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Abstract

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JEL Classification: E40, E41, E44, E52, E58 and G10
Keywords: inflation and monetary policy

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1. Introduction

In his Nobel Prize Lecture, Lucas (1995) presents a simple (full information) overlapping generations model with elastic labor supply (to be called, henceforth, the *Lucas Model*) to show that anticipated monetary expansions are, in general, non neutral. In his model, money is a pure store of value and inflation has a negative effect on output by diluting the return from working. According to Lucas' interpretation, the rate of money creation is a distortionary tax and, as such, it is unlikely to be expansionary. The example allows him to propose imperfect information as a more promising, alternative way to explain positive, albeit temporary, effects of monetary expansions on output. In this note I want to offer a different interpretation of the effects of monetary expansions in the class of models used in his lecture, that may be useful under a more general definition of central banks liabilities and instruments.

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The negative effect of money growth in the Lucas Model arises from a restriction on the policies that central banks are allowed to use, such as paying a nominal interest rate on money or, equivalently, making unlimited open market operations on government securities. When we remove these restrictions, money growth becomes expansionary. In particular, a rise in money transfers would generate a reallocation of consumption to the old individual away from the young and a rising labor supply. One way to see this is to note that the inability of the central bank to exploit all available instruments implies that the higher inflation caused by the money transfers has a negative effect on the real interest rate and, then, no expansion in the real money stock can effectively take place in equilibrium. As a consequence, money transfers turn out to reduce old individuals' consumption and decrease labor supply. If we abstract from this narrow definition of monetary policy, we can say that the Lucas Model provides an example where outside assets, when traded at a discount, may increase output in an economy where the market allocation of life-time consumption does not provide enough incentives for work. I call this a *public liquidity effect*, a term that I borrow from Woodford (1990) and Holmstrom and Tirole (1998).

To be sure, any discussion of the effects of money growth in a model with zero transaction costs (or liquidity services) is incomplete. Although studying the effects of monetary policy in a cashless economy is possible and relevant[1], it is certainly true that cash plays an important role in monetary policy and paying a nominal rate on very liquid assets may be unfeasible for legal or technical reasons. Hence, one may wonder whether the idea that monetary expansions are discouraging labor supply may gain some validity when cash must be exchanged at par value for legal or practical reasons. In fact, when money provides transaction services, the nominal rate generates distortions in the allocation of resources. For example, if one assumes a standard cash-in-advance constraint in the Lucas Model, it is immediately verified that a rise of the nominal rate may discourage labor supply. This effect, however, interacts with the public liquidity effect of money creation that I was referring to above in this introduction, and it may or may not override the positive effects of liquidity provision on the inter-temporal allocation of resources. If

[1]For instance, Woodford (2003) considers this type of economy to study the effects of interest rate target policies and open market operations.

the distortionary effect of the nominal rate on the consumption-labor choices is not "too large", monetary expansions may still increase output and improve the intertemporal allocation of consumption in the Lucas Model (*i.e.*, the public liquidity effect is prevalent with respect to the effect arising from the distortionary tax).

A reason why a re-interpretation of the Lucas Model along the lines considered in this note may be relevant is that central banks have recently moved away from traditional policies (direct control of money supply and an exclusive focus on price stabilization) and cash is being progressively replaced by electronic money and other forms of exchange[2]. On the other hand, asset trading and liquidity provision for the purpose of financial stabilization has been an important focus of central banks operations. In other words, well developed financial systems are likely to be characterized by small liquidity frictions and, based on recent experience, relatively large financial frictions. Hence, understanding why money growth is non neutral in the absence of the distortions generated by the inflation tax is a key question.

One should take the present note as a comment on Lucas' lecture in light of well established views about the role of money. In fact, my observations are certainly not new, and some may consider them largely settled in the existing literature on monetary theory. In particular, it is well known that, except for Arrow-Debreu economies, money may affect output and incentives to work because it creates an opportunity for individuals to reallocate consumption across states or periods of time that the market is unable to offer, or because money is a substitute for (inoperative) private insurance. Examples where outside assets help to undue the negative effects of market imperfections have been provided in Bewley (1986), Levine (1988) and Woodford (1990). More recently, Brunnermeier and Sannikov (2015) have introduced the idea (called the *I Theory of Money*) that monetary policy can be effective (and expansionary) because it redistributes wealth across agents and affects asset values in economies characterized by financial frictions. In their model,

[2] The possibility and relevance of a scenario in which an electronic payment system would replace currency, and eliminate the advantage of clearing payments through accounts at the central bank, are suggested by King (1999) and Woodford (2003). According to Cole and Ohanian (1997), the ratio of nominal GDP to M1 has risen by a factor of about three between 1950 and 1980 in the US.

money creation mitigates overhang problems following excessive private debt accumulation and risk exposure. In some sense, the Lucas Model falls in this class of examples, but, differently from most of them, contains no market imperfection or financial friction. In fact, although the specific example considered by Lucas generates a unique inefficient *laissez fair* equilibrium, the model can be extended to the case of market efficiency by endowing old individuals with a sufficient amount of resources. In particular, a positive correlation between money and output does not require the failure of the first welfare theorem. This implies that, within the Lucas' economy, the claim that a monetary expansion is welfare improving cannot be based on a Pareto criterion, as a higher output increases consumption at the cost of a higher labor effort and it may reduce old individuals' welfare. However, public liquidity may be welfare improving when the Planner puts enough weight on future generations' utility. In any case, it is possible to argue that the overlapping generations economies and the economies characterized by financial frictions share a common feature: the fact that changes in outside assets have a non neutral effect on allocations through redistributions of wealth across heterogeneous individuals. This may be considered a consequence of demographics (overlapping generations) or financial frictions.

