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Abstract

The uniform pricing puzzle for vertically differentiated products states that a monopolist sells high and low quality products at the same price despite the fact that quality is perfectly observable and that there are no significant costs of adjusting prices. The puzzle is relevant for movies, books, music, and mobile apps, among others. We show that the puzzle can be resolved by accounting for consumer loss aversion in monetary and consumption utilities and by assuming that consumers face a random utility shock. The novelty of our approach is that the reference transaction is endogenously set as part of a `personal equilibrium' and includes only past purchases of products of the same quality.

JEL Classification: D03, D21, L1 and L2
Keywords: expectations-based loss aversion, personal equilibrium, uniform pricing puzzle and vertically differentiated products

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1 Introduction

High quality products sometimes sell at prices which are the same as or similar to those of low quality products. For instance, the price of a movie ticket does not depend on observable signals that predict gross revenue, such as the movie's ratings or popularity, or whether the movie is a sequel or a new release (Orbach and Einav 2007, Thompson 2012). Similar pricing patterns for vertically differentiated products also exist for sports games and theater season tickets (Chu et al. 2011), music, including CDs or downloadable tracks (Shiller and Waldfogel 2011), books, mobile apps, and apparel (Eckert and West 2013). A considerable amount of work addresses price uniformity, but the puzzle remains for vertically differentiated products. Under standard demand specifications, the profit maximizing price should increase with quality. The puzzle is that observed prices do not respond to quality (price uniformity) or else they respond insufficiently (price compression).

Behavioral economics offers a persuasive rationale for uniform pricing. Kahneman et al. (1994) propose to address consumers' fairness concerns by incorporating loss aversion in the utility function. Consumers have a 'reference transaction' (which may separately keep track of consumption utility and monetary payment) and react more strongly if the outcomes fall short of this reference than otherwise. Although the loss aversion argument explains price uniformity in some contexts, whereby prices do not respond to cost (Heidhues and Kőszegi 2008) or demand shocks (Chen and Cui 2013), Orbach and Einav (2007), among others, argue that in the case of vertically differentiated products, the loss aversion argument requires the assumption that there is a single reference transaction for all purchases, which is unrealistic when consumers observe quality.[1] Thus, consumers should have different reference points corresponding to different product quality classes, in which case the simple argument in Kahneman et al. (1994) does not apply. This paper adopts the behavioral approach and offers a rigorous analysis of the impact of loss aversion when consumers have a reference transaction for each quality class.

Following Mussa and Rosen (1978), we assume that a representative consumer's valuation for a product of quality q is $v = v_0 + q\theta$, where v_0 is the baseline value and $\theta \in [\theta_0, \theta_1]$ is an idiosyncratic random preference shock that captures the consumer's taste uncertainty. In this formulation, the taste shock and product quality are complements; this is an assumption that plays a key role in the analysis and that is natural for the products we study in this paper.

[1] Orbach and Einav (2007) write, "All consumers are familiar with the concept of different prices for different products. [...] Thus, charging premiums or giving discounts for unique categories of movies is unlikely to be perceived as unfair. For example, given the unique characteristics and highly publicized production budgets of event movies, charging premiums for such movies probably would not violate fairness perceptions."

For example, in the movie context, all consumers prefer popular movies, but a consumer may have idiosyncratic preferences for each movie. The quality component captures the fact that some movies are (on average) preferred by all consumers. As we argued before, it is plausible to assume that consumers make comparisons between similar movies but not between movies of different quality classes. To set a benchmark, we focus on distributions of the preference shock where the profit maximizing price for loss neutral consumers increases with quality.

Given the price, the consumer decides on which taste realisations to consume using the notion of *personal equilibrium* (hereafter PE) developed in Kőszegi and Rabin (2006, 2009). A loss averse consumer compares current purchases with a reference point, which can be described as her rational expectations about purchase outcomes, based on the recent past. A series of experimental and empirical papers provide evidence for reference points as expectations held under uncertainty.[2] We show that any PE has a threshold form; that is, if the consumer consumes for a taste realization, then she consumes for any larger taste realization as well. We characterize all the consumer equilibria and derive the *preferred personal equilibrium* (hereafter PPE). This is the PE that yields the highest expected utility. We establish that for moderate gain-loss parameter values and when quality is not too high, full consumption, whereby the consumer consumes for all taste realizations, solves the firm's revenue maximization problem.

Two forces favor the full consumption equilibrium. While, under loss neutrality, expected utility is an inverse U-shaped function of the threshold, small amounts of loss aversion transform it into a U-shaped function, for which extreme consumption plans dominate intermediate ones. This is because the net effect of gain-loss aversion is significant losses associated with comparing consumption and monetary outcomes across taste draws. These losses are minimized when the consumer consumes independently of the taste draw. Second, the firm's profits decrease with the consumption threshold when quality is not too high. While full consumption is a local minimum under risk neutrality, it is a local maximum when monetary loss aversion is large enough, and this is true for any distribution of taste draw. Under additional conditions, full consumption is the global maximum.

The optimal price responds to changes in quality under loss aversion less than it does under loss neutrality, which we interpret as loss aversion being a source of price compression. When the lower bound of the taste shock support is zero, i.e., $\theta_0 = 0$, the optimal price does not respond to quality at all. Price compression, however, is optimal only up to a threshold level of quality.

[2] Abeler et al. (2011) in effort provision experiments, Ericson and Fuster (2011) in exchange experiments, Pope and Schweitzer (2011) in professional golf, Card and Dahl (2011) in violent behavior, Bartling et al. (2015) in players' and coaches' behavior in professional soccer.

When this is the case, the consumer's surplus increases with quality, a feature consistent with the applications discussed earlier. For product qualities above that threshold, full consumption is not optimal anymore and prices respond to quality.

When uniform pricing is optimal, deviating from it imposes a first order loss on profits. To demonstrate this point, we show that the losses of wrongly assuming loss neutrality when the consumer is actually loss averse, dominate the loss of making the opposite mistake. This is because the profit function is flat around the optimal price under loss neutrality. Under loss aversion, however, the optimal profit is achieved at the full consumption corner, and a deviation from the optimal policy imposes a first order loss. Shiller and Waldfogel (2011) estimate that iTunes could increase its revenue by a sixth to a third by moving away from uniform pricing at $0.99 per song. These figures, which assume consumers are loss neutral, will change once the loss aversion costs associated with differential pricing of quality classes are taken into account.

This work contributes to a growing literature that applies endogenous reference points, under the PPE concept proposed by Kőszegi and Rabin (2006), to study the firms' pricing behavior.[3] Although various approaches have been proposed to model the reference point (Carbajal and Ely 2012, Eliaz and Spiegler 2014, Zhou 2011), we adopt the framework in Kőszegi and Rabin (2006) because it is natural to assume that consumers form lagged expectations using only past purchases of products of that quality class.[4] Our model is closely related to Herweg and Mierendorff (2013)'s study of the optimality of two-part tariffs under multi-unit demand for a single product. In contrast, we restrict our analysis to a unit-demand model to tackle the case of vertically differentiated products. This allows us to discuss interesting interactions in monetary and consumption loss aversion, an issue that has rarely been tackled in the literature due to obvious tractability challenges.

There are several explanations for uniform pricing for vertically differentiated products, including perceived fairness, stigma for cheap products, demand uncertainty, menu and monitoring costs, and tacit coordination on price (for example Orbach and Einav (2007), Eckert and West (2013), Chen and Cui (2013); see also *ArtsJournal* (2013)). The stigma and demand uncertainty explanations do not apply in our application because quality is objectively known. The menu/monitoring cost is an unlikely explanation for several product categories where the

[3]For example, Heidhues and Kőszegi (2005), Heidhues and Kőszegi (2008), Karle and Peitz (2014), and Lindsey (2011).

[4]In Heidhues and Kőszegi (2008)'s model of horizontally differentiated products, all consumers share the same reference point and it corresponds to a random purchase. Although this approach is reasonable for horizontally differentiated products that are hedonistically substitutable, it is less so for vertically differentiated products because quality draws natural boundaries between product classes.

deviations in demand are systematic, as is argued by (Orbach and Einav 2007). Tacit collusion cannot explain why uniform pricing persists even in markets where firms have significant market power. This paper offers a formal model of loss aversion that does not assume the answer by choosing arbitrary reference transactions.

The rest of the paper is organized as follows. Section 2 presents the model and notations. Section 3 solves the consumer decision-making problem and the firm revenue maximization problem. Section 4 presents our main results on the uniform pricing puzzle and discusses an application with uniform distribution of consumer valuation. Section 5 concludes.

