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CROSS-LICENSING AND COMPETITION†

Abstract

We study bilateral cross-licensing agreements among N (>2) competing firms. We find that the industry-profit-maximizing royalty can be sustained as the outcome of bilaterally efficient agreements. This holds regardless of whether agreements are public or private and whether firms compete in quantities or prices. We extend this monopolization result to a general class of two-stage games in which firms bilaterally agree in the first stage to make each other payments that depend on their second-stage non-cooperative actions. Policy implications regarding the antitrust treatment of cross-licensing agreements are derived.

JEL Classification: D14, F13, L24, L41 and O34
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1 Introduction

A cross-license is an agreement between two firms that allows each to practice the other's patents (Shapiro, 2001, and Régibeau and Rockett, 2011). Cross-licensing has been a widespread practice for a long time. For instance, Taylor and Silberston (1973) report that cross-licensing accounts for a significant share of all licensing arrangements in many industries: 50% in the telecommunications and broadcasting industry, 25% in the electronic components industry, 23% in the pharmaceutical industry, etc.[1] Cross-licensing is therefore likely to have an impact on competition in a large number of sectors.

Cross-licensing agreements involve both technological and monetary transfers. Technological transfers are generally perceived as pro-competitive: they can result in goods being produced at lower costs by potentially more firms. These transfers are particularly useful in Information Technology (IT) industries, such as the semiconductor and mobile phone industries, where the intellectual property rights necessary to market a product are typically held by a large number of parties, a situation known as a *patent thicket* (Shapiro, 2001; U.S. DOJ and FTC, 2007; Galasso and Schankerman, 2010).[2] Monetary transfers, however, can be anticompetitive. More specifically, high per-unit royalties can allow firms to sustain high prices.

A natural question arises: do cross-licensing partners have incentives to agree on high royalties? The existing literature (discussed below) provides an answer to this question in a duopoly setting: in such environment, firms sign a cross-licensing agreement with royalties high enough to replicate the monopoly profit (see e.g., Fershtman and Kamien, 1992). This monopolization result can be generalized in a straightforward way to a setting with more than two firms signing a multilateral agreement involving all of them (see Section 2.2).

However, in the typical scenario observed in practice, i.e., *bilateral* cross-licensing in industries comprised of *more than two* firms, it is unclear whether two given firms would agree on high royalties. First, this might weaken their competitive positions with respect to their rivals. Second, if the terms of the cross-licenses are publicly observable and firms

[1] In particular, cross-licensing in the semiconductor industry has received much attention in the literature (Grindley and Teece, 1997; Hall and Ziedonis, 2001; Galasso, 2012).

[2] According to FTC (2011, pp.55-56), "The IT patent landscape involves products containing a multitude of components, each covered by numerous patents. ... This contrasts with the relationship between products and patents in the pharmaceutical and biotech industries where innovation is generally directed at producing a discrete product covered by a small number of patents." Patent thickets raise many concerns and are considered as one of the most crucial intellectual property issues of the day (Shapiro, 2007; Régibeau and Rockett, 2011).

compete in quantities, they have incentives to agree on low royalties to be perceived as efficient and, therefore, aggressive by their competitors. We build a model to investigate whether bilateral cross-licensing agreements can still allow firms to sustain the monopoly outcome in the presence of these countervailing effects.

We consider $N(> 2)$ competing firms owning one patent each. Firms can get access to the technologies covered by their rivals' patents through cross-licensing agreements, before competing in the product market. We suppose that the larger the set of patents to which a firm has access, the lower its marginal cost.[3] In the baseline model, we assume that firms are symmetric and engage in Cournot competition and focus on symmetric equilibria where all cross-licensing agreements specify the same royalty. We show the robustness of our main results by considering a number of extensions of our basic setup.

We focus on *bilaterally efficient* agreements. A set of cross-licensing agreements is said to be bilaterally efficient if each agreement maximizes the joint profit of the pair of firms who signed it, given all other agreements. Note that a firm's overall profit is composed of its *downstream profit*, i.e., the profit it makes from selling its product, and the *upstream profit* generated by patent licensing. We distinguish between public and private cross-licensing agreements: the terms of a private agreement are observable only to the parties who sign the agreement while in the case of a public agreement, the terms are observable to all the firms in the industry.

Consider first the case of public cross-licensing agreements. We show that the royalties that two firms in a coalition charge each other have two opposite effects on their joint downstream profit: the *Stackelberg effect*, which captures the fact that the firms can influence their rivals' outputs through their choice of royalties, and the *coordination effect*, which refers to the idea that two firms can restrict their joint output by increasing the royalties they charge each other. We also show that the royalties have two opposite effects on the firms' joint upstream profit: the *royalty-saving effect* and the *licensing revenue effect*. The former refers to the simple fact that the royalties paid to each other are transfers within the coalition and hence do not count as a cost in the joint profit, while the latter captures the idea that these royalties influence the licensing revenues collected from the rivals by affecting their output choices.

We show that a two-firm coalition's deviation in the (public) cross-licensing stage always has opposite effects on its downstream and upstream profits. It turns out that

[3] Alternatively, we can assume that the larger the set of patents to which a firm has access, the higher the value of its product. We show in Appendix B that our model of cost-reducing technologies can be equivalently interpreted as a model of value-increasing technologies.

these two effects cancel out when the (symmetric) per-unit royalty charged by firms is the one that *maximizes the industry profit.* This implies that the monopoly outcome can always be sustained through bilaterally efficient agreements. Moreover, we establish that, when the Stackelberg effect dominates the coordination effect, the situation in which all firms charge each other zero per-unit royalty (i.e., the most competitive outcome) is also sustainable as a bilaterally efficient outcome.

Consider now the case of private cross-licensing agreements. The only difference with the scenario of public cross-licensing is that the Stackelberg effect and the licensing revenue effect are now absent. Using this, we show that the royalty allowing to achieve the monopoly outcome is the unique bilaterally efficient royalty under private cross-licensing.

Thus, the monopoly outcome can be sustained when cross-licensing agreements are decided *bilaterally,* independent of whether the agreements are public or private. We show that this monopolization result extends to i) a setting in which the terms of the cross-licensing agreements are agreed upon by coalitions of any size and ii) an environment in which firms compete in prices rather than quantities. We also establish that this finding holds in a general two-stage model that allows for all kinds of asymmetries and applies to any situation in which firms that have downstream interactions sell inputs to each other through bilateral agreements. Examples include cross-licensing of patents, two-way access pricing in telecommunications (Armstrong, 1998; Laffont, Rey, Tirole, 1998a,b), interconnection among Internet backbone companies (Crémer, Rey and Tirole, 2000) and interbank payments for the use of ATMs (Donze and Dubec, 2006).

Finally, our analysis generates policy implications for the antitrust treatment of cross-licensing. Both American and European competition authorities grant antitrust safety zone to (cross-) licensing agreements signed by firms whose combined market share is below a certain threshold.[4] These policies are implicitly based on the presumption that market forces can discipline cross-licensing partners regarding the level of royalties they agree on: firms with relatively low market power are expected to find it unprofitable to charge each other high per-unit royalties. However, the existing theoretical literature on cross-licensing does not allow to analyze the relevance of such exemptions since it studies bilateral cross-licensing only in a duopoly setting. Our findings question those

[4] For instance, Article 10 of the EC Technology Transfer Block Exemption Regulation provides antitrust exemption to bilateral licensing agreements between competitors if their combined market share does not exceed 20%. Similarly, according to the US guidelines (U.S. DOJ and FTC, 1995, p.22), "... the Agencies will not challenge a restraint in an intellectual property licensing arrangement if (1) the restraint is not facially anticompetitive and (2) the licensor and its licensees collectively account for no more than twenty percent of each relevant market significantly affected by the restraint."

exemptions since they show that cross-licensing can actually allow firms in a given industry to implement the monopoly outcome regardless of their number and their market shares.

Our paper contributes to the literature on the competitive effects of cross-licensing agreements and patent pools. In a pioneering paper, Priest (1977) shows how these practices can be used as a disguise for cartel arrangements. Fershtman and Kamien (1992) develop a model in which cross-licensing improves the efficiency of R&D investments by eliminating effort duplication but favors price collusion between firms. Eswaran (1994) argues that cross-licensing can enhance the degree of collusion achieved in a repeated game by credibly introducing the threat of increased rivalry in the market for each firm's product.[5] Shapiro (2001) shows that patent pools tend to raise (lower) welfare when patents are perfect complements (substitutes), an idea which is generalized to intermediate levels of substitutability/complementarity by Lerner and Tirole (2004), and further explored in the case of uncertain patents by Choi (2010) and Choi and Gerlach (2014).[6] To the best of our knowledge, our paper is the first formalized study of the competitive effects of *bilateral* cross-licensing agreements in an industry comprised of *more than two* firms.[7]

The remainder of the paper is organized as follows. Section 2 describes the baseline model with symmetric firms and Cournot competition. Section 3 (Section 4) characterizes bilaterally efficient public (private) cross-licensing agreements. Section 5 discusses two extensions of the basic setup. Section 6 introduces a general model and shows that our monopolization result holds in a wide range of circumstances. Section 7 derives policy implications regarding the current antitrust treatment of cross-licensing agreements in the U.S. and EU. Section 8 gathers concluding remarks. All the proofs are relegated to Appendix A. Appendix B provides an interpretation of the baseline model in terms of value-increasing (instead of cost-reducing) innovations and two detailed extensions of that model.

[5] Relatedly, Kultti, Takalo and Toikka (2006) also view cross-licensing as mechanism to establish multimarket contact but argue that it can only facilitate collusion in so far as multimarket contact does.

[6] Choi and Gerlach (2014) show that in the case of uncertain patents, pooling complementary patents can decrease welfare by shielding them from potential litigation.

[7] In our companion paper (Jeon and Lefouili, 2014), we study bilateral licensing agreements in an industry with more than two competitors. However, in that paper, licensing contracts are assumed to involve fixed fees only and we focus on how equilibrium networks of bilateral licensing contracts affect market structure. That paper is complementary to the current one as it shows that even when royalties cannot be used to raise prices, bilateral licensing agreements can reduce welfare by inducing exclusion of some firms.

2 Baseline Model

2.1 Setting

Consider an industry consisting of $N \geq 3$ symmetric firms producing a homogeneous good. Each firm owns one patent covering a cost-reducing technology and can get access to its rivals' patents through cross-licensing agreements. We assume that the patents are symmetric in the sense that the marginal cost of a firm only depends on the number of patents it has access to. Let $c(n)$ be a firm's marginal cost when it has access to a number $n \in \{1, ..., N\}$ of patents with $c(N)(\equiv \underline{c}) \leq c(N-1) \leq ... \leq c(1)(\equiv \bar{c})$.

We consider a two-stage game in which, prior to engaging in Cournot competition, each pair of firms can sign a cross-licensing agreement whereby each party gets access to the patented technology of the other one. More precisely, the two-stage game is described as follows:

- Stage 1: *Cross-licensing*

Each pair of firms $(i, j) \in \{1, ..., N\}^2$ with $i \neq j$ decide whether to sign a cross-licensing agreement and determine the terms of the agreement if any. We assume that a bilateral cross-licensing agreement between firm i and firm j specifies a pair of royalties $(r_{i \to j}, r_{j \to i}) \in [0, +\infty)^2$ and a lump-sum transfer[8] $F_{i \to j} \in (-\infty, +\infty)$, where $r_{i \to j}$ (resp. $r_{j \to i}$) is the per-unit royalty[9] paid by firm i (resp. firm j) to firm j (resp. firm i) and $F_{i \to j}$ is a transfer made by firm i to firm j (which is negative if firm i receives a transfer from firm j). All bilateral negotiations occur simultaneously.

- Stage 2: *Cournot competition*

Firms compete à la Cournot with the cost structure inherited from Stage 1.

Depending on the observability of the terms of the agreement between two firms to their rivals, we distinguish *public* cross-licensing from *private* cross-licensing. In the case of public cross-licensing, all firms observe the terms of all the cross-licensing agreements signed at stage 1 before they engage in Cournot competition. In contrast, in the case

[8] The lump-sum transfers $F_{i \to j}$ make it possible to separate internal distribution from joint profit maximization, which justifies the solution concept we use later (see Definition 1). However, given the nature of that concept, these transfers will not matter in the analysis.

[9] We restrict attention to non-negative royalties for two reasons. First, negative royalties are rarely observed (Liao and Sen, 2005, p. 291). Second, in the presence of monitoring costs, at least small negative royalties are dominated by zero royalty since zero royalty does not require any monitoring of rivals' output to enforce the licensing contract (Katz and Shapiro, 1985, p. 508).

of private cross-licensing, the terms of the cross-licensing agreement between firms i and j are known only to these two firms and, therefore, each firm $k \neq i, j$ should form a conjecture about those terms.

We assume that the firms face an inverse demand function $P(\cdot)$ satisfying the following standard conditions (Novshek, 1985):

A1 $P(\cdot)$ is twice continuously differentiable and $P'(\cdot) < 0$ whenever $P(\cdot) > 0$.

A2 $P(0) > \bar{c} > \underline{c} > P(Q)$ for Q sufficiently high.

A3 $P'(Q) + QP''(Q) < 0$ for all $Q \geq 0$ with $P(Q) > 0$.

These mild assumptions ensure the existence and uniqueness of a Cournot equilibrium $(q_i^*)_{i=1,\ldots,n}$ satisfying the following (intuitive) comparative statics properties, where c_i denote firm i's marginal cost (see e.g., Amir, Encaoua and Lefouili, 2014):

i) $\frac{\partial q_i^*}{\partial c_i} < 0$ and $\frac{\partial q_i^*}{\partial c_j} > 0$ for any $j \neq i$; $\frac{\partial Q^*}{\partial c_i} < 0$ for any i, where $Q^* = \sum_i q_i^*$ is the total equilibrium output;

ii) $\frac{\partial \pi_i^*}{\partial c_i} < 0$ and $\frac{\partial \pi_i^*}{\partial c_j} > 0$ for any $j \neq i$, where π_i^* is firm i's equilibrium profit.

2.2 Benchmark: multilateral cross-licensing agreement

We consider here as a benchmark the case of a multilateral licensing agreement among all firms. This corresponds to a *closed* patent pool (Lerner and Tirole, 2004), i.e., a patent pool whose only customers are its contributors. We focus on a symmetric outcome where all firms pay the same royalty r to each other.

Let $P^m(\underline{c})$ be the monopoly price when each firm's marginal cost is $\underline{c}$. It is characterized by

$$\frac{P^m(\underline{c}) - \underline{c}}{P^m(\underline{c})} = \frac{1}{\varepsilon(P^m(\underline{c}))}. \tag{1}$$

where $\varepsilon(.)$ is the elasticity of demand.

