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A MECHANISM DESIGN APPROACH TO THE TIEBOUT HYPOTHESIS[†]

Abstract

We revisit the Tiebout hypothesis in a world in which agents may possess private information as to how they value the various public goods in the various locations, and jurisdictions are free to choose whatever mechanism to attract citizens possibly after making some investments. It is shown that efficiency can be achieved as a competitive equilibrium when jurisdictions seek to maximize local revenues but not necessarily when they seek to maximize local welfare. Limitations of the result are discussed.

JEL Classification: D82 and H4
Keywords: competing exchange platforms, competing mechanisms, endogenous entry, free riding, local public goods, mechanism design and tiebout hypothesis

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1 Introduction

A well-known source of inefficiency related to public goods is the so called free riding problem (Samuelson 1954). An informational version of the free riding problem can be described as follows. Agents interested in the construction a public good may pretend they are less so in an attempt to reduce the price they have to pay for it, relying on others to contribute more to the costs associated to the public good.[1]

Tiebout (1956) suggested that if public goods are provided locally and if agents can freely choose their jurisdictions, the free riding problem should be alleviated presumably because the choice of jurisdictions would somehow reveal the preference of the citizen:[2] According to the Tiebout hypothesis, the option of citizens to vote with their feet would eliminate the inefficiencies attached to the provision of public goods. Or to put it differently, the competition between jurisdictions to attract citizens would be viewed as restoring the efficiency of the economy despite the presence of public goods.

There have been several attempts to formalize the Tiebout hypothesis most of them embedded in the theory of general equilibrium. Typically, a price is attached to the membership in any community of any characteristic in an addition to the traditional prices of private goods, and the question is whether there exists a system of prices that allows to implement the first-best.[3] That literature has mostly ignored the possibility that citizens would have private information as to how much they value the various public goods (an incomplete literature review of this strand includes Bewley (1981), Ellickson et al. (1999), Cole and Prescott (1997), Prescott and Townsend (2006), see Scotchmer (2002) and Wooders (2012) for surveys).

We take a different route in this paper. We explicitly allow for asymmetric information in public good contexts. The private information may take a possibly complex form (for example multi-dimensional signals with different privately known values attached to the various possible public goods in the various jurisdictions), but we assume that citizens have quasi-linear preferences (i.e., taxes enter in an additive form in the utility). That is, we consider public good environments of the sort considered in Clarke (1971) and Groves (1973) in the early mechanism design literature. We also assume that jurisdictions are

[1] This implies that the choice of public goods has to be sufficiently responsive to reports. Mailath and Postlewaite (1990) provide a striking illustration of the inefficiency this may cause in a context with a large number of agents.

[2] It is not so clear however if there are several types of citizens who would join the same jurisdiction how the free riding problem would be completely eliminated.

[3] Such approaches have built on the insight of Samuelson (1994) and the general equilibrium version of it, the Lindhal equilibrium (see Mas Colell et al, 1995).

subject to some congestion, implying that it would not be optimal to have just one active jurisdiction. This is a simple way to ensure that in our world with endogenous participation, there is effective competition between jurisdictions. It should also be mentioned that we do not allow for externalities between jurisdictions. That is, if a citizen joins jurisdiction j, this citizen is indifferent as to what happens in other jurisdictions $j' \neq j$. Furthermore, we do not allow for interdependent valuations (or informational externalities). That is, how a citizen values the various public goods does not depend on the information held by other citizens. Other than that, our framework is very permissive, in particular allowing inside jurisdictions for any kind of externalities that would depend on publicly observable characteristics of citizens.

Jurisdictions which may differ in many characteristics, including their cost structure regarding the provision of public goods, post mechanisms that govern how public goods and taxes are chosen locally (typically after citizens have reported how they value the various possible public goods as in the standard mechanism design literature). After jurisdictions have posted their mechanisms, citizens optimally choose which jurisdiction to go to depending on their (rational) expectations as to how citizens adjust their participation decision as a function of the posted mechanism and how public goods and taxes are decided as a function of the profile of participation.

We have in mind contexts with a large number of citizens and jurisdictions. This leads us to assume that there is a continuum of citizens and jurisdictions in the global economy so that a single choice of mechanism by one jurisdiction (and more generally by a measure 0 of jurisdictions) would not affect the equilibrium utilities of the different types of citizens. That is, we assume that jurisdictions are utility-takers and anticipate that when they choose a mechanism, the expected participation rate from the various groups of citizens will adjust so that on average, the utilities of the various types of citizens coincide with their equilibrium utilities.[4] It should be mentioned that the number of citizens entering a given jurisdiction will be finite and stochastic in our model: this is a departure from traditional general equilibrium models to the extent that in the mechanisms governing the choice of public goods, each individual citizen has a non-negligible impact on the outcome (i.e., citizens have market power unlike in the usual general equilibrium models).[5]

One missing element in the above description is the objective of jurisdictions. The

[4] In some way, this can be viewed as the analog of the price-taking behavior in traditional general equilibrium models, even if here the game theoretic motivation for the assumption should sound simpler.

[5] See Ellickson et al. (1999), Prescott and Townsend (2006) and Zame (2007), and Scotchmer and Shannon (2010) for an exception.

main objective function that we consider is the local revenues defined as the sum of local taxes from which the cost of the public good is deducted. Alternative objectives such as local welfare are also discussed.

Our main result is that if jurisdictions seek to maximize local revenues, one equilibrium outcome of the above competitive environment is the first-best outcome in which (1) public goods are efficiently chosen in each jurisdiction (which corresponds to an ex-post efficiency criterion), and (2) citizens are efficiently distributed across jurisdictions (from an ex ante perspective). Interestingly, if jurisdictions are instructed to maximize local welfare, inefficiencies may necessarily arise. This will be illustrated through an example.

It should be emphasized that our analysis allows for the formation of heterogenous communities with citizens having different tastes. Thus, it is not the case that efficiency is achieved here because the choice of the jurisdiction reveals the preference as some of the previous literature has suggested. It should also be mentioned that whenever all jurisdictions receive positive participation in equilibrium, jurisdictions typically make strictly positive expected profits. Thus, our efficiency result is not driven by a cutting price argument as in the Bertrand model.

Roughly, efficiency is achieved for the following reasons. First, because jurisdictions are utility-takers, their objective coincides with the net local welfare defined as the local welfare from which the opportunity costs of the various participating citizens should be deducted. This in turn explains why there is some form of alignment between the objective of jurisdictions and local welfare despite the fact that jurisdictions seek to maximize local revenues. It should be stressed though that jurisdictions are also interested in attracting the best participation profile as this would affect their expected local revenues. As it turns out, when jurisdictions post pivot mechanisms defined as Clarke-Groves mechanisms in which every agent pays the externality he imposes on others, an efficient choice of public good is achieved and efficient participation profile is part of an equilibrium (because in a pivot mechanism, citizens get their contribution to local welfare so that their decision to participate is aligned with the objective of the jurisdiction). This in turn ensures that the first-best can be obtained as a competitive equilibrium in which jurisdictions post pivot mechanisms and citizens are sorted efficiently in the various jurisdictions.

Our main result bears some resemblance with results obtained in the literature on competing auctions (a seminal contribution is McAfee (1993), Levin and Smith (1994) being a partial equilibrium counterpart of it with ex-ante symmetric buyers), which can be viewed as a special case of our setup in which jurisdictions correspond to sellers and citizens correspond to buyers. The insight of this literature[6] is that competing auctioneers would find it optimal to post second price auctions with reserve prices set at their valuation, i.e. the pivot mechanism. By contrast with auction environments, an important specificity arising in our general public good environment is the need for a public device to be observed by citizens after jurisdictions have posted their mechanisms. This is so because in the context of public goods, economies of scale would justify not spreading (ex ante similar) citizens uniformly over all (ex ante similar) jurisdictions, and the public devices are required for an optimal determination of the participation decisions.

We consider various extensions of the basic setup. We allow jurisdictions to make decisions affecting their cost structure before the interactions start, and we obtain the same efficiency result when jurisdictions seek to maximize local revenues. We consider the case in which jurisdictions seek to maximize local welfare, and we illustrate that with such objectives it may be impossible to avoid inefficiencies.[7] We consider the case in which in each jurisdiction there are some non-mobile citizens in which case, to obtain efficiency, it should be that the objective of jurisdictions is the local revenues augmented by the welfare of the non-mobile citizens in the jurisdiction. We also consider the case in which there would be several public goods provided by different revenue maximizers within the jurisdiction, and we show how this can be a source of inefficiency.

Overall, we believe that embedding the competition between jurisdictions into a mechanism design framework allows us to shed new light on an old problem that the general equilibrium approach did not permit.[8] Besides, the generality of our setup allows us to revisit applications that go beyond the ones usually considered in the context of local public goods: in particular pricing in two-sided markets and market design for exchange platforms. From a regulatory perspective, our analysis can be used to suggest the potential advantage of letting agents freely choose where to trade and letting market platforms freely choose how to organize and tax trades.

The rest of the paper is structured as follows. In Section 2 we describe the economic environment. In Section 3, we describe the competitive equilibrium. Our main decentralization result appears in Section 4. In Section 5, we discuss various extensions. Section 6

[6]See Jehiel and Lamy (2015) for the most general treatment allowing for both ex ante and ex post asymmetries between buyers and covering general information structures.

[7]When jurisdictions seek to maximize local welfare, it is as if a double weight were put on the well being of citizens as compared to the cost of the public good, thereby giving jurisdictions excessive incentive to attract too many citizens.

[8]For example, the inefficiency highlighted in Benabou (1993) is due to the constraint on the set of mechanisms used to internalize the externalities that does not allow for the use of the pivot mechanism.

concludes.

2 The economic environment

We present the model in several steps. First, we introduce the various agents in the economy as well as their information and preferences. We then provide a number of leading applications. Then we describe the strategic choice made by jurisdictions, which amounts to a choice of mechanism specifying the choice of public goods and individual taxes for all possible profiles of participants and announcements.

2.1 Preliminaries

Jurisdictions

We consider finitely many types of jurisdictions belonging to the set $\mathcal{K}_J$. Any given jurisdiction j is characterized by a publicly observed type $k_j \in \mathcal{K}_J$ which determines the private cost of the jurisdiction j of providing the various local public goods.[9] The mass of type k_j jurisdictions is denoted by $f_J(k_j)$. In an extension, we will allow jurisdictions to make some investment in k_j before the interaction starts. But, for the time being, the distribution of k_j is given exogenously.

Citizens

We consider finitely many groups of citizens. We wish to allow for situations in which some characteristics of the groups are publicly observable (such as physical attributes) whereas others are not (such as tastes). We wish also to allow that citizens upon joining a jurisdiction would learn more about how they fully value the public goods locally provided.

This leads us to adopt the following formalism. Prior to the location choice, any given citizen i is characterized by a group $k_i = (k_i^p, k_i^s) \in \mathcal{K}_C$, where k_i^p is publicly observable and k_i^s is private information to citizen i. The set of publicly observable groups is denoted by $\mathcal{K}_C^p := \{k^p | \exists \widehat{k} = (\widehat{k}^p, \widehat{k}^s) \in \mathcal{K}_C \text{ such that } \widehat{k}^p = k^p\}$. Groups are also referred to by an index $k \in \{1, \ldots, K\}$ where K denotes the cardinality of $\mathcal{K}_C$. The mass of group k citizens is denoted by $f_C(k)$.

After joining a jurisdiction, any given citizen i from group k learns his type $\theta_i \in \Theta$ which fully characterizes his preferences. Without loss of generality, the type θ_i includes

[9] We could allow jurisdictions to observe privately some parts of their costs. Our decentralisation result would still hold, as long as the private information held by jurisdictions does not affect citizens' preferences (for the same reasons as those developed in Jehiel and Lamy (2015)).

a specification of the group to which i belongs. Type θ_i can be decomposed into (θ_i^p, θ_i^s) where θ_i^p is publicly observable and θ_i^s is private information to citizen i.

Let $\mathbf{k} : \Theta \to \{1, \ldots, K\}$ [resp. $\mathbf{k}^p : \Theta \to \mathcal{K}_C^p$] denote the function that maps any citizen's type to his group [resp. what is publicly observable from his group]. We wish to allow for correlations among the various θ_i of citizens of the same group k. We model this by introducing an extra variable y taking values in Y. Conditional on the realization y of the variable Y, the type of a citizen from a given group k is assumed to be distributed according to the measure $f_k(.|y)$ and the types of the various citizens are assumed to be distributed independently (given y). In a jurisdiction with type k_j, the variable Y is distributed according to the measure $f_Y(.|k_j)$ and the variables Y are distributed independently across jurisdictions.[10]

Consider a given jurisdiction with n citizens, the profile of types of the n citizens is denoted by $\theta = (\theta^1, \ldots, \theta^n) \in \Theta^n$. We let $\overline{\Theta} := \bigcup_{n \in \mathbb{N}} \Theta^n$ denote the set of all such vectors of types allowing for any composition of jurisdictions. Similarly, we let θ^p denote the vector of public types, and we let $\overline{\Theta}^p$ denote the set of θ^p. For any $k \in \mathcal{K}_C$, we let $\theta^{[k]}$ be the subvector of types in θ from group k, namely the vector of the types θ_i, $i \in \{1, \ldots, n\}$, such that $\mathbf{k}(\theta_i) = k$. The length of the vector θ^k is denoted by $n_k(\theta)$. The vector of $n_k(\theta)$ for all groups k is denoted by $n(\theta) := (n_1(\theta), \ldots, n_K(\theta))$. We also let $\widetilde{n}(\theta) := \sum_{k=1}^K n_k(\theta)$ denote the total number of citizens in the jurisdiction as a function of θ. For a given vector θ and a given citizen $i \in \{1, \ldots, \widetilde{n}(\theta)\}$, we adopt the convention $\theta_{-i} = (\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_{\widetilde{n}(\theta)})$. With a slight abuse of notation we let $\theta = \theta_i \cup \theta_{-i}$. We use the notation $\mathbf{k}(\theta)$ [resp. $\mathbf{k}^p(\theta)$] for the vector of group membership [resp. the observable part of the profile of group membership] of the citizens with the vector of types θ, i.e. $\mathbf{k}(\theta) := (\mathbf{k}(\theta_1), \ldots, \mathbf{k}(\theta_{\widetilde{n}(\theta)}))$ [resp. $\mathbf{k}^p(\theta) := (\mathbf{k}^p(\theta_1), \ldots, \mathbf{k}^p(\theta_{\widetilde{n}(\theta)}))$]. Similarly, for a given citizen i, we refer to $\mathbf{k}(\theta_{-i})$ (resp. $\mathbf{k}^p(\theta_{-i})$) as the vector of (resp. public) group membership of citizens other than i.

For $N = (n_1, \ldots, n_K) \in \mathbb{N}^K$ and $k_j \in \mathcal{K}_J$, we let $q(\theta|N, k_j) = \int_{y \in Y} \prod_{k=1}^K \prod_{i=1}^{n_k(\theta)} f_k(\theta_i^{[k]}|y) \cdot f_Y(y|k_j) dy$ denote the measure of the distribution of types θ in a jurisdiction with type k_j conditional on $n(\theta) = N$ (or equivalently conditional on the corresponding realization of $\mathbf{k}(\theta)$ so that we sometimes use the notation $q(\theta|\mathbf{k}(\theta), k_j)$).[11]

[10] To alleviate notation, we do not allow for the possibility of ex post publicly observable types for either the jurisdiction or the citizens. It should be mentioned though that our main efficiency result does not carry over to environments in which jurisdictions receive additional private information about her private costs at the ex post stage.

[11] Whenever $n(\theta) \neq N$, then we have $q(\theta|N, k_j) = 0$. Similarly we use the notation $q(\theta_{-i}|N, k_j, \theta_i)$ when we take the point of view of agent i with type θ_i (whose type realization gives some information about

Local public goods

Each jurisdiction will be providing a local public good z in exchange for taxes which may be citizen specific. The set of feasible public goods will be allowed to depend on the public characteristics of the citizens joining the jurisdiction so that we can accommodate situations with externalities such as crowding effects or any other externality based on the publicly observable characteristics of citizens.

Formally, let $\mathcal{Z}$ denote the set of all possible public goods (allowing for all possible configurations of participants). We define $Z : \overline{\Theta}^p \to 2^{\mathcal{Z}}$ as the function which assigns to any vector of observable characteristics a non-empty and finite set of feasible social alternatives.[12,13] For any $\theta^p \in \overline{\Theta}^p$, we let $\mathcal{T}^Z_{\theta^p}$ denote the set of probability distributions on the set $Z(\theta^p)$, and we let $\mathcal{T}^Z := \bigcup_{\theta^p \in \overline{\Theta}^p} \mathcal{T}^Z_{\theta^p}$.

Citizens' preferences are assumed to be quasi-linear. That is, citizen i with type θ_i getting the public good z in exchange for a tax $t_i \in \mathbb{R}$ gets an overall payoff of

$$v(z, \theta_i) - t_i$$

where $v(\cdot, \cdot)$ is a common function that applies to all.

Observe that we assume that the payoff of a citizen depends exclusively on the public good in the jurisdiction where he lives as well as the tax he pays in that jurisdiction and not on the public goods provided in other jurisdictions. Thus, we are ruling out externalities across jurisdictions. Note however that our formalism allows us to capture any kind of externality that would depend on the public characteristics of the participating citizens (and also implicitly on the type of the jurisdiction) insofar as z may contain a description of the profile of the public characteristics of participants.

Jurisdictions are characterized by a cost function $C : \mathcal{Z} \times \mathcal{K}_J \to \mathbb{R}$ where $C(z, k_j)$ should be interpreted as the cost of providing the public good z when the jurisdiction is of type k_j. We assume that there exists $z^0 \in \mathcal{Z}$ such that for all $\theta^p \in \overline{\Theta}^p$ (including $\theta^p = \emptyset$, i.e. when there is nobody in the jurisdiction) and $k_j \in \mathcal{K}_J$, $z^0 \in Z(\theta^p)$ and $C(z^0, k_j) = 0$. This will ensure that any jurisdiction can propose a mechanism that guarantees that her budget

the realization of Y and thus indirectly about the distribution of the types of other citizens).