2 Model

We first consider a firm that sells a single good of quality q to a representative loss averse consumer. In Section 4, the firm sells multiple quality classes $q \in [q_0, q_1]$. We assume that there is no marginal cost associated with serving the consumer, which is consistent with the applications discussed in the Introduction. The consumer's valuation for the product has two components. The first is that the consumer is willing to pay $v_0 \geq 0$ for any product, independent of its quality. In the case of movies, this could capture the inherent value of a night out with family and friends to watch a movie. In addition, the consumer has a random marginal willingness to pay for quality. Let $\theta \in \boldsymbol{\Theta} = [\theta_0, \theta_1]$ denote the consumer's taste draw. The marginal impact of quality on the consumer's valuation is always positive, $\theta_0 \geq 0$. We also refer to θ as the state of nature. As in Mussa and Rosen (1978), we assume that the consumer's intrinsic utility for a product of quality q in state θ is $v = v_0 + q\theta$, which implies quality q and the taste parameter θ are complements. The same structure of consumer preferences is used by Leslie and Sorensen (2014) in an empirical application to live concerts. Likewise, Herweg and Mierendorff (2013) consider a similar structure in a model of loss aversion to study different issues than the ones considered here. Finally, θ is distributed with density $g(\theta)$, cumulative distribution $G(\theta)$, and survival function $\overline{G}(\theta)$. The average of θ is denoted $\overline{\theta} = \int_{\boldsymbol{\Theta}} \theta dG(\theta)$. We make the following assumption:

Assumption 1. *$g(\theta)$ is increasing for $\theta < \overline{\theta}$ and $g(\overline{\theta} - x) = g(\overline{\theta} + x)$ for $x \in [0, \overline{\theta}]$.*

Assumption 1 implies that $g(\cdot)$ is single peaked and symmetric. This assumption holds for many distributions, such as truncated normal, uniform, and any tent-shaped distribution.

If consumers are loss neutral, the firm's revenue is $R^{\mathrm{LN}}(\theta; q) = (v_0 + q\theta)\overline{G}(\theta)$. Denote the threshold θ that maximizes revenue, $\theta^{\mathrm{LN}}(q) = \operatorname{argmax}_\theta R^{\mathrm{LN}}(\theta, q)$, and the optimal price

$p^{\text{LN}}(q) = v_0 + q\theta^{\text{LN}}(q)$. Define $\epsilon_0 = (v_0 + q\theta_0)\frac{g(\theta_0)}{q}$ as the price elasticity of demand at the corner $p = v_0 + q\theta_0$. Assumption 2 implies that there is a unique interior solution to the firm revenue maximization problem when the consumer is loss averse:

Assumption 2. $R^{LN}_{\theta\theta} < 0$, $\epsilon_0 < 1$, *and* $g(\theta_1) > 0$.

The concavity of R^{LN} in θ, i.e., $R^{\text{LN}}_{\theta\theta} < 0$, is implied by log-concavity of $g(\cdot)$, which is a standard assumption. In the loss neutral case, the firm does not serve the consumer for all taste draws. Moreover, the optimal price increases with product's quality ($p^{\text{LN}}_q(q) > 0$)[5]. In the uniform case, $\epsilon_0 < 1$ is equivalent to $q > v_0$.

Our goal is to incorporate consumer loss aversion in money and consumption, where we use the Kőszegi and Rabin (2006) conception of a PE to characterize the consumer's reference point. The consumer compares her consumption outcome with a 'reference transaction' and experiences feelings of gain and loss depending on how the actual outcome compares to the reference transaction. Consistent with the extant literature, we assume that the gain-loss utility is piece-wise linear (for example Kőszegi and Rabin (2006), Heidhues and Kőszegi (2008), Herweg and Mierendorff (2013)). We consider loss aversion in both consumption utility and monetary utility because there is evidence that both could matter (Carbajal and Ely 2012). While past studies have typically focused on a single dimension, we study the interaction between the two dimensions of loss aversion.

The consumer experiences a loss in consumption utility equal to $\lambda_c(v'-v)$ when her valuation v is lower than her reference valuation v', and a loss in monetary utility equal to $\lambda_p(p - p')$ if she pays a price p greater than the reference price p'. Likewise, the consumer experiences gains $\beta_c(v - v')$ when $v > v'$ and $\beta_p(p' - p)$ when $p < p'$. We denote $\gamma_c = \lambda_c - \beta_c$ and $\gamma_p = \lambda_p - \beta_p$ and assume $\gamma_c > 0$, $\gamma_p > 0$. Consumers are loss averse; that is, they dislike losses more than they like equal-sized gains.

From Kőszegi and Rabin (2006), a PE is a consumption plan such that the decision in each state is optimal given the reference point. Formally, denote a consumption plan $\overline{\pi} = \{\pi(\theta)\}_{\theta\in\mathbf{\Theta}}$ where $\pi(\theta)$ is the probability that the consumer consumes in state θ. The 'full consumption' plan is defined as $\pi(\theta) = 1$ for all θ.

The expected utility from following consumption plan $\overline{\pi}$ in state θ is presented in Table 1. The first line corresponds to the standard consumption utility and the other terms to the consumption and monetary gain-loss utility. The gain-loss terms compare what actually happens

[5] $p^{\text{LN}}_q(q) = \theta^{\text{LN}}(q) + q\theta^{\text{LN}}_q(q)$ and $\theta_q = -\frac{R^{\text{LN}}_{\theta q}}{R^{\text{LN}}_{\theta\theta}} > 0$.

Table 1: Utility $u(\pi(\theta)|\overline{\pi},\theta)$

Consumption Utility	$\pi(\theta)(v_0 + q\theta - p)$	
	Gain-Loss from:	
	Consumption Utility	Monetary Utility
Case 1: Consume (probability $\pi(\theta)$)	$-\lambda_c q \int_{\theta}^{\theta_1} \pi(\theta')(\theta' - \theta)dG(\theta')$ $+\beta_c(v_0 + q\theta) \int_{\mathbf{\Theta}}(1 - \pi(\theta'))dG(\theta')$ $+\beta_c q \int_{\theta_0}^{\theta} \pi(\theta')(\theta - \theta')dG(\theta')$	$-\lambda_p p \int_{\mathbf{\Theta}}(1 - \pi(\theta'))dG(\theta')$
Case 2: Not Consume (probability $1 - \pi(\theta)$)	$-\lambda_c \int_{\mathbf{\Theta}} \pi(\theta')(v_0 + q\theta')dG(\theta')$	$\beta_p p \int_{\mathbf{\Theta}} \pi(\theta')dG(\theta')$

in state θ (consume with probability $\pi(\theta)$) with what the consumer expects to happen in her reference point (consume with probability $\pi(\theta')$ in state θ', which occurs with density $g(\theta')$). The consumption utility simplifies to

$$\begin{aligned} u(\pi(\theta)|\overline{\pi},\theta) =& \pi(\theta)(v_0 + q\theta - p) - (1 - \pi(\theta)) \int_{\mathbf{\Theta}} \pi(\theta') \left(\lambda_c(v_0 + q\theta') - p\beta_p\right) dG(\theta') \\ &+ \pi(\theta) \int_{\mathbf{\Theta}} \left(\pi(\theta') \left(\beta_c(\theta - \theta')^+ - \lambda_c(\theta' - \theta)^+\right) q + (1 - \pi(\theta')) \left(\beta_c(v_0 + q\theta) - \lambda_p p\right)\right) dG(\theta'). \end{aligned} \tag{1}$$

This definition implies consumption complementarity across states; that is, consuming more in one state increases the marginal utility of consumption in other states. From Kőszegi and Rabin (2006), $\overline{\pi}$ is a PE if and only if

$$u(\pi(\theta)|\overline{\pi},\theta) \geq u(x|\overline{\pi},\theta)\ \forall x \in [0,1],\ \forall \theta \in \mathbf{\Theta}. \tag{2}$$

The consumer's overall expected utility is

$$\mathrm{EU}(\overline{\pi}) = \int_{\mathbf{\Theta}} u(\pi(\theta)|\overline{\pi},\theta)dG(\theta) \tag{3}$$

The set of PEs is not necessarily singleton. A preferred personal equilibrium (PPE) is a PE that yields the highest utility.

Finally, we follow Herweg and Mierendorff (2013) in assuming that the consumer has the option not to participate in which case she receives a utility normalized to zero. The participation constraint (PC) says that the consumer adopts PPE $\overline{\pi}$ only if her expected utility is non-negative ($\mathrm{EU}(\overline{\pi}) \geq 0$). Adding a PC is standard in models of price discrimination in contract theory (Bolton and Dewatripont 2005).[6]

[6] A challenge with the concept of personal equilibrium is that the PPE may yield a lower expected utility

To summarize, the timing of events is as follows. The firm sets the price p for a product of quality q. The consumer decides to participate. If she does, she forms expectations $\overline{\pi}$ about her probability to consume in each state. Then, the consumer discovers her taste draw θ. Finally, the consumer makes her consumption decision $\pi(\theta)$ using reference transaction $\overline{\pi}$. To solve the model, we impose three conditions: (1) PE Requirement: The consumer carries through her expectations i.e., expectation $\overline{\pi}$ and actual consumption rule $\pi(\theta)$ coincide. (2) PPE Requirement: The consumer selects the PE with the highest utility. (3) PC requirement: The consumer's expected utility is non-negative. The firm maximizes revenue, $p \int_{\mathbf{\Theta}} \pi(\theta) dG(\theta)$, subject to PE, PPE, and PC requirements. Table 3 in Appendix A summarizes the notation we use in the paper. Throughout this paper, we illustrate the results assuming a uniform distribution over $\mathbf{\Theta} = [0, 1]$ and setting $\beta_c = \beta_p = 0$.

