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DESIGNING INSTITUTIONS FOR DIVERSITY[†]

Abstract

This paper analyzes the design of innovation contests when the quality of an innovation depends on the research approach of the supplier, but the best approach is unknown. Diversity of approaches is beneficial because of the resulting option value. An auction induces the social optimum, while a fixed-prize tournament induces insufficient diversity. The optimal contest for the buyer is an augmented fixed-prize tournament, where suppliers can choose from a set of at most two prizes. This allows the buyer to implement any level of diversity at the lowest cost.

JEL Classification: D02, D21, D44, L14, L22 and L23
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The use of contests to procure innovations has a long history, and it is becoming ever more popular. Recently, private buyers have awarded the Netflix Prize, the Ansari X Prize, and the InnoCentive prizes. Public agencies have organized, for instance, the DARPA Grand Challenges, the Lunar Lander Challenge and the EU Vaccine Prize.[1] The literature on contest design deals with the problem of providing incentives for costly innovation effort.[2] However, effort is by no means the only important requirement for a successful innovation. Ex ante, there are often many potentially optimal approaches to solving an innovation problem. Contest design should therefore encourage innovators to take different approaches. In this paper, we thus ask how contest design influences the diversity of research approaches.

Many practical examples illustrate the importance of the issue. First, the often cited Longitude Prize of 1714 for a method to determine a ship's longitude at sea featured two competing approaches.[3] The lunar method was an attempt to use the position of the moon to calculate the position of the ship. The alternative, ultimately successful, approach relied on a clock which accurately kept Greenwich time at sea, thus allowing estimation of longitude by comparison with the local time (measured by the position of the sun). Second, when the Yom Kippur War in 1973 revealed the vulnerability of the US aircraft to Soviet-made radar-guided missiles, General Dynamics sought to resolve the issue through electronic countermeasures, while McDonnell Douglas, Northrop, and eventually Lockheed, attempted to build planes with small radar cross-section.[4] Third, the EU Vaccine Prize was announced in 2012 with the goal of improving the so-called cold-chain vaccine technology. Interestingly, the competition rules explicitly stated that diverse innovation approaches were conceivable: "It is important to note that approaches to be taken by the participants in the competition are not prescribed and may include alternate formulations, novel packaging and/or transportation techniques, or significant improvements over existing technologies, amongst others."[5] Finally, the announcement of the 2015 Horizon Prize for better use of antibiotics contains a similar statement.[6]

Thus, in many innovation contests both the buyer (the contest designer) and the suppliers (contestants) are aware that there are multiple conceivable approaches to innovation. Furthermore, none of the participants knows the best approach beforehand. However, after the suppliers have followed a particular approach, it is often possible to assess the quality of innovations, for instance, by looking at prototypes or detailed descriptions of research projects. In the following, we will ask whether buyers can and should do more than to appeal to suppliers to pursue diverse approaches: Can they design institutions in such a way that suppliers have incentives to provide diversity? And will they benefit from doing so?

Architectural contests share some important properties with innovation contests. A buyer who thinks about procuring a new building usually does not know what exactly the ideal building would look like, but once she examines the submitted plans, she can choose the one she prefers. Guidelines for architectural competitions explicitly recognize the need for diversity. For example, the Royal Institute of British Architects states: "Competitions enable a

[1] See "Innovation: And the winner is...", *The Economist.* Aug 5, 2010.

[2] Section 7 discusses this literature.

[3] See, e.g., Che and Gale (2003) for a discussion of the Longitude Prize.

[4] See Paul Crickmore (2003), *Nighthawk F-117: Stealth Fighter.* Airlife Publishing Ltd.

[5] European Comission (2012), "Prize Competition Rules." August 28, 2012 (accessed on April 3, 2015). http://ec.europa.eu/research/health/pdf/prize-competition-rules_en.pdf

[6] European Commision (2015), "Better use of antibiotics." March 24, 2015 (accessed on April 3, 2015). http://ec.europa.eu/research/horizonprize/index.cfm?prize=better-use-antibiotics

wide variety of approaches to be explored simultaneously with a number of designers."[7] Two important features are common in innovation contests and architecture contests. First, the buyer often specifies a fixed prize ex ante. Second, the number of contestants may be large. There is a tension between these observations and the results of an interesting theoretical literature that investigates the optimal design of procurement contests.[8] This literature focuses on incentives for efforts rather than for diversity. Most papers conclude that (i) buyers should use auctions rather than fixed-prize tournaments and (ii) it is optimal for a buyer to invite only two participants. Our paper argues that neither of these conclusions can be taken for granted when the diversity of approaches is an issue.

To our knowledge, our paper is the first analysis of the design of innovation contests with multiple conceivable research approaches. We develop a model with a buyer and several suppliers. Crucially, the value of an innovation does not depend on effort, but on the difference between the chosen research approach and an ideal, but initially unknown approach. The suppliers and the buyer agree about the distribution of the ideal approach. It seems plausible that the buyer would like to induce the suppliers to choose different approaches to gain from the resulting option value. This paper studies which contests can induce such diversity. More broadly, we analyze the consequences of different institutions on buyer payoffs and total welfare.

In line with the literature on innovation contests, we assume that neither research inputs nor research outputs are verifiable,[9] because they are both difficult to evaluate, and the relation between them is stochastic. Clearly, this assumption is more palatable in some cases than in others. For instance, in the 2005 Grand Challenge, DARPA asked participants to build an unmanned vehicle that could complete a 212 kilometer course in rugged territory as fast as possible. In principle, DARPA could have rewarded participants according to the exact time needed to complete the course, a verifiable signal of quality.[10] In other contexts, for instance, when basic research (or architectural style) is considered, more judgment is necessary, and it is much harder to come up with verifiable measures of quality and, in particular, of the monetary value of quality differences. In these cases, non-verifiability seems highly plausible.

The lack of verifiability of research activity precludes any kind of contract that could be implemented by a court, which leads to a hold-up problem. One solution to this problem is to use what Che and Gale (2003) define as a contest mechanism (or simply contest). Such a contest prescribes a possible set of prices and commits the buyer to paying the price chosen by the supplier from which the innovation is procured. Examples are fixed-prize tournaments as well as auctions with or without reserve price. As argued by Che and Gale (2003), a court can easily enforce the rules of a contest, since it only needs to verify if the correct price was paid out to one of the participants. In line with the literature, we focus on contests as means of procuring innovation.

In our benchmark model, there are two homogeneous suppliers who choose a research

[7] See Royal Institute of British Architects (2013), "Design competitions guidance for clients." (accessed on Apr 3, 2015)
http://competitions.architecture.com/requestform.aspx.

[8] See Section 7 for details on the related literature.

[9] For an extensive discussion see Che and Gale (2003) and Taylor (1995).

[10] Instead, DARPA only used the speed criterion to rank participants (see Section 1.6 of the DARPA Grand Challenge Rules (2004, accessed on June 24, 2015). http://archive.darpa.mil/grandchallenge05/Rules_8oct04.pdf).

approach, modeled as a point on the unit interval. The distribution of the ideal approach has a symmetric and single-peaked density. Quality depends linearly on the distance between the approach taken and the ideal approach. After the buyer has communicated the rules of the game, the suppliers choose their approaches, and qualities become common knowledge. We abstract from cost considerations, so as to focus on suppliers' incentives to diversify.

In the benchmark model, we obtain the following results.[11] First, maximizing the expected total payoffs of buyers and suppliers (henceforth, the social optimum) involves diversity of approaches. Second, an auction mechanism implements the social optimum. Third, fixed-prize tournaments do not induce any diversity, but nevertheless yield higher expected buyer payoffs than auctions. Fourth, the optimal contest for the buyer is an "augmented fixed-prize tournament", in which the suppliers can choose from at most two prizes; these contests allow the buyer to finetune the degree of diversification. Finally, for a uniform state distribution, the fixed-prize tournament maximizes the expected buyer payoffs.