[12] The finiteness restriction is here only for technical reasons (some of our examples involve indeed a continuous set of alternatives). We could easily deal with a continuum if the set of feasible social alternatives were assumed to be a compact set and the underlying payoff functions were assumed to be continuous w.r.t. the chosen alternative so that the social alternative that maximizes the welfare would be well-defined. Note however that we do not assume that the set $\mathcal{Z}$ is finite, as the set of participant profiles is infinite.

[13] The assumption that $Z(\theta^p)$ does not depend on the public type of the jurisdiction is without loss of generality since it can be put into the citizens' type. We made it to alleviate notations.

is non-negative (think of such a z^0 as a default null public good). Various objectives will be considered for jurisdictions. Our main result will assume that jurisdictions are interested in local revenues, i.e., jurisdiction j seeks to maximize $\Sigma_{i=1}^{\widetilde{n}(\theta)} t_i - C(z, k_j)$.

We will denote by $\underline{V}_k > 0$ the default expected utility that a citizen of group k can guarantee by himself (say by being in a vacuous jurisdiction consisting of himself only). Later when we introduce our equilibrium concept, we will require that citizens of group k get no less than $\underline{V}_k$ in expectation, implicitly assuming that citizens always have the option to remain on their own away from any inhabited jurisdiction.

Throughout the paper, we make several implicit "anonymity" restrictions. Jurisdictions are assumed not to be allowed to discriminate (even through indirect discrimination policies) between citizens from the same group. Relatedly, we assume that the set of feasible alternatives treats citizens with the same observable characteristics in an anonymous way. Formally, for any $\theta^p \in \Theta^p$ and any pair of citizens i and j with $\theta_i^p = \theta_j^p$, then for any $z \in Z(\theta^p)$, there exists a social alternative denoted by $z_{i \leftrightarrows j}^{\theta^p} \in Z(\theta^p)$ that exchanges the role of i and j in such a way that $v(z, (\theta_k^p, .)) = v(z_{i \leftrightarrows j}^{\theta^p}, (\theta_k^p, .))$ for any $k \neq i, j$, $v(z, (\theta_i^p, .)) = v(z_{i \leftrightarrows j}^{\theta^p}, (\theta_j^p, .))$ and $v(z, (\theta_j^p, .)) = v(z_{i \leftrightarrows j}^{\theta^p}, (\theta_i^p, .))$.

In some cases, it may be that a resident coming from group k can exclude himself ex post from the jurisdiction (i.e., after having made the choice to participate in this jurisdiction). We assume such a group k resident then gets payoff $\underline{\underline{V}}_k < \underline{V}_k$, where the inequality may be thought of as reflecting mobility (sunk) costs. More importantly (and irrespective of whether citizens have a right to exclude themselves -as will be noted this plays no role in our setup), we assume that when one more citizen joins, the jurisdiction has the option to put this citizen aside and to implement the public good among the remaining citizens. This is formalized by assuming that for any $\theta \in \overline{\Theta}$ and any $i \in \{1, \ldots, \widetilde{n}(\theta)\}$, we have

$$Z(\theta_{-i}^p) \subseteq Z(\theta^p) \tag{1}$$

and if $z \in Z(\theta_{-i}^p)$, then the left aside citizen i gets $v(z, \theta_i) = \underline{\underline{V}}_{\mathbf{k}(\theta_i)}$. With that interpretation in mind, the public goods we are considering are excludable local public goods. The allocation z^0 corresponds then to providing no public good (or equivalently to the exclusion of all participants from the public good).

Local welfare

The associated (ex post local) welfare function depends on the public good z, the type of the jurisdiction k_j and the profile of residents' types. It is formally defined by $w : \mathcal{Z} \times \mathcal{K}_J \times \overline{\Theta} \to \mathbb{R}$ where

$$w(z, k_j, \theta) := \sum_{i=1}^{\widetilde{n}(\theta)} v(z, \theta_i) - C(z, k_j). \tag{2}$$

Let $z^* : \mathcal{K}_J \times \overline{\Theta} \to 2^{\mathcal{Z}}$ denote the efficient choice correspondence, i.e. $z^*(k_j, \theta) := \operatorname{Arg\,max}_{z \in Z(\theta^p)} w(z, k_j, \theta)$ and $w^*(k_j, \theta) := \max_{z \in Z(\theta^p)} w(z, k_j, \theta)$ the associated welfare.

The next assumption is used to guarantee that it cannot be optimal that an infinite number of citizens joins a single jurisdiction.[14]

Assumption A 1 *There exists $H \in \mathbb{R}$ such that $w^*(k_j, \theta) \leq H + \sum_{k=1}^{K} n_k(\theta) \cdot \underline{\underline{V}}_k$ for any $(k_j, \theta) \in \mathcal{K}_j \times \Theta$.*

Comment: We have not yet modelled entry, but the intuition as to why the above assumption implies that too large jurisdictions would run into deficit should be clear. Given that citizens from group k get no less than $\underline{V}_k$, the revenue of a jurisdiction of type k_j is bounded from above by $H + \sum_{k=1}^{K} n_k \cdot [\underline{\underline{V}}_k - \underline{V}_k]$, which is negative once any n_k gets large enough.

2.2 Applications

A notable feature of our setup is that it is flexible enough to cover many applications.

Example 0: Reduced form club good

Citizens and jurisdictions are homogeneous. The local welfare generated in a jurisdiction of size n is $w(n)$ where it is assumed that $w(n)/n$ is strictly smaller than $w(1)$ when n is large enough. Natural specifications would assume that $w(n)/n$ is concave in n with $\operatorname{Arg\,max} \frac{w(n)}{n} = n^* > 1$. Managers of club goods can charge fees. A question arises as to whether citizens split optimally in equilibrium, and what impact the objective of the club good managers has on the equilibrium splitting.

Example 1: Sharing a natural resource.

Citizens are homogenous and the set of feasible alternatives is a singleton for each jurisdiction but jurisdictions may be heterogenous. Each jurisdiction is characterized by $k_j \in \mathbb{R}_+$ which corresponds to a limited resource of total value k_j which is extracted

[14] A1 is much stronger than actually needed for our main decentralization result to hold. When $K = 1$, A1 can be replaced by the weaker condition $\lim_{n \to +\infty} E_{\theta | \widetilde{n}(\theta) = n}[\frac{w^*(k_j, \theta)}{n}] \leq \underline{\underline{V}}$ for any $k_j \in K_J$.

at no cost and then shared equally among the residents.[15] The welfare is thus given by $w(k_j) = w^*(k_j) = k_j$ if at least one citizen joins the jurisdiction and 0 otherwise. Jurisdictions can charge fees to affect the local revenues, and the question is whether the induced participation decisions are made efficiently.

Example 2: Selection of the users of a (local) public good under complete information.

Citizens and jurisdictions are homogenous and the set of feasible alternatives is the subset of participants, namely $Z(n) = \{S|S \subseteq \{1, ...n\}\}$ when there are n participants, where S corresponds to the subset of users. The utility of a citizen as a user of the public good is denoted by $\widehat{\theta} > 0$ while the utility of a non-user is zero (i.e. $\underline{\underline{V}} = 0$). The cost function as a function of the number of users is given by $C(n)$, a non-decreasing function. We assume furthermore that the cost function $C(n)$ is convex with $C(1) > \widehat{\theta}$, the average cost $A_c(n) := \frac{C(n)}{n}$ is convex and minimized at the mode n^* and finally that $\lim_{n\to+\infty} \frac{C(n)}{n} > \widehat{\theta}$ and $\frac{C(n^*)}{n^*} < \widehat{\theta} - \underline{\underline{V}}$. The welfare within a jurisdiction in which the number of users is n is thus given by $n \cdot \widehat{\theta} - C(n)$. What is at stake is thus to coordinate entry in jurisdictions. If entry could be coordinated "perfectly" (which is not so in our equilibrium analysis because we restrict attention to symmetric equilibria in which ex ante symmetric citizens adopt the same participation strategy), what would be optimal (if $n^* \cdot f_J \geq f_C$, that is if there is no shortage of jurisdictions) would be to have either jurisdictions with n^* citizens or jurisdictions with no citizen at all.[16]

Example 3: Intensity/quality of a (local) public good with asymmetric information.

Each jurisdiction chooses the characteristic q of the public good and a subset of the participants as the users of the public good denoted by S where the cardinality of S is denoted n_S. The utility of a user of the public good with type θ_i is given by $v(q, \theta_i)$ while the cost of the jurisdiction is given by $C(q, n_S, k_j)$ with $\lim_{n\to\infty} \frac{C(q,n,k_j)}{n} = +\infty$ for any q and k_j. Next, we will consider the special case in which $v(q, \theta_i) = q \cdot \theta_i$, which corresponds to Mussa and Rosen's (1978) classic specification.

[15] To simplify, we consider here implicitly that excluding some citizens is impossible. This is completely innocuous since citizens are assumed to be anonymous. Considering that the set of jurisdictions' type is a continuum will later simplify our calculations.

[16] Let $n^{**} \geq n^*$ denote the largest integer n such that $C(n+1) - C(n) > \widehat{\theta} - \underline{\underline{V}}$. If $n^* \cdot f_J < f_C < n^{**} \cdot f_J$, the optimal solution consists in splitting uniformly the citizens across jurisdictions. If $f_C \geq n^{**} \cdot f_J$, the optimal solution consists in assigning n^{**} citizens to each jurisdiction and then to attribute the default outside option $\underline{\underline{V}}$ to the other citizens.

Example 4: Competition between exchange platforms.

There are various groups of economic agents differentiated by their initial endowments and their preferences (still sticking to our assumption that preferences are quasi-linear in money). The issue for jurisdictions (or market places) is to propose exchange mechanisms that govern how goods are assigned and which monetary payments are made. E.g., we can think of (heterogenous) web-sites running double auctions to sell ad slots to advertisers as an illustration of competing exchanges. To fix ideas, we will later discuss a two-sided environment in which one homogenous indivisible good is exchanged on the trading platforms proposed by jurisdictions, agents are either (unit-demand) buyers characterized by a valuation or (unit-supply) sellers characterized by a production cost. Consider a platform with n_B buyers having valuations $v_1 \geq v_2 \geq \ldots \geq v_{n_B}$ and n_S sellers having costs $c_1 \leq c_2 \leq \ldots \leq c_{n_S}$ and let n^* the largest integer such that $v_{n^*} \geq c_{n^*}$.[17] If the platform has no friction, then the efficient allocation consists in n^* transactions: the n^* buyers with the highest valuation buyers purchase one unit of good from the n^* sellers with the lowest costs. The welfare is then $\sum_{i=1}^{n^*}(v_i - c_i)$. For our main result to apply, we also need a trading congestion that guarantees that it is suboptimal to have an infinite number of citizens joining a given jurisdiction (see Assumption A1). Such a congestion may be the result of land scarcity when goods have to be delivered at the market place. To fix ideas, we will later discuss the stylized case in which at most $\overline{n}$ transactions can arise in the platform and in which each transaction is costless. For the optimal assignment, we simply have to replace n^* above by $\min\{n^*, \overline{n}\}$.

Example 5: Competition between auctioneers/sellers.

A variant of Example 4 is one in which sellers are non-mobile citizens attached to auctioneers and only buyers have to decide which auction to go to. More specifically, we will discuss the following two examples: 1) If each jurisdiction corresponds to a seller who has a fixed number of goods for sale (which guarantees that A1 is satisfied), then this corresponds to the environment analyzed in Jehiel and Lamy (2015)).[18] 2) If each jurisdiction corresponds to a local market that has a captive set of sellers having private information about their reservation value, this corresponds to the environment studied in Gomes and Mirrokni (2014).

[17] We let $n^* = 0$ if $v_1 < c_1$.

[18] The previous literature modelling auctions with endogenous entry (see e.g. Levin and Smith (1994) and McAfee (1993)) imposes that potential entrants are either not informed at all or fully informed about their valuation. This contrasts with our framework in which buyers may have initial information, which is further refined upon participation.

Example 6: School assignments.

In the problem of assigning students to schools/universities, the set of alternatives is the set of students enrolled in the school. The feasible alternatives are subsets of applicants. The difference with Example 2 is the presence of peer effects, namely that the utility function of a student may depend not solely on whether or not he is accepted in the school but also on the composition of the school (e.g. social diversity could be beneficial). What is at stake is how to optimize the composition of the school through the choice of appropriate tuition fees charged to applicants.[19] Peers effect and free mobility are much discussed in the literature about segregation (see e.g. Benabou (1993)). Our approach differs from most of the previous literature in that we allow jurisdictions to use any mechanism (as opposed to less flexible price mechanisms).

Example 7: Two-sided markets with externalities.

Another application is one in which various platforms (jurisdictions) offer two-sided markets (Rochet and Tirole (2003), Caillaud and Jullien (2004), Armstrong (2006)). Externalities of various sorts may arise -for example in the context of newspapers with readers and advertisers as the two sides of the market, it is typically the case that advertisers impose negative externalities on themselves and on the readers, while readers impose positive externalities on advertisers. Besides, in some of these markets such as nightclubs (with men and women being the two sides of the market), the congestion hypothesis seems natural (because of limited capacity).

Example 8: Competition to attract workers.

There are different groups of workers with different skill attributes. Firms which possess a production technology that takes the profile of workers' skill as input (with possibly some complementarities) compete to attract workers. As long as workers have quasi-linear preferences and as long as their (ex-post) private information concerns their private costs of productivity, our setup applies to describe the competition between firms to attract workers. The so-called directed search literature (Eeckoudt and Kircher, 2010) often analyzes such environments but usually with assumptions on how to model free mobility that differ from the ones developed in Section 3.[20]

[19] This could, for example, call for subsidizing those students who exert positive externalities on their peers.

[20] Peters (2010) is an exception. He also considers a model in which firms are interested in hiring only one worker and hire only the worker with the highest productivity among the candidates who apply: the welfare is then a function of the private type of the firm and the type of the worker who is hired.

2.3 The mechanism design setup

A key innovation of our approach with respect to the literature on local public goods is that we allow citizens to have private information, and, in line with this informational environment, we allow jurisdictions to post the mechanisms of their choice where a mechanism determines for all possible scenarios which public good is chosen in the jurisdiction as well as the taxes paid by the various residents of the jurisdictions. Our only restriction is that we do not allow the mechanism chosen by a jurisdiction to depend on the mechanisms chosen by other jurisdictions.[21] We believe the mechanisms we consider are easy to implement (as they do not require the fine observation of the mechanisms chosen by competing jurisdictions which may not be accessible). It should also be mentioned that in our environment with a continuum of jurisdictions our analysis would not be altered if we allowed the mechanism in a given jurisdiction to depend in an anonymous way on the distribution of mechanisms chosen by other jurisdictions.[22]

To simplify the presentation, we assume that jurisdictions can only choose direct mechanisms in which residents are asked to report their type and public goods and taxes are implemented based on the reports.[23] Since the set of participants in the jurisdiction is not known in advance, a mechanism should consider all possible realizations of who joins the jurisdiction. Formally, a direct mechanism, denoted by $(\widetilde{z}, \widetilde{t}) : \overline{\Theta} \to \mathcal{T}^{Z} \times \bigcup_{n\in\mathbb{N}} \mathbb{R}^n$, is of the following form: Each citizen i is asked to report his type $\widehat{\theta}_i \in \Theta$ with the constraint that $\widehat{\theta}^p_i = \theta^p_i$. Based on the profile of reports $\widehat{\theta}$,[24] the probability distribution $\widetilde{z}(\widehat{\theta})$ on the set of feasible alternative $Z(\theta^p)$ is implemented while $\widetilde{t}(\widehat{\theta}) := (\widetilde{t}_1(\widehat{\theta}), \cdots, \widetilde{t}_{\widetilde{n}(\theta)}(\widehat{\theta})) \in R^{\widetilde{n}(\theta)}$ represents the vector of transfers requested from citizens $i = 1, \ldots, \widetilde{n}(\theta)$. We further assume that the jurisdiction collects the monetary transfers from the citizens and that there is no waste of monetary resources so that the revenue of the jurisdiction (with type k_j) is $\sum_{i=1}^{\widetilde{n}(\theta)} t_i(\widehat{\theta}) - E_{z\sim\widetilde{z}(\theta)}[C(z, k_j)]$. We let $\mathrm{Supp}(\widetilde{z}(\widehat{\theta})) := \{z \in \mathcal{Z} | \mathrm{Prob}(\widetilde{z}(\widehat{\theta}) = z) > 0\}$ denote the support of the distribution $\widetilde{z}(\widehat{\theta})$ where $\mathrm{Prob}(\widetilde{z}(\widehat{\theta}) = z)$ denotes the corresponding

[21] This matters mainly when we discuss equilibrium uniqueness. By contrast, our decentralization result does not depend on this restriction.

[22] In general multi-principal settings, the dependence of the mechanism on other principals' mechanisms allow for folk theorem type results as a deviation by one principal may trigger an undesirable choice of mechanism by the competitor (see Peters and Troncoso-Valverde, 2013). With a continuum of principals as considered here, mechanisms even if contingent on the whole distribution of mechanisms do not allow for such folk theorem features because a deviation by a single principal does not affect the distribution of mechanisms.

[23] As argued later when we invoke the revelation principle, this restriction is without loss of generality.

[24] To simplify the notation and since the preferences of the jurisdiction are fully characterized at the ex-ante stage, we do not consider mechanisms where the jurisdiction is active in the mechanism (as it is the case in auctions with secret reserve price for example). This would not change our results.

probability that the local public good z is implemented when the profile of report is $\widehat{\theta}$. Note that at this stage, we do not require that citizens report truthfully their type in the mechanism, but of course we will later on require that citizens employ reporting strategies that constitute a Nash-Bayes equilibrium. In some applications, it is meaningful to allow citizens to exclude themselves from the mechanism, namely not to participate. Such a possibility is discussed in Section 4.5.

