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THE HOUSING COST DISEASE[†]

Abstract

We use a simple two-sector, life-cycle economy with bequests to explain the increasing wealth to income ratio, housing wealth and wealth inequality that have been observed in several countries over the long-run as a consequence of a rising labor efficiency in manufacturing (housing cost disease). When consumption inequality across households is sufficiently large, the housing cost disease has adverse effects on a measure of social welfare based on an egalitarian principle: the higher the housing's value appreciation, the lower the welfare benefit of a rising labor efficiency in manufacturing.

JEL Classification: D9, E2 and O4
Keywords: capital, housing, productivity, wealth and wealth inequality

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1 Introduction

The households' wealth-to-income ratio has been steadily rising in the last forty years in most advanced countries; and, in many of them, this phenomenon has gone along with a rise in the share of housing in total wealth. We build a frictionless life-cycle model able to replicate these stylized facts as a result of an exogenous improvement in the efficiency of labor in manufacturing and we show that, under certain conditions, this explains a rising inequality in wealth and consumption across individuals.

The long-run increase in total and housing wealth to income ratios in advanced economies have been recently emphasized and widely discussed by economists and commentators. In particular, Piketty (2014) and Piketty and Zucman (2014) have claimed that these trends have implications for the distribution of income and wealth across individuals. According to the data collected by Piketty and Zucman (2014), between 1970 and 2010, the private wealth-to-income ratio increased in most advanced countries, by a minimum of 67 percentage points, in the US, to more than 300 in Japan, and 400 in Italy. The housing share in private wealth increased on average by 12 percentage points in the main European economies, *i.e.*, France, Germany, Italy and the UK. It dropped, instead, by 10 percentage points in the US. However, in the shorter sample 1970 to 2006, that excludes the observations after the Great Recession, the housing share increased also in the US, by 3 percentage points. In fact, for the median household, housing has gradually become the main component of wealth reaching 80% of net wealth in the Eurozone, and 60% in the US (Iacoviello, 2010).

In this paper, we follow up on a claim, suggested in Bonnet et al. (2014), Piketty and Zucman (2014), Rognlie (2014) and Summers (2014), that, to understand the existing trends in wealth inequality, wealth-to-output ratios and income shares, it is crucial to analyze the dynamics of housing. In particular, much of the long-term dynamics of the capital-income ratio, as well as the net capital share of income, is accounted for by housing wealth and, in particular, by capital gains[1].

[1] According to Bonnet et al. (2014), housing should not be evaluated in terms of market prices, but by

We model an economy with sectors: manufacturing and construction. A multi-sector approach is important to study the evolution of wealth's composition in advanced economies (vs. economies in the early stage of the development process) since housing replaced land in households' assets and because factor price equalization across sectors generates interesting linkages between the dynamics of productivity and asset prices. The model works around a version of the Baumol's cost disease problem. In the seminal work by Baumol (1967), a market economy has two sectors producing two goods using labor as the only input and enjoying different patterns of technological progress. Under perfect labor mobility and wage equalization, a rising labor productivity in the dynamic sector generates a higher production cost and, then, a rising relative output price in the stagnant sector (for example, to use one of Baumol's examples, music played by a horn quintet). If the demand of the stagnant sector output is sufficiently inelastic, labor will move to this sector and aggregate output growth may decline. We extend Baumol (1967)'s analysis to a model with capital, where construction plays the role of the stagnant sector, while manufacturing experiences labor-augmenting technological progress, and we provide a set of conditions under which a rise in the efficiency of labor in manufacturing generates a strong housing price appreciation, a rise in the wealth-to-income ratio (mostly driven by higher housing wealth and a week dynamics in average labor productivity) and in the size of bequests (*i.e.*, a rising wealth inequality). We label this set of phenomena a *housing cost disease*.

More specifically, we consider an economy with overlapping generations of heterogeneous altruistic households living for two periods only, supplying their labor inelastically when young, and deriving utility from housing services. The only source of heterogeneity between individuals is the degree of parental altruism, represented by a discount rate applied to the next generation's utility. Consistently with the assumption of one-sided altruism we assume that parents cannot force gifts on their children. Hence, this heterogeneity generates a par-

the present value of future rents and, under this alternative rule, the share of housing in private wealth is much smaller. Rognlie (2014) claims, instead, that housing accounts for nearly 100% of the long-term capital-income ratio.

tition of the set of households at steady states into a subset of *rich* individuals receiving bequests from their parents and a subset of *poor* individuals receiving (and giving) no bequests. We show that the housing cost disease is most likely under the assumptions that manufacturing is more capital intensive than construction, the construction sector displays a sufficiently small elasticity of substitution between capital and labor and housing demand is sufficiently inelastic with respect to its own price. The robustness of these results is studied numerically simulating the model for a specific and rather general specification of preferences (CES) and technology (Cobb-Douglas production function in manufacturing and CES in construction).

An additional contribution of the paper is to clarify the effect of the housing cost disease on social welfare. This may be important in order to assess the merits of public policy in the face of a rising wealth-to-income ratio. By assuming that this is not a desirable outcome because of the implications in terms of possible unequal distribution of wealth across households, Piketty (2014) advocates the institution of a wealth tax. However, if the increase in housing prices depends on the increasing relative productivity of non-construction sectors, then policies targeting specifically the housing sector are probably more appropriate, as noted for example by Auerbach and Hasset (2015)[2]. We leave to future research the evaluation of such policies, and we instead use the model to analyze whether a change in the composition of wealth toward housing, following a rise of efficiency in manufacturing, may not be desirable from a welfare point of view. Deaton and Laroque (2001) investigated a similar question and found that the presence of a market for housing determines a portfolio reallocation away from capital towards housing, causing the accumulation of capital to fall short of the *Golden Rule* level. Our welfare criterion is based on an egalitarian welfare function that takes into account the households heterogeneous degrees of altruism with respect to the next generations and allows for negative bequests. We conclude that, when housing appreciation

[2]Note that housing taxation is, in any case, very controversial, since housing is a consumption good, as well as an asset, and home ownership is much more evenly distributed across individuals than stocks and other financial assets.

is sufficiently strong and consumption inequality sufficiently large, the steady state welfare benefit of a rising labor efficiency in manufacturing is lower than it would be in the planning optimum. In fact, a housing appreciation has two opposing effects on welfare. On the one hand, it raises the wealth of the initial poor old households so as to relax the non-negativity constraint on bequest values. On the other hand, it makes housing less affordable. Our analysis shows that the latter effect is stronger than the former.

The remainder of the paper is organized as follows. Section 2 present some stylized facts, section 3 introduces the model, section 3.5 characterizes a steady state equilibrium, section 4 illustrates the effects of improvements in relative labor productivity, section 5 discusses the welfare implications and section 6 concludes.

2 Stylized Facts

In this section, we present two sets of stylized facts. First, we look at the evolution of households' total, and housing, wealth using data from Piketty and Zucman (2014). Second, we look at the evolution of wealth inequality. We use data from Piketty and Saez (2014), who have put together an incredibly rich dataset, starting from national accounts data, for the period 1970-2010, for the United States, Germany, the United Kingdom, Canada, Japan, France, Italy and Australia[3]. Private wealth is net wealth of households, and assets include all non-financial and financial assets. All assets and liabilities are valued at market prices. Housing wealth is one of the components of private wealth, and it measures the net value of households' real-estate holdings.

Figure 1 considers the evolution of wealth, for the period 1970-2010. Total private wealth, as a fraction of income, increased tremendously over the period 1970-2010 (top panel). The average wealth to income ratio was equal to 2.8 in 1970 and 5.1 in 2010, when it reached high values of 6.8 in Italy, and 6 in Japan. Italy and Japan are also the countries where the private

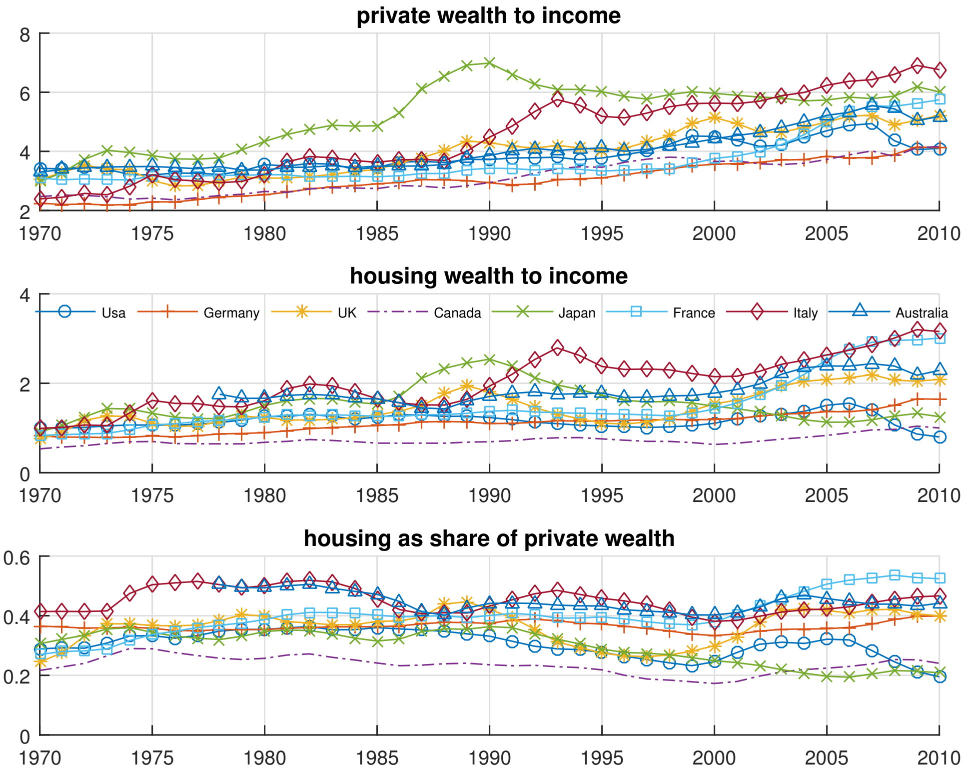
[3]For a smaller subset of countries, Piketty and Zucman (2014) provide longer time-series. However, we restrict our focus on a time-period for which we could maximize the number of countries in the sample. For a detailed description of the data refer to the Appendix A.

wealth to income ratio increased the most (437 and 302 percentage points respectively). On the contrary, the US is the country with the smallest increase in this ratio: only 67 percentage points, with a value in 2010 of about 4. Housing wealth increased along with total wealth in all countries, with the exception of the US (middle panel). The average housing wealth was equal to 0.8 in 1970 and 1.9 in 2010. Interestingly, the evolution of housing wealth is not uniform across the countries in the sample. For example, it increased by about 200 percentage points in Italy and France, but only by 40 percentage points in Canada and Japan[4]. The United States are the only country where the housing wealth to income dropped (by about 19 percentage points) over the period 1970 to 2010. However, it is worth noticing that the United States, in 1970, had the second highest housing wealth to income ratio among the countries in the sample and this ratio is characterized by an upper trend if we end the sample in 2007, at the onset of the Great Recession. The bottom panel of figure 1 plots housing as a share of private wealth. In 1970, the average housing share of wealth was 30 percent. In 2010, the same share was about 35.9 percent, and it was just a bit higher (*i.e.*, 36.6 percent) if we end the same in 2007. Over the period 1970 to 2010, the evolution of the housing share of wealth was very different across the countries in the sample: for example, it increased by 25 percentage points in France and by 15 percentage points in the UK. On the other hand, it dropped by 9 percentage points in both the United States and Japan. Italy is the country with the largest housing wealth share both at the beginning (41 percent) and at the end (47 percent) of the sample.

Figure 2 plots the wealth share of the top 10 and 1 percent of the wealth distribution. With respect to figure 1, the sample includes different countries, or geographical areas (*i.e.*, France, the UK, the United States, Sweden and Europe), and for a longer time-period that goes from 1950 to 2010. Over the full sample, wealth inequality decreased: the average wealth share of the top 10 (top 1) percent was 73.4 (36.1) percent in 1950 and 65.4 (26.3) percent in 2010. However, if we split this time interval, we see that wealth inequality declined

[4]Even though Japan experienced a housing boom followed by a crash in the sample (at the peak, in 1990, the housing wealth to income in Japan reached the value of 2.5).

Figure 1: Households wealth



Notes: This figure plots the evolution over time of private wealth to income (top panel); housing wealth to income (middle panel); housing as a share of private wealth (bottom panel). The countries in the sample are the United States, Germany, the UK, Canada, Japan, France, Italy and Australia. Data are annual for the period 1970-2010 from Piketty and Zucman (2014). Additional details on the series are available on a separate Appendix (section A).

steeply from 1950 to 1970 in all countries (with the exception of the United States, where it did not change much), and increased at a slower pace since 1980, or earlier, from then on. In particular, the average wealth share of both the top 10 and 1 percent of the wealth distribution increased by about 5 percent since 1970. Using a different set of data, Cragg and Ghayad (2015) find similar results for the United States where median wealth is below mean wealth by a factor of 5, and mean wealth is above the wealth of the 80th percentile of the wealth distribution.