Quite remarkably, this view about the role of monetary policy is absent in the prevalent mainstream *sticky-price approach*, according to which central banks' primary target should not be the level of economic activity, but, instead, price stabilization. This approach is, for instance, supported in Woodford (2003)[3] and, with some analogy with the views expressed in Lucas (1995) and by the rational expectations school, it is based on the idea that output deviations from the natural level can only be transitory. Ultimately, these different views about monetary policy depend on an empirical evaluation of the relative importance of three different frictions: price stickiness, lack of information and the type of frictions that are considered

[3]According to Woodford, his theory *"implies not only that price stability should matter in addition to stability of the output gap, but also that, at least under certain circumstances, inflation stabilization eliminates any need for further concern with the level of real economic activity"* (Woodford (2003), p. 13).

in this paper, *i.e.*, the existence of important heterogeneities between market participants, due to financial imperfections (such as borrowing limits) or demographic characteristics (such as those assumed in overlapping generations economies).

This note is organized as follows. In section 2 I briefly review the literature related to the issues raised in the present note. In section 3 I set-up a slight modification of the Lucas model with and without interest bearing money. In section 4 I provide a re-interpretation of the model as an economy with tight debt limits similar to Bewley (1986). The use of this type of economy, as opposed to the overlapping generations framework used in Lucas (1995) is useful only because it may provide a more practical interpretation of the insurance role of money in the short run, as the equations characterizing competitive equilibria in this note are essentially equivalent to the ones shown by Lucas. In section 5 I consider the effects of monetary policies at stationary equilibria with and without interest bearing money. In section 6 I will reconsider the model with liquidity services generated by a cash-in-advance constraint. Section 7 concludes.

2. Related Literature

The notion of *public liquidity* has been explored in Woodford (1990) in a model with Bewley-type of equilibria, *i.e.*, symmetric equilibria with two infinitely lived individuals subject to alternating binding debt limits every other period. In this type of economy, equilibria are Pareto sub-optimal, as it is possible to increase *ex-ante* social welfare[4] by reallocating consumption to the constrained individual away from the unconstrained. As shown by Woodford, increasing government debt is one way to perform this transfer. In his own words, *"increased government borrowing can benefit [borrowers and lenders], insofar as they effectively receive a highly liquid asset, government debt, in exchange for giving the government an increased claim on their future income, their own claim to which represented a highly illiquid asset"* (Woodford (1990), p. 382). In a nutshell, public debt serves the role of *public liquidity.* An other way to describe this effects is that a rise in government debt, by squeezing the amount of loanable funds available for borrowers, rises the real interest rate and, then, it lowers the wedge between the market price of existing liabilities

[4]By *ex ante* I mean social welfare before individuals know in what state they will end up at the beginning of time.

and the borrowers' willingness to pay for an extra unit of debt. Ignoring second-order effects, the rate of inflation is greater than the nominal rate of interest minus the rate of time preference at any equilibrium where increasing public liquidity is (ex-ante) welfare enhancing. Hence, one may get closer to the first best (which is equivalent to the Friedman Rule) by lowering inflation for given nominal rate or by increasing the nominal rate for given inflation.

Although the issue discussed in Woodford (1990) is normative, and his model does not allow for an elastic labor supply, one may easily modify the model by introducing a leisure-consumption choice along the lines of the Lucas Model and show that a higher public liquidity has a positive effect on output. The reason is that, as it happens in the Lucas Model, a higher real rate of interest increases individuals' incentives to work. The mapping between Bewley's and Lucas's models is only based on a formal analogy (*i.e.*, on a similar analytical representation of equilibrium conditions) and money creation has different effects in the two models (more consumption smoothing across periods of time in the Bewley model and more old age consumption in Lucas Model). However, both models are examples of the benefits of public liquidity provision in their own way. If we allow for a free readjustment of the nominal interest rate on a government liability (be it money or bonds), there is no reason why a rising inflation may have negative effects on the economy.

The idea that monetary policy may be effective and welfare improving by allowing for a market nominal rate on monetary assets or through open market operations have been thoroughly scrutinized in the literature. Friedman (1969) famously advocated the first policy on a normative ground. In his view, Pareto optimal monetary equilibrium allocations must be such that no one pays a positive price for transaction services, *i.e.*, in Bewley's words, no one "economizes on money". Using a model with debt limits, Bewley (1986) shows the existence of equilibria where this condition can only be implemented under the Friedman Rule, *i.e.*, when the real rate of return on money equals the rate of time preference. In this case, nobody is subject to binding debt limits and the equilibrium allocation reaches a First Best. Regarding open market policies, Fama (1980) showed that, in the absence of financial frictions, and with an unregulated, competitive banking system, *"the portfolio management activities of banks are the type of pure financing decisions covered by the Modigliani-Miller (1958) theorem"*. Then, in this case, monetary policy is ineffective when it operates through changes in the composition of banks assets and liabilities (*i.e.*, in the absence of currency). More specifically, the effectiveness of open market operations (size and the composition of the central banks balance sheet) has been denied by Wallace (1981) as an application of the Modigliani-Miller theorem in the absence of specific frictions. However, his result depends on the assumption that fiscal policy responds to open market operations so as to neutralize any redistribution.

This paper suggests that paying a nominal interest rate on monetary assets may give central banks more power to implement stimulating monetary expansions in certain environments. However, there may be other reasons why this policy may not be advisable. The most obvious is that this prevents governments from generating revenue through the inflation tax, somewhat less obvious reasons are based on the stability of the banking system. The "legal restriction theory" proposed by Bryant and Wallace (1980) may be an answer to the question why the coexistence of interest-bearing perfectly safe bonds and non interest-bering money is possible, but it does not provide any argument why legal restrictions are desirable, a part from granting central banks a monopoly on a specific liability. In fact, Sargent and Wallace (1982) show that, in the absence of a proper central bank open market policy, it is optimal to remove any restriction on the use of interest-bearing safe assets for small transactions.

