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PRICES? LOSS LEADING IN A MODEL OF
SALES WITH SALIENT THINKERS**

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TOO MUCH ATTENTION ON LOW PRICES? LOSS LEADING IN A MODEL OF SALES WITH SALIENT THINKERS[†]

Abstract

Loss leading is analyzed in a model of promotions (as in Varian 1980) with limited consumer attention: (i) Consumers only compare prices of a selected number of products and (ii) they may pay more attention either to price or quality, depending on the salience of the respective attributes. When consumers have standard preferences, which is our benchmark case, manufacturers benefit when one-stop shopping induces retailers to discount their products, as this expands demand. Results are strikingly different when consumers are salient thinkers. When one-stop shopping or retail competition increases the scope for loss leading, manufacturers' profits decline and there may be an inefficient substitution to lower-quality products. In particular, shoppers who compare products may end up with a choice that is strictly inferior to that of non-shoppers who are locked in to a (local) retailer. Our analysis has implications both for competition policy, as we analyze the implications of a ban on loss leading, and for marketing, as we also analyze how salience affects retailers' product and promotion strategies.

JEL Classification: D21, D43, D83, L11, L13 and L15

Keywords: limited attention, loss leading, manufacturer profits, product choice, promotions, quality choice, retailing, sales and salience

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*“Farmers fuming as supermarkets sell milk at half price of water.”*¹

1 Introduction

The initial quote is not unusual as milk is an important “loss leader” in many countries.^{2,3} In Germany, Wal-Mart’s attempt to gain market share through heavy discounts on milk in particular has even led to a landmark decision of the highest court in 2002 and subsequently to a change of the national competition law: it now bans loss leading explicitly in the food retailing industry.⁴ But why are manufacturers opposing deep discounts, which after all expand demand? And why are their demands supported by policymakers and consumer interest groups, even though consumers should benefit from lower prices?⁵ Our paper provides answers to these questions. Manufacturers’ fears of lower profits and consumer interest groups’ concerns about a reduction in quality both prove to be unfounded in our baseline model with fully rational consumers, but they receive support when consumers are salient thinkers. Still, even in the latter case we show how the prohibition of below-cost pricing can backfire.

Our model is chosen so as to capture important features of modern retailing compe-

¹Quote taken from MailOnline for the UK’s Daily Mail, 15 January 2011.

²We refer to loss leaders as (retail) products that are sold at low prices, and in particular below cost, to attract consumers who generate additional sales from more profitable products or services.

³Depending on the national cuisine, also other staple goods are common loss leaders. For instance, in Spain the government repeatedly undertakes efforts to curb discounts on olive oil (e.g., “Deal Could Stop Use of Olive Oil as Loss Leader”, OliveOilTimes, 4 March, 2013). These efforts are helped by Spain’s legal prohibition of loss leading, of which recently the German discounter Lidl fell foul when selling wine at a discount (“Lidl rapped for selling wines at a loss”, TheDrinksBusiness, 16 October 2014). Wine is also an example of a loss leader in a category with more branded goods, such as also coffee.

⁴This goes back to a decision of the German competition authority in 2000 to prohibit below-cost selling by Wal-Mart, as well as Germany’s two leading discounters Aldi and Lidl. Wal-Mart challenged this decision and lost on appeal. Wal-Mart exited the German market in 2006. While we discuss further policy experience below, in light of the introductory quote it is interesting that in the UK, heavy discounting of dairy products seems to persist and to still raise the same concerns. E.g., “Farmers’ fury at 89p supermarket milk: Blockade threat as price of four pints is slashed” (MailOnline for the UK’s Daily Mail, 14 October 2014). Notably, this article states: “Farmers are threatening protests after Iceland cut the price of a four-pint milk carton from £1 to 89p. The budget store is using milk as a loss leader – selling below cost price to lure in customers – with the result it is even undercutting discount chains Aldi and Lidl.”

⁵To our knowledge, European countries where concerns have led to a full or partial (e.g., sector specific) prohibition of loss leading include, next to Germany, Belgium, France, and Ireland. While U.S. federal law does not forbid below-cost selling or loss leading *per se*, several states have enacted below-cost selling laws. California goes even further and rules in its Business and Professions Code Section 17044 that “[i]t is unlawful for any person engaged in business within this State to sell or use any article or product as a ‘loss leader’ as defined in Section 17030 of this chapter.”

tition: competition on a selected number of items and competition through promotions.⁶ For this we employ Varian’s seminal model of sales (Varian 1980) and let consumers focus on a selected number of items (in our case, one) when comparing retailers. Consumers’ willingness to pay for all other products in their basket thus captures the extent of one-stop shopping, and competition on the selected (potentially loss-leading) items occurs through promotional discounts. Retailers can choose between high-quality and low-quality products. While we consider all possible cases, our focus is on that where the provision of the high-quality product is more efficient. Then, in our baseline analysis where consumers have standard preferences, always the high-quality product is supplied and the respective manufacturers indeed gain when an increase in one-stop shopping or more intense retail competition induce retailers to further discount their products.⁷ Results are however strikingly different when consumers are salient thinkers.

A guiding theme of our analysis is that of consumers’ limited or biased attention. This is one way to motivate consumers’ focus on comparing only the prices of the promoted (loss-leading) products. And, at the same time, such limited or biased attention may make consumers focus on particular attributes of individual products. Here, we follow Bordalo et al. (2013) and stipulate that consumers may overweigh quality or price when making comparisons, depending on whether the offer’s price or quality differs more compared to the respective market average. While thus, in principle, both quality and price of a particular offer can be salient, we show that it is only the salience of a low price, but not that of a high quality, that becomes relevant. When consumers are salient thinkers, manufacturers have reason to resist deep discounts in their product category, just as in our introductory example. When their product category is chosen as a loss leader to attract one-stop shoppers, the lower price level makes it more attractive for retailers to offer cheaper substitutes of lower quality. Even when such substitution is not yet observed, the threat reduces manufacturers’ profitability. Also policymakers’ and consumer groups’ concerns about quality are no longer unfounded when consumers are salient thinkers, as the same forces, that is one-stop shopping and retail competition, can indeed induce retailers to switch to lower-quality (possibly private-label) variants. As we explain below, however, a ban of below-cost pricing may backfire.

⁶Promotions (sales) are a defining feature of modern retailing competition and account for a large share of the observed price variation in retailing. Among the numerous empirical studies documenting the vast prevalence of retail promotions are Volpe (2013), Nakamura and Steinsson (2008), Berck et al. (2008), Hosken and Reiffen (2004), Pesendorfer (2002), and Villas-Boas (1995).

⁷Incidentally, as part of our analysis we characterize the mixed-strategy (“sales”) equilibrium for N retailers and elastic total demand.

With salience, but not without it, we can identify circumstances when, with positive probability, products of both high and low quality are on the market. Shoppers will then more often buy low-quality products compared to non-shoppers and, even though they search for the most attractive offer, shoppers may end up with an overall inferior choice compared to non-shoppers. When this outcome materializes, ex-post it would have been better for the respective consumer not to have compared offers: Having had at his disposition a larger choice set has then hurt the consumer.⁸ We also compare the respective pricing strategies for high-quality and low-quality offers and analyze how “deep” the respective promotion discount (in the mixed-strategy model of sales) becomes and how prices respond to changes in market structure, such as in the number of retailers or the presence of (more) shoppers. The pricing strategies of high-quality retailers and those of low-quality retailers differ markedly, as, for instance, low-quality retailers respond to increasing competition with deeper discounts, while this is not the case for high-quality retailers.

Various policy reports, such as OECD (2007), have stressed that loss leading for promoted products is an essential part of competition in the retailing industry. We do not dispute this, but our analysis also draws attention to potential detrimental effects, notably through a deterioration of product quality. While such claims are frequently heard in the policy debate, to our knowledge this paper is the first to motivate them formally. But it is not the first paper that shows potential drawbacks of loss leading. Chen and Rey (2012) show how, in the presence of asymmetric retailers, loss leading can be used to extract more of consumer rents, as pricing below cost for a competitive segment of “leading” products can be used to screen consumers according to their shopping cost. In our model, deep discounts on one product arise as (multi-product) retailers make a positive margin on other products that consumers purchase on their shopping trip and for which consumers do not compare prices.⁹ An interesting aspect of our analysis and an important difference to Chen and Rey (2012) is that detriments from loss leading arise particularly when there is intense competition between retailers.¹⁰

⁸It should be noted that in Varian’s model of sales shoppers do not incur any costs from searching and comparing products.

⁹This broadly follows Lal and Matutes (1994), albeit we do not endogenize advertising in our model. There is also a small literature that analyzes the possible incidence of below-cost pricing when consumers are equally informed about all prices, e.g., due to differences in demand elasticities (see Bliss (1988), Ambrus and Weinstein (2008)).

¹⁰This does not mean, however, that increased concentration in the food retailing sector, as witnessed in many European countries, makes our analysis less relevant, as this has not necessarily reduced the intensity of competition (but is possibly even the result of more intense competition). OECD (2013)

A policy that bans loss leading does however not always lead to higher quality and may even hasten a shift towards the provision of the less efficient, lower-quality product.¹¹ While it often makes a low-quality deviation by a retailer less attractive, as the lower bound on the respective discount may prevent price from becoming salient, below-cost pricing also limits the extent to which retailers can make a high-quality product attractive to consumers.¹² In our model, we obtain clear-cut conditions on when such a policy backfires. A prohibition of below-cost pricing can, positively or negatively, affect efficiency only when consumers are salient thinkers and only when the extent of one-stop shopping is sufficiently large.

We also provide additional insights on private labels.¹³ Our analysis suggests different rationales for the spread of (especially lower-quality) private labels, and in particular how consumers' limited attention could contribute to this.

Further Literature. In the fields of psychology, marketing, and behavioral economics, researchers have long documented consumers' tendency to overweigh particularly salient characteristics, tilting their choice towards items that a rational consumer may not select. Important papers in this vein include Huber et al. (1982), Thaler (1985), Simonson (1989), and Tversky and Simonson (1993).¹⁴ A formalization of salience, on which we

contains a broad discussion of various developments across different countries.

¹¹Note that we abstract from theories of harm that focus on other, potentially smaller and less competitive retailers. Absent an outright prohibition by law, a prosecution of loss leading along such lines would typically require to find predatory behavior. For a detailed treatment of predatory behavior see Bolton et al. (2000). Chen and Rey (2012) discuss in some detail relevant case material and findings from recent sector inquiries.

¹²While we focus on efficiency, by putting a limit on price competition a ban on below-cost selling can directly harm consumers. Allain and Chambolle (2005) and Rey and Vergé (2010) show how also intrabrand, next to interbrand competition, can thereby be dampened, as below-cost pricing regulations can allow manufacturers to impose price floors.

¹³The market share of private labels in European food retailing has risen significantly, with now more than 40% in some countries, such as the UK. Frequently, but not exclusively, private labels are positioned at the lower end of the quality and price range. See European Commission (2011) for an overview across Europe. Bergès-Sennou et al. (2004) provide an earlier guide to the academic literature, stressing different roles for the introduction and positioning of private labels. Especially in Europe, concerns about a potentially inefficient proliferation of private labels, combined with an exercise of buyer power, have recently resurfaced in sector inquiries and have led to various policy studies (e.g., European Commission 2014).

¹⁴For example, Huber et al. (1982) show that the choice among two alternatives can crucially be affected if a third, dominated alternative is added (the so-called "attraction effect"). Similarly, Simonson (1989) demonstrates that adding an alternative that is particularly good on one dimension, but bad on another (e.g., a product with very high quality, but also a very high price) may tilt consumers' choice among the initially available alternatives ("compromise effect"). Overall, the literature stresses the importance of the choice context for the weights consumers put on different product attributes. Wedell (1991) provides a psychological analysis of the associated "weight-shift" of attributes.

rely on, has recently been provided by Bordalo et al. (2013). As we discuss in detail below, what drives our results is the specification that salience is determined by how different a product’s attribute is relative to the market average. The motivation for our contribution is different from Bordalo et al. (2014), which considers a model of undifferentiated, duopolistic competition. We focus instead on changes in the degree of retail competition and one-stop shopping, as we are interested in their impacts on manufacturer profits and retailers’ product choice. Interestingly, in our model salience matters not only off-equilibrium (and thereby undermines manufacturers’ profits in their negotiations with retailers), but sometimes price, rather than quality, is salient on-equilibrium when both low-quality and high-quality products are offered and actually bought by different consumers in the market. This allows us to compare the outcome for consumers for whom, given that they compare offers, salience matters and for consumers for whom this is not the case as their choice set is limited. In these circumstances, we can also analyze the different price (promotion) strategies of retailers with high-quality and low-quality offers.

Related is also Armstrong and Chen (2009), which develops a model of product pricing in the spirit of Varian (1980), augmented by a discrete choice of product quality. In their model, all consumers observe all retailers’ prices, but some consumers are inattentive to product quality. Our model is different in at least two dimensions. First, we analyze a vertical supply chain, which allows to derive implications for manufacturer profits. Second, we endogenize consumers’ attention to product attributes, so that in our model salience is only relevant under particular market conditions, notably when retail competition is intense and the extent of one-stop shopping large.¹⁵

Our paper further contributes to the large literature on “models of sales”. We consider retailers’ decision which products to stock and at what conditions. Thereby, we extend Varian’s (1980) classic setup and account for vertical contracting, which has mostly been neglected.¹⁶ As in the influential work by Narasimhan (1988), we study the depth of

¹⁵Other relevant models in the developing field of behavioral industrial organization include Ellison and Ellison (2009), Spiegler and Eliaz (2011), Hefti (2012), and de Clippel et al. (2013). In all of these papers, consumers’ attention is limited to a subset of alternative choices. Although this “consideration set” can be manipulated by firms’ strategic decisions (such as advertising), the evaluation of options within a given consideration set is exogenous and does not depend on the attributes of available options. In Koszegi and Szeidl (2013), consumers’ utility weighting of product attributes increases in the range of the respective attributes across all choices in consumers’ consideration set. Their analysis focuses on (distortions in) consumers’ intertemporal choice. Consumers’ inattention to certain product characteristics may also be driven by rational consumers’ efficient allocation of attention. Such models of “rational inattention” include Matějka and McKay (2012), Persson (2013), and Gabaix (2014).

¹⁶One recent exception is Janssen and Shelegia (2015), who study pricing with homogeneous qualities and in costly sequential search.

promotions for both low- and high-quality products, as a function of market fundamentals, such as the intensity of competition and the degree of one-stop shopping, and with special consideration of salience.

Organization. The remainder of this article is organized as follows. Section 2 sets out the model for the baseline case where consumers have standard preferences. This baseline case is analyzed in Section 3. Section 4 contains the main analysis with salient thinkers. We are interested, in particular, in how the presence of salient thinkers interacts with one-stop shopping and competition. Section 5 looks in more detail at retailers’ product choice and pricing decision when both high-quality and low-quality products are in the market. We turn to policy implications in Section 6 and conclude in Section 7. All proofs are relegated to the appendix.

2 The Baseline Model

We model a market where consumers, albeit they purchase a basket of products, focus their attention both on a particular product as well as on particular (“salient”) attributes. In the baseline model, we first provide a benchmark by abstracting from the role of salient product attributes. This allows us subsequently to isolate the role that salience plays for firms’ pricing and product choice. Still, as we are interested in the impact of retailers’ pricing strategy on manufacturer profits and retailers’ choice of products, the model comprises various decisions. We now introduce these elements in several steps and then discuss the market game at the end of this section.

Products. Products are sold through $n = 1, \dots, N$ retailers, each of which stocks $i = 1, \dots, I$ products in different categories. It is convenient to also denote the respective sets by N and I . The price of product i at retailer n is p_n^i and we suppose that the respective quality can be described by a real-valued variable q_n^i . Our analysis focuses on the provision of product $i = 1$. At each retailer, this can be provided either at a low quality q_L by several, say for specificity $M > 1$, manufacturers or at a high quality $q_H > q_L$ by a single manufacturer. The respective constant product costs are denoted by $c_L \leq c_H$.¹⁷ The quality of all other products $i > 1$ is fixed and for convenience symmetric at all retailers: $q_n^i = q^i$ for $i > 1$ and $n \in N$. As our focus is on the choice of product $i = 1$, as well

¹⁷As we discuss below, we can also think of the low-cost, low-quality variant as a retailer’s own private label.

as on the respective wholesale and retail prices, we simplify the specification for all other products as follows. Each retailer can procure products in categories $i > 1$ at a wholesale price equal to the respective constant marginal costs $c_n^i = c^i$.¹⁸

Consumers. There is a mass one of consumers indexed by $r \in R$. Each consumer has a reservation value θ_r , which is distributed according to $H(\theta_r)$. Hence, θ_r denotes a consumer's utility from taking up an outside option, instead of turning to either one of the N retailers. Once in a shop, a consumer decides whether he wants to purchase some of the $i \in I$ independent products (in different categories). If he purchases the subset $I' \subseteq I$, the respective utility is $\sum_{i \in I'} (q_n^i - p_n^i)$.

Our analysis is simplified by the specification that a consumer visits at most one shop, so that he indeed buys any of the I products either at the same shop or not at all.¹⁹ Following Varian (1980), a fraction $(1 - \lambda)/N$ of consumers can only shop at their "local" retailer n , so that these consumers do not consider other retailers' offers. In contrast, the fraction λ of consumers, called "shoppers", is free to choose any retailer, but they make their decision dependent only on a comparison of the offer of product $i = 1$. It is this feature of the model that will possibly give rise to "loss leading".²⁰

Market Game. To conclude the specification of the model, the contracts for the supply of product $i = 1$ at each retailer n are determined as follows. We suppose that at each retailer n , the respective $M > 1$ providers of the low-quality variant and the respective single provider of the high-quality variant make simultaneous competing offers. In our model, it is without loss of generality that, among the class of all supply contracts, we restrict consideration to so-called two-part tariff contracts, prescribing a fixed fee T and a constant wholesale price w .²¹

Given the offered contracts, retailers then simultaneously undertake the following

¹⁸This would hold in particular if there was perfect competition by manufacturers of products $i > 1$. This specification is however not needed for our analysis. In particular, results would be unaffected if manufacturers in other categories would earn a strictly positive margin. The assumption of symmetry across retailers, which is in line with the literature, heavily simplifies the analysis.

¹⁹However, this outcome would also arise endogenously when we specified some arbitrary small costs $\varepsilon > 0$ of visiting (other) shops. Monopoly prices for products $i > 1$ would then be an implication of the well-known "Diamond paradox" in markets with search costs.

²⁰Next to consumer limited attention, this would arise when only the offer of product $i = 1$ was advertised, while consumers form rational expectations about all other prices. We do not endogenize advertising in our already rich analysis.

²¹Note that for convenience we suppress here all subscripts denoting the respective manufacturer and retailer.

choices: They choose which offer to accept (and thus which quality q_n^1 to supply) and they choose all prices p_n^i for $i \geq 1$. After this, all shoppers (of mass λ) choose which (if any) retailer $n \in N$ to visit and which products $I' \subseteq I$ to purchase in the shop, while all non-shoppers (of mass $1 - \lambda$) decide whether to visit their “local” retailer n and which products to purchase there.

3 Baseline Analysis

For the baseline model (that is, without salience) we can be somewhat short as part of the analysis follows well-established results, though all remaining gaps are filled in the proofs in the appendix. We thus first establish these preliminary results.

Supply Contracts. So as to avoid double-marginalization, each product quality is provided at a marginal wholesale price that is equal to the respective constant marginal costs of production (i.e., c_L or c_H).²² Further, given competition for the provision of the low-quality product, the respective offers will not contain a positive fixed part.²³ In contrast, the offer of the respective high-quality manufacturer of product $i = 1$ at retailer n may contain a fixed part $T_n \geq 0$. When this offer is accepted, T_n is thus the profit of the high-quality manufacturer. The determination of T_n will be part of our equilibrium characterization, and it is also an essential part of our comparison between the baseline model and the analysis with salience. Note further that, since in any equilibrium in which retailers accept high-quality manufacturers’ offers it must be the case that retailers are exactly indifferent between accepting or rejecting (as otherwise, T_n could be increased profitably), we can already pin down that each retailer’s equilibrium expected profit will equal its maximal profit when stocking q_L (which can be done without incurring any fixed fee) and pricing optimally, given the strategies of other retailers.

To save on notation we will subsequently use that, as we just noted, a retailer’s marginal cost reflects that of the respective manufacturer: Depending on the chosen quality q_n^1 , the retailer’s marginal cost of offering product $i = 1$ is thus either c_L or c_H .²⁴

²²This holds even when total demand is inelastic (i.e., if the distribution of the outside option θ_r is degenerate).

²³We focus on non-dominated strategies. Also, all our results hold qualitatively when we consider simultaneous pairwise negotiations of retailer n with the respective manufacturers, by applying the axiomatic Nash bargaining solution to each pairwise negotiation. The presently considered case then corresponds to the corner solution where manufacturers have all the bargaining power.