Given a symmetric royalty r, each firm's marginal cost is $\underline{c} + (N-1)r$. The firms will agree on a royalty to achieve the monopoly price. Given a symmetric royalty r, firm i chooses its output q_i in the second stage to maximize $[P(Q_{-i} + q_i) - \underline{c} - (N-1)r]\, q_i + rQ_{-i}$ where $Q_{-i} \equiv Q - q_i$ is the quantity chosen by all other firms. Let r^m be the royalty that allows to achieve the monopoly price $P^m(\underline{c})$. Then, from the first-order condition associated with firm i's maximization program, we have

$$\frac{P^m(\underline{c}) - \underline{c} - (N-1)r^m}{P^m(\underline{c})} = \frac{1}{\varepsilon(P^m(\underline{c}))N}. \tag{2}$$

From (1) and (2), r^m is determined by

$$\frac{P^m(\underline{c}) - \underline{c}}{N} = r^m. \tag{3}$$

Proposition 1 *(Multilateral cross-licensing). Suppose that all firms in the industry jointly agree on a symmetric royalty. Then they agree on* $r^m = (P^m(\underline{c}) - \underline{c})/N$*, which allows them to achieve the monopoly price* $P^m(\underline{c})$ *as an equilibrium price.*

3 Public cross-licensing agreements

3.1 Preliminaries

We first define our solution concept.

Definition 1 *A set of public cross-licensing agreements is bilaterally efficient if, for any pair of firms* (i, j)*, the bilateral agreement between* i *and* j *maximizes their joint profit, given all other bilateral agreements and the (anticipated) equilibrium outcome of the competition stage.*

Since any bilateral agreement can include the payment of a fixed fee, we argue that it is reasonable to assume that a bilateral agreement signed between a pair of firms maximizes their joint profit.[10]

Notice first that any given pair of firms find it (jointly) optimal to sell a license to each other. To see why, assume that firm i does not license its patent to firm j. These two firms can (weakly) increase their joint profit if i licenses its patent to j by specifying the payment of a per-unit royalty $r_{j \to i}$ equal to the reduction in marginal cost allowed by j's use of the technology covered by i's patent. Such licensing agreement would not affect the level of joint output but will (weakly) decrease the cost of firm j. It will therefore (weakly) increase their joint profit.

In what follows, we consider a symmetric situation where all firms sign bilateral cross-licensing agreements. Let r denote the (common) per-unit royalty paid by any firm i to have access to the patent of any firm $j \neq i$ with $i, j = 1, ..., N$, and $S(r, N)$ denote

[10] Our solution concept is similar to the concept of *contract equilibrium* (Crémer and Riordan, 1987; O'Brien and Shaffer, 1992) and *pairwise-proof contracts* (McAfee and Schwartz, 1994). It is also closely related to the concept of *Nash equilibrium in Nash bargains* used in the bilateral monopoly/oligopoly literature (Horn and Wolinsky, 1988; Inderst and Wey, 2003; Collard-Wexler, Gowrisankaran and Lee, 2014).

the corresponding set of cross-licensing agreements. We below study the incentives of a two-firm coalition to deviate from the symmetric royalty r under the assumption that all firms are active (i.e., produce a positive output) no matter what the royalties the deviating coalition chooses.[11]

The next lemma shows that it is sufficient to focus on deviations such that firms in the deviating coalition pay the same royalty to each other. Indeed, the joint payoff from any asymmetric deviation can be replicated by a symmetric one because the joint payoff depends on the royalties paid by each firm to the other only through their *sum*.

Lemma 1 *Consider a symmetric set of cross-licensing agreements $S(r, N)$. The joint payoff a coalition $\{i, j\}$ gets from a deviation to a cross-licensing agreement in which firm i (resp. firm j) pays a royalty $r_{i\to j}$ (resp. $r_{j\to i}$) to firm j (resp. firm i) depends on $(r_{i\to j}, r_{j\to i})$ only through the sum $r_{i\to j} + r_{j\to i}$.*

Proof. See Appendix A. ■

Consider a deviation by the coalition formed by firms 1 and 2, which we denote by $\{1,2\}$. Let $\hat{r}$ be the royalty that these firms charge to each other. For given $(r, \hat{r})$, let $Q^*(r, \hat{r})$ denote the total industry output and $Q^*_{12}(r, \hat{r})$ denote the sum of the outputs of firms 1 and 2 in the (second-stage) Cournot equilibrium. Let $Q^*_{-12}(r, \hat{r}) \equiv Q^*(r, \hat{r}) - Q^*_{12}(r, \hat{r})$. Then, the considered set of symmetric agreements is bilaterally efficient if and only if:

$$r \in Arg \max_{\hat{r}\geq 0} \ \pi_{12}(r, \hat{r})$$

where

$$\pi_{12}(r, \hat{r}) = [P(Q^*_{12}(r, \hat{r}) + BR_{-12}(Q^*_{12}(r, \hat{r}))) - (\underline{c} + (N-2)r)]\, Q^*_{12}(r, \hat{r}) + 2rBR_{-12}(Q^*_{12}(r, \hat{r}))$$

is the joint profit of the coalition and $BR_{-12}(.)$ is defined as follows. If $N = 3$, then $BR_{-12}(.)$ is the best-response function of firm 3. If $N \geq 4$, then $BR_{-12}(.)$ is the *aggregate response* of the coalition's rivals: for any joint output $Q_{12} = q_1 + q_2$ of firms 1 and 2, $BR_{-12}(Q_{12})$ is the (unique) real number such that $\frac{BR_{-12}(Q_{12})}{N-2}$ is the best-response of any firm $i \in \{3, ..., N\}$ to each firm $k \in \{1, 2\}$ producing q_k and each firm $j \in \{3, ..., N\} \setminus \{i\}$ producing $\frac{BR_{-12}(Q_{12})}{N-2}$.

After observing the coalition's deviation to $\hat{r} \neq r$, its rivals expect it to produce $Q^*_{12}(r, \hat{r})$ and will best respond to this quantity by producing $BR_{-12}(Q^*_{12}(r, \hat{r}))$ in aggre-

[11] We plan to relax this assumption in another project which focuses on the relationship between cross-licensing and barriers to entry.

gate. In other words, from a strategic point of view, the coalition's deviation to $\hat{r} \neq r$ is equivalent to its commitment to produce $Q^*_{12}(r, \hat{r})$ as a Stackelberg leader. For this reason, we below define the following two-stage Stackelberg game.

Definition 2 *For any $r \geq 0$ and $N \geq 3$, the Stackelberg game $G(r, N)$ is defined by the following elements:*

Players: The game involves $N - 1$ players : coalition {1,2} and firms $i = 3, ..., N$. Each player has a marginal production cost $\underline{c}$ (excluding royalties). The coalition pays a per-unit royalty r to each of the other players. Each firm $i \in \{3, ..., N\}$ pays a per-unit royalty $2r$ to the coalition and a per-unit royalty r to each firm $j \in \{3, ..., N\} \setminus \{i\}$.

Actions: Coalition {1,2} chooses its (total) output Q_{12} within the interval $I(r, N) \equiv [0, Q^*_{12}(r, 0)]$ and each firm $i \in \{3, ..., N\}$ chooses an output level $q_i \geq 0$.

Timing: There are two stages:

- Stage 1: The coalition {1,2} acts as a Stackelberg leader and chooses Q_{12} within the interval $[0, Q^*_{12}(r, 0)]$.

- Stage 2: If $N = 3$, then firm 3 chooses its output q_3. If $N \geq 4$, then all firms $i = 3, ..., N$ choose simultaneously (and non-cooperatively) their outputs q_i.

The following lemma provides an equivalence result that is useful for the subsequent analysis.

Lemma 2 *A symmetric set of cross-licensing agreements $S(r, N)$ is bilaterally efficient if and only if choosing $Q^*_{12}(r, r)$ is optimal for the coalition in the Stackelberg game $G(r, N)$.*

Proof. See Appendix A. ■

3.2 Incentives to deviate: downstream and upstream profits

We now study the incentives of coalition $\{1, 2\}$ to marginally expand or contract its output with respect to $Q^*_{12}(r, r)$ in the game $G(r, N)$. Note that the coalition's marginal cost in that game is $\underline{c} + (N - 2) r$ whereas each of its member's marginal cost at the Cournot stage of the original game presented in Section 2 is $\underline{c} + (N - 1) r$. The difference between the two has to do with the royalty payment between firms 1 and 2. In what follows, we call rQ_{12} the *royalty saving* of the coalition (as compared to a single firm producing the same quantity Q_{12}).

The coalition's profit can be rewritten as

$$\pi_{12}\left(Q_{12}, r\right) = \underbrace{\left[P\left(Q_{12} + BR_{-12}(Q_{12})\right) - \left(\underline{c} + (N-1)\, r\right)\right] Q_{12}}_{\pi_{12}^{D}(Q_{12}, r)} + \underbrace{r\left[Q_{12} + 2BR_{-12}(Q_{12})\right]}_{\pi_{12}^{U}(Q_{12}, r)}. \tag{4}$$

The term $\pi_{12}^{D}\left(Q_{12}, r\right)$ represents the coalition's profit in the downstream product market.[12] The term $\pi_{12}^{U}(Q_{12}, r)$ represents the coalition's profit in the upstream market of patent licensing. This profit is composed of the royalty saving and the licensing revenues received from all firms outside the coalition. We below study the effect of a (local) variation of Q_{12} on each of the two sources of profit.

- Effect on the downstream profit

The partial derivative of $\pi_{12}^{D}\left(Q_{12}, r\right)$ with respect to Q_{12}, when evaluated at $Q_{12}^{*}(r, r)$, is given by

$$\begin{aligned} \frac{\partial \pi_{12}^{D}}{\partial Q_{12}}(Q_{12}^{*}(r,r), r) \quad &= \quad P'\left(Q^{*}(r,r)\right) Q_{12}^{*}(r,r) BR'_{-12}(Q_{12}^{*}(r,r)) \qquad (5) \\ &\quad + P'\left(Q^{*}(r,r)\right) Q_{12}^{*}(r,r) + \left[P\left(Q^{*}(r,r)\right) - \left(\underline{c} + (N-1)\, r\right)\right]. \end{aligned}$$

The term $P'\left(Q^{*}(r,r)\right) Q_{12}^{*}(r,r) BR'_{-12}(Q_{12}^{*}(r,r)) > 0$ in (5) captures the (usual) *Stackelberg effect*: the leader has an incentive to increase its output Q_{12} above the Cournot level $Q_{12}^{*}(r,r)$ because such an increase will be met with a decrease in the aggregate output of the followers (one can easily check that $BR'_{-12}(Q_{12}^{*}(r,r)) < 0$). It is very useful to rewrite this term as $-2\left[P\left(Q^{*}(r,r)\right) - \left(\underline{c} + (N-1)\, r\right)\right] BR'_{-12}(Q_{12})$, which is obtained from the F.O.C. of firm i (with $i = 1, 2$) in the Cournot game:

$$P'\left(Q^{*}(r,r)\right) \frac{Q_{12}^{*}(r,r)}{2} + P\left(Q^{*}(r,r)\right) - \left(\underline{c} + (N-1)\, r\right) = 0. \tag{6}$$

The term $P'\left(Q^{*}(r,r)\right) Q_{12}^{*}(r,r) + \left[P\left(Q^{*}(r,r)\right) - \left(\underline{c} + (N-1)\, r\right)\right]$ in (5) represents the marginal downstream profit of the coalition in a setting where it would play a simultaneous quantity-setting game with its rivals. This term captures a *coordination effect*: the coalition has an incentive to reduce output below the Cournot level $Q_{12}^{*}(r,r)$ since the joint output of the coalition when each member chooses its quantity in a non-cooperative way

[12] The downstream profit is defined with respect to the individual marginal cost of each member of the coalition at the second stage of the original game. This facilitates our analysis because we can use each firm's first order condition at the Cournot competition stage (see (6)).

is too high with respect to what maximizes its joint downstream profit (in a simultaneous quantity-setting game). Indeed, using (6), we have:

$$P'\left(Q^*(r,r)\right)Q_{12}^*(r,r)+\left[P\left(Q^*(r,r)\right)-\left(\underline{c}+(N-1)\,r\right)\right]=-\left[P\left(Q^*(r,r)\right)-\left(\underline{c}+(N-1)\,r\right)\right]<0.$$

Therefore, the overall marginal effect of a local increase of Q_{12} (above the Cournot level $Q_{12}^*(r,r)$) on the coalition's downstream profit is given by:

$$\frac{\partial\pi_{12}^D}{\partial Q_{12}}(Q_{12}^*(r,r),r)=-\left[P\left(Q^*(r,r)\right)-\left(\underline{c}+(N-1)\,r\right)\right]\left[1+2BR'_{-12}(Q_{12}^*(r,r))\right]. \tag{7}$$

The first term between brackets in (7) is positive while the sign of the second term between brackets in (7) can be either positive or negative.

- Effect on the upstream profit

Let us now turn to the effect of a local variation in Q_{12} on the coalition's upstream profit $\pi_{12}^U(Q_{12})$. We have:

$$\frac{\partial\pi_{12}^U}{\partial Q_{12}}(Q_{12}^*(r,r),r)=r\left(1+2BR'_{-12}(Q_{12}^*(r,r))\right). \tag{8}$$

We identify two opposite effects on the upstream profit. On the one hand, a marginal increase in Q_{12} results in a larger royalty saving. We call this the *royalty-saving effect*. On the other hand, a marginal increase in Q_{12} induces the rivals of the coalition to reduce their output by $\left|BR'_{-12}(Q_{12})\right|$ and hence results in a reduction of $2r\left|BR'_{-12}(Q_{12})\right|$ in the licensing revenues that the coalition gets from its rivals. We call this the *licensing revenue effect*.

Even if the effect of an increase in Q_{12} (above the Cournot level $Q_{12}^*(r,r)$) on each of the coalition's two sources of profits is ambiguous, it follows from (7) and (8) that *the sign of the effect on the downstream profit is always opposite to the sign of the effect on the upstream profit.*

3.3 Incentives to deviate

By summing up (7) and (8), the total effect of a marginal increase in Q_{12} on the coalition's profit can be described in a simple way as:

$$\frac{\partial\pi_{12}}{\partial Q_{12}}(Q_{12}^*(r,r),r)=\left[\underline{c}+Nr-P\left(Q^*(r,r)\right)\right]\left[1+2BR'_{-12}(Q_{12}^*(r,r))\right]. \tag{9}$$

We distinguish four cases depending on whether the Stackelberg effect is stronger or weaker than the coordination effect[13] and whether the effect of a local deviation on the downstream profit dominates or is dominated by its effect on the upstream profit.

- Stackelberg effect vs. coordination effect

Let us first examine the term $1 + 2BR'_{-12}(Q^*_{12}(r,r))$ in (9) which determines whether the Stackelberg effect is stronger or weaker than the coordination effect. The F.O.C. for the maximization program of any firm i ($i = 3, ..., N$), when the coalition produces a given quantity Q_{12}, can be written as:

$$P'\left(Q_{12} + BR_{-12}(Q_{12})\right)\frac{BR_{-12}(Q_{12})}{N-2} + P\left(Q_{12} + BR_{-12}(Q_{12}) - \left(\underline{c} + (N-1)\,r\right) = 0.$$

Differentiating the latter with respect to Q_{12} (and dropping the argument (r,r))) yields

$$BR'_{-12}(Q_{12}) = -\frac{P''(Q)BR_{-12}\left(Q_{12}\right) + (N-2)P'(Q)}{P''(Q)BR_{-12}\left(Q_{12}\right) + (N-1)P'(Q)}.$$

This, combined with $P'(Q) < 0$, proves that $-1 < BR'_{-12}(Q_{12}) < 0$ (a result that will be useful later), and, when evaluated at $Q_{12} = Q^*_{12}$, yields

$$1 + 2BR'_{-12}(Q^*_{12}) = -\frac{\frac{N-2}{N}P''(Q^*)Q^* + (N-3)P'(Q^*)}{\frac{N-2}{N}P''(Q^*)Q^* + (N-1)P'(Q^*)}$$

because the equality $BR_{-12}\left(Q^*_{12}\right) = \frac{N-2}{N}Q^*$ holds as the corresponding equilibrium is symmetric. Distinguishing between the two scenarios $P''(Q^*) \leq 0$ and $P''(Q^*) > 0$ and using the fact that $BR_{-12}\left(Q^*_{12}\right) \leq Q^*$, one can easily show that **A3** implies that the denominator is always negative. Therefore, from $P'(Q^*) < 0$, it follows that

$$1 + 2BR'_{-12}(Q^*_{12}) \lesseqgtr 0 \Longleftrightarrow \frac{Q^*P''(Q^*)}{P'(Q^*)} \gtreqless -\frac{N(N-3)}{N-2}. \tag{10}$$

This shows that the coordination effect dominates the Stackelberg effect ($1+2BR'_{-12}(Q^*_{12}) > 0$) if and only if the slope of the inverse demand is sufficiently elastic ($-\frac{Q^*P''(Q^*)}{P'(Q^*)} > \frac{N(N-3)}{N-2}$). In particular, for $N \geq 4$, under **A3**, the Stackelberg effect always dominates the coordination effect.