3. A Modified Lucas Model

I start by presenting a slight modification of the Lucas's set-up, to be called the Modified Lucas Model (MLM). Each generation's utility is represented by

$$u(x_t^1) + \beta u(x_{t+1}^2) + v(1 - y_t),$$

where $\beta \in (0, 1)$, x_t^1 and x_{t+1}^2 denote consumption at t and $t + 1$ of the individual in generation t, and y_t denotes labor effort. Individuals preferences satisfy the following assumption.

Assumption 1. *The utility functions $u(.)$ and $v(.)$ are strictly increasing, strictly concave, continuously differentiable functions, satisfying the Inada conditions:*

$$\lim_{x\to 0} u'(x) = \lim_{y\to 1} v'(1-y) = \infty, \qquad \lim_{x\to\infty} u'(x) = \lim_{y\to 0} v'(1-y) = 0.$$

The single good is produced by identical competitive firms with a technology defined by a linear production function such that, at all time $t \geq 0$, y_t units of the young individual's labor effort generates y_t units of output. Then, positivity of output and profit maximization imply a unitary real wage.

Some public authority (the central bank or the government) issues, at all $t \geq 0$, a stock of nominal liabilities, B_t, and makes a money transfer, $H_t/2$, to old and young individuals. The fiscal authority levies a real lump-sum tax, τ_t, on the young at every period $t \geq 0$. Letting p_t be the price level at t and i_{t+1} the nominal interest rate on a unit of the government liabilities maturing at time $t+1$, an individual born at time t faces the following sequence of budget constraints,

$$p_t x_t^1 = p_t y_t - p_t \tau_t + H_t/2 - B_{t+1}/(1+i_{t+1}), \tag{1}$$

$$p_{t+1} x_{t+1}^2 = B_{t+1} + H_{t+1}/2. \tag{2}$$

Market clearing and the government (period-by-period) budget constraint provide

$$x_t^1 + x_t^2 = y_t, \tag{3}$$

$$B_{t+1} = (1+i_{t+1})(B_t - p_t\tau_t + H_t). \tag{4}$$

Since this is a cashless economy (*i.e.*, an economy where money generates no specific transaction services), B_t may represent circulating money (*i.e.*, checking accounts and money certificates) or government bonds yielding a common nominal rate of return i_{t+1}. If we rule out the existence of money, B_t should be thought of as nominal government debt. However, this does not prevents the existence of a central bank conducting monetary policy through open market operations. In particular, let B_t^g denote the total liabilities of the government, B_t the government debt held by households and B_t^b the government debt held by the Central Bank. Then,

$$B_t + B_t^b = B_t^g.$$

The Central Bank provides "liquidity" by injecting the monetary transfer, H_t, and the budget constraints of the government and the central bank are, respectively,

$$B^g_{t+1}/(1+i_{t+1}) \quad = \quad B^g_t - p_t\tau, \tag{5}$$

$$-B^b_{t+1}/(1+i_{t+1}) \quad = \quad -B^b_t + H_t. \tag{6}$$

By consolidating the above two budget constraints we obtain the (consolidated) public budget constraint represented by (4).

By utility maximization, we derive the first order conditions

$$u'(x^1_t) \quad = \quad v'(1-y_t), \tag{7}$$

$$(1+i_{t+1})p_t/p_{t+1} \quad = \quad u'(x^1_t)/\beta u'(x^2_{t+1}). \tag{8}$$

Now define the real value of public debt plus transfer, which we may call *real public liquidity*, as

$$\lambda_t = (B_t + H_t/2)/p_t. \tag{9}$$

Then, using the budget constraints (1) and (2), and Walras law, we can provide the following definition. Then, the sequence $\{\tau_t\}_{t=0}^{\infty}$ defines a *fiscal policy* and the sequence $\{i_{t+1}, \lambda_t, B_{t+1}\}_{t=0}^{\infty}$ defines a *monetary policy*. Form now on we will assume that the government sets the fiscal policy independently. This justifies the following definition.

Definition 1. *For a given initial public debt, $B_0 > 0$, and a given fiscal policy, $\{\tau_t\}_{t=0}^{\infty}$, a competitive equilibrium for the MLM is a positive sequence,*

$$\{y_t, p_t, i_{t+1}, \lambda_t, B_{t+1}\}_{t=0}^{\infty},$$

satisfying $y_t \in [0, \lambda_t] \subset [0, 1]$ and

$$u'(y_t - \lambda_t) \quad = \quad v'(1-y_t), \tag{10}$$

$$(1+i_{t+1})p_t/p_{t+1} \quad = \quad u'(y_t - \lambda_t)/\beta u'(\lambda_{t+1}), \tag{11}$$

$$B_{t+1} \quad = \quad (1+i_{t+1})(2p_t\lambda_t - B_t - p_t\tau_t). \tag{12}$$

Observe that the sequence of real public liquidity levels, $\{\lambda_t\}_{t=0}^{\infty}$, produce a reallocation of consumptions across generations and they determine the output levels, $\{\lambda_t\}_{t=0}^{\infty}$. In particular, by the concavity of the utility functions, u and

v, equation (7) defines, implicitly, a strictly increasing continuous function, $\phi : (0,1) \to (0,1)$, such that, for all (y_t, λ_t) satisfying equation (10),

$$y_t = \phi\left(\lambda_t\right), \tag{13}$$

with $\phi(0) > 0$, $\lim_{\lambda \to 1} \phi(\lambda) = 1$, $\phi'(\lambda) \in (0,1)$.

Following Lucas (1985), I now characterize a stationary equilibrium, *i.e.*, an equilibrium as defined in 1, with constant values of output, consumption, real public liquidity. This is accomplished by imposing a constant given tax rate, τ, and a constant growth rate of the nominal debt. More specifically, we define a *monetary policy* as a pair, (λ, μ), with $\mu = (B_{t+1} - B_t)/B_t$. Then, following definition 1, we can provide the following.