3 Model Analysis

To illustrate how loss aversion influences the consumer's decision making process, we start with the benchmark case where $\theta_0 = \theta_1 = 0$, which implies that the consumer's valuation is a constant v_0 and independent of the quality. This benchmark is a generalization of the shopping example in Kőszegi and Rabin (2006) to non-identical gain-loss parameters in money and consumption dimensions ($\lambda_c \neq \lambda_p$ and $\beta_c \neq \beta_p$). In Section 3.2, we consider the general case with valuation uncertainty. All proofs are presented in Appendix B.

3.1 Benchmark: No Valuation Uncertainty

In this section, we illustrate the procedure to characterize the PEs and PEE. As such we ignore mixed strategies ($\pi \in (0, 1)$),[7] which means that is the consumer may either consume or not. When is consuming a PE? To answer this question, we take consuming as the reference point and compute the utility from consuming as $v_0 - p$. Because consuming is the reference point, if the consumer consumes, there is no gain-loss utility. The utility from not consuming is $-\lambda_c v_0 + \beta_p p$. The consumer expects to consume, and hence not consuming imposes a consumption loss of $-\lambda_c v_0$. However, the consumer saves the price that she expected to pay and therefore feels a monetary gain of $\beta_p p$; see the left column in Table 2. Consuming is indeed a PE if 'consume' dominates 'not consume', or $p \leq \frac{1+\lambda_c}{1+\beta_p} v_0$. Similarly, we compute the utility of not consuming and

than never consuming $\pi(\theta) = 0$. Adding a PC then is equivalent to assuming that $\pi(\theta) = 0$ is always a PE. See Heidhues and Kőszegi (2014), Eliaz and Spiegler (2014) for models that do not impose PC.

[7]In Section 3.2, we show that this is without loss of generality.

consuming when not consuming is the reference point. Not consuming is a PE if $p \geq \frac{1+\beta_c}{1+\lambda_p} v_0$; see the right column in Table 2.

Table 2: The PPE without taste uncertainty

	Payoff when:	
	Reference is to consume	Reference is not to consume
Consume	$v_0 - p$	$v_0(1+\beta_c) - (1+\lambda_p)p$
Not Consume	$-\lambda_c v_0 + \beta_p p$	0
PE Condition	$p \leq \frac{1+\lambda_c}{1+\beta_p} v_0$	$p \geq \frac{1+\beta_c}{1+\lambda_p} v_0$

Observe that loss aversion implies $\frac{1+\lambda_c}{1+\beta_p} > \frac{1+\beta_c}{1+\lambda_p}$. Consider first the case $\frac{1+\lambda_c}{1+\beta_p} \geq 1 \geq \frac{1+\beta_c}{1+\lambda_p}$. We distinguish four cases. If $p > \frac{1+\lambda_c}{1+\beta_p} v_0$, not consuming is the unique PE. If $p \in [v_0, \frac{1+\lambda_c}{1+\beta_p} v_0]$ and both consuming and not consuming are PEs however, not consuming is the PPE because it yields a higher utility. If $p \in [\frac{1+\beta_c}{1+\lambda_p} v_0, v_0]$, again both consuming and not consuming are PEs and consuming is the PPE. If $p < \frac{1+\beta_c}{1+\lambda_p} v_0$, consuming is the only PE. Putting all cases together, there is a unique PPE: the consumer consumes if and only if $p \leq v_0$, which coincides with the loss neutral case. In other words, the consumer's demand function does not change with the introduction of loss aversion.

The analysis changes slightly if $\frac{1+\lambda_c}{1+\beta_p} \leq 1$ or $\frac{1+\beta_c}{1+\lambda_p} > 1$. The loss averse optimal consumption rule does not always match the loss neutral one. Take the case $\frac{1+\lambda_c}{1+\beta_p} \leq 1$. Not consuming is the PPE (only PE) for $p \in (\frac{1+\lambda_c}{1+\beta_p} v_0, v_0]$. A loss neutral consumer would consume for prices in that interval. In the case $\frac{1+\beta_c}{1+\lambda_p} > 1$, the consumer consumes for $p \in [v_0, \frac{1+\beta_c}{1+\lambda_p} v_0)$, although she wouldn't under loss neutrality.[8]

3.2 The Consumer Problem

This section characterizes the set of PEs, shows that a PPE always exists, and derives sufficient conditions under which full consumption is the only PPE. The analysis with loss aversion differs in several ways from the loss neutral case. Under loss neutrality, the consumer consumes if her valuation is greater than the price. This delivers the consumption threshold $\theta = \frac{p-v_0}{q}$ if $\frac{p-v_0}{q} \in \boldsymbol{\Theta}$ and a corner at θ_0 or θ_1 otherwise. This threshold rule is identical to maximizing the utility that is expected from consuming when the taste draw is greater than θ, and following that rule always gives non-negative expected utility (PC holds). Under loss aversion, the consumer still uses a threshold rule in any PE (Lemma 1), but the PE threshold rule does not necessarily maximize expected utility, and the PE threshold rule that gives the highest expected utility

[8]As a caveat, we note that this generalization to $\lambda_p \neq \lambda_c$ and $\beta_p \neq \beta_c$ demonstrates that Proposition 3 in Kőszegi and Rabin (2006) holds only if $\frac{1+\lambda_c}{1+\beta_p} \geq 1 \geq \frac{1+\beta_c}{1+\lambda_p}$.

(PPE requirement) does not necessarily satisfy PC. Thus, the analysis of the consumer problem proceeds sequentially: (a) analyze the consumption decision (the PE thresholds), (b) derive the optimal consumption rule (the PPE threshold), and (c) impose the participation constraint.

Because $u(\pi(\theta)|\overline{\pi},\theta)$ is linear in $\pi(\theta)$, it follows that the consumer never randomizes consumption.

Lemma 1. *In a PE, $\pi(\theta) \in \{0,1\}$ almost everywhere and is non-decreasing in θ.*

Lemma 1 implies that the optimal consumption plan takes a threshold form; that is, do not consume if $\theta < \theta^*$ and consume if $\theta > \theta^*$, where $\theta^* \in \mathbf{\Theta}$. Define $u^1(\theta,\theta^*)$ as the utility of consuming and $u^0(\theta,\theta^*)$ as the utility of not consuming when the consumer's taste draw is θ and the threshold is θ^*. Since the utility from not consuming is independent of the taste draw (see equation (1)), we denote it simply $u^0(\theta^*)$.

$$u^0(\theta^*) = -\lambda_c \int_{\theta^*}^{\theta_1} (v_0 + q\theta')dG(\theta') + \beta_p p\overline{G}(\theta^*), \quad \text{for}\, \theta \leq \theta^* \tag{4}$$

$$\begin{aligned} u^1(\theta,\theta^*) =& v_0 + q\theta - p - \lambda_c q \int_{\theta}^{\theta_1} (\theta' - \theta)dG(\theta') + \beta_c \left((v_0 + q\theta)G(\theta^*) + q\int_{\theta^*}^{\theta} (\theta - \theta')dG(\theta') \right) \\ & - \lambda_p G(\theta^*)p, \quad \text{for}\, \theta \geq \theta^*. \end{aligned} \tag{5}$$

The net utility of consuming over not consuming, $\Delta u(\theta,\theta^*) \triangleq u^1(\theta,\theta^*) - u^0(\theta^*)$, is increasing in θ. For interior equilibrium, the equilibrium condition is $\Delta u(\theta,\theta) = 0$ (we drop the $*$-superscript for ease of exposition and without loss of clarity). It follows that $\theta \in (\theta_0,\theta_1)$ is an interior PE if and only if

$$V(\theta) = p \quad \text{where,} \quad V(\theta) \triangleq (v_0 + q\theta)\frac{1 + \beta_c + \gamma_c \bar{G}(\theta)}{1 + \beta_p + \gamma_p G(\theta)}. \tag{6}$$

Here $V(\theta)$ is the implied willingness to pay which we write as $V(\theta) = (v_0 + q\theta)L(\theta)$ where

$$L(\theta) \triangleq \frac{1 + \beta_c + \gamma_c \bar{G}(\theta)}{1 + \beta_p + \gamma_p G(\theta)}. \tag{7}$$

The function $L(\theta)$ is positive and decreasing such that $L(\theta_0) = \frac{1+\lambda_c}{1+\beta_p} > 1$ and $L(\theta_1) = \frac{1+\beta_c}{1+\lambda_p} < 1$. The numerator of $L(\theta)$ increases the consumer's willingness to pay and is called the 'attachment effect'. If the consumer consumes in states greater than θ, she suffers an attachment in states lower than θ for which she doesn't consume. Consumption loss aversion alone pushes toward over consumption relative to the loss neutral case. The denominator decreases the consumer's willingness to pay and is called the 'comparison effect'. The consumer receives a net monetary benefit in states lower than θ because she saves p relative to the states above θ in which she consumes. Price loss aversion alone pushes toward under consumption relative to the loss neutral case. Define $\tilde{\theta} = G^{-1}(\frac{\lambda_c - \beta_p}{\gamma_c + \gamma_p})$ as the valuation for which the comparison and attachment effects are equal; that is, $L(\tilde{\theta}) = 1$.

