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## SEARCH FOR YIELD

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# SEARCH FOR YIELD<sup>†</sup>

## Abstract

We present a model of the connection between real interest rates, credit spreads, and the structure and the risk of the banking system. Banks intermediate between entrepreneurs and investors, and choose the monitoring intensity on entrepreneurs' projects. We characterize the equilibrium for a fixed aggregate supply of savings, showing that safer entrepreneurs will be funded by nonmonitoring (shadow) banks and riskier entrepreneurs by monitoring (traditional) banks. We also show that a savings glut reduces interest rates and spreads, increases the relative size of the shadow banking system and the probability of failure of the traditional banks. The model provides a framework for understanding the emergence of endogenous boom and bust cycles, as well as the procyclical nature of the shadow banking system, the existence of countercyclical risk premia, and the low levels of interest rates and spreads leading to the buildup of risks during booms.

JEL Classification: E44, G21 and G23

Keywords: bank monitoring, banking crises, boom and bust cycles, credit spreads, financial stability, real interest rates, savings glut and shadow banks

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# 1 Introduction

The connection between interest rates and financial stability has been the subject of extensive discussions and a significant amount of (mostly empirical) research. This paper contributes to this literature by constructing a theoretical model of the relationship between real interest rates, credit spreads, and the structure and the risk of the banking system. It thus provides a framework to understand how a “global savings glut” that reduces the level of long-term real interest rates, noted by Bernanke (2005) and Caballero, Fahri, and Gourinchas (2008), can generate incentives to “search for yield” and increases of risk-taking that lead to financial instability, as noted by Rajan (2005) and Summers (2014).

We show that a savings glut reduces interest rates and interest rate spreads, increases the relative size of the originate-to-distribute (shadow) banking system, and increases the probability of failure of the originate-to-hold (traditional) banks.<sup>1</sup> Moreover, the model generates endogenous boom and bust cycles: the accumulation of savings leads to a reduction in rates and spreads and an increase in risk-taking that eventually materializes in a bust, which reduces savings, starting again the process of wealth accumulation that leads to a boom. The model also yields a number of empirically relevant results such as the procyclical nature of the shadow banking system and the existence of countercyclical risk premia. These findings contribute to our understanding of the role of financial factors in economic fluctuations.

The paper starts with a simple partial equilibrium model of bank lending with three types of risk-neutral agents: entrepreneurs, investors, and a bank. Entrepreneurs seek bank finance for their risky investment projects. The bank, in turn, needs to raise funds from a set of (uninsured) investors. Banks can monitor entrepreneurs’ projects, which reduces the probability of default but entails a cost for the bank. As monitoring is not contractible there is a moral hazard problem à la Holmström and Tirole (1997). Assuming that entrepreneurs are in the short side of the market, so they will only be able to borrow at a rate that

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<sup>1</sup>Our use of the term shadow banking follows the Financial Stability Board (2014): “The shadow banking system can broadly be described as credit intermediation involving entities and activities outside of the regular banking system.” They note that some authorities and market participants prefer to use other terms such as “market-based financing” instead of “shadow banking.”

leaves them no surplus, we characterize the optimal contract between the bank and the investors. We show that there are circumstances in which the bank chooses not to monitor entrepreneurs and others in which it chooses to monitor them. We associate the first case to (shadow) banks that originate-to-distribute, and the second case to (traditional) banks that originate-to-hold.

The partial equilibrium results show that which case obtains depends on the spread between the bank's lending rate and the expected return required by the investors, which equals the safe rate. In particular, a reduction in this spread reduces monitoring, and makes it more likely that the bank will find it optimal to originate-to-distribute.

To endogenize interest rates and interest rate spreads we embed our model of bank finance into a general equilibrium model in which a large set of heterogeneous entrepreneurs, that differ in their observable risk type, seek bank finance for their investment projects from a competitive banking sector. We assume that the higher the total investment in projects of a particular risk type the lower the return, and characterize the equilibrium for a fixed aggregate supply of savings. We show that safer entrepreneurs will borrow from originate-to-distribute banks while riskier entrepreneurs will borrow from originate-to-hold banks.

We then analyze the effects of an increase in the aggregate supply of savings, showing that it will lead to a reduction in interest rates and interest rate spreads, an increase in investment and in the size of banks' lending to all types of entrepreneurs, an expansion of the relative size of the shadow banking system, and a reduction in the monitoring intensity and hence an increase in the probability of failure of the traditional banks. These results provide a consistent explanation of a number of stylized facts of the period preceding the 2007-2009 financial crisis; see, for example, Brunnermeier (2009).

Although we focus on the effects of an exogenous increase in the supply of savings, the same effects obtain when there is an exogenous decrease in the demand for investment, due for example to a negative productivity shock. Thus, the model provides an explanation of the way in which changes leading to a reduction in the equilibrium real rate of interest, as those noted by Summers (2014), can be linked to an increase in financial instability.

Next we consider three interesting extensions. First, we show that the effect of a savings glut on financial stability critically depends on the increase in the size of the traditional banks. When banks that originate-to-hold cannot increase their balance sheet (and adjust their loan rates), there will be a greater increase in the size of the shadow banking system, a greater reduction in the safe rate, and wider spreads for the traditional banks, so they will become safer. The assumption of a fixed size may be rationalized in terms of some capacity constraint that cannot be immediately relaxed. But the effect will only be temporary, and as soon as originate-to-hold banks are able to relax the constraint they will become riskier. This result allows us to distinguish between the short- and the long-run effects of a savings glut, and provides a rationale for the idea that the buildup of risks happens when (real) interest rates are “too-low for too-long.”

The second extension deals with the case where investors are risk-averse. We show that a reduction in risk aversion has similar effects as a savings glut except for the level of the safe rate, which goes up instead of down. This provides a simple way to empirically distinguish a savings glut from a reduction in investors’ risk appetite. The intuition is that when investors are less risk-averse, there is a shift in investment toward riskier entrepreneurs that reduces the funds available for safer ones. This leads to a reduction in loan rates for the former and an increase in loan rates for the latter, which reduces spreads and hence banks’ monitoring incentives.

The third extension analyses a model with bounded project returns where high risk projects will not be undertaken. In such case, a savings glut will expand the set of (riskier) entrepreneurs that get funded.

Finally, we consider a dynamic version of our model in which the aggregate supply of savings is endogenous. Specifically, the supply of savings at any date is the outcome of agents’ decisions at the previous date together with the realization of a systematic risk factor that affects the return of entrepreneurs’ projects. For good realizations of the risk factor, aggregate savings will accumulate (the boom state) leading to lower interest rates and spreads, which translate into higher risk-taking. In this situation the economy is especially vulnerable to a bad realization of the risk factor, which can lead to a crisis (the bust state).

The associated reduction in aggregate savings leads to higher interest rates and spreads, which translate into lower risk-taking and a safer financial system. Then savings will grow, restarting the process that leads to another boom and a fragile financial system. In this manner, we can generate endogenous boom and bust cycles.

The dynamic model yields other interesting and potentially testable results. First, interest rates and interest rate spreads are countercyclical. Second, during booms the safe rate may be below investors' subjective discount rate, and it may even be negative. Third, the shadow banking system is highly procyclical. Fourth, even though investors are risk-neutral, they behave as if they were risk-averse, so risky assets have positive risk premia. Fifth, even though investors' preferences do not change over time, such risk premia are countercyclical.

The brief review of the literature that follows discusses the relation to previous studies and the evidence on some of these predictions.

**Literature review** This paper is linked to different strands of the (theoretical and empirical) literature on the relationship between interest rates, financial frictions and financial structure, and the business cycle.

Our interest in the effects of financial frictions on macroeconomic activity relates to numerous studies following the seminal papers of Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1996), and Kiyotaki and Moore (1997).<sup>2</sup> We have chosen to introduce these frictions using the moral hazard setup of Holmström and Tirole (1997). We depart from their model by focussing exclusively on the banks' moral hazard problem, endogeneizing the return structure that entrepreneurial projects offer in a competitive setup, and introducing heterogeneity in the ex-ante risk profile of entrepreneurs instead of in their net worth. In their characterization of equilibrium, entrepreneurs with low net worth borrow from monitoring banks while those with high net worth are directly funded by the market. In contrast, in our setup riskier entrepreneurs borrow from monitoring (originate-to-hold) banks while safer entrepreneurs borrow from nonmonitoring (originate-to-distribute) banks, which could be

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<sup>2</sup>See Quadrini (2011) and Brunnermeier, Eisenbach, and Sannikov (2012) for surveys of macroeconomic models with financial frictions, and Adrian, Colla, and Shin (2013) for a review of the performance of these models in explaining key features of the 2007-2009 financial crisis.

interpreted as market funding.

Most papers that analyze the role of financial intermediaries in economic fluctuations focus on leverage; see, for example, Gertler and Kiyotaki (2010), Repullo and Suarez (2013), and Adrian and Shin (2014). We depart from this literature by considering a model in which banks have no equity capital. Our focus on the effect of endogenously determined interest rates on banks' decisions in a general equilibrium setting links our findings to those of Boissay, Collard, and Smets (2015). They analyze a model with an interbank market where lower interest rates make riskier banks more prone to borrow from safer banks. Their paper, like ours, generates endogenous boom and bust cycles which are driven by banks' strategic responses to changes in interest rates. But we ignore the interbank market, and focus on the effect of interest rates on banks' monitoring and risk-taking decisions.

It should be noted that, as in Brunnermeier and Sannikov (2012) or He and Krishnamurthy (2012), we depart from previous studies by not analyzing a linearized version of the model but instead solving the full equilibrium dynamics.

Our work is related to a large volume of research spurred following the 2007-2009 financial crisis. On the one hand, our paper provides a theoretical framework that links a savings glut with the level of interest rates and the increases in risk-taking noted by Rajan (2005) and Summers (2014) among many others. On the other hand, it yields some predictions regarding the behavior of interest rates and spreads, risk premia, and the structure and the risk of the banking system that are in line with recent empirical findings. For example, Lopez-Salido, Stein and Zakrajšek (2015) show that the widening of credit spreads following a period of low spreads is closely tied to a contraction in economic activity.<sup>3</sup> Our results on risk premia are also in line with Gilchrist and Zakrajšek (2012), who find a negative relationship between risk premia and economic activity, and Muir (2014), who finds that risk premia increase substantially in financial crises. Finally, our results on the procyclicality of shadow banking are consistent with the evidence in Pozsar et al. (2012).

Many empirical papers analyzing the link between interest rates and banks' risk-taking

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<sup>3</sup>They interpret this result in behavioral terms (a change in "credit market sentiment"), whereas our story does not rely on changes in investors' preferences.

focus on monetary policy issues. Although we have a real model without nominal frictions, some of this evidence is also in line with our predictions; see, for example, Jimenez et al. (2014), Altunbas, Gambacorta, and Marques-Ibanez (2014), Dell’Ariccia, Laeven, and Suarez (2014), and Ioannidou, Ongena and Peydro (2015). Interestingly, our paper provides a rationale for the idea that low interest rates are dangerous from a financial stability perspective when they are low for a long period of time. However, our story is driven by the behavior of real interest rates, and is therefore not related to the stance of monetary policy.

**Structure of the paper** Section 2 presents the partial equilibrium model of bank finance under moral hazard. Section 3 embeds the partial equilibrium model into a general equilibrium setup, characterizing the equilibrium for a fixed aggregate supply of savings and analyzing the effects of an increase in the supply of savings. Section 4 considers three extensions of the general equilibrium model, which allow us to discuss the possible differences between the short- and long-run effects of a savings glut, the effect of having risk-averse instead of risk-neutral investors, and the way in which a savings glut can expand the set of (riskier) entrepreneurs that get funded. Section 5 analyzes a dynamic version of the model that generates endogenous booms and busts, and Section 6 concludes. The proofs of the analytical results are in the Appendix.

