section focuses on the impact of anticipated shocks to bank equity. We employ a simple but general formulation which can be traced back to particular foundations of such shocks. Suppose that in each period production is deterministic as in our baseline, but the returns of bankers are affected by an additional exogenous shock. Specifically, let

$$r_t^B = \begin{cases} \theta\lambda(r_t^M, r_t^I) - \delta - 1 & \text{with probability } 1-\nu \\ \kappa(\theta\lambda(r_t^M, r_t^I) - \delta) - 1 & \text{with probability } \nu \end{cases}$$

where $0 < \kappa < 1$ and $0 < \nu < 1$. The shocks to bank equity are independently and identically distributed across time and realized before the bankers make their consumption-saving decision. Typically, ν is assumed to be rather small and thus such shocks occur infrequently. The bankers' Euler equation reads

$$\frac{1}{C_t^B} = \beta_B \mathbb{E}_t \left[\frac{1 + r_{t+1}^B}{C_{t+1}^B} \right]$$

where $\mathbb{E}_t$ denotes the expectation taken at the end of period t. As current shocks are realized when the consumption-saving decision is made, the functional form of the saving policy remains unaffected compared to the non-stochastic case. Specifically, $C_t^B = (1 - \beta_B)(1 + r_{t+1}^B)E_t$ and $E_{t+1} = \beta_B(1 + r_{t+1}^B)E_t$, i.e. saving rates remain constant with respect to end-of-period equity. Hence, with high probability, the dynamics associated with anticipated but rare negative shocks to bank equity are approximately the same as if such shocks occur unanticipated at the deterministic steady state if the shocks are sufficiently rare. The reason is that with very high probability, the economy is close to its steady state, when a negative bank equity shock occurs.[28]

9.2 Stochastic Productivity Shocks

In analogy to the real business cycle literature, we introduce stochastic shocks to technology. For this purpose we reintroduce the index for total factor productivity which follows a stochastic process. Typically, we assume

$$A_t = \overline{A} \mathrm{e}^{a_t}$$
$$a_t = \rho a_{t-1} + \eta_t$$

where η_t is an independent and identically normally distributed random variable with mean zero and variance σ_η^2. The parameter ρ $(0 < \rho < 1)$ measures the persistence of the shock. By using the same argument as in the previous section we note that the saving rates with respect to end-of-period wealth of bankers and investors remain unaffected by uncertainty about future returns to capital and bank equity. Hence, the laws of motion essentially remain the same, but with stochastic time paths of all economic variables.

9.3 Costs of Intermediation

So far, we have assumed that banks do not incur real costs when they monitor entrepreneurs. Typically, however, commercial or universal banks have to spend considerable resources on such activities. Such costs can easily be integrated in our model. Suppose e.g. that banks incur a cost c $(c > 0)$ per unit of loans they monitor. Then,

[28] Assuming larger probabilities for negative bank equity shocks would allow to investigate limit distributions and stochastic steady states. For instance, leverage is distributed symmetrically for our calibration when we set $\kappa = 0.8$ and $\nu = 0.2$.

the non-pledgeable part increases to $(c+\theta)K_t^I$ while bankers only obtain θK_t^I from their lending activities. The market-imposed leverage constraint, equation (5), adjusts to

$$K_t^I = \frac{1+r_t^M}{r_t^M - r_t^I + \theta + c} E_t.$$

The analysis can be performed in the same way for this version of the model as in the baseline version. Specifically, similar to Section 5.1, we obtain

$$\begin{aligned}
\hat{r}^M &= \delta + \rho_H \\
\theta\hat{\lambda} &= \delta + \rho_B + 1 \\
\hat{\lambda} &= \frac{1+\delta+\rho_H}{\delta+\rho_H-\hat{r}^I+\theta+c} = \frac{1+\delta+\rho_B}{\theta} \\
\hat{r}^I &= \hat{r}^M + \frac{\theta(\rho_B-\rho_H)}{1+\delta+\rho_B} + c.
\end{aligned}$$

Hence, steady state leverage $\hat{\lambda}$ and the return on equity $\theta\hat{\lambda}$ are unaffected by costs of intermediation. However, less capital can be invested in sector I which reduces both, level of bank equity and investor wealth in the steady state.