²⁴We use here that each retailer is supplied by a different high-quality manufacturer. Still, when manufacturers suffer from a problem of opportunism, our results extend as follows to the situation where

Demand. As shoppers only compare retailers on the basis of product $i = 1$, optimally retailers set $p_n^i = q_n^i$ for products $i > 1$.²⁵ Aggregating over all products $i > 1$, denote the respective (per-visit) retailer profits from these products by

$$v = \sum_{i \in I \setminus \{1\}} (q^i - c^i).$$

The variable v captures the extent of one-stop shopping, and v increases as more products I are added to the set of products (“basket”) that are purchased at each shopping trip. By restricting comparison to the case where $v < c_L$ we ensure that also below cost prices will remain strictly positive in what follows.²⁶

A consumer only (weakly) benefits from visiting retailer n when, with respect to product $i = 1$, it holds that $q_n - p_n \geq \theta_r$, where we have now abbreviated the notation to $q_n = q_n^1$ and $p_n = p_n^1$. Shoppers have also attractive options $n' \neq n$, provided again that $q_{n'} - p_{n'} \geq \theta_r$. For a given realization of qualities and prices, the demand D_n for retailer n is thus composed as follows. Denote the consumer surplus at retailer n by $s_n = q_n - p_n$, the respective maximum across all retailers by $s^{\max} = \max_{n' \in N} (q_{n'} - p_{n'})$, and by N^{\max} the number of retailers for which $s_n = s^{\max}$. Then demand is given by

$$D_n = H(s_n) \left(\frac{1 - \lambda}{N} \right) \text{ if } s_n < s^{\max} \quad (1)$$

and by

$$D_n = H(s_n) \left[\frac{1 - \lambda}{N} + \frac{\lambda}{N^{\max}} \right] \text{ if } s_n = s^{\max}, \quad (2)$$

where we assume that shoppers randomize with equal probability when indifferent, albeit this specific tie-breaking rule is inconsequential for the subsequent analysis.

Monopoly Profits. We next introduce some additional notation. To do so, denote for now a retailer’s cost of providing product $i = 1$ by c and the respective quality of the

there is a single high-quality manufacturer. Results extend fully when we consider, as in the preceding footnote, pairwise negotiations and apply there the Nash bargaining solution. As this essentially presumes that the manufacturer is represented by different agents in each bilateral negotiation, this immediately implies that contracts maximize the bilateral surplus (so that the marginal wholesale price equals marginal cost of production). As is well known, such an outcome also arises when non-observable supply contracts are simultaneously offered by a single manufacturer to all retailers and when these hold passive beliefs – provided that such an equilibrium indeed exists. As this is not the focus of this paper, we do not further discuss these alternative specifications.

²⁵Note that at this stage the respective consumer has already foregone his outside option of value θ_r .

²⁶Negative (or even very low but positive) prices may also attract commercial buyers interested in reselling, which is why retailers frequently impose volume restrictions on their promotions. For our analysis of policy intervention below we also solve for the case with a binding lower boundary on the price of product $i = 1$.

product by q . When p is the product's price, $p - c + v$ is the respective margin from a consumer visiting the retailer, given that the consumer then also purchases all other products $i > 1$ at the shop. Further, when a consumer decides only between whether to visit the retailer or to take up his outside option, he will visit the retailer when $\theta_r < q - p$ and thus with probability $H(q - p)$. A retailer's expected profits with any of the non-shopping local consumers is thus

$$\Pi(q, c; p) = (p - c + v)H(q - p).$$

When H is non-degenerate, we suppose for convenience that h/H is weakly increasing in its argument, so that the respective (“monopoly”) price

$$p^m(q, c) = \arg \max_p \Pi(q, c; p)$$

is uniquely determined and results in (“monopoly”) profits of

$$\Pi^m(q, c) = \Pi(q, c; p^m(q, c)).$$

Clearly, when only non-shoppers are targeted, then the maximum profit is $\frac{1-\lambda}{N}\Pi^m(q, c)$. When H is degenerate, it is convenient to set $\theta_r = \theta = 0$, so that then $p^m(q, c) = q$ and $\Pi^m(q, c) = q - c + v$.

3.1 Illustration of Pricing Equilibrium

To illustrate the working of retail competition in our model, we first provide a short characterization of an equilibrium where all retailers stock the same product with quality $q_n = q$ and choose the same (mixed-strategy) pricing. Below we will derive conditions on when an equilibrium with $q_n = q$ for either low or high quality exists. Also, we comment subsequently on the existence of non-symmetric pricing equilibria. Our results hold irrespective of which pricing equilibrium is chosen. The following characterization in Lemma 1 follows from standard arguments.

Lemma 1 *Setting $q_n = q$ and $c_n = c$ for all retailers, in the unique symmetric pricing equilibrium all retailers randomize over prices $p \in [\underline{p}, p^m(q, c)]$ according to the atomless distribution function*

$$G(p) = 1 - \sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left[\frac{\Pi^m(q, c)}{(p - c + v)H(q - p)} - 1 \right]}, \quad (3)$$

where the lower boundary \underline{p} of the support is implicitly defined by

$$\left(\frac{1-\lambda}{N} + \lambda\right) (\underline{p} - c + v) H(q - \underline{p}) = \frac{1-\lambda}{N} \Pi^m(q, c). \quad (4)$$

A retailer choosing $p_n = \underline{p}$ can be certain to sell to all shoppers, provided their outside option is sufficiently low. The determination of \underline{p} in (4) ensures that the respective expected profits are the same as when selling instead only to locked-in consumers at the price $p^m(q, c)$, thus realizing the profit $\frac{1-\lambda}{N} \Pi^m(q, c)$. The choice of the distribution $G(p)$ in (3) ensures that the expected demand for each retailer is such that the retailer is indeed indifferent between all prices $p \in [\underline{p}, p^m(q, c)]$.

It is well known (cf. Baye et al. 1992) that with $N > 2$ there exists a continuum of asymmetric pricing equilibria, even when retailers are symmetric. For our results it is sufficient that all lead to the same profits (cf. Proposition 1 and its proof).

3.2 Equilibrium Product Choice and Manufacturer Profits

Denote $\Delta_q = q_H - q_L$ and $\Delta_c = c_H - c_L$. We say that the high-quality product is superior when $\Delta_q > \Delta_c$ and that the low-quality product is superior when the converse holds strictly. That is, the superior product delivers the highest level of social surplus.

Proposition 1 *In the baseline model (without salience), when one type of product $i = 1$ is strictly superior to the other, then this is supplied by all retailers in equilibrium, i.e., the high-quality product when $\Delta_q > \Delta_c$ and the low-quality product when $\Delta_q < \Delta_c$. Further, when $\Delta_q > \Delta_c$ the high-quality manufacturers make total profits of*

$$\Pi_M = (1 - \lambda) [\Pi^m(q_H, c_H) - \Pi^m(q_L, c_L)]. \quad (5)$$

Proof. See Appendix.

When $\Delta_q > \Delta_c$ holds, so that the high-quality product is superior, the high-quality manufacturers' total profits are thus given by (5), which is the difference between (N times) profits if the high-quality product is supplied at a retailer and the profits that a retailer would make when choosing instead the low-quality product. It is interesting to note, also in light of the subsequent differences in the case with salient thinkers, that a retailer deviating to a low-quality product would optimally choose the ‘‘monopoly price’’ $p^m(q = q_L, c = c_L)$, thus selling only to non-shoppers. This is why the retailer’s (deviating) profit with the low-quality product is $\Pi^m(q_L, c_L)$ times the number of locked-in consumers.

We next derive from Proposition 1 comparative statics results, which we will subsequently compare to the case with salience. For ease of reference, we first state separately the following immediate implication, which will be strikingly different when consumers are salient thinkers.

Corollary 1 *In the baseline model, retailers' product choice depends only on the comparison of the benefits of higher quality, Δ_q , with the respective costs, Δ_c , and thus not on any other parameter of the model, in particular not on the extent of one-stop shopping, v .*

When the high-quality product is superior, as $\Delta_q > \Delta_c$, high-quality manufacturers' profits are strictly positive and given by (5). We can derive the following comparative results:

Corollary 2 *When the high-quality product is chosen as it is superior from $\Delta_q > \Delta_c$, the high-quality manufacturers' total profit Π_M is weakly increasing in the extent of one-stop shopping (captured by v): Π_M is independent of the extent of one-stop shopping v when total demand is inelastic but strictly increasing in v when total demand is elastic.*

Proof. See Appendix.

The intuition for the comparative analysis in v is straightforward. Higher additional profits v per customer induce more aggressive pricing for product $i = 1$ by all retailers, which expands demand when total demand is elastic. The high-quality manufacturer can then extract the incremental benefits from its superior product for a larger quantity. From Corollary 2 it thus follows that high-quality manufacturers benefit when the extent of one-stop shopping increases, and, therefore, retailers offer the manufacturer's product at a greater discount.²⁷

One motivation for our analysis, as given in the introduction, is the claim by manufacturers that their profits decline when retail pricing in their category becomes more aggressive due to intensified retail competition or one-stop shopping. Corollaries 1 and 2 refute this claim and also the concern that this negatively affects the quality of supplied products, at least in the baseline model. The outcome will be strikingly different in case consumers are salient thinkers.

²⁷While a comparative analysis of prices is not reported in Corollary 2, it is straightforward to derive this, e.g., in terms of the lower boundary \underline{p} , which applies uniformly for all (symmetric or asymmetric) pricing equilibria.

4 The Model when Consumers are Salient Thinkers

As discussed in the introduction, the main theme of our analysis is that consumers' attention is limited and selective. We now extend this insight to consumers' comparison of the different attributes, price and quality, which clearly is only relevant for the offering of product $i = 1$ and for shoppers. Price or quality become salient when the respective attribute differs relatively more compared to the average of all products in the market that a consumer may consider buying. This is now formalized by relying on the approach in Bordalo et al. (2013).

A consumer r who shops around compares all alternatives $n \in N$ that are not strictly dominated, including by his outside option. Denote this set by N_r , which may be empty, in which case the consumer chooses his outside option.²⁸ Clearly, the interesting case, where salience will play a role, is that where N_r is neither empty nor a singleton. For this case, define $P_r = \frac{1}{|N_r|} \sum_{n \in N_r} p_n$ and $Q_r = \frac{1}{|N_r|} \sum_{n \in N_r} q_n$. We ask when, for the offer of retailer n and the respective consumer r , price is salient and when quality is salient. Suppose that the retailer's price is below the average price, $p_n < P_r$, and also quality is below the average quality, $q_n < Q_r$. Then, for consumer r , price is salient when

$$\frac{p_n}{P_r} < \frac{q_n}{Q_r} \quad (6)$$

and quality is salient when the converse holds strictly.²⁹ That is, the lower price, but not the lower quality, is salient when price is relatively lower (that is, in percentage terms), compared to the average of considered offers, than quality, again compared to the average. Suppose next that the retailer's price and quality are both higher than the respective average, $p_n > P_r$ and $q_n > Q_r$. Then price is salient when now $\frac{p_n}{P_r} > \frac{q_n}{Q_r}$, while when the converse holds strictly, quality is salient.

When assessing the offer of a given retailer, a consumer discounts a non-salient attribute by some factor $\delta \in [0, 1]$, which measures the importance of salience. Using also that $p_n^i = q_n^i$ for all $i > 1$ and all $n \in N_r$, the perceived surplus of retailer n 's offer to consumer

²⁸Precisely, using already that $p_n^i = q_n^i$ for all $i > 1$, N_r consists of all $n \in N$ such that $q_n - p_n \geq \theta_r$ and that there does not exist some $n' \in N$ with both $q_{n'} \geq q_n$ and $p_{n'} \leq p_n$ (one strictly). See Bordalo et al. (2013) for a discussion and motivation of the removal of strictly dominated alternatives ("editing").

²⁹Bordalo et al. (2013) consider more general salience functions that satisfy what they call "diminishing sensitivity". They motivate this on the basis of results from cognitive psychology, whereby stimuli are perceived with diminishing sensitivity. As we point out below, this feature is key for our results to hold. They show further how the simple expression in terms of ratios arises when one imposes, in addition, an assumption of homogeneity of degree zero.

$r, s_{n,r}$, is given as follows:³⁰

$$s_{n,r} = \begin{cases} \delta q_n - p_n & \text{if } \left\{ p_n < P_r \wedge \frac{p_n}{P_r} < \frac{q_n}{Q_r} \right\} \vee \left\{ p_n > P_r \wedge \frac{p_n}{P_r} > \frac{q_n}{Q_r} \right\} \\ q_n - \delta p_n & \text{if } \left\{ p_n < P_r \wedge \frac{p_n}{P_r} > \frac{q_n}{Q_r} \right\} \vee \left\{ p_n > P_r \wedge \frac{p_n}{P_r} < \frac{q_n}{Q_r} \right\} \\ q_n - p_n & \text{if } \frac{p_n}{q_n} = \frac{P_r}{Q_r} \end{cases} . \quad (7)$$

Take the first line in (7). If the respective conditions hold, then price is salient. In this case, as already observed, quality is discounted by the factor δ . The second line in (7) captures the case where quality is salient, so that now price is discounted. The third line captures the case where no attribute is discounted. Applying now (7) to all offers, the decision of consumer r is as follows. The consumer takes his outside option when N_r is empty. When N_r is instead not empty, so that $\max_{n' \in N_r} s_{n',r} \geq \theta_r$, he randomizes with equal probability over all choices $n \in N_r$ that satisfy $s_{n,r} = \max_{n' \in N_r} s_{n',r}$. Again, as noted previously for the case without salience, the symmetric tie-breaking rule is without consequences for our results.

The subsequent analysis in this section focuses on the case where, in equilibrium, all retailers choose the same product quality. Recall that only this case applied without salience.³¹ Still, salience will crucially affect both the conditions for when the high-quality equilibrium exists and the respective profits of high-quality manufacturers, as both depend on retailers' deviation profits. Subsequently, we will consider the case where not all retailers stock the same product with probability one. Then, even though the high-quality product is superior, with positive probability retailers will stock the low-quality product, so that salience will matter on equilibrium and shoppers will buy different qualities as non-shoppers. Note also that our notion of efficiency and (consumer) welfare in what follows reflect consumers' true utility from consumption. Throughout this section we restrict consideration to the standard case where total demand is inelastic (as $H(\cdot) = 1$ is degenerate).

4.1 High-Quality Equilibrium

In this section, we ask when there exists an equilibrium where only high-quality products are offered and we analyze the respective manufacturer profits. As noted above, with $q_n = q_H$ for all $n \in N$, the issue of salience does not arise on equilibrium. Still, salience affects the profitability of a retailer's strategy to deviate and stock the low-quality product. This

³⁰Bordalo et al. (2013) make use of a scaling factor, $\frac{2}{1+\delta}$, which we drop, albeit this is without relevance for our analysis.

³¹Strictly speaking, this holds generically, that is when $\Delta_q \neq \Delta_c$.

affects both manufacturer profits and the condition for when the high-quality equilibrium exists.

Existence. Consider thus a deviating retailer n choosing $q_n = q_L$ and some price p_n . Given that $q_{n'} = q_H$ holds for all other retailers n' , shoppers who decide not to take up their outside option will decide between the deviating offer of retailer n and that of another retailer m with the lowest price $p_m := \min_{n' \neq n} p_{n'}$. The respective comparison of some consumer r between $(q_n = q_L, p_n)$ and $(q_m = q_H, p_m)$ depends now on whether price or quality is salient.³² To make a lower price $p_n < p_m$ salient, it must not exceed the fraction $\frac{q_L}{q_H}$ of p_m (cf. condition (6)). That is, to make price salient, the price decrease must be larger in percentage terms compared to the difference in qualities: With $\Delta_p = p_m - p_n$, it must hold that $\frac{\Delta_p}{p_m} > \frac{\Delta_q}{q_H}$.³³ The key observation is now that, so as to achieve this, retailer n needs to undercut p_m by less in absolute terms (Δ_p) when p_m is lower. This is precisely how competition and the extent of one-stop shopping affect the profitability of a low-quality deviation under salience: When the extent of one stop shopping is larger, it is more likely that p_m is lower, and the same applies when competition is more intense. By the preceding argument, this allows the deviating retailer n , who offers a low-quality product, to make price salient with a smaller absolute discount (Δ_p) than what would be necessary when p_m was larger.

When a deviating low-quality retailer achieves price salience with his low price, this attenuates the quality difference (precisely, by the factor δ). Then, delivering a perceived higher net surplus to consumers is more profitable with the low-quality product than with the high-quality product when the respective cost savings Δ_c exceed the loss in perceived value, $\delta\Delta_q$. This observation implies in turn that $\Delta_c \leq \Delta_q$ is now no longer sufficient to establish existence of the high-quality equilibrium.

³²When p_m was chosen by more than one firm, N_r would, for given r , contain more than two elements, which would affect the formation of the averages P_r and Q_r . This will however be a zero-probability event. To streamline the exposition in the main text, we restrict the discussion to the case where N_r contains at most two elements. However, note as well that when we discuss restrictions on below-cost pricing below, retailers will choose the same price with strictly positive probability. The analysis then still proves to be tractable.

³³For $N_r = \{m, n\}$ this is obtained by transforming for both m and n the respective conditions $\frac{p_n}{P} < \frac{q_n}{Q}$ and $\frac{p_m}{P} > \frac{q_m}{Q}$ (where for the averages we have dropped the subscript r for the respective consumer). As $P = (p_n + p_m)/2$ and $Q = (q_n + q_m)/2$, the two conditions $\frac{p_n}{P} < \frac{q_n}{Q}$ and $\frac{p_m}{P} > \frac{q_m}{Q}$ coincide. Both become, after substitution, $\frac{p_n}{p_n + p_m} < \frac{q_n}{q_n + q_m}$. This transforms to $\frac{p_n}{p_m} < \frac{q_n}{q_m}$. Using $\Delta_q = q_H - q_L$, $q_m = q_H$, and $q_n = q_L$, while defining $\Delta_p = p_m - p_n$, we finally have the (percentage) requirement that $\frac{\Delta_p}{p_m} > \frac{\Delta_q}{q_H}$.

Proposition 2 *When consumers discount non-salient attributes, an equilibrium where all retailers offer the high-quality product exists if and only if*

$$\Delta_c \leq \max\{\delta\Delta_q, \frac{\Delta_q}{q_H}\underline{p}\}, \quad (8)$$

where

$$\underline{p} = \frac{\lambda N(c_H - v) + (1 - \lambda)q_H}{1 - \lambda + \lambda N}. \quad (9)$$

As a consequence, even when the high-quality product is superior as $\Delta_q > \Delta_c$, there is no longer an equilibrium where this is offered by all retailers if

- i) consumers sufficiently discount the non-salient attribute (low δ) and*
- ii) (high-quality) equilibrium prices (precisely, the low boundary of the support \underline{p}) are low as either competition is intense (high N or λ) or the extent of one-stop shopping is large (large v).*

Proof. See Appendix.

The two parts in condition (8) formalize the preceding discussion. First, when a deviating retailer discounts the price sufficiently so as to make price salient, $\Delta_c \leq \Delta_q$ is no longer sufficient to ensure that a high-quality equilibrium exists. Then, only the stronger condition $\Delta_c \leq \delta\Delta_q$ is sufficient, as this ensures that the difference between perceived valuation and cost is still larger with the high-quality product, even when quality differences are discounted. Second, whether it is indeed attractive for a deviating retailer to lower the price sufficiently so as to achieve price salience depends on the prevailing price level. We find that to establish this, it is sufficient to consider the lower boundary of the support of high-quality prices, \underline{p} .³⁴ As stated in assertion ii) of Proposition 2, it is through this channel that the parameters capturing retail competition and one-stop shopping (N , λ , and v) determine whether there still exists an equilibrium where only high-quality products are offered.

Manufacturer Profits. Even when the high-quality equilibrium exists, as the conditions of Proposition 2 hold, salience affects a high-quality manufacturer's profits. To pin down precisely the importance of salience, we now need to select a particular pricing strategy that is played in a high-quality equilibrium. We choose the symmetric equilibrium, as described by $G(p)$ in (3).

³⁴Note that the same lower boundary applies irrespective of whether for $N > 2$ we choose a symmetric or an asymmetric pricing equilibrium when $q_n = q_H$.

Proposition 3 *Suppose that condition (8) holds, so that an equilibrium exists where all retailers choose the high-quality product, and suppose retailers choose symmetric pricing strategies $G(p)$ (as given in (3) with $q = q_H$, $c = c_H$, and H degenerate). Then a high-quality manufacturer's profit is strictly lower when consumers are salient thinkers ($\delta < 1$) if and only if the extent of one-stop shopping is sufficiently large with*

$$v > \frac{q_H c_L - q_L c_H}{\Delta_q}, \quad (10)$$

while otherwise the profit does not change. If condition (10) holds, profits of high-quality manufacturers are

- i) weakly lower when non-salient attributes are discounted more (low δ),*
- ii) strictly lower when the extent of one-stop shopping is larger (higher v), and*
- iii) lower when retail competition is more intense (strictly so when λ increases and only weakly so when N increases).*

Proof. See Appendix.

Propositions 2 and 3 speak right to the concerns of manufacturers, consumer interest groups, and policymakers, as discussed in the introduction. Recall that without salience, Corollaries 1 and 2 together refuted the respective claims. In the baseline model, neither retail competition nor the extent of one-stop shopping affected the provision of high-quality products, while manufacturers were surely not negatively affected by the larger discounts on their products. When consumers are salient thinkers, however, one-stop shopping makes it more likely that the high-quality equilibrium no longer exists, even though the provision of high-quality products is strictly efficient (Proposition 2). And even when this equilibrium exists, an increase in the extent of one-stop shopping as well as more intense retail competition reduce the profits of high-quality manufacturers (Proposition 3). In our model, this is so as more intense retail competition and a greater extent of one-stop shopping make a deviation to a low-price, low-quality offer more profitable, and even more so when consumers discount non-salient attributes more strongly (lower δ).

