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Abstract

Many financial assets are disseminated to final investors via chains of over-the-counter transactions between intermediaries (dealers). We model such an intermediation process as a game with successive take-it-or-leave-it offers: An agent buying some units of an asset can offer to sell part of the volume to another OTC partner, and so on. At the equilibrium of this game, the length of intermediation chains, the terms of trade (prices and units sold) and their pattern along a chain are endogenously determined. Our model thus gives a framework to analyze how intermediation chains impact an asset's market liquidity, its issuance, and who ultimately holds the asset.

JEL Classification: C78, D85, G21 and G23
Keywords: dealer markets, intermediation chains, liquidity and otc markets

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1 Introduction

Many financial assets are sold by their issuer to intermediaries (e.g., dealers), who sell them either to customers, who typically hold them to maturity, or to other dealers. The asset gradually reaches final investors through chains of intermediaries on an over-the-counter market. The issued volume and the price depend on the asset's liquidity on this market, that is, on the availability and the resources of intermediaries and their access to final investors, either directly, or indirectly through intermediation chains.

Our objective in this paper is to analyze the formation of intermediation chains and their impact on asset origination. We provide a strategic model of the building up of chains based on the search for agents with cash that needs to be invested (for instance, on behalf of customers). We characterize equilibrium strategies, both for the origination of an asset and the terms of trade (prices and units sold), in a game with possibly several successive intermediaries. We thus jointly analyze the asset's market liquidity, intermediation, in particular the number of intermediaries involved in a chain of transactions, and the volume of the asset disseminated across this chain. We perform a comparative statics analysis on the asset return, the interest rate, and the funding liquidity of intermediaries, described by the distribution of cash.

We model the intermediation process as a game with successive take-it-or-leave-it offers. Importantly, intermediaries have incomplete information on the cash endowment of their partners. The game starts with an initiator who needs to finance the purchase of several units of an asset. He can borrow the needed amount from external financiers, or sell some of the units of the asset on an OTC market by making an offer to a trading partner. A partner who accepts an offer can finance the purchase with cash, borrow the amount needed, or sell some of the units to another partner, who again needs to finance his purchase. Transactions continue until an offer is turned down or until no new offer is made. At the end of the process, an intermediation chain has been formed in which assets are disseminated from one partner to the next, while cash "travels" in the opposite direction.

The equilibrium determines how the offers are chosen, accepted or refused, and how the intermediation chains depend on the initiator's financing needs. The initiator can make a tough offer by setting the unit price equal to the asset's fundamental value, so that only a partner with enough cash to cover the offer will accept it. He can instead choose an offer with a lower price and a higher volume and obtain the same amount of cash if the offer is accepted. Although the initiator's profit conditional on the offer being accepted is decreased because of the discount, the acceptance probability is increased: A partner whose cash endowment is not enough to cover the purchase may still accept the offer, as she can resell some of the units, typically at a higher price, to her own partner. A softer offer thus lowers the risk that the initiator will have to borrow and may be more profitable: this is the source of intermediation chains.

The optimal offer is characterized by a "target" partner, rich enough to be indifferent between accepting and rejecting the offer. The target strikes a balance between acceptance probability and profit conditional on acceptance. The same trade-off arises for the successive partners. The exact offer and target that an intermediary chooses depend on the offer he receives and on his financing needs, i.e., the amount of cash needed to finance the purchase. They also depend on how he expects his partner to behave, which, in turn, depends on the partner's expectations about her own partners, and so on. At equilibrium, these expectations must be correct. A difficulty is that the number of intermediaries involved and their needs are variable and a priori unknown, so that the game cannot be solved by backward induction. This problem is solved by showing that, ultimately, expectations bear on the total amount of cash (or funding) that an intermediary expects to be collected, up to his financing needs, from the chain of intermediaries. This equilibrium *collected cash*, which is a function of the financing needs, plays a crucial role. The function solves a fixed point equation entirely determined by the funding liquidity, specifically by the distribution of the cash endowment of each partner. Once the equilibrium collected cash function is known, an intermediary's optimal offer can be computed as a function of his financing needs, the asset's value, and the interest rate.

We derive a pattern for the successive optimal offers along a chain of intermediaries. As intermediaries' financing needs decrease at each step of the chain, the price tends to increase along the chain, while volume decreases: Each intermediary with positive financing needs proposes to sell at a discount, which decreases over time as the asset is sold to buy-and-hold investors, the time to convergence being endogenous.[1]

The impact of the asset's value and of the interest rate on the optimal offers can be analyzed. For instance, an increase in the interest rate leads intermediaries to sell more units of the asset at a lower price: Their partners face higher costs if they have to borrow, and thus ask for a larger discount when buying the asset. This results in more dissemination - the initiator needs to sell more units - a lower profit for the initiator, and a general increase in the profit of the subsequent intermediaries who have more cash than the target.

The average number of intermediaries along a chain depends on the initiator's financing needs and the distribution of cash among intermediaries. This distribution has a direct effect, since it affects the available liquidity, and an indirect effect, since it affects the chosen targets. The impact of the distribution is thus not straightforward, due to the strategic reaction of intermediaries at each level of the intermediation chain. We provide simulations to study this impact, showing that a distribution with a higher variance leads intermediaries to make offers with larger discounts and shortens intermediation chains. The asset return and the interest rate have no impact on the intermediation level, but they have an impact on volumes, hence on dissemination.

Asymmetric information on the partner's cash plays a crucial role in our model. Being privately informed about her cash endowment, a buyer who accepts an offer makes a positive profit in equilibrium, which corresponds to an informational rent. Because of this rent, it is optimal for an intermediary to make an offer that targets rich enough intermediaries only. As a result, an intermediary sometimes sees his offer rejected and needs to borrow in equilibrium, even though his partner has some cash. Total losses are equal (up to r) to the expected amount of cash that is thus "wasted" due to strategic behavior; we show that these losses depend only on the distribution of cash, and provide simulations showing that they increase with the variance of the distribution.

We finally consider the situation in which the initiator determines the volume of his initial purchase. This can model either a dealer who buys a volume from a large customer, the negotiation of a new issuance, or a bank securitizing mortgages. The presence of more liquidity in intermediation chains leads to a higher originated volume, which affects the

[1] The asset is thus temporarily illiquid due to a form of "slow-moving capital" phenomenon (Duffie (2010)).

entire chain of intermediaries through what we call an *origination effect*. A decrease in interest rates will spur more origination, so that intermediaries have larger quantities to sell and need to concede larger discounts.[2] This goes against the direct effect of a decrease in the interest rate, which is to increase the price offered by an intermediary for a given volume. Combining the two effects, different markets thus react differently to changes in the funding conditions of intermediaries, depending on how sensitive origination is to funding costs. Factors that increase the available liquidity in the OTC market allow the originator to issue a larger quantity; the total volume to be distributed increases and possibly leads to longer intermediation chains.

Our model captures some important stylized facts. First, securing a cheap access to funding has been an increasingly important determinant of profits for financial firms (see e.g. Hanson, Kashyap, and Stein (2011)). When cash is in scarce supply, reducing funding costs is a powerful motive to trade besides traditional ones such as diversification. Second, intermediaries in our model can be interpreted as dealers who buy the asset from other dealers, sell some units to their customers, and sell the rest to other dealers. The demand coming from his customers is each dealer's private information, which justifies our asymmetric information setting.[3] Third, intermediation chains are a well-documented feature of dealer markets. Li and Schuerhoff (2014) for instance study the market for municipal bonds in the U.S. and find chains of up to 7 dealers. Our framework could be used to analyze data on transactions that are not independent but are part of the same chain triggered by a large order, as in their paper. A typical example is the issuance of a new asset, which is gradually sold to end investors, typically over a few days or weeks.

The end of this section relates our work to the theoretical literature. Section 2 describes the model, Section 3 solves for the equilibrium, Section 4 derives predictions of the model for OTC trading, and Section 5 introduces endogenous origination.

[2] Newman and Rierson (2004) document the impact of new issuances on the liquidity of OTC markets and show that the magnitude of our origination effect can be large.

[3] As in Green (2007), the access to the final investors is a source of rents for intermediaries. However, in our model this rent is informational.

Related literature. Our approach to OTC markets is related to the literature that analyzes the terms of trade and the role of intermediaries in a decentralized market. Limited access to potential trading partners and asymmetric information constitute the main sources of departure from the centralized market paradigm. Two approaches have been followed to model limited market access in a tractable way, namely bilateral matching and networks.

Bilateral random matching with search, initiated by Rubinstein and Wolinsky (1987), is the foundation of recent models of OTC markets starting with Duffie, Garleanu, and Pedersen (2005). Traders meet randomly in pairs, bargain and either reach an agreement, or fail to do so and search for another partner. Applied to OTC markets, such an analysis sheds light on how the distribution of traders' valuations and option values affects the pricing and the liquidity of assets. Two recent papers in this literature, Hugonnier, Lester, and Weill (2014) and Shen, Wei, and Yan (2015), have introduced the possibility of intermediation chains by allowing traders to have a continuum of valuations for the asset traded. As a result, traders with intermediate valuations can both sell to high-valuation traders and buy from low-valuation traders, thus generating intermediation (see also Atkeson, Eisfeldt, and Weill (2015) for a model with endogenous intermediation). Afonso and Lagos (2015) study a similar environment in the context of money markets, and develop a model that can be calibrated using data on the US Fed funds market.

In our model, instead of engaging in Nash bargaining with symmetric information, traders make successive offers without knowing their partners' cash. As a result, trading inefficiencies arise in equilibrium. Furthermore, our environment is not stationary, as the traders' financing needs decrease along a chain. Gains from trade, which are proportional to the collected cash, thus endogenously decrease over time. Our model applies particularly well to certain environments such as interdealer trades around the issuance of a new asset and allow us to replicate some stylized facts (for instance, on prices), thus complementing the search literature for the study of these markets.

Networks also model restrictions to trade, as transactions are only possible between pairs of linked agents. Such a modeling is convenient to understand the impact of "local" monopoly power, held by an intermediary who is the only one to connect some agents (see for example Blume *et al.* (2009) in a two-sided context). Most papers study the trading of a single indivisible object. The main questions are whether the owner of this object is able to reach a buyer with whom exchange is beneficial, whether this buyer is the one with the highest valuation, and how much the intermediaries will extract from the sale. This approach is followed by Condorelli and Galeotti (2012), Manea (2013), or Gofman (2011) on an OTC market. We instead assume a perfectly divisible asset in order to study the dissemination of the asset across various intermediaries and their customers. Finally, Malamud and Rostek (2012) develop a very general model of decentralized trading and mainly study the implications on an asset's liquidity, but not the build-up of intermediation chains per se. Another important departure from these papers is that the volume of assets to be sold can be endogenously chosen by the initiator, which enables us to study the impact of the OTC market on investment.

Asymmetric information is also an important concern in a decentralized trading environment and has been recently introduced in search or network models. In contrast to our private-value environment, Glode and Opp (2014) study a common-value model à la Akerlof in which traders are heterogeneously informed about the value of the asset. In their model, intermediation chains mitigate an adverse selection problem as the overall informational gap between the first seller and the final buyer is "spread" along the chain. Our rationale for intermediation chains differs: Chains are formed to collect cash between intermediaries who share the same belief on the asset value but are privately informed on their cash holdings. The two models deliver opposite predictions regarding the role of informational asymmetries: In Glode and Opp (2014) informational asymmetries about the asset's value generate longer intermediation chains, whereas in our model informational asymmetry about intermediaries' endowments make chains shorter than would be optimal. We see the common-value model as applying better to assets more subject to asymmetric information such as stocks or some structured products, whereas our private-value model may fit bond and other fixed-income markets better.[4] In line with our model, Gabrieli and Georg (2014) report that the failure of Lehman Brothers and the subsequent increase in uncertainty led to shorter intermediation chains on the European interbank market.[5]

[4]The common value environment also differentiates our paper from Golosov, Lorenzoni, and Tsyvinski (2014), who introduce adverse selection and take-it-or-leave-it offers in a search model, and Zhu (2012), who studies strategic bargaining between a customer and two dealers.

[5]See also Di Maggio and Tahbaz-Alehi (2015) for a model of interbank chains with moral hazard in lending decisions.

2 The model

This section presents our modeling assumptions and how we define an equilibrium.

2.1 The game

The economy. We consider the trading of an asset that pays a sure return ρ per unit. There is a set of intermediaries denoted $\{D_n\}_{n\in\mathbb{N}^*}$. Each intermediary D_n has a cash endowment denoted ω_n, the ω_n being independently drawn from the same distribution $G(.)$. D_1 has the opportunity to buy v_0 units of the asset at a unit price p_0; if he decides to realize the purchase, he needs to find $\max(p_0 v_0 - \omega_1, 0)$ in cash. There is no centralized market for the asset: D_1 can trade only with D_2, who can also trade with D_3, etc.

In addition to their cash, whose return is normalized to zero, intermediaries can borrow any amount at the interest rate $r > 0$. r can be interpreted as the spread between the borrowing and the lending rates of a financial intermediary and measures (unmodelled) frictions in the financial sector.[6] Both ρ and r are exogenous parameters.

The OTC chain and the trading process. Trade takes place sequentially. Intermediary D_1 starts by making a take-it-or-leave-it offer (p, v) to intermediary D_2, where v is the volume and p the price per unit. If D_2 rejects the offer, the trading process stops and D_1 has to borrow at rate r to finance his investment. If D_2 accepts the offer, she can make a new take-it-or-leave-it offer to D_3, and so on. Accepting an offer is a commitment to buy, so that D_1 is sure to be paid: If D_3 rejects D_2's offer, then D_2 has to borrow at rate r to finance the purchase. Importantly, I_n has to answer I_{n-1}'s offer before knowing whether his own offer to I_{n+1} will be accepted. This models the fact that finding a counterparty takes time and intermediaries have to bear some risk of not being able to quickly offload their inventory.

At each step in the chain, there is a positive probability $1-q$ that the next intermediary is not available to trade, so that any offer has a maximum probability q of being accepted. The game continues until an offer is turned down, or until no new offer is made. Then all trades are settled, the asset's payoff is realized and loans are reimbursed.

Figure 1 summarizes the trading process and gives two examples of realized chains after

[6] See Heider, Hoerova, and Holthausen (2015) for a model of interbank spreads under asymmetric information.

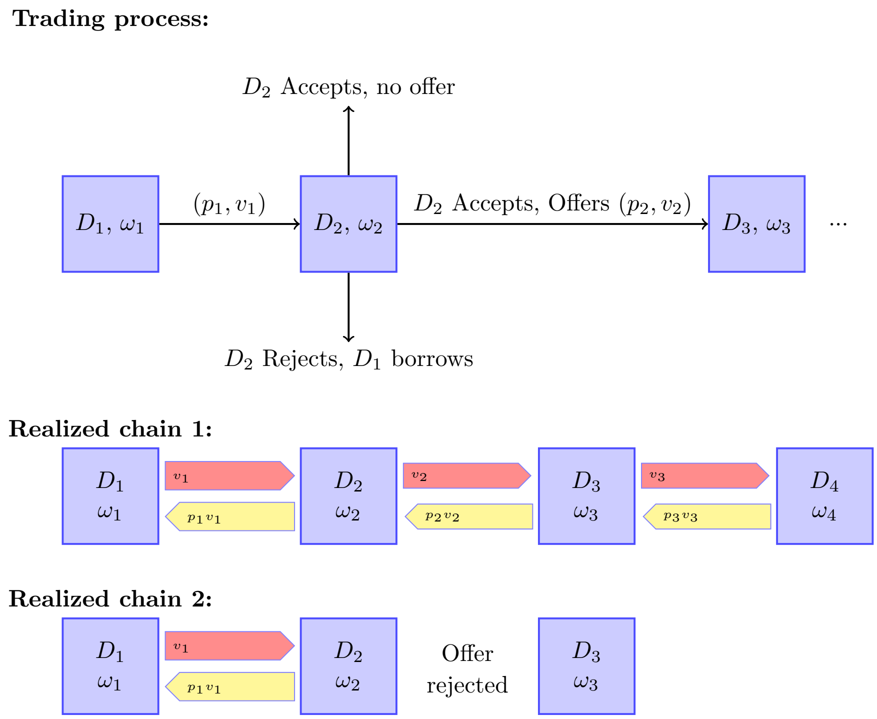

Figure 1: Trading process and two examples. The first part of the figure summarizes the trading process: Intermediary D_n makes an offer (p_n, v_n) to D_{n+1}, who either accepts or rejects, and can make a new offer in case of acceptance. The two chains below show two possible outcomes of this process. Red arrows represent transfers of assets, D_n sells v_n to D_{n+1}, while yellow arrows represent transfers of cash, as D_{n+1} gives $p_n v_n$ to D_n in exchange for v_n units of the asset.

D_1's offer. In the first example, all offers are accepted and the chain stops at D_4, who does not make a new offer (as we will show, this occurs if her cash is large enough to finance the received offer without borrowing); the length of the chain is 4. In the second example, D_2's offer is rejected, so that the length of the chain is 2.

Assumptions. All agents are risk-neutral and have unlimited liability. The return ρ, the interest rate r, the parameter q and the distribution G are common knowledge. The amount of cash ω_n is privately known by D_n. In particular, when making an offer, intermediary D_{n-1} does not know how much cash D_n has. Finally, we make the following technical assumption, standard in auction theory, on the distribution of cash G:[7]

(A1): G admits a continuously differentiable density g, with $g/(1-G)$ increasing.