We then provide several extensions, including the following. First, we show that, when the buyer can charge sufficiently large participation fees, she can implement the social optimum and appropriate the resulting rents completely. Second, we allow for multiple suppliers. The social optimum continues to involve diversity, and it can be implemented with an auction. Fixed-prize tournaments now induce some diversity, but less than socially optimal. However, as the number of suppliers increases, the difference between fixed-prize tournaments and auctions becomes smaller. The buyer continues to prefer tournaments to auctions. She may benefit from inviting a large number of suppliers, which is a straightforward implication of the option value provided by additional suppliers. Third, we consider exogenously heterogeneous suppliers, reflecting differences in technology or style (as in the architectural example). For substantial heterogeneity, active diversification is no longer optimal. Otherwise the main results carry through: Auctions implement the social optimum; tournaments do not; buyers nevertheless prefer fixed-prize tournaments unless there are substantial fixed costs or participation fees. Moreover, with heterogeneous suppliers, it is possible to analyze the impact of the revelation of quality information on the incentives of contestants to diversify. Revelation is crucial: Without it, the unique equilibrium of an auction is the one without any diversification.

The contest metaphor has useful interpretations beyond the procurement context. As we discuss briefly in the conclusion, our model also applies when suppliers choose products in the face of uncertain demand by a potentially large number of homogeneous buyers. Contest design then corresponds to the choice of alternative regulatory institutions. Our approach shows that unregulated markets provide incentives for suppliers to choose the socially optimal products, but at the cost of endowing them with ex-post market power. As a result, regulation may yield higher expected consumer surplus, even though it does not induce the optimal expected product quality.

In Section 1, we introduce the benchmark model with two homogeneous suppliers. In Section 2, we introduce innovation contests. In Section 3, we show that an auction induces the social optimum. Section 4 deals with the optimal design of contests from the buyer's perspective, while Section 5 extends the analysis of optimal contests beyond the benchmark model by considering fixed costs and unconditional transfers. Section 6 presents extensions

[11] In addition, we allow for more general distribution functions and transportation costs, and we consider negotiations, contests with multiple prizes and contests with multiple approaches developed by the same supplier as alternative procurement institutions.

of the model. Section 7 discusses our paper in the context of related literature. Section 8 concludes, pointing in particular to the above-mentioned re-interpretation of our model for a world with many buyers. Short and instructive proofs are in the Appendix. The Online Appendix (available at https://sites.google.com/site/iletina/did) contains technical notation and omitted proofs, as well as more details about the extensions.

1 The Benchmark Model

We now describe the assumptions of our benchmark model. Its simple setting brings out the intuition most clearly. We will see in Section 6 that the main insights are much more general.

A risk-neutral buyer B needs an innovation that two risk-neutral suppliers ($i \in \{1, 2\}$) can provide. Each supplier chooses an approach $v_i \in [0, 1]$ at cost $C(v_i)$. The quality q_i of the resulting innovation depends on a state $\sigma \in [0, 1]$, which is distributed with density $f(\sigma)$, and corresponds to an (ex-post) ideal approach. We thus assume that $q_i = \Psi - \delta(|v_i - \sigma|)$, where $\Psi > 0$ and δ is an increasing function. Unless specified otherwise, we will maintain assumptions (A1)-(A3) below.

Assumption (A1) *The density function $f(\sigma)$ is (i) symmetric: $f(1/2 - \varepsilon) = f(1/2 + \varepsilon)$ $\forall \varepsilon \in [0, 1/2]$, (ii) single-peaked: $f(\sigma) \leq f(\sigma') \; \forall \sigma < \sigma' < 1/2$ and (iii) has full support: $f(\sigma) > 0 \; \forall \sigma \in [0, 1]$.*

Assumption (A2) *All approaches have the same fixed costs $C(v_i) \equiv C \geq 0$.*

Assumption (A3) $\delta(v_i, \sigma) = b|v_i - \sigma|$ *with* $b \in (0, \Psi]$.

Using (A3), we denote the quality resulting from approach v_i in state σ as $q(v_i, \sigma) = \Psi - b|v_i - \sigma|$ for some $\Psi > 0$. Thus quality is bounded below by $\Psi - b$ and bounded above by Ψ. Though most of the results are more general, we will specify the density function as follows for some specific results:

Assumption (A1)' *The density function $f(\sigma)$ is uniform.*

When (A1)' holds, all approaches are equally likely to be ideal and, by (A2), they are equally costly. Nevertheless, the market participants agree ex ante that the central approach maximizes expected quality and thus expected total profits.

As we show in the Proposition 9 in the Online Appendix, a vector maximizing social welfare (v_1^*, v_2^*) always exists and always features diversity, that is $v_1^* \neq v_2^*$. Intuitively, without diversity, the expected minimal distance to the ideal approach can be reduced by arbitrarily moving one of the two approaches away from the other. Depending on the distribution of σ, diversity can be substantial. For instance, when (A1)' holds, the social optimum is $(v_1^*, v_2^*) = (1/4, 3/4)$.[12]

[12] See Proposition 9 in the online appendix.

Period 1: Suppliers simultaneously select approaches $v_i \in [0,1]$.
Period 2: The state is realized. All players observe $v_i \in [0,1]$; thus q_1 and q_2 become common knowledge.
Period 3: Suppliers simultaneously choose prices $p_i \in \mathcal{P}$.
Period 4: The buyer observes prices; then she chooses a supplier $i \in \{1,2\}$. She pays p_i to the chosen supplier and 0 to the other supplier.

2 Innovation Contests

The buyer can choose an innovation contest determining the procedure for choosing and remunerating suppliers. These contests are closely related to the procurement contests analyzed by Che and Gale (2003), where suppliers choose efforts rather than approaches.[13] In line with these authors, we assume that neither v_i nor q_i is contractible.[14] The environment (b, Ψ, C) of a contest consists of the utility and cost parameters. For now, we set $C = 0$, so as to separate the suppliers' decisions on which approach to choose from the decision whether to produce. In Section 5, we allow for positive fixed costs so as to analyze the entry decision.

The buyer chooses a set $\mathcal{P}$ of allowable prices (bids), where $\mathcal{P}$ is an arbitrary finite union of closed subintervals of $\mathbb{R}^+$.[15] We denote the minimum of $\mathcal{P}$ as $\underline{P}$ and the maximum, if it exists, as $\overline{P}$. An *innovation contest* is the extensive-form game between the buyer and the suppliers defined by the buyer's choice of $\mathcal{P}$ and the following rules:

The assumption that all players observe v_i and σ is convenient, as it allows us to apply the subgame perfect equilibrium (SPE). However, the assumption is more restrictive than necessary. As long as all players can observe qualities, all results still hold with the SPE replaced by a Perfect Bayesian Equilibrium with suitably specified beliefs.[16] Moreover, in the Subsection 6.2 we modify the observability assumption by imposing that the suppliers have to submit bids when they choose approaches; thus they cannot observe qualities when they choose prices.[17]

The following are examples of innovation contests:

1. $\mathcal{P} = \mathbb{R}^+$: a *procurement auction without reserve price.*

2. $\mathcal{P} = \left[0, \overline{P}\right]$: a *procurement auction with reserve price* $\overline{P}$.

3. $\mathcal{P} = \{A\}$, where $A \geq 0$: a *fixed-prize tournament* (FPT) with A set by the buyer.

4. $\mathcal{P} = \{A, 0\}$, where $A \geq 0$: an *augmented fixed-prize tournament.*

[13] In Section 6, we will briefly discuss an alternative class of institutions.

