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INVESTING IN SYSTEMATIC FACTOR PREMIUMS

Abstract

In this paper we investigate and evaluate factor investing in the United States and Europe for equities and bonds. We show that factor-based portfolios generally produce comparable or better portfolios than market indices. We expand the analysis to other asset classes and factors, work with other optimization methods and add a basic liability structure. The results remain robust when we add real estate and commodities to equities and bonds. Also, the results are not dependent to the removal of a specific factor. Finally, we study the results for a worldwide investor who invests beyond the US and Europe. Over the longer term and with consistently applied factor diversification, factor investing appears to be advantageous.

JEL Classification: G11, G12, G15 and G23
Keywords: European data, factor investing, optimization, timing

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1 Introduction

Diversification is one of the key fundamental investment concepts, both in theory and in practice. It even has the status of being the only truly free lunch in the world of investment (Ilmanen, 2011). The idea is that by combining investment categories, you can limit a portfolio's price volatility without affecting expected returns. However, since the 2007–2009 financial crisis, when almost all investment categories declined en masse, this received wisdom is being increasingly criticized. In short, the investments turned out to be not as diversified as expected. And that begs the question: Is it then possible to effectively diversify a portfolio? Does this mean a portfolio has to be structured differently, or even that another type of portfolio altogether needs to be constructed?

Investing using factors is one such approach. Investors and researchers were already aware of this approach, and have identified many factors over the years. The interest in factors only got the boost it needed when in 2009 the Norwegian Government Pension Fund requested researchers Ang, Goetzmann and Schaefer study the results of the fund's active management. The results of active management turn out to be more closely related to market developments than had previously been thought. The researchers analyzed this effect, ascribing the results for a large part to factors that demonstrate a more systematic character, rather than to factors of a more transient nature to which active managers were more likely to allocate. This result triggered other investors to question whether they themselves were actually aware of the systematic factors behind their investments, and how a greater understanding of these factors could improve the investment process and results. This report reflects that line of research.

The sections are structured as follows. Section 2 places the attention to factor investing in a wider context. The aim of this section is to examine the relevant literature, and to answer the question as to why this subject is such a live one. Section 3 then introduces the factors. What is the theoretical support for them, and what are the pros and cons? Section 4 practically fleshes out the theoretical view of Section 2 with historical data series. We compare in plain terms how a portfolio based on investment categories performs differently than a portfolio using factors. Section 5 assesses the robustness of the results. Sections 6 and 7 aim at embedding the factor approach in the portfolio management process. Various optimization methods are compared in Section 6, and Section 7 then places this optimization study in the context of regulatory requirements. Section 8 summarizes the key findings.

2 Factor investing in perspective

Factor investing is increasingly in the spotlights. The concept is also known as "risk-premia investing", or investing in risk premiums. Other investors call it smart beta. What all these terms have in common is that investors try to identify sources of return and then attempt to define how they can capitalize on them effectively. One new aspect of this development is that it goes further than active versus passive, adjustment to benchmark or the development of an effective investment strategy. So what is it then? It revolves around an advanced understanding of the key choices faced by investors.

To handle this subject properly it is worth taking a step back and assessing how and why investing using factors has gained so much notoriety of late. We can identify three distinct periods in this process: the foundation of the investment profession, the quest for diversification, and the subsequent emerging role of factor investing.

2.1 Period 1: Foundation

We can comfortably place the first period in the founding years of the profession. Over the course of the seventies, academic insights gained greater acceptance among investors. The growing awareness of the increasing complexity and of complicated market conditions demanded that investing become a serious discipline (for an overview, see Bernstein, 2007). For example, Markowitz (1952) developed the concept that divides investing into three elements: return, risk and correlation. There is a connection between expected returns and risk exposure. Higher returns can only be realized with the acceptance of greater risk. Total portfolio risk can be reduced by spreading the portfolio over investments that are not fully correlated. If investments in price movements show limited correlation (in statistical terms: a correlation of less than 1), then that creates opportunities for a portfolio to realize a higher expected return despite the risk. Sharpe adapted this model to make it suitable for investing. They divided risk into unavoidable risk and avoidable risk. Part of the risk, the so-called systematic risk, of each individual investment is unavoidable. It is caused by general economic developments in the market, or the common characteristics of the investment itself: equity investors run different risks than bond holders. Systematic risk cannot be limited by diversification, so this part of the portfolio's risk profile should be compensated in its expected return requirements. It is a different matter with the other part of the risk, so-called specific risk, as that can be reduced using diversification.

In 1976, Stephen Ross further expounded systematic risk in his Arbitrage Pricing Theory (see Ross, 1976). If the investor bases his expected risk on a systematic risk he is exposed to, and that systematic risk changes due to various economic influences, is it then not more practical to relate returns directly to those various economic influences? In short, changes in expectations can be explained by the sensitivity to different economic factors. But what are these then? Chen, Roll and Ross (1986) find that macroeconomic factors – unexpected inflation, economic growth, investor confidence and changes in the yield curve – have a clear influence on equity returns. In their view, a good factor is one that is primarily expressed in unexpected movements, has a distinctly non-diversifiable effect, for which information is rapidly and easily accessible, and for which a solid economic argument can be put forward. Because this approach offers all sorts of benefits (such as the supposed linear relationship between factors and returns), for many asset managers it forms the basis for their portfolio and risk-management systems.

2.2 Period 2: Refinement and diversification

Model development goes hand in hand with the growth of financial markets, especially since the end of the eighties. For various reasons, equities were becoming increasingly attractive. However, from the end of the nineties the rise of the second period occurs, with diversification playing a key role. Investors look back to the nineties as the period of equity investment. At the same time, there was a growing awareness that this was not sustainable – as with any investment the risk of a bubble increased over time. The first omens of this were visible in 1997–1998 when, as a result of different crises (the collapse of the LTCM hedge fund, capital outflows from developing countries and Russia's refusal to pay its debts), equities fell sharply over a short period of time. Diversification in the classic sense did not appear to be working – equities declined in all regions of the globe. It became clear that proper diversification required a different approach.

The solution to this has been known among investors for some time already. David Swensen, head of investment at Yale since the eighties, heralded a new phase of development by posing the question as to which investments would actually be most suitable for a university fund. As with pension funds, the assets of such funds are to be paid out over the longer term. In short, his answer is greater illiquidity and diversification in the investment portfolio (Swensen, 2000). In the same way an institutional investor can add further (new) investment categories to his portfolio, an institutional investor who only has to pay out over the longer term is in a position to earn liquidity premiums where other investors are unable. Whereas in 1988 his portfolio comprised US treasuries and a few stocks, ten years later the Yale fund invested in private equity, emerging-market equities and high-yield bonds. And the results were impressive. In the mid-nineties, advisers adopted this model, with the expression “be like Yale” going mainstream in 1998. The dependency on equities in particular, and the increasing awareness that this investment category appeared to have reached its zenith (for the time being), made many managers question whether a widely diversified portfolio was not perhaps the solution they were looking for.

This development gained momentum specifically after the technology bubble burst at the beginning of this century, as investors raced towards diversification. Initially, diversification involved dividing investment categories into subcategories. Investors also increasingly applied other investment styles. From the nineties onwards, it also became clear that there were further factors behind equity returns, with a closer focus on value, growth and market cap. These corrections were called “anomalies”, as they influenced returns and risk but did not fit into the standard models. The next step in diversification was that the results of active management – alpha – were being seen as a separate investment style or category, and being included in portfolio optimization. Investment restrictions seen as holding back the portfolio manager were relaxed, giving rise to investment models like the 130/30.

2.3 Period 3: Towards effective diversification

The litmus test for the refinement and diversification described above came in late 2007. Portfolios were more sophisticated, more widely diversified than ever, and should have been better able to absorb shocks. But the reality was different. Over the course of 2007, active managers and hedge-fund managers struggled with evaporating liquidity and disappointing results. Submarkets were “drying up”. This turned out to be a warning shot for the crisis beginning in the fourth quarter of 2008. After a number of bailouts, Lehman Brothers investment bank went bankrupt. This firm had a pivotal role in derivative transactions, and therefore the financial structuring of institutional investors. The crisis was severe, but ran along familiar lines: declining trading volumes, increasing volatility and a greater degree of risk aversion. Equity and bond markets responded in general as can be expected. Due to a combination of overreaction and a deteriorating outlook, investors sold stocks en masse and bought bonds.

However, many pension funds were surprised by the development of alternative investments, with the returns of hedge funds, for instance, disappointing. Despite their ‘skill’, hedge-fund managers in particular do not always seem able to keep their heads above water in challenging market conditions. Examined more closely, these investments often turned out not to be quite as unique as had been thought. Some strategies are based on stocks, and thus correlate strongly with the equity portfolio. Furthermore, many strategies are leveraged – built up using borrowed capital – linking them closely to the credit issuer and interest rates. Exposure to equity risk is not just found in stocks, but also in corporate bonds, hedge funds and private equity. The idea behind diversification was: the greater the number of different investment categories, the better, while in reality this was just compounding risk. In other words, gradually the realization took hold that investments must not be seen in isolation, but as a bundling of factors that have an impact on risk and return. Investments turn out to have all sorts of unexpected connections, or underlying factors that drive otherwise unrelated investments. Identifying these, and then managing them accordingly is the first important development.

This discussion shifted up a gear when the Norwegian Government Pension Fund launched an investigation into its active management policy. After years of stable outperformance, the negative results of 2008 destroyed the cumulative active return of the previous ten years, triggering the fund to investigate whether it was of any value to this part of the portfolio. Andrew Ang, William Goetzmann and Stephen Schaefer investigated why the active return had behaved in this way. They concluded that 70 percent of the active return could be explained by exposure to systematic factors. They therefore advised structuring the portfolio using 10 factors relevant to the fund.

Diversification across factors, instead of across investment categories, can have major benefits. The correlation between the factors is much lower than between investment categories. This reduces the total correlation between the elements that make up the portfolio. But this raises further questions. What is the origin of these factor premiums exactly? Only when you are able to discover where these premiums actually originate, are you able to achieve the optimal undiluted exposure. This is especially relevant as some factors generate greater returns that their associated risk might at first suggest.

Ilmanen and Kizer (2012) compared factors with the ingredients are used to prepare food. The idea is that an investment category – equities, for example – can be unraveled into a number of factors that influence return and risk. Share prices can be affected by changes in oil prices, interest-rate movements, general sentiment in the market, and by investor earnings estimates or the likelihood of bankruptcy. Under certain circumstances, it could even be possible to avoid investing in equities entirely. The overlap with other factors can then also be identified and used to manage the investing process. This makes excessive concentrations of one sort – like an emphasis on equities – a thing of the past. However, investing on the basis of factors has even more potentially radical consequences for the investing process. Firstly, the emphasis shifts from weights in euros to weights based on risk (further defined in Section 6). The focus then goes from investment categories to factors for investment styles and/or other more fundamental factors.

3 Factor premiums

3.1 Introduction

This section gives an overview of the available factor premiums. When considering investing using factors, the question arises as to which factors to use. In conventional investing, this issue is easy to assess. The characteristics of the various familiar asset categories that are crucial to asset allocation, such as equities and bonds, are more or less a known quantity these days. Depending on (your) expected returns and risks on the assets in question, any funds to be invested are given a certain allocation.

Many different factors are identified and used, both in the academic literature and in practice, yet there is no agreement about what constitutes a good overview of these factors and, more importantly, which factors lend themselves to portfolio management. The Norwegian Government Pension Fund's ‘Strategy Council’ puts it more succinctly: “No consensus has yet been formed on which factors, besides the equity premium, drive asset prices” (Dimson et al, 2010, p. 10). In short, there is no handbook that investors can refer to that shows them which factors should or should not be used. In this section, we therefore first describe the many different types of factor, considering which criteria are used in the literature for the applicability of those criteria. Based on that we then make a selection with which we create portfolios in the following sections.

There is also no consensus in the literature on nomenclature. The terms ‘risk factor’, ‘factor’ and ‘factor premium’ are used interchangeably and the definitions are not uniform. We have therefore decided to use the term factor or factor premium instead of the term risk factor. The reason for this choice is that the term risk factor suggests that the factor derives its rationale from offering compensation for exposure to a certain risk. That is certainly not the case for certain factors, or the empirically positive premium is even associated with a *lower* risk (the LowVol factor or the Short Credit factor, for instance). However, other factors may indeed originate from investor sentiment driven effects, such as the momentum factor. That is why we opted for the more neutral term “factor”.