In the next section, we present a simple overlapping generation life-cycle growth model with two sectors that qualitatively accounts for the stylized facts outlined in this section.

Figure 2: Wealth distribution

Notes: The top (bottom) panel of this figure plots the evolution of the wealth share of the top 10 (top 1) percent of the wealth distribution. The countries in the sample are the France, the UK, the United States, Sweden and Eueope. Data are decennial for the period 1970-2010 from Piketty and Zucman (2014). Additional details on the series are available on a separate Appendix (section A).

3 The Model

In this section we present a simple life-cycle model of a competitive economy, exogenous technical progress and two sectors, construction and manufacturing. Under factor price equalization, a rise in labor efficiency in manufacturing generates, *ceteris paribus*, a rise in the relative price of construction. To evaluate whether this leads to higher housing values and wealth to income ratios, we have to consider the response of housing demand and the reallocation of inputs across the two sectors in equilibrium. For example, the rise in housing prices may reduce the demand for construction so that labor may shift back to manufacturing. In addition, a higher housing price and a reallocation of capital and labor may reduce business capital and affect the wage rate and the interest rate. The purpose of the model is to identify conditions on technologies and households' preferences under which some of the stylized facts mentioned above are possible.

3.1 Households

A set L_t of households are born every period $t = 0, 1, 2, ...$ They live for two periods, supply labor time inelastically, in young age only, and have identical time-invariant preferences for manufacturing consumption and housing services, the latter being measured by the housing stock. Households are characterized by some degree of altruism with respect to their offsprings, which is defined by an individual specific discount rate of the next generation's utility. In particular, households born at time t belong to different types, indexed by i, with i in a finite set $\mathcal{I}$, each type i composed of a mass m_i of individuals (*i.e.*, a collection of positive numbers, $(m_i)_{i\in\mathcal{I}}$, such that $\sum_{i\in\mathcal{I}} m_i = 1$), with life-time utility defined by:

$$V^{t,i} = u(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^i) + \theta_i(1+n)V^{t+1,i},$$

for all $t \geq 0$, where $(c_t^{i,t}, c_{t+1}^{i,t})$ are age-contingent consumptions, h_{t+1}^i is the housing stock acquired by the household in young age and n is the population growth rate.

Assumption 1. *The (inter-generational) discount factors satisfy $\theta^i(1+n) < 1$ for all i and the utility function u is increasing, strictly concave, twice continuously differentiable and satisfying Inada-type conditions (*i.e.*, the marginal utility of each argument diverges to infinity as it converges to zero).*

We assume perfect financial markets allowing for unlimited lending and borrowing and, for simplicity, we ignore the housing rental market[5]. Any household born at time t acquires residential property when young, enjoys the housing services generated by it, resells the

[5]Households are assumed to derive some, however small, satisfaction from ownership, so that, in the absence of market frictions, ownership is a dominant choice relative to renting. The average home ownership rate across OECD countries is approximately 67%. At the top of the distribution are countries like Greece (87%) and Spain with high rates of more than 80%; while at the bottom countries like Germany (43%), Japan (36%) and Switzerland (35%). Data are from the OECD (2012).

property when old and leaves some bequests to the offsprings. Hence, the households' inter-temporal budget constraints are:

$$c_t^{t,i} + c_{t+1}^{t,i}/R_{t+1} + \pi_t h_{t+1}^i + (1+n) b_{t+1}^i / R_{t+1} = W_t + b_t^i, \tag{1}$$

where b_t^i denotes the bequests, W_t is the time-t real wage, q_t is the time-t housing price,

$$\pi_t = q_t - (1-\delta) q_{t+1}/R_{t+1}$$

denotes the user cost of housing, *i.e.*, the cost of a unit of housing net of the present value of selling the same un-depreciated unit the next period and $\delta \in (0,1)$ is the housing depreciation rate. Since parental altruism is one-sided, we rule out forced gifts from children to parents, and impose the non-negativity constraint

$$b_{t+1}^i \geq 0.$$

Denoting with $u_{j,t}^i$ the partial derivative of $u(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^i)$ with respect to the j-th argument, the first order conditions characterizing the young household's optimal consumption choice subject to the budget constraint, (1), are

$$\begin{aligned} u_{1,t}^i &= R_{t+1} u_{2,t}^i, & (2) \\ u_{1,t}^i \pi_t &= u_{3,t}^i, & (3) \\ u_{2,t}^i &\geq \theta_i u_{1,t+1}^i, & (4) \end{aligned}$$

together with the complementary slackness condition

$$b_{t+1}^i \left(u_{2,t}^i - \theta_i u_{1,t+1}^i \right) = 0. \tag{5}$$

From the above first order conditions we derive the time-t households saving, s_t^i, the demand for housing, h^i_{t+1}, and the supply of bequests, b^i_{t+1}. These are specified as:

$$
\begin{aligned}
s^i_t &= S^i(W_t + b^i_t, \pi_t, R_{t+1}), && (6)\\
h^i_{t+1} &= H^i(W_t + b^i_t, \pi_t, R_{t+1}), && (7)\\
b^i_{t+1} &= B^i(W_t + b^i_t, \pi_t, R_{t+1}). && (8)
\end{aligned}
$$

3.2 Production

Manufacturing output (Y^m) can be consumed or used as capital in both sectors and construction output (Y^h) corresponds to investment in new housing. Technology is represented by the production functions

$$Y^h_t = F_h(K^h_t, A^h_t L^h_t), \quad Y^m_t = F_m(K^m_t, A^m_t L^m_t),$$

where, for $j = h, m$, K^j and L^j are the amounts of capital and labor employed in the two sectors and A^j is a labor-augmenting technological level. Production functions satisfy standard properties, *i.e.*, they are increasing, concave and satisfy constant returns to scale. We will specify in the next sections some additional assumptions for analytical tractability.

It is convenient to provide a more compact notation by normalizing variables with respect to the level of labor efficiency. In particular, for $j = h, m$, let $k^j = K^j/A^jL^j$ be the sector-specific capital intensities and y^j the sector-specific labor productivities in efficiency units. By constant returns to scale, we can write

$$y^h = F_h(k^h, 1) \equiv f_h(k^h) \qquad y^m = F_m(k^m, 1) \equiv f_m(k^m),$$

where f_j denotes the intensive-form production functions.

Firms in construction and manufacturing are price-takers and labor and capital are fully mobile across the two sectors. Then, when both manufacturing and new housing are produced, full employment and profit maximization imply that labor and capital stocks satisfy the following conditions:

$$
\begin{aligned}
L_t &= L_t^h + L_t^m, && (9)\\
K_t &= k_t^h A_t^h L_t^h + k_t^m A_t^m L_t^m, && (10)\\
R_t &= f_m'(k_t^m) = q_t f_h'(k_t^h), && (11)\\
W_t &= [f_m(k_t^m) - k_t^m f_m'(k_t^m)]A_t^m = q_t[f_h(k_t^h) - k_t^h f_h'(k_t^h)]A_t^h, && (12)
\end{aligned}
$$

for all $t \geq 0$ and a given level of the time-t aggregate capital stock, K_t, and labor supply, L_t. Equations (9) and (10) imply full employment of labor and capital; equations (11) and (12) imply profit maximization. In the remainder of the paper we say that an allocation is *interior* if (k_t^j, L_t^j, y_t^j) is strictly positive for $j = h, m$ and all $t \geq 0$.

3.3 Relative Prices and Labor Efficiency

Let $a = A^m/A^h$ be the labor augmenting efficiency in manufacturing relative to construction (henceforth *relative productivity*) and $w = W/A^m$ the wage rate per units of efficiency in the manufacturing sector, where for convenience we do not specify the time subscript when not essential. In this section, we analyze the link between housing and factors prices, (q, w, R), and the relative labor efficiency in manufacturing, a. Our goal is to come up with a well behaved map from the pair (w, a) into R and q and the capital labor ratios, k^m and k^h. In particular, we set the stage for the analysis of the general equilibrium effects of a rise in labor efficiency in manufacturing by showing that the housing price q is an increasing function of a and it may be increasing or decreasing in w according to the relative capital intensities in the two sectors.

By the wage equalization condition, in (12), we derive

$$w = f_m(k^m) - k^m f_m'(k^m) = q(f_h(k^h) - k^h f_h'(k^h))/a. \qquad (13)$$

The relation between w, a and q is trivial if the housing sector has zero capital intensity. In this case $y^h = 1$ and $f_h'(k^h) = 0$. Therefore, by wage equalization across sectors (cf. equation (13)), we would get $q = aw$, *i.e.*, the housing price is unit elastic with respect to the relative labor efficiency in manufacturing for any given wage. This is the classic Baumol's cost disease effect. If we consider instead the more general case $f_h'(k^h) > 0$, the relation between w, a and q is more complicated.

To simplify the exposition, and if not otherwise specified, from now on we analyze the model under the following assumption.

Assumption 2. *The intensive form production functions, f_j, are strictly increasing, strictly concave, continuously differentiable and satisfy*

(a1) $\lim_{k^j \to 0}(f_j(k^j) - k^j f_j'(k^j))/f_j'(k^j) = 0$, $\lim_{k^j \to \infty}(f_j(k^j) - k^j f_j'(k^j))/f_j'(k^j) = \infty$.

(a2) $\lim_{k^m \to 0} f_m'(k^m) > \min\{1/\theta_i; i \in \mathcal{I}\} > \lim_{k^m \to \infty} f_m'(k^m)$.

Assumption 2 simplifies the analysis and guarantees the existence of a steady state equilibrium with some convenient properties. In particular, by (a1), we can express k^m, k^h, R and q as differentiable functions of (w, a) for all w in some interval $\mathcal{I} = [\underline{w}, \bar{w}]$ and for all $a \geq 0$, as stated in the following claim.

Proposition 1. *Under assumption 2, for any given strictly positive pair, (w, a), with $w \in \mathcal{I} = [\underline{w}, \bar{w}]$ and $a > 0$, there is a unique solution, $(k^h(w, a), k^m(w), q(w, a), R(w))$, to equations (11), (13). Moreover, the functions $k^m = k^m(w)$, $k^h = k^h(w, a)$, $R = R(w)$, $q = q(w, a)$ are differentiable, with partial derivatives $k_w^m(w) > 0$, $k_w^h(w, a) > 0$, $R'(w) < 0$, $k_a^h(w, a) > 0$, $q_a(w, a) > 0$.*

A sketch of the proof is the following. First of all, by a trivial application of the implicit function theorem and by the assumption $f_m'' < 0$, the marginal productivity condition

$$w = f_m(k^m) - k^m f_m'(k^m)$$

can be locally inverted in some interval $\mathcal{I} = [\underline{w}, \bar{w}]$ to obtain $k^m = k^m(w)$, with $k^m(.)$ increasing in w and such that $k^m(\underline{w}) = 0$, $k^m(\bar{w}) = \infty$. Existence of the function $R(w)$ follows from (11) and, in particular, by defining $R(w) \equiv f'_m(k^m(w))$, which proves $R'(w) < 0$. Then, let $\omega(w) \equiv w/R(w)$ and observe that $\omega'(w) > 0$ and, by equations (11) and (13),

$$\omega_g(k^h) \equiv f_h(k^h)/f'_h(k^h) - k^h = a\omega(w).$$

Since $f''_h(k^h) < 0$, the above is strictly increasing in w. In a second step, we use again equations (11), (13) and the above proposition to obtain the function

$$q = q(w,a) \equiv \frac{f'_m\left(k^m(w)\right)}{f'_h\left(k^h(w,a)\right)}$$

for all (w,a) in $\mathcal{I} \times \mathbb{R}_+$. Notice that $k^h(w,a)$ and $q(w,a)$ are both increasing in a. In other words, a rise in the labor efficiency in manufacturing (relative to construction) has a positive effect on the manufacturing labor intensity and on the housing price, for given w. In particular, for $j = h, m$, let

$$\mu_j^L = 1 - k^j f'_j(k^j)/f_j(k^j),$$

be the labor shares in manufacturing and construction and

$$\sigma_j(k^h) = \frac{\partial \ln k^j}{\partial \ln(F_{j,L}/F_{j,K})}$$

the elasticity of substitution between capital and labor in the construction sector. By direct computation of the partial derivatives, we obtain the elasticities of k^h and q with respect to a as

$$k_a^h(w,a)a/k^h = \sigma_h(k^h), \qquad q_a(w,a)a/q = \mu_h^L(k^h). \tag{14}$$

Therefore, a 1% rise in relative productivity increases the housing price *directly* (*i.e.*, for given w) by less than a percentage point, and the magnitude of this effect is higher the lower is the labor share in construction.