Discussion. The model is kept quite general so as to accommodate a number of applications. Intermediary D_1 can for instance be a dealer who bought the asset from a customer and then

[7]Standard distributions (uniform, beta, or exponential) satisfy (A1).

sells it to other dealers. Each subsequent intermediary's ω can be interpreted as a demand for the asset coming from his customers, so that a dealer can buy assets for the amount ω without incurring funding costs. Intermediaries know the demand from their own customers, but not the demand faced by their partners, which generates an informational asymmetry. D_1 can also be an "originator", for instance a bank extending and securitizing loans, who chooses the initial volume k. This will be considered in Section 5. Finally, while we model the problem of a seller offering assets against cash, the case of a buyer offering cash against assets would be symmetric.[8]

The trading process we consider models two frictions. First, an intermediary can only contact a limited number of partners (here a single agent, but the analysis carries through to the case of a finite number) who have a finite amount of cash, making access to the chain of intermediaries beneficial. This friction is a fundamental feature of OTC markets: Each trader knows and trusts only a subset of the other market participants, which gives rise to the intermediation chains we are interested in.

Second, there is two-sided asymmetric information between any buyer and seller on their respective cash holdings and their availability. This is a private value model in which the gains from trade come from differences in funding costs and in available cash across different agents, and other elements like diversification or differences of opinion on the asset return are kept out of the model.[9] This second friction is a source of potential inefficiencies, as some opportunities might not be exploited. The optimal mechanism that an intermediary should use in this setup is an open question, but we show in Section 3.4 that no mechanism can achieve the first-best outcome. The take-it-or-leave-it protocol we consider, which is known to be optimal for the seller in the case of a single round and an indivisible object (Myerson (1981)), may not be optimal in our multi-unit and dynamic environment. However, it is a natural and tractable way to model OTC transactions while allowing for inefficiencies due to informational rents.[10]

[8] In that case uncertainty would bear on the asset endowment of intermediaries.

[9] Other sources of heterogeneity could lead to similar results, for instance if intermediaries faced different r. However, heterogeneity in cash endowments appears to be particularly tractable.

[10] The search literature typically adopts the diametrically opposite assumption that two traders immediately learn each other's types when they meet and bargain under full information.

2.2 Equilibrium

We now precisely define the strategies and the equilibrium of the game.

2.2.1 The intermediation decision

Consider an intermediary D, with a cash endowment ω, who receives an offer (p^-, v^-). D evaluates how much profit can be achieved by accepting the offer, and accepts if her profit is non-negative. D can make a new offer to finance the purchase, so that her expected profit depends on her belief that the offer is accepted, which will be endogenized in equilibrium.

Financing needs. Assume D accepts the offer (p^-, v^-). We define D's *financing needs* as:

$$y = \max(p^- v^- - \omega, 0). \tag{1}$$

If D makes no offer, she needs to borrow y and her profit is thus:

$$(\rho - p^-)v^- - ry. \tag{2}$$

Offers. Let D make an offer (p, v) to the next intermediary. D cannot sell more units than he buys, so that v has to be lower than v^-. If (p, v) is not accepted, D is in the same situation as if she had made no offer: y must be borrowed on the market at the cost ry, and the expected profit is as in (2). If (p, v) is accepted, the profit is equal to:

$$(\rho - p^-)v^- - (\rho - p)v - r \max(y - pv, 0). \tag{3}$$

$(\rho - p^-)v^-$ is the *net value* of the offer received by D. Symmetrically, $(\rho - p)v$ is the net value of the offer made by D. The last term corresponds to D's financing costs, taking into account that D's financing needs are decreased by pv. We call pv the *cash transfer* associated to the offer (p, v).

Beliefs on acceptance. An offer may be turned down because no partner is available or no available partner finds it attractive. An intermediary forms some belief $\Phi(p, v)$ that an offer (p, v) is accepted. As we will explain below, we focus on equilibria in which the function Φ is the same for every intermediary D_n. The probability of acceptance is surely bounded by the probability that at least one intermediary is available: $\Phi(p, v) \leq q$. We assume that Φ is decreasing in p (this will be satisfied at equilibrium). Finally, a natural property is that offers at a price larger than ρ are not accepted: we assume $\Phi(p, v) = 0$ for $p > \rho$.[11]

Profit and optimal behavior. Given the belief described by Φ, using (3) and rearranging, the intermediary D's expected profit from offer (p, v) is:

$$\begin{aligned} \pi_\phi(p^-, v^-, y; p, v) &= (\rho - p^-)v^- - ry + \Phi(p, v)Q(y; p, v), \text{ where} && (4) \\ Q(y; p, v) &= r \min(pv, y) - (\rho - p)v. && (5) \end{aligned}$$

Comparing (4) with (2), $Q(y; p, v)$ is the *transaction benefit* for D if the offer is accepted. Since the probability of acceptance is $\Phi(p, v)$, the expected gain from the offer (p, v) is equal to $\Phi(p, v)Q(y; p, v)$.

Upon the receipt of offer (p^-, v^-), intermediary D determines her optimal behavior in two steps. First, D chooses an optimal offer maximizing the expected transaction benefit over the feasible offers. Second, D accepts (p^-, v^-) if this optimal offer yields a non-negative profit. Formally, an *optimal offer* solves

$$\pi^*_\phi(p^-, v^-, y) = \max_{(p,v), v \leq v^-} \pi_\phi(p^-, v^-, y; p, v). \tag{6}$$

Offer (p^-, v^-) is thus accepted if the value $\pi^*_\phi(p^-, v^-, y)$ is non-negative.

At an optimal offer, D surely chooses a price between $\frac{\rho}{1+r}$ and ρ: For a price less than $\frac{\rho}{1+r}$, the transaction benefit is smaller than $-(\rho - p)v + rpv$, from (5), which is negative for $p < \frac{\rho}{1+r}$. For a price larger than ρ, the offer is never accepted. Thus D faces an intuitive trade-off: For a given volume, decreasing the price lowers the transaction benefit but also lowers the risk of refusal and the need for costly borrowing. The exact price solving this trade-off depends on the acceptance probability Φ.

Let us now show that D accepts the offer (p^-, v^-) if she has enough cash. First, if her financing needs are null, she surely makes a non-negative profit by accepting, as can be seen from (4) using $p^- \leq \rho$ and $y = 0$. Under assumption (A1), this is a possibility. Second, D's

[11] This assumption only rules out bubble equilibria, in which intermediaries always resell the asset at a higher price, resulting in an infinite sequence of trades with a price going to infinity.

profit is decreasing in y and thus increasing in D's cash ω. This implies the following lemma.

Lemma 1. *There is a threshold level for D's cash, denoted by $W_\Phi(p^-, v^-)$, above which D accepts the offer (p^-, v^-) :*

$$W_\Phi(p^-, v^-) = \inf\{\omega \text{ such that } \pi^*_\phi(p^-, v^-, max[p^-v^- - \omega, 0]) \geq 0\}. \tag{7}$$

Surely $W_\Phi(\frac{\rho}{1+r}, v^-) = 0$, $W_\Phi(p^-, v^-) \leq p^-v^-$ and $W_\Phi(\rho, v^-) = \rho v^-$.

The last statements are easily proved. They mean that D always accepts an offer at price $\frac{\rho}{1+r}$. D also accepts any offer if her financing needs are null, even at price ρ (and this is the only case in which such an offer is accepted). This last point uses the fact that there is a risk of refusal, i.e., that the partner is unavailable ($\Phi(p, v) \leq q$): From (4) and (5) the profit for $p^- = \rho$ is bounded by $-r(1-q)y$, which is negative.[12]

2.2.2 Equilibrium definition

The previous section derived the optimal behavior of an intermediary, say D_n, who anticipates an acceptance probability, say Φ_n. Equilibrium requires that this anticipation is correct for each intermediary, i.e., the receiver's optimal response indeed induces Φ_n. The receiver's optimal response depends, in turn, on Φ_{n+1}, and so on. In our model, the gains from trade come from differences in funding costs and in available cash across different agents. The knowledge of who originated the assets and how many units were issued does not bring relevant information to an intermediary who faces an offer. The setting is thus Markovian: The situation faced by an intermediary is entirely described by the received offer (p^-, v^-) and his endowment ω, but not by n. We look for a stationary equilibrium in which two intermediaries in the same situation behave identically. In such an equilibrium, $\Phi_n = \Phi$ for every n.

To make this precise, let us introduce the probability, denoted by $H(\omega)$, that an interme-

[12]If $q = 1$, not only the equality does not hold but also there is a simple equilibrium in which intermediaries always accept an offer and, if needed, make an offer at a price equal to ρ and a volume v that covers their needs, i.e. $\rho v = y$. Hence borrowing is completely avoided. Such an equilibrium does not exist whenever $q < 1$ and there is a risk of refusal.

diary is available and has more cash than ω:

$$H(\omega) = q(1 - G(\omega)). \tag{8}$$

Intermediary D facing offer (p^-, v^-) expects a receiver to accept an offer (p, v) with probability $\Phi(p, v)$. D accepts (p^-, v^-) if his cash is above the threshold $W_\Phi(p^-, v^-)$ defined by (7). Hence the probability that (p^-, v^-) is accepted is $H(W_\Phi(p^-, v^-))$. A stationary equilibrium requires that this probability is equal to $\Phi(p^-, v^-)$.

Definition 1. *An equilibrium is characterized by a threshold* W *and an acceptance probability* Φ *such that* $W(p^-, v^-) = W_\Phi(p^-, v^-)$ *and* $\Phi(p^-, v^-) = H(W(p^-, v^-))$ *for any* (p^-, v^-).

Given the equilibrium W (or Φ), the intermediaries' behavior is entirely determined: Given an offer (p^-, v^-), the cash level determines the financing needs y, an intermediary computes an optimal offer according to (6) and decides whether to accept the offer he received. It remains to find an equilibrium W.

3 Solving the game

Before solving the game, we consider a simple example that can be solved explicitly.

3.1 Example without asymmetric information

Assume that all intermediaries are endowed for sure with the same level ω^*: There is no asymmetric information. As a result, the game is simple to solve by induction. To simplify, let the financing needs y_1 of D_1 be a multiple of ω^*.

Consider $y_1 = \omega^*$. D_1 knows that D_2 has enough cash to cover his financing needs if she is available. Thus D_1 makes the best offer that covers his needs and is accepted by D_2, i.e., that provides her with a non-negative profit. This optimal offer has a price equal to ρ:

$$\rho v_1 = \omega^* \text{ and } \rho - p_1 = 0.$$

Surely $v_1 \leq v_0$ since $y_1 = \omega^* \leq p_0 v_0$. Thus D_1 collects ω^* and his transaction benefit is equal to $r\omega^*$. This benefit is equal to the surplus to share between D_1 and D_2. Thus, as is standard in a take-it-or-leave-it-offer game under symmetric information, the proposer extracts all the surplus.

Now consider $y_1 = 2\omega^*$. Again, D_1 makes an offer that is just accepted by D_2 if she is available. D_1 can make an offer with a cash transfer $pv = \omega^*$, as above, or $2\omega^*$. In the latter case, D_2 will make a new offer herself: She only has ω^* in cash, her financing needs are thus equal to ω^* and she is in the same situation as analyzed above. Playing optimally, she expects to collect ω^* with probability q. Knowing that, D_1's optimal offer (p_1, v_1) is characterized by the same two conditions, namely the offer covers D_1's financing needs and is just accepted by D_2:

$$p_1 v_1 = 2\omega^* \text{ and } (\rho - p_1)v_1 - r\omega^* + rq\omega^* = 0.$$

The corresponding offer is feasible, i.e., $v_1 \leq v_0$, provided that D_1 accepts (p_0, v_0) and indeed makes an offer. To see this, observe that D_1's profit is less than $(\rho - p_0)v_0 - (\rho - p_1)v_1$. Hence D_1 accepts (p_0, v_0) if the net value $(\rho - p_0)v_0$ of the offer she receives is larger than the net value $(\rho - p_1)v_1$ of the offer she makes. As furthermore $p_1 v_1 \leq p_0 v_0$, this implies $v_1 \leq v_0$.

When D_2 is available, D_1 collects $(1 + q)\omega^*$ and D_1's transaction benefit is equal to $r(1 + q)\omega^*$. Since this benefit is larger than $r\omega^*$, the benefit achieved by making an offer with a cash transfer ω^*, D_1 prefers the former. Accounting for the non-availability of D_2, D_1 expects to collect $q\omega^* + q^2\omega^*$ on average.

We can proceed by induction. If D_1's financing needs are $(n + 1)\omega^*$, his optimal offer is characterized by the same two conditions:[13] the offer covers D_1's financing needs and leaves no profit to D_2, whose needs are $n\omega^*$. D_1's expected transaction benefit is equal to r multiplied by $\Omega((n + 1)\omega^*)$, where

$$\Omega((n + 1)\omega^*) = q[1 + q + q^2 + ... + q^n]\omega^* \tag{9}$$

[13]They write $p_1 v_1 = (n + 1)\omega^*$ and $(\rho - p_1)v_1 - rn\omega^* + rq\omega^*[1 + q + ... + q^{n-1}] = 0$. This offer is feasible, i.e., $v_1 \leq v_0$, when D_1 accepts (p_0, v_0).

is the total expected cash along the chain of length $n+1$. The price is equal to

$$\frac{\rho}{1 + r\left[\frac{n - q(1+q+q^2+\ldots+q^{n-1})}{n+1}\right]}.$$

Notice that D_1's optimal offer depends on his financing needs, even though the probability of acceptance is always equal to q. Indeed, in equilibrium D_1 anticipates that each partner along the chain bears the risk of having to rely on costly borrowing. To be accepted, D_1's offer must compensate for the risk his partner bears, and for the discount she herself will have to offer to her partners, and so on. As a result, the offered price is decreasing in D_1's financing needs. Furthermore, along the chain, each intermediary offers a price that depends on her own needs, which are lower than the previous intermediary's. Hence prices increase along the chain.

3.2 Collected cash, targets and equilibrium offers

Auxiliary games. In the general case with asymmetric information, such an analysis by backward induction is not possible. We know that offers are accepted by intermediaries whose cash is above a certain threshold. However, the threshold levels, the number of intermediaries and the probability of acceptance are endogenously determined in equilibrium. Some features of the above example extend: An intermediary makes an optimal offer that just covers his financing needs and the optimal strategy can be interpreted as extracting cash along the chain of *targeted* intermediaries, those who make a null profit.

In order to solve the model, we introduce a sequence of auxiliary games. Define $\mathcal{G}_n(p_0, v_0, y)$ the game starting with an intermediary who accepted offer (p_0, v_0) and has financing needs y, with the restriction that the game stops after at most n offers are accepted. We solve the games $\mathcal{G}_n$ by backward induction, and we prove that the general game with an unbounded number of layers actually coincides with a game $\mathcal{G}_n$, for n high enough but finite. We sketch the reasoning in this section, the complete proof being in the Appendix B.1 and B.2.

Solving $\mathcal{G}_1$. Consider the game $\mathcal{G}_1(p_0, v_0, y)$ first. In this simple game, intermediary D makes an offer (p, v) to a receiver R with a cash endowment ω, who cannot make a new offer. R's reaction is easily determined and the game is solved by backward induction. R's profit if she accepts D's offer is $(\rho - p)v - r\max(y - \omega, 0)$. This profit is null at the threshold $W(p, v)$.

An important point is that, when the offer is accepted, D's transaction benefit $Q(y; p, v) = r\min(pv, y) - (\rho - p)v$ is exactly equal to $rW(p, v)$ if the cash transfer of D's offer is less than y and the threshold is positive, which is indeed the case at equilibrium. Thus, D's expected transaction benefit is equal to $rW(p, v)H(W(p, v))$, since the offer (p, v) is accepted with probability $H(W(p, v))$. As a result, maximizing D's profit is equivalent to maximizing $W(p, v)H(W(p, v))$ over offers with a cash transfer less than y.

Moreover, when D accepts the offer (p^-, v^-), there surely exists a feasible offer for which $W(p, v)$ is equal to the maximizer of $WH(W)$. This further simplifies the problem: The threshold at the optimal offer maximizes $\omega H(\omega)$ over $\omega \leq y$.

To summarize, the situation is as if D collected the cash endowment of the targeted type, and faced a trade-off between getting more cash in case of acceptance, and having a higher acceptance probability. The offer is derived by two conditions similar to the case of symmetric information: The offer covers D's financing needs and the profit of the targeted receiver is null. The solution of $\mathcal{G}_1(p_0, v_0, y)$ is thus characterized by:

$$\Omega_1(y) = \max_{0 \leq \omega \leq y} H(\omega)\omega; \; T_1(y) = arg \max_{0 \leq \omega \leq y} H(\omega)\omega. \tag{10}$$

We call $T_1(y)$ the *target* of D and $\Omega_1(y)$ his (expected) *collected cash*. Assumption (A1) ensures that $\omega H(\omega)$ is maximized at some positive ω^*. We have $T_1(y) = \min(y, \omega^*)$ and $\Omega_1(y) = T_1(y)H(T_1(y))$.

