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Abstract

The Great Moderation (GM) is widely documented in the literature as one of the most important changes in the US business cycle. All the papers that analyze it use post WWII data. In this paper, for the first time we place the GM in a long historical perspective, stretching back a century and a half, which includes secular changes in the economic structure and a substantial reduction of output volatility. We find two robust structural breaks in volatility at the end of WWII and in the mid-eighties, showing that the GM still holds in the longer perspective. Furthermore, we show that GM volatility reduction is only linked to expansion features. We also date the US business cycle in the long run, finding that volatility plays a primary role in the definition of the business cycle, which has important consequences for econometricians and forecasters.

JEL Classification: C22 and E32
Keywords: business cycle, secular changes, structural breaks and volatility

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1 Introduction

The significant decline in macroeconomic volatility that began in the mid-1980s in the US economy, known as the Great Moderation (GM, henceforth), is one of the most widely documented stylized facts of modern macroeconomy (both in the academic and non-academic literature).

Academic references to the GM are endless. A great deal of empirical work has appeared since the seminal papers of Kim and Nelson (1999) and McConnell and Perez-Quiros (2000).[1] One of the most recent notes on the GM is Gadea-Rivas et al. (2014) where they show, contrary to what is stated in various papers, that the Great Recession (GR, henceforth) would not represent the end of the GM, even if the GR and its subsequent recovery lasted for fifteen years or even more. The explanations of the GM fall into three categories. First, the decrease of US GDP fluctuations could be due to changes in the structure of production; in particular, the improvement in inventory investment and in supply chain management, the tertiarization of the economy and increased competition in products markets. The second explanation emphasizes the improvement of economic policy as a result of, on the one hand, the implementation of monetary stabilization policies with an inflation target and, on the other, of independent monetary authorities. Third, good luck —that is, the absence of significant exogenous shocks—, might also have played an important role in tempering economic fluctuations.[2]

All the papers that analyze the GM use post-World War II (WWII, henceforth) data. Most of these papers argue that the heterogeneity of the sample and the incomparability of the quality of the data are the main reasons for initiating the sample after WWII. Romer (1986a,b, 1991) explains that this incomparability is due to the way this data is constructed, especially when it includes the sample previous to the publication of the industrial production series by the Federal Reserve in 1919.[3] Diebold and Rudebusch (1992), when studying the pre- and post-WWII business

[1] See, amongst others, Stock and Watson (2002). More recently, see Bean (2010) for a review. Blanchard and Simon (2001) show that the GM was not a uniquely US phenomenon but occurred around the same time in many other advanced economies.

[2] There is no consensus on the relative importance of the causes of GM. Examples of this debate can be found in the literature, starting with the papers by Stock and Watson (2002) and Ahmed et al. (2004) until the more recent evidence in Giannone et al. (2008), Canova (2009), Gali and Gambetti (2009), Canova and Gambetti (2010) and Inoue and Rossi (2011), to name just a few.

[3] She states that the prewar economy, due to data construction, was even more volatile that the postwar one, which would make it more difficult to find a structural break associated which the GM, precisely what we want to prove in this paper.

cycle avoid comparing volatility and concentrate only on the duration of recession periods. However, it is clear that, apart from changes in the quality of data, there are major historical changes in the structure of the US economy when using a large sample that should affect the volatility of output.

Indeed, there are some papers, prior to the identification of the GM, that use a long sample and document —subject to caveats regarding data quality— secular changes in US volatility[4] that were located around the mid XX century.[5] Early works like Burns (1960) and Basu and Taylor (1999) already mention a set of major changes in the US and other advanced economies that modified the business cycle. In particular, they highlight the diminished importance of agriculture due to a process of industrialization and tertiarization, the increase in corporate profits, the huge expansion of government enterprises, the increase in the protection of unemployed workers, the increase in financial organization, the implementation of countercyclical monetary policy and the degree of openness of the economy, among others, as the main causes of the changing pattern of the business cycle. On the contrary, DeLong and Summers (1986) consider that the decline in variability since WWII can be explained neither by changes in the composition of economic activity nor by the avoidance of financial panics. Even so, they acknowledge the role played by increased automatic stabilization by the government and also mention increased availability of private credit.

When looking at these major changes, the GM and its possible causes, namely, changes in the structure of production, improved policy and good luck, seem to be a minor change. However, the GM has never before been studied in a long historical perspective which includes secular changes in US economic structure. In this paper, we want to add to the evidence of the structural break of the GM, questioning whether it would hold when considering a longer dataset than that used in the original papers.[6] We use quarterly data starting in 1875.

Enlarging the sample allows us not only to relate the GM with such intense episodes as the Great Depression of the 1930s, World War I (WWI, henceforth) and

[4]These papers pay special attention to the volatility periods linked to the two World Wars.

[5]Blanchard and Simon (2001) consider that the decline in output volatility started in the 1950s, although they acknowledge the difficulties to exactly establish the date due to lack of consistent data.

[6]Recently, a broad new agenda in empirical macroeconomics is being developed to better understand the surge of the GR and the influence of financial factors in macroeconomic outcomes (see Jordà et al. (2011, 2010)). We follow this approach to review in deep the robustness of the GM in an historical framework.

WWII, but also to provide better empirical evidence on what should be expected in the future. Additionally, if the GM still holds in the longer perspective, this will have some implications. First, on the importance of its possible causes and, second, on the perception of the GM as a permanent phenomenon.

We analyze the US GDP growth rate in the long term and look for the presence of structural breaks (both in the mean and the variance), obtaining different volatility periods that show a secular reduction in the variability of output (Section 2). We identify two structural breaks approximately located at the end of WWII and the at beginning of the GM. Once the presence of structural breaks has been documented, we present the main features that characterize each of the historical periods found in our statistical analysis (Section 3). A thorough study of volatility changes suggests that they respond to different causes related to the features of the cyclical phases. for instance, the WWII volatility reduction is due to changes in both cyclical phases, expansions and recessions, while that of the GM is associated with the early stages of expansions.

Bearing these results in mind, we illustrate the risks of turning a blind eye to volatility by carrying out a business cycle dating of the US economy in the long run (Section 4). We use the Markov Switching (MS) non-parametric methodology and we propose, as a novelty in modeling US economic growth, both a MS model with endogenous structural breaks in volatility and a MS model with up to three variance regimes. We find a better performance in the models that control for volatility with respect to those that do not. Additionally, we take care to check the robustness of our results with a method that allows some flexibility in the choice of the business cycle states, that is, Finite Mixtures Markov Switching. We find that volatility plays a primary role in the definition of the US business cycle.

Overall, empirical evidence in this paper suggests that the GM holds in this longer perspective and valuable information would be missed if we ignored this pattern in the output. We also find it necessary to include the changes in volatility experienced in the US economy in order to date the US business cycle correctly because the changes in the signal-to-noise ratio associated with the change in the volatility of the series makes standard procedures of dating seriously misleading. Our findings have important implications for econometricians in the provision of a precise dating of the business cycle and for forecasters in properly capturing the size of future fluctuations. Both clearly affect the process of policy decision-making.

2 Analysis of the series. Blending the GM with pre-WWII data

The seminal papers of Kim and Nelson (1999) and McConnell and Perez-Quiros (2000) and, as far as we know, the academic work that analyzes the phenomenon of volatility reduction, have focused on the postwar period, usually from 1952 until before the GR. Only Gadea-Rivas et al. (2014) introduce the period of the GR into the sample, concluding that it was not able to break the stability of the GM and would not do so, even if the GM and its recovery had lasted for a significant period of time (15 years or even more).

However, we believe that properly understanding the magnitude and the main features of the GM requires considering a broad historical context. To that end, we employ quarterly real US GDP data from the NBER, covering the sample period 1875.1-2014.2. This approach, that considers historical data from the nineteenth century as well as more recent data, will let us to know the importance of the GM in relation to other major historical macroeconomic events and enable us to construct a precise long-term business cycle chronology using different methodologies. The series are displayed both in levels and growth rates in Figure 1.[7] A simple look at GDP data reveals that the intensity of the shocks and the shape of business cycles have changed over 140 years of macroeconomic history. At the beginning of this period, the US economy had to cope with the panics of 1873 and 1893. In the twentieth century, the US economy suffered the effects of WWI. However, some of the most influential economic events of the past century were the Great Depression of the 30s, with devastating economic effects, and WWII. Then, there was a post-war economic boom until the 70s with the end of the Bretton Woods system and substantial oil price shocks, economic growth became stagnant and inflation grew. In the 80s, a reversal of these disequilibria was achieved and the US economy experienced a reduction in the volatility of the business cycle. During this period, called the GM, the US enjoyed long economic expansions only interrupted by three recessions. The first two, 1990-91 and 2001, were mild by historical standards. However, following the burst of the housing bubble, the US economy entered into a severe recession (2007-2009) that has been followed by a weak recovery.