For future reference, it is useful to introduce a measure of the labor share in construction relative to manufacturing (*relative labor share*),

$$\Delta = \frac{\mu_h^L - \mu_m^L}{\mu_m^L} = (1 - \mu_h^L)\left(\frac{ak^m - k^h}{k^h}\right). \tag{15}$$

Notice that the relative labor share is positive if and only if capital intensity in manufacturing is larger than in construction and the sign of this variable determines whether q is an increasing or decreasing function of w. In fact, it can be shown that q is increasing in w when manufacturing is sufficiently capital intensive relative to construction or, equivalently, when the labor share in construction is higher than the one in manufacturing. Furthermore, we should mention that Δ is a function of (w, a). In particular, observing that, for $j = h, m$,

$$\frac{\partial \mu_j^L}{\partial k^j}\frac{k^j}{\mu_j^L} = (1 - \mu_j^L)\left(\frac{1 - \sigma_j}{\sigma_j}\right), \qquad \text{for } j = h, m,$$

we derive

$$\Delta_a(w, a)a = (1 - \sigma_h)(1 - \mu_h^L)(1 + \Delta), \tag{16}$$

i.e., $\Delta_a \geq 0$ if and only if $\sigma_h \leq 1$. In other words, a low elasticity of substitution between capital and labor in the construction sector implies that a higher relative productivity in manufacturing generates a higher labor share in construction relative to manufacturing.

When production functions are Cobb-Douglas, μ_m^L and μ_h^L are constants, so that Δ is independent of a. In the slightly more general case of f_m and f_h displaying the CES property with a common elasticity of substitution, $\sigma \neq 1$, *i.e.*,

$$f_j(k^j) = \left[\alpha_j (k^j)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha^j)\right]^{\frac{\sigma}{\sigma-1}},$$

we have that $\Delta(w, a) > 0$ if and only if

$$a^{\frac{\sigma-1}{\sigma}} < \left(\frac{\alpha^m}{1-\alpha^m}\right)\left(\frac{1-\alpha^h}{\alpha^h}\right).$$

Notice that the above inequality is verified for a high enough a if $\sigma < 1$ and for a low enough a if $\sigma > 1$.

3.4 Equilibrium

For simplicity, we assume that labor efficiency in construction is time invariant and normalized to one and, from now on, we set $A_t^m = a_t$, $A_t^h = 1$. Expressing all equilibrium restrictions and all the relevant variables in per capita terms, we set

$$k_t = K_t/L_t, \qquad \lambda_t = L_t^h/L_t.$$

Then, the full employment conditions (9), (10) are replaced by

$$k_t = \lambda_t k_t^h + (1-\lambda_t) a_t k_t^m, \qquad \lambda_t \in [0,1]. \tag{17}$$

To close the model we impose the market clearing conditions in manufacturing and construction. In particular, setting

$$c_t^{t-1} = \sum_i m_i c_t^{t-1,i}, \quad c_t^t = \sum_i m_i c_t^{t,i}, \quad h_t = \sum_i h_t^i, \quad b_t = \sum_i m_i b_t^i,$$

we derive the following equilibrium restrictions

$$c_t^{t-1}/(1+n) + c_t^{i,t} + (1+n)k_{t+1} = y_t^m a_t (1-\lambda_t), \tag{18}$$

$$h_{t+1} = y_t^h \lambda_t + (1-\delta)h_t/(1+n), \tag{19}$$

with $y^m = f_m(k^m)$, $y^h = f_h(k^h)$. Notice that, by the individuals' budget constraints and the market clearing conditions (18), (19), we derive the asset market equilibrium condition:

$$(1+n)k_{t+1} + q_t h_{t+1} = s_t, \tag{20}$$

where $s_t \equiv \sum_i m_i s_t^i$. By Walras Law, (18) is redundant, and the minimum set of equilibrium restrictions may be represented by equations (19), (20) and

$$\begin{aligned} s_t &= \sum_i m_i S^i(W_t + b_t^i, \pi_t, R_{t+1}), && (21) \\ h_{t+1} &= \sum_i m_i H^i(W_t + b_t^i, \pi_t, R_{t+1}), && (22) \\ b_{t+1} &= \sum_i m_i B^i(W_t + b_t^i, \pi_t, R_{t+1}). && (23) \end{aligned}$$

By assumption 2 and proposition 1, we have $k^m = k^m(w)$, $k^h = k^h(w,a)$, $q = q(w,a)$, $R = R(w)$, where $k^m(.)$, $k^h(.)$, $q(.)$ and $R(.)$ are positive and continuous functions for w in $\mathcal{I} = [\underline{w}, \bar{w})$ and for all $a > 0$. Then, an *interior competitive equilibrium* is a strictly positive sequence, $\{k_t, \lambda_t, h_t, w_t\}_{t=0}^{\infty}$, of average capital stocks, shares of labor in construction, per capita housing stocks and wage rates in units of efficiency, satisfying equations (11), (13), (7), (17), (19), (20), (21), (22), (23), along with $w_t \in \mathcal{I}$, $k_t^m = k^m(w_t)$, $k_t^h = k^h(w_t, a_t)$, $q_t = q(w_t, a_t)$, $R_{t+1} = R(w_{t+1})$ and

$$\pi_t = q(w_t, a_t) - (1-\delta)q(w_{t+1}, a_{t+1})/R(w_{t+1}) > 0,$$

for all $t \geq 0$, for a given sequence of relative productivities, $\{a_t\}_{t=0}^{\infty}$, and some initial conditions, $(k_0, h_0) > 0$.

3.5 Steady States with Bequests

From now on we concentrate on a steady state equilibrium with two types of households. In particular, let $\mathcal{I} = \{p, r\}$ and $\theta_r > \theta_p$. We say that household type r is *rich* and household type p is *poor*, although one may also say that the former is more altruistic than the latter.

It is clear from the first order conditions (2)-(5) that we may have two type of steady states. One is such that $R \leq 1/\theta_r$ and no individual leaves any bequests, so that the resulting equilibrium is equivalent to the one that would take place in a *canonical* overlapping generations economy. The other is such that the rich individuals leave positive bequests, whereas the poor leave zero bequests at any time. In this case we have $R = 1/\theta_r < 1/\theta_p$. We refer to the first type of equilibrium as a *zero bequests steady state* (ZBSS) and the second type a *positive bequests steady state* (PBSS). Notice that, since R is a decreasing function of w, by assumption 2, the ZBSS occurs for $w \geq R^{-1}(1/\theta_r) \equiv w^o$, where $R^{-1}(.)$ denotes the inverse of $R(.)$, whereas $w = w^o$ at a PBSS. Hence, in a PBSS, the real interest rate and the wage rate in units of efficiency do not depend on the relative productivity parameter, a.

In what follows, we only study a PBSS, mostly because this type of equilibrium allows for a sharp characterization of intra-generational inequality. Observe that assumption 2 guarantees positivity of the user cost of housing, *i.e.*, since $R(w^o) = 1/\theta_r > 1 - \delta$, at any PBSS,

$$\pi = q(w^o, a)\,(1 - (1-\delta)\theta_r) \equiv \pi(a) > 0.$$

Let $c^{y,i}$ and $c^{o,i}$ be the levels of consumption of the young and the old individual at steady state, respectively. By the life-time budget constraint of type i defined in (1) and evaluated at steady state, we derive

$$c^{y,i} + c^{o,i}/R + \pi h^i = aw^o + (1 - (1+n)/R)\, b^i. \tag{24}$$

By the first order conditions (2)-(5) and assumption 1, we derive $(1+n)/R < 1$ for all i, so that the present value of households' lifetime income is increasing in bequests. In particular, the bequest motive provides more wealth (*i.e.*, more consumption opportunities) to the r-type individual relative to the p-type at the PBSS. This implies that, if housing demand is a normal good, *i.e.*, it is increasing in the present value of income, housing is an increasing function of steady state bequests at equilibrium.

Now, for given prices and bequests, (W, b^i, π, R), and $i = p, r$, we define the stationary saving and housing demand functions from the first order conditions (2)-(4) and budget constraint (24), as

$$s^i = \tilde{S}(W, b^i, \pi, R), \qquad h^i = \tilde{H}(W, b^i, \pi, R),$$

where b^i is a variable to be determined in equilibrium subject to the complementary slackness conditions $b^i(1 - R\theta_i) = 0$, $R\theta_i \leq 1$. Since $b^r \geq b^p = 0$, we can save notation by setting $b^r = b$, since no confusion can arise regarding type specification, and define the aggregate saving and housing demand functions as

$$s(W, b, \pi, R) = \sum_i m_i \tilde{S}(W, b^i, \pi, R), \qquad h(W, b, \pi, R) = \sum_i m_i \tilde{H}(W, b^i, \pi, R).$$

A convenient *reduced-form* characterization of a PBSS is in terms of just two variables and two equations, after we set $R = 1/\theta_r$, $w = w^o$. The two variables are the (per-capita) rich household bequest, b, and the (per-capita) *housing wealth* $v = qh$, and the two equations are the demand and supply of housing wealth, to be defined shortly. In particular, considering (19) at a stationary allocation, we derive

$$\lambda = \left(\frac{\delta + n}{1 + n}\right) \frac{v}{qy^h}. \tag{25}$$

Using the above in (17) and recalling (15), we derive the average business capital as

$$(1 + n)k = (1 + n)ak^m - \left(\frac{\delta + n}{R}\right) \Delta v. \tag{26}$$

Observe that average business capital falls short of (exceeds) the amount of capital employed in manufacturing if $\Delta > 0$ ($\Delta < 0$), *i.e.*, if manufacturing displays a higher (lower) capital intensity than construction. Finally, plugging this into (20) and imposing stationarity, we rewrite the capital market equilibrium as

$$(1+n)ak^m + v\left(1 - \left(\frac{\delta+n}{R}\right)\Delta\right) = s(W, b, \pi, R). \tag{27}$$

Then, we define the (aggregate) *demand of housing wealth* as

$$v^d(b, a) = q(w^o, a)h(aw^o, b, \pi(a), 1/\theta_r), \tag{28}$$

and the (aggregate) *supply of housing wealth*, as any value $v^s(b, a) > 0$ solving equation (27) for some given $(b, a) > 0$, $W = aw^o$, $\pi = \pi(a)$, $R = 1/\theta_r$ and verifying

$$\lambda = \left(\frac{\delta+n}{1+n}\right)\frac{v^s}{qy^h} \in [0, 1].$$

A PBSS is a nonnegative pair, $(b^*(a), v^*(a))$, such that

$$v^*(a) = v^d(b^*(a), a) = v^s(b^*(a), a). \tag{29}$$

From now on, existence of a PBSS will be simply assumed, although we should mention that this requires that the parameter space is subject to the following restriction.

Assumption 3. $(\delta+n)\theta_r\Delta\left(w^o, a\right) < 1,\ s(aw^o, b^*, \pi(a), 1/\theta_r) > (1+n)ak^m(w^o)$.

In fact, proposition 4 in appendix B shows that the above inequalities are necessary conditions to have $v^s > 0$ and $\lambda \in [0, 1]$ at any steady state equilibrium, so that we can set

$$v^s(b, a) = \frac{s(aw^o, b, \pi(a), 1/\theta_r) - (1+n)ak^m(w^o)}{1 - (\delta+n)\theta_r\Delta(w^o, a)}. \tag{30}$$

Intuitively, the function $v^s(b, a)$ defines the amount of housing wealth that is consistent with a capital market equilibrium, *i.e.*, with the available amount of savings and business capital employed in manufacturing and construction. Observe that v^s is increasing in saving, decreasing in manufacturing investment, ak^m, and increasing in the relative labor share, Δ. The latter effect occurs because a higher Δ implies lower investment in business capital, k, as it is apparent from equation (26), and, then, it generates more resources for housing investment.

4 Productivity Improvements

4.1 The Housing Cost Disease

In this section we will study the comparative statics of the model generated by exogenous productivity improvements, *i.e.*, an increase in the value of the parameter a. One objective of this analysis is to examine the effects of a on the equilibrium bequest, b^*, which, in our model, is a measure of wealth inequality across (rich and poor) households. More generally, we are going to show that the model may replicate some of the features of a two-sector economy studied by Baumol (1967), with construction of housing playing the role of the *stagnant sector* and manufacturing the role of the *progressive sector*. We recall that the *Baumol's cost disease* holds if, following a rise in productivity in the dynamic sector, (a) the relative price of the stagnant sector output increases (*price increase*), (b) the stagnant industry takes a rising share of nominal output (*unbalanced growth*) and (c) the changing composition of output across stagnant and dynamic industries reduces the effect of the productivity improvement on the average productivity (*adverse effect on productivity*).