There may also exist corner equilibria. A corner PE at $\theta = \theta_0$ exists if $\Delta u(\theta_0, \theta_0) \geq 0$ or

$$p \leq \frac{1 + \lambda_c}{1 + \beta_p}(v_0 + q\theta_0). \tag{8}$$

A corner PE at $\theta = \theta_1$ exists if $\Delta u(\theta_1, \theta_1) \leq 0$ or

$$p \geq \frac{1 + \beta_c}{1 + \lambda_p}(v_0 + q\theta_1). \tag{9}$$

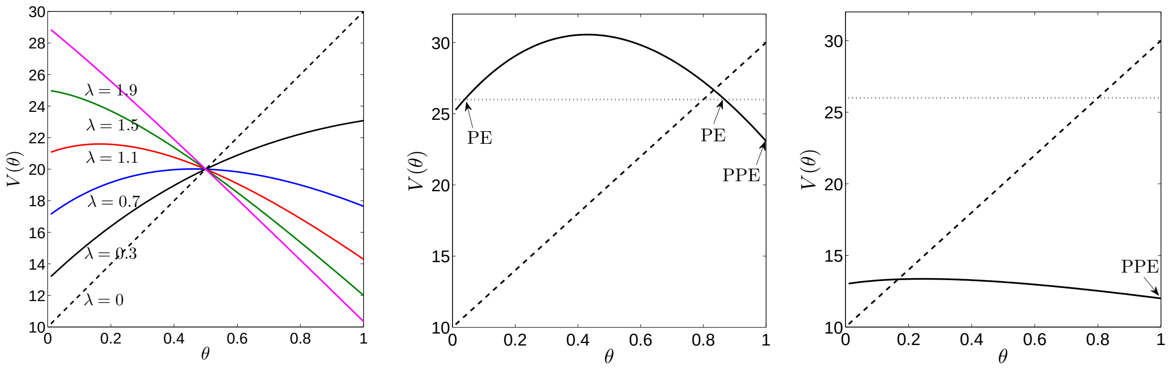
Denote $\mathbf{\Theta}^{\mathrm{PE}}(p)$ as the set of PEs associated with price p. It includes the interior PEs (θs that satisfy equation (6)) and corner PEs (corner $\theta = \theta_0$ if p satisfies (8) and corner $\theta = \theta_1$ if p satisfies (9)). In absence of loss aversion ($\lambda = \beta = 0$) the PE (both interior and corner) boil down to the standard consumption decision rule (consume if and only if $\theta \geq \frac{p - v_0}{q}$).

Lemma 2. *A PPE always exists.*

Multiple PEs might exist. Equilibrium multiplicity arises when a corner PE exists simultaneously with another corner or an interior PE. Multiple interior PEs may also exist because equation (6) may admit multiple solutions. In the absence of loss aversion $V(\theta) = v_0 + q\theta$ is linear in θ and there is a unique solution to $V(\theta) = p$. With loss aversion, however, there is no standard restriction on $g(\cdot)$ to impose a regular behavior on the function $V(\theta)$. As a result, a PE exists each time $V(\theta)$ crosses p, which may happen multiple times.

When θ is uniformly distributed over $[\theta_0, \theta_1]$, there are at most two interior PEs in addition to two possible corner PEs. Figure 1 plots $V(\theta)$ for given values of loss aversion coefficients. The dashed line plots $V(\theta)$ for loss-neutral consumers. Surprisingly, even intermediate values of loss aversion can have a significant impact on the shape of the implied willingness to pay (the difference between the solid curve and dashed line). Figure 1(a) shows that loss aversion rotates $V(.)$ around $\bar{\theta}$. For λ greater than one, the implied willingness to pay is decreasing in θ, instead of increasing, over most of the support of θ. The dotted lines on Figures 1(b) and 1(c) illustrate the interior PEs corresponding to hypothetical price $p = \$26$. For sufficiently large loss aversion coefficients, there are multiple interior PEs (Figure 1(b)) or there are no interior PEs (Figure 1(c)). In the latter case, only the corner θ_1 is a PE.

A consumer who consumes when her taste draw is above threshold θ obtains the expected



(a) $\lambda_c = \lambda_p = \lambda$ (b) $\lambda_c = 1.5, \lambda_p = 0.3$ (c) $\lambda_c = 0.3, \lambda_p = 1.5$

Figure 1: Implied willingness to pay, $V(\theta)$. The dashed line is $V(\theta)$ for loss neutral consumers. In this figure, $v_0 = 10$, $q = 20$, $\beta_c = \beta_p = 0$, and $\theta \sim U[0, 1]$.

utility

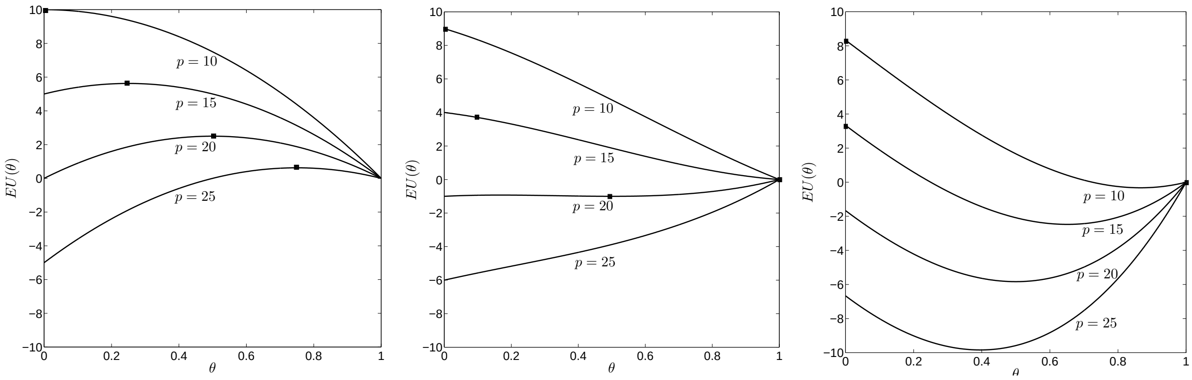
$$\mathrm{EU}(\theta, p) = u^0(\theta)G(\theta) + \int_{\theta}^{\theta_1} u^1(\theta', \theta)dG(\theta'). \tag{10}$$

Our next result simplifies $\mathrm{EU}(\theta, p)$ and helps distinguish its components.

Lemma 3. *The consumer's expected utility in following threshold consumption rule θ is*

$$EU(\theta, p) = \int_{\theta}^{\theta_1} (v_0 + q\theta' - p)dG(\theta') - \gamma_c \int_{\theta}^{\theta_1} (G(\theta') - \bar{G}(\theta'))(v_0 + q\theta')dG(\theta') - \gamma_p pG(\theta)\bar{G}(\theta). \tag{11}$$

The expected utility in (11) has three components. The first term is the standard expected utility without loss aversion. The second term captures consumption loss aversion and is negative. The third term is the monetary loss aversion and is also negative. While this last term is zero under full consumption, the consumption loss aversion term is not, and this is because the consumer compares consumption utility across valuation draws. Figure 2 depicts the consumer expected utility for a uniform taste distribution for three pairs of (λ_c, λ_p). The black dots represent the PPEs. The loss neutral consumer ($\lambda_p = \lambda_c = 0$) has a concave expected utility. On Figure 2(a) the θ that maximizes expected utility is also the marginal threshold (θ such that $v_0 + q\theta = p$) from Figure 1. Small values of loss aversion eliminate the curvature of the expected utility (Figure 2(b)). For sufficiently large values of loss aversion coefficients, the expected utility is convex (Figure 2(c)) and the PPE is achieved at a corner. In contrast with the loss neutral case, the PPE under loss aversion does not necessarily maximize expected utility over all possible thresholds.



(a) $\lambda_c = 0, \lambda_p = 0$ (b) $\lambda_c = 0.3, \lambda_p = 0.3$ (c) $\lambda_c = 0.5, \lambda_p = 1$

Figure 2: Consumers' expected utility under loss neutrality and loss aversion. The PPEs are shown on the graphs with black rectangles. In this figure, $v_0 = 10$, $q = 20$, $\beta_c = \beta_p = 0$ and $\theta \sim U[0,1]$. The black dots represent the PPE θ corresponding to price $p = 10, 15, 20, 25$.

The set of PPEs associated with price p is

$$\mathbf{\Theta}^{\text{PPE}}(p) = \left\{\theta \in \mathbf{\Theta}^{\text{PE}}(p) \text{ s.t. } \text{EU}(\theta, p) \geq \text{EU}(\theta', p) \text{ for any } \theta' \in \mathbf{\Theta}^{\text{PE}}(p)\right\}.$$

By Lemma 2, the set $\mathbf{\Theta}^{\text{PPE}}(p)$ is non-empty and could have multiple elements if the consumer receives the same expected utility in multiple non-dominated PEs. When this is the case, we use the tie-breaking rule that the consumer selects the lowest PPE with non-negative utility, which is the PPE that maximizes firm's profits. With this convention, $\theta^{\text{PPE}}(p)$ has a unique element. A PPE exists for any price. The converse is not true: there may exist thresholds that are not a PPE for any price. Define the set of implementable consumption thresholds $\mathbf{\Theta}^{\text{PPE}} = \{\mathbf{\Theta}^{\text{PPE}}(p) \text{ for } p \geq 0\}$.