4.2 Low-Quality Equilibrium

The presence of salient thinkers does not affect the conditions for when a low-quality equilibrium exists.

Proposition 4 *Irrespective of whether consumers discount non-salient attributes or not, the condition $\Delta_q \leq \Delta_c$ is both necessary and sufficient so that an equilibrium exists where all retailers stock the low-quality product.*

Proof. See Appendix.

While Proposition 2 provides conditions for when salience leads to non-existence of the high-quality equilibrium and thus to an underprovision of the high-quality product, from Proposition 4 such an underprovision can not occur for the low-quality product. We now offer some intuition for this discrepancy in results.

The following discussion is not meant to provide a formal proof, as it shortcuts parts of the analysis. We consider for both the high-quality and the low-quality (candidate) equilibrium a potential deviation and compare the role of salience. Consider first the case of a high-quality equilibrium (as for Proposition 2). Suppose that instead of choosing some p when $q_n = q_H$, retailer n deviates to p_{dev} and $q_n = q_L$. When the new offer should make price salient, compared to the offer p and q_H by other retailers, the price discount $\Delta_p = p - p_{dev}$ must satisfy³⁵

$$\Delta_p > \frac{\Delta_q}{q_H} p, \quad (11)$$

while to be more profitable, it must hold that

$$\Delta_p < \Delta_c. \quad (12)$$

Our key observation was that the constraint to ensure price-salience for the deviation (11) is compatible with the profitability requirement (12) when p is sufficiently small: When p is low, already a small discount Δ_p generates a large relative discount. We compare this now with the case of a low-quality equilibrium, where a retailer is supposed to choose some p when $q_n = q_L$ and now deviates to p_{dev} and $q_n = q_H$. Now the retailer wants to prevent the price from becoming salient, so that the price increase must satisfy

$$\Delta_p < \frac{\Delta_q}{q_L} p, \quad (13)$$

while the price increase should cover the additional costs:

$$\Delta_p > \Delta_c. \quad (14)$$

³⁵This argument is admittedly incomplete as other retailers randomize over a set of prices. For the derivation of (11) see footnote 33.

As $p \leq q_L$ holds surely, as otherwise nobody would purchase, this sufficiently constrains the price increase Δ_p that is compatible with condition (13), so that condition (14) can not hold.³⁶ That is, for a retailer deviating to a high-quality offer when $\Delta_q < \Delta_c$, it is now not possible to raise the price sufficiently so as to cover the additional costs of the high-quality product (condition (14)), while at the same time ensuring that quality and not price becomes salient (condition (13)).

5 Different Offers On Equilibrium

Recall that without salience quality was either high or low at *all* retailers. While with salient thinkers the condition for the existence of the low-quality equilibrium was unchanged, compared to the case without salience, the condition for the existence of the high-quality equilibrium turned out to be stricter with salience. When the high-quality equilibrium fails to exist with salience, but exists without salience, with positive probability some retailers must choose the inferior low-quality product, even though $\Delta_q > \Delta_c$. Salience then matters on equilibrium, as with positive probability products of different quality are offered.

In this section, we characterize a symmetric equilibrium, so that each retailer chooses the high-quality product with probability $\alpha \in (0, 1)$ and consequently the low-quality product with probability $1 - \alpha$. When a retailer chooses the high-quality product, its pricing strategy is characterized by some distribution function G_H , while with the low-quality choice the pricing strategy is given by some distribution G_L . Though we are able to characterize the equilibrium explicitly, it is instructive to first focus in Section 5.1 on the equilibrium product choice, α^* , and to discuss in Section 5.2 the pricing strategy.

5.1 Provision of Products

The subsequent Proposition 5 covers the so far excluded case where $\Delta_q > \Delta_c$ but where also condition (8) does not hold.

Proposition 5 *Suppose*

$$\max\{\delta\Delta_q, \frac{\Delta_q}{q_H}p\} < \Delta_c < \Delta_q, \quad (15)$$

³⁶To see this, substitute $p_{dev} - p = \frac{\Delta_q}{q_L}p$ into (14) to obtain the requirement $p > \frac{\Delta_c}{\Delta_q}q_L$, which can not hold from $p \leq q_L$ and $\frac{\Delta_c}{\Delta_q} > 1$.

so that there does not exist an equilibrium where all retailers choose either high or low quality. Then there exists a symmetric equilibrium where each retailer chooses the high-quality product with probability

$$\alpha^* = \sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left[\frac{\Delta_q - \Delta_c}{\frac{q_L}{q_H}(c_H - v) - (c_L - v)} \right]} \quad (16)$$

and the low-quality product with probability $1 - \alpha^*$. The corresponding pricing strategies G_L and G_H are such that price is always salient if both qualities coexist, and all shoppers surely choose the low-quality product unless all retailers stock the high-quality product. High-quality manufacturers make zero profits.

Proof. See Appendix.

The characterization in Proposition 5 entails the following implications for the qualities that consumers ultimately purchase. A non-shopper chooses the more efficient low-quality product with probability α^* . The respective probability is strictly lower for a shopper: It is $(\alpha^*)^N$, as, whenever any of the retailers offers the low-quality product (at a much discounted price; see below), all shoppers will turn to this retailer. Inspection of expression (16) yields next the following striking difference between the behavior of the respective probabilities, that is of α^* for non-shoppers and of $(\alpha^*)^N$ for shoppers, as the market becomes increasingly fragmented with $N \rightarrow \infty$. Then, while each retailer will almost surely offer the high-quality product, as $\alpha^* \rightarrow 1$, so that any non-shopper almost surely obtains the high quality, we can show that $(\alpha^*)^N \rightarrow 0$, so that any shopper almost surely obtains the low quality. We further comment on the different purchasing behavior of shoppers and non-shoppers below.

When we consider consumers' (true) consumption utility, note from the preceding observations that total welfare from production and consumption of product $i = 1$ is

$$W^* = (q_L - c_L) + \lambda(\alpha^*)^N(\Delta_q - \Delta_c) + (1 - \lambda)\alpha^*(\Delta_q - \Delta_c).$$

This clearly does not depend directly on the extent of one-stop shopping, but only through the effect that this has on α^* . Also, the fraction of shoppers, λ , would not affect welfare when shoppers always consumed the same product as non-shoppers, which is however no longer the case with salience.

Corollary 3 *Both the likelihood with which for $\Delta_q > \Delta_c$ the high-quality product is chosen and total welfare W^* are lower when*

- i) the extent of one-stop shopping v is higher and when*
- ii) the fraction of shoppers λ is higher.*

Proof. See Appendix.

Recall that both when one-stop shopping becomes more extensive and when the share of the market that is competitive increases, it becomes less likely that condition (8) for the existence of a high-quality equilibrium is satisfied. Corollary 3 extends this result: When the high-quality product is superior, with salience but not without it, the likelihood that indeed the high-quality product is offered and purchased always (weakly) decreases with one-stop shopping and with more shoppers in the market.

Our model allows for consumers with different choice sets: shoppers and non-shoppers. Salience is only relevant for shoppers as non-shoppers choose only between purchasing at a particular (“local”) retailer or not purchasing at all. Further, in the presently analyzed case different retailers may end up with different price and quality offers, even though the high-quality product is superior. We already noted above that, as a consequence, shoppers will be more likely to buy the discounted low-quality product, and for $N \rightarrow \infty$ the difference becomes particularly transparent as shoppers then end up buying the low-quality product with probability one and non-shoppers with probability zero. When analyzing retailers’ pricing in what follows, we will show that shoppers’ access to more offers can lead to lower true utility.

5.2 Salient Pricing and Depth of Discounts

From the characterization of the equilibrium in Proposition 5 we know that when products of different quality are offered, then always price will be salient. Hence, in equilibrium it happens (with positive probability) that consumers place too much weight on a low price. But there is never too much weight placed on a high quality. This finding mirrors the lopsided results with respect to quality and price throughout this paper. The presence of salient thinkers makes it more attractive for retailers to deviate away from a high-quality equilibrium through making price salient, while the opposite is not true. A strategy that would make quality salient does not become profitable.

The further characterization of retailers’ pricing strategy in the case where they offer different qualities now follows broadly the previous intuition. The presence of shoppers, for whom retailers are homogeneous, leads again to mixed pricing. Further, when a retailer offers the less efficient low-quality product, then it will optimally do so only when its

pricing strategy ensures that price is indeed salient in case both high-quality and low-quality products are offered. This implies that there must be a sufficiently large gap between the two supports, which we denote by $[\underline{p}_L, \bar{p}_L]$ and $[\underline{p}_H, \bar{p}_H]$ for the low-quality and the high-quality pricing strategies. We next state explicitly the respective distributions (as obtained in the proof of Proposition 5) and derive additional comparative results.

Corollary 4 *Suppose the conditions of Proposition 5 apply. Then a retailer with a high-quality product chooses the price distribution function*

$$G_H(p) := 1 - \frac{1}{\alpha^*} \sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_H + v} - 1 \right)}$$

with support

$$[\underline{p}_H, \bar{p}_H] = \left[\frac{\Delta_c}{\Delta_q} q_H, q_H \right]$$

and a retailer with a low-quality product chooses

$$G_L(p) := 1 - \frac{\sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left[\frac{q_H - c_H + v}{p - c_L + v} - 1 \right]} - \alpha^*}{1 - \alpha^*}$$

with support

$$[\underline{p}_L, \bar{p}_L] = \left[c_L - v + (q_H - c_H + v) \frac{1 - \lambda}{1 - \lambda + \lambda N}, \frac{\Delta_c}{\Delta_q} q_L \right].$$

Proof. See Appendix.

We consider the support of the two pricing functions. When both the high-quality and low-quality products are offered, we know that shoppers always choose the low quality, i.e., even when the lowest price at which this is offered equals \bar{p}_L and when they could buy the high quality at price \underline{p}_H (i.e., at the lowest price at which it may be offered). Substituting the respective values, a shopper's true utility from the low-quality product then equals $q_L \frac{\Delta_q - \Delta_c}{\Delta_q}$, while the true utility from the offer of the high-quality product would be $q_H \frac{\Delta_q - \Delta_c}{\Delta_q}$, so that the shopper loses in this case exactly the utility $\Delta_q - \Delta_c$. The larger choice set of a shopper can thus induce a shopper to make the wrong decision and lead to an outcome that is notably worse than what a non-shopper realizes, even though the latter's choice set is strictly smaller. We thus have the following result:

Corollary 5 *When both high-quality and low-quality products are offered, with positive probability a shopper makes the wrong choice and is thus, in particular, worse off than at least some non-shoppers.*

We next focus on retailers' pricing strategies, for which we can obtain the following results:

Corollary 6 *From the characterization of retailers' pricing strategy in Corollary 4, the following limit results follow:*

i) As $\lambda \rightarrow 1$, so that only shoppers remain in the market, G_H is not affected whereas G_L converges (in distribution) to the strategy of choosing $p = c_L - v$ with probability one.

ii) As $N \rightarrow \infty$, so that each retailer's locked-in fraction of consumers goes to zero, G_H converges (in distribution) to the strategy of choosing $p = q_H$ with probability one and G_L converges (in distribution) to that of choosing $p = c_L - v$ with probability one.

Proof. See Appendix.

Irrespective of whether we consider the limit as all consumers turn into shoppers, $\lambda \rightarrow 1$, or that where for each retailer the own monopolistic market becomes arbitrarily small compared to the competitive market, $N \rightarrow \infty$, all retailers offering a low-quality product will offer (in the limit) the competitive price $p = c_L - v$.³⁷ The pricing strategy of high-quality retailers is markedly different. In both cases, where $\lambda \rightarrow 1$ or $N \rightarrow \infty$, the price remains bounded away from the respective competitive price, $c_H - v$, as the price distribution is not affected by changes in λ and converges to $p = q_H$ as $N \rightarrow \infty$. Thus the margins of retailers offering the low-quality product converge to zero, while those offering a high-quality product remain bounded away from zero.³⁸

In the marketing literature, notably in Narasimhan's (1988) extension of Varian's model of sales, the respective differences in the supports, $\bar{p}_L - \underline{p}_L$ and $\bar{p}_H - \underline{p}_H$, are frequently interpreted as the "depth" of promotion discounts. In this sense, the promotion discount for a low-quality product becomes increasingly deep when competition intensifies, but this does not hold for the high-quality product. There, the respective promotion discount, i.e., the difference $\bar{p}_H - \underline{p}_H$, remains constant. As our focus in this paper is on manufacturers' profits and the choice of products, we leave a further derivation of positive implications for pricing and promotions to further research. We turn instead to normative (policy) implications.

³⁷Note that when $N \rightarrow \infty$, deviating to serve only the own locked-in segment of consumers does not yield higher profits as the respective mass of consumers, $(1 - \lambda)/N$, converges to zero. Interestingly, the limit for $N \rightarrow \infty$ is just the opposite when retailers offer symmetric qualities either in the high-quality or low-quality equilibrium, as then the respective strategies converge towards charging the monopoly price (i.e., either q_H or q_L ; cf. equation (3)).

³⁸Still, both choices yield the same profit level as α^* adjusts accordingly (cf. Proposition 5). In the limit, all shoppers buy the low-quality product.

6 Policy Implications

In light of the discussion in the introduction, in this section we focus on a prohibition of below-cost pricing. As discussed above, the extant literature has mainly discussed the negative implications that this has on consumer welfare through increasing prices, though Chen and Rey (2012) have shown that it may protect consumer rent as it restricts retailers' scope for price discrimination (according to shopping costs). We noted that a proclaimed main objective of such a policy is to prevent an inefficient provision of low-quality products (e.g., as an outcome of a "race to the bottom") and another objective is the protection of manufacturers.³⁹ We analyze under which circumstances these goals can be achieved by a prohibition of below-cost pricing. Our definition of such an intervention is that the retail price must not fall below the respective cost of production, $p_n \leq c_n$.⁴⁰ Throughout the subsequent analysis we focus on the case with $\Delta_q > \Delta_c$ and ask, in particular, when a prohibition of below-cost pricing makes it more likely that any given retailer indeed offers the more efficient high-quality product – and when such a prohibition backfires and decreases the respective likelihood.

6.1 Prohibition of Below-Cost Pricing in the Baseline Model

Recall that in the baseline model we abstract from salience. We focus on the choice of products and on manufacturer profits, for which we have the following results:

Proposition 6 *In the baseline model (without salient thinkers), prohibiting below-cost pricing has no impact on the provision of products ($q_n = q_H$ for all n , as $\Delta_q > \Delta_c$). Further, with*

$$\bar{v} := \frac{1 - \lambda}{\lambda}(q_H - c_H) > 0,$$

the prohibition does not affect manufacturer profits as long as $v \leq \bar{v}$ and it strictly increases manufacturer profits otherwise.

³⁹The objective to protect small manufacturers, in particular those that are deemed dependent, is even enshrined in some competition laws.

⁴⁰We are aware that vertical contracts could, in principle, be used to partially circumvent restrictions on below-cost pricing, which is a feature that does not arise in other contributions as they do not explicitly consider the retailing sector. For instance, if we were to consider only two-part tariff contracts, a prohibition that targeted only the marginal wholesale price could induce firms to reduce this price component and simultaneously increase the fixed part (e.g., through a higher slotting allowance). Other regimes could, for instance, compare average wholesale and retail prices, which would generate additional complications in a setting as ours, with (random) promotions. We must leave a detailed comparison of various approaches to future research.

Proof. See Appendix.

Crucially, without salient thinkers the prohibition of below-cost pricing does not affect product choice. Still, manufacturers of high-quality products will benefit, at least when the extent of one-stop shopping is large (but not so otherwise), which follows simply from the resulting increase in prices to the detriment of consumers. In short, when in the baseline case manufacturers lobby for a prohibition of below-cost pricing, then this should not be supported by consumers and their advocates.

Before moving on to the case with salience, we should note that the characterization of a pricing equilibrium when below-cost pricing is prohibited is of interest in its own right. The proof of Proposition 6 contains the respective details. When the extent of one-stop shopping v becomes sufficiently large, as $v \geq \bar{v}$, all retailers choose the lowest possible price $p_n = c_H$. When instead v is sufficiently small, as

$$v \leq \underline{v} := \frac{1 - \lambda}{\lambda N} (q_H - c_H),$$

the prohibition does not affect the outcome. In the proof of Proposition 6 we characterize also the outcome in the intermediate range, where $\underline{v} < v < \bar{v}$. There, in a symmetric equilibrium each retailer chooses the lowest possible price $p_n = c_H$ with positive probability and mixes over some support $[\underline{p}_r, q_H]$, where \underline{p}_r lies strictly above c_H . Hence, when retailers face a binding lower boundary, they will choose the lowest possible price with positive probability or choose prices that are strictly bounded away.

As noted above, while we do not incorporate such a feature in our analysis, even absent the prohibition of below-cost pricing retailers may face a lower boundary on the price of any particular good, as choosing a price below this boundary may trigger (too many) purchases from reselling intermediaries (e.g., small shops) that will not purchase any other products and will thus not generate the additional value v . Our characterization shows that this may give rise to an interesting “quasi-bimodal” distribution of prices.

6.2 Prohibition of Below-Cost Pricing with Salient Thinkers

The introduction of price floors gives rise to the possibility that two or more firms tie for the best offer. In fact, in light of our previous results it may then occur that, with a prohibition of below-cost pricing, a number of retailers tie at the price floor for the low-quality product and a number of retailers tie at the price floor of the high-quality product. It turns out however, that in this case still the same attribute is salient for all

(non-dominated) offers in the market.⁴¹ We can thus still say that either price or quality is salient.

Turning to the impact that policy intervention has on product choice, we first slightly restate the outcome without such intervention. Rewriting condition (8), without a prohibition of below-cost pricing, retailers always choose the high-quality product if either $\delta\Delta_q \geq \Delta_c$ or

$$v \leq \tilde{v} := c_H + \frac{(1 - \lambda)q_H - \frac{\Delta_c}{\Delta_q}q_H(1 - \lambda + \lambda N)}{\lambda N},$$

while otherwise, in the considered symmetric equilibrium, they choose the high-quality product with probability $\alpha^* < 1$ (as defined in Proposition 5). Policy intervention changes the outcome as follows:

Proposition 7 *If consumers are salient thinkers, a prohibition of below-cost pricing has an impact on the provision of product quality only if condition (8) does not hold, so that, even though it is more efficient, without intervention there is no (all) high-quality equilibrium. Then, the intervention*

i) backfires and strictly reduces the likelihood α^ with which each retailer chooses the high-quality product if $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and if the extent of one-stop shopping v exceeds some threshold $\hat{v} > \tilde{v}$;*

ii) has the desired positive effect and increases the likelihood α^ with which each retailer chooses the high quality product if $\frac{q_H}{q_L} > \frac{c_H}{c_L}$;*

iii) has no effect otherwise (i.e., if $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and $v < \hat{v}$).

Proof. See Appendix.

Note first again the role of the extent of one-stop shopping v . From condition (8) and Proposition 7, the intervention can only have a positive or negative impact when v is sufficiently large ($v > \tilde{v}$ or $v > \hat{v}$). And it backfires when, in addition, $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ holds. That is, even though the high-quality product generates strictly more net utility than the low-quality product, the respective percentage increase in quality is below the respective

⁴¹This can be seen as follows. Suppose that $L \geq 1$ retailers charge the lowest price p_L for a low-quality product, whereas $H \geq 1$ retailers charge the lowest price p_H for a high-quality product. Clearly, all other offers are then dominated, so that the average price is thus $P = \frac{Lp_L + Hp_H}{L+H}$ and the average quality $Q = \frac{Lq_L + Hq_H}{L+H}$. Then for a low-quality offer price is salient if $\frac{P}{p_L} > \frac{Q}{q_L}$, which again becomes $\frac{p_H}{p_L} > \frac{q_H}{q_L}$ – or quality is salient if $\frac{P}{p_L} < \frac{Q}{q_L}$, which again becomes $\frac{p_H}{p_L} < \frac{q_H}{q_L}$. And for a high-quality offer $\frac{p_H}{P} > \frac{q_H}{Q}$ and $\frac{p_H}{P} < \frac{q_H}{Q}$ transform to the same conditions. Note that this argument also holds when the non-dominated offer for a particular quality (e.g., the low quality) is a single deviating price (such that, in this case, $p_L = p_{dev}$ and $L = 1$).

percentage increase in costs. In this paper, we do not want to speculate for which products this is more or less likely. Our focus is instead on identifying a channel through which, with salient thinkers but not without, a prohibition of below-cost pricing can indeed affect welfare, though the effect may not always be as desired.⁴² We finally turn to profits. As without salience, policy intervention may raise firms' joint profits. However, when the prohibition backfires with salience and decreases welfare (cf. Proposition 7), all of the incremental profits go to retailers and not manufacturers.

Proposition 8 *A prohibition of below-cost pricing has the following implications on firm profits:*

i) If $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and $\delta\Delta_q < \Delta_c$, it does not affect manufacturer profits, but weakly increases retailer profits (they remain unchanged for $v \leq \bar{v}$, and strictly increase for $v > \bar{v}$).

ii) If $\frac{q_H}{q_L} > \frac{c_H}{c_L}$ or $\delta\Delta_q \geq \Delta_c$, it weakly increases (decreases) manufacturer (retailer) profits (strictly so if $v > \tilde{v}$ and $\delta\Delta_q < \Delta_c$).