9.4 Workers also Save

We consider a variant when households are not separated into investors and workers such that there are only two types of agents: households and bankers. The former own the capital stock Ω and supply labor inelastically. The problem of the household is the same as described in Subsection 3.3, except for the budget constraint (3) which now includes labor income

$$C_t^H + \Omega_{t+1} = w_t^I L^I + w_t^M L^M + \Omega_t(r_t^M + 1 - \delta).$$

Optimization yields the standard Euler equation for the household problem

$$C_{t+1}^H = \beta_H(1 + r_{t+1}^M - \delta)C_t^H. \tag{21}$$

The steady state condition derived from the households' necessary condition, remains unaffected. As there is no change in the banker's first-order-condition as well, the following proposition ensues:

Proposition 8 (Steady State of Two-Type Economy).
The steady state in the two-type economy is the same as in the three-type economy.

However, transitional dynamics in the two-agent economy cannot be made explicit as there are no closed form solutions for the consumption-saving policies. Quantitatively, the results are of a similar order of magnitude, given the calibration targets and strategy as illustrated in Table 9).[29]

Table 9: Capital Shocks: Multipliers and Welfare Costs

Parameter	Output Multipliers	Welfare Costs		
		Investor	Worker	Banker
3-Type Economy				
Bank Equity Shock	5.2073	0.3504	0.1696	3.0894
Investor Wealth Shock	4.9553	0.4002	0.1543	0.5522
2-Type Economy				
Bank Equity Shock	4.2997	0.2053		3.0838
Household Wealth Shock	3.3685	0.2031		0.4750

10 Summary and Outlook

We have presented a simple model of capital accumulation in which financial intermediaries are essential for small and medium firms to invest. The model delivers a set of insights into the underlying shock propagation mechanism, is consistent with various stylized facts and allows to study policy responses to downturns associated with a decline of bank equity. Numerous generalizations and extensions can be pursued. We just outline here three promising avenues for further research.

First, as the Eurozone and much of Asia rely heavily on bank loans while corporate bonds are much more dominant in the US,[30] our framework can help to investigate which type of economic structure is more resilient to adverse shocks.

Second, apart from monitoring firms, banks also perform risk sharing and maturity transformation. Including these functions into our banking model with capital accumulation is challenging but can provide further valuable insights. For instance, maturity transformation can play a crucial role for the amplification and persistence of shocks,

[29] A quantitative assessment on the impulse responses to the different shocks is available upon request.
[30] See e.g. De Fiore and Uhlig (2011) and Ghosh (2006).

making it more difficult to recover after a banking crisis.

Third, introducing frictional labor markets with extensive employment margin and imperfect labor transition between production sectors can shed light on the interaction of frictional labor markets and financial frictions and their role for amplification and persistence of adverse shocks. In particular, as mentioned previously, it may happen that the more mobile labor between the sectors impaired with search frictions, the more persistent are shocks and the mis-allocation between sectors.

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A Appendix – Global Stability

First, we show by contradiction that for any (E_0, Ω_0) with $E_0 \geq \bar{E}(K_0)$, there exists a period τ at which financial frictions start to matter, for all $t > \tau$. Suppose the assertion is not true and the economy remains in the frictionless case at all times. In the frictionless case, Proposition 2 delivers

$$\frac{E_{t+1}}{\Omega_{t+1}} = \frac{\beta_B}{\beta_H}\frac{E_t}{\Omega_t} < \frac{E_t}{\Omega_t}. \tag{A1}$$

Hence, the ratio $\frac{E_t}{\Omega_t}$ declines from period 0 on at rate $1 - \frac{\beta_B}{\beta_H}$. We next note that investor wealth will not decline whenever K_t satisfies

$$\beta_H \left[1 + \alpha z^M \left(\frac{K_t}{L^M(1+z\ell)}\right)^{\alpha-1} - \delta\right] > 1$$

and thus

$$K_t < (1+z\ell)L^M \left(\frac{\alpha z^M}{\delta + \rho_H}\right)^{\frac{1}{1-\alpha}} \doteq \tilde{K}.$$

For any initial values $(E_0,\ K_0)$, household wealth Ω_t is bounded from above and below by strictly positive numbers. Thus, equation (A1) implies

$$\lim_{t\to\infty} E_t = 0$$

which violates the no-friction condition (9) and thus establishes the contradiction.

Second, we show that for any (E_τ, Ω_τ) with $E_\tau \leq \bar{E}(K_\tau)$, the system converges towards its steady state. For this purpose, it is useful to start with the phase diagram. As financial frictions matter in the only steady state of the economy and the area where financial frictions matter is open in $\mathbb{R}^2_+$, financial frictions matter in a neighborhood of $(\hat{E}, \hat{\Omega})$.