Although we focus on breaks in the variance, we first examine the possibility of changes in the mean, as their existence could affect the identification of the former.

[7] The growth rate is calculated as the first logarithmic difference.

2.1 Stability of GDP growth in the long run

First, we would like to let the data speak for themselves without imposing any parametric structure on them. The sample is so long, and the economy has been exposed to so many changes, that it is useful to start the analysis of the data without any priors about which parametric model should be the one driving the data. Hence, we use a non-parametric approach, namely, a rank-type test, the Wilcoxon test, to compare differences between different subsamples of the data.

The first question to solve is how many local changes in the mean could potentially affect the changes in the variance. The idea is the following. Are we looking for just one change in the mean or several small changes? What size of window should be used to build subsamples? Suppose that we consider the possibility of a major change in the whole sample. We would choose a window that was the size of the sample and simply test for a change in the ranks of the observations before and after the proposed break. However, we could choose windows of smaller size and check for the presence of changes in the mean in those smaller windows. This is what we do. We apply a rolling window of size M and test for the presence of a structural break in the mean of the observations in that subsample.

As can be seen in Table 1, when the window is sufficiently long (60 quarters, 15 years), there are basically no structural breaks in the sample. Only when the window is smaller (2.5 years) do we detect a higher proportion of structural breaks. However, periods where the mean change test is significant are distributed throughout the whole sample and not concentrated at the end or at any other time. Thus, they are rather due to the instability of the mean in the short term and should be interpreted as a volatility phenomenon instead of a structural change in the long-run growth.

This result is tested by applying a more sophisticated approach for the existence of an unknown number of structural breaks in the mean of GDP growth. We apply the methodology of Bai and Perron (1998, 2003a,b) (BP), which is based on the principle of global minimizers of the sum of squared residuals. To our knowledge, we are the first to formally test for changes in the mean in such a long sample.[8]

The BP methodology consistently determines the number of break points over all possible partitions as well as their location. We consider two models, Model 1, a model with just one variant constant representing the mean of growth rate, and

[8]This methodology is also applied in Gadea-Rivas et al. (2014) with post-WWII data. They do not find any structural break in mean.

Model 2, which adds an invariant standard autoregressive model of order 1.[9]

Bai and Perron (1998) suggest three types of tests to select the number and location of breaks.[10] A maximum number of 5 breaks has been considered, which, along with the sample size, T=549, supposes a trimming of $\epsilon = 0.15$. The process is allowed to present autocorrelation and heteroskedasticity. A non-parametric correction has been employed to consider these effects. As can be seen in Table 2, there is no structural break in the mean; all the statistics are way below the critical value. So, we can safely assume that possible breaks in variance can not be driven by the misspecification of the mean parameters.

2.2 Volatility developments

To analyze the evolution of the volatility of US GDP growth rate series, we first present evidence of a rolling estimation of the volatility (see Figure 2a) and the conditional-GARCH variance (see Figure 2b). An examination of the two measures shows that this series is very volatile until the end of the forties, with a peak in the inter-war period, after which the volatility decreases, although it remains quite high until the mid eighties approximately, when it begins to become even lower. Looking at the whole period, it seems that the stability achieved during the GM is relatively small compared with past decreases of US GDP volatility, particularly when looking at the dramatic decrease of the post-WWII period. In the following paragraphs, we treat the data more rigorously to test the significance of these changes.

We propose several methods to test for the significance of changes in the variance, as in Gadea-Rivas et al. (2014). First, we use Inclán and Tiao (1994)'s (IT) test to detect changes in the unconditional variance of the series, which has been used extensively, especially in financial series. This test uses an Iterated Cumulative Sum of Squares (ICSS) and assumes that the innovations are zero-mean normally, i.i.d. random variables. However, the IT test has big size distortions when the assumption of normally distributed innovations fails in the fourth order moment or for heteroskedastic conditional variance processes and, consequently, it tends to overestimate the number of breaks. Given that Figure 2 shows clearly that the

[9] We verify, through a battery of unit root tests, that the series is stationary.

[10] The $supF\_(k)$ test considers the null hypothesis of no breaks against the alternative of k breaks. The $supF\_(l+1/l)$ test takes the existence of l breaks, with $l = 0, 1, ...$, as its H_0, against the alternative of $l+1$ changes. Finally, the $UDmax$ and $WDmax$ tests consider the null of the absence of structural breaks against the existence of an unknown number of breaks. Additionally, the SBIC criterion is used to select the number of break points.

variance is not constant over time and the distribution of GDP growth is non-mesokurtic with a fat right tail, we apply the corrections proposed by Sanso et al. (2004) for high kurtosis and heteroskedastic conditional variance processes. We denote this corrected IT tests as $IT(\kappa_2)$. Table 3 shows the results of applying this test to US GDP growth and conclude that there are three breaks in the variance, chronologically located in 1917.4, 1946.2 and 1984.1, that roughly correspond to the end of each of the world wars and the beginning of the GM.

These results are confirmed with an analysis within the parametric framework which consists of applying the BP test to the mean of the absolute value of the estimated residuals.[11] We obtain the same break points that with the IT test, 1947.1, 1984.2 and 1917.3 in this order.

Finally, we compute the method used in McConnell and Perez-Quiros (2000). This is a parametric test of the changes in volatility when the mean and variance parameters are estimated jointly. The test only allows one break but we apply it in a sequential procedure, only finding two breaks. A striking result is that the first break we find is that associated with the GM (1984.3). Notice that it is the most likely break because it maximizes the distance between the two subsamples when the full data is considered. Conditional on this break, the other break that appears is close to the end of WWII. The break in 1917 is not even found.[12]

3 Fathoming volatility changes

Results in previous sections robustly confirm the existence of different periods in the US business cycle. In this section, we intend to shed some light on the characteristics of these different volatility periods.

Given that the break in 1917 is less robust and to make the discussion clearer, we divide the sample into three subsamples. Period 1, covering up to the end of WWII; period 2, from the end of WWII to the beginning of the GM and; period 3, from the beginning of the GM to the end of the sample. To offer some evidence on the idiosyncrasy of each of the three periods, we compute the well-known Kolmogorov-

[11] Zhou and Perron (2008) show that, if there are changes that are not taken into account in the mean of the series, the test suffers from serious size distortions. However, our series do not have any changes in the mean as we have found previously. This method has been used by Herrera and Pesavento (2005), Stock and Watson (2002), and Gadea-Rivas et al. (2014) among others.

[12] Not even looking at the longer sample, do we find a significant break associated with the GR. Using post-WWII data, Gadea-Rivas et al. (2014) do neither identify this break.

Smirnoff test through which we test for equal distribution across subsamples. With this non-parametric test, we clearly reject that the post-WWII subsample is statistically equal to the pre-WWII sample, and also reject that the GM is statistically equal to both (Table 4).

In Table 5, we show some descriptive statistics that provide some interesting facts about the nature of the secular volatility reduction. All the three subsamples present similar means while the reduction in volatility is clearer between subsamples 1 and 2 than between 2 and 3. However, the difference between periods 2 and 3 (the second break) seems to be not only associated with a change in volatility but also with changes in higher moments of the distribution. Indeed, skewness and kurtosis change.[13]

To see whether these changes are associated with changes in the shape of the business cycle, as Gadea-Rivas et al. (2014) state, we describe business cycle characteristics using the NBER dating that pinpoints the turning points. Following Harding and Pagan (2002), we dissect the business cycle and calculate some outcomes for each phase, trough-to-peak for expansions and peak-to-trough for recessions. We obtain the mean *duration* (in quarters), *amplitude* (this compares the log level of GDP at the turning points), *cumulation* (this is the cumulated gain or loss and consists of the sum of the amplitudes of each cyclical phase) and *excess* (this refers to the difference between the hypothetical triangle which would have formed with a uniform growth rate throughout the whole phase and the real area drawn by the path of the GDP growth)[14] of recessions and expansions.

