DISCUSSION PAPER SERIES

No. 10619

THE DEMOGRAPHIC TRANSITION AND THE POSITION OF WOMEN: A MARRIAGE MARKET PERSPECTIVE

V Bhaskar

DEVELOPMENT ECONOMICS

Centre for Economic Policy Research

ISSN 0265-8003

THE DEMOGRAPHIC TRANSITION AND THE POSITION OF WOMEN: A MARRIAGE MARKET PERSPECTIVE

V Bhaskar

Discussion Paper No. 10619
May 2015
Submitted 13 May 2015

Centre for Economic Policy Research
77 Bastwick Street, London EC1V 3PZ, UK
Tel: (44 20) 7183 8801
www.cepr.org

THE DEMOGRAPHIC TRANSITION AND THE POSITION OF WOMEN: A MARRIAGE MARKET PERSPECTIVE†

Abstract

We present international evidence on the marriage market implications of cohort size growth, and set out a theoretical model of how marriage markets adjust to imbalances. Since men marry younger women, secular growth in cohort size worsens the position of women. This effect has been substantial in many Asian countries, and in sub-Saharan Africa. Secular decline in cohort sizes, as is happening in East Asia, improves the position of women. We show that the age gap at marriage will not adjust in order to equilibrate the marriage market in response to persistent imbalances, even though it accommodates transitory shocks. This is true under transferable utility even if age preferences are relatively minor, as well as under non-transferable utility. We examine the distributional consequences on the sexes, and on dowry payments.

JEL Classification: J12, J13 and J16
Keywords: dowry, sex ratio, marriage markets, marriage squeeze and stable matching

V Bhaskar v.bhaskar@austin.utexas.edu
Department of Economics, University of Texas at Austin, and CEPR

---

† I am grateful for comments from Tim Dyson and many seminar audiences, and to Dhruva Bhaskar and Ahmed-Al-Khwaja for research assistance. I am grateful to the British Academy-Leverhume Trust for its support, and to the Institute for Advanced Study Princeton for its hospitality.

1 Introduction

The demographic transition has major implications for the position of women within the family, and in society. Fertility declines as parents trade "quantity for quality" in their children, and investment in human capital becomes increasingly important. Doepke and Tertilt (2009) argue that the increased importance of human capital investments played an important role in the advancement of women's rights, by making men became more willing to relinquish their monopoly on property rights. Fernandez (2010) sets out a model where the decline in fertility plays an important role in the advancement of women's rights, and uses data from the state-wise variation in the granting of property rights in the US to test this model. While the broad brush picture of the world in the last two centuries indicates a dramatic expansion in women's rights, within marriage and more generally in society, this trend has not been as universal or comprehensive as might be expected. The developing world offers a more nuanced picture. In India, concerns have been raised about "missing women", as well as the trend of rising dowries. In sub-Saharan Africa, polygyny continues to flourish despite modernization and economic development.

The paper focuses on the implications of the demographic transition for the marriage market, and consequently, upon the position of women. This appears not to have been previously been appreciated, although demographers have pointed out some aspects of this picture. Our focus is on the rate at which marriage cohorts grow, in the different phases of the demographic transition.[1] Given that men marry younger women, systematic growth in cohort sizes implies that each cohort of men is matched with a larger cohort of women, giving rise to a *marriage squeeze* on women, i.e. their excess supply.[2] Similarly, systematic decline in cohort sizes imply a reverse marriage squeeze, where men are in excess supply. The consequent marriage market effects are substantial. Deferring the detailed evidence to section II, the stylized facts are as follows. In phase I, which covers most human history, cohort sizes are basically static or growing very slowly, and effective sex ratios are balanced. In phase II, with the decline in mortality, especially infant mortality, cohorts grow rapidly, at 2-3% per annum in many developing countries. With an age gap at marriage of 4-5 years, this translates into a 8-15% increase in the effective supply of women, as compared to phase

[1] While the demographic transition is normally phrased in terms of population growth, the relevant variable for the marriage market is the rate of growth in marriage cohort size, which often differs quite substantially from population growth. For example, the Chinese population is still growing but marriage cohorts are rapidly shrinking.

[2] Demographers use the term marriage squeeze (see Akers, 1977; Schoen, 1984) to denote a marriage market imbalance, due to the effective excess of women or men. This may arise due to shocks to the marriage market sex ratios, e.g. due to wars or due to variations in the sex ratio at birth, or transitory shocks to cohort size, due to baby booms or famines, give rise to imbalances.

I. In phase III, cohort size growth becomes either zero or significantly negative, at -1 to -2% per annum, implying an increase in the supply of men of 4-10% as compared to phase I. As compared to phase II, the change can be as much as 25%.

How does the marriage market adjust to such large imbalances. One obvious mechanism is polygamy – i.e. polygyny in phase II, when there is an excess of women, and polyandry in phase III, when there is an excess of men. This can be an important factor, especially in societies where legal and social sanctions against polygamy are absent. For example, in sub-Saharan Africa, cohort size growth continues to be substantial, implying an significant excess of women in the marriage market relative to the number of men. Thus demographic trends could explain the surprising persistence of polygyny in sub-Saharan Africa, despite modernization. Polygamy may not, however, be very appealing for the more abundant sex, and in this case, it utility consequences may not be very different from non-marriage.

If we abstract from polygamy each man can be matched to at most one woman, and vice-versa. The key potential margin of adjustment is via the age gap at marriage. If the age gap at marriage were to adjust in response to marriage market imbalances, these imbalances could be reduced, and even substantially eliminated. A reduction in the age gap at marriage reduces both the excess of women in phase II and the excess of men in phase III. While the age gap at marriage tends to fall in the process of development, as women become more educated, our question is whether, the age gap falls *endogenously*, in response to marriage market imbalance. The existing literature suggests that the age gap does adjust to reduce imbalances. Bergstrom and Lam (1989a, 1989b) and Brandt, Siow and Vogel (2008) have examined the effects of temporary shocks to birth cohort size, using data from Sweden and from the Chinese famine of 1959-61 respectively. These papers use a transferable utility assignment model in the tradition of Shapley and Shubik (1972) and Becker (1981), and find that marriage markets display considerable flexibility – the age gap at marriage adjusts in order to accommodate large shocks to cohort size. The empirical findings of Brandt et al. are particularly noteworthy, given that the Chinese famine reduced cohort sizes by 75%.

In the light of these results, one might be sanguine about the how the marriage market adjusts to secular growth in cohorts, in the demographic transition. However, our analysis shows that there is a considerable difference between the marriage squeeze due to temporary shocks, and that arising from *systematic growth* in cohort size. We find that with transferable utility, the age gap at marriage is *completely* insensitive to systematic marriage market imbalances. Moreover, this is true no matter how small the relative weight of age preferences is relative to other considerations such as wealth or attractiveness. If age related preferences play a relatively minor role in men's and women's utility, a social planner could greatly reduce imbalances by mandating a reduced age gap, with only minor negative utility consequences.

However, such an outcome would not arise in a marriage market equilibrium.

Why is it that the age gap adjusts to transitory shocks in cohort size, but not to secular growth? Suppose that a shock reduces the size of the cohort indexed by date t. If the age gap at marriage is τ, this results in an excess supply of men at date $t-\tau$ (and an excess supply of women at $t+\tau$). Thus attractive men from date $t-\tau$ now become available. Women from cohorts that are adjacent to that affected by the shock (i.e. $t-1$ and $t+1$) now

have more attractive options, and would prefer to match with these men rather than their "customary" match. Thus the age gap adjusts, and the magnitude of adjustment is greater the smaller the weight of age related preference relative to other considerations. Thus if considerations such as wealth, education, attractiveness or agreeability are more important in marriage than considerations of age, we should expect that marriage market imbalances will be considerably mitigated by age gap adjustments. In consequences, the distributional consequences of cohort size fluctuations upon the sexes will be small.

Consider now secular positive cohort growth, at some rate $g > 0$. The equilibrium age gap will be the preferred age gap τ^*,[3] no matter how large g is, and no matter how much of a marriage market imbalance that this causes. Indeed, the equilibrium age gap will be τ^* even if growth were to be negative. The basic reason is that there is no margin for profitable adjustment available. In a steady state equilibrium, there will be an excess of women in every cohort. So the unmatched women from cohort t are as attractive as unmatched women from any other cohort $\tilde{t}$. This implies that there is no incentive to choose a woman other than from ideal age gap, τ^*. This is true *no matter how weak the preferences for a specific age gap are*, relative to other characteristics. While our main analysis assumes transferable utility, our substantive finding, that the age gap does not necessarily adjust to reduce imbalances, is robust. With non-transferable utility, the age gap that is preferred by the short-side of the market will prevail. Thus, the equilibrium age gap may well respond to changes in marriage market conditions, but there is no guarantee that the direction of adjustment will such that imbalances are reduced – they may well be aggravated.

Given that quantities do not adjust, the burden of adjustment falls entirely on prices. That is, the imbalances improve the position of the scarcer sex, and harm that of the abundant sex. Thus cohort growth has adverse consequences for the position of women in phase II of the demographic transition. Conversely, our analysis shows that this picture will soon be reversed in many developing countries (and this is already the case in East Asia), since falling cohort growth and the shortage of women improves their position. Thus our analysis highlights that the demographic transition has important implications for the balance

[3]More precisely, τ^* is defined as the age gap that maximizes the sum of payoffs in the match.

of power and resources between the sexes. It suggests that the cause of women's equality has been hindered by high cohort size growth, but that this may now be reversed in many countries, as cohort growth becomes negative.

Changes in the effective excess supply of women have major implications for the balance of power between the sexes and for the allocation of resources within the household. Angrist (2002) and Chiappori, Fortin and Lacroix (2001) find important effects on female labor supply and household allocation for smaller changes in marriage market balance, arising from shocks. In the Indian context, demographers such as Bhatt and Halli (1999) have argued that the marriage squeeze is responsible for the deterioration of the position of women in India, and replacement of the institution of bride price in many regions and communities by dowries (payment from the bride's family to the groom). Rao (1993) analyzes data on dowries from a sample of Indian villages and attributes the increase in dowries in India to the marriage squeeze. [4] Anderson (2007a) examines the time-path of dowry payments in response to one-time shock to cohort size. Our analysis suggests that the utility consequences of imbalances on the sexes are large. However, we also show that the effects on dowries may be more nuanced than has hitherto been suggested, since the change in the effective sex ratio improves the match quality of the partner for any given type of man, and the extent of this improvement is not uniform across the types of men.

Our focus on marriage market flows differs from that of the large literature on the number of "missing women" in the population stock (see Sen (1990), Coale (1991) and Anderson and Ray (2010), for a very partial list). This is not to deny the importance of missing women overall, but rather because they are unlikely to have similar behavioral consequences.[5]

This paper is related to two main strands of literature. First, there is the literature on marriage markets, following Gale and Shapley (1962) who assume non-transferable utility and Shapley and Shubik (1972) and Becker (1981), who assume transferable utility. More specifically, there is the literature on the marriage squeeze, by demographers as well as economists, including Akers (1977), Schoen (1983), Bergstrom and Lam (1989a, 1989b), Bhatt and Halli (1999), Anderson (2007a), Brandt, Siow and Vogel (2008). Second, our work is related to a large volume of work on empirical work on the sex ratio. Apart from the literature on missing women, Neelakantan and Tertilt (2008) is particularly relevant since they focus on the marriage market.

The organization of the rest of the paper is as follows. Section 2 sets out the empirical evidence on cohort growth and marriage market balance in number of developing countries

[4] One problem that empirical researchers face is that dowry payments are illegal in India, even though there are widespread reports of the law being flouted.

[5] One could argue that if women are more abundant, their greater political voice may ensure that public policy is more female friendly; however, this mechanism reinforces imbalances rather than correcting them.

in Asia and Africa. Section 3 sets out a transferable utility model of the marriage market, and examines whether the marriage market permits an adjustment mechanism, via the endogeneity of the age gap. Section 4 considers non-transferable utility. Section 5 examines the distributional consequences of marriage market imbalances in a transferable utility setting. It also examines the effects on dowries, and suggests a cautionary note – the effect on dowries may not be the identical with the effect on payoffs, since the consequent change in matching patterns may directly affect premuneration values. The appendix provides details of the formal proofs that are not dealt with in the body of the paper.