[14] For example, Che and Gale (2003) and Taylor (1995) assume that neither inputs nor outputs of innovative activity are verifiable. As an example of the verifiability problem, Che and Gale (2003) point to the protracted battle between John Harrison, the inventor of the marine chronometer, and the Board of Longitude, over whether his invention met the requirements of the 1714 Longitude Prize. See also references in Taylor (1995).

[15] Formally, $\mathcal{I}(\mathbb{R}^+) := \{\mathcal{P} \subseteq \mathbb{R}^+ : \mathcal{P} = \cup_{k=1}^{\bar{k}} [a_k, b_k] \text{ or } \mathcal{P} = \cup_{k=1}^{\bar{k}} [a_k, b_k] \cup \left[a_{\bar{k}+1}, \infty\right) \text{ for } a_k \leq b_k \in \mathbb{R}^+, \bar{k} \in \mathbb{N}\}$.

[16] Of course, if the suppliers could develop a design anew, they could achieve a better quality by incorporating what they have learned. We exclude this possibility and focus on the highest quality that can be produced at the end of the contest. One justification is that, due to unmodelled dynamic considerations, the good has to be produced in the current period.

[17] We cannot pursue this issue with homogeneous suppliers, as we would run into existence problems.

The first three examples are well-known. The last example will turn out to be a useful alternative for the buyer. We apply the following tie-breaking rules.

(T1) If suppliers offer the same surplus, the buyer prefers the higher quality one. If both have the same quality, the tie is randomly broken.

(T2) Given equal monetary payoffs, the suppliers prefer to win the contest.

(T1) and (T2) will be shown to guarantee that the outcomes are robust to infinitesimal changes in the reward structure.[18]

3 Implementing the Social Optimum

Proposition 9 in the Online Appendix demonstrates that the social optimum requires at least some diversity under very general conditions. Moreover, this diversity can be substantial, as the case of the uniform state distribution shows. We now show that a simple innovation contest implements the social optimum. We require the following notation:

Notation 1 $\overline{p}(\sigma) \equiv \max\{p \in \mathcal{P} | \, p \leq |q(v_1,\sigma) - q(v_2,\sigma)| + \underline{P}\}$.

The following result formalizes the familiar "asymmetric Bertrand" logic that low-quality firms choose minimal prices, whereas high-quality firms translate the quality differential into a price differential.

Lemma 1 *The subgame of an innovation contest corresponding to (q_i, q_j) has an equilibrium such that $p_i(q_i, q_j) = \overline{p}(\sigma)$ if $q_i \geq q_j$ and $p_i(q_i, q_j) = \underline{P}$ if $q_i < q_j$. In any SPE of any contest, $p_i(q_i, q_j) = \overline{p}(\sigma)$ if $q_i \geq q_j$.*

Lemma 1 sharpens the Bertrand logic: The price differential will only fully reflect the quality differential when this is feasible for the high-quality supplier. In many cases, the equilibrium described in Lemma 1 is unique.[19] Lemma 1 is essential in the following result.

Proposition 1 *(i) The auction mechanism ($\mathcal{P} = \mathbb{R}^+$) implements the social optimum. (ii) If f is uniform ((A1)' holds), then any equilibrium of an auction is socially optimal.*

The logic of the direct proof that we provide in the Online Appendix is very similar to the reasoning behind the optimality of Vickrey-Clarke-Groves mechanisms.[20] Claim (i) also follows from the more general results in Lemma 5. Here, we therefore merely explain why the auction mechanism induces diversity, as required for the social optimum. Intuitively, if the approaches are identical, the suppliers have the same quality in all states. Hence, by Lemma 1, subgame equilibrium prices are zero, and suppliers will not earn positive profits. A deviation to any other approach is thus profitable.

[18] For example, in an auction (T1) ensures that the higher quality supplier wins by offering the same surplus to the seller as his competitor. Even without (T1), the higher quality supplier could ensure that he wins by offering a slightly higher surplus. Similarly, (T2) will allow us to avoid the consideration of limits in the discussion of the optimal tournament.

[19] If P is convex and $\sup P > \bar{p}(\sigma)$ for all σ, then $p_i(q_i, q_j) = \underline{P}$ for $q_i < q_j$ in every equilibrium. To see this, note that, according to the lemma, $p_i = \overline{p}(\sigma) = \underline{P} + q(v_i, \sigma) - q(v_j, \sigma)$ in any equilibrium. If $p_j > \underline{P}$, then player i can choose a slightly higher prize, and he still wins. Hence, this is a profitable deviation.

[20] See, e.g., Mas-Colell et al. (1995).

4 The Optimal Contest for the Buyer

The auction mechanism induces the efficient amount of diversity, but leaves rents to the successful supplier. It will turn out that, because it avoids such rents, a suitable FPT is preferable for the buyer, even though it does not induce any diversity in a two-player setting. The remaining results of the section show that the optimal innovation contest for the buyer is a (possibly augmented) FPT. For the special case of the uniform state distribution, there are no contests leading to higher buyer payoffs than the FPT with the minimal prize $A = 0$. An FPT never induces diversity:

Proposition 2 *For any $A \geq 0$, the unique equilibrium of an FPT is $(v_1, v_2) = (1/2, 1/2)$.*

The intuition is straightforward. As the size of the prize is independent of quality differences, the suppliers care only about maximizing the expected winning probability. By (A1), this requires moving to the center.[21] We need further notation:

Notation 2 $\Delta q(v_i, v_j) \equiv |q(v_i, v_i) - q(v_j, v_i)|$ *is the maximum quality difference given (v_i, v_j).*

By Lemma 1, in any subgame the successful supplier chooses the highest available price below the sum of the quality differential and the minimum bid in any subgame. We now show that, for the equilibrium choice of approaches, the bid corresponds *exactly* to the sum of the maximum quality differential and the minimum bid. This is also the price paid in all other states resulting in the maximum quality difference.

Lemma 2 *Let $v_1 \leq v_2$. (i) If a contest implements (v_1, v_2), then $\Delta q(v_1, v_2) + \underline{P} \in \mathcal{P}$. (ii) If $\sigma \in [0, v_1] \cup [v_2, 1]$, the successful supplier bids $p_i(q_i, q_j) = \Delta q(v_i, v_j) + \underline{P}$.*

Intuitively, (i) if $\Delta q(v_1, v_2) + \underline{P} \notin \mathcal{P}$, suppliers could increase their chances of winning by small moves towards the approach of the other party, without reducing the price in those cases where they win. (ii) shows that in all states outside the interval (v_1, v_2) the buyer pays a constant price, reflecting the (maximal) quality difference between the two suppliers.

Lemma 2(i) has an immediate implication.

Corollary 1 *In an auction with a reserve price $\bar{P}$, the diversity in any (pure strategy) equilibrium (v_i, v_j) is bounded by the reserve price: $\bar{P} \geq \Delta q(v_i, v_j)$.*

If the maximal quality difference between the two suppliers were above the maximum feasible bid, the supplier could not charge the buyer for this quality difference. He could thus choose an approach slightly closer to the competitor to increase the chances of winning without reducing the price.

Corollary 1 embeds the polar cases treated so far, the auction without reserve price and the FPT. In an auction without reserve price, suppliers are free to choose the bid and thus

[21] For $A = 0$, uniqueness requires the tie-breaking rule (T2). Otherwise, any design choice is an equilibrium. An alternative justification of the equilibrium $(1/2, 1/2)$ is that it is the limit of the equilibria for $A \to 0$.

capture the benefits of diversification. This results in optimal diversity. By Corollary 1, reserve prices limit this possibility: They determine an upper bound on equilibrium diversity. As the reserve price approaches zero, so does the equilibrium diversity, as for an FPT with prize zero. Thus, the choice of reserve price involves a tradeoff between efficiency-increasing diversity and market power for the suppliers. As the next result shows, the buyer never resolves this tradeoff in favor of efficiency, as she prefers fixed prize tournaments to any auction with or without reserve price.