Proof. See Appendix.

7 Conclusion

Our choice of model in this paper is motivated by the following two observations. First, one-stop shopping leads consumers to base their choice of retailers only on a comparison of a selected number of products, while retailers typically promote only a selected number of products. These are consequently the products on which price competition is fiercest, leading potentially to loss leading. Second, consumers' attention to different attributes of a product, notably price and quality, may not be "fixed", but may depend instead on market circumstances, precisely on whether a particular offer is "saliently different" along the respective attribute, compared to the other offers in the market. Our analysis captures these two features by combining a model of one-stop stopping and limited information about prices, set into a model of sales (Varian 1980), with recent developments in behavioral economics (precisely, the formalization of salience in Bordalo et al. (2013)).

We derive novel normative and positive implications from this model, showing in particular the differences that the presence of salient thinkers makes, compared to a baseline

⁴²In addition, note that when total demand was elastic, an increase in prices would generate deadweight loss. Such an extended analysis could only be obtained in numerical examples.

case where consumers have rational attention. Our first implications concern manufacturers' profits. The role of salient thinkers is crucial, as an increase in the extent of one-stop shopping will only then negatively affect manufacturer profits. Thus, when consumers are salient thinkers, but not so otherwise, we can indeed support manufacturers' concerns when retailers use the respective product category for loss leading. The second set of implications that we derive concerns the choice of product quality. We show how high-quality products may be crowded out inefficiently when these are used for promotions and when competition is fierce or the extent of one-stop shopping is large. The converse is not true, however, so that there is never an overprovision of high-quality products, regardless of circumstances in the market.

As discussed in the introduction, these implications directly relate to the ongoing policy debate about possible detrimental effects of retailers' deep discounting in loss-leading product categories. We thus explicitly consider a policy of prohibiting below-cost pricing. This only affects product choice when consumers are salient thinkers, but not so otherwise. We identify the precise circumstances when this intervention increases or decreases overall efficiency.

Our third set of implications relates to pricing. In line with the (marketing) literature, we interpret retailers' mixed-pricing strategies as promotions and show, amongst other things, how the respective promotions of retailers with low-quality and high-quality offers respond differently to changes in competition. As we noted above, future research may also explore more fully how promotions of low-quality and high-quality retailers change with the model's primitives.

8 References

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9 Appendix: Proofs

Proof of Proposition 1. We first introduce some additional notation. Allowing for mixed strategies, we denote by α_n the likelihood that $q_n = q_H$. Recall that $G_n(p_n)$ denotes the respective distribution over prices p_n . Each retailer can offer the low-quality product at marginal cost c_L , while the retailer can offer the quantity x_n of the high-quality product at total costs $W_n(x_n) = T_n + x_n w_n$, according to the contract offered by the respective high-quality manufacturer. $W_n(x_n)$ will be determined as part of the equilibrium characterization.

Next, for each retailer we write demand as a function of consumers' perceived net surplus s_n . Recall first that $s_n = q_n - p_n$, given the anticipated monopolistic pricing at all products $i > 1$. Given the anticipated strategies $\alpha_{n'}$ and $G_{n'}$ at all other retailers $n' \neq n$, in an equilibrium retailer n faces some expected demand $X_n(s_n)$.⁴³ We first show the implication for retailers' choice of products, then the implication for the profit of high-quality manufacturers.

Suppose first that $\Delta_q > \Delta_c$. We show by contradiction that then $\alpha_n = 0$ is not part of an equilibrium. Take a price in the support of $G_n(p_n)$, which gives rise to expected demand $X_n(q_L - p_n)$. Consider a deviation to $q_n = q_H$ and the choice of a price $\hat{p}_n = p_n + \Delta_q$, which thus realizes the same expected demand. In case the high-quality manufacturer offered his product at the wholesale price $w_n = c_H$, the retailer's increase in profit (gross of T_n) would then be at least

$$\varkappa = \frac{1 - \lambda}{N} (\Delta_q - \Delta_c) > 0.$$

By setting $T_n = \varkappa/2$ the respective high-quality manufacturer can thus ensure that its (deviating) offer is accepted for sure and that it generates strictly positive profits, so that $\alpha_n = 0$ is indeed not an equilibrium when $\Delta_q > \Delta_c$. It is next immediate to extend this argument. When $\Delta_q > \Delta_c$, we can also rule out that $0 < \alpha_n < 1$, next to $\alpha_n = 0$, using the retailer's indifference in this case. Thus, $\alpha_n = 1$ must hold.

Suppose next that $\Delta_q < \Delta_c$. We argue again to a contradiction and now suppose first that $\alpha_n = 1$. Once more, take some p_n in the respective support of $G_n(p_n)$, which induces a net utility $s_n = q_H - p_n$ and the expected demand $X_n(s_n)$. As before, the same expected demand could be achieved when the retailer offered the low-quality product at a price $\hat{p}_n = p_n - \Delta_q$, though the respective cost of delivering this net utility would drop by

⁴³While it is straightforward to derive this explicitly, based on the function $D_n(s_n)$ that was introduced in the main text, this is not needed for what follows.

$\Delta_c - \Delta_q > 0$. Hence, even when $w_n = c_H$ and $T_n = 0$, the increase in the retailer's profit would then be at least $\frac{1-\lambda}{N}(\Delta_c - \Delta_q) > 0$, yielding a contradiction. Again, the argument can be extended to rule out $0 < \alpha_n < 1$, so that $\Delta_q < \Delta_c$ implies $\alpha_n = 0$ for all $n \in N$.

For what follows, we suppose that $\Delta_q > \Delta_c$. We now argue that $w_n = c_H$ as this uniquely maximizes the respective joint profits of retailer n and the high-quality manufacturer. We first rule out that $w_n > c_H$, noting that the argument contradicting $w_n < c_H$ is analogous. Notice that any price in the support of $G_n(p_n)$ maximizes $(p_n - w_n)X_n(q_n - p_n)$, while the joint-profit maximizing choice maximizes $(p_n - c_n)X_n(q_n - p_n)$. Clearly, when the support of $G_n(p_n)$ is not degenerate, containing notably two prices $p'_n < p''_n$ that must then give rise to strictly different expected demand realizations $X'_n > X''_n$, then $w_n > c_H$ can not be optimal. This is so as at a marginally lower wholesale price the manufacturer could induce a price $\hat{p}_n \leq p'_n$, which would also (marginally) increase the retailer's profit, while from $w_n > c_H$ and $X'_n > X''_n$ it would have a first-order impact on the manufacturer's profit and thus altogether a positive first-order impact on joint surplus, yielding a contradiction. This argument extends to the case where $G_n(p_n)$ is degenerate (i.e., with all mass on one price p_n). When $\Delta_q > \Delta_c$, we thus have for all n that $q_n = q_H$ and that retailers face constant marginal costs of $w_n = c_H$.

We next know from Baye et al. (1992) that in all pricing equilibria each retailer's gross profit (i.e., without consideration of T_n) is equal to

$$\frac{1-\lambda}{N}\Pi^m(q_H, c_H). \quad (17)$$

A high-quality manufacturer optimally extracts the difference between the jointly realized surplus and a retailer's reservation value, so that T_n is determined as the difference between (17) and a retailer's (maximum) deviation profit when it chooses instead the low-quality product. We now show that the deviation profit is equal to

$$\frac{1-\lambda}{N}\Pi^m(q_L, c_L), \quad (18)$$

i.e., to offering q_L at $p_n = p^m(q_L, c_L)$ to consumers in the local market only. Multiplying then the difference between (17) and (18) by N yields expression (5). To prove the claim we make the following argument. Consider any two levels of net utility that a retailer may offer to consumers, $s' < s''$. Then, when offering s' is weakly preferred for a retailer that (on-equilibrium) chooses $q_n = q_H$, offering the lower net utility is strictly preferred when the retailer deviates to $q_n = q_L$. Formally, using that in the baseline model demand $X_n(s)$ is a function of only the respective net utility, and as the respective prices are $p'_H = q_H - s'$

and $p''_H = q_H - s''$ with high quality and $p'_L = q_L - s'$ and $p''_L = q_L - s''$ with low quality, we claim that

$$(q_H - s' + v - c_H)X_n(s') \geq (q_H - s'' + v - c_H)X_n(s'')$$

implies

$$(q_L - s' + v - c_L)X_n(s') > (q_L - s'' + v - c_L)X_n(s'').$$

This indeed holds from $\Delta_q > \Delta_c$.⁴⁴ From this we know that the deviating retailer optimally offers a price $p_n \geq p^m(q_H, c_H) - \Delta_q$, which does not attract shoppers. The highest profits when no shoppers are attracted are indeed given by (18).⁴⁵ **Q.E.D.**

Proof of Corollary 2. The only assertion that is not immediate from expression (5) relates to the comparative statics in v when $H(\cdot)$ is non-degenerate. Using uniqueness of $p^m(q_H, c_H)$ and from this continuous differentiability of Π_M , we have

$$\frac{d\Pi_M}{dv} = (1 - \lambda) [H(q_H - p^m(q_H, c_H)) - H(q_L - p^m(q_L, c_L))],$$

so that it remains to prove that

$$p^m(q_H, c_H) - p^m(q_L, c_L) < \Delta_q. \quad (19)$$

We now argue that this is implied by $\Delta_q > \Delta_c$. To see this, we can rewrite

$$\begin{aligned} p^m(q_L, c_L) &= \arg \max_p [(p - c_L + v)H(q_L - p)] \\ &= \arg \max_p ((p - \Delta_q) - c_L + v)H(q_L - (p - \Delta_q)) - \Delta_q \\ &= p^m(q_H, \Delta_q + c_L) - \Delta_q. \end{aligned}$$

Hence, the requirement (19) transforms to

$$p^m(q_H, c_H) < p^m(q_H, \Delta_q + c_L),$$

which follows as, when this is interior for a non-degenerate $H(\cdot)$, $\partial p^m / \partial c > 0$ and as $\Delta_q + c_L > c_H$ from $\Delta_q > \Delta_c$. **Q.E.D.**

⁴⁴More precisely, when we impose equality on the first condition and use this to substitute out the terms relating to s' and s'' , the second condition translates to

$$X_n(s')[(q_H - c_H) - (q_L - c_L)] < X_n(s'')[(q_H - c_H) - (q_L - c_L)],$$

which holds as $\Delta_q > \Delta_c$ and as clearly $X_n(s') < X_n(s'')$.

⁴⁵That indeed $p^m(q_H, c_H) - \Delta_q < p^m(q_L, c_L)$ when demand is elastic is also shown directly in the proof of Corollary 2. When demand is inelastic, this holds with equality as $p^m(q_H, c_H) = q_H$ and $p^m(q_L, c_L) = q_L$.

Proof of Proposition 2. We start by proving that condition (8) is necessary for a high-quality equilibrium to exist. We show that when condition (8) does not hold, then a retailer n could profitably deviate and offer instead a low-quality product. For this we first derive some characteristics that apply to any (candidate) high-quality equilibrium. The first is that

$$\tilde{\Pi} = \frac{1 - \lambda}{N}(q_H - c_H + v) \quad (20)$$

provides an upper boundary for each retailer's profits. This is obtained by setting $T_n = 0$ and using that in any pricing equilibrium with $q_n = q_H$ and symmetric retailer costs c_H , all retailers must realize gross profits of $\tilde{\Pi}$. A formal proof for this can be found in Baye et al. (1992), Lemmas 7 and 8, as stated within their proof of Theorem 1. We next derive, across all equilibria, the lowest price that any retailer may charge. As the retailer can ensure the profit (20), gross of any T_n , by charging $p_n = q_H$, we can obtain this price p from the requirement

$$(p - c_H + v) \left(\frac{1 - \lambda}{N} + \lambda \right) = \tilde{\Pi}.$$

The respective minimum price is thus \underline{p} , as given in (9).

Consider now a deviation of retailer n to stocking $q_n = q_L$. This deviating retailer can attract all shoppers with probability one by pricing at the minimum of $\underline{p} \frac{q_L}{q_H}$ (which guarantees that price is salient) and $\underline{p} - \delta \Delta_q$ (which guarantees that the retailer attracts the shoppers, provided that price is salient). If the minimum is given by the latter condition ($\underline{p} > \delta q_H$), the deviating retailer's profit with this price satisfies

$$\left(\underline{p} \frac{q_L}{q_H} - c_L + v \right) \left(\frac{1 - \lambda}{N} + \lambda \right) > \tilde{\Pi} \text{ if } \Delta_c > \frac{\Delta_q}{q_H} \underline{p}. \quad (21)$$

If instead $\underline{p} \leq \delta q_H$, the deviating retailer's profit satisfies

$$(\underline{p} - \delta \Delta_q - c_L + v) \left(\frac{1 - \lambda}{N} + \lambda \right) > \tilde{\Pi} \text{ if } \Delta_c > \delta \Delta_q. \quad (22)$$

Hence, no matter whether $\underline{p} > \delta q_H$ or $\underline{p} \leq \delta q_H$, if it holds that

$$\Delta_c > \max \left\{ \delta \Delta_q, \frac{\Delta_q}{q_H} \underline{p} \right\}, \quad (23)$$

a deviation to choosing $q_n = q_L$ is strictly profitable. This proves that condition (8) (the converse of (23)) is indeed necessary.

We now turn to sufficiency. We show that when (8) holds, then there exists an equilibrium with $q_n = q_H$ and where retailers choose a symmetric pricing strategy, as given

by

$$G(p) = 1 - \sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_H + v} - 1 \right)},$$

which is obtained from (3) by using that $H(\cdot)$ is now degenerate. We show this by comparing the respective (candidate) equilibrium profit, which is $\tilde{\Pi} - T_n$, with the maximum profit of a deviating retailer choosing $q_n = q_L$. Denote the latter by Π_{dev}^* . Clearly, as a high-quality manufacturer would otherwise reduce T_n , we can restrict attention to showing that

$$\tilde{\Pi} \geq \Pi_{dev}^* \quad (24)$$

if condition (8) holds. This is what we do in the rest of this proof. For this, but also for the subsequently proven comparative results, we proceed as follows. We first derive the optimal price that a deviating retailer would choose, which we denote by p_{dev}^* . We then characterize the deviation profits Π_{dev}^* explicitly for different “types” of optimal deviation prices p_{dev}^* . Finally, we show that indeed $\Pi_{dev}^* \leq \tilde{\Pi}$ whenever condition (8) holds.

Lemma 2 *Define $\underline{\delta} := \frac{p}{q_H}$ and consider a (candidate) equilibrium in which all retailers choose the high-quality product and a symmetric pricing strategy. Then, the optimal deviation price p_{dev}^* of a retailer offering a low-quality product is*

$$p_{dev}^* = \begin{cases} q_L & \text{if } \frac{q_H}{q_L} > \frac{c_H - v}{c_L - v} \\ \{p_{dev} \mid p_{dev} \in [\underline{p}_{q_H}, q_L]\} & \text{if } \delta \leq \underline{\delta} \text{ and } \frac{q_H}{q_L} = \frac{c_H - v}{c_L - v} \\ \underline{p}_{q_H} & \text{if } \delta \leq \underline{\delta} \text{ and } \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \\ \{p_{dev} \mid p_{dev} \in [\delta q_L, q_L]\} & \text{if } \delta > \underline{\delta} \text{ and } \frac{q_H}{q_L} = \frac{c_H - v}{c_L - v} \\ \delta q_L & \text{if } \delta > \underline{\delta} \text{ and } \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \text{ and } \delta > \frac{c_H - c_L}{q_H - q_L} \\ \{p_{dev} \mid p_{dev} \in [p - \delta(q_H - q_L), \delta q_L]\} & \text{if } \delta > \underline{\delta} \text{ and } \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \text{ and } \delta = \frac{c_H - c_L}{q_H - q_L} \\ p - \delta(q_H - q_L) & \text{if } \delta > \underline{\delta} \text{ and } \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \text{ and } \delta < \frac{c_H - c_L}{q_H - q_L}. \end{cases}$$

Proof. Denote the (random) minimum price of all $N-1$ rivals by \tilde{p}_{min} , which is distributed according to

$$G_{min}(p_{min}) = 1 - \frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p_{min} - c_H + v} - 1 \right).$$

We now consider two possibilities. In the first case, the deviating retailer’s price p_{dev} is such that quality is salient with $\frac{q_H}{q_L} > \frac{\tilde{p}_{min}}{p_{dev}}$. Note that then the retailer can only attract the shoppers when $q_L - \delta p_{dev} \geq q_H - \delta \tilde{p}_{min}$. But this condition, together with that of

“quality salience”, can only hold jointly if both $\tilde{p}_{min} < p_{dev} \frac{q_H}{q_L}$ and $\tilde{p}_{min} \geq \frac{q_H - q_L}{\delta} + p_{dev}$, which requires that

$$p_{dev} \frac{q_H}{q_L} > \frac{q_H - q_L}{\delta} + p_{dev}.$$

Solving this for p_{dev} , this condition is clearly incompatible with $p_{dev} \leq q_L$, which is required to ensure that the offer is accepted by any consumer. It thus follows that the deviating retailer can only attract the shoppers with its low-quality product if price is salient. Precisely, then two conditions must hold jointly: The “salience constraint”

$$\tilde{p}_{min} \geq p_{dev} \frac{q_H}{q_L} \tag{25}$$

and the “competition constraint”

$$\tilde{p}_{min} \geq p_{dev} + \delta(q_H - q_L). \tag{26}$$

The salience constraint binds if $p_{dev} \geq \delta q_L$ and the competition constraint binds if $p_{dev} \leq \delta q_L$.

Clearly, the competition constraint (26) is irrelevant if the deviating retailer can already guarantee to win all shoppers for a price larger than δq_L . This is the case if the price which deterministically wins all shoppers under a binding salience constraint (25), $p_{dev} = \underline{p}_{\frac{q_L}{q_H}}$, exceeds δq_L . Solving the requirement yields $\delta \leq \underline{\delta} = \frac{p}{q_H} < 1$. Consider this case first (Case A) and then the complementary case (Case B).

Case A: $\delta \leq \underline{\delta}$. For all (relevant) values of $p_{dev} \geq \delta q_L$, the retailer’s expected profit is then given by

$$\begin{aligned} \Pi(p_{dev}) &= (p_{dev} - c_L + v) \left(\frac{1 - \lambda}{N} + \lambda \Pr \left\{ \tilde{p}_{min} \geq p_{dev} \frac{q_H}{q_L} \right\} \right) \\ &= (p_{dev} - c_L + v) \left(\frac{1 - \lambda}{N} \right) \frac{q_H - c_H + v}{p_{dev} \frac{q_H}{q_L} - c_H + v} \\ &\propto \frac{p_{dev} - c_L + v}{p_{dev} \frac{q_H}{q_L} - c_H + v}. \end{aligned}$$

Taking the derivative with respect to p_{dev} , its sign is equal to the sign of $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v}$. Hence, for $\delta \leq \underline{\delta}$, there are three cases. If $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v} > 0$, the retailer’s optimal deviation price is q_L . If $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v} < 0$, its optimal deviation price is $\underline{p}_{\frac{q_L}{q_H}}$. And finally, if it holds (non-generically) that $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v} = 0$, the deviating retailer is indifferent between choosing any price in the interval $[\underline{p}_{\frac{q_L}{q_H}}, q_L]$.

Case B: $\delta > \underline{\delta}$. Recall that in this case it is not possible for the deviating retailer to attract all shoppers for sure while the salience constraint (25) binds. Hence, pricing at or below δq_L may become optimal. Before turning to this possibility, note that for prices $p_{dev} \geq \delta q_L$ the findings from Case A immediately carry over, so that the deviating retailer's expected profit behaves monotonically in p_{dev} . Repeating now the same exercise for $p_{dev} < \delta q_L$, the deviating retailer's expected profit can be written as

$$\begin{aligned}\Pi(p_{dev}) &= (p_{dev} - c_L + v) \left(\frac{1 - \lambda}{N} + \lambda \Pr \{ \tilde{p}_{min} \geq p_{dev} + \delta(q_H - q_L) \} \right) \\ &= (p_{dev} - c_L + v) \left(\frac{1 - \lambda}{N} \right) \frac{q_H - c_H + v}{p_{dev} + \delta(q_H - q_L) - c_H + v} \\ &\propto \frac{p_{dev} - c_L + v}{p_{dev} + \delta(q_H - q_L) - c_H + v}.\end{aligned}$$

Taking the derivative with respect to p_{dev} , its sign is equal to the sign of $\delta(q_H - q_L) - (c_H - c_L)$. Combining this with the observation from Case A that the deviating retailer's expected profit for prices above δq_L is increasing (decreasing) if and only if $\frac{q_H}{q_L} > \frac{c_H - v}{c_L - v}$ ($\frac{q_H}{q_L} < \frac{c_H - v}{c_L - v}$). We have the following cases:⁴⁶

- i) if $\delta(q_H - q_L) - (c_H - c_L) > 0$ and $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v} > 0$, the optimal deviation price is q_L ,
- i') if $\delta(q_H - q_L) - (c_H - c_L) > 0$ and $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v} = 0$, the optimal deviation price is any price in the interval $[\delta q_L, q_L]$,
- ii) if $\delta(q_H - q_L) - (c_H - c_L) > 0$ and $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v} < 0$, the optimal deviation price is δq_L ,
- ii') if $\delta(q_H - q_L) - (c_H - c_L) = 0$ and $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v} < 0$, the optimal deviation price is any price in the interval $[\underline{p} - \delta(q_H - q_L), \delta q_L]$,
- iii) and if $\delta(q_H - q_L) - (c_H - c_L) < 0$ and $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v} < 0$, the optimal deviation price is $\underline{p} - \delta(q_H - q_L)$.