To characterize $\mathbf{\Theta}^{\text{PPE}}(p)$ for any p, one needs to rank any pair of PEs according to the expected utility criteria. Such ranking will depend on the shape of $\text{EU}(\theta, p)$. Figure 2 hints that simple and general ranking rules may not exist. Take the case $\lambda_p = \lambda_c = 0.3$ (Figure 2(b)) and $p = 20$. The expected utility is almost flat. If there were multiple interior PE, the PPE selection could change for arbitrarily small changes in p or λ. That being said, an interesting pattern appears in Figure 2(c): extreme consumption thresholds ($\theta_0 = 0$ or $\theta_1 = 1$) dominate intermediate ones. In fact, we can derive fairly general conditions such that when the full consumption corner θ_0 is a PE, it is also the PPE. To proceed, we use the notation $\hat{\theta}(\theta)$ to denote the symmetric value of $\theta \in [\theta_0, \bar{\theta}]$ relative to $\bar{\theta}$, that is, $\frac{\hat{\theta}(\theta)+\theta}{2} = \bar{\theta}$. For the sake of exposition, we drop the argument and we use notation $\hat{\theta}$ instead of $\hat{\theta}(\theta)$.

Lemma 4. *Assume Assumption 1 holds and θ is a PE in $[\theta_0, \bar{\theta}]$ such that $\frac{\partial}{\partial\theta}EU(\theta) < 0$ and*

$EU(\theta) \geq 0$. *Then, θ dominates any PE in $(\theta, \hat{\theta}]$.*

Lemma 4 establishes that when the expected utility is decreasing at a PE for which consumption happens frequently ($\theta < \bar{\theta}$), that PE dominates any PE with intermediate consumption frequencies. Our next result establishes when $\frac{\partial}{\partial\theta}\mathrm{EU}(\theta) < 0$.

Lemma 5. *Assume θ is a PE. Then $\frac{\partial}{\partial\theta}EU(\theta) < 0$ if and only if $G(\theta) < \frac{\lambda_p(1+\lambda_c)-\beta_c(1+\beta_p)}{\gamma_c+\gamma_p+2(\lambda_p\lambda_c-\beta_c\beta_p)}$.*

The condition in Lemma 5 simplifies to the following assumption when it is applied to the corner PE at θ_0.

Assumption 3. $\lambda_p(1 + \lambda_c) > \beta_c(1 + \beta_p)$.

This assumption is not restrictive. It is implied, for example, by $\lambda_p \geq \beta_c$, which is itself implied by Kőszegi and Rabin (2006)'s assumption that gain and loss coefficients are the same for consumption and money ($\lambda_c = \lambda_p$ and $\beta_c = \beta_p$). The assumption also holds if the gain part of the value function is assumed to be flat ($\beta_c = \beta_p = 0$). Our next result applies Lemmas 4 and 5 to establish sufficient conditions under which θ_0 is a PPE.

Proposition 1. *Assume Assumptions 1 and 3 hold and θ_0 is a PE such that $EU(\theta_0) \geq 0$. Then θ_0 is a PPE.*

The proof shows that the expected utility decreases at $\theta = \theta_0$ and never increases back to that level for $\theta > \theta_0$. To see why this is the case, consider the three terms in equation (11). The first term is inverse U-shaped with a peak at $\theta = \frac{p-v_0}{q}$. The last two terms are negative and have a unique minimum at $\overline{\theta}$. If the last two terms are sufficiently large, the expected utility is initially decreasing and has a U-like shape; see Figure 2(c).

3.3 The Firm's Problem

The firm's revenue can be written as a function of the consumption threshold θ. Consider the case of interior values $\theta \in (\theta_0, \theta_1)$. Threshold θ is a PE for price $p = V(\theta)$, but this is not sufficient to ensure that the firm sells with probability $\bar{G}(\theta)$. In addition, θ must also be a PPE (i.e., $\theta \in \mathbf{\Theta}^{\mathrm{PPE}}$) and satisfy the PC (i.e., $\mathrm{EU}(\theta, p) \geq 0$). Define $\mathbf{\Theta}^U \triangleq \{\theta \in (\theta_0, \theta_1) |\ \mathrm{EU}(\theta, V(\theta)) \geq 0\}$ as the set of interior PEs with non-negative utility. The firm's revenues are

$$R^{\mathrm{LA}}(\theta) = V(\theta)\bar{G}(\theta) = R^{\mathrm{LN}}(\theta)L(\theta) \tag{12}$$

The firm maximizes $R^{\mathrm{LA}}(\theta)$ subject to $\theta \in \mathbf{\Theta}^U \cap \mathbf{\Theta}^{\mathrm{PPE}}$. In addition to choosing interior thresholds, the firm can choose the full consumption corner θ_0, in which case the revenue is $p_0^{\mathrm{LA}} \triangleq R^{\mathrm{LA}}(\theta_0) = \frac{1+\lambda_c}{1+\beta_p}(v_0 + q\theta_0)$ if $\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) \geq 0$, and the highest price such that $\mathrm{EU}(\theta_0, p) = 0$ otherwise. The corner θ_1 is ignored because the firm earns zero revenue. Denote θ^{LA} the threshold that maximizes the firm's revenue.

Loss aversion changes the firm's objective function relative to the loss neutral case in two ways: (a) the objective function is weighted by $L(\theta)$, and (b) not all consumption thresholds are feasible. The function $R^{\mathrm{LA}}(\theta)$ is not necessarily concave, and the set $\mathbf{\Theta}^U \cap \mathbf{\Theta}^{\mathrm{PPE}}$ is not necessarily convex. We could not find general conditions to characterize the optimal solution for all parameter values of loss aversion. Our main result rests on the observation that the revenue function reaches a maximum at θ_0 for a general subset of parameter values. In particular, this will be the case when $R^{\mathrm{LA}}(\theta)$ is decreasing, which is equivalent to

$$\frac{R_\theta^{\mathrm{LN}}(\theta)}{R^{\mathrm{LN}}(\theta)} \leq -\frac{L_\theta(\theta)}{L(\theta)} \tag{13}$$

Since $L(\theta)$ is decreasing, inequality (13) always holds for $\theta \geq \theta^{\mathrm{LN}}$. We derive a sufficient condition such that $R^{\mathrm{LA}}(\theta)$ is decreasing for any θ.

Assumption 4. $1 + \bar{G}(\theta^{LN})\left(1 - \frac{1+\beta_c\beta_p}{(1+\lambda_c)(1+\lambda_p)}\right) \geq \epsilon_0^{-1}$

As expected, this assumption contradicts Assumption 2 in the loss-neutral case. In that case, $L(\theta) = 1$ and $R^{\mathrm{LN}}(\theta)$ is increasing up to θ^{LN}. Since ϵ_0^{-1} increases with $\frac{q}{v_0}$, assumption 4 is less likely to hold for high quality products.

Lemma 6. *Assumption 4 implies that $R^{LA}(\theta)$ is decreasing. In the uniform case, a necessary and sufficient condition for $R^{LA}(\theta)$ decreasing is $\frac{q}{v_0} < 1 + \lambda_p + \frac{\lambda_c}{1+\lambda_c}$.*

Assumption 4 provides a sufficient condition and $R^{\mathrm{LA}}(\theta)$ can be decreasing more generally. $R^{\mathrm{LA}}(\theta)$ is decreasing at θ_0 if the price elasticity of demand at θ_0, denoted ϵ_0^{LA}, is greater than 1. We have

$$\epsilon_0^{\mathrm{LA}} = \frac{\epsilon_0}{1 - \kappa\epsilon_0},$$

where $\kappa = \frac{\lambda_c + \lambda_p + \lambda_c\lambda_p - \beta_c\beta_p}{(1+\beta_p)(1+\lambda_c)}$. Loss aversion increases the consumer's price elasticity, although monetary and utility loss aversion do not have the same effects: κ increases to infinity with λ_p while $\lim_{\lambda_c \to \infty} \kappa = \frac{1+\lambda_p}{1+\beta_p}$. There always exists a sufficiently large λ_p such that $R^{\mathrm{LA}}(\theta)$ is decreasing at θ_0. Loss aversion transforms a standard inverse U-shaped revenue function into a function that is decreasing at θ_0 when $1 + \kappa > \epsilon_0^{-1}$ and decreasing everywhere when Assumption

4 holds. In the uniform case with $\beta_c = \beta_p = 0$, $1 + \kappa > \epsilon_0^{-1}$ is equivalent to the condition stated in the Lemma and $R^{\mathrm{LA}}(\theta)$ decreasing at θ_0 implies that it is decreasing everywhere. This will be the case when monetary loss aversion is high and when product quality is low.