Proposition 3 *Among all contests where $\mathcal{P}$ is convex, the buyer's payoff is maximal in an FPT with $A = 0$.*

In auctions with and without reserve price, $\mathcal{P}$ is convex. Thus, according to Proposition 3, the buyer prefers the socially inefficient tournament without diversity to the unrestricted auction inducing socially optimal diversity and to to auctions with reserve price inducing intermediate levels of diversity. The proof of Proposition 3 relies heavily on the fact that higher quality suppliers bid the sum of the quality differential and the minimum $\underline{P}$ whenever available (Lemma 1). Thus the buyer's expected payoff is the difference between the expectation of the minimum quality and the minimum bid. The best she can do is to choose an FPT with $A = 0$, because this maximizes the minimum quality and minimizes the minimum bid.

Proposition 3 highlights an important point: Contrary to the case of effort-inducing contests (Che and Gale, 2003), the buyer can never benefit from using auctions in diversity-inducing contests. Intuitively, while diversity is desirable because of the option value resulting from different approaches, it also leads to greater quality differences ex post. The resulting increase in market power eliminates all benefits for the buyer, which has no counterpart in effort-inducing contests.

We now ask whether the buyer can do even better by using non-convex price sets. The following implementation result shows how the buyer can use augmented tournaments to fine-tune diversity.

Proposition 4 *Suppose $A = \Delta q(v_1, v_2)$ for some (v_1, v_2) such that $0 < v_1 \leq 1/2 \leq v_2 < 1$. In the augmented FPT with $\mathcal{P} = \{A, 0\}$, the strategy profiles $(v_1, v_2, p_1(), p_2())$ such that $p_i(q_i, q_j) = A$ if $q_i - q_j \geq A$ and 0 otherwise, form an equilibrium.*

Thus, the buyer can implement any desired diversity in an augmented FPT with A as the corresponding maximal quality difference. For instance, to induce the social optimum, the buyer has to set $A = \Delta q(v_i^*, v_j^*)$. The resulting payments to the successful supplier are as low as possible: In states where the quality difference is maximal, the buyer has to pay this quality difference, which is the minimum payment consistent with Lemma 2. In all other states, the buyer pays 0, which is obviously minimal. The flexibility of the augmented FPT in inducing diversity and the low buyer payments suggest that the optimal contest is in this class. The following result confirms this intuition.

Proposition 5 *The buyer can achieve the highest possible payoff among all innovation contest equilibria with an augmented FPT.*

Proposition 5 is the central result of this section. Like the class of convex contests analyzed above, the class of augmented FPTs contains the FPT with $A = 0$. As the next result shows, this FPT is the optimal contest if the state distribution is uniform.

Corollary 2 *If (A1)' holds, the FPT with $A = 0$ is an optimal contest.*

To sum up, the buyer always prefers the optimal FPT to the auction. The class of augmented FPTs contains her optimal innovation contest. In the special case of the uniform state distribution, the FPT with $A = 0$ is optimal.

5 Fixed Costs and Participation Fees

Our analysis without fixed costs and participation fees has identified a conflict between efficiency and rent extraction. We now show to which extent the buyer can resolve this conflict when there are fixed costs and/or she can charge participation fees. For simplicity, we will assume (A1)'. Denote with $W(v_1, v_2) = E_\sigma [\max\{q(v_1, \sigma), q(v_2, \sigma)\}]$ the gross welfare generated by (v_1, v_2). To avoid uninteresting cases, we assume that total net profits are non-negative even when the suppliers select $(v_1, v_2) = (1/2, 1/2)$.

Assumption (A4) $W(1/2, 1/2) \geq 2C$.

Suppose the buyer can charge participation fees of at most $T \geq 0$. We do not allow for discriminatory participation fees. The fee can be negative, corresponding to a subsidy. The buyer thus chooses a participation fee $t \in (-\infty, T]$ which is the same for each supplier.[22] An innovation contest with participation fees consists of a pair $(\mathcal{P}, t)$ with an environment (b, Ψ, C, T) that is common knowledge. The rules are as before, with two modifications. First, when the buyer decides on $\mathcal{P}$, she also fixes t. Second, after the buyer has chosen the rules of the innovation contest, the suppliers decide whether to enter, with an outside option of zero. Upon entering, C and t are sunk,[23] and the suppliers choose their approaches based only on $\mathcal{P}$, that is, exactly as described in Sections 2-4. The buyer chooses $\mathcal{P}$ and t to maximize her expected payoff, which is equal to the total expected surplus, net of supplier payoffs and (possibly negative) transfers. Thereby, she indirectly selects the equilibrium levels of v_1 and v_2 and the functions $p_1(\cdot, \cdot)$ and $p_2(\cdot, \cdot)$, assigning prices to qualities. Compared with Section 2, she also has to take the participation constraints into account.

Proposition 6 *Suppose (A1)' and (A4) hold and let $v_1^* \leq v_2^*$.*

(i) If $T \geq F(v_1^*)\,\Delta q(v_i^*, v_j^*) - C$, the optimal contest is given by $t = F(v_1^*)\,\Delta q(v_i^*, v_j^*) - C$ and $\mathcal{P} = \{\bar{p}, 0\}$ where $\bar{p} = \Delta q(v_i^*, v_j^*)$. In equilibrium, $v_i = v_i^*$; and $p_i = \bar{p}$ if and only if $q_i - q_j \geq \bar{p}$ $(i = 1, 2;\ j \neq i)$.

(ii) If $T < F(v_1^*)\,\Delta q(v_i^*, v_j^*) - C$, the optimal contest is given by $t = T$ and $\mathcal{P} = \{\bar{p}, 0\}$ where $\bar{p} = \Delta q(\widetilde{v}_i, 1 - \widetilde{v}_i)$ for $\widetilde{v}_i \in (v_i^*, 1/2]$ defined by $T = F(\widetilde{v}_i)\,\Delta q(\widetilde{v}_i, 1 - \widetilde{v}_i) - C$. In equilibrium, $v_i = \widetilde{v}_i$, $v_j = 1 - \widetilde{v}_i$, and $p_i = \bar{p}$ if and only if $q_i - q_j \geq \bar{p}$ $(i = 1, 2;\ j \neq i)$.

If $T \geq F(v_1^*)\,\Delta q(v_i^*, v_j^*) - C$, the maximal participation fee is at least as high as the expected net profit that each supplier earns in the equilibrium of an augmented tournament

[22] The assumption on feasible transfers is slightly more restrictive than necessary. First, T does not need to be finite. Second, the maximal possible subsidy does not need to be infinite. The result holds whenever the maximal subsidy is at least C.

[23] The participation decision obliges the supplier to produce one design at cost C and to pay participation fees to the buyer. If participation fees are negative (i.e. if the buyer offers a subsidy), the participation decision obliges the buyer to pay the subsidy.

that induces efficient diversity. The buyer uses this augmented tournament, appropriating the total net surplus by charging the maximal participation fee such that each supplier expects to break even. If $T < F\left(v_1^*\right) \Delta q(v_i^*, v_j^*) - C$, the buyer charges the maximal participation fee and only induces enough diversity for suppliers to break even on expectation.[24] Again, she opts for rent extraction rather than efficiency.

Summing up, by Corollary 2, without fixed costs and participation fees, the buyer cannot use an appropriate choice of $\mathcal{P}$ to increase surplus and to capture some of the increase herself. Participation fees help to capture the surplus increase. Proposition 6 shows that the buyer optimally induces the amount of diversity which increases the surplus exactly by the amount that can be captured through the participation fees.

6 Extensions

We now show that our main findings still hold for multiple suppliers, heterogeneous suppliers, and for general state distribution and distance measures. We also show that the buyer can potentially do better than in the optimal FPT by not committing to an ex-ante price policy, but instead relying on ex-post negotiations. On the contrary, FPTs with multiple prizes and contests where each buyer can submit multiple designs do not improve the outcome.