[13] This determines not only the sign of the effect of an increase in Q_{12} on the downstream profit but also the sign of the effect on the upstream profit because these two signs are always opposite.

- Downstream profit vs. upstream profit

Let us now examine the term $\underline{c} + Nr - P\left(Q^*(r,r)\right) \equiv f(r,N)$ in (9). The effect of an increase in Q_{12} on the downstream profit (strictly) dominates the effect on the upstream profit if and only if $f(r,N) < 0$. For instance, when $r = 0$, there is no upstream profit and we have $f(0,N) = \underline{c} - P\left(Q^*(0,0)\right) < 0$. Intuitively, we expect that the upstream profit becomes more important as r increases, which turns out to be true as we below show that $\frac{\partial f}{\partial r}(r,N) = N - \frac{dQ^*}{dr} P'(Q^*) > 0$. Summing the F.O.C. for each firm i' s maximization program from $i = 1$ to $i = N$ yields

$$P'(Q^*)\, Q^* + NP(Q^*) - N\left(\underline{c} + (N-1)r\right) = 0.$$

Differentiating the latter with respect to r leads to

$$\frac{dQ^*}{dr}\left[P'(Q^*) + P''(Q^*)\, Q^*\right] + N\left[P'(Q^*)\frac{dQ^*}{dr} - (N-1)\right] = 0.$$

From $P'(Q^*) + P''(Q^*)\, Q^* < 0$ (by **A3**) and $\frac{dQ^*}{dr} < 0$, it follows that $P'(Q^*)\frac{dQ^*}{dr} - (N-1) < 0$, which implies that $\frac{\partial f}{\partial r}(r,N) > 0$ for any $N \geq 3$. Since $f(r,N)$ strictly increases with r, the solution in r to $f(r,N) = 0$ is unique whenever it exists.

Surprisingly, it turns out that the unique royalty r for which $f(r,N) = 0$ is the fully cooperative royalty r^m, as defined in (3). At $r = r^m$, we have that $P\left(Q^*(r^m, r^m)\right) = P^m(\underline{c})$ and, therefore, $\underline{c} + Nr^m - P\left(Q^*(r^m, r^m)\right) = 0$. Thus, for $r < r^m$, the effect on the downstream profit dominates the effect on the upstream profit and the reverse holds for $r > r^m$.

3.4 Bilaterally efficient royalties

From the previous analysis of local deviations, we know that there are four possible cases depending on which of the Stackelberg effect and the coordination effect is stronger and which of the downstream profit effect and the upstream profit effect dominates.

Consider first the case in which the Stackelberg effect is stronger than the coordination effect. Then, if the downstream profit effect dominates the upstream profit effect (i.e., r belongs to $[0, r^m)$), the coalition has an incentive to decrease its royalty[14] in order to induce the rivals to reduce their outputs, which generates $r = 0$ as the unique potential bilateral efficient royalty among royalties r in $[0, r^m)$. If the upstream profit effect dominates the

[14]By this we mean the royalty that the members of the coalition pay each other.

downstream profit effect (i.e., $r > r^m$), the coalition has an incentive to increase its royalty to boost the rivals' production and thereby the licensing revenues it receives from them. At $r = r^m$, the downstream profit effect is equal to the upstream profit effect and the coalition has no incentive to deviate locally. In summary, when the Stackelberg effect dominates the coordination effect, there are two potential bilaterally efficient royalties: $r = 0$ and $r = r^m$.

Now let us consider the case in which the coordination effect is stronger than the Stackelberg effect. Then, if the downstream profit effect dominates the upstream profit effect (i.e., r belongs to $[0, r^m)$), the coalition has an incentive to increase its royalty in order to boost its downstream prices. In a symmetric way, if the upstream profit effect dominates the downstream profit effect (i.e., $r > r^m$), the coalition has an incentive to decrease its royalty. In summary, when the coordination effect is stronger than the Stackelberg effect, we have a unique potential bilaterally efficient royalty: $r = r^m$.

The local analysis above allowed us to identify candidates for bilaterally efficient royalties. It remains to perform a global analysis (i.e., to examine global and not only local deviations) in order to confirm that the candidates are indeed bilaterally efficient. The following proposition follows from our global analysis:

Proposition 2 *(public bilateral cross-licensing) Consider the two-stage game of public cross-licensing followed by Cournot competition.*

(i) If the Stackelberg effect dominates the coordination effect $\left(\frac{QP''(Q)}{P'(Q)} > -\frac{N(N-3)}{N-2}\right)$, *then* $S(r, N)$ *is bilaterally efficient if and only if* $r \in \{0, r^m\}$.

(ii) If the coordination effect dominates the Stackelberg effect $\left(\frac{QP''(Q)}{P'(Q)} < -\frac{N(N-3)}{N-2}\right)$, *then* $S(r, N)$ *is bilaterally efficient if and only if* $r = r^m$.

Proof. See Appendix A. ■

This proposition shows that the monopoly outcome is always sustainable through bilaterally efficient public agreements. Moreover, if the Stackelberg effect is stronger than the coordination effect (which happens in our setting whenever $N \geq 4$) then the most competitive outcome (i.e., the one corresponding to $r = 0$) is also sustainable through bilaterally efficient public agreements.

4 Private cross-licensing agreements

We now consider private cross-licensing: each bilateral agreement is only observable to the two firms involved in it. Hence, at the beginning of the competition stage, each firm is only aware of the terms of the contracts it signed itself and should form expectations regarding the terms agreed on by its competitors. As in the public cross-licensing case, we define a bilaterally efficient set of agreements as one such that the agreement between any pair of firms maximizes their joint profits given all other agreements. However, in contrast to the public cross-licensing case, a deviation by a two-firm coalition in the cross-licensing stage is not observed by its rivals who keep the same beliefs about the agreements made by their competitors. Moreover, when a coalition of two firms deviates by changing the terms of the agreement between them, each of these two firms maintains the same beliefs about the agreements signed by its rivals. This assumption is in a sense the counterpart in our setting of the usual passive-belief assumption in the literature on vertical contracting (Hart and Tirole, 1990; McAfee and Schwartz, 1994).

We can show that Lemma 1 continues to hold in the case of private cross-licensing and, therefore, restrict attention to deviations involving a symmetric royalty (within the deviating coalition). Moreover, a result similar to Lemma 2 holds: the symmetric set of cross-licensing agreements $S(r, N)$ is bilaterally efficient if and only if choosing to produce $Q_{12}^*(r, r)$ is optimal for coalition $\{1, 2\}$ when all other firms produce the individual equilibrium output corresponding to a symmetric royalty r. The major difference between public cross-licensing and private cross-licensing is that the Stackelberg effect and the licensing revenue effect are absent under the latter because of the unobservability of the deviations in the private cross-licensing stage: formally speaking, the analysis under private cross-licensing can be derived from that under public cross-licensing by setting $BR'_{-12}(Q_{12})$ equal to zero when considering the effect of a coalition's deviation (in the first stage of the game) on its profit. This implies that $S(r^m, N)$ is the unique bilaterally efficient set of symmetric agreements.

Proposition 3 *(private bilateral cross-licensing) In the two-stage game of private cross-licensing followed by Cournot competition, $S(r^m, N)$ is the unique bilaterally efficient set of symmetric cross-licensing agreements.*

Therefore, the result that firms are able to sustain the monopoly outcome through bilateral agreements is even stronger in the case of private cross-licensing: in this scenario, the monopoly outcome is always the unique symmetric outcome.

Before discussing the robustness of our monopolization result, we provide the main intuitions behind it. Consider first the case of private cross-licensing agreements. The monopoly output Q^m is defined by the following first-order condition:

$$P'(Q^m)Q^m + P(Q^m) = \underline{c}. \tag{11}$$

Moreover, since r^m allows to achieve the monopoly outcome, the first-order condition with respect to q_i for a single firm i is given by:

$$P'(Q^m)\frac{Q^m}{N} + P(Q^m) = \underline{c} + (N-1)\,r^m. \tag{12}$$

Thus, an increase in a firm's perceived marginal cost by $(N-1)\,r^m$ makes it act as if it were internalizing the effects of its decision on its $(N-1)$ rivals. Therefore, the payment of a per-unit royalty r^m to each rival has the same effect as internalizing the impact of a price reduction on that rival. Formally, this amounts to writing

$$-P'(Q^m)\frac{Q^m}{N} = r^m,$$

which follows immediately from (11) and (12).

Suppose now that two firms (i,j) contemplate a joint deviation in the cross-licensing stage. By agreeing on some royalties $(r_{i\to j}, r_{j\to i})$, they can choose a joint output $q_i + q_j$ different from $2Q^m/N$. However, it turns out that the first-order condition for the coalition's maximization program is satisfied exactly at $q_i + q_j = 2Q^m/N$:

$$P'(Q^m)\frac{2Q^m}{N} + P(Q^m) = \underline{c} + (N-2)\,r^m, \tag{13}$$

which is easily derived from (12) by adding $P'(Q^m)\frac{Q^m}{N}$ to its LHS and substracting r^m from its RHS. The intuition behind this result is as follows. On the one hand, a two-firm coalition's marginal cost is lower than a single firm's marginal cost by r^m, which gives the coalition an incentive to increase its output. This is the royalty-saving effect. On the other hand, when two firms decide jointly the royalties they charge each other, they internalize the competitive externalities they exert on each other in the competition stage. This is the coordination effect. Since the payment of r^m is equivalent to internalizing the effect of price reduction on one rival, these two effects cancel out.

Consider now the case of public cross-licensing agreements. The discussion above shows that the coordination effect and the royalty-saving effect cancel out when all firms charge each other $r = r^m$. It remains to provide an intuition for why the two other effects that appear under public cross-licensing, i.e., the Stackelberg effect and the licensing revenue effect also cancel out.

Consider the coalition comprised of firms 1 and 2. When all firms charge each other $r = r^m$, a marginal decrease $dQ_{-12} < 0$ in the joint output Q_{-12} of the firms outside the coalition (due to an increase in the coalition's output Q_{12}) induces a variation in the coalition's profit by $P'(Q)\frac{2Q^m}{N}dQ_{-12} - 2r^m dQ_{-12}$. The first term captures the Stackelberg effect: a marginal decrease in Q_{-12} increases the market price by $P'(Q)dQ_{-12}$ which induces an increase $P'(Q)\frac{2Q^m}{N}dQ_{-12}$ in the coalition's downstream profit. The second term captures the licensing revenue effect: a marginal decrease in Q_{-12} results in a reduction in the coalition's licensing revenues by $2r^m dQ_{-12}$. Thus, the coalition internalizes both the positive impact of a decrease in its rivals' output on its downstream profit but also the negative impact of such decrease on its upstream profit. The reason why these two effects cancel out can be seen as the dual of the reason why the coordination effect and the royalty-saving effect cancel out: charging a per-unit royalty equal to r^m is an indirect way for the members of the coalition to fully internalize the effect of their joint decision on their rivals.

5 Extensions

In Appendix B, we show that firms are able to sustain the fully cooperative outcome in two extensions of our baseline model which we briefly discuss below.

5.1 k-efficient agreements

In this extension, we investigate cross-licensing agreements that are *k-efficient* in the sense that no coalition of size $k \in \{3, ..., N-1\}$ finds it optimal to change the terms of the cross-licensing agreements among its members. Note that the special case $k = 2$ corresponds to the bilateral efficiency criterion while $k = N$ corresponds to industry-profit maximization.

Consider first the case of public cross-licensing agreements. We show that the set of (symmetric) *k-efficient* royalties depends again on the magnitude of the Stackelberg effect relative to the coordination effect. If the Stackelberg effect dominates the coordination effect, then there are two *k-efficient* royalties: $r = 0$ and $r = r^m$. However, if the Stackelberg effect is dominated by the coordination effect, then the unique *k-efficient* royalty is $r = r^m$. Therefore, in both cases, the fully cooperative outcome is sustainable through *k-efficient* cross-licensing agreements. Moreover, our analysis shows that the relative magnitude of the Stackelberg effect with respect to the coordination effect decreases in the size of the coalition k, which makes the scenario in which $r = 0$ arises as a sustainable outcome less likely as k increases. The intuition behind this result is that, for a given number of firms in the industry, the magnitude of the Stackelberg effect tends to decrease when the size of the coalition increases (because the number of firms outside the coalition decreases) while the magnitude of the coordination effect increases (since it increases in the number of firms inside the coalition). In the limit case of $k = N$, the Stackelberg effect completely disappears.

Consider now the case of private cross-licensing agreements. As in the baseline model, it turns out that the outcome in this scenario can be formally derived from the one under public cross-licensing by considering the special case in which the Stackelberg effect and the licensing revenue effect would be absent: the fully cooperative royalty is the unique symmetric royalty sustainable through *k-efficient* agreements.

5.2 Bertrand competition

In this extension, we assume that firms produce differentiated goods and compete in prices. Again we consider the effects of a deviation by a two-firm coalition in the first stage of the game on both the downstream and the upstream profits.

Consider first the case of public cross-licensing agreements. As in the Cournot case, a variation in the royalties charged by two firms to each other has two effects on their joint downstream profit: a Stackelberg effect and a coordination effect. Moreover, the effect of such variation on the coalition's upstream profits can also be divided into a royalty-saving effect and a licensing-revenue effect. However, in the Bertrand case, these two effects are more subtle than in the Cournot case: beside the *direct* royalty saving effect and the *indirect* licensing-revenue effect (i.e., through the change in the rivals' prices) that appear under Cournot competition, a change in the royalties also has an *indirect* royalty saving effect (since the change in the rivals' prices affects the demand for the products of the firms in the coalition) and a *direct* licensing-revenue effect (since the change in the prices charged by the firms in the coalition affects each rival's demand).

Our analysis shows that the fully cooperative royalty is the unique symmetric royalty sustainable through bilaterally efficient agreements. In sharp contrast to the Cournot case, $r = 0$ is never a bilaterally efficient royalty under Bertrand competition. The intuition behind this stems from the fact that the Bertrand game features strategic complementarity (under the standard assumptions we make) while the Cournot game is a game of strategic substitutes. This entails that the Stackelberg effect and the coordination effect are reinforcing each other under the Bertrand case. Therefore, considering a situation in which all firms charge a royalty of $r = 0$ to their competitors, a coalition of two firms would increase its downstream profit by increasing the royalties they charge each other. This, combined with the fact that upstream profits are zero when all firms charge $r = 0$, implies that a coalition of two firms has an incentive to deviate in the first stage of the game, meaning that $r = 0$ is not bilaterally efficient.