When Lemma 6 holds, the firm charges $p_0^{\mathrm{LA}} = \frac{1+\lambda_c}{1+\beta_p}(v_0 + q\theta_0)$ and implements the full consumption corner as long as the participation constraint holds. The consumer's expected utility from full consumption is obtained from equation (11)

$$\mathrm{EU}\left(\theta_0, p_0^{\mathrm{LA}}\right) = v_0 + q\bar{\theta} - p_0^{\mathrm{LA}} - \gamma_c q \int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta' dG(\theta') \tag{14}$$

Assumption 5 states a sufficient condition for $\mathrm{EU}\left(\theta_0, p_0^{\mathrm{LA}}\right) \geq 0$ for any distribution function $G(\cdot)$.

Assumption 5. $q(\bar{\theta} - \theta_0)(1 - \gamma_c/2) \geq \frac{\lambda_c - \beta_p}{1+\beta_p}(v_0 + q\theta_0)$.

The consumer participates if the expected consumption utility dominates the net loss from comparing consumption utility across θs, which is always true when there is no loss aversion in consumption ($\lambda_c = \beta_c = 0$). More generally, Assumption 5 is more likely to hold for $\bar{\theta} - \theta_0$ large and λ_c small. When Assumption 5 holds, the participation constraint is satisfied at the full consumption corner.

Lemma 7. *Assumption 5 implies that $EU(\theta_0, p_0^{LA}) \geq 0$.*

Assumption 5 is independent of $G(\cdot)$, but clearly PC holds more generally.

Proposition 2. *Assume Assumptions 1 and 3-5 hold. The full consumption corner with associated price p_0^{LA} solves the firm's revenue maximization problem ($\theta^{LA} = \theta_0$).*

The intuition for this proposition is as follows. Under loss neutrality, the firm has to lower price to $p_0^{\mathrm{LN}} = v_0 + \theta_0 q$ to induce the consumer to always buy. This is not the case under loss aversion. The firm charges $p_0^{\mathrm{LA}} > p_0^{\mathrm{LN}}$ as long as the consumer participates, which is the case under Assumption 5. It also has to be the case that the firm does not want to increase the price above p_0^{LA}. This will be the case if the demand elasticity is not too large at θ_0 (Assumption 4).

4 The Uniform Pricing Puzzle

We assume that the firm sells multiple quality classes and we study how the optimal price depends on quality. In doing so, we follow Shiller and Waldfogel (2011) in that we ignore demand interactions across quality classes. They argue that this approach is correct when the products' demands are 'independent' and offer evidence suggesting that this assumption is not unrealistic in the context of their application to the pricing of music songs. Denote the optimal firm price for a product of quality q by $p^{\mathrm{LA}}(q)$. Figure 3(a) plots $p^{\mathrm{LA}}(q)$ for $\lambda_p = 0.6$ and for different values of λ_c. The figure illustrates a wide range of parameter values (q, λ) that cover the analysis presented above and more. The dotted line plots the optimal price schedule in the loss neutral case, $p^{\mathrm{LN}}(q)$. The pricing schedules $p^{\mathrm{LA}}(q)$ have at most two kinks. The flat segments correspond to the case discussed above: full consumption and the participation constraint holds. The parts to the right of flat segments correspond to interior equilibria. The part to the left of the flat segments corresponds to the case where the participation constraint binds.

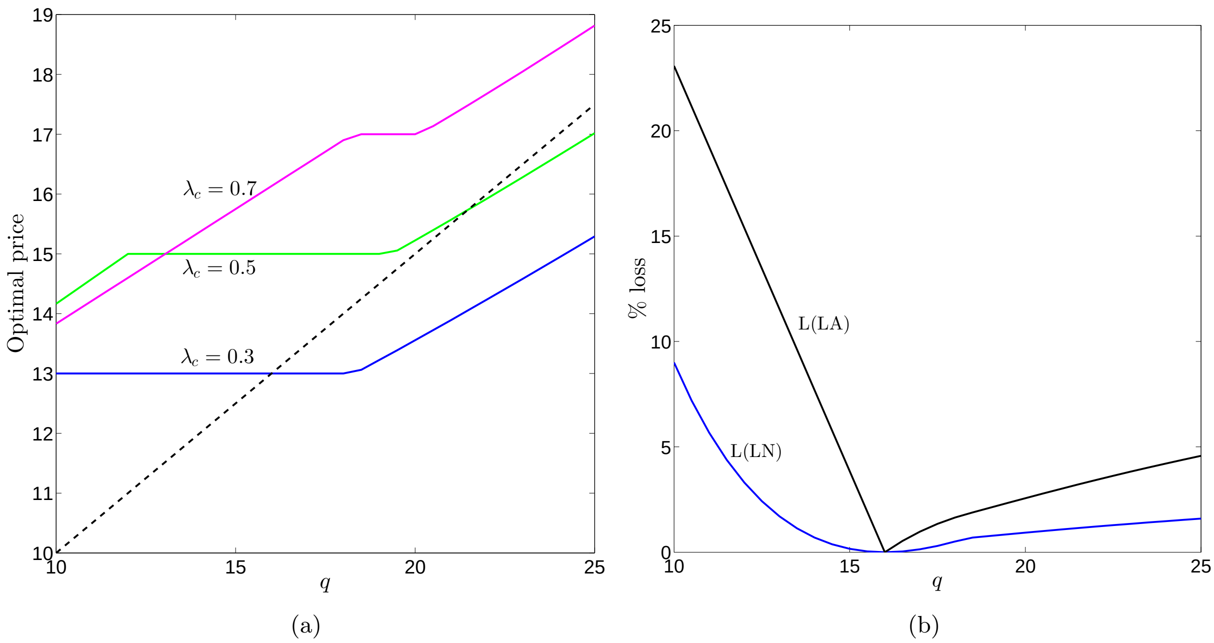


Figure 3: In both panels, we have $\lambda_p = 0.6$, $v_0 = 10$ and $\theta \sim U[0, 1]$. Panel (a) plots the optimal price as a function of q for different values of λ_c. The dashed line represents the optimal price when $\lambda_c = \lambda_p = 0$. The top curve $L(LA)$ on Panel (b) plots the percentage profit loss from wrongly assuming that the consumer is loss neutral when she is truely LA with $\lambda_c = 0.3$ and the lower curve $L(LN)$ plots the same loss from wrongly assuming that the consumer is loss averse.

Clearly, the pricing schedules under loss aversion are less steep than under loss neutrality: loss aversion generates price compression. To formalize this observation, we need workable definitions of uniform pricing and price compression applicable in our setting. A strict interpretation of the uniform pricing puzzle is that the price does not respond to quality $p_q^{\mathrm{LA}}(q) = 0$. We denote this property (P1). Figure 3(a) reveals that (P1) holds on the flat segments. More generally, loss aversion may decrease the responsiveness of price to quality relative to loss neutrality. A lower responsiveness implies price compression. A second property, denoted (P2), is $p_q^{\mathrm{LA}}(q) < p_q^{\mathrm{LN}}(q)$. This property holds everywhere in Figure 3(a): for a given q, the loss neutral price schedule is steeper than any loss averse price schedule.

We cover here the case corresponding to Assumptions 1 and 3-5 (the flat portion of the pricing schedules). Appendix C derives additional results when these conditions do not hold. The main result of the paper on price compression and price uniformity is stated next.

Proposition 3. *Assume Assumptions 1-5 hold. (a) If $\theta_0 = 0$, then (P1) and (P2) hold. (b) If $\theta_0 > 0$, then (P2) holds as long as $\frac{1+\beta_p}{1+\lambda_c}\theta^{LN} > \theta_0$.*

Proposition 3(a) explains why vertically differentiated products with a wide range of quality may sell at the same price: we have $p_q^{\mathrm{LA}}(q) = 0$ (P1 holds). Since, under loss neutrality, $p^{\mathrm{LN}}(q) = v_0 + q\theta^{\mathrm{LN}}(q) > 0$, we conclude that $p_q^{\mathrm{LN}}(q) > p_q^{\mathrm{LA}}(q)$ and (P2) hold. The price responds less to quality under loss aversion than under loss neutrality. We illustrate the relevance of Proposition 3(a) when the consumer's taste is uniformly distributed.

Corollary 1. *Consider the uniform case with $\lambda_c < 3$. If $\frac{6\lambda_c}{3-\lambda_c} \leq \frac{q}{v_0} < 1 + \lambda_p + \frac{\lambda_c}{1+\lambda_c}$, then $p(q) = (1+\lambda_c)v_0$ and consumption plan $\theta = 0$ maximize the firm's profits.*

The conditions stated in Corollary 1 are tight bounds for $p_q = 0$ (the flat segments on Figure 3(a)). Uniform pricing can be optimal with consumption loss aversion alone or monetary loss aversion alone. Still, the two sources of loss aversion play asymmetric roles: monetary loss aversion can only increase the chance that uniform pricing be optimal, and for high quality products ($q > 2v_0$) only monetary loss aversion can make uniform pricing optimal. Consumption loss aversion cannot be too strong. An increase in consumption loss aversion lowers the expected utility, and PC is violated for high enough λ_c. When $\frac{6\lambda_c}{3-\lambda_c} > \frac{q}{v_0}$, the full consumption price has to be set below p_0^{LA} in order not to bind PC.