## 2 Partial Equilibrium

Consider an economy with two dates ( $t = 0, 1$ ), a large set of potential *entrepreneurs*, a large set of risk-neutral *investors*, and a single risk-neutral *bank*. Entrepreneurs have investment projects that require external finance, which can only come from the bank. The bank, in turn, needs to raise funds from the investors, which are characterized by an infinitely elastic supply of funds at an expected return equal to  $R_0$ .

Each entrepreneur has a project that requires a unit investment at  $t = 0$  and yields a stochastic return  $\tilde{R}$  at  $t = 1$  given by

$$\tilde{R} = \begin{cases} R, & \text{with probability } 1 - p + m, \\ 0, & \text{with probability } p - m, \end{cases} \quad (1)$$

where  $R$  and  $p$  are constant parameters, and  $m \in [0, p]$  is a variable that captures the bank's monitoring intensity. Monitoring increases the probability of getting the high return  $R$ , but entails a cost  $c(m)$ . The monitoring cost function  $c(m)$  satisfies  $c(0) = c'(0) = 0$ ,  $c'(m) \geq 0$ ,  $c''(m) > 0$ , and  $c'''(m) \geq 0$ . A special case that satisfies these assumptions and will be used for our numerical results is the quadratic function

$$c(m) = \frac{\gamma}{2}m^2, \quad (2)$$

where  $\gamma > 0$ . We assume that monitoring is not observed by the investors, so there is a moral hazard problem.

The bank can only fund a limited set of projects, taken to be just one for simplicity. Thus, entrepreneurs will be in the short side of the market and so they will only be able to borrow at the rate  $R$  that leaves them no surplus.

There are two possible modes of finance. The bank can keep the loan until maturity (*originate-to-hold*) or sell it to the investors (*originate-to-distribute*). We assume that the bank sells the loan when it is indifferent between keeping and selling it. Since monitoring is costly, and it is not observed by the investors, the bank will never monitor the entrepreneur when it is going to sell the loan, because it will get no compensation for its monitoring. Hence, *originate-to-hold* obtains when it is optimal for the bank to monitor the borrower (i.e. set  $m > 0$ ), and *originate-to-distribute* obtains when the bank prefers to do no monitoring (i.e. set  $m = 0$ ).

To characterize the optimal mode of finance, suppose that the bank borrows from the investors at a rate  $B$ , chooses a monitoring intensity  $m \in [0, p]$ , and lends to the entrepreneur at the rate  $R$ .

An *optimal contract* between the bank and the investors is a pair  $(B^*, m^*)$  that solves

$$\max_{(B,m)} [(1 - p + m)(R - B) - c(m)] \quad (3)$$

subject to the bank's incentive compatibility constraint

$$m^* = \arg \max_m [(1 - p + m)(R - B^*) - c(m)], \quad (4)$$

the bank's participation constraint

$$(1 - p + m^*)(R - B^*) - c(m^*) \geq 0, \quad (5)$$

and the investors' participation constraint

$$(1 - p + m^*)B^* = R_0. \quad (6)$$

The incentive compatibility constraint (4) characterizes the bank's choice of monitoring  $m^*$  given the promised repayment  $B^*$ , and the participation constraints (5) and (6) ensure that the bank makes nonnegative profits, net of the monitoring cost, and that the investors get the required expected return on their investment.

An interior solution to (4) is characterized by the first-order condition

$$R - B^* - c'(m^*) = 0. \quad (7)$$

Solving for  $B^*$  in the participation constraint (6), substituting it into the first-order condition (7), and rearranging gives the equation

$$c'(m) + \frac{R_0}{1 - p + m} = R. \quad (8)$$

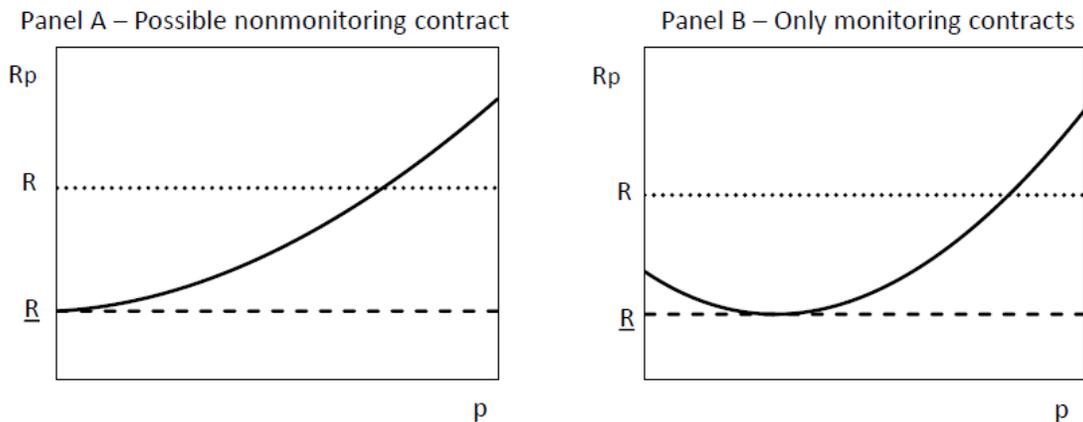
Since we have assumed  $c'''(m) \geq 0$ , the function in left-hand side of this equation is convex in  $m$ . Let  $\underline{R}$  denote the minimum value of this function in the feasible range  $[0, p]$ , that is

$$\underline{R} = \min_{m \in [0, p]} \left( c'(m) + \frac{R_0}{1 - p + m} \right). \quad (9)$$

The following result shows the condition under which bank finance is feasible and characterizes the corresponding optimal contract between the bank and the investors.

**Proposition 1** *Bank finance is feasible if  $R \geq \underline{R}$ , in which case the optimal contract between the bank and the investors is given by*

$$m^* = \max \left\{ m \in [0, p] \mid c'(m) + \frac{R_0}{1 - p + m} \leq R \right\} \text{ and } B^* = \frac{R_0}{1 - p + m^*}. \quad (10)$$



**Figure 1. Characterization of the optimal contract**

Panel A shows a case in which the optimal contract may entail zero monitoring (dashed line), and Panel B a case where the optimal contract always has positive monitoring.

Proposition 1 shows that if the minimum value  $\underline{R}$  defined by (9) is smaller than or equal to the lending rate  $R$ , bank finance is feasible and the optimal contract is characterized by the highest value of  $m$  that satisfies

$$c'(m) + \frac{R_0}{1 - p + m} \leq R.$$

Monitoring in the optimal contract may be at the corner with zero monitoring  $m^* = 0$ , at the corner with full monitoring  $m^* = p$ , or it may be interior  $m^* \in (0, p)$ . The first case corresponds to the originate-to-distribute mode of finance, while the second and third cases correspond to the originate-to-hold mode of finance.

Figure 1 illustrates the originate-to-distribute and the originate-to-hold modes of finance for the quadratic monitoring cost function. Panel A shows a case where the slope of the function in left-hand side of (8) is positive at the origin, in which case the optimal contract may entail  $m^* = 0$  (for  $R = \underline{R}$ ). Panel B shows a case where the slope of this function is negative at the origin, in which case the optimal contract always entails  $m^* > 0$ .

We next derive some interesting comparative static results on the optimal contract, assuming that it involves an interior level of monitoring.

**Proposition 2** *If  $R > \underline{R}$  we have*

$$\frac{\partial m^*}{\partial R_0} < 0 \text{ and } \frac{\partial m^*}{\partial R} > 0.$$

Thus, a reduction in the spread  $R - R_0$ , due to either an increase in the funding cost  $R_0$  or a decrease in the lending rate  $R$ , reduces optimal monitoring, thereby increasing the bank's portfolio risk. For sufficiently low spreads, the bank may find it optimal to choose zero monitoring, switching from originate-to-hold to originate-to-distribute.

Figure 1 illustrates the second result in Proposition 2: whenever bank finance is feasible, a reduction in the lending rate  $R$  (from the dotted to the dashed lines) always reduces monitoring  $m^*$ .

Summing up, we have presented a partial equilibrium model of bank finance under moral hazard that shows that *banks' monitoring incentives and hence banks' portfolio risk depends on the spread between lending and borrowing rates*. A reduction in the spread reduces monitoring, and makes it more likely that the bank will find it optimal to originate-to-distribute. However, the model assumes that interest rates are exogenous. To construct a model of the search for yield phenomenon we need to endogenize these rates, to which we turn now.

### 3 General Equilibrium

This section embeds our partial equilibrium model of bank finance into a general equilibrium setup in which a set of heterogeneous entrepreneurs seek finance for their risky projects. We characterize the equilibrium for a fixed aggregate supply of savings, showing that safer entrepreneurs will borrow from originate-to-distribute (shadow) banks while riskier entrepreneurs will borrow from originate-to-hold (traditional) banks. We then analyze the effects of an increase in the supply of savings, showing that it will lead to a reduction in interest rates and interest rate spreads, and an increase in the risk of the banking system.

Consider an economy with two dates ( $t = 0, 1$ ) and a large set of potential *entrepreneurs with observable types*  $p \in [0, 1]$ . Entrepreneurs have investment projects that require external finance, which can only come from banks. Banks are risk-neutral agents that specialize in

lending to specific types of entrepreneurs. To simplify the presentation, we will assume that for each type  $p$  of entrepreneurs there is a single bank that lends to them.<sup>4</sup> Banks, in turn, need to raise funds from a set of investors, which are characterized by a *fixed aggregate supply of savings*  $w$ .

Each entrepreneur of type  $p$  has a project that requires a unit investment at  $t = 0$  and yields a stochastic return  $\tilde{R}_p$  at  $t = 1$  given by

$$\tilde{R}_p = \begin{cases} R_p, & \text{with probability } 1 - p + m, \\ 0, & \text{with probability } p - m, \end{cases} \quad (11)$$

where  $m \in [0, p]$  is monitoring intensity of its bank. As before, monitoring is costly and the monitoring cost  $c(m)$  satisfies our previous assumptions. We assume that the success return  $R_p$  is a decreasing function  $R(x_p)$  of the aggregate investment of entrepreneurs of type  $p$ , denoted  $x_p$ . Thus, the higher the aggregate investment  $x_p$  the lower the return  $R_p$ .

This assumption may be rationalized by introducing a representative consumer with a CES utility function over the continuum of goods produced by entrepreneurs of types  $[0, 1]$ . As originally shown by Dixit and Stiglitz (1977), in this case the demand of goods of type  $p$  takes a simple functional form. Specifically, assuming a linear production function that transforms (in case of success) a unit of investment into  $\mu$  units of output, the equilibrium price  $R_p$  is determined by the condition  $\mu x_p = (R_p)^{-\sigma}$ , where  $\sigma > 1$  denotes the (constant) elasticity of substitution between any two goods.<sup>5</sup> From here it follows that

$$R(x_p) = (\mu x_p)^{-1/\sigma}. \quad (12)$$

This function will be used to derive the numerical results of the paper.

If the bank lending to entrepreneurs of type  $p$  sets a loan rate  $L_p$ , then a measure  $x_p$  of such entrepreneurs will enter the market until  $L_p = R_p = R(x_p)$ . Thus, as in the partial equilibrium setup, entrepreneurs will only be able to borrow at a rate that leaves them no surplus.

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<sup>4</sup>As will be clear below, this assumption is made without loss of generality. We could equally have many banks lending to each type of entrepreneur.

<sup>5</sup>We are setting equal to 1 the proportional term in the demand function that depends on the income of the representative consumer and the aggregate price index.

To pin down equilibrium loan rates, we assume that the market for lending to entrepreneurs of each type  $p$  is *contestable*. Thus, although there is a single bank that lends to each type, the incumbent could be undercut by an entrant if it were profitable to do so.

Finally, to simplify the presentation, we assume that the returns of the projects of entrepreneurs of each type  $p$  are perfectly correlated. This implies that the bank's return per unit of loans is identical to the individual project return, which is given by (11).