Definition 2. *For a given stationary fiscal policy, τ, a stationary competitive equilibrium of the MLM is an array, $\{y, \lambda, \mu, p_t, i, B_{t+1}\}_{t=0}^{\infty}$, such that*

$$\begin{aligned}
y &= \phi(\lambda), && (14)\\
p_{t+1}/p_t &= B_{t+1}/B_t = (1+\mu), && (15)\\
(1+i) &= (1+\mu)u'(y-\lambda)/\beta u'(\lambda), && (16)\\
b((1+i)+(1+\mu)) &= (1+i)(2\lambda - \tau). && (17)
\end{aligned}$$

4. Financial Frictions

To show that some of the characteristics of the Modified Lucas Model are shared by economies with financial frictions, we consider a special Bewley-type of economy. There are two type of infinite lived individuals subject to a no-borrowing constraint. The two individuals, $i = e, o$ (even, odd), have identical life-time utility function

$$U = \sum_{t=0}^{\infty} \beta^t (u(c_t^i) + v(1 - y_t^i)),$$

where c^i stands for consumption, $y^i \in [0,1]$ is labor effort and u and v satisfy assumption 1.

The technology for the production of the unique consumption good is the same define in the MLM, but the two individuals have a non constant labor productivity. In particular, we consider the extreme case in which individual e (o) is able to produce a unit of output for each unit of her labor time at even (odd) periods and she is totally unproductive at odd (even) periods. Labor is the only source of income in this economy. Then, by the linearity of the production function, we derive that the real wage rate is $w_t = 1$ and output is

$$y_t = \begin{cases} y_t^e & \text{for } t \text{ even,} \\ y_t^o & \text{for } t \text{ odd.} \end{cases}$$

We assume that no borrowing is allowed and that the government, at all time $t \geq 0$, imposes a time-independent real lump-sum tax, τ_t, on the employed worker (high-productivity individual) and makes a money transfer, H_t, to be distributed equally across individuals, *i.e.*, both individuals receive $H_t/2$ at all periods. The latter assumption insures that monetary policy is "blind" with respect to individuals liquidity needs, reflecting the inability to discriminate due to informational or legal constraints.

We seek equilibrium configurations such that, at each time period, the low-productivity individual hits the debt limit. In particular, let (x_t^1, A_{t+1}^1) $((x_t^2, A_{t+1}^2))$ denote the level of consumption and the net nominal claims acquired at time t by the employed high-productivity (unemployed low-productivity) individual at time t. Then, the high and low-productivity individuals' budget constraints at time t can be written as

$$\frac{1}{1+i_{t+1}} A_{t+1}^1 + p_t(x_t^1 + \tau - y_t) - H_t/2 = 0, \tag{18}$$

$$p_t x_t^2 - H_t/2 = A_t^1. \tag{19}$$

Since the only individual who has a leisure-consumption choice is the high-endowment individual and the low-productivity individual has a (possibly) binding debt limit, the first order conditions characterizing the optimal consumption-leisure plan of each individual are defined by (7), (8) and

$$(1+i_{t+1})\frac{p_t}{p_{t+1}} \leq \frac{u'(x_t^2)}{\beta u'(x_{t+1}^1)}. \tag{20}$$

The above inequality guarantees that the the low-productivity individual's debt limit may be binding. By market clearing in the good and asset markets we derive $A_t^1 = B_t$ and equations (3), (4).

Using the market clearing condition, (3), and the government budget constraint, (4), an equilibrium of the economy with tight debt limits, for some initial debt level,

$B_0 > 0$, and a given fiscal policy, $\{\tau_t\}_{t=0}^{\infty}$, is a positive sequence,

$$\{y_t, p_t, i_{t+1}, \lambda_t, B_{t+1}\}_{t=0}^{\infty},$$

verifying equations (10), (11), (12), together with (20). Hence, an equilibrium with tight debt limits correspond to an equilibrium for the MLM in definition 1, when we add the additional restriction (20), *i.e.*,

$$(1 + i_{t+1})\frac{p_t}{p_{t+1}} \leq \frac{u'(\lambda_t)}{\beta u'(y_{t+1} - \lambda_{t+1})}. \tag{21}$$

Notice that the First Best cannot be supported as a *laissez faire equilibrium*, *i.e.*, an equilibrium with zero outside money, taxes and transfers, under the constrained $A_{t+1}^i \geq 0$ for all i and $t \geq 0$, since, in a First Best *laissez faire* equilibrium, the low-productivity individual would have to borrow.

A key observation is that λ_t plays the role of a sort of public insurance: if it goes up, individuals are able to benefit from more consumption smoothing across periods of time. By equation (11), the real interest rate rises, so that borrowing becomes more expensive and, then, excess demand for borrowing falls (reducing the wedge between the market rate and the borrowers' willingness to pay). At the same time, by equation (10), the fall in x^1 induced by the rising λ_t, increases the marginal rate of substitution between labor and consumption, *i.e.*, individuals increase labor supply.

Following from the above observations, we define a stationary equilibrium for the economy with tight debt limits, for a given fiscal policy, τ, as an array,

$$\{y, \lambda, \mu, p_t, i, B_{t+1}\}_{t=0}^{\infty},$$

satisfying equations (14), (15), (16), (17) in definition 2, together with

$$\lambda \leq y/2. \tag{22}$$

In fact, by equation (22), the low-productivity individual has less consumption than the high-productivity individual, so that the assumption that the former may have a binding debt limit is satisfied. By (14) and (15), this condition implies $(1 + \mu)v'(1 - y) \geq (1 + i)\beta v'(1 - y)$, *i.e.*,

$$1 + \mu \geq \beta(1 + i). \tag{23}$$

Observe that a *full insurance* (First Best) equilibrium allocation is a stationary equilibrium, $(y^f, \lambda^f, i^f, \mu^f, h^f)$ characterized by equal individual consumption across periods, so that