Consider now the case of private cross-licensing agreements. Again the analysis of private agreements can be derived from that of public agreements by putting aside all the indirect effects. It turns out, as in the Cournot case, that the unique (symmetric) bilaterally efficient royalty is the fully cooperative royalty.

6 A general class of games

We now develop a general model that can be applied to situations different from cross-licensing of patents and shows the robustness of our main result. Consider the following N-firm two-stage game:

- Stage 1 (*upstream bilateral agreements*): Firms agree *bilaterally* on transfers they make to each other. More specifically, all pairs of firms (i, j) simultaneously choose a pair of (real-valued) *input prices* $(r_{i\to j}, r_{j\to i})$ as well as (real-valued) fixed transfers $(F_{i\to j}, F_{j\to i})$.

- Stage 2 (*downstream non-cooperative actions*): Firms choose non-cooperatively and simultaneously (real-valued) actions x_i.

Let $\boldsymbol{r} = ((r_{i\to j}, r_{j\to i}))_{1\leq i<j<N}$, $\boldsymbol{F} = ((F_{i\to j}, F_{j\to i}))_{i\neq j}$, $\mathbf{x} = (x_i)_i$. Let $\mathbf{x}_{-ij}$ denote the vector obtained from vector $\mathbf{x}$ by removing x_i and x_j and $\Pi_i(\mathbf{x},\boldsymbol{r},\boldsymbol{F})$ player i's payoff function.

We set the following assumptions regarding the effects of transfers on payoffs:

G1 For any i, there exists a function π_i such that, for any $(\mathbf{x},\boldsymbol{r},\boldsymbol{F})$, $\Pi_i(\mathbf{x},\boldsymbol{r},\boldsymbol{F}) = \pi_i(\mathbf{x},\boldsymbol{r}) + \sum\limits_{j\neq i}(F_{j\to i} - F_{i\to j})$

G2 For any i, j such that $i \neq j$, and any $\mathbf{x}$, $\pi_i(\mathbf{x},\boldsymbol{r}) + \pi_j(\mathbf{x},\boldsymbol{r})$ does not depend on $r_{i\to j}$.

G3 For any i, j, k such that $k \notin \{i, j\}$, and any $\mathbf{x}$, $\pi_k(\mathbf{x},\boldsymbol{r})$ does not depend on $r_{i\to j}$.

We also make the following technical assumptions:

G4 For any $\boldsymbol{r}$, there exists a unique Nash equilibrium $\mathbf{x}^*(\boldsymbol{r})$ to the second-stage subgame.

G5 For any $\boldsymbol{r}$, any (i,j) and any $\left(r'_{i\to j}, r'_{j\to i}\right)$, the two-player game derived from the N-player downstream game by fixing the action of each player $k \notin \{i,j\}$ to $x_k^*(\boldsymbol{r})$ has a unique Nash equilibrium $\left(\tilde{x}_i^*\left(r'_{i\to j}, r'_{j\to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right), \tilde{x}_j^*\left(r'_{i\to j}, r'_{j\to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right)\right)$.

G6 There exists a unique vector $\mathbf{x}^{\mathbf{m}}$ of downstream actions that maximizes the (downstream) joint payoff of all players; moreover the joint payoff function is differentiable at $\mathbf{x}^{\mathbf{m}}$ and the latter is the unique solution to the corresponding system of FOCs.

This general model can be applied to many economic situations, including:

- *Cross-licensing:* $r_{i\to j}$ is a per-unit royalty paid by patent holder i to patent holder j and x_i is a price or a quantity. A major difference with our baseline model is that the general model allows for asymmetric cost functions and asymmetric patents (e.g., with different marginal values to the firms) which is particularly important for the policy implications we present later. Moreover, note that the general model applies not only to the case in which cross-licensing partners produce substitutable goods but also to the case in which they produce complementary goods.

- *Two-way access pricing in telecommunication networks:* $r_{i\to j}$ is the access charge paid by network i to network j and x_i is the linear retail price charged by network i to its customers (see Armstrong, 1998, Laffont, Rey and Tirole, 1998a, 1998b for a duopolistic setting)

- *Interconnection among Internet backbone companies*: $r_{i\to j}$ is the access charge paid by backbone company i to j in a transit agreement and x_i is the capacity choice made by i (see Crémer, Rey and Tirole, 2000).

- *Interbank payments for the use of ATMs:* $r_{i\to j}$ is the interchange fee paid by bank i to bank j and x_i is the number of ATMs deployed by bank i (see Donze and Dubec, 2006, for a setting with multilateral negotiation of the interchange fee).

Note that the general model introduced above is not a generalization of our cross-licensing model *stricto sensu*. First, in contrast to the cross-licensing model, input prices can take positive as well as negative values. This rules out non-interior equilibria, which simplifies the analysis by making it possible to rely on first-order conditions. Another difference with the baseline model is that the first stage of our general model is slightly different from that of the cross-licensing model: there, we assumed that firms can decide not to sign an agreement in the first stage while in the current model, it is implicitly assumed that each pair of firms sign an agreement (the only decision variable is the terms of their agreement). However, this restriction does not entail any loss of generality when firms' incentives are such that each pair of firms find it jointly profitable to sign a bilateral upstream agreement, as is the case in our cross-licensing model. Moreover, this assumption is satisfied for upstream agreements that are made mandatory by regulators as is typically the case for instance with interconnection among telecommunication companies.

We now introduce the following definitions which generalize those adopted in our cross-licensing model:

Definition 3 *A vector* $\mathbf{r}$ *of input prices is fully cooperative if*

$$\boldsymbol{r} \in Arg \max_{\boldsymbol{r}'} \sum_{i=1}^{N} \pi_i \left(\mathbf{x}^* \left(\boldsymbol{r}' \right), \boldsymbol{r}' \right).$$

Definition 4 *A vector* $\mathbf{r}$ *of privately observable input prices is bilaterally efficient if for any* (i, j) *with* $i \neq j$*, the following holds:*

$$(r_{i\to j}, r_{j\to i}) \in Arg \max_{\left(r'_{i\to j}, r'_{j\to i}\right)} \left[\pi_{ij} \left(\tilde{x}_i^* \left(r'_{i\to j}, r'_{j\to i}, \mathbf{x}^*_{-ij} \left(\boldsymbol{r} \right) \right), \tilde{x}_j^* \left(r'_{i\to j}, r'_{j\to i}, \mathbf{x}^*_{-ij} \left(\boldsymbol{r} \right) \right), \mathbf{x}^*_{-ij} \left(\boldsymbol{r} \right), \boldsymbol{r}_{-ij} \right) \right]$$

where $\boldsymbol{r}_{-ij}$ denotes the vector obtained from $\boldsymbol{r}$ by removing $r_{i\to j}$ and $r_{j\to i}$.

Definition 5 *A vector* $\boldsymbol{r}$ *of publicly observable input prices is bilaterally efficient if for any* (i, j) *with* $i \neq j$*, the following holds:*

$$(r_{i\to j}, r_{j\to i}) \in Arg \max_{\left(r'_{i\to j}, r'_{j\to i}\right)} \left[(\pi_i + \pi_j) \left(\mathbf{x}^* \left(r'_{i\to j}, r'_{j\to i}, \boldsymbol{r}_{-ij} \right), \boldsymbol{r}_{-ij} \right) \right]$$

Let $\mathbf{D}$ denote the set of vectors $\mathbf{r}$ of input prices such that for any (i, j), $x_j^* (.)$ and $\tilde{x}_j^* (.)$ are differentiable with respect to all their arguments at $\boldsymbol{r}$ and $\pi_i (., \boldsymbol{r})$ is differentiable with respect to all its arguments at $\mathbf{x}^* (\boldsymbol{r})$.[15] The following provides a sufficient condition for a vector $\boldsymbol{r} \in \mathbf{D}$ of input prices to be fully cooperative. This condition also ensures that a multilateral agreement involving all firms allows them to achieve the monopoly outcome in the downstream market.

[15]Note that in applications where the second-stage subgame is a competition game, the subset of $\mathbf{r} \notin \mathbf{D}$ is typically of zero measure.

Lemma 3 *A sufficient condition for a vector $\boldsymbol{r} \in \mathbf{D}$ of input prices to be fully cooperative is that for any $j \in \{1, ..., N\}$,*

$$\sum_{i=1}^{N} \frac{\partial \pi_i}{\partial x_j} \left(\mathbf{x}^* \left(\boldsymbol{r}\right), \boldsymbol{r}\right) = 0. \tag{14}$$

Moreover, when this condition is met, the fully cooperative upstream agreements allow firms to achieve the fully cooperative downstream outcome.

Proof. See Appendix A. ■

6.1 Private agreements

We now provide a necessary condition for a vector of privately observable input prices in $\mathbf{D}$ to be bilaterally efficient.

Lemma 4 *Assume that, for any $\boldsymbol{r} \in \mathbf{D}$ and any $(i, j) \in \{1, ..., N\}^2$ with $i \neq j$, we have*

$$\begin{vmatrix} \frac{\partial \tilde{x}_i^*}{\partial r_{i\to j}} & \frac{\partial \tilde{x}_j^*}{\partial r_{i\to j}} \\ \frac{\partial \tilde{x}_i^*}{\partial r_{j\to i}} & \frac{\partial \tilde{x}_j^*}{\partial r_{j\to i}} \end{vmatrix} \neq 0, \tag{15}$$

where the argument $\left(r_{i\to j}, r_{j\to i}, \mathbf{x}_{-ij}^* \left(\boldsymbol{r}\right)\right)$ is omitted. Then a necessary condition for a vector of privately observable input prices $\boldsymbol{r} \in \mathbf{D}$ to be bilaterally efficient is that

$$\frac{\partial \pi_i}{\partial x_j} \left(\mathbf{x}^* \left(\boldsymbol{r}\right), \boldsymbol{r}\right) = 0,$$

for any $(i, j) \in \{1, ..., N\}^2$ such that $i \neq j$.

Proof. See Appendix A. ■

The rank condition (15) means that $r_{i\to j}$ and $r_{j\to i}$ are *independent* instruments in the sense that any local downstream deviation can be obtained through a local upstream deviation. This condition ensures that the set of instruments is rich enough to implement any desired agreement. Let us show that it is satisfied, for instance, in the simple context of the previous cross-licensing model with a downstream Cournot oligopoly featuring (potentially asymmetric) linear costs and linear (inverse) demand $p = a - Q$. Then we have

$$\begin{vmatrix} \frac{\partial \tilde{x}_i^*}{\partial r_{i\to j}} & \frac{\partial \tilde{x}_j^*}{\partial r_{i\to j}} \\ \frac{\partial \tilde{x}_i^*}{\partial r_{j\to i}} & \frac{\partial \tilde{x}_j^*}{\partial r_{j\to i}} \end{vmatrix} = \begin{vmatrix} \frac{\partial \tilde{x}_i^*}{\partial c_i} & \frac{\partial \tilde{x}_j^*}{\partial c_i} \\ \frac{\partial \tilde{x}_i^*}{\partial c_j} & \frac{\partial \tilde{x}_j^*}{\partial c_j} \end{vmatrix} = \frac{1}{9} > 0.$$

Note that in environments in which the second stage takes the form of a Cournot game and the input prices affect only the marginal cost of production (such as our cross-licensing example), Condition (15) means that own cost effects (on output) are not equal to cross cost effects. In fact, in imperfect competition models, the property that own cost effects strictly dominate cross cost effects is quite standard (see e.g., Vives, 1999).

Using the previous two lemmas, it is straightforward to get the following result about the cooperative potential of private bilateral agreements.

Proposition 4 *(privately observable input prices) Under Condition (15), a bilaterally efficient vector of privately observable input prices* $\boldsymbol{r} \in \mathbf{D}$ *is necessarily fully cooperative.*

In contrast to the baseline cross-licensing model, we do not establish the existence of at least one bilaterally efficient vector of input prices in the current general framework.[16] We however show that whenever a bilaterally efficient vector of input prices exists, it will maximize the N firms' joint profits.

6.2 Public agreements

We now provide a necessary condition for a vector of publicly observable input prices in $\mathbf{D}$ to be bilaterally efficient.

Lemma 5 *Assume that, for any* $\boldsymbol{r} \in \mathbf{D}$,

$$\det M^{public} \neq 0, \tag{16}$$

where M^{public} *is a* $N^2 \times N^2$ *matrix whose elements are defined as follows*

$$M^{public}_{N(i-1)+j,N(l-1)+k} = \begin{cases} \frac{\partial x_k^*}{\partial r_{i\to j}} & \text{if} \quad l = i \text{ and } i \neq j \\ \frac{\partial x_k^*}{\partial r_{i\to j}} & \text{if} \quad l = j \text{ and } i \neq j \\ 1 & \text{if} \quad l = k = i = j \\ 0 & \text{otherwise} \end{cases}$$

for any $i, j, l, k \in \{1, ..., N\}$. *In the matrix, the subscripts* (i, j) *refer to* $r_{i\to j}$ *and the subscripts* (l, k) *refer to* $\frac{\partial \pi_l}{\partial x_k}$.

[16] This would essentially amount to showing that a given system of $N(N-1)$ first-order equations with $N(N-1)$ unknowns has at least one solution. This turns out to be complicated because of the (fully) asymmetric nature of the equations.

Then a necessary condition for a vector of publicly observable input prices $\boldsymbol{r} \in \mathbf{D}$ *to be bilaterally efficient is that*

$$\frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\mathbf{r}\right), \mathbf{r}\right) = 0,$$

for any $(i, j) \in \{1, ..., N\}^2$ *such that* $i \neq j$.

Proof. See Appendix A. ■

Condition (16) is the counterpart of Condition (15) for publicly observable input prices. Similarly to the case of privately observable input prices, this condition ensures that any local downstream deviation can be obtained through a local upstream deviation. The reason why Condition (16) has a less simple form than Condition (15) is that now a coalition that contemplates a deviation has to take into account the responses of its rivals in the second stage of the game.

Combining Lemmas 3 and 5, we get the following result about the cooperative potential of public bilateral agreements.

Proposition 5 *(publicly observable input prices) Under Condition (16), a bilaterally efficient vector of publicly observable input prices* $\boldsymbol{r} \in \mathbf{D}$ *is necessarily fully cooperative.*

Proposition 2 shows that the most competitive outcome, i.e. the one corresponding to zero royalties, can arise as a bilaterally efficient outcome in our cross-licensing model (when agreements are public). Proposition 5 shows however that this does not happen in the general model. The reason behind it is that the non-negativity restriction on royalties in the baseline cross-licensing model is key to get that all firms charging no royalties can be a bilaterally efficient outcome. If this restriction is relaxed, as is done in the general model, then a pair of firms will find it optimal to deviate to an agreement with negative royalties.

7 Policy implications

We now discuss the policy implications of our results regarding the antitrust treatment of bilateral cross-licensing agreements between competitors.