The strategy for the analysis is going to be as follows. First, we characterize the investment allocation corresponding to any given safe rate  $R_0$ , which is derived from the condition that investors must get the same expected return by funding a bank lending to risky entrepreneurs of type  $p > 0$  than by funding a bank lending to safe entrepreneurs of type  $p = 0$ . Then we introduce the market clearing condition that equates the aggregate demand for investment to the aggregate supply of savings to determine the equilibrium safe rate  $R_0^*$ .

By contestability, a bank lending to entrepreneurs of type  $p = 0$  will set the rate  $R_0$ , since at a lower rate it will make negative profits and at a higher rate it will be undercut by another bank. Similarly, banks lending to entrepreneurs of types  $p > 0$  will set the lowest feasible rate, which by Proposition 1 (together with the perfect correlation assumption) is given by

$$\underline{R}_p = \min_{m \in [0, p]} \left( c'(m) + \frac{R_0}{1 - p + m} \right). \quad (13)$$

The assumptions on the monitoring cost function  $c(m)$  imply that we have a corner solution with zero monitoring if and only if

$$c''(0) - \frac{R_0}{(1 - p)^2} \geq 0,$$

which gives  $p \leq \hat{p}$ , where

$$\hat{p} = 1 - \sqrt{\frac{R_0}{c''(0)}}. \quad (14)$$

Thus, banks lending to (safer) entrepreneurs of types  $p \leq \hat{p}$  will originate-to-distribute, and banks lending to (riskier) entrepreneurs of types  $p > \hat{p}$  will originate-to-hold. In what follows we will assume that  $R_0 < c''(0)$ , so  $\hat{p} \in (0, 1)$ .<sup>6</sup>

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<sup>6</sup>The model also works with  $R_0 \geq c''(0)$ , but in this case monitoring is so profitable that no bank would originate-to-distribute.

The intuition for this result is that since monitoring is especially useful for riskier entrepreneurs, they will have an incentive to borrow from originate-to-hold (monitoring) banks, and since monitoring is less useful for safer entrepreneurs (and useless for those with  $p = 0$ ), they will borrow from originate-to-distribute (nonmonitoring) banks.

For banks that originate-to-distribute ( $p \leq \hat{p}$ ) loan rates are given by

$$R_p = \underline{R}_p = \frac{R_0}{1 - p}, \quad (15)$$

where we have used the assumption  $c'(0) = 0$ . This result implies  $(1 - p)R_p - R_0 = 0$ , so the expected return of the banks' investments equals the funding cost. Thus, profits in the originate-to-distribute mode of finance will always be zero.

For banks that originate-to-hold ( $p > \hat{p}$ ) loan rates are given by

$$R_p = \underline{R}_p = c'(m_p) + \frac{R_0}{1 - p + m_p}, \quad (16)$$

where the monitoring intensity  $m_p$  satisfies the first-order condition<sup>7</sup>

$$c''(m_p) - \frac{R_0}{(1 - p + m_p)^2} = 0. \quad (17)$$

This result implies

$$(1 - p + m_p)R_p - R_0 - c(m_p) > (1 - p + m_p)c'(m_p) - m_p c'(m_p) = (1 - p)c'(m_p) > 0,$$

where we have used (16) and the fact that  $c(m_p) < m_p c'(m_p)$  by the convexity of the monitoring cost function. Thus, profits in the originate-to-distribute mode of finance will always be positive.

Differentiating (15) and applying the envelope theorem to (16) implies that loan rates  $R_p$  (and hence spreads  $R_p - R_0$ ) are increasing in the risk type  $p$ . Moreover, for originate-to-hold banks monitoring  $m_p$  is increasing in the risk type  $p$ , which can be proved by differentiating the first-order condition (17) and taking into account that  $c'''(m) \geq 0$ .

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<sup>7</sup>Notice that we cannot have a corner solution with  $m_p = p$  since the slope of the function in the right-hand-side of (13), evaluated at  $m_p = p$ , satisfies  $c''(p) - R_0 \geq c''(0) - R_0 > 0$ , where we have used  $c'''(m) \geq 0$  and  $R_0 < c''(0)$ .

Increases in the safe rate  $R_0$  lead to an increase in the spreads  $R_p - R_0$ . For originate-to-distribute banks this follows from the zero profit condition (15), which implies

$$R_p - R_0 = \frac{pR_0}{1-p}, \quad (18)$$

so spreads are linear in the safe rate  $R_0$ . For originate-to-hold banks we can apply the envelope theorem to (16), which gives

$$\frac{d(R_p - R_0)}{dR_0} = \frac{1}{1-p+m_p} - 1 = \frac{p-m_p}{1-p+m_p} > 0.$$

The positive effect of the safe rate  $R_0$  on the spread  $R_p - R_0$  leads to an increase in the monitoring intensity  $m_p$  of originate-to-hold banks, which can be proved by differentiating the first-order condition (17) and taking into account that  $c'''(m) \geq 0$ .

We can summarize the preceding results as follows.

**Proposition 3** *For any given safe rate  $R_0 < c''(0)$ , there exists a marginal type  $\hat{p} \in (0, 1)$  given by (14) such that banks lending to entrepreneurs of types  $p \leq \hat{p}$  will originate-to-distribute, and banks lending to entrepreneurs of types  $p > \hat{p}$  will originate-to-hold. Interest rate spreads  $R_p - R_0$  are increasing in the risk type  $p$  and satisfy*

$$\frac{d(R_p - R_0)}{dR_0} > 0.$$

*For banks that originate-to-hold, monitoring  $m_p$  is increasing in the risk type  $p$  and satisfies*

$$\frac{dm_p}{dR_0} > 0.$$

We are now ready to define an equilibrium, which requires to specify the investment  $x_p$  of the different types of entrepreneurs, and hence the rates  $R_p = R(x_p)$  at which they will borrow. By our previous results, both will be a function of the equilibrium safe rate  $R_0^*$ .

Formally, an *equilibrium* is an investment allocation  $\{x_p^*\}_{p \in [0,1]}$  and corresponding loan interest rates  $R_p^* = R(x_p^*)$  such that loan rates satisfy

$$R_p^* = \underline{R}_p^* = \min_{m \in [0,p]} \left( c'(m) + \frac{R_0^*}{1-p+m} \right), \text{ for all } p \in [0, 1], \quad (19)$$

and the market clears

$$\int_0^1 x_p^* dp = w. \quad (20)$$

Condition (19) follows from the assumption that the market for lending to entrepreneurs of each type  $p$  is contestable, so equilibrium loan rates will be at the lowest feasible level  $\underline{R}_p^*$  implied by the equilibrium safe rate  $R_0^*$ . Condition (20) ensures that the aggregate demand for investment is equal to the aggregate supply of savings  $w$ . Notice that the investors' participation constraint ensures that they all get the same expected return  $R_0^*$ , regardless of the type of bank they fund.

By Proposition 3, there will be an equilibrium marginal type

$$p^* = 1 - \sqrt{\frac{R_0^*}{c''(0)}} \quad (21)$$

such that banks lending to entrepreneurs of types  $p \leq p^*$  will originate-to-distribute, and banks lending to entrepreneurs of types  $p > p^*$  will originate-to-hold. We will restrict attention to (sufficiently high) values of  $w$  so that  $R_0^* < c''(0)$  and  $p^* \in (0, 1)$ .

### 3.1 An increase in the supply of savings

To analyze the effects of an exogenous increase in the supply of savings  $w$  notice that the market clearing condition (20) may be written as

$$F(R_0^*) = \int_0^1 R^{-1}(R_p^*) dp = w, \quad (22)$$

where  $x_p^* = R^{-1}(R_p^*)$  is the inverse function of  $R_p^* = R(x_p^*)$ . Since we have assumed  $R'(x_p) < 0$ , and Proposition 3 implies that  $R_p^*$  is increasing in  $R_0^*$ , we have  $F'(R_0^*) < 0$ , which implies

$$\frac{dR_0^*}{dw} = \frac{1}{F'(R_0^*)} < 0.$$

Hence, an increase in the aggregate supply of savings  $w$  leads to a decrease in the safe rate  $R_0^*$  and consequently in the rates  $R_p^*$  charged to entrepreneurs of all types  $p$ . This, in turn, implies a higher investment  $x_p^*$  for all types  $p$ .

Since the marginal type  $p^*$  in (21) is decreasing in the equilibrium safe rate  $R_0^*$ , the originate-to-distribute region will be larger. Moreover, by Proposition 3, the increase in  $w$

will reduce interest rate spreads  $R_p^* - R_0^*$  and the monitoring intensity  $m_p^*$  of originate-to-hold banks, so they will be riskier.

We can summarize these results as follows.

**Proposition 4** *An increase in the aggregate supply of savings  $w$  leads to*

1. *A reduction in the safe rate  $R_0^*$  and in the loan rates  $R_p^*$  of all types of entrepreneurs.*
2. *An increase in investment  $x_p^*$  and hence in the size of banks' lending to all types of entrepreneurs.*
3. *An expansion of the range  $[0, p^*]$  of entrepreneurs that borrow from banks that originate-to-distribute, and a shrinkage of the range  $[p^*, 1]$  of entrepreneurs that borrow from banks that originate-to-hold.*
4. *A reduction in interest rate spreads  $R_p^* - R_0^*$ .*
5. *An reduction in the monitoring intensity  $m_p^*$  (and hence an increase in the probability of failure  $p - m_p^*$ ) of originate-to-hold banks.*

We can illustrate these results for the case where the monitoring cost function is given by (2) and the relationship between the success return  $R_p$  and the aggregate investment of entrepreneurs of type  $p$  is given by (12).

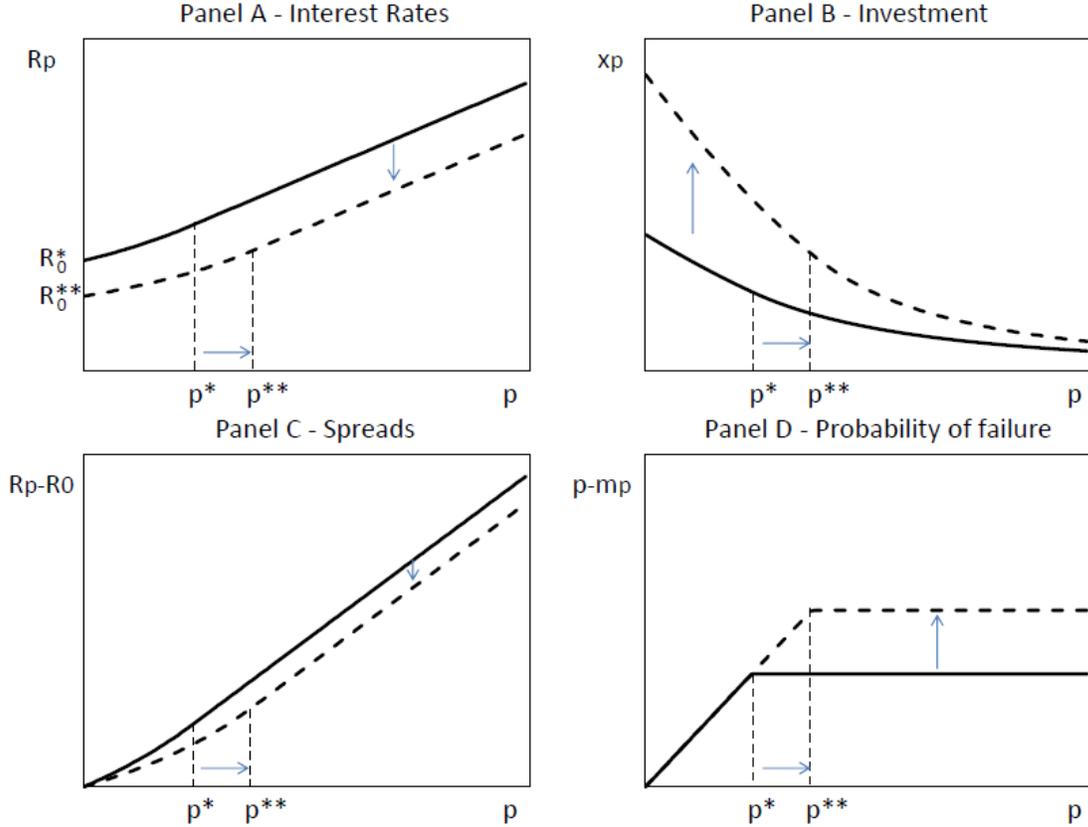
When the monitoring cost function is quadratic, solving the first-order condition (17) we obtain the following equilibrium monitoring intensity of originate-to-hold banks

$$m_p^* = p - \left(1 - \sqrt{\frac{R_0^*}{\gamma}}\right) = p - p^*, \text{ for } p > p^*.$$

This implies  $p - m_p^* = p^*$ , so *all banks that originate-to-hold have the same probability of failure*, which equals the type  $p^*$  of the marginal entrepreneur. Thus, in this case  $p^*$  is a sufficient statistic for the risk of the banking system.