For $t \geq \tau$, the financial friction matters. Recall that leverage λ_t is a function of E_t and Ω_t and solves

$$\lambda_t \varphi(\lambda_t) = \alpha z^M(\lambda_t - 1)\left(\frac{E_t + \Omega_t - \lambda E_t}{L^M}\right)^{\alpha-1} - 1 + \theta\lambda_t - \alpha z^I \lambda_t \left(\frac{\lambda E_t}{L^I}\right)^{\alpha-1} = 0.$$

Rewriting the laws of motion as

$$E_{t+1} = \beta_B E_t(\theta\lambda(E_t,\Omega_t) - \delta)$$
$$\Omega_{t+1} = \beta_H \Omega_t\left(1 - \delta + \alpha z^M\left(\frac{E_t + \Omega_t - \lambda(E_t,\Omega_t)E_t}{L^M}\right)^{\alpha-1}\right)$$

the $\Delta E = 0$ - locus, i.e. the location for E_t, and Ω_t such that $E_{t+1} = E_t$, is characterized by

$$\theta\lambda(E_t,\Omega_t) = \delta + \rho_B + 1. \tag{A2}$$

We recall that $\frac{\partial\lambda_t}{\partial E_t} < 0$ and $\frac{\partial\lambda_t}{\partial\Omega_t} > 0$. Hence, the function $\Omega^{\Delta E}(E_t)$, defined implicitly by (A2), has a positive slope. Moreover,

$$E_{t+1} < E_t \Leftrightarrow \theta\lambda(E,\Omega) < \delta + \rho_B + 1.$$

The function $\Omega^{\Delta\Omega}(E_t)$ associated with the locus $\Delta\Omega_t = 0$ - locus is defined implicitly by

$$\Omega_t + E_t - \lambda(E_t,\Omega_t)E_t = \left(\frac{\alpha z^M}{\delta + \rho_H}\right)^{\frac{1}{1-\alpha}} L^M. \tag{A3}$$

Moreover,

$$\Omega_{t+1} < \Omega_t \quad\Leftrightarrow\quad \Omega_t + E_t - \lambda(E_t,\Omega_t)E_t > \left(\frac{\alpha z^M}{\delta + \rho_H}\right)^{\frac{1}{1-\alpha}} L^M. \tag{A4}$$

Characterizing the slope of $\Omega^{\Delta\Omega}(E_t)$ is more involved. Consider the intraperiod equilibrium associated with a level of bank equity E_t, that satisfies $E_t < \hat{E}$, and investor wealth $\Omega_t = \Omega^{\Delta E}(E_t)$. According to equation (A2), λ_t is equal to the steady-state value. Hence, $K_t^I < \hat{K}^I$ and $r_t^I > \hat{r}^I$. As leverage is equal to the steady-state value and $(1 + r_t^M)/(r_t^M - r_t^I + \theta)$, we conclude that $r_t^M > \hat{r}^M$. Hence, $K_t^M < \hat{K}^M$. According to equation (A3), on the $\Delta\Omega = 0$ - locus, capital invested in sector M is equal to its steady-state value. Corollary 4 implies that, for a given E_t, Ω_t has to rise from $\Omega^{\Delta E}(E_t)$ to $\Omega^{\Delta\Omega}(E_t)$ to attain the $\Delta\Omega = 0$ - locus from the $\Delta E = 0$ - locus. Similarly, $E_t > \hat{E}$ implies $\Omega^{\Delta\Omega}(E_t) < \Omega^{\Delta E}(E_t)$. In essence, this yields

$$\Omega^{\Delta\Omega}(E_t) > \Omega^{\Delta E}(E_t) \quad\Leftrightarrow\quad E_t < \hat{E}_t.$$

From Corollaries 1 and 4 and the preceding consideration we obtain

Corollary 6. *When financial frictions are binding,* *(i)* $\Omega^{\Delta E}(E_t) < \Omega_t \Leftrightarrow E_t < E_{t+1}$,

(ii) $\Omega_t < \Omega^{\Delta\Omega}(E_t) \Leftrightarrow E_{t+1} > E_t$.

