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Abstract

This paper addresses issues related to industry heterogeneity, default clustering and parameter uncertainty of capital requirements in US retail loan portfolios. Using a multi-factor model of credit risk, we show that the Basel II capital requirements overstate the riskiness of small businesses. Retail exposures are a much safer investment than the regulator would suggest. We find that sensitivity to the common risk factors is low and that small business risk is predominantly a reflection of idiosyncratic risk. Our results show that only 0.00-3.39% of the asset variability is explained by economy-wide risk factors. The remaining 96.61%-100.00% of small business risk is due to changes in the firm-specific characteristics. Moreover, both expected and unexpected losses are time dependent. Their shifts over the course of financial crisis cause uncertainty in the provisions level and capital requirements. Importantly, our estimates of asset correlations are significantly lower than any available estimates for corporate firms. Our results are based on a new, representative dataset of US retail businesses from 2005 to 2011 and give fundamental insights into the US economy.

JEL Classification: G17, L14 and L25
Keywords: capital requirements

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THE CONTINUOUSLY EVOLVING ECONOMIC CONDITIONS in which firms operate make it hard to expect that the loss distribution in a commercial and industrial loan portfolio remains constant over the years. But typically the expected losses covered by loan pricing and provisioning are considered not to have uncertainty. This is in turn associated with the unexpected losses for which financial institutions hold regulatory capital. However, when new information becomes available the expectation about the losses shifts. We build on the existing literature to include aspects of loan portfolio diversification, dynamics of default risk and capital requirements. Our interest lies in an empirical study of credit risk in US retail loan portfolios that consist of loans granted to small businesses.

The principal aim of this paper is to provide empirical insights into risk management of US retail loan portfolios. Our contribution to the existing literature is threefold. First, we focus our attention on privately held firms which, although they are very central and important to the US economy, remain an opaque research area due to lack of financial statements and market trading. As small businesses represent an engine of economic growth and job creation, our findings give fundamental insights into risk sources and dynamics of the US economy. This study employs a unique panel of loan exposures to US private firms from 2005 to 2011, which captures, among other things, the evolution of small business risk during the turmoil of 2007 to 2009. Secondly, this paper discusses whether the regulatory formula accurately captures the underlying small business credit risk, or whether it distorts the risk management practices in financial institutions which hold such portfolios. We confirm the existence of capital allocation inefficiencies in US retail loan portfolios arising from the Basel II formula for asset correlation.[1] Thirdly, we overcome the limited information availability in small businesses by deriving a simple yet effective estimation technique of joint default risks

[1]The Basel II asset correlation formula for retail exposures is used both for the foundation and the advanced IRB approach, in which certain banks can develop their own credit risk models for estimation of probability of default, exposure at default and loss given default, respectively. This formula was not subject to any change in the Basel 2.5 nor in Basel III. For further details please refer to Basel Committee on Banking Supervision (2006), Basel Committee on Banking Supervision (2009) and Basel Committee on Banking Supervision (2011).

in retail loan portfolios. Importantly for corporate debt portfolios, our estimation technique yields results which are coherent with Basel II capital requirements. Thus, the results for retail loan portfolios can be positioned next to the regulatory ones.

Small businesses in the US are not as well-researched as their larger counterparts. Although they contribute about 50% to US GDP and employ about 50% of the private workforce, the available financial information is rather limited. This lack of information stems from the absence of publicly available financial statements, as well as absence of market trading. Also, until recently most of the information available about this significant segment of the US economy was based on estimates rather than hard data. While some efforts were undertaken to shed light in the area of default dependency in the US retail portfolios, these efforts were limited to aggregate measures of small business credit risk (Lee, Wang, and Zhang (2009)) or to loans originated under the US Small Business Administration (SBA) guarantee program (Glennon and Nigro (2005)). Unlike these earlier studies, our study performs an empirical analysis on a new and comprehensive dataset on defaults of US private firms, covering a period of seven years from 2005 to 2011. Our panel contains quarterly observations on small and medium sized firms across all credit ratings and industries in the US, with an average of nearly 240,000 obligors per time period. It provides a unique opportunity to analyze credit risk in retail loans before, during and after the crisis. Note that several non-US studies of small businesses are available: Carling, Rönnegård, and Roszbach (2004) who analyze Swedish retail loan market and Düllmann and Scheule (2003) with their study on German small and large firms. Unlike these studies, we are able to pay attention to the evolution of portfolio risk as well as to changes in expected and unexpected losses.

Credit risk in small businesses is of particular interest to US financial institutions. As the FDIC reports, US commercial banks' exposure to loans granted to small businesses is significant and amounts to 24.90% of the commercial and industrial loans (June, 2011). The large size of the retail loan portfolios and the limited information available on borrower credit worthiness make small businesses objects of particular relevance for Basel II capital requirements. A discussion of the Basel II capital requirement can also be found in Botha

and van Vuuren (2010) and Lopez (2004). The former study asks how asset correlations derived from loss data relate to Basel II and its corresponding capital charges. The latter study reports empirical asset correlations for US, Japanese, and European corporate and private firms. Both studies, however, do not pay attention to possible parameter uncertainty in the asset correlation and capital requirement estimates. In the context of our study such uncertainty provides a basis for a prudential approach to capital requirements.

An important aim of our study is to verify the validity of the Basel II minimum capital formulas for the US retail loan portfolios. The general setting for our analysis is a multi-factor model. This choice allows us to compare our estimates with the outcomes of the Basel II single-factor model and at the same time allows the economy to have a more advanced structure. In such a multi-factor economy risks can be industry- or firm-size-specific, which is not possible in the context of a single global risk factor in the Basel II model. It becomes unnecessary to impose a strong assumption of a single-factor economy. Our multi-factor setting incorporates possible heterogeneity of obligors and risk factors as testable hypotheses. The studies by Dietsch and Petey (2002, 2004) also belong to the sizable family of single-factor models. The former study explores capital requirements in the context of probit and gamma models, as well as deviations from the Basel II Accord, but focuses on French small businesses. The latter study focuses on the nature of asset correlation in small businesses in the French and German markets. Dietsch and Petey (2004) employs a single-factor model. This was extended to multiple common risk factors in 2009. Their generalized linear mixed model assumes that financial institutions possess a considerable set of information on their borrowers, which is typically not available for US small business.

As Jorion and Zhang (2009) observe, calibration of portfolio credit risk models from single-factor family is notoriously difficult. However, we propose a simple estimation technique in which we demonstrate that the observed default frequencies per homogeneous obligor class are sufficient to estimate the joint default risk in a retail loan portfolio. To model and estimate the default dependencies, we begin with the Vasicek (1987) firm value model, elaborated in Bharath and Shumway (2008) and Gordy (2003), who shows its applicability to

banks' capital requirements. This type of models finds its roots in the work of Merton (1974) and is applied in practice by Credit Metrics (Gupton, Finger, and Bhatia (1997)) and KMV (Crosbie and Bohn (2003)). The advantage of the estimator proposed lies in the minimal information required to assess the joint default risk in a retail loan portfolio. In fact, our model is of an *incomplete information* type, as described by Giesecke (2006), in which the investors observe a default barrier and obtain noisy reports on a firm's asset value. And although there exist more sophisticated empirical models of joint default risk, including Duffie, Saita, and Wang (2007), McNeil and Wendin (2007), Duffie et al. (2009), Berndt, Ritchken, and Sun (2010) and Azizpour, Giesecke, and Schwenkler (2012), limited data availability precludes their use for informationally opaque small businesses loans.

In our empirical analysis we address some fundamental questions about how common risk factors are distributed across the economy and which firm characteristics are relevant for diversification. We first select the dimensions to partition obligors into homogenous classes. Here industries and credit ratings play important role in portfolio partitioning. In general, we find that sensitivity to obligor-class-specific common risk factors remains low and varies between 0.00-18.41% with only 0.00-3.39% of the asset variability explained by the common risk factors. The remaining 96.61-100.00% of small business risk is due to changes in the firm-specific characteristics. During the whole period analyzed the implied asset correlation averages around 0.41%. Regardless of the small business' riskiness, industry or firm size our estimates are significantly lower than any available estimates for corporate firms. Our estimates imply that a single-factor model, as assumed by Basel II capital calculations, is too simplistic to summarize the entire structure of the US economy. In fact the US economy shows more complexity and has more relevant sources of risk than a nation-wide single-factor.

Next, we analyze how the riskiness of US small business has evolved over the course of the financial crisis. Two important elements of default risk are present in a portfolio of loans: expected and unexpected losses. We find that the firms which withstood the crisis showed less sensitivity to economic conditions, a substantial reliance on the firm characteristics and lower default clustering from macroeconomic exposures. The importance of firm-specific risk

as a source of default risk was also discussed in Jarrow and Yu (2001), who link it to the individual business connections of a firm.

Lastly, we compare our results with Basel II capital requirement calculations, which imply a substantially larger exposure of retail loan portfolios to common risk factors. What we observe is a sizeable overstatement of retail debt risk as perceived by the Basel II compared to our method. In our view this difference stems from the overly simplistic way in which Basel II models and estimates the asset correlations in retail loan portfolios. In fact, our results show that, from a credit risk perspective, retail exposures are safer investments than the regulator would suggest. We summarize the empirical results by discussing the parameter uncertainty of our estimates. A prudential adjustment of the capital requirements can be achieved by accounting for parameter uncertainty, but also by allowing for fat-tail distributed risk factors. Such adjustment aims to provide a better understanding of the results presented in this study for risk management purposes.

The paper is organized as follows. The next section introduces a probabilistic model of joint default risk and its proposed estimators. Section III outlines the D&B dataset of small US businesses. The empirical results for the pre-, during and post-crisis phases are presented in section IV, which also summarizes the implications of our findings for risk management and capital requirements in financial institutions. Section V concludes.