Substituting this result in (16) gives the following equilibrium loan rates

$$R_p^* = \gamma(p - p^*) + \frac{R_0^*}{1 - p^*}. \tag{23}$$



**Figure 2. Effects of an increase in the supply of savings**

This figure shows the effects of an increase in the supply of savings on equilibrium loan rates (Panel A), investment (Panel B), spreads (Panel C), and the probability of failure (Panel D) for different types of entrepreneurs. Solid (dashed) lines represent equilibrium values before (after) the increase in savings.

Thus, with the quadratic monitoring cost function, equilibrium loan rates  $R_p^*$  (and spreads  $R_p^* - R_0^*$ ) for originate-to-hold banks are linear in the risk type  $p$ .

Figure 2 shows the effects of an increase in the aggregate supply of savings  $w$ . Equilibrium variables before the change are indicated with a star and represented by solid lines, while equilibrium variables after the change are indicated with two stars and represented by dashed lines. The horizontal axis of the four panels represents the entrepreneurs' types  $p$ . They all show the shift in the position of the marginal type from  $p^*$  to  $p^{**}$ . The intuition for this shift

is straightforward. The reduction in interest rate spreads associated with the increase in  $w$  implies that banks lending to entrepreneurs of types slightly above  $p^*$  will have an incentive to reduce their monitoring. But since  $m_p^*$  is close to zero they will move to a corner solution with  $m_p^{**} = 0$ , so the originate-to-distribute region will expand.

Panel A shows the effect on equilibrium loan rates. The increase in  $w$  shifts downwards the function  $R_p^*$  to  $R_p^{**}$ . The intercept of these functions is the interest rate charged to entrepreneurs of type  $p = 0$  (the safe rate), which goes down from  $R_0^*$  to  $R_0^{**}$ . To the left of the marginal types  $p^*$  and  $p^{**}$ , loan rates are convex in  $p$  (and given by (15)), while to the right of these points they are linear (and given by (23)).

Panel B shows the effect on equilibrium investment allocations. The increase in  $w$  shifts upwards the function  $x_p^*$  to  $x_p^{**}$ . The total amount of lending by banks that originate-to-distribute is clearly increasing, since banks that were initially using this mode of finance will increase their lending, and some banks that were monitoring their borrowers are now originating-to-distribute. The effect on the total amount of lending by banks that originate-to-hold is in principle ambiguous, because fewer banks monitor their borrowers although they become bigger. In our parameterization, lending by banks that originate-to-hold is also increasing, but the proportion of total lending that is accounted for by them goes down. In other words, these results show that *a savings glut increases the relative size of the originate-to-distribute (shadow) banking system*.

Panel C shows the effects on equilibrium spreads. As stated in Proposition 4, interest rate spreads go down from  $R_p^* - R_0^*$  to  $R_p^{**} - R_0^{**}$ . Since equilibrium loan rates for originate-to-hold banks are linear in  $p$  with a slope equal to  $\gamma$  (see (23)), it follows that for types riskier than  $p^{**}$  spreads will be reduced by a constant amount.<sup>8</sup>

Finally, Panel D shows the effect on equilibrium probabilities of bank failure. The shift of entrepreneurs with types in the interval between  $p^*$  and  $p^{**}$  from monitoring to nonmonitoring banks means that their probability of default will go up. Also, banks that originate-to-hold will increase their probability of failure from  $p - m_p^* = p^*$  to  $p - m_p^{**} = p^{**} > p^*$ . Thus, the increase in the aggregate supply of savings  $w$  has an *extensive margin effect* due to the

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<sup>8</sup>Substituting (21) into (23), we get  $R_p^* = \gamma p - \gamma + 2\sqrt{\gamma R_0^*}$ , so the constant is  $2(\sqrt{\gamma R_0^*} - \sqrt{\gamma R_0^{**}}) > 0$ .

shift of originate-to-distribute banks toward riskier entrepreneurs (shown by the horizontal arrows), and an *intensive margin effect* due to the reduction in the intensity of monitoring by originate-to-hold banks (shown by the vertical arrows). Hence, we conclude that *a savings glut increases the risk of the banking system*.

We have so far analyzed the effects of an exogenous shock to the supply of savings  $w$ . However, one can show that the same effects obtain when there is an exogenous decrease in the demand for investment, which in the context of our model could be simply captured by an increase in parameter  $\mu$  of the inverse demand for loans (12). Substituting this function into the market clearing condition (20) gives

$$\int_0^1 x_p^* dp = \frac{1}{\mu} \int_0^1 (R_p^*)^{-\sigma} dp = w.$$

Clearly, equilibrium allocations depend on the product  $\mu w$ , so we conclude that the effects of a savings glut are identical to the effects of a proportional fall in the demand for investment. Thus, we provide *a theoretical explanation of the way in which changes leading to a reduction in the equilibrium real rate of interest, as those noted by Summers (2014), can be linked to an increase in financial instability*.

Summing up, we have embedded a partial equilibrium (moral hazard) model of bank finance into a simple general equilibrium model of the determination of equilibrium interest rates. The results show that a savings glut (or a fall in the demand for investment) reduces interest rates and interest rate spreads, increases the relative size of the originate-to-distribute (shadow) banking system, and increases the probability of failure of the originate-to-hold (traditional) banks. These results provide a consistent explanation of a number of stylized facts of the period preceding the 2007-2009 financial crisis; see, for example, Brunnermeier (2009).

## 4 Extensions

This section analyzes three extensions of our general equilibrium model. First, we consider what happens if only originate-to-distribute banks can expand following the increase in the aggregate supply of savings. The implicit assumption is that originate-to-hold banks are

subject to some constraints that slow down their adjustment to the new environment. In this way, we intend to shed light on the possible differences between the short- and long-run effects of a savings glut. Second, we introduce risk-averse instead of risk-neutral investors. This will allow us to distinguish the effects of a change in the supply of savings from those of a change in investors' risk appetite. Finally, we consider a variation of the model in which high risk entrepreneurs may not be able to fund their projects. In this setup, a savings glut will have a new extensive margin effect, due to the shift in the upper bound of the range of entrepreneurs that get funded.

#### 4.1 Short- vs long-run effects of a savings glut

Suppose that we start from an initial equilibrium corresponding to an aggregate supply of savings  $w$ , and consider the effect of an increase  $\Delta w$  in  $w$  when originate-to-hold banks cannot increase the size  $x_p^*$  of their lending. Thus, the increase in savings will have to be accommodated by originate-to-distribute banks. This assumption may be justified by introducing some (unmodeled) adjustment costs that make it difficult for originate-to-hold banks to quickly increase their size.

The increase in the size of originate-to-distribute banks leads to a reduction in their loan rates, while the rates charged by originate-to-hold banks remain fixed at  $R_p^* = R(x_p^*)$ . Since the safe rate goes down, interest rates spreads increase for originate-to-hold banks. This will induce them to choose a higher monitoring intensity, which will reduce the risk of their portfolio. Thus, *in the short-run originate-to-hold banks will be safer*.

The new marginal type  $p^{**}$  will be determined by the condition

$$\frac{R_0^{**}}{1-p} = R(x_p^*). \quad (24)$$

The left-hand-side of this expression is the originate-to-distribute loan rate for entrepreneurs of type  $p$  corresponding to the new safe rate  $R_0^{**}$ , while the right-hand-side is the fixed originate-to-hold loan rate for entrepreneurs of type  $p$ . Since  $R(x_p^*)$  is increasing in  $p$ , it follows that the fall in the safe rate  $R_0^{**}$  will lead to a shift to the right in the position of the marginal type that separates the originate-to-distribute from the originate-to-hold regions.

Moreover,  $p^{**}$  will be higher than in the baseline model where originate-to-hold banks can increase their size, because in this model the loan rates in the right hand side of (24) will be lower.

From here it follows that the new equilibrium safe rate  $R_0^{**}$  is obtained by solving the market-clearing condition

$$\int_0^{p^{**}} x_p^{**} dp + \int_{p^{**}}^1 x_p^* dp = w + \Delta w. \quad (25)$$

The first term in the left-hand-side of (25) is the total amount of lending by banks that originate-to-distribute, where  $x_p^{**}$  solves the zero profit condition

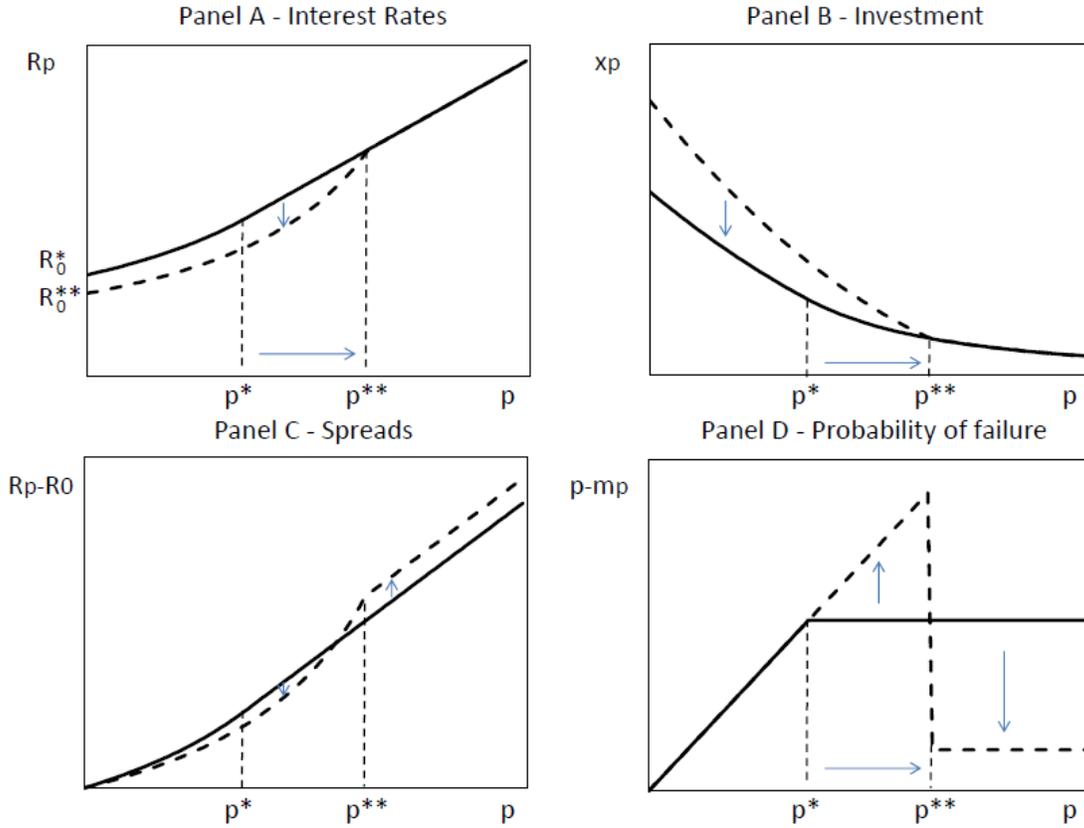
$$R(x_p^{**}) = \frac{R_0^{**}}{1-p}.$$

The second term in the left-hand-side of (25) is the total amount of lending by originate-to-hold banks, which by assumption maintain their initial lending  $x_p^*$ . The right-hand-side of (25) is the increased aggregate supply of savings.