Note next that for $\delta > \underline{\delta}$, there are no other cases (i.e., cases where $\delta(q_H - q_L) - (c_H - c_L) \leq 0$ and $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v} \geq 0$). To see this, it suffices to show the converse, namely that for $\delta > \underline{\delta}$, $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v} \geq 0$ implies $\delta(q_H - q_L) - (c_H - c_L) > 0$. For a proof, observe that from

⁴⁶A prime indicates non-generic parameter combinations.

$\delta > \underline{\delta}$, $\delta(q_H - q_L) - (c_H - c_L)$ is bounded from below by

$$\begin{aligned}
& \underline{\delta}(q_H - q_L) - (c_H - c_L) \\
&= \frac{1 - \lambda + \lambda N \left(\frac{c_H - v}{q_H} \right)}{1 - \lambda + \lambda N} (q_H - q_L) - (c_H - c_L) \\
&> \frac{(1 - \lambda)(c_H - c_L) + \lambda N(c_H - v) - \lambda N(c_H - v) \frac{q_L}{q_H} - (c_H - c_L)(1 - \lambda + \lambda N)}{1 - \lambda + \lambda N} \\
&= \frac{\lambda N}{1 - \lambda + \lambda N} \left[c_L - v - (c_H - v) \frac{q_L}{q_H} \right] \geq 0,
\end{aligned}$$

where the first inequality follows from $\Delta_q > \Delta_c$ and the second from $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v} \geq 0$.

Finally, combining Case A and Case B and noting that q_L constitutes the best deviation price whenever $\frac{q_H}{q_L} - \frac{c_H - v}{c_L - v} > 0$, the lemma immediately follows. **Q.E.D. (Lemma 2).**

Using the optimal deviation prices characterized in Lemma 2 (and inserting $p_{dev}^* = \delta q_L$ into the retailer's expected profit function when this price is optimal), the following lemma is immediate.

Lemma 3 *Consider a (candidate) equilibrium in which all retailers choose the high-quality product and a symmetric pricing strategy. Then, the optimal deviation profit Π_{dev}^* of a retailer offering a low-quality product is*

$$\Pi_{dev}^* = \begin{cases} (q_L - c_L + v) \frac{1 - \lambda}{N} & \text{if } \frac{q_H}{q_L} \geq \frac{c_H - v}{c_L - v} \\ \left(\underline{p} \frac{q_L}{q_H} - c_L + v \right) \left(\frac{1 - \lambda}{N} + \lambda \right) & \text{if } \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \text{ and } \delta \leq \underline{\delta} \\ \frac{\delta q_L - c_L + v}{\delta q_H - c_H + v} (q_H - c_H + v) \frac{1 - \lambda}{N} & \text{if } \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \text{ and } \delta > \underline{\delta} \text{ and } \delta \geq \frac{\Delta_c}{\Delta_q} \\ \left(\underline{p} - \delta(q_H - q_L) - c_L + v \right) \left(\frac{1 - \lambda}{N} + \lambda \right) & \text{if } \frac{q_H}{q_L} < \frac{c_H - v}{c_L - v} \text{ and } \delta > \underline{\delta} \text{ and } \delta < \frac{\Delta_c}{\Delta_q} \end{cases}$$

Building on Lemma 3, it is now straightforward to conclude the proof of Proposition 2. Clearly, a high-quality equilibrium can only exist if a retailer's maximal deviation profit does not exceed its (candidate equilibrium) profit of $\tilde{\Pi} = (q_H - c_H + v) \frac{1 - \lambda}{N}$. We now consider the different possible cases. First, since $(q_L - c_L + v) \frac{1 - \lambda}{N} < \tilde{\Pi}$ due to $\Delta_q > \Delta_c$, a high-quality equilibrium exists whenever $\frac{q_H}{q_L} \geq \frac{c_H - v}{c_L - v}$. Second, if it holds that $\frac{q_H}{q_L} < \frac{c_H - v}{c_L - v}$ and $\delta \leq \underline{\delta}$, a high-quality equilibrium exists if and only if

$$\left(\underline{p} \frac{q_L}{q_H} - c_L + v \right) \left(\frac{1 - \lambda}{N} + \lambda \right) \leq \tilde{\Pi}. \quad (27)$$

Using the fact that $\tilde{\Pi}$ can also be written as $(\underline{p} - c_H + v) \left(\frac{1 - \lambda}{N} + \lambda \right)$, (27) becomes

$$\frac{\underline{p}}{q_H} = \underline{\delta} \geq \frac{\Delta_c}{\Delta_q}.$$

Hence, for $\frac{q_H}{q_L} < \frac{c_H-v}{c_L-v}$ and $\delta \leq \underline{\delta}$, a high-quality equilibrium exists if and only if $\underline{\delta} \geq \frac{\Delta_c}{\Delta_q}$. Third, if $\frac{q_H}{q_L} < \frac{c_H-v}{c_L-v}$ and $\delta > \underline{\delta}$ and $\delta \geq \frac{\Delta_c}{\Delta_q}$, a high-quality equilibrium exists if and only if

$$\frac{\delta q_L - c_L + v}{\delta q_H - c_H + v} (q_H - c_H + v) \frac{1 - \lambda}{N} \leq \tilde{\Pi}.$$

A straightforward comparison reveals that this is the case if and only if $\delta \geq \frac{\Delta_c}{\Delta_q}$, which is true by assumption. And fourth, if $\frac{q_H}{q_L} < \frac{c_H-v}{c_L-v}$ and $\delta > \underline{\delta}$ and $\delta < \frac{\Delta_c}{\Delta_q}$, a high-quality equilibrium can not exist. To see this, observe that in this case we can write

$$\begin{aligned} \Pi_{dev}^* &= (\underline{p} - \delta(q_H - q_L) - c_L + v) \left(\frac{1 - \lambda}{N} + \lambda \right) \\ &= \tilde{\Pi} + (\Delta_c - \delta \Delta_q) \left(\frac{1 - \lambda}{N} + \lambda \right) > \tilde{\Pi}, \end{aligned}$$

where the last inequality comes from $\delta < \frac{\Delta_c}{\Delta_q}$. To sum up all four cases, a high-quality equilibrium thus exists if either $\frac{q_H}{q_L} \geq \frac{c_H-v}{c_L-v}$, or $\delta \leq \underline{\delta}$ and $\underline{\delta} \geq \frac{\Delta_c}{\Delta_q}$, or $\delta > \underline{\delta}$ and $\delta \geq \frac{\Delta_c}{\Delta_q}$. These conditions are now finally simplified.

Note for this that condition $\frac{q_H}{q_L} \geq \frac{c_H-v}{c_L-v}$ is redundant as it implies $\underline{\delta} \geq \frac{\Delta_c}{\Delta_q}$ (but not vice versa).⁴⁷ Focusing on the two remaining conditions (a) $\delta \leq \underline{\delta}$ and $\underline{\delta} \geq \frac{\Delta_c}{\Delta_q}$ and (b) $\delta > \underline{\delta}$ and $\delta \geq \frac{\Delta_c}{\Delta_q}$, the respective parameter space can be separated into two regimes. First, if it holds that $\underline{\delta} \geq \frac{\Delta_c}{\Delta_q}$, the considered high-quality equilibrium exists for all values of δ , as either (a) or (b) must be satisfied. Second, if it holds that $\underline{\delta} < \frac{\Delta_c}{\Delta_q}$, the candidate equilibrium only exists if $\delta \geq \frac{\Delta_c}{\Delta_q}$, as (a) can not be satisfied and $\delta \geq \frac{\Delta_c}{\Delta_q}$ becomes the binding condition in (b). This concludes the proof of the proposition. **Q.E.D.**

Proof of Proposition 3. Note that from Lemma 2 in the proof of Proposition 2, a deviating retailer under salience will only find it optimal to choose the same price as a deviating retailer without salience if

$$\frac{q_H}{q_L} \geq \frac{c_H - v}{c_L - v}. \quad (28)$$

In all other cases, facing salient thinkers strictly increases each retailer's outside option value of deviating, thereby also reducing the profit that a high-quality manufacturer can extract. Rearranging the converse of (28), we obtain the required condition.

⁴⁷One possibility to show this is by isolating $\frac{q_H}{q_L} \geq \frac{c_H-v}{c_L-v}$ for c_L , which gives $c_L \geq v + \frac{q_L}{q_H}(c_H - v)$. Inserting this into $\frac{\Delta_c}{\Delta_q}$ gives an upper bound for the latter. One can then prove via direct manipulation that even at this upper bound, it must hold that $\frac{\Delta_c}{\Delta_q} \leq \underline{\delta}$.

To prove the comparative statics claims, we make use of Lemma 3 to express each retailer's (maximal) deviation profits given that the converse of (28) holds and given that a high-quality equilibrium exists. We obtain

$$\Pi_{dev}^* = \begin{cases} \left(\underline{p} \frac{q_L}{q_H} - c_L + v \right) \left(\frac{1-\lambda}{N} + \lambda \right) & \text{if } \delta \leq \frac{p}{q_H} \text{ and } \Delta_c \leq \frac{\Delta_q}{q_H} \underline{p} \\ \frac{\delta q_L - c_L + v}{\delta q_H - c_H + v} (q_H - c_H + v) \frac{1-\lambda}{N} & \text{if } \delta > \frac{p}{q_H} \text{ and } \Delta_c \leq \delta \Delta_q \end{cases}$$

Subtracting Π_{dev}^* from $\tilde{\Pi}$, after some manipulation (and noting that $(q_H - c_H + v) \frac{1-\lambda}{N} = (\underline{p} - c_H + v) \left(\frac{1-\lambda}{N} + \lambda \right)$), we arrive at the following total high-quality manufacturer profits:

$$\Pi_M = \begin{cases} (1-\lambda)(\Delta_q - \Delta_c) - \lambda N \left[(c_H - v) \frac{q_L}{q_H} - (c_L - v) \right] & \text{if } \delta \leq \frac{p}{q_H} \text{ and } \Delta_c \leq \frac{\Delta_q}{q_H} \underline{p} \\ (1-\lambda)(\Delta_q - \Delta_c) - (1-\lambda) \frac{(1-\delta)[q_L(c_H - v) - q_H(c_L - v)]}{\delta q_H - (c_H - v)} & \text{if } \delta > \frac{p}{q_H} \text{ and } \Delta_c \leq \delta \Delta_q \end{cases}$$

Since $(c_H - v) \frac{q_L}{q_H} - (c_L - v) > 0$, it can be seen directly that Π_M is strictly decreasing in N and λ in the first case, while it is constant in N in the second. Moreover, unless $\Delta_c = \delta \Delta_q$, Π_M is also strictly decreasing in λ in the second case, since $\Delta_q - \Delta_c > \frac{(1-\delta)[q_L(c_H - v) - q_H(c_L - v)]}{\delta q_H - (c_H - v)}$ whenever $\Delta_c < \delta \Delta_q$ ⁴⁸ (if $\Delta_c = \delta \Delta_q$, Π_M is equal to zero and is therefore not affected by changes in λ). It is further immediate that Π_M is strictly decreasing in v in the first case. Analyzing its derivative with respect to v in the second case, it is strictly decreasing in v if and only if $\Delta_c < \delta \Delta_q$ (while $\Pi_M = 0$ for $\Delta_c = \delta \Delta_q$). Finally, in the first case Π_M is independent of δ , whereas in the second, it is strictly increasing in δ . This completes the comparative statics claims in the proposition. **Q.E.D.**

Proof of Proposition 4. Analogous to the proof of Proposition 2, we start by proving that the condition $\Delta_q \leq \Delta_c$ is necessary for a low-quality equilibrium to exist. We show that when this condition does not hold, a retailer n could profitably deviate and offer instead a high-quality product. For this, note that in any low-quality equilibrium, each retailer makes expected profits of now (in a slight abuse of notation compared to (20))

$$\tilde{\Pi} = \frac{1-\lambda}{N} (q_L - c_L + v). \quad (29)$$

Again, a formal proof for this can be found in Baye et al. (1992), Lemmas 7 and 8, as stated within their proof of Theorem 1. Consider now a deviation to $(q_H, p_{dev} = q_H)$. This would generate a deviation profit of at least $\frac{1-\lambda}{N} (q_H - c_H + v) > \tilde{\Pi}$, where the last inequality follows from $\Delta_q > \Delta_c$.

⁴⁸A straightforward way to show this is by isolating δ in the respective inequality. Note for this that $\delta q_H - (c_H - v) > 0$ follows from $\delta > \frac{p}{q_H}$ and $\underline{p} > c_H - v$.

We now turn to showing that $\Delta_q \leq \Delta_c$ is also sufficient for a low-quality equilibrium to exist. We prove existence of a low-quality equilibrium where retailers use symmetric strategies, as characterized in expression (3), with $q = q_L$, $c = c_L$, and degenerate $H(\cdot)$:

$$G(p) = 1 - \sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left(\frac{q_L - c_L + v}{p - c_L + v} - 1 \right)},$$

with $\bar{p} = q_L$ and $\underline{p} = c_L - v + \frac{1-\lambda}{\lambda N + 1 - \lambda}(q_L - c_L + v)$. The distribution of the (random) minimum price \tilde{p}_{min} out of $N - 1$ rival firms can be written as

$$G_{min}(p_{min}) = 1 - \frac{1-\lambda}{\lambda N} \left(\frac{q_L - c_L + v}{p_{min} - c_L + v} - 1 \right).$$

We now consider a deviation to $q_n = q_H$, instead of $q_n = q_L$, where we can use $T_n = 0$.

A deviating price p_{dev} attracts shoppers either if (1) price is salient, i.e., $\frac{q_H}{q_L} < \frac{p_{dev}}{\tilde{p}_{min}}$, and the deviating retailer's offer is more preferable when price is salient, $\delta q_H - p_{dev} \geq \delta q_L - \tilde{p}_{min}$, or if (2) quality is salient, i.e., $\frac{q_H}{q_L} > \frac{p_{dev}}{\tilde{p}_{min}}$, and the offer is more preferable when quality is salient, $q_H - \delta p_{dev} \geq q_L - \delta \tilde{p}_{min}$. Note that (1) requires that $\tilde{p}_{min} < p_{dev} \frac{q_L}{q_H}$ and $\tilde{p}_{min} \geq p_{dev} - \delta(q_H - q_L)$, which is only possible if $p_{dev} < \delta q_H$. On the other hand, (2) requires that both (2a) $\tilde{p}_{min} > p_{dev} \frac{q_L}{q_H}$ and (2b) $\tilde{p}_{min} \geq p_{dev} - \frac{q_H - q_L}{\delta}$. Note here that (2b) is implied by (2a), as (2a) is binding whenever $p_{dev} < \frac{q_H}{\delta}$, which is always the case. Combining these conditions, that is from (1) and (2a), the deviation will attract shoppers if either $\tilde{p}_{min} \geq p_{dev} - \delta(q_H - q_L)$ (if $p_{dev} < \delta q_H$) or $\tilde{p}_{min} > p_{dev} \frac{q_L}{q_H}$ (if $p_{dev} \geq \delta q_H$). In the first case, with $p_{dev} < \delta q_H$, the deviating retailer's expected profit is thus

$$\Pi(p_{dev}) = (p_{dev} - c_H + v) \left(\frac{1-\lambda}{N} + \lambda \Pr \left\{ \tilde{p}_{min} \geq p_{dev} - \delta(q_H - q_L) \right\} \right), \quad (30)$$

while in the second case, with $p_{dev} \geq \delta q_H$, it is

$$\Pi(p_{dev}) = (p_{dev} - c_H + v) \left(\frac{1-\lambda}{N} + \lambda \Pr \left\{ \tilde{p}_{min} \geq p_{dev} \frac{q_L}{q_H} \right\} \right). \quad (31)$$

Inserting the rivaling retailers' minimum price distribution function $G_{min}(\cdot)$, it can easily be verified that the sign of the derivative $\Pi'(p_{dev})$ in (30) equals that of $\Delta_c - \delta \Delta_q$, which is strictly positive as $\Delta_c \geq \Delta_q$. Next, the sign of the derivative $\Pi'(p_{dev})$ in (31) equals that of $\frac{q_L}{q_H} - \frac{c_L - v}{c_H - v}$. To see that this is strictly positive as well, observe that

$$\frac{q_L}{q_H} - \frac{c_L - v}{c_H - v} \geq \frac{q_H - c_H + c_L}{q_H} - \frac{c_L - v}{c_H - v} > 0,$$

where we use from $\Delta_c \geq \Delta_q$ that $q_L \geq q_H - c_H + c_L$ and where the second inequality is obtained from straightforward manipulation. We have thus established that $\Pi'(p_{dev}) > 0$

holds everywhere, so that $\Pi_{dev}^* = \Pi(q_H)$, which is $(q_H - c_H + v) \frac{1-\lambda}{N}$ and thus weakly lower than $\tilde{\Pi}$ in (29) as $\Delta_c \geq \Delta_q$. **Q.E.D.**

Proof of Proposition 5. We first characterize the symmetric (candidate) equilibrium and then show that retailers have indeed no incentive to deviate.

We claim that under the given parameter constraints, a mixed-strategy equilibrium exists in which each retailer stocks q_H (q_L) with probability α^* ($1 - \alpha^*$), where for convenience we restate

$$\alpha^* = \sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left[\frac{\Delta_q - \Delta_c}{\frac{q_L}{q_H}(c_H - v) - (c_L - v)} \right]}.$$

Conditional on stocking q_H , retailers draw prices from the CDF

$$G_H(p) := 1 - \frac{1}{\alpha^*} \sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_H + v} - 1 \right)}$$

with support

$$[\underline{p}_H, \bar{p}_H] = \left[\frac{\Delta_c}{\Delta_q} q_H, q_H \right].$$

Conditional on stocking q_L , retailers draw prices from the CDF

$$G_L(p) := 1 - \frac{\sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left[\frac{q_H - c_H + v}{p - c_L + v} - 1 \right]} - \alpha^*}{1 - \alpha^*}$$

with support

$$[\underline{p}_L, \bar{p}_L] = \left[c_L - v + (q_H - c_H + v) \frac{1-\lambda}{1-\lambda + \lambda N}, \frac{\Delta_c}{\Delta_q} q_L \right].$$

Note first that the defined supports are such that $\frac{\underline{p}_H}{\bar{p}_L} = \frac{q_H}{q_L}$ and $\delta q_L - \bar{p}_L > \delta q_H - \underline{p}_H$, where the latter follows from the proposition's parameter constraint $\Delta_c > \delta \Delta_q$. This implies that price is always salient if high- and low-quality products coexist in the market, and that price salience ensures that shoppers receive a higher perceived utility from buying the low-quality product, no matter which prices are drawn. Hence, for any price p in the support of $G_H(\cdot)$, a high-quality firm's probability of attracting shoppers is

$$(\alpha^* [1 - G_H(p)])^{N-1}, \tag{32}$$

while for any price p in the support of $G_L(\cdot)$, a low-quality firm's probability of attracting shoppers is

$$(\alpha^* + (1 - \alpha^*) [1 - G_L(p)])^{N-1}. \tag{33}$$

With this we can confirm that retailers are indeed indifferent between choosing low or high quality (where $T_n = 0$) and choosing a price in the respective support $[\underline{p}_H, \bar{p}_H]$ and $[\underline{p}_L, \bar{p}_L]$. Precisely, using (32) we have

$$\Pi_H(p) = (p - c_H + v) \left\{ \frac{1 - \lambda}{N} + \lambda [\alpha^* (1 - G_H(p))]^{N-1} \right\} = \Pi^* := \frac{1 - \lambda}{N} (q_H - c_H + v),$$

while using (33) we have

$$\Pi_L(p) = (p - c_L + v) \left\{ \frac{1 - \lambda}{N} + \lambda [\alpha^* + (1 - \alpha^*) (1 - G_L(p))]^{N-1} \right\} = \Pi^*.$$

Below we show that there is also no profitable deviation to prices $p \notin [\underline{p}_H, \bar{p}_H]$ and $p \notin [\underline{p}_L, \bar{p}_L]$. Before doing so, we show that the distribution functions are well-behaved. For this note first that $\alpha^* \in (0, 1)$, i.e., that

$$\frac{1 - \lambda}{\lambda N} \left[\frac{\Delta_q - \Delta_c}{\frac{q_L}{q_H} (c_H - v) - (c_L - v)} \right] \in (0, 1).$$

That the expression is indeed positive follows from $\frac{q_L}{q_H} (c_H - v) - (c_L - v) > 0$, which is obtained from using $\Delta_c > \frac{\Delta_q}{q_H} \underline{p}$ and $\underline{p} > c_H - v$. To see that the expression is smaller than 1, we obtain from substituting for \underline{p} that this is equivalent to $\Delta_c > \frac{\Delta_q}{q_H} \underline{p}$, again as required by the proposition. Both $G_H(\cdot)$ and $G_L(\cdot)$ are clearly (strictly) increasing over the respective supports and substitution of α^* reveals that they are also well-behaved at the boundaries.