Competition authorities usually prohibit the use of royalties that are disproportionate with respect to the market value of the license.[17] For instance, according to the Guidelines

[17] This would amount in our model to setting an upper bound on the level of royalties that cross-licensing partners can agree on.

on the application of Article 81 of the EC Treaty to technology transfer agreements (European Commission, 2014), "... Article 101(1) may be applicable where competitors cross license and impose running royalties that are clearly disproportionate compared to the market value of the licence and where such royalties have a significant impact on market prices." However, the Technology Transfer Block Exemption Regulation (TTBER) of the European Commission grants antitrust exemption to bilateral cross-licensing agreements between competitors if their joint market share does not exceed 20%. In a similar vein, the competition authorities in the U.S. grant a safe harbor to cross-licensing agreements (not necessarily bilateral) among partners whose joint market share is below 20% (U.S. DOJ and FTC, 1995, p.22). This policy relies on market forces to discipline cross-licensing partners regarding the level of royalties they agree on and presumes that firms with relatively low market power will not find it profitable to charge each other high per-unit royalties.

Our analysis shows that this disciplining effect of market forces can arise only under a limited range of circumstances: the most competitive outcome (i.e., $r = 0$) is one of the sustainable outcomes only if i) agreements are public[18], ii) the Stackelberg effect dominates the coordination effect, and iii) firms compete in quantities. Moreover, we show that firms can always sustain royalties that are high enough to implement the monopoly outcome (i.e., $r = r^m$), regardless of the information structure (i.e., whether the licensing terms are public or private) and the type of downstream competition (i.e., Cournot or Bertrand). This result clearly questions the current antitrust treatment of cross-licensing agreements in the U.S. and the EU. Consider for instance the specific example of an industry comprised of ten symmetric firms. In such setting, any bilateral cross-licensing agreements would benefit from an antitrust exemption since the joint market share criterion used by American and European antitrust authorities would be satisfied. However, our findings show that such *legal* agreements allow firms to achieve the monopoly outcome, which will make consumers worse off if the cross-licensed technologies are relatively substitutable. This suggests in particular that multiple bilateral cross-licensing agreements involving the same firm should not be treated separately by antitrust authorities.

[18] Anecdotical evidence strongly suggests that the vast majority of cross-licensing agreements are private.

8 Concluding remarks

The main message of this paper is as follows: under a wide range of circumstances, bilateral upstream agreements among competing firms can allow them to achieve the same outcome as under full upstream cooperation. This result has been shown to hold independently of the nature of downstream interactions, and regardless of whether the agreements are public or private and whether firms are symmetric or not. Our result does not necessarily imply that bilateral upstream agreements reduce social welfare. First, if firms produce complements rather than substitutes, full upstream cooperation and hence bilateral upstream agreements are socially desirable. Second, even if firms produce substitutes, the outcome of full upstream cooperation can be superior to the outcome of no upstream agreement at all. For instance, cross-licensing of patents can reduce firms' costs such that the final price can be lower than the price without cross-licensing. Third, in the case of cross-licensing of patents, one should also take into account how cross-licensing affects firms' incentives to invest in innovation.

Our setting can be extended to study other policy issues related to cross-licensing. First, we can introduce, in addition to incumbent firms, entrants with no (or weak) patent portfolios. This would allow us to study whether cross-licensing can be used to raise barriers to entry (U.S. DOJ and FTC, 2007). Second, we can include in the set of players non-practicing entities (NPEs) which do not compete in the downstream market. This would allow us to study the conditions under which NPEs weaken competition and (when these conditions are met) to isolate the anticompetitive effects generated by NPEs from the effects resulting from cross-licensing in the absence of them.[19] Note that NPEs and entrants involve completely opposite asymmetries. The former are present in the upstream market of patent licensing but are absent in the downstream (product) market while the second are absent (or have very weak presence) in the upstream market but are present in the downstream market.

9 References

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[19]The issue of how NPEs affect competition and innovation is of substantial current interest to policy makers (Scott Morton and Shapiro, 2014).

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10 Appendix A: Proofs

Proof of Lemma 1

Assume, without loss of generality, that $(i,j)=(1,2)$. The joint payoff that firms 1 and 2 derive from a deviation to a licensing agreement involving the payment of $(r_{1\to 2}, r_{2\to 1})$ is:

$$\begin{aligned}\pi_1^* + \pi_2^* &= \left[P\left(Q^*\right) - \underline{c} - r_{1\to 2} - (N-2)\, r\right] q_1^* + r_{2\to 1} q_2^* + r Q_{-12}^* \\ &\quad + \left[P\left(Q^*\right) - \underline{c} - r_{2\to 1} - (N-2)\, r\right] q_2^* + r_{1\to 2} q_1^* + r Q_{-12}^* \\ &= \left[P\left(Q^*\right) - \underline{c} - (N-2)\, r\right] \left(q_1^* + q_2^*\right) + 2rQ_{-12}^*.\end{aligned}$$

which can be rewritten as

$$\pi_1^* + \pi_2^* = \left[P\left(Q^*\right) - \underline{c} - (N-2)\, r\right] \left(Q^* - Q_{-12}^*\right) + 2rQ_{-12}^*. \tag{17}$$

Denoting c_i the marginal cost of firm i (including the royalties paid to the other firms), the F.O.C. for firm i' s maximization program is:

$$P\left(Q^*\right) - c_i + q_i^* P'\left(Q^*\right) = 0.$$

Summing the F.O.C.s for $i = 1, 2, .., N$ yields:

$$NP\left(Q^*\right) - \sum_{i\geq 1} c_i + Q^* P'\left(Q^*\right) = 0,$$

which shows that Q^* depends on $(c_1, c_2, .., c_N)$ only through $\sum_{i\geq 1} c_i$. Moreover, summing the F.O.C.s for $i = 3, .., N$ yields

$$(N-2)\, P\left(Q^*\right) - \sum_{i\geq 3} c_i + Q_{-12}^* P'\left(Q^*\right) = 0,$$

which implies that Q_{-12}^* depends on $(c_1, c_2, ..., c_N)$ only through $\sum_{i\geq 1} c_i$ and $\sum_{i\geq 3} c_i$. From $(c_1, c_2, c_3, ..., c_N) = (\underline{c} + r_{1\to 2} + (N-2)\, r, \underline{c} + r_{2\to 1} + (N-2)\, r, \underline{c} + (N-1)\, r, ..., \underline{c} + (N-1)\, r)$, it then follows that both Q^* and Q_{-12}^* depend on $(r_{1\to 2}, r_{2\to 1})$ only through $r_{1\to 2} + r_{2\to 1}$, which, combined with (17), implies that $\pi_1^* + \pi_2^*$ depends on $(r_{1\to 2}, r_{2\to 1})$ only through $r_{1\to 2} + r_{2\to 1}$.

Proof of Lemma 2

It is straightforward that $r \in \underset{\hat{r}\geq 0}{Arg\max}\left(\pi_1^*+\pi_2^*\right)(r,\hat{r})$ if and only if:

$$\begin{aligned} Q_{12}^*(r,r) \quad &\in \underset{Q_{12}\in\left[0,Q_{12}^*(r,0)\right]}{Arg\max} \quad \pi_{12}\left(Q_{12}\right)\equiv \\ &\left[P\left(Q_{12}+BR_{-12}(Q_{12})\right)-\left(\underline{c}+\left(N-2\right)r\right)\right]Q_{12}+2rBR_{-12}(Q_{12}) \end{aligned} \tag{18}$$

which means that $Q_{12}^*(r,r)$ is a subgame-perfect equilibrium strategy of the coalition $\{1,2\}$ in the game $G(r,N)$.

Proof of Proposition 2

Let us first show some general preliminary results which will be useful for the subsequent specific analysis of the four considered scenarios. We have:

$$\begin{aligned} \frac{\partial\pi_{12}}{\partial Q_{12}}\left(Q_{12},r\right) \quad &= \quad P'\left(Q_{12}+BR_{-12}(Q_{12})\right)Q_{12}(1+BR'_{-12}(Q_{12}))+ \\ &\quad \left[P\left(Q_{12}+BR_{-12}(Q_{12})\right)-\left(\underline{c}+\left(N-1\right)r\right)\right]+r\left(1+2BR'_{-12}(Q_{12})\right) \\ &= \quad \left[\underline{c}+Nr-P\left(Q_{12}+BR_{-12}(Q_{12})\right)\right]\left[1+2BR'_{-12}(Q_{12})\right]+ \end{aligned}$$

$$\underbrace{2\underbrace{\left[P'\left(Q_{12}+BR_{-12}(Q_{12})\right)\frac{Q_{12}}{2}+P\left(Q_{12}+BR_{-12}(Q_{12})\right)-\left(\underline{c}+\left(N-1\right)r\right)\right]}_{\equiv J(Q_{12},r)}\left[1+BR'_{-12}(Q_{12})\right]}_{\equiv D((Q_{12},r)}$$

Let us show that $J(Q_{12},r)$ is decreasing in Q_{12}. We have

$$\frac{\partial J(Q_{12},r)}{\partial Q_{12}}=\left[P''\left(Q\right)\frac{Q_{12}}{2}+P'\left(Q\right)\right]\underbrace{\left(1+BR'_{-12}(Q_{12})\right)}_{>0}+\underbrace{\frac{P'\left(Q\right)}{2}}_{<0}.$$

Since $P''\left(Q\right)\frac{Q_{12}}{2}+P'\left(Q\right)<\max\left[P''\left(Q\right)Q+P'\left(Q\right),P'\left(Q\right)\right]<0$ then $J(Q_{12},r)$ is decreasing in Q_{12}. This, combined with the fact that the F.O.C. for each firm $i=1,2$, satisfied at the symmetric Cournot equilibrium, is given by $J(Q_{12}^*\left(r,r\right),r)=0$, yields that

$$J(Q_{12},r)\lesseqgtr 0 \Longleftrightarrow Q_{12}\gtreqless Q_{12}^*\left(r,r\right). \tag{19}$$

Since $1+BR'_{-1,2}(Q_{12})>0$, it follows that

$$D(Q_{12},r)\lesseqgtr 0 \Longleftrightarrow Q_{12}\gtreqless Q_{12}^*\left(r,r\right). \tag{20}$$

Moreover, from

$$BR'_{-12}(Q_{12}) = -\frac{P''(Q)BR_{-12}(Q_{12}) + (N-2)P'(Q)}{P''(Q)BR_{-12}(Q_{12}) + (N-1)P'(Q)},$$

it follows that

$$1 + 2BR'_{-12}(Q_{12}) = -\frac{P''(Q)BR_{-12}(Q_{12}) + (N-3)P'(Q)}{P''(Q)BR_{-12}(Q_{12}) + (N-1)P'(Q)}, \tag{21}$$

which can be rewritten as

$$\begin{aligned} & 1 + 2BR'_{-12}(Q_{12}) \\ = \quad & -\frac{\frac{N-2}{N}P'(Q)}{P''(Q)BR_{-12}(Q_{12}) + (N-1)P'(Q)}\left[\frac{QP''(Q)}{P'(Q)}\left(\frac{N}{N-2}.\frac{BR_{-12}(Q_{12})}{Q}\right) + \frac{N(N-3)}{N-2}\right]. \end{aligned}$$

Since $P''(Q)BR_{-12}(Q_{12})+(N-1)P'(Q) \leq \max\left((N-1)P'(Q), P''(Q)Q + (N-1)P'(Q)\right) < 0$, it follows that

$$1 + 2BR'_{-12}(Q_{12}) \lesseqqgtr 0 \Longleftrightarrow \frac{QP''(Q)}{P'(Q)} \gtreqless -\frac{N(N-3)}{N-2}\left[\frac{N-2}{N}.\frac{Q}{BR_{-12}(Q_{12})}\right], \tag{22}$$

(for any Q_{12} such that $BR_{-12}(Q_{12}) \neq 0$). From the fact that $BR_{-12}(Q_{12}) - \frac{N-2}{N}Q = BR_{-12}(Q_{12}) - \frac{N-2}{N}(Q_{12} + BR_{-12}(Q_{12}))$ is decreasing in Q_{12} and $BR_{-12}(Q^*_{12}(r,r)) = \frac{N-2}{N}Q^*(r,r)$ (by symmetry of the considered Cournot equilibrium), it follows that

$$\frac{N-2}{N}\frac{Q}{BR_{-12}(Q_{12})} \gtreqless 1 \Longleftrightarrow Q_{12} \gtreqless Q^*_{12}(r,r).$$

In particular we obtain the following result which will be useful for the next steps of the proof: If $\frac{QP''(Q)}{P'(Q)} > -\frac{N(N-3)}{N-2}$ (for any Q such that $P'(Q) \neq 0$) then $1 + 2BR'_{-12}(Q_{12}) < 0$ for any $Q_{12} \geq Q^*_{12}(r,r)$.

- Let us now show that $r = r^m$ is bilaterally efficient regardless of whether the Stackelberg effect dominates or is dominated by the coordination effect.

$$\begin{aligned}\frac{\partial \pi_{12}}{\partial Q_{12}}(Q_{12}, r^m) &= P'\left(Q_{12} + BR_{-12}(Q_{12})\right) Q_{12}(1 + BR'_{-12}(Q_{12})) + \\ &\quad \left[P\left(Q_{12} + BR_{-12}(Q_{12})\right) - \left(\underline{c} + (N-1)\, r^m\right)\right] + r^m \left(1 + 2BR'_{-12}(Q_{12})\right) \\ &= 2\left(P'\left(Q_{12} + BR_{-12}(Q_{12})\right) \frac{Q_{12}}{2} + r^m\right)(1 + BR'_{-12}(Q_{12})) + \\ &\quad \left[P\left(Q_{12} + BR_{-12}(Q_{12})\right) - \left(\underline{c} + N r^m\right)\right]\end{aligned}$$

Since $\underline{c} + N r^m - P\left(Q^*(r^m, r^m)\right) = 0$ then

$$\begin{aligned}P\left(Q_{12} + BR_{-12}(Q_{12})\right) - \left(\underline{c} + N r^m\right) &= P\left(Q_{12} + BR_{-1,2}(Q_{12})\right) \\ &\quad - P\left(Q^*_{12}(r^m, r^m) + BR_{-12}(Q^*_{12}(r^m, r^m))\right)\end{aligned}$$

Moreover, combining $\underline{c} + N r^m - P\left(Q^*(r^m, r^m)\right) = 0$ with the F.O.C.

$$\begin{gathered}P'\left(Q^*_{12}(r^m, r^m) + BR_{-12}(Q^*_{12}(r^m, r^m))\right) \tfrac{Q^*_{12}(r^m, r^m)}{2} + \\ P\left(Q^*_{12}(r^m, r^m) + BR_{-12}(Q^*_{12}(r^m, r^m))\right) - \underline{c} - (N-1)\, r^m = 0\end{gathered}$$

yields

$$\begin{aligned}P'\left(Q_{12} + BR_{-12}(Q_{12})\right) \frac{Q_{12}}{2} + r^m &= P'\left(Q_{12} + BR_{-12}(Q_{12})\right) \frac{Q_{12}}{2} - \\ &\quad P'\left(Q^*_{12}(r^m, r^m) + BR_{-12}(Q^*_{12}(r^m, r^m))\right) \frac{Q^*_{12}(r^m, r^m)}{2}\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial \pi_{12}}{\partial Q_{12}}(Q_{12}, r^m) &= [2P'\left(Q_{12} + BR_{-12}(Q_{12})\right) \frac{Q_{12}}{2} \\ &\quad -2P'\left(Q^*_{12}(r^m, r^m + BR_{-12}(Q^*_{12}(r^m, r^m))\right) \frac{Q^*_{12}(r^m, r^m)}{2}] \left(1 + BR'_{-12}(Q^*_{12}(r^m, r^m)\right) + \\ &\quad \left[P\left(Q_{12} + BR_{-12}(Q_{12})\right) - P\left(Q^*_{12}(r^m, r^m) + BR_{-12}(Q^*_{12}(r^m, r^m))\right)\right]\end{aligned}$$

Using the fact that $P'(Q) < 0$ and $1 + BR'_{-12}(Q_{12}) > 0$, it is straightforward to show that both functions $P'\left(Q_{12} + BR_{-12}(Q_{12})\right) \frac{Q_{12}}{2}$ and $P\left(Q_{12} + BR_{-12}(Q_{12})\right)$ are decreasing in Q_{12}, which implies that

$$\frac{\partial \pi_{12}}{\partial Q_{12}}(Q_{12}, r^m) \gtreqless 0 \Longleftrightarrow Q_{12} \lesseqgtr Q^*_{12}(r^m, r^m)$$

Therefore, $Q^*_{12}(r^m, r^m)$ maximizes $\pi_{12}(Q_{12}, r^m)$ over $[0, Q^*_{12}(r^m, 0)]$, which is equivalent to the fact that $S(r^m, N)$ is bilaterally efficient.