Now, suppose that financial frictions do not matter. To examine this case, it is useful to define $(E^{\Delta E}, \Omega^{\Delta E}(E^{\Delta E}))$ by the point in the prolongation of $\Omega^{\Delta E}(E_t)$ for which financial frictions start not to matter. We similarly define $(E^{\Delta\Omega}, \Omega^{\Delta\Omega}(E^{\Delta\Omega}))$. When financial frictions do not matter.

$$
\begin{aligned}
r_t^M = r_t^I &= \alpha z^M \left(\frac{K_t}{L^M(1+z\ell)}\right)^{\alpha-1} = \alpha z^I \left(\frac{z\ell K_t}{L^I(1+z\ell)}\right)^{\alpha-1}, \\
E_{t+1} &= \beta_B \left(1 + \alpha z^M \left(\frac{K_t}{L^M(1+z\ell)}\right)^{\alpha-1} - \delta\right) E_t, \\
\Omega_{t+1} &= \beta_H \left(1 + \alpha z^M \left(\frac{K_t}{L^M(1+z\ell)}\right)^{\alpha-1} - \delta\right) \Omega_t.
\end{aligned}
$$

Furthermore, by continuity at $(E^{\Delta E}, \Omega^{\Delta E}(E^{\Delta E}))$, $E_{t+1} = E_t$ and thus $r_t^{M,\Delta E} = \delta + \rho_B$. Similarly, at $(E^{\Delta\Omega}, \Omega^{\Delta\Omega}(E^{\Delta\Omega}))$, $\Omega_{t+1} = \Omega_t$ and thus $r_t^{M,\Delta\Omega} = \delta + \rho_H$. With obvious notations, we obtain

$$
K^{\Delta E} \doteq (1+z\ell)L^M \left(\frac{\alpha z^M}{\delta + \rho_B}\right)^{\frac{1}{1-\alpha}} < (1+z\ell)L^M \left(\frac{\alpha z^M}{\delta + \rho_H}\right)^{\frac{1}{1-\alpha}} \doteq K^{\Delta\Omega}.
$$

We can now easily derive the following corollary:

Corollary 7. *When financial frictions do not matter,*

(i) $K_t < K^{\Delta E} \Leftrightarrow E_{t+1} > E_t$,

(ii) $K_t < K^{\Delta\Omega} \Leftrightarrow \Omega_{t+1} > \Omega_t$.

Corollaries 6 and 7 allow to draw the phase diagram in Figure 2 and to derive convergence towards the steady state whatever the initial capital endowment in the economy (E_τ, Ω_τ), which finally completes the proof.

B Appendix – Dividend Payout Restrictions and Capital Injections

Consumption of bankers in period $\tau = 0$ is unchanged if

$$(1 - d_0)(\hat{R}_0^B \hat{E} - \Delta E + \Delta T) = (1 - \beta_B)(\hat{R}_0^B \hat{E} - \Delta E).$$

Therefore,

$$d_0 = 1 - (1 - \beta_B)\frac{\hat{R}_0^B \hat{E} - \Delta E}{\hat{R}_0^B \hat{E} - \Delta E + \Delta T}. \tag{B1}$$

We next observe that

$$E_1^{bb} = d_0(\hat{R}_0^B \hat{E} - \Delta E + \Delta T) = \beta_B(\hat{R}_0^B \hat{E} - \Delta E) + \Delta T = E_1^{lf} + \Delta T > E_1^{lf}. \tag{B2}$$

E_1^{bb} and E_1^{lf} denote the bank equity levels with balanced bailout and laissez-faire, respectively. Property (B2) is a direct consequence of the scheme enacted in period 0. We next observe that

$$E_1^{bb} + \Omega_1^{bb} > E_1^{lf} + \Omega_1^{lf}. \tag{B3}$$

The reason why inequality (B3) holds is that $E_1^{bb} = E_1^{lf} + \Delta T$ while households suffering a loss of ΔT will reduce savings by $\beta_H(\Delta T)$ which is less than ΔT.

As a consequence of Corollary 1, $\lambda_1^{bb} < \lambda_1^{lf}$. However, we have

$$\lambda_1^{bb} E_1^{bb} > \lambda_1^{lf} E_1^{lf}. \tag{B4}$$

The latter property follows from the following contradiction. Suppose that $\lambda_1^{bb} E_1^{bb} = K_1^{I,bb} \leq K_1^{I,lf} = \lambda_1^{lf} E_1^{lf}$. Hence, $K_1^{M,bb} \geq K_1^{M,lf}$. As a consequence, $r_1^{I,bb} \geq r_1^{I,lf}$ and $r_1^{M,bb} \leq r_1^{M,lf}$. This leads to $\lambda_1^{bb} \geq \lambda_1^{lf}$ which is a contradiction.