2 Marriage market balance

The effective supply-demand situation in the marriage market depends upon the sex ratio at birth, and upon rate of growth of female cohorts relative to the males that they are matched with. Consider a society where cohort sizes are growing at an annual rate g. Let the age gap at marriage, between men and women, be τ years. The *required number of boys*, R, per 100 girls, is given by

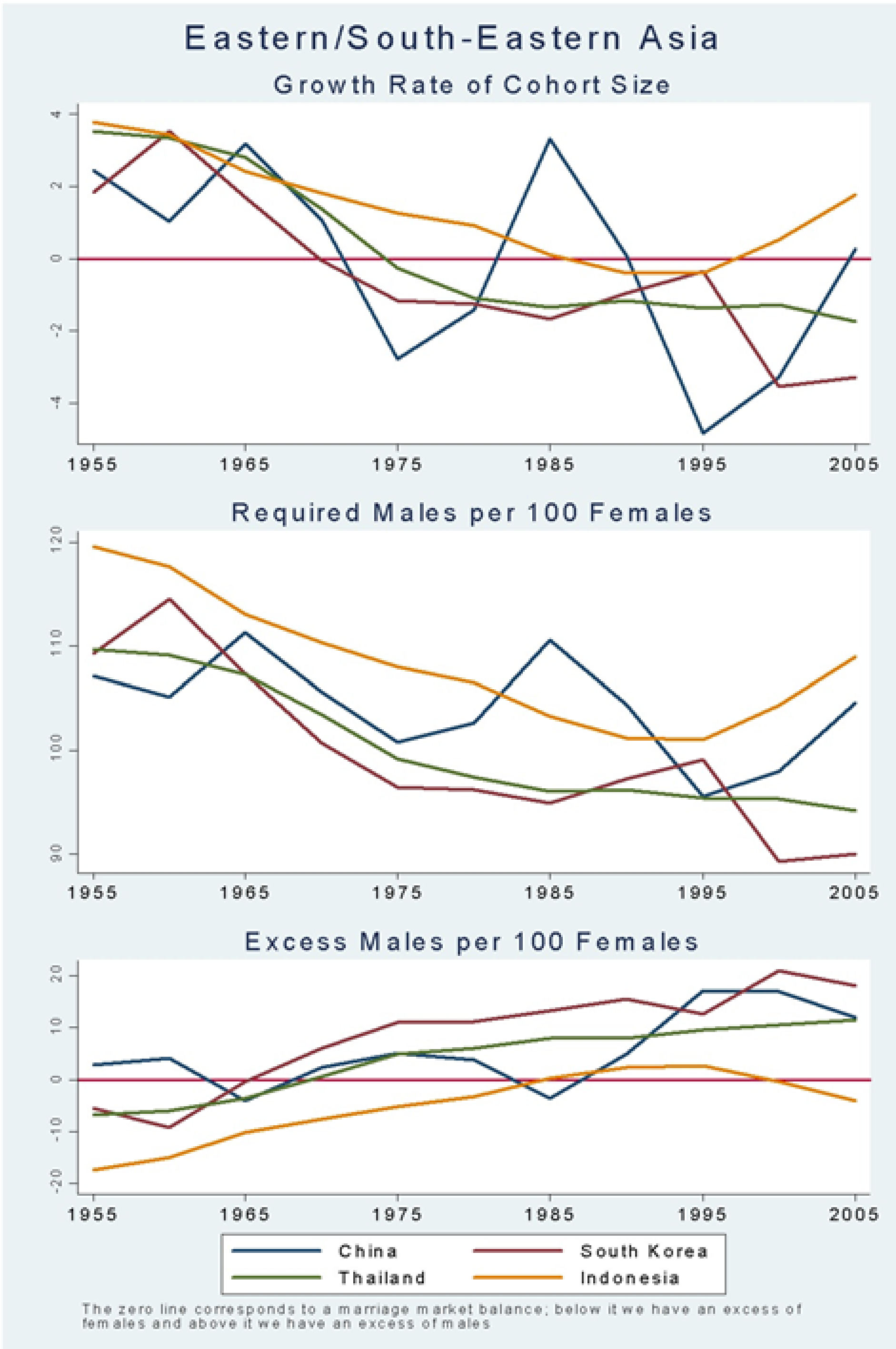
$$R = 100(1+g)^{\tau}\frac{\Delta_G}{\Delta_B}\frac{\lambda_G}{\lambda_B},$$

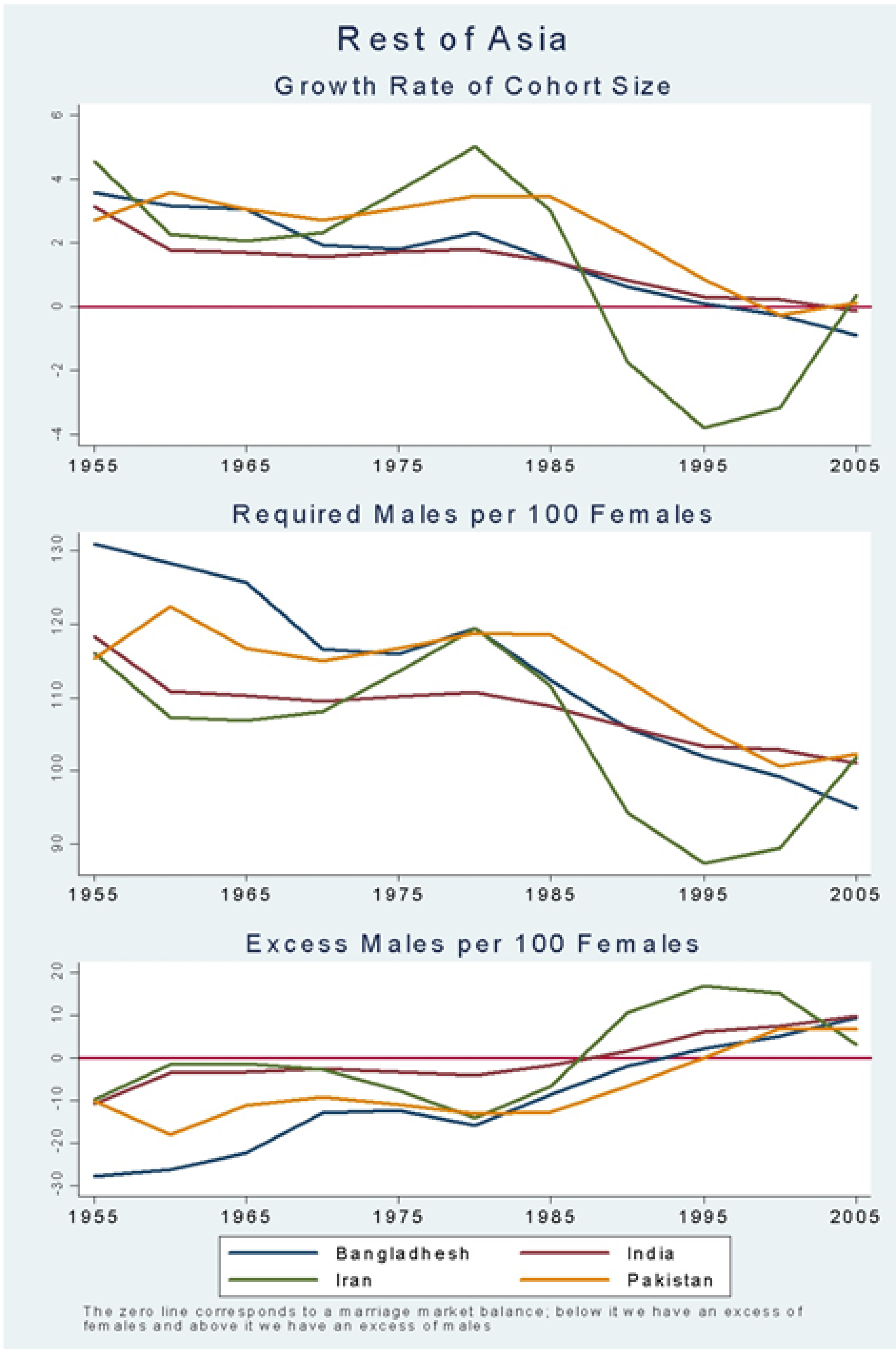
where Δ_B is the survival rate for boys, between infancy and the age of marriage, and Δ_G is that for girls. λ_G (resp. λ_B) is the proportion of girls (resp. boys) who would like to marry.[6] Thus if the marriage market is to be balanced, the actual number of boys per hundred girls, A, must be close to or equal to R. The difference $G = A - R$ measures the extent to which there is an excess of men (or missing women) on the marriage market.

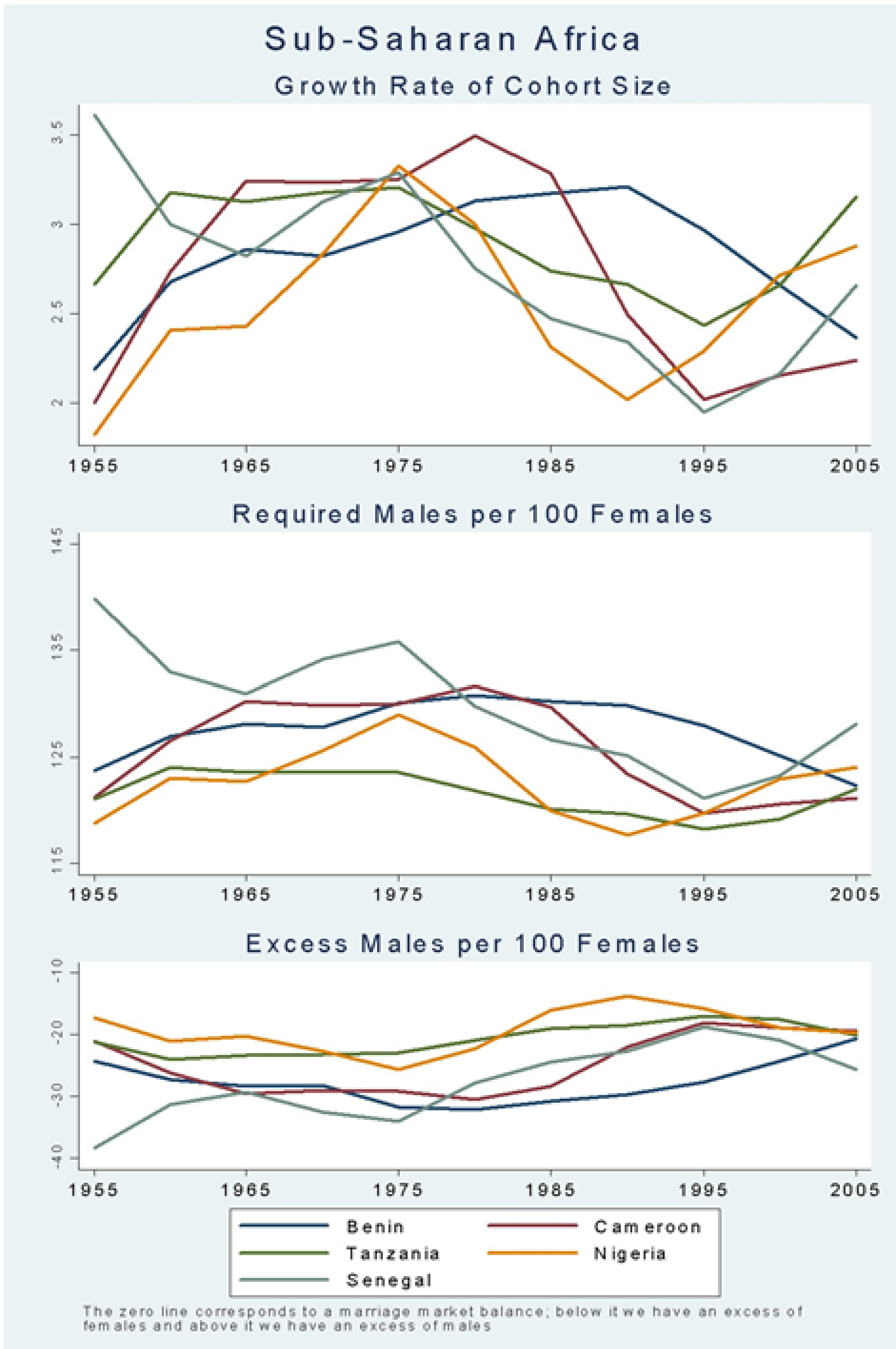
We now present estimates from a range of countries in Asia and Africa on g, R, and G. Population growth is estimated using the UN's "World Population Prospects: The 2012 Revision" data on population by sex in the age group 0-4, at five yearly intervals starting 1950. Since cohort data is noisy, g for any year t is based on a regression estimate of the growth rate using observations at t, $t-5$ and $t+5$. To estimate survival rates, we use infant mortality data for each year from the same data set, since this is the main component of mortality. Since it is problematic to use observed data on the proportions marrying in order to estimate λ_G and λ_B, we use the singles rate in the age group 50-54 from the UN's "World Marriage Data 2012". The age gap is also from this data set, and based on the difference between the singulate mean ages at marriage for men and women. The singles rate and the