As we mentioned earlier, the changes from the first to the second subsample are of a different nature than those from the second to the third subsample. From pre-WWII to post WWII, the changes in the characteristics can be directly linked to changes in the features corresponding to recession phases (Table 6). For example, there are shorter and shallower recessions while the expansions are similar in both subsamples. On the contrary, the difference between periods 2 and 3 are basically due to changes in the shape of expansion phases whose duration is strikingly higher and whose excess has fallen dramatically in the last period. Changes in recession

[13] According to a bootstrap exercise, there are no significant differences in kurtosis between periods 1 and 2, while 3 is different from both. Regarding skewness, the results are less clear: the three periods are significantly different, but periods 1 and 2 are more similar.

[14] In the case of expansions, a negative Excess (concave path) means that the recovery starts with a high growth rate and goes on slowly, whereas a positive Excess (convex path) is produced when the economy has a smooth beginning that becomes sharper at the end.

periods such as those in the pre-WWII to post-WWII period reduce volatility to a pure accounting exercise. Recessions are, in general, more volatile than expansions.[15] Therefore, less frequent and shorter recession periods reduce volatility. On the contrary, the second reduction in volatility, that associated with the GM, is more subtle. Recession features before and after the GM are basically equal. The biggest change in the characteristics associated with the GM is in the shape of expansion periods, that is, in the excess.

To develop this idea in greater depth, we carry out an experiment on the characteristics of each of the business cycle phases. We replace the recessions (expansions) of each period with others taken randomly from a different period, while keeping the expansions (recessions) unchanged. Thus, first, we select each recession (expansion) of period "x" and of period "y", according to the NBER chronology. Second, we remove each recession (expansion) of period "x" and replace it with a recession (expansion) of period "y", chosen randomly.[16] We repeat this experiment for 10,000 iterations, estimate the number and location of volatility structural breaks and compute the percentage of times that the structural breaks associated with WWII and the GM are found. Results are displayed in Table 7.

The idea of the exercise is the following. If, for a given period "x", we substitute its recessions (or expansions) by those of the other period "y" and we do not find a structural break from "x" to "y" anymore, we could conclude that these recessions (expansions) explain the changes in volatility and, when looking for an explanation, we should concentrate on understanding the characteristics in those recession (expansion) periods.[17]

Concerning the empirical results, first, we change the recessions of period 2 for the recessions of 1, finding that the structural break of WWII holds only in 48% of the cases. This result could mean that the nature of the WWII volatility break in more than half of the cases is associated with the characteristics of recession phases. Second, if we change the recessions of period 3 for the recessions of 2, we obtain the GM structural break in 100% of the cases. Thus, we can state that, in no case, were recessions characteristics behind the GM break in volatility because despite using

[15] See, for example, French and Sichel (1993).

[16] In particular, we replace each recession from peak to through (or expansion from through to peak) independently of their duration.

[17] Notice that, in this exercise, we substitute all the business cycle features (duration, amplitude, cumulation and excess) of the exchanged cyclical phase (expansion/recession). This implies that only duration and the sign of excess of the other cyclical phase (recession/expansion) remain unchanged.

recessions (duration, amplitude and excess) of the previous period, the break is still identified.

So, is it characteristics of expansion periods that explain the lower volatility from one period to the next? To find out, first, we change the expansions of period 2 for those of period 1 and find that the structural break of WWII is identified in 31% of the cases. Adding these results to the previous ones about recessions, we conclude that it is a mixture of the features of the expansions and recessions that explains the volatility reduction of WWII. However, if we change the expansions of period 3 for those of 2, in no case does the structural break of the GM hold. Again, combining these results with the previous ones about recessions, we can conclude that the nature of the GM structural break is entirely explained by the features of expansion phases.

These findings reinforce those obtained by Gadea-Rivas et al. (2014) who show that the decrease in output growth volatility seems to be clearly associated with the shape of expansions and, specifically, with the disappearance of high-growth recoveries. Broadening our perspective with a longer dataset, the results of the experiment suggest that the secular fall in volatility responds to two structural changes of a different nature. The WWII volatility reduction is related to changes in both cyclical phases, while that of the GM is primarily due to the characteristics of expansions.

4 Chronology of the US business cycle. The risks of turning a blind eye to volatility

After having identified the presence of structural volatility breaks in the series and their different natures, we show the risks of not taking them into account when modeling the business cycle. In this section, we analyze the hidden business cycle of the US GDP growth in the long run. The most popular method that allows us both to date the cycle and to make inferences about future periods is the Markov Switching approach developed by Hamilton (1989).[18]

This approach characterizes the evolution of a variable through a process of conditioned mean to a state of a specific nature. The changes in value in this dynamic process will allow us to distinguish between periods of expansion and contraction.

[18]For a comparison of different business cycle dating methods, see Layton and Katsuura (2001) and Chauvet and Piger (2008).

Regime shifts are governed by a stochastic and unobservable variable which follows a Markov chain. In general, we consider the following process for the growth of GDP, calculated as first log difference:

$$y_t = \mu_{S_j} + \epsilon_t \tag{1}$$

where y_t is US GDP growth rate, μ_{S_j} is the vector of MS intercepts and $\epsilon_t/_{S_j}$ $\sim$N(0, σ). It is standard to assume that these varying parameters depend on an unobservable state variable S_j that evolves according to an irreducible m-state Markov process where p_{ij} controls the probability of a switch from state j to state i.

In this framework, we estimate a MS model with 2 states ($j = 1, 2$) and a constant variance for the full period:

$$\begin{aligned} y_t &= \mu_1 + \epsilon_t \text{ for state 1} \\ y_t &= \mu_2 + \epsilon_t \text{ for state 2} \end{aligned} \tag{2}$$

Assuming a classical cycle, μ_1 and μ_2 are associated with expansion and recession phases, respectively, and $p_{11} = p$ and $p_{22} = q$ represent the probability of being in expansion/recession and staying in the same state. The estimated parameters are displayed at the top of Table 8 and Figure 3 presents the model-estimated probability of being in recession, which is compared with the NBER official chronology. We find the surprising result that the MS model identifies no recession after WWII, which highlights the lack of robustness of the method in the presence of structural changes. Unlike non-parametric methods, which look for turning points locally, the MS provides an overall estimate of the economic cycle. Therefore, a period of high volatility and high fluctuations in output, such as the one that occurred before WWII, may obscure the identification of the business cycle phases in the rest of the sample. It is clear that ignoring the existence of volatility changes in the series leads to model misspecifications. McConnell and Perez-Quiros (2000) show the difficulties of this method in describing turning points in the presence of structural breaks in volatility. If these breaks exist, μ_1 and μ_2 should change proportionally to capture business cycle characteristics.[19] In our paper, the volatility changes are

[19] If we estimate the model without taking the structural breaks into account, its performance is very poor because, when volatility decreases, the business cycle means drop and the parameter estimates are useless to describe the business cycle.

more frequent and dramatic than in McConnell and Perez-Quiros (2000). To capture this phenomenon, we have to modify Hamilton (1989) methodology even more. So, we adopt a dual strategy in the MS estimation: introducing structural changes in the variance and considering different variance regimes.

4.1 A MS model with structural breaks in variance

We extend the previous approach to consider the presence of a maximum of M structural breaks in (2) which defines $M+1$ regimes with different means and variances but maintaining the same transition matrix across periods. Although, on the basis of previous results, we are particularly interested in changes in variance, we also allow the mean of both states to differ in each regime,

$$\begin{aligned} &\mu_1^1,\ \mu_2^1, \sigma_1 \text{ for regime } 1 \\ &\ldots \\ &\mu_1^{M+1},\ \mu_2^{M+1}, \sigma_{M+1} \text{ for regime } M+1 \end{aligned} \tag{3}$$

We propose this innovative strategy as a methodological alternative to include changes in variance in MS frameworks. We take a maximum number of 3 breaks and estimate all possible break points in volatility (with a minimum interval of 5 years), selecting their location in accordance with the SBIC. Table 8 shows the results of the estimation for the cases of 1, 2 or 3 structural breaks and Figure 3 displays the different probabilities. We find that considering structural changes greatly improves the precision with which the turning points are located, as evidenced by the value of the SBIC, which selects a model with 2 breaks and 3 regimes (Table 10). The two breaks are located in 1946.2 and 1984.2, almost exactly the same points detected by the test of changes in volatility.[20] The period up to the first break is characterized by a very high variance and heavy falls during recessions. The period between the end of WWII and the onset of the GM shows a significant reduction in volatility with sustained expansions and less pronounced downturns.[21] Finally, the GM period has a low variance and moderate growth rates.