We impose an extra anonymity constraint on mechanisms stipulating that citizens from the same group should be treated alike (from an ex-ante point of view). We will also assume that citizens from the same group follow the same strategy in equilibrium. Formally, let $\mathcal{M}^*$ denote the set of all feasible direct and anonymous mechanisms. Feasibility requires that $\text{Supp}(\widetilde{z}(\widehat{\theta})) \subseteq Z(\theta^p)$ and anonymity requires that for any $\widehat{\theta} \in \overline{\Theta}$, for any pair i, j such that $\widehat{\theta}_i = \widehat{\theta}_j$ and for any $z \in \mathcal{Z}$, then $\text{Prob}(\widetilde{z}(\widehat{\theta}) = z) = \text{Prob}(\widetilde{z}(\widehat{\theta}) = z^{\widehat{\theta}^p}_{i\leftrightarrows j})$ and $\widetilde{t}_i(\widehat{\theta}) = \widetilde{t}_j(\widehat{\theta})$.

Citizens who join jurisdiction j with mechanism m are assumed to play a Bayes-Nash equilibrium (if one exists) of the induced announcement game. Individual incentives in the game typically depend on the citizen's belief about the types of the other citizens who join the jurisdiction. Those beliefs do not depend solely on the primitives about the distribution of preferences of citizens in the full economy but also on the way the various groups of citizens self-select in the jurisdiction, which is endogenously determined in equilibrium as detailed in the next Section.

Comments: 1) In our mechanism design approach, we assume implicitly that residents learn their type before interacting in the mechanism proposed by the jurisdiction. Otherwise, we should consider two-stage mechanisms: in a first stage, each citizen i reports his pre-entry type k_i and then learns more finely his type. In a second stage, each citizen reports his type θ_i. 2) Even though, anonymity is imposed at the mechanism level and symmetry is imposed on the equilibrium behavior of citizens (see also below the modelling of participation), our model allows for the possibility of having different groups with the same characteristics, thereby mitigating the severity of the constraint imposed by this restriction.

3 Free mobility equilibrium

The timing of events is as follows. First, each jurisdiction $j \in \mathcal{K}_J$ simultaneously posts a mechanism $m \in \mathcal{M} \subseteq \mathcal{M}^*$ as described above. Second, a random device ζ is simultaneously and independently drawn from a uniform distribution on $[0,1]$ for each jurisdiction. The realizations of ζ are publicly observed by all citizens and will typically help the citizens coordinate their participation decisions. Note that ζ does not enter the utility functions and can be thought of as a public correlation device similar to the one considered in (public) correlated equilibria in game theory (Aumann, 1974) or in sunspot equilibria (Cass and Shell, 1983). The introduction of ζ which we think is natural from a descriptive viewpoint will help establish that the welfare efficient solution can be decentralized as an equilibrium where jurisdictions use pure and symmetric strategies (see elaborations on this below). Third, citizens of the various groups $\mathcal{K}_C$ simultaneously decide which jurisdiction to participate to. In line with the above symmetry assumptions, we require that citizens of the same type adopt the same participation strategy. Finally, once in a jurisdiction with mechanism m, citizens play a (symmetric) Nash-Bayes equilibrium as induced by m.

Poisson distributions

In equilibrium, according to their group, citizens distribute themselves across the various jurisdictions with various intensities. This results in a random number of participants in each individual jurisdiction. Specifically, we adopt the view that if a mass x of citizens from group k distributes itself uniformly over a mass y of jurisdictions, each individual jurisdiction receives a random number n of citizens from group k that is distributed according to a Poisson distribution with mean $\mu_k = \frac{x}{y}$. That is,

$$\Pr(\textit{number of group } k \textit{ citizens} = n) = e^{-\mu_k} \frac{\mu_k^n}{n!}.$$

This formulation, which is popular in the search literature (see Rogerson et al's (2005) survey or more recently Peters (2010)), corresponds to the limit distribution obtained in a setting with n_B potential citizens and n_S jurisdictions, each potential citizen choosing (independently) a given jurisdiction with probability $\frac{1}{n_S}$, where n_B and n_S would tend to infinity and $\frac{n_B}{n_S}$ would be equal (or asymptotically close) to μ_k.[25] Our equilibrium concept will work directly in the limit case with a continuum of agents (see Jehiel and Lamy (2015) for elaborations in the auction application as to why this corresponds to the limit (when the number of agents goes to infinity) of equilibria with a finite sets of agents).

[25] The directed search literature (see Eeckoudt and Kircher, 2010) analyzes different classes of "matching technologies" which rely implicitly on some coordination between the entrants (e.g. if entry decisions are sequential). The Poisson distribution has a clear interpretation. It assumes entry decisions are made independently. Our decentralization result depends on this assumption.

In order to define formally the free mobility equilibrium, we introduce some additional notation where $\mu = (\mu_k)_{k\in\mathcal{K}_C} \in \mathbb{R}_+^K$ refers to a profile of Poisson parameters for each group $k \in \mathcal{K}_C$ and $\sigma = (\sigma_k)_{k\in\mathcal{K}_C}$ refers to a reporting strategy profile in a jurisdiction, where $\sigma^{\theta_i}_{\mathbf{k}(\theta_i)}(.)$ is the reporting strategy of a type θ_i-citizen.[26] Under truthful reporting, $\sigma^{\theta}_k(.)$ is a mass point on θ for each $\theta \in \Theta$ and each $k \in \mathcal{K}_C$. Under truthful reporting, we drop σ from the subsequent expressions to alleviate notation.

- We let $P(N|\mu) = e^{-\sum_{k=1}^K \mu_k} \cdot \prod_{k=1}^K \frac{[\mu_k]^{n_k}}{n_k!}$ denote the probability that $n(\theta) = N$ when the profile of participation rate is μ. From the perspective of any citizen in the economy, the probability that he faces the set of entrants N (not counting himself) is also equal to $P(N|\mu)$. This property of Poisson distributions is sometimes referred to as the *environmental equivalence* property in the literature (see Myerson (1998)).

- We let $g(.|\mu, k_j)$ denote the measure of the vector of types of the residents of a type $k_j \in \mathcal{K}_J$ jurisdiction when the profile of participation rates is $\mu \in \mathbb{R}_+^K$. By iterated expectation, we have $g(\theta|\mu, k_j) = \sum_{\substack{N\in\mathbb{N}^K: \\ n(\theta)=N}} P(N|\mu) \cdot q(\theta|N, k_j)$. From the environmental equivalence property, we have that $g(.|\mu, k_j)$ corresponds also to the measure of the profile of types of citizens other than i in a type $k_j \in \mathcal{K}_J$ jurisdiction when the profile of participation rates is $\mu \in \mathbb{R}_+^K$.

- We let $u_J(m, k_j, \mu, \sigma)$ denote the expected utility of a jurisdiction with type k_j having selected the mechanism $m = (\widetilde{z}, \widetilde{t})$ when the profile of participation rate is given by the Poisson distributions μ and residents are assumed to follow strategy σ. When the jurisdiction's objective is to maximize revenues, we have:

$$u_J(m, k_j, \mu, \sigma) = \int_{\Theta} \left[\int_{\Theta^{\widetilde{n}(\theta)}} \Big(\sum_{i'=1}^{\widetilde{n}(\theta)} \widetilde{t}_{i'}(\theta') - E_{z\sim\widetilde{z}(\theta')}[C(z, k_j)] \Big) \cdot \prod_{i=1}^{\widetilde{n}(\theta)} \sigma^{\theta_i}_{\mathbf{k}(\theta_i)}(\theta'_i) d\theta'_i \right] \cdot g(\theta|\mu, k_j) d\theta.$$

Under truthful reporting ($\sigma^{\theta_i}_{\mathbf{k}(\theta_i)}(.)$ is a dirac distribution concentrated on θ_i), the

[26] When citizens do not have the possibility to exclude themselves from the jurisdiction after joining it (as assumed implicitly in the subsequent formulas in order to alleviate notation), then for any type $\theta \in \Theta$, $\sigma^{\theta}_k(.)$ corresponds to a measure on Θ. By contrast, if they can exclude themselves, then we will add a "non participation" decision in the set of possible reports as discussed in Section 4.5.

expression simplifies into

$$u_J(m, k_j, \mu) = \int_{\overline{\Theta}} \underbrace{\Big[\sum_{i=1}^{\widetilde{n}(\theta)} \widetilde{t}_i(\theta) - E_{z\sim\widetilde{z}(\theta)}[C(z, k_j)]\Big]}_{:=\widetilde{R}(m,k_j,\theta)} \cdot g(\theta|\mu, k_j) d\theta,$$

where the function $\widetilde{R} : \mathcal{M}^* \times \mathcal{K}_J \times \overline{\Theta} \to \mathbb{R}$ is such that $\widetilde{R}(m, k_j, \theta)$ corresponds to the expected revenue (also referred to as the budget) of the jurisdiction of type k_j proposing the mechanism m when the vector of reported types is θ.

- Similarly we let $u(m, k_j, \mu, \sigma, \theta_i, \widehat{\theta}_i)$ denote the expected (interim) utility of a type θ_i citizen entering a type k_j jurisdiction with the direct mechanism $m = (\widetilde{z}, \widetilde{t})$ when he reports $\widehat{\theta}_i \in \Theta$, the distribution of participation is governed by μ and other citizens are assumed to follow the reporting strategy σ. We have:

$$u(m, k_j, \mu, \sigma, \theta_i, \widehat{\theta}_i) = \int_{\overline{\Theta}} \Bigg[\int_{\Theta^{\widetilde{n}(\theta_{-i})}} \Big[E_{z\sim\widetilde{z}(\widehat{\theta}_i \cup \theta'_{-i})}[v(z, \theta_i)] - \widetilde{t}_i(\widehat{\theta}_i \cup \theta'_{-i})\Big] \cdot \prod_{\substack{l=1 \\ l\neq i}}^{\widetilde{n}(\theta)} \sigma^{\theta_l}_{\mathbf{k}(\theta_l)}(\theta'_l) d\theta'_l \Bigg] \cdot g(\theta_{-i}|\mu, k_j, \theta_i) d\theta_{-i}.$$

When other citizens report truthfully, the expression simplifies into

$$u(m, k_j, \mu, \theta_i, \widehat{\theta}_i) = \int_{\overline{\Theta}} \underbrace{\Big[E_{z\sim\widetilde{z}(\widehat{\theta}_i \cup \theta_{-i})}[v(z, \theta_i)] - \widetilde{t}_i(\widehat{\theta}_i \cup \theta_{-i})\Big]}_{:=\widetilde{u}(m,\theta_{-i},\theta_i,\widehat{\theta}_i)} \cdot g(\theta_{-i}|\mu, k_j, \theta_i) d\theta_{-i},$$

where the function $\widetilde{u} : \mathcal{M}^* \times \overline{\Theta} \times \Theta \times \Theta \to \mathbb{R}$ is such that $\widetilde{u}(m, \theta_{-i}, \theta_i, \widehat{\theta}_i)$ corresponds to the expected payoff, in a jurisdiction proposing mechanism m, of citizen i having type θ_i and reporting type $\widehat{\theta}_i$, while the vector of reports of the other citizens in the jurisdiction is θ_{-i}. In the sequel (and with some abuse of notation), we will use the shortcut notation $\widetilde{u}(m, \theta_{-i}, \theta_i) = \widetilde{u}(m, \theta_{-i}, \theta_i, \theta_i)$.

- We also let $\widetilde{W}(m, k_j, \theta) := \sum_{i=1}^{\widetilde{n}(\theta)} \widetilde{u}(m, \theta_{-i}, \theta_i) + \widetilde{R}(m, k_j, \theta) = E_{z\sim\widetilde{z}(\theta)}[w(z, k_j, \theta)]$ denote the corresponding welfare under truthful reporting.

- Finally, we let $u_k(m, k_j, \mu, \sigma) := E_{\theta_i|k,k_j}\Big[\int_{\Theta} u(m, k_j, \mu, \sigma, \theta_i, \widehat{\theta}_i)\sigma^{\theta_i}_k(\widehat{\theta}_i) d\widehat{\theta}_i\Big]$ denote the expected (ex ante) utility of a group k citizen entering the type k_j jurisdiction with mechanism m when the distribution of participation is governed by μ and the strategy inside the mechanism is given by σ.

Competitive equilibrium

We are now ready to define our notion of equilibrium. First, an equilibrium specifies a choice of mechanism for every type k_j jurisdiction. We denote the choice of type k_j jurisdiction by $m^*_{k_j}$. Second, an equilibrium specifies a profile of participation rate functions $\mu^* = (\mu^*_k)_{k \in \mathcal{K}_C}$ and a profile of reporting strategies $\sigma^* = (\sigma^*_k)_{k \in \mathcal{K}_C}$ that determines for each mechanism m, each type k_j jurisdiction and each realization ζ: a) a Poisson distribution for group k with mean $\mu_k(m, k_j, \zeta)$ and b) a reporting strategy at the mechanism stage. We also let V_k denote the expected equilibrium payoff of a group k citizen. Formally,

Definition 1 *A free mobility equilibrium is defined as a quadruple* $m^* = (m^*_{k_j})_{k_j \in \mathcal{K}_J}$ *(where* $m_{k_j} \in \mathcal{M}$*),* $\mathcal{V}^* = (V_k)_{k \in \mathcal{K}_C} \in \mathbb{R}^K_+$*,* $\mu^* = (\mu^*_k)_{k \in \mathcal{K}_C}$ *with* $\mu^*_k : \mathcal{M} \times \mathcal{K}_j \times [0,1] \to \mathbb{R}_+$ *and* $\sigma^* = (\sigma^*_k)_{k \in \mathcal{K}_C}$ *where* $[\sigma^*_k(m, k_j, \zeta)]^\theta(.)$ *is a measure over* Θ *(for a type* θ *of citizen from group* k*) which represents the strategy of type* k *citizens in a* $k_j \in \mathcal{K}_J$ *jurisdiction proposing mechanism* $m \in \mathcal{M}$ *with public signal* $\zeta \in [0,1]$*, such that*

1. *(Utility maximization for jurisdictions) For any* $k_j \in \mathcal{K}_j$*,*

$$m^*_{k_j} \in Arg \max_{m \in \mathcal{M}} \int_0^1 u_J(m, k_j, \mu^*(m, k_j, \zeta), \sigma^*(m, k_j, \zeta)) d\zeta. \tag{3}$$

2. *(Utility maximization for citizens at the entry stage) For any* $(m, k_j, \zeta) \in \mathcal{M} \times \mathcal{K}_J \times [0,1]$ *and for all* $k \in \mathcal{K}_C$*,*[27]

$$\mu^*_k(m, k_j, \zeta) \underset{(resp.=)}{>} 0 \Rightarrow u_k(m, k_j, \mu^*(m, k_j, \zeta), \sigma^*(m, k_j, \zeta)) \underset{(resp.\leq)}{=} V_k. \tag{4}$$

3. *(Individual rationality at the entry stage) For all* $k \in \mathcal{K}_C$*,*

$$V_k \geq \underline{V}_k. \tag{5}$$

[27] To alleviate notation, we omit the case $u_k(m, k_j, \mu^*(m, k_j, \zeta), \sigma^*(m, k_j, \zeta)) > V_k$ for which we should set $\mu^*_k(m, k_j, \zeta) = \infty$. This could happen for example if a type k_j jurisdiction decided to pay a subsidy greater than V_k to all residents from type k and to provide no public good. As discussed above (and formalized in the proof of Lemma 4.1), this cannot happen on the equilibrium path. Out of the equilibrium path, this could occur but we omit that possibility because such mechanisms can never be profitable deviations (given A1).

4. (Matching conditions) For any $k \in \mathcal{K}_C$,

$$V_k \underset{(resp.=)}{>} \underline{V}_k \Rightarrow \int_0^1 \sum_{k_j \in \mathcal{K}_J} \mu_k^*(m_{k_j}^*, k_j, \zeta) \cdot f_J(k_j) d\zeta \underset{(resp. \leq)}{=} f_C(k). \tag{6}$$

5. (Equilibrium within jurisdictions) For any $(m, k_j, \zeta) \in \mathcal{M} \times \mathcal{K}_J \times [0,1]$ *and for all* $k \in \mathcal{K}_C$*, the strategy profile* $\sigma^*(m, k_j, \zeta)$ *is a Bayes-Nash equilibrium given the entry profile* $\mu^*(m, k_j, \zeta)$*.*[28]

Condition (1) says that jurisdictions (which must choose their mechanism before the realizations of ζ are known) pick mechanisms that maximize their expected utility given their type and their (correct) expectation concerning both the participation rates and the reporting strategies for every possible choice of mechanism m and any realization of ζ.

Condition (2) which is the fundamental free mobility condition says that group k citizens adjust their participation rate to any possible mechanism so that, if the participation rate is positive, citizens of group k get their equilibrium payoff V_k in expectation, and, if the participation rate is null, they get a non-larger payoff. There are several implicit assumptions here. First, the equilibrium notion defines participation rates and reporting strategies even for mechanisms that are not proposed in equilibrium. This is required so as to define how jurisdictions would assess the effect of choosing any possible mechanism. Second, when a given jurisdiction contemplates the effect of varying the proposed mechanism, it is assumed that the equilibrium utilities of the various group k citizens is unaffected by the mechanism. This is justified by our assumption that each individual jurisdiction is infinitesimal and it would not be a valid assumption if jurisdictions had market power. In this sense, our utility-taking assumption captures situations with perfect competition at the entry stage.[29]

Condition (3) is a classic individual rationality assumption. In our environment, it captures the idea that every citizen should get in equilibrium (in expectation) at least what he can get by living on his own.

Condition (4) ensures that in equilibrium all type k citizens are assigned to at most one jurisdiction and that all type k citizens are assigned to one jurisdiction if their expected payoff is strictly larger than the expected payoff they can derive on their own.

[28] This implicitly requires that an equilibrium exists. A simple way of ensuring this would be to discretize the type space as well as the set of mechanisms. We avoid getting into such technicalities here, as they are irrelevant for our insight.