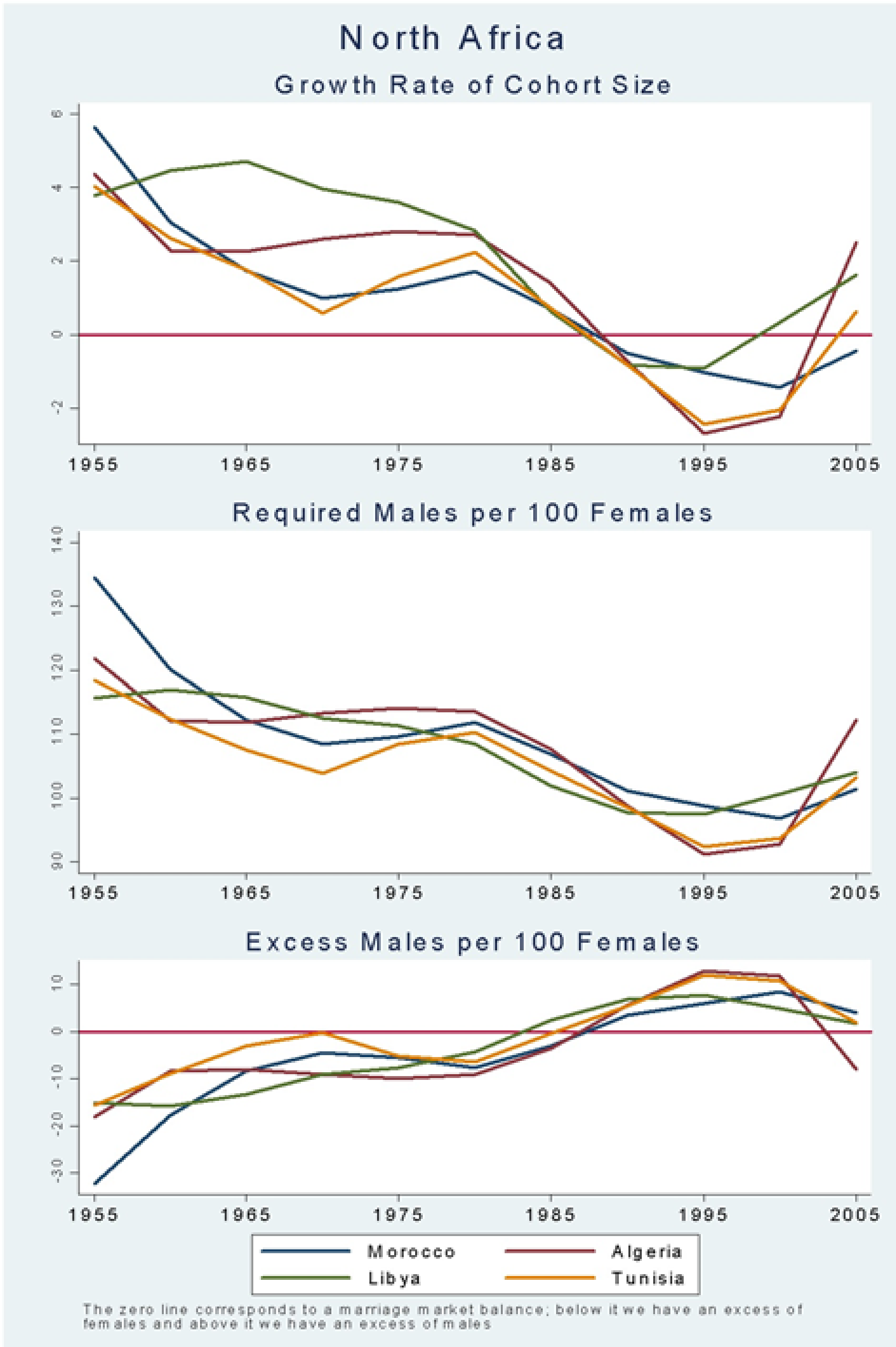
[6]This is valid if λ_G and λ_B are the proportions who *desire* marriage. However, in empirical work, demographers use the observed proportions, and we have some reservations about this, since the actual proportions of women and men that marry will reflect marriage market conditions, i.e. will be endogenous. As we shall see, our substantive conclusions are not much affected by this adjustment.

age gap is taken at a fixed year – 1990, or the closest year with available data.




Rest of Asia
Growth Rate of Cohort Size
Required Males per 100 Females
Excess Males per 100 Females
Bangladhesh
India
Iran
Pakistan
The zero line corresponds to a marriage market balance; below it we have an excess of females and above it we have an excess of males


Sub-Saharan Africa
Growth Rate of Cohort Size
3.5
3
2.5
2
1955
1965
1975
1985
1995
2005
Required Males per 100 Females
145
135
125
115
1955
1965
1975
1985
1995
2005
Excess Males per 100 Females
-10
-20
-30
-40
1955
1965
1975
1985
1995
2005
Benin
Cameroon
Tanzania
Nigeria
Senegal
The zero line corresponds to a marriage market balance; below it we have an excess of females and above it we have an excess of males


North Africa
Growth Rate of Cohort Size
Required Males per 100 Females
Excess Males per 100 Females
1955
1965
1975
1985
1995
2005
Morocco
Algeria
Libya
Tunisia
The zero line corresponds to a marriage market balance; below it we have an excess of females and above it we have an excess of males

The charts in figures 1-4 present the evolution of g, R and G in four groups of countries: East/South East Asia, South Asia and Iran, North Africa, and sub-Saharan Africa. Within each group we present a selection of countries, mainly to economize on space – we have constructed these measures for all countries in Asia and Africa, and the trends there are similar. The main features that these charts illustrate are as follows:

- Cohorts were growing rapidly in the 1950s and 1960s, in almost all developing counties, and this resulted in a large excess supply of women in the 1970s and 1980s.
- This phase of cohort growth ended around the 1970s (i.e. among the marriage cohorts of the 1990s) in East Asia. In other Asian countries such as India and Pakistan, and in North Africa (e.g. Egypt), it continued into the 1990s, and appears to be ending only now. Thus the marriage markets today, in South Asia and North Africa, are still in a situation where there is an excess supply of women.
- Most striking is the continuation of extremely large cohort growth in sub-Saharan Africa to the present day, giving rise to a large excess of women in the next 10-20 years.
- Cohorts are shrinking rapidly in East Asia, especially in China and South Korea. This is aggravating the problem of sex ratio imbalances at birth in China. While this trend is most pronounced in East Asia, it is also apparent elsewhere, e.g. in Tunisia.

3 A model of the equilibrium age gap

Our empirical findings raise several questions. The first relates to the endogeneity of the age gap. Our estimates in the previous section assume that the age gap at marriage, τ, is exogenous. Clearly, τ may change, for exogenous reasons. As women become more educated, their marriage age increases, while that of men does not increase by the same amount. Does the age gap at marriage adjust in order to help "clear" the marriage market? Economic "intuition" suggests that it should do so, at least on reading much of the literature on the subject. If this is the case, this could restore balance in the marriage market. A reduction in the age gap would help reduce the excess supply of women that is predicted to prevail in most parts of India. Similarly, even in China, the large excess in the actual number of boys could be reduced if the age gap fell, and even more if men began marrying older women. This point assumes greater relevance in view of the existing literature on how the age gap responds to the marriage squeeze. The pioneering work in this regard is by Bergstrom and

Lam (1989a, 1989b), who study 19th century Sweden, where there were large fluctuations in the effective sex ratios across marriage cohorts, due to the age gap at marriage and the fluctuations in the size of birth cohorts. They set out an assignment model of the marriage market with transferable utility, in the style of Shapley-Shubik (1972) and Becker (1981), and conclude that marriage markets showed considerable flexibility, in the sense that age gaps at marriage adjusted relatively easily in order to clear marriage markets. More recently, Brandt, Siow and Vogel (2008) use a similar transferable utility framework to examine the effects of the large shocks to cohort size arising from the Chinese famine of 1959-61. They find that participants in the marriage market showed considerable flexibility, so that the marriage rates of the cohorts who were matched with the famine affected cohorts were damaged, but not to the extent that one might imagine.

A positive age gap seems a very pervasive phenomenon – data from the United Nations (1990), from over 90 countries, show that the difference between the ages at first marriage for men and women is positive in every single country, for every time period. This suggests that both men and women prefer a positive age gap,[7] and such preferences may have an evolutionary foundation. Kaplan and Robson (2003) present evidence from hunter-gatherer societies on age-productivity profiles. The productivity of women in gathering is relatively constant over time, while the productivity of men in hunting rises sharply from a low initial base, so that between the ages of 25 and 50, men produce a large surplus relative to their consumption requirements. In addition, women reach peak fertility relatively quickly, and their fertility declines more rapidly than that of men. In this paper, we shall assume that the pervasiveness of a positive age gap is grounded in preferences, of men as well as women. This may have evolutionary foundations. With development, as women become more educated and take on paid employment, they will prefer to delay marriage, while the preferred age of marriage for men may not rise as much. Thus the preferred age gap may fall, for exogenous reasons. An alternative approach is set out by Bergstrom and Bagnoli (1990) who attribute the age gap to the differential roles of women and men in marriage, with the suitability of women for their role (the production of offspring) being revealed earlier, while that of men in their role (bread-winning) being revealed only later. We have also briefly examined how the age gap in the Bergstrom-Bagnoli model adjusts to imbalances, and find results that are broadly consistent with those reported here.

[7] If men preferred a positive age gap while women preferred a negative age gap, then one would expect a negative age gap to emerge in marriage markets where women are in short supply if there are limitations on the transferability of utility – see section 4.

3.1 The model

Let time be discrete, and index it by the integers, positive and negative.[8] We assume a continuum population, that grows at a constant rate g per year. At each date, the relative measure of women to men and women equals $\bar{r}$, i.e. $\bar{r}$ is the sex ratio at birth or within each cohort. There are two dimensions to an individual's marriage market characteristics and preferences. Age constitutes a "horizontal dimension of differentiation, and "quality" is the vertical dimension. We assume that preferences are stationary, i.e. they do not depend on the time index of the individuals.

In a transferable utility environment, what matters is the total payoff (or marital surplus), Q, that can be generated by a couple. This depends on the qualities of the two individuals and the age gap between them. Assume that the quality of a man in any cohort, ε, is distributed with a continuous, strictly increasing cumulative distribution function $F(.)$ on $[\varepsilon_{\min}, \varepsilon_{\max}]$.Similarly, the quality of a woman, η, is distributed with with a continuous, strictly increasing cumulative distribution function $G(.)$ on $[\eta_{\min}, \eta_{\max}]$. Let τ denote the age gap, i.e. the difference between the man's age and the woman's age. Thus Q is a function of $(\varepsilon, \eta, \tau)$. We assume that it is continuous and strictly increasing in the two quality dimensions (ε and η). We also assume that for any (ε, η), Q is maximized at a single value of τ, denoted by τ^*, where $\tau^* > 0$. That is, match surplus is maximized at a positive age gap. [9] This assumption is compatible with men and women having different preferences over τ, with differing ideal points τ_B and τ_G, with τ^* lying intermediate between these ideal points. In a transferable utility setting, what matters is the total match surplus Q, since this can be split up between the two parties in any way they choose. The payoffs from being single are $\bar{u}$ for a man and $\bar{v}$ for a woman. We may, without loss of generality, assume that these reservation values do not depend on the type of the individual.[10] We state our main assumptions as follows:

Assumption 1. $Q(\varepsilon, \eta, \tau)$ is continuous and strictly increasing in ε and η, and is maximized at a single value τ^* for any (ε, η).

Assumption 2. $Q(\varepsilon_{\min}, \eta_{\min}, \tau^*) < \bar{u} + \bar{v} < Q(\varepsilon_{\max}, \eta_{\max}, \tau^*)$.

Assumption 1 is required for our results. Assumption 2 is not essential, but is invoked in our examples – since there are matched as well as unmatched individuals in equilibrium, it ensures that equilibrium payoffs are unique.

Let M be the set of men with generic element m, and let W be the set of women with

[8] Since we are focusing on steady states, we assume that time extends indefinitely, backwards and forwards, thus avoiding any initial date effects that would preclude existence of steady state.

[9] Given the symmetric role of the sexes in our model, it is clear that our formal results would not differ if τ^* was strictly negative.

[10] If reservation values are type dependent, we may re-define the match payoff as Q minus the sum of reservation values.

generic element w. Each element of M has a pair of characteristics, (ε, t), i.e. has a quality type and a cohort date. A matching ϕ is a one-to-one correspondence from the set $\mathsf{M} \cup \mathsf{W}$ to itself that satisfies:

i) $\phi(\phi(x)) = x$,

ii) $\phi(m) \neq m \Rightarrow \phi(m) \in \mathsf{W}$ and $\phi(w) \neq w \Rightarrow \phi(w) \in \mathsf{M}$.

Further, the matching is measure preserving: if X is any finite Lebesgue measure subset of $\mathsf{M} \cup \mathsf{W}$, the Lebesgue measure of X equals the Lebesgue measure of $\phi(\mathsf{X})$.

Associated with any matching is a payoff allocation, $(u : \mathsf{M} \to \mathbb{R}, v : \mathsf{W} \to \mathbb{R})$. The payoff allocation must be feasible, i.e. for any matched pair (m, w),

$$u(m) + v(w) = Q(\varepsilon(m), \eta(w), \tau(m, w)).$$

For an unmatched individual, the payoff allocation equals his/her payoff from being single.

We now turn to equilibrium matchings. We focus on *stable, steady-state* matchings. Stability implies that $\forall m \in \mathsf{M}$, $u(m) \geq \bar{u}$ and $\forall w \in \mathsf{W}, v(w) \geq \bar{v}$, i.e. any individual gets at least the payoff from being single. Secondly, for any (m, w) who are not matched to each other, $u(m) + v(w) \geq Q(\varepsilon(m), \eta(w), \tau(m, w))$. That is, the payoff allocations of this pair must be weakly greater than the total match value that they could generate by matching together, since otherwise, this pair would have a profitable pair-wise deviation from the matching. A *stable payoff allocation* is the allocation associated with a stable match.

Our final requirement is that we shall restrict attention, in stable matchings, to steady states, where the matching pattern is stationary. Let x and x' be individuals of the same sex that are of identical quality, and differ only on the cohort dimension (for example, x is a man of type (ε, t) and x' has type (ε, t')). The steady state assumption requires that $\phi(x)$ and $\phi(x')$ only differ on the cohort dimension, and the difference equals $t - t'$: if $\phi(x)$ has type $(\eta, \hat{t})$ then $\phi(x')$ has type $(\eta, \hat{t} + t - t')$. Also, if $\phi(x) = x$ then $\phi(x') = x'$. Note that the steady state requirement applies only to equilibrium matches – deviating couples could have an arbitrary age gap.