Iteration and conclusion. Knowing the solution of $\mathcal{G}_1(p_0, v_0, y)$, we can iterate and consider the game $\mathcal{G}_2(p_0, v_0, y)$. In this game, an intermediary D makes an offer (p, v) to an intermediary R with cash ω, who will play $\mathcal{G}_1(p, v, \max(pv - \omega, 0))$ if she accepts the offer. Since we know R's payoff in the latter game, we can solve $\mathcal{G}_2(p_0, v_0, y)$ by backward induction. More generally, we can iterate and solve for the target T_n and the collected cash Ω_n of a proposer in any of the games $\mathcal{G}_n$.

Finally, we can connect the auxiliary games $\mathcal{G}_n$ to the unbounded game described in Section 2. In the Appendix, we show that there exists N such that the unbounded game is equivalent to $\mathcal{G}_n(p_0, v_0, p_0v_0 - \omega_1)$ for $n \geq N$. The equilibrium of the unbounded game is then characterized by Ω_N and T_N, which we simply denote Ω and T.

Theorem 1. *The collected cash Ω and the target T are characterized by:*

$$\Omega(y) = \max_{\omega \leq y} H(\omega)(\omega + \Omega(y-\omega)) \tag{11}$$

$$T(y) = arg \max_{\omega \leq y} H(\omega)(\omega + \Omega(y-\omega)). \tag{12}$$

Ω and T do not depend on ρ and r. Ω is bounded above by some Ω^∞, increasing in y, and $\Omega(y) \leq qy$.

Let intermediary D face offer (p^-, v^-) and have positive financing needs y. D accepts the offer if and only if $\pi^*(p^-, v^-, y) = (\rho - p^-)v^- - ry + r\Omega(y) \geq 0$. In that case, D chooses a target ω in $T(y)$ and makes a new offer (p, v) that satisfies:

$$pv = y \text{ and } (\rho - p)v = r(y - \omega - \Omega(y-\omega)). \tag{13}$$

There exists $\underline{y}_1 \in (0, \omega^*)$ such that for $y < \underline{y}_1$ we have $\Omega(y) = \Omega_1(y)$ and $T(y) = y$; while for $y > \underline{y}_1$ we have $\Omega(y) > \Omega_1(y)$, $T(y) < y$ and $T(y) < \omega^*$. Furthermore, the financing needs of a targeted intermediary, $Z(y) = y - T(y)$, are weakly increasing in y.

The proof is given in the Appendix B.1 and B.2. Figure 3 shows plots of T and Ω in an example, as well as T_n and Ω_n for the first iterations.[14]

According to Theorem 1, the properties that are relevant to an intermediary are summarized in the collected cash Ω, which includes both the direct cash an intermediary draws from having access to one intermediary, and the indirect effect of this intermediary having access to other intermediaries, and so on. The target of an intermediary D with financing needs y maximizes $H(\omega)(\omega + \Omega(y-\omega))$. Choosing a higher target ω decreases the probability of acceptance $H(\omega)$, but increases D's transaction benefit in case of acceptance. The impact of a marginal increase in the target is decomposed into these two effects:

$$-\frac{g(\omega)}{1 - G(\omega)} + \frac{1 - \Omega'(y-\omega)}{(\omega + \Omega(y-\omega))}. \tag{14}$$

[14] All figures not in the text are in the Appendix A.

The first term reflects the relative decrease in the acceptance probability and the second term represents the relative gain in the transaction benefit when choosing a higher target. The marginal transaction benefit, the numerator $1 - \Omega'(y - \omega)$, is less than 1 because a targeted intermediary who has more cash collects less cash from her own partner. At the optimal target, expression (14) is null.

Finally, the values of T and Ω can be determined when financing needs are large:

Property 1. *The collected cash $\Omega(y)$ converges to a finite value Ω^∞ and the target $T(y)$ to ω^∞ as $y \to +\infty$, where Ω^∞, ω^∞ satisfy the joint equations:*[15]

$$\omega^\infty \textit{ solves } \max_\omega q(1 - G(\omega))(\omega + \Omega^\infty) \textit{ and } \Omega^\infty = \max_\omega q(1 - G(\omega))(\omega + \Omega^\infty). \tag{15}$$

Let us give an intuition for expression (15) that characterizes the upper bound Ω^∞ and the limit target ω^∞. Consider a hypothetical situation in which intermediary D expects the receiver's collected cash to be Ω^∞, independently of her offer. Then, according to the left hand side of (15), it is optimal for D to target ω^∞. According to the right hand side, the collected cash is also equal to Ω^∞. This situation is roughly the one faced by a proposer D who has very large financing needs, because the financing needs of D's partner are also large with a high probability. They remain large down the chain, but with a probability decreasing with the number of intermediaries. As $q < 1$, intermediaries further down the chain have a diminishing importance. This explains why Ω^∞ is the limit value for the collected cash. When y is finite, D cannot achieve Ω^∞ because the receivers' financing needs vary with their cash endowment and decrease along the chain, so that Ω^∞ is the maximum value of the collected cash.

Once Ω and T are known, D's optimal behavior is characterized by the second part of the theorem. To determine whether to accept the offer (p^-, v^-) when his financing needs y are positive, D computes the cash $\Omega(y)$ he can collect by making a further offer, derives the net expected financing costs, $y - \Omega(y)$, and accepts the offer if its net value $(\rho - p^-)v^-$ is larger than r times these costs. In that case, the optimal offer is accepted by the receiver if

[15] Put differently, Ω^∞ is the fixed point of the function $x \mapsto \max_\omega q(1 - G(\omega))(\omega + x)$, and ω^∞ is the maximizer of $(1 - G(\omega))(\omega + \Omega^\infty)$.

his cash endowment ω is larger than the target $T(y)$.

In case of acceptance, (13) and the target $T(y)$ determine D's equilibrium offer. The offer depends on the financial parameters ρ and r, although Ω and T do not, as will be analyzed in Section 4.2. Let us examine (13). The first equation requires that the offer exactly covers D's financing needs. The second equation states that the profit of a receiver R whose cash is exactly equal to the chosen target ω is null: The left hand side, $(\rho - p)v$ is the net value of the offer, whereas the right hand side, $r(y - \omega - \Omega(y - \omega))$, is equal to the expected financing costs of the next intermediary. Indeed, $y - \omega$ is the next intermediary's financing needs and she can collect $\Omega(y - \omega)$ in cash. Hence the nullity of R's profit says that the net value of the offer is equal to R's expected financing costs.

3.3 Intermediation chains

Given the intermediaries' cash levels, the sequence of offers, their acceptance or rejection are determined by repeated application of the theorem to the sequence of financing needs. Let us first illustrate this in the example depicted in Figure 2.

Example. Unless stated otherwise, all numerical exercises use the following baseline parameters: $\rho = 100$, $r = 0.05$, $q = 0.9$. For this example G is a Gamma distribution with parameters $k = 5$ and $\theta = 10$. D_1 can buy 5 units at a price of \$90 and needs to find \$450 to finance the purchase. As his profit is positive even with borrowing, equal to \$27.5 (500-450 (1+0.05)), D_1 surely accepts the offer. He makes an offer $(p_1, v_1) = (96.54, 4.67)$, which is accepted only if D_2's cash endowment is larger than 18.2, i.e., $T(450) = 18.2$. If the offer is accepted, D_1 keeps 0.33 units, hence makes a profit equal to \$33, which is of course larger than \$27.5. $\omega_2 = 50$, so that D_2 accepts the offer. She buys 4.67 units of the asset at price \$96.54 and thus gives exactly \$450 to D_1, but needs to find $450 - 50 = 400$. She makes a new offer $(p_2, v_2) = (96.71, 4.14)$, which is accepted if $\omega_3 \geq 16.4$, i.e. $T(400) = 16.7$. Since $\omega_3 = 200$, D_2's offer is accepted. D_2 keeps 0.53 units that she has bought for 50 (since she has bought for 450 and sold for 400) and thus makes a profit of \$3. If her offer is refused instead, that is if D_3 has less then 16.4, then D_2 makes a loss: She bought 4.67 units at a total price of 450, hence with a net value of 17, but she incurs a borrowing cost of \$20 (400×0.05). The remaining steps are obtained in a similar way. Since D_3's financing needs are equal to 200 and D_4 has $\omega_4 = 250$, D_4 accepts D_3's offer and does not need to sell any unit of the asset, so the chain stops. In the end $\omega_1 + \omega_2 + \omega_3 + \omega_4 \geq 450$, which is enough to finance the initial purchase of D_1.

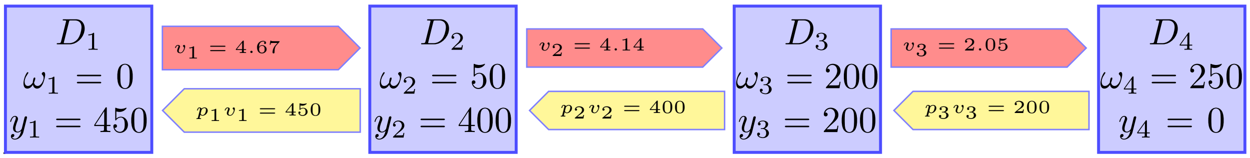

Figure 2: First layers of an intermediation chain. Example.

Targeted chains. Starting with an intermediary D with financing needs y, the previous example extends to define the *realized chain* given the cash levels. A particularly interesting case of chain is when the receiver's ω is equal to the target at each chain. We call it the *targeted chain*, which we define as follows: D targets an intermediary with cash $T(y)$ and financing needs $Z(y)$. This targeted intermediary chooses a target $T(Z(y))$, and this target will have financing needs $Z(y) - T(Z(y)) = Z^2(y)$. We obtain a sequence $Z^j(y)$ until the smallest integer n such that $Z^n(y) = 0$.[16] The length of this chain, n, is denoted by $N(y)$. $N(y)$ is non-decreasing because Z is non-decreasing in y.[17]

When $y \to +\infty$, we know that $T(y)$ converges (Property 1) and successive intermediaries along a chain thus approximately make the same offers. To study this asymptotic behavior, we consider the following *simple process* associated with a fixed value ω. The process starts with D_1 and stops as soon as an intermediary is inactive or has a cash endowment below ω. The process thus stops at each round with a constant probability $\mu = q(1 - G(\omega))$. The expected length of a chain following this process is $\ell_\omega = \mu + \mu^2 + \mu^3 + \cdots = \frac{\mu}{1-\mu}$, or

$$\ell_\omega = \frac{q(1 - G(\omega))}{1 - q(1 - G(\omega))}. \tag{16}$$

There are two differences relative to the equilibrium process: The simple process does not stop when the sum of cash endowments reaches a value y, and the target is fixed at ω.

[16] Or, equivalently, $Z^{n-1}(y) \leq \underline{y}_1$. Such an integer exists because there is a positive lower bound to the threshold when it is positive, that is for $y \leq \underline{y}_1$. See the proof of Lemma B.1 where the threshold is shown to be above $\omega_{min} = H(\underline{y}_1)\underline{y}_1(1 - H(0))/H(0)$.

[17] Moreover, one can obtain a lower bound on N using that $T(y) \leq \omega^*$ (see Theorem 1): The financing needs of an intermediary along the targeted chain decrease by at most ω^* at each step. To reach financing needs of $\underline{y}_1$ starting with some y, one needs $N(y)$ to be larger than $\lceil (y - \underline{y}_1)/\omega^* \rceil + 1$.

Heuristically, when the financing needs are large, these differences vanish and the equilibrium process is close to the simple process associated with ω^∞. More formally:

Property 2. *As y tends to $+\infty$, we have $\Omega^\infty = \omega^\infty \times \ell_{\omega^\infty}$ and $N(y) \sim \frac{y}{\omega^\infty}$.*

We compute the formula for Ω^∞ from (15) and (16). The intuition is as follows: for large financing needs the target is roughly equal to ω^∞ so that the collected amount is roughly equal to ω^∞ multiplied by the average length of the chain in the simple process associated with ω^∞. As for the limit of $N(y)$, we know from Property 1 that $T(y)$ converges to ω^∞. When y is large, the length of the targeted chain increases by one unit for every increment of y of length ω^∞. As a result, $N(y)$ goes to infinity and is asymptotically equivalent to $\frac{y}{\omega^\infty}$.

Realized chains differ from targeted chains for two reasons. First, some offers are not accepted, either because the receiver is inactive or because she lacks cash. Second, a receiver accepting an offer (almost surely) has more cash than the target, and hence lower financing needs. $N(y)$ is thus an upper bound on the length of realized chains. Moreover, the expected length of a realized chain in necessarily lower than $\ell_0 = \frac{q}{1-q}$.

3.4 Trading inefficiencies and rents

There are foregone opportunities to use available cash when an offer is turned down. Asymmetric information - more precisely, the lack of information on the partner's cash - is at the root of this trading inefficiency. Conversely, receivers who are endowed with more cash than the target receive a positive profit, an informational rent. To assess the impact of asymmetric information, this section introduces a full information benchmark, keeping the same sequential trading constraint. Then we study the equilibrium rents and measure inefficiencies by comparing total profits in equilibrium and in the full-information benchmark.

Total profits and gains from trade. Consider a realized chain starting with D_1 and ending with D_n. Denote by y_j and π_j the financing needs and profit of D_j. There are two cases to consider for D_n: either $y_n = 0$ and D_n completes the volume, or $y_n > 0$ and D_n makes a new offer, which is rejected; in both cases D_n's profit writes $\pi_n = (\rho - p_{n-1})v_{n-1} - ry_n$.

We compute the total profits $\sum_{j=1}^{n} \pi_j$. Consider $j \in [1, n-1]$. Using equation (3), the profit of D_j is equal to $\pi_j = (\rho - p_{j-1})v_{j-1} - (\rho - p_j)v_j$. Summing from $j = 1$ up to

$n-1$ and adding π_n, the total profits in the chain are thus equal to $(\rho - p_0)v_0 - ry_n$. The financing needs for D_j, $j < n$, are surely positive and satisfy $y_j = y_{j-1} - \omega_j$. We thus have $y_n = max(y_1 - \sum_{j=2}^{n} \omega_j, 0)$ or, equivalently $y_n = y_1 - \min(y_1, \sum_{j=2}^{n} \omega_j)$. We can write total profits as:

$$\sum_{j=1}^{n} \pi_j = (\rho - p_0)v_0 - ry_1 + r \times \min\left(y_1, \sum_{j=2}^{n} \omega_j\right) \tag{17}$$

The total profits in the chain are equal to the profits that D_1 would make by borrowing if his financing needs were y_n, or equivalently if he had the total $\sum_{j=1}^{n} \omega_j$ in cash. The total gains from trade in the chain can be measured by $r \times \min\left(y_1, \sum_{j=2}^{n} \omega_j\right)$, which is the difference between total profits and the profit D_1 would achieve on his own. The gains from trade are thus equal to r times the total cash collected in the chain, with a cap equal to the first intermediary's financing needs.

Full-information benchmark. To measure inefficiencies, we compare the total gains from trade achieved in the game to what would be obtained under full information. In this case, the offer of D_j always makes D_{j+1} exactly indifferent between accepting and rejecting, so that each offer has a probability q of being accepted and the total cash drawn from the chain is maximized. The expected gains from trade are equal to the expectation of $r \times \min\left(y_1, \sum_{j=2}^{n} \omega_j\right)$. They are given by $r\Omega_F(y_1)$, with Ω_F defined recursively by:

$$\Omega_F(y) = q\left[y(1 - G(y)) + \int_0^y (\omega + \Omega_F(y - \omega))dG(\omega)\right], \text{ and } \Omega_F(0) = 0. \tag{18}$$

Expression (18) computes the maximum total cash (capped by y) that can be collected in the chain starting with an intermediary D with financing needs y. With probability $1 - q$ the chain ends with D and the amount collected is zero. With probability $q(1 - G(y))$, the next intermediary has a cash endowment $\omega \geq y$, so that the maximal amount y is collected. Then, for all values $\omega \in [0, y]$, the next intermediary will provide his own cash ω, plus the cash $\Omega_F(y - \omega)$ that he can indirectly collect from further intermediaries.

Equilibrium inefficiencies. Under asymmetric information, the situation is different as an intermediary does not capture the entire gains from trade in the chain below him, and thus fails to internalize the following intermediaries' profits. Consider an intermediary D

with financing needs y, making an offer (p, v) corresponding to a target $T(y)$. If she accepts the offer, a receiver R with cash ω makes a profit $\pi_R(\omega) = (\rho - p)v - rz + r\Omega(z)$, where $z = \max(y - \omega, 0)$ are R's financing needs. Using equation (13) that defines the target, we can rewrite π_R as:

$$\pi_R(\omega) = r[\omega - T(y) + \Omega(y - \omega)) - \Omega(y - T(y))] \text{ for } \omega \leq y. \tag{19}$$

By definition, $\pi_R(T(y)) = 0$: The offer (p, v) exactly compensates the target. Then, each marginal unit of cash brings an additional profit of $r(1 - \Omega'(y - \omega))$ at the margin, as long as $\omega \leq y$. Thus, due to asymmetric information, cash provides an informational rent for those intermediaries who receive an offer and have more than the targeted level. In equilibrium, the presence of this rent introduces a wedge between an intermediary's maximization program and the efficient full information benchmark. Denoting $\Omega_E(y)$ the expected total cash collected in equilibrium, starting with financing needs y, we have a recursive definition close to formula (18):

$$\Omega_E(y) = q\left[y(1 - G(y)) + \int_{T(y)}^{y} (\omega + \Omega_E(y - \omega))dG(\omega)\right], \text{ and } \Omega_E(0) = 0. \tag{20}$$

The comparison with (18) identifies the source of the inefficiency: To implement the efficient outcome, each offer should be accepted with a probability q. However, at each level in the chain, an intermediary with financing needs y uses a target $T(y) > 0$, so that offers are sometimes rejected, which implies a "waste" of cash. The total costs generated by this inefficiency are given by $r(\Omega_F(y) - \Omega_E(y))$. Equation (12) shows that the target $T(y)$ does not depend on r, and inspecting (20) and (18) shows that Ω_E and Ω_F do not depend on r either. The total costs are thus made of the difference $\Omega_F(y) - \Omega_E(y)$ which is entirely determined by q and G, multiplied by r, which acts as a shadow price of cash.