II Methodology

We generalize the existing asymptotic single-factor model to a multi-factor one which includes aspects of diversification and segmentation. The model used departs from the Basel II asymptotic single risk factor in that we allow flexibility in the number of risk factors in the economy (i.e. a common factor per obligor class) as opposed to a single global risk factor. This general framework finds empirical support in the next section. Industry-related heterogeneity and multiple common risk factors in the US economy find support in the data. Our model is equivalent to the regulatory one if we observe perfectly correlated common risk factors and yields estimates consistent with the regulatory ones.

Consider a portfolio of N small obligors which are ordered into homogeneous obligor classes $k \in \{1, \ldots, K\}$. This set of homogeneous obligor classes is categorized with respect to firm's credit worthiness, industry, etc. Let latent variable $A_{i,t}$ denote the asset value of obligor i in obligor class k at time t which without loss of generality is standardized and centered around zero. The asset value is driven by two independent components: a common risk factor $x_{k,t}$ per obligor class k and an idiosyncratic risk factor $\epsilon_{i,t}$ per obligor i:

$$A_{i,t} = w_k x_{k,t} + \sqrt{1 - w_k^2}\epsilon_{i,t} \qquad i \in k \qquad t = 1, \ldots, T \tag{1}$$

where $\mathrm{E}[x_{k,t}\epsilon_{i,t}] = 0$. The class-specific common risk factor $x_{k,t}$ represents changes in the economic conditions common to all obligors in obligor class k and the idiosyncratic risk factor $\epsilon_{i,t}$ stands for firm-specific risk attributed to each obligor. The weight w_k of the common risk factor measures the sensitivity of obligor i to its economic conditions. Given that any two firms classified into the same obligor class are sufficiently homogeneous, it is customary to assume that the class-specific factor has an identical effect on these firms' asset values (McNeil and Wendin (2007), Gordy (2000)). It follows that the weight w_k is the same for obligors in one obligor class. Credit portfolio concentration risk depends heavily upon the magnitude with which obligors' asset values respond to the common risk factor. The higher the firm's sensitivity to the common risk factor, the more responsive the asset value to unanticipated changes in the economic environment. In fact, the default dependency in a loan portfolio arises from co-movements in asset values that is induced by those common risk factors' correlation across obligor classes, with a correlation matrix Ω, where:

$$\Omega_{kl} = \mathrm{Corr}[x_k, x_l] \tag{2}$$

Empirically, time variation in the sensitivity w_k can be achieved by applying a moving-time-window technique. By shifting the time windows for pre-, during- and post-crisis phases we are able to investigate changes in the sensitivity w_k over time.

At the beginning of each period t, the cohort consists of $N_{k,t}$ obligors in non-defaulted

state. However, this state of obligor i can be subject to change and it depends on a relative distance of obligor's asset value (1) to a threshold that defines the default event. We assume that $x_{k,t}$ and $\epsilon_{i,t}$, and hence the asset value, are standard normally distributed. The default threshold is equal to $\Phi^{-1}(\bar{p}_k)$ where $\Phi^{-1}(\cdot)$ denotes the standard normal CDF and $\bar{p}_k$ stands for the unconditional probability of default in obligor class k. Our model shares the definition of the default event with the structural models that date back to work of Merton (1974) and Black and Cox (1976). In this framework an obligor i defaults if:

$$w_k x_{k,t+1} + \sqrt{1 - w_k^2}\epsilon_{i,t+1} < \Phi^{-1}(\bar{p}_k) \quad \Leftrightarrow \quad D_{i,t+1} = 1 \tag{3}$$

where $D_{i,t+1}$ denotes a default indicator of firm i. By definition $D_{i,t+1}$ takes value 1 if firm i defaults at time $t+1$ and 0 otherwise. Because we are interested in joint default occurrences, i.e. obligors simultaneously going into default, for the default assessment at time $t+1$ we exclude contracts that are in default at time t. From (3) it follows that if the economic conditions $x_{k,t}$ are good, a firm defaults only if the realization of the idiosyncratic risk factor $\epsilon_{i,t}$ is worse. Also, the asset correlation between two obligors i and j is:

$$\rho_{ij} = \mathrm{Corr}[A_{i,t}, A_{j,t}] = w_k w_l \Omega_{kl} \qquad i \in k, j \in l \tag{4}$$

From the above relationship one can see that, keeping Ω constant, with an increase in w the obligors become more strongly correlated, but with a decrease in the sensitivity parameter it is the idiosyncratic risk that dominates.

In this setup we derive the theoretical moment for the joint probability of default to be equal to the probability of two obligors being simultaneously below the default threshold (for a derivation, please refer to Appendix A). Hence, the joint probability of default of obligors

i and j follows as:

$$\begin{aligned} p_{kl} &\equiv P[D_{i,t+1} = 1, D_{j,t+1} = 1] \\ &= \int_{-\infty}^{\Phi^{-1}(\bar{p}_l)} \Phi\left(\frac{\Phi^{-1}(\bar{p}_k) - \Omega_{kl} w_k w_l y}{\sqrt{1 - \Omega_{kl}^2 w_k^2 w_l^2}}\right) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} y^2\right) dy \end{aligned} \tag{5}$$

where obligor i belongs to obligor class k and obligor j belongs to obligor class l. The empirical analysis focuses on estimation of the parameter set $\theta \equiv (w, \Omega)$ where $w \equiv (w_1, \ldots, w_K)$ denotes the vector of common risk factors sensitivities and Ω represents the matrix of common risk factors correlations. The proposed method of moments estimation for credit risk is compatible with a statistical analysis of obligors clustered into obligor classes.

Equation (5) is at the center of the estimation procedure. The left hand side of the equation gives the theoretical moment for the joint probability of default that is caused by the aggregate behavior of obligors in an obligor class. Next, we minimize the distance between this theoretical moment and its sample counterpart. Denote an observed default frequency in obligor class k at time t by $ODF_{k,t}$. It follows that the observed default frequency is equal to a ratio of all default events in obligor class k to the total number of obligors in this class $ODF_{k,t} = \sum_{i \in k}^{N_{k,t}} D_{i,t+1}/N_{k,t}$. It can be shown (see Appendix A) that for two obligor classes k and l, the sample joint probability of default corresponds to a historical average of products of their observed default frequencies. As a result, the following relationship holds for the joint probability of default for two obligors i and j in obligor classes k and l respectively:

$$\hat{p}_{kl} = \frac{1}{T} \sum_{t=1}^{T} (ODF_{k,t} \cdot ODF_{l,t}) \tag{6}$$

We refer to the expression in (6) as the *between obligor class* sample moment since it depicts the joint probability of default for obligors in two different obligor classes. By analogy, the *within obligor class* sample moment for the joint probability of default for two

obligors in the same obligor class k follows as:

$$\hat{p}_{kk} = \frac{1}{T}\sum_{t=1}^{T}(ODF_{k,t} \cdot ODF_{k,t}) \tag{7}$$

Our estimate of w follows from method of moments applied to equation (5) using the sample moments in (7). The vector w that is obtained is used in the next step to estimate Ω from equations (5) and (6).[2] Importantly, only minimal information on the obligor class level is required to estimate the relevant parameter vector θ, namely the observed default frequencies per obligor class. Moreover, this information is usually readily available within a loan-granting financial institution, which facilitates easy application of the approach proposed by small business finance providers. The advantage of the multi-factor model over a single-factor one is more realistic modeling of portfolio risk. Thus, by estimating Ω from the *between obligor class* moments one obtains a more comprehensive view of the portfolio risk, its diversification possibilities and a more informed segmentation of exposures. Interestingly, the single-factor model is estimated based solely on sub-portfolios composed of homogeneous obligors, which is equivalent to estimation of the *within obligor class* moments (see Gordy (2000), Dietsch and Petey (2002, 2004)).

Furthermore, this multi-factor model collapses into a single-factor in case of perfectly correlated common risk factors. It follows that the common risk factor x is one-dimensional (as assumed in Gordy (2003)). In other words, the perfect correlation imposes a single common risk factor as the sole external source of default correlation. The above property can be used to test the single-factor assumption and homogeneity of obligors with obligor class.

III Data

In this section we outline the main characteristics of a unique dataset provided by Dun & Bradstreet. The dataset contains about 500,000 US small firms that were active between

[2]Essentially it is a numerical optimization which minimizes the sum of squared errors between the population and sample moments over a domain θ.

2005 and 2011 at different points in time. Our analysis is free of selection bias and includes on average nearly 240,000 firms across all the credit ratings, industries and firm sizes in the US, which represent a cross section of the US economy.

The D&B dataset contains rich quarterly information on firms' actual borrowing and payment behavior, public detrimental information such as county court judgments, legal pre-failure events (receivership, bankruptcy, etc.), credit ratings but also legal form, age, industry and firm's location. The sample covers about $19 billion of small business financial activity annually, providing a representative outlook on the economy. The average credit outstanding per firm is $31,860.33 with 24.49% of the exposures below $1,000 and 99.75% below $1 million. Informational coverage of the US economy is substantial with about 6,000 major firms (both financial and non-financial) reporting to D&B. It includes loan and trade records stored by financial institutions and vendors. We adopt the Basel Accords view in which a default takes place if a payment is 90 days past due or unlikely to be paid. Thus, at the end of each quarter and for each active non-defaulted ïňĄrm, we observe the firm's characteristics and whether within one year the firm has payments 90 days past due or written off, goes bankrupt or is downgraded to credit score 0 (default).