Figure 3 shows the short-run effects of an increase in the aggregate supply of savings  $w$  for the same parameterization used in Section 3. As before, equilibrium variables before the change are indicated with a star and represented by solid lines, while equilibrium variables after the change are indicated with two stars and represented by dashed lines. The horizontal axis of the four panels represents the entrepreneurs' types  $p$ .

Panel A shows the effect on equilibrium loan rates. The increase in  $w$  leads to a reduction in rates from  $R_p^*$  to  $R_p^{**}$  but only for entrepreneurs funded by banks that originate-to-distribute. Entrepreneurs funded by banks that originate-to-hold will not experience any change in their loan rates.

Panel B shows the effect on equilibrium investment allocations. The increase in  $w$  shifts the position of the marginal type from  $p^*$  to  $p^{**}$ , and shifts upwards the function  $x_p^*$  to  $x_p^{**}$  for entrepreneurs in the new originate-to-distribute region. The total amount of lending by banks that originate-to-distribute increases by more than the increase in the aggregate supply of savings, while the total amount of lending by banks that originate-to-hold decreases. Hence, in the short-run a savings glut leads to a large expansion of the shadow banking system and a contraction of the traditional banking system.



**Figure 3. Short-run effects of an increase in the supply of savings**

This figure shows the effects of an increase in the supply of savings when traditional banks cannot expand their balance sheet on equilibrium loan rates (Panel A), investment (Panel B), spreads (Panel C), and the probability of failure (Panel D) for different types of entrepreneurs. Solid (dashed) lines represent equilibrium values before (after) the increase in savings.

Panel C shows the effects on equilibrium spreads. The results on loan rates imply that interest rate spreads will go down for types below  $p^*$  that were originally borrowing from originate-to-distribute banks, and will go up for types above  $p^{**}$  that remain borrowing from originate-to-hold banks. Consequently, spreads in the middle range that moves from originate-to-hold to originate-to-distribute will switch from lower to higher at some point in this range.

Finally, Panel D shows the effect on equilibrium probabilities of bank failure. Banks that originate-to-distribute will be lending to some riskier entrepreneurs that were funded before by originate-to-hold banks, so they will be originating riskier loans. On the other hand, banks that originate-to-hold will be able to borrow at the lower rate  $R_0^{**}$ , so they will enjoy higher spreads,  $R_p^* - R_0^{**}$ . This will induce them to choose a higher monitoring intensity  $m_p^{**}$ . Consequently their probability of failure will go down.

Thus, when banks that originate-to-hold cannot increase their balance sheet (and adjust their loan rates) in response to a savings glut, they will be a greater increase in the size of the shadow banking system, a greater reduction in the safe rate, and wider spreads for the traditional banks, so they will become safer. The assumption of a fixed size may be rationalized in terms of some capacity constraint that cannot be immediately relaxed. For example, we could assume that originate-to-hold banks are subject to a regulation that requires them to fund a fraction of their lending with equity capital, and that it takes some time to raise the capital required for any additional lending. But the effect will only be temporary, and as soon as they are able to relax the constraint they will have a much higher probability of failure, as shown by our previous results.

The results in this section show that to get an increases in the risk of failure of originate-to-hold banks it is essential that the savings glut be accompanied by what Shin (2012) calls a *banking glut*, that is an increase in the size of the traditional banking system. This provides a possible rationale for the use of (macroprudential) policies that slow down credit growth by traditional banks in order to deal with the impact on financial stability of changes in equilibrium interest rates. However, such policies should take into account the impact they might have on the shadow banking system.

## 4.2 Risk-averse investors

Our modeling so far has assumed that investors are risk-neutral. We now consider what happens when they are risk-averse. Specifically, consider a simple setup in which there is a continuum of measure  $w$  of atomistic investors with unit wealth that can be invested in only one bank (so we do not allow any portfolio diversification). Since each investor has a unit

wealth, the measure of investors  $w$  is equal to the aggregate supply of savings.

We assume that investors have a constant relative risk aversion utility function. Given that bank assets can yield a zero return, the coefficient of relative risk aversion is restricted to be between zero and one. Thus, we have

$$u(c) = c^{1/\alpha}, \quad (26)$$

where  $\alpha \geq 1$ . Risk-neutrality corresponds to the limit case  $\alpha = 1$ .

As in our baseline model, in equilibrium investors have to be indifferent between funding banks lending to different types of entrepreneurs. This implies that in the definition of an optimal contract between a bank lending to entrepreneurs of type  $p$  and the risk-averse investors, the participation constraint (6) becomes

$$(1 - p + m_p^*) (B_p^*)^{1/\alpha} = R_0^{1/\alpha},$$

which may be rewritten as

$$B_p^* = \frac{R_0}{(1 - p + m_p^*)^\alpha}. \quad (27)$$

Notice that the investors' expected payoff satisfies

$$(1 - p + m_p^*) B_p^* = \frac{R_0}{(1 - p + m_p^*)^{\alpha-1}} > R_0,$$

so they require positive risk premia.

Substituting (27) into the first-order condition (7) gives

$$c'(m_p) + \frac{R_0}{(1 - p + m_p)^\alpha} = R. \quad (28)$$

As before, the function in left-hand side of (28) is convex in  $m_p$ . Let  $\underline{R}_p^\alpha$  denote the minimum value of this function. Then, we can follow the same steps as in the proof of Proposition 1 to show that bank finance is feasible if  $R \geq \underline{R}_p^\alpha$ . In such case, the optimal contract between the bank and the investors is given by

$$m_p^* = \max \left\{ m \in [0, p] \mid c'(m) + \frac{R_0}{(1 - p + m)^\alpha} \leq R \right\}.$$

Thus, we have essentially the same characterization of the optimal contract as in the risk-neutral case analyzed in Section 2. The difference is that the convex function in the left-hand side of (28) is increasing in  $\alpha$ , so risk aversion makes it more difficult to ensure the feasibility of bank finance.

Following the same steps as in Section 3, we can characterize the equilibrium of the model with risk-averse investors. In this equilibrium, the marginal type is given by

$$p^* = 1 - \left( \frac{\alpha R_0^*}{c''(0)} \right)^{\frac{1}{1+\alpha}}.$$

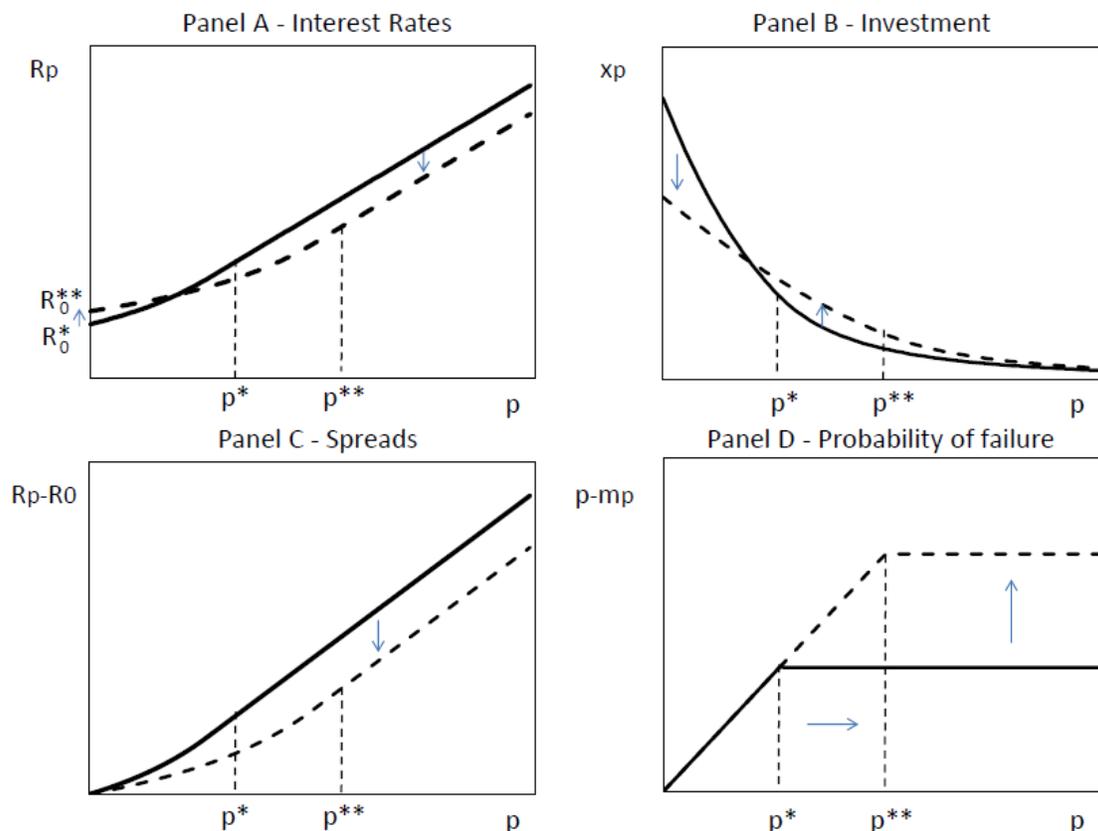
Notice that  $p^*$  is decreasing in the safe rate  $R_0^*$  (as before) and also in the risk-aversion parameter  $\alpha$ . Thus, risk-aversion increases the value of monitoring and consequently the comparative advantage of originate-to-hold banks.

Figure 4 shows the effect of a reduction in the investors' risk aversion from  $\alpha = 2$  to  $\alpha = 1$  (risk-neutrality) for the same parameterization used in Section 3. As before, equilibrium variables before the change are indicated with a star and represented by solid lines, while equilibrium variables after the change are indicated with two stars and represented by dashed lines. The horizontal axis of the four panels represents the entrepreneurs' types  $p$ .

Panel A shows the effect on equilibrium loan rates. The reduction in risk aversion shifts the investors' preferences toward riskier assets, so loan rates go down for riskier entrepreneurs and increase for safer entrepreneurs. In particular, the safe rate will go up from  $R_0^*$  to  $R_0^{**}$ . The increase in the safe rate further reduces the comparative advantage of originate-to-hold banks, which explains the shift the position of the marginal type from  $p^*$  to  $p^{**}$ .

Panel B shows the effect on equilibrium investment allocations. The reduction in risk aversion produces a redistribution in the allocation of savings toward riskier entrepreneurs. Since the aggregate supply of savings is fixed, this means that investment in safer projects falls. However, the shift in the position of the marginal type from  $p^*$  to  $p^{**}$  implies that the effect on the relative size of the shadow banking system is ambiguous.

Panel C shows the effects on equilibrium spreads. The results on loan rates imply that interest rate spreads go down from  $R_p^* - R_0^*$  to  $R_p^{**} - R_0^{**}$ . This reduces the incentives to monitor and hence the probability of failure of originate-to-hold banks, which is shown in



**Figure 4. Effects of a reduction in investors' risk aversion**

This figure shows the effects of a change in investors' preferences from risk aversion to risk neutrality on equilibrium loan rates (Panel A), investment (Panel B), spreads (Panel C), and the probability of failure (Panel D) for different types of entrepreneurs. Solid (dashed) lines represent equilibrium values before (after) the change in preferences.

Panel D. As in the case of a savings glut, a reduction in risk-aversion has an extensive margin effect due to the shift of originate-to-distribute banks toward riskier borrowers (shown by the horizontal arrows), and an intensive margin effect due to the reduction in the intensity of monitoring by originate-to-hold banks (shown by the vertical arrows).