The limits of $\Omega_F(y)$ and $\Omega_E(y)$ for $y \to +\infty$ are related to the expected length of the chain in each case. Starting with an intermediary with financing needs y, the realized chain depends on the realized cash endowments of successive intermediaries. The expected length of this chain, denoted $L_E(y)$, satisfies the recursive expression:

$$L_E(y) = q(1 - G(T(y)) + q\int_{T(y)}^{y} L_E(y - \omega)dG(\omega), \text{ and } L_E(0) = 0. \tag{21}$$

In the full-information case, the expected length $L_F(y)$ of a chain satisfies the same formula, replacing $T(y)$ with 0. This directly implies that $L_F(y) \geq L_E(y)$: equilibrium chains stop earlier than in the full information benchmark because offers are rejected with a non-zero probability. We can compute the limits of $L_F(y)$ and $\Omega_F(y)$ for large values of y by using the simple process associated with 0. Similarly, the limits of $L_E(y)$ and $\Omega_E(y)$ are obtained by using the simple process associated with ω^∞.

Property 3. *We have the following limits for the expected chain length and collected cash in the full information benchmark and in equilibrium, respectively:*

$$\lim_{y \to +\infty} L_F(y) = \ell_0 = \frac{q}{1-q} \text{ and } \lim_{y \to +\infty} \Omega_F(y) = \ell_0 \mathbb{E}[\omega],$$

$$\lim_{y \to +\infty} L_E(y) = \ell_{\omega^\infty} \text{ and } \lim_{y \to +\infty} \Omega_E(y) = \ell_{\omega^\infty} \mathbb{E}[\omega | \omega \geq \omega^\infty].$$

See the Appendix C for the proof. In both cases, the limit of the total collected cash is equal to the expected length of the chain times the expected cash at each level of the chain. The expected lengths are bounded, even though the targeted chains are not (Property 2). In the full information case, the latter quantity is simply the mean of the distribution G. In the equilibrium case, this quantity is the expected cash conditional on being above ω^∞. Here the shape of the distribution G matters.

The extent of asymmetric information depends on how concentrated the distribution of G is. To illustrate, we consider the example of a normal distribution G with parameters 0.5 and σ, truncated at zero, with σ between 0.01 and 0.2. When σ is equal to 0.2, an intermediary with large financing needs chooses $\omega^\infty = 0.21$; a low ω^∞ is optimal because many types of intermediaries have low cash endowments. As σ decreases, there are two effects: for a given ω^∞, there is a higher proportion of types above this level, which increases the total collected cash; ω^∞ increases as intermediaries adapt their behavior to the new distribution, which goes in the other direction. Fig. 4 plots the limits of $\Omega_F(y)$ and $\Omega_E(y)$ and shows that the first effect dominates. When $\sigma = 0.2$, almost half of the cash that can be collected in the network is lost due to the strategic behavior of intermediaries. As the variance decreases, the game becomes close to the symmetric information case. For $\sigma = 0.01$ we have a high value of ω^∞ (0.47), but since more than 99.9% of intermediaries are above this threshold the inefficiency approaches zero.

Remark on the trading protocol. Given the full-information benchmark, we can show that no trading protocol would allow to implement the efficient solution. Consider an intermediary D with financing needs y. Any trading protocol he can use can be described by functions $p(.)$ and $v(.)$ so that a receiver R of type ω would select the offer $(p(\omega), v(\omega))$. The efficient solution requires that $p(\omega)v(\omega) = y$ for any ω. By contradiction, assume $p(\omega)v(\omega) < y$ for a given ω. D has to borrow at rate r, even though there is a non-zero probability that the sum of cash endowments in the chain starting with R is larger than y, which is inefficient. Conversely, if $p(\omega)v(\omega) > y$, there is a non-zero probability that R does not have an active partner and needs to borrow $p(\omega)v(\omega) - \omega$ instead of $y - \omega$, which is inefficient. Thus, necessarily $p(\omega)v(\omega) = y$. R then faces a menu of offers that all imply the same payment y to be made to D, but R receives different volumes $v(\omega)$ of the asset in exchange. The only way to obtain R to truthfully reveal her ω is thus to have $v(\omega)$ constant. Thus both p and v are the same for all types, and we are back to our take-it-or-leave it mechanism, which is inefficient. Inefficiencies are thus a necessary feature of the environment we consider.

4 Implications on OTC trading and dealer networks

We first derive the implications of our model on trading dynamics and compare some of our results to the empirical literature on dealer networks. Then we perform comparative statics exercises and analyze the impact of the financial parameters of the model, ρ and r, and the funding liquidity, as described by the probability q to find a counterpart and the distribution of cash G. The analysis substantially differs in the two situations since the collected cash Ω and the target T do not depend on ρ and r. This has important implications in particular on the length of the chains.

4.1 Prices and volumes

Once the equilibrium collected cash Ω is known, the price and volume offered by each intermediary, given his financing needs, can be computed from (13) in Theorem 1:

Implication 1. *Optimal offers are given by*

$$P(y) = \frac{\rho}{1 + r\kappa(y)},\ V(y) = \frac{y(1 + r\kappa(y))}{\rho}, \tag{22}$$

$$\text{with } \kappa(y) = \frac{Z(y) - \Omega(Z(y))}{y}. \tag{23}$$

V is weakly increasing in y. For $y \leq \underline{y}_1$, $P(y) = \rho$ and $V(y) = y/\rho$, and

$$\lim_{y \to +\infty} P(y) = \frac{\rho}{1 + r},\ V(y) \sim \frac{(1 + r)y}{\rho}.$$

When the intermediary has small financing needs, smaller than $\underline{y}_1$, the target is y and the price is the maximum price ρ; in that case the game ends after this offer. Instead, for large enough financing needs, the price becomes close to its minimum value $\frac{\rho}{1+r}$. In between, prices and volumes depend on $\kappa(y)$, which can be interpreted as a *bargaining ratio*, as it is a measure of the relative positions of the two parties: $r\kappa(y)$ is the ratio of the expected financing costs of the target due to borrowing, $r(Z(y) - \Omega(Z(y)))$, relative to the proposer's financing needs y. Since an offer satisfies $pv = y$, the ratio determines how 'tough' the offer is: When the ratio is low, the proposer can make a tough offer with a high price and a small volume; instead, when the ratio is high, the price has to be low, hence the volume to be high, for the partner to accept the offer.

The impact of an increase in y on volume is always positive. The impact on the offered price is more ambiguous and depends on whether κ is increasing. This occurs if the expected financing costs of the target increases more than the financing needs, which in turn depends on the target's choice. When the target is fixed, as in the case without asymmetric information, the bargaining ratio increases. This may not always be true in the general case, so that P may not be decreasing for all y, although this does not occur in the simulations we have performed (see below).

Notice that, starting with D_1, the financing needs y decrease at each step in the chain. In a very long chain, the prices offered at the beginning of the game are close to $\rho/(1 + r)$, while at the end they are closer to ρ. In other words, the illiquidity of the asset, measured by $\rho - P(y)$, tends to decrease over time as the financing needs of intermediaries and traded volumes become smaller. This is illustrated in the bottom panel of Fig. 6, which plots $P(y)$

for different distributions. The bottom panel of Fig. 7 shows how the behavior of $P(y)$ translates into prices that increase along an intermediation chain. Starting with D_1 who has financing needs $y_1 = 3$, we simulate the model $5,000$ times. We report the price p_1, the average price p_2 across all chains with two intermediaries or more, the average price p_3 across all chains with 3 intermediaries or more, etc.

The tendency of prices to come closer to the fundamental value of the asset over time while volumes become lower matches a stylized fact from the empirical literature. Green, Hollifield, and Schuerhoff (2007) for instance study the interdealer transactions on municipal bonds and document that the bonds are traded at a discount in the first five days after issuance, a mispricing that decreases over time while interdealer transactions and volumes simultaneously decrease. In our model, this is explained by the fact that the asset is gradually redistributed to final investors, so that intermediaries have lower inventories and financing needs as this process goes on. An asset is thus typically underpriced at issuance, and the price mean-reverts afterwards (see, e.g., Bao, Pan, and Wang (2011)).

Li and Schuerhoff (2014) show the existence of intermediation chains including up to 7 dealers with an average length of 1.5, which matches the typical lengths of 1-3 we obtain in simulations. Interestingly, they find that the costs paid by the dealers' customers increase with the length of the chain, a result that can be obtained in our model. Since the length of the targeted chain $N(y)$ increases in y, a long chain can be obtained in our model only when the financing needs of the first intermediary are large. An intermediary who needs to use a long intermediation chain has to compensate the other intermediaries in the chain through an important discount (a low $P(y)$), which decrease his profit. A dealer whose trade with a customer generates large financing needs will naturally pass on part of the associated costs to the customer.

4.2 The impact of ρ and r

This section studies how prices, volumes and profits are affected when the parameters ρ and r vary. As shown in Theorem 1, neither T nor Ω depend on these parameters. They thus leave D_1's target unaffected. Furthermore, D_1's partner's financing needs are unaffected as well, since the cash transfer of D_1's offer, equal to his financing needs, is unchanged. The same property extends to all the intermediaries in a realized chain: Given the realized cash levels, the targets and the financing needs are not affected by ρ and r. However, these financial parameters affect the offers, in particular they affect the breakdown between the price $P(y)$ and the volume $V(y)$, as can be seen from Implication 1. They also affect D_1's profit, hence whether he accepts the offer he receives, as well as the following intermediaries' profits (except for targeted intermediaries, whose profits are null by construction). Assuming that D_1 finds it profitable to make an offer under the parameters considered, we easily derive the following results:

Implication 2. *For given financing needs of D_1, the targets, the length of the targeted chain, the acceptance probability of the offers made and the financing needs of all intermediaries following D_1 are unaffected by ρ and r. An increase in ρ or a decrease in r lead to higher prices and lower volumes at each step of the intermediation chain.*

To understand the impact of ρ on the offers, consider two assets with different values for ρ, denoted ρ_A and ρ_B, with $\rho_B > \rho_A$. Consider two intermediaries D_A and D_B who have the same financing needs y and hold asset A or B. They target the same $T(y)$ by making an offer $P(y, \rho_A), V(y, \rho_A)$ or $P(y, \rho_B), V(y, \rho_B)$. One easily checks that

$$0 < P(y, \rho_B) - P(y, \rho_A) < \rho_B - \rho_A \text{ and } V(y, \rho_B) < V(y, \rho_A).$$

In both situations, the receiver accepts the offer if her cash is above $T(y)$, and she will have the same financing needs $y - \omega$. The receiver will thus select the same target $T(y - \omega)$, using either the offer $(P(y - \omega, \rho_A), V(y - \omega, \rho_A))$ or $(P(y - \omega, \rho_B), V(y - \omega, \rho_B))$. These offers satisfy the same properties as above, and so on along the chain.

Even though the cash transfer is the same in both cases, the price and the volume differ. Intermediaries sell asset B at a higher price, and sell a lower volume. Indeed, recall that they choose a price and a quantity such that the cash transfer covers their financing needs in case of acceptance. As asset B is more valuable, the receiver of an offer is ready to accept a higher price. It is thus optimal to adjust to the higher value of the asset by raising the price and decreasing the quantity. However, the price increase is moderate enough to keep the probability that the offer is accepted unchanged. As a result, the discount is increased.

The same reasoning applies to the interest rate r. Given an asset and financing needs, an increase in r leads to offers with a lower price: The receiver loses more on each unit she buys if her own offer is not accepted, hence she requires a lower price to accept. The proposer then optimally chooses to sell more units at a lower price.

These comparative statics results give interesting implications on asset prices and traded volumes along intermediation chains, for instance in interdealer markets. When r increases and accessing funding is more costly, intermediaries have to concede larger discounts and to sell more units to their partners. The price of the asset decreases and volumes are higher at each level of the chain. The total trading volume thus increases. These effects are typically more important at the beginning, when $\kappa(y)$ is high, than at the end of the chain.

Let us now consider profits. An increase in r leads the first intermediary D_1 to sell more units at a lower price, which implies a loss for D_1 and a gain for the next intermediary D_2. D_2 also has to sell at a lower price when she makes an offer to D_3, but the price she receives from D_1 precisely compensates her for this loss. As a result, we have the following asymmetry between D_1 and the subsequent intermediaries along the chain:

Implication 3. *D_1's profit is decreasing in the interest rate r and the subsequent intermediaries' profits are linearly increasing in r.*

The implication follows directly from the expression (19) for the profit, which is simply proportional to r. As we have just seen, the offer is adjusted to an increase in r so as to compensate the targeted intermediary for the increase in her funding costs. Any additional unit of cash above the target is then rewarded at the interest rate r. The reasoning extends to all the subsequent layers. Ultimately, cash provides a rent for those intermediaries who receive an offer and have more than the targeted level, and this rent increases with r.[18] For D_1, who starts with a given y, an increase in r only affects his funding cost and reduces his profit.

Intermediaries' profits can be observed empirically. Li and Schuerhoff (2014) observe that on dealer markets the dealers at the end of an intermediation chain make more profits than those at the beginning. Interestingly, this is a natural feature of our model, since we

[18]Cash, the rare resource in our model, can also be interpreted as the demand coming from a dealer's customers. See Green (2007) for a model in which a dealer's capacity is a source of profits.

do not assume that the "last" intermediaries are in any way different from the first ones. Endogenously, some chains end with an intermediary who has enough cash to cover the entire offer. Since, in our model, profit comes from a rent earned on cash above the target, such an intermediary completing a chain receives the maximal rent.

4.3 Cash distribution and chain length

The availability of a partner, as reflected by q, and the cash she might have, as reflected by the distribution G, influence behaviors and the length of intermediation chains. In contrast with the financial parameters ρ and r considered in the previous section, the collected cash and target functions Ω and T do depend on q and G. As a result, the analysis is more complex and possibly ambiguous.

Consider an increase in the probability q that an intermediary is active. There are two effects, a direct effect and an indirect, strategic, effect. The direct effect arises for a fixed T, i.e., in the absence of any change in the intermediaries' strategy: As q increases, a chain is less likely to stop and chains are on average longer. However, because of the higher probability of being accepted, intermediaries may have an incentive to choose a higher target, a strategic effect which goes in the other direction. To understand this strategic effect, consider two target policies T and T' with $T \geq T'$. In that case, for each realization of cash levels, the length of the chain can only be smaller for T than for T' because a refusal can only happen earlier. The reason is that, as long as there is no refusal, the intermediaries' financing needs are unchanged under the two processes: They are equal to $y_2 = y - \omega_2$ for D_2, $y_3 = y - \omega_2 - \omega_3$ for D_3, and so on. The first step, say n, at which the processes might differ arises because D_n's cash is below the threshold T but not below T': $T(y_n) > \omega_n > T'(y_n)$. Thus, the length of the intermediation chain under T is equal or smaller than under T'.

Both effects may not go in the same direction, but we can predict the impact of q for extreme values of y:[19]

[19]The combination of the two effects would be unambiguous in general if the target T were decreasing in q. Denoting T_q the target function associated to a particular q, the fist order equation derived from (14) gives

$$-\frac{g(\omega)}{1 - G(\omega)} + \frac{1 - \Omega_q'(y - \omega)}{(\omega + \Omega_q(y - \omega))} = 0 \text{ at } \omega = T_q(y). \tag{24}$$

Implication 4. *ω^∞ decreases in q, so that when $y \to \infty$ the limit ℓ_{ω^∞} of the average chain length $L_E(y)$ increases in q.*

$\underline{y}_1$ decreases in q, so that for any $y \leq \underline{y}_1$ the average chain length $L_E(y)$ increases in q.

Differentiating condition (15) shows that ω^∞ decreases in q. Using equation (16), we see that a larger q leads to a larger expected chain length for a given target. Since the target is lower, both effects go in the direction of longer chains. We thus obtain that a larger q increases the average chain length when y is large enough. Similarly, $\underline{y}_1$ solves the equation $yg(y)/(1 - G(y)) = 1 - q$, and implicit differentiation directly shows that $\underline{y}_1$ decreases in q. Starting with y close to but below $\underline{y}_1$, an increase in q typically implies that an intermediary will target a chain of length 2 instead of 1.