We restate the Baumol's cost disease result in the present framework as follows. Define the *average labor productivity*

$$y = (1 - \lambda)ay^m + \lambda qy^h = w + Rk, \tag{31}$$

and the *wealth to income ratio*,

$$\xi = \frac{(1+n)k + v}{y} = \xi^k + \xi^h,$$

where $\xi^k = (1+n)k/y$ is the business capital component and $\xi^h = v/y$ the housing component, and let $(q^*(a), y^*(a), \xi^{h,*}(a), \xi^*(a), b^*(a))$ be the steady state equilibrium values of the housing price, average productivity, housing wealth and total wealth to output ratio and bequests. Then, we say that there exists a *housing cost disease* if a rise in the efficiency of labor in manufacturing, a, increases the equilibrium values of the housing price, q, the housing wealth to income ratio, ξ^h, the rich households' bequests, b^r, and if average labor productivity, y, increases by a small margin or falls (*i.e.*, it increases by less than a percentage point for any percentage increase in a).

We impose, now, two key restrictions on parameters that may be responsible for the housing cost disease. In particular, we assume that the labor share in construction is larger than in manufacturing (or, equivalently, that capital intensity in manufacturing is higher than in construction) and that the demand for housing is sufficiently inelastic.

Assumption 4. *The production functions and households preferences satisfy:*

1. $\mu_h^L \geq \mu_m^L$, i.e., $\Delta \geq 0$,
2. *inelastic demand for housing,* i.e., *for all household type, i, and all (b^i, a), the own price elasticity of the demand for housing is less than one.*

The motivations for these assumptions are based on the logic of the Baumol's disease proposition and on the available empirical evidence.

First of all, observe that the relative labor share plays a key role in the comparative statics of the model. If $\Delta > 0$, the average capital and output per worker fall short of their respective values in the manufacturing sector by a margin that depends on the size of housing wealth. In particular, by plugging the expression for λ provided by (25) in (31), exploiting wage equalization, (13), and the definition of relative labor share, (15), we obtain

$$y = ay^m - \left(\frac{\delta + n}{1 + n}\right) v\Delta. \tag{32}$$

Hence, the total wealth to income and housing wealth to income ratios are

$$\xi = \frac{(1+n)ak^m + v(1-(\delta+n)\theta_r\Delta)}{ay^m - \left(\frac{\delta+n}{1+n}\right) v\Delta}, \qquad \xi^h = \frac{v}{ay^m - \left(\frac{\delta+n}{1+n}\right) v\Delta}. \tag{33}$$

Then, if $\Delta > 0$, the higher is the housing wealth (v), the lower is the average labor productivity (y), relative to its level in the manufacturing sector (y^m), and the higher is the housing wealth to income ratio, when all else is unchanged. We will see in a moment that $\Delta > 0$ contributes to generate a positive response of wealth and bequests to a rising a. Recalling that $\Delta > 0$ if and only if $\mu_h^L \geq \mu_m^L$, this inequality appears to be in line with the evidence provided in Valentinyi and Herrendorf (2008), which sets the capital share in manufacturing and construction at 0.4 and 0.2, respectively.

The assumption of a low own price elasticity of housing demand follows the logic underpinning the Baumol's cost disease proposition (low enough price elasticity for the demand of the stagnant sector output). The basic idea is that a rise in the efficiency of labor in manufacturing, a, by rising the relative price of housing, generates a relatively small effect on the demand of this good. Then, the demand for housing wealth ($v = qh$) rises with a and generates a reallocation of production and labor to the less productive sector. There exists strong evidence that housing demand responds by less than a percentage point to a one per cent rise in price. In particular, Hanushek and Quigley (1980), Mayo (1981) and Ermisch et al. (1996) provide estimates of the housing demand elasticity in the range $(-0.8, -0.5)$.

Although our results are mostly based on a set of simulations that are presented in section 4.4, here we provide some analytical results and intuition for the model with positive bequests and a CES specification of households' preferences. From now on, we will simplify the exposition by imposing CES preferences, *i.e.*, for some positive parameters, $(\alpha, \beta, \chi, \gamma)$, with $\alpha + \beta + \chi = 1$, we set

$$U(c^{y,i}, c^{o,i}, h^i) = \begin{cases} \left[\alpha (c^{y,i})^{\frac{\gamma-1}{\gamma}} + \beta (c^{o,i})^{\frac{\gamma-1}{\gamma}} + \chi (h^i)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} & \text{if } \gamma \neq 1, \\ \alpha \log c^{y,i} + \beta \log c^{i,o} + \chi \log h^i & \text{otherwise.} \end{cases} \tag{34}$$

Notice that the parameter γ represents the elasticity of substitution between each pair of goods. This specification has some advantage in terms of aggregation across individuals and, most importantly, implies normality of housing and consumption demand, as well as the validity of the law of demand (*i.e.*, housing and age-contingent consumptions are decreasing in their own price).

With CES utility, the demands for age-contingent consumptions and housing are all unit elastic with respect to the steady state present value of income. Hence, defining the latter as

$$I_a^i = aw^o + (1 - (1+n)\theta_r)b^i,$$

we can write

$$c^{y,i}/I_a^i = \phi^y, \qquad \theta_r c^{o,i}/I_a^i = \phi^o, \qquad \pi h^i/I_a^i = \phi^h,$$

for all $i = p, r$, where, for $j = y, o, h$, $\phi^j = \phi^j(\pi)$ define the expenditure shares, all positive and continuous functions of π, and such that $\sum_{j=y,o,h} \phi^j = 1$.

Notice that the size of the own price elasticity of housing demand depends on the elasticity of substitution, γ. More precisely, from now on, for any real function $g(x_1, ..x_n)$, we let $\hat{g}_{x_j}$ be the elasticity of g with respect to x_j, *i.e.*, $\hat{g}_{x_j} = x_j \partial \log g(x)/\partial x_j$. Then, from straightforward calculations, we get

$$\hat{h}_b^i = (1 - (1+n)\theta_r)b^i/I_a^i, \tag{35}$$

$$\hat{h}_\pi^i = -(\gamma + (1-\gamma)\phi^h), \tag{36}$$

where

$$\phi^h = \frac{\chi^\gamma \pi^{1-\gamma}}{\alpha^\gamma + \beta^\gamma \theta_r^{1-\gamma} + \chi^\gamma \pi^{1-\gamma}}.$$

Notice that the own price elasticity, $-\hat{h}^i_\pi$, is smaller than one if and only if $\gamma < 1$. In other words, the latter inequality verifies assumption 4-2 and implies that a rise in the user cost of housing generates a rise in housing demand. Furthermore, with $\gamma < 1$, the expenditure shares ϕ^y and ϕ^o are decreasing and ϕ^h is increasing in π. In this case, we say that housing demand is inelastic.

4.2 Unbalanced Growth and Wealth Inequality

The housing cost disease defined earlier can be restated in terms of elasticities through the conditions:

(i) $\hat{q}_a > 0$ (price increase),

(ii) $\hat{\xi}^h_a > 0$ (unbalanced growth),

(iii) $\hat{y}_a < 1$ (adverse effect on productivity),

(iv) $\hat{b}_a > 0$ (increasing wealth inequality).

Observe that the unbalanced growth phenomenon may be defined, alternatively, as an increase in the share of labor employed in construction. However, this phenomenon and the one defined in (ii) are strictly related, as one can see from equation (25). In particular, using this equation together with (14), we obtain

$$\hat{\lambda}_a = (\hat{v}_a - 1) + (1 - \mu^L_h)(1 - \sigma_h). \tag{37}$$

A first observation is that a price increase at a PBSS follows directly from (14) and it is unrelated with the behavior of equilibrium bequests. In particular, the model predicts that a rise in a by a percentage point generates an increase of μ^h_L percentage points in q

at equilibrium. The effects of a on ξ^h and y depend, instead, on how relative productivity modifies housing wealth. In particular, using (32), (33), recalling (16), and computing elasticities for given w, we obtain

$$\hat{y}_a = 1 - \left(\frac{\delta + n}{1+n}\right) \xi^h \left[\Delta\left(\hat{v}_a - 1\right) + (1-\sigma_h)(1-\mu_h^L)(1+\Delta)\right], \tag{38}$$

$$\hat{\xi}_a^h = (\hat{v}_a - 1) + (1 - \hat{y}_a). \tag{39}$$

Inspection of (38) and (39) reveals that an adverse effect on productivity can only occur when $\Delta \neq 0$. If, on the other hand, $\Delta = 0$, y grows on a one-to-one percentage points basis with a and unbalanced growth occurs only if $\hat{v}_a > 1$. Turning, now, to the more general case $\Delta > 0$, a sufficient condition for (ii) and (iii) (and, as a consequence, of a positive impact of a on the share of labor employed in construction) is

$$\sigma_h < 1, \qquad \hat{v}_a > 1. \tag{40}$$

How likely are these condition? The restriction on σ_h is supported by most estimates. In particular, Chirinko (2008) provides a comprehensive survey of the available empirical evidence about the elasticity of substitution between capital and labor, as well as his own estimates, and puts the most likely range for σ_j between 0.4 and 0.6. Turning to the elasticity of housing wealth, $\hat{v}_a$, we observe that

$$\hat{v}_a = \hat{v}_a^d + \hat{v}_b^d \hat{b}_a, \tag{41}$$

i.e., it can be evaluated through the sum of the direct and indirect effect of a on the demand for housing wealth at equilibrium. The first effect is positive by (i) and the assumption that the own price elasticity of housing demand is less than one (cf. assumption 4). To check the indirect effect (*i.e.*, the one generated by a change in equilibrium bequests), we need to study how equilibrium bequests respond to a.

A key step in our comparative statics exercise is, then, checking for the uniqueness of a PBSS, *i.e.*, uniqueness of a solution $b^*(a)$ for all $a > 0$ to (29). This is established in appendix B (proposition 5), where it is shown that, with CES preferences and $\Delta \geq 0$,

$$v_b^s(b^*, a) > v_b^d(b^*, a), \tag{42}$$

i.e., the supply of housing wealth, $v^s(b, a)$, is steeper than the demand of housing wealth, $v^d(b, a)$, as a function of the bequest value, b. Essentially, the model predicts that a rise in bequests has a strong effect on saving, relative to the effect on housing demand.

In appendix C we show that

$$\hat{v}_a = 1 + \mu_h^L(1-\gamma)(1-\phi^h) + \hat{v}_b^d(\hat{b}_a - 1), \tag{43}$$

where $\hat{b}_a$ is defined by a complicated expression involving various key parameters of the model, the most important of which are the elasticities of substitutions (in utility and construction technology), γ and σ_h, and the relative labor share, Δ. Referring the interested reader to the appendix C for the general case, we provide here the most relevant insights.

First of all, $\hat{b}_a = 1$ for $\gamma = \sigma_h = 1$ (e.g., when utility is logarithmic and the production function in construction is Cobb-Douglas). Hence, the case of unit elasticities (in preferences and housing production) represents a sort of benchmark for our economy, in which

$$\hat{b}_a = \hat{v}_a = \hat{y}_a = 1, \qquad \hat{\xi}_a^h = \hat{\lambda}_a = 0.$$

A second observation is that

$$\lim_{\Delta \to 0}(\hat{b}_a - 1) = \frac{\mu_h^L(1-\gamma)\left(\phi^o + \theta_r\phi^y(1-\delta)\right) - (1-\mu_h^L)(1-\sigma_h)(\delta+n)\theta_r}{\hat{v}_b^s - \hat{v}_b^d}, \tag{44}$$

so that $\hat{b}_a$ is positive and close to one for Δ small enough, σ_h and/or μ_h^L close enough to one and $\gamma < 1$. By inspection of (43), these conditions imply $\hat{v}_a > 1$. A more precise statement, which can be derived from the analysis in appendix C, is the following.

Proposition 2. *Assume 4 and let*

$$(1-\delta)/(\delta+n) \geq \Delta > 0, \qquad \sigma_h \leq 1,$$

γ sufficiently small and σ_h and/or μ_L^h close enough to one. Then,

$$\hat{v}_a > 1, \qquad \hat{b}_a > 1, \qquad \hat{y}_a < 1, \qquad \hat{\xi}^h > 0.$$

We may get some intuition about the above findings through a demand-supply type of analysis. What makes the effect of a on bequests ambiguous is that a rise in a produces an upward shift in both demand and supply of housing wealth. In fact, a higher a generates higher wages and higher housing prices. In turn, since $\gamma < 1$, a housing price appreciation increases the expenditure share on housing and lowers the one on young age consumption, so that households save more. Now recall that, by (42), the demand for housing wealth crosses the supply from above at b^*. Then, a higher a has a positive effect on bequests only if demand shifts up by more than supply. The above discussion shows that, for Δ small, the upward shift in the demand for housing wealth generated by higher wages and higher housing prices, is greater than the upward shift in supply. However, when $\sigma_h < 1$, the increase in relative productivity, a, generates a higher supply of housing wealth, because a larger Δ lowers investment in business capital and it generates more resources for housing investment. Hence, provided that Δ falls or it does not change much with a, bequests will be increase with relative productivity.