Consider now deviations to prices outside the respective supports. If some retailer n chooses $q_n = q_H$ but deviates to a price $p_{dev} < \underline{p}_H$, its offer is clearly preferred to any other offer of a high-quality product and it is also preferred to the lowest-price offer of a low-quality product if, for the respective minimum \tilde{p}_{min} , it holds that $\tilde{p}_{min} > p_{dev} \frac{q_L}{q_H}$, so that quality becomes salient, or $\tilde{p}_{min} > p_{dev} - \delta \Delta_q$, so that the deviating offer is preferred even if price is salient. It thus follows for the expected deviation profits that

$$\begin{aligned} \Pi_H(p_{dev}) &= (p_{dev} - c_H + v) \cdot \\ &\quad \left\{ \frac{1 - \lambda}{N} + \lambda \left[\alpha^* + (1 - \alpha^*) \left[1 - G_L \left(\min \left\{ p_{dev} \frac{q_L}{q_H}, p_{dev} - \delta \Delta_q \right\} \right) \right] \right]^{N-1} \right\}, \end{aligned}$$

which from inserting $G_L(\cdot)$ transforms to

$$\Pi_H(p_{dev}) = \frac{1 - \lambda}{N} (q_H - c_H + v) \left(\frac{p_{dev} - c_H + v}{\min \left\{ p_{dev} \frac{q_L}{q_H}, p_{dev} - \delta \Delta_q \right\} - c_L + v} \right).$$

The sign of the function's derivative is equal to the sign of $(c_H - v)\frac{q_L}{q_H} - (c_L - v)$ if $p_{dev}\frac{q_L}{q_H} \leq p_{dev} - \delta\Delta_q$ or the sign of $\Delta_c - \delta\Delta_q$ if $p_{dev}\frac{q_L}{q_H} > p_{dev} - \delta\Delta_q$, respectively. The former is strictly positive due to $\Delta_c > \frac{\Delta_q}{q_H}\underline{p}$, as we have already shown, while the latter is strictly positive as $\Delta_c > \delta\Delta_q$. Thus, we have shown that $\Pi_H(p_{dev}) < \Pi^*$ if $p_{dev} < \underline{p}_H$ for a high-quality firm.

Consider finally deviations by low-quality firms, where we need to consider deviations $p_{dev} > \bar{p}_L$. Clearly, by construction the profit can only exceed Π^* if it still attracts shoppers, for which it is in turn necessary that all other retailers choose high quality and that price remains salient. Thus, even when we only consider these constraints, the deviation profit is bounded by

$$\Pi_L(p_{dev}) = (p_{dev} - c_L + v) \left\{ \frac{1 - \lambda}{N} + \lambda \left[\alpha^* \left(1 - G_H \left(\frac{q_H}{q_L} p_{dev} \right) \right) \right]^{N-1} \right\},$$

which from inserting $G_H(\cdot)$ transforms to

$$\Pi_L(p_{dev}) = \frac{1 - \lambda}{N} (q_H - c_H + v) \left(\frac{p_{dev} - c_L + v}{p_{dev}\frac{q_L}{q_H} - c_H + v} \right).$$

The sign of the derivative is determined by $-\left[(c_H - v)\frac{q_L}{q_H} - (c_L - v) \right]$, which is strictly negative due to $\Delta_c > \frac{\Delta_q}{q_H}\underline{p}$ (see above). Thus, we have also shown that $\Pi_L(p_{dev}) < \Pi^*$ if $p_{dev} > \bar{p}_L$ for a low-quality firm. **Q.E.D.**

Proof of Corollary 3. It is immediate to see that α^* is strictly decreasing in v and λ . Moreover, since W^* only depends on v via α^* , in which it is strictly increasing, W^* is strictly decreasing in v . To see that W^* is also strictly decreasing in λ , note that

$$\begin{aligned} \frac{dW^*}{d\lambda} &= (\Delta_q - \Delta_c) \left[(\alpha^*)^N + \lambda N (\alpha^*)^{N-1} \frac{\partial \alpha^*}{\partial \lambda} - \alpha^* + (1 - \lambda) \frac{\partial \alpha^*}{\partial \lambda} \right] \\ &< (\Delta_q - \Delta_c) \left[\lambda N (\alpha^*)^{N-1} \frac{\partial \alpha^*}{\partial \lambda} + (1 - \lambda) \frac{\partial \alpha^*}{\partial \lambda} \right] < 0. \end{aligned}$$

Q.E.D.

Proof of Corollaries 4 and 6. The respective distribution functions have already been derived in the proof of Proposition 5. It remains to prove the limit results. Inserting the equilibrium α^* in $G_H(p)$, we have more explicitly

$$G_H(p) = 1 - \sqrt[N-1]{\left(\frac{\frac{q_L}{q_H}(c_H - v) - (c_L - v)}{\Delta_q - \Delta_c} \right) \left(\frac{q_H - c_H + v}{p - c_H + v} - 1 \right)}.$$

Clearly, this function does not depend on λ . Moreover, as the expression under the root is strictly positive for any $p < q_H$ (as established in the proof of Proposition 5), letting N tend towards infinity leads to $G_H(p) = 0$ for every $p < q_H$ (this is because $\lim_{N \rightarrow \infty} z^{1/N} = 1$ for all $z > 0$). In contrast, since the expression under the root is equal to zero for $p = q_H$, we have that $G_H(q_H) = 1$. This, in conjunction with the fact that $\lim_{N \rightarrow \infty} \alpha^* = 1$ shows that the firms' pricing strategy converges in distribution to the pure strategy of pricing at q_H as N tends to infinity.

Consider $G_L(p)$ next. As $\lim_{\lambda \rightarrow 1} \alpha^* = 0$, we have that $\lim_{\lambda \rightarrow 1} G_L(p) = 1$ for all $p > \underline{p}_L$. This implies that conditional on stocking q_L , a firms' pricing strategy converges in distribution to the pure strategy of pricing at $\lim_{\lambda \rightarrow 1} \underline{p}_L = c_L - v$ as the number of informed consumers in the market tends to one. Finally, by extending its denominator and inserting α^* , we can rewrite $G_L(p)$ as

$$G_L(p) = \frac{1 - \sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left[\frac{q_H - c_H + v}{p - c_L + v} - 1 \right]}}{1 - \sqrt[N-1]{\frac{1-\lambda}{\lambda N} \left[\frac{\Delta_q - \Delta_c}{\frac{q_L}{q_H} (c_H - v) - (c_L - v)} \right]}} = \frac{1 - \sqrt[N-1]{\frac{x}{N}}}{1 - \sqrt[N-1]{\frac{y}{N}}},$$

where $x > 0$ follows from $p \leq \bar{p}_L$ and $\Delta_q > \Delta_c$, and $y > 0$ was already shown in the proof of Proposition 5. Applying L'Hôpital's rule we find that

$$\lim_{N \rightarrow \infty} G_L(p) = \lim_{N \rightarrow \infty} \left(\frac{x}{y} \right)^{\frac{1}{N-1}} \left[\frac{N - 1 + N \log \left(\frac{x}{N} \right)}{N - 1 + N \log \left(\frac{y}{N} \right)} \right].$$

By further applying L'Hôpital's rule twice to $\lim_{N \rightarrow \infty} \frac{N-1+N \log \left(\frac{x}{N} \right)}{N-1+N \log \left(\frac{y}{N} \right)}$, we can establish that the limit is 1. As also $\lim_{N \rightarrow \infty} \left(\frac{x}{y} \right)^{\frac{1}{N-1}} = 1$, we can therefore conclude that $\lim_{N \rightarrow \infty} G_L(p) = 1$ for all $p > \underline{p}_L$ in a low-quality firm's support. Hence, as the number of firms in the market tends to infinity, a low-quality retailer's pricing strategy converges in distribution to choosing $\lim_{N \rightarrow \infty} \underline{p}_L = c_L - v$ with probability one **Q.E.D.**

Proof of Proposition 6. Consider first product choice. The result follows immediately after recognizing that the intervention constrains the price choice for the high-quality product less than that of the low-quality product, i.e., the respective retailer can always still make a more attractive offer when it sells the high-quality product than when it sells the low-quality product.

We turn now to manufacturer profits, noting that below, where we characterize fully also the retailers' pricing equilibrium, we will add additional details. Note first that a

retailer's deviation profit (from not choosing high quality) is still $\frac{1-\lambda}{N}(q_L - c_L + v)$, as without policy intervention. The (on-equilibrium) profit that the retailer makes (gross of the fixed fee that it pays to the manufacturer) is only affected by the intervention when this indeed constrains the lowest price. By standard arguments, there are then two cases to consider. The first case is that where the retailer is indifferent between choosing a lower price (selling also with some probability to shoppers) and $p_n = q_H$ (selling only to non-shoppers). In this case, the manufacturer profit is clearly the same as without intervention. The second case is that where all retailers choose the lowest possible price $p_n = c_H$ with probability one. The resulting profit $\frac{v}{N}$ exceeds $(q_H - c_H + v)\frac{1-\lambda}{N}$ if and only if $v \geq \bar{v}$. Also in light of the proof of subsequent propositions, we add more structure to these results by characterizing the pricing equilibrium:

Lemma 4 *Take the baseline model without salient thinkers and suppose that below-cost pricing is prohibited. Then for $v \leq \underline{v}$, the prohibition does not bind and thus does not affect the set of pricing equilibria. For $v > \underline{v}$, we have the following characterization:*

i) When $\underline{v} < v < \bar{v}$, then there exists a (for $N = 2$ unique) symmetric pricing equilibrium in which retailers sample c_H with probability $\beta^ \in (0, 1)$, whereas with the remaining probability, they sample prices continuously from the CDF*

$$G_r(p) := 1 - \frac{N^{-1} \sqrt{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_H + v} - 1 \right)}}{1 - \beta^*} \quad (34)$$

with support $[\underline{p}_r, q_H]$, where β^ is defined implicitly by the unique solution to*

$$\frac{1 - (1 - \beta)^N}{\beta} = \frac{1 - \lambda}{\lambda v} (q_H - c_H), \quad (35)$$

and

$$\underline{p}_r := c_H - v + \frac{q_H - c_H + v}{1 + \frac{\lambda N}{1-\lambda} (1 - \beta^*)^{N-1}} \in (c_H, q_H). \quad (36)$$

ii) When $v \geq \bar{v}$, all retailers set $p_n = c_H$.

Proof of Lemma 4. Recall that $\underline{p} < c_H$ holds if and only if $v > \underline{v}$. Since in any pricing equilibrium with high-quality products, no matter whether symmetric or asymmetric, no firm samples prices below \underline{p} , the set of pricing equilibria is clearly not affected if $v \leq \underline{v}$.

Suppose next that $v \geq \bar{v}$. If all retailers choose $p_n = c_H$, each makes a profit of $\frac{v}{N}$. Note that here and in what follows, the considered profits are gross of the fixed payment to the manufacturer. Deviations downward are impossible due to the price floor and the

best upward deviation is to q_H , which realizes $(q_H - c_H + v)\frac{1-\lambda}{N}$ and does not exceed $\frac{v}{N}$ if and only if $v \geq \bar{v}$.

To show uniqueness of this equilibrium, assume the contrary, such that at least one retailer's (possibly mixed) pricing strategy does not put all probability on c_H . Denote the largest upper support of retailers' pricing strategy by $\hat{p} \in (c_H, q_H]$. Clearly, it cannot be part of an equilibrium that multiple retailers have a mass point at \hat{p} , which implies that there must be at least one retailer that never attracts the shoppers when sampling \hat{p} , thereby realizing profits

$$(\hat{p} - c_H + v)\frac{1-\lambda}{N} \leq (q_H - c_H + v)\frac{1-\lambda}{N}.$$

Given this inequality, by deviating to c_H this retailer could for $v > \bar{v}$ strictly increase its profit to at least $\frac{v}{N}$ (as at worst, all other retailers charge c_H for sure).

Suppose finally that $v \in (\underline{v}, \bar{v})$. It is straightforward to verify that $p_n = c_H$ and $p_n \in [\underline{p}_r, q_H]$ all yield the same profit of $\frac{1-\lambda}{N}(q_H - c_H + v)$. Precisely, for \underline{p}_r note that the expected profit is $(\underline{p}_r - c_H + v) \left[\frac{1-\lambda}{N} + \lambda(1-\beta)^{N-1} \right]$, as shoppers can only be attracted if none of the $N-1$ rivals samples c_H . And with $p \in (\underline{p}_r, q_H)$, the expected profit is $(p - c_H + v) \left\{ \frac{1-\lambda}{N} + \lambda [(1-\beta)(1-G_r(p))]^{N-1} \right\}$, as shoppers can only be attracted if all rivals sample a price above p . Provided that $\beta \in (0, 1)$, which will be verified below, $G_r(p)$ in (34) is strictly increasing in p , with $G_r(\underline{p}_r) = 0$ and $G_r(q_H) = 1$. Finally, if a retailer samples c_H , its expected profit can be written as

$$\Pi_i(c_H) = (c_H - c_H + v) \left[\frac{1-\lambda}{N} + \sum_{j=0}^{N-1} \binom{N-1}{j} \beta^j (1-\beta)^{N-1-j} \frac{\lambda}{j+1} \right],$$

as it has to share the shoppers with $j \in \{0, \dots, N-1\}$ rivals, which happens with probability $\binom{N-1}{j} \beta^j (1-\beta)^{N-1-j}$, respectively. Using that

$$\sum_{j=0}^{N-1} \binom{N-1}{j} \beta^j (1-\beta)^{N-1-j} \frac{1}{j+1} = \frac{1}{\beta N} [1 - (1-\beta)^N],$$

which follows from the binomial theorem, this uniquely pins down $\beta = \beta^*$. (Note that the left-hand side of (35) is strictly decreasing in β for $\beta \in (0, 1)$, as it can be rewritten as $f(\beta) = \sum_{k=0}^{N-1} (1-\beta)^k$.)

Note finally that no retailer can profitably deviate to $p_n \in (c_H, \underline{p}_r)$ as, by construction, a strictly higher profit is realized with \underline{p}_r instead. **Q.E.D.**

Proof of Proposition 7. We prove instead the following result, which describes in more detail the different regimes. The respective details will be used below.

Claim. *If consumers are salient thinkers, a prohibition of below-cost pricing has the following consequences for retailers' product choice:*

(I) *If $\frac{q_H}{q_L} > \frac{c_H}{c_L}$, in equilibrium all retailers stock the high-quality product.*

(II) *If instead $\frac{q_H}{q_L} < \frac{c_H}{c_L}$, there are four different subcases:*

(IIa) *If $\delta\Delta_q \geq \Delta_c$ or $v \leq \tilde{v} := c_H + \frac{(1-\lambda)q_H - \frac{\Delta_c}{\lambda}q_H(1-\lambda+\lambda N)}{\lambda N} < \underline{v}$ (or both), retailers always stock the high-quality product.*

(IIb) *If $\delta\Delta_q < \Delta_c$ and $\tilde{v} < v \leq \hat{v}$, where $\hat{v} \in (\underline{v}, \bar{v})$ is defined implicitly by the unique solution to*

$$v \left(\frac{1 - [\alpha^*(v)]^N}{1 - \alpha^*(v)} \right) = \frac{1 - \lambda}{\lambda} (q_H - c_H),$$

retailers stock the high-quality product with probability $\alpha^(v) \in (0, 1)$ (as defined in Proposition 5), while they stock the low-quality product with complementary probability. Moreover, $\alpha^*(v)$ is strictly decreasing in v .*

(IIc) *If $\delta\Delta_q < \Delta_c$ and $\hat{v} < v < \bar{v}$, retailers stock the high-quality product with probability $\tilde{\alpha}(v) \in (0, \alpha^*(v))$, where $\tilde{\alpha}(v)$ is defined implicitly by the unique solution to*

$$\frac{1 - \alpha^N}{1 - \alpha} = \frac{1 - \lambda}{\lambda v} (q_H - c_H),$$

while they stock the low-quality product with complementary probability. With $\tilde{\alpha}(\hat{v}) = \alpha^(\hat{v})$ and $\tilde{\alpha}(\bar{v}) = 0$, it holds that $\tilde{\alpha}(v) < \alpha^*(v)$ for all $v \in (\hat{v}, \bar{v}]$.*

(IId) *Finally, if $\delta\Delta_q < \Delta_c$ and $v \geq \bar{v}$, retailers always stock the low-quality product.*

The claim is proven by a series of lemmas.

Lemma 5 *If $\frac{q_H}{q_L} > \frac{c_H}{c_L}$ and below-cost pricing is prohibited, a high-quality equilibrium always exists.*

Proof of Lemma 5. In what follows, we assume that the high-quality equilibrium of Lemma 4 is played, and then prove that deviating to low quality, combined with an optimal deviation price, does not pay. Note that since a deviating low-quality firm's offer can never attract the shoppers if quality is salient, it is sufficient to check that no retailer has an incentive to deviate if it only needs to make price salient in order to attract the shoppers. In order to show this, we have to consider three cases: (i) $v \geq \bar{v}$, (ii) $v \in (\underline{v}, \bar{v})$, and (iii) $v \leq \underline{v}$.

(i) If $v \geq \bar{v}$, any high-quality equilibrium is characterized by all firms charging c_H deterministically, giving rise to an equilibrium profit of $\frac{v}{N}$. A deviating retailer can only

attract the shoppers with his deviation price p_{dev} if $\frac{c_H}{p_{dev}} > \frac{q_H}{q_L}$ (in order to make price salient). This requires $p_{dev} < \frac{q_L}{q_H}c_H$, which is however no longer feasible as $\frac{q_L}{q_H}c_H < c_L$ (as follows from $\frac{q_H}{q_L} > \frac{c_H}{c_L}$). A deviating retailer can therefore only target non-shoppers, which yields less than $\frac{v}{N}$ due to $v \geq \bar{v}$.

(ii) If $v \in (\underline{v}, \bar{v})$, symmetric high-quality equilibria are characterized by retailers sampling c_H with positive probability $\beta^* \in (0, 1)$, while they draw prices continuously from a compact interval $[\underline{p}_r, q_H]$ ($\underline{p}_r > c_H$) with remaining probability. Now it is possible for a deviating retailer to attract shoppers with positive probability. More precisely, a deviating retailer's expected profit when charging a deviation price $p_{dev} \in [\frac{q_L}{q_H}\underline{p}_r, q_L]$ (provided that this is not below costs of c_L) is given by

$$\Pi_{dev}(p_{dev}) = (p_{dev} - c_L + v) \left\{ \frac{1 - \lambda}{N} + \lambda \left[(1 - \beta^*) \left(1 - G_r \left(\frac{q_H}{q_L} p_{dev} \right) \right) \right]^{N-1} \right\},$$

as the shoppers can only be attracted (price can only be made salient) if all rivals price above $\frac{q_H}{q_L} p_{dev}$, which happens with probability $\left[(1 - \beta^*) (1 - G_r(\frac{q_H}{q_L} p_{dev})) \right]^{N-1}$. Inserting $G_r(\cdot)$ from Lemma 4, the above profit function simplifies to

$$\Pi_{dev}(p_{dev}) = (p_{dev} - c_L + v) \frac{1 - \lambda}{N} \left(\frac{q_H - c_H + v}{\frac{q_H}{q_L} p_{dev} - c_H + v} \right),$$

which is strictly increasing (decreasing) in p_{dev} if $v < \frac{q_H c_L - c_H q_L}{q_H - q_L}$ ($v > \frac{q_H c_L - c_H q_L}{q_H - q_L}$). If it is strictly increasing in p_{dev} , the optimal deviation price is q_L for a maximal deviation profit of $\frac{1-\lambda}{N}(q_L - c_L + v)$, which is less than the candidate equilibrium profit. If it is strictly decreasing in p_{dev} , the optimal deviation price is $\frac{q_L}{q_H}\underline{p}_r$ and yields an expected profit of

$$\left(\frac{q_L}{q_H}\underline{p}_r - c_L + v \right) \frac{1 - \lambda}{N} \left(\frac{q_H - c_H + v}{\underline{p}_r - c_H + v} \right).$$

Comparing this with the candidate equilibrium, deviating would only pay if $\underline{p}_r < \frac{q_H(c_H - c_L)}{q_H - q_L}$. However, this cannot be the case, as $\underline{p}_r > c_H$, but $\frac{q_H(c_H - c_L)}{q_H - q_L} < c_H$ for $\frac{q_H}{q_L} > \frac{c_H}{c_L}$.

(iii) If $v \leq \underline{v}$, the regulation does not affect on-equilibrium pricing. Furthermore, from Proposition 2 we know that if any deviation was possible, the high-quality equilibrium would exist if $\Delta_c \leq \frac{\Delta_q}{q_H}\underline{p}_r$, which is equivalent to

$$v \leq \tilde{v} := c_H + \frac{(1 - \lambda)q_H - \frac{\Delta_c}{\Delta_q}q_H(1 - \lambda + \lambda N)}{\lambda N}.$$

But since $\underline{v} < \tilde{v}$ for $\frac{q_H}{q_L} > \frac{c_H}{c_L}$, this must always be satisfied in the considered case. **Q.E.D.**

We proceed with the complimentary case where $\frac{q_H}{q_L} < \frac{c_H}{c_L}$, and first show under which circumstances a high-quality equilibrium still exists.