These results illustrate the differences between the effects of a savings glut from the effects of a reduction in the investors' risk appetite. Both changes lead to a reduction in

interest rate spreads and an increase in the probability of failure of originate-to-hold banks, but there are some significant differences. A savings glut increases funding for all types of entrepreneurs and the size of the shadow banking system, while a fall in risk aversion reduces funding for safer entrepreneurs and has an ambiguous effect on the size of the shadow banking system. A simple way to empirically distinguish the two changes is to look at the effect on the equilibrium safe rate: it goes down in the case of a savings glut and it goes up in the case of a reduction in risk aversion.

### 4.3 Endogenous range of entrepreneurs' types

One feature of the numerical illustration of the baseline model and the extensions considered so far is that the full range  $[0, 1]$  of entrepreneurs' types is funded by either originate-to-distribute or originate-to-hold banks. This follows from the fact that the inverse demand for loans  $R(x_p) = (\mu x_p)^{-1/\sigma}$  is such that  $\lim_{x_p \rightarrow 0} R(x_p) = +\infty$ , so funding very risky projects may be profitable when investment in them is arbitrarily small.

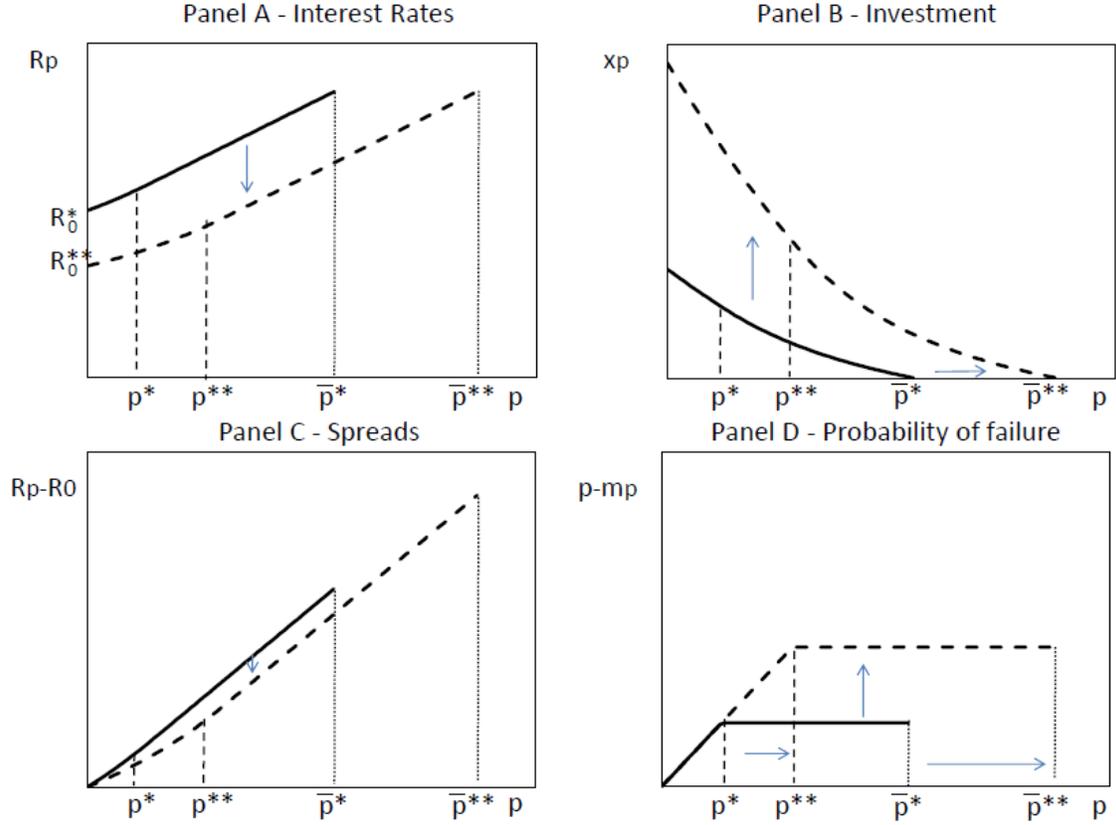
We next consider what happens when the inverse demand for loans  $R(x_p)$  is bounded. A simple way to do this is by changing the previous functional to

$$R(x_p) = (\mu x_p + \theta)^{-1/\sigma}, \quad (29)$$

where  $\theta > 0$ . In this case, one can show that there is a critical type  $\bar{p}$  such that entrepreneurs of types riskier than  $\bar{p}$  will not be able to borrow.

Figure 5 shows the effects of an increase in the aggregate supply of savings  $w$  for the case in which the relationship between the success return  $R_p$  and the aggregate investment of entrepreneurs of type  $p$  is given by (29). As before, equilibrium variables before the change are indicated with a star and represented by solid lines, while equilibrium variables after the change are indicated with two stars and represented by dashed lines. The horizontal axis of the four panels represents the entrepreneurs' types  $p$ .

The results in Panels A-D are similar to those in the corresponding panels of Figure 2, except for the fact that the increase in the supply of savings has a new extensive margin effect (shown by the second set of horizontal arrows), moving the riskiest type that will be



**Figure 5. Effects of an increase in the supply of savings with bounded project returns**

This figure shows the effects of an increase in the supply of savings when project returns are bounded on equilibrium loan rates (Panel A), investment (Panel B), spreads (Panel C), and the probability of failure (Panel D) for different types of entrepreneurs. Solid (dashed) lines represent equilibrium values before (after) the increase in savings.

funded from  $\bar{p}^*$  to  $\bar{p}^{**}$ . Thus, in this setup a savings glut makes it possible for some riskier entrepreneurs to be funded.

## 5 Endogenous Booms and Busts

We have so far analyzed the equilibrium of a static model for a given aggregate supply of savings and shown how an exogenous change in this supply affects the risk of the banking

system. This section analyzes a dynamic extension of our static model in which the aggregate supply of savings at any date is the outcome of agents' decisions at the previous date together with the realization of a systematic shock that affects the return of entrepreneurs' projects.

The dynamic model generates endogenous booms and busts. The intuition is straightforward: the accumulation of savings leads to a reduction in interest rates and interest rate spreads and an increase in risk-taking that eventually materializes in a bust, which reduces savings, increasing interest rates and interest rate spreads and reducing risk-taking, starting again the process of wealth accumulation that leads to a boom.

Suppose that at each date  $t$  we have a continuum of one-period-lived entrepreneurs of types  $p \in [0, 1]$  that have investment projects that can only be funded by banks. As before, we assume that banking sector is contestable and that there is a single bank that lends to entrepreneurs of type  $p$  at date  $t$ , choosing the monitoring intensity  $m_{pt} \in [0, p]$ .

The project of an entrepreneur of type  $p$  yields at date  $t$  a return  $R_{pt} = R(x_{pt})$  with probability  $p$  and zero with probability  $1-p+m_{pt}$ , where  $x_{pt}$  denotes the aggregate investment of entrepreneurs of type  $p$  at date  $t$ , and  $R(x)$  is given by (12).

To simplify the presentation, we assume that banks are run by penniless one-period-lived bankers that consume the profits that they may obtain before they die. This implies that banks have no equity capital and bank profits do not contribute to the accumulation of wealth.

Banks need to raise from investors the funds lent to entrepreneurs. At each date  $t$ , there is a continuum of measure  $w_t$  of infinitely-lived risk-neutral atomistic investors with unit wealth. Investors have a discount factor  $\beta \in (0, 1)$  and the period utility function is given by  $u(c_t) = c_t$ .

To describe the dynamics of wealth accumulation we need a model of the realization of project returns. We will maintain the assumption that the returns of the projects of entrepreneurs of each type  $p$  are perfectly correlated, but will assume that project returns are correlated across types. Specifically, we will use the *single risk factor model* of Vasicek (2002) in which the outcome of the projects of entrepreneurs of type  $p$  is driven by the

realization of a latent random variable

$$y_{pt} = -\Phi^{-1}(p - m_{pt}) + \sqrt{\rho} z_t + \sqrt{1 - \rho} \varepsilon_{pt}, \quad (30)$$

where  $z$  is a *systematic risk factor* that affects all types of entrepreneurs,  $\varepsilon_{pt}$  is an *idiosyncratic risk factor* that only affects the projects of entrepreneurs of type  $p$ ,  $\rho \in (0, 1)$  is a parameter that determines the extent of correlation in the returns of the projects of entrepreneurs of different types,  $\Phi(\cdot)$  denotes the cdf of a standard normal random variable and  $\Phi^{-1}(\cdot)$  its inverse. It is assumed that  $z_t$  and  $\varepsilon_{pt}$  are standard normal random variables, independently distributed from each other as well as, in the case of  $\varepsilon_{pt}$ , across types. They are also independent over time.

The projects of entrepreneurs of type  $p$  fail at date  $t$  when  $y_{pt} < 0$ . The (unconditional) probability of failure is

$$\Pr(y_{pt} < 0) = \Pr \left[ \sqrt{\rho} z_t + \sqrt{1 - \rho} \varepsilon_{pt} < \Phi^{-1}(p - m_{pt}) \right] = p - m_{pt}.$$

The probability of failure conditional on the realization of the systematic risk factor  $z_t$  is

$$\Pr(y_{pt} < 0 \mid z_t) = \Pr \left[ \sqrt{\rho} z_t + \sqrt{1 - \rho} \varepsilon_{pt} < \Phi^{-1}(p - m_{pt}) \mid z_t \right] = \Phi \left( \frac{\Phi^{-1}(p - m_{pt}) - \sqrt{\rho} z_t}{\sqrt{1 - \rho}} \right).$$

The dynamic behavior of aggregate wealth is then given by

$$w_{t+1} = g(w_t, z_t) = \int_0^1 \delta(p - m_{pt}, z_t) x_{pt} B_{pt} dp, \quad (31)$$

where  $\delta(p - m_{pt}, z_t)$  denotes the conditional probability of success defined by

$$\delta(p - m_{pt}, z_t) = \Pr(y_{pt} \geq 0 \mid z_t) = \Phi \left( \frac{\sqrt{\rho} z_t - \Phi^{-1}(p - m_{pt})}{\sqrt{1 - \rho}} \right). \quad (32)$$

The integrand of this expression is the product of the conditional (on the realization of the systematic risk factor  $z_t$ ) probability of success of the projects of entrepreneurs of type  $p$  at date  $t$ , multiplied by the payment to investors in case of success, which is equal to the the product of the investment  $x_{pt}$  by the interest rate at which they lend to the corresponding bank,  $B_{pt}$ . Since the systematic risk factor  $z_t$  is a random variable, the dynamic behavior of aggregate wealth will also be random.

We assume that investors can either consume their unit wealth, invest it in the bank lending to entrepreneurs of type  $p = 0$ , or invest it in the bank lending to entrepreneurs of a type  $p > 0$ . Let  $s_0 \in [0, 1]$  and  $s_p \in [0, 1]$  denote the amounts invested in the two banks, and  $c = 1 - s_0 - s_p \in [0, 1]$  the amount consumed. The Bellman equation is then given by

$$v(w_t) = \max_{(s_0, s_p)} \{1 - s_0 - s_p + \beta [s_0 R_{0t} E [v(w_{t+1})] + s_p B_{pt} E [\delta(p - m_{pt}, z_t) v(w_{t+1})]]\}. \quad (33)$$

Since  $\lim_{x \rightarrow 0} R(x) = \infty$ , in equilibrium we must have  $s_0 > 0$  and  $s_p > 0$ . Then, differentiating the right-hand side of (33) with respect to  $s_0$  and  $s_1$ , equating to zero the resulting expressions, and subtracting one from the other, gives the following condition

$$R_{0t} E [v(w_{t+1})] = B_{pt} E [\delta(p - m_{pt}, z_t) v(w_{t+1})]. \quad (34)$$

This condition states that the investor must be indifferent between lending to the two banks. Substituting (34) into the Bellman equation (33) gives

$$v(w_t) = \beta R_{0t} E [v(w_{t+1})] = \beta B_{pt} E [\delta(p - m_{pt}, z_t) v(w_{t+1})]. \quad (35)$$

Investors will distribute themselves over the set of risky types  $p > 0$ , so this condition holds for all  $p$ .<sup>9</sup>

In equilibrium it must be the case that  $v(w) \geq 1$ , since the investor can always set  $s_0 = s_p = 0$ , consuming all her wealth, which gives  $u(1) = 1$ . It must also be the case that  $v(w) > 1$  only if  $c = 1 - s_0 - s_p = 0$ , since if  $c > 0$  the marginal utility of lending to any of the two banks must be equal to the marginal utility of consumption which is one. Let us now define

$$\hat{w} = \inf \{w \mid v(w) = 1\}. \quad (36)$$

Clearly, we have  $v(w) = 1$  for all  $w \geq \hat{w}$ . Thus, when  $w < \hat{w}$  the value of one unit of wealth is greater than one and investors do not consume, while when  $w \geq \hat{w}$  the value of one unit of wealth is equal to one and they invest  $\hat{w}$  and devote the difference  $w - \hat{w}$  to consumption.