3.2 Categories

There is no generally accepted categorization of the various factor premiums. For this study, we concur for practical reasons with the categorization of Bender et al. (2010). They distinguish three types of factor premium that arise from exposure to the risk of a broad (1) asset class, (2) style or (3) strategy. The *asset class* factor premium comes from passive investing in the traditional sources of risk – equities, bonds, real estate or commodities. The *style* factor premium covers the expected returns from assets with comparable fundamental or technical characteristics. In terms of equities, examples include value, smallcap and momentum. For bonds, examples are yield-curve spread, credit spread and high-yield spread. Liquidity and volatility could also be factor premiums in this category. The third type, the *strategy* factor premium, is generated by implementing a certain strategy, such as merger arbitrage, convertible arbitrage or 'carry'. In addition to systematic market risks (equities, yield, credit) and security characteristics (size, value, momentum), Clarke et al. (2005) advise on the contrary to also include certain 'global market factors' in factor-oriented investing. This approach takes explicit account of the differences that exist between countries and regions in terms of relative attractiveness. Further, Ilmanen and Kizer (2012) use the terms "static asset-class premia" and "more-dynamic style premia" or static and dynamic factor premiums. The authors recognize the common factors value, momentum and carry, but add trend-style premium. Value and momentum are calculated on equities, while carry and trend premiums are calculated on liquid macro-asset instruments.

3.3 Most common factors

Even though there is no consensus over which factors offer investors the best opportunities, some factors have clearly garnered greater attention in both academic research and in practice. Table 3.1 summarizes the key factors.

Table 3.1: Factors identified in the investment literature.

Type	Factor in the literature	Applicable to
Asset	Inflation	Equities, Bonds, Currencies, Commodities
	Economic growth	Equities, Bonds, Currencies, Commodities
Style	Value	Equities, Bonds, Currencies, Commodities
	Growth	Equities, Bonds, Currencies, Commodities
	Momentum	Equities, Bonds, Currencies, Commodities
	Low Volatility	Equities, Commodities
	Term	Bonds
	Credit	Bonds
	Short Treasury, Short Credit	Bonds
	Short Term and Long Term Reversal	Equities
	Volatility	Equities
	Liquidity	Equities, Bonds
	Emerging Equity Market	Equities
	Convexity	Bonds
Strategy	Carry	Equities, Bonds, Currencies, Commodities
	Trending	Equities, Bonds, Currencies, Commodities
	Anomaly factors, such as the 'accruals anomaly', 'IPO-anomaly', 'index change anomaly' or 'calendar	Mainly equities

	effects', including the 'Turn-of-the-Month' effect and the 'pre-holiday' effect	

The asset-class factor premium comes from passive investing in the traditional source of risk – equities. The style factor premium covers the expected returns from assets with comparable fundamental or technical characteristics. The strategy factor premium is generated by implementing a certain strategy.

For equities, the factors Market, Value, Size and Momentum are the most well-known among investors. Mainly due to the Fama and French (1992, 1993) studies into Value and Size and the Jegadeesh, Titman (1993) and Carhart (1997) studies into Momentum.[4] See also Blitz, Huij and Martens (2011) for a recent study. Norges bank (2009, p. 10) also asserts "The size, value and momentum effects were the first anomalies to be identified and are therefore also the most widely cited in the debate on efficient markets". These factors are also investigated by Ang et al. (2009), Bender et al. (2010), Blitz and Van Vliet (2008), Clarke et al. (2005) and Huij and Verbeek (2007, 2009). Indeed, these articles also mention Market, Value, Size and Momentum as factors. As do Ilmanen and Kizer (2012), who calculated the factors themselves. Subsequent to the initial description in academic articles, these factors continue to generate positive returns. They have not been arbitrated away. The only exception to this is possibly the Size factor premium, where there is a lack of clarity as to whether this effect is still generating mark-up. Finally, we also mention here the LowVol factor. This factor premium has been identified in the academic literature for a long time, but is less well-known than the Value, Size and Momentum factors. We refer to Haugen and Baker (1991), Ang et al. (2006), Blitz and Van Vliet (2007, 2011) and Van Vliet (2011) for further underpinning. The 'leverage aversion' theory serves as an explanation of this anomaly (see Asness, Frazzini and Pedersen (2012) for a good description).

In bond markets, the factors Term and Credit are nearly always mentioned. References include Ang et al. (2009), Bender et al. (2010), Clarke et al. (2005), Fama and French (1989, 1993), Gebhardt, Hvidkjaer and Swaminathan (2005) and Ilmanen and Kizer (2012). In many of these studies, these are the only two factors mentioned, with other factors barely getting a mention, if at all. After all, most attention in factor investing is focused on equity markets, or all financial assets taken together. Bond markets are the subject of less research, which is strange given the size of bond markets and the importance of fixed-income investments for pension funds, banks and asset managers. Only recently are more studies appearing, mostly practice-based, into the use of factor premiums such as carry, momentum and value in bond markets. Furthermore, there are a couple of academic studies into the most suitable term to invest in. In

[4] The website of Kenneth French (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html) provides a detailed data library that includes the factors Value, Size and Momentum. This website only gives equity factor data, and no factor data for bonds or other asset classes.

these studies, the outcome is that the short end of the curve, around two to three years, provides the best risk-return ratios. We hereby refer to Bieri and Chincarini (2005, p. 21), Derwall, Huij and De Zwart (2009), Frazzini and Pedersen (2010), Ilmanen (1996) and Pilotte and Sterbenz (2006). Our study therefore uses Short Credit and Short Treasury to make use of these factor premiums.

The factor Credit is calculated in various ways. Some studies use just a single criterion, for instance the spread between long government bonds and long (with the same duration) investment-grade corporate bonds. But other studies add the Credit spread between bonds with an investment grade and bonds with a B-rating, and even include the spread between bonds with a B-rating and high-yield bonds. The spread on Residential Mortgage Backed Securities is also used, or the spread on sovereign bonds.

Some studies use a total of just three ‘premia’, namely Value, Momentum and Carry (see Mesomeris et al. (2012), for instance). These three premiums are then consistently calculated for four asset classes, namely equities, bonds, currencies and commodities. In total, this approach thus covers 4 x 3 = 12 factor premiums. The empirical results confirm that these three factors provide ‘excess return’ and that the underlying correlations are low. Investing in factors, both by asset class and, in particular, across the four different asset classes, leads on average to a strong improvement in the risk-return ratios of the investment. The factor premium Carry is sometimes also mentioned under a different name. This factor is best known for currencies, with reference to Hodrick (1987) and Froot and Thaler (1990), for example.

To complete the overview, we mention other, less well-known factors:

Short Term Reversal and *Long term Reversal*. Both factors are explained on the Kenneth French website, which also includes the data. See also De Groot, Huij and Zhou (2012), for example.

Volatility. We refer to Ang et al. (2006, 2009). In studies, this factor premium is measured using a variance swap, or the difference between the VIX and volatility actually realized. The idea behind this is that institutional and private investors are often buyers of either puts or calls. Dealers are compensated for exposure to volatility risk by earning a risk premium in the implied volatility sold. The VIX Futures Index is used, too.

Liquidity. We refer to Ang et al. (2009), Li, Mooradian and Zhang (2007), Pástor and Stambaugh (2003). There is still no consensus in the literature on how this factor should be measured. Various articles suggest that market-wide systematic liquidity is a factor to be taken into account. The Norwegian Government Pension Fund is also of the opinion that the liquidity premium should be ‘harvested’ (see Government Pension Fund Global (2011, p. 5)).

Trending. We refer to Moskowitz, Ooi and Pedersen (2011) or to Ilmanen and Kizer (2012). This anomaly is also called time series momentum, referring to both the regular momentum factor and cross-sectional momentum. Fung and Hsieh (2004) also use a Trending factor premium, which they calculate as the return made on look back straddles on both bonds and currencies, as well as on commodities.

Emerging Equity Markets. See for example Bhansali (2011) and David Hsieh's website. The IFC Emerging Markets Index is often used here, or in Datastream the MSCI Emerging Markets Index. See also Government Pension Fund Global (2011, p. 12), which argues that this factor premium is suitable for long-term investors in particular.

Convexity. This cross-sectional criterion only applies to bonds. Bonds with higher convexity show favorable price movement in volatile markets. This potential factor premium is thus closely related to the potential Volatility factor.

Inflation. This factor has been measurable and tradable for some time now, via euro inflation-linked bonds minus nominal government bonds, as in France, for example, or TIPS in the US. See also Elton, Gruber and Blake (1995) or Derwall, Huij and De Zwart (2009).

Economic growth. There is no 'natural' way to invest in this factor. How do you gain exposure to GDP growth? Callan (2012) takes the MSCI World Index as a 'proxy', but this is of course only a partial solution. Peters (2008) also mentions equities as the asset of choice to gain exposure to long-term worldwide economic growth.

Finally, various *anomalies* can also be used, such as the 'accruals anomaly', 'IPO-anomaly', 'index change anomaly' or 'calendar effects' (such as the 'turn-of-the-month' effect and the 'pre-holiday' effect). This is not the place to investigate these possible factors further. A bibliography covering this field is available from the authors on request.

3.4 Alternative approach

It is worth noting that a different approach could be considered. The factor premiums mentioned above originate in empirical research. Indeed, it turns out that these properties or strategies lead to positive returns. But this approach has limitations. Fama and French, who were involved when factor premiums were first being identified, discovered that "five factors do a good job […] but the choice of factors […] is motivated by empirical experience. Without a theory that specifies the exact form of the state variables or common factors in returns, the choice of any particular version of the factors is somewhat arbitrary." (Fama and French, 1993, p. 53).

Another approach that to a degree answers the criticisms of Fama and French is to identify factor premiums from a macroeconomic perspective, and to make that as robust as possible under different scenarios. In that case, inflation and economic growth in particular are taken as the two key 'drivers', or factor premiums. Ang and Kjaer (2011) for instance identify inflation and economic growth, but also political risk as so-called 'fundamental factors'. The aim is then to put together a portfolio that performs well under as many scenarios as possible, so under diverse combinations of inflation and economic growth. This approach can be partly coupled to the factor-oriented investment approach in this study. The idea is then that the two 'drivers' inflation and economic growth translate into the more conventional risk factors term spread and interest rate. Regression analysis can be used to determine to what degree a change in the macro drivers translates into changes in the factor premiums. For example, the credit spread generally increases when economic growth is below expectations, but credit spread has a low correlation with unexpected changes in inflation. Translating macro-scenario analyses into factor premiums in this way can help determine robust asset allocation. However, not all possible future scenarios can be predicted, so in the event of so-called 'regime shifts' the intended diversification could turn out to be substantially less effective than expected.

3.5 Choice of factor premiums

When choosing between factors, all sorts of criteria are important. These are concentrated around questions like rationality, persistence, implementability, manageability, and explainability. We then couple this to the advancing insights into putting together portfolios. In the European context, the Alternative Investment Fund Managers Directive is a major criterion. For our study, we base the selection of factor premiums on the following criteria:

- Rationality – Examples include the requirement that the factor premium must have a strong theoretical basis and a clearly defensible economic intuition. Chen, Roll and Ross (1986) add that this can include a non-diversifiable factor. In other words the factor cannot be unraveled further. Only after a structural change in the market would the factor premium disappear.
- Persistence – Otherwise the factors must also have been known for some time, with a thorough research history. This means that they have been realizing positive returns for a longer period, though this may of course be negative in cyclical periods.
- Implementability – Finally, the factor premium must have sufficient liquidity, be scalable and accessible at acceptable costs. The way to implement the factor premium must be clear and involve as few financial instruments as possible.

- Manageability – The pension fund must apply a consistent strategy to the factor premium. Results must not, for example, cause an excessive turnover rate, with major changes in weights due to (risk) rebalancing. Furthermore, a combination of factors should not cause undesirable secondary effects. These might be higher leverage, especially where the factor premiums are used for simultaneous buying and selling, or counterparty risk where factor premiums involve relatively high numbers of (OTC) derivatives.
- Explainability – The factors must be easy to explain. This is closely related to the points above, and reflects what many pension-fund managers are increasingly expressing as policy principle: if we don't understand it, we're not investing in it.

Evident factor premium choices for equities are thus: Market, Value, Momentum, and Size. The factor LowVol is also included for equities. We leave aside the other potential equity factor premiums for the time being. For bonds, the logical choices are: Market, TermSpread and CreditSpread. We add to this the Short factor (or short terms) owing to the study results in this area mentioned above. We thereby distinguish between ShortCredit and Short Treasury factors. For bonds we initially leave aside the factors Value, Momentum and Size, because we did not find good data series for these factors. Further, given that we are working with unleveraged long-only portfolios, we initially leave aside the factors Carry and Trending. Later in this study, in the robustness test, we include the Carry factor in the analysis. The rationales for each of the factors used in this study are given briefly below. Or: why does it work? In this explanation we use the recent and very detailed report by MSCI (2013).