At this stage, we would also like to mention one natural extension that appears less tractable. One might want to combine our analysis which focuses on diversity of approaches with the previous literature that focuses on costly effort. Preliminary considerations suggest that such a convex combination is not straightforward, and it is not obvious that it would lead to interesting insights. While we can identify multiple equilibrium candidates as (fairly intransparent) functions of parameters, verifying that these candidates satisfy second-order conditions is an arduous task in general. The complications are at least partly due to the fact that equilibrium existence problems arise with homogeneous suppliers, so that the analysis has to be carried out in the heterogeneous supplier framework described below, which already is quite complex in itself. We thus refrained from treating this case.

6.1 Number of Suppliers

In innovation contests there are usually more than two suppliers. For example, there were 49 registered competitors in the EU Vaccine Prize, 12 of which submitted final designs for evaluation.[25] Our main results hold even when there are many suppliers. For simplicity, we assume that (A1)' holds.[26] The social optimum and the equilibria in the FPT are as follows.

Lemma 3 *Suppose there are $n > 3$ suppliers and (A1)', (A2) and (A3) hold.*
(i) The social optimum is $(v_1^*, ..., v_n^*) = (1/2n, 3/2n, 5/2n, ..., (2n-1)/2n)$; in particular, there is no duplication.
(ii) For any $k \in \{1, ..., n\}$, an outcome with k active approaches $(r_1, ..., r_k)$ can be supported in an equilibrium if

[24] This case contains the results of Proposition 2 as a special case (where $T = C = 0$).

[25] European Commision (2014), "German company has won the EU's € 2 million vaccine prize." March 10, 2014 (accessed on April 3, 2015). http://ec.europa.eu/research/health/vaccine-prize_en.html

[26] We do not consider the case $n = 3$. In this case, an FPT does not have a pure strategy equilibrium, so that comparison with the other cases would be difficult.

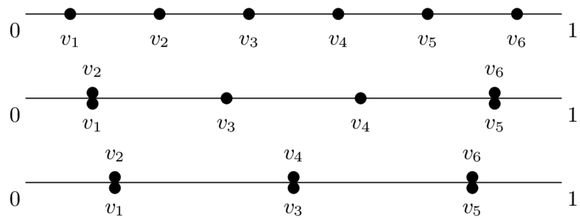

Figure 1: Equilibria when $n = 6$.
The first constellation represents the socially optimal outcome of an auction. The other two constellations represent two possible outcomes of an FPT.

(a) $k \in \{\underline{k}, ..., \bar{k}\}$*, where* $\bar{k} = n - 2$ *and* $\underline{k} = n/2$ *if* n *is even and* $\underline{k} = (n+1)/2$ *if* n *is odd and*
(b) $(r_1, ..., r_k) = ((1/2k, 3/2k, 5/2k, ..., (2k-1)/2k))$.
Two suppliers choose the extreme approaches r_1 *and* r_k*; each of the intermediate approaches* $r_2, ..., r_{k-1}$ *is chosen by one or two suppliers.*

Figure 1 illustrates the two previous lemmas for $n = 6$. In line with Lemma 3(i), the social optimum (depicted in the first constellation) involves evenly spread approaches. Proposition 10 (i) in the Online Appendix shows that the auction mechanism implements the social optimum. The two other constellations describing the equilibria of the FPT highlight implications of Lemma 3(ii), which are stated more generally in Proposition 10 (ii) and (iii) in the Online Appendix. First, the two most extreme approaches are not as far apart as the most extreme approaches of the social optimum; in this sense, there is less than optimal diversity. Second, as $\bar{k} < n$, there is duplication; moreover, this duplication always affects the two most extreme approaches chosen by the suppliers. Finally, depending on the specific equilibrium, there may be additional duplication for intermediate approaches.

It is important to note that the difference between the FPT and the social optimum decreases as the number of suppliers increases.[27]

Lemma 3 has another simple but important implication when we allow for fixed costs and transfers: The optimal innovation contest may involve an arbitrarily large number of suppliers. This differs from several results for the case of contests that merely influence the suppliers' efforts, where the optimal number of participants is typically two.

Corollary 3 *Suppose the fixed costs are* $C > 0$ *and that (A1)' holds. Define* $n_-(C) = \max\left\{n \in \mathbb{N} | \, n \leq \sqrt{b} \Big/ 2\sqrt{C}\right\}$ *and* $n_+(C) = n_-(C) + 1$*. An innovation contest that maximizes total net profits is an auction with* $n_-(C)$ *or* $n_+(C)$ *suppliers.*

The result is a straightforward implication of the previous results. Lemma 3(i) characterizes the socially optimal allocation, and the auction mechanism implements this allocation. The condition in the Corollary describes the number of suppliers that optimally balances the

[27] As $r_k - r_1 = (k-1)/k$, and the minimum for k on $\{\underline{k}, ..., \bar{k}\}$ is $\underline{k}$, $r_k - r_1$ attains its minimum for even n at $k = n/2$, where it becomes $(n-2)/n$ rather than the socially optimal maximal diversity of $(n-1)/n$. Hence, the difference between the actual diversity and the social optimum is bounded above by $1/n$. The argument for uneven n is analogous.

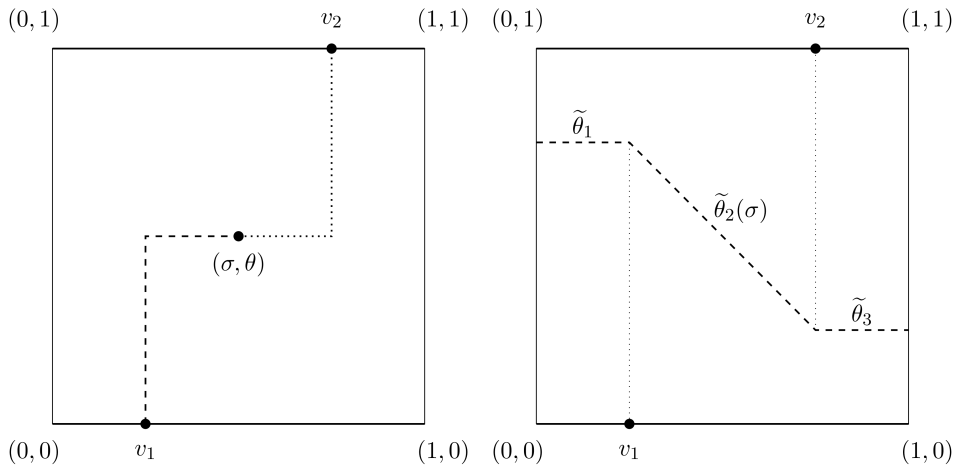

Figure 2: Quality from approaches v_1 and v_2.
(Left panel) The transportation costs correspond to a weighted sum of the length of the vertical and the horizontal dashed (dotted) line for supplier 1(2). (Right panel) For a given realization (σ, θ) of the state, supplier 1 is the producer of higher (lower) quality if θ is below (above) some critical value $\widetilde{\theta}(\sigma)$.

gains from higher expected quality against the losses from higher fixed costs. While the corollary is stated for the socially optimal contest, it is clear that the buyer can also often benefit from inviting more than two suppliers.

6.2 Heterogeneous Suppliers

The assumption of homogeneous suppliers simplifies the analysis. In many contexts, it is nevertheless natural to allow for heterogeneity: Suppliers may differ in their approaches to innovation, because they have different kinds of expertise or research capabilities. Architects may have different and essentially fixed styles. We now show that the main results hold with heterogeneous suppliers.