$$x^1 = y - \lambda = \lambda = x^2.$$

By the first order conditions, the above implies that λ^f satisfies

$$\lambda^f = \phi(\lambda^f)/2, \tag{24}$$

and $i^f = (1+\mu)/\beta - 1$ (*i.e.*, the *Friedman Rule*). Evidently, there exists a unique value, λ^f, satisfying (24) and $\lambda \leq \phi(\lambda)/2$ (*i.e.*, $x^2 \leq x^1$) for all $\lambda \leq \lambda^f$. This follows by observing that, for all $\lambda \in (0,1)$, we have $\phi(\lambda) > \lambda$, $\phi'(\lambda) < 1$ and $\phi(0) > 0$, $\lim_{\lambda \to 1} \phi(\lambda) = 1$. By the above discussion, this allocation is a stationary equilibrium of a monetary economy, *i.e.*, it satisfies $(h, i) \geq 0$, if (τ, μ) are such that

$$\lambda^f \leq \frac{\tau}{1-\beta}, \qquad \mu \geq -(1-\beta).$$

Notice the different role played by the real public liquidity level, λ, in the overlapping generations model and in the model with tight debt limits. In the former, a rise in λ implies a transfer of resources from young to old age (as in the case of social security) and, through the first order condition (10), it induces the young to work more. In the latter, as we have said above, a rise in λ generates more consumption smoothing across periods, and attains full consumption smoothing at λ^f. This allocation is optimal from an *ex ante* perspective, *i.e.*, when individuals' welfare is maximized before they know their labor productivity. On the other hand, under the overlapping generations interpretation of the equilibrium conditions, the case $\lambda^f = \phi(\lambda^f)/2$ may be justified on the basis of some specific welfare function. In particular, one can easily show that $x_t^1 = x_t^2$ is optimal when the welfare function is a discounted sum of all generations' utility functions with the social discount rate equal to the subjective discount rate, β. More generally, a rise in the real interest rate is Pareto improving in the overlapping generations model as long the existing rate is negative (or below the population growth rate). Bloise and Reichlin (2011) provide a notion of *ex-post* constrained efficiency in economies with tight debt limits and show that, similarly to what happens in the overlapping generations model, *ex-post* constrained inefficiency occurs for negative real interest rates.

5. Monetary Policy

In this section I consider the effects of monetary policy in a stationary equilibrium, operating by managing (λ, μ, i, b). The equilibrium I refer to is the one characterized in definition 2 if we stick to the overlapping generations interpretation. If, on the other hand, we are considering a stationary equilibrium of the economy with tight debt limits, we add the restriction $\lambda \leq y/2$. From now on, any one of these equilibria will be called a stationary *Modified Lucas Model* (MLM) equilibrium.

A further restriction may be required to guarantee that the stationary equilibrium implies non-negative nominal interest rate, *i.e.*, $i \geq 0$. We can take this as a requirement for any monetary economy under the plausible assumption that some sort of cash is ready to be used for transactions. The following proposition show that this requirement is satisfied for a large enough value of λ, conditional on any given inflation rate μ.

Proposition 1. *A stationary MLM equilibrium generates a non negative nominal rate of interest, i.e.,* $i \geq 0$, *and a non negative monetary transfer if and only if, for any given* $\mu \geq 0$, $\lambda \geq \lambda(\mu)$, *where* $\lambda(\mu) \in (0,1)$ *is the unique value of* λ *in* $(0,1)$ *such that* $i = 0$, *i.e.,*

$$u'(\phi(\lambda(\mu)) - \lambda(\mu)) = \beta(1+\mu)u'(\lambda(\mu)).$$

Furthermore, $\lambda(\mu)$ *is a continuous decreasing function of* μ *for all* $\mu \geq 0$.

Proof. For $i \geq 0$ we need

$$\psi(\lambda, \mu) \equiv u(\phi(\lambda) - \lambda) - \beta(1+\mu)u'(\lambda) \geq 0.$$

By the properties of the function $\phi(\lambda)$, we know that

$$\lim_{\lambda \to 0} \psi(\lambda, \mu) = -\infty, \qquad \lim_{\lambda \to 1} \psi(\lambda, \mu) = +\infty, \qquad \psi_\lambda(\lambda, \mu) > 0.$$

Then, there exists a unique value, $\lambda(\mu) \in (0,1)$ such that $i = 0$. Finally, observing that $\psi_\mu(\lambda, \mu) < 0$ for all $\lambda > 0$, the proposition follows. □

The case in which money earns no nominal rate of return, *i.e.*, $i = 0$, because of transaction costs or a legal constraint, is the one considered in Lucas (1995). We call this the *pure currency model*. By proposition 1, in this case we have $\lambda = \lambda(\mu)$, *i.e.*, the central bank (or government) loses the ability to set the level of real public liquidity independently.

In a pure currency model, the equilibrium conditions (14)-(17) reduce to the following pair of restriction on y and b:

$$\frac{1}{1+\mu} = \frac{u'(y-\lambda(\mu))}{\beta u'(\lambda(\mu))}, \qquad y = \phi(\lambda(\mu)), \qquad b(2+\mu) = 2\lambda(\mu) - \tau.$$

Noticing that

$$\frac{\partial y}{\partial \mu} = \phi'(\lambda)\lambda'(\mu) < 0,$$

we conclude that output is a decreasing function of μ.

In other words, by imposing $i_{t+1} = 0$ and $B_{t+1}/B_t = 1 + \mu$ at all $t \geq 0$, the monetary authority is effectively fixing the real interest rate at $1/(1+\mu)$. By the first order conditions for individual optimality, the real interest rate and labor supply, y, are positively related. Then, a rise in inflation, by reducing the real rate, causes a fall in output and less consumption smoothing. It is important to notice that, in this case, real public liquidity, or individuals' real net financial wealth, λ, has to adjust to a change in target inflation. In other words, causality runs from inflation to output and real private wealth.