Part (b) of the proposition shows that (P2) holds for $\theta_0 > 0$ when θ^{LN} is sufficiently large. The intuition is that under loss neutrality there are two effects of increasing quality on price $p_q^{\mathrm{LN}}(q) = \theta^{\mathrm{LN}}(q) + q\theta_q^{\mathrm{LN}}(q)$, where $\theta^{\mathrm{LN}}(q)$ is a direct effect; that is, willingness to pay increases with quality. The term $q\theta_q^{\mathrm{LN}}(q)$ is an indirect effect due to re-optimizing price. Because the indirect effect is positive, a lower bound for $p_q^{\mathrm{LN}}(q)$ is $\theta^{\mathrm{LN}}(q)$. Under loss aversion, we have $p_q^{\mathrm{LA}}(q) = \theta^{\mathrm{LA}}(q)L(\theta^{\mathrm{LA}}(q)) = \frac{1+\lambda_c}{1+\beta_p}\theta_0$, since $\theta_q^{\mathrm{LA}}(q) = 0$ (the indirect effect disappears when θ is a corner). When $\theta^{\mathrm{LN}} > \frac{1+\lambda_c}{1+\beta_p}\theta_0$, the direct effect under loss aversion is smaller than the direct effect under loss neutrality.[9]

[9] It is also possible to compare the curvatures of the price schedules under mild additional assumptions. Under

The loss from deviating from uniform pricing under loss aversion can be substantial. This is because the firm's optimal profits are achieved at the full consumption corner. The firm's profits are not flat at the optimal price. Any deviation from the optimal price imposes a first order loss. Figure 3(b) illustrates this point. The top curve (L(LA)) plots the percentage profit loss from wrongly assuming that the consumer is loss neutral when she is loss averse with value $\lambda_c = 0.3$ (and charging $p^{\mathrm{LN}}(q)$ instead of $p^{\mathrm{LA}}(q)$). The bottom curve (L(LN)) plots the loss from making the symmetric mistake. For $\lambda_c = 0.3$ we have $p^{\mathrm{LN}}(16) = p^{\mathrm{LA}}(16)$ and L(LA) = L(LN) = 0 for $q = 16$. This establishes a benchmark case for which there is no cost of not knowing whether the consumer is loss neutral or averse. L(LN) stays close to zero for small deviations from $q = 16$, and that is because making small pricing mistakes has only a second order effect on the loss neutral profits. This is not the case for L(LA), however. Even small mistakes can have a large negative impact on profits when the consumer is loss averse.

The assumption that the taste draw θ be complement with product quality in the consumption utility is essential. To see why, assume instead that the taste draw and product quality are additive: the consumer valuation is $\tilde{q} + \theta$ (instead of $v_0 + q\theta$), where $\tilde{q}$ is an additive quality component. The analysis follows after the change of variable $v_0 = \tilde{q}$ and $q = 1$. The optimal price under loss aversion is $p^{\mathrm{LA}}(\tilde{q}) = (\tilde{q} + \theta_0)\frac{1+\lambda_c}{1+\beta_p}$ and $p^{\mathrm{LA}}_{\tilde{q}}(\tilde{q}) = \frac{1+\lambda_c}{1+\beta_p}$. Under loss neutrality, the price schedule is such that $p^{\mathrm{LN}}_{\tilde{q}}(\tilde{q}) < 1$.[10] The price schedule is steeper under loss aversion than under loss neutrality: $p^{\mathrm{LA}}_{\tilde{q}}(\tilde{q}) > p^{\mathrm{LN}}_{\tilde{q}}(\tilde{q})$. In both the additive and multiplicative cases, loss aversion increases consumption: the firms sells the product to the consumer for all taste draws. But this alone is not sufficient to make the price schedule flatter. Product quality and the taste draw have to be complement. When this is the case, the firm's price, which is equal to the lower bound of the valuation support, responds little to a change in quality.

Under general conditions, the consumer's expected utility increases with product quality. Denote the consumer's expected utility from product q when the price is $p^{\mathrm{LA}}_0(q)$, $\tilde{\mathrm{EU}}(q) = \mathrm{EU}(\theta_0, p^{\mathrm{LA}}_0(q))$. We have

$$\frac{\partial}{\partial q}\tilde{\mathrm{EU}}(q) = \bar{\theta} - \frac{1+\lambda_c}{1+\beta_p}\theta_0 - \gamma_c \int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta' dG(\theta').$$

Lemma 8. *Assume $\lambda_c \geq \beta_p$ and Assumption 5 holds.* $\frac{\partial}{\partial q}\tilde{EU}(q) \geq (\bar{\theta} - \theta_0)(1 - \gamma_c(\bar{\theta} - \theta_0)) -$

loss aversion, the price increases linearly with quality. Under loss neutrality the price is a convex function of quality as long as $qR^{\mathrm{LN}}_{\theta\theta\theta} - 2R^{\mathrm{LN}}_{\theta\theta} > 0$. Thus loss aversion transforms an increasing convex price schedule into a linear one, or constant in the case ($\theta_0 = 0$).

[10] $p^{\mathrm{LN}}(\tilde{q}) = \tilde{q} + \theta^{\mathrm{LN}}(\tilde{q})$ and $p^{\mathrm{LN}}_{\tilde{q}}(\tilde{q}) = 1 + \theta^{\mathrm{LN}}_{\tilde{q}}(\tilde{q}) < 1$ because $\theta^{\mathrm{LN}}_{\tilde{q}}(\tilde{q}) = \frac{g(\theta)}{R^{\mathrm{LN}}_{\theta\theta}(\theta)} < 0$.

$\frac{\lambda_c-\beta_p}{1+\beta_p}\theta_0 \geq 0$.

Lemma 8 says that consumers receive a larger surplus from better products. Consumers also suffer a greater consumption gain-loss utility from better products. But the overall effect, taking into account direct consumption utility, is such that overall utility increases with quality. This establishes a key feature of the uniform pricing puzzle: price compression is observed despite the fact that consumers receive strictly larger surpluses from better products.

5 Conclusions

The uniform pricing puzzle has not been resolved for the case of vertically differentiated products. We present a model of monopoly pricing with vertically differentiated products and consumer loss aversion, in which the consumer compares current purchases using a lagged expectation of transactions of products of the same quality. Thus, loss aversion applies within a class of products of the same quality, but not across quality classes. We show that uniform pricing can be optimal across quality classes up to a quality threshold. This will be the case if the consumer is sufficiently loss averse in monetary utility, but not too much in consumption utility and if product quality and the consumer's idiosyncratic taste draw are complement.

The consumer consumes for all taste draws and quality influences price only through its impact on the consumer's lowest possible valuation. Price compression happens because the consumer's lowest valuation responds little to quality. Price uniformity is optimal when the consumer's lowest valuation does not depend on quality. In both cases, the price differences across quality classes is smaller with loss aversion relative to loss neutrality, and the consumer surplus increases with quality, which is consistent with casual observation. Finally, the loss from mistakenly assuming that the consumer is loss neutral, when she is truly loss averse, dominates the cost of making the opposite mistake. This implication is relevant to the empirical literature that compares the profitability of price uniformity and variable pricing (e.g. Chu et al. (2011), Shiller and Waldfogel (2011)).

For the sake of exposition, we have ignored cost issues. Although we assumed no fixed costs, the analysis naturally follows when there is a fixed cost of producing each product that could be increasing with the product's quality. We also assumed a negligible or no variable cost of producing each unit. This is the case for movies, mobile apps, and music tracks, among other items. The analysis could include a marginal cost of serving the consumer, and the optimal price would not depend on the marginal cost when full consumption is optimal. For high quality products, interior consumption is optimal and the marginal cost influences the optimal price.

The main point of this paper is to demonstrate that loss aversion with random utility can explain price compression and price uniformity for vertically differentiated products. As mentioned in the Introduction, this does not rule out other explanations based on menu cost, contractual constraints, or other rationales. The main message of this work is that even small values of loss aversion can have important implications for a firm's optimal price when the consumer faces random valuation shocks.