[20]The gain with respect to the model with 3 breaks (the other one located in 1917.3) is very small (-2.0859x10^3 in the case of two breaks against -2.0860x10^3 in the case of three breaks).

[21]Keating and Valcarcel (2012) analyze the annual US GDP series from 1790 to 2009 and also find there was a much greater decline of volatility after WWII than during the GM.

4.2 A MS model with variance regimes

In this subsection, we augment the standard specification of the MS model given in (2) by including two or three states for the variance which are driven by a Markov chain that is independent of the one that drives the states of the mean. This is the approach followed by McConnell and Perez-Quiros (2000). The advantage of this specification, compared to considering volatility breaks, is that the same variance regime can be identified in different periods, capturing volatility more accurately. In addition, as we do for the mean, we date the states of the variance endogenously and can make inferences about future states of volatility because we have an estimate of the transition matrices for the volatility states as well. We consider two specifications. The first yields two possible states for the variance, σ_1 being the variance in the high-variance state and σ_2 the variance in the low-variance state, along with four possible states for the mean of output growth corresponding to different variances and cyclical phases.

$$\begin{array}{l} \mu_1^1,\ \sigma_1 \text{ for expansion and high variance regime} \\ \mu_2^1,\ \sigma_1 \text{ for recession and high-variance regime} \\ \mu_1^2,\ \sigma_2 \text{ for expansion and low-variance regime} \\ \mu_2^2,\ \sigma_2 \text{ for recession and low-variance regime} \end{array} \tag{4}$$

However, we have a longer dataset, in which volatility changes are more frequent and dramatic, than that of McConnell and Perez-Quiros (2000). So, in the second specification, we extend, for the first time, the McConnell and Perez-Quiros (2000) proposal by allowing three possible regimes for the Markov chain that governs the variance, σ_1 for the high-variance regime, σ_2 for the medium-variance regime and σ_3 for the low-variance regime. Consequently, the number of means increases to six, depending on the two values of S_t and the three variance regimes.

$$\begin{array}{l} \mu_1^1,\ \sigma_1 \text{ for expansion and high-variance regime} \\ \mu_2^1,\ \sigma_1 \text{ for recession and high-variance regime} \\ \mu_1^2,\ \sigma_2 \text{ for expansion and medium-variance regime} \\ \mu_2^2,\ \sigma_2 \text{ for recession and medium-variance regime} \\ \mu_1^3,\ \sigma_3 \text{ for expansion and low-variance regime} \\ \mu_2^3,\ \sigma_3 \text{ for recession and low-variance regime} \end{array} \tag{5}$$

The estimated parameters appear in Table 9. Both models improve their per-

formance with respect to the MS with structural breaks (Table 10). Furthermore, the model with three variance regimes has the highest SBIC (-1.9629x10^3). The results with three regimes are shown in Figure 4. Regime 1 (high-variance) lasts until 1951.1, although some periods of medium variance also appear interspersed. Regime 2 (medium-variance) is detected in the interval 1950.2-1984.2 as well as during the recent GR.[22] Moreover, it is also found occasionally in the pre-WWII period, precisely when a high-variance regime is not detected. Regime 3 (low-variance) corresponds to what we know as the GM.

Overall, we have illustrated how to date the US business cycle, taking into account differences in volatility, extending the traditional MS approach. A MS model with two structural breaks in variance performs better than a traditional MS and we are even able to compute a model with three endogenous variance regimes which slightly improves the results of the MS model with structural breaks.

4.3 Robustness analysis. Finite Markov Mixture Models

In this subsection, with a view to assessing the robustness of our results, we consider an alternative strategy —applied, for the first time, to model US GDP in the long run— that allows us flexibility in the characterization of the states and to endogenously determine their number. It combines both the possibility of different volatility and mean regimes and a business cycle dating without imposing restrictions on the number of these regimes/states. We can model the random variable y_t as a mixture of univariate normal distributions, each of them representing the characteristics and distribution of each state.

The Finite Markov Mixture Models (FMMM) (see Frühwirth-Schnatter (2006) for a detailed revision) combine clustering techniques, finite mixtures and Bayesian approaches, which leads to a rich class of non-linear models.[23] Hence, they are a good choice when the unobservable latent indicator that drives the process is a Markov chain. FMMM are often used for the purpose of clustering. This approach assumes the existence of K hidden groups and intends to reconstruct them by fitting a K component mixture density. The time series $\{y_1, ..., y_t\}$ is assumed to be a realization of a stochastic process Y_t generated by a finite Markov mixture from a

[22]The change to the medium-variance regime occurs between 2008.3 and 2009.2 and then there is a return to the low-variance regime (in line with the volatility increase of the rolling estimation).

[23]Some applications of these techniques to the analysis of the business cycle can be found in Fruhwirth-Schnatter and Kaufmann (2008) and Hamilton and Owyang (2012).

specific distribution family:

$$Y_t|S_t \sim \Upsilon(\theta_{S_t}) \tag{6}$$

Y is said to arise from a finite mixture distribution if the probability density function $p(y)$ of this distribution takes the form of a mixture density for all $y \in Y$

$$p(y) = \eta_1 p_1(y) + ... + \eta_K p_K(y) \tag{7}$$

where S_t is an unobservable K-states ergodic Markov chain, and the random variables Y_1,...,Y_T are stochastically independent, conditional on knowing S. For each $t \succeq 1$, the distribution of Y_t arises from one of K distributions $\Upsilon(\theta_1), ..., \Upsilon(\theta_K)$, depending on the state of S_t

$$Y_t|S_t = k \sim \Upsilon(\theta_k) \tag{8}$$

The stochastic properties of S_t are described by the KxK transition matrix ξ where

$$\xi_{jk} = P(S_t = k|S_{t-1} = j), \forall\ j, k \in \{1, ..., K\} \tag{9}$$

For the doubly stochastic process $\{S_t, Y_t\}_{t=1}^T$, the marginal distribution of Y_t is

$$p(y_t|\varphi) = \sum_{k=1}^{K} p(y_t|S_t = k, \varphi)p(S_t = k|\varphi) \tag{10}$$

with $\varphi = (\theta_1, ..., \theta_k, \eta)$. Because S_t is a stationary Markov chain, we can obtain the unconditional distribution of Y_t as a finite mixture of $\Upsilon(\theta)$

$$p(y_t|\varphi) = \sum_{k=1}^{K} p(y_t|\theta_k)\eta_k \tag{11}$$

In our case, where GDP growth y_t is an observed discrete signal with noises, we can define the Markov mixture of normal distributions as follows,

$$y_t = \begin{cases} \mu_1 + \varepsilon_t,\ \varepsilon_t \sim N(0, \sigma_1^2),\ S_t = 1 \\ ... \\ \mu_k + \varepsilon_t,\ \varepsilon_t \sim N(0, \sigma_k^2),\ S_t = K \end{cases} \tag{12}$$

So, we estimate K, the number of states of the hidden Markov chain, the state-specific parameter and the transition matrix: $\varphi = (\theta_1, ..., \theta_k, \eta)$. We use a Bayesian

approach with Markov Chain Monte Carlo (MCMC) methods and Gibbs Sampling to estimate the posterior probability $p(\varphi|S, y)$; 5,000 draws and non-informative priors are considered.[24] The number of components K can be selected by informal methods such as the point process representation or according to the maximum likelihood. Details of the results appear in Table 11 and Figures 5-7.[25]

Firstly, we apply the FMMM to the whole sample (Figure 5) identifying four clusters of states/regimes. The first is characterized by high growth and high variance, the second by negative growth and high variance, and the other two states present moderate growth with different variances. These two states are basically concentrated before WWII. The third state (moderate growth and relatively high variance) is located mainly from WWII up to 1984.3, although it appears occasionally before WWII and in 1990.4 and 2008.1-2009.2.[26] After this date, we find a fourth period of moderate growth and low variance. Therefore, this method leads to a classification dominated by volatility and sacrifices a plausible business cycle chronology.