Proposition 2. *In any stationary MLM equilibrium with pure currency, stationary output is decreasing in the inflation rate and the central bank has no power to use public liquidity to affect the incentives to work.*

Now consider the case of interest-bearing currency (or unlimited open market operations), *i.e.*, i positive and endogenously determined by demand and supply of public liabilities. In this case, we can fix two variables, for example λ and μ, and find the remaining variables, (y, i, b), that solve (14)-(17) and belong to the appropriate range. It follows that the monetary authority can generate a target value of output, say y^* (this should satisfy $y \leq y^f$ in the case of tight debt limits), by fixing inflation and public liquidity appropriately and letting the nominal interest rate and the real money balances be set by market forces. In particular, the desired output level y^* is generated by any policy (λ^*, μ^*) such that

$$\lambda^* = \phi^{-1}(y^*) \geq \max\{\lambda(\mu^*), \tau/2\}, \tag{25}$$

and it generates an equilibrium pair (i^*, b^*) such that

$$i^* = (1+\mu^*)\frac{u'(y^* - \lambda^*)}{\beta u'(\lambda^*)} - 1, \qquad b^* = \frac{(1+i^*)}{(1+\mu^*) + (1+i^*)}(2\lambda^* - \tau).$$

As can it be seen by the above characterization, the central bank can reach two objectives simultaneously, output and inflation, provided that inflation, μ^*, is large enough relative to the output target y^* (otherwise the nominal rate would be negative) and the tax rate, τ, selected by the fiscal authority, is low enough (to guarantee positivity of public outstanding liabilities).

Proposition 3. *In a stationary MLM equilibrium with interest-bearing money, the central bank can generate a rise in output and select the inflation rate independently in some appropriate range by raising the level of real public liquidity, provided that the inflation rate and the tax rate are not too large.*

How does the central bank engineer a rising real public liquidity? Is this equivalent to a monetary expansion? Assume that the monetary authority follows the following rules

$$B_{t+1} = (1+\mu)B_t, \tag{MR}$$

$$H_t = 2(\lambda^* p_t - B_t), \tag{TR}$$

for some fixed coefficients (μ, z). The question is whether this is feasible at equilibrium and what level of output does this policy generate. By following these rules, the monetary authority is able to generate a constant value of real public liquidity, λ, exactly equal to λ^* at any period. By the government budget constraint, we derive the evolution of nominal debt, B_t, consistent with this rule, for a given tax $\tau \geq 0$, as:

$$B_{t+1} = (1+i_{t+1})(p_t(2\lambda^* - \tau) - B_t). \tag{26}$$

By the discussion in the previous section, one can immediately verify that the monetary authority can implement any arbitrary output level $y^* \in (0,1)$ by choosing some inflation rate

$$p_{t+1}/p_t = B_{t+1}/B_t = \mu,$$

and the rule (TR) with $\lambda^* \equiv \phi^{-1}(y^*)$, provided that the conditions in (25) are verified.

Evidently, this is not the only policy that may successfully implement a given output. An alternative policy could be based on a *nominal interest rate rule*. However, this policy may not be as effective and simple as the one just described. Let, for instance, the interest rate rule be

$$(1 + i_{t+1}) = \rho p_{t+1}/p_t, \tag{IR}$$

with $\rho > 0$ predetermined. Then, the monetary authority is effectively targeting the real rate to ρ. By the first order conditions, this policy generates a sequence of output levels, $\{y_t\}_{t=0}^{\infty}$, such that

$$\rho\beta u'(\phi^{-1}(y_{t+1})) = u'(y_t - \phi^{-1}(y_t)),$$

and a sequence of prices and money stocks such that

$$B_{t+1}/p_{t+1} = \rho(2\phi^{-1}(y_t) - \tau) - \rho B_t/p_t.$$

Whether the simple rule (IR) is able to generate a desired output is open to question.

6. The Economy with Cash

I now assume that transactions can only be carried out by using cash, a non-interest bearing public liability, to be denoted by M. In particular, I consider the tight-debt-limit model and impose a slightly modified version of the standard cash-in-advance constraint. Then, for all (poor and rich) individuals $j = 1, 2$, the following inequality must be verified

$$M_t^j \geq \max\{0, p_t(x_t^j - \gamma y_t^j) - H_t^j\}, \tag{27}$$

where the superscripts 1 and 2 identify the high and low-productivity individuals, respectively, M_t^j is the stock of cash carried over to period t and H_t^j a cash transfer from the monetary authority.

Constraint (27) states that the cash carried over at time t from period $t-1$ by individual j must be sufficient to cover the value of the consumption good bought at t less the cash transfer received from the monetary authority and some fraction, $\gamma \in [0, 1]$, of the total labor income earned in the course of the same period. The case $\gamma = 0$ corresponds to the standard cash-in-advance constraint considered in most of the literature. As I will show in a moment, any value of γ smaller than one implies that the nominal interest rate generates a distortion in the allocation of consumption and labor.

As in the previous section, we assume that high-productivity individuals are subject to a real lump-sum tax $\tau \geq 0$, and that $H_t^j = H_t/2$ for $j = 1, 2$. The assumptions about individuals labor productivity made in the previous section are maintained, so that $y_t^1 = y_t$, $y_t^2 = 0$ and no individual is allowed to borrow at any time period, *i.e.*,

$$A_{t+1}^j \geq 0 \tag{28}$$

for $j = 1, 2$ and $t \geq 0$.