4.3 Wealth to Income Ratios and Capital Shares

In this section we show that the housing cost disease may be responsible for a higher capital to income ratio and a higher capital share of income, when the latter is affected by imputed rents. In a series of influential papers, Piketty and Zucman (2014), Piketty (2015a) and Piketty (2015b) have argued that the rising capital share of income is a consequence of the joint hypothesis of a steadily rising capital-output ratio and low diminishing returns to capital. Most of the evidence on the rising capital-output ratios and capital shares provided in this literature reflects the role of housing. In particular, housing values are included in the definition of capital (and represent a big part of it) and capital shares are evaluated based on a definition of income that includes imputed rents. For instance, Rognlie (2014), using Picketty's data, claims that housing "accounts for nearly 100% of the long-term increase in the capital/income ratio, and more than 100% of the long-term increase in the net capital share of income." (Rognlie (2014), p. 3). On a similar ground, according to Bonnet et al. (2014), the capital income ratio has dropped or remained roughly constant, when we take housing capital aside. The question we raise here is whether the phenomena highlighted by Piketty and Zucman (2014), Piketty (2015a) and Piketty (2015b) may be a consequence of a rising efficiency of labor in manufacturing, *i.e.*, as a byproduct of a housing cost disease.

The effect of a on the capital to income ratio is measured by

$$\hat{\xi}_a = \xi^h \left(\hat{v}_a - 1\right) + \left(\frac{\xi - (1+n)\theta_r}{\xi}\right)(1 - \hat{y}_a).$$

Then, if $\xi \geq (1+n)\theta_r$, the same sufficient assumptions for a housing cost disease stated in proposition 2 imply $\hat{\xi}_a > 0$.

Now consider the effect of a on the capital share of income. A natural definition of this variable is

$$\zeta^k = Rk/y.$$

Then, assuming $R = 1/\theta_r$, consistently with a PBSS, and using equations (26) and (32), we derive

$$\hat{\zeta}^k_a = \left(\frac{1-\zeta^k}{\zeta^k}\right)(\hat{y}_a - 1),$$

i.e., the existence of the housing cost disease as defined above and, in particular, $\hat{y}_a < 1$, implies a declining capital share of income at steady state. In other words, the housing cost disease dampens the dynamics of business capital more than labor productivity. However, suppose that, following the prevailing practice, we include imputed rents in the definition of GDP. This would provide the following alternative definition of capital share

$$\zeta^{k,h} = \frac{Rk + \pi h}{y + \pi h} = \frac{Rk/y + (1-(1-\delta)/R)\xi^h}{1 + (1-(1-\delta)/R)\xi^h}.$$

Hence, $\zeta^{k,h}$ is typically larger than ζ^k and it is increasing in ξ^h for any given k, y and R. By straightforward computations we derive that

$$\hat{\zeta}^{k,h}_a = \frac{1-\zeta^k}{\zeta^k + (\pi/q)\xi^h}\left[\left(\frac{(\pi/q)\xi^h}{1+(\pi/q)\xi^h}\right)\hat{\xi}^h_a - (1-\hat{y}_a)\right].$$

Hence, when the definition of income includes imputed rents, a rising relative productivity may generate a rising capital share if the growth in housing wealth is sufficiently strong.

4.4 Numerical Simulation

In this section we study numerically the consequences of a rising relative productivity on stationary equilibrium variables. We assume that labor efficiency in construction is time invariant and normalized to one so that $A^m = a$ and $A^h = 1$; and that population growth is constant and equal to $n = 1\%$. We then compare different steady states for different values of $a = 1, \ldots, 3$. In what follows, we assume that preferences have the CES representation specified in (34). On the production side, we assume a Cobb-Douglas production function for the manufacturing sector (f_m), and a CES production function for the housing sector

(f_h):

$$f_m(k^m) = (k^m)^{\alpha_m}, \qquad f_h(k^h) = \left[\alpha_h (k^h)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_h)\right]^{\frac{\sigma}{\sigma-1}}.$$

Table 1 reports all the parameters of the model. We set the preference parameters to $\alpha = 0.2$, $\beta = 0.5$, $\chi = 0.2$ and $\gamma = 0.5$. We set the inter-generational discount factors of the rich households to $\theta_r = 0.75$. Note that the gross interest rate is constant and equal to $1/\theta_r$. As a result, we implicitly set a mean annual real interest rate of about 1% over a generation[6]. We also set the fraction of rich households $m_r = 10\%$. The depreciation of the housing stock is set equal to $\delta = 20\%$, implying complete depletion over five generations as in Deaton and Laroque (2001). The elasticity of substitution between capital and labor is equal to one in the manufacturing sector, given the assumption of Cobb-Douglas production function. On the contrary, for the housing sector, we assume a CES production function with elasticity of substitution σ equal to 0.5, in the range of values estimated by Chirinko (2008). Note that Karabarbounis and Neiman (2014) and Piketty and Zucman (2014) argue instead for an elasticity of substitution between capital and labor greater than one. We also assume that the manufacturing sector is more capital intensive than the housing sector, and set $\alpha_m = 0.5$ and $\alpha_h = 0.5$.

We summarize the main results under the proposed parametrization in figures 3 and 4. Several of the relevant variables of the model are either constant and determined by our parametrization (*i.e.*, R, w) or have a straightforward relation with a (for example, q is necessarily increasing in a). Therefore, in this section we focus on the variables that are instead determined in general equilibrium. In particular, in figure 3, we plot the changes of steady-state values of three key variables, the equilibrium supply of bequests (b), the total wealth to income ratio (ξ), and the housing wealth to income ratio (ξ^h), with respect to their initial values when $a = 1$. For variables expressed in percentage, like the total wealth to income ratio, we report the change in percentage points. On the other hand, for variables in

[6] As a back of the envelope calculation, assume that a generation lasts twenty-five years. In this case, the net annual real interest rate is equal to $\bar{r}_a = R^{1/25} - 1$. If $\theta_r = 0.75$, then $\bar{r}_a = 1.16\%$.

Table 1: Model parameters

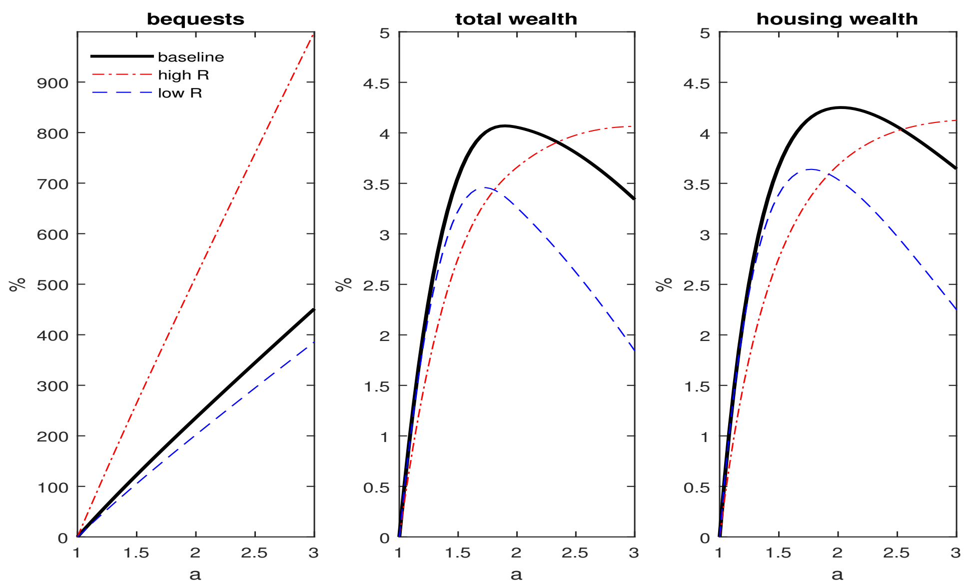
Parameter	
Weigth consumption young: α	0.20
Weigth consumption old: β	0.60
Weight housing: χ	0.20
Elasticity of substitution preferences: γ	0.50
Inter-generational discount factor rich: θ_r	0.75
Housing depreciation: δ	0.20
Capital share in housing: α_h	0.20
Capital share in manufacturing: α_m	0.67
Elasticity of substitution housing: σ	0.50
Fraction of rich households: mr	0.10
Population growth rate: n	0.01

Notes: In this table we report the parameters used to simulate different steady-states for different values of the exogenous parameter a. Note that the elasticity of substitution implied by the CES production function of housing is equal to σ.

levels, like bequests, we report the corresponding percentage change. The black solid lines correspond to the baseline specification of table 1. To check the robustness of the results, we include also results for the case of higher (red dot-dashed line) and lower (blue dashed line) interest rates. Bequests (left panel) respond strongly, and more than proportionally, to an increase in the relative productivity. The effect is stronger the higher the equilibrium level of the interest rate. The total, and housing, wealth to income ratios initially increase with a, but then stablilize and eventually decline for even higher (but less plausible) levels of the relative productivity parameter a. The effect of a 100 percent increase in relative productivity is to increase the ratios of total, and housing, wealth to income by about 3 percentage points. In both cases, the initial effect is stronger under our baseline value for the interest rate, and more persistent the higher the interest rate.

In figure 4, we examine the effect of rise in relative productivity on additional key variables of the model. As a result of the increase in a, we observe an increase in the labor share in construction (λ), average labor productivity (y), and per-capita housing stock (h). The effect of a 100 percent increase in relative productivity is to increase the share of labor in construction by about 1 percentage points. On the contrary, both the average labor productivity and the per-capita housing stock respond strongly to an increase in a. However, the response of y is less than proportional. Therefore, we conclude that, under our preferred parametrization, and over a reasonable range of values for a, the *housing cost disease* proposition is satisfied.

Figure 3: Simulation results (I/III)

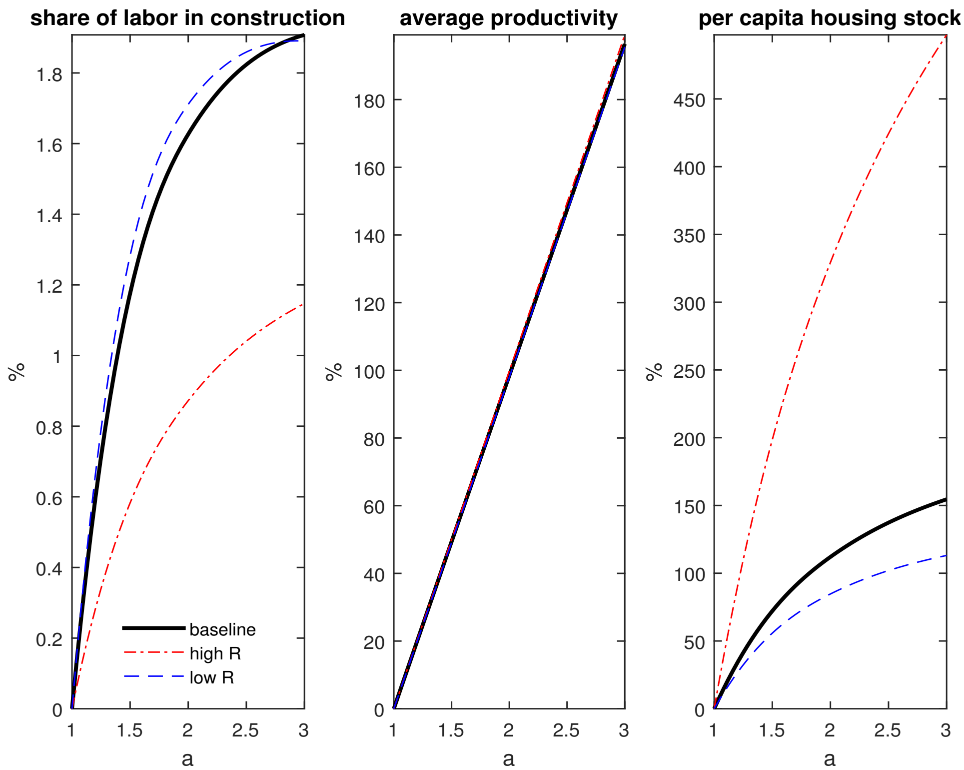


Notes: This figure plots the change in the steady-state values of the level of bequests (b), and the total (ξ), and housing (ξ^h), wealth to income ratios for different values of the relative productivity sector $a = 1, \ldots, 3$, with respect to their value for $a = 1$. For b, we report the percentage change. For ξ and ξ^h, we report the change in percentage points. All variables are in intensive form. The back solid lines correspond to the baseline parametrization. The blue dashed line is plotted for a lower interest rate ($R = 1/\theta_r = 1.25$). The red dot-dashed line is plotted for a higher interest rate ($R = 1/\theta_r = 1.66$).