For equities, we distinguish between:

Value. Equities with a low price relative to their fundamental value – book value, sales, income, earnings, dividend, etc. – are overweighed. With an efficient market approach, ‘Value’ companies would be more risky and therefore offer greater compensation to investors. A common argument here is that Value companies are less able to adjust to unfavorable economic conditions. From a ‘behavioral finance’ point of view, it can be asserted that this factor premium exists because investors are ‘loss averse’ and have ‘mental accounting biases’. Such behavioral predispositions lead to the presence of a value premium. We refer to Basu (1977) for a very early study into the Value factor premium.

Momentum. Equities that have shown relatively high returns in the recent past are overweighed. The most common explanations come from the ‘behavioral finance’ theory. Momentum can be caused by the way in which investors interpret and process news. The market can overreact, but also under-react. Both responses can lead to the existence of momentum, under certain assumptions. The best known behavioral predispositions that lead to momentum in the market are overconfidence, conservatism, and loss-aversion. The ‘career risk’ of major investors is also mentioned as a cause of momentum. We refer to Jegadeesh and Titman (1993) and Rouwenhorst (1998) for early studies into the Momentum factor premium.

Size. Stocks of relatively small companies are overweighed. In an efficient market approach, investing in this sort of stock carries a higher risk and therefore offers a premium. Size could function as an approach to certain types of risk related to the size of the company, such as the liquidity of the shares, the uncertainty of the information on the business and the chance of financial risks, including bankruptcy. From a ‘behavioral finance’ point of view, the tendency to extrapolate the recent past could mean that smallcaps that grow fast are assessed and evaluated over-optimistically. We refer to Banz (1981) and Fama and French (1992, 1993) for early studies on the Size factor premium.

LowVol. Low-volatility stocks, mostly with low beta, are overweighed. Commonly used explanations come from ‘behavioral finance’ theory. Investors are too focused on tracking error instead of on actual risk, making low-risk stocks less attractive and generating a premium. Another possible reason is not daring to take on leverage in the portfolio. Greater leverage on the balance sheet leads to higher expected returns and, in the search for returns, there is greater demand for companies with relatively high leverage. The ‘lottery ticket’ effect is also mentioned as a reason: investors love to buy volatile stocks in the hope of earning big bucks in a short time. Furthermore, there are other ‘behavioral finance’ explanations for the existence of this premium, such as ‘attention bias’ and the ‘winner’s curse’. We refer to Haugen and Baker (1991) and Van Vliet (2011) for further studies into the LowVol factor premium.

For bonds, we distinguish between, in order:

TermSpread. Bonds with a long maturity are overweighed. Such bonds have higher price volatility, so greater risk. This factor premium represents the compensation for taking interest-rate risk by consciously not investing in short-dated paper. See Dyl and Joehnk (1981) and Fama and French (1989, 1993) for early studies into the TermSpread factor premium.

CreditSpread. Bonds with a higher credit risk are overweighed. This factor premium represents the compensation for taking credit risk by consciously not investing in very creditworthy bonds. See Fama and French (1989, 1993) and Elton, Gruber, Agrawal and Mann (2001) for studies into the CreditSpread factor premium.

Short Treasury and Short Credit. Short-dated bonds of both governments and other issuing bodies are overweighed. Empirical evidence shows that lengthening duration to two years in particular leads to higher returns. Historically, the most favorable ratio between risk and return in maturities is two to three years. Aversion to the use of leverage, or simply its prohibition, is often mentioned as a reason for the existence of this factor premium. Furthermore, it is mentioned that short-dated bonds are relatively less sensitive to business-cycle changes and thus have a different source of systematic risk than long-dated paper. We refer to Ilmanen (1996) and Bieri and Chincarini (2005) for studies into the Short Bonds factor premium.

4 Empirical Results

4.1 Approach

The first empirical results are described below. To ascertain what the discrete and combined effects are of the factor premiums, we compile portfolios on the basis of the 1/N principle: each factor is allocated equal importance in terms of weight, with the weights being regularly rebalanced. This means we keep the effect of the portfolio optimization technique (thus excluded) separate from the effect of the factor premiums. Section 5 goes into more detail, examining to what degree the results change if we use other portfolio optimization techniques.

Appendix I describes the data series in detail. For the record, in the tables concerned, we split the factors into 'market-cap weighted' and 'alternative' factors. The first category relates to the factors for which the underlying building blocks (the 'constituents') are assigned the same weights as the conventional market indices. This is the case for regular indices for equities, bonds, real estate and commodities. In the second category, such as Momentum, Value or Size, we use a different (hence 'alternative') criterion to ascertain the factor.

The reported returns relate to annualized geometric averages. The standard deviation is calculated annually to reflect returns and standard market practices. We have not taken any account of transaction costs, bid-offer spreads, 'slippage' and such matters. The number of observations in Table 4.1 is 1,003 for all series. The maximum drawdown was calculated as the maximum percentage 'peak-to-trough' loss suffered over the entire period. To illustrate: if the maximum drawdown ends up at 50 percent, this means that at a certain point an intermediate peak is reached in the cumulative return, after which a 50 percent loss arises relative to that peak. The percentage loss was not higher either before or after. The reported Calmar ratio is calculated as the average return (i.e. 'cumulative average gross return') divided by the maximum drawdown. The drawdown used when calculating the Calmar ratio is often measured in the literature over a period of three years. The example below uses the maximum drawdown over the *entire* period as input for the calculation and is thus a conservative interpretation of this financial risk indicator. However, in the end the Sharpe ratio takes precedence when assessing the various portfolios, as in practice this criterion is the most commonly used. The Sharpe ratio is calculated as the ratio of the average portfolio return minus the risk-free rate (the numerator), both annualized, to the standard deviation of the same data series (the denominator). For further qualification on using the Sharpe ratio, we refer to Ledoit and Wolf (2008), Lo (2002) and Memmel (2003).[5]

4.2 Results for equity portfolios

Table 4.1 gives the details of various 'long-only' equity portfolios for the period January 1929 to July 2012. The column 'EW' stands for 'Equally Weighted'. This column gives the results for an equally weighted portfolio made up of the four Size, Value, Momentum and LowVol portfolios, each 25% and reweighted monthly. See Appendix I for more details on the data source.

Table 4.1. Factor premiums for US equities, January 1929 – July 2012

Factor premium type	Market-cap weighted	Alternative factor premiums				
	Market	Size	Value	Momentum	LowVol	EW
Average return	8.73%	11.86%	11.14%	12.53%	9.69%	11.09%
Standard deviation	19.12%	26.49%	25.37%	19.40%	16.22%	20.29%
Skew	0.12	1.47	1.54	-0.33	0.12	0.54
Kurtosis	7,21	15,19	17,86	2,92	7,94	9,51
Maximum drawdown	83.7%	87.4%	89.7%	77.3%	78.9%	83.5%
Calmar ratio	0.10	0.14	0.12	0.16	0.12	0.13
Sharpe ratio	0.35	0.42	0.40	0.53	0.44	0.47

The results in Table 4.1 demonstrate that average market returns are lower than the four discrete factor portfolios Size, Value, Momentum and LowVol. Apparently, these four factor premiums have on average outperformed the market average in the US since 1929. Strikingly, the Momentum portfolio performed best – at 12.53% the result is (12.53 - 8.73)/8.73 = 44 percent higher than that of the market portfolio. Since 1929, the equally weighted factor portfolio returned 11.09% on average, placing it (11.09 - 8.73)/8.73 = 26 percent higher than the market average.

An investment in one of the factor portfolios would have given two factors (Size and Value) a higher standard deviation than that of the market. In these two cases, higher returns thus appear correlated to higher risk, insofar as this is measured by standard deviation. The standard deviation of the Momentum

[5] Lo (2002) suggests that one should be cautious when using Sharpe ratios. In practice this criterion is very popular. However, Lo states that the degree of variability and the underlying non-normality of the returns (including series correlation) can result in the Sharpe ratio giving a distorted picture of the attractiveness of a portfolio return distribution. See also the footnote on the Jobson-Korkie test.

portfolio is roughly equal to that of the market portfolio. However, the LowVol portfolio has a clearly lower standard deviation, which corresponds with the underlying idea of this strategy.
The skews in Table 4.1 show that neither the factor portfolios, nor the market have a very skewed return distribution. A skew of zero indicates that the observations are equally distributed on either side of the mean and that the distribution is probably (though not necessarily) symmetrical. The skews found in Table 4.1 have a slight tendency towards positive values. The kurtosis estimates in Table 4.1 show that the return distribution has a fatter tail than would be the case for a normal distribution, but this is usual in financial markets. It is also striking that the Momentum portfolio clearly has less kurtosis than the other factor portfolios, which have fatter tails than the market portfolio. Nevertheless, this high degree of fat tails is not expressed in higher drawdown. Indeed, in Table 4.1 the drawdowns are very high for all portfolios, varying from a minimum of 77.3 percent for the Momentum portfolio to as much as 87.4 percent for the Size portfolio. These record losses were all suffered in the early thirties of the previous century.

The results in Table 4.1 indicate that the Calmar ratio for all factor portfolios is higher than for the market portfolio. The difference in Calmar ratio varies from 18 percent for the LowVol portfolio to 56 percent for the Momentum portfolio. Finally, Table 4.1 shows that the Sharpe ratio for all factor portfolios is also higher than for the market. The relative differences amount to at least (0.40 - 0.35)/0.35 = 14 percent for the Value portfolio and rise to (0.53 - 0.35)/0.35 = 51 percent for the Momentum portfolio. The equally weighted factor portfolio also has distinctly higher Sharpe ratios than the market, with a (0.47 - 0.35)/0.35 = 33 percent improvement. The Jobson-Korkie (1981) test, developed to determine whether two Sharpe ratios are statistically significantly different, confirms that this difference is significant to a confidence interval of more than 99%.[6]

The above results indicate that these four factor premiums have performed well since 1929. An equally weighted portfolio of the four factor premiums would have been a clear improvement on the market. For today's investor, the long history of the equity series since 1929 is crucial, as it provides a basis for tomorrow's expectations. Indeed, as the effects observed become stronger and more consistent over time, they generate greater confidence that one should take account of them in future too. At the same time,

[6] This test was developed by Jobson and Korkie (1981) to determine whether two Sharpe ratios were statistically significantly different. This test was further improved upon in later studies, among others by Memmel (2003) and Ledoit and Wolf (2008). This last article used time-series bootstrap-based confidence intervals, assuming the underlying Student (t) distributed returns. In this article we use the original version of the test, as it is the better-known variant. However, the ‘power’ of this test is quite low and the underlying assumption of normally distributed returns is not correct. When using the Jobson-Korkie test, one must therefore remain non-committal.

today's investor must also be aware of what the recent past has demonstrated. Have the long-term effects continued over the last couple of decades? To answer this question, see Table 4.3 below that gives the results from January 1990, thus representing a recent sub-period from the results in Table 4.1. Studying the details of Table 4.3, one comes to the general conclusion that the results over the last two decades are clearly comparable in qualitative terms to those in Table 4.1. Although the ultimate returns are slightly different in Table 4.3, the underlying classification is very similar, which tends to confirm the previous conclusions. Evidently, the results are fairly stable over time. Once again the four factor portfolios and the equally weighted portfolio give a higher mean return than the market. The standard deviation of most of the factor portfolios is higher than the market, except in the case of the LowVol portfolio.

Table 4.3. US equities, January 1990 – July 2012

Factor premium type	**Market-cap weighted**	**Alternative factor premiums**				
	Market	**Size**	**Value**	**Momentum**	**LowVol**	**EW**
Average return	8.82%	13.06%	9.89%	12.33%	9.68%	11.10%
Standard deviation	16.28%	19.78%	16.78%	16.22%	12.34%	14.90%
Skew	-0.76	-0.48	-0.89	-0.49	-0.70	-0.83
Kurtosis	1.56	1.26	2.84	0.99	2.14	1.80
Maximum drawdown	51.6%	54.1%	59.6%	48.1%	44.3%	51.0%
Calmar ratio	0.17	0.24	0.17	0.26	0.22	0.22
Sharpe ratio	0.40	0.55	0.46	0.61	0.56	0.60

Further, though all skews in Table 4.3 are negative, this is only the case to a limited degree. The kurtosis since 1990 is lower than the measurement over the entire period from 1929, with the Great Depression having a major impact. The Calmar ratio confirms this picture further: the Calmar ratio of all the factor portfolios is higher than that of the market. The only exception is the Value portfolio, for which this indicator is roughly equal. Finally, the Sharpe ratio shows that it would have been very lucrative to invest in factor portfolios over the last two decades, too. The Sharpe ratios for all factor portfolios are higher, with a relative improvement for the equally weighted factor portfolio of no less than (0.60 - 0.40)/0.40 = 49 percent. The Jobson-Korkie test confirms that this difference is significant to a confidence interval of over 99%.