- Consider now the case $\frac{QP''(Q)}{P'(Q)} > -\frac{N(N-3)}{N-2}$ and let us show that $r = 0$ is bilaterally efficient, that is $Q^*_{12}(0,0)$ maximizes $\pi_{12}(Q_{12}, 0)$ over $[0, Q^*_{12}(0,0)]$. We have

$$\frac{\partial \pi_{12}}{\partial Q_{12}}(Q_{12}, 0) = (\underline{c} - P(Q_{12} + BR_{-1,2}(Q_{12})))(1 + 2BR'_{-12}(Q_{12})) + D(Q_{12}, 0)$$

where $D(Q_{12}, r)$ is defined in the beginning of the proof.

Let us now show that $P''(Q_{12}+BR_{-12}(Q_{12}))BR_{-12}(Q_{12})+(N-3)P'(Q_{12}+BR_{-12}(Q_{12})) < 0$ for any $Q_{12} \in [0, Q^*_{12}(0,0)]$, which, by (21), is sufficient to state that $1+2BR'_{-12}(Q_{12}) < 0$ for any $Q_{12} \in [0, Q^*_{12}(0,0)]$. On the one hand, if $P''(Q_{12} + BR_{-1,2}(Q_{12})) < 0$ then it follows from $BR_{-12}(Q_{12}) \geq 0$ and $P'(Q) < 0$ that $1 + 2BR'_{-12}(Q_{12}) < 0$. On the other hand, if $P''(Q_{12} + BR_{-12}(Q_{12})) \geq 0$ then from $BR_{-12}(Q_{12}) \leq Q$, it follows that $P''(Q_{12} + BR_{-12}(Q_{12}))BR_{-12}(Q_{12})+(N-3)P'(Q_{12}+BR_{-12}(Q_{12})) \leq P''(Q)Q+(N-3)P'(Q)$. Note first that if $P''(Q_{12} + BR_{-12}(Q_{12})) \geq 0$, it must hold that $N \geq 4$; otherwise the condition $\frac{QP''(Q)}{P'(Q)} > -\frac{N(N-3)}{N-2}$ (which is one of the two conditions defining the present scenario) would be violated. This implies that $P''(Q)Q+(N-3)P'(Q) \leq P''(Q)Q+P'(Q)$, which combined with **A3** yields $P''(Q)Q + (N-3)P'(Q) < 0$ and, therefore, $1 + 2BR'_{-12}(Q_{12}) < 0$. We are now in position to state that, for any $Q_{12} \in [0, Q^*_{12}(0,0)]$, the latter inequality holds independently of whether $P''(Q_{12} + BR_{-12}(Q_{12})) < 0$ or $P''(Q_{12} + BR_{-12}(Q_{12})) \geq 0$. Combining that with the fact that the two inequalities $\underline{c} - P(Q_{12} + BR_{-1,2}(Q_{12})) \leq \underline{c} - P(Q^*_{12}(0,0) + BR_{-1,2}(Q^*_{12}(0,0))) < 0$ and $D(Q_{12}, 0) \geq 0$ (from (20)) hold for any $Q_{12} \in [0, Q^*_{12}(0,0)]$, we get that $\frac{\partial \pi_{12}}{\partial Q_{12}}(Q_{12}, 0) \geq 0$ for any $Q_{12} \in [0, Q^*_{12}(0,0)]$. This implies that $Q^*_{12}(0,0)$ maximizes $\pi_{12}(Q_{12}, 0)$ over $[0, Q^*_{12}(0,0)]$. Therefore, $S(0, N)$ is bilaterally efficient.

Proof of Lemma 3

By **G1, G2** and **G3** it holds that $\sum_{i=1}^{N} \Pi_i(\mathbf{x}, \boldsymbol{r}, \boldsymbol{F}) = \sum_{i=1}^{N} \pi_i(\mathbf{x}, \mathbf{r})$ does not depend on $\mathbf{r}$ for any $\mathbf{x}$. By **G6**, $\mathbf{x^m}$ is then the unique solution to the system of N equations:

$$\sum_{i=1}^{N} \frac{\partial \pi_i}{\partial x_j}(\mathbf{x}, \mathbf{r}) = 0 \text{ for } j \in \{1, ..., N\},$$

for any $\mathbf{r}$. Therefore, if a vector $\mathbf{r} \in D$ is such that $\sum_{i=1}^{N} \frac{\partial \pi_i}{\partial x_j}(\mathbf{x}^*(\mathbf{r}), \mathbf{r}) = 0$ for any

$j \in \{1, ..., N\}$, then it must be that $\mathbf{x}^*(\mathbf{r}) = \mathbf{x^m}$, which implies that (i) $\sum_{i=1}^{N} \pi_i(\mathbf{x}^*(\mathbf{r}), \mathbf{r}) \geq \sum_{i=1}^{N} \pi_i(\mathbf{x}^*(\mathbf{r}'), \mathbf{r}')$ for any $\mathbf{r}'$, that is, $\mathbf{r}$ is fully cooperative, and (ii) the fully cooperative upstream agreements allows the agent to achieve the fully cooperative downstream outcome.

Proof of Lemma 4

Assume that $\boldsymbol{r} \in \mathbf{D}$ is bilaterally efficient. Then for any $(i, j) \in \{1, ..., N\}^2$ with $i \neq j$, it must hold that

$$\frac{\partial}{\partial r_{i \to j}} (\pi_i + \pi_j) \left(\tilde{x}_i^* \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right), \tilde{x}_j^* \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right), \mathbf{x}_{-ij}^*(\boldsymbol{r}), \boldsymbol{r}_{-ij}\right) = 0,$$

which can be rewritten as

$$\begin{aligned}
& \frac{\partial \tilde{x}_i^*}{\partial r_{i\to j}} \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right) \frac{\partial \pi_i}{\partial x_i} \left(\tilde{x}_i^* \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right), \tilde{x}_j^* \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right), \mathbf{x}_{-ij}^*(\boldsymbol{r}), \boldsymbol{r}_{-ij}\right) + \\
& \frac{\partial \tilde{x}_j^*}{\partial r_{i\to j}} \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right) \frac{\partial \pi_i}{\partial x_j} \left(\tilde{x}_i^* \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right), \tilde{x}_j^* \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right), \mathbf{x}_{-ij}^*(\boldsymbol{r}), \boldsymbol{r}_{-ij}\right) + \\
& \frac{\partial \tilde{x}_i^*}{\partial r_{i\to j}} \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right) \frac{\partial \pi_j}{\partial x_i} \left(\tilde{x}_i^* \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right), \tilde{x}_j^* \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right), \mathbf{x}_{-ij}^*(\boldsymbol{r}), \boldsymbol{r}_{-ij}\right) + \\
& \frac{\partial \tilde{x}_j^*}{\partial r_{i\to j}} \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right) \frac{\partial \pi_j}{\partial x_j} \left(\tilde{x}_i^* \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right), \tilde{x}_j^* \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right), \mathbf{x}_{-ij}^*(\boldsymbol{r}), \boldsymbol{r}_{-ij}\right) \\
= {} & 0.
\end{aligned}$$

Using the definition of a Nash equilibrium , it is straightforward to see that $\tilde{x}_i^* \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right) = x_i^*(\boldsymbol{r})$ and that $\tilde{x}_j^* \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right) = x_j^*(\boldsymbol{r})$. Therefore, it holds that

$$\begin{aligned}
& \frac{\partial \tilde{x}_i^*}{\partial r_{i\to j}} \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right) \frac{\partial \pi_i}{\partial x_i} (\mathbf{x}^*(\boldsymbol{r}), \boldsymbol{r}) + \frac{\partial \tilde{x}_j^*}{\partial r_{i\to j}} \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right) \frac{\partial \pi_i}{\partial x_j} (\mathbf{x}^*(\boldsymbol{r}), \boldsymbol{r}) + \\
& \frac{\partial \tilde{x}_i^*}{\partial r_{i\to j}} \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right) \frac{\partial \pi_j}{\partial x_i} (\mathbf{x}^*(\boldsymbol{r}), \boldsymbol{r}) + \frac{\partial \tilde{x}_j^*}{\partial r_{i\to j}} \left(r_{i\to j}, r_{j \to i}, \mathbf{x}_{-ij}^*(\boldsymbol{r})\right) \frac{\partial \pi_j}{\partial x_j} (\mathbf{x}^*(\boldsymbol{r}), \boldsymbol{r}) = 0.
\end{aligned}$$

By definition of the downstream Nash equilibrium $\mathbf{x}^*(\mathbf{r})$, it holds that

$$\frac{\partial \pi_i}{\partial x_i} (\mathbf{x}^*(\boldsymbol{r}), \boldsymbol{r}) = \frac{\partial \pi_j}{\partial x_j} (\mathbf{x}^*(\boldsymbol{r}), \boldsymbol{r}) = 0.$$

This yields

$$\frac{\partial \tilde{x}_j^*}{\partial r_{i\to j}}\left(r_{i\to j}, r_{j\to i}, \mathbf{x}_{-ij}^*\left(\boldsymbol{r}\right)\right)\frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)+\frac{\partial \tilde{x}_i^*}{\partial r_{i\to j}}\left(r_{i\to j}, r_{j\to i}, \mathbf{x}_{-ij}^*\left(\boldsymbol{r}\right)\right)\frac{\partial \pi_j}{\partial x_i}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)=0.$$

By symmetry we also have

$$\frac{\partial \tilde{x}_i^*}{\partial r_{j\to i}}\left(r_{i\to j}, r_{j\to i}, \mathbf{x}_{-ij}^*\left(\boldsymbol{r}\right)\right)\frac{\partial \pi_j}{\partial x_i}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)+\frac{\partial \tilde{x}_j^*}{\partial r_{j\to i}}\left(r_{i\to j}, r_{j\to i}, \mathbf{x}_{-ij}^*\left(\boldsymbol{r}\right)\right)\frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)=0.$$

Denoting $y_{ij} = \frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)$ and $y_{ji} = \frac{\partial \pi_j}{\partial x_i}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)$, and omitting the argument $\left(r_{i\to j}, r_{j\to i}, \mathbf{x}_{-ij}^*\left(\boldsymbol{r}\right)\right)$, the latter two equations can be rewritten as a two-equation *linear* system in y_{ji} and y_{ij}:

$$\begin{cases} \frac{\partial \tilde{x}_i^*}{\partial r_{i\to j}}.y_{ji}+\frac{\partial \tilde{x}_j^*}{\partial r_{i\to j}}.y_{ij}=0 \\ \frac{\partial \tilde{x}_i^*}{\partial r_{j\to i}}.y_{ji}+\frac{\partial \tilde{x}_j^*}{\partial r_{j\to i}}.y_{ij}=0 \end{cases}$$

If $\begin{vmatrix} \frac{\partial \tilde{x}_i^*}{\partial r_{i\to j}} & \frac{\partial \tilde{x}_j^*}{\partial r_{i\to j}} \\ \frac{\partial \tilde{x}_i^*}{\partial r_{j\to i}} & \frac{\partial \tilde{x}_j^*}{\partial r_{j\to i}} \end{vmatrix} \neq 0$, then the latter system has a unique solution, given by $y_{ji} = y_{ij} = 0$. Hence, we get the following: for any $(i,j) \in \{1,...,N\}^2$ with $i \neq j$, the following equation must hold

$$\frac{\partial \pi_i}{\partial x_j}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)=0.$$

Proof of Lemma 5

Assume that a given vector of public input prices $\mathbf{r} \in D$ is bilaterally efficient. The FOCs associated with

$$\left(r_{i\to j}, r_{j\to i}\right) \in \underset{\left(r'_{i\to j}, r'_{j\to i}\right)}{Arg\max}\left[\left(\pi_i+\pi_j\right)\left(\mathbf{x}^*\left(r'_{i\to j}, r'_{j\to i}, \boldsymbol{r}_{-ij}\right),\boldsymbol{r}_{-ij}\right)\right]$$

are

$$\sum_{k=1}^{n}\frac{\partial x_k^*}{\partial r_{i\to j}}\left(\boldsymbol{r}\right)\frac{\partial \pi_i}{\partial x_k}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)+\sum_{k=1}^{n}\frac{\partial x_k^*}{\partial r_{i\to j}}\left(\boldsymbol{r}\right)\frac{\partial \pi_j}{\partial x_k}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)=0;$$

and

$$\sum_{k=1}^{n}\frac{\partial x_k^*}{\partial r_{j\to i}}\left(\boldsymbol{r}\right)\frac{\partial \pi_i}{\partial x_k}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)+\sum_{k=1}^{n}\frac{\partial x_k^*}{\partial r_{j\to i}}\left(\boldsymbol{r}\right)\frac{\partial \pi_j}{\partial x_k}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)=0.$$

Hence, the FOCs associated with the bilateral efficiency of the upstream agreements give rise to $N(N-1)$ conditions. Adding these to the N FOCs $\frac{\partial \pi_i}{\partial x_i}\left(\mathbf{x}^*\left(\boldsymbol{r}\right),\boldsymbol{r}\right)=0$ associated to the downstream Nash equilibrium, we end up with a system of N^2 equations. The latter can be represented as a *linear* system $M^{public}Y = 0$ where M^{public} is a $N^2 \times N^2$ matrix whose elements are defined as follows:

$$M^{public}_{N(i-1)+j,N(l-1)+k} = \begin{cases} \frac{\partial x_k^*}{\partial r_{i\to j}} & \text{if} \quad l = i \text{ and } i \neq j \\ \frac{\partial x_k^*}{\partial r_{i\to j}} & \text{if} \quad l = j \text{ and } i \neq j \\ 1 & \text{if} \quad l = k = i = j \\ 0 & \text{otherwise} \end{cases}$$

for any $i, j, l, k \in \{1, ..., N\}$; and Y is a $N^2 \times 1$ matrix whose elements (which are the "unknown variables") are defined as follows

$$Y_{N(l-1)+k} = \frac{\partial \pi_l}{\partial x_k} \left(\mathbf{x}^* \left(\boldsymbol{r}\right), \boldsymbol{r}\right).$$

If $\det M^{public} \neq 0$, then this linear system has a unique solution given by

$$\frac{\partial \pi_l}{\partial x_k} \left(\mathbf{x}^* \left(\boldsymbol{r}\right), \boldsymbol{r}\right) = 0,$$

for any $k, l \in \{1, ..., N\}$, which completes the proof.