The simplest steady state matchings are monomorphic steady states, where every couple that is matched has the same age gap τ. In a monomorphic steady state with age gap τ, men born at date t will be matched with women born at $t + \tau$, and the ratio of the latter to the former will equal $r = \bar{r}(1 + g)^{\tau}$. Since the matching must be measure preserving, if $r > 1$, a there will be more unmatched females than males in every cohort, while if $r < 1$, the reverse is the case.

We are now in a position to state the main result of this section, but before doing this, let us put it in context. Shapley and Shubik (1972) analyze the assignment problem when there are finitely many agents – i.e. the problem of finding the matching that maximizes total surplus. They formulate this as a linear-programming problem, and show that the dual of this linear programming problem gives the payoffs corresponding to a stable matching. See Roth and Sotomayor (1992) for a lucid exposition. Gretsky, Ostroy and Zame (1992) extend this result to continuum economies with a finite measure of agents. Our society has a countable infinity of dates, so that the total measure is not finite. In consequence, total surplus is not well defined, since the infinite sum of payoffs associated with an allocation is not well defined, the series being non-convergent. This precludes using the existing results, and our proof of existence and uniqueness must be done directly. By using the results of Gretsky, Ostroy and Zame, we construct a stable allocation in the static context, and use this to show the existence of a stable matching in the dynamic setting. We also verify that no other steady state age gap is stable.

Proposition 1 *Under assumption 1, a steady state stable match exists; the age gap equals τ^*, independent of g, in any steady state match.*

Proof. Consider a static matching problem where men from fixed cohort $\hat{t}$ are matched with women from cohort $\hat{t}+\tau^*$, so that the measure of women relative to men equals $r^* = \bar{r}(1+g)^{\tau^*}$. By Gretsky, Ostroy and Zame (1992), there exists a stable match $\tilde{\phi}$, with associated payoffs $u^*(\varepsilon)$ for every type ε of man and $v^*(\eta)$ for every type η of woman. Now consider the dynamic matching problem, and construct the matching ϕ from $\tilde{\phi}$ as follows. For any man of type (ε, t), let this man be matched with a woman of type $(\tilde{\phi}(\varepsilon), t + \tau^*)$. If the man is unmatched under $\tilde{\phi}$, he is also unmatched under ϕ. Let the payoff allocation be $u^*(\varepsilon), v^*(\eta)$, i.e. it identical with the payoff allocation in the static matching. It is straightforward to verify that this payoff allocation is feasible. We now show that this steady state is stable. Consider any pair of qualities (ε, η) with age gap τ^*. Since the static allocation is stable, $Q(\varepsilon, \eta, \tau^*) \leq u^*(\varepsilon) + v^*(\eta)$, implying that the dynamic allocation cannot be blocked by this pair. Now consider any pair of qualities (ε, η) with age gap $\tau \neq \tau^*$. Since $Q(\varepsilon, \eta, \tau) < Q(\varepsilon, \eta, \tau^*) \leq u^*(\varepsilon) + v^*(\eta)$, this is also not a blocking pair. Thus the allocation is stable.

To show that there is no other stable steady state, suppose that there is a polymorphic steady state where some type ε is matched to type η with age gap $\tau \neq \tau^*$. Let the payoff allocation for this pair be $u(\varepsilon), v(\eta)$, where $u(\varepsilon) + v(\eta) = Q(\varepsilon, \eta, \tau) < Q(\varepsilon, \eta, \tau^*)$. Suppose that a man of quality ε is matched with a woman of quality η under this matching, and earn payoffs $u(\varepsilon)$ and $v(\eta)$ respectively, where $u(\varepsilon)+v(\eta) = Q(\varepsilon, \eta, \tau) < Q(\varepsilon, \eta, \tau^*)$. Then the man of quality ε in an arbitrary cohort can propose a match to the woman of the same quality

η, but with age gap τ^*. Since the total payoff of this match is $Q(\varepsilon, \eta, \tau^*) > u(\varepsilon) + v(\eta)$, the candidate matching and allocation is not stable. ■

It is noteworthy that this invariance result does not depend upon how important age related preferences are relative to the quality dimension (ε, η) in the overall match payoff function $Q(.)$. For instance, considerations of age could be arbitrarily unimportant, so that deviations from τ^* reduce Q very little. However, a reduction in τ below τ^* could significantly reduce the number of matches.

While this model predicts complete uniformity in the age gap at marriage, more realistic results can be obtained by allowing heterogeneity in preferences. If we assume that age preferences are uncorrelated with quality, then there would be a distribution of age gaps in the steady state, and similar conclusions would follow. Choo and Siow (2006) set out a model where the underlying preferences are perturbed by shocks that have an extreme value distribution.

The main content of proposition 1 is that the age gap *does not adjust* to changes in g or $\bar{r}$ that result in a marriage market imbalance. That is, the simple intuition of supply and demand does not operate in the context of the marriage market, due to the indivisibility in the assignment problem, whereby one man can be assigned to at most one woman, and vice versa. Under transferable utility – i.e. precisely the assumptions made by Bergstrom-Lam and Brandt et al. – the age gap does not adjust in response to changes either in the sex ratio at birth or changes in the rate of growth of cohort size. Indeed, this result applies even if preferences regarding the age gap have relatively little weight in the utility functions of men and women. Things are quite different if we consider a *transitory shock* to cohort size, as we will see in the next sub-section.

3.2 Some examples

An example, with positive assortative matching on the quality dimension, is illustrative. Let Q be strictly supermodular in (ε, η), and let assumptions 1 and 2 hold. Recall that $r^* = \bar{r}(1+g)^{\tau^*}$ is the equilibrium sex ratio in the marriage market. Let $\tilde{\varepsilon}$ and $\tilde{\eta}$ denote the lowest matched types of men and women respectively, where $\tilde{\phi}(\tilde{\varepsilon}) = \tilde{\eta}$. $\tilde{\varepsilon}$ and $\tilde{\eta}$ are the unique solutions to the pair of equations:

$$1 - F(\tilde{\varepsilon}) = r^*[1 - G(\tilde{\eta})]), \tag{1}$$

$$Q(\tilde{\varepsilon}, \tilde{\eta}, \tau^*) = \bar{u} + \bar{v}. \tag{2}$$

Let matching be assortative, with $\tilde{\phi}(\varepsilon) \geq \tilde{\eta}$ being defined, for $\varepsilon \geq \tilde{\varepsilon}$ by

$$1 - F(\varepsilon) = r^*[1 - G(\tilde{\phi}(\varepsilon))]. \tag{3}$$

Having defined payoffs for the marginal types in the market, we now turn to the rest. Let $v^*(\eta)$ denote the payoff of a woman of type η. Let $\tilde{\phi}(\varepsilon)$ denote the match of a man of type ε. Stability implies that $\tilde{\phi}(\varepsilon)$ is the optimal choice for a man of type ε, i.e.

$$\arg\max_{\eta} [Q(\varepsilon, \eta, \tau^*) - v^*(\eta)] = \tilde{\phi}(\varepsilon).$$

The first order condition for this maximization problem gives us the differential equation

$$Q_\eta(\varepsilon, \tilde{\phi}(\varepsilon), \tau^*) = v^{*\prime}(\eta).$$

The solution to this differential equation, in conjunction with the boundary condition $v^*(\tilde{\eta}) = \bar{v}$, gives us the payoffs all types of matched women. The payoff to men equals the residual, $Q(\varepsilon, \tilde{\phi}(\varepsilon), \tau^*) - v(\tilde{\phi}(\varepsilon))$. The payoffs to unmatched men, of type $\varepsilon < \tilde{\varepsilon}$, equals $\bar{u}$. The payoff to unmatched women, of type less than $\tilde{\eta}$, equals $\bar{v}$. This defines a payoff allocation, for every type of man and woman. Let us denote this payoff allocation by (u^*, v^*). Note that this payoff allocation depends upon the sex ratio r^*, which in turn depends upon the age gap. Finally, note that supermodularity, in conjunction with assumption 1 (which ensures that there are always unmatched types on both sides) ensures an essentially unique steady state stable matching, and a unique steady state payoff allocation, (u^*, v^*).

3.3 Adjustment to transitory shocks

We now consider imbalances arising from transitory shocks and show that the pattern of adjustment is very different, as compared to the case of trend growth. In particular, if there is a transitory imbalance in one cohort - e.g. because there is a surplus of women in that cohort – then the age gap will adjust away from desired age gap τ^*, both in the directly affected cohort and nearby cohorts. In consequence, the adverse effect on the marriage rate for women in the affected cohort will be mitigated. The positive effect on the marriage rate for men who are of the ideal age gap from the cohort of affected women will also be mitigated. In consequence, the distributional consequences of the shock on the sexes will be mitigated. If preferences regarding age are less important, relative to considerations of quality, then the shock is spread over a larger number of cohorts, reducing its distributional impact. Thus, transitory shocks have very different effects are compared to secular cohort growth.

For simplicity, consider an infinite horizon economy without any trend growth, so that $g = 0$. Thus there are equal measures of men and women at every date, with the quality of men, ε, distributed according to $F(.)$, while that of the women, η, is distributed according to G. Now let us consider a shock, where the measure of women at date 0 equals $1 + \Delta$, where $\Delta > 0$.

The first question is, how does one solve for a stable matching in this infinite horizon economy? Clearly, the method we used in order to solve for the steady state cannot be directly applied. However, there is generalization of this method that can be used. Let us assume that the matching pattern is affected by this shock only for finitely many cohorts of women, those in cohorts lying between $-K$ and K. Similarly, assume that only men in cohorts ranging between $\tau^* - K$ and $\tau^* + K$ are affected. Assume that the matching is unaffected by the shock for cohorts lying outside this range, i.e. any cohort of women t such that $t < -K$ or $t > K$ is matched with the cohort of men $t + \tau^*$ according to the steady state matching ϕ. The remaining affected cohorts are of finite measure, and the stable matching when restricted to this cohorts is given by the solution to linear programming assignment problem. Finally, one needs to verify that the resulting allocation is stable even when the set of affected cohorts is allowed to match outside the specified range.

To solve for the resulting equilibrium explicitly, assume that Q is given by

$$Q(\varepsilon, \eta, \tau) = \varepsilon + \eta - h(|\tau - \tau^*|),$$

where h is a non-negative function defined on the non-negative integers, with $h(0) = 0$, $h(1) > 0$ and with $h(n) > n.h(1)$. The last assumption is weaker than requiring that $h(.)$ be strictly convex. Let the payoffs from being single be $\bar{u} = \bar{v} \in (0, 1)$. In the absence of a shock, the marginal types are given by stationary values $\tilde{\varepsilon}$ and $\tilde{\eta}$, so that all types above the respective thresholds are matched. We also assume that the distributions of types for men and women are uniform on $[0, 1]$ – this assumption allows us to solve explicitly for the equilibrium but is not required for the qualitative results.

Consider a shock so that the measure of women in cohort 0 is $1 + \Delta$, while leaving the measure of either sex in every cohort at 1. The direct effect of the shock would be a fall in marriage rates for women in cohort 0 and an increase in the marriage rate for men in cohort τ^*. This implies that attractive women from cohort 0 are available and need only be provided a small utility price in order to be married. Thus, if age preferences are not very strong (i.e. if $h(1)$ is small relative to quality considerations) it becomes advantageous for men in cohorts $\tau^* + 1$ to marry them, rather than women from their ideal cohort, 1. This in turn induces lower marriage rates in the women of cohort 1, who now become attractive to men from cohorts $\tau^* + 2$. Thus the effect cascades for a finite number of cohorts, K^*, where K^* is larger if age preferences are less important. Since the model is symmetric, similar adjustments take place in the marriage rates of women of cohort $-1, -2$ etc, and in the marriage rates of men in cohorts $\tau^* - 1, \tau^* - 2$.

Fig. 5 illustrates the pattern of adjustment, depicting cohort specific marriage rates for women, in the left-panel, and for men. Marriage rates for women increase the further one moves from affected cohorts, while those for men display the opposite pattern (here τ^* is the affected cohort, since it is the one directly affected by the shock to women of cohort 0). The distributional consequences are the mirror opposite of the effects on marriage rates, since payoffs are inversely related to marriage rates. This follows from the fact that the equilibrium payoffs for any type of woman, η_t, is linearly increasing in the difference between the type and the marginal type of woman in that cohort, $\tilde{\eta}_t$. Thus a higher marriage rate corresponds to lower marginal type, and thus a higher payoff for every woman in the cohort. We conclude therefore that if age preferences are small, then the distributional consequences of a shock are small, both for the directly affected women and for the directly affected men.

Effect on marriage rates of a shock

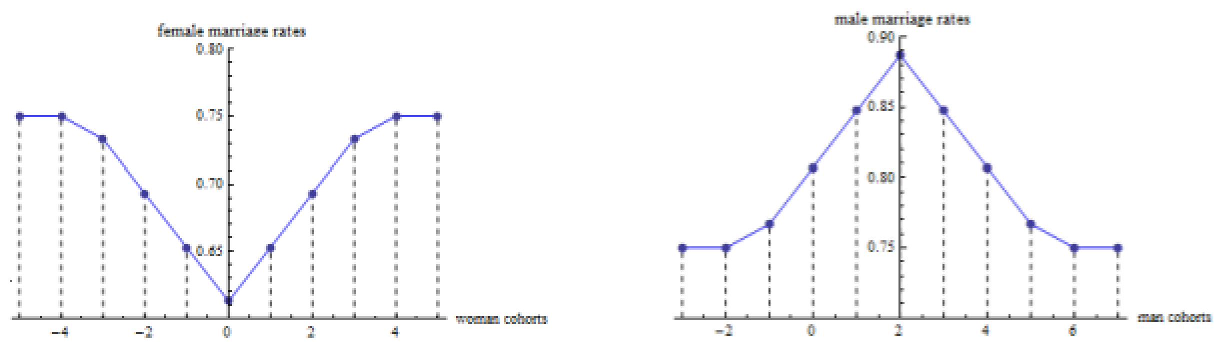


Fig.5a Fig.5b

Our main findings are summarized in the following proposition.