4.3 Results for bond portfolios

Below we examine the use of factor portfolios for bonds instead of equities. The data series available for bond factors are shorter than for equities. The data originate from Datastream and the St. Louis Federal Reserve website. See Appendix I for more details on the data source. The data series has a start date of

January 1976 and runs to August 2012, with 439 observations in total. Table 4.5 shows the characteristics of the various bond-portfolio series. The returns are high, varying from 6.6 percent for the Short Treasury series to 9.9 percent for High Yield. Over the last two to three decades, this means that bond returns have been almost as high as equity returns. The protracted decline in interest rates to very low levels as of late 2012 is the main reason these returns have reached such high levels. The standard deviations of bond returns are clearly lower than for equities, which fits this less risky investment category. The Short Treasuries and Short Credit bonds show distinctly lower standard deviations than the other series. After all, the prices of these bonds are less sensitive to interest-rate movements.

Table 4.5. Factor premiums for US bonds, January 1976 – August 2012

Factor premium type	Market-cap weighted factor premiums				Alternative factor premiums	
	Market	Term	Credit	High Yield	Short Credit	Short Treasury
Average return	8.28%	9.55%	8.75%	9.88%	7.56%	6.57%
Stand. dev.	5.58%	10.01%	7.01%	9.11%	2.76%	2.86%
Skew	0.68	0.46	0.37	-0.33	1.02	1.80
Kurtosis	6.66	2.80	5.79	5.94	12.30	15.04
Max. drawdown	12.7%	21.0%	19.3%	27.2%	5.2%	3.9%
Calmar ratio	0.65	0.46	0.45	0.36	1.47	1.77
Sharpe ratio	0.54	0.45	0.50	0.54	0.83	0.57

Further, in Table 4.5, most series show a slightly positive skew and a higher-than-average kurtosis than is the case for equities. Apparently the distribution of bond returns tends to have relatively fat tails. On balance, the risks are clearly lower, a fact also expressed in the lower drawdowns that vary from 3.9 percent for Short Treasuries to 27.2 percent for the High Yield series. These drawdown levels run from less than half to even less than a tenth of those shown by the equity portfolios. Ultimately, the combination of relatively high returns and low drawdowns, especially in the two short bond series, leads to Calmar ratios that are distinctly higher than for the equity portfolios. The Sharpe ratio is then less sensitive to drawdowns as this criterion uses standard deviations as input parameter. At 0.45 and 0.50, the Sharpe ratios of the Term and Credit series are slightly lower than that of the Market series (0.54). In the High Yield series, both return and standard deviation are higher, ultimately giving this series the same Sharpe ratio as the Market series. However, it should be noted here that return data for the High Yield series was only available from January 1980, making proper comparison to this series more difficult.

For Short Treasuries, the reverse is the case. For this series, returns and risks are both clearly lower, with the Sharpe ratio at about the same level (actually marginally higher) as that of the Market series. The

Short Credit series clearly has the highest Sharpe ratio of 0.83, which is (0.83 - 0.54)/0.54 = 53 percent higher than the Market series. Although not reported, we find that other bond return series with short maturities and low ratings also have relatively high Sharpe ratios. We find that, where maturity increases or rating decreases, the Sharpe ratio also decreases. Indirectly, this result supports including both short-dated bonds and high-yield bonds in the factor portfolio.

Further, it is also interesting to compare the various equally weighted factor portfolios to the market portfolio. See Table 4.6 that includes three different equally weighted factor portfolios. The Eq. Weighted market-cap weighted factor portfolio is made up of the factors Term, Credit and High Yield. The Eq. Weighted alternative factor portfolio is made up to the two factors Short Credit and Short Treasury. The last factor portfolio, Eq. Weighted all factor premiums, is made up of the five factor premiums, all equally weighted.

Table 4.6. Factor premiums vs US bond market, January 1976 – August 2012

Factor premium type	Market	Eq. Weighted market-cap weighted factor premiums	Eq. Weighted alternative factor premiums	Eq. Weighted all factor premiums
Average return	8.28%	9.08%	7.20%	8.52%
Stand. dev.	5.58%	6.61%	2.71%	5.16%
Skew	0.68	0.41	1.74	0.64
Kurtosis	6.66	6.15	14.45	7.44
Max. drawdown	12.7%	18.0%	3.1%	12.5%
Calmar ratio	0.65	0.50	2.35	0.68
Sharpe ratio	0.54	0.57	0.73	0.62

The first factor portfolio, with only market-cap weighted factors, performs (9.08 - 8.28) = 0.80 percentage points better than the market. But, at the same time, there is also clearly higher risk, with a higher standard deviation and drawdown. The Sharpe ratio of the first factor portfolio is 0.57, only marginally higher than that of the market (0.54). The potential benefits of factor-oriented investing become clearer when we further consider the second factor portfolio. Although this portfolio, made up of the two alternative factors, realizes a (8.28 - 7.20) = 1.08 percent lower return, the risk is comparatively much lower. The standard deviation is (5.58 - 2.71)/5.58 = 51 percent lower and drawdown also decreases sharply. As a result of the lower risk, the Sharpe ratio increases by (0.73 - 0.54)/0.54 = 35 percent. For the factor portfolio representing all factors, the return increases (8.52 - 8.28) = 0.24 percentage points relative to the market, with standard deviation and maximum drawdown decreasing somewhat. This increases the Sharpe ratio by 0.62, higher than stated earlier for the equally weighted factor portfolios for equities from

1927 (= 0.47) and from 1990 (= 0.60) (see Table 4.3). This is an increase of (0.62 - 0.54)/0.54 = 14 percent. According to the Jobson-Korkie test, this difference is not significant.

Table 4.8 then combines the two assets, equities and bonds, into a new multi-asset portfolio with a start date of January 1976. The first column (‘2-Assets’) shows the results for a portfolio comprising 50 percent equity markets (as used earlier) and 50 percent bond markets (the Barclays Capital US Aggregate Total Return Index). The second column (‘Factors’) is 50 percent of the equally weighted equity-factor portfolio from Table 4.1 and 50 percent of the equally weighted bond-factor portfolio from Table 4.5. Chart 4.6 shows the cumulative returns.

Table 4.8. Asset and factor portfolios in the US for January 1976 – August 2012

Factor premium portfolio	2-Assets	Factors
Average return	9.83%	11.20%
Standard deviation	9.15%	7.02%
Skew	-0.39	-0.76
Kurtosis	1.39	3.22
Maximum drawdown	27.5%	26.1%
Calmar ratio	0.36	0.43
Sharpe ratio	0.52	0.83

Table 4.8 shows that in the US, the factor premium portfolio realized both a higher return and a lower risk than the 2-Assets portfolio over the period from January 1976. Rounded off, the return is (11.20 - 9.83) = 1.37 percentage points higher. However, the biggest improvement is in the lower risk realized by the factor portfolio. The standard deviation of the factor portfolio is 7.02 percent, or (9.15 - 7.02)/9.15 = 23 percent lower than the Assets portfolio. The Calmar ratio improves significantly, but the Sharpe ratio in particular shows a sharp rise to as much as 0.83. Relative to the Assets portfolio, the Sharpe ratio increases by (0.83 - 0.52)/0.52 = 60 percent. The Jobson-Korkie test confirms that this difference is significant to a confidence interval of over 99%. This outcome forms the basis of the empirical findings: A diversified factor portfolio has a distinctly higher favorable risk-return ratio than a diversified asset portfolio.

4.4 Results for European data

The analyses have up to this point been carried out using US data, which provide longer time series and are widely available. These analyses confirm earlier studies. Below we report the outcome of a comparable analysis for European equities and bonds. We use the same factor premiums as for the United States. The only exception relates to the LowVol factor, which is not included in the analysis because we did not find data for this factor. Because the approach to European equities is very similar to that used for the United States, we will keep the explanation of the European results brief. In Table 4.9 it is striking that the Momentum portfolio realizes the highest returns. Also, with a value of 0.48, the Sharpe ratio of this factor portfolio is the highest of all portfolios in Table 4.9. In Table 4.9, for the period from November 1990 (with 262 observations), the Sharpe ratio of an equally weighted factor portfolio is 0.35. This is a rise of (0.35 - 0.25)/0.25 = 38 percent versus the market portfolio. The Jobson-Korkie test cannot confirm whether that difference is significant, probably due to the limited number of observations.

Table 4.9. European equities for November 1990 – September 2012

Factor premium type	**Market-cap weighted**	**Alternative factor premiums**			
	Market	**Size**	**Value**	**Momentum**	**EW**
Average return	7.04%	5.38%	9.26%	13.69%	9.60%
Standard deviation	24.45%	24.78%	26.58%	26.38%	25.19%
Skew	-0.43	-0.56	-0.45	-0.47	-0.54
Kurtosis	1.60	2.24	1.80	1.37	1.90
Maximum drawdown	63.9%	67.8%	63.5%	60.3%	63.9%
Calmar ratio	0.11	0.08	0.15	0.23	0.15
Sharpe ratio	0.25	0.19	0.33	0.48	0.35

Tables 4.11 and 4.13 then state the results for European bond portfolios. Reliable data are only available for these portfolios from February 1998, with 175 observations per series. In Table 4.11 it turns out that the market-cap weighted factor premiums Term and High Yield Credit realized a higher return than the market. The Term portfolio in particular realized high returns. However, the Credit factor premium performed 0.16 percentage point (5.24 - 5.08) lower than the market. The Term and High Yield Credit series have distinctly higher standard deviations and higher drawdowns than the market. Both the Calmar ratio and the Sharpe ratio of the market are higher than for the discrete market-cap weighted factor premiums.

However, for the alternative factor premiums in Table 4.11, things are clearly different. Although returns here are lower than for the market, risks are a great deal lower. The standard deviation of the Short Credit series is (3.71 - 1.63)/3.71 = 56 percent lower than the market. For the Short Treasury series, the decline is (3.71 - 1.56)/3.71 = 58 percent. For maximum drawdowns, the difference with the market is even higher in relative terms. Due to the lower risks, both the Calmar ratio and the Sharpe ratio of both ‘Short’ bond series are higher than for the market.

Table 4.11. European bonds for February 1998 – September 2012

Factor premium type	Market-cap weighted				Alternative factor premiums	
	Market	**Term**	**Credit**	**HYCred**	**Short Cred**	**Short Tsy**
Average return	5.24%	8.40%	5.08%	5.73%	4.27%	3.90%
Standard deviation	3.71%	11.05%	3.48%	13.55%	1.63%	1.56%
Skew	0.18	0.76	-0.52	-0.33	0.13	0.39
Kurtosis	-0.01	2.80	2.01	3.00	0.97	1.17
Maximum drawdown	6.4%	16.9%	6.8%	39.9%	1.7%	1.3%
Calmar ratio	0.82	0.50	0.75	0.14	2.59	2.97
Sharpe ratio	0.70	0.55	0.68	0.28	0.95	0.81

Table 4.13 then documents to what degree the factor portfolios are comparable with the market. The market-cap weighted factor portfolio returned (6.74 - 5.24) = 1.50 percentage points higher than the market, but also has a clearly higher risk. The Sharpe ratio of this portfolio is 0.65, which is slightly lower than the market's Sharpe ratio of 0.70. The equally weighted alternative-factor portfolio generates a return that is (5.24 - 4.09) = 1.15 percentage points lower than the market. However, the standard deviation falls even more in comparison, as does maximum drawdown. This means the Sharpe ratio rises sharply from 0.70 to 1.00, a rise of 43 percent. The Jobson-Korkie test confirms that this difference is significant to a confidence interval of over 99%.

The last column in Table 4.13 shows that an equally weighted portfolio of all factor premiums, so including both market-cap weighted and alternative factor premiums, generates to a limited degree a higher Sharpe ratio than the market. The rise for this last portfolio is (0.73 - 0.70)/0.70 = 5 percent relative to market, which is not significant according to the Jobson-Korkie test.

Table 4.13. European bonds for February 1998 – September 2012

Factor premium portfolio	**Market**	**Eq. Weighted market-cap weighted factor premiums**	**Eq. Weighted Alternative factor premiums**	**Eq. Weighted All factor premiums**
Average return	5.24%	6.74%	4.09%	5.71%
Standard deviation	3.71%	6.27%	1.40%	4.08%
Skew	0.18	0.17	0.11	0.26
Kurtosis	-0.01	1.09	0.24	0.85
Maximum drawdown	6.4%	16.9%	1.2%	8.4%
Calmar ratio	0.82	0.40	3.28	0.68

Sharpe ratio	0.70	0.65	1.00	0.73

Finally, Table 4.14 compares the European 2-Assets portfolio with the Multi–factor portfolio. The 2-Assets portfolio is 50 percent equity markets and 50 percent bond markets, with a monthly reweighting. The factor premium portfolio has 50 percent from the equity factor premium portfolio and 50 percent from the bond factor premium portfolio, once again reweighted monthly. The results in Table 4.14 for European investments are directly comparable with those for US investments in Table 4.7, although the latter does cover a longer period.