Section D.2 in the Online Appendix introduces the modified set-up and the results in detail. We suppose that the state space is now a unit square, with elements (σ, θ). The set of feasible approaches of supplier 1 is the lower edge of the square; the set of supplier 2 is the upper edge (see Figure 2). We specify the quality functions as

$$q_1(v_1, \varphi) = \Psi - a\theta - b|v_1 - \sigma| \tag{1}$$
$$q_2(v_2, \varphi) = \Psi - a(1-\theta) - b|v_2 - \sigma| \tag{2}$$

The optimal diversification depends on the ratio $\beta \equiv b/a$.

Proposition 7 *(i) Suppose without loss of generality that $v_1 \leq v_2$. Then the social optimum* *with heterogeneous suppliers is:*

$$(v_1^*, v_2^*) = \begin{cases} \left(\frac{1}{2}, \frac{1}{2}\right) & \textit{if} \quad \beta < 1 \\ \left(\frac{1}{2\beta}, 1 - \frac{1}{2\beta}\right) & \textit{if} \quad 1 \leq \beta < 2 \\ \left(\frac{1}{4}, \frac{3}{4}\right) & \textit{if} \quad 2 \leq \beta \end{cases}.$$

(ii) An auction weakly implements the social optimum.

The intuition for (i) is simple. Whenever $\beta < 1$, exogenous differences between suppliers matter more than differences in approaches. The social optimum (which minimizes the expected horizontal transportation costs) is $(v_1, v_2) = (1/2, 1/2)$. As β increases, so does the expected value of diversity. The expected quality is maximal for $(v_1, v_2) = (1/4, 3/4)$, which is the social optimum for homogeneous suppliers. Interestingly, the auction implements the social optimum even when this does not require diversification.

As for homogeneous suppliers, we can show that fixed-prize tournaments do not induce any diversification (Proposition 11), but that buyers prefer them to auctions (Proposition 12(i)). Moreover, buyers prefer symmetric auction equilibria to asymmetric auction equilibria (Proposition 12(ii)).

The modified framework allows us to use the alternative informational assumption that suppliers cannot observe qualities when they submit bids, which is intractable for homogeneous suppliers (see Proposition 8). Thus, we can also ask: Should buyers reveal quality information to suppliers if they are the only ones who observe quality? Information revelation is indeed essential for the positive result of Proposition 7(ii). To see this, we modify the innovation contest by assuming that suppliers simultaneously choose research approaches and bids. In such an auction without information revelation, suppliers thus commit to their bids before they learn quality. We assume that β is sufficiently low, that is, suppliers are sufficiently heterogeneous.

Assumption (A5): $\beta < 2\sqrt{2} + 3$

When this assumption is violated, in particular for homogeneous suppliers, existence problems arise in auctions without information revelation. Note, however, that the upper bound on β does not rule out any of the cases discussed in Proposition 7.[28]

Proposition 8 *For heterogeneous suppliers, an auction without information revelation has a unique equilibrium. In this equilibrium,*$(v_i, p_i) = (1/2, a)$ *for* $i = 1, 2$.

The degree of exogenous differentiation a fully determines prices. Moreover, the equilibrium involves minimum differentiation. Whenever $\beta > 1$, such an equilibrium is not socially optimal according to Proposition 7(ii), reflecting inefficiently low expected quality.

[28] In particular, Assumption (A5) is consistent with the case $\beta \geq 2$ in which the social optimum is $(v_1, v_2) = (1/4, 3/4)$, the maximum diversity that can be optimal for any set of parameters.

6.3 Generalized distributions and transportation costs

First, we show that the main results from Sections 3 and 4 are neither sensitive to the assumptions on the distribution of the optimal approach σ nor to the details of the quality function $q(v_i, \sigma)$. For simplicity, we confine the analysis to the game without transfers or fixed costs.[29] Proposition 13 in the Online Appendix applies to arbitrary state distributions with positive density and arbitrary quality functions that are decreasing in the distance between a given approach and the ideal one. It shows that (i) the social optimum still requires diversity of approaches, (ii) the auction mechanism implements the social optimum; and (iii) in any FPT, there is no diversity in the unique equilibrium, and both suppliers choose the approach corresponding to the median of the state distribution. The result that buyers prefer the optimal FPT to an auction does not hold under these very general conditions. However, we show in Proposition 14 in the Online Appendix that it is still valid if we relax only the assumption on state distributions (A1) or the assumption on quality functions (A3).

6.4 Multiple Prizes

In the 2005 DARPA Grand Challenge, only the winner of the contest was eligible for the prize ($2 million), while the other contestants received nothing. This corresponds to an FPT as introduced above. However, in the subsequent DARPA contest, known as the 2007 Urban Challenge, rules specified that not only would the winner receive a prize (which was again $2 million), but the next two participants would also receive prizes ($1 million and $0.5 million).[30] In this section, we show that a buyer is worse off in an FPT with two prizes than with a single prize.[31]

Clearly, when there are only two suppliers, a second prize has no effect, as the suppliers would consider it as an unconditional transfer, and the effective prize would be the difference between the first and the second prize. Hence, as in Section 6.1, we suppose $n > 3$. We assume that (A1)', (A2) and (A3) hold, $C = 0$, and that the two prizes are $A_1 > A_2 > 0$. In Online Appendix D.4, we prove the following result:

Lemma 4 *For any (pure strategy) equilibrium in an FPT with two prizes, there exists an equilibrium in an FPT with a single prize which makes the buyer strictly better off.*

The proof shows that any equilibrium of an FPT with two prizes involves more duplication than the chosen equilibrium of an FPT with a single prize, which leads to a lower buyer payoff. This result suggests that multiple prizes do not improve diversity.[32]

6.5 Multiple Designs by the Same Supplier

We have assumed so far that each supplier can only develop a single approach. However, in the 2005 DARPA Grand Challenge, vehicles designed by the Red Team from Carnegie

[29] This game is equivalent to the subgame following entry of both suppliers in the game with arbitrary levels of transfers and costs.

[30] See Section 1.4 of the DARPA Urban Challenge Rules (2007) (accessed on June 24, 2015). http://archive.darpa.mil/grandchallenge/docs/Urban_Challenge_Rules_102707.pdf

[31] The results can be extended to more than two prizes.

[32] Of course, there may be reasons outside of the model which would make multiple prizes a desirable choice for a contest designer. For example, if suppliers are risk averse, providing multiple prizes may be a way of increasing their expected utility.

Mellon University took the second and third place. By developing multiple designs, a supplier internalizes some of the resulting option value. It is thus natural to ask if our results still hold when suppliers develop multiple approaches. The modified model is analytically intractable, but a numerical analysis suggest that our main results are robust. We study the cases with $n \in \{2, ..., 5\}$ suppliers, each of which can develop $m = 2$ approaches, and the case with $n = 2$ suppliers, each of which can develop $m = 3$ approaches. We assume that (A1)', (A2) and (A3) hold and that $C = 0$. We also fix values of Ψ and b.[33]

Result 1 *If there are n suppliers and each develops m approaches, then: (i) in an auction there exists an equilibrium which is equivalent to the socially optimal equilibrium of an auction with $n \cdot m$ suppliers, each of which develops one approach. (ii) In an FPT, there exists an equilibrium which is identical to the maximally duplicative equilibrium of an FPT with $n \cdot m$ suppliers, each of which develops one approach.*

Proposition 10 in Online Appendix D.1 proves that the buyer prefers an FPT to an auction inducing the socially optimal outcome when there are multiple suppliers and each develops one approach. Moreover, Result 1 suggests that the case where n suppliers each develop m approaches corresponds to the case where $n \cdot m$ suppliers each develop one approach (see Section 6.1). This suggests that the buyer is better off holding an FPT than an auction also in the case when suppliers can develop multiple approaches.