[29] This consideration appears in the same way in the direct search literature (Guerrieri et al. (2010) and Peters (2010)).

Condition (5) ensures that given the equilibrium participation rates μ^*, citizens play according to a Nash-Bayes equilibrium of the game governed by the mechanism posted in the jurisdiction.

From the revelation principle, if we have a free mobility equilibrium then for any (direct) mechanism $m \in \mathcal{M}$ and any pair (μ, σ) of corresponding participation rates and reporting strategies, we can define a direct truthful mechanism, i.e. where truthful reporting is an equilibrium under the same (equilibrium) entry rate μ. This is the reason why in our subsequent analysis without loss of generality, we restrict ourselves to free mobility equilibria in which truthful reporting is an equilibrium for any feasible mechanism.[30] To alleviate notation, we then drop the dependence on σ^* in our notation.

When truth-telling is an equilibrium, then condition (5) requires that

$$\theta_i \in \operatorname{Arg}\max_{\widehat{\theta}_i \in \Theta} E_{\theta_{-i}|\mu_k^*(m,k_j,\zeta),\theta_i,\theta^p}\left[\widetilde{u}(m, \theta_{-i}, \theta_i, \widehat{\theta}_i)\right] \tag{7}$$

for any $(m, k_j, \zeta, \theta_i) \in \mathcal{M} \times \mathcal{K}_J \times [0, 1] \times \Theta$, where the expectations bear over realizations of θ conditional on θ_i and θ^p (we implicitly assume that once in a jurisdiction, citizens only observe the public characteristics of other citizens additionally to their private type θ_i).[31]

Let $m^0 = (\widetilde{z}^0, \widetilde{t}^0)$ denote the "default" mechanism that consists in providing no public good and charging no tax.[32] Observe that the "default" mechanism trivially satisfies the incentive compatibility constraints (7), and it guarantees budget balancedness, since $\widetilde{R}(m^0, k_j, \theta) = 0$ for any $\theta \in \overline{\Theta}$. Thus, when the objective of a jurisdiction is to maximize her revenue and when $m^0 \in \mathcal{M}$, the budget of this jurisdiction should be balanced in any free mobility equilibrium insofar as the jurisdiction can guarantee herself a null budget by posting the default mechanism m^0.[33]

[30] It should be noted that the revelation principle also allows us to have that the restriction to direct mechanisms is without loss of generality to the extent that mechanisms can not be made contingent on other jurisdictions' mechanisms.

[31] We are agnostic as to what those public characteristics may consist of, and we take them as exogenous. Note that the disclosure about fellow citizens characteristics could be part of the mechanism itself. This plays no role in our analysis.

[32] Formally, $\widetilde{z}^0(\theta) = z^0$ and $\widetilde{t}_i^0(\theta) = 0$ for any $\theta \in \overline{\Theta}$ and $i \in \{1, \ldots, \widetilde{n}(\theta)\}$.

[33] Note also that since $\underline{\underline{V}}_k < \underline{V}_k$ for any $k \in \mathcal{K}_C$, (4) and (5) imply that $\mu_k^*(m^0, k_j, \zeta) = 0$.

4 The main result

This Section is decomposed into several subsections. We first define a notion of (global) welfare efficiency in our economy. Next, we introduce a special mechanism called the pivot mechanism that is familiar in the mechanism design literature and that will play a key role in our main result. Then we state our main decentralization result and provide the main line of arguments as to why the result holds.

4.1 Efficiency

In this subsection we define the (unconstrained) solution that would maximize the global welfare in the economy which is also referred to as the global first-best. For any given profile of mechanisms and participation rate functions (m^*, μ^*), let $GW(m^*, \mu^*)$ denote the global welfare (when citizens report their type truthfully) which is formally expressed as

$$\begin{aligned} GW(m^*, \mu^*) \quad : \quad &= \sum_{k_j \in \mathcal{K}_J} \int_0^1 \left[\int_{\overline{\Theta}} \widetilde{W}(m, k_j, \theta) \cdot g(\theta \mid \mu^*(m, k_j, \zeta), k_j) d\theta \right] \cdot f_J(k_j) d\zeta \\ &+ \sum_{k=1}^{K} \left[f_C(k) - \sum_{k_j \in \mathcal{K}_J} \int_0^1 \mu_k^*(m, k_j, \zeta) \cdot f_J(k_j) d\zeta \right] \cdot \underline{V}_k. \end{aligned} \tag{8}$$

$GW(m^*, \mu^*)$ corresponds to the integral of the welfare over all jurisdictions for those citizens joining a non-empty jurisdiction augmented by the welfare of those citizens of the various groups k remaining on their own who then obtain $\underline{V}_k$.

The welfare maximizing solution seeks to maximize $GW(m^*, \mu^*)$ subject to the matching constraint that every citizen whatever his group k can belong to at most one jurisdiction.

It is straightforward that $\widetilde{W}(m, k_j, \theta)$ is bounded from above by the efficient ex post welfare $w^*(k_j, \theta)$ and that $\widetilde{W}(m, k_j, \theta) = w^*(k_j, \theta)$ if the mechanism m implements a public good in $z^*(k_j, \theta)$ which maximizes the welfare. Let $m_{k_j}^{eff}$ be such a mechanism (which is obtained by taking $\mathrm{Supp}(\widetilde{z}_{k_j}^{eff}(\theta)) \subseteq z^*(k_j, \theta)$ for any $\theta \in \overline{\Theta}$) and denote the profile of such mechanisms by $m^{eff} = (m_{k_j}^{eff})_{k_j \in \mathcal{K}_J}$.

Seeking for a welfare maximizing solution boils down to finding a solution to:

$$\max_{\widehat{\mu} \in \mathbb{R}_+^{\mathcal{K}_C \times [0,1] \times \mathcal{K}_J}} GW(m^{eff}, \mu^{opt}[\widehat{\mu}]) \tag{9}$$

subject to the matching constraints

$$\sum_{k_j \in \mathcal{K}_J} \int_0^1 \widehat{\mu}_k(k_j, \zeta) \cdot f_J(k_j) d\zeta \leq f_C(k) \tag{10}$$

for $k \in \mathcal{K}_C$, and where $\mu^{opt}[\widehat{\mu}]$ is defined by $\mu^{opt}[\widehat{\mu}](m_{k_j}^{eff}, k_j, \zeta) = \widehat{\mu}(k_j, \zeta)$.

The following existence result is proven in the Appendix.

Lemma 4.1 *Let assumption A1 hold. A solution exists to the maximization program (9).*

Call $\lambda = (\lambda_k)_{k \in \mathcal{K}_C} \in \mathbb{R}_+^K$ the vector of the Lagrange multipliers λ_k of the constraints (10). The corresponding Lagrangian writes:

$$\begin{aligned}
\mathcal{L}(\widehat{\mu}, \lambda) &= \sum_{k_j \in \mathcal{K}_J} \int_0^1 \left[\int_{\underline{\Theta}} w^*(k_j, \theta) g(\theta \mid \widehat{\mu}(k_j, \zeta), k_j) d\theta \right] \cdot f_J(k_j) d\zeta \\
&\quad + \sum_{k=1}^{K} \left[f_C(k) - \sum_{k_j \in \mathcal{K}_J} \int_0^1 \widehat{\mu}_k(k_j, \zeta) \cdot f_J(k_j) d\zeta \right] \cdot (\lambda_k + \underline{V}_k).
\end{aligned}$$

Define $V_k = \lambda_k + \underline{V}_k$, which is no smaller than $\underline{V}_k$ since $\lambda_k \geq 0$, and let $\mathcal{V} = (V_k)_{k \in \mathcal{K}}$. For now, V_k can be interpreted as the marginal welfare gain brought by an extra citizen of type k. Later on, in our main decentralization result, V_k will be interpreted as the equilibrium utility of group k citizens as appearing in the definition of the competitive equilibrium.

For a given profile of utilities $\mathcal{V}$, a given profile of participation rates $\mu = (\mu_k)_{k \in \mathcal{K}}$, a given direct truthful mechanism $m \in \mathcal{M}$, a given jurisdiction of type $k_j \in \mathcal{K}_j$, define the net local welfare as

$$\begin{aligned}
LNW(m, k_j, \mu; \mathcal{V}) &= \int_{\underline{\Theta}} \widetilde{W}(m, k_j, \theta) g(\theta \mid \mu, k_j) d\theta - \sum_{k=1}^{K} \mu_k \cdot V_k \qquad (11) \\
&= \sum_{N \in \mathbb{N}^K} P(N \mid \mu) \cdot \int_{\underline{\Theta}} \widetilde{W}(m, k_j, \theta) q(\theta \mid N, k_j) d\theta - \sum_{k=1}^{K} \mu_k \cdot V_k.
\end{aligned}$$

The Lagrangian can be re-written as a function of $(\widehat{\mu}, \mathcal{V})$ as

$$\mathcal{L}(\widehat{\mu}, \mathcal{V}) = \sum_{k_j \in \mathcal{K}_J} \int_0^1 LNW(m_{k_j}^{eff}, k_j, \widehat{\mu}(k_j, \zeta); \mathcal{V}) \ \cdot f_J(k_j) d\zeta + \sum_{k=1}^{K} f_C(k) \cdot V_k. \tag{12}$$

Moreover, let

$$J(m,k_j;\mathcal{V}) := \left\{\mu \in \mathbb{R}_+^K \mid \text{for all } k \in \mathcal{K}_C, \frac{\partial LNW(m,k_j,\mu;\mathcal{V})}{\partial \mu_k} \underset{(resp. \leq)}{=} 0 \text{ if } \mu_k \underset{(resp =)}{>} 0\right\}.$$

The maximization of the Lagrangian $\mathcal{L}$ implies that for any optimum $(\widehat{\mu}^{opt}, \mathcal{V}^{opt})$, we have

$$\widehat{\mu}^{opt}(k_j,\zeta) \in \operatorname{Arg}\max_{\mu \in \mathbb{R}_+^K} LNW(m_{k_j}^{eff}, k_j, \mu; \mathcal{V}) \tag{13}$$

for any $k_j \in \mathcal{K}_J$ and $\zeta \in [0,1]$, which further implies from the first-order conditions that

$$\widehat{\mu}^{opt}(k_j,\zeta) \in J(m_{k_j}^{eff}, k_j; \mathcal{V}^{opt}).$$

In order to illustrate the role of the public devices ζ in the derivation of the first-best solution, we build on Example 2. Intuitively, ζ is required because it need not be optimal to spread uniformly citizens from the same group over jurisdictions of the same type, especially when economies of scale would dictate it is optimal to have some empty jurisdictions.

Example 2 (continued): Let $n_l^* := \inf\{n \in \mathbb{N} | n \cdot \widehat{\theta} - C(n) > 0\}$ and $n_u^* := \inf\{n \in \mathbb{N} | C(n+1) - C(n) - \widehat{\theta} > 0\}$. We can check that $1 < n_l^* \leq n^* < n_u^*$. The efficient alternative is given by : $Z(n) = \emptyset$ if $n < n_l^*$, $Z(n) = \{1, ...n\}$ if $n_l^* \leq n < n_u^*$ and $Z(n) = \{1, ...n_u^* - 1\}$ if $n \geq n_u^*$. If there are not enough citizens, the public good is not provided. If there are too many citizens, it would be too costly to provide the public good to everybody and there is some exclusion. For intermediate numbers of citizens, the public good is provided to all citizens.

Let $\overline{\mu} = \operatorname{Arg}\max_{\mu \geq 0} LNW(m^{eff}, \mu; \underline{V})$. Let us assume that building the public good can be strictly profitable, namely $LNW(m^{eff}, \overline{\mu}; \underline{V}) > 0$ and also that the mass of jurisdictions is sufficiently large so that $LNW(m^{eff}, \mu; \underline{V}) < 0$ for any $0 < \mu \leq \frac{f_C}{f_J}$.

The maximization problem (9) reduces to the choice of the function $\mu : [0,1] \to \mathbb{R}_+$, i.e. how to share the homogenous citizens among the homogenous jurisdictions. The maximization program is then:

$$\max_{\substack{\mu(.):\\ \int_0^1 \mu(\zeta)d\zeta \leq \frac{f_C}{f_J}}} \int_0^1 \sum_{n=n_l^*}^{\infty} e^{-\mu(\zeta)} \frac{[\mu(\zeta)]^n}{n!} \cdot \underbrace{\left[\min\{n, n_u^* - 1\} \cdot \widehat{\theta} - C(\min\{n, n_u^* - 1\})\right]}_{\equiv \phi_n} d\zeta + \left[\frac{f_C}{f_J} - \int_0^1 \mu(\zeta)d\zeta\right] \cdot \underline{V}.$$

First we note that we must have $\mu(\zeta) = 0$ for a positive mass ζ because otherwise we would have $0 < \mu(\zeta) \leq \frac{f_C}{f_J}$ for a positive mass of ζ which would be incompatible with optimality (LNW would then be negative, see above). Then we obtain as a by-product that the matching condition is necessarily binding (otherwise it would be strictly profitable to create extra surplus by filling empty jurisdictions up to the level $\overline{\mu}$). Last, without loss of generality, we can seek for solutions with $\mu(\zeta) = \widehat{\mu} \geq \frac{f_C}{f_J}$ for $\zeta \leq \frac{f_C}{\widehat{\mu}}$ and $\mu(\zeta) = 0$ where $\widehat{\mu} \in [\frac{f_C}{f_J}, \infty]$.[34]

In this case, the global welfare as a function of $\widehat{\mu}$ denoted by $GW(\widehat{\mu})$ is then given by

$$GW(\widehat{\mu}) = f_C \cdot \sum_{n=n_l^*}^{\infty} e^{-\widehat{\mu}} \frac{\widehat{\mu}^{n-1}}{n!} \cdot \phi_n = \frac{f_C}{\widehat{\mu}} \cdot LNW(m^{eff}, \widehat{\mu}; 0). \tag{14}$$

If there exists $\mu^{opt} \geq \frac{f_C}{f_J}$ such that $\mu^{opt} \in \operatorname{Arg\,max}_{\widehat{\mu} \geq 0} GW(\widehat{\mu})$, then $\widehat{\mu} = \mu^{opt}$ is a solution that maximizes the global welfare.[35] If $n^* = n_u^*$, then $\widehat{\mu} \to GW(\widehat{\mu})$ is concave which further implies that $\widehat{\mu} = \frac{f_C}{f_J}$ is the solution that maximizes the global welfare for $\widehat{\mu} \in [\frac{f_C}{f_J}, \infty)$.

4.2 Pivot mechanism

For a given type k_j of jurisdiction, we say that a mechanism $m = (\widetilde{z}, \widetilde{t})$ is a (generalized) Groves mechanism[36] if it is an efficient mechanism, i.e. for every θ,

$$Supp(\widetilde{z}(\theta)) \subseteq z^*(k_j, \theta)$$

and if the transfers are of the form

$$\widetilde{t}_i(\theta) = -w^*(k_j, \theta) + E_{z \sim \widetilde{z}(\theta)}[v(z, \theta_i)] + h_i(k_j, \theta_i^p, \theta_{-i}), \tag{15}$$

for $i = 1, \ldots, \widetilde{n}(\theta)$ and for any $\theta \in \Theta$, where h_i is an arbitrary function of $k_j \in \mathcal{K}_J$, $\theta_i^p \in \Theta^p$ and $\theta_{-i} \in \overline{\Theta}$.

Observe that compared to the textbook description of Groves mechanism, our setting is slightly more general given that the set of feasible alternatives is allowed to depend on the set of public characteristics of the participants. This extension is required given

[34] If $\mu(.)$ can take several positive values, then we can raise at least the same global welfare by assigning the citizens homogenously to jurisdiction with the entry rate $\operatorname{Arg\,max}_{\mu \geq 0} \frac{LNW(m^{eff}, \mu; 0)}{\mu}$.

[35] By contrast, if $\mu^{opt} \in \operatorname{Arg\,max}_{\widehat{\mu} \geq 0} GW(\widehat{\mu})$ implies that $\mu^{opt} < \frac{f_C}{f_J}$, then we need an extra-assumption to characterize the optimum.

[36] See Groves (1973).

that we wish to allow for externalities that depend on the public characteristics of the residents.[37]

Despite this difference, it is readily verified that the maximization program of any resident i of a type k_j jurisdiction coincides with the maximization of the (local) welfare (formally, $\widetilde{u}(m, \theta_{-i}, \theta_i, \widehat{\theta}_i) = E_{z \sim \widetilde{z}(\theta_{-i} \cup \widehat{\theta}_i)}[w(z, k_j, \theta)] - h_i(k_j, \theta_i^p, \theta_{-i})$) so that it is a weakly dominant strategy for each citizen to report truthfully the private part of his type, and therefore truthful reporting constitutes an equilibrium of the Groves mechanism whatever resident i's expectation about the vector of types θ_{-i} of other residents in the jurisdiction (which also depends on the equilibrium participation rates).

In the class of Groves mechanisms, we introduce a special mechanism which plays a central role in our main result. For any $k_j \in \mathcal{K}_J$, the pivot mechanism that we denote by $m_{k_j}^{piv} = (\widetilde{z}_{k_j}^{piv}, \widetilde{t}_{k_j}^{piv})$ is the Groves mechanism in which

$$h_i(k_j, \theta_i^p, \theta_{-i}) = w^*(k_j, \theta_{-i}).$$

In other words, in the pivot mechanism each resident pays the externality that he imposes due to his presence. Such a definition implies that the payoff of each resident is equal to the net welfare contribution he brings to the jurisdiction due to his presence: for each $\theta \in \overline{\Theta}$ and $k_j \in \mathcal{K}_J$,

$$\widetilde{u}(m_{k_j}^{piv}, \theta_{-i}, \theta_i) = w^*(k_j, \theta) - w^*(k_j, \theta_{-i}). \tag{16}$$

Comments: 1) Formally, the possibility that $z^*(k_j, \theta)$ is not always a singleton opens the door to multiple specifications of what we refer to as the pivot mechanism. Nevertheless, the payoffs of citizens (as specified in (16)) do not depend on the selection. 2) The pivot mechanism satisfies the anonymity restriction we impose throughout the paper: Identical citizens have the same payoff and so a fortiori the same expected payoff.