Table 4.14. Asset and factor portfolios in Europe for February 1998 – September 2012

Factor premium portfolio	**2-Assets**	**Factors**
Average return	6.82%	9.03%
Standard deviation	13.11%	14.46%
Skew	-0.42	-0.66
Kurtosis	1.31	2.15
Maximum drawdown	35.8%	38.8%
Calmar ratio	0.19	0.23
Sharpe ratio	0.37	0.49

The factor portfolio shows a distinctly higher return with just a slightly higher standard deviation. Drawdown too is only slightly higher. All things considered, the risk-weighted performance of the factor portfolio is clearly better than that of the 2-Assets portfolio. The Sharpe ratio rises by (0.49 - 0.37)/0.37 = 32 percent. However, the Jobson-Korkie test cannot confirm that this difference is significant by more than 99% confidence.

5 Addition of investment categories and factors

Table 5.1 combines the assets, equities and bonds, into a new multi-asset portfolio starting January 1976. The columns “2-Assets” and “Factors” are taken from Table 4.8 above. The first column (‘2-Assets’) shows the results for a portfolio with 50 percent US equity market and 50 percent US bond market, as used in Section 4. The second column (‘4-Assets’) shows the results for a portfolio with 25 percent equities, 25 percent bonds, 25 percent commodities (the GSCI Commodity Total Return Index) and 25 percent real estate (the FTSE/NAREIT ALL REITS $ Total Return Index). The last column (‘Factors’) is 50 percent of the equally weighted US equity-factor portfolio from Table 4.1 and 50 percent of the equally weighted US bond-factor portfolio from Table 4.6. Chart 5.1 shows the cumulative returns.

Table 5.1. Asset and factor portfolios in the US for January 1976 – August 2012

Factor premium portfolio	2-Assets	4-Assets	Factors
Average return	9.83%	10.25%	11.20%
Standard deviation	9.15%	10.04%	7.02%
Skew	-0.39	-1.34	-0.76
Kurtosis	1.39	7.12	3.22
Maximum drawdown	27.5%	45.9%	26.1%
Calmar ratio	0.36	0.22	0.43
Sharpe ratio	0.52	0.52	0.83

Cumulative returns for 2, 4 Equally Weighted Asset Portfolio’s, and the Equally Weighted Factor Portfolio, January 1976 – August 2012

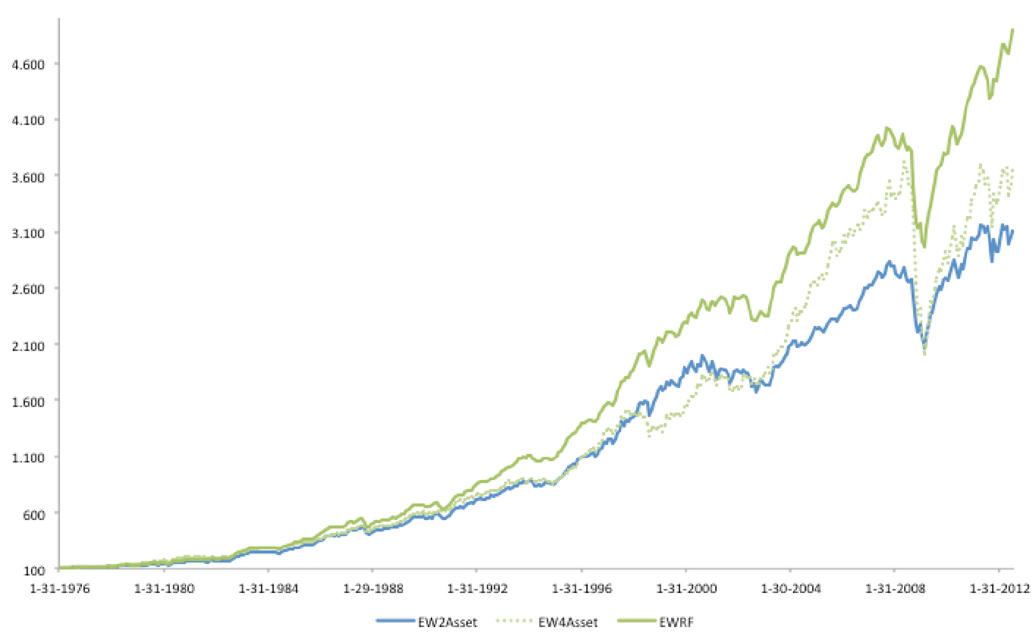


Chart 5.1 shows that since 1976 the 4-Assets portfolio has generated a slightly higher return than the 2-Assets portfolio. The average return of the four-assets based portfolio is clearly lower than that of the factor portfolio. The difference is (11.20 - 10.25) = 0.95 percentage point (see Table 5.1). When we evaluate the risks, we see that the standard deviation of the 4-Assets portfolio is higher than that of the 2-Assets portfolio. Although it is diversified across four assets instead of two, the risk does not decline. The value of the maximum drawdown further reinforces this picture, as this criterion rises from 27.5 percent to 45.9 percent. This brings the drawdown on the new 4-Assets portfolio to no less than (45.9 - 26.1)/26.1 = 76 percent higher than for the factor portfolio. Finally, the Sharpe ratio of the 4-Assets portfolio is 0.52. This means this Sharpe ratio is precisely the same as that of the 2-Assets portfolio, which is (0.83 - 0.52)/0.83 = 37 percent lower than that of the factor portfolio.

The conclusion of this first robustness check is that further diversification of the US 2-Assets portfolio to an US 4-Assets portfolio does not influence the results. The factor approach clearly shows better results than either the 2-Assets or the 4-Assets portfolio. To test whether these outcomes depend on the selection of the US asset portfolios, we will also examine the European asset portfolios below. Table 5.2 shows the outcome of a European robustness check.

Table 5.2. Asset and factor portfolios in Europe for February 1998 – September 2012

Factor premium portfolio	**2-Assets**	**4-Assets**	**Factors**
Average return	6.82%	7.78%	9.03%
Standard deviation	13.11%	14.89%	14.46%
Skew	-0.42	-0.99	-0.66
Kurtosis	1.31	4.13	2.15
Maximum drawdown	35.8%	49.0%	38.8%
Calmar ratio	0.19	0.16	0.23
Sharpe ratio	0.37	0.40	0.49

The columns “2-Assets” and “Factors” are taken from Table 4.14 above. The “4-Assets” column is new. This column shows the outcome of a portfolio made up of equities, bonds, commodities and real estate. Each of those assets is weighted 25%, with monthly reweighting. We apply the same methodology here as previously used for the US 4-Assets portfolio. Investments are now made in European equities, bonds and real estate proxied by the FTSE/EPRA NAREIT Eurozone Index, while the GSCI Index in euros is used for the commodities investment.

The results in Table 5.2 show that both return and risk increase relative to the 2-Assets portfolio. Return rises from 6.82% to 7.78%, but is still lower than the return on the factor portfolio. The risk also increases, due in particular to the addition of the volatile commodities prices. The diversification advantage of real-estate investments is also limited, as it is strongly correlated to equity prices. The result of this is that the standard deviation of the return increases, from 13.11% to 14.89%. The kurtosis and maximum drawdown also rise sharply. Due to the substantial increase in risk, the advantage of the performance increase is limited. The total effect is that the Sharpe ratio of the 4-Assets portfolio rises to 0.40, from 0.37 for the 2-Assets portfolio. Both asset portfolios have a Sharpe ratio that is clearly lower than the outcome for the factor portfolio.

Finally, we have carried out an analysis for an investor who is not restricted to the geographical areas of Europe or US, but who invests in the whole world. The period we have analyzed, runs from November 1990 to September 2012. The 'World' equity-factor series originates from the above-mentioned Kenneth French website, so they are constructed using the same procedures as we used for the Europe and US empirical studies above. Sound and long-term 'World' bond-factor series are difficult to come by. We have opted for a pragmatic solution – the average of the European and US bond factors (Term, Market, Credit, High Yield, Short Credit and Short Treasury). For bond markets we used the JPM Global Broad Total Return Index in USD. For equity investments we selected the MSCI World USD Index. The outcomes are stated in Table 5.3.

Table 5.3. World asset and factor portfolios for November 1990 – September 2012

Factor premium portfolio	**Assets**	**Factors**
Average return	7.13%	9.95%
Standard deviation	11.55%	8.12%
Skew	-1.09	-0.97
Kurtosis	5.11	2.41
Maximum drawdown	47.8%	30.1%
Calmar ratio	0.15	0.33
Sharpe ratio	0.38	0.82

The results in table 5.3 are in line with the earlier empirical findings. The world factor portfolio generated better performance over roughly the last two decades than the world asset portfolio. The combination of a higher return and a lower standard deviation (as well as drawdown and kurtosis) leads to a clearly higher Sharpe ratio. The change in Sharpe ratio is significant to a 1 percent confidence interval according to the Jobson-Korkie test. These results reinforce the conclusions drawn earlier.

As an extra robustness check we then added a new factor to the factor portfolio used above. In many studies, Carry is used as one of the factor premiums. We added the Carry factor to the "world factor portfolio" with a weight of 20% in the total portfolio. The way in which we calculate the value of the

Carry is explained in the Appendix. Our results show that the addition of a simple Carry component to the factor portfolio clearly improves performance. The average return rises, while the standard deviation remains almost unchanged. The maximum drawdown drops sharply and the skew and kurtosis values of the distribution of portfolio returns become more favorable. The Sharpe ratio increases further due to this simple approach.

However it should be taken into account that this calculation is oversimplified, and that the specific selection of the carry factor was not motivated from any theoretical framework. The aim of this addition was mainly to examine whether the outcomes of the empirical studies are robust to limited changes in the asset and factor portfolios, and that is in fact the case. Indeed, there appears to be room for further improvement of performance of the factor portfolios when other factor premiums are added, such as Carry. Various studies suggest that adding a trending factor can also further improve the performance of a factor portfolio (see for example Moskowitz, Ooi and Pedersen (2011)).

It can be concluded that the basic empirical findings in this section's robustness analyses remain unchanged. The results demonstrate that there are periods in which the advantage of factor-oriented investing is not directly reflected in the numbers. But over the longer term, and if factor diversification is applied consistently, the empirical findings convincingly support factor investing.

6 Optimization of the portfolio

6.1 The equally weighted method

In the previous sections, we have consistently used equally weighted portfolios that are rebalanced monthly. This 1/N approach is not only very popular in practice, but is also used a great deal in academic research (see for example Bernartzi and Thaler (2001) and Windcliffe and Boyle (2004)). In the first study, the authors show that more than a third of the participants of a 'direct contribution' pension plan opt for an equally weighted diversification of the investments. DeMiguel, Garlappi and Uppal (2009) evaluate the performance of 14 allocation models on seven different datasets. The models vary from simple to very complex and include Bayesian estimates and moment restrictions. Many of the econometric model refinements are designed to keep the error of estimation of future expectations to a minimum. We refer to Kritzman, Page and Turkington (2010) for an extensive overview of progress on this optimization research in the literature. The article by DeMiguel, Garlappi and Uppal (2009) finds that none of the models examined by them performs any better than the naive 1/N approach. Other studies are also producing similar results, such as Duchin and Levy (2009).

In addition to this empirical support of the 1/N approach, there are also other reasons that explain the popularity of this approach. Partly based on Kritzman, Page and Turkington (2010), we have compiled the following list with additional arguments:

- It prevents strongly concentrated positions.
- Rebalancing involves selling high and buying low, making use of any "mean-reversion" effect that may be present.
- You are always invested in the best-performing asset as part of the portfolio.
- The Size factor premium helps explain the returns, as smallcaps are overweighed and largecaps underweighted.
- The method is simple, transparent and does not require complex models or theories.

Against this list of points that argue for the 1/N approach are some important counter arguments. The first objection is that the 1/N approach assumes de facto that the investor does not possess any useful knowledge or information about the component parts that make up the portfolio. After all, even if only limited information was available on expected returns, risks or correlations, then it would be logical to use this information to optimize the portfolio. The question of course is what information is available and how certain it is when making decisions on allocation. Kritzman, Page and Turkington (2010) argue forcefully that the study by DeMiguel, Garlappi and Uppal (2009) can be rightly criticized. Indeed the latter authors use a (too) short historical period of just five or ten years to determine expected returns for the different assets. It does not appear to be sensible as investor to put so much weight in historical returns of the last five or ten years. For example, as of February 2009 the five-year historical return on the S&P 500 was -5.7 percent versus 3.8 percent on the Barclays US Aggregate Bond Index over the same period. But it would probably be not smart to base future returns on that same -5.7 percent and 3.8 percent? Indeed, as stated above, there is relatively strong empirical evidence that a positive equity-market factor premium exists and that this premium is higher than that of the bond market. DeMiguel, Garlappi and Uppal (2009) also demonstrate that very long data series are necessary to arrive at useful estimates. Even the period from 1928 used in this study is still on the short side for this purpose.