Proposition 2 *A shock that increases the number of women in a single cohort* 0 *will affect matching patterns in a finite number of cohorts,* K^**, around the affected cohort. If age considerations are less important (i.e.* $h(1)$ *is smaller), then* K^* *is larger, and the adjustment is spread over more cohorts. If* $h(1)$ *is smaller, women in cohort 0 are affected less, and men in cohort* τ^* *benefit less. As* $h(1) \to 0, K^* \to \infty$*, and the shock has negligible effects on the directly affected cohort.*

Proof. See appendix. ■

The contrast between propositions 1 and 2 highlights the difference between secular trends and transitory shocks. With a transitory shock, women in some cohort become more abundant, inducing other men to marry them. Conversely, with cohort growth, all women become more plentiful. Thus there are no relative price effects across cohorts that induce an adjustment in the age gap.

3.4 Welfare considerations

In marriage markets with finitely many agents and transferable utility, the Shapley-Shubik results imply that the first and second welfare propositions hold. With transferable utility, welfare equals the sum of payoffs across all agents, and a stable matching maximizes this, implying the first welfare proposition. Furthermore, if lump sum transfers across agents are feasible, they can be used to redistribute payoffs, without any consequences for the matching pattern. Thus the second welfare proposition also holds.

In the present context, the sum of payoffs across all agents is not well defined since the infinite series is necessarily non-convergent. If $g \geq 0$, so that cohort sizes are growing or constant, then the infinite series is clearly non-convergent. However, this is true even if $g < 0$, since the infinite sum going backwards in time is non-convergent. So we must adopt alternative welfare criterion. One possibility is to consider average per-capita payoffs. However, even here there is some ambiguity. Consider a steady state where the age gap is τ, not necessarily equal to τ^*, and where the associated marriage market sex ratio equals $r = \bar{r}(1+g)^{\tau}$. Assume that payoffs are weakly supermodular, so that matching on the quality dimension is assortative. The lowest types on the long side of the market are unmatched, and for a man of type ε who is matched, his match depends on τ and is written $\phi(\varepsilon, \tau)$. This satisfies

$$[1 - F(\varepsilon] = r[1 - G(\phi(\varepsilon,\, \tau))].$$

The total surplus associated with matches between men at date t and women at date $t + \tau$ equals

$$y(\tau) \equiv \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} Q(\varepsilon, \phi(\varepsilon, \tau), \tau) dF(\varepsilon).$$

Now consider the welfare per man at date t. The surplus generated per man equals $y(\tau)$ if all men are matched, i.e. if $r \geq 1$. However, if $r < 1$, then the surplus per man equals $ry(\tau)$. That is,

$$W_m(\tau) = \min \left\{ y(\tau), ry(\tau) \right\}.$$

Consider instead the surplus the surplus generated per woman at date t. This is given by

$$W_f(\tau) = \min\left\{y(\tau), \frac{1}{r}y(\tau)\right\}.$$

A third possible welfare criterion is the surplus generated per person in each cohort. Since the proportion of women to men within cohort is $\frac{1}{1+\bar{r}}$, this is equal to $\frac{1}{1+\bar{r}}W_m(\tau) + \frac{\bar{r}}{1+\bar{r}}W_f(\tau)$, i.e. it is a convex combination of the previous two welfare measures.

The steady state stable age gap τ^* does not, in general, maximize any of these welfare criteria. To examine this issue further, let us consider the expression $Q(\varepsilon, \phi(\varepsilon,\tau)), \tau)$ at any realization of ε. Since $\phi(\varepsilon)$ is increasing in r (i.e. a man of a given quality gets a better match if the sex ratio increases), this implies that Q is increasing in r. The second argument of Q is τ and Q is single peaked around τ^*. This establishes that at τ^*, a change in τ reduces $W_m(\tau)$ if this change in τ reduces r; however, this change in τ may increase $W_m(\tau)$ if the consequence is to increase r, and thereby $\phi(\varepsilon)$ sufficiently. This argument is summarized in the following proposition.

Proposition 3 *The stable age gap τ^* does not, in general, maximize $W_m(\tau)$ or $W_f(\tau)$. If $r(\tau^*) = \bar{r}(1+g)^{\tau^*} < 1$ and age preferences are sufficiently weak, then $W_m(\tau)$ is maximized at a value of τ that increases r relative to $r(\tau^*)$. If $r(\tau^*) = \bar{r}(1+g)^{\tau^*} > 1$ and age preferences are sufficiently weak, then $W_f(\tau)$ is maximized at a value of τ that reduces r relative to $r(\tau^*)$.*

3.5 Explaining the age gap

Our analysis presumes that preferences for a positive age gap are given, and examines its consequences. Such preferences may have an evolutionary foundation – this seems plausible given the universality of a positive age gap across the globe. Bergstrom and Bagnoli (1990) do provide an economic model of the age gap. Their model assumes that women and men have different roles in marriage, with the suitability of women for their role (the production of offspring) being revealed earlier, while that of men in their role (bread-winning) being revealed only later. We have briefly examined how the age gap in the Bergstrom-Bagnoli model adjusts to secular imbalances, and find results that are broadly consistent with those reported here, i.e. it does not adjust in order to eliminate imbalances.

Our model based on preferences shows that the age gap does not adjust secular imbalances. However, this does not mean that it is immutable and that it cannot change in response to other factors. Indeed, the age gap at marriage has declined with development in many countries, and it would be interesting to explain this decline. While an examination of the possible reasons for this decline would take us far from our present focus, it is possible that the increased role of human capital in household production can play a role. With faster human capital accumulation, and complementarity between the human capital endowments of the spouses, "compatibility" on this dimension may become more important. An examination of these issues requires a growth model, and is the subject of our current research.

In this context, recent empirical work (Anderberg et al, 2015) examines the trade off between age preferences and the human capital. The raising of the school leaving age in England by one year in 1972, in conjunction with the age-gap at marriage, caused a temporary imbalance in the marriage market, since women in the first two years in the affected cohorts were more educated than their "customary partners". The desire to match assortatively on the educational qualification dimension caused a change in the equilibrium matching patterns, implying that age preferences were relatively less important.

Finally, we should mention the work of Sautmann (2010), who argues that the marriage squeeze (caused by cohort growth) is responsible for the decline in the age gap at marriage. She uses a search model, and the model is complex and since it cannot be solved analytically, her arguments are based on numerical simulations. In consequence, it is difficult to pinpoint the exact mechanism that gives rise to this effect.

4 Non-transferable utility

Now let us consider the other polar extreme, of non-transferable utility. Since our model has two dimensions of preferences – quality and age – the non-transferable utility model is hard to analyze in full generality. If men and women have different ideal age gaps, trading "quality" for the age gap dimension becomes complicated, and characterizing equilibrium allocations becomes very difficult. To avoid this, we assume that the quality dimension is absent, so that only the age gap matters for preferences. Assume that men (resp. women) have single-peaked preferences with an ideal age gap τ_B (resp. τ_G). [11] Assume also that a man (resp. woman) prefers to be matched with a woman (resp. man) with her (his) preferred age gap, to remaining single.

Let the sex ratio in any cohort, $\bar{r}$, be given. Notice first that the stable age gap must lie between τ_B and τ_G. Let $r(\tau) := \bar{r}(1+g)^\tau$ be the marriage market sex ratio when the age gap is τ. Consider first the case where $r(\tau_B) > 1$ and $r(\tau_G) > 1$, so that there are excess women in the marriage market for any age gap τ that lies between the two ideal points. We

[11] Allowing for the quality dimension is possible if we assume that men and women agree on the ideal age gap. In this case, it is straightforward to show that the equilibrium age gap corresponds to the common ideal age gap, and is invariant to cohort size growth.

claim that then the equilibrium age gap τ must equal τ_B, the ideal point of men, who are on the short side of the market. Consider a matching, where every man is matched with a woman τ_B years younger, and where there are unmatched women. This matching is clearly unblocked, since each man is getting his best possible partner. To show uniqueness, consider any alternative age gap τ that lies between τ_B and τ_G. This has unmatched women in every cohort, and thus a man can propose to an unmatched women with his ideal age gap, and this pair can block this matching. Finally, any age gap $\tau > \max\{\tau_G, \tau_B\}$ or $\tau < \min\{\tau_G, \tau_B\}$ cannot be stable, since a couple with an age gap τ' lying between the two ideal points is a blocking pair. Similarly, if $r(\tau_B) < 1$ and $r(\tau_G) < 1$, the unique stable matching is one where the equilibrium age gap equals τ_G.

Consider finally the case where $r = 1$ so that the marriage market is balanced, e.g. because $\bar{r} = 1$ and $g = 0$ – this situation is a good approximation to marriage market conditions for much of human history. In this case, any age gap $\hat{\tau}$ that lies between τ_G and τ_B is an equilibrium steady state age gap. Consider a steady state where every man is matched to a woman with age gap $\hat{\tau}$. Since $r = 1$, every woman is also matched with age gap $\hat{\tau}$.Suppose that a man who proposes to woman he prefers – such a woman must have an age gap relative to him of τ' that is closer to τ_B than $\hat{\tau}$ is. However, since $\hat{\tau}$ lies between τ_B and τ_G, this implies that τ' is further away from this woman's ideal point τ_G than $\hat{\tau}$ is, and so this proposal is not acceptable to the woman. The following proposition summarizes and slightly generalizes these results.

Proposition 4 *There exists a stable steady state matching, for all parameter values. i) If $\bar{r}(1+g)^{\tau_G} > 1$ and $\bar{r}(1+g)^{\tau_B} > 1$, the unique equilibrium age gap equals τ_B. ii) If $\bar{r}(1+g)^{\tau_G} < 1$ and $\bar{r}(1+g)^{\tau_B} < 1$, the unique equilibrium age gap equals τ_G. iii) If neither the two above conditions hold, so that $\left(\bar{r}(1+g)^{\tau_G} - 1\right)\left(\bar{r}(1+g)^{\tau_B} - 1\right) \leq 0$, there exists a steady state where all women and all men are matched. In particular, if $\bar{r} = 1$ and $g = 0$, then every age gap $\hat{\tau}$ between τ_G and τ_B is a stable age gap.*

Proof. Parts (i) and (ii) have been proven in the text, so we turn to (iii). Let $\hat{\tau}$ be

any value of τ that solves the equation $\bar{r}(1+g)^{\hat{\tau}} = 1$, so that if the age gap is $\hat{\tau}$, then the marriage market sex ratio r equals one. We show now that if $\hat{\tau}$ is an integer between τ_G and τ_B, then $\hat{\tau}$ will be a stable steady state age gap. Consider a steady state where every man is matched to a woman with age gap $\hat{\tau}$, and where every individual is matched – from the definition of $\hat{\tau}$ such a matching is measure preserving and thus feasible.Suppose that a man who proposes to woman he prefers – such a woman must have an age gap relative to him of τ' that is closer to τ_B than $\hat{\tau}$ is. However, since $\hat{\tau}$ lies between τ_B and τ_G, this implies that τ' is further away from this woman's ideal point τ_G than $\hat{\tau}$ is, and so this proposal is not acceptable to the woman. Suppose that no integer value of $\hat{\tau}$ that solves the equation $\bar{r}(1+g)^{\hat{\tau}} = 1$. Consider the expression

$$h(\lambda) \equiv \lambda \bar{r}(1+g)^{\tau_G} + (1-\lambda)\bar{r}(1+g)^{\tau_B} - 1.$$

Since h is continuous, $h(0) > 0$ and $h(1) < 0$, there exists a value $\hat{\lambda}$ such that $h(\hat{\lambda}) = 0$. Let every individual be matched, with a fraction $\hat{\lambda}$ in each cohort having an age gap τ_G with the remainder having an age gap τ_B. This matching is constructed to be measure preserving, and also, no individual is left unmatched. It is also stable, since no woman with matched age gap τ_G will accept a different match, and no man with age gap τ_B will also accept a different match. ■

This proposition illustrates why the age gap does not necessarily adjust endogenously in order to equilibrate demand and supply in the marriage market, even with non-transferable utility. Suppose that the initial situation is one where the sex ratio at birth is the normal one, i.e. $\bar{r} = 1$ and where $g = 0$. In this case, the equilibrium age gap will be some $\hat{\tau}$ between τ_B and τ_G. Now if g becomes positive, there will be an excess supply of girls, and so the equilibrium age gap will shift to τ_B. So the age gap increases, aggravating the marriage market imbalance if $\tau_B > \tau_G$, if men prefer a larger age gap than women. The age gap will decrease if $\tau_G > \tau_B$. Similarly, the effect of sex selection for males, that reduce r, is to make the age gap that preferred by women – this may reduce marriage market imbalance, or aggravate it.

We can generalize the results of this section to the case of imperfectly transferable utility, as set out by Legros and Newman (2007).[12] Let us specialize to the case where there is an excess of women, for any τ between τ_G and τ_B. Let $U(\tau)$ (resp. $V(\tau)$) denote the single peaked payoff function of a man (resp. woman) who is matched with age gap τ. If a woman and man with age gap τ are matched, and the woman pays a dowry d to the man, their respective payoffs are

$$V(\tau) - d,$$

$$U(\tau) + \varphi(d),$$

where $\varphi(d)$ is real valued, strictly increasing and strictly concave function, with $\varphi(0) = 0$.