If we apply this method to post-WWII in accordance with the date of the volatility break identified in previous sections, we find three cluster-states. The first corresponds to recession and high variance, the second to high growth and high variance and the third to moderate growth and low variance (Figure 6). The first and second are detected mainly before the GM, while the third is located from 1984.3 on. However, the first, which captures well-known crisis episodes, also identifies the 90s recession and the GR. This classification is closer to the NBER official chronology than that of the whole sample. It quite accurately identifies recession periods as well as two different types of expansions, according to their volatility.

Finally, with data after the GM break, this method identifies three regimes (Figure 7). The first two present low variance, the first is characterized by negative or low growth, corresponding to 1990.3-1991.1 and 2008.1-2009.2. It also shows a peak of probability during the deceleration of the 2000's. The second corresponds

[24] All the calculations of this section have been done using the Matlab Toolbox provided by Fruhwirth-Schnatter and Kaufmann (2008).

[25] Each figure includes three panels. In panel (a), the MCMC draws will scatter around the points corresponding to the true point process representation, with the spread of the clouds representing the uncertainty of estimating the points. Panel (b) displays the probability of being in each state. Finally, panel (c) shows the classification of the time series according to state probabilities.

[26] The method clearly finds the GR (which is a transient episode of volatility, similar to the pre-GM) and, more weakly, the slowdown of the 90s. The NBER identifies another recession period during the last three quarters of 2001. However, it is not detected by any dating method.

to moderate growth. The last one has zero probability throughout most of the period considered.[27] In this sample, where the volatility is less heterogeneous, the chronology is more accurate than in the two previous exercises.

To sum up, as if we used a zoom lens, with the whole sample the two phases of the business cycle are not to be differentiated because the weight to select states is driven by changes in volatility instead of growth differences. When we reduce the zoom to the post-WWII period, the chronology is closer to that of the NBER. When we focus on the calm times of the GM, we obtain quite a precise dating.

Notice that all these results are consistent with our previous findings and also highlight the difficulties of the traditional MS method to accurately date when there are significant changes in volatility. In fact, volatility appears to play a key role in the building of clusters and in the posterior classification of each observation coming from a mixture distribution, sacrificing, in some cases, a more precise chronology. Only in periods with similar variance, such as the post-WWII and, even more, the GM, do we obtain the established cyclical dating.

5 Conclusions

Since the end of the 19th century, the US economy has experienced major economic changes that have affected the characteristics of its business cycle. To quote a few, the diminished importance of agriculture in favor of industry and services, the increase in corporate profits, the huge expansion of government enterprise, the increase in the protection of unemployed workers, the increase in financial organization or the implementation of countercyclical monetary policy and the degree of openness of the economy. Against this background, the GM might seem a minor phenomenon hidden among all the major changes in the US economy. However, in this paper, we show that this is not the case. The GM had as great an impact on the characteristics of the series as the end of WWII had. Even though these impacts are comparable, we show that the nature of the volatility reduction is very different. The WWII reduction affects both business cycle, recessions and expansion, while the GM is only linked to the characteristics of expansion periods.

In addition, after conducting a long run business cycle dating of the US economy, we show that we must not ignore changes in volatility, such as the ones linked to WWII or the GM, to accurately dating, defining and forecasting the US business

[27] In fact, the probability is only higher than 0.5 in 2008.4.

cycle.

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Tables

TABLE 1
Structural breaks in the mean
(Rolling Wilcoxon Sum Rank test)

Years	Window size	% rejections
10	40	10.50
20	80	4.29
40	160	3.28
60	240	0.03

Notes: We apply the Wilcoxon rank sum test by using a rolling matrix. That is, we take rolling pieces of the sample with different window size, build a matrix and apply the test.

TABLE 2
Multiple structural breaks (Bai-Perron methodology)

	Model 1	Model 2	Critical values	
			5%	1%
supF_(k)				
k=1	1.74	1.37	8.58	12.29
k=2	1.71	1.59	7.22	9.36
k=3	2.14	1.63	5.96	7.60
k=4	1.97	1.52	4.99	6.19
k=5	1.06	0.91	3.91	4.19
supF_(l+1/l)				
l=0	2.50	2.42	8.58	12.29
l=1	2.60	1.84	10.13	13.89
l=2	1.35	1.29	11.14	14.80
l=3	–	–	11.83	15.28
l=4	–	–	12.25	15.76
UDmax	2.14	1.63	8.88	12.37
WDmax	3.39/3.91(∗)	2.61/3.01(∗)	9.91	13.07
SBIC	0	0		
LWZ	0	0		
Sequential	0	0		

Notes: We look for changes in the mean in a pure structural model (Model 1) and including an autoregressive (Model 2). The trimming parameter is $\epsilon = 0.10$ and the maximum number of breaks is 3. Serial correlation and heterogeneity in the errors are allowed. The consistent covariance matrix is constructed using the Andrews (1991) method.
(*) values at 5% and 1% respectively.

TABLE 3

Detecting changes in variance

Panel A. Inclan-Tiao test				
$IT(\kappa_2)$	1917.4	1946.2	1984.1	
Panel B. Bai-Perron methodology				
	Model 1	Model 2	Critical values	
			5%	1%
supF_(k)				
k=1	75.27	58.13	8.58	12.29
k=2	48.68	39.42	7.22	9.36
k=3	48.13	31.92	5.96	7.60
k=4	36.18	23.39	4.99	6.19
k=5	26.01	17.68	3.91	4.19
supF_(l+1/l)				
l=0	75.27	58.13	8.58	12.29
l=1	32.37	24.96	10.13	13.89
l=2	32.37	24.96	11.14	14.80
l=3	0.02	-	11.83	15.28
l=4	–	–	12.25	15.76
UDmax	75.27	58.13	8.88	12.37
WDmax	75.27/77.83*	58.13/58.13*	9.91	13.07
SBIC	2	2		
LWZ	2	2		
Sequential	3(1917.3, 1947.1, 1984.2)	3(1917.2, 1946.4, 1984.1)		
Panel C. McConnell-Perez-Quiros methodology				
	Sup	Exp	Ave	Tb
1875.1-2014.2	69.49 (0.000)	31.08 (0.000)	31.29 (0.000)	1984.3
1875.1-1984.3	31.63 (0.000)	12.95 (0.000)	10.12 (0.000)	1951.1
1875.1-1951.1	6.42 (0.224)	1.79 (0.071)	2.83 (0.038)	1929.4
Summary of panels				
ICSS algorithm	Bai-Perron	McConnell-Perez-Quiros		
$IT(\kappa_2)$	Model 1			
1917.4	1917.3	1951.1		
1946.2	1947.1	1984.3		
1984.1	1984.2			

Notes: See Table 2 for details of Bai-Perron procedure. $IT(\kappa_2)$ refers to Inclán and Tiao (1994) test with the correction proposed by Sanso et al. (2004) for heteroskedastic conditional variance processes.

TABLE 4

Kolmogorov-Smirnoff

	1875.2-1946.2	1946.3-1984.2	1984.3-2014.2
1875.2-1946.2		0.217 (0.000)	0.358 (0.000)
1946.3-1984.2			0.300 (0.000)
1984.3-2014.2			

Notes: We test whether the two samples came from the same distribution.

TABLE 5

Descriptive statistics

	1875.2-2014.2	1875.2-1946.2	1946.3-1984.2	1984.3-2014.2
mean	0.81	0.84	0.87	0.66
median	0.81	1.04	0.86	0.74
max	7.97	7.97	3.91	1.87
min	−8.76	−8.76	−2.62	−2.14
st.dev.	2.19	2.92	1.18	0.60
skewness	−0.57	−0.50	−0.16	−1.32
kurtosis	6.05	3.76	3.11	6.87

Notes: Computed on US GDP growth rate.