Throughout our analysis, we assume that jurisdictions can use the pivot mechanism:

Assumption A 2 *For any $k_j \in \mathcal{K}_J$, we have $m_{k_j}^{\mathrm{piv}} \in \mathcal{M}$.*

We illustrate in the various examples what the pivot mechanism implies.

[37] One place in the mechanism design literature in which such dependence appears is Jehiel et al. (1996) who require in the context of a sale with externalities that the good cannot be transferred to a potential buyer who would decide not to participate in the mechanism.

Example 1 (continued): In the pivot mechanism as defined above, we have $\widetilde{t}_i(\theta) = \frac{k_j}{\widetilde{n}(\theta)}$ if $\widetilde{n}(\theta) > 1$ and $\widetilde{t}_i(\theta) = 0$ otherwise. The payoff of a citizen is strictly positive and it is equal to k_j only in the case when the citizen is the only citizen.

Example 2 (continued): In the pivot mechanism, the transfer paid by a citizen who has access to the public good is given by $C(\widetilde{n}(\theta)) - C(\widetilde{n}(\theta) - 1)$ if $\widetilde{n}(\theta) < n_u^*$ and $\widehat{\theta}$ if $\widetilde{n}(\theta) \geq n_u^*$. Note that the ex post payoff of a citizen is strictly positive only when $n_l^* \leq \widetilde{n}(\theta) < n_u^*$. Concerning the budget of the jurisdiction, it is equal to 0 when $\widetilde{n}(\theta) < n_l^*$, $(\widetilde{n}(\theta)-1) \cdot C(\widetilde{n}(\theta)) - \widetilde{n}(\theta) \cdot C(\widetilde{n}(\theta)-1)$ when $n_l^* \leq \widetilde{n}(\theta) < n_u^*$ and $(n_u^* - 1) \cdot \widehat{\theta} - C(n_u^* - 1)$. When the public good is provided, the budget is positive when $\widetilde{n}(\theta) > n^*$ and negative otherwise.

Example 4 (continued): For a realization of the type of the entrants such that $n^* \leq \overline{n}$, the pivot mechanism is characterized by: the n^* buyers with the highest valuation buy a unit at price $\max\{v_{n^*+1}, c_{n^*}\}$ and the n^* sellers with the lowest costs sell a unit at price $\min\{v_{n^*}, c_{n^*+1}\}$. The revenue of the platform is then $n^* \cdot [\max\{v_{n^*+1}, c_{n^*}\} - \min\{v_{n^*}, c_{n^*+1}\}] \leq 0$ (where the inequality is obtained by noting that n^* is such that $v_{n^*} \geq c_{n^*}$ and $v_{n^*+1} < c_{n^*+1}$). In this example, we see that the pivot mechanism runs some deficit except when the congestion constraint is binding ($n^* > \overline{n}$) in which case revenue can be expressed as $\overline{n} \cdot [\max\{v_{\overline{n}+1}, c_{\overline{n}}\} - \min\{v_{\overline{n}}, c_{\overline{n}+1}\}]$ which is typically positive.

Example 5 (continued): When the jurisdiction is not a trading platform but rather a seller who owns K homogenous goods and when citizens are unit-demand buyers, the pivot mechanism corresponds to the $(K+1)^{th}$ price auction with the reserve price set at the seller's reservation value.

4.3 Decentralization result

Theorem 1 *Let assumptions A1 and A2 hold. When jurisdictions are revenue-maximizers, there is a competitive equilibrium in which all jurisdictions propose the pivot mechanism and the global first-best is achieved.*

Note that the efficiency achieved by the equilibrium considered in the Theorem concerns both the efficient choice of public good in every jurisdiction for any constituency (this is achieved by the use of Groves mechanisms) and the efficient profile of participation in the various jurisdictions, which is achieved thanks to the use of the specific selection of the pivot mechanism among Groves mechanisms. Indeed, as we will see in more details in the next Section, the pivot mechanism precisely aligns the incentives to participate of the citizens of the various groups with the net local welfare in the jurisdiction, which in turn gives rise to the overall efficiency result.

Perhaps it may come as a surprise that jurisdictions interested in revenue maximization would pick mechanisms that maximize global welfare. There are several reasons for that. First, jurisdictions are utility-takers, which implies that local revenue coincides with net local welfare. Second, given the re-writing of the objective of jurisdictions as net local welfare, it may then sound less surprising that jurisdictions would pick an efficient mechanism. But, jurisdictions also care about the participation rates of the various groups. A fundamental property of the pivot mechanism is that it aligns citizens' incentives with the net local welfare, which in turn ensures that an optimal participation rate can be part of an equilibrium when jurisdictions post the pivot mechanism.

Interestingly, as we show in the extension section, if jurisdictions seek to maximize local welfare (of some sort), then inefficiencies may be inevitable. So the lesson from our main decentralization result is that somehow privatizing the management of jurisdictions need not be a bad idea even if one is interested in the maximization of global welfare. In the extension section, we somehow mitigate or refine this conclusion in the light of various elaborations such as the consideration of non-mobile citizens or the consideration of multiple public goods within a single jurisdiction where decentralization would be done at the level of the public good.

Example 7 (continued): One of the key questions posed by the two-sided markets literature concerns the puzzle as to why it is often the case that only one side of the market is requested to pay a fee (see Caillaud and Jullien (2004), Rochet and Tirole (2003), or Armstrong (2006)) while the other side is even sometimes subsidized. Our analysis suggests a clear-cut answer that does not rely on coordination issues: In the revenue-maximizing equilibrium, the equilibrium fee/subsidy of each side reflects the externality imposed marginally by this agent on other agents (this is the definition of the pivot mechanism). Given that the externality is not necessarily negative in these markets, such a mechanism may, in some cases, lead to subsidize one side of the market (e.g. women in a nightclub).

Example 4 (continued): Concerning the development of E-commerce where economies of scale seem unlimited, assumption A1 need not be relevant. However, in the presence of some degree of specialization/differentiation or when trade requires space and space is scarce, then efficiency would dictate that each single platform attracts finitely many economic agents, and our decentralization result then applies.

Example 8 (continued): Our efficiency result contrasts with the inefficiency highlighted in Peters (2010). The key difference is that firms post wages in Peters (2010) and not general mechanisms. By contrast, in the pivot mechanism, the wage to be received by a worker depends on the profile of candidates applying to the jobs in the firm (and would not in general be a sole function of the own characteristics of the worker).

4.4 The main arguments

We decompose the main arguments for the proof of the Theorem into several steps. The technical details are given in the Appendix.

First, in a given free mobility equilibrium (with truthful reporting), a type k_j jurisdiction seeks to maximize with respect to $m \in \mathcal{M}$ local expected revenues which can be re-written as

$$\int_0^1 u_J(m, k_j, \mu^*(m, k_j, \zeta))d\zeta = \int_0^1 LNW(m, k_j, \mu^*(m, k_j, \zeta); \mathcal{V}^*)d\zeta. \tag{17}$$

Indeed, given that the utilities of group k citizens are fixed at V_k independently of m, and given that in expectation there are $\mu_k^*(m, k_j, \zeta)$ group k citizens joining jurisdiction with type k_j when m is proposed and ζ is drawn, local revenues are simply the total local welfare net of $\sum_{k=1}^K \mu_k^*(m, k_j, \zeta) \cdot V_k$, which corresponds to the sum of the expected utilities of participating citizens. This is precisely the definition of $LNW(m, k_j, \mu^*(m, k_j, \zeta); \mathcal{V}^*)$, thereby explaining (17).

Maximizing (17) with respect to m is seemingly a complex problem given that the dependence of $\mu^*(m, k_j, \zeta)$ in m is not a priori known. For any type k_j jurisdiction, consider instead the relaxed maximization program that requires maximizing

$$\int_0^1 LNW(m, k_j, \mu(\zeta); \mathcal{V}^*)d\zeta \tag{18}$$

with respect to $m \in \mathcal{M}$ and $\mu : [0,1] \to \mathbb{R}_+^K$ where μ can be chosen independently of m. This is obviously an upperbound on what type k_j jurisdiction can achieve in equilibrium. However, we will show that this upperbound is obtained in our proposed candidate equilibrium, thereby paving the way to establishing the Theorem.

To see this, observe that by offering the pivot mechanism $m_{k_j}^{piv}$ and for any participation rate profile function $\mu : [0,1] \to \mathbb{R}_+^K$, truth-telling is an equilibrium and the outcome resulting from truth-telling maximizes $LNW(m, k_j, \mu(\zeta); \mathcal{V}^*)$ with respect to the mechanism

m. This is because the pivot mechanism is an efficient mechanism in which truth-telling is an equilibrium.

But, observe also - as a result of (16) the second key property of the pivot mechanism - that the expected utility derived by a group k citizen is precisely his contribution to the local welfare so that the equilibrium condition (4) for any k_j, ζ (and $m = m_{k_j}^{piv}$) can be re-written as

$$\frac{\partial LNW}{\partial \mu_k}(m_{k_j}^{piv}, k_j, \mu^*(\zeta); \mathcal{V}^*) \underset{(resp.\leq)}{=} 0 \text{ if } \mu_k^*(\zeta) \underset{(resp.=)}{>} 0 \tag{19}$$

which is equivalent to saying that $\mu^*(\zeta) \in J(m_{k_j}^{piv}, k_j; \mathcal{V}^*)$ for any $\zeta \in [0,1]$ (see the definition of J in subsection 4.1).

Since the solution $(m_{k_j}^{piv},\ \mu(\cdot))$ to the unconstrained program (18) satisfies $\mu(\zeta) \in J(m_{k_j}^{piv}, k_j; \mathcal{V}^*)$ for any ζ (those are the first-order conditions), it follows that the choice of the pivot mechanism $m_{k_j}^{piv}$ together with the efficient participation rate profiles constitute an equilibrium and achieves the first-best solution. In light of Rogerson (1992) who was the first to stress that in the pivot mechanism, any efficient pre-participation private investment on one's own type is sustainable in equilibrium,[38] this second step of our argument is to be expected, once we note that entry decisions can be viewed as a specific private investment on one own's type (not entering corresponds equivalently to having a zero valuation type and imposing no externalities on the fellow citizens).

One may wonder about the necessity in general of having the random devices ζ in the above arguments and whether it is needed on efficiency grounds. The following proposition establishes that in the absence of ζ, there may be no equilibrium that achieves the global first-best solution.[39]

Proposition 4.2 *When jurisdictions are revenue-maximizers, it may happen that there is no competitive equilibrium that implements the global-first best in which the function $\mu(m, k_j, \zeta)$ does not depend on ζ.*

The proof relies on Example 2.

Example 2 (continued): Suppose that $\text{Arg}\max_{\widehat{\mu}\geq 0} GW(\widehat{\mu}) = \{\mu^*\} \subset (f_C, \infty)$, then having the mass $\frac{f_C}{\mu^*}$ of jurisdictions with the mass μ^* of citizens in each while leaving the remaining jurisdictions empty is the solution to the first-best and thus cannot be

[38] See Hatfield et al. (2015) for a converse.

[39] We stress that our notion of equilibrium in Definition 1 assumes implicitly that jurisdictions of the same type should use symmetric pure strategies.

implemented with jurisdictions posting the same mechanism in which participation rate would be uniform.

In the context of Example 2, it may be noted that in the equilibrium considered in the decentralization theorem, the expected revenue of a jurisdiction having positive participation is null, thereby ensuring that our equilibrium construction can be interpreted as a competitive equilibrium in which there would be no public device on which citizens could coordinate their participation decisions but instead jurisdictions would at the decentralized level post (m, ζ) where ζ would help citizens coordinate their participation decision.

To see that jurisdictions with positive participation make zero revenues in this context, observe that the first-order condition for the global first-best leads to

$$LNW(m^{eff}, \mu^*; 0) = \mu^* \cdot \frac{\partial LNW(m^{eff}, \mu^*; 0)}{\partial \mu}.$$

Furthermore, in the pivot mechanism, we have that the utility of citizens is equal to

$$u(m^{piv}, \mu^*) = \frac{\partial LNW(m^{eff}, \mu^*; 0)}{\partial \mu}.$$

Hence, the expected revenue of a non-empty jurisdiction can be written as

$$LNW(m^{eff}, \mu^*; u(m^{piv}, \mu^*)) = LNW(m^{eff}, \mu^*; 0) - \mu^* \cdot u(m^{piv}, \mu^*) = 0. \tag{20}$$

More generally, we show in the proof of Theorem 1 that in the equilibrium that implements the first-best, all type k_j jurisdictions make the same expected profit irrespective of ζ, which thus allows us more generally to reinterpret the equilibrium of the Theorem as one in which there is no public device ζ, but jurisdictions post ζ in addition to the mechanism $m_{k_j}^{piv}$. When some k_j jurisdictions are empty at the optimum, then this means that non-empty jurisdictions also make zero revenues, which is somehow reminiscent of the insight discussed about contestable markets in IO that the pressure from potential non-present rivals may drive profits down to zero (see Baumol, Panzar and Willig (1982) for a discussion of this in the context of Bertrand competition with constant marginal costs).

4.5 Participation constraints

We discuss in this subsection the case in which citizens can exclude themselves from the jurisdiction after entering it. In the language of the mechanism design literature, this corresponds to imposing some individual rationality constraints. We will argue that our analysis is unchanged independently of how those constraints are specified in our environment with excludable local public goods.

In our mechanism design framework, (interim) participation constraints can be formalized as adding a possible report $\widehat{\theta} = NP \notin \Theta$ (where NP stands for non-participation) such that $u(m, k_j, \mu, \sigma, \theta_i, NP) = \underline{\underline{V}}_{\mathbf{k}(\theta_i)}$. Interim participation constraints can be formalized as:

$$E_{\theta_{-i}|\mu_k^*(m,k_j,\zeta),\theta_i,\theta^p}\left[\widetilde{u}(m, \theta_{-i}, \theta_i)\right] \geq \underline{\underline{V}}_{\mathbf{k}(\theta_i)} \tag{21}$$

for any $\theta_i \in \Theta$, $\theta^p \in \overline{\Theta}^p$ and $i \in \{1, \ldots, \widetilde{n}(\theta)\}$.

An alternative approach is to consider ex post participation constraints,[40] which can be formalized as:

$$\widetilde{u}(m, \theta_{-i}, \theta_i) \geq \underline{\underline{V}}_{\mathbf{k}(\theta_i)} \tag{22}$$

for any $\theta \in \overline{\Theta}$ and $i \in \{1, \ldots, \widetilde{n}(\theta)\}$. It is straightforward that the ex post constraints (22) are stronger than the interim ones (21).

Since we have assumed that jurisdictions can exclude whoever they wish from group k , thereby giving a payoff $\underline{\underline{V}}_k$ to such a group k citizen, the participation constraints are automatically satisfied in the pivot mechanism. Formally, (1) implies that $w^*(k_j, \theta) \geq w^*(k_j, \theta_{-i}) + \underline{\underline{V}}_{\mathbf{k}(\theta_i)}$ for any $k_j \in \mathcal{K}_j$, $\theta \in \overline{\Theta}$ and any citizen $i = 1, ..., \widetilde{n}(\theta)$, which further implies that under the pivot mechanism, the strongest form of participation constraints (22) is satisfied:

$$v(\widetilde{z}_{k_j}^{piv}(\theta), \theta_i) - [\widetilde{t}_{k_j}^{piv}]_i(\theta) \geq \underline{\underline{V}}_{\mathbf{k}(\theta_i)}. \tag{23}$$

That is, even if citizen i could decide ex post to be on his own (and get $\underline{\underline{V}}_{\mathbf{k}(\theta_i)}$) he would be no worse off sticking to the outcome of the pivot mechanism (and get $v(\widetilde{z}_{k_j}^{piv}(\theta), \theta_i) - [\widetilde{t}_{k_j}^{piv}]_i(\theta)$).

[40] Ex post participation constraints differ slightly from ex post quitting rights (Compte and Jehiel, 2009), which have more practical relevance. However, in our "first-best equilibrium" that relies on the pivot mechanism, ex post quitting rights will not affect our results either: it is still a (weakly) dominant strategy to report its true type even when one can exert quitting rights after the choice of public good and tax is implemented.

If we had not allowed jurisdictions to exclude citizens, then ex post participation constraints would not necessarily be satisfied in the pivot mechanism. However, our decentralization result would still hold in this case provided jurisdictions can charge entry fees right at the time citizens make their participation decision so that participation constraints are required only ex ante, as reflected in (5).

4.6 Related literature

Mechanism design

An interesting by-product of our decentralization result is that the equilibrium shown in the decentralization theorem is such that local budgets are ex ante balanced (given that the default mechanism m^0 is available to jurisdictions and m^0 guarantees null revenues). This observation together with the observations that public decisions are ex post efficient and citizens' individual participation constraints are satisfied in the pivot mechanism seems at odds with the vast literature that has followed the introduction of the VCG mechanism and which has found in the vein of Myerson and Satterthwaite (1983) that it was impossible to satisfy simultaneously ex post efficiency, individual participation constraints and budget balancedness in contexts with fixed sets of participants. Of course, a key difference is that the set of participants is endogenous in our setting, and our congestion assumption leads the equilibrium utilities of agents to be fixed independently of the chosen mechanism. As a matter of fact, considering the exchange platform application in Example 4, the congestion assumption is key for the derivation of our result. If there were no congestion, the pivot mechanism would lead to budget deficits (as reminded above). It is only thanks to the presence of congestion that the pivot mechanism may in some cases result in budget surpluses, and as it turns out the participation rates in the profile that maximizes global welfare are such that indeed there are expected budget surpluses.