[12] Legros and Newman focus on how imperfect transferability affects the matching pattern and provide conditions under which one has positive or negative assortative matching.

A steady state equilibrium consists on an age gap and transfer, $(\hat{\tau}, \hat{d})$,that must satisfy

$$V(\hat{\tau}) - \hat{d} = \bar{v}.$$

$$U(\tau) + \varphi(V(\tau) - \bar{v}) \leq U(\hat{\tau}) + \varphi(\hat{d}).$$

In other words, $\hat{\tau}$ must maximize $U(\tau) + \varphi(V(\tau) - \bar{v})$. Thus generically, there will be a unique value $\hat{\tau}$ that lies between τ_B and τ^*, the value of τ that maximizes the sum of payoffs of the two sexes. We conclude therefore that the equilibrium age gap $\hat{\tau}$ will be insensitive to changes in the rate growth as long as there continues to be an excess of women. Similar arguments apply when there is an excess of men – $\hat{\tau}$ will lie between τ_G and τ^*. Limited transferability is similar to non-transferability in that the equilibrium age gap only changes when the imbalance in the sex ratio changes from an excess of men to an excess of women. The extent of the change is less, since the abundant sex can compensate the scarcer sex partially via transfers.

5 Distributional effects on the sexes

What are distributional effects of changes in the effective sex ratio in the marriage market, whether due to the marriage squeeze or due to sex selective abortions? Clearly, if r increases, men benefit and women lose out, due to a worsened competitive position. This occurs regardless of whether there is transferable utility or non-transferable utility, since more men will be able to marry, while less women are able to. However, if utility is transferable, there will be *distributional* effects, even on women who are able to find partners. We now show that the *magnitude* of distributional effects depends upon the nature of the marriage market. Specifically, the distributional effects of sex ratio imbalances will be large in Asian societies, while they will be more muted in North European societies.

To elucidate the reasons for this difference, one must consider the difference between marriage institutions across cultures. In his seminal work, Hajnal (1965) pointed out the difference between marriage patterns in Northern Europe (NE) and Southern or Eastern European society.[13] As Hajnal (1982) observes, these differences are accentuated all the more when one compares Northern Europe with Asia – i.e. Southern or Eastern Europe lie somewhere in between the Asian and Northern European marriage pattern. The salient features are as follows:

[13]Hajnal's notion of a Northern European pattern is cultural, and also extends former colonies that were settled by Northern Europeans, such as Australia, Canada, New Zealand and the United States.

- A high age at marriage for both men and women (NE), as compared to earlier marriage.
- A small age gap at marriage (NE).
- A large fraction of the population who never married (NE), with a never married rate of about % (NE), as compared to a 98 or 99% marriage rate in Asia. [14]

Clearly, a large age gap at marriage magnifies the marriage squeeze, both due to temporary shocks and due to secular growth. For example, a 3.5% rate of growth translates into a 19% excess supply of women if the age gap is five years, but only a 7% excess supply if the age gap is 2 years – such are the dramatic effects of compound growth.

More subtle is the effect of the last factor – the distributional effects of the marriage squeeze will be large in societies where marriage is near universal, but much smaller in North European type societies, where marriage rates are lower.

Since we have established that the age gap will be τ^*, we define

$$q(\varepsilon, \eta) := Q(\varepsilon, \eta, \tau^*).$$

Our analysis may be conducted in terms of the quality function $q(.)$ that fixes $\tau = \tau^*$. Let us now consider the effects of a change in r^*, that may arise either due to a worsening of the sex ratio at birth, or due to a change in the age gap An increase in r^* raises the relative supply of women. From the conditions for the marginal type (1) and (2), one may verify that this increases $\tilde{\eta}$ and reduces $\tilde{\varepsilon}$. That is, the the singles rate for men declines and that of women increases. Turning to payoffs, consider the payoffs of a man of a given type, ε. This can be written as

$$u^*(\varepsilon, r) = \bar{u} + \int_{\tilde{\varepsilon}(r)}^{\varepsilon} u'(x)dx.$$

Recall that the derivative, $u'(\varepsilon)$, is given by

$$u'(\varepsilon) = q_\varepsilon(\varepsilon, \phi(\varepsilon, r)).$$

Thus the payoff can be written as

$$u^*(\varepsilon, r) = \bar{u} + \int_{\tilde{\varepsilon}(r)}^{\varepsilon} q_x(x, \phi(x, r))dx.$$

[14]Nonetheless, there are variations in marriage rates, as shown by Gupta (2014), who uses historical data from the Indian censuses to examine the effect of sex ratio imbalances at region and community level upon marriage rates.

The derivative of this payoff with respect to a change in the equilibrium sex ratio, r, is given by

$$\frac{\partial u(\varepsilon)}{\partial r} = \int_{\tilde{\varepsilon}(r)}^{\varepsilon} q_{xy}(x, \phi(x,r)) \frac{\partial \phi(x,r)}{\partial r} dx - q_x(x, \phi(x))|_{x=\tilde{\varepsilon}(r)} \frac{\partial \tilde{\varepsilon}}{\partial r}.$$

Suppose now that the payoff function is strictly supermodular. Then $q_{xy} > 0$, and $\frac{\partial \phi(x,r)}{\partial r} > 0$, i.e. an increase in the supply of women improves the match quality of any type of man. Thus the first term is strictly positive. Furthermore, $\frac{\partial \tilde{\varepsilon}}{\partial r} < 0$, i.e. the marginal type of man worsens. Since q_x is positive, the second term is also positive, and the payoff of any type of man increases. The same argument implies that the payoff of any type of woman worsens.

If the payoff if weakly supermodular, so that $q_{xy} = 0$, then the first term is zero, but the second effect persists, so that an increase in the sex ratio worsens the position of women and increases that of men. In this case, we may write the payoff $q(x, y) = x + y$, so that

$$u^*(\varepsilon, r) = \bar{u} + [\varepsilon - \tilde{\varepsilon}(r)].$$

This verifies that the payoff of men increases at a rate that depends upon how quickly $\tilde{\varepsilon}$ declines as r increases. Thus in the case where the payoff function is additive, distributional effects will be larger, the greater the change in the marginal type induced by the sex ratio. From the conditions (1) and (2), the derivative of the marginal type of woman, as a function of the sex ratio, is given by

$$\frac{d\tilde{\eta}}{dr} = -\frac{1 - G(\tilde{\eta})}{f(\tilde{\varepsilon})\frac{q_\eta}{q_\varepsilon} + rg(\tilde{\eta})} = -\frac{1 - G(\tilde{\eta})}{f(\tilde{\varepsilon}) + rg(\tilde{\eta})}.$$

Thus the distributional effects are larger if the densities associated with the marginal types, $f(\tilde{\varepsilon})$ and $g(\tilde{\eta})$, are small. Suppose that ε and η are given by single peaked distributions, where f and g are strictly increasing up to a point and then strictly decreasing. Consider two different societies. First, an Asian one where $\bar{u}$ and $\bar{v}$ are low, so that the equilibrium marriage rate is high, over 95%. Second, a North European one where $\bar{u}$ and $\bar{v}$ are high, so that the equilibrium marriage rate is low, say below 90%. The numerator will be smaller in the latter, and the denominator will be larger. We summarize our results in the following proposition.

Proposition 5 *An increase in r^*, induced by a change in the sex ratio or in cohort size growth, increases the singles rate for women and reduces that for women. The payoff for a non-marginal type of man strictly increases, and that for a non-marginal woman falls. If* *payoffs are additive, the magnitude of distributional effects is larger in societies where the density of marginal types is sparse.*

5.1 The effect on dowries

The phenomenon of dowries, i.e. payments made by the family of the bride to the groom or his family, is widely prevalent in South Asia. One puzzle is that this seems to becoming more widespread, with the process of development (Anderson, 2003). Demographers such as Bhatt and Halli (1999) have argued that the marriage squeeze is responsible for the deterioration of the position of women in India, and replacement of the institution of bride price in many regions and communities by dowries (payment from the bride's family to the groom). Rao (1993) provides some empirical evidence in support, although one should note that data on dowry payments is hard to get since the practice is deemed illegal in India. Anderson (2007a) examines the effect of transitory cohort shocks upon dowry payments, and shows that transitory shocks cannot explain persistent levels or trends in dowries.

The purpose of this section is to provide a cautionary note: while the effects of a change in the effective sex ratio on equilibrium payoffs of the two sexes may be unambiguous, the effects on dowry payments may be more nuanced. Becker has argued that dowries arise due to the inflexibility in the division of the marital surplus within the marriage.[15] Under this interpretation, the total marital output, $q(\varepsilon, \eta)$, is divided into *premuneration values* $U(\varepsilon, \eta)$ and $V(\varepsilon, \eta)$, for the man and woman respectively.[16] These will, in general, differ from the payoffs in the stable allocations $u^*(\varepsilon)$ and $v^*(\eta)$, and a dowry or brideprice is paid to reconcile the two. Our analysis so far speaks to the equilibrium payoffs, not dowries per se. Indeed, much of the literature on dowries argues that dowries move one-to-one with equilibrium payoffs. Thus it is argued that an increase in r, the proportion of women, will increase dowries, just as it raises the equilibrium payoffs of men, and reduces that of women.

Let us briefly explain why the link between equilibrium payoffs of men and the dowries that they receive may not be so immediate. Consider a larger rate of (steady state) cohort growth, that gives rise to an excess of women, so that $r > 1$. As we have established, this increases the steady state marriage rate for men, while reducing that of women, thereby increasing the equilibrium payoff for any type of men. Furthermore, it increases the relative supply of *every type of woman.* Consequently, the matching ϕ changes – if payoffs are

[15]Dowries may also play a different role, as a form of inheritance by daughters (Botticini and Siow, 2003). We abstract from this consideration, in line with our focus on how dowry payments equilibrate the marriage market.

[16]The term is introduced by Mailath, Samuelson and Postelwaite (2014), who use the concept to examine exante investments in a matching context where there are constraints on how prices/transfers may depend on characteristics.

supermodular in the quality dimension, then each type of men, ε, is matched to a better quality woman, i.e. $\phi(\varepsilon)$ is larger in the new equilibrium. If the man's premuneration value, $U(\varepsilon, \eta)$, is increasing in the woman's quality η, then the dowry need not increase as much as the man's equilibrium payoff does. It turns out that the increase in equilibrium premuneration values will be type dependent – it may increase greatly for some types of men, but not for others. Thus the effect on dowries depends upon the type of man – similarly, the dowry paid by any type of woman will also depend on her type or quality.[17]

This point is illustrated most clearly by a set of examples where individual quality types are two-dimensional. Let a man's quality type be $\varepsilon = (\varepsilon_1, \varepsilon_2)$, and similarly the woman's type be $\eta = (\eta_1, \eta_2)$. The first dimension, ε_1 (or η_1) measures the attractiveness of the individual to the match partner. The second dimension, ε_2 (or η_2) measures the intensity of the individual's desire for marriage. We shall assume that total match quality q is given by

$$q = \lambda(\varepsilon_1 + \eta_1) + ((1 - \lambda)(\varepsilon_2 + \eta_2) + \rho\varepsilon_1\eta_1,$$

where $\rho > 0$ so that there is supermodularity in the attractiveness dimension. Indeed, we shall focus on the limit case as $\rho \to 0$, so that payoffs are additive in the limit. Assume that attractiveness and the desire for marriage are independently and uniformly distributed on the interval $[0, 1]$, and that $\bar{u} = \bar{v}$.

Define $\varepsilon := \lambda\varepsilon_1 + (1 - \lambda)\varepsilon_2$, $\eta := \lambda\eta_1 + (1 - \lambda)\eta_2$. Clearly, selection into matching when $\rho = 0$ only depends upon the one dimensional variables ε and η. Similarly, the equilibrium payoffs only depend upon ε and η, and may therefore be written as $u^*(\varepsilon)$ and $v^*(\eta)$. However, since matching is assortative based on attractiveness, as long as $\rho > 0$, we shall assume that this is the case in the limit as well. This is a stable matching when $\rho = 0$, but there is a multiplicity of stable matchings when payoffs are additive in types. The matching patterns and dowry payments we derive will be close to the equilibrium payments and matching patterns for ρ strictly positive but sufficiently small.

Consider first the benchmark case where the sex ratio is balanced, i.e. when $r = 1$. Given the symmetry of the model, all men with $\varepsilon \geq \bar{u}$ will be matched, as will all women with $\eta \geq \bar{u}$, and the equilibrium payoffs will be $u^* = \varepsilon$ and $v^* = \eta$. Consider a matched pair, with characteristics $(\varepsilon_1, \varepsilon_2)$ and (η_1, η_2). Since any matched pair will have identical levels of attractiveness, $\varepsilon_1 = \eta_1$ in any such pair, although ε_2 need not equal η_2 in the pair. In consequence, the equilibrium payoff of the man, $u^* = \varepsilon$, need not equal the equilibrium payoff of his partner, $v^* = \eta$. Nonetheless, the premuneration value of the man in any such

[17]Note that this caveat applies specifically to the comparative statics of the sex ratio. It does not apply when we consider a change that does not affect the matching pattern, e.g. as in Ambrus, Field and Torreo (2010), who examine the effects of mandated alimony payments on dowries in Bangladesh.

match will equal ε, since it equals $\lambda\eta_1 + (1-\lambda)\varepsilon_2$, and $\eta_1 = \varepsilon_1$, and thus the transfers/dowry payments are zero. Similarly, the premuneration value of the woman will also equal η.