We now consider the impact on the offers. From (23), the price is increasing in q if the bargaining ratio $\kappa(y)$ is decreasing, which is not necessarily true. Fig. 5 gives an example with different values for q. In particular, we observe that higher values of q lead to higher prices for large financing needs y and to a smaller price for smaller financing needs. We now turn to the impact of the distribution G. The simplest comparative statics exercise is to scale up or down the cash of all intermediaries. We define a scale parameter θ and the new distribution $G_\theta(\omega)$ such that $G_\theta(\theta\omega) = G(\omega)$. G_θ represents a situation in which intermediaries at all points of the distribution G see their cash endowments multiplied by θ. Writing that $G_\theta(\omega) = G(\omega/\theta)$ and $g_\theta(\omega) = g(\omega/\theta)/\theta$, we can easily solve the new game and find the functions Ω_θ and T_θ. Indeed, it is readily checked that if Ω and T satisfy equations (11) and (12), then $\Omega_\theta(y) = \theta\Omega(y/\theta)$ and $T_\theta(y) = \theta T(y/\theta)$ satisfy the same conditions. Using the formulas of Implication 1, it is straightforward to show that $\kappa_\theta(\theta y) = \kappa(y)$, so that we have $P_\theta(y) = P(y/\theta)$ and $V_\theta(y) = \theta V(y/\theta)$. This gives the following result:

Implication 5. *If $P_\theta(y)$ is decreasing in y, then the price $P_\theta(y)$ increases and the volume $P_\theta(y)$ decreases in θ.*

When P is decreasing, injecting more cash in the economy leads to the intuitive result that for given financing needs the price will increase and be closer to the fundamental value

Only the second term depends on q. While it is easy to show that Ω_q increases in q, Ω'_q may be decreasing or increasing in q, hence the ambiguity.

of the asset. Since $P(y)V(y) = y$, the offered volume will decrease. Choi and Shachar (2013) show that the lower demand for corporate bonds in 2007-2009 led dealers to accumulate larger inventories and trade at lower prices. In line with this result, in our model an intermediary who accepts an offer when demand is low knows that she has a high probability of keeping the assets and thus asks for a larger discount.

Finally, we consider the impact of increasing the uncertainty of the cash distribution. It is difficult to obtain a general result, but some insight can be gained by considering changes in stochastic dominance in the second order sense. Recall that the target $T(y)$ is bounded. Choose ω_0 an upper bound. Change the distribution and make it dominated in the second order sense as follows: Let $G(\omega)$ increase for $\omega \leq \omega_0$ and decrease for ω larger than ω_0, keeping the mean constant. This change results in a direct effect (keeping T fixed) that increases the probability of refusal and decreases the collected cash, since all targets are below ω_0. This change is likely to induce a strategic reaction to increase the acceptance, i.e., to decrease the threshold.

This reaction is confirmed by Fig. 6 and 7, that show a comparative statics exercise performed with truncated Gaussian distributions, with $\mu = 0.5$ and σ between 0.01 and 0.2. When the standard deviation is small, the target $T(y)$ is close to $\min(0.5, y)$, the price is close to ρ. As the variance of the distribution increases, an intermediary making an offer optimally chooses a lower target, a lower price and a larger volume; hence there is more dissemination. However, this is not enough to maintain the same acceptance probability, so that chains are typically shorter. Finally, the bottom panel of Fig. 7 shows the average price at different levels of realized intermediation chains. When the variance is large the asset initially trades at a discount of close to 3%, and the price increases at each level in the chain, until reaching values close to ρ after 9 steps. For lower variances, the initial discount is lower and convergence of the price to ρ happens after fewer steps.

5 Implications on investment and origination

In this section we endogenize the volume and the financing needs of the first intermediary, whom we call the "originator".[20] The originator needs cash to finance an investment opportunity, and will choose the scale of the investment depending on the cost at which she can access funding on the OTC market.

Specifically, intermediary D_1 chooses an optimal investment volume k instead of starting with an exogenous quantity v_0 bought at price p_0. This is the only departure we make from the initial model. Without loss of generality, we focus on the case in which $\omega_1 = 0$ in order to avoid conditioning on ω_1. We assume that the originator can choose any volume k, but originating k units has a cost $C(k)$. This cost must be financed either via selling some of the units to the next intermediary or by borrowing at rate r. Thus, once k is chosen, the game is exactly as before, with an endogenous starting point $(p_0, v_0) = (C(k)/k, k)$. We make the following technical assumptions on C:

(A2): The originator's cost function C is differentiable, strictly increasing and convex: $C' > 0, C'' \geq 0$. Furthermore, $C(0) = 0$ and $C'(0) \leq \frac{\rho}{1+r}$.

The monotonicity and convexity assumptions on C are standard. The last assumption simplifies the presentation by avoiding to consider the case in which the originator does not invest: She chooses to issue a positive volume, even without access to the OTC market.

Our formulation of the origination problem is quite general and can accommodate many applications. The originator can still be a dealer, who for instance faces a large customer who wants to sell an asset and will sell more if he receives a better price from the dealer. The large customer could be the issuer of an asset, for instance a municipality issuing new bonds, that are bought by a dealer who then disseminates them to other counterparties. The "originator" can also be a bank extending and securitizing loans, in which case C is for instance the cost of processing loan applications, extending and packaging the loans.

Optimal origination and implications on investment. Let us derive the optimal investment when the originator has access to a network of intermediaries. As she expects to

[20]The originator will be referred to with the female pronoun "she".

collect $\Omega(C(k))$ in cash when originating k units, the originator's profit is:

$$\pi_O(k) = \rho k - C(k) - rC(k) + r\Omega(C(k)). \tag{25}$$

It is useful to consider two extreme cases. Denoting k^{min} the optimal level when the originator has no access to a network ($\Omega = 0$), and k^{max} the level when she does not need the network because she can borrow at a null interest rate ($r = 0$), due to Assumption (A2) both levels are positive and characterized by the first order conditions:

$$C'(k^{min}) = \frac{\rho}{1+r} \text{ and } C'(k^{max}) = \rho. \tag{26}$$

Since marginal cost C' is increasing, we have $k^{min} \leq k^{max}$. With access to a chain of intermediaries, the originator can use some of cash resources and finance investment at a possibly lower cost than on her own, but at a higher cost than when the interest rate is null.

Proposition 1. *Let k be the optimal investment for the originator. It satisfies:*

$$C'(k) = \frac{\rho}{1 + r(1 - \Omega'(C(k)))}. \tag{27}$$

k is between k^{min} and k^{max}, respectively the optimal levels without access to a chain and with a null rate r. k is weakly increasing in ρ and weakly decreasing in r.

For ρ large enough, the originator's financing needs are larger than $\underline{y}_1$ so that she chooses an offer with $p < \rho$, and chains have more than two intermediaries with positive probability.

See the Appendix C for the proof. The comparison of (26) and (27) reveals that the marginal collected cash Ω' acts like a reduction in the interest rate r, which measures the shadow cost of cash in this model. In the extreme case in which q, hence $\Omega'(y)$, is close to 1, condition (27) becomes equivalent to the second equation of (26) and the originated volume is maximal. More generally, if it is easier to extract cash from the intermediation chain at the margin, the initial investment will be higher. Conversely, a change in the distribution of cash among intermediaries or in their availability that reduces the marginal collected cash Ω' will decrease investment, and the larger r, the greater the impact. In stressed times with high interest rates, investment will be adversely affected by the fluctuations in the endowments of intermediaries. These properties illustrate that borrowing and selling assets are substitute sources of funding for the originator.

An implication is that the trading inefficiency highlighted in Section 3.4 lowers investment:

Corollary 1. *Denoting k_F the origination level obtained under the full information benchmark, we have $k \leq k_F \leq k^{max}$ if Ω is concave.*

See the Appendix C for the proof of the corollary, which is illustrated by Fig. 8. Corollary 1 shows that the equilibrium investment level is reduced by the combination of several frictions. In a centralized market, the originator could simultaneously sell to all the other intermediaries and, provided that the total amount of cash available on the market were sufficient, she could invest up to the maximum level k^{max}. This is not possible when $q < 1$, because with some probability it will be impossible to form a chain, even of length 1. This friction, due to the decentralized nature of the market, reduces investment to k_F. In the presence of asymmetric information, the originator and each subsequent intermediary need to leave rents to the next level in the chain, which further reduces the marginal collected cash and thus leads to a lower origination level k.

The origination effect on intermediation and dissemination. We now study how a change in the different parameters will affect trading along an intermediation chain, taking into account that the initial investment k endogenously adjusts to a change in the parameters, a mechanism we call an *origination effect.* Recall that the crucial state variable of the model is an intermediary's financing needs, y. The originator's financing needs are determined by her investment k, and the financing needs of all the subsequent intermediaries are endogenously determined by the chain of offers they receive, starting with the originator.

Consider an increase in ρ. It follows from Proposition 1 that the originated volume k is increasing in ρ, and so do the financing needs $C(k)$ of the originator. The length $N(y)$ of the targeted chain by the originator is thus longer (recall that N itself does not depend on ρ). Furthermore, given the realized cash endowments, D_2 financing needs are equal to $\max(y_1 - \omega_2, 0) = \max(C(k) - \omega_2, 0)$, hence are increased as $C(k)$. Arguing recursively, a higher ρ leads to a higher k and hence to higher financing needs at all steps in the chain. The exact same reasoning can be made about a decrease in r, which gives us:

Implication 6. *The investment of the originator, the length of the targeted chains, and the financing needs of the intermediaries receiving an offer are weakly increasing in the expected return ρ and weakly decreasing in the interest rate r.*

The implication is intuitive: Both a higher return and a lower interest rate increase the initial volume of investment and hence the total amount of funding needed. The originator needs to find more cash in the intermediation chain, so that targeted chains are longer. However, some chains may be shorter: This occurs if the targets are increased so that the chain is stopped earlier for some realized endowments due to refusal.

The impact of ρ and r on traded volumes is more ambiguous. As ρ and the originated volume k increase, all the financing needs increase, which leads all intermediaries to sell more units (Implication 1). At the same time, for given financing needs an intermediary offers lower volumes when ρ is higher (Corollary 2). The direct effect of ρ and its origination effects thus go in opposite direction. The overall impact will depend on how strong is the origination effect, which is linked in particular to the convexity of C. If for a given asset C is close to being linear, a small change in ρ or in r has a large impact on k, so that the origination effect dominates and a higher ρ (a lower r) leads to higher volumes at all levels in the chain. Conversely, if C is very convex so that k is not very flexible, a higher ρ will translate into lower volumes. Assuming that prices are always decreasing in y, they will move in a direction opposite to volumes. A lower r for instance increases the price $P(y)$, but since the originated volume is higher the financing needs are larger, and intermediaries typically concede a larger discount to be able to sell the asset. These observations suggest that it is important for empirical analyses to control for the total volume to be distributed, in particular when looking at prices and volumes on dates shortly after issuance.

6 Conclusion

We propose a model in which financial intermediaries differ only in their access to liquidity and show how their positions in an OTC market for an asset are determined by their liquidity needs and the ease with which they can sell to other intermediaries. Intermediation chains arise naturally from such a model and assets get disseminated among many intermediaries, depending on how cash is distributed among them. This approach delivers implications on OTC markets, in particular for settings in which a large block trade or a new issuance lead to a sequence of trades. We show how traded volumes decrease and prices tend to increase over time. As the asset gradually gets disseminated to end investors, intermediaries have lower inventories and thus concede lower discounts when selling the asset. The speed at which the price converges to the asset's fundamental value depends on how the network of intermediaries is structured.

The model can be extended in several directions. One extension is to consider a different network structure, for example with a stylized core-periphery network, a structure often highlighted by empirical studies on financial networks (see e.g. Boss, Elsinger, Summer, and Thurner (2004)). This can be modeled for instance by having D_1, the core dealer, running an auction between n other dealers who each have access to a chain with one intermediary at each step. A second extension is to introduce the possibility to use the asset as collateral as a cheap alternative to unsecured borrowing, and to analyze the impact of the collateral value on origination. A particularly interesting application bears on banks originating loans, securitizing them and selling them to other financial intermediaries who then disseminate the securitized products through the shadow banking sector. The length of the intermediation chain can then be interpreted as a measure of how widely an asset gets disseminated in the financial system. We leave these extensions for future research.

A Figures

Unless stated otherwise, the parameters are equal to their baseline values: $\rho = 100$, $r = 0.05$, $q = 0.9$.

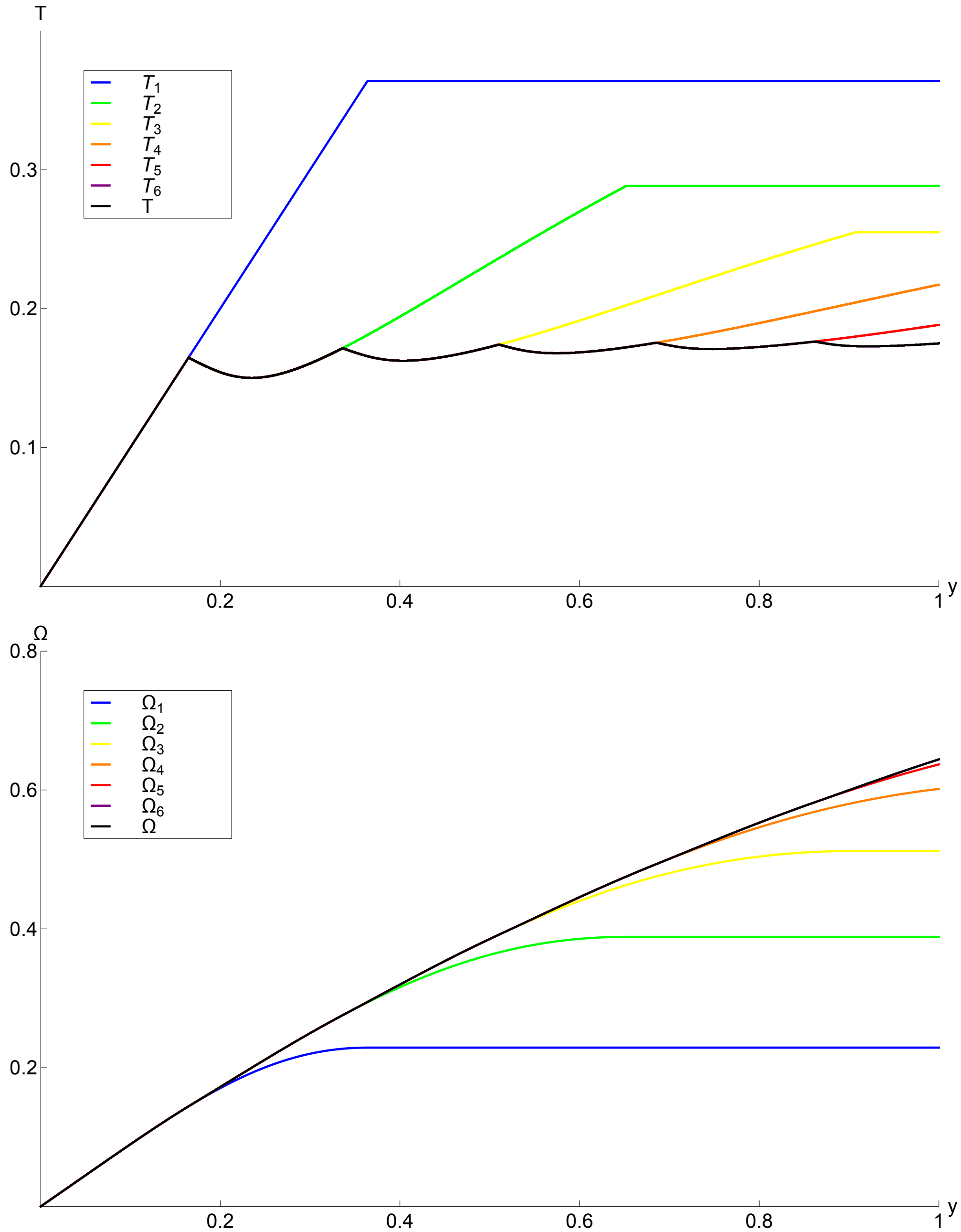

Figure 3: Equilibrium target T and T_n (top) and collected cash Ω and Ω_n (bottom). Given the financing needs y, $T_n(y)$ and $\Omega_n(y)$ are the equilibrium target and collected cash in the auxiliary game $\mathcal{G}_n$, in which there is a maximum of n layers. The figure illustrates how each T_n and Ω_n are obtained from the previous ones, and shows that on any finite interval T and Ω are equal to particular T_n and Ω_n. In this example, $T = T_7$ and $\Omega = \Omega_7$. G is a Gamma distribution with parameters $k = 5, \theta = 0.1$.