TABLE 6

Business cycle characteristics by periods

	1875.2-2014.2	1875.2-1946.2	1946.3-1984.2	1984.3-2014.2
DURATION				
Expansions	13.69	9.22	13.44	27.00
Recesions	5.52	6.61	3.75	3.67
AMPLITUDE				
Expansions	19.51	18.59	16.16	20.76
Recesions	−4.02	−5.33	−1.92	−1.72
CUMULATION				
Expansions	198.47	142.03	179.30	319.28
Recesions	−31.22	−47.73	−3.61	−5.81
EXCESS				
Expansions	-15.42	-6.95	-24.49	-4.54
Recesions	−2.60	−4.10	0.10	−0.78

Notes: We use the NBER chronology as reference.

TABLE 7

Changing the cyclical phases characteristics

RECESSIONS	
Recessions of period 2 for recessions of period 1	48% SWW
Recessions of period 3 for recessions of period 2	100% GM
EXPANSIONS	
Expansions of period 2 for expansions of period 1	31% SWW
Expansions of period 3 for expansions of period 2	0% GM

Notes: Period 1: pre-WWII; Period 2: pre-GM; Period 3: GM.

TABLE 8

MS Estimation with Structural breaks

	μ_1	μ_2	σ	p	q
without breaks					
1875.2-2014.2	1.15 (0.08)	−4.62 (0.35)	2.92 (0.19)	0.98 (0.00)	0.61 (0.09)
with 1 break					
1875.2-1950.2	1.52 (0.14)	−4.30 (0.49)	4.66 (0.45)	0.95 (0.001)	0.65 (0.007)
1950.3-2014.2	0.97 (0.06)	−0.55 (0.24)	0.57 (0.06)		
with 2 breaks					
1875.2-1946.1	1.58 (0.15)	−4.21 (0.51)	4.75 (0.48)	0.95 (0.001)	0.71 (0.007)
1946.2-1984.1	1.29 (0.14)	−0.27 (0.22)	0.90 (0.14)		
1984.2-2014.2	0.77 (0.05)	−0.66 (0.25)	0.22 (0.03)		
with 3 breaks					
1875.2-1917.2	1.21 (0.18)	−4.04 (1.86)	3.83 (0.49)	0.95 (0.001)	0.72 (0.007)
1917.3-1946.1	2.36 (0.29)	−4.02 (0.63)	5.49 (0.97)		
1946.2-1984.1	1.30 (0.14)	−0.26 (0.21)	0.89 (0.14)		
1984.2-.2014.2	0.77 (0.05)	−0.65 (0.25)	0.22 (0.03)		

Notes: We introduce structural breaks in variance in a two-states Markov-Switching model.

TABLE 9

MS Estimation with variance regimes

	μ_1	μ_2	σ	p	q
Model with 2 variance regimes					
Regime 1	2.01 (0.21)	−3.84 (0.61)	5.51 (0.70)	0.93 (0.001)	0.67 (0.007)
Regime 2	0.98 (0.06)	−0.58 (0.19)	0.56 (0.06)		
Matrix of Markov transition probabilities of variance					
$\begin{bmatrix} 0.967 & 0.021 \\ 0.033 & 0.979 \end{bmatrix}$					
Model with 3 variance regimes					
Regime 1	2.17 (0.23)	−3.42 (0.57)	5.74 (0.76)		
Regime 2	1.24 (0.09)	−0.46 (0.17)	0.74 (0.12)	0.91 (0.002)	0.73 (0.005)
Regime 3	0.90 (0.06)	0.33 (0.01)	0.18 (0.04)		
Matrix of Markov transition probabilities of variance					
$\begin{bmatrix} 0.963 & 0.036 & 0.000 \\ 0.037 & 0.953 & 0.015 \\ 0.000 & 0.011 & 0.985 \end{bmatrix}$					

Notes: We have estimated a two-states Markov-Switching model with 2 and 3 variance regimes.

TABLE 10

Model selection

	dates	SBIC
without breaks	-	-2.3524x10^3
with one break	1950.3	-2.1293x10^3
with two breaks	1946.2, 1984.2	-2.0859x10^3
with three breaks	1917.3, 1946.2, 1984.2	-2.0860x10^3
with two variance regimes		-2.0374x10^3
with three variance regimes		-1.9629x10^3

Notes: SBIC denotes the Schwartz Information Criterion.

TABLE 11

MS Estimation with Finite mixtures

	μ_{S_t}	σ_{S_t}
1875.2-2014.2 Full sample		
S_t=1	0.79 (0.101)	0.54 (0.072)
S_t=2	2.24 (0.367)	2.31 (0.890)
S_t=3	0.66 (0.242)	1.20 (0.234)
S_t=4	−3.70 (0.775)	2.39 (2.154)

Matrix of transition probabilities ξ_{ij}

$$\begin{bmatrix} 0.955 & 0.004 & 0.004 & 0.004 \\ 0.004 & 0.887 & 0.053 & 0.056 \\ 0.033 & 0.017 & 0.920 & 0.031 \\ 0.011 & 0.275 & 0.045 & 0.668 \end{bmatrix}$$

1946.3-2014.2 Postwar period		
S_t=1	−0.15 (0.239)	0.96 (0.255)
S_t=2	1.59 (0.208)	0.86 (0.194)
S_t=3	0.77 (0.055)	0.48 (0.042)

Matrix of transition probabilities ξ_{ij}

$$\begin{bmatrix} 0.777 & 0.180 & 0.043 \\ 0.120 & 0.813 & 0.068 \\ 0.049 & 0.015 & 0.936 \end{bmatrix}$$

1984.3-2014.2 Post-GM period		
S_t=1	−0.29 (0.296)	0.67 (0.288)
S_t=2	0.79 (0.054)	0.44 (0.036)
S_t=3	−2.80 (1.797)	0.76 (2.563)

Matrix of transition probabilities ξ_{ij}

$$\begin{bmatrix} 0.693 & 0.241 & 0.066 \\ 0.038 & 0.957 & 0.005 \\ 0.154 & 0.116 & 0.730 \end{bmatrix}$$

Notes: Standard errors in parentheses.

Figures

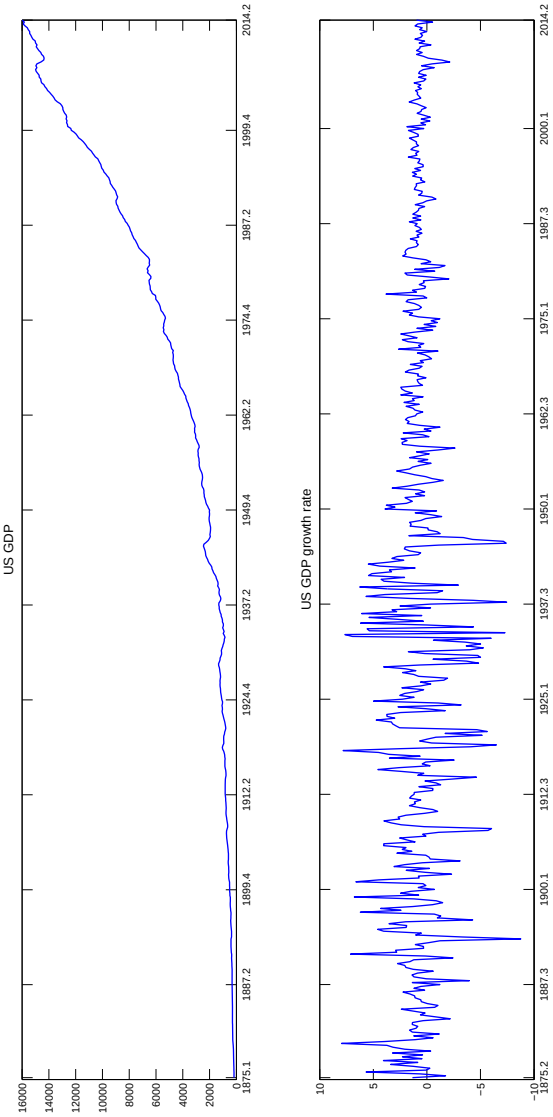
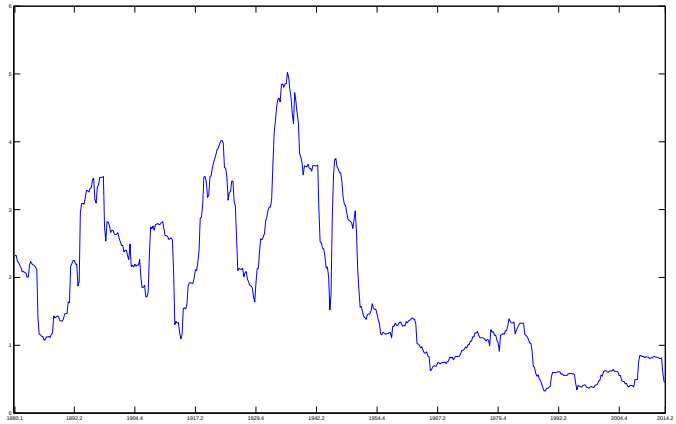
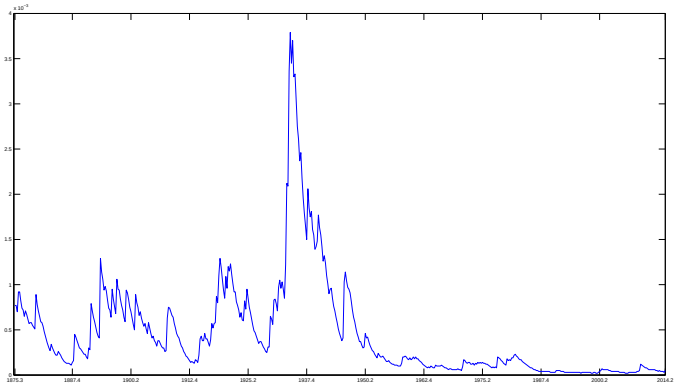