Now consider a sex ratio $\hat{r} > 1$, so that there is an excess of women. Equilibrium is characterized by new thresholds $\tilde{\varepsilon}(\hat{r}) < \bar{u} < \tilde{\eta}(\hat{r})$, so that the marriage rate of men increases while that of women falls. In consequence, equilibrium payoffs for each type of man increases, while that of each type of woman decreases. Indeed, for any type that was previously matched, the change in payoff equals $\bar{u} - \tilde{\varepsilon}(\hat{r})$ in absolute value, being positive for a man and negative for a woman. Thus the change in equilibrium payoffs does not depend upon λ, the relative weight of attractiveness and desire for marriage in the contribution to the type.

However, equilibrium premuneration values do depend upon λ, and thus the dowry function, $d(\varepsilon_1)$, varies with λ. This is illustrated in Figs. 6, which graph the dowry function for three distinct values of λ $(0, 0.5$ and $1)$. The appendix provides full details of the analysis that underlies the figures. The figures are drawn for the case where $\hat{r} = 2$ and $\bar{u} = \bar{v} = 0.25$, so that all men are matched and women with attractiveness above the median are matched, but our main conclusions are robust, and also hold under different parameter values. Figure 6a shows that when $\lambda = 0$, so that attractiveness is not a consideration and only the desire for marriage matters, the dowry is a constant function, i.e. every man receives the same payment from his partner. Thus the excess of women raises the dowry for every type of man. This follows from the fact that the premuneration value of any man is unaffected by the change in the sex ratio, and thus the dowry increases to reflect the rise in $u^*(\varepsilon)$, the equilibrium payoff.

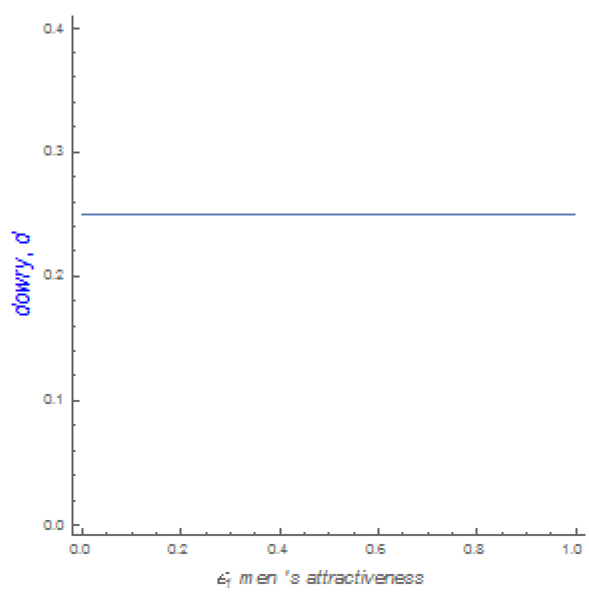


Fig. 6a: λ = 0, attractiveness does not matter

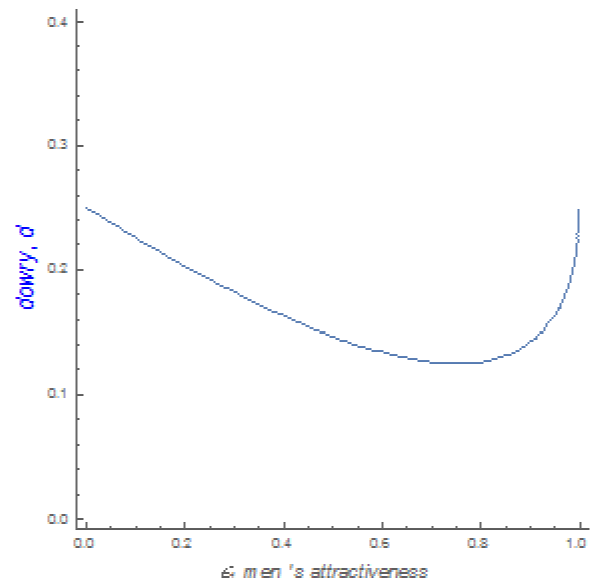


Fig. 6b: λ = 0.5, attractiveness matters partially

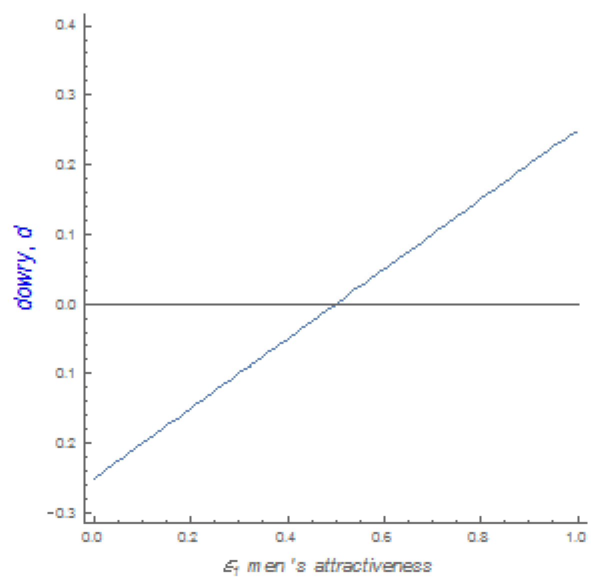


Fig. 6c: λ = 1, only attractiveness matters

Figure 6b considers the case where $\lambda = 0.5$, so that both attractiveness and desire for marriage matter. We see that the dowry received by a man is not monotone in his own attractiveness. Indeed, a man with an intermediate level of attractiveness may well receive a lower dowry than one who is less attractive. The underlying reason for this is that with a sex ratio imbalance, the marginal distribution of attractiveness among matched women differs from the marginal distribution of attractiveness for matched men. Whereas the latter is uniform (since all men are matched), the marginal distribution of attractiveness for women is increasing in η_1. In consequence, it turns out that men of intermediate attractiveness experience the largest increase in match quality and thus they need to be compensated less via a dowry.

Figure 6c considers the case where only attractiveness matters. In this case, we see that the dowry is linearly increasing in ε_1, with the least attractive men paying a bride price! This is even though their equilibrium payoff rises. Intuitively, the imbalance causes the least attractive men to be matched with relatively attractive women, and they need to compensate the latter partially so that they are willing to marry them.

To summarize, while an increase in the ratio of women to men always worsens the payoffs of women, the effects on dowries may be more nuanced. This depends on how premuneration values change. If the attractiveness of the partner is an important component of an individual's premuneration value, a part of the increased payoff of a man will come from being matched with more attractive women. Thus the effects on dowries may depend on the quality of the man. Indeed, if partner attractiveness is sufficiently important in determining match quality, it is possible that dowries may fall for some types of men, even as their equilibrium payoffs increase.

To conclude, this paper has provided empirical evidence on the effects of cohort size growth on marriage market balance. Birth cohort growth, in conjunction with the age gap at marriage between men and women, gives rise to large imbalances. Cohorts have been growing rapidly in many countries, including those in South Asia and sub-Saharan Africa. Conversely cohort sizes are shrinking in many East Asian countries and these countries may well be the mirror of the future for other developing economies. Our theoretical analysis demonstrates that the age gap at marriage does not adjust in order to eliminate imbalances arising from secular growth, even though it does adjust when there are transitory shocks. In consequence, the distributional consequences of these supply imbalances on the sexes is considerable. This is particularly so in the countries where marriage rates are very high, and where the non-participation margin is thin. However, this evidence also points to a more favorable environment for women in these countries in the coming years – as cohort growth declines and becomes negative, their bargaining power is likely to improve considerably.

6 Appendix

Proof of Proposition 2

Recall that every cohort has unit measures of men and women, except for the women of cohort 0 that are of measure $1 + \Delta$, where $\Delta > 0$. Furthermore, in every cohort, the payoff of the marginal individual of either sex must equal the payoff of from being single. That is $u(\tilde{\varepsilon}_t) = \bar{u} \forall t$ and $v(\tilde{\eta}_t) = \bar{u}\ \forall t$. At this point it will be convenient to re-number the cohorts of men, so that cohort τ^* is normalized to zero, leaving the numbering of women cohorts unchanged. Under this re-numbering, the ideal difference in cohort dates is zero, and our condition can be written as

$$\tilde{\varepsilon}_t + \tilde{\eta}_t = 2\bar{u}, \forall t. \tag{4}$$

Thus for any pair of marginal individuals with the ideal age gap, τ^*, since the total surplus generated by this pair equals the sum of their types, this must equal $2\bar{u}$. Also, for any non-marginal man of type $\varepsilon_t > \tilde{\varepsilon}_t$,

$$u(\varepsilon_t) = \varepsilon_t - \tilde{\varepsilon}_t + \bar{u}. \tag{5}$$

For any $\eta_t > \tilde{\eta}_t$,

$$v(\eta_t) = \eta_t - \tilde{\eta}_t + \bar{u}. \tag{6}$$

To see this, consider an arbitrary matched pair, (ε_t, η_t). The sum of their payoffs must equal the total surplus they generate, i.e.

$$u(\varepsilon_t) + v(\eta_t) = \varepsilon_t + \eta_t.$$

Furthermore, stability requires

$$u(\varepsilon_t) + v(\tilde{\eta}_t) \geq \varepsilon_t + \tilde{\eta}_t,$$

$$u(\tilde{\varepsilon}_t) + v(\eta_t) \geq \tilde{\varepsilon}_t + \eta_t.$$

These inequalities, in conjunction with the conditions $u(\tilde{\varepsilon}_t) = \bar{u}$ and $v(\tilde{\eta}_t) = \bar{u}$ imply the conditions 5 and 6.

If the shock Δ is large enough, and $h(1)$, the penalty from deviating from the ideal age gap by one year, is not too large, then women from cohort 0 will match with men from cohorts 1 and -1, as well as cohort 0. This will induce some women from cohort 1 to match with men from 2, and some women from cohort 2 to match with men from cohort 3, and so on. This will terminate in some women of cohort $K-1$ matching with men from cohort K, leaving women from cohort K onwards and men from cohort $K+1$ onwards unaffected. Symmetrically, matching patterns will be affected for women in cohorts between $-(K-1)$ and 0, and for men between cohorts $-K$ and -1. Lemma 6 shows that age gaps that deviate from the ideal gap $\tau^*$by more than 1 cannot be optimal, given our assumption on the $h(.)$ function ($h(n) > n.h(1)$) and the additive preferences for quality.

We now claim that the total surplus generated by matches between the marginal types that are one year apart equal the sum of their single payoffs. That is, we must have

$$\tilde{\varepsilon}_{t+1} + \tilde{\eta}_t - h(1) = 2\bar{u}, \text{if} 0 \leq t \leq K-1, \tag{7}$$

$$\tilde{\varepsilon}_{t-1} + \tilde{\eta}_t - h(1) = 2\bar{u}, \text{if} 0 \geq t \geq -(K-1). \tag{8}$$

This is immediate if these marginal types are actually matched to each other, since the payoff to each marginal type equals $\bar{u}$. This is also true if for example type $\tilde{\varepsilon}_{t+1}$ is not matched to type $\tilde{\eta}_t$, as long as individuals from these cohorts are matched. Given the additivity of payoffs, the total surplus generated is unchanged if the matching is modified so that $\tilde{\varepsilon}_{t+1}$ is indeed matched to type $\tilde{\eta}_t$, and since the stable payoffs are unique, we must have $\tilde{\varepsilon}_{t+1} + \tilde{\eta}_t - h(1) = 2\bar{u}$.