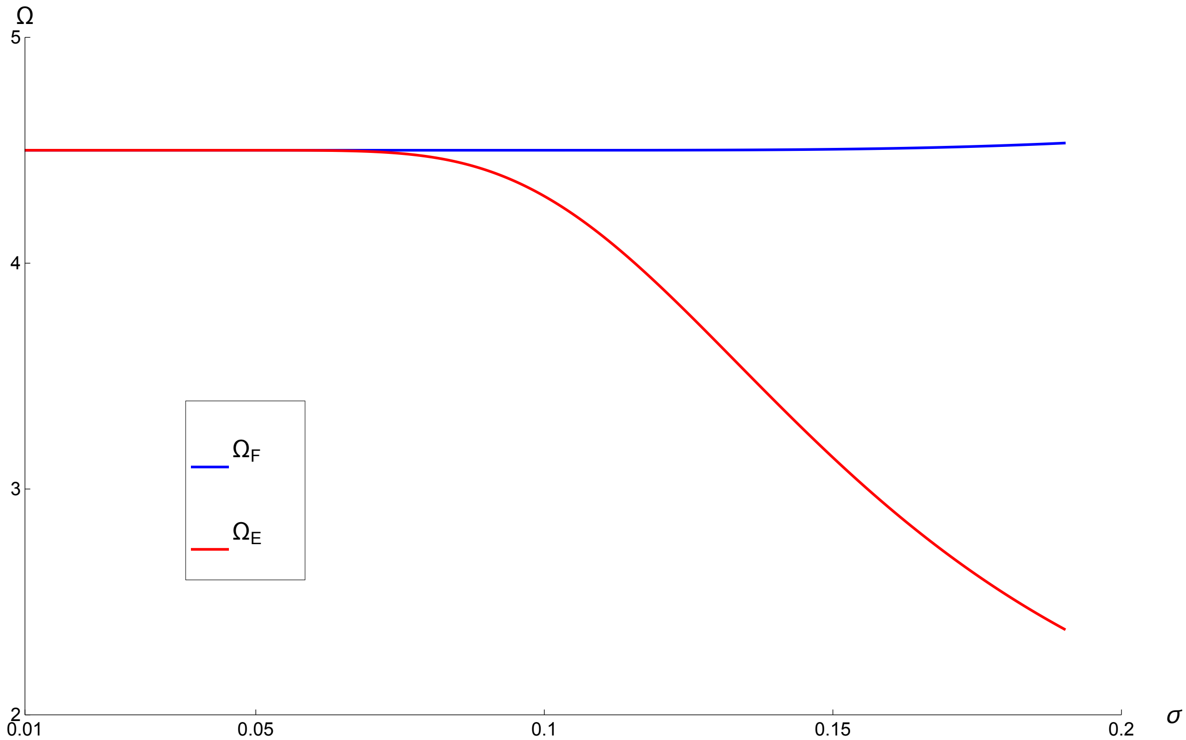

Figure 4: Trading inefficiencies. This figure plots the limit when $y \to \infty$ of the efficient level of collected cash Ω_F (in blue) and of the level Ω_E collected in equilibrium (in red), for different distributions G. G is a normal distribution with parameters 0.5 and σ, truncated at zero, with σ between 0.01 and 0.2.

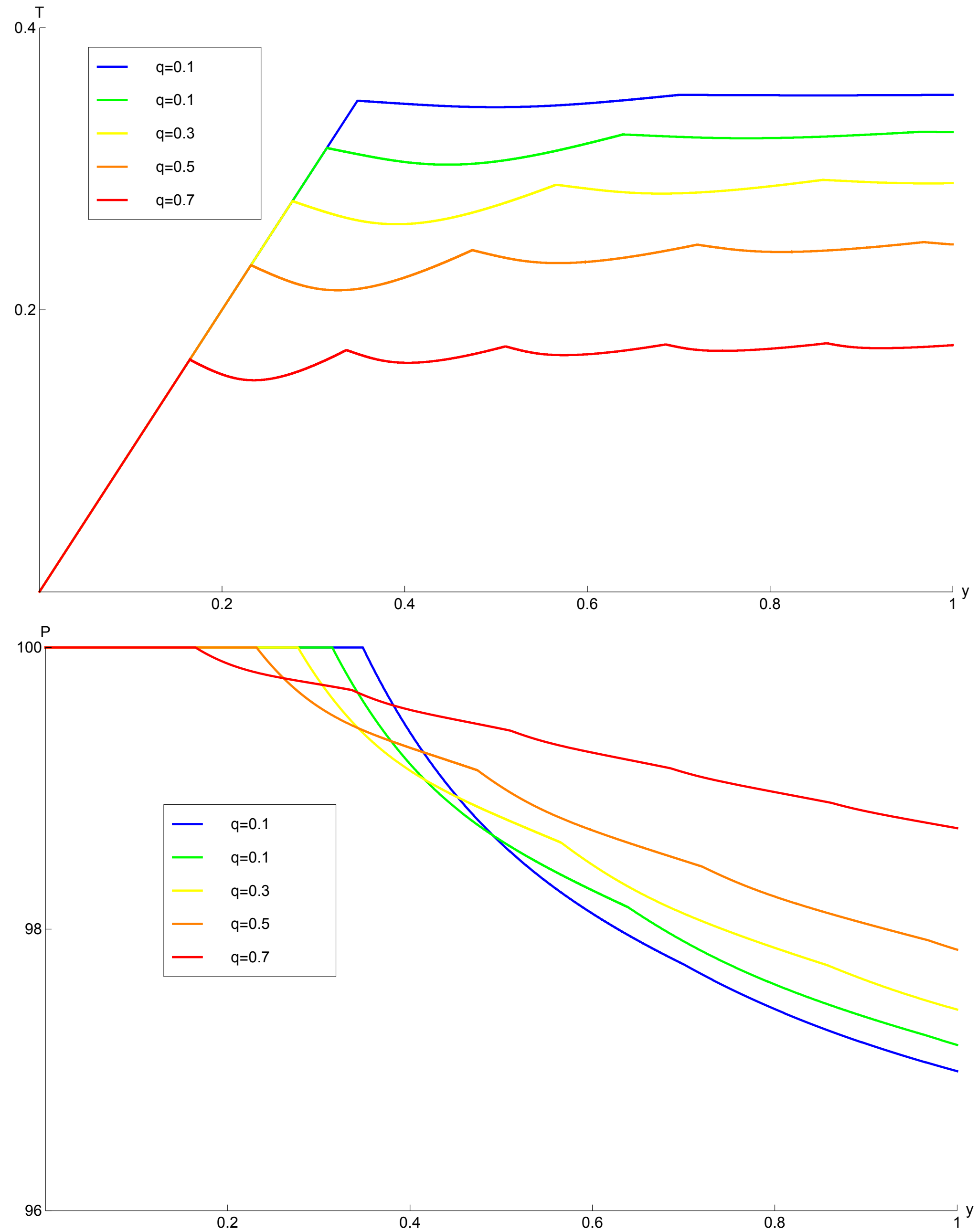

Figure 5: Target $T(y)$ (top) and offered price $P(y)$ (bottom), for different values of q. The top panel plots the target $T(y)$ of an intermediary with financing needs y, for different values of the probability to find a partner q. The bottom panel plots the offered price $P(y)$ for the same values of q. G is a Gamma distribution with parameters $k = 5, \theta = 0.1$.

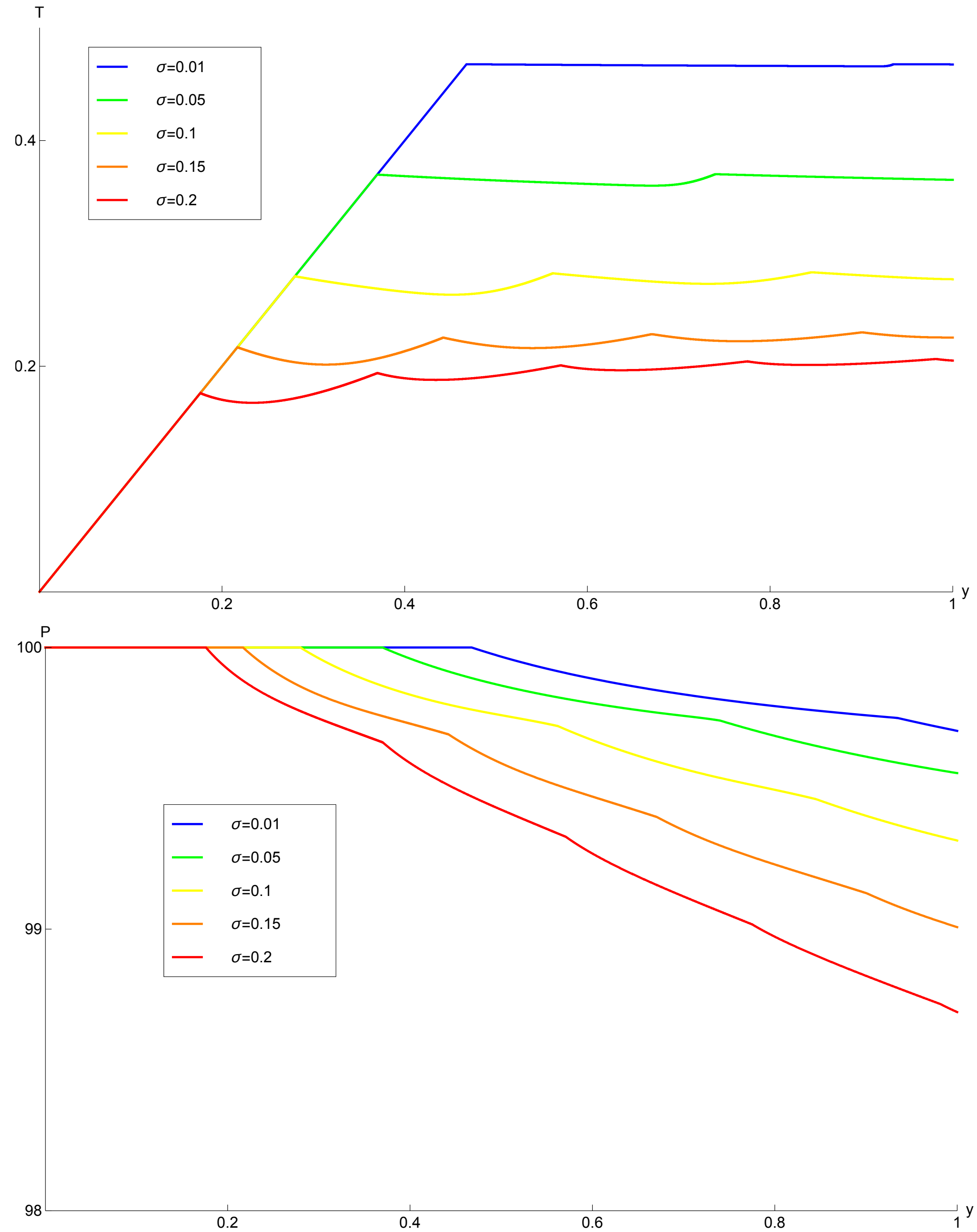

Figure 6: Target $T(y)$ (top) and offered price $P(y)$ (bottom), for different values of σ. G is a normal distribution with parameters 0.5 and σ, truncated at zero, with σ between 0.01 and 0.2. The top panel plots the target $T(y)$ of an intermediary with financing needs y, for different values of σ. The bottom panel plots the price $P(y)$ offered by an intermediary with financing needs y.

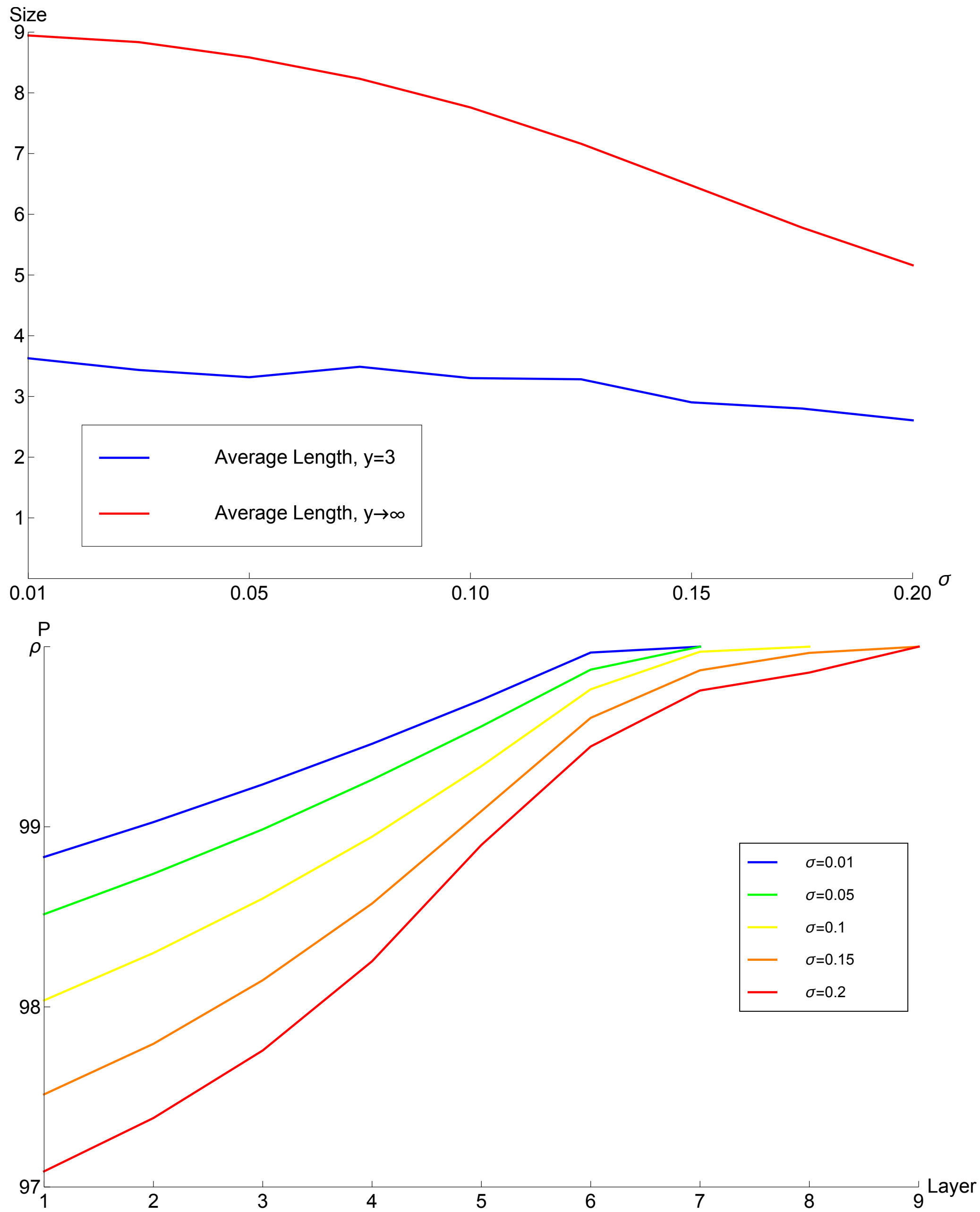

Figure 7: Chain size (top) and average price at different layers of the chain (bottom), for different values of σ. This figure illustrates the outcome of the model for different distributions G. G is a normal distribution with parameters 0.5 and σ, truncated at zero, with σ between 0.01 and 0.2. For each value of σ, we run 5,000 simulations of the model. The top panel plots the average length of the realized chain $L_E(y)$ for each parameter (in blue) when D_1 has $y = 3$, and the limit value of $L_E(y)$ for $y \to \infty$, ℓ_{ω^∞} (in red). The bottom panel shows the average price offered at a given layer, conditional on this level being reached, for each value of σ, starting with $y = 3$. For $\sigma = 0.20$ for instance, D_1 offers a price of 97.09, D_2 a price of 97.38 on average, and so on.

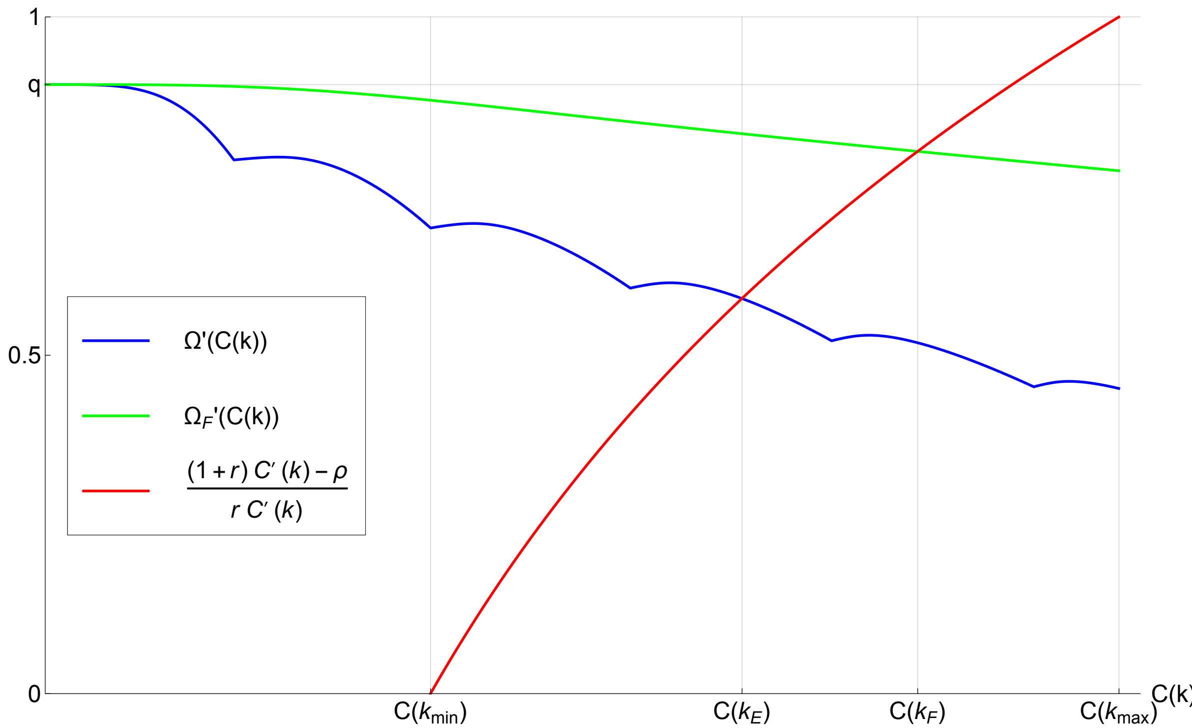

Figure 8: Originated volume in equilibrium, and in the full information benchmark. This figure illustrates Proposition 1 and Corollary 1. The first two curves are the marginal expected cash for $y = C(k)$ in equilibrium and in the full information benchmark, respectively. The third curve crosses the first one for $k = k_E$, the second one for $k = k_F$, the x-axis for $k = k_{min}$, and is equal to 1 when evaluated at $k = k_{max}$. The ordering $k_{min} \leq k_E \leq k_F \leq k_{max}$ follows from $0 \leq \Omega' \leq \Omega'_F \leq q \leq 1$ and the third curve being increasing. G is a Gamma distribution with parameters $k = 5, \theta = 0.1$, and we use the cost function $C(x) = c_1 x^{c_2}$, with $c_1 = 120, c_2 = 1.05$.