In figure 5 we plot the change in capital shares of income. Following the discussion of section 4.3, we consider both the standard definition of capital share defined by $\zeta = Rk/y$ (left panel), and the definition, more commonly used in the empirical literature, that includes imputed rent and defined by $\zeta^{k,h} = (Rk + \pi h)/(y + \pi h)$. When a increases, ζ declines, and the decline would be larger for lower values of the interest rate. On the contrary, $\zeta^{k,h}$ first increases, and then declines.

In this section, we have presented results of a numerical simulation of the model and

Figure 4: Simulation results (II/III)



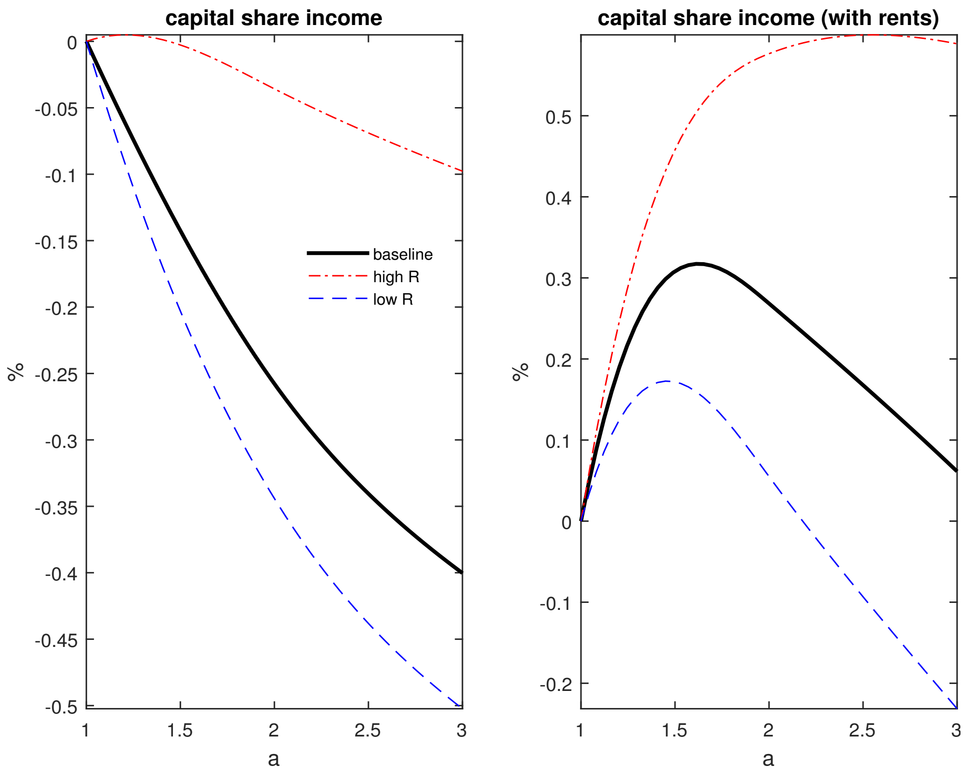
Notes: This figure plots the change in the steady-state values of the labor share in construction (λ), the average productivity (y), and the per capita housing stock (h) for different values of the relative productivity, $a = 1, \ldots, 3$, with respect to their value for $a = 1$. For λ, we report the change in percentage points. For y and h, we report the percentage change. All variables are in intensive form.

showed that, under our parametrization, it generates a large *housing cost disease*. In the next section, we look at data on productivity and wealth, for several developed countries, to find empirical support for our numerical results.

4.5 Empirical Estimation

In this section we provide some empirical support for the main conjecture investigated in this paper, *i.e.*, the positive effect of relative productivity with respect to the construction sector on the housing wealth to income ratio. The analysis is based on two different dataset: the time series for the wealth to income ratios collected in Piketty and Zucman (2014), and discussed in section 2, and the EU KLEMS Growth and Productivity Accounts (O'Mahony

Figure 5: Simulation results (III/III)



Notes: This figure plots the change in capital share of income for different values of the relative productivity, $a = 1, \ldots, 3$, with respect to their initial value when $a = 1$. We consider two alternative definition: the left panel corresponds to the standard definition of capital share $\zeta^k = Rk/y$; the right panel corresponds instead to a definition that includes imputed rents as in $\zeta^{k,h} = (Rk + \pi h)/(y + \pi h)$. All variables are in intensive form.

and Timmer, 2009) providing sectoral data on gross value added, capital and labor inputs and capital and labor shares, that we use to build a measure of relative productivity for the manufacturing with respect to construction sector. In particular, for each country in the Piketty and Zucman (2014)'s sample, we follow the assumptions discussed in section 4.4 and generate annual time-series of labor-augmenting productivity in manufacturing relative to construction based on Cobb-Douglas production function for the manufacturing sector, and CES production function for the construction sector[7].

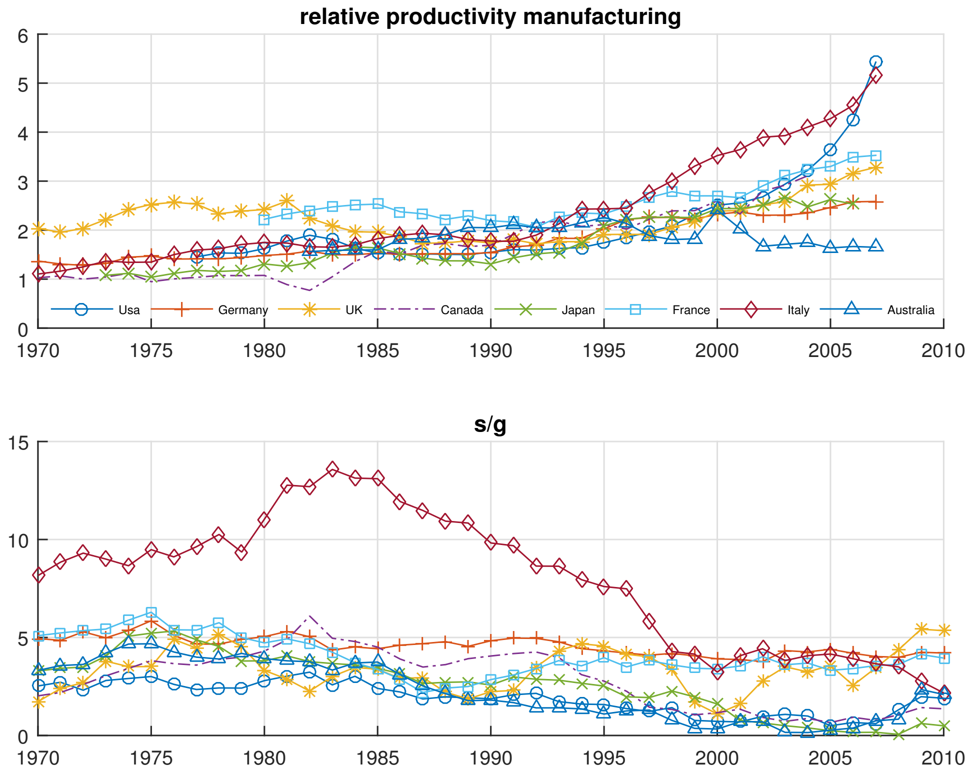
For most of the countries in the sample, we find that relative productivity in manufactur-

[7]KLEMS data are currently available up to 2007. Therefore, we consider the sample period 1970-2007. For some countries, the sample period is shorter due to lack of data. A separate appendix (section A) contains additional details on the data and on the construction of the indices of relative productivity.

ing, with respect to construction increased over the 1970-2007 period. Note that according to the classic Solow's formula, the steady state wealth to income ratio should equal the ratio between the (net of depreciation) saving rate and the income growth rate. This latter ratio did not increase over time, and in fact declined significantly, for countries like Italy or Japan, that coincidentally also have the highest ratios of private wealth to income at the end of the sample. Figure 6 provides support to these statements. In the top panel, we plot relative productivity in manufacturing with respect to construction, and, in the bottom, private saving as a fraction of income (s) divided by the average income growth (g). Italy and the US are the two countries that experienced the sharpest increase in relative productivity: in these countries relative productivity approximately tripled, and reached a value of about 5.5 in 2007. In 2007, in all the remaining countries in the sample, relative productivity was lower than 3.5. If relative productivity on average increased, the ratio between the saving rate and the growth rate of income declined in most countries. In Italy, this ratio was approximately 8 at the beginning of the sample, then reached a high value of more than 12 in the early eighties, and since then has been declining reaching a low value of about 2 in 2010. Ratios between saving rates and income growth rates declined also in other countries, with Japan, Canada and the US reaching low values of less than 1.5 in 2010.

To test the positive effect of relative productivity in manufacturing on total and housing wealth to income ratios we start by computing simple correlation coefficients, and continue with the estimation of a linear regression model. In table 2, we report sample correlation coefficients, for the countries in the sample, between wealth to income ratios and relative productivity in manufacturing with respect to construction. We compute correlations between both the series in levels, and their HP-filtered trends and cycles. The top panel takes into account private wealth to income, while the bottom housing wealth to income. First, correlations computed between the levels, or HP-filtered trends, are on average very large and positive both for private wealth and housing wealth ratios. Japan is the only country with a positive correlation between private wealth and relative productivity, but a negative

Figure 6: Relative productivity and saving rate



Notes: The top panel of the figure reports the relative labor-augmenting productivity of manufacturing with respect to construction. The bottom panel reports the ratio between private household (net of depreciation) savings to income and the average growth rate of real income (s/g). The countries in the sample are: the US, Germany, the UK, Canada, Japan, France, Italy and Australia. Productivity data are for the sample 1970-2007. Due to lack of data, for some countries the sample is shorter. Additional details on the series and on their time-length are available on a separate Appendix (section A). Saving to income ratios are for the sample 1970-2010. Data are from Piketty and Zucman (2014) and O'Mahony and Timmer (2009).

correlation between housing wealth and relative productivity. Australia is the only country with negative correlations for both private and housing wealth ratio. If correlations between levels or HP-filtered trend are large and positive, correlations between HP-filtered cycles do not show any clear pattern. The prediction of the model are about long-run trends, and not at business cycle frequency. Therefore, results from table 2 confirm that the series share a trend, without any implications in terms of possible causation.

Clearly, the high co-movement, for several of the countries in our sample, between private and housing wealth to income ratios and relative labor productivity in manufacturing, can be simply interpreted as the result of a spurious relationship. In section 3 we present a

Table 2: Correlations: wealth ratios and relative productivity

	US	DE	UK	CA	JP	FR	IT	AU
	private wealth and relative productivity							
levels	0.87	0.93	0.24	0.91	0.50	0.91	0.88	-0.10
HP-trends	0.91	0.96	0.34	0.98	0.66	0.98	0.92	-0.14
HP-cycles	0.43	0.07	-0.14	-0.24	-0.56	-0.16	-0.29	0.21
	housing wealth and relative productivity							
levels	0.60	0.88	0.42	0.49	-0.30	0.89	0.77	-0.33
HP-trends	0.74	0.92	0.55	0.75	-0.24	0.98	0.88	-0.40
HP-cycles	-0.30	-0.11	-0.03	-0.01	-0.63	-0.02	-0.34	-0.04

Notes: This table reports correlation coefficients between wealth to income ratios and relative productivity in manufacturing for the US, Germany, the UK, Canada, Japan, France, Italy and Australia. The two panels consider different definition of wealth: private (top) and housing (bottom). We report both correlations between the effective wealth to income ratios and relative productivities ("corr") and between HP-filtered trends of the two series ("HP-trend corr"). Data are annual from Piketty and Zucman (2014) and O'Mahony and Timmer (2009) for the period 1970-2007. For some countries the time-sample is shorter due to lack of data. Additional details on data are available in a separate Appendix (section A).

model that shows that a possible interpretation of the data is that the common trend *is* relative productivity itself. In table 3 we report some additional empirical results. We run country-level linear regressions of private wealth to income on the saving rate (s/y), real income growth (g) and relative manufacturing labor productivity productivity (a). Note that according to Solow's formula, we should expect a positive coefficient on s/y and a negative coefficient on g. Solow's is a one sector model, as a result there is no role for relative productivity growth. In contrast to Solow's predictions, we find negative coefficients, mostly significant, associated to the saving rate. On the contrary, the coefficients associated to the growth rate of real income are, as expected, mostly negative, but not always significantly different from zero. The coefficients associated to relative productivity are positive and significant for the US, Germany, France and Italy. On the contrary, the coefficients for Japan and Australia are instead negative, and significant. Note that in the regressions all the variables are in levels, with the exception of the real income growth. We follow this empirical strategy since the model presented in section 3 is about the long-run, rather than business cycle relationships.