6.2 Optimization methods

It is therefore probably better to find other ways to generate an outlook on future returns. Kritzman, Page and Turkington (2010, p. 3) use constant long-term averages for the various asset classes or percentages assessed as probable by the authors, “simply because they seem reasonable”. Blitz (2011) takes a similar approach and uses conservative expectations, instead of the average historical returns. These expectations are related in his study not to assets, but to factor premiums. Moreover, the maximum allocation per factor is limited in order to preventing excessive concentrations. The core idea behind this approach is to make use of a reasonable prediction in combination with economic intuition. Kritzman, Page and Turkington (2010) conclude that optimization generally offers better outcomes that the 1/N method, as long as such ‘reasonable’ inputs are used. Blitz (2011) and Clarke, De Silva and Thorley (2006) also report that optimized portfolios perform better than the standard 1/N approach.

One of the most-used optimization techniques is *mean variance optimization*. This method is based on the work of Markowitz (1952) for equity portfolios. However, this method has the disadvantage as mentioned above that future expected returns, correlations and volatilities are uncertain. Furthermore, the allocation outcomes are strongly dependent on small changes in the input parameters. A small change in the level of the expected returns can in some cases lead to disproportionately high or low allocations, resulting in a very unstable portfolio. A different yet related disadvantage is that such portfolios tend to be strongly concentrated in a limited number of investments (see Maillard, Roncalli and Teïletche (2010)). In practice, many use the standard 60 percent / 40 percent distribution between equities and bonds. We refer to Chaves et al (2011) for further discussion of the mean variance approach.

An alternative to the mean variance approach is the so-called *risk parity approach*. Indeed, with a 60/40 equity and bond portfolio, more than 90 percent of the portfolio risk is caused by the equity component (see Thiagarajan and Schachter (2011)). Moreover, Bhansali (2011) shows that 81 percent of the risk of a nine-asset portfolio, with 35 percent invested in equities, is equity risk. In the risk parity approach, each part of the allocation is weighted in the same way to the volatility (i.e. risk) contribution to the portfolio. Each asset (or factor) is given the same "risk budget". There is no consensus on this approach in the market. A simplified version of the risk parity, which often serves as benchmark, includes the monthly reweighting of the allocations with the inverse of the volatility.

Regardless of the risk parity approach, the result of this is generally that a relatively large amount is invested in bonds and less in equities. A simple risk-parity portfolio is invested on average for around 15 percent in equities and 85 percent in bonds (Asness, Frazzini and Pedersen, 2012, p. 51). The required target return may be pursued by applying leverage to the portfolio. The weights can be multiplied by a constant, depending on the willingness to engage in leverage and the desired degree of volatility of the portfolio returns. If leverage is not accepted, the weights will be lowered so that together they add up to one. Such leverage of course brings new risks, such as changes in the funding costs and their availability. This also increases the so-called tail risks. A major advantage of the risk parity approach is that the investor does not need to draw up any expectations in relation to future returns. But there are still covariance estimates required at implementation.

It is also possible to work with *macroeconomic scenarios*, in which economic growth and inflation, for instance, change. These scenarios are then translated into variance – and covariance – estimates that are dependent on the regime that arises (see Thiagarajan and Schachter [2011]). Some assets are expected to do better in a scenario with high economic growth and high inflation, while other assets actually do better when both these 'drivers' have low values. Chaves et al (2011) find that, over the period 1980–2010, the Sharpe ratio of a 60/40 percent equity-bond allocation of 0.50 rises to 0.62 when a risk parity approach is followed. Also Peters (2009) and Asness, Frazzini and Pedersen (2012) report higher returns and lower volatility of the risk parity portfolio compared to a 60/40 percent portfolio, or a market-capitalization weighted portfolio. Using their data sets, the Sharpe ratio rises about as much as reported in Chaves et al (2011).

Another much-used optimization approach is *'minimum variance'*, requiring covariance information, but no assumption is necessary in relation to expected returns. This method determines the portfolio allocations in a way that pursues the lowest possible volatility of the portfolio. Various methods are possible here, including the use of constant or time-varying correlations, single-index approach, principal-component approaches, or a 'statistical shrinkage' approach. Otherwise, Luo et al (2011) suggest the minimum-variance approach should not be the preferred method for factor-oriented investment.

Chaves et al (2011) and Maillard, Roncalli and Teïletche (2010) both compare various allocation methods. The risk parity approach is favored in both studies. Furthermore, this type of studies shows clearly that the results depend strongly on the time period and assets used. It is hard to arrive at generally sound statements due to data dependency. Finally, there is an empirical finding that low-volatility assets tend to have higher Sharpe ratios (see also the LowVol factor premium for equities and the results relating to short term bonds). Adding such investments to the portfolio will in general lead to higher Sharpe ratios. According to Peters (2011) this is one of the reasons behind the success of the risk parity approach: the portfolios allocate more (or less) to equities in times of lower (or greater) volatility. In turbulent markets, bonds perform better than equities and that is the basis for the success of the risk parity approach. We refer to the 'leverage aversion' theory of Asness, Frazzini and Pedersen (2012). This theory states that the aversion to leverage leads to amendments to the Modern Portfolio Theory. Due to the presence of leverage aversion, low beta assets offer high for risk corrected expected returns. The standard CAPM would have to be adapted for this aspect. The highest risk-corrected returns are determined by investments in relatively safe assets. The empirical findings support this theory (see for example Frazzini and Pedersen [2010]).

Finally, it is worth noting that in addition to the already mentioned allocation optimization methods, there are also more (complex) variants. Recent research, for example, suggests that certain factor premiums tend towards trending, as do most assets (see Moskowitz, Ooi and Pedersen [2011], or Mesomeris et al [2012]). Ergun and Stork (2013) and Faber (2010) show how this trending can be used in determining the allocations. The application of a simple moving average buy/sell rule will in most markets lead to significantly lower risks for the same expected returns. There are also studies that show that, with a simple 'regime shift' model, the allocation performance could be improved. In this approach, two economic situations, or regimes, can be identified, namely 'risk on' and 'risk off'. A determining variable here is the height of the VIX. The covariance matrix and expected return is very different in one regime than in the other. See Luo et al (2011), Peters (2008, 2011, p. 130) and Kritzman, Page and Turkington (2010, p. 7).

In conclusion it can be stated that knowledge in the field of portfolio optimization is developing rapidly. Both in the field of risk parity in general, and the way to apply factor-oriented investing, there appear to be strong opportunities for improvement. The application of complex methods also seems to have potential.

6.3 Risk parity optimization results

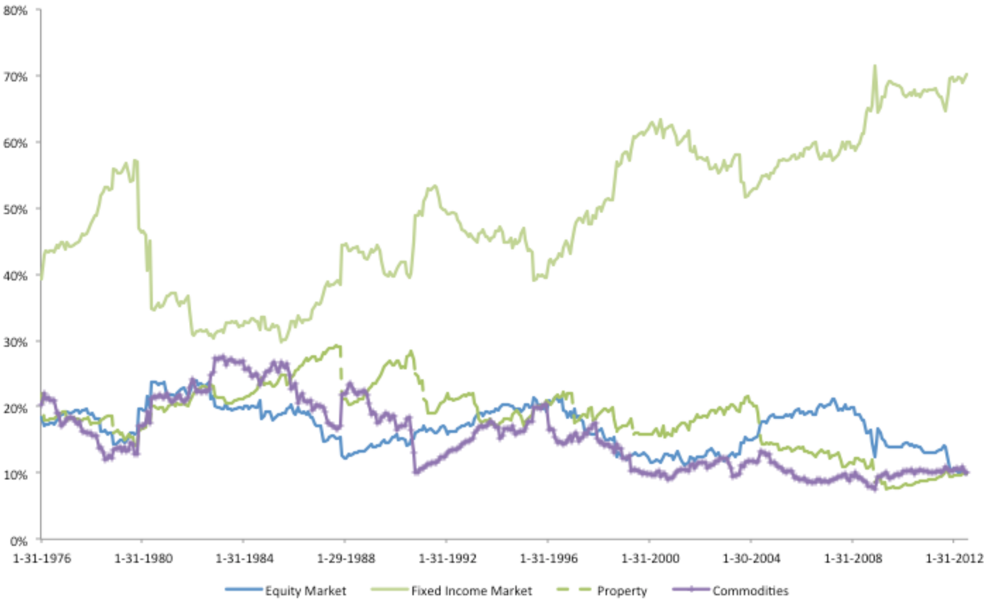
In this section, we carry out a number of optimizations on the portfolio allocations. Compared with the (1/N) allocation, we initially focus on a ‘risk parity’ optimization. This approach comes favorably to the fore from various studies (for example, Chaves et al [2011] and Maillard, Roncalli and Teïletche [2010]). However, various critiques have also been written, see e.g. the article by Inker (2011). He argues that standard deviation is only a limited part of the actual risk generated by an investment in a certain asset class. Besides interest rates are now extremely low compared with recent decades. This could have distorted the investment results realized with risk parity.

We consciously do not choose to use the ‘mean variance’ optimizations due to the disadvantages stated in the previous section, such as the unstable portfolio weights and the tendency to strong concentrations in a limited number of investments. In the risk parity approach used here, the allocations are weighted to the inverse of the volatility. We use for this an Exponentially Weighted Moving Average (EWMA) of the volatility. We set the lambda (λ) in the standard RiskMetrics EWMA comparison at a value of 0.94, in line with the original RiskMetrics recommendation. As an initial value for volatility, we take the average standard deviation over the entire observation period. Although deliberate, the selection of initial volatility and the choice of the lambda are not ‘hard’. That is to say that other values could have been chosen in our empirical calculations. We have done a couple of robustness tests on both choices and find that the ultimate conclusions do not depend on these choices – we have the same qualitative findings for other values of lambda and initial volatility.

To remain consistent with the study, leverage is not permitted. As described in the previous section, the result of the risk parity optimization is that the weight of low-risk investments, such as bonds, increases in the portfolio. This drives down the average return of the portfolio as bonds underperformed equities on average. By using leverage, the expected return can be raised further, but in this study we do not use this technique. We begin by applying risk parity to the three US asset and factor portfolios also used in Table 5.1 using EWMA volatilities of the equity and bond markets over time.

Unreported results show that the weight of the equity allocation is clearly lower than that of the bond investments. On average, over the entire period, 26% was invested in equities and 74% in bonds. The weights fluctuate considerably over the years. At the end of the observation period, for example, just 12% was invested in equities and 88% in bonds. The rise of bond weights is caused on the one hand by constant good performance of the bond market, with only a few severe shocks, and on the other hand the increased volatility of the equity market, especially during the credit crisis and the years thereafter.

Weights of 4-Asset U.S. portfolio, January 1976 – August 2012



For the 4-Assets portfolio, the weights will change. Chart 6.3 shows these allocations over time. One aspect jumps out – the weight of the bond investments is relatively high. Moreover, after the interest-rate shock in the early eighties, this allocation rose further to peak at nearly 70 percent. The weights in the three other categories fell by some 10 percent per category, or 30 percent taken together.

The risk-parity optimization can of course also be applied to the factor portfolios. There is not much empirical research published in this field up until now. Over the coming years, knowledge of factor optimization is expected to increase rapidly. Below we use two, slightly different risk-parity optimizations to the US factor portfolio. In the first approach, we apply risk parity to all factors in the portfolio. All equity factors and all bond factors are thus treated equally. The disadvantage of this approach is that it does not take account of the number of factors per investment category. The equity-factor portfolio for example has four factors (Size, Value, Momentum and LowVol), while the bond-factor portfolio uses six (Market, Term, Credit, High Yield, Short Credit and Short Treasury). The last two factors are also strongly correlated and both factors have very low volatility. The result is that these two factors are very strongly represented in the resulting portfolio. To illustrate this, at the end of the observation period, as of July 2012, the four equity factors had an allocation of some 2–3 percent each. Even the LowVol equity factor does not exceed the four percent allocation. The rest of the allocation belongs to bond factors, with Short Treasury being allocated as much as 40 percent weight. Although it is an interesting exercise, this allocation cannot be called especially realistic.

However, it is easy to construct a more logical approach. In the second risk parity approach we leave the factor distribution within the equity portfolio unchanged (1/N). Also within the bond factor portfolio, the weights remain at (1/N). The risk parity approach is then applied to the two separate factor portfolios. Naturally, the result of this is that the allocation to bonds (factors in the bond market) increases strongly at the cost of the allocation to equities (factors in the equity market). The weight of the bond factor portfolio begins at a level of around 70 percent and rises to 78 percent as of July 2012. However, this weight distribution is less extreme than in the first version of the risk parity approach to the factor portfolios, and is more in line with the distribution found in the two asset portfolios.