B Appendix: Proof of Theorem 1

We prove Theorem 1 by considering the games $\mathcal{G}_n$ and taking the limit. We first consider an auxiliary take-it-or-leave-it offer game. The main task is to prove Proposition B.1 in Section B.1, which will be used in B.2 to prove the theorem by induction on n.

B.1 An auxiliary game

The take-it-or-leave-it offer game has two players, D and R (for receiver). Intermediary D faces an offer (p^-, v^-) and has positive financing needs y. R has an uncertain cash endowment as in the main game, which is larger than ω with probability $H(\omega)$. The game depends on a function β. We denote this game $\Gamma_\beta(p^-, v^-, y)$.

D plays first. She decides whether to refuse or accept the offer (p^-, v^-). In case of acceptance, D can make an offer (p, v) to R. In that case, R accepts or rejects the offer. In case of acceptance, R's must finance $z = \max(pv - \omega, 0)$ by borrowing, where ω is his cash endowment, and receives an additional benefit $r\beta(z)$. Thus R's expected profit if he accepts (p, v) is

$$\pi_R(p, v, \omega) = (\rho - p)v - rz + r\beta(z) \text{ with } z = \max(pv - \omega, 0). \tag{28}$$

We assume that β is equal to zero when financing needs are null and is increasing in z with a slope less than q:

$$\beta(0) = 0 \text{ and there is } \overline{z} \geq 0 \text{ such that } \beta \text{ increases for } z \leq \overline{z} \text{ and } \beta(z) = \overline{\beta} \text{ for } z > \overline{z}. \tag{29}$$

$$\beta(z) - \beta(z') \leq q(z - z') \text{ for } z' < z. \tag{30}$$

The game is easily solved by backward induction. R optimally accepts the offer if his profit is non-negative. π_R is increasing in ω up to $\omega = pv$ and then constant equal to the non-negative value $(\rho - p)v$. There is thus a threshold value $W(p, v)$ for R's cash endowment above which R accepts the offer (p, v), which gives the acceptance probability $\Phi(p, v) = H(W(p, v))$. We can then rewrite (4) as:

$$\begin{aligned} \pi_\phi(p^-, v^-, y; p, v) &= (\rho - p^-)v^- - ry + \Phi(p, v)Q(y; p, v) \text{ where } \Phi(p, v) = H(W(p, v)) \quad (31) \\ Q(y; p, v) &= r\max(pv, y) - (\rho - p)v \quad (32) \end{aligned}$$

We are now ready to state Proposition B.1. It first shows that D's optimal offer is characterized by a target τ and D's acceptance behavior by π^* given by the following expressions:

$$\beta^*(y) = \max_{0\leq\omega\leq y} H(\omega)(\omega+\beta(y-\omega)) \tag{33}$$
$$\tau(y) = arg \max_{0\leq\omega\leq y} H(\omega)(\omega+\beta(y-\omega)) \tag{34}$$
$$\text{and } \pi^*(p^-,v^-,y) = (\rho-p^-)v^- - ry + r\beta^*(y). \tag{35}$$

The second statement provides a bound for the target and states a monotonicity property with respect to D' s financing needs. The third and fourth statements gather properties that are useful to run the induction on the games $\mathcal{G}_n$.

Proposition B.1. *Let β satisfy (29) and (30). Consider $\Gamma_\beta(p^-,v^-,y)$:*

1. *In equilibrium, D accepts the offer if and only if $\pi^*(p^-,v^-,y) \geq 0$. D then makes a new offer (p,v) with $W(p,v) \in \tau(y)$. D's expected transaction benefit is equal to $r\beta^*(y) \geq 0$ and D's expected profit is $\pi^*(p^-,v^-,y)$. p and v satisfy:*

$$pv = y \text{ and } (\rho-p)v = r(y-\omega-\beta(y-\omega)), \omega \in \tau(y). \tag{36}$$

2. *A target τ is bounded above by ω^*, and the financing needs of the target are increasing in y:*

   if $y-\omega > 0$ and $y < y'$, then $y-\omega < y'-\omega'$ for $\omega \in \tau(y)$ and $\omega' \in \tau(y')$.

3. *The function β^* satisfies assumptions (29) and (30). In particular, let $\overline{\omega}$ be the (unique) value that maximizes $H(\omega)(\omega+\overline{\beta})$ where $\overline{\beta}$ denotes the maximum of $\beta(z)$. Define $\overline{y} = \overline{z} + \overline{\omega}$. We have $\tau(\overline{y}) = \overline{\omega}$. β^* is increasing for y less than $\overline{y}$ and constant equal to $\overline{\beta}^* = H(\overline{\omega})(\overline{\omega}+\overline{\beta})$ for y larger than $\overline{y}$.*

4. *$\overline{\omega}$ is positive if $\frac{H'}{H}(0) + \frac{1}{\beta} > 0$; in that case $\frac{H'}{H}(0) + \frac{1}{\beta^*} > 0$.*

Proof: The proof has several steps. We first prove that R's profit at the threshold level $W(p,v)$ is null (Step 1) and that D makes an offer with a cash transfer lower than D's financing needs. (Step 2). At such an offer, D's benefit is equal to the saving on borrowing costs due to R's endowment at the threshold and R's extra benefit associated to β: I extracts the receiver's benefit at the threshold. This implies that the expected transaction benefit is bounded by $r\beta^*(y)$ and D's profit by $\pi^*(p^-,v^-,y)$ (Step 3); and finally we prove that these upper bounds can be reached by a feasible offer (Step 4).

Step 1. $\pi_R(p,v,W(p,v)) = 0$.

Consider two cases. In the first case, $W(p, v) > 0$. Hence $\pi_R(p, v, 0) < 0$ by definition of the threshold; it follows that $\pi_R(p, v, W(p, v)) = 0$ by continuity of β and H. In the second case, $W(p, v) = 0$. Assume by contradiction $\pi_R(p, v, 0) > 0$; then surely $p < \rho$. D can thus increase the price by a small amount so that R's profit is still positive for any level of cash: D's transaction benefit is increased and the probability of acceptance is kept equal to $H(0)$: A contradiction. ■

Step 2. *An optimal offer (p, v) satisfies $pv \leq y$.*

An optimal offer maximizes $\Phi(p, v)Q(y; p, v)$, with $\Phi(p, v) = H(W(p, v))$. By contradiction, assume that $pv > y$, so that $Q(y; p, v) = ry - (\rho - p)v$. There are two cases to consider.

If $p = \rho$, then $Q(y; p, v) = ry$ and $W(p, v) = \rho v$. Consider an alternative offer (p', v'), with $p' = \rho$, $v' < v$ and $\rho v' > y$. We then have $Q(y; p, v) = Q(y; p', v')$ and $W(p', v') \leq W(p, v)$. We thus obtain $\Phi(p', v') > \Phi(p, v)$ and $Q(y; p', v') = Q(y; p, v)$: (p, v) is not optimal.

Consider now $p < \rho$. By marginally decreasing v and increasing p, we can find an alternative offer (p', v') satisfying $(\rho - p')v' = (\rho - p)v$, $v' < v$ and $p'v' > y$. These conditions imply that the transaction benefit is unchanged, $Q(y; p, v) = Q(y; p', v')$, and $p'v' < pv$. We show that the threshold is decreased. Consider the offer (p, v) and $\omega = W(p, v)$. Since $p < \rho$ and R's profit is null, R's financing needs must be positive (from (28)): Denote $z = pv - W(p, v)$. Surely the financing needs z' at (p', v') are lower since $p'v' < pv$: $z' < z$. Since the function $z - \beta(z)$ is increasing by (30), R's profit satisfies:

$$\pi_R(p', v', W(p, v)) = (\rho - p)v - rz' + r\beta(z') > (\rho - p)v - rz + r\beta(z) = 0$$

This inequality proves that $W(p', v') < W(p, v)$. We thus obtain that $\Phi(p', v') > \Phi(p, v)$ and $Q(y; p', v') = Q(y; p, v)$: (p, v) is not optimal. ■

Step 3. *D's transaction benefit is equal to the saving on borrowing costs due to R's endowment and R's extra benefit associated to β:*

$$Q(y; p, v) = r\omega + r\beta(pv - \omega) \text{ at } \omega = W(p, v). \tag{37}$$

D's expected transaction benefit $H(W(p, v))Q(y; p, v)$ is bounded above by $r\beta^*(y)$, and D's profit by $\pi^*(p^-, v^-, y)$.

For $pv \leq y$, the sum of the benefits that D and R draw from the transaction takes a simple form. Using expressions (28) and (32), the sum is equal to $Q(y; p, v) + \pi_R(p, v, \omega) = rpv - rz + r\beta(z)$. At the threshold $W(p, v)$, R's profit is null and $W(p, v) \leq pv$: (37) immediately follows.

Since $pv \leq y$ and β is nondecreasing, it follows: $Q(y;p,v) \leq r(W(p,v)+\beta(y-W(p,v)))$. The bounds on D's expected transaction benefit and profit then follow from the definition (33) of β^* and (35). ■

Step 4. *When $\pi^*(p^-,v^-,y) < 0$, D refuses (p^-,v^-). When $\pi^*(p^-,v^-,y)$ is non-negative, there is a feasible offer, $p \leq \rho$ and $v \leq v^-$, that yields the expected transaction benefit $r\beta^*(y)$ to I, and D's profit is $\pi^*(p^-,v^-,y)$.*

The first statement straightforwardly follows from Step 3. Now, consider the case $\pi^*(p^-,v^-,y) \geq 0$. The offer that yields the expected maximal benefit $r\beta^*(y)$ is characterized by:

$$pv = y \text{ and } (\rho - p)v = r(y - \tau(y) - \beta(y-\tau(y)). \tag{38}$$

(38) says that the offer exactly finances the needs and that $W(p,v) = \tau(y)$. We first show that this offer (p,v) yields the profit $\pi^*(p^-,v^-,y)$ to D; then, using that this profit is non-negative, we prove that the offer is feasible.

Using (37) and $W(p,v) = \tau(y)$, D's expected benefit at (p,v) is equal to $H(\tau(y))(\tau(y) + \beta(y-\tau(y))$, which is equal to $\beta^*(y)$ by the definitions (33) and (34). D's expected profit when offering (p,v) is thus $\pi^*(p^-,v^-,y)$, by (35).

Let us show that (p,v) is feasible. Solving for (38),we have

$$p = \frac{\rho y + r(z-\beta(z))}{y + r(z-\beta(z))}, v = \frac{y + r(z-\beta(z))}{\rho},$$

where $z = y - \tau(y)$. These equations imply $p \leq \rho$ and $v \geq 0$ since $z - \beta(z) \geq 0$. The offer is thus feasible if furthermore $v \leq v^-$. Profit is equal to:

$$\pi^*(p^-,v^-,y) = (\rho - p^-)v^- - ry + H(\tau(y))Q(y;p,v).$$

By contradiction, assume that $v > v^-$. Since $H(\tau(y)) < 1$ and $Q(y;p,v) = ry - (\rho - p)v$, we have $0 \leq \pi^*(p^-,v^-,y) \leq (\rho - p^-)v^- - (\rho - p)v$. If $v > v^-$, this inequality implies that $p > p^-$. Then $y = p^-v^- - \omega_- < pv$, which contradicts Step 2.

This shows the desired result: D can achieve $\pi^*(p^-,v^-,y)$ when it is non-negative, hence D accepts the offer in that case. ■

Proof of *2*: The target τ is bounded above by ω^*: As β is non negative and non decreasing, the derivative of $H(\omega)(\omega + \beta(y-\omega))$ with respect to ω is bounded by $H'(\omega)\omega + H(\omega)$, which is strictly negative for $\omega > \omega^*$.

The problem (33) of maximizing $H(\omega)(\omega + \beta(y-\omega))$ with respect to ω, with $0 \leq \omega \leq y$, can be stated equivalently in terms of the financing needs $z = y - \omega$, as that of maximizing $F(z, y) = H(y-z)[y - z + \beta(z)]$ with respect to $z, 0 \leq z \leq y$. Denoting the maximizers by ζ, one has $\zeta(y) = y - \tau(y)$. The monotonicity of ζ follows from the (ordinal) single-crossing property satisfied by F:

$$F(z, y) - F(z', y) \geq 0 \text{ for } y \geq z > z' \Rightarrow F(z, y') - F(z', y') > 0 \text{ for } y' > y. \tag{39}$$

Indeed, suppose (39). Let $z \in \zeta(y)$; then $z \leq y$ and $F(z, y) - F(z', y) \geq 0$ for any $z' \leq y$, in particular for any $z' < z$. Hence (39) implies $F(z, y') - F(z', y') > 0$ for any $y' > y$ and $z' < z$: z' is surely not in $\zeta(y')$.

It remains to prove that F satisfies (39). Observe that

$$F(z, y) - F(z', y) \geq 0 \Leftrightarrow \frac{y - z + \beta(z)}{y - z' + \beta(z')} - \frac{H(y - z')}{H(y - z)} \geq 0.$$

Thus it suffices to show that the function on the right hand side is increasing in y. The first term writes as

$$\frac{y - z + \beta(z)}{y - z' + \beta(z')} = 1 + \frac{z' - \beta(z') - z + \beta(z)}{y - z' + \beta(z')},$$

hence is increasing in y because $z' - \beta(z') < z - \beta(z)$ for $z' < z$ (by (30)). The second term is nondecreasing in y by the log-concavity of H (Assumption (A1)): The log derivative w.r.t. y of $-\frac{H(y-z')}{H(y-z)}$ is $-\frac{H'}{H}(y - z') + \frac{H'}{H}(y - z)$, which is non-negative since $z' < z$. This proves (39). ■

Proof of *3*: Let $\overline{\omega}$ maximize $H(\omega)(\omega + \beta)$ over $\omega \geq 0$. As H is log-concave, the log of $H(\omega)(\omega + \beta)$ is strictly concave with a derivative equal to $\frac{H'}{H}(\omega) + \frac{1}{\omega+\beta}$, which is negative for $\omega > \omega^*$. It follows that $\overline{\omega}$ is unique and not larger than ω^*.

β^* **satisfies (29).** Clearly, $\beta^*(0) = 0$ and β^* is nondecreasing. Observe that $\overline{\beta}^*$ is an upper bound to β^*: Using $\beta(z) \leq \overline{\beta}$ implies

$$\beta^*(y) = \max_{\omega \leq y} H(\omega)(\omega + \beta(y - \omega)) \leq \max_{\omega} H(\omega)(\omega + \overline{\beta}) = \overline{\beta}^*.$$

Furthermore $\beta^*(y)$ is equal to $\overline{\beta}^*$ only if $\beta(y - \omega) = \overline{\beta}$ and $\omega = \overline{\omega}$. This is possible for $y \geq \overline{y}$ where $\overline{y} = \overline{z} + \overline{\omega}$ and only for these. This proves that $\beta^*(y)$ is less than $\overline{\beta}^*$ for $y \leq \overline{y}$ and constant equal to the maximum for $y \geq \overline{y}$.

It remains to show that β^* is increasing in y for $y < \overline{y}$. Pick $y < \overline{y}$; the targeted financing needs, $y - \omega$ for $\omega \in \tau(y)$, are lower than those for $\overline{y}$, by *2*, hence lower than $\overline{z}$. This implies that the targeted financing needs are in the range of values for which β is increasing. It follows that for $\omega \in \tau(y)$ and $y' > y$:

$$\beta^*(y) = H(\omega)(\omega + \beta(y-\omega)) < H(\omega)(\omega + \beta(y'-\omega)).$$

As $\omega \le y < y'$, the right hand side is less than or equal to $\beta^*(y')$, which proves the desired result.

β^* **satisfies (30).** Let $y' < y$. Let $\omega \in \tau(y)$ and $\omega' \in \tau(y')$.