Table 3: Regression: Housing wealth to income

	US	DE	UK	CA	JP	FR	IT	AU
cost	0.82	0.84	0.82	0.44	5.17	-0.53	1.03	3.56
	(0.13)	(0.37)	(0.73)	(0.07)	(1.18)	(0.56)	(0.66)	(0.40)
s/y	2.95	-3.63	-5.45	1.14	-14.76	-6.52	1.12	-6.18
	(1.21)	(2.83)	(3.22)	(0.34)	(5.64)	(3.68)	(1.70)	(1.15)
g	-0.91	0.10	2.68	-0.20	-4.16	-7.34	-7.75	-2.91
	(0.67)	(0.86)	(2.23)	(0.34)	(2.01)	(4.84)	(3.72)	(1.42)
a	0.13	0.32	0.41	0.09	-1.32	1.07	0.39	-0.70
	(0.04)	(0.07)	(0.24)	(0.02)	(0.43)	(0.19)	(0.12)	(0.18)
R^2	0.51	0.78	0.31	0.61	0.45	0.82	0.70	0.71
obs	31	38	38	35	34	28	38	26

Notes: This table reports, for each country in the sample, results from linear regressions of housing wealth to income ratios on the saving to income ratio, the growth rate of real income and relative productivity in manufacturing. In parenthesis we report HAC standard errors. Data are annual from Piketty and Zucman (2014) and O'Mahony and Timmer (2009) for the period 1970-2007. For some countries the sample is shorter due to lack of data. Additional details on data are available in a separate Appendix (section A).

5 Welfare

In this section we address the housing cost disease problem from a welfare point of view. Is the fact that housing takes a large share of private wealth undesirable?

Within a similar overlapping generations model, Deaton and Laroque (2001) find that the presence of a demand for housing generates a portfolio reallocation away from capital towards housing, causing the accumulation of capital to fall short of the *Golden Rule* level. They take this results as possible reason for confiscating property and giving it to consumers at no charge. An other reason to argue in favor of a reduction in housing wealth may be based on Piketty's argument that a rising wealth to income ratio, especially when caused by a rising housing stock appreciation, may lead to increasing inequality, especially when housing takes a sizable share of intergenerational bequests. For example, Auerbach and Hasset (2014) note that reducing the tax benefits for owner-occupied housing in a progressive manner (or a deregulation in land use) may be more effective than a wealth tax in addressing the inequality problem (as proposed by Piketty).

However, this conclusions must be taken with some caution. Regarding Deaton and

Laroque (2001)'s analysis, it should be noted that allocations departing from the Golden Rule are inconsistent with a social optimum only if we endorse a specific social welfare criterion, such as a weighted sum of all generations' utilities with rate of time preference equal to the population growth rate. In fact, any market allocation at which the rate of interest is larger than the population growth rate is Pareto optimal and, in these cases, reducing the value of the housing stock may have adverse effects on some generation's welfare[8]. Conversely, when the real interest rate falls short of the population growth rate, a case that can only occur when both type of households leave zero bequests, Pareto improvements can be obtained by decreasing investment in housing as well as in the capital stock. In other words, crowding out of capital induced by housing demand and exchange across generations may, in fact, be desirable to avoid an over-accumulation of capital. Regarding the role of housing in generating more inequality within generations through a rise in bequest levels, one should note that a rise in housing prices may increase the relatively poor old generations' wealth and, in this way, allow them to leave positive bequests.

In this section, we examine how a change in the reallocation of resources induced by a rise in relative productivity affects social welfare in a market economy, under an egalitarian criterion.

Recall that, by forward iteration, the initial old type-i household's utility can be written as

$$V^{-1,i} = \beta u(c_{-1}^{-1,i}, c_0^{-1,i}, h_0^{-1,i}) + \sum_{t=0}^{\infty}(\theta_i(1+n))^{t+1}u(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^{i}).$$

Then, the objective of the Egalitarian Planner is

$$\mathcal{U} = \sum_i m_i V^{-1,i}.$$

[8] Using standard arguments, and recalling that population is stationary, one can show that the equilibrium allocation derived in the above sections is Pareto efficient if and only there exists no bounded sequence, $\{\xi_t\}_{t=0}^{\infty}$, such that $\xi_{t+1} \geq R_{t+1}\xi_t$, $\xi_0 > 0$ for all $t \geq 0$ and $\xi_0 > 0$. Evidently, this requires that the sequence of gross interest rates is below one for an infinite number of periods. In particular, at the steady state, we need $R < 1$, which can only verified in a ZBSS.

It is easily shown that there are only two conditions characterizing a social optimum that may not be implemented in equilibrium: the optimal allocation of consumption across generations and the transversality condition. In particular, letting the (shadow) present value prices be defined by

$$p_t = \theta_i^t u_{1,t}^i / u_{1,0}^i,$$

for $i = p, r$, and setting $q_t = f_m'(k_t^m)/f_h'(k_t^h)$, a Planner optimum is such that, for all $i = p, r$,

$$\begin{aligned}
f_m'(k_{t+1}^m) &= p_t/p_{t+1}, && (45)\\
a_{t+1}(f_m(k_{t+1}^m) - f_m'(k_{t+1}^m)k_{t+1}^m) &= q_{t+1}(f_h(k_{t+1}^h) - f_h'(k_{t+1}^h)k_t^h), && (46)\\
u_{1,t}^i/u_{2,t}^i &= p_t/p_{t+1}, && (47)\\
u_{1,t}^i(q_t - (1-\delta)p_{t+1}/p_t) &= u_{3,t}^i, && (48)\\
u_{2,t}^i &= \theta_i u_{1,t+1}^i, && (49)\\
\lim_{t\to\infty} p_t(1+n)^t k_{t+1} &= \lim_{t\to\infty} p_{t+1}(1+n)^{t+1} q_{t+1} h_{t+1} = 0. && (50)
\end{aligned}$$

Comparing the above conditions characterizing the Planner's optimum with the market allocation defined by profit maximization, (11), (12), and the households optimality conditions (2)-(4), we can immediately verify that the only possible differences are in the allocation of bequests and the fact that a market allocation does not explicitly provide a transversality condition. However, the latter condition is always verified at any equilibrium such that $R_{t+1} > (1+n)$. In turn, this inequality is always verified when at least one type of households leaves positive bequests at all periods.

Notice that, although the Planner is egalitarian, she takes into account the subjective discount factors, θ_i, representing their degree of altruism with respect to the offsprings, in allocating resources. In fact, by conditions (47), (48) and (49) and the assumption $\theta_r > \theta_p$, we get

$$u_{1,t}^r/u_{1,t+1}^r > u_{1,t}^p/u_{1,t+1}^p,$$

i.e., the Planner's allocation is such that the poor (less altruistic) type young households end up with a lower consumption than the rich (more altruistic) type.

Now we consider the effect on the Planner's welfare function, $\mathcal{U}$, of an unanticipated rise in the level of the relative labor efficiency, $a = A^m/A^h$, at $t = 0$, for a constant labor efficiency in construction, $A^h_t = 1$ at a competitive equilibrium such that the poor-type households leave zero bequests and the rich-type leave positive bequests at all periods. Later on we will make some comments on the case of zero bequests across all households.

By exploiting the equilibrium conditions (*i.e.*, resource feasibility, first order conditions for individual optimality at equilibrium and budget constraints), and defining subjective prices, for $i = p, r$, as

$$\rho^i_t = (\theta_i(1+n))^{t+1} u^i_{1,t},$$

we derive that the welfare effect of a rise in a at time zero at equilibrium is given by the following expression:

$$\partial\mathcal{U}/\partial a = \sum_{t=0}^{\infty} \rho^i_t w_t(1-\lambda_t) + m_p\Gamma, \tag{51}$$

where

$$\Gamma = \sum_{t=0}^{\infty}(\rho^p_t - \rho^r_t)\left(\frac{\partial c^{t,p}_t}{\partial a} + \frac{1}{1+n}\frac{\partial c^{t-1,p}_t}{\partial a} + \pi_t\frac{\partial h^p_t}{\partial a}\right) + \sum_{t=0}^{\infty}\rho^p_t\left(\frac{u^p_{2,t-1}}{\theta_p u^p_{1,t}} - 1\right)\frac{1}{1+n}\frac{\partial c^{t-1,p}_t}{\partial a}.$$

Since a First Best allocation is such that

$$\rho^r_t = \rho^p_t, \qquad \frac{u^p_{2,t-1}}{\theta_p u^p_{1,t}} = 1,$$

for all $t \geq 0$, the first summation on the right hand side of (51) represents the *undistorted* component of the welfare effect of a rising a, whereas Γ represents two possible distortions: the first one arising from the possibility that consumption is not allocated as dictated by the Planner across individuals of the same generation, and the second from the fact that the poor-type households are unable to leave positive bequests to their offsprings. Observe that, at equilibrium, $u^p_{2,t-1}/\theta_p u^p_{1,t} > 1$, whereas the sign of $\rho^p_t - \rho^r_t$ is ambiguous, since it depends on the differences between consumptions across the two type of households (in the same age and time) and the discount factors. If the discount factors are very similar and the rich household has a much larger consumption compared with the poor household, we have $\rho^p_t - \rho^r_t > 0$. A further observation is that the second summation in Γ would vanish in the case of an equilibrium where both households are identical and leave zero bequests (the *canonical* overlapping generations model). In this case, equation (51) would imply that a rise in a generates a welfare benefit exceeding the undistorted (First Best) value if this parameter shift generates a higher consumption for the olds. In turn, the latter consumption is likely to rise if the increase in a has a positive effect on housing wealth (since old individuals are net suppliers of housing).

We provide now an equivalent more intuitive expression for $\partial \mathcal{U}/\partial a$ at a steady state. In particular, assume that $R_t = 1/\theta_r$ for all $t \geq 0$ and observe that, in this case, the budget constraints of the poor households provide

$$\frac{\partial c^{t,p}_t}{\partial a} + \theta_r \frac{\partial c^{t,p}_{t+1}}{\partial a} + \pi_t \frac{\partial h^p_t}{\partial a} = w_t - \frac{\partial \pi_t}{\partial a} h^p_{t+1}, \qquad \frac{\partial c^{-1,p}_0}{\partial a} = (1-\delta) h^p_0 \frac{\partial q_0}{\partial a}.$$

Then, by rearranging terms, we derive

$$\Gamma = \beta \left(u'(c^{-1,p}_0) - u'(c^{-1,r}_0) \right) (1-\delta) h^p_0 \frac{\partial q_0}{\partial a} + \sum_{t=0}^{\infty} (\rho^p_t - \rho^r_t) \left(w_t - \frac{\partial \pi_t}{\partial a} h^p_{t+1} \right). \tag{52}$$

In other words, the distortionary component of the welfare effect of a rising a depends on how consumption is allocated across households and on the impact of a on wages and housing prices. The latter are important because they alter the old individuals' housing wealth and the affordability of housing (measured by π) for the young households.

Now consider now a steady state, where $c^{t,i}_t = c^{y,i}$, $c^{t-1,i}_t = c^{o,i}$ for all $t \geq 0$ are the stationary young and old age consumptions of the two types, and

$$z^i \equiv \sum_{t=0}^{\infty} \rho_t^i = \frac{\theta_i(1+n)}{1-\theta_i(1+n)} u_1^i.$$

Using the above findings and evaluating the expression for Γ in (52) at a steady state, we can decompose $\partial\mathcal{U}/\partial a$ into two components: the effect arising from a change in w and the effect arising from the housing appreciation, *i.e.*, from a change in q. In particular, we have

$$a\frac{\partial\mathcal{U}}{\partial a} = A_w W + A_q \left(\frac{\partial q/q}{\partial a/a}\right),$$

with

$$\begin{aligned} A_w &= (1-\lambda)z^r + m_p(z^p - z^r), \\ A_q &= -m_p q h^p \left(\left(\frac{\delta+n}{1+n}\right)(z^p - z^r) + \left(\frac{1-\delta}{1+n}\right)\left(\frac{\theta_r - \theta_p}{\theta_r}\right) z^p\right). \end{aligned}$$

Remember that $\theta_r > \theta_p$ and $\partial q/\partial a > 0$ at equilibrium. Then, the signs of the terms A_w and A_q are, respectively, positive and negative when $z^p \geq z^r$, *i.e.*, when

$$\frac{u_1^p}{u_1^r} > \frac{\theta_r(1-(1+n)\theta_p)}{\theta_p(1-(1+n)\theta_r)}, \tag{53}$$

and they are ambiguous otherwise. In turn, the above inequality holds when the poor household's consumption is sufficiently lower than the rich one, relative to the size of the two discount rates, θ_r, θ_p. For example, in the log-utility model, equation (53) is verified for

$$\frac{b}{W} > \frac{\theta_r - \theta_p}{(1-(1+n)\theta_r)^2}.$$

Proposition 3. *Suppose the economy exhibits an improvement in labor efficiency in manufacturing at a PBSS characterized by condition (53). Then, this is welfare reducing if and only if it induces a too strong housing appreciation.*

6 Conclusions

We propose a Baumol's cost disease interpretation of the increasing role of housing in private wealth over the long-run and show that this change in wealth composition may affect the allocation of wealth and the income distribution in a simple two-sector growth model with no financial frictions and bequests motivated by parental altruism. If housing construction is less capital intensive than manufacturing, and the elasticity of substitution between capital and labor in construction is not too large, a rise in labor efficiency in manufacturing may produce a strong upward pressure on housing prices, a rise in private wealth driven by the increase in the share of housing wealth and a rise in bequests. This housing appreciation may be social welfare reducing under the assumptions that the Planner is egalitarian and the market allocation assigns too little consumption to the households that are unable to live and receive bequests.