Table 6.2. Optimized asset and factor portfolios in the US for January 1976 – August 2012

Factor premium portfolio	2 Assets	4 Assets	Factors Version 1	Factors Version 2
Average return	9.38%	9.72%	9.08%	10.33%
Standard deviation	6.44%	6.84%	5.01%	5.70%
Skew	0.33	-0.65	-0.51	-0.18
Kurtosis	2.72	3.57	4.80	3.57
Maximum drawdown	10.5%	21.4%	8.7%	13.6
Calmar ratio	0.89	0.45	1.04	0.76
Sharpe ratio	0.63	0.64	0.75	0.86

The results of the various risk parity optimizations are stated in Table 6.2. In Table 6.2 it is striking that the factor-driven approach still generates higher Sharpe ratios than the pure asset-driven approach. It is thus not the case that application of risk parity optimization offsets the benefits of factor-oriented investing. However, for a more thorough analysis of the results, the outcomes in Table 6.2 should be compared with those in Table 5.1 (without optimization). The following conclusions can then be made:

- Average returns of the risk parity portfolios are lower. The 2-asset, 4-asset and factor portfolios fell from 9.83% to 9.38%, 10.25% to 9.72% and 11.20% to 10.33%, respectively. Please note, we discuss below only version 2 of the risk parity factor portfolios, as this version is more realistic than version 1, the results of which we report in Table 6.2 for the sake of completeness.
- The standard deviations of the risk parity portfolios are much lower. The volatilities of the 2-asset, 4-asset and actor portfolios fell from 9.15% to 6.44%, 10.04% to 6.84% and 7.02% to 5.70% respectively. Relatively speaking, these declines are stronger than the declines in average return.
- The maximum drawdowns of the risk parity portfolios are much smaller. The drawdowns of the 2-asset, 4-asset and factor portfolios fell from 27.5% to 10.5%, 45.9% to 21.4% and 26.6% to 13.6% respectively. The maximum drawdowns are halved, or decrease even further.

- The Sharpe ratios rise as a result of the changes outlined above. The Sharpe ratios of the 2-asset, 4-asset and factor portfolios rose from 0.52 to 0.63, 0.53 to 0.64 and 0.83 to 0.86 respectively. The rise in the Sharpe ratios is the least impressive for the factor portfolio, although the difference with both asset portfolios remains considerable.

A logical next step is to analyze whether the same conclusions can be drawn with the application of risk parity optimization on the European portfolios. Table 6.3 reports these results.

Table 6.3. Optimized asset and factor portfolios in Europe for February 1998 – August 2012

Factor premium portfolio	2 Assets	4 Assets	Factors Version 1	Factors Version 2
Average return	5.85%	7.18%	5.32%	6.98%
Standard deviation	4.02%	5.90%	2.82%	5.64%
Skew	-0.27	-1.20	-0.11	-0.56
Kurtosis	0.44	4.19	1.59	2.10
Maximum drawdown	8.3%	19.9%	6.0%	14.20%
Calmar ratio	0.70	0.36	0.88	0.49
Sharpe ratio	0.78	0.76	0.91	0.74

To arrive at a sound assessment of the European risk parity results, these should be compared with the non-optimized outcomes in Table 5.2. We do not discuss the results for the European portfolios in detail, as most of the conclusions for US portfolios also apply to the European portfolios. Once again, average returns, standard deviations and maximum drawdowns all declined. Because the risk measures fell more strongly than the average returns, the Sharpe ratios rose versus the non-optimized portfolios. However in contrast to what we observed with the US portfolios, the Sharpe ratio of the European risk parity factor portfolio is not higher than that of the 2-asset and 4-asset portfolios. The Sharpe ratios are about the same, all between 0.74 and 0.78. Strikingly, Version 1 of the risk parity factor portfolios does lead to a distinctly higher Sharpe ratio of 0.91.

6.4 Optimization using a Trending Rule

In this section, we focus on one alternative optimization possibility. In Section 6.1, we already briefly mentioned using the promising factor of 'trending' to reduce portfolio risk. We refer to Ergun and Stork (2013) and to Moskowitz, Ooi and Pedersen (2011). If markets have a tendency towards long-term trending, then a moving-average rule could take advantage of that. We have tested this possibility using the following approach. We first apply the risk parity optimization discussed in the previous section. We then use the rule that an investment can only be retained as long as the last value of the relevant Total

Return Index is higher at the end of the month than the average of the ten previous months. There must be a rising trend in the result, roughly measured using the average of the previous ten Total Return Index values. This measure is thus an approximation of the 200-day moving average commonly used in practice. When the last value of the Total Return Index ends below the moving average, the entire position is sold and the proceeds are held in cash for a return that corresponds to short-term rates. We recognize that this basic rule is difficult for a major investor to apply in practice, as large portfolio components cannot be sold from one day to the next. However, there are hybrid, watered-down versions of the rule available, with less rigorous allocation changes, that are feasible in practice. Table 6.4 reports the outcomes of the moving average rule. To facilitate comparison, the 'pure' risk parity results from Table 6.2 are also included.

Table 6.4. Use of moving average rule on portfolios in the US for January 1976 – August 2012

Factor premium portfolio	2 Assets		4 Assets		Factors	
	RP	RP + MA	RP	RP + MA	RP	RP + MA
Average return	9.38%	8.84%	9.72%	9.63%	10.33%	10.22%
Standard deviation	6.44%	5.48%	6.84%	5.68%	5.70%	4.70%
Skew	0.33	-0.33	-0.65	-0.66	-0.18	0.16
Kurtosis	2.72	2.12	3.57	2.47	3.57	2.95
Maximum drawdown	10.5%	11.0%	21.4%	8.7%	13.6%	9.2%
Calmar ratio	0.89	0.81	0.45	1.11	0.76	1.11
Sharpe ratio	0.63	0.64	0.64	0.75	0.86	1.01

Note: The abbreviation 'RP' stands for risk parity portfolio, while the abbreviation 'RP + MA' stands for risk parity portfolio to which the moving average rule is applied.

For the 2-asset portfolio, the moving average rule is of little added value, and the Sharpe ratio hardly changes. For the 4-asset and factor portfolios on the other hand however, application of the moving average rule leads to a strong improvement of the results. Average returns barely fall, while the standard deviations are much lower. The biggest improvement is in the decline of the maximum drawdowns, from 21.4% to 8.7% for the 4-asset portfolio and from 13.6% to 9.2% for the factor portfolio. This 'tail-risk reduction' corresponds closely with the outcomes in Ergun and Stork (2013).

For the last two portfolios, the Sharpe ratios rise markedly, from 0.64 to 0.75 and from 0.86 to 1.01 respectively. Note that the outcomes in Table 6.4 are only based on strongly simplified rules applied to a single data set. The outcomes are therefore not necessarily indicative. However, the improvement relative to the non-optimized results in Table 5.1 is very substantial. Moreover, there is probably room for improvement of the rules used here. Our conclusion is that optimization relative to an equally weighted portfolio should be rewarding, both for the asset portfolios and for the factor portfolios. Application of a few simple rules offers major advantages here, particularly in lowering risk.

7 Factor-oriented investing if liabilities are present

7.1 Role of liabilities

Only a few studies have so far been published on applying factor-oriented investing in combination with the presence of liabilities. This gap in research is striking given that many investors have not only an asset side but also a liabilities side. Several articles describe the basic principles of matching assets and liabilities and how the presence of liabilities could affect allocation of the assets (see for example, Elton and Gruber [1992], Peters [2011], Petrakis [2011], and Stockton, Donaldson and Shtekhman [2008]). For other recent work in this field, we refer to Campbell and Viceira (2002), Hoevenaars, Molenaar, Schotman and Steenkamp (2008) and to De Jong, Schotman and Werker (2008). Our analysis below is limited to an initial effort. Further research is certainly desirable and new insights in this area are of direct importance to many pension funds and insurers. However, the follow-up questions are beyond the scope of this current study. We hope in future to cover this subject in further detail.

The effects of wage and price indexation and an increasing life expectancy are not included as these liabilities are difficult or impossible to hedge. Interest-rate risk on the other hand can relatively easily be hedged, even for the long terms typical of pension funds. For the US portfolios, we take the Barclays US Treasury 20+ Years Total Return Index as a proxy for the liabilities side. We also used the Barclays US Government Long Total Return Index for control purposes, but the results do not differ much from the results from using the first index choice and are therefore not reported separately. We assume that the assets and liabilities of investors in US securities were of equal value on the observation period start date (January 1976, as with our analyses above); i.e. a funding ratio equal to 100%. This assumption does not have many consequences. We have also looked at what the effects are of assuming higher (or lower) funding ratios, but the conclusions do not change much in quantitative terms. We limit ourselves to three portfolios, the 2-asset, 4-asset and factor portfolios. After all, this is about obtaining initial insights and therefore these three portfolios are an excellent starting point for analysis.

7.2 When liabilities are not additionally hedged

We begin the analysis with the assumption that the investor does not have an additional interest-rate hedge. The assets side of the balance sheet thus remains unchanged relative to the analyses applied in the sections above. Since the assets have a certain interest-rate sensitivity, there is already a limited interest-rate hedge in the portfolio that is not increased in this section. In the following section, we do increase the interest-rate hedge. The reported measures in Table 7.1 relate to the percentage changes of the ratio between the assets and the liabilities, multiplied by 100 – further referred to as (A/L). This ratio may be interpreted as the so-called funding ratio return, which is different from the returns on only the asset side as in previous sections.

Table 7.1. Asset and factor portfolios with liabilities in the US for January 1976 – August 2012

Factor premium portfolio	**2 Assets**	**4Assets**	**Factors**
Average A/L change	0.25%	0.68%	1.50%
Standard deviation	12.36%	14.35%	11.40%
Skew	-0.58	-0.66	-0.34
Kurtosis	3.82	4.86	2.47
Maximum drawdown	54.4%	53.4%	47.8%
Correl. assets and liabilities	0.30	0.11	0.33
Modified Sharpe ratio	0.02	0.05	0.13

The Calmar ratio and the Sharpe ratio are not reported, as they have little importance to this analysis. We have added a 'Modified Sharpe ratio' in which the 'excess returns' are calculated as the returns of the assets minus those of the liabilities (instead of the risk-free rate). The average value of these excess returns is divided by their standard deviation to arrive at a 'Modified Sharpe ratio'.

The results in Table 7.1 show that the average change in the A/L ratio is highest for the factor portfolio. At the same time, this portfolio shows the lowest volatility, as well as a lower kurtosis, a lower maximum drawdown and a higher modified Sharpe ratio. The differences are slightly in favor of the factor portfolios relative to the two asset portfolios. The correlation between the returns on the assets and the liabilities is highest for the factor portfolio, although the difference with the 2-asset portfolio is only minimal. For the 4-asset portfolio the correlation is quite a bit lower, as a result of the commodities and real-estate diversification effects. Results not reported here show that the correlations between the asset returns and the liability returns fluctuate strongly. Over the period starting in February 1998 (the period available for the European data), correlations are negative (-0.17 and -0.16) for the 2-asset and 4-asset portfolios, while for the factor portfolio, the correlation between the assets and liabilities has a value of 0.01.

Next to the analysis described above for US portfolios, we also looked at the results for European portfolios. The results are given in Table 7.2..

Table 7.2. Asset and factor portfolios with liabilities in Europe for February 1998 – September 2012

Factor premium portfolio	**2 Assets**	**4 Assets**	**Factors**
Average A/L change	-1.46%	-0.57%	0.58%

Standard deviation	18.09%	20.23%	19.24%
Skew	-0.22	-0.50	-0.43
Kurtosis	1.84	3.07	1.90
Maximum drawdown	49.3%	58.8%	48.0%
Correl. assets and liabilities	-0.15	-0.25	-0.16
Modified Sharpe ratio	-0.08	-0.03	0.03

Comparison with the US results in Table 7.1 is not very useful, as European data covers a much shorter period. What is striking about the European results in Table 7.2 is that, for both asset portfolios, the changes in the A/L ratio are negative on average. The long-dated bonds (used to model the liabilities) apparently have outperformed the asset portfolios. The reason for this is the gradual decline in bond yields, at the same time as the equity price declines in the periods 2000–2002 and 2007–2009. The factor portfolio has realized a slightly higher return than the 2-asset and 4-asset portfolios. The standard deviation of the factor portfolio is comparable to those of the asset portfolios. The correlations are now negative, but the US results already indicated that these estimates can be quite volatile and depend strongly on the period studied. The modified Sharpe ratio of the factor portfolio is highest. All things considered, the results in Table 7.2 turn out slightly positive for the factor portfolio (compared with both asset portfolios). This conclusion reflects the findings for US data series, as reported in Table 7.1.