6.6 Negotiations

Finally, we suppose that the buyer decides not to commit to a design contest, but instead leaves the determination of payments to negotiations, which split the surplus of the relationship, i.e., the quality differential Δq.[34] A share $\tau \in (0, 1)$ accrues to the buyer, the remainder to the successful supplier. This introduces a potential hold-up problem when there are fixed costs. If the suppliers are not convinced that they will have sufficient bargaining power to break even, they will not invest. If the suppliers are not deterred from investing, negotiations implement the social optimum, because they share an important feature with auctions: Suppliers are rewarded for higher quality relative to the other supplier. The distributional properties depend on τ. Independent of its value, the buyer prefers negotiations to auctions, because the latter give the entire surplus Δq to the supplier. Holding an auction thus represents a transfer of market power from the buyer to the suppliers. For sufficiently high bargaining power of the buyer, negotiations are preferable to the optimal FPT for the buyer, provided suppliers expect to break even.

[33] For details and the code used to obtain numerical results, see Supplementary Material for Section VI.E, available at https://sites.google.com/site/iletina/did.

[34] Somewhat relatedly, Ding and Wolfstetter (2011) consider a case where a supplier can choose to bypass the contest and negotiate with the buyer directly in an environment where innovation quality is obtained by expending costly effort.

7 Relation to the Literature

This paper contributes to the literature on optimal contest design, especially the design of innovation contests. In models of fixed-prize tournaments, Taylor (1995) shows that free entry is undesirable, and Fullerton and McAfee (1999) show that the optimal number of participants is two. Fullerton et al. (2002) find that buyers are better off using auctions than fixed-prize tournaments. In a very general framework, Che and Gale (2003) show that an auction with two suppliers is the optimal contest. Contrary to the previous literature, our paper focuses on inducing the suppliers to choose the right approaches rather than to exert as much effort as possible. We find that it can be optimal for the buyer to invite a large number of suppliers. Moreover, for any given number of suppliers, we show that the buyer often prefers fixed-prize tournaments to auctions, even though the latter implement the socially optimal diversity of research approaches.

While we are not aware of any paper that considers optimal contest design when diversity plays a role, some authors obtain related results. For instance, Schöttner (2008) considers two contestants who influence quality stochastically by exerting effort. While she does not deal directly with optimal contest design, she finds that, for large random shocks, the buyer prefers to hold a fixed-prize tournament rather than an auction to avoid the market power of a lucky seller in an auction. In our setting, the correlation of outcomes is endogenous, and diversity of approaches implies that, whenever one supplier is subject to a negative shock, the other one will likely be subject to a positive shock. Thereby, diversification generates an option value which is desirable from a social point of view. However, as we saw above, in an auction, the option value is often captured by the suppliers and not by the buyer. Thus, the buyer is better off not inducing diversity through an auction at all.

In Ganuza and Hauk (2006), suppliers choose both an approach to innovation and a costly effort.[35] However, these authors focus on fixed-prize tournaments, while we study the optimal contest design.[36] Erat and Krishnan (2012) study a fixed-prize tournament where suppliers can choose from a discrete set of approaches.[37] Each approach is successful with some probability that is independent of success or failure of any other approach. The qualities of successful approaches can vary. The authors find that suppliers cluster on approaches delivering the highest quality. This result is similar to our result that fixed-prize tournaments lead to a duplication of approaches in equilibrium. In addition to considering alternative contests, our model also considers correlated rather than independent qualities; it is thus meaningful to speak of similar approaches.[38]

More broadly, our paper is related to the literature on innovation contests with exponential-bandit experimentation (see Halac, Kartik and Liu (2014) and references therein). In these

[35] In Ganuza and Pechlivanos (2000), Ganuza (2007) and Kaplan (2012), the buyer has to choose the design or alternatively can reveal information about the preferred design.

[36] More broadly related is Bajari and Tadelis (2001) who do not deal with innovations, but with construction projects. The issue of the right approach to the problem arises in such settings as well. The supplier obtains new information during the period when the contract is being executed, which allows him to adapt the original approach at some cost. Since the relationship is between a buyer and only one supplier, the question of diversity of approaches does not arise. This is also true for the related work by Arve and Martimort (2015) who study risk-sharing considerations in the design of contracts with ex-post adaptation.

[37] See also Terwiesch and Xu (2008) for the effect of number of suppliers when exogeneous random shocks are large. For empirical evidence see Boudreau, Lacetera and Lakhani (2011).

[38] See also Konrad (2014) for a variant of Erat and Krishnan's model where first best is restored if the tie-breaking is decided via costly competition (for example lobbying) as opposed to randomly.

models, it is uncertain whether the innovation is feasible. Suppliers participating in the contest expend costly effort to learn the state, and they also learn from the experimentation of their opponents. The goal of the contest is to induce experimentation. However, each supplier experiments in the same way. In our model, experimentation arises at the industry level for suitable contests, as the heterogeneity of approaches allows the buyer to pick the best available choice.

Finally, our paper is related to the literature on policy experimentation. For instance, Callander and Harstad (2015) show that decentralized policy experimentation yields too much diversity. In their model, the success probabilities of different experiments are independent, no matter how similar the policies are. This assumption removes the option value of having different experiments, which is central in our model. If there existed an ideal policy (in terms of quality) as in our model, then the option value would have to be traded off against the benefits of convergence emphasized by Callander and Harstad (2015). It would be interesting to see whether and how centralization would help to resolve this trade off.

8 Conclusions and Discussion

The ideal approach to solving an innovation procurement problem is usually unknown when suppliers are asked to choose research approaches. The paper investigates the implications of this uncertainty for contest design. Under very general conditions, it is socially optimal to induce suppliers to take diverse research approaches, and the social optimum can be obtained with an auction mechanism, provided the bidding takes place after qualities are commonly known. To reduce supplier rents, the buyer nevertheless prefers a fixed-prize tournament that induces less diversification. In a two-player setting, diversification decreases with the reserve price in an auction. Moreover, the optimal contest is an augmented fixed prize tournament, which induces diversity at the lowest possible cost to the buyer.

Our paper offers a rationale for the frequent use of fixed-prize tournaments in complex procurement tasks, as in innovation and architecture. Even though other mechanisms may yield higher option value and thus higher expected quality, with suitable fixed-prize tournaments the buyer can extract all rents from the suppliers. This trade-off between efficiency and rent appropriation vanishes when the buyer can charge sufficiently high participation fees. Then the buyer can induce the social optimum, and use transfer payments or participation fees to make sure that the suppliers just break even. Moreover, when the exogenous differentiation of supplier characteristics is sufficiently pronounced, it may not even be necessary to provide additional incentives for diversification from a social point of view.

Our model can be used to analyze how institutions affect the incentives of individuals to experiment in domains where the optimal approach to solving a given problem is not known. A particularly promising application would be to think of our model as capturing product choice in markets with a unit mass of homogeneous buyers, each of which has unit demand. We can then interpret the uncertainty about the ideal state in two ways. First, it may capture uncertainty about the buyers' taste. Second, it may capture an "engineering uncertainty" where the suppliers know what the buyers would like, but are uncertain about how to achieve this. Either way, the rules of the contest translate directly into a description of the regulatory constraints in a market environment. For instance, the auction corresponds to an unregulated market environment where suppliers choose products under uncertainty about the preferred product and charge prices once qualities are realized. The fixed prize tournament can be interpreted as a regulated market where prices are fixed ex ante: The prize A is then the profit that the firm earns in the market from selling at the regulated price to a unit mass of consumers. Similarly, auctions with reservation prices have a natural interpretation as markets where there is a price cap.