Finally, it must also be the case that it is optimal that cohorts greater than K or less than $-K$ are unaffected. That is, we must have the symmetric boundary conditions.

$$\tilde{\varepsilon}_{K+1} + \tilde{\eta}_t - h(1) < 2\bar{u}, \tag{9}$$

$$\tilde{\varepsilon}_{-K-1} + \tilde{\eta}_{-K} - h(1) < 2\bar{u}. \tag{10}$$

In view of the fact that the problem is symmetric with respect to cohorts t and $-t$, we focus on positive values of t. Equations 4 and 7 yield first order difference equations for the marginal types:

$$\tilde{\eta}_{t+1} - \eta_t = -h(1), 0 \leq t \leq K-2,$$

$$\tilde{\varepsilon}_{t+1} - \tilde{\varepsilon}_t = h(1), 0 \leq t \leq K-1.$$

Similarly, equations 4 and 8 yield the difference equations

$$\tilde{\eta}_{t-1} - \eta_t = -h(1), 0 \geq t \geq -(K-2),$$

$$\tilde{\varepsilon}_{t-1} - \tilde{\varepsilon}_t = h(1), 0 \geq t \geq -(K-1).$$

Finally, one must have marriage market balance in the affected cohorts — the measure of matched men (the left-hand side of the equation below) must equal the measure of matched women (the right-hand side):

$$\sum_{t=-K}^{K}\left[1-F(\tilde{\varepsilon}_t)\right]=\sum_{t=-K}^{K}\left[1-G(\tilde{\eta}_t)\right]+\Delta\left[1-G(\tilde{\eta}_0)\right].$$

At this point we assume that $F(.)$ and $G(.)$ are both uniform on $[0,1]$, and explicitly solve this system (of difference equations and marriage market balance) for the equilibrium threshold values. Using the symmetry in the time index, , the measure of matched men can be written as

$$\sum_{t=-K}^{K}\left[1-F(\tilde{\varepsilon}_t)\right]=(1-\tilde{\varepsilon}_0)+2\sum_{t=1}^{K}\left[1-\tilde{\varepsilon}_t\right] \tag{11}$$

Using the fact that $\tilde{\varepsilon}_t=\varepsilon_0+th(1)$, this equals

$$(2K+1)(1-\tilde{\varepsilon}_0)-K(K+1)h(1). \tag{12}$$

Similarly, the measure of matched women can be written as

$$\sum_{t=-K}^{K}\left[1-G(\tilde{\eta}_t)\right]+\Delta\left[1-G(\tilde{\eta}_0)\right]=(2K+1+\Delta)(1-\tilde{\eta}_0)+K(K+1)h(1), \tag{13}$$

where the difference in the sign of the term $K(K+1)h(1)$, as compared to the case of men, is due to the fact that $\tilde{\eta}_t$ decreases with the time index, $\tilde{\eta}_t=\tilde{\eta}_0-th(1)$. Equating the expressions in (11) and (13), and utilizing the equilibrium condition $\tilde{\varepsilon}_0+\tilde{\eta}_0=2\bar{u}$, we solve for $\tilde{\eta}_0$ as a function of K to obtain

$$\tilde{\eta}_0=\frac{\Delta+2K(K+1)h(1)+2(2K+1)\bar{u}}{2(2K+1)+\Delta}. \tag{14}$$

The equilibrium value K^* is the largest integer K such that $\tilde{\eta}_K>\bar{u}$. Since $\tilde{\eta}_K=\tilde{\eta}_0-Kh(1)$, we may use the expression for $\tilde{\eta}_0$ in 14 to express this condition only in terms of K and the exogenous parameters of the problem. Thus we deduce that K^* is the largest integer K such that the quadratic function $\rho(K)$ defined below is strictly positive:

$$\rho(K):=\Delta(1-\bar{u})-2K^2-h(1)\Delta K.$$

Since $\rho(0)>0$, the quadratic has a unique positive root κ, where

$$\kappa=-\frac{\Delta}{2}+\sqrt{\frac{\Delta^2}{4}+\frac{2\Delta(1-\bar{u})}{h(1)}}.$$

Thus $K^*=[\kappa]$, the integer part of κ if κ is not an integer; otherwise, $K^*=\kappa-1$. Since

κ is decreasing in $h(1)$ and $\kappa \to \infty$ as $h(1) \to 0$, we see that the shock is spread over more and more cohorts as age preferences become less important.

Lemma 6 *With additive preferences, in response to a shock, the equilibrium age gaps lie in the set* $\{\tau^* - 1, \tau^*, \tau^* + 1\}$, *if* $h(n) > n.h(1)$.[18]

Proof. As in the proof of the main proposition, above, we normalize the numbering of males so that the ideal age gap equals 0. Consider two types ε_t and η_{t+n}, with $|n| > 1$. Without loss of generality, we may assume that $\varepsilon_t \geq \tilde{\varepsilon}_t$ and $\eta_{t+n} \geq \tilde{\eta}_{t+n}$. This follows from the fact that any marginal man is available at the minimum utility price of $\bar{u}$, and so any deviating woman need not consider a sub-marginal man as a partner. Thus the sum of the equilibrium payoffs of this couple equals

$$u(\varepsilon_t) + v(\eta_{t+n}) = \bar{u} + (\varepsilon_t - \tilde{\varepsilon}_t) + \bar{u} + (\eta_{t+n} - \tilde{\eta}_{t+n}),$$

while the surplus they generate equals

$$\varepsilon_t + \eta_{t+n} - h(|n|).$$

So the difference between equilibrium payoffs and surplus equals

$$2\bar{u} - (\tilde{\varepsilon}_t + \tilde{\eta}_{t+n}) - h(|n|).$$

Since $\tilde{\eta}_{t+n} \geq \tilde{\eta}_t - |n|\,.h(1)$, and since $\tilde{\varepsilon}_t + \tilde{\eta}_t = 2\bar{u}$, the above expression is no greater than $n.h(1) - h(|n|)$, which is strictly negative for $n > 1$. ∎

Derivations underlying the dowry function (see Fig 6):

For the $\lambda = 0$, premuneration values are constant for any matched man, and do not depend on the type of his partner. Thus any man who is matched must be paid an equilibrium dowry of $\bar{u} - \tilde{\varepsilon}(\hat{r})$, and the dowry is a constant function.

Now consider the case where $\lambda \in (0, 1)$. When $r > 1$, recall that $\tilde{\varepsilon}(\hat{r}) < \tilde{\eta}(\hat{r})$. Since men and women are matched according to attractiveness, one needs to consider the marginal distribution of attractiveness, conditional on the total match value being greater than the threshold. The marginal distributions are given by

$$\hat{F}(\varepsilon_1; \tilde{\varepsilon}(\hat{r})) := \int_{\varepsilon_2 = \max\{\tilde{\varepsilon}(\hat{r}) - \frac{\lambda}{1-\lambda}\varepsilon_1, 0\}}^{1} f(\varepsilon_1, \varepsilon_2) d\varepsilon_2,$$

[18] The result that age gaps can deviate by no more than 1 from the optimum is specific to the model where preferences for quality are additive. With strict supermodularity in the quality dimension, one could well have a wider distribution of age gaps.

$$\hat{G}(\eta_1;\tilde{\eta}(\hat{r})) := \int_{\eta_2=\max\{\tilde{\eta}(\hat{r})-\frac{\lambda}{1-\lambda}\eta_1,0\}}^{1} g(\eta_1,\eta_2)d\eta_2.$$

Since the matching $\tilde{\phi}(\varepsilon_1)$ is assortative on the attractiveness dimension, for $\varepsilon_1 \geq \tilde{\varepsilon}(\hat{r})$, it is given by

$$1-\hat{F}(\varepsilon_1;\tilde{\varepsilon}(\hat{r})) = r[1-\hat{G}(\tilde{\phi}(\varepsilon_1);\tilde{\eta}(\hat{r}))].$$

On solving these equations for $\lambda = 0.5$, $\bar{u} = \bar{v} = 0.25$ and $r = 2$, we obtain

$$\tilde{\phi}(\varepsilon_1) = 1-\sqrt{1-\varepsilon_1}.$$

Since the dowry equals the equilibrium payoff minus the premuneration value, this gives rise to the dowry schedule

$$d(\varepsilon_1) = -\frac{1}{4} + \frac{\sqrt{1-\varepsilon_1}-\varepsilon_1}{2}.$$

Finally, when $\lambda = 1$ and only attractiveness matters, let $\hat{F}$ and $\hat{G}$ denote the distributions of attractiveness in the two sexes. The matching $\tilde{\phi}(\varepsilon_1)$ is given by

$$1-\hat{F}(\varepsilon_1) = r[1-\hat{G}(\tilde{\phi}(\varepsilon_1))].$$

Thus, under the parameters of the example,

$$\tilde{\phi}(\varepsilon_1) = \frac{1+\varepsilon_1}{2},$$

and this also equals the equilibrium premuneration value. Since the equilibrium payoff $u^*(\varepsilon_1) = \varepsilon_1 + \frac{1}{4}$,

$$d(\varepsilon_1) = -\frac{1}{4} + \frac{\varepsilon_1}{2}.$$

References

[1] Akers, D.S., 1967, On Measuring the Marriage Squeeze, *Demography* 4, 907-24.

[2] Ambrus, A., E. Field and M. Torreo, 2010, Muslim Family Law, Prenuptial Agreements and the Emergence of Dowry in Bangladesh, *Quarterly Journal of Economics* 125 (3), 1349-1397.

[3] Anderberg, D., J. Bagger, V. Bhaskar and T. Wilson, 2015, General Equilibrium Effects of a Reform: The Raising of the School Leaving Age and the Marriage Market, mimeo.

[4] Anderson, S., 2003, Why Dowry Payments Declined with Modernization in Europe but Are Rising in India, *Journal of Political Economy*, 111 (2), 269-310.

[5] Anderson, S., 2007a, Why the Marriage Squeeze cannot cause Dowry Inflation, *Journal of Economic Theory*, 137(1), 140-152.

[6] Anderson, S., 2007b, Economics of Brideprice and Dowry, *Journal of Economic Perspectives*, 21 (4), 151-174.

[7] Anderson, S., and D. Ray, 2010, Missing Women: Age and Disease, *Review of Economic Studies*, 77, 1262-1300.

[8] Angrist, J., 2002, How do Sex Ratios Affect Marriage and Labor Markets? Evidence from America's Second Generation, *Quarterly Journal of Economics* 117, 997-1038.

[9] Becker, G., 1981, *A Treatise on Marriage,* Cambridge: Harvard University Press.

[10] Bergstrom, T., and M. Bagnoli, 1990, Courtship as a waiting game, *Journal of Political Economy*, 101, 185-202.

[11] Bergstrom, T., and D. Lam, 1989a, The effects of cohort size on marriage markets in twentieth century Sweden, in T. Bengtsson (ed.), *The Family, the Market and the State in Industrialized Countries.*

[12] Bergstrom, T., and D. Lam, 1989b, The Two-Sex Problem and the Marriage Squeeze in an Equilibrium Model of Marriage Markets, mimeo.

[13] Bhaskar, V., 2011, Sex Selection and Gender Balance, *American Economic Journal: Microeconomics* 3, 214-244.

[14] Bhatt P.M., and S. Halli, 1999, Demography of Brideprice and Dowry: Causes and Consequences of the Indian Marriage Squeeze, *Population Studies* 53, 129-148.

[15] Botticini, M., and A. Siow, 2003, Why Dowries?, *American Economic Review* 93, 1385-1398.

[16] Brandt, L, A. Siow and C. Vogel, 2008, Large Shocks and Small Changes in the Marriage Market for Famine Born Cohorts in China, mimeo.

[17] Census of India, 1991, Female Age at Marriage: An Analysis of 1991 Census Data, New Delhi: Registrar General of India.

[18] Chiappori, P-A., B. Fortin and G. Lacroix, 2002, Marriage Market, Divorce Legislation and Household Labor Supply, *Journal of Political Economy* 110, 37-72.

[19] Coale, A., 1991, Excess Female Mortality and the Balance of the Sexes in the Population: An Estimate of the Number of 'Missing Females', *Population and Development Review* 17, 517-523.

[20] Choo and A. Siow, 2006, Who Marries Whom and Why, *Journal of Political Economy* 114 (1), 75–201.

[21] Doepke, M., and M. Tertilt, 2009, Women's Liberation: What's in it for Men, *Quarterly Journal of Economics* 124 (4), 1541–1591.

[22] Doepke, M., and M. Tertilt, 2011, Does Female Empowerment Promote Economic Development, mimeo.

[23] Fernandez, R., 2010, Womens' Rights and Development, mimeo, New York University.

[24] Gale, D., and L. Shapley, 1962, College Admissions and the Stability of Marriage, *American Mathematical Monthly* 69, 9-15.

[25] Gretsky, N. E., J.M. Ostroy & W.R. Zame, 1992, The Nonatomic Assignment Model, *Economic Theory*, 2(1), 103-127.

[26] Gupta, B., 2014, Where have all the Brides Gone? Son Preference and Marriage in India over the 20th Century, *Economic History Review,* 67(1), 1-24.

[27] Hajnal, J., 1965, European Marriage Patterns in Perspective, in D. V. Glass and D.E.C. Eversley, *Population in History*, London.

[28] Hajnal, J., 1982, Two Kinds of Preindustrial Household Formation System, *Population and Development Review* 8 (3), 449-494.

[29] Kaplan, H., and A. Robson, 2003, The Evolution of Human Longevity and Intelligence in Hunter-Gatherer Economies, *American Economic Review* 93, 150-169.

[30] Legros, P., and A. Newman, 2007, Beauty is a Beast, Frog is a Prince: Assortative Matching under Non-Transferabilities, *Econometrica* 75(4), 1073-1102.

[31] Lundberg, S., and R. Pollak, 2003, Efficiency in Marriage, *Review of Economics of the Household* 1, 153-167.

[32] Mailath, G., A. Postelwaite and L. Samuelson, 2013, Pricing and Investments in Matching Markets, *Theoretical Economics*, 8(2), 535-590.

[33] Neelakantan, U., and M. Tertilt, A Note on Marriage Market Clearing, *Economics Letters* 101, 103-105.

[34] Rao, V., 1993, The Rising Price of Husbands: A Hedonic Analysis of Dowry Increases in Rural India, *Journal of Political Economy* 101, 666-77.

[35] Roth, A.,and M. Sotamayor, 1992, *Two-Sided Matching: A Study in Game-Theoretic Modelling and Analysis,* (No. 18), Cambridge University Press.

[36] Sautmann, A., 2010, Partner Search and Demographics: The Marriage Squeeze in India, mimeo, Brown University.

[37] Schoen, R., 1983, Measuring the Tightness of a Marriage Squeeze, *Demography* 20, 61-78.

[38] Sen, A., 1990, More than 100 Million Women are Missing, *New York Review of Books,* 37 (20), 61-66.

[39] Shapley, L., and M. Shubik, 1972, The Assignment Game I: The Core, *International Journal of Game Theory* 5, 111-130.

[40] United Nations, 1990, *Patterns of First Marriage: Timing and Prevalence.* New York: United Nations.