In what follows I am restricting attention to equilibria such that $i_{t+1} > 0$ and $x_t^1 > \gamma y_t + H_t/2$ for all $t \geq 0$, so that the cash-in-advance constraint (27) is binding at all $t \geq 0$. Then, similarly to the procedure followed in section 3, we seek equilibrium configurations such that, at each time period, the low-productivity individual hits the debt limit. In particular, $A_t^2 = 0$ at all $t \geq 0$, so that the budget constraints and cash-in-advance constraints of the two individuals can be written as follows

$$\begin{aligned}
\frac{A_{t+1}^1}{1+i_{t+1}} + M_{t+1}^1 + p_t(x_t^1 + \tau - y_t) &= M_t^2 + H_t/2, & (29)\\
M_{t+1}^2 + p_t x_t^2 &= A_t^1 + M_t^1 + H_t/2, & (30)\\
H_t/2 + M_t^2 &= p_t(x_t^1 - \gamma y_t), & (31)\\
H_t/2 + M_t^1 &= p_t x_t^2. & (32)
\end{aligned}$$

Since I am assuming that the low-productivity individual's debt limit is binding, the first order condition for (x^j, A^j, M^j) to be individually optimal for $j = 1, 2$ are defined by

$$\begin{aligned}
u'(x_t^1) &= \left(\frac{1+i_t}{1+\gamma i_t}\right) v'(1-y_t), & (33)\\
\frac{p_t}{p_{t+1}}(1+i_t) &= \frac{u'(x_t^1)}{\beta u'(x_{t+1}^2)} \leq \frac{u'(x_t^2)}{\beta u'(x_{t+1}^1)}. & (34)
\end{aligned}$$

The above conditions replace the analogous condition (10), (11), (20) for the cashless economy establishing the optimal trade-off between labor and consumption with cash-in-advance. Contrary to the case in which money does not provide transaction services, a rising nominal interest rate (*i.e.*, the opportunity cost of holding cash) induces individuals to substitute labor for consumption since leisure is not a *cash good*.

Observe that equations (30) and (32) imply $M^2_{t+1} = A^1_t$, *i.e.*, the low-productivity individual uses non cash net assets to acquire the amount of cash that she will need to buy goods next period. Then, by asset market clearing, we derive

$$A^1_t = M^2_{t+1} = B_t, \quad M^1_t = M_t - M^2_t = M_t - B_{t-1}$$

at all $t \geq 0$. Using the above with equations (29), (31), together with the consolidated public budget constraint

$$B_{t+1}/(1+i_{t+1}) + M_{t+1} = B_t + M_t + H_t - p_t\tau, \tag{35}$$

and resource feasibility, we derive

$$M_t + H_t = (1-\gamma)p_t y_t \tag{36}$$

$$x^1_t = y_t - \theta_t, \qquad x^2_t = \theta_t, \tag{37}$$

where

$$\theta_t \equiv \frac{M_t + H_t/2 - B_{t-1}}{p_t}.$$

The variable θ plays the role of real public liquidity that was represented by λ in the model with no transaction services. The reason why the two variables differ is the following. Recall that, under the assumption $A^2_t = 0$, the low-productivity individual's budget constraint implies

$$x^2_t = \frac{B_t + M^1_t + H_t/2}{p_t} - \frac{M^2_{t+1}}{p_t}. \tag{38}$$

Then, this individual's consumption falls short of her initial real claims by the amount of cash that she needs to purchase goods in the next period. By the central bank's balance sheet, we derive $M^1_t = M_t - M^2_t$. Then, equation (40) can restated as

$$x^2_t = \left(\frac{B_t + M_t + H_t/2}{p_t}\right) - \left(\frac{M^2_t + M^2_{t+1}}{p_t}\right). \tag{39}$$

The first expression in parenthesis in the right hand side of the above equation represents aggregate real public liquidity, and the second expression in parenthesis is the demand for cash of the low-productivity individual in the two consecutive periods. In the preceding discussion we have shown that the low productivity individual uses her interest bearing asset holdings to buy cash, *i.e.*, $M_{t+1}^2 = A_t^1 = B_t$. Then, substituting in equation (41), we derive $x_t^2 = \theta_t$.

I define a *symmetric equilibrium* of the cash-in-advance economy, for a given fiscal policy, τ, and some initial public liabilities, $(B_0, M_0) > 0$, as a non-negative sequence, $\{y_t, p_t, B_{t+1}, M_{t+1}, i_{t+1}, H_t\}_{t=0}^{\infty}$, verifying equations (33), (34), (35), (36), (37) with x_t^1 and x_t^2 non-negative for all $t \geq 0$.

A *stationary symmetric equilibrium* for some tax, $\tau \geq 0$, is an equilibrium such that consumption, output, interest rate, real cash transfers and real debt are time-invariant and prices and nominal debt grow at a constant rate, μ. In particular, at a stationary equilibrium, we let

$$p_{t+1}/p_t = B_{t+1}/B_t = M_{t+1}/M_t = 1 + \mu,$$

and define $m = M_t/p_t$, $h = H_t/p_t$. Then, by (35) and (36), a stationary symmetric equilibrium can be represented as an array, (y, θ, i, μ, h), such that $(i, h) > 0$ and,

$$u'(y-\theta) = \left(\frac{1+i}{1+\gamma i}\right) v'(1-y), \tag{40}$$

$$\frac{u'(y-\theta)}{\beta u'(\theta)} = \left(\frac{1+i}{1+\mu}\right), \tag{41}$$

$$(i-\mu)b = (1+i)(\tau + \mu m - h), \tag{42}$$

$$m + h = (1-\gamma)y, \tag{43}$$

$$\theta \leq y/2 \tag{44}$$

and

$$y \in [0,1], \qquad \theta = m + h/2 - b/(1+\mu) \in [0,1].$$

As in the case with no cash-in-advance, constraint (44) insures that $x^1 \geq x^2$, and it is verified for any (i, μ) satisfying (23).

The analysis of the symmetric steady state equilibrium characterized by the above set of equations reveals that the effect of public liquidity, θ, on output, y, is ambiguous. This is because the nominal rate, i, has a negative effect on labor supply for any given θ, for all $\gamma < 1$ (*i.e.*, when labor income cannot be used for transactions). In turn, since θ is positively correlated with i, a rising public liquidity may or may not increase output for any given inflation, μ.