11 Appendix B: Extensions

11.1 Alternative equivalent formulation

Instead of assuming that access to more patents reduces a firm's marginal cost, we can assume that access to more patents increases the value of the product produced by the firm. We below show that our model of cost-reducing innovations can be equivalently interpreted as a model of value-increasing innovations.

We consider a constant symmetric marginal cost c for all firms. Each firm has one patent. Let $v(n)$ represent the value of the product produced by a firm when the firm has access to $n \in \{1, ..., N\}$ number of distinct patents with $v(N) \geq v(N-1) \geq ... \geq v(1)(\equiv \underline{v})$. Let $\mathbf{v} \equiv (v_1, ..., v_N)$ be the vector representing the value of each firm's product after the licensing stage.

We define Cournot competition for given $\mathbf{v} \equiv (v_1, ..., v_N)$ as follows. Each firm i simultaneously chooses its quantity q_i. Given $\mathbf{v} \equiv (v_1, ..., v_N)$, $\mathbf{q} \equiv (q_1, ..., q_N)$ and $Q = q_1 + ... + q_N$, the quality-adjusted equilibrium prices are determined by the following two conditions:

- an indifference condition:

$$v_i - p_i = v_j - p_j \qquad \text{for all } (i,j) \in \{1,...,N\}^2;$$

- a market-clearing condition:

$$Q = D(p) \text{ where } p_i = p + v_i - \underline{v}.$$

In other words, p is the price for the product of a firm which has access to its own patent only. The market clearing condition means that this price is adjusted to make the total supply equal to the demand. The indifference condition implies that the price each firm charges is adjusted such that all consumers who buy any product are indifferent among all products. A micro-foundation of this setup can be provided as follows. There is a mass one of consumers. Each consumer has a unit demand and hence buys at most one unit among all products. A consumer's gross utility from having a unit of product of firm i is given by $u+v_i$: u is specific to the consumer while v_i is common to all consumers. Let $F(u)$ represent the cumulative distribution function of u. Then, by construction of quality-adjusted prices, any consumer is indifferent among all products and the marginal consumer indifferent between buying any product and not buying is characterized by $u + \underline{v} - p = 0$, implying

$$D(p) = 1 - F(p - \underline{v}).$$

In equilibrium, p is adjusted such that $1-F(p-\underline{v}) = Q$. Let $P(Q)$ be the inverse demand function. In equilibrium, a firm's profit is given by

$$\pi_i = \left(P(Q) + v_i - \underline{v} - c - \sum_{j \neq i} r_{i \to j} \right) q_i + \sum_{j \neq i} r_{j \to i} q_j.$$

After making the following change of variables

$$c - (v_i - \underline{v}) = c_i,$$

the profit can be equivalently written as

$$\pi_i = \left(P(Q) - c_i - \sum_{j \neq i} r_{i \to j} \right) q_i + \sum_{j \neq i} r_{j \to i} q_j,$$

which is the profit expression in our original model of cost-reducing patents. Therefore, our model of cost-reducing innovations can be equivalently interpreted as a model of value-increasing innovations.

11.2 k-efficient agreements

In this extension, we show that the main results previously obtained by considering a coalition of size 2 extend to a coalition of any given size k (with $3 \leq k \leq N-1$). We will say that a set of cross-licensing agreements is *k-efficient* if no coalition of size k finds it optimal to change the terms of the cross-licensing agreements between the members of the coalition. We consider first the case of public cross-licensing and then the case of private cross-licensing.

Suppose that cross-licensing agreements are public. Consider the deviation of a coalition composed of $\{1, ..., k\}$ in the licensing stage. Lemma 1 continues to hold in the case of coalition of size k and hence, without loss of generality, we can restrict attention to deviations involving a symmetric royalty $\hat{r}$. For given $(r, \hat{r})$, let $Q_k^*(r, \hat{r})$ denote the sum of the outputs of the firms in the coalition in the (second-stage) equilibrium of Cournot competition. Let $Q_{-k}^*(r, \hat{r}) \equiv Q^*(r, \hat{r}) - Q_k^*(r, \hat{r})$.

Denoting Q_k the total quantity produced by the considered coalition and r the common royalty paid to the firms outside the coalition, the coalition's profit can be rewritten as

$$\pi_k\left(Q_k, r\right) = \underbrace{\left[P\left(Q_k + BR_{-k}(Q_k)\right) - \left(\underline{c} + \left(N-1\right)r\right)\right] Q_k}_{\pi_k^D(Q_k, r)} + \underbrace{r\left[(k-1)Q_k + kBR_{-k}(Q_k)\right]}_{\pi_k^U(Q_k, r)}. \tag{23}$$

Equation (23) generalizes (4). Suppose that the coalition marginally expands or contracts its output Q_k with respect to $Q_k^*(r, r)$. Then, we have

$$\frac{\partial \pi_k^D}{\partial Q_k}(Q_k^*(r,r), r) = -\left[P\left(Q^*(r,r)\right) - \left(\underline{c} + \left(N-1\right)r\right)\right]\left[k - 1 + kBR'_{-k}(Q_k^*(r,r))\right].$$

$$\frac{\partial \pi_k^U}{\partial Q_k}(Q_k^*(r,r), r) = r\left(k - 1 + kBR'_{-k}(Q_k^*(r,r))\right).$$

Summing up the two terms leads to

$$\frac{\partial \pi_k}{\partial Q_k}(Q_k^*(r,r), r) = \left[\underline{c} + Nr - P\left(Q^*(r,r)\right)\right]\left[k - 1 + kBR'_{-k}(Q_k^*(r,r))\right]. \tag{24}$$

Equation (24) generalizes (9). In particular, the first bracket term is the same in both equations and does not depend on k while the second bracket term in (24) depends on k. The Stackelberg effect dominates the coordination effect if and only if $k-1+kBR'_{-k}(Q_k^*(r,r))<0$. We have

$$k-1+kBR'_{-k}(Q_k^*(r,r)) \lesseqgtr 0 \text{ iff } \frac{QP''(Q)}{P'(Q)} \gtreqless -\frac{N(N-2k+1)}{N-k}.$$

The important point is that at $r=r^m$, the first bracket term in (24) is zero *regardless of the coalition size*: $\underline{c}+Nr^m-P\left(Q^*(r^m,r^m)\right)=0$. Therefore, we have the following result

Proposition 6 *(public cross-licensing) Consider the two-stage game of public cross-licensing followed by Cournot competition.*

(i) If the Stackelberg effect dominates the coordination effect $\left(\frac{QP''(Q)}{P'(Q)}>-\frac{N(N-2k+1)}{N-k}\right)$, *then* $S(r,N)$ *is k-efficient if and only if* $r\in\{0,r^m\}$.

(ii) If the coordination effect dominates the Stackelberg effect $\left(\frac{QP''(Q)}{P'(Q)}<-\frac{N(N-2k+1)}{N-k}\right)$, *then* $S(r,N)$ *is k-efficient if and only if* $r=r^m$.

Proposition 6 generalizes Proposition 2 to any given size of coalition. In particular, this proposition shows that the monopoly outcome is obtained for any size of coalition.

Proposition 6 also implies that Proposition 3 of private cross-licensing generalizes to coalitions of any size since private cross-licensing is formally a particular case of public cross-licensing in which the Stackelberg effect is absent (i.e., $BR'_{-k}=0$).

Proposition 7 *(private cross-licensing) In the two-stage game of private cross-licensing followed by Cournot competition,* $S(r^m,N)$ *is the unique k-efficient set of symmetric agreements.*

11.3 Bertrand competition with differentiated products

In this section we extend our analysis to a different mode of downstream competition. Consider the same game as before but assume now that firms produce differentiated goods and compete in prices in the second stage of the game. Let p_j denote the price of product $j\in\{1,2,...,N\}$, $D_i(p_1,p_2,...,p_N)$ the demand for product $i\in\{1,2,...,N\}$, and $S_i\equiv\left\{(p_1,p_2,...,p_n)\in R_+^n \mid D_i(p_1,p_2,...,p_n)>0\right\}$. We make the following assumptions for each $i\in\{1,2,...,N\}$:

B1 D_i is twice continuously differentiable on S_i.

B2 (i)$\frac{\partial D_i}{\partial p_i}<0$, (ii)$\frac{\partial D_i}{\partial p_j}>0$ for any $j\neq i$, and (iii) $\sum_{j=1}^n \frac{\partial D_i(p,p,...,p)}{\partial p_j}<0$ over the set S_i.

B3 (i) $\frac{\partial^2 D_i}{\partial p_i \partial p_j} > 0$ for any $j \neq i$ and (ii) $\sum_{j=1}^{n} \frac{\partial^2 D_i}{\partial p_i \partial p_j} < 0$ over the set S_i.

B4 $\sum_{j=1}^{n} \frac{\partial^2 D_i}{\partial p_k \partial p_j} \leq 0$ for any $k \neq i$ on S_i.

Conditions **B1-B3** are quite general, and are commonly invoked for differentiated-good demand systems to guarantee that the standard Bertrand game with linear cost is supermodular and has a unique equilibrium (see e.g., Vives, 1999). They have the following meanings and economic interpretations. For **B2,** part (i) is just the ordinary law of demand; part (ii) says that goods i and j are substitutes; and part (iii) is a dominant diagonal condition for the Jacobian of the demand system, which is required to hold only at equal prices. It says that, along the diagonal, own price effect on demand exceeds the total cross-price effects. For **B3,** part (i) says that demand has strictly increasing differences in own price and any rival's price, and part (ii) says that the Hessian of the demand system has a dominant diagonal.

Moreover, assume that the demand system is symmetric (i.e., products are symmetrically differentiated), and that a unique second-stage equilibrium exists for any first-stage cross-licensing agreements. Assumption **B4** is a technical assumption ensuring the monotonicity of the second-stage equilibrium price with respect to first-stage royalties.

Note that, while Assumption **B3** (i) guarantees that the second-stage pricing game features strategic complementarity when there is no cross-licensing agreement involving the payment of a strictly positive per-unit royalty (i.e., when the second-stage game is a standard Bertrand game), it does not imply that this property holds for all first-stage agreements.

The concept of bilateral efficiency extends in a very natural way to the current setting, both for private and public agreements. As in the cournot case, we focus on the scenario in which all firms license their patents to each other.

11.3.1 Fully cooperative royalty

Let us first characterize the fully cooperative downstream price when all firms license their patents to each other. The latter, which we assume to be unique, is given as follows (by symmetry) by

$$p^m \equiv \arg\max\,(p - \underline{c}) \sum_{i=1}^{N} D_i(p, p, ..., p) = \arg\max\,(p - \underline{c})\, D_1(p, p, ..., p).$$

The corresponding F.O.C. is given by

$$D_1(p^m,...,p^m) + (p^m - \underline{c}) \left[\frac{\partial D_1}{\partial p_1}(p^m,...,p^m) + (N-1)\frac{\partial D_2}{\partial p_1}(p^m,...,p^m) \right] = 0. \tag{25}$$

When all cross-licensing agreements involve the payment of the same royalty r, firm 1's profit function in the second-stage subgame is given by

$$\pi_1(p_1,...,p_N) = (p_1 - (\underline{c} + (N-1)r))\, D_1(p_1,...,p_N) + r\sum_{j=2}^{N} D_j(p_1,...,p_N).$$

Using the symmetry of the problem, the second-stage equilibrium $p^*(r)$ is given by

$$D_1(p^*(r),...,p^*(r)) + \frac{\partial D_1}{\partial p_1}(p^*(r) - (\underline{c} + (N-1)r)) + r(N-1)\frac{\partial D_2}{\partial p_1} = 0, \tag{26}$$

where $\frac{\partial D_1}{\partial p_1} = \frac{\partial D_1}{\partial p_1}(p^*(r),...,p^*(r))$ and similarly for $\frac{\partial D_2}{\partial p_1}$. From (25) and (26), one derives the monopoly royalty satisfying $p^*(r^m) = p^m$:

$$r^m = \frac{\frac{\partial D_2}{\partial p_1}(p^m,...,p^m)}{\frac{\partial D_2}{\partial p_1}(p^m,...,p^m) - \frac{\partial D_1}{\partial p_1}(p^m,...,p^m)}(p^m - \underline{c}).$$

11.3.2 Public agreements

Let us first introduce some useful notations. Let $p_1^*(r, r_{1\to2}, r_{2\to1})$ and $p_2^*(r, r_{1\to2}, r_{2\to1})$ denote the second-stage equilibrium prices when the royalty paid by firm 1 to 2 is $r_{1\to2}$, the royalty paid by firm 2 to 1 is $r_{2\to1}$, and all other cross-licensing agreements involve the payment of the same royalty r. With a slight abuse of notation, we will denote $p^*(r) \equiv p_1^*(r,r,r) = p_2^*(r,r,r)$. Furthermore, considering the special case where all firms license their patents each other at the same royalty $r > 0$, define $R_{-12}(p_1, p_2, r)$ as follows: if $N = 3$, then $R_{-12}(p_1, p_2, r)$ is the best response of firm 3 to firms 1 and 2 setting prices p_1 and p_2; if $N > 3$, then $R_{-12}(p_1, p_2, r)$ is the (assumed unique) number such that for any $i \in \{3,...,N\}$, $R_{-12}(p_1, p_2, r)$ is firm i's best-response to firms 1 and 2 setting prices p_1 and p_2 and all other firms setting a price $R_{-12}(p_1, p_2, r)$.[20] Note that $R_{-12}(p^*(r), p^*(r), r) = p^*(r)$.

We now define the counterpart of the game $G(r, N)$ when firms set prices instead of quantities.

[20] Here we make the implicit assumption that the best-response correspondence is indeed a function.

Definition 6 *For any $r \geq 0$ and $N \geq 3$, the game $G^B(r,N)$ is defined by the following elements:*

Players: There are $N-1$ players: coalition {1,2} and firms $i = 3, ..., N$. Each player has a marginal production cost $\underline{c}$ (excluding royalties). The coalition pays a per-unit royalty r to each of the other players. Each firm $i \in \{3, ..., N\}$ pays a per-unit royalty $2r$ to the coalition and a per-unit royalty r to each firm $j \in \{3, ..., N\} \setminus \{i\}$.

Actions: The coalition {1,2} chooses the price pair (p_1, p_2) within the interval $[p_1^*(r,0,0), +\infty) \times [p_2^*(r,0,0), +\infty)$ and each firm $i \in \{3, ..., N\}$ chooses a price $p_i \geq 0$.

Timing: There are two stages.

- Stage 1: The coalition {1,2} acts as a Stackelberg leader and chooses (p_1, p_2) within $[p_1^*(r,0,0), +\infty) \times [p_2^*(r,0,0), +\infty)$.

- Stage 2: If $N = 3$, then firm 3 chooses its price p_3. If $N \geq 4$, then all firms $i = 3, ..., N$ choose simultaneously (and non-cooperatively) their prices p_i.

The following lemma is the counterpart in a Bertrand setting of Lemma 2.