A review of the business size reveals that firms represent all the major US industries with a high concentration in services (40.78%), retail trade (14.82%) and construction (13.61%). Aside from the non-classified firms, it is manufacturing that experiences the highest default rate of 17.48%, which is also illustrated in Figure 1. In the context of recession this high default rate is explained by the fact that consumers tend to abstain from new purchases and to repair the equipment they already own (consistent with lower default rate in services). In this sample 56.59% of firms have fewer than 5 employees and 98.29% have fewer than 100 employees. Surprisingly, the very small firms seem to perform on average better than small or medium-sized firms. Table I reports that the annual default rate increases with firm size from 9.67% for very small firms (up to 5 employees) to 35.98% for those which employ more than 100 people. Similar result can be found in Glennon and Nigro (2005) who also report higher default rates for larger firms. The observed regularity can be due to higher

cash holdings in very small businesses which create a buffer for financial distress (Steijvers and Niskanen (2009)).

[Table 1 about here.]

[Figure 1 about here.]

With the vast majority of records containing information on privately held firms (99.97%) this study sheds light on the private small business economy. The firms analyzed are located in all major US regions with a higher concentration in California in the West, Texas in the Southwest and New York in the Northeast, representing 12.09%, 6.74% and 6.56% of the population, respectively.

The homogeneous obligor classes are differentiated with respect to three criteria: credit rating, industry and firm size. For purpose of our study we adopt the D&B credit evaluation points (CPOINTS) as an indicator of firm's credit worthiness. On this basis we construct the credit ratings as percentiles of the whole distribution such that the credit rating "1" contains the 10% most credit worthy obligors and credit rating "10" the 10% least credit worthy obligors. The accuracy ratio of the credit rating is 19.20%. The discriminatory power of this rating is highly significant, which is confirmed by Kolmogorov-Smirnov and Mann-Whitney U tests (not shown).

We categorize the firms into sets of homogeneous obligor classes based on their credit ratings and ten major SIC industry divisions. But in the absence of industry classification, financial institutions may turn to other available information to classify their exposures. Hence we conduct the analysis for credit ratings and seven firm size classes which are differentiated with respect to the number of employees. Those seven firm size classes include very small firms with less than 5 employees, or those which employ 6-10, 11-20, 21-30, 31-50, 51-100 or more than 100 individuals.

IV Results

In this section the estimator we propose is applied to the dataset described in the previous section. Particular interest is paid to the role of industry and firm size in shaping the risk in loan portfolios. We test the validity of the single-factor assumption in retail portfolios. Also, we discuss aspects of expected and unexpected losses, default clustering and compare our results to the Basel II minimum capital requirements.

The empirical analysis begins by identifying the relevant risk factors in loan portfolios. We look at conventional factors such as credit rating and firm size (Basel II) and some unconventional ones such as industry. Next, firms' reactions to the factor determine whether it is a relevant risk factor. If firms react in a homogeneous way to the factor, it does not have significant additional risk content. If, however, the reaction is heterogeneous, we uncover a relevant additional risk factor. In the multi-factor setting homogeneity is defined as a situation in which firms from same obligor class have equal sensitivity parameters w even if further segmented into smaller subclasses.

In practice credit ratings often serve to identify the homogenous obligor classes. It is a procedure which separates the firms according to their distribution with respect to risk. Hence for purpose of the study we select the credit rating as the primary dimension of the analysis, and subsequently subdivide into industry or firm-size categories. Dietsch and Petey (2009) and McNeil and Wendin (2007) underline the relevance of other sources of heterogeneity than credit rating such as industry. Their claim is that a specialization in financing to specific industry may question the capital requirements based solely on credit rating and hence should include industry characteristics as well. The results of their study are based on corporate exposures. In small businesses we find some support for this hypothesis, which can be seen in Table II. The table reports point estimates of the sensitivities to the common risk factors w_k for firms classified with respect to both credit rating and industry, but also estimates for credit ratings only and for industries only. Indeed, the sensitivities to common risk factors per credit rating are not affected by the industry related heterogeneity. The inverse holds true as well. Thus, all credit ratings in one industry react in a similar

fashion to a change in common risk factors.

[Table 2 about here.]

[Table 3 about here.]

On the other hand, the industry related heterogeneity in credit ratings is revealed as different common risk factors per obligor class. Table IV presents Jennrich (1970) tests for equality of correlation matrices where the reference matrix is equal to a matrix of ones hence a perfect correlation matrix. Panel A shows significant evidence of industry heterogeneity in the common risk factors. Only credit rating 4 and 9 remain robust to the industry related heterogeneity. This is good news for portfolio risk management as the industry-related heterogeneity gives rise to diversification benefits that stem from lower correlations between the common risk factors. Consider for a moment the whole economy categorized into industries. Each of those industries consists of firms from various credit ratings. Panel C in Table IV shows this credit rating has a significant source of heterogeneity within a given industry. Interestingly, only the finance industry remains homogeneous. This homogeneity is present notwithstanding the very distinctive risk factors which influence firms in the finance industry and regardless of the credit rating, i.e. money provision, regulation or credit cycle.

[Table 4 about here.]

In addition to industry, heterogeneity within credit rating arises typically from firm size. Intuitively micro-firms which are great in number should operate in an almost perfectly competitive environment while the larger firms could benefit from market power. Table III shows the sensitivity to common risk factors for obligor classes separated with respect to credit rating and firm size. With respect to these sensitivity parameters we find strong evidence of firm-size related homogeneity in credit ratings. On the other hand, if the segmentation would have been implemented only according to firm size, the assumption of class homogeneity would have been violated. Thus, the credit rating contains significant information about the sensitivities to common risk factors, but not the firm size. Panels B and

panel D in Table IV, however, find only moderate support for the homogeneity across the credit rating and firm size common risk factors. From a risk management perspective this allows for some diversification benefits.

This redundancy of firm size factor favors credit rating and industry as relevant risk factors. Moreover, our results do not support the conventional wisdom of Basel II according to which firm size determines the level of risk. Instead, we find that firms react to the size factor in a homogenous way, making this dimension redundant. Also, Basel II assumes a strictly decreasing asset correlation function in the dimensions of credit rating and firm size (Basel Committee on Banking Supervision (2005)). To the contrary, we find a non-monotonic relationship between firm size and asset correlation that is inconsistent with the Basel II formula for minimum capital requirements.

A large proportion of the existing literature (i.e. Gordy (2000, 2003), Lopez (2004)) and regulatory frameworks such as Basel II (Basel Committee on Banking Supervision (2005)) assume a single-factor model. This assumption translates into a situation in which only a single economy-wide common risk factor exists and all obligors are subject to changes therein. It is counterintuitive that all industries would depend on identical risk factors which strike at the same time, with the same strength. It is hard to believe that weather risk associated with agriculture industry, demographic risk with construction industry, oil price risk with transportation industry, or liquidity risk associated with the finance industry are all perfectly correlated.

Next we formally test for the presence of one common factor. In terms of our model in which the correlation matrix Ω is estimated in an unconstrained manner, we statistically test for a single risk factor if all common risk factors were perfectly correlated. In order to test the validity of this simplifying assumption for US small businesses we use Jennrich (1970) test for equality of correlation matrices. The test results are shown in Table V. Our results call into question the assumption of a single common risk factor in US retail portfolios. This assumption is violated for the obligors segmented according to their credit rating and industry. As expected, those two dimensions capture some of information differentiating

obligors' risk types. However, there is no empirical evidence in favor of the second type of segmentation organized according to credit rating and firm size. We find those dimensions redundant where risk factors are perfectly correlated. In view of these results we create homogenous obligor classes with respect to two criteria: credit rating and industry.

[Table 5 about here.]

Next, we attempt to answer the question which risk dominates in small businesses: systematic or idiosyncratic. Given that small businesses correspond to a significant part of the US economy, one might expect that their aggregate behavior follows the swings in the economy as a whole. On the other hand each small business has individual qualities and attributes, such as location, business network, faithful clients etc. that are relatively stable over the business cycle and often decisive. A bakery on the corner or a dentist downtown can do fine even during recession. Table II shows that across the whole sample period small businesses have a tendency to manifest significant idiosyncratic risk. The direct obligor's characteristics that often decide on the success or failure of the small business (see also Phelan (2011)) show up significantly in Table II. Even though the sample period covers a whole business cycle, with expansion in 2005 to 2006, recession in 2007 to 2009 and recovery in 2010, we observe that the estimated sensitivities to the common risk factors remain low and vary between 0.00-18.41% explaining only 0.00-3.39% of the asset variability. The remaining 96.61-100.00% of small business risk is due to changes in the firm-specific characteristics.[3] These results are striking, especially in the light of the crisis which affected the whole economy with very few exceptions. Although the probabilities of default were on average at a high level during crisis, the uncertainty about default decreased in the sense that it almost became more of a certainty. A second reason for finding low sensitivities of US small business to systemic risk factor stems from the fact that the dataset is quasi-exhaustive and

[3]The low values of sensitivity parameters w_k remain robust to changes in the default definition to a less conservative one which considers only events of losses acquired by a debt holder. Also for US geographic regions the values of sensitivity parameters w_k remain low. Intuitively, it is expected that geographic proximity in the activity of small businesses would cause them to be more susceptive to common risk factors. However, the results for US states show that the idiosyncratic risk in small business loans prevails.

approaches the limits of diversification in the US economy. As Dietsch and Petey (2004) we believe financial institutions observe higher asset correlations in their portfolios due to possible further diversification within their books.

On the asset correlation side, presented in Table VI, we find that during the whole period analyzed the implied asset correlation averaged around 0.41%, with the lowest values of 0.00% for mining and agriculture and a statistically significant highest value of 0.78% for the retail trade businesses. Most importantly, regardless of the small business' riskiness and industry, our estimates are significantly lower than any available estimates for corporate firms. For example, McNeil and Wendin (2007) report asset correlations between corporate firms ranging from 6.30%-10.90%. That is fifteen to twenty-six times larger than our average estimate for small businesses. These considerably lower asset correlations between small businesses have important regulatory consequences, which are discussed further in this section.

[Table 6 about here.]