More precisely, let $\eta(i) = (1+i)/(1+\gamma i)$. By equation (40) we derive

$$y = \psi(\theta, i),$$

where ψ is a continuously differentiable function such that

$$\begin{aligned}\psi_\theta &= \frac{u''(y-\theta)}{u''(y-\theta) + \eta(i)v''(1-y)},\\ \psi_i &= \frac{\eta'(i)v'(1-y)}{u''(y-\theta) + \eta(i)v''(1-y)}.\end{aligned}$$

Observe that, if $\gamma \neq 1$ and $\theta = y/2$ (full insurance), we have $1+\mu = \beta(1+i)$ and $y = \psi(y/2, i) < y^f$, unless $\gamma = 1$ or $i = 0$. In other words, no amount of public liquidity is enough to achieve the First Best.

Using $y = \psi(\theta, i)$ and (40) in (41), and assuming that $\gamma = 0$ and $v'' = 0$ are not simultaneously verified, we derive

$$i = i(\theta, \mu), \qquad y = F(\theta, \mu) \equiv \psi(\theta, i(\theta, \mu)),$$

with $i(\theta, \mu)$ implicitly defined by

$$(1+\gamma i)\beta u'(\theta) = (1+\mu)v''(1 - \psi(\theta, i)).$$

By total differentiation, we derive

$$i_\theta = -\frac{(1+\gamma i)\beta u''(\theta) + (1+\mu)v''(1-y)\psi_\theta}{\gamma\beta u'(\theta) + (1+\mu)v''(1-y)\psi_i} > 0.$$

Taking derivatives and exploiting condition (40), we obtain

$$F_\theta(\theta, \mu) = \psi_\theta(\theta, i) + \psi_i(\theta, i)i_\theta = Z\left((1-\gamma)\frac{\sigma(\theta)}{1+i} - \gamma\sigma(y-\theta)\right), \tag{45}$$

where $\sigma(x) = -u''(x)/u'(x)$ defines the absolute degree of risk aversion and

$$Z = \frac{\beta u'(y-\theta)u'(\theta)}{(u''(y-\theta) + \eta(i)v''(1-y))(\gamma\beta u'(\theta) + (1+\mu)v''(1-y)\psi_i)} < 0.$$

Regarding equation (45), we can say that $\psi_\theta > 0$ represents the direct effect on output of a rising public liquidity and $\psi_i i_\theta < 0$ the indirect effect due to the distortion arising from the transaction technology. Observe that, if $\gamma = 0$, as in the standard cash-in-advance model, we have

$$\frac{\partial y}{\partial \theta} = F_\theta(\theta, \mu) < 0.$$

In words, if no part of an individual's labor income can be used to save on cash, the negative distortionary effect of a rising nominal rate on labor supply is stronger than the positive direct effect of a rising public liquidity. By (45), we derive that $F_\theta(\theta, \mu) > 0$ (the direct effect overcomes the indirect effect) if and only if

$$\gamma(1+i) > \frac{1}{1+\sigma(y-\theta)/\sigma(\theta)}. \tag{46}$$

Proposition 4. *Assume that, at a symmetric steady state, (y, θ, i, μ, h), $v'' \neq 0$, (46) and $1 + \mu > \beta(1+i)$ are verified. Then,*

- *a policy producing a rise in θ for given (μ, τ) generates a rise in y and a rise in i,*
- *and the negative effect on output of a (small enough) rise in inflation, μ, can be offset by a large enough rise in public liquidity, θ.*

Following the discussion in the previous section, the monetary authority may be able to generate some arbitrary level of output (as well as state contingent consumption) by choosing a suitable pattern for the monetary transfers. In particular, a policy achieving a target level of output is

$$H_t = 2\theta p_t + 2B_{t-1} - 2M_t \tag{47}$$

and $p_{t+1}/p_t = M_{t+1}/M_t = B_{t+1}/B_t = 1 + \mu$.

7. Conclusion

In his Nobel Prize lecture, Lucas emphasizes a "tension" between two incompatible ideas: *"that changes in money are neutral units changes, and that they induce movements in employment and production in the same direction"* (Lucas (1995), p. 248). Although these two ideas may be incompatible in economies with "perfect markets" (such as the Arrow-Debreu set-up), they are not incompatible in economies with limited participation (overlapping generations economy) or tight debt limits, provided that the monetary authority is allowed to use a sufficient number of instruments, such as paying a nominal rate on money or using open market purchases of government bonds. Because of these restrictions, the monetary authority is unable to induce a higher labor effort through an increase in real public liquidity. In fact, in order for a higher public liquidity to stimulate output, we need a higher real interest rate and, in the absence of an opportunity to adjust the nominal rate, a higher real rate is not compatible with a rising inflation. In my view, the above argument implies that, in more general institutional settings (where cash coexists with other type of public liabilities and open market operations are allowed), the consequences of money growth are twofold: there is a benefit coming from enhanced public liquidity, and a cost due to liquidity distortions. Which one prevails is a function of the degree of development of the payments system and the characteristics of individuals' preferences.

Hence, this note is essentially raising two related questions. The first is what specific feature of a monetary economy is responsible for a contractionary effect of money growth in the Lucas Model. The second (more general) question is why do central banks provide liquidity by issuing non interest bearing liabilities instead of ordinary debt instruments or make limited recourse to open market operations. This second question is not addressed in this paper, but follows directly from the first.

It has been observed that central banks policies may run into a problem of incompatible objectives, *i.e.*, financial stability (through liquidity provision) and macroeconomic stabilization (through interest rate targeting). For instance Keister et al. (2008) observe that a *"central bank's payments policy, liquidity policy, and desire to promote efficient allocation may all come into conflict with its monetary policy objectives."* In particular, the first set of objectives (financial stability) may require remuneration of banks reserves and the minimization of the distortions generated by the inflation tax. The relevance of this problem is testified by the observation that reserve balances at commercial banks have been rising substantially since the US Federal Reserve has started to pay a nominal interest rate in 2008. In this paper I have shown that financial stability may not be the only reason why a central bank may use monetary transfers and liability management as policy instruments. In fact, the class of models considered in this paper are such that these instruments may affect macroeconomic variables, such as output and employment.

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