Lemma 6 *If $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and below-cost pricing is prohibited, a high-quality equilibrium, as characterized by Lemma 4, exists if and only if $\delta\Delta_q \geq \Delta_c$ or $v \leq \tilde{v}$.*

Proof of Lemma 6. We first prove existence of a high-quality equilibrium if either $\delta\Delta_q \geq \Delta_c$ or $v \leq \tilde{v}$. Start with the former case, such that v can take an arbitrary value. If $v \geq \bar{v}$, we know from Lemma 4 that in a high-quality equilibrium, all retailers must charge c_H deterministically, for an equilibrium profit of $\frac{v}{N}$. From $\delta\Delta_q \geq \Delta_c$, it follows immediately that deviations which attract the shoppers are impossible, since doing so requires pricing below $c_H - \delta\Delta_q$ (in order for the competition constraint to be satisfied), which falls short of c_L and is therefore prohibited. Hence, the optimal deviation price is q_L , which yields lower profits due to $v \geq \bar{v}$. If $v \leq \underline{v}$, price setting in a high-quality equilibrium is not constrained. Then, at worst, the optimal deviation price is not constrained either. But even if this is true, we know from Proposition 2 that a high-quality equilibrium exists for $\delta\Delta_q \geq \Delta_c$.

Consider finally the intermediate case in which $v \in (\underline{v}, \bar{v})$. Lemma 4 characterizes a hypothetical high-quality equilibrium in this case. A retailer's expected deviation profit when charging a (feasible) deviation price p_{dev} is given by

$$\begin{aligned} \Pi_{dev}(p_{dev}) &= (p_{dev} - c_L + v) \cdot \\ &\quad \left\{ \frac{1 - \lambda}{N} + \lambda \left[(1 - \beta^*) \left(1 - G_r \left(\max \left\{ \frac{q_H}{q_L} p_{dev}, p_{dev} + \delta\Delta_q \right\} \right) \right) \right]^{N-1} \right\} \\ &= (p_{dev} - c_L + v) \frac{1 - \lambda}{N} \left(\frac{q_H - c_H + v}{\max \left\{ \frac{q_H}{q_L} p_{dev}, p_{dev} + \delta\Delta_q \right\} - c_H + v} \right), \end{aligned}$$

where the maximum operator ensures that both the salience constraint and competition constraint are satisfied, and the second line follows from inserting $G_r(\cdot)$ and simplifying. Using this, it is not hard to show that, for the given parameter range, the optimal deviation price is interior and equals δq_L . Hence, if this price is even feasible in face of the regulation, the optimal deviation profit is given by

$$(\delta q_L - c_L + v) \frac{1 - \lambda}{N} \left(\frac{q_H - c_H + v}{\delta q_H - c_H + v} \right).$$

It is then straightforward to show that this optimal deviation profit falls short of the candidate equilibrium's profit of $\frac{1-\lambda}{N}(q_H - c_H + v)$ as $\delta\Delta_q \geq \Delta_c$.

We check next possible deviations when $v \leq \tilde{v}$. Note then that since $\tilde{v} < \underline{v}$ for $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ (compare with the opposite case in the proof of Lemma 5 above), on-equilibrium pricing is not affected under the regulation. Hence, at worst the regulation also doesn't restrict the set of optimal deviations, as was true in the baseline model with salience. From Proposition 2 it thus follows that a high-quality equilibrium exists if $\Delta_c \leq \frac{\Delta_q}{q_H} \underline{p}$, which is equivalent to $v \leq \tilde{v}$.

We will proceed to show that no high-quality equilibrium can exist in the complementary case where both $\delta\Delta_q < \Delta_c$ and $v > \tilde{v}$. If $v \in (\tilde{v}, \underline{v}]$, the equilibrium pricing of a high-quality equilibrium would still be unaffected, as high-quality firms' lowest price \underline{p} would satisfy $\underline{p} \geq c_H$. Then there are two cases. If $\delta \leq \frac{p}{q_H}$, a deviating retailer can attract shoppers by pricing at $\underline{p} \frac{q_L}{q_H} - \epsilon$ (as the salience constraint is binding for low δ). This price is feasible because $\underline{p} \frac{q_L}{q_H} > c_L$ due to $\underline{p} \geq c_H$ and $\frac{q_H}{q_L} < \frac{c_H}{c_L}$. It is then easy to show that the resulting deviation profit $\Pi_{dev} = \left(\underline{p} \frac{q_L}{q_H} - c_L + v \right) \left(\frac{1-\lambda}{N} + \lambda \right)$ exceeds the profit $(q_H - c_H + v) \frac{1-\lambda}{N} = (\underline{p} - c_H + v) \left(\frac{1-\lambda}{N} + \lambda \right)$ in a hypothetical high-quality equilibrium if $\Delta_c > \underline{p} \frac{\Delta_q}{q_H}$, i.e., if $v > \tilde{v}$. Second, if it instead holds that $\delta > \frac{p}{q_H}$, a deviating retailer can attract all shoppers by pricing at $\underline{p} - \delta\Delta_q - \epsilon$ (as the competition constraint is binding for high δ). This price is feasible because $\underline{p} - \delta\Delta_q > c_L$ due to $\underline{p} \geq c_H$ and $\delta\Delta_q < \Delta_c$. The corresponding deviation profit of $\Pi_{dev} = (\underline{p} - \delta\Delta_q - c_L + v) \left(\frac{1-\lambda}{N} + \lambda \right)$ again exceeds the hypothetical equilibrium profit in a high-quality equilibrium, provided that $\delta\Delta_q < \Delta_c$.

We now show that no high-quality equilibrium can exist for $v \in (\underline{v}, \bar{v})$. This is because, although the regulation becomes binding and retailers' pricing in a hypothetical high-quality equilibrium becomes restricted to prices at or above c_H , retailers' expected profit stays at $(q_H - c_H + v) \frac{1-\lambda}{N}$ (compare with Lemma 4 above). Hence, similar to the case where $v \in (\tilde{v}, \underline{v}]$ discussed before, if $\delta \leq \frac{c_H}{q_H}$, a deviating retailer can attract all shoppers by pricing at $c_H \frac{q_L}{q_H} - \epsilon$, which is permissible as $c_H \frac{q_L}{q_H} > c_L$. It is then easy to show that the resulting deviation profit of $\left(c_H \frac{q_L}{q_H} - c_L + v \right) \left(\frac{1-\lambda}{N} + \lambda \right)$ strictly exceeds $(q_H - c_H + v) \frac{1-\lambda}{N}$ for $v > \underline{v}$. If it holds in contrast that $\delta > \frac{c_H}{q_H}$, a deviating retailer can attract all shoppers by pricing at $c_H - \delta\Delta_q - \epsilon$, which is feasible because $c_H - \delta\Delta_q > c_L$. The corresponding deviation profit of $(c_H - \delta\Delta_q - c_L + v) \left(\frac{1-\lambda}{N} + \lambda \right)$ also strictly exceeds $(q_H - c_H + v) \frac{1-\lambda}{N}$, as follows from $\delta\Delta_q < \Delta_c$ and $v > \underline{v}$.

Observe finally that a high-quality equilibrium also cannot exist for $v \geq \bar{v}$ and $\delta\Delta_q < \Delta_c$. In this case, the high-quality retailers would charge c_H deterministically for a profit of $\frac{v}{N}$ (see Lemma 4). If $\delta \leq \frac{c_H}{q_H}$, deviating to low quality with a deviation price of $\frac{q_L}{q_H} c_H - \epsilon > c_L$ would attract all shoppers and give a deviation profit of $\left(\frac{q_L}{q_H} c_H - c_L + v \right) \left(\frac{1-\lambda}{N} + \lambda \right)$, which

clearly exceeds $\frac{v}{N}$. If instead $\delta > \frac{c_H}{q_H}$, deviating to low quality with a deviation price of $c_H - \delta\Delta_q - \epsilon > c_L$ would result in a deviation profit of $(c_H - \delta\Delta_q - c_L + v) \left(\frac{1-\lambda}{N} + \lambda\right)$, which is also larger than $\frac{v}{N}$ because $\delta\Delta_q < \Delta_c$. **Q.E.D.**

The following sequence of lemmas characterizes the symmetric equilibrium in product choice and pricing under the regulation if $\frac{q_H}{q_L} < \frac{c_H}{c_L}$, $\delta\Delta_q < \Delta_c$, and $v > \tilde{v}$ such that no high-quality equilibrium exists.

Lemma 7 *If $\frac{q_H}{q_L} < \frac{c_H}{c_L}$, $\delta\Delta_q < \Delta_c$, and $v \in (\tilde{v}, \underline{v}]$, the equilibrium is still characterized by Proposition 5 and Corollary 4.*

Proof of Lemma 7. Recall that in this candidate equilibrium, the lowest price a low-quality firm samples is $\underline{p}_L = c_L - v + (q_H - c_H + v) \frac{1-\lambda}{1-\lambda+\lambda N}$, while the lowest price a high-quality firm samples is $\underline{p}_H = \frac{\Delta c}{\Delta q} q_H$. Observe first that \underline{p}_H exceeds c_H for every v , so the regulation clearly does not bind for high-quality firms. And it also does not bind for low-quality firms, provided that $v \leq \underline{v}$. Hence, as the equilibrium pricing is not affected for $v \in (\tilde{v}, \underline{v}]$, which was part of an equilibrium without the regulation (see the proof of Proposition 5), the corresponding strategy-combination still constitutes an equilibrium with the regulation. **Q.E.D.**

Lemma 8 *If $\frac{q_H}{q_L} < \frac{c_H}{c_L}$, $\delta\Delta_q < \Delta_c$, and $v \geq \bar{v}$, all retailers choose the low-quality product and set $p_n = c_L$.*

Proof of Lemma 8. Note that each retailer makes a profit of $\frac{v}{N}$ in this candidate equilibrium. As further undercutting is impossible, the best possible deviation while keeping low-quality is to price at q_L for a profit of $(q_L - c_L + v) \frac{1-\lambda}{N}$, which falls short of $\frac{v}{N}$ for $v \geq \bar{v}$. Deviating to high-quality while rendering quality salient is impossible, as the highest such price, $p_{dev} = \frac{q_H}{q_L} c_L$ is below c_H . With a price of q_H (in order to fully exploit its loyal consumers) the expected profit $(q_H - c_H + v) \frac{1-\lambda}{N}$ does not exceed $\frac{v}{N}$ if $v \geq \bar{v}$. The other possibly optimal deviation is to charge $c_L + \delta\Delta_q$, which is the highest price that attracts shoppers although price, rather than quality, is salient. However, this price is below c_H if $\delta\Delta_q < \Delta_c$. **Q.E.D.**

The following technical lemma is needed for a characterization of the remaining case where $\frac{q_H}{q_L} < \frac{c_H}{c_L}$, $\delta\Delta_q < \Delta_c$, and $v \in (\underline{v}, \bar{v})$.

Lemma 9 *There exists a unique $\hat{v} \in (\underline{v}, \bar{v})$ such that*

$$v \left(\frac{1 - [\alpha^*(v)]^N}{1 - \alpha^*(v)} \right) = \frac{1 - \lambda}{\lambda} (q_H - c_H), \quad (37)$$

where

$$\alpha^*(v) = \sqrt[N-1]{\frac{1 - \lambda}{\lambda N} \left[\frac{\Delta_q - \Delta_c}{\frac{q_L}{q_H} (c_H - v) - (c_L - v)} \right]},$$

as defined in Proposition 5.

Proof of Lemma 9. Note first that the RHS of equation (37) is independent of v . Hence, it is sufficient to prove that (1) $\underline{v} \left(\frac{1 - [\alpha^*(\underline{v})]^N}{1 - \alpha^*(\underline{v})} \right) < \frac{1 - \lambda}{\lambda} (q_H - c_H)$, (2) $\bar{v} \left(\frac{1 - [\alpha^*(\bar{v})]^N}{1 - \alpha^*(\bar{v})} \right) > \frac{1 - \lambda}{\lambda} (q_H - c_H)$, and (3) $v \left(\frac{1 - [\alpha^*(v)]^N}{1 - \alpha^*(v)} \right)$ is strictly increasing in v over the relevant range. For (1), note that since $\tilde{v} < \underline{v}$, $\alpha^*(\tilde{v}) = 1$, $\alpha^*(v)$ is strictly decreasing in v , and $\frac{1 - \alpha^N}{1 - \alpha}$ is strictly increasing in α , the LHS for $v = \underline{v}$ must fall short of $\underline{v} \left(\lim_{\alpha \rightarrow 1} \frac{1 - \alpha^N}{1 - \alpha} \right) = \underline{v}N$, which is the RHS of equation (37). For (2), note that $\frac{1 - \alpha^N}{1 - \alpha}$ strictly exceeds 1 for all $\alpha \in (0, 1)$. Hence, the LHS of equation (37) for $v = \bar{v}$ must exceed \bar{v} , which is the RHS of the equation.

For (3), it is first useful to recall the definition of $\alpha^*(v)$, which is given by the solution of

$$\Pi_H(\underline{p}_H) = \left(\frac{\Delta_c}{\Delta_q} q_H - c_H + v \right) \left(\frac{1 - \lambda}{N} + \lambda \alpha^{N-1} \right) \stackrel{!}{=} \frac{1 - \lambda}{N} (q_H - c_H + v).$$

Implicit differentiation establishes that

$$\frac{d\alpha^*(v)}{dv} = - \frac{\alpha^*(v)}{\left(\frac{\Delta_c}{\Delta_q} q_H - c_H + v \right) (N - 1)}.$$

As $v \left(\frac{1 - [\alpha^*(v)]^N}{1 - \alpha^*(v)} \right)$ can be rewritten as $v \sum_{j=0}^{N-1} \alpha^*(v)^j$, the derivative of the latter with respect to v is then given by

$$\begin{aligned} & \sum_{j=0}^{N-1} \alpha^*(v)^j + v \sum_{j=0}^{N-1} j \alpha^*(v)^{j-1} \frac{d\alpha^*(v)}{dv} \\ &= \sum_{j=0}^{N-1} \alpha^*(v)^j - v \sum_{j=0}^{N-1} j \alpha^*(v)^{j-1} \left(\frac{\alpha^*(v)}{\left(\frac{\Delta_c}{\Delta_q} q_H - c_H + v \right) (N - 1)} \right) \\ &> \sum_{j=0}^{N-1} \alpha^*(v)^j - v \sum_{j=0}^{N-1} j \alpha^*(v)^j \left(\frac{1}{v(N - 1)} \right) \\ &> \sum_{j=0}^{N-1} \alpha^*(v)^j - \sum_{j=0}^{N-1} (N - 1) \alpha^*(v)^j \left(\frac{1}{N - 1} \right) = 0, \end{aligned}$$

where the third line follows from $\frac{q_H}{q_L} < \frac{c_H}{c_L}$. This finalizes the proof of Lemma 9. **Q.E.D.**

Lemma 10 *If $\frac{q_H}{q_L} < \frac{c_H}{c_L}$, $\delta\Delta_q < \Delta_c$, and $v \in (\underline{v}, \hat{v})$, the following constitutes an equilibrium. With probability $\alpha^* \in (0, 1)$, as defined in Proposition 5, retailers choose the high-quality product and sample prices according to the CDF $G_H(p)$, again as defined in Proposition 5. With probability $1 - \alpha^*$, retailers choose the low-quality product. If they do so, they choose $p_n = c_L$ with probability $\tilde{\beta}(v) \in (0, 1)$, where $\tilde{\beta}(v)$ is defined implicitly by*

$$v \left(\frac{1 - \left[1 - (1 - \alpha^*(v))\tilde{\beta} \right]^N}{(1 - \alpha^*(v))\tilde{\beta}} \right) = \frac{1 - \lambda}{\lambda} (q_H - c_H), \quad (38)$$

with $\tilde{\beta}(\underline{v}) = 0$, $\tilde{\beta}(\hat{v}) = 1$, and $\tilde{\beta}'(v) > 0$. With the remaining probability $1 - \tilde{\beta}(v)$, low-quality retailers sample prices continuously from a CDF $G_{L,r}(p)$ with support $[\underline{p}_{L,r}, \frac{\Delta_c}{\Delta_q} q_L]$, where

$$\underline{p}_{L,r} := c_L - v + \frac{(q_H - c_H + v) \frac{1-\lambda}{N}}{\frac{1-\lambda}{N} + \lambda \left[1 - (1 - \alpha^*(v))\tilde{\beta}(v) \right]^{N-1}} \in \left(c_L, \frac{\Delta_c}{\Delta_q} q_L \right),$$

and

$$G_{L,r}(p) := 1 - \frac{N^{-1} \sqrt{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_L + v} - 1 \right)} - \alpha^*(v)}{(1 - \alpha^*(v)) (1 - \tilde{\beta}(v))}.$$

Proof of Lemma 10. Because the lower support bound of high-quality firms (which is given by $\frac{\Delta_c}{\Delta_q} q_H$ and thus strictly exceeds c_H , as follows from $\frac{q_H}{q_L} < \frac{c_H}{c_L}$) and the upper support bound of low-quality firms are the same as in the mixed-product equilibrium without the regulation (see Proposition 5 and Corollary 4), price will always be salient and high-quality firms cannot serve shoppers if both low-quality and high-quality products are introduced to the market. Hence, by the construction in Proposition 5 and Corollary 4, high-quality firms are still indifferent between sampling any price in their specified support, and make an expected profit of $(q_H - c_H + v) \frac{1-\lambda}{N}$. A low-quality firm's expected profit when sampling $\bar{p}_L = \frac{\Delta_c}{\Delta_q} q_L$ is $(\frac{\Delta_c}{\Delta_q} q_L - c_L + v) \left(\frac{1-\lambda}{N} + \lambda(\alpha^*)^{N-1} \right)$, as it can only attract the shoppers if all of its rivals stock q_H . By construction of α^* , this also yields an expected profit of $(q_H - c_H + v) \frac{1-\lambda}{N}$. If a low-quality firm samples $\underline{p}_{L,r}$, its expected profit is given by

$$\Pi_L(\underline{p}_{L,r}) = (\underline{p}_{L,r} - c_L + v) \left\{ \frac{1 - \lambda}{N} + \lambda \left[\alpha^* + (1 - \alpha^*) (1 - \tilde{\beta}) \right]^{N-1} \right\},$$

as it can only attract the shoppers if all rivals either stock q_H , or stock q_L , but do not sample c_L . Setting this equal to $(q_H - c_H + v) \frac{1-\lambda}{N}$, we find $\underline{p}_{L,r}$. If a low-quality firm

samples an arbitrary price p in its support, its expected profit is given by

$$(p - c_L + v) \left\{ \frac{1 - \lambda}{N} + \lambda \left[\alpha^* + (1 - \alpha^*) (1 - \tilde{\beta}) (1 - G_{L,r}(p)) \right]^{N-1} \right\},$$

as it can only attract shoppers if all of its rivals either choose q_H , or choose q_L but do not charge a lower price than p . Setting this equal to $(q_H - c_H + v) \frac{1-\lambda}{N}$, we find the CDF $G_{L,r}(p)$ reported in the lemma.

If a low-quality firm samples c_L , its expected profit is

$$\begin{aligned} \Pi_L(c_L) &= (c_L - c_L + v) \left\{ \frac{1 - \lambda}{N} + \sum_{j=0}^{N-1} \binom{N-1}{j} \left[(1 - \alpha^*) \tilde{\beta} \right]^j \left[1 - (1 - \alpha^*) \tilde{\beta} \right]^{N-1-j} \frac{\lambda}{j+1} \right\} \\ &= v \left[\frac{1 - \lambda}{N} + \lambda \left(\frac{1 - \left[1 - (1 - \alpha^*) \tilde{\beta} \right]^N}{N(1 - \alpha^*) \tilde{\beta}} \right) \right], \end{aligned}$$

as it has to share the shoppers with $j \in \{0, \dots, N-1\}$ rivals which also sample c_L with probability $\binom{N-1}{j} \left[(1 - \alpha^*) \tilde{\beta} \right]^j \left[1 - (1 - \alpha^*) \tilde{\beta} \right]^{N-1-j}$, and where the second line is obtained by proper manipulation and making use of the binomial theorem. Setting this equal to $(q_H - c_H + v) \frac{1-\lambda}{N}$ and simplifying, we find the implicit definition of $\tilde{\beta}(v)$ as provided in the lemma.

Taken together, each price in retailers' support thus indeed yields the same expected profit. Further, low-quality retailers have no profitable deviating price as pricing below c_L is prohibited, pricing between c_L and $\underline{p}_{L,r}$ is strictly dominated by pricing at $\underline{p}_{L,r}$, and pricing above their upper support bound $\frac{\Delta_c}{\Delta_q} q_L$ was already shown to be inferior in the proof of Proposition 5, where the high-quality product is stocked with the same probability and high-quality firms use the same strategy as in the present candidate equilibrium. Deviating high-quality firms can never guarantee to attract all shoppers, as they would have to price below $c_L \frac{q_H}{q_L}$, which falls short of c_H . If a high-quality firm deviates to $p_{dev} \in [\underline{p}_{L,r} \frac{q_H}{q_L}, \frac{\Delta_c}{\Delta_q} q_H)$, its expected profit is given by

$$\begin{aligned} \Pi_H(p_{dev}) &= (p_{dev} - c_H + v) \cdot \\ &\quad \left\{ \frac{1 - \lambda}{N} + \lambda \left[\alpha^* + (1 - \alpha^*) (1 - \tilde{\beta}) \left(1 - G_{L,r} \left(\min \left\{ p_{dev} \frac{q_L}{q_H}, p_{dev} - \delta \Delta_q \right\} \right) \right) \right]^{N-1} \right\} \\ &= (p_{dev} - c_H + v) \frac{1 - \lambda}{N} \left(\frac{q_H - c_H + v}{\min \left\{ p_{dev} \frac{q_L}{q_H}, p_{dev} - \delta \Delta_q \right\} - c_L + v} \right), \end{aligned}$$

where the second line follows from inserting $G_{L,r}(\cdot)$, as found in the lemma, and simplifying.