We now turn our attention to the development of small business riskiness over the course of the recent financial crisis. For this purpose we apply a moving window technique in which the overall sample period is subdivided into three phases according to NBER business cycle reference dates. First phase: pre-crisis, covers June 2005 till September 2007. The crisis phase is from December 2007 until June 2009 and the post-crisis phase covers September 2009 until December 2010. This subdivision allows us to estimate the model separately in those three phases and to focus on changes of joint default risk. Figure 2 addresses two important elements of default risk in a portfolio of loans: expected and unexpected losses. Both of these are expected to vary over the different phases of the crisis. Expected losses are associated with the probabilities of default, as illustrated in Panels a, c and e. Unexpected losses associated with the asset correlations, as is illustrated in Panels b, d and f. Clustering of defaults can be linked to both of these channels, either through increased default frequencies or by high asset correlation and thus higher uncertainty of defaults. Before the crisis the

probabilities of default were high, with average of 13.47% (Panel a), and they fell slightly to 13.14% on average during crisis (Panel c). It is after the trough that the probability of default fell considerably to a low average value of 11.23% (Panel e). We also observe that before the economic turmoil the asset correlation was on average at a low level (0.25%). It declined even further to 0.13% during the crisis when the mass of the asset correlations moved to the left, with many obligors exhibiting virtually no correlation with each other. Those firms which withstood the crisis showed little sensitivity to economic conditions but instead a substantial reliance on the firm characteristics. On the other hand, after the economic turmoil the average asset correlation increased to 0.68% on average exposing the dependencies between small businesses.

[Figure 2 about here.]

[Figure 3 about here.]

Figure 3 sheds a different light on the evolution of portfolio default risk. It is a comparison of Monte-Carlo-generated default distributions plotted for the three separate phases defined above and for the entire period analyzed. We simulate panels of default indicators for a portfolio of 10,000 firms distributed across credit ratings and industries proportionally to the historical data. To that end we use the estimates of θ and of default thresholds $\Phi^{-1}(\bar{p}_k)$ which are phase-specific. The density estimates are obtained by Gaussian kernel smoothing (with interval length of 10). From a Basel perspective, the pricing of loan exposures and provisions should cover losses up to the expected losses. On the other hand, if there are any losses associated with the unexpected losses they should be covered by the capital requirements.

The results show that default frequency distributions were quite different during the different phases. We observe shifts in both expected and unexpected losses as we move through different phases of the crisis. The pre-crisis phase was characterized by a relatively high expected losses and moderate unexpected losses. In this phase the average realized number of defaults was 1,345 with 99.9^{th} percentile of losses at 1,560 defaults. Interestingly, the

least uncertain level of defaults occurred during crisis. At that time the distance between the realized number of defaults (1,314) and 99.9th percentile (1,450) reached its minimum, pointing at low capital requirement but nevertheless high provisions. The post-crisis phase was characterized by low expected losses (1,121 defaults), signaling an economic recovery which, however, was accompanied by high uncertainty with 99.9th percentile at 1,453 defaults. Typically, the expected losses (thus provisions level) are considered not to embody uncertainty, which instead is associated with the unexpected losses. However, what we observe are considerable shifts in the expected losses related to provisions levels. In practice, from a risk management perspective, this should mean that the capital a financial institution holds accounts for shifts in expected losses as well.

[Table 7 about here.]

Next, we employ our estimator to a portfolio of *corporate* debt. We show that the estimator proposed produces results for corporate exposures that are similar to the outcomes of the Basel II regulatory framework, which illustrates its reliability. To this end we use the public information on US corporate default rates per credit rating provided by S&P. S&P reports payment history of about 3,000 US firms during a period of six years from 2005 through 2010 and cover a broad range of industries. Both S&P and our study weigh the default events by the number of obligors rather than the nominal value of default. We exclude AAA and AA+ ratings from the analysis due to lack of defaults in those rating categories during the period analyzed. For consistency with the Basel methodology (Gordy (2000, 2003)) the estimation of our model follows per sub-portfolio composed of obligors from one homogenous obligor class. This procedure is equivalent to estimation of a single-factor model.

Panel A in Table VII shows the resulting asset correlation estimates, together with the default rates, capital requirements for corporate exposures and the difference between our model and the Basel approach. In general the results show that although corporate firms exhibit low probabilities of default relative to retail debt, they are heavily exposed to changes

in economic conditions. The asset correlations vary between 3.83% and 22.18% and average at 15.01%, which confirms a substantial interdependence in corporate exposures. Most importantly, the capital requirements for corporate exposures implied by our estimates of asset correlations are in line with the regulatory ones, which is shown both in Panels A and B in Table VII. The paired difference test reported confirms that our model and Basel II formula produce on average similar outcomes. We find no significant difference between the capital requirements computed according to regulatory formula and the ones computed using our estimates of asset correlation.

[Table 8 about here.]

Given the consistency of the Basel II and the proposed model in corporate portfolios one could perhaps expect to find matching estimates in case of retail portfolios as well. To illustrate the implication of the model on capital requirements in financial institutions holding retail portfolios, we use the results from Table VI and contrast them with outcomes of the Basel II regulatory formula. Table VIII suggests that small businesses are subject to inefficient capital allocations imposed by the regulator. The results show significant discrepancies in capital requirements implied by Basel II and the proposed model. For all levels of credit worthiness of the obligor, the Basel II formula significantly overstates the asset correlations and thus the capital requirements for sub-portfolios of small businesses, which is shown by the highly significant paired difference test. Indeed, we observe that the capital requirement is on average almost four times higher than the data suggest. And it is the more creditworthy obligors that suffer the highest capital charges relative to their riskiness. For these firms the regulatory formula overestimates the capital requirement even by factor of about ten. As a result these more creditworthy obligors pay for the credit risk of their less creditworthy peers. It also creates inverse incentives for financial institutions, which may flee to other obligor classes in which loans originated are less costly to hold. Similarly, we compute the 'aggregated' capital requirement on a portfolio level which is composed of all the obligor classes in the historical proportions. Here the regulatory capital requirement

amounts to 7.31%, which is almost four times more than our multi-factor model implies (2.01%).

As the Basel Committee on Banking Supervision (2005) suggests, the overly high capital requirements for US retail loan portfolios may stem from a need for constructing a uniform framework applicable to a wider range of countries. The regulatory formula for retail asset correlation was not fitted to historical loan data. Instead the Basel Committee on Banking Supervision reverse-engineered the asset correlation from the information on historical capital that banks held. Our results suggest that the retail asset correlation function obtained and imposed by regulator is far from accurate. Moreover, the resulting inefficiencies in capital allocations encourage more financing in the corporate sector rather than in the small business economy, an undesirable outcome, also from the point of view of policy makers.

From a risk management perspective an important feature of the approach proposed in this study is given by a possibility to assess the parameter uncertainty of the capital requirements. For example, if we take the prudential value of the capital requirement equal to its estimate plus its uncertainty (here the standard error), the capital requirement increases even by 10.80% (from 9.56% to 20.36%) for least creditworthy obligors from mining industry. But on average the prudential financial would hold 1.87% above the model's requirement.

This leads us to another aspect of parameter uncertainty, namely uncertainty which stems from normality assumption of common risk factors. Although the normality of risk factors is not a necessity to construct the multi-factor model, we derive our estimates of asset correlation for the case in which the common and idiosyncratic risk factors are normally distributed. We conducted simulations to explore the effects of implementing student-t and gamma distributed risk factors. The estimates remain close to the values under normality. In spite of this, any difference in the distribution of the risk factors may have an impact on the portfolio risk, see Schönbucher (2000). Thus if a financial institution believes the common risk factors of its portfolio follow a fat-tailed distribution, the proper response is to implement a corresponding prudential approach to asset correlation estimates and following capital requirements.

V Concluding remarks

This paper compares the minimum capital requirements implied by the Basel II Accord and those implied by a multi-factor firm value model for an extensive dataset of US small businesses. We find, firstly, that for retail loan portfolios the Basel II formula overestimates economic capital. Moreover, it is the most creditworthy small obligors that suffer the highest capital charges relative to their riskiness. These most creditworthy obligors essentially pay for their riskier peers. This can result in distorted lending and risk management practices by financial institutions which hold retail loan portfolios. Our empirical results show that, from a credit risk perspective, retail exposures are a much safer investment than the regulator would suggest. In our view this regulatory flaw results from the overly-simplistic way in which Basel II models and estimates the asset correlations in retail loan portfolios.

Secondly, we trace the evolution of two important elements of default risk in a portfolio of loans: expected and unexpected losses. Interestingly, the crisis eliminated many uncertainties about defaults in a retail loan portfolio. Thus, the firms which withstood the deterioration of macroeconomic conditions did not go systematically into default.

Lastly, equipped with a simple yet effective estimation technique, we provide an empirical analysis of a representative panel of exposures to US small businesses between 2005 and 2011. We find that in general sensitivity to the common risk factors remains low and small business risk is predominantly resulting from idiosyncratic risk, even when controlling for different definitions of the default event, geographical proximity, as well as industry and firm size heterogeneity. Our results show that only 0.00-3.39% of the asset variability is explained by economy-wide risk factors. The remaining 96.61%-100.00% of small business risk is due to changes in the firm-specific characteristics. Importantly, regardless of small businesses' riskiness, industry or firm size, our estimates of asset correlations are significantly lower than any available estimates for corporate firms.