The negative development of the combined portfolio suggests once again that, for an investor with liabilities, the degree of matching is at least as important an issue as the question of what the most optimal setup is of the asset side alone. One conclusion might be that the asset allocation should be changed to take greater account of the intended matching of the liabilities. For such an investor, raising the interest-rate sensitivity or duration of the asset side would be a logical idea and would support the results in this section. The decision of the fictional investor in this section to not further hedge the interest-rate risks appears to have been relatively more important than whether to opt for a conventional asset portfolio or a factor portfolio. In Koedijk, Slager and Stork (2013), we further investigate the effectiveness of the factor based investing approach, while hedging 50 percent of the interest rate risk present in the liabilities. For the sake of brevity, we do not report those results here. In short, we find that the addition of the interest-rate hedge sharply improves the outcome. The average return turns positive for all three portfolios, with the factor portfolio realizing a clearly higher return than the two asset portfolios. The main findings remain unaltered, both for the US and the European data.

8 Conclusions

Investors appear to be placing greater importance on factor investing in the portfolio management process. But what is factor investing precisely? The theoretical support for this is that the price movements of financial instruments like equities or bonds are driven by underlying factors such as interest rates, currencies, economic growth or corporate earnings. The idea is that if these underlying factors can be identified, then a portfolio structure based on these factors provides a more robust and diversified portfolio.

We construct simple portfolios with the key factors for equities and bonds, and compare the progress of these portfolios with widely accepted indices. We do this initially with US data, as most research is based on that data. We now extend this to include European data, though the time series available for these data are shorter. Using easy-to-understand analyses, we see that factor-based portfolios generally produce comparable or better portfolios than market indices. This gives us room for higher returns or lower risk in line with the relevant objectives.

We then subject these analyses to three further in-depth steps designed to translate the results of the analyses to the practical situation of the institutional investor. We expand the asset classes and factors, work with other optimization methods and, finally, add a basic liability structure.

The results remain robust when we add real estate and commodities to equities and bonds, making the factor portfolio more comparable to the main investment categories applied by institutional investors. We also see that the results do not "depend" on the addition or removal of a specific factor. Finally, we study the results for a worldwide investor who invests beyond the US or Europe. The findings remain unchanged. Over the longer term and with consistently applied factor diversification, factor investing appears to be advantageous.

Furthermore, we analyze both equally weighing factors in the portfolio (the 1/N approach), as well as other portfolio optimization techniques. Optimizing an equally weighted portfolio appears to be a rewarding approach. Applying good rules already contributes significantly to the result, which suggests that the (positive) results revealed earlier on portfolio composition using factor premiums are conservative.

The results are promising as they are practical and relevant, but they also generate the necessary questions. One question rises throughout the study: why do the factors behave like this? An overview of the literature shows that there is no shortage of studied and potential factors. There is extensive literature identifying factors that are more or less persistent. Some of these have been known since the early nineties, with researchers Fama and French finding that factors such as Size, Momentum and Value contribute to the explanatory potential of a portfolio's systematic risk. This list is being constantly expanded and assessed by multiple researchers. However, one question often not answered in the literature is, what do we actually find when a factor is identified? It is difficult to provide a good economic explanation for a number of factors. Yet this is important from a portfolio management perspective – if we do not know why a factor exists, we will not know when it will disappear. And that also makes this a question for institutional investors who, after all, must be able to account for their investment choices.

Use of the term "factors" also suggests a form of persistence, but investors should not always take that for granted. In terms of expected premium, factors can be out of kilter for long periods. One example was commodities. The S&P GSCI Index generated a positive 'roll yield' on average over the period 1970–1992. However, with growing interest from institutional investors and with the arrival of trackers on commodities, the market went through a structural change. Over the period 1992–2002, the roll yield switched to a slightly negative value and over the following ten years fell further to a significantly negative level. Another frequently stated factor is the term premium, the difference between long and short rates. The market can structurally change here too. New regulatory rules are pushing towards hedging long-term risk by banks, insurers and pension funds. There is greater pressure to invest in very long-dated bonds. Due to greater demand, yields are falling – institutional investors are relatively insensitive to prices as they are partly motivated by new regulations. In short, factor premiums fluctuate over time and individual premiums can disappoint for longer periods.

The research in this first paper shows that investing in systematic factors offers many benefits above and beyond conventional asset-oriented investment. We see that various institutional investors have also now realized the potential benefits of factor investing and are currently implementing organizational changes in their investment process. A decision by an investor to invest in factors should be taken strategically with a long-term horizon within the framework of a strong governance structure.

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Appendix I. Data description

This appendix describes the data used. We distinguish between data series denominated in US dollars and series listed in euros. For each series, we state in order the name, the description including method of calculation, the source and the start date. The data series below relate to returns as measured monthly on portfolios with equities and bonds, using so-called ‘long-only’ portfolios. That is to say, that no short positions are permitted in the calculation of returns. This approach was chosen as it more closely reflects the day-to-day practice and emulates the way in which major investors are structuring their investments at the moment. Moreover, going short is often not permitted.

I.1 Equities, in dollar denominated series.

1. **Market.** The monthly return on a portfolio comprises all NYSE, AMEX and Nasdaq stocks, weighted to value. They are constructed by adding the risk-free rate to the Market factor. Available from January 1927. The source is Robeco, which downloaded the data from the Kenneth French library, see http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
2. **Size.** The average monthly return of three smallcap portfolios, comprising equities with a low, medium and high book-to-market ratio. This relates only to stocks below the NYSE median market-value threshold of the 2 x 3 Size/Value portfolios from the Kenneth French library. The portfolios are compiled as of end June and comprise the intersections of two portfolios formed on the basis of Size (i.e. market value) and three portfolios formed on the basis of the ratio of book value to market value. The Size threshold value is determined by median NYSE market value as at end June of year t. The book-value / market-value ratio as at June of year t is calculated as the book value of the previous fiscal year-end in t-1 divided by the market value as at December of year t-1. The book-value / market-value ratio thresholds are equal to the 30^{th} and 70^{th} percentiles. Available from January 1927. The source is Robeco, which downloaded the data from the Kenneth French library, see http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
3. **Value.** The monthly return of a portfolio comprising equities with both a high ratio of book value to market value and a large Size (i.e. market value). This largecap / value portfolio is constructed from the 2 x 3 Size / Value portfolios from the Kenneth French library. The portfolios are compiled as of end June and comprise the intersections of two portfolios formed on the basis of Size (i.e. market value) and three portfolios formed on the basis of the ratio of book value to

market value. The Size threshold value is determined by median NYSE market value as at end June of year t. The book-value / market-value ratio as at June of year t is calculated as the book value of the previous fiscal year-end in t-1 divided by the market value as at December of year t-1. The book-value / market-value ratio thresholds are equal to the 30th and 70th percentiles. Available from January 1927. The source is Robeco, which downloaded the data from the Kenneth French library, see http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

4. **Momentum.** The monthly return of a portfolio comprising equities with both a high 'Momentum' (i.e. the 'winners') and a large Size (i.e. the market value). This largecap / momentum portfolio is constructed from the 2 x 3 Size / Momentum portfolios from the Kenneth French library. The portfolios are compiled as of end June and comprise the intersections of two portfolios formed on the basis of Size (i.e. market value) and three portfolios formed on the basis of historical returns. The return is measured as the return realized over an eleven-month period, namely the period between twelve months ago and one month ago. The return over the most recent month is thus not used in the portfolio construction. The Size threshold value is determined by median NYSE market value as at end June of year t. The historic return thresholds are equal to the 30th and 70th NYSE percentiles. Available from January 1927. The source is Robeco, which downloaded the data from the Kenneth French library, see http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
5. **LowVol.** The monthly return of a portfolio comprising equities with both a low Volatility and a large Size (i.e. the market value). This large cap / LowVol portfolio is constructed by Robeco in line with the procedure also used to construct the Size, Value and Momentum data series in the Kenneth French library, as described above. The portfolios comprise the intersections of two portfolios formed on the basis of Size (i.e. market value) and three portfolios formed on the basis of the historical 36-month Volatilities. The Size threshold value is determined by the median NYSE market value as at end June of year t. The historical volatility thresholds are equal to the 30th and 70th NYSE percentiles. The source is Robeco. Available from January 1929.

I.2 Bonds, in dollar denominated series.

1. **Market**. The monthly return on a portfolio that comprises the series Barclays Capital US Dollar denominated Aggregate Total Return Index (Mnemonic LHAGGBD) from Datastream.
2. **Term.** The monthly return on a portfolio comprising long-dated Treasury bonds. Uses 20-year and 30-year Treasury Constant Maturity Rates. For both series, Robeco has calculated what a fictional investment would have generated. The 'Term' portfolio is then calculated by us as the

average of the two series, so as if as much were invested in both 20-year as in 30-year Treasury bonds. The monthly returns are calculated as the average of the 'business days' during the month. Available from January 1973. The source of the Constant Maturity series is the St. Louis Federal Reserve website, see http://research.stlouisfed.org/fred2/categories/115.

3. **Credit.** The monthly return on a portfolio that comprises the series Barclays Capital US Credit Bond Total Return Index (Mnemonic LHCRPBD) downloaded from Datastream.
4. **High Yield Credit.** The monthly return on a portfolio that comprises the Bank of America Merrill Lynch US High Yield Total Return Index (Mnemonic MLH100$) downloaded from Datastream.
5. **Short Credit.** The monthly return on a portfolio that comprises the 1–3 Year Credit series, obtained by Robeco from Barclays Capital.
6. **Short Treasury.** The monthly return on a portfolio comprising the Bank of America Merrill Lynch US Treasury 1-3 Year Total Return Index (Mnemonic MLUS1T3) from Datastream.

I.3 Other series denominated in dollars

1. **Risk-free rate.** The 1-month Treasury Bill rate from Ibbotson Associates. Available from January 1927. The source is Robeco, which downloaded the data from the Kenneth French library, see http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
2. **Commodities.** The monthly returns of the S&P GSCI Commodity Total Return Index. Covers all commodities insofar as there is sufficient liquidity in the futures market and it is actually possible to invest in it, in line with the rules drawn up by Goldman Sachs. See http://www.goldmansachs.com/what-we-do/securities/products-and-business-groups/products/gsci/ Source: Datastream. Available from January 1970.
1. **Real estate.** The monthly returns of the FTSE/NAREIT All Reits $ Total Return Index from Datastream. This index is 'free float adjusted', weighted to market capitalization and covers all 'tax qualified' REITS with stocks listed on the NYSE, AMEX and NASDAQ National Market.

I.4 Equities, in euro denominated series.

The **Market**, **Value**, **Size** and **Momentum** series are constructed in similar way as above for the US data. They cover monthly returns on portfolios with all equities from Belgium, Denmark, Germany, the UK, Finland, France, Greece, Ireland, Italy, the Netherlands, Norway, Austria, Portugal, Spain, Sweden and Switzerland, weighted to value. For underlying research, the source is the Kenneth French library, see http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. The source for the French library is Morgan Stanley Capital International for the period 1975–2006 and Bloomberg for the period from 2007. The Market Index is constructed by adding the risk-free rate to the Market factor. For the risk-free rate, we used the Money-market 1-month Frankfurt Interbanking Middle Rate from Datastream (Mnemonic BDMNY1M(IR)). The series are originally denominated in US dollars and are converted by us to euro denomination using the end-of-month euro/dollar exchange rate. Source of exchange rate: Datastream (Mnemonic USEURSP(ER)). Available from November 1998.

I.5 Bonds, in euro denominated series.

1. **Market.** This relates to the monthly return on the Bank of America Merrill Lynch Pan European Government Total Return Index from Datastream (Mnemonic MLPEALE(RI)).
2. **Term.** This relates to the average of the monthly returns on the German and French Benchmark 30-year Government Indices from Datastream (Mnemonics BMBD30Y(RI) and BMFR30Y(RI)).
3. **Credit.** The monthly return on a portfolio that comprises the series Bank of America Merrill Lynch EMU Corporate Total Return Index (Mnemonic MLEXPTE) from Datastream.
4. **High Yield Credit.** The monthly return on a portfolio that comprises the series Bank of America Merrill Lynch European Currency High Yield Total Return Index (Mnemonic MLHECCU) from Datastream.
5. **Short Credit.** The monthly return on a portfolio that comprises the series Bank of America Merrill Lynch EMU Corporate 1-3 Year (Mnemonic MLHEXPAE) from Datastream.
6. **Short Treasury.** The monthly return on a portfolio that comprises the series Bank of America Merrill Lynch Pan European Government 1-3 Year Total Return Index (Mnemonic MLPE13E) from Datastream.

The bond series above are partly only available from February 1998, so this date is used as start date for all bond calculations that use the euro bond series.

I.6 Other series denominated in euros

1. Real estate. The Global Property Research 250 REIT Europe Total Return Index is used, from Datastream (Mnemonic GPRREUE(RI)).
2. Commodities. The S&P GSCI Commodity Total Return Index, so the same series as used for the US, but then denominated in euros.