Another important difference with the literature on budget-balanced mechanism pioneered by d'Aspremont and Gerard-Varet (1979) is that here, jurisdictions play the role of residual claimant and full efficiency does not require that the budget be entirely redistributed to citizens. Had we forced jurisdictions to redistribute the entire budget surplus, then some inefficiencies would be inevitable (see section 5 on local welfare objective for an illustration of this).

It should be stressed that our decentralization result has a robust mechanism design flavor. On the one hand, the equilibrium shown above does not require that jurisdictions have any knowledge about how information is distributed given that the pivot mechanism is prior free. As such it is robust to the exact specifications of what jurisdictions observe about the types of the various agents in the economy. On the other hand, our equilibrium is also robust to what citizens observe about other citizens' preferences.

Auctions

Our decentralization result has some similarity with the observation made in competing auction environments that sellers would set reserve prices at their valuations in second-price auctions (see McAfee (1993), Levin and Smith (1994), Peters (1997, 2001) and the extension to discrimination issues considered by Jehiel and Lamy, 2015).[41] Indeed, in the auction environment the pivot mechanism takes the form of a second price auction (or equivalently an ascending/English auction) with a reserve price set at the seller's valuation, and thus our decentralization result can be viewed as embedding as a special case the auction setup previously considered in the literature. It should be mentioned that with the exception of Kim and Kircher (2014) who consider a stylized version of McAfee (1993), the previous literature does not address the issue of global efficiency as defined in section 4.1.

An important difference with auction environments and our general treatment is that the public device ζ is not needed in the context of auctions so as to help coordinate citizens on unbalanced participation decisions among homogenous jurisdictions. This is so because, in the auction setup, efficiency requires that (homogenous) buyers should split uniformly across homogenous sellers (that post the pivot mechanism), which comes from the fact that buyers are substitutes (formally the functions $\mu \to LNW(m_{k_j}^{piv}, k_j, \mu; \mathcal{V})$ are concave as shown in Jehiel and Lamy (2015)).

Jehiel and Lamy (2015) also strengthens the argument in favor of the pivot mechanism by showing that the efficient entry profile is the only equilibrium profile when entry is modelled with Poisson distributions (as in the present paper). This means that all equilibria implement the first-best and are payoff-equivalent to the efficient equilibrium with the pivot mechanism. We will discuss equilibrium uniqueness later in Section 5.2.

[41] There is technically a small difference with McAfee's (1993) model and ours: we deal with a discrete set of ex-ante types for which we impose no structure while McAfee (1993) and later developments of his model deal with a (uni-dimensional) continuum and consider specific payoff functions (allowing the use of Myerson's (1981) techniques to express the rents of the agents). Fundamentally, what is required for our decentralization insight is that Lemma 4.1 holds.

4.7 An explicit equilibrium construction

Example 1 (continued): To simplify the algebra, we assume that the type k_j is distributed according to a distribution with positive density on $[0, \overline{k}]$, namely we are dealing with continuous instead of discrete types. For each jurisdiction, the problem reduces to one of setting the average number of entrants as a function of her type k_j.

For an exogenously given $V \equiv V_1 \geq \underline{V}_1 \equiv \underline{V} > 0$, the local welfare maximization problem corresponds to

$$\max_{m \in \mathcal{M}, \mu \geq 0} LNW(m, k_j, \mu; \mathcal{V}) = (1 - e^{-\mu}) \cdot k_j - \mu \cdot V.$$

This leads to the optimal solution $\mu^{opt}(k_j) = \log[\frac{k_j}{V}]$ if $k_j \geq V$ or $\mu^{opt}(k_j) = 0$ otherwise. In the optimal solution, the matching condition is then given by

$$V \underset{(resp.\ =)}{>} \underline{V} \Longrightarrow \underbrace{\int_V^\infty \log[\frac{k_j}{V}] f_J(k_j) dk_j}_{\equiv \psi(V)} \underset{(resp.\ \leq)}{=} f_C. \tag{24}$$

The function ψ is strictly monotonic on $[0, \overline{k}]$ with $\psi(\overline{k}) = 0$ so that the matching condition (24) has a unique solution in $[\underline{V}, \infty)$, with $V = \underline{V}$ if and only if $\psi(\underline{V}) \leq f_C$. Let V^{opt} be the solution to (24).

In this setup, a mechanism reduces to a fee $f(n)$ as a function of the total number of participants n. The equilibrium equation (4) reduces to

$$\mu(f, k_j) \underset{(resp.\ =)}{>} 0 \Longrightarrow \sum_{n=0}^{\infty} e^{-\mu(f,k_j)} \frac{[\mu^*(f, k_j)]^n}{n!} \cdot [\frac{k_j}{n+1} - f(n+1)] \underset{(resp.\ \leq)}{=} V. \tag{25}$$

When solving the maximization program of a given jurisdiction, we can restrict ourselves without loss of generality to positive fixed fees.[42] Let $\xi : [0, \infty] \to [1, \infty]$ be the bijection defined by $\xi(x) = \frac{x}{1-e^{-x}}$ for $x > 0$ and $\xi(0) = 1$. From (25), the equilibrium participation rate for a fixed fee $\widehat{f}$ is thus given by $\mu(\widehat{f}, k_j) = \xi^{-1}(\max\{\frac{k_j}{V+\widehat{f}}, 1\})$.

[42] The argument is the following: for a given jurisdiction k_j, consider a fee schedule $f : \mathbb{N} \to \mathbb{R}$ and a corresponding equilibrium participation rate $\widetilde{\mu}$ solving (25). Then we can replace f by the fixed fee $\widehat{f} = \sum_{n=0}^{\infty} e^{-\widetilde{\mu}} \frac{\widetilde{\mu}^n}{n!} f(n+1)$ for which $\widetilde{\mu}$ is still an equilibrium participation rate (but not necessarily the unique one). Furthermore, the budget constraint requires that the fee is positive.

As implied by our general Theorem, we can check that each jurisdiction proposing the pivot mechanism does implement the first best: it leads both to $\mu(m_{k_j}^{piv}, k_j) = \mu^{opt}(k_j)$ and $V = V^{opt}$.

5 Extensions

In this Section, we consider various extensions of the basic setup. Some of these extensions lead to the same conclusion as in our main decentralization result. Other extensions require amending the objective of the jurisdictions so as to achieve the first-best solution. We also consider in the setting described above the case in which jurisdictions would seek to maximize local welfare instead of revenues and suggest that inefficiencies must then arise. Finally, we consider situations in which decentralization would be made at the level of the local public goods when several public goods are provided in each jurisdiction and suggest that some inefficiencies must then arise.

5.1 Pre-investment of jurisdictions

We start with the case in which prior to the interaction, jurisdictions j engage in some investment regarding their characteristic $k_j \in \mathcal{K}_j$. Formally, we assume there are different ex ante types of jurisdictions referred to as $\kappa_j \in \varkappa_j$ and each κ_j jurisdiction invests to get some k_j according to some distribution $\widetilde{p}$ at a cost $\gamma(\kappa_j, \widetilde{p})$ that depends both on κ_j and $\widetilde{p}$. The global first-best should now optimize over the investments of jurisdictions as well as the choice of public goods in every jurisdiction and the participation rates in the various jurisdictions. In the following result, we note that our basic decentralization result extends to this situation:

Proposition 5.1 *In the model with pre-investment, if jurisdictions are revenue-maximizers, one equilibrium is such that jurisdictions invest optimally, they chose the pivot mechanism and participation rates as well as the choices of public goods are those corresponding to the global first-best.*

The main reason why Proposition 5.1 holds is that as in the main model the objective of the jurisdictions can be shown to coincide with the net local welfare as defined above. Thus assuming participations rates are determined optimally (as was shown to be a possible equilibrium outcome when jurisdictions choose the pivot mechanism), the incentives of jurisdictions are aligned with the global welfare at the investment stage, thereby explaining why the basic decentralization result extends to this environment with pre-investment. We note that a similar observation appears in the context of auctions in Albrecht et al. (2014).

In a different vein, our main decentralization result would also hold if the type k_j of jurisdiction j were not publicly observed as long as the density $f_Y(y|k_j)$ does not depend on k_j as we noted in footnote 9: this is so because disclosing k_j for the jurisdiction through the choice of the pivot mechanism would not be detrimental to the jurisdiction, and such a choice of mechanism would still allow to implement the first-best.

Comments: 1) It should be mentioned that the result of Proposition 5.1 does not depend on the details of the investment game (e.g. whether it is simultaneous versus sequential) since each jurisdiction is assumed to be atomistic and thus does not expect that her individual choice would change the decisions of other jurisdictions. 2) In a model in which citizens can also invest in their type at the same time or before they make their participation decision, there is still an equilibrium which implements the global first-best because in the pivot mechanism the investment incentive of citizens is aligned with local welfare. 3) We note however that if investment decisions are made after citizens' choices of jurisdictions, then inefficiencies may arise due to the well known hold-up problem (Rogerson, 1992). This echoes another general observation that if the choice of mechanism is made after the participation decision, then inefficiencies would arise in the same vein as in Myerson's (1981) analysis of optimal auctions with exogenous participation. 4) Our observation about the efficiency of investment may be related to the literature on investment in matching (see Mailath et al. (2013) for a recent account of this literature). Indeed, in that literature, when prices are fully flexible, the transfers made in their general equilibrium formulation correspond to the transfers in the pivot mechanism, thereby explaining why efficiency obtains.

5.2 Other equilibria

The main reason why we need not have a unique equilibrium outcome is that when jurisdictions propose the pivot mechanism, it is not a priori guaranteed that the participation rates of the various groups of citizens in the various jurisdictions will be determined so as to maximize the global welfare (even if that is a possibility, as the Theorem demonstrates).

In some instances though such as the auction setup studied in Jehiel and Lamy (2015), one can ensure a uniqueness result building on some concavity properties. As an extension of this, we can establish:

Proposition 5.2 *If the functions $\mu \to LNW(m_{k_j}^{eff}, k_j, \mu; \mathcal{V})$ are strictly quasi-concave on R_+^K for any $k_j \in \mathcal{K}_J$, then the global first-best is achieved in any equilibrium.*

Proof On the one hand, in any equilibrium with the vector $\mathcal{V}$ for citizens' expected payoffs, the revenue obtained by the jurisdiction k_j is equal to $\max_{\mu \in \mathbb{R}_+^K} LNW(m_{k_j}^{eff}, k_j, \mu; \mathcal{V})$ because proposing the pivot mechanism guarantees to reach this value which is an upper-bound on what she can raise (because for concave functions, first-order conditions imply that we are at an global optimum, i.e. formally $J(m_{k_j}^{eff}, k_j; \mathcal{V}) = \operatorname{Arg\,max}_{\mu \in \mathbb{R}_+^K} LNW(m_{k_j}^{eff}, k_j, \mu; \mathcal{V}))$.

Furthermore, when we wish to maximize the Lagrangian $\mathcal{L}(\widehat{\mu}, \mathcal{V})$, the quasi-concavity of the local welfare implies that we can look without loss of generality for a solution such that $\widehat{\mu}(m_{k_j}^{eff}, k_j, \zeta)$ does not depend on ζ (because for a quasi-concave functions the set of optima is convex such that we can replace the entry rate function $\zeta \to \widehat{\mu}(m_{k_j}^{eff}, k_j, \zeta)$ by the constant vector $\int_0^1 \widehat{\mu}(m_{k_j}^{eff}, k_j, \zeta) d\zeta$).

We are thus left with maximizing the mass of citizens entering each kind of jurisdiction given the matching constraints. The corresponding Lagrangian is concave and the first-order conditions with respect to those masses [resp. to the vector $\mathcal{V}$] coincide with the equilibrium conditions (4) [resp. the matching conditions (6)]. For quasi-concave functions, first-order conditions imply that we are at a global optimum, which completes the proof. **Q.E.D.**

The strict quasi-concavity of the net local welfare function is a strong assumption, even if as proven in Jehiel and Lamy (2015), it holds in single good auction environments (and also in some multi-object environments). While Jehiel and Lamy (2015) establishes a partial efficiency property, we obtain as a corollary of Proposition 5.2 a slightly stronger global efficiency property.

Remark. When the net local welfare function $LNW(m, k_j, \mu; \mathcal{V})$ is a (strictly) concave function of the participation rate μ_k, we have that the functions $\mu_k^*(\cdot)$ must be constant across jurisdictions that are "payoff-equivalent"[43] from their point of view in the optimum but also in any equilibrium.[44] Put differently, in the concave case, agents of a given

[43]We say that two jurisdictions characterized by (m, k_j, μ_{-k}) and (m', k_j', μ_{-k}') are payoff equivalent from the point of view of group k agents if $u_k(m, k_j, (\mu_k, \mu_{-k})) = u_k(m', k_j', (\mu_k, \mu_{-k}'))$ for any $\mu_k \geq 0$.

[44]Otherwise there would be a contradiction with implementing the first-best because the global welfare could be raised by a uniform splitting of those group k citizens to those payoff-equivalent jurisdictions (from the point of view of those group k citizens).

type should spread uniformly across jurisdictions which propose pivot mechanisms that are payoff equivalent to them. This observation explains why McAfee (1993) obtains, in a competing auction model, the property that buyers with the same valuation should spread uniformly across those auctions they participate to because in this case, the net local welfare function is concave.[45]

When concavity does not hold, the first-best optimality is not guaranteed in all equilibria, but it may be argued that it gives then a potential role to the federal government in coordinating the economic agents on the best equilibrium (the federal government could suggest the (best) equilibrium and given that economic agents would have no reason not to follow the prescribed play, they could coordinate on this equilibrium).[46]

5.3 What if other local objectives?

In this subsection we investigate how the equilibrium would look like in particular regarding its efficiency properties if jurisdictions were instructed to maximize objective functions other than the local revenue.[47]

We consider two alternative objective functions for jurisdictions that represent different modelling of local welfare. In the first case, we assume that jurisdictions are instructed to maximize local welfare defined as the sum of utilities of all participating agents. In the second case, we assume that jurisdictions seek to maximize per capita welfare (implicitly assuming that citizens are homogenous).

Local welfare as objective

For a given vector of utility profile $\mathcal{V} = (V_k)_{k=1}^{K}$, a jurisdiction j of type k_j now seeks to maximize

$$\int_0^1 \int_{\Theta} \widetilde{W}(m, k_j, \theta) g(\theta \mid \mu^*(m, k_j, \zeta), k_j) d\theta d\zeta$$

[45] It also explains why Peters (2010) obtains the same property in a seemingly completely different environment, in which firms compete for workers by posting wages (see the Appendix for details).

[46] It is also possible that a clever use of fees (depending finely on the number of participants) that come on the top of the pivot mechanism (which would thus require to strengthen assumption A2) would allow to get uniqueness beyond the concavity case covered by Proposition 5.2. In this vein, Weyl (2010) proposes "insulating tariffs" in a two-sided framework in order to avoid multiple equilibria: those pricing schemes correspond to an insurance fee with respect to the number of entrants from the other side in the platform. In a framework with asymmetric information and participation costs, Lu (2009) proposes a modified second-price auction in which participants are fully reimbursed from their entry costs which guarantees equilibrium uniqueness.

[47] These alternative objective functions may also be the result of political economy considerations (voting procedures and desire to be reelected), see Bierbrauer and Boyer (2014) for a recent formal model giving some support to this when voters are privately informed and with candidates posting general public good mechanisms with bribes.

In the special case in which the group k from which the citizen comes is publicly observable, easy adaptations of our main decentralization reveal shows that one equilibrium is such that each jurisdiction proposes the pivot mechanism augmented with a subsidy fee of V_k for each citizen k that joins the jurisdiction (note that such a mechanism is a Groves mechanism) and the participation rate profiles are determined so as to maximize a pseudo global welfare in which the utility of citizens would count twice as much as the cost of public goods (since their opportunity costs of not participating are not taken into account). That is, such an equilibrium fails to maximize global welfare as a result of jurisdictions' distorted incentives to attract too many citizens.

It may be argued that it would be hard to sustain situations in which budget is not balanced ex ante (which unlike in the revenue maximizing case could now arise). Assume as an alternative that jurisdictions seek to maximize local welfare subject to the constraint that the budget should be balanced, and consider Example 1.

Example 1 (continued): Assume that the jurisdiction maximizes the gross welfare of its citizens subject to the constraint that the budget be balanced ex post. The maximization program (3) reduces to

$$\max_{\widehat{f}\in\mathbb{R}_+}\sum_{n=1}^{\infty} e^{-\mu^*(\widehat{f},k_j)}\frac{[\mu^*(\widehat{f},k_j)]^n}{n!}k_j.$$

The optimal mechanism consists thus in setting no fees at all in order to attract a number of citizens as large as possible. The equilibrium payoff $V^*\geq \underline{V}$ is then characterized by the matching condition

$$V^* \underset{(resp.\ =)}{>} \underline{V} \Longrightarrow \int_{V^*}^{\infty}\xi^{-1}[\frac{k_j}{V^*}]f_J(k_j)dk_j \underset{(resp.\ \leq)}{=} f_C \tag{26}$$

where the function ξ has been defined in subsection 4.5.

Since $e^x > \xi(x)$ on $(0,\infty)$, (24) and (26) lead to $V^*\geq V^{opt}$ with $V^* > V^{opt}$ if $\int_{\underline{V}}^{\infty}\xi^{-1}[\frac{k_j}{\underline{V}}]f_J(k_j)dk_j > f_C$. Inefficiency comes from the fact that there are too many jurisdictions (those with $V < V^*$) that will never attract any citizen, which reflects the property that the expected payoff of citizens is too high. This latter feature could suggest that this is beneficial for the global welfare. However the global welfare is limited to V^* since the profit of each jurisdiction is null. More generally, even fixing the expected citizens'payoff V, citizens do not participate efficiently in the various jurisdictions: The loss in welfare comes from the fact that citizens participate too much in jurisdictions with high value, since they do not internalize the externality imposed to the other residents.

Local welfare per capita as objective

Assume now that jurisdictions maximize the welfare per capita of its citizens still under the constraint that the budget should be balanced ex post. We continue with Example 1.