References

Auerbach, Alan J. and Kevin Hasset, "Capital Taxation in the 21st Century," 2015. mimeo.

Baumol, William, "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis," *American Economic Review*, June 1967, *57* (3), 415–426.

Bonnet, Odran, Pierre-Henri Bono, Guillaume Chapelle, and Etienne Wasmer, "Le capital lodgement contribute-t-il aux inégalités?," April 2014. LIEPP Working Paper.

Chirinko, Robert S., "The Long and Short of It," 2008. mimeo.

Cragg, Michael and Rand Ghayad, "Growing Apart: The Evolution of Income vs. Wealth Inequality," *The Economists' Voice*, 2015, *12* (1), 1–12.

Deaton, Angus and Guy Laroque, "Housing, Land Prices, and Growth," *Journal of Economic Growth*, June 2001, *6*, 87–105.

Ermisch, John F., Jeanette Findlay, and Kenneth Gibb, "The Price Elasticity of Housing Demand in Britain: Issues of Sample Selection," *Journal of Housing Economics*, 1996, *5*, 64–86.

Hanushek, Eric A. and John M. Quigley, "What is the Price Elasticity of Housing Demand?," *Review of Economics and Statistics*, 1980, *62*, 449–454.

Iacoviello, Matteo, "Housing Wealth and Consumption," 2010. International Finance Discussion Papers No. 1027.

Karabarbounis, Loukas and Brent Neiman, "The Global Decline of the Labor Share," *Quarterly Journal of Economics*, February 2014, *129*, 61–103.

Mayo, Stephen K., "Theory and Estimation in the Economics of Housing Demand," *Journal of Urban Economics*, 1981, *10*, 95–116.

OECD, *OECD Economic Surveys*, Paris: OECD Publishing, 2012.

O'Mahony, Mary and Marcel P. Timmer, "Output, Input and Productivity Measures at the Industry Level: the EU KLEMS Database," *Economic Journal*, 2009, *119* (538), F374–F403.

Piketty, Thomas, *Capital in the Twenty-First Century*, Cambridge, Massachusetts: Belknap Press - Harvard, 2014.

_ , “About Capital in the 21st Century,” *American Economic Review*, 2015, *105* (5), 1–6.

_ , “Putting Distribution Back at the Centre of Economics,” *Journal of Economic Perspectives*, 2015, *29* (1), 67–88.

_ **and Emmanuel Saez**, “Inequality in the long run,” *Science*, May 2014, *344* (6186).

_ **and Gabriel Zucman**, “Capital is Back: Wealth Income Ratios in Rick Countries 1700-2010,” *Quarterly Journal of Economics*, 2014, *129* (3), 1255–1311.

Rognlie, Matthew, “A Note on Piketty and diminishing returns to capital,” 2014. mimeo.

Summers, Larry, “The Inequality Puzzle,” Democracy A Journal of Ideas 2014. Summer.

Valentinyi, Akos and Berthed Herrendorf, “Measuring Factor Income Shares at the Sector Level,” *Review of Economics Dynamics*, October 2008, *11*, 820–835.

A Data

We use the KLEMS database, available at http://www.euklems.net/, and organized by O'Mahony and Timmer (2009), to construct a measure of relative labor efficiency in manufacturing. In particular, we use the ISIC Rev. 3 version of the KLEMS database (March, 2011), which reports data from 1970 up to 2007. For each of the countries in the sample, and for different sectors, we collect data on real gross valued added (Y^i); real capital (rK^i) and labor compensation (wL^i); hours worked by employee; real capital (K^i) and labor inputs (L^i), where i is an index denoting the different sectors (we consider KLEMS sectors "total industries," as a proxy for manufacturing, and "construction"). Only for Canada, we consider the earlier March 2008 release which covers the sample 1970-2004; and use "wood and product of wood" as proxy for the manufacturing sector. In order to compute the relative labor efficiency, we compute labor productivities in manufacturing under the assumption of Cobb-Douglas technology, and in construction under the assumption of CES technology. For manufacturing, we first derive the capital share parameter in the production function (α_m) by dividing real capital compensation, in this sector, by the corresponding real gross value added. Given the assumption of constant returns to scale, we assume that the labor share parameter is simply equal to $1 - \alpha_m$. We then derive the labor-augmented productivity A^m as:

$$A^m = \frac{1}{L^m}[Y^m(K^m)^{-\alpha_m}]^{\frac{1}{1-\alpha_m}}. \tag{54}$$

For construction, we assume that the elasticity of substitution between capital and labor is equal to $\sigma = 2$, and that the parameter α_h in the CES production function is the same as for the case of a Cobb-Douglas specification (i.e., it is equal to the ratio between real capital compensation in construction and real gross value added in the same sector). Then, the labor-augmented productivity A^h is derived as:

$$A^h = [\frac{(Y^h)^{\frac{\sigma-1}{\sigma}} - \alpha_h(K^h)^{\frac{\sigma-1}{\sigma}}}{(1-\alpha_h)(L^h)^{\frac{\sigma-1}{\sigma}}}]^{\frac{\sigma}{\sigma-1}} \tag{55}$$

Relative labor efficiency in manufacturing in then simply equal to $a_t = A_t^m/A_t^h$.

In table 4 we report the length of the sample periods for each of the variables used in the empirical estimation of section 4.5.

Table 4: Sample: Details

Countries	PWI	HWI	s/y	g	a
US	1970-2010	1970-2010	1970-2010	1970-2010	1977-2007
DE	1970-2010	1970-2010	1970-2010	1970-2010	1970-2007
UK	1970-2010	1970-2010	1970-2010	1970-2010	1970-2007
CA	1970-2010	1970-2010	1970-2010	1970-2010	1970-2004
JP	1970-2010	1970-2010	1970-2010	1970-2010	1973-2006
FR	1970-2010	1970-2010	1970-2010	1970-2010	1980-2007
IT	1970-2010	1970-2010	1970-2010	1970-2010	1977-2007
AU	1970-2010	1978-2010	1970-2010	1970-2010	1982-2007

Notes: This table reports the available samples for the variables used in the empirical estimation of section 4.5. PWI is private wealth to income; HWI is housing wealth to income; s/y is saving as a fraction of net income; g is the real income growth; a is relative labor efficiency in manufacturing. The countries in the sample are: the US, Germany, the UK, Canada, Japan, France, Italy and Australia. Data are from Piketty and Zucman (2014) and O'Mahony and Timmer (2009).

B Propositions and Proofs

Proposition 4. *Any equilibrium steady state must be such that*

$$s - (1+n)ak^m > 0, \qquad 1 - (\delta + n)\Delta/R > 0.$$

Proof. Since $\Delta = (\mu_h^L - \mu_m^L)/\mu_m^L$ and $qf_h(k^h) = a(w + Rk^h)$, we have

$$1 - (\delta+n)\frac{\Delta}{R} = \frac{aw - (\delta+n)ak^m + k^h(aR + (\delta+n))}{qy^h} \geq \frac{aw - (1+n)ak^m}{qy^h}. \tag{56}$$

Then, $v^s > 0$ if either $s - (1+n)ak^m > 0$ or $1 - (\delta+n)\Delta/R < 0$. By (56) we derive that

$$v^s = qy^h \left(\frac{s - (1+n)ak^m}{aw - (\delta+n)ak^m + k^h(aR + (\delta+n))} \right),$$

and, then,

$$v < \left(\frac{1+n}{\delta+n}\right) qy^h \quad \text{if} \quad s - (1+n)ak^m > 0,$$
$$v > \left(\frac{1+n}{\delta+n}\right) qy^h \quad \text{if} \quad 1 - (\delta+n)\Delta/R < 0.$$

By equation (25) and the above inequalities, we derive

$$\lambda(w, a, v^s) = \left(\frac{\delta+n}{1+n}\right)\frac{v^s}{qy^h} \leq 1 \qquad \text{if and only if} \qquad s - (1+n)ak^m > 0.$$

Now observe that

$$s - (1+n)ak^m \le a(w - (1+n)k^m) = ak^m \left(\frac{f(k^m)}{k^m} - f'(k^m) - (1+n) \right).$$

Since $\lim_{k^m \to \infty} f(k^m)/k^m \le \lim_{k^m \to \infty} f'(k^m)$, $s - (1+n)ak^m < 0$ for k^m large enough and, hence, the function

$$v^s(w,a) = \frac{s - (1+\rho)ak^m}{1 - (\delta+n)\Delta/R} \tag{57}$$

is bounded above by $qf_h(k^h(w,a))$ and positive when $\lambda(w,a) \le 1$. □

Proposition 5. *Assume $\Delta > 0$ and let households' preferences have the CES representation (34). Then, if it exists, the PBSS value, $b^*(a)$ is unique and the demand for housing wealth as function of bequests crosses the supply of housing wealth from above at $b^*(a)$,* i.e., *$v_b^d(b^*, a) < v_b^s(b^*, a)$.*

Proof. Since demand functions are unit elastic with respect to I_a^i, the impact of a rising bequest on saving is measured by

$$s_b^i = 1 - (1 - (1+n)\theta_r)\phi^y, \tag{58}$$

so that, by (28), (30), and considering (35), we have

$$\begin{aligned} v_b^d(.) &= m_r \left(1 - (1+n)\theta_r\right) (q/\pi)\phi^h, &(59)\\ v_b^s(.) &= \frac{m_r}{1 - (\delta+n)\theta_r\Delta} \left(1 - (1 - (1+n)\theta_r)\phi^y\right). &(60) \end{aligned}$$

By straightforward manipulations of the expressions (59), (60), we obtain

$$v_b^s - v_b^d = m_r \frac{(\delta+n)\theta_r + (1 - (1+n)\theta_r)((1-\delta)\theta_r\phi^y + \phi^o + (\delta+n)\theta_r\Delta\phi^h)}{(1 - (1-\delta)\theta_r)(1 - (\delta+n)\theta_r\Delta)} > 0.$$

□

C Comparative Statics

By (14), (35) and (36), and, using the equilibrium condition (29), we derive the following expression for the elasticity of b^* with respect to a:

$$\hat{b}_a = \frac{\hat{v}_a^d - \hat{v}_a^s}{\hat{v}_b^s - \hat{v}_b^d}. \tag{61}$$

By direct computation, the elasticities of the demand and supply of housing wealth with respect to a and b satisfy the relations

$$\hat{v}_a^d + \hat{v}_b^d = 1 + \Sigma^d, \qquad \hat{v}_a^s + \hat{v}_b^s = 1 + \Sigma^s, \tag{62}$$

with

$$\Sigma^d = \mu_h^L(1-\gamma)(1-\phi^h), \qquad \Sigma^s = \frac{\mu_h^L(1-\gamma)(1-(1-\delta)\theta_r)\phi^y + (\delta+n)\theta_r a\Delta_a}{1-(\delta+n)\theta_r\Delta}.$$

Then, using (62), we obtain

$$\hat{b}_a = 1 + \frac{\Sigma^d - \Sigma^s}{\hat{v}_b^s - \hat{v}_b^d}, \tag{63}$$

where

$$\begin{aligned}\Sigma^d - \Sigma^s = {} & \mu_h^L(1-\gamma)\left(\phi^o + \theta_r\phi^y\left(\frac{(1-\delta)-(\delta+n)\Delta}{1-(\delta+n)\theta_r\Delta}\right)\right) \\ & - (1-\mu_h^L)(1-\sigma_h)\frac{(\delta+n)\theta_r}{1-(\delta+n)\theta_r\Delta}(1+\Delta).\end{aligned} \tag{64}$$

From (41), (61) and (62) it follows that

$$\hat{v}_a - 1 = \Sigma^d + \frac{\hat{v}_b^d}{\hat{v}_b^s - \hat{v}_b^d}\left(\Sigma^d - \Sigma^s\right).$$

Then, $\Delta < (1-\delta)/(\delta+n)$, $(1-\mu_L^h)(1-\sigma_h) = 1$ imply $\hat{v}_a > 1$.