Appendix A Parameter estimation

Given the vector of sensitivity parameters w, the distribution of a single default event in a obligor class k is given by:

$$p_i = P[D_{i,t+1} = 1] = P\left[A_{i,t+1} < \Phi^{-1}\left(\bar{p}_k\right)\right] = \int_{-\infty}^{\Phi^{-1}(\bar{p}_k)} f(A_{i,t+1}) dA_{i,t+1} \tag{A1}$$

where $f(\cdot)$ is a density function and in our application of the model takes the form of normal probability distribution function and $\Phi(\cdot)$ denotes the cumulative standard normal distribution function. By design for any i and j where $i \neq j$ the probability distribution of a default event in which two obligors fail to meet their payments is modeled as a bivariate normal distribution:

$$f_{ij}(A_{i,t}; A_{j,t}) = \frac{1}{2\pi|\Sigma|^{1/2}} \exp\left\{-\frac{1}{2} A^T \Sigma^{-1} A\right\} \tag{A2}$$

$$\text{where} \quad A = \begin{bmatrix} A_{i,t} \\ A_{j,t} \end{bmatrix} \tag{A3}$$

$$\text{and} \quad \Sigma = \begin{bmatrix} 1 & w_k w_l \Omega_{kl} \\ w_k w_l \Omega_{kl} & 1 \end{bmatrix} \tag{A4}$$

The above joint density of $A_{i,t}$ and $A_{i,t}$ can be transformed by standardizing the vector A and integrating out the effects of the risk factors. Consequently one will obtain the probability of an event in which both obligors default at once:

$$\begin{aligned} p_{kl} &\equiv P[D_{i,t+1} = 1, D_{j,t+1} = 1] \\ &= \int_{-\infty}^{\Phi^{-1}(\bar{p}_l)} \Phi\left(\frac{\Phi^{-1}(\bar{p}_k) - \Omega_{kl} w_k w_l y}{\sqrt{1 - \Omega_{kl}^2 w_k^2 w_l^2}}\right) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} y^2\right) dy \end{aligned} \tag{A5}$$

The expression in (A5) gives the population moment for joint probability of default. The

sample moment is derived in the following way. We take the joint probability of default for two firms i and j from two different obligor classes k and l to be an average of all occasions in which both firms are simultaneously in default:

$$\hat{p}_{ij} = \frac{1}{T}\sum_{t=1}^{T}(D_{i,t+1} \cdot D_{j,t+1}) \tag{A6}$$

Next, to arrive at sample moment of joint probability of default for two obligor classes, we need to take an average over all possible pairs of firms in both obligor classes:

$$\hat{p}_{kl} = \frac{1}{N_{k,t}N_{l,t}}\sum_{i\in k}^{N_{k,t}}\sum_{j\in l}^{N_{l,t}}\frac{1}{T}\sum_{t=1}^{T}(D_{i,t+1} \cdot D_{j,t+1}) \tag{A7}$$

where N_{kt} and N_{lt} is the number of firms in obligor class and respectively. Now we change the order of summation which gives us that the sample moment for joint probability of default is an average over time of the product of observed default frequencies:

$$\begin{aligned} \hat{p}_{kl} &= \frac{1}{T}\sum_{t=1}^{T}\frac{\sum_{i\in k}^{N_{k,t}} D_{i,t+1}}{N_{k,t}}\frac{\sum_{j\in l}^{N_{l,t}} D_{j,t+1}}{N_{l,t}} \\ \Rightarrow \hat{p}_{kl} &= \frac{1}{T}\sum_{t=1}^{T}(ODF_{k,t} \cdot ODF_{l,t}) \end{aligned} \tag{A8}$$

The GMM estimator proposed minimizes the distance between the population and sample moments with respect to the parameter vector θ.

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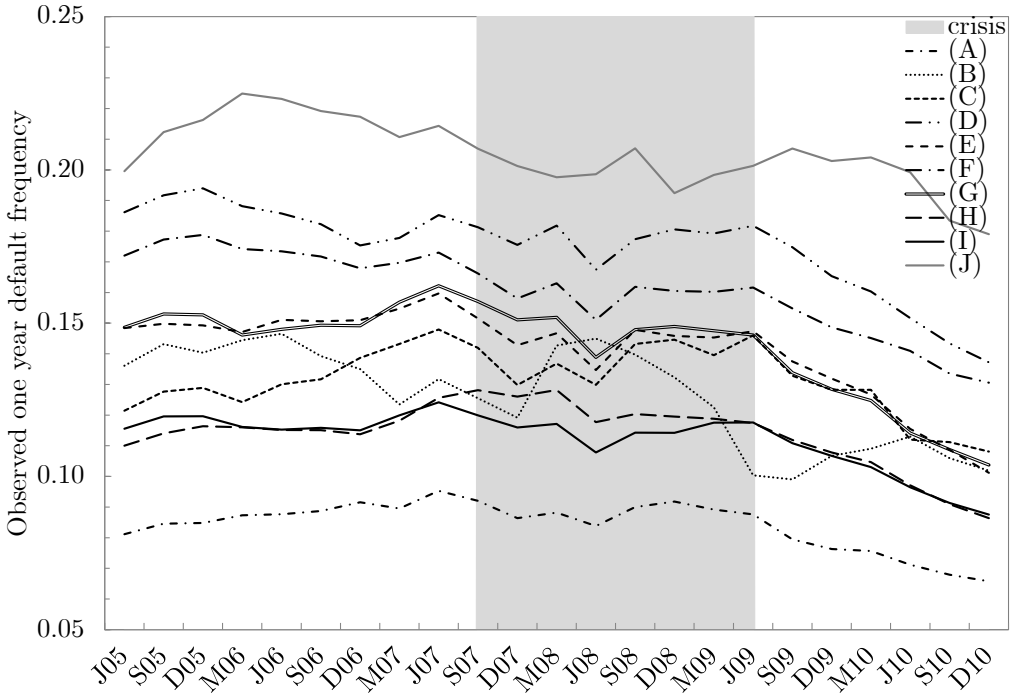

Figure 1: Observed default frequencies per industry classes. The shadowed area illustrates the crisis phase as defined by NBER business cycle reference dates. The pre-crisis phase covers June 2005 till September 2007; crisis is from December 2007 until June 2009 and post-crisis phase covers September 2009 until December 2010.

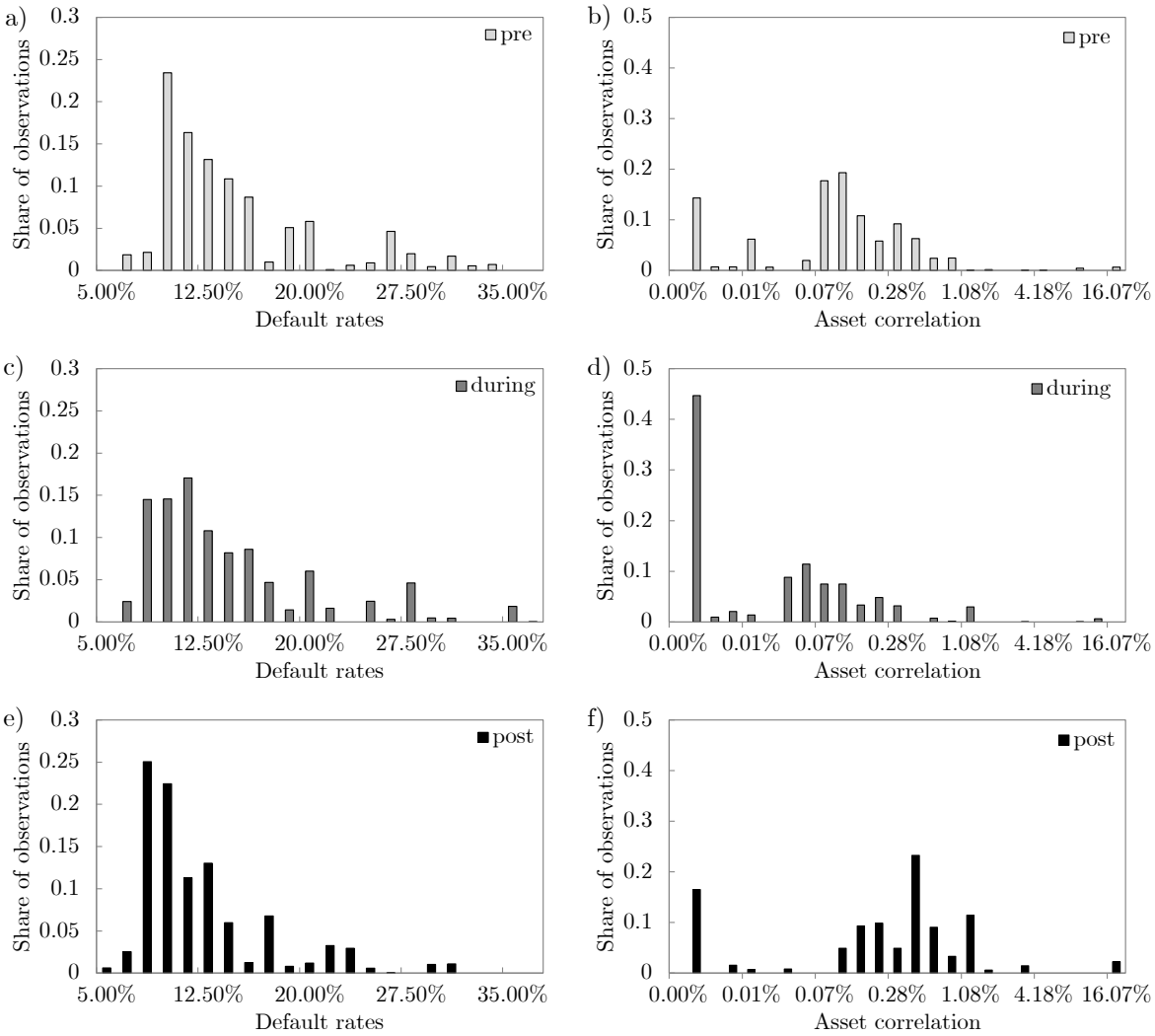

Figure 2: Default rates $\bar{p}$ and asset correlation ρ_{ii} per credit rating & industry in the pre-, during and post-crisis phases. The pre-crisis phase covers June 2005 till September 2007; crisis is from December 2007 until June 2009 and post-crisis phase covers September 2009 until December 2010.