Example 1 (continued): The maximization program (3) reduces now to $\max_{\widehat{f}\in\mathbb{R}_+}\sum_{n=1}^{\infty}e^{-\mu(\widehat{f},k_j)}\frac{[\mu(\widehat{f},k_j)]^n}{n!}\frac{k_j}{n}$. For a given V, this program is equivalent to $\max_{\mu\leq\xi^{-1}(\frac{k_j}{V})}\sum_{n=1}^{\infty}e^{-\mu}\frac{\mu^n}{n!}\frac{k_j}{n}$. Let μ^{**} be the solution of this program without the constraint $\mu\leq\xi^{-1}(\frac{k_j}{V})$ (which reflects the budget constraint - note that μ^{**} depends neither on k_j nor on V). The optimal fee is then $f^{**}(k_j)=\max\{(\frac{k_j}{\xi(\mu^{**})}-V),0\}$. The equilibrium payoff $V^{**}\geq\underline{V}$ is then characterized by the matching condition

$$V^{**}\underset{(resp.\ =)}{>}\underline{V}\Longrightarrow\int_{V^{**}}^{\infty}\min\{\xi^{-1}[\frac{k_j}{V^{**}}],\mu^{**}\}f_J(k_j)dk_j\underset{(resp.\ \leq)}{=}f_C. \tag{27}$$

(26) and (27) lead to $V^*\geq V^{**}$ with $V^*>V^{**}$ if $\int_{\underline{V}}^{\infty}\xi^{-1}[\frac{k_j}{\underline{V}}]f_J(k_j)dk_j>f_C$. The loss in welfare comes now from the fact that citizens participate too little in jurisdictions with high values and too much in jurisdictions with low values.[48]

The inefficiencies highlighted when jurisdictions seek to maximize some form of local welfare should be contrasted with our main decentralization result saying that when the objective is local revenues, there is a globally efficient equilibrium.

5.4 What if some residents are not mobile?

It is natural to consider situations in which some citizens would not be mobile: they would already be attached to some jurisdictions and high switching costs would deter them from moving to other jurisdictions. If jurisdictions continue to seek to maximize revenues, then some inefficiencies may arise in the presence of non-mobile citizens in the same way as inefficiencies arise in the optimal auction with exogenous set of bidders as analyzed by Myerson (1981): The designer in an attempt to reduce the rents of non-mobile citizens (which unlike those of mobile citizens depend on the choice of mechanism) will have an incentive to distort the choice of the allocation rule typically by providing less public good than is socially desirable (see the next subsection for a more structured example of public good and the kind of distortion one might expect).

[48] It is not clear how V^{**} and V^{opt} compare.

But, there is a simple fix to these inefficiencies to the extent that one can impose the objective function to jurisdictions: Call $R^{NM}(m, k_j, \mu)$ the (expected) utility of the non-mobile residents in a given jurisdiction and suppose every jurisdiction is instructed to maximize its revenue augmented with the utility of the non-mobile residents, namely $u_j(m, k_j, \mu) = E_{\theta|\mu,k_j}[\widetilde{R}(m, k_j, \theta)] + R^{NM}(m, k_j, \mu)$. Then it is straightforward that jurisdictions seek to maximize the local welfare net of the utilities of the mobile citizens, i.e. that the equality (17) still holds in this environment. Our main result easily adapts to such a case: the global first-best solution can be obtained as an equilibrium outcome in which jurisdictions choose the pivot mechanism (so that public goods are efficiently chosen given the constituency) and mobile citizens distribute themselves efficiently across jurisdictions. This can be summarized in the following result:

Proposition 5.3 *In the presence of non-mobile residents, if jurisdictions seek to maximize local revenues augmented with the utility of the local non-mobile residents, the global first-best can be achieved as an equilibrium in which jurisdictions all choose the pivot mechanism.*

Thus, in a world with a mix of non-mobile and mobile citizens, our analysis suggests that jurisdictions should be incentivized in some way to internalize the well-being of non-mobile citizens together with the profit they can make from all citizens (whether mobile or non-mobile). Otherwise, inefficiencies may arise as shown in Jehiel and Lamy (2015) for auctions.

5.5 Multiple public good jurisdictions

It is typically the case that different types of public goods are provided within each jurisdiction: parks, museums, transportation service, schooling, etc. Our main analysis carries over to the extent that we interpret the public good z in the main model as the profile of public goods in the more usual sense. But, once one acknowledges the presence of multiple goods, it becomes natural to reconsider the above analysis assuming instead that each local public good is managed separately by a separate body. As we will see, a decentralization/privatization of this sort may lead to inefficiencies if as the word privatization suggests each management body seeks to maximize its own revenue and even if there are no direct interactions of any sort (crowding out or complementarity) between the various public goods and even if all citizens are mobile as considered in the main model above.

The rough intuition for this is as follows. The attractiveness of a jurisdiction will now be determined by the profile of mechanisms provided by the different public good managers. Thus, from the viewpoint of one such manager, the perceived effect of the choice of mechanism on the participation rate of who joins the jurisdiction will be less than when this is the only public good provided. As a result, there is free-riding w.r.t. to the incentives to attract citizens: Every public good manager has an incentive to reduce part of the rent related to her own public good of those citizens joining the jurisdiction, which as in the model with non-mobile citizens results in some inefficiencies.[49]

While a full analysis of such a model is somehow complicated, we illustrate now some of the insights that we expect would arise in the context of two examples.

We first consider a framework without ex-post asymmetric information that will serve the purpose of showing how the rent is shared between the citizens and public good managers. We will next consider a setup with asymmetric information in which we will comment on the form of the inefficiencies that arise in equilibrium.

Example 0 (continued): We assume that both jurisdictions and citizens are homogeneous and that there is no private information. We will derive a symmetric equilibrium in which the expected number of citizens in each jurisdiction is b where b is the ratio of the total mass of citizens to the total mass of jurisdictions. Within each jurisdiction, there are D principals providing various public goods $z_1, \ldots, z_D$. We let $\widehat{w}_d(n) > 0$ denote the total ex post surplus related to principal d when the number of citizens in the jurisdiction is $n \geq 1$ (we assume $\widehat{w}_d(n)$ is non-decreasing in n) and $W_d(\mu) := \sum_{n=1}^{\infty} P(n \mid \mu) \cdot \widehat{w}_d(n)$ denote the corresponding expected surplus when the participation rate is μ.

We may assume without loss of generality that the instrument (or mechanism) of each principal is π_d, the share of citizens' contribution to the local welfare that is redistributed to the citizen (this is equivalent to a user fee but it will be more convenient to present the analysis). The vector $\pi = (\pi_1, \ldots, \pi_D)$ corresponds then to the complete mechanism posted by a jurisdiction. Based on π, citizens decide which jurisdiction to go to. Using our previous notation, total local welfare can be written as $w(n) = \sum_{d=1}^{D} \widehat{w}_d(n)$.

If there are n participants, the payoff of each participant is then $\sum_{d=1}^{D} \pi_d \cdot \left[\widehat{w}_d(n) - \widehat{w}_d(n-1)\right]$ in the jurisdiction with the mechanism profile π. The equilibrium condition for participation reduces then to:

[49] Of course, if the different managers could freely negotiate on the choice of mechanisms in exchange for side-payments, we would expect them to behave as if there were a unique manager in charge of all public goods and the same decentralization result as the one arising above would arise.

$$\sum_{d=1}^{D} \pi_d \cdot \frac{\partial W_d}{\partial \mu}(\mu^*(\pi)) = V \tag{28}$$

where V is the (endogenously determined) expected ex ante utility of citizens. After basic calculations, the expected payoff of the principal d when the participation rate is μ can be written as:

$$W_d(\mu) - \pi_d \cdot \mu \cdot \frac{\partial W_d(\mu)}{\partial \mu} = W_d(\mu) + \sum_{d' \neq d} \pi_{d'} \cdot \mu \cdot \frac{\partial W_d(\mu)}{\partial \mu} - \mu \cdot V. \tag{29}$$

In equilibrium, maximizing (29) with respect to μ leads to first-order conditions, which after simple transformations can be written as

$$\pi_d = 1 - \sum_{d' \neq d} \pi_{d'} \cdot \frac{-b \cdot \frac{\partial^2 W_{d'}}{\partial^2 \mu}(b)}{\frac{\partial W_d}{\partial \mu}(b)}. \tag{30}$$

Assuming symmetry ($W_d = W$ for all d) allows us to simplify (30) into:

$$\pi_d = \frac{1}{1 + (D-1) \cdot RRAC(b)} \tag{31}$$

where $RRAC(b) := \frac{-b \cdot \frac{\partial^2 W}{\partial^2 \mu}(b)}{\frac{\partial W}{\partial \mu}(b)}$ should be interpreted as the relative risk aversion coefficient of the surplus with respect to the entry rate.

To the extent that $RRAC(b) > 0$ (which holds when the various public goods are in fact auctions for private goods, as shown in Jehiel and Lamy (2015)), we have that π_d is decreasing in D, which formalizes that free-riding is more severe when there are more principals in each jurisdiction. In the limit with many principals, every principal takes participation as exogenous, thereby leading her to extract all the surplus from citizens ($\pi_d = 0$) as expected. The effect of the risk aversion coefficient $RRAC(b)$ is the following: when $RRAC(b)$ increases, then participation becomes less elastic (entrants are somehow more risk-averse when they consider switching to the mechanism of a jurisdiction where someone deviates from the equilibrium), which encourages principals to be more aggressive, i.e. to leave less surplus to citizens.

In this example with multiple public goods, there are no inefficiencies: neither ex post because there is no asymmetric information, nor ex ante because uniform splitting across jurisdiction is optimal. To discuss the ex-post inefficiencies associated to multiple public goods in the context of homogeneous jurisdictions, we now develop a model with asymmetric information in the vein of Example 3 above.

Example 3 (continued): Within each jurisdiction, there are D public goods $z_1, \ldots, z_D$. Citizens are homogeneous and have no ex ante private information. Their reservation utility is 0. Upon joining a jurisdiction, a citizen i learns his type $\theta_i = (\theta_i^d)_{d=1}^D$. If he has access to the profile of public goods $z = (z_d)_{d=1}^D$ and pays t, he enjoys utility $\sum_{d=1}^D \theta_i^d \cdot z_d - t$. Each citizen may unilaterally decide to exclude himself from any public good d in which case he does not enjoy the corresponding public good z_d and has nothing to pay for it. The cost of providing z_d when n citizens are served is denoted $C(z_d, n)$ and is assumed to be convex in z_d with $\lim_{n\to\infty} \frac{C(z_d,n)}{n} = \infty$ for all z_d. We assume that θ_i^d are independently distributed across citizens and across public goods according to a density function $f(\cdot)$ with support $[0,1]$ and cumulative denoted by $F(\cdot)$. We assume F is regular in the sense that $\theta \to \theta - \frac{1-F(\theta)}{f(\theta)}$ is increasing in θ.

Manager d chooses a direct truthful mechanism m^d of a form similar to the one described above except that he only tries to elicit θ_i^d from every citizen i and not $\theta_i^{d'}$ for $d' \neq d$ and as a result of the profile of announcements $\widehat{\theta}^d$ proposes a choice $z^d(\widehat{\theta}^d)$ of public good, a probability $p_i^d(\widehat{\theta}^d)$ that i has access to the public good d and a transfer $t_i^d(\widehat{\theta}^d)$. We let $\mu(m^d)$ denote the participation rate (of the unique group of citizens) as a function of the mechanism m^d chosen by manager d (assuming other managers $d' \neq d$ pick their equilibrium mechanism) and we let V denote the ex ante equilibrium utility of citizens (V is endogenously determined in equilibrium but for the point we wish to make it suffices to take it as given).

We let $Z_i^{d,n}(\theta_i^d) := \int\limits_{\theta_{-i}^d} z^d(\widehat{\theta}^d) \cdot p_i^d(\widehat{\theta}^d) \cdot \prod_{\substack{i'=1 \\ i'\neq i}}^n f(\theta_{i'}^d) d\theta_{i'}^d$ and $T_i^{d,n}(\theta_i^d) := \int\limits_{\theta_{-i}^d} t_i^d(\widehat{\theta}^d) \cdot \prod_{\substack{i'=1 \\ i'\neq i}}^n f(\theta_{i'}^d) d\theta_{i'}^d$ denote the expected public good d citizen i has access to when i reports θ_i^d and other citizens $-i$ truthfully report their type, the mechanism is m^d and there are n citizens. The expected utility of citizen i with type θ_i^d derives from public good d by announcing $\widehat{\theta}_i^d$ can be written as $u^{d,n}(\theta_i^d; \widehat{\theta}_i^d) = \theta_i^d \cdot Z_i^{d,n}(\widehat{\theta}_i^d) - T_i^{d,n}(\widehat{\theta}_i^d)$. Citizens' incentives to report their type truthfully then imply that for all $\theta_i^d; \widehat{\theta}_i^d$,

$$u^{d,n}(\theta_i^d; \theta_i^d) \geq u^{d,n}(\theta_i^d; \widehat{\theta}_i^d). \tag{32}$$

The participation constraints (we assume here that citizens are free ex-post to quit any mechanism with no cost) reduce to

$$u^{d,n}(0;0) \geq 0. \tag{33}$$

It is by now routine calculation (see Myerson, 1981) to write the expected revenue of manager d conditional on n citizens joining as:

$$R^{d,n} = \int\limits_{\theta^d} \Big[\sum_{i=1}^{n}[\theta_i^d - \frac{1-F(\theta_i^d)}{f(\theta_i^d)}]\cdot z^d(\theta^d) - C(z^d(\theta^d),n)\Big]\cdot\prod_{i=1}^{n} f(\theta_i^d)d\theta_i^d - n\cdot u^{d,n}(0;0). \tag{34}$$

We let $P(n \mid \mu) = e^{-\mu}\frac{\mu^n}{n!}$ denote the probability that n citizens show up when the participation follows a Poisson distribution with mean μ. For any $d = 1,\ldots,D$, let $u^d(\mu, m^d)$ denote the expected utility derived by a citizen from public good d when the participation rate is μ. It is readily verified from standard calculus that

$$u^d(\mu, m^d) = \sum_{0}^{\infty}\Big[P(n \mid \mu)\big[u^{d,n}(0;0) + \int\limits_{n=0}^{1}(1-F(\theta_i^d))\cdot Z_i^{d,n}(\theta_i^d)d\theta_i^d\big]\Big]. \tag{35}$$

The maximization program of manager d amounts to maximizing

$$\sum_{n=0}^{\infty} P(n \mid \mu)\cdot R^{d,n}$$

with respect to m^d and μ subject to

$$\sum_{d=1}^{D} u^d(\mu, m^d) = V. \tag{36}$$

Calling λ the Lagrange multiplier of (36) and using the expressions (34) and (35), the first order condition with respect to m^d implies that up to the constraint that $Z_i^{d,n}(\theta_i^d)$ should be non-decreasing in θ_i^d, the implemented amount of public good $z^d(\theta^d)$ should be such that it maximizes the virtual welfare defined as

$$\sum_{i=1}^{n}\max\{\theta_i^{d,virtual}, 0\}\cdot z - C(z, \widehat{n}(\theta^d)),$$

and

$$p_i^d(\theta^d) = \underset{(resp.0)}{1} \text{ if } \theta_i^{d,virtual} \underset{(resp.<)}{\geq} 0$$

where

$$\theta_i^{d,virtual} = \theta_i^d - (1-\lambda)\frac{1-F(\theta_i^d)}{f(\theta_i^d)}$$

and $\widehat{n}(\theta^d) := \sum_{i=1}^n p_i^d(\theta^d)$ denotes the number of citizens that have access to the public good.[50]

When there is a single public good $D = 1$ our previous analysis shows that $\lambda = 1$ as the equilibrium shown in the main Theorem induces an efficient choice of public good. However, when there is more than one public good $D > 1$, some inefficiencies would typically arise and take the form shown above (which is parameterized by a single parameter λ). When the number of public goods tends to infinity (and payoffs are say divided by the number of public goods to ensure all expressions remain finite), then the single choice of mechanism by the manager of one public good would not affect participation and thus the above solution would require that $\lambda = 0$ so that we find the usual textbook definition of virtual valuation. For intermediate number of public goods, we would expect the solution to be in between the efficient solution and the Myersonian solution, i.e. we would expect that $0 < \lambda < 1$ but more work is required to establish this formally.

With multiple public good providers inside a jurisdiction, a natural question is then how to restore efficiency (or limit free riding) in a decentralized way. While we leave that general question for future research, we wish to suggest that in some special cases efficiency can be restored simply by giving all the bargaining power to citizens while public good providers would only be allowed to accept or reject deals proposed by citizens.[51] Specifically, assume that once citizens join a jurisdiction, all private information becomes public, and suppose that the pivot outcome lies in the core.[52] Then, if only citizens make

[50] Without ex post participation constraints, no ex post inefficiencies would arise because entry fees could be used to affect participation without any additional gain induced by ex post distortions. We would then be back to Example 0 and we would thus obtain that expected entry fees would be larger than in the pivot mechanism so that the citizens with 0 type would not participate. In light of our calculation,, we would obtain that the Lagrange multiplier λ would be 1 so that virtual valuations would coincide with valuations in such a case. When ex post participation constraints are required as assumed in the main text, ex post inefficiencies would typically occur whenever (33) is binding (because then we would expect lambda to be different from 1).

[51] This kind of solutions appears in the holdout problem (Kominers and Weyl, 2012). This idea also appears in Lamy (2013) to avoid the hold-up problem (with respect to the entry costs) in auctions with entry costs.