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<sup>9</sup>Notice that (35) implies that the debt of all the banks is priced using a stochastic discount factor equal to  $\beta v(w_{t+1})/v(w_t)$ .

Hence, the aggregate consumption of investors is given by

$$c(w_t) = \begin{cases} w_t - \widehat{w}, & \text{for } w_t \geq \widehat{w}, \\ 0, & \text{for } w_t < \widehat{w}. \end{cases} \quad (37)$$

Following the same steps as in the analysis of the static model in Section 3, and solving for  $B_{pt}$  in (34), one can show that banks lending to entrepreneurs of types  $p > 0$  will set the lowest feasible rate, which is given by

$$\underline{R}_{pt} = \min_{m_{pt} \in [0, p]} \left( c'(m_{pt}) + \frac{R_{0t} E[v(w_{t+1})]}{E[\delta(p - m_{pt}, z_t)v(w_{t+1})]} \right). \quad (38)$$

Notice that in the static model we have  $v(w_{t+1}) = 1$ , which implies  $E[\delta(p - m_{pt}, z_t)v(w_{t+1})] = 1 - p + m_{pt}$ , so (38) becomes (13).

We are now ready to define an equilibrium, which requires to specify the investment  $x_{pt}$  of the different types of entrepreneurs, and hence the rates  $R_{pt} = R(x_{pt})$  at which they will borrow from the banks, the rates  $B_{pt}$  at which the banks will borrow from the investors, and their monitoring intensity  $m_{pt}$ . All these variables depend on the wealth  $w_t$  of the investors, which is the state variable of the dynamic model. The equilibrium also requires to specify the value function of the investors  $v(w_t)$ , their aggregate consumption decision  $c(w_t)$ , and the dynamics of wealth accumulation.

Formally, an *equilibrium* is an array  $\{x_p^*(w_t), R_p^*(w_t), B_p^*(w_t), m_p^*(w_t)\}_{p \in [0, 1]}$ ,  $v(w_t)$ ,  $c(w_t)$ ,  $g(w_t, z_t)$  such that

1. Entrepreneurs' investment decisions satisfy  $R_p^*(w_t) = R(x_p^*(w_t))$ ,
2. Banks' lending rates  $R_p^*(w_t)$  equal  $\underline{R}_{pt}$  in (38),
3. Banks' borrowing rates  $B_p^*(w_t)$  satisfy (34),
4. Banks' monitoring intensity  $m_p^*(w_t)$  solves (38),
5. The value function  $v(w_t)$  satisfies (35),
6. The consumption function  $c(w_t)$  satisfies (37),

7. The investors' wealth  $w_t$  evolves according to (31), and

8. The market clears

$$\int_0^1 x_p^*(w_t) dp = w_t - c(w_t).$$

It should be noticed that the indifference condition (34) implies that the investors' expected payoff satisfies

$$(1-p+m_{pt})B_{pt} = \frac{(1-p+m_p)E[v(w_{t+1})]}{E[\delta(p-m_{pt}, z_t)v(w_{t+1})]}R_{0t} = \left[1 + \frac{Cov(\delta(p-m_{pt}, z_t), v(w_{t+1}))}{E[v(w_{t+1})]}\right]^{-1} R_{0t} > R_{0t},$$

for all  $p > 0$ , where we have used

$$E[\delta(p-m_{pt}, z_t)v(w_{t+1})] = (1-p+m_p)E[v(w_{t+1})] + Cov(\delta(p-m_{pt}, z_t), v(w_{t+1}))$$

and  $Cov(\delta(p-m_{pt}, z_t), v(w_{t+1})) < 0$ .<sup>10</sup> Thus, *investors require positive risk premia for funding the risky banks*. In other words, they behave as if they were risk-averse.

We can illustrate the equilibrium of the dynamic model using the parameterization in Section 3.<sup>11</sup> An interesting result that also obtains here with the quadratic monitoring cost function (2) is that there exists a marginal type  $p_t^*$  such that banks lending to entrepreneurs of types  $p \leq p_t^*$  will originate-to-distribute, setting  $m_{pt}^* = 0$ , and banks lending to types  $p > p_t^*$  will originate-to-hold, setting  $m_{pt}^* = p - p_t^*$ . This implies  $p - m_{pt}^* = p_t^*$ , so *all banks that originate-to-hold have the same probability of failure*, which equals the type  $p_t^*$  of the marginal entrepreneur.<sup>12</sup>

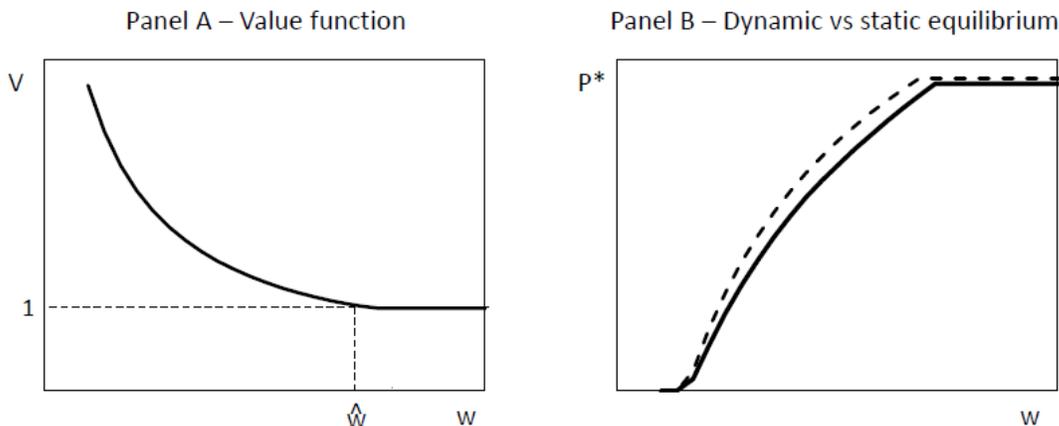
<sup>10</sup>This follows from the fact that both  $\delta(p-m_{pt}, z_t)$  and  $w_{t+1} = g(w_t, z_t)$  are increasing in  $z_t$ , and  $v'(w_{t+1}) \leq 0$ , with strict inequality for low values of  $w_{t+1}$ .

<sup>11</sup>We assume a discount factor  $\beta = 0.96$  and a correlation parameter  $\rho = 0.15$ .

<sup>12</sup>To prove this result, suppose that the solution to (38) for some  $p$  satisfies  $m_{pt}^* > 0$ . Then,  $m_{pt}^*$  satisfies the first-order condition

$$\gamma + \frac{d}{dm_{pt}} \left( \frac{R_{0t}^* E[v(w_{t+1})]}{E[\delta(p-m_{pt}, z_t)v(w_{t+1})]} \right) \Big|_{m_{pt}=p-p_t^*} = 0.$$

But given the functional form of  $\delta(p-m_{pt}, z_t)$  in (32), it follows that this condition is also satisfied for any  $p \geq p_t^*$  (so that the corresponding  $m_{pt}^* = p - p_t^* \geq 0$ ). Moreover, for the same reason, there cannot be an interior solution for  $p < p_t^*$ , which proves the result.



**Figure 6. Value function and dynamic vs static equilibrium**

This figure shows the value of one unit of wealth in the dynamic model (Panel A), and compares the dynamic (solid) with static (dashed) probability of failure of the traditional banks (Panel B) for a range of values of the state variable  $w$ .

Since (34) and (38) imply

$$R_{pt}^* = \gamma m_{pt}^* + B_{pt}^*,$$

it follows that banks' lending and borrowing rates,  $R_{pt}^*$  and  $B_{pt}^*$ , coincide for  $p \leq p_t^*$ , that is for banks that originate-to-distribute, and satisfy  $R_{pt}^* > B_{pt}^*$  for  $p > p_t^*$ , that is for banks that originate-to-hold. In this case, as discussed in Section 3, the intermediation margin  $R_{pt}^* - B_{pt}^*$  covers the monitoring cost and leaves some positive profits for the banks.

Panel A of Figure 6 shows the value function  $v(w_t)$ , which is decreasing and convex for  $w_t < \hat{w}$ , and satisfies  $v(w_t) = 1$  for  $w_t \geq \hat{w}$ . By (32) and (31), for sufficiently low values of the systematic risk factor  $z_t$  we have  $w_{t+1} < \hat{w}$ , in which case wealth will be very valuable since it will be scarce. As noted above, this implies  $Cov(\delta(p - m_{pt}, z_t), v(w_{t+1})) < 0$ , so investors will require positive risk premia for funding the risky banks. This opens up the spreads and increases the incentives to monitor entrepreneurs, and hence the comparative advantage of originate-to-hold banks. Thus, in this regard *the dynamic model behaves as the static model with risk-averse investors*.

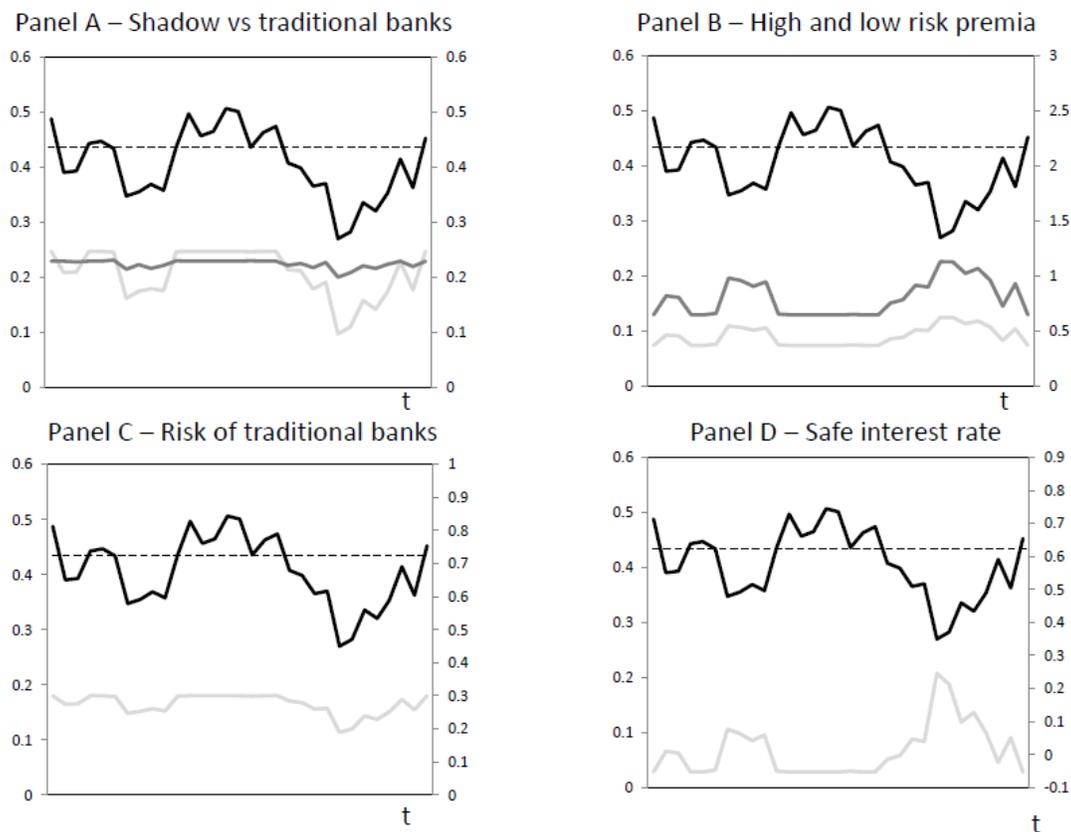
Panel B of Figure 6 illustrates this effect by comparing, for different values of the state variable  $w_t$ , the marginal type  $p_t^*$  for the dynamic model (solid line) with that of the static model with risk-neutral investors (dashed line). The former is everywhere below the latter, except for low values of  $w_t$  when  $p_t^* = 0$  in both models. This means that the dynamic model features a smaller relative size of the originate-to-distribute banking system, and a lower probability of failure of the originate-to-hold banks. Thus, the forward-looking behavior of investors contributes to the stability of the banking system.