Our results suggest that an unregulated market maximizes expected total surplus, whereas the regulated market maximizes the expected consumer surplus. The unregulated market gives incentives for firms to diversify, but leaves them with market power. The trade-off resembles the trade-off between ex-ante incentives and ex-post monopoly power in the innovation literature. In our case, however, the higher expected quality from the unregulated market does not result from higher innovation incentives at the individual firm level, but rather from the higher diversification incentives at the market level. Price caps strike a balance between the goals of maximizing consumer surplus and total surplus. Augmented FPTs have no obvious counterpart in reality: They would correspond to a regulated environment where firms can select between offering two different prices depending on the realized quality levels. Our analysis suggests that, in some markets, such augmented FPTs may even be better for consumers than full price regulation. These simple considerations clearly have limitations resulting from the rather special market environment. However, the arguments suggest that the contest approach may potentially be valuable to analyze product innovations (or product selection) in market environments. A full analysis of this topic is left for future research.

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9 Appendix: Main Proofs

9.1 Proof of Proposition 2

Using (T1), in an FPT, the expected profit of supplier i, given v_j, is

$$\Pi_i(v_i|v_j) = \begin{cases} A\Pr(|v_i-\sigma| \leq |v_j-\sigma|) & \text{if} \quad v_i \neq v_j \\ A/2 & \text{if} \quad \mathrm{v}_i = v_j \end{cases}.$$

Since $\Pr(|v_i-\sigma| \leq |1/2-\sigma|) < 1/2$ for all $v_i \neq 1/2$, $(v_1,v_2) = (1/2,1/2)$ is an equilibrium. For uniqueness, suppose an equilibrium with $v_i < v_j$ exists. Then $\Pi_i(v_i|v_j) = A\int_0^{(v_i+v_j)/2} dF(\sigma)$. Thus supplier i can increase his expected profit by marginally increasing v_i, since $d\Pi_i(v_i|v_j)/dv_i = Af((v_i+v_j)/2)/2 > 0$. The argument for $v_i > v_j$ is analogous. There can be no equilibrium with $v_i = v_j \neq 1/2$, because $\Pi_i(1/2|v_j) > A/2 = \Pi_i(v_i|v_j)$.

9.2 Proof of Lemma 2

(i) The result is trivial for $v_1 = v_2$. For $v_1 < v_2$, we show that supplier 1 can profitably deviate to some $v_1' > v_1$ if $\Delta q(v_1,v_2) + \underline{P} \notin \mathcal{P}$. Before the deviation, by Lemma 1, if $\sigma \in [0,v_1]$, supplier 1 wins and $\overline{p}(\sigma) < \Delta q(v_1,v_2) + \underline{P}$. By continuity, $\exists\ v_1' \in (v_1,v_2]$ such that $\overline{p}(\sigma) < \Delta q(v_1',v_2) + \underline{P} < \Delta q(v_1,v_2) + \underline{P}$. By deviating to v_1', supplier 1 wins whenever $\sigma < (v_1'+v_2)/2$ rather than when $\sigma < (v_1+v_2)/2$. The set of states in which supplier 1 wins after the deviation thus is a strict superset of the set of states in which the supplier wins before the deviation. For $\sigma \in [0,v_1]$, the price is unaffected. For $\sigma \in (v_1,(v_1'+v_2)/2]$, the price is at least as high as before the deviation. Thus, v_1' is a profitable deviation by (T2).
(ii) follows directly from Lemmas 1 and 2 (i).

9.3 Proof of Proposition 3

Denote the minimum allowable price with $\underline{P}$. If $v_1 \neq v_2$ in equilibrium, by Proposition 2, the contest is not an FPT. Suppose that $v_1 < v_2$. By Lemmas 1 and 2, the buyer pays $q_i - q_j + \underline{P}$ to the supplier with $q_i \geq q_j$ in equilibrium. Thus, for any σ, the buyer payoff is $\min\{q_1,q_2\} - \underline{P}$. Hence, the expected payoff of a buyer who induces $v_1 < v_2$ with $\underline{P}$ is

$$\begin{aligned} \Pi_b(v_1,v_2\,;P) &= \int_0^1 \min\{q_i(v_i,\sigma), q_j(v_j,\sigma)\} dF(\sigma) - \underline{P} \\ &= \int_0^{\frac{v_1+v_2}{2}} q_2(v_2,\sigma)\,dF(\sigma) + \int_{\frac{v_1+v_2}{2}}^1 q_1(v_1,\sigma)\,dF(\sigma) - \underline{P} \end{aligned}$$

Thus

$$\frac{d\Pi_b}{dv_1} = \int_{\frac{v_1+v_2}{2}}^1 \frac{\partial q_1}{\partial v_1} dF(\sigma) > 0;\ \frac{d\Pi_b}{dv_2} = \int_0^{\frac{v_1+v_2}{2}} \frac{\partial q_2}{\partial v_2} dF(\sigma) < 0.$$

Thus, the buyer payoff is maximal for $v_1 = v_2$ and $\underline{P} = 0$. Given $v_1 = v_2$, the buyer payoff is maximal for $v_1 = v_2 = 1/2$, the unique equilibrium of an FPT with A arbitrarily close to 0. Given (T2), it is an equilibrium for $A = 0$.

9.4 Proof of Proposition 4

Sequential rationality of $p_i\left(\right)$ follows from Lemma 1. We now show that $(v_1, p_1\left(\right))$ is a best response of player 1 to $(v_2, p_2\left(\right))$; the argument for player 2 is analogous. For $A = 0$, only $(v_1, v_2) = (1/2, 1/2)$ satisfies the above conditions. Thus, the statement for $A = 0$ follows from Proposition 2. If $v_1 < v_2$, $\Delta q\left(v_1, v_2\right) > 0$, and the probability that player 1 wins with a positive prize is $F\left(v_1\right)$. Deviating to $v_1' < v_1$ is not profitable, because the winning probability falls to $F\left(\widehat{v}_1\right)$, with $\widehat{v}_1 < v_1$ implicitly defined by $q\left(v_1', \widehat{v}_1\right) - q\left(v_2, \widehat{v}_1\right) = \Delta q\left(v_1, v_2\right)$, and the prize does not rise. It is not profitable to deviate to $v_1'' \in (v_1, \widetilde{v})$, where $\widetilde{v} = \min\left(2v_2 - v_1, 1\right) \geq 1/2$: For such deviations, $\Delta q\left(v_1'', v_2\right) < \Delta q\left(\widetilde{v}, v_2\right) \leq \Delta q\left(v_1, v_2\right) \forall \sigma$, so that the probability of winning a positive prize is 0. Finally, if $\widetilde{v} < 1$, deviating to $v_1''' \in [\widetilde{v}, 1]$ is not profitable, because $\widetilde{v} \geq 1/2 + v_2 - v_1$ implies $1 - \widetilde{v} \leq 1/2 - (v_2 - v_1) \leq v_2 - (v_2 - v_1) = v_1$ and therefore, by symmetry of the state distribution, $1 - F\left(v_1'''\right) \leq 1 - F\left(\widetilde{v}\right) \leq F\left(v_1\right)$. By analogous arguments, there are no profitable deviations for supplier 2.

9.5 Proof of Proposition 5

We provide a sketch of the proof here. The complete proof is in the Online Appendix. The result follows from two lemmas. First we show that approaches maximizing buyer's payoffs satisfy $0 < v_1^B \leq \frac{1}{2} \leq v_2^B < 1$. Hence by Proposition 4, they can be implemented with an augmented FPT. To prove this claim, we show that $v_1^B \leq \frac{1}{2} \leq v_2^B$ must hold in any equilibrium, as otherwise the supplier with the lower winning probability can profitably deviate to the uncontested half of $[0, 1]$. $0 < v_1^B \leq v_2^B < 1$ must hold since approaches on the boundary are socially inefficient, and the buyer can either match or increase the total payoff without increasing the transfers to the suppliers. The second lemma shows that a suitable augmented FTP implements (v_1^B, v_2^B) at the lowest expected cost for the buyer, hence maximizing the buyer payoff. The intuition for this follows from Lemma 2, as the payments are minimal in each state.