[52] This is so when for each public good provider, the welfare associated to the ex-post efficient allocation is submodular with respect to the set of residents. When public good providers propose private goods,

offers at the bargaining stage, one can conjecture based on the bargaining model considered in Compte and Jehiel (2010) that the outcome of the bargaining as parties get infinitely patient will be the selection from the core that maximizes a generalized Nash product putting 0 weight on the designer's payoff (since the designer makes no offer). Given that the pivot outcome would maximize such a Nash product whatever the relative bargaining powers of citizens, one can conjecture that the outcome of the bargaining game would be the pivot outcome, and thus our decentralization result would extend to such a scenario, despite the presence of multiple public good providers.[53]

6 Conclusion

This paper has revisited the classic Tiebout hypothesis assuming that competing jurisdictions can post the mechanism of their choice as in the tradition of the mechanism design literature. To the extent that citizens have quasi-linear preferences, that there are no informational externalities (private value setting), that there are no externalities between jurisdictions and that jurisdictions control all local public goods, it has established that profit maximizing jurisdictions may lead to a globally efficient outcome both in terms of the choice of public goods and the allocation of citizens into jurisdictions. Such a decentralization result together with a good understanding of its limits may lead to re-thinking the role of public interventions in the context of local public goods, suggesting for example, in the benchmark scenario (with only mobile citizens) that the role of the federal government should be reduced to that of coordinating agents on the good equilibrium.

An interesting avenue for future research is to analyze environments with multiple public good providers within jurisdictions where our analysis suggests that it would be a good idea for jurisdictions to incentivize public good providers to post mechanisms that implement payoffs that are close to the pivot payoffs. An illustration of this is provided by auction houses (in the role of jurisdictions) that typically try to deter sellers (in the role of public good providers) from posting reserve prices above their valuation. More generally, the fee structure plays an important role in shaping the success of market places: Taxing unsold goods instead of charging fees only on the final transactions is a way to incentivize

this is so when goods are substitutes (Ausubel and Milgrom, 2002).

[53]If citizens hold asymmetric information but are able to guess the true reservation values of each public good provider inside the jurisdiction, then if jurisdictions force public good providers to use the Ausubel-Milgrom's proxy core selecting auction but with jump bids as in Lamy (2013) and with public good providers having the right to pick the combination of offers they like best among the final offers, then the pivot payoffs are implemented for the same reasons as in Lamy (2013).

sellers to choose lower reserve prices, which may increase the efficiency of the system.[54]

From the perspective of local public economics, it would be of interest to extend the analysis to the case in which there could be externalities across jurisdictions and in particular investigate whether bargaining between jurisdictions could eliminate the potential inefficiencies resulting from these externalities (this would parallel the question addressed in Jehiel (1997), but allowing for much more general competitive equilibrium environments). It would also be of interest to embed the present model into a dynamic model in which (some) citizens would be subject to moving costs and the admission of new residents would be subject to the approval of past residents, in the vein of Jehiel and Scotchmer (2001). How our analysis should be modified to cope with such dynamic considerations as well as modifications of the admission rule should be the subject of future research.

From a mechanism design perspective, the literature concerned with the elicitation of private information has stressed in contexts with exogenous sets of participants that it would be impossible to achieve efficiency while at the same time ensuring the participation constraints and budget-balancedness in models in which private information is independently distributed across agents (Myerson and Satterthwaite (1983) and Mailath and Postelwaite (1990)). The literature has also had a hard time dealing with multidimensional information and it has suggested that if private information is correlated across agents, a designer can easily extract the full surplus if the exact shape of the correlation is available to the designer. Our analysis reveals that when there is competition across designers/jurisdictions and when participation is endogenous, the insights previously highlighted are deeply affected: 1. Equilibrium does not require that designers have any knowledge about how private information is distributed (a robust mechanism design insight addressing Wilson's (1987) critique); 2. Correlation of private information across agents plays no role. 3. Private information may be multi-dimensional. 4. Efficiency, participation constraints and budget balancedness may simultaneously be satisfied. From that perspective, the next step would be to extend the analysis to non-private value settings (in such a case, ex post inefficiencies may necessarily arise, see Jehiel and Moldovanu, 2001) and to cases in which utilities are not quasi-linear in money.

[54]Engelbrecht-Wiggans and Sonnenmacher (1999) formalize what is considered nowadays by historians as one of the main explanations for the spectacular development of the Port of New-York in the early nineteenth century (which started before the completion of the Erie Canal): drastic institutional changes in the design of auctions for imported goods. Several innovations in the auction law in New-York encouraged sellers to lower their reserve price.

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Appendix

Proof of Lemma 4.1

As a preliminary, note that A1 implies that for a given jurisdiction k_j,

$$\int_{\Theta} \widetilde{W}(m_{k_j}^{eff}, k_j, \theta) \cdot g(\theta \mid \mu, k_j) d\theta - \sum_{k=1}^{K} \mu_k \cdot \underline{V}_k \geq H - \sum_{k=1}^{K} \mu_k \cdot \left[\underline{V}_k - \underline{\underline{V}}_k \right]$$

for any vector $\mu \in R_+^K$. As a corollary, if $\mu_k > \frac{H}{[\underline{V}_k - \underline{\underline{V}}_k]}$ for some k, then the participation to the global welfare of the jurisdiction is negative, meaning that the welfare would be larger with no participants at all. Let $\overline{\mu} := \max_{k=1,\ldots,K} \frac{H}{[\underline{V}_k - \underline{\underline{V}}_k]}$.

When we seek to maximize $GW(m^{eff}, \mu^*[\widehat{\mu}])$ over $\widehat{\mu} \in \mathbb{R}_+^{K \times [0,1] \times \mathcal{K}_J}$ under the constraints (10), we can thus restrict ourselves without loss of generality to $\widehat{\mu} \in [0, \overline{\mu}]^{K \times [0,1] \times \mathcal{K}_J}$ and impose that $\widehat{\mu}_k(k_j, .)$ is nondecreasing on $[0,1]$ for any $k = 1, \ldots, K$ and $k_j \in \mathcal{K}_j$, and that the constraints (10) hold, a set denoted as Ω_μ. Let us equip ourselves with the product topology. From Lebesgue Dominated Converge Theorem (see Aliprantis and Border, 2006), both the functions $\widehat{\mu} \to GW(m^{eff}, \mu^*[\widehat{\mu}])$ and $\widehat{\mu} \to \int_0^1 \sum_{k_j \in \mathcal{K}_J} \widehat{\mu}_k(k_j, \zeta) \cdot f_J(k_j) d\zeta$ (for any $k \in \mathcal{K}_C$) are continuous on Ω_μ (w.r.t. the product topology). In order to prove the existence of a solution to the maximization program (9), it is thus sufficient to show that the set Ω_μ is compact (w.r.t. the product topology). From Tychonoff Product Theorem (see Aliprantis and Border, 2006), the set $[0, \overline{\mu}]^{K \times [0,1] \times \mathcal{K}_J}$ is compact. Then the subset of $[0, \overline{\mu}]^{K \times [0,1] \times \mathcal{K}_J}$ such that the constraints (10) hold is closed since those constraints are continuous w.r.t. $\widehat{\mu}$. Furthermore, any limit of nondecreasing functions is also nondecreasing guaranteeing that the subset of $[0, \overline{\mu}]^{K \times [0,1] \times \mathcal{K}_J}$ such that $\widehat{\mu}_k(k_j, .)$ is nondecreasing on $[0,1]$ for any $k \in \mathcal{K}_C$ and $k_j \in \mathcal{K}_j$ is also a closed set. The intersection of two closed sets is also closed. On the whole, Ω_μ is compact as a closed subset of a compact set, which completes the proof.

Proof of Theorem 1

From Lemma 4.1, a solution of the maximization program (9) exists and then a maximum exists to the corresponding Lagrangian. Let $(\widehat{\mu}^{opt}, \mathcal{V}^{opt})$ denote such a solution, where we have seen that the Lagrange multiplier $\mathcal{V}_k^{opt}$ is larger than $\underline{V}_k$ the payoff derived by a group k citizen when he stays on his own. We build then a free mobility equilibrium where each jurisdiction posts the pivot mechanism (i.e. $m^*_{k_j} = m^{piv}_{k_j}$), where group k citizens expected payoff is equal to the Lagrange multiplier at the optimum (i.e. $V_k = V_k^{opt}$), where the equilibrium entry rates at the pivot mechanism match those at the optimum (i.e. $\mu^*(m^{piv}_{k_j}, k_j, \zeta) = \widehat{\mu}^{opt}(k_j, \zeta)$), where citizens report truthfully their type in the pivot mechanism, and last where for any other mechanisms $m \in \mathcal{M}$ the entry profile in the mechanism satisfies condition (2) of Definition 1 and truthful reporting is an equilibrium (as argued earlier this is without loss of generality).

We now have to check that it is an equilibrium. More precisely, what remains to be checked is: 1/ that jurisdictions find it optimal to post the pivot mechanism, i.e.

$$m^{piv}_{k_j} \in Arg \max_{m \in \mathcal{M}} \int_0^1 u_J(m, k_j, \mu^*(m, k_j, \zeta), \sigma^*(m, k_j, \zeta)) d\zeta \tag{37}$$

and that the optimal entry profiles are equilibrium profiles in the pivot mechanism, i.e.

$$\widehat{\mu}^{opt}_k(k_j, \zeta) \underset{(resp.=)}{>} 0 \Rightarrow u_k(m^{piv}_{k_j}, k_j, \widehat{\mu}^{opt}(k_j, \zeta)) \underset{(resp.\leq)}{=} V_k \tag{38}$$

for any $k \in \mathcal{K}_C$ and $\zeta \in [0, 1]$.

The expected welfare in a jurisdiction corresponds to the sum of all agents' expected rents,[55] namely

$$\int_{\Theta} \widetilde{W}(m, k_j, \theta) g(\theta \mid \mu, k_j) d\theta = E_{\theta|\mu,k_j}[\widetilde{R}(m, k_j, \theta)] + \sum_{k=1}^{K} \mu_k \cdot u_k(m, k_j, \mu)$$

and then by integrating w.r.t. ζ and for equilibrium entry rates, we obtain that for any type k_j jurisdiction and any mechanism $m \in \mathcal{M}$

$$\int_0^1 u_J(m, k_j, \mu^*(m, k_j, \zeta)) d\zeta = \int_0^1 \Big(\int_{\Theta} \widetilde{W}(m, k_j, \theta) g(\theta \mid \mu^*(m, k_j, \zeta), k_j) d\theta \Big) d\zeta - \sum_{k=1}^{K} \int_0^1 \mu^*_k(m, k_j, \zeta) \cdot u_k(m, \mu^*(m, k_j, \zeta)) d\zeta$$
$$\int_0^1 \Big(\int_{\Theta} \widetilde{W}(m, k_j, \theta) g(\theta \mid \mu^*(m, k_j, \zeta), k_j) d\theta \Big) d\zeta - \sum_{k=1}^{K} \Big[\int_0^1 \mu^*_k(m, k_j, \zeta) d\zeta \Big] \cdot V_k,$$

[55] We do not allow jurisdictions to burn money. If they could, then the local net welfare will an upperbound of the seller's revenue and it would not change our result.

where the last equality comes from the equilibrium equations (4). On the whole, we obtain (17).

The revenue of type k_j jurisdictions is thus bounded by

$$\text{Arg}\max_{\mu\in\mathbb{R}_+^K} LNW(m_{k_j}^{piv}, k_j, \mu; \mathcal{V}^{opt}). \tag{39}$$

Furthermore, this bound (which does not depend on ζ) is attained for the pivot mechanism since

$$\mu^*(m_{k_j}^{piv}, k_j, \zeta) = \widehat{\mu}^{opt}(k_j, \zeta) \in \text{Arg}\max_{\mu\in\mathbb{R}_+^K} LNW(m_{k_j}^{piv}, k_j, \mu; \mathcal{V}^{opt}) \tag{40}$$

for any $\zeta \in [0,1]$, which proves (6).

Remark: We see that in this equilibrium candidate, then the revenue of the seller is the same for any ζ. If the optimum requires that $\mu^{opt}(k_j, \zeta) = 0$ for some ζ, then the revenue of type k_j jurisdiction is null in equilibrium.

What remains to be shown is that the entry rates $\widehat{\mu}^{opt}(k_j, \zeta)$ are equilibrium profiles.

For any vector $N \in \mathbb{N}^K$, let $N_{-k} = (n_1, \ldots, n_{k-1}, n_k - 1, n_{k+1}, \ldots, n_K)$. Similarly we let $N_{+k} = (n_1, \ldots, n_{k-1}, n_k + 1, n_{k+1}, \ldots, n_K)$. As a preliminary, note that $\frac{\partial P(N|\mu)}{\partial \mu_k} = P(N_{-k}|\mu) - P(N|\mu)$ if $n_k \geq 1$ and $\frac{\partial P(N|\mu)}{\partial \mu_k} = -P(N|\mu)$ if $n_k = 0$.

For any $(k, k_j) \in \mathcal{K}_C \times \mathcal{K}_J$ and m, we have

$$\begin{aligned}
\frac{\partial LNW(m, k_j, \mu; \mathcal{V}^*)}{\partial \mu_k} &= \sum_{N\in\mathbb{N}^K} \frac{\partial P(N \mid \mu)}{\partial \mu_k} \cdot \Big[\int_{\overline{\Theta}} \widetilde{W}(m, k_j, \theta) q(\theta \mid N, k_j) d\theta \Big] - V_k \\
&= \sum_{N\in\mathbb{N}^K} P(N \mid \mu) \cdot \Big[\int_{\overline{\Theta}} \widetilde{W}(m, k_j, \theta) q(\theta \mid N_{+k}, k_j) d\theta - \int_{\overline{\Theta}} \widetilde{W}(m, k_j, \theta) q(\theta \mid N, k_j) d\theta \Big] - V_k \\
&= \sum_{N\in\mathbb{N}^K} P(N \mid \mu) \cdot \int_Y \int_{\overline{\Theta}} \int_{\Theta} \widetilde{W}(m, k_j, \theta \cup \widetilde{\theta}) f_k(\widetilde{\theta}|y) d\widetilde{\theta} \prod_{k'=1}^{K} \prod_{i_{k'}=1}^{n_{k'}} f_{k'}(\theta_{i_{k'}}^{[k']}|y) d\theta f_Y(y|k_j) dy \\
&\qquad - \sum_{N\in\mathbb{N}^K} P(N \mid \mu) \cdot \int_Y \int_{\overline{\Theta}} \widetilde{W}(m, k_j, \theta) \prod_{k'=1}^{K} \prod_{i_{k'}=1}^{n_{k'}} f_{k'}(\theta_{i_{k'}}^{[k']}|y) d\theta f_Y(y|k_j) dy - V_k. \\
&= \sum_{N\in\mathbb{N}^K} P(N \mid \mu) \cdot \int_Y \int_{\overline{\Theta}} \int_{\Theta} [\widetilde{W}(m, k_j, \theta \cup \widetilde{\theta}) - \widetilde{W}(m, k_j, \theta)] \\
&\qquad \times f_k(\widetilde{\theta}|y) d\widetilde{\theta} \prod_{k'=1}^{K} \prod_{i_{k'}=1}^{n_{k'}} f_{k'}(\theta_{i_{k'}}^{[k']}|y) d\theta f_Y(y|k_j) dy - V_k
\end{aligned} \tag{41}$$

For the pivot mechanism, we have from its fundamental property (16) that

$$\widetilde{W}(m_{k_j}^{piv}, k_j, \theta \cup \widetilde{\theta}) - \widetilde{W}(m_{k_j}^{piv}, k_j, \theta) = w^*(k_j, \theta \cup \widetilde{\theta}) - w^*(k_j, \theta) = \widetilde{u}(m_{k_j}^{piv}, \theta, \widetilde{\theta}). \tag{42}$$

We have also that

$$u_k(m, k_j, \mu) = E_{\theta_i|k,k_j}\Big[\int_\Theta u(m, k_j, \mu, \sigma, \theta_i, \widehat{\theta}_i)\sigma_k^{\theta_i}(\widehat{\theta}_i)d\widehat{\theta}_i\Big] \tag{43}$$

and thus

$$\frac{\partial LNW(m_{k_j}^{piv}, k_j, \mu; \mathcal{V}^*)}{\partial \mu_k} = u_k(m_{k_j}^{piv}, k_j, \mu) - V_k. \tag{44}$$

We conclude by noting that $\mu^*(m_{k_j}^{piv}, k_j, \zeta) = \widehat{\mu}^{opt}(k_j, \zeta) \in J(m_{k_j}^{eff}, k_j; \mathcal{V}^{opt})$ implies thus that the equilibrium conditions (4) for the vector $\mu^*(m_{k_j}^{piv}, k_j, \zeta)$ for any $k_j \in \mathcal{K}_J$ and $\zeta \in [0, 1]$.

Peters' (2010) model and its relation with ours and McAfee (1993)

Peters (2010) considers a model where firms (jurisdiction in our terminology) post a job offer (a mechanism) with a predetermined wage $w \in \mathbb{R}_0$ and where each worker (citizen) i has a known productivity $\theta_i \geq 0$ and where their payoff corresponds to their wage if they get the job (and zero otherwise) while the payoff of the firm is a function of the productivity of the worker and its wage. Peters (2010) considers a free mobility equilibrium with respect to workers' choice to apply to a given firm. Once the workers profile vector is $\theta = (\theta_1, \ldots, \theta_n)$ with $\theta_1 \leq \theta_2 \leq \ldots \leq \theta_n$, the worker with the highest productivity θ_n gets the job. Next we adopt the convention $\theta_0 = 0$.

We can reinterpret this model through the lens of the following auction model: each job with wage w' corresponds to a good which is valued $w' - \phi(\theta_i) + \phi(\theta_{n-1})$ for worker i if he gets it and zero otherwise where ϕ is some increasing function on $\mathbb{R}$. The preferences of the firms are also specified so that the welfare of a firm attracting the set of workers θ is given by $\phi(\theta_n)$. In this model, posting the wage w corresponds to the pivot mechanism and the equilibrium satisfies thus the uniform "spread property" as in McAfee (1993). To match this equilibrium with the one in Peters (2010) we have to guarantee that the entry incentives for the workers are the same, which is the case if the expectation of $-\phi(\theta_n) + \phi(\theta_{n-1})$ conditional on θ_n is a constant (as a function of θ_n). This property characterizes indeed the function ϕ.