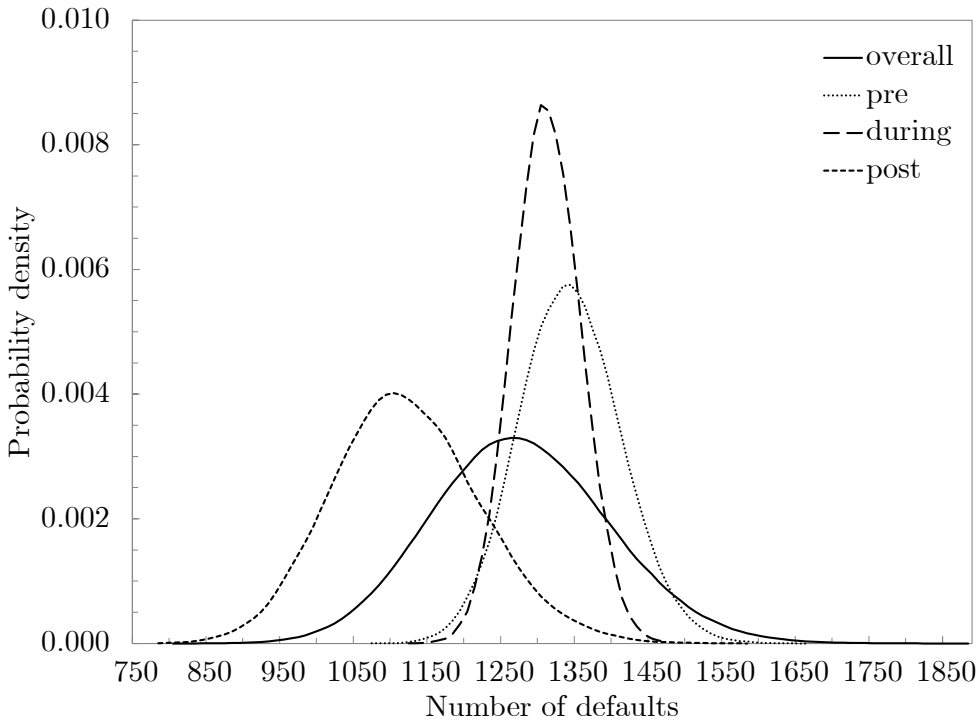

Figure 3: Evolution of portfolio default distribution. We observe considerable shifts in both expected and unexpected losses over the course of the crisis. Also, the default distribution narrows in the crisis. Density of number of defaults for the pre-crisis (dotted line), in crisis (dashed line), post-crisis (square-dotted line) phases and over the whole analyzed period (solid line). The density estimates are given by Gaussian kernel smoothing (with interval length of 10) of the Monte Carlo generated default distribution. The pre-crisis phase covers June 2005 till September 2007; crisis is from December 2007 until June 2009 and post-crisis phase covers September 2009 until December 2010.

Table I

Small businesses in the US

Descriptive statistics for US small businesses in the D&B dataset covering period from 2005 to 2011. The values: number of firms, % total and default rate (%) represent a historical average. Geographic regions are defined as: Central: IA, KS, MN, MO, NE, ND, SD; West: AK, AZ, CA, CO, HI, ID, MT, NV, NM, OR, UT, WA, WY; Northeast: CT, ME, MA, NH, NJ, NY, PA, RI, VT; Midwest: IL, IN, MI, OH, WI; Southeast: DE, DC, FL, GA, MD, VA, NC, SC, WV, AL, KY, MS, TN; Southwest: AR, LA, TX, OK.

	# firms	% total	min	max	defaults (%)
1. SIC industry division					
Agriculture, Forestry, Fishing	9,902	4.19	9,340	10,188	8.39
Mining	825	0.35	758	872	12.55
Construction	32,180	13.61	27,048	36,275	13.13
Manufacturing	16,382	6.93	14,155	18,278	17.48
Transportation, Communications, Electric, Gas, and Sanitary Services	8,123	3.44	6,963	9,046	14.12
Wholesale Trade	16,048	6.79	14,063	17,836	16.02
Retail Trade	35,032	14.82	29,552	39,993	14.19
Finance, Insurance, and Real Estate	20,020	8.47	17,170	22,310	11.34
Services	96,379	40.78	85,672	104,065	11.19
Public Administration and non-classified	1,467	0.62	1,358	1,831	23.88
2. Firm size					
1-5	133,755	56.59	115,434	147,547	9.67
6-10	44,125	18.67	38,308	49,158	12.89
11-20	28,244	11.95	24,731	31,174	15.82
21-30	10,890	4.61	9,778	11,867	18.53
31-50	9,150	3.87	8,344	9,904	21.16
51-100	6,149	2.60	5,670	6,751	26.42
>100	4,043	1.71	3,700	4,446	35.98
3. $ outstanding					
$0-500	38,530	16.30	29,436	48,676	5.78
$501-1,000	18,648	7.89	15,510	24,119	7.57
$1,001-2,000	22,880	9.68	19,990	27,531	9.15
$2,001-5,000	32,174	13.61	29,208	35,538	10.94
$5,000-15,000	48,536	20.54	42,366	52,458	12.67
$15,001-30,000	28,001	11.85	24,930	31,288	14.73
>$30,001	47,589	20.13	38,951	53,303	22.23
4. Region					
Central	17,512	7.41	16,135	18,876	10.65
West	53,754	22.74	45,590	59,743	12.84
Northeast	49,437	20.92	43,212	54,240	12.37
Midwest	36,319	15.37	32,368	39,741	12.31
Southeast	55,219	23.36	47,174	61,552	14.00
Southwest	24,118	10.20	21,533	26,437	12.62
5. Private					
Yes	236,284	99.97	206,140	260,471	12.74
No	74	0.03	50	117	42.58
Total	236,358	100.00	206,196	260,590	12.74

Table II

Sensitivity to the credit rating & industry common risk factors

The industry dimension adds little information to the sensitivity parameter. The credit rating is constructed to represent deciles of the firms' risk distribution where 1 represents the lowest and 10 the highest credit risk. Sensitivity w_k without the industry/credit rating related heterogeneity is reported at the bottom of the table. Significant difference to w_k without the credit rating related heterogeneity (Panel C) is denoted by * at the 90% level, ** at the 95% level and *** at 99% level. Significant difference to w_k without the industry related heterogeneity (Panel B) is denoted by † at the 90% level, †† at the 95% level and ††† at 99% level. Bootstrapped S.E. in parenthesis.

	Sensitivity w_k (%)									
Industry	Credit rating 1	2	3	4	5	6	7	8	9	10
Panel A: w_k to the credit rating & industry common risk factors										
Agri	3.70	0.00	5.54	0.00	7.74	8.39	7.61	6.86	6.34	6.14
	(4.05)	(3.73)	(3.96)	(3.9)	(4.27)	(4.04)	(4.53)	(4.52)	(4.54)	(4.80)
Mining	0.00	18.41	2.76	0.00	6.58	0.00	0.00	0.00	0.00	16.07
	(7.81)	(12.86)	(11.23)	(11.87)	(13.11)	(12.78)	(13.53)	(15.53)	(15.59)	(18.79)
Constr	5.49	5.59	4.84	5.00	6.44	5.94	6.14	6.37	7.82*	8.72**
	(2.92)	(2.23)	(2.16)	(2.2)	(1.79)	(1.83)	(1.71)	(1.61)	(1.47)	(1.53)
Mfg	5.85	4.46	5.43	6.15	4.59	4.68	6.69	6.10	7.06	6.31
	(2.76)	(3.07)	(2.76)	(2.6)	(2.84)	(2.7)	(2.7)	(2.37)	(2.25)	(2.64)
Trans	0.00	2.33	6.52	8.11	0.00	5.61	5.15	7.41	7.78	8.97
	(4.21)	(4.67)	(4.57)	(4.15)	(3.94)	(4.04)	(3.88)	(3.51)	(3.82)	(3.36)
Wholes	4.34	6.45	2.19	5.74	6.43	2.57	6.49	4.61	7.17	6.29
	(2.7)	(2.71)	(2.66)	(2.98)	(2.77)	(2.78)	(2.73)	(2.87)	(2.54)	(3.00)
RetlTrd	3.81	6.71	6.30	7.10	7.70	6.77	7.14	6.37	6.59	8.85
	(2.62)	(2.33)	(1.95)	(1.69)	(1.67)	(1.67)	(1.57)	(1.41)	(1.33)	(1.45)
FIRE	3.77	4.75	2.62	6.72	4.63	4.94	8.68	6.92	7.63	7.58
	(2.67)	(2.73)	(2.47)	(2.22)	(2.73)	(2.73)	(2.43)	(2.52)	(2.89)	(3.01)
Service	3.68	3.68	4.19	5.03	4.99	4.96	5.40	5.40	5.68	7.22**
	(1.23)	(1.17)	(1.03)	(1.01)	(0.99)	(0.94)	(0.91)	(0.93)	(0.94)	(1.00)
PA	7.36	0.00	0.00	5.77	5.56	5.93	11.83	12.00	6.18	9.69
	(4.77)	(5.65)	(6.59)	(7.77)	(8.59)	(8.58)	(8.39)	(8.68)	(8.45)	(7.46)
Panel B: w_k per credit rating only (no industry related heterogeneity)										
Industries	4.31	4.73	4.51	5.41	5.20	5.02	5.93	5.82	6.14	7.07
	(0.64)	(0.66)	(0.60)	(0.56)	(0.55)	(0.55)	(0.53)	(0.53)	(0.55)	(0.62)

Panel C: w_k per industry only (no credit rating related heterogeneity)

	Industry									
	Agri	Mining	Constr	Mfg	Trans	Wholes	Retl-Trd	FIRE	Service	PA
Credit ratings	4.71	5.28	5.02	5.45	6.33	5.32	6.82	5.34	4.81	2.52
	(1.45)	(4.81)	(0.58)	(0.81)	(1.22)	(0.83)	(0.59)	(0.81)	(0.34)	(2.49)

Table III

Sensitivity to the credit rating & firm size common risk factors

The firm size dimension adds information to the sensitivity parameter. The credit rating is constructed to represent deciles of the firms' risk distribution where 1 represents the lowest and 10 the highest credit risk. The firm size stands for number of employees in a firm. Sensitivity w_k without the firm size/credit rating related heterogeneity is reported at the bottom of the table. Significant difference to w_k without the credit rating related heterogeneity (Panel C) is denoted by * at the 90% level, ** at the 95% level and *** at 99% level. Significant difference to w_k without the firm size related heterogeneity (Panel B) is denoted by † at the 90% level, †† at the 95% level and ††† at 99% level. Bootstrapped S.E. in parenthesis.