The dynamic model yields a number of empirically relevant and potentially testable relationships between aggregate variables. In order to highlight some of these relationships, in what follows we consider a sample realization of the (iid) systematic risk factor  $z_t$  and look at the corresponding evolution of investors' wealth  $w_t$  over time  $t$  together with some interesting variables. The black line of Panels A-D of Figure 7 represents  $w_t$ , and is measured in the left-hand-side axis, and the dashed line shows the value  $\hat{w}$  above which investors consume  $w_t - \hat{w}$ .

In Panel A the dark grey line plots the total amount of lending by (traditional) banks that originate-to-hold, that is  $\int_{p_t^*}^1 x_p^*(w_t) dp$ , and the light grey line plots the total amount of lending by (shadow) banks that originate-to-distribute, that is  $\int_0^{p_t^*} x_p^*(w_t) dp$ . Although lending by both traditional and shadow banks is positively correlated with investors' wealth, most of the variation in  $w_t$  is channeled through shadow banks. In other words, *the shadow banking system is highly procyclical*, a result that is consistent with the evidence in Pozsar et al. (2012) that shows that shadow bank liabilities grew much faster than traditional banking liabilities in the run-up to the crisis, and contracted substantially since the peak in 2007.

The grey lines in Panel B show the evolution of the risk premia  $(1 - p + m_{pt}^*)B_{pt}^* - R_{0t}^*$ , measured in percentage points in the right-hand axis, for two types that are always funded by shadow banks.<sup>13</sup> The dark (light) grey corresponds to a higher (lower) type  $p$ . Risk premia are negatively correlated with  $w_t$ , and are higher for riskier types. The fact that risk premia go down when wealth accumulates means that the dynamic model yields *countercyclical risk*

<sup>13</sup>Specifically, we take  $p = 0.05$  (light grey) and  $p = 0.1$  (dark grey) such that they are below the marginal type  $p_t^*$  for all  $t$ .



**Figure 7. Model dynamics**

The black line in all panels shows the dynamic evolution of investors' wealth for a sample realization of the systematic risk factor, and the dashed line shows the threshold level of wealth above which investors consume. Panel A plots the total amount of lending by traditional banks (dark grey) and shadow banks (light grey), Panel B the risk premia for investors in two different shadow banks, Panel C the probability of failure of the traditional banks, and Panel D the safe rate (in net terms).

*premia*.<sup>14</sup> In other words, investors behave as if they were less risk-averse (or have greater risk appetite) during financial booms compared to busts, although their underlying (risk-neutral) preferences do not change.

The grey line in Panel C shows the evolution of the type  $p_t^*$  of the marginal entrepreneur, measured in the right-hand axis. This variable is positively correlated with  $w_t$ . Thus, in line

<sup>14</sup>This result becomes weaker for types that are funded by traditional banks, since in this case higher wealth leads to lower monitoring and hence riskier banks.

with the results of the static model, higher wealth increases the relative size of the shadow banking system, and increases the probability of failure of the traditional banks.

Finally, the grey line in Panel D shows the evolution of the safe rate  $R_{0t}^*$ , measured in net terms in the right-hand axis. This variable is negatively correlated with  $w_t$ . Moreover, our numerical results show that *the safe rate can be below the discount rate  $1/\beta$  and even become negative in boom states.*<sup>15</sup> The intuition is that the expectation of positive returns in the future (when the economy is hit by a negative shock and wealth is very valuable) makes investors willing to forgo current consumption, which lowers the safe rate.

As previously explained, the rationale underlying the observed dynamics is as follows: good realizations of the systematic risk factor result in higher aggregate wealth which reduces interest rates and interest rate spreads, increases the size of the shadow banking system, and increases the probability of failure of the traditional banks, making the banking system especially vulnerable to a bad realization of the systematic risk factor. Therefore, our model provides a framework for understanding the emergence of endogenous boom and bust cycles, as well as the procyclical nature of the shadow banking system, the existence of counter-cyclical risk premia, and the low levels of interest rates and interest rate spreads during booms.

## 6 Conclusion

This paper presents a general equilibrium model of the connection between real interest rates, credit spreads, and the structure and the risk of the banking system. Banks intermediate between a heterogeneous set of entrepreneurs and a set of investors characterized by a fixed aggregate supply of savings. We assume that all agents are risk-neutral and that banks can monitor entrepreneurs' projects at a cost, but this is not observed by investors. This moral hazard problem is the key friction that drives the results of the model. We also assume that project returns are decreasing in the aggregate investment of entrepreneurs of each type, and that the market for lending to entrepreneurs is contestable.

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<sup>15</sup>Notice that for  $w_t \geq \hat{w}$  we have  $v(w_t) = 1$  and  $E[v(w_{t+1})] > 1$ , so (35) implies  $\beta R_{0t} < 1$ , i.e.  $R_{0t} < 1/\beta$ .

We first characterize the equilibrium of the model, showing that safer entrepreneurs will be funded by banks that do not monitor their projects and riskier entrepreneurs by banks that monitor them. We assume that monitoring requires keeping the loans in the banks' books, and for this reason we associate nonmonitoring banks with intermediaries that originate-to-distribute (called shadow banks) and monitoring banks with intermediaries that originate-to-hold (called traditional banks). We then analyze the effects of an increase in the supply of savings, showing that it will lead to a reduction in interest rates and interest rate spreads, an expansion of the relative size of the shadow banking system, and a reduction in the monitoring intensity and hence an increase in the probability of failure of the traditional banks.

We extend our basic static framework to a dynamic setup in which the aggregate supply of savings at any date is the outcome of agents' decisions at the previous date together with the realization of a systematic risk factor that affects the return of entrepreneurs' projects. The dynamic model generates endogenous booms and busts: the accumulation of savings leads to a reduction in interest rates and interest rate spreads and an increase in risk-taking that eventually materializes in a bust, which reduces savings, starting again the process of wealth accumulation that leads to a boom. The model also yields a number of empirically relevant results such as the procyclical nature of the shadow banking system, the existence of countercyclical risk premia, and the low levels of interest rates and interest rate spreads leading to the buildup of risks during booms.

These results provide a theoretical explanation for the facts noted by Bernanke (2005) and Rajan (2005) that link the global savings glut with the low level of real interest rates, and the incentives to search for yield by financial intermediaries. They also rationalize the way in which changes leading to a reduction in real interest rates, as those noted by Summers (2014), can be linked to an increase in financial instability. Moreover, the results provide a rationale for a number of empirical facts in the run-up of the 2007-2009 financial crisis.

It should be noted that although the model focuses on bank monitoring, a similar story could be constructed if banks could increase the quality its pool of loan applicants by screen-

ing them at a cost.<sup>16</sup> It should also be noted that the paper entirely focuses on debt finance, abstracting from the possibility of funding the banks with (inside) equity. Since equity finance would strengthen the banks' monitoring incentives, adding this possibility would require the introduction of a differential cost of equity.<sup>17</sup> Finally, it is important to stress that this is a real model without any nominal frictions, so monetary policy is completely absent from our story of search for yield. Introducing nominal frictions would allow to study the connection between monetary policy and financial stability, a topic that would merit a separate paper.

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<sup>16</sup>This setup would be similar to the one in Helpman, Itskhoki and Redding (2010), where firms can increase the average ability of the workers they hire by paying a screening cost.

<sup>17</sup>See Martinez-Miera and Repullo (2015) for a model along these lines that is used to discuss the effects of different types of bank capital requirements.

## Appendix

**Proof of Proposition 1** If  $R < \underline{R}$ , for any  $m \in (0, p]$  we have

$$R - \frac{R_0}{1 - p + m} - c'(m) < 0,$$

which implies that the bank has an incentive to reduce monitoring  $m$ . But for  $m = 0$  we have

$$R - \frac{R_0}{1 - p} - c'(0) < 0,$$

which, using the assumption  $c(0) = c'(0) = 0$ , implies  $(1 - p)R - R_0 - c(0) < 0$ , which violates the bank's participation constraint (5).

If  $R \geq \underline{R}$ , by the convexity of the function in the left-hand side of (8) there exist an interval  $[m^-, m^*] \subset [0, p]$  such that

$$R - \frac{R_0}{1 - p + m} - c'(m) \geq 0 \quad \text{if and only if} \quad m \in [m^-, m^*].$$

By our previous argument, for any  $m \in (0, p]$  for which

$$R - \frac{R_0}{1 - p + m} - c'(m) < 0,$$

the bank has an incentive to reduce monitoring  $m$ . Similarly, for any  $m \in [0, p)$  for which

$$R - \frac{R_0}{1 - p + m} - c'(m) > 0,$$

the bank has an incentive to increase monitoring  $m$ . Hence, there are three possible candidates for monitoring in the optimal contract:  $m = m^*$ ,  $m = m^-$ , and  $m = 0$  (when  $m^- > 0$ ).

To prove that the bank prefers  $m = m^*$ , let

$$B^* = \frac{R_0}{1 - p + m^*} \quad \text{and} \quad B = \frac{R_0}{1 - p + m}$$

for either  $m = m^-$  or  $m = 0$  (if applicable). Since  $m^* \geq m$  implies  $B^* \leq B$  and  $c(m^*) \geq c(m)$  we conclude

$$(1 - p + m^*)(R - B^*) - c(m^*) \geq (1 - p + m^*)(R - B) - c(m),$$

with strict inequality when  $m^* > m$ . Finally, to prove that the bank's participation constraint (5) is satisfied notice that

$$\begin{aligned}
(1 - p + m^*)(R - B^*) - c(m^*) &= (1 - p + m^*)R - R_0 - c(m^*) \\
&\geq (1 - p + m^*)c'(m^*) - m^*c'(m^*) \\
&= (1 - p)c'(m^*) \geq 0,
\end{aligned}$$

where we have used the investors' participation constraint (6), the condition

$$c'(m^*) + \frac{R_0}{1 - p + m^*} \leq R$$

that characterizes the optimal contract, and the fact that the convexity of the monitoring cost function implies  $m^*c'(m^*) \geq c(m^*)$ .  $\square$

**Proof of Proposition 2** Differentiating condition (8) for an interior level of monitoring  $m^* \in (0, p)$  gives

$$\left( c''(m^*) - \frac{R_0}{(1 - p + m^*)^2} \right) dm^* + \frac{1}{1 - p + m^*} dR_0 - dR = 0.$$

By Proposition 1, when  $R > \underline{R}$  and  $m^* < p$  the slope of the function

$$c'(m) + \frac{R_0}{1 - p + m}$$

evaluated at  $m = m^*$  must be positive, which implies

$$\begin{aligned}
\frac{\partial m^*}{\partial R_0} &= -\frac{1}{1 - p + m^*} \left( c''(m^*) - \frac{R_0}{(1 - p + m^*)^2} \right)^{-1} < 0, \\
\frac{\partial m^*}{\partial R} &= \left( c''(m^*) - \frac{R_0}{(1 - p + m^*)^2} \right)^{-1} > 0. \quad \square
\end{aligned}$$

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