Assume $\omega \le y'$. Then ω is a feasible target for y'; hence, by definition of β^*, $\beta^*(y') \ge H(\omega)(\omega + \beta(y'-\omega))$. Thus

$$0 \le \beta^*(y) - \beta^*(y') \le H(\omega)(\beta(y-\omega) - \beta(y'-\omega)) \le H(0)^2(y-y').$$

Assume $\omega > y'$. Then $\omega > \omega'$; it follows that $H(\omega) \le H(\omega')$ and

$$0 \le \beta^*(y) - \beta^*(y') \le H(\omega')[\omega + \beta(y-\omega) - \omega' - \beta(y'-\omega')].$$

Since $y - \omega \ge y' - \omega'$ by Point *2*, $\beta(y-\omega) - \beta(y'-\omega') \le y - \omega - y' + \omega'$ by the contraction property (30), hence the term in square brackets is less than $y - y'$. This finally gives $0 \le \beta^*(y) - \beta^*(y') \le H(0)(y-y')$. ▪

Proof of *4*: The maximizer $\overline{\omega}$ of $H(\omega)(\omega+\beta)$ is characterized by the first order condition. Thus $\overline{\omega}$ is null if $\frac{H'}{H}(0) + \frac{1}{\beta} \le 0$. Otherwise, it is positive, characterized by:

$$\frac{H'}{H}(\overline{\omega}) + \frac{1}{\overline{\omega}+\beta} = 0. \tag{40}$$

As $\overline{\beta} = H(\overline{\omega})(\overline{\omega}+\beta) \le \overline{\omega}+\beta$, this implies $\frac{H'}{H}(\overline{\omega}) + \frac{1}{\overline{\beta}} > 0$ and finally $\frac{H'}{H}(0) + \frac{1}{\overline{\beta}} > 0$ by the log-concavity of H. ▪

B.2 Proof of Theorem 1

We now prove Theorem 1 by considering the games $\mathcal{G}_n$ and using Proposition B.1 recursively. Starting with $\Omega_0 = 0$, let us define the functions Ω_n recursively for $n \ge 1$ by:

$$\Omega_n(y) = \max_{\omega \le y} H(\omega)(\omega + \Omega_{n-1}(y-\omega)) \tag{41}$$

and let $T_n(y)$ denote the cash level(s) that solve the above maximization program. As already observed, the game $\mathcal{G}_1$ coincides with the auxiliary game Γ_β when β is equal to

0. In particular,

$$\Omega_1(y) = H(y)y, \text{ for } y \le \omega^*, \; H(\omega^*)\omega^* \text{ for } y \ge \omega^*. \tag{42}$$

In $\mathcal{G}_2$, if D makes an offer (p, v), then a receiver R is playing game $\mathcal{G}_1$ as a proposer. Thus, if R accepts and has positive financing needs z, he anticipates the collected cash $\Omega_1(z)$. Note that this quantity depends only on his financing needs and satisfies assumptions (29) and (30) (Proposition B.1). Anticipating R's collected cash and behavior, D plays the game Γ_β in which $\beta = \Omega_1$. Hence D's optimal behavior yields him the cash $\Omega_2(y)$, which also satisfies assumptions (29) and (30). We can repeat the argument and obtain that $\Omega_n(y)$ is the maximal collected cash that an intermediary with financing needs y can have in $\mathcal{G}_n$. Furthermore, the financing needs of the target Z_{n+1} are increasing in y.

The sequence Ω_n is nondecreasing, $\Omega_n(y) \le \Omega_{n+1}(y)$ for each y, because a player with needs y playing $\mathcal{G}_{n+1}$ can always play as in $\mathcal{G}_n$ and hence secure at least $\Omega_n(y)$ (alternatively, this is shown by induction using (41)). To prove the Theorem, we show that the limit Ω satisfies equation (11), the rest of the Theorem then follows from point *1* of Proposition B.1, applied to $\beta = \Omega$.

Let us denote by $\overline{\Omega}_n$ the maximum value of $\Omega_n(y)$, and by $\overline{\omega}_n$ the maximizer of $H(\omega)(\omega + \overline{\Omega}_{n-1})$. By point *3* of Proposition B.1, we know that Ω_n is constant for the values of y above some threshold $\overline{y}_n$, where $\overline{y}_n = \overline{y}_{n-1} + \overline{\omega}_n$ and $\overline{y}_1 = \omega^*$. We prove that the sequence $\overline{\omega}_n$ has a positive lower bound, which implies that the sequence $\overline{y}_n$ is strictly increasing and diverges to $+\infty$.

$\overline{\omega}_n$ is strictly positive: For $n = 1$, we have $\overline{\omega}_1 = \omega^* > 0$ and $\overline{\Omega}_0 = 0$, so that $\frac{H'}{H}(0) + \frac{1}{\overline{\Omega}_1} > 0$ by point *4* of Proposition B.1. It suffices then to use point *4* repeatedly to obtain that $\frac{H'}{H}(0) + \frac{1}{\overline{\Omega}_n} > 0$. Hence $\overline{\omega}_n$ is positive for any n and the sequence $(\overline{\Omega}_n, \overline{\omega}_n)$ is characterized by

$$\overline{\Omega}_{n+1} = H(\overline{\omega}_{n+1})(\overline{\omega}_{n+1} + \overline{\Omega}_n) \text{ and } \frac{H'}{H}(\overline{\omega}_{n+1}) + \frac{1}{\overline{\omega}_{n+1} + \overline{\Omega}_n} = 0. \tag{43}$$

$\overline{\omega}_n$ is larger than ω^∞: It is easy to check that the sequence $\overline{\Omega}_n$ is increasing, $\overline{\omega}_n$ is decreasing and the limit $(\Omega^\infty, \omega^\infty)$ satisfies:

$$\Omega^\infty = H(\omega^\infty)(\omega^\infty + \Omega^\infty) \text{ and } \frac{H'}{H}(\omega^\infty) + \frac{1}{\omega^\infty + \Omega^\infty} = 0, \tag{44}$$

which also proves Property 1.

Proof of equation (11): Lemma B.1 below shows that, for each n, there is a positive $\underline{y}_n$ such that D with financing needs less than $\underline{y}_n$ behaves as if he had access to at most n

layers even if he has access to more. Furthermore, the sequence $\underline{y}_n$ increases to $+\infty$. Thus the collected cash $\Omega_m(y)$ and the optimal target $T_m(y)$ stay constant on the interval $[0, \underline{y}_n]$ as more rounds are possible (m increases).

We denote $\Omega(y)$ and $T(y)$ these values on this interval, which thus gives $\Omega(y) = \Omega_m(y)$, $T(y) = T_m(y)$. In particular, $\Omega(y) = \Omega_n(y)$ and $\Omega(y - \omega) = \Omega_n(y - \omega)$ for any $\omega \leq y$. Thus, applying (41) we obtain $\Omega(y) = \max_{\omega \leq y} H(\omega)(\omega + \Omega(y - \omega))$, i.e., (11) holds for y in the interval $[0, \underline{y}_n)$. Since the sequence $\underline{y}_n$ increases to $+\infty$, this proves that (11) is valid for each y. ∎

Lemma B.1. *For $n \geq 2$, there exists $\underline{y}_n$, $\underline{y}_n < \overline{y}_n$, such that $\Omega_n(y) = \Omega_{n-1}(y)$ for $y \leq \underline{y}_n$ and $\Omega_n(y) > \Omega_{n-1}(y)$ for $y > \underline{y}_n$. For $y < \underline{y}_n$, D's strategy in $\mathcal{G}_m$, $m \geq n$, is the same as in $\mathcal{G}_n$. The sequence $\underline{y}_n$ is increasing and goes to $+\infty$.*

Proof of Lemma B.1: The proof is by induction.

n = 2: We have to show that for some positive $\underline{y}_1$, $\Omega_2(y) = \Omega_1(y)$ if $y \leq \underline{y}_1$, in which case she plays as in $\mathcal{G}_1$, and $\Omega_2(y) > \Omega_1(y)$ if $y > \underline{y}_1$. By the definition (41), Ω_2 satisfies

$$\Omega_2(y) = \max_{\omega \leq y} H(\omega)(\omega + \Omega_1(y - \omega)). \tag{45}$$

$\Omega_2(y) = \Omega_1(y)$ if the target $T_2(y)$ is equal to y or equivalently if $Z_2(y) = 0$. Conversely, if $T_2(y) < y$, or equivalently $Z_2(y) > 0$, surely $\Omega_2(y) > \Omega_1(y)$.

Define $\underline{y}_1$ as the infimum value such that $Z_2(y) > 0$. It suffices to show that $\underline{y}_1$ is finite, in particular, smaller than $\overline{y}_1$, and positive. $\underline{y}_1$ is smaller than $\overline{y}_1$: Recall that $\overline{y}_1$ satisfies $H'(\overline{y}_1)\overline{y}_1 + H(\overline{y}_1) = 0$. The derivative of $H(\omega)(\omega + \Omega_1(y - \omega))$ at $\omega = y$ is $H'(y)y + H(y)(1 - H(0))$, hence is negative for $y = \overline{y}_1$. Thus, decreasing ω improves D's profit.

$\underline{y}_1$ is positive: Since $\Omega_1(z) \leq H(0)z$ we obtain $\Omega_2(y) \leq \max_{\omega \leq y} H(\omega)(\omega + H(0)(y - \omega))$. For y small enough, the maximum over $\omega \leq y$ of the function $H(\omega)(\omega + H(0)(y - \omega))$ is reached at $\omega = y$: The function is log-concave with a derivative at $\omega = y$ equal to $H'(y)y + H(y)(1 - H(0))$, which is positive for y small enough. This implies $\Omega_2(y) \leq yH(y)$. Now recall that $yH(y) \leq \Omega_1(y)$ and $\Omega_1(y) \leq \Omega_2(y)$. We thus obtain that for y small enough $\Omega_2(y) = \Omega_1(y) = yH(y)$, which implies $T_2(y) = T_1(y) = y$ and $Z_2(y) = 0$. Hence $\underline{y}_1 > 0$.

Finally, we show that $\Omega_m(y) = \Omega_1(y)$ for $y \leq \underline{y}_1$ and any positive m. For $m = 3$ the result follows from the definition of Ω_3: Since $\Omega_3(y) = \max_{\omega \leq y} H(\omega)(\omega + \Omega_2(y - \omega))$ and $\Omega_2(y - \omega) = \Omega_1(y - \omega)$ for $y - \omega \leq y \leq \underline{y}_1$, surely $\Omega_3(y) = \Omega_2(y)$. The argument can be repeated for any m.

Induction argument: Assume the property is true for a given n. Define $\underline{y}_{n+1}$ as the infimum of the y such that $Z_{n+1}(y) \geq \underline{y}_n$ in the sense that $z > \underline{y}_n$ for any $z \in Z_{n+1}(y)$. For

$y \le \underline{y}_{n+1}$, $\Omega_{n+1}(y) = \Omega_n(y)$ because $\Omega_n(z) = \Omega_{n-1}(z)$ for financing needs z less than $\underline{y}_n$.

$\overline{y}_{n+1} > \underline{y}_{n+1}$: Recall that $\overline{y}_{n+1} = \overline{y}_n + \overline{\omega}_n$. By point *3* of Proposition B.1, this implies that $Z_{n+1}(\overline{y}_{n+1}) = \overline{y}_n$. Using the definition of $\underline{y}_{n+1}$ and the fact that Z_{n+1} is increasing, we obtain $\underline{y}_{n+1} < \overline{y}_{n+1}$.

The sequence $\underline{y}_n$ strictly increases and goes to infinity: For $y \ge \underline{y}_1$, $\Omega_n(y) \ge H(\underline{y}_1)\underline{y}_1$. This provides a lower bound ω_{min} to any target: $\omega_{min} = H(\underline{y}_1)\underline{y}_1(1 - H(0))/H(0)$. Thus, $z \le y - \omega_{min}$ for $z \in Z_{n+1}(y)$. As a result, surely $z \le y - \omega \le \underline{y}_n$ for $y \le \underline{y}_n + \omega_{min}$. This implies $\underline{y}_n + \omega_{min} \le \underline{y}_{n+1}$, which yields the desired result. ∎

C Appendix-Other proofs

Proof of Property 3. Let us prove that $\lim_{y \to +\infty} L_E(y) = \ell_{\omega^\infty}$. We already showed that $L_E(y)$ is bounded above, hence it converges as y increases. Let us denote the limit by ℓ^∞. We show $\ell^\infty = \ell_{\omega^\infty}$. Remember that L_E satisfies the recursive expression (21).

$\ell^\infty \ge \ell_{\omega^\infty}$. Let $\ell < \ell^\infty$. By definition of ℓ^∞, there is z such that $L_E(z) \ge \ell$. This implies that $L_E(y - \omega) \ge s$ for any ω such that $y - \omega \ge z$. Since $T(y) \le \omega^*$, for any y such that $y - z > \omega^*$, we have:

$$\int_{T(y)}^{y} L_E(y-\omega)dG(\omega) \ge \int_{T(y)}^{y-z} \ell dG(\omega) \ge (G(y-z) - G(T(y))\ell.$$

Plugging this inequality into (**??**), we obtain : For any $y > z$, $L_E(y) \ge q(1 - G(T(y)) + q(G(y-z) - G(T(y))\ell$. Taking the limit in y yields $\ell^\infty \ge q(1 - G(\omega^\infty))(1 + \ell)$. Since this is true for any ℓ, $\ell < \ell^\infty$, we obtain $\ell^\infty \ge q(1 - G(\omega^\infty))(1 + \ell^\infty)$, which is equivalent to $\ell^\infty \ge \ell_{\omega^\infty}$.

$\ell^\infty \le \ell_{\omega^\infty}$. By definition $L_E(z) \le \ell^\infty$ for any z. Using this inequality to bound the integral in (**??**), we obtain $L_E(y) \le q(1 - G(T(y))(1 + \ell^\infty)$. Taking the limit in y yields $\ell^\infty \le q(1 - G(\omega^\infty))(1 + \ell^\infty)$, which is equivalent to $\ell^\infty \le \ell_{\omega^\infty}$.

Since L_F is given by a similar expression as L_E, replacing $T(y)$ by 0, the same argument shows that $\lim_{y \to +\infty} L_F(y) = \ell_0$.

The proofs for Ω_E and Ω_F follow the same lines and are omitted. To prove that Ω_F (hence Ω_E) is bounded above, observe that $\Omega_F(y)$ is surely less than the expected amount collected in the simple process associated to 0; this amount is equal to $\ell_0 \mathbb{E}[\omega]$ since $\mathbb{E}[\omega]$ is collected at each step. The remainder of the proof follows, using that $y(1 - G(y))$ necessarily goes to zero when y tends to $+\infty$ because $\mathbb{E}[\omega]$ exists. ∎

Proof of Proposition 1. The originator's profit if she chooses k is $\pi_O(k) = \rho k - C(k) - rC(k) - \Omega(C(k))$, which gives the derivative:

$$\frac{\partial \pi_O}{\partial k} = \rho - C'(k) - rC'(k)[1 - \Omega'(C(k))].$$

Thus, the first-order condition associated with the optimal k satisfies (27). The second derivative is negative at the optimal k. The cross derivative $\frac{\partial^2 \pi_O}{\partial k \partial \rho}$ is equal to 1. The monotonicity of the optimal k with respect to ρ follows.

The derivative $\frac{\partial \pi_O}{\partial k}$ decreases with r, hence the optimal investment is nonincreasing in r.

Since an optimal k is larger than k^{min}, the financing needs at k are larger than at k^{min}. Thus it suffices to show that $C(k^{min})$ is larger than $\underline{y}_1$ for ρ large enough. According to condition (26), k^{min} increases in ρ without bounds due to the convexity of C, whereas $\underline{y}_1$ does not depend on ρ, which completes the proof.

Proof of Corollary 1. We can rewrite condition (27) as $\Omega'(C(k)) = \frac{(1+r)C'(k)-\rho}{rC'(k)}$. Under full information, the collected cash that the originator would expect would not be Ω but the Ω_F studied in 3.4. Thus we only need to show that $\Omega'(y) \leq \Omega'_F(y)$ for any y. Using equation (18), we have, for any $y \geq 0$:

$$\Omega'_F(y) = q\left(1 - G(y) + \int_0^y \Omega'_F(y-\omega)g(\omega)d\omega\right).$$

Using (11) and the envelope theorem, we have $\Omega'(y) = q(1 - G(T(y)))(T(y) + \Omega'(y - T(y)))$ for any $y \geq 0$, which we can rewrite as:

$$\Omega'(y) = q\int_{T(y)}^y \Omega'(y - T(y))g(\omega)d\omega \leq q\int_{T(y)}^y \Omega'(y-\omega)g(\omega)d\omega,$$

where the inequality comes from the concavity of Ω. We can thus write:

$$\Omega'_F(y) - \Omega'(y) \geq q\left(1 - G(y) + \int_0^{T(y)} \Omega'_F(y-\omega)g(\omega)d\omega + \int_{T(y)}^y [\Omega'_F(y-\omega) - \Omega'(y-\omega)]g(\omega)d\omega\right)$$

As $\Omega'(0) = \Omega'_F(0)$, this recursive inequality implies that $\Omega'_F(y) - \Omega'(y) \geq 0$ for any positive y, proving the corollary.

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