At the end of period 1 before consumption/savings decisions are made, the banker's wealth $\theta \lambda_1^{bb} E_1^{bb}$ with the balanced bailout is higher than under laissez-faire. Hence, the policy-maker can again impose a dividend payout restriction $1 - d_1 < 1 - \beta_B$ that satisfies

$$(1 - d_1)\theta \lambda_1^{bb} E_1^{bb} = (1 - \beta_B)\theta \lambda_1^{lf} - E_1^{lf}.$$

We next show that $r_1^{M,bb} > r_1^{M,lf}$ for β_H sufficiently high. Suppose to the contrary that

$r_1^{M,bb} < r_1^{M,lf}$, i.e.

$$K_1^{I,bb} < K_1^{I,lf} + \Delta T(1 - \beta_H).$$

Hence,

$$\frac{1 + r_1^{M,bb}}{r_1^{M,bb} - r_1^{I,bb} + \theta}(E^{lf} + \Delta T) < \frac{1 + r_1^{M,lf}}{r_1^{M,lf} - r_1^{I,lf} + \theta}E^{lf} + \Delta T(1 - \beta_H).$$

We decrease the left-hand side by using $r_1^{M,lf}$ instead of $r_1^{M,bb}$ and thus the above inequality implies

$$E_1^{lf}(-r_1^{I,lf}) + \Delta T(1 + r_1^{M,lf})(r_1^{M,lf} - r_1^{I,lf} + \theta) < \\ E_1^{lf}(-r_1^{I,bb}) + \Delta T(1 - \beta_H)(r_1^{M,lf} - r_1^{I,bb} + \theta).$$

For β_H sufficiently close to 1, the relationship can only hold if

$$K_1^{I,bb} > K_1^{I,lf} + \Delta T(1 - \beta_H)$$

which is a contradiction.

Hence, $r_1^{M,bb} > r_1^{M,lf}$, $\lambda_1^{bb}E_1^{bb} > \lambda_1^{lf}E_1^{lf}$, and $E_1^{bb} + \Omega_1^{bb} > E_1^{lf} + \Omega_1^{lf}$. We obtain

$$\Omega_1^{bb}(1 + r_1^{M,bb}) + \theta\lambda_1^{b}bE_1^{bb} = \alpha\left\{z^M\left(\frac{K_1^{M,bb}}{L^M}\right)^\alpha + z^I\left(\frac{K_1^{I,bb}}{L^I}\right)^\alpha\right\} > \\ \alpha\left\{z^M\left(\frac{K_1^{M,lf}}{L^M}\right)^\alpha + z^I\left(\frac{K_1^{I,lf}}{L^I}\right)^\alpha\right\} = \Omega_1^{lf}(1 + r_1^{M,lf}) + \theta\lambda_1^{lf}E_1^{lf}, \quad \text{(B5)}$$

as $r^{I,bb} > r_1^{M,bb} > r_1^{M,lf}$ and $K_1^{M,bb} + K_1^{I,bb} > K_1^{M,lf} + K_1^{I,lf}$ and more capital is invested in the more productive sector.

At the end of period 1, households save $\beta_H(1 + r_1^{M,bb})\Omega_1^{bb}$ and bankers are forced to save via the dividend payout restriction

$$d_1\theta\lambda_1^{bb}E_1^{bb} = \theta\lambda_1^{bb}E_1^{bb} - \theta\lambda_1^{lf}E_1^{lf} + \beta_B\theta\lambda_1^{lf}E_1^{lf}.$$

In combination with (B5), we get

$$\beta_H(1 + r_1^{M,bb})\Omega_1^{bb} + d_1\theta\lambda_1^{bb}E_1^{bb} > \beta_H(1 + r_1^{M,lf})\Omega_1^{lf} + \beta_B\theta\lambda_1^{lf}E_1^{lf}.$$

Hence, after policy-makers impose a dividend payout restriction at the end of period 1 on bank equity – thereby keeping consumption of bankers unchanged – we obtain

$$E_2^{bb} > E_2^{lf} \quad \text{and} \quad E_2^{bb} + \Omega_2^{bb} > E_2^{lf} + \Omega_2^{lf}.$$

A similar logic applies for the second and all subsequent periods.