Lemma 6 *A symmetric set of cross-licensing agreements $S(r,N)$ is bilaterally efficient if and only if choosing $(p^*(r), p^*(r))$ is optimal for the coalition in the Stackelberg game $G^B(r,N)$.*

Proof. Similar to the proof of Lemma 2. ■

We will assume in what follows that the coalition's maximization problem is well-behaved in the following sense: it has a unique solution and when the latter is interior, it is fully characterized by the corresponding first-order condition.

Again we can split the coalition's joint profit into a downstream profit and an upstream profit:

$$\pi_{12}(p_1, p_2, r) = \pi_{12}^D(p_1, p_2, r) + \pi_{12}^U(p_1, p_2, r)$$

where

$$\pi_{12}^D(p_1, p_2, r) \equiv \sum_{i=1,2} D_i(p_1, p_2, R_{-12}(p_1, p_2, r), ..., R_{-12}(p_1, p_2, r))(p_i - (\underline{c} + (N-1)r))$$

and

$$\pi_{12}^U(p_1, p_2, r) \equiv r \begin{bmatrix} \sum_{i=1,2} D_i(p_1, p_2, R_{-12}(p_1, p_2, r), ..., R_{-12}(p_1, p_2, r)) + \\ 2\sum_{j=3}^{N} D_j(p_1, p_2, R_{-12}(p_1, p_2, r), ..., R_{-12}(p_1, p_2, r)) \end{bmatrix}$$

For a symmetric set of cross-licensing agreements $S(r,N)$, with $r>0$, to be bilaterally efficient, it must hold that

$$\frac{\partial \pi_{12}}{\partial p_1}((p^*(r),p^*(r),r)=\frac{\partial \pi_{12}}{\partial p_2}((p^*(r),p^*(r),r)=0.$$

Our assumption that the coalition's problem is well-behaved ensures that the latter necessary first-order conditions are sufficient to characterize a bilaterally efficient positive symmetric royalty.

Let us now investigate the effects of a (local) variation of p_1 (or symmetrically p_2) on both sources of profit.

Considering first the coalition's downstream profit, we have

$$\begin{aligned}\frac{\partial \pi_{12}^D}{\partial p_1}((p^*(r),p^*(r),r) &= \left(\frac{\partial D_1}{\partial p_1}+\frac{\partial D_2}{\partial p_1}\right)(p^*(r)-(\underline{c}+(N-1)r))+D_1(p^*(r),...,p^*(r))+\\ &\quad (p^*(r)-(\underline{c}+(N-1)r))\left(\sum_{j=3}^{N}\frac{\partial\left(D_1+D_2\right)}{\partial p_j}\right)\frac{\partial R_{-12}}{\partial p_1}((p^*(r),p^*(r),r)\,.\end{aligned}$$

From (26) it follows that:

$$\begin{aligned}\frac{\partial \pi_{12}^D}{\partial p_1}((p^*(r),p^*(r),r) &= \frac{\partial D_2}{\partial p_1}(p^*(r)-(\underline{c}+2(N-1)r))+\\ &\quad (p^*(r)-(\underline{c}+(N-1)r))\left(\sum_{j=3}^{N}\frac{\partial\left(D_1+D_2\right)}{\partial p_j}\right)\frac{\partial R_{-12}}{\partial p_1}((p^*(r),p^*(r),r)\,.\end{aligned}$$

The first term, i.e., $\frac{\partial D_2}{\partial p_1}(p^*(r)-(\underline{c}+2(N-1)r))$, captures the *coordination effect* in this setting. In contrast to the Cournot setting, the sign of this effect is ambiguous. Note however that it is positive in the special case of $r=0$, which captures the usual collusive incentive to increase prices in the standard Bertrand model. The remaining term captures the Stackelberg effect. Its sign is solely determined by the sign of $\frac{\partial R_{-12}}{\partial p_1}((p^*(r),p^*(r),r)$. If the second stage pricing game features strategic complementarity[21], then $\frac{\partial R_{-12}}{\partial p_1}((p^*(r),p^*(r),r)>0$ and hence the sign of the Stackelberg effect is positive (as in the standard Bertrand game).

[21] We assumed this property to hold in the case $r=0$ (corresponding to the standard Bertrand game) but we do not impose it for $r>0$.

Considering now the coalition's upstream profit, we have:

$$\begin{aligned}\frac{\partial\pi_{12}^U}{\partial p_1}((p^*(r),p^*(r),r) &= r\left(\frac{\partial D_1}{\partial p_1}+\frac{\partial D_2}{\partial p_1}\right)+2r\sum_{j=3}^N\frac{\partial D_j}{\partial p_1}+r\left(\sum_{j=3}^N\frac{\partial\left(D_1+D_2\right)}{\partial p_j}\right)\frac{\partial R_{-12}}{\partial p_1} \\ &\quad +2r\sum_{k=3}^N\left(\sum_{j=3}^N\frac{\partial D_j}{\partial p_k}\right)\frac{\partial R_{-12}}{\partial p_1}((p^*(r),p^*(r),r)\end{aligned}$$

The first term, which is negative (by **B2** (ii) and (iii)), captures a *direct royalty-saving effect*. The second term, which is positive, can be interpreted as the *direct* effect of a price increase on licensing revenues. The third term, which has the same sign as $\frac{\partial R_{-12}}{\partial p_1}((p^*(r),p^*(r),r)$, captures an *indirect royalty-saving effect*. Finally, the fourth term, which has the same sign as $-\frac{\partial R_{-12}}{\partial p_1}((p^*(r),p^*(r),r)$, can be interpreted as the *indirect* effect of a price increase on licensing revenues. Note that under Cournot setting, we have only the first and the last effects.

Before stating the main result of this section, let us show why $r=0$ cannot be a bilaterally efficient symmetric royalty. In this special case, the upstream profits are zero and

$$\begin{aligned}\frac{\partial\pi_{12}}{\partial p_1}((p^*(0),p^*(0),0) &= \frac{\partial\pi_{12}^D}{\partial p_1}((p^*(0),p^*(0),0) \\ &= \underbrace{(p^*(0)-\underline{c})}_{>0}\left[\underbrace{\frac{\partial D_2}{\partial p_1}}_{>0}+\underbrace{\left(\sum_{j=3}^N\frac{\partial\left(D_1+D_2\right)}{\partial p_j}\right)}_{>0\text{ by }\mathbf{B2}\text{ (ii)}}\underbrace{\frac{\partial R_{-12}}{\partial p_1}((p^*(0),p^*(0),0)}_{>0}\right]\end{aligned}$$

Hence $\frac{\partial\pi_{12}}{\partial p_1}((p^*(0),p^*(0),0)>0$, which shows that it is not optimal for the coalition to set $(p_1,p_2)=(p^*(0),p^*(0))$ when $r=0$. Lemma 6 then implies that $S(0,N)$ is not a set of bilaterally efficient agreements. This contrasts with the case of Cournot competition. The reason for this is that, when $r=0$, prices are strategic complements. Therefore, the Stackelberg effect has a positive sign and thus reinforces the coordination effect (while it mitigates it under Cournot competition).

The following proposition characterizes the bilaterally efficient symmetric public agreements.

Proposition 8 *(public bilateral cross-licensing under Bertrand) In the two-stage game of public cross-licensing followed by Bertrand competition, $S(r,N)$ is bilaterally efficient* *if and only if* $r = r^m$.

Proof. We have already shown that $r = 0$ is not a bilaterally efficient symmetric royalty. We can therefore focus on interior royalties $r > 0$. The F.O.C. characterizing $p^*(r)$ is given by

$$D_1(p^*(r), ..., p^*(r)) + \frac{\partial D_1}{\partial p_1}.\left(p^*(r) - (\underline{c} + (N-1)r)\right) + r(N-1)\frac{\partial D_2}{\partial p_1} = 0. \tag{27}$$

Assume that $S(r, N)$ is bilaterally efficient. Then

$$\frac{\partial \pi_{12}^D}{\partial p_1}((p^*(r), p^*(r), r) + \frac{\partial \pi_{12}^U}{\partial p_1}((p^*(r), p^*(r), r) = 0,$$

which is the same as

$$\begin{aligned}
&\left(\tfrac{\partial D_1}{\partial p_1} + \tfrac{\partial D_2}{\partial p_1}\right).\left(p^*(r) - (\underline{c} + (N-1)r)\right) + D_1(p^*(r), ..., p^*(r)) + r\left(\tfrac{\partial D_1}{\partial p_1} + \tfrac{\partial D_2}{\partial p_1}\right) + \\
&2r\sum_{j=3}^{N} \tfrac{\partial D_j}{\partial p_1} + \left(p^*(r) - (\underline{c} + (N-1)r)\right)\left(\sum_{j=3}^{N} \tfrac{\partial (D_1 + D_2)}{\partial p_j}\right) \tfrac{\partial R_{-12}}{\partial p_1}((p^*(r), p^*(r), r) + \\
&r\left(\sum_{j=3}^{N} \tfrac{\partial (D_1 + D_2)}{\partial p_j}\right) \tfrac{\partial R_{-12}}{\partial p_1} + 2r\sum_{k=3}^{N}\left(\sum_{j=3}^{N} \tfrac{\partial D_j}{\partial p_k}\right) \tfrac{\partial R_{-12}}{\partial p_1}((p^*(r), p^*(r), r) = 0.
\end{aligned}$$

Using the symmetry of the problem the latter can be rewritten as

$$\begin{aligned}
&D_1(p^*(r), ..., p^*(r)) + \tfrac{\partial D_1}{\partial p_1}\left(p^*(r) - (\underline{c} + (N-2)r)\right) + \tfrac{\partial D_2}{\partial p_1}\left(p^*(r) - (\underline{c} - (N-2)r)\right) \\
&+2(N-2)\tfrac{\partial R_{-12}}{\partial p_1}((p^*(r), p^*(r), r)\left[r\tfrac{\partial D_1}{\partial p_1} + \left(p^*(r) - (\underline{c} + r)\right)\tfrac{\partial D_2}{\partial p_1}\right] = 0.
\end{aligned}$$

Combining the latter and (27) yields

$$\left(1 + 2(N-2)\frac{\partial R_{-12}}{\partial p_1}((p^*(r), p^*(r), r)\right)\left[r\frac{\partial D_1}{\partial p_1} + \left(p^*(r) - (\underline{c} + r)\right)\frac{\partial D_2}{\partial p_1}\right] = 0,$$

which implies

$$r\frac{\partial D_1}{\partial p_1} + \left(p^*(r) - (\underline{c} + r)\right)\frac{\partial D_2}{\partial p_1} = 0.$$

Adding the latter multiplied by $(N-1)$ to (27) yields

$$D_1(p^*(r), ..., p^*(r)) + \frac{\partial D_1}{\partial p_1}.\left(p^*(r) - \underline{c}\right) + (N-1)\frac{\partial D_2}{\partial p_1}\left(p^*(r) - \underline{c}\right) = 0. \tag{28}$$

The latter condition characterizes uniquely p^m. Therefore $p^*(r) = p^m$.

Thus, a necessary condition for $S(r, N)$ to be bilaterally efficient is that $p^*(r) = p^m$. This condition can be easily shown to be sufficient by checking that

$$\frac{\partial \pi_{12}^D}{\partial p_1}(p^m, p^m, r^m) + \frac{\partial \pi_{12}^U}{\partial p_1}(p^m, p^m, r^m) = 0$$

is satisfied (this is readily obtained by following the previous steps in reverse order) and using the fact the coalition's maximization problem in the game $G^B(r, N)$ is well behaved. We can therefore conclude that $S(r, N)$ is bilaterally efficient if and only if $p^*(r) = p^m$

To complete the proof it remains to show that $p^*(r) = p^m$ if and only if $r = r^m$. Since $p^*(r^m) = p^m$, it is sufficient to establish the strict monotonicity of $p^*(.)$ to prove the latter equivalence result. Differentiating (26) with respect to r, we get

$$\frac{dp^*}{dr} = \frac{(N-1)\left[\frac{\partial D_1}{\partial p_1} - \frac{\partial D_1}{\partial p_2}\right]}{\sum_{j=1}^{N}\frac{\partial D_1}{\partial p_j} + \sum_{j=1}^{N}\frac{\partial^2 D_1}{\partial p_1 \partial p_j}\left(p^*(r) - (\underline{c} + (N-1)r)\right) + \frac{\partial D_1}{\partial p_1} + r(N-1)\sum_{j=1}^{N}\frac{\partial^2 D_2}{\partial p_1 \partial p_j}}$$

The numerator is strictly negative by **B2** (i) and (ii), and the denominator is strictly negative by **B2** (iii), **B3** (ii), **B2** (i) and **B4.** We can therefore state that $p^*(r)$ is strictly increasing in r, which completes the proof. ■

11.3.3 Private agreements

As in the Cournot competition case, the analysis of private agreements can be derived from that of public agreements by putting aside all the indirect (or strategic) effects. More precisely, the Stackelberg effect, the indirect royalty-saving effect and the indirect effect of a price increase on licensing revenues are absent under private cross-licensing. The following proposition characterizes the bilaterally efficient symmetric private agreements.

Proposition 9 *(private bilateral cross-licensing under Bertrand) In the two-stage game of private cross-licensing followed by Bertrand competition, $S(r, N)$ is bilaterally efficient if and only if $r = r^m$.*

Proof. It is easy to show that $r = 0$ is not a bilaterally efficient symmetric royalty: as in the Cournot case with private cross-licensing, when $r = 0$, the only effect at work is the coordination effect (which gives incentives to increase royalties). We can therefore focus on $r > 0$.

We follow the same steps as in the proof of Proposition 8. More specifically, we show that $S(r,N)$ is bilaterally efficient if and only if $p^*(r)$ satisfies the F.O.C. uniquely defining p^m and then conclude that $S(r,N)$ is bilaterally efficient if and only if $r = r^m$.

In the case of private cross-licensing, the condition $\frac{\partial \pi_{12}^D}{\partial p_1}((p^*(r), p^*(r), r) + \frac{\partial \pi_{12}^D}{\partial p_1}((p^*(r), p^*(r), r) = 0$ writes (using the symmetry of the problem)

$$D_1(p^*(r), ..., p^*(r)) + \left(\frac{\partial D_1}{\partial p_1} + \frac{\partial D_2}{\partial p_1}\right)(p^*(r) - (\underline{c} + (N-2)r)) + 2r(N-2)\frac{\partial D_2}{\partial p_1} = 0,$$

which can be rewritten as

$$D_1(p^*(r), ..., p^*(r)) + \frac{\partial D_1}{\partial p_1}(p^*(r) - (\underline{c} + (N-2)r)) + \frac{\partial D_2}{\partial p_1}(p^*(r) - (\underline{c} - (N-2)r)) = 0.$$

Multiplying the latter by $(N-1)$ and substracting (26) multiplied by $(N-2)$ yields (after some straightforward algebraic manipulations):

$$D_1(p^*(r), ..., p^*(r)) + \frac{\partial D_1}{\partial p_1}.(p^*(r) - \underline{c}) + (N-1)\frac{\partial D_2}{\partial p_1}(p^*(r) - \underline{c}) = 0,$$

which is the equation defining p^m.

Thus, $S(r,N)$ is bilaterally efficient if and only if $p^*(r) = p^m = p^*(r^m)$, which is the same as $r = r^m$ because $p^*(.)$ is strictly monotonic (see the proof of Proposition 8). ■