	Sensitivity w_k (%)									
Firm	Credit rating									
size	1	2	3	4	5	6	7	8	9	10
Panel A: w_k to the credit rating & firm size common risk factors										
≤5	4.69***	5.43***	5.69***	5.69***	6.07***	6.61***††	6.85***	6.99***	7.55***†	7.53***
	(1.62)	(1.07)	(0.92)	(0.81)	(0.88)	(0.78)	(0.80)	(0.75)	(0.77)	(0.84)
6-10	3.20***	4.83	5.15***	6.20***	6.16***	5.37***	7.34***	7.39***	6.49***	7.32***
	(2.15)	(1.64)	(1.54)	(1.42)	(1.44)	(1.58)	(1.41)	(1.40)	(1.43)	(1.69)
11-20	2.49***	2.55***	2.54***	3.66***	5.05***	4.22***	6.50***	5.82**	4.91***	6.22***
	(2.02)	(2.06)	(2.02)	(2.11)	(2.01)	(2.08)	(1.83)	(2.00)	(2.20)	(2.23)
21-30	4.58*	6.27***	2.27***	5.66	6.72***	3.16***	5.07***	4.55***	5.89***	9.68***
	(2.68)	(3.04)	(3.24)	(3.58)	(3.70)	(3.73)	(3.82)	(3.87)	(3.54)	(3.21)
31-50	3.82***	5.77***	0.00***	6.26***	6.92***	6.98***	6.72***	5.33***	5.72***	5.82***
	(2.70)	(3.57)	(3.27)	(3.79)	(3.89)	(4.04)	(3.81)	(3.87)	(3.75)	(3.83)
51-100	6.59***	6.66***	7.74***	6.43***	8.52***	7.62***	8.14***	6.01	10.83***	7.91*
	(4.31)	(4.53)	(4.04)	(3.93)	(3.79)	(4.04)	(4.18)	(4.23)	(4.14)	(4.86)
>100	12.59***†	10.54***	5.79***	3.53***	5.12**	10.20***	7.68***	9.52***	11.95***	10.00***
	(4.76)	(4.56)	(4.38)	(4.39)	(4.63)	(4.75)	(5.16)	(5.50)	(6.07)	(7.18)
Panel B: w_k per credit rating only (no firm size related heterogeneity)										
Firm	4.31	4.73	4.51	5.41	5.20	5.02	5.93	5.82	6.14	7.07
sizes	(0.64)	(0.66)	(0.60)	(0.56)	(0.55)	(0.55)	(0.53)	(0.53)	(0.55)	(0.62)

Panel C: w_k per firm size only (no credit rating related heterogeneity)

	Firm size						
	≤5	6-10	11-20	21-30	31-50	51-100	>100
Credit	6.10	5.24	4.21	5.57	5.69	7.72	8.53
ratings	(0.30)	(0.50)	(0.60)	(1.03)	(1.12)	(1.30)	(1.53)

Table IV

Homogeneity of credit rating/industry/firm size common risk factors

Jennrich (1970) test for equality of correlation matrices. It tests the difference between an estimate of a partition of (common risk factors) correlation matrix Ω and a matrix of ones. The partitioning is done according to the dimension tested for homogeneity. Thus if the homogeneity within credit rating is analyzed, the Ω is broken in such way that only the correlations within a given credit rating remain.

	Credit rating									
	1	2	3	4	5	6	7	8	9	10
Panel A: Industry related homogeneity in credit rating										
χ^2	138.37	$21{\cdot}10^{28}$	110.28	29.03	104.40	1,953.50	228.35	2,785.00	35.98	168.43
df	45	45	45	45	45	45	45	45	45	45
p-value	0.00	0.00	0.00	0.97	0.00	0.00	0.00	0.00	0.83	0.00
Panel B: Firm size related homogeneity in credit rating										
χ^2	24.29	15.62	771.52	68.69	44.80	1,061.80	214.94	113.01	189.32	14.88
df	21	21	21	21	21	21	21	21	21	21
p-value	0.28	0.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.83
Panel C: Credit rating related homogeneity in industry										
	Industry									
	Agri	Mining	Constr	Mfg	Trans	Wholes	RetlTrd	FIRE	Service	PA
χ^2	448.66	621.77	770.89	678.61	6,098.10	$253{\cdot}10^3$	$7{,}805{\cdot}10^4$	11.35	2,429.40	2.42
df	45	45	45	45	45	45	45	45	45	45
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	1.00
Panel D: Credit rating related homogeneity in firm size										
	Firm size									
	$\leq$5	6-10	11-20	21-30	31-50	51-100	>100			
χ^2	149.81	480.08	$13{\cdot}10^5$	4.62	179.24	47.09	4.48			
df	45	45	45	45	45	45	45			
p-value	0.00	0.00	0.00	1.00	0.00	0.39	1.00			

Table V

Single vs. multi-factor model

There is support for industry to be a relevant risk factor but not for firm size. The firm size factor collapses to a single factor. This result questions Basel II approach featuring firm size as a risk factor. The table reports Jennrich (1970) test for equality of correlation matrices. It tests the difference between an estimate of common risk factors correlation matrix Ω and a matrix of ones. The obligor classes are divided with respect to credit rating and industry, or credit rating and firm size.

	Credit rating & industry	Credit rating & firm size
χ^2	63,086.00	982.53
df	4950	2415
p-value	0.00	1.00

Table VI

Asset correlation and default rates per credit rating & industry

Small businesses are subject mainly to idiosyncratic risk with low asset correlation. The values reported cover period from June 2005 to December 2010. The credit rating is constructed to represent deciles of the firms' risk distribution where 1 represents the lowest and 10 the highest credit risk. Bootstrapped S.E. in parenthesis.

		Asset correlation ρ_{ii} (%) *within obligor class* and default rates $\bar{p}$ (%)									
Industry		Credit rating 1	2	3	4	5	6	7	8	9	10
Agri	ρ_{ii}	0.14	0.00	0.31	0.00	0.60	0.70	0.58	0.47	0.40	0.38
		(0.43)	(0.36)	(0.50)	(0.40)	(0.68)	(0.67)	(0.72)	(0.65)	(0.63)	(0.68)
	$\bar{p}$	5.94	5.68	5.50	6.17	6.52	8.09	9.99	11.15	16.49	23.49
Mining	ρ_{ii}	0.00	3.39	0.08	0.00	0.43	0.00	0.00	0.00	0.00	2.58
		(1.62)	(5.12)	(3.36)	(3.78)	(4.54)	(4.32)	(4.77)	(6.65)	(6.83)	(9.73)
	$\bar{p}$	9.16	9.05	8.71	11.96	13.34	15.03	18.33	17.21	23.32	31.57
Constr	ρ_{ii}	0.30	0.31	0.23	0.25	0.41	0.35	0.38	0.41	0.61	0.76
		(0.30)	(0.24)	(0.20)	(0.22)	(0.23)	(0.21)	(0.21)	(0.21)	(0.23)	(0.27)
	$\bar{p}$	8.65	7.30	8.06	8.84	9.53	10.82	12.19	14.56	20.46	30.82
Mfg	ρ_{ii}	0.34	0.20	0.29	0.38	0.21	0.22	0.45	0.37	0.50	0.40
		(0.30)	(0.30)	(0.30)	(0.32)	(0.27)	(0.25)	(0.35)	(0.28)	(0.31)	(0.31)
	$\bar{p}$	13.38	12.62	12.99	13.79	14.37	15.56	17.39	19.55	24.11	32.36
Trans	ρ_{ii}	0.00	0.05	0.43	0.66	0.00	0.32	0.27	0.55	0.60	0.80
		(0.48)	(0.59)	(0.65)	(0.67)	(0.42)	(0.51)	(0.47)	(0.50)	(0.57)	(0.57)
	$\bar{p}$	10.32	10.65	10.71	10.81	11.06	11.67	13.34	15.01	18.06	26.01
Wholes	ρ_{ii}	0.19	0.42	0.05	0.33	0.41	0.07	0.42	0.21	0.51	0.40
		(0.24)	(0.34)	(0.20)	(0.34)	(0.35)	(0.23)	(0.34)	(0.27)	(0.35)	(0.34)
	$\bar{p}$	13.11	11.89	12.77	13.86	14.10	14.76	16.49	18.49	22.90	30.26
RetlTrd	ρ_{ii}	0.14	0.45	0.40	0.50	0.59	0.46	0.51	0.41	0.43	0.78
		(0.21)	(0.30)	(0.24)	(0.24)	(0.26)	(0.22)	(0.22)	(0.18)	(0.18)	(0.26)
	$\bar{p}$	10.67	9.84	10.50	11.08	11.14	12.21	13.10	14.63	18.21	25.68
FIRE	ρ_{ii}	0.14	0.23	0.07	0.45	0.21	0.24	0.75	0.48	0.58	0.58
		(0.21)	(0.26)	(0.17)	(0.29)	(0.26)	(0.28)	(0.41)	(0.34)	(0.42)	(0.43)
	$\bar{p}$	9.25	8.09	8.94	9.21	9.46	10.63	11.63	14.42	18.40	25.63
Service	ρ_{ii}	0.14	0.14	0.18	0.25	0.25	0.25	0.29	0.29	0.32	0.52
		(0.09)	(0.09)	(0.09)	(0.10)	(0.10)	(0.09)	(0.10)	(0.10)	(0.11)	(0.15)
	$\bar{p}$	7.93	7.62	8.17	8.99	9.46	10.29	11.56	13.30	17.40	24.73
PA	ρ_{ii}	0.54	0.00	0.00	0.33	0.31	0.35	1.40	1.44	0.38	0.94
		(0.72)	(0.83)	(1.14)	(1.64)	(2.02)	(1.95)	(2.18)	(2.29)	(1.97)	(1.61)
	$\bar{p}$	21.29	21.90	21.47	23.51	21.83	19.56	15.72	16.47	17.42	22.28