(Note that the term $\min \{ p_{dev} \frac{q_L}{q_H}, p_{dev} - \delta \Delta_q \}$ appears because a deviating high-quality firm

can win the shoppers if either quality is salient, or price is salient but its offer still provides a higher perceived utility). The derivative of $\Pi_H(p_{dev})$ with respect to p_{dev} is strictly positive for all p_{dev} , which implies that also firms with q_H have no profitable deviation.

In the proof of Proposition 5, it has already been established that α^* and $G_H(p)$ are well-behaved. Furthermore, provided that $\tilde{\beta}(v) \in [0, 1)$, $G_{L,r}(p)$ is also well-behaved, as it is strictly increasing in p , with $G_{L,r}(\underline{p}_{L,r}) = 0$ and $G_{L,r}\left(\frac{\Delta_c}{\Delta_q} q_L\right) = 1$. It remains to show that $\tilde{\beta}(v)$ and $\underline{p}_{L,r}$ are well-behaved, where for convenience we restate the implicit definition of $\tilde{\beta}(v)$:

$$v \left(\frac{1 - [1 - (1 - \alpha^*(v))\tilde{\beta}]^N}{(1 - \alpha^*(v))\tilde{\beta}} \right) = \frac{1 - \lambda}{\lambda}(q_H - c_H).$$

Note that for $v = \underline{v}$, the above equation can only be satisfied if $(1 - \alpha^*)\tilde{\beta} = 0$, which, as $\alpha^* \in (0, 1)$ for $v > \tilde{v}$, implies that $\tilde{\beta}(\underline{v}) = 0$. Setting $\tilde{\beta} = 1$, this becomes

$$v \left(\frac{1 - \alpha^*(v)^N}{1 - \alpha^*(v)} \right) = \frac{1 - \lambda}{\lambda}(q_H - c_H),$$

where the unique solution is given by $\hat{v} \in (\underline{v}, \bar{v})$ (see Lemma 9 above). We will now prove that $\tilde{\beta}(v)$ is strictly increasing in v . To see this, note first that

$$v \left(\frac{1 - [1 - (1 - \alpha^*(v))\tilde{\beta}]^N}{(1 - \alpha^*(v))\tilde{\beta}} \right) = v \sum_{j=0}^{N-1} [1 - (1 - \alpha^*(v))\tilde{\beta}]^j.$$

Plugging this into the above implicit definition of $\tilde{\beta}(v)$ and differentiating implicitly, we find that $\tilde{\beta}(v)$ is strictly increasing in v if

$$\sum_{j=0}^{N-1} [1 - (1 - \alpha^*(v))\tilde{\beta}]^j + v \sum_{j=0}^{N-1} [1 - (1 - \alpha^*(v))\tilde{\beta}]^{j-1} \tilde{\beta} \left(\frac{d\alpha^*(v)}{dv} \right) > 0.$$

To see that this is indeed the case, note that

$$\begin{aligned}
& \sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta}\right]^j + v \sum_{j=0}^{N-1} j \left[1 - (1 - \alpha^*(v)) \tilde{\beta}\right]^{j-1} \tilde{\beta} \left(\frac{d\alpha^*(v)}{dv}\right) \\
= & \sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta}\right]^j + v \sum_{j=0}^{N-1} j \left[1 - (1 - \alpha^*(v)) \tilde{\beta}\right]^{j-1} \tilde{\beta} \left(-\frac{\alpha^*(v)}{\left(\frac{\Delta_c}{\Delta_q} q_H - c_H + v\right) (N-1)}\right) \\
> & \sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta}\right]^j + v \sum_{j=0}^{N-1} j \left[1 - (1 - \alpha^*(v)) \tilde{\beta}\right]^{j-1} \tilde{\beta} \left(-\frac{\alpha^*(v)}{v(N-1)}\right) \\
> & \sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta}\right]^j - \sum_{j=0}^{N-1} (N-1) \left[1 - (1 - \alpha^*(v)) \tilde{\beta}\right]^j \tilde{\beta} \left(\frac{\alpha^*(v)}{N-1}\right) \\
= & \sum_{j=0}^{N-1} \left[1 - (1 - \alpha^*(v)) \tilde{\beta}\right]^j \left(1 - \alpha^*(v) \tilde{\beta}\right) \geq 0,
\end{aligned}$$

where the second line follows from implicitly differentiating the definition of $\alpha^*(v)$, and the third line follows from $\frac{q_H}{q_L} < \frac{c_H}{c_L}$. Hence, $\tilde{\beta}(v)$ is strictly increasing in v , with $\tilde{\beta}(\underline{v}) = 0$ and $\tilde{\beta}(\hat{v}) = 1$. From this it follows directly that $\underline{p}_{L,r}(\underline{v}) = c_L$, whereas using the definition of $\alpha^*(v)$ yields $\underline{p}_{L,r}(\hat{v}) = \frac{\Delta_c}{\Delta_q} q_L$. A proof that $\underline{p}_{L,r}(v)$ is strictly increasing in v is omitted for brevity. **Q.E.D.**

Lemma 11 *If $\frac{q_H}{q_L} < \frac{c_H}{c_L}$, $\delta\Delta_q < \Delta_c$, and $v \in (\hat{v}, \bar{v})$, the following constitutes an equilibrium. Retailers choose the high-quality product with probability $\tilde{\alpha}(v)$, where $\tilde{\alpha}(v)$ is the unique solution to*

$$\frac{1 - \alpha^N}{1 - \alpha} = \frac{1 - \lambda}{\lambda v} (q_H - c_H),$$

with $\tilde{\alpha}(\hat{v}) = \alpha^*(\hat{v})$, $\tilde{\alpha}(\bar{v}) = 0$, and $\tilde{\alpha}'(v) < 0$. High-quality retailers sample prices continuously from a CDF $G_{H,r}(p)$ with support $[\underline{p}_{H,r}, q_H]$, where

$$\underline{p}_{H,r} := c_H - v + \frac{(q_H - c_H + v) \frac{1-\lambda}{N}}{\frac{1-\lambda}{N} + \lambda \tilde{\alpha}^{N-1}} \in \left(\frac{\Delta_c}{\Delta_q} q_H, q_H\right)$$

and

$$G_{H,r}(p) := 1 - \frac{N^{-1} \sqrt{\frac{1-\lambda}{\lambda N} \left(\frac{q_H - c_H + v}{p - c_H + v} - 1\right)}}{\tilde{\alpha}(v)}.$$

With the remaining probability $1 - \tilde{\alpha}(v)$, retailers choose q_L and $p_n = c_L$.

Proof of Lemma 11. As $\underline{p}_{H,r} > \frac{\Delta_c}{\Delta_q} q_H > c_L \frac{q_H}{q_L}$, price is always salient if both high-quality and low-quality products are in the market. Then, all shoppers prefer a low-quality firm's offer, as $\delta q_H - \underline{p}_{H,r} < \delta q_L - c_L$ due to $\underline{p}_{H,r} > c_H$ and $\delta \Delta_q < \Delta_c$. If a high-quality firm samples an arbitrary price p in its support, its expected profit is given by

$$(p - c_H + v) \left\{ \frac{1 - \lambda}{N} + \lambda [\tilde{\alpha} (1 - G_{H,r}(p))]^{N-1} \right\},$$

as it can only attract the shopper if all of its rivals stock q_H and sample a higher price than p . Setting this equal to $(q_H - c_H + v) \frac{1 - \lambda}{N}$, we find the CDF $G_{H,r}(p)$ and $\underline{p}_{H,r}$. Clearly, provided that $\tilde{\alpha}(v) > 0$, $G_{H,r}(p)$ is well-behaved, as it is strictly increasing in p , with $G_{H,r}(\underline{p}_{H,r}) = 0$ and $G_{H,r}(q_H) = 1$.

If a retailer chooses q_L and $p_n = c_L$, its expected profit is given by

$$\begin{aligned} \Pi_L(c_L) &= (c_L - c_L + v) \left[\frac{1 - \lambda}{N} + \sum_{j=0}^{N-1} \binom{N-1}{j} (1 - \tilde{\alpha})^j \tilde{\alpha}^{N-1-j} \frac{\lambda}{j+1} \right] \\ &= v \left[\frac{1 - \lambda}{N} + \lambda \frac{1 - \tilde{\alpha}^N}{(1 - \tilde{\alpha}) N} \right], \end{aligned}$$

as it has to share the shoppers with $j \in \{0, \dots, N-1\}$ rivals, which happens with probability $\binom{N-1}{j} (1 - \tilde{\alpha})^j \tilde{\alpha}^{N-1-j}$, and where the second line again follows from the binomial theorem. Setting this equal to $(q_H - c_H + v) \frac{1 - \lambda}{N}$ yields the implicit definition of $\tilde{\alpha}(v)$ in the lemma. Note that the LHS of this is strictly increasing in α , which implies that $\tilde{\alpha}(v)$ must be strictly decreasing in v . It is also easy to check that $\tilde{\alpha}(\bar{v}) = 0$. Furthermore, comparing the above equation with the definition of \hat{v} , which is the unique v that satisfies

$$\frac{1 - \alpha^*(\hat{v})^N}{1 - \alpha^*(\hat{v})} = \frac{1 - \lambda}{\lambda \hat{v}} (q_H - c_H),$$

it is apparent that $\tilde{\alpha}(\hat{v}) = \alpha^*(\hat{v})$. Using the latter two results, from the definition of $\underline{p}_{H,r}$ it immediately follows that $\underline{p}_{H,r}(\bar{v}) = q_H$, whereas also using the definition of $\alpha^*(v)$ yields $\underline{p}_{H,r}(\hat{v}) = \frac{\Delta_c}{\Delta_q} q_H$. It can likewise be established that $\underline{p}_{H,r}(v)$ is strictly increasing in v .

It remains to show that no firm can have a profitable deviation. Note first that due to the regulation, it is impossible for deviating high-quality firms to render quality salient if any rival stocks q_L . Hence, the best deviation a high-quality retailer can make is to charge the highest price for which its offer wins although price, rather than quality, is salient. But this price, $p_{dev} = c_L + \delta \Delta_q$, is prohibited due to $\delta \Delta_q < \Delta_c$. If a low-quality firm chooses

$p_{dev} > c_L$, its expected profit is at best (that is, for $\delta = 0$) given by

$$\begin{aligned}\Pi_L(p_{dev}) &= (p_{dev} - c_L + v) \left\{ \frac{1 - \lambda}{N} + \lambda \left[\tilde{\alpha} \left(1 - G_{H,r} \left(p_{dev} \frac{q_H}{q_L} \right) \right) \right]^{N-1} \right\} \\ &= (p_{dev} - c_L + v) \frac{1 - \lambda}{N} \left(\frac{q_H - c_H + v}{p_{dev} \frac{q_H}{q_L} - c_H + v} \right),\end{aligned}$$

where the second line follows from inserting $G_{H,r}(\cdot)$ and simplifying. From this, it is easy to show that $\Pi_L(p_{dev})$ is strictly decreasing in p_{dev} , from which it follows that low-quality firms' optimal deviation price is $\underline{p}_{H,r} \frac{q_L}{q_H} > c_L$ for a maximal deviation profit of

$$\left(\underline{p}_{H,r} \frac{q_L}{q_H} - c_L + v \right) \frac{1 - \lambda}{N} \left(\frac{q_H - c_H + v}{\underline{p}_{H,r} - c_H + v} \right).$$

This could only exceed the candidate equilibrium's profit $(q_H - c_H + v) \frac{1 - \lambda}{N}$ if $\underline{p}_{H,r} < \frac{\Delta_c}{\Delta_q} q_H$, which is not satisfied. Hence, also low-quality firms do not have a profitable deviation.

Q.E.D.

For the comparison of $\tilde{\alpha}(v)$ and $\alpha^*(v)$, we finally establish the following.

Lemma 12 $\tilde{\alpha}'(v) < \alpha^{*'}(v)$ for all $v \in [\hat{v}, \bar{v}]$.

Proof of Lemma 12. We first note that the implicit definition of $\tilde{\alpha}(v)$ can be rewritten as

$$\sum_{j=0}^{N-1} \alpha^j = \frac{1 - \lambda}{\lambda v} (q_H - c_H).$$

Implicitly differentiating this, we obtain that

$$\begin{aligned}|\tilde{\alpha}'(v)| &= \frac{1 - \lambda}{\lambda v^2} (q_H - c_H) \left(\frac{1}{\sum_{j=0}^{N-1} j \tilde{\alpha}(v)^{j-1}} \right) \\ &= \frac{1}{v} \left(\frac{\sum_{j=0}^{N-1} \tilde{\alpha}(v)^j}{\sum_{j=0}^{N-1} j \tilde{\alpha}(v)^{j-1}} \right) \\ &> \frac{1}{v} \left(\frac{\sum_{j=0}^{N-1} \tilde{\alpha}(v)^j}{\sum_{j=0}^{N-1} (N-1) \tilde{\alpha}(v)^j} \right) = \frac{1}{v(N-1)},\end{aligned}$$

where the second line follows from using $\frac{1 - \lambda}{\lambda v} (q_H - c_H) = \sum_{j=0}^{N-1} \tilde{\alpha}(v)^j$ by the above definition. We have next that

$$|\alpha^{*'}(v)| = \frac{\alpha^*(v)}{\left(\frac{\Delta_c}{\Delta_q} q_H - c_H + v \right) (N-1)} < \frac{\alpha^*(v)}{v(N-1)},$$

where the expression for $\alpha^{*'}(v)$ has already been established in the proof of Lemma 9, and the inequality follows from $\frac{q_H}{q_L} < \frac{c_H}{c_L}$. Comparing the lower bound of $|\tilde{\alpha}'(v)|$ with the upper bound of $|\alpha^{*'}(v)|$, it is clear that $\tilde{\alpha}(v)$ must have a larger absolute slope than $\alpha^*(v)$ if $\alpha^*(v) \leq 1$, which is indeed the case for all $v \in [\tilde{v}, \bar{v}]$. **Q.E.D.**

Taken together, Lemmas 5 to 12 prove Proposition 7. **Q.E.D.**

Proof of Proposition 8. Start with the case where $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and $\delta\Delta_q < \Delta_c$. Without the regulation, we can infer from Proposition 2 that a high-quality equilibrium exists if and only if $\Delta_c \leq \frac{\Delta_q}{q_H} \underline{p}$, which is equivalent to $v \leq \tilde{v}$. Similarly, Proposition 7 establishes that also under the regulation, a high-quality equilibrium exists if and only if $v \leq \tilde{v}$. If no high-quality equilibrium exists (and therefore, retailers either mix between stocking the high and low-quality product, or deterministically stock the low-quality product), manufacturers cannot extract a positive profit, as retailers are at best indifferent between choosing high quality or low quality. Hence, both under the regulation and without it, manufacturers make zero profits for $v \geq \tilde{v}$.

If $v < \tilde{v}$ such that a high-quality equilibrium exists (and manufacturers' profits are strictly positive), manufacturer profits do not depend on the regulation. To see this, note first that for $\frac{q_H}{q_L} < \frac{c_H}{c_L}$, it holds that $\tilde{v} < \underline{v}$, which implies that whenever a high-quality equilibrium exists, retailers' on-equilibrium pricing is not affected by the pricing regulation (since $\underline{p} > c_H$). But also the maximum deviation profit of a retailer is not affected, since deviating retailers can guarantee to attract all shoppers by pricing at $\min\{\underline{p}_{q_H}, \underline{p} - \delta\Delta_q\} > \min\{c_H \frac{q_L}{q_H}, c_H - \delta\Delta_q\}$, which does not fall short of c_L and therefore is not prohibited due to $\frac{q_H}{q_L} < \frac{c_H}{c_L}$ and $\delta\Delta_q < \Delta_c$. Hence, for $v \leq \tilde{v}$, the regulation does not affect retailers' on-equilibrium gross profits and their deviation profits. Note finally that without the regulation, for any $v > \tilde{v}$, retailers make an expected profit of $\frac{1-\lambda}{N}(q_H - c_H + v)$. Under the regulation, they also make an expected profit of $\frac{1-\lambda}{N}(q_H - c_H + v)$ for $v \in (\tilde{v}, \bar{v}]$, but $\frac{v}{N} > \frac{1-\lambda}{N}(q_H - c_H + v)$ for $v > \bar{v}$. Hence, retailers' expected profits strictly increase for $v > \bar{v}$, but are unaffected for any $v \leq \bar{v}$.

Proceed with the complimentary case where $\frac{q_H}{q_L} > \frac{c_H}{c_L}$ or $\delta\Delta_q \geq \Delta_c$ (or both). For this, we will consider three different subcases: $v \leq \underline{v}$, $v \geq \bar{v}$, and $v \in (\underline{v}, \bar{v})$. If $v \leq \underline{v}$, a high-quality equilibrium exists both under the regulation and without it (as a high-quality equilibrium always exists if either $\delta\Delta_q \geq \Delta_c$ or $v < \tilde{v}$, and the latter is implied by $v \leq \underline{v}$ and $\frac{q_H}{q_L} > \frac{c_H}{c_L}$). Moreover, the regulation does not affect retailers' equilibrium pricing and gross profits. But it may reduce their scope for deviations, as it is possible that $\underline{p}_{q_H} < c_L$

even though $\underline{p} \geq c_H$ (if $\frac{q_H}{q_L} > \frac{c_H}{c_L}$), or alternatively that $\underline{p} - \delta\Delta_q < c_L$ even though $\underline{p} \geq c_H$ (if $\delta\Delta_q \geq \Delta_c$). Hence, under the considered parameters, retailers' equilibrium profits weakly decrease under the regulation for $v \leq \underline{v}$, while manufacturers' equilibrium profits weakly increase.

If $v > \bar{v}$, under the regulation all retailers choose $p_n = c_H$ for a gross profit of $\frac{v}{N}$. Because they find it impossible to attract shoppers when deviating (as pricing below $\min\{\frac{q_L}{q_H}c_H, c_H - \delta\Delta_q\}$ is prohibited if either $\frac{q_H}{q_L} > \frac{c_H}{c_L}$ or $\delta\Delta_q \geq \Delta_c$), their optimal deviation price is q_L for a maximal deviation profit of $\frac{1-\lambda}{N}(q_L - c_L + v)$. Hence, retailer (manufacturer) profits weakly decrease (increase) in the specified parameter region. Note moreover that if $v > \bar{v}$, $v \geq \tilde{v}$ and $\delta\Delta_q < \Delta_c$, manufacturer (retailer) profits strictly increase (decrease), as no high-quality equilibrium would exist without the regulation,⁴⁹ while manufacturers can appropriate a strictly positive profit of $\frac{v}{N} - \frac{1-\lambda}{N}(q_L - c_L + v)$ with the regulation (and retailer profits strictly decrease from $\frac{1-\lambda}{N}(q_H - c_H + v)$ to $\frac{1-\lambda}{N}(q_L - c_L + v)$).

Finally, if $v \in (\underline{v}, \bar{v})$, retailers choose c_H with positive probability and sample prices up to q_H with remaining probability. Again, due to $\frac{q_H}{q_L} > \frac{c_H}{c_L}$ or $\delta\Delta_q \geq \Delta_c$, it is impossible for deviating retailers to choose a price that captures all shoppers deterministically. If a deviating retailer chooses a (feasible) deviation price p_{dev} , it makes an expected profit of

$$\begin{aligned} \Pi_{dev}(p_{dev}) &= (p_{dev} - c_L + v) \cdot \\ &\quad \left\{ \frac{1-\lambda}{N} + \lambda \left[(1 - \beta^*) \left(1 - G_r \left(\max \left\{ \frac{q_H}{q_L} p_{dev}, p_{dev} + \delta\Delta_q \right\} \right) \right) \right]^{N-1} \right\} \\ &= (p_{dev} - c_L + v) \frac{1-\lambda}{N} \left(\frac{q_H - c_H + v}{\max \left\{ \frac{q_H}{q_L} p_{dev}, p_{dev} + \delta\Delta_q \right\} - c_H + v} \right), \end{aligned}$$

where the maximum operator follows from the fact that shoppers can only be attracted if both price is made salient and a higher perceived utility under salient prices is offered, and the second line follows from inserting $G_r(\cdot)$ and simplifying (compare also with the proof of Lemma 5). If we contrast this with the deviation profit without the regulation (cf. the proof of Lemma 2), we note that the two functions are identical. Hence, under the pricing regulation, a retailer's optimal deviation price (if it could freely choose any price) remains unaffected, but it may not be allowed to set such a price because of the prohibition of below-cost pricing, such that its deviation profit decreases. Hence, also for $v \in (\underline{v}, \bar{v})$, retailers' profits weakly decrease, while manufacturers' profits weakly increase (and strictly

⁴⁹Strictly speaking, a high-quality equilibrium would still exist for $v = \tilde{v}$, but with manufacturers making zero profits.

so if $v \geq \tilde{v}$ and $\delta\Delta_q < \Delta_c$, as without the regulation, no high-quality equilibrium would exist⁵⁰). **Q.E.D.**

⁵⁰Apart from the borderline case where $v = \tilde{v}$, in which manufacturers make zero profits.