Figure 1: US GDP



(a) Rolling volatility (window=20)



(b) Conditional variance from GARCH estimation (model AR(1)-GARCH(1,1))

Figure 2. Analysis of volatility

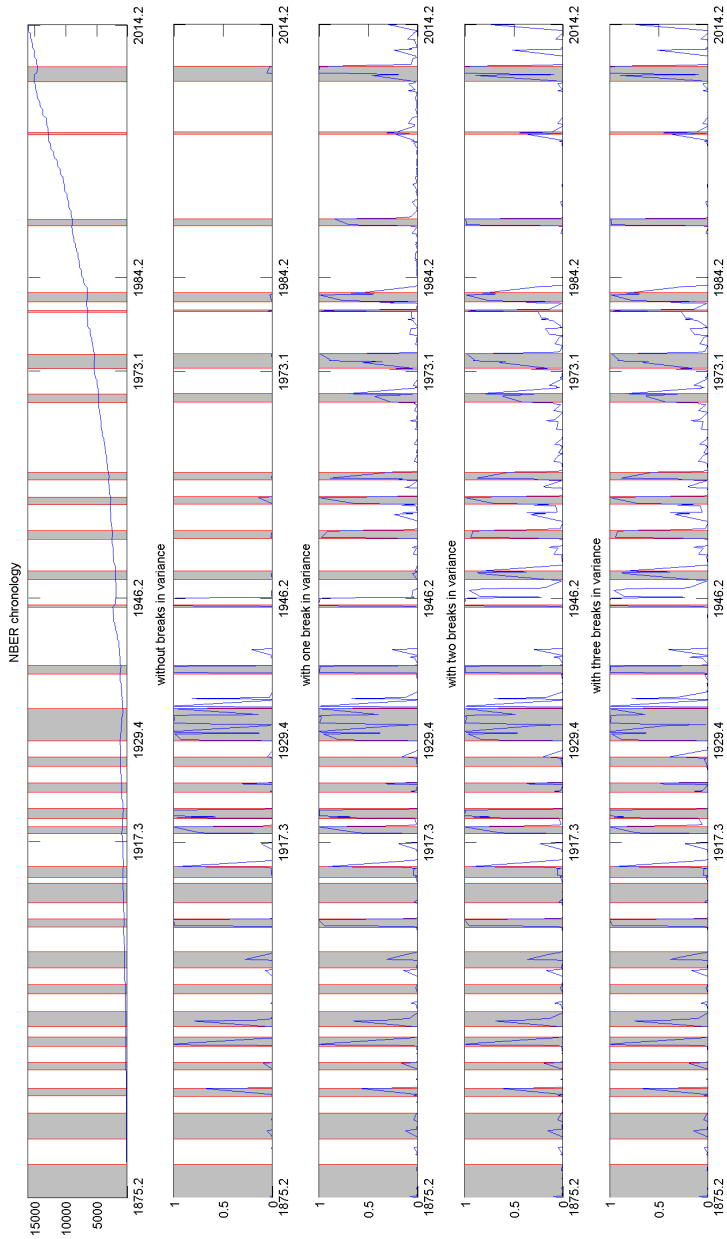


Figure 3. US business cycle and estimated probabilities of a MS model with structural breaks in the variance

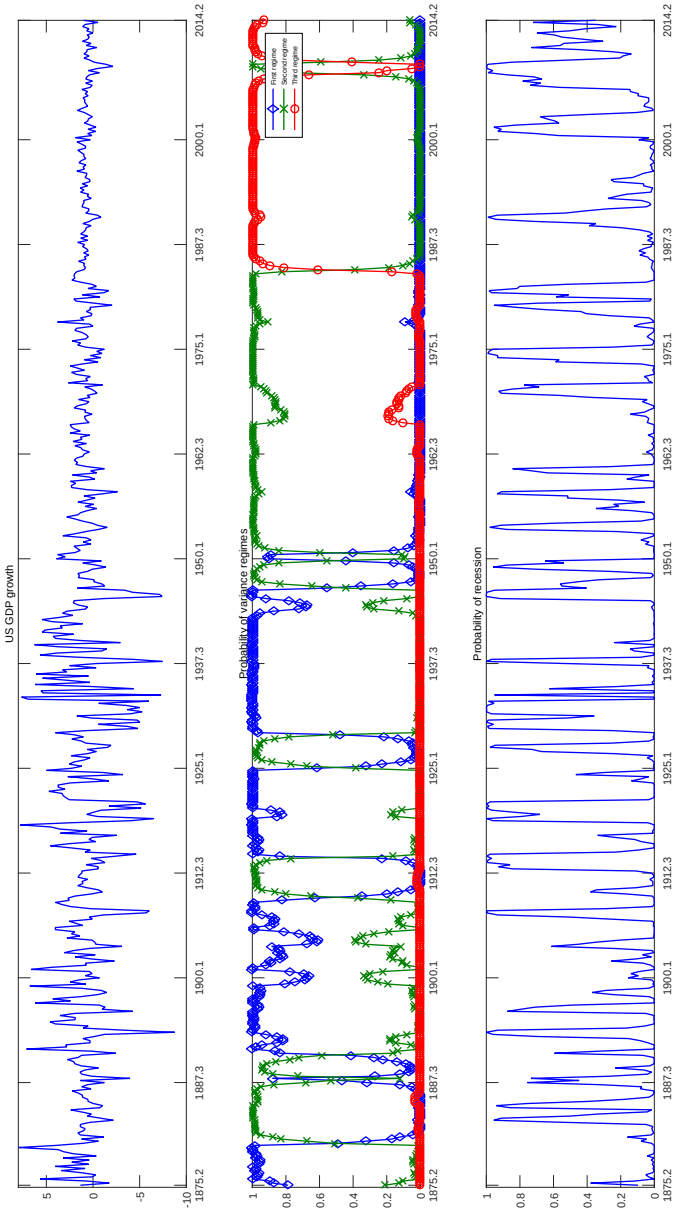
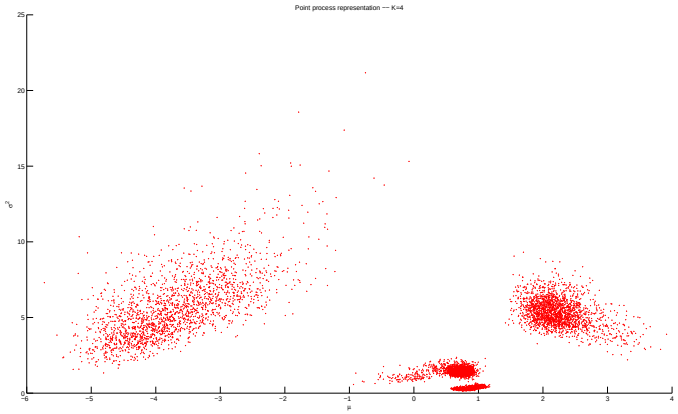
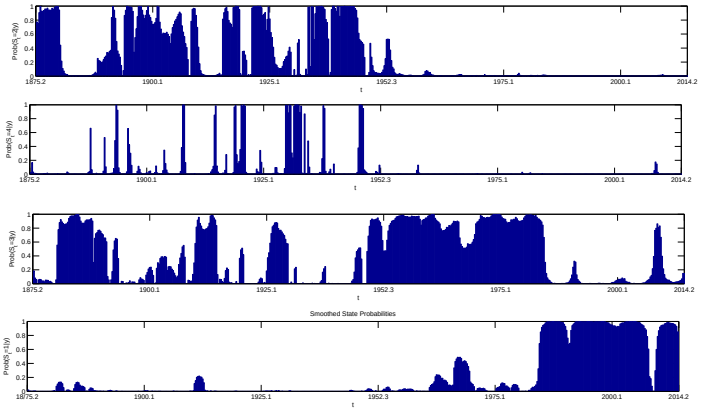