Table VII

Capital requirement for corporate debt in the US

Our estimates of capital requirement in corporate portfolios are in line with Basel II. The default rates $\bar{p}$ are an average over time of observed default frequencies. Estimation of asset correlation ρ_{ii} within obligor class is based on sample of annual default rates provided by S&P. The time span is 2005-2010. Monte Carlo S.E. in parenthesis. K_m stands for capital requirement computed with the regulatory formula but with our estimates of asset correlation, K_r is the regulatory one. In computation of capital requirement we assume $LGD = 0.50$ and effective maturity $M = 3$. Significant difference to K_r is denoted by * at the 90% level, ** at the 95% level and *** at 99% level. Panel B displays tests for a difference between the K_r and K_m.

Panel A: Capital requirement for corporate sub-portfolios

	AA	AA-	A+	A	A-	BBB+	BBB	BBB-
$\bar{p}$ (%)	0.19	0.17	0.13	0.16	0.16	0.16	0.24	0.30
ρ_{ii} (%)	20.72	20.28	19.34	11.57	20.11	11.54	9.57	13.20
	(6.33)	(6.31)	(6.40)	(4.60)	(6.46)	(4.40)	(3.78)	(4.60)
K_r (%)	4.14	3.92	3.45	3.83	3.83	3.79	4.72	5.31
K_m (%)	3.64	3.33	2.73	1.69	3.22	1.66	1.77	2.89
	(1.25)	(1.17)	(1.00)	(0.72)	(1.16)	(0.68)	(0.73)	(1.09)
Difference	0.50	0.58	0.72	2.15	0.61	2.13	2.95	2.42
t-statistic	0.40	0.50	0.71	2.99***	0.53	3.13***	4.06***	2.23**

	BB+	BB	BB-	B+	B	B-	CCC/C
$\bar{p}$ (%)	0.68	0.44	0.47	1.61	2.92	6.24	23.97
ρ_{ii} (%)	13.80	11.31	3.83	15.02	16.81	22.18	15.92
	(4.50)	(4.05)	(1.86)	(4.52)	(5.14)	(5.45)	(6.00)
K_r (%)	7.59	6.30	6.55	10.16	11.88	14.92	22.24
K_m (%)	4.93	3.08	1.20	8.82	13.33	23.38	25.81
	(1.66)	(1.14)	(0.50)	(2.56)	(3.71)	(4.72)	(5.09)
Difference	2.66	3.23	5.35	1.34	-1.45	-8.47	-3.57
t-statistic	1.60	2.82***	10.67***	0.52	0.39	1.79*	0.70

Panel B: Test for difference between K_r and K_m)

	LGD	M	Test statistics	p-value
Paired t-test	0.5	3	0.88	0.39
Wilcoxon signed-rank test	0.5	3	-1.48	0.14
Sign test	0.5	3	-	0.04

Table VIII

Capital requirement for small business portfolios

Basel II significantly overstates the small business risk and introduces distortions. Credit rating buckets are constructed to represent deciles of the firms' risk distribution where 1 represents the lowest and 10 the highest credit risk level. The time period is 2005 to 2010. K_m (%) stands for capital requirement computed with the regulatory formula but with our estimates of asset correlation, K_r (%) is the regulatory requirement. We take the asset correlation as in Table VI and assume $LGD = 0.50$. The latter parameter does not affect the ratio K_r/K_m. Bootstrapped S.E. are in parentheses.

Panel A: Capital requirement for small businesses sub-portfolios)

	Credit rating:	1	2	3	4	5	6	7	8	9	10
Agri	K_r	6.01	5.98	5.96	6.04	6.09	6.33	6.71	6.97	8.21	9.48
	K_m	0.73	0.00	1.07	0.00	1.78	2.27	2.35	2.25	2.64	3.09
		(0.91)	(0.80)	(0.88)	(0.90)	(1.13)	(1.25)	(1.57)	(1.65)	(2.05)	(2.56)
	K_r/K_m	8.23	NA	5.55	NA	3.43	2.79	2.86	3.10	3.11	3.07
Mining	K_r	6.54	6.52	6.45	7.16	7.49	7.88	8.59	8.36	9.46	10.34
	K_m	0.00	6.33	0.71	0.00	2.41	0.00	0.00	0.00	0.00	9.56
		(2.76)	(5.77)	(4.39)	(5.50)	(6.51)	(6.57)	(7.48)	(8.68)	(9.10)	(10.80)
	K_r/K_m	NA	1.03	9.06	NA	3.10	NA	NA	NA	NA	1.08
Constr	K_r	6.44	6.20	6.33	6.48	6.61	6.90	7.22	7.78	9.00	10.28
	K_m	1.48	1.34	1.23	1.36	1.89	1.88	2.11	2.46	3.73	5.00
		(0.85)	(0.58)	(0.59)	(0.65)	(0.58)	(0.63)	(0.64)	(0.67)	(0.75)	(0.92)
	K_r/K_m	4.35	4.64	5.16	4.77	3.51	3.67	3.42	3.16	2.41	2.06
Mfg	K_r	7.50	7.32	7.40	7.59	7.73	8.00	8.40	8.83	9.57	10.39
	K_m	2.13	1.53	1.92	2.29	1.71	1.84	2.88	2.79	3.62	3.63
		(1.07)	(1.13)	(1.05)	(1.05)	(1.13)	(1.12)	(1.24)	(1.14)	(1.21)	(1.55)
	K_r/K_m	3.52	4.79	3.85	3.32	4.51	4.35	2.92	3.17	2.64	2.86
Trans	K_r	6.78	6.86	6.87	6.89	6.95	7.09	7.49	7.88	8.54	9.82
	K_m	0.00	0.69	2.07	2.66	0.00	1.86	1.85	2.96	3.46	4.83
		(1.40)	(1.61)	(1.62)	(1.52)	(1.35)	(1.47)	(1.52)	(1.51)	(1.82)	(1.90)
	K_r/K_m	NA	9.98	3.32	2.60	NA	3.81	4.05	2.66	2.46	2.03
Wholes	K_r	7.43	7.15	7.35	7.61	7.67	7.82	8.21	8.63	9.40	10.24
	K_m	1.52	2.19	0.73	2.13	2.44	0.95	2.71	2.01	3.59	3.53
		(1.00)	(1.00)	(0.96)	(1.19)	(1.13)	(1.10)	(1.22)	(1.31)	(1.34)	(1.73)
	K_r/K_m	4.88	3.26	10.06	3.57	3.15	8.24	3.03	4.30	2.62	2.90
RetlTrd	K_r	6.86	6.68	6.83	6.96	6.97	7.22	7.43	7.79	8.57	9.78
	K_m	1.15	2.02	1.97	2.33	2.56	2.35	2.61	2.47	2.91	4.73
		(0.84)	(0.77)	(0.67)	(0.61)	(0.62)	(0.64)	(0.63)	(0.59)	(0.63)	(0.82)
	K_r/K_m	5.95	3.31	3.47	2.99	2.73	3.07	2.85	3.16	2.95	2.07
FIRE	K_r	6.56	6.33	6.49	6.55	6.60	6.85	7.08	7.74	8.61	9.77
	K_m	1.03	1.20	0.69	1.93	1.31	1.52	3.01	2.67	3.43	4.02
		(0.78)	(0.75)	(0.70)	(0.70)	(0.83)	(0.90)	(0.94)	(1.06)	(1.39)	(1.67)
	K_r/K_m	6.35	5.26	9.45	3.39	5.04	4.51	2.36	2.90	2.51	2.43
Service	K_r	6.31	6.25	6.35	6.50	6.60	6.78	7.07	7.48	8.40	9.66
	K_m	0.90	0.87	1.06	1.38	1.42	1.49	1.78	1.94	2.42	3.75
		(0.32)	(0.30)	(0.28)	(0.30)	(0.31)	(0.31)	(0.32)	(0.36)	(0.43)	(0.55)
	K_r/K_m	7.00	7.15	6.00	4.71	4.65	4.53	3.98	3.85	3.47	2.57
PA	K_r	9.14	9.24	9.17	9.49	9.23	8.83	8.04	8.20	8.41	9.30
	K_m	3.56	0.00	0.00	2.89	2.68	2.71	5.11	5.33	2.65	4.90
		(2.45)	(2.93)	(3.46)	(4.32)	(4.72)	(4.51)	(4.15)	(4.39)	(4.25)	(4.08)
	K_r/K_m	2.56	NA	NA	3.28	3.44	3.26	1.57	1.54	3.17	1.90

Panel B: Test for difference between K_r and K_m

	LGD	Test statistics	p-value
Paired t-test	0.5	41.65	0.00
Wilcoxon signed-rank test	0.5	-8.68	0.00
Sign test	0.5	-9.90	0.00