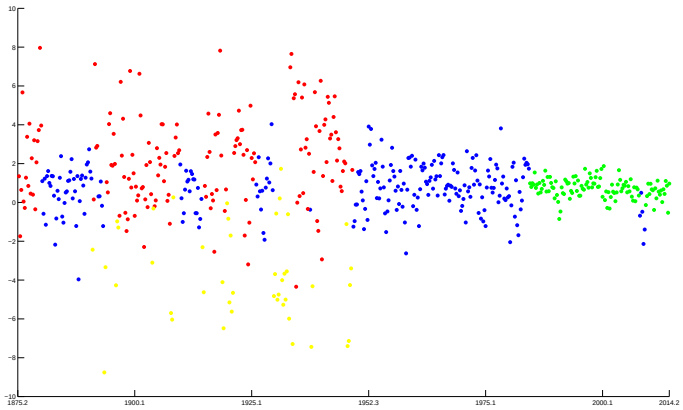
Figure 4. US business cycle and estimated probabilities of a MS with three variance regimes



(a) Draw point process representation K=4

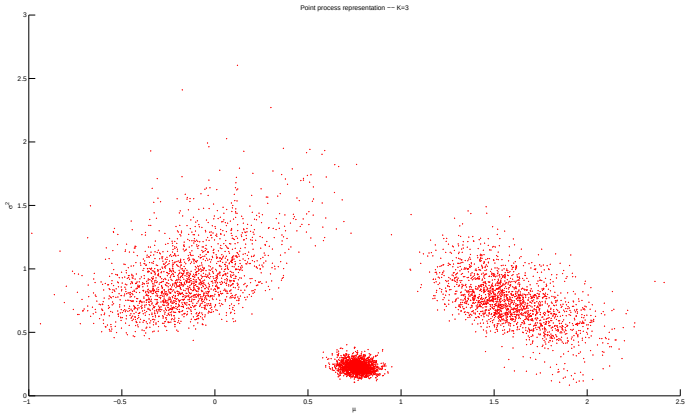


(b) Smoothed states probabilities

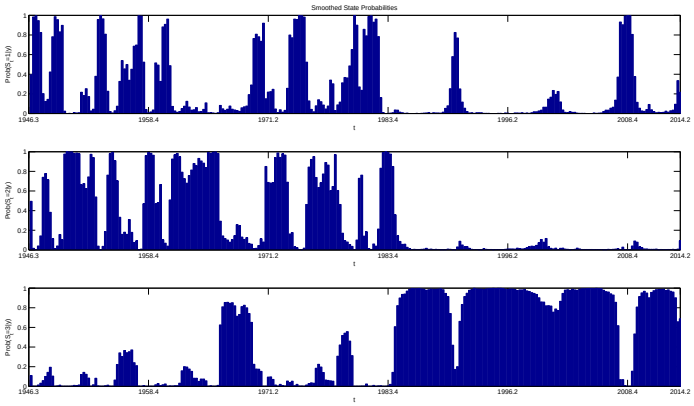


(c) Time series segmentation based on smoothed states probabilities

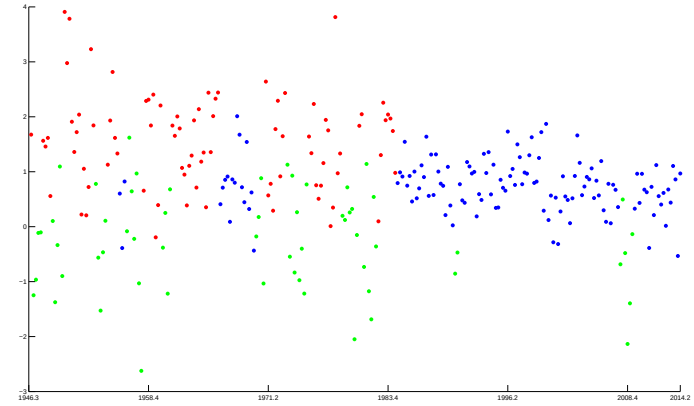
FIGURE 5. Results of estimating a Markov mixture with Kmax=4 1875.2-2014.2



(a) Draw point process representation K=3

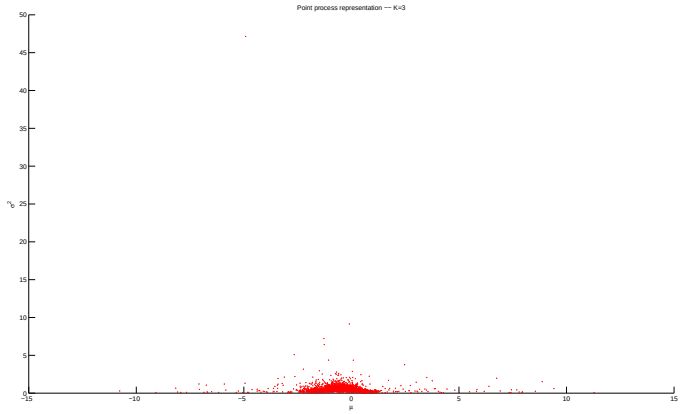


(b) Smoothed states probabilities

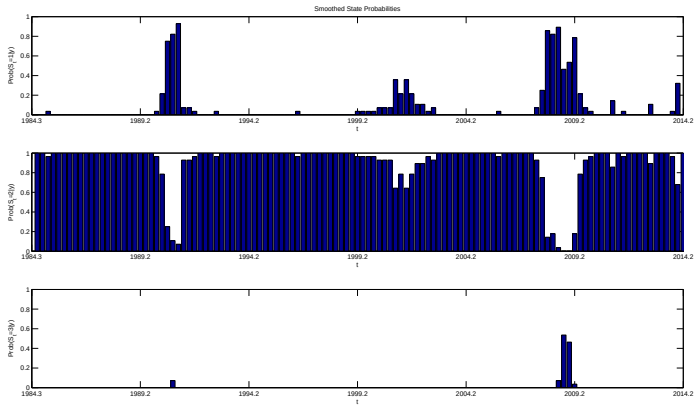


(c) Time series segmentation based on smoothed states probabilities

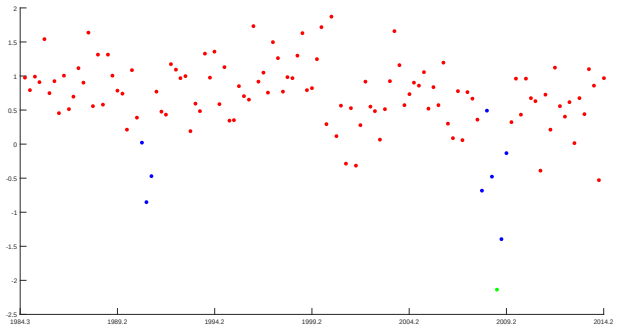
FIGURE 6. Results of estimating a Markov mixture with Kmax=4 1946.3-2014.2



(a) Draw point process representation K=3



(b) Smoothed states probabilities



(c) Time series segmentation based on smoothed states probabilities

FIGURE 7. Results of estimating a Markov mixture with Kmax=4 1984.3-2014.2