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Appendices

A Notation

Table 3: Notations

q	product quality
$\theta \in \mathbf{\Theta} = [\theta_0, \theta_1]$	consumer's taste draw
$g(\theta)$, $G(\theta)$	probability distribution function and cumulative distribution function of θ
$\bar{\theta}$	mean θ
$\hat{\theta}(\theta)$	symmetric value of $\theta \in [\theta_0, \bar{\theta}]$ relative to $\bar{\theta}$ $((\hat{\theta} + \theta)/2 = \bar{\theta})$
$v = v_0 + q\theta$	consumer valuation
ϵ_0	price elasticity at $p = v_0 + q\theta_0$
λ_c, β_c	loss-gain parameters in consumption utility
γ_c	$\lambda_c - \beta_c$
λ_p, β_p	loss-gain parameters in monetary utility
γ_p	$\lambda_p - \beta_p$
$\pi(\theta)$	consumption probability is state θ
$\bar{\pi} = \{\pi(\theta)\}_{\theta \in \mathbf{\Theta}}$	consumption plan
$u(\bar{\pi}\|\bar{\pi}, \theta)$	utility from consumption plan $\bar{\pi}$ in state θ (equation 1)
$\mathrm{EU}(\bar{\pi})$	expected utility of consumption plan $\bar{\pi}$ (equation 3)
$u^0(\theta^*)$	utility of not consuming for threshold θ^* (equation 4)
$u^1(\theta, \theta^*)$	utility of consuming for taste draw θ and threshold θ^* (equation 5)
$\Delta u(\theta, \theta^*)$	net utility of consuming over not consuming $(\Delta u(\theta, \theta^*) = u^1(\theta, \theta^*) - u^0(\theta^*))$
$V(\theta)$	implied willingness to pay (equation 6)
$L(\theta)$	ration of attachment effect to comparison effect (equation 7)
$\mathrm{EU}(\theta, p)$	expected utility from threshold θ (equation 10 and 11)
$\tilde{\theta}$	state θ at which $L(\theta) = 1$
$\mathbf{\Theta}^{PE}(p)$	set of interior PEs associated to a price p
$\mathbf{\Theta}^{U}$	set of PEs with non-negative utility
$\mathbf{\Theta}^{\mathrm{PPE}}(p)$	PPE associated to price p
$\mathbf{\Theta}^{\mathrm{PPE}}$	set of implementable consumption thresholds
$R^{\mathrm{LN}}(\theta; q)$	firm's revenue when consumers are loss neutral
$\theta^{\mathrm{LN}}(q)$	threshold θ maximizing $R^{\mathrm{LN}}(\theta; q)$
$p^{\mathrm{LN}}(q)$	price maximizing $R^{\mathrm{LN}}(\theta; q)$
$R^{\mathrm{LA}}(\theta; q)$	firm's revenue when consumers are loss averse (equation 12)
$\theta^{\mathrm{LA}}(q)$	optimal consumption threshold for product q
$p^{\mathrm{LA}}(q)$	optimal price for product q
p_0^{LA}	revenue under full consumption $(p_0^{\mathrm{LA}} = R^{\mathrm{LA}}(\theta_0))$

B Proofs

Proof of Lemma 1: We first show that $\pi(\theta)$ is non-decreasing in θ. The proof goes by contradiction. Assume there exist $\theta_i < \theta_j$ such that $\pi(\theta_i) > \pi(\theta_j)$. We have

$$u(\pi(\theta_i)|\overline{\pi}, \theta_i) \geq u(\pi(\theta_j)|\overline{\pi}, \theta_i)$$

$$u(\pi(\theta_j)|\overline{\pi}, \theta_j) \geq u(\pi(\theta_i)|\overline{\pi}, \theta_j)$$

Summing up these two inequalities

$$(u(\pi(\theta_i)|\overline{\pi}, \theta_j) - u(\pi(\theta_i)|\overline{\pi}, \theta_i)) - (u(\pi(\theta_j)|\overline{\pi}, \theta_j) - u(\pi(\theta_j)|\overline{\pi}, \theta_i)) \leq 0$$

which contradicts the fact that

$$\frac{\partial^2}{\partial\theta\partial\pi(\theta)} u(\pi(\theta)|\overline{\pi}, \theta) = q + \lambda_c q \int_{\theta}^{\theta_1} \pi(\theta')dG(\theta') + \beta_c q \int_{\theta_0}^{\theta} \pi(\theta')dG(\theta') + \beta_c q \int_{\mathbf{\Theta}} (1-\pi(\theta'))dG(\theta') > 0$$

Next we show that $\pi(\theta) \in \{0, 1\}$ almost everywhere. Assume this is not the case. There exists an interval $[\theta_a, \theta_b]$ such that $\pi(\theta) \in (0, 1)$ for $\theta \in [\theta_a, \theta_b]$. Thus, $u(1|\overline{\pi}, \theta) = u(0|\overline{\pi}, \theta)$ for $\theta \in [\theta_a, \theta_b]$. But we have

$$u(0|\overline{\pi}, \theta) = -\lambda_c \int_{\mathbf{\Theta}} \pi(\theta')(v_0 + q\theta')dG(\theta') + \beta_p p \int_{\mathbf{\Theta}} \pi(\theta')dG(\theta')$$

and $\frac{\partial}{\partial\theta} u(0|\overline{\pi}, \theta) = 0$ for $\theta \in [\theta_a, \theta_b]$ while

$$\frac{\partial}{\partial\theta} u(1|\overline{\pi}, \theta) = q + \lambda_c q \int_{\theta}^{\theta_1} \pi(\theta')dG(\theta') + \beta_c q \int_{\theta_0}^{\theta} \pi(\theta')dG(\theta') + \beta_c q \int_{\mathbf{\Theta}} (1 - \pi(\theta'))dG(\theta') > 0.$$

A contradiction. □

Proof of Lemma 2: If $\Delta u(\theta_0, \theta_0) > 0$, then $\theta = \theta_0$ is a corner PE. If $\Delta u(\theta_1, \theta_1) < 0$ then $\theta = \theta_1$ is a corner PE. If neither holds, then by continuity of the function $\Delta u(x, x)$, there exists an interior PE $\theta \in (\theta_0, \theta_1)$ such that $\Delta u(\theta, \theta) = 0$. Thus a PE always exists. The existence of a PPE follows. □

Proof of Lemma 3: Plug into equation (10) the values for

$$u^0(\theta) = -\lambda_c \int_{\theta}^{\theta_1} (v_0 + q\theta')dG(\theta') + \beta_p p\bar{G}(\theta)$$

$$u^1(\theta', \theta) = v_0 + q\theta' - p - \lambda_c q \int_{\theta'}^{\theta_1} (\theta'' - \theta')dG(\theta'') + \beta_c \left((v_0 + q\theta')G(\theta) + q \int_{\theta}^{\theta'} (\theta' - \theta'')dG(\theta'') \right) - \lambda_p pG(\theta)$$

for $\theta' > \theta$ and after replacement, we obtain $\mathrm{EU}(\theta)$ as

$$\begin{aligned}\mathrm{EU}(\theta, p) =& (1 - \gamma_c G(\theta)) \int_{\theta}^{\theta_1} (v_0 + q\theta')dG(\theta') - \gamma_c q \int_{\theta}^{\theta_1} \int_{\theta'}^{\theta_1} (\theta'' - \theta')dG(\theta'')dG(\theta') \\ &- p\overline{G}(\theta)(1 + \gamma_p G(\theta))\end{aligned} \tag{15}$$

Note that

$$\int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}(\theta''-\theta')dG(\theta'')dG(\theta')=\int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}\theta''dG(\theta'')dG(\theta')-\int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}\theta'dG(\theta'')dG(\theta').$$

Applying integration by parts to the first term yields $-G(\theta)\int_{\theta}^{\theta_1}\theta'dG(\theta')+\int_{\theta}^{\theta_1}G(\theta')\theta'dG(\theta')$. Collecting terms, we obtain

$$\int_{\theta}^{\theta_1}\int_{\theta'}^{\theta_1}(\theta''-\theta')dG(\theta'')dG(\theta')=-\int_{\theta}^{\theta_1}(\bar{G}(\theta')-G(\theta'))\theta'dG(\theta')-G(\theta)\int_{\theta}^{\theta_1}\theta'dG(\theta')$$

Plugging this expression in (15), we obtain

$$\begin{aligned}\mathrm{EU}(\theta,p)=&(1-\gamma_c G(\theta))\int_{\theta}^{\theta_1}(v_0+q\theta')dG(\theta')+\gamma_c q\left(\int_{\theta}^{\theta_1}(\bar{G}(\theta')-G(\theta'))\theta'dG(\theta')+G(\theta)\int_{\theta}^{\theta_1}\theta'dG(\theta')\right)\\&-p\overline{G}(\theta)(1+\gamma_p G(\theta))\\=&\int_{\theta}^{\theta_1}(v_0+q\theta'-p)dG(\theta')-\gamma_c v_0 G(\theta)\bar{G}(\theta)+\gamma_c q\int_{\theta}^{\theta_1}(\bar{G}(\theta')-G(\theta'))\theta'dG(\theta')-\gamma_p pG(\theta)\bar{G}(\theta).\end{aligned}$$

Because $G(\theta)\bar{G}(\theta)=-\int_{\theta}^{\theta_1}(\bar{G}(\theta')-G(\theta'))dG(\theta')$, we can write $\mathrm{EU}(\theta,p)$ as

$$\mathrm{EU}(\theta,p)=\int_{\theta}^{\theta_1}(v_0+q\theta'-p)dG(\theta')-\gamma_c\int_{\theta}^{\theta_1}(G(\theta')-\bar{G}(\theta'))(v_0+q\theta')dG(\theta')-\gamma_p pG(\theta)\bar{G}(\theta).$$

□

Proof of Lemma 4 We state and prove two results that together prove Lemma 4.

Claim 1. *Assume θ^* is a PE in $[\theta_0,\bar{\theta}]$ such that $\frac{\partial}{\partial\theta}EU(\theta^*)<0$ and $EU(\theta^*)\geq 0$. Then $\frac{d}{d\theta}EU(\theta)<0$ for $\theta\in[\theta^*,\bar{\theta})$.*

Proof of Claim 1: From Lemma 3, we obtain

$$\frac{d}{d\theta}\mathrm{EU}(\theta)=g(\theta)(M(\theta)-F(\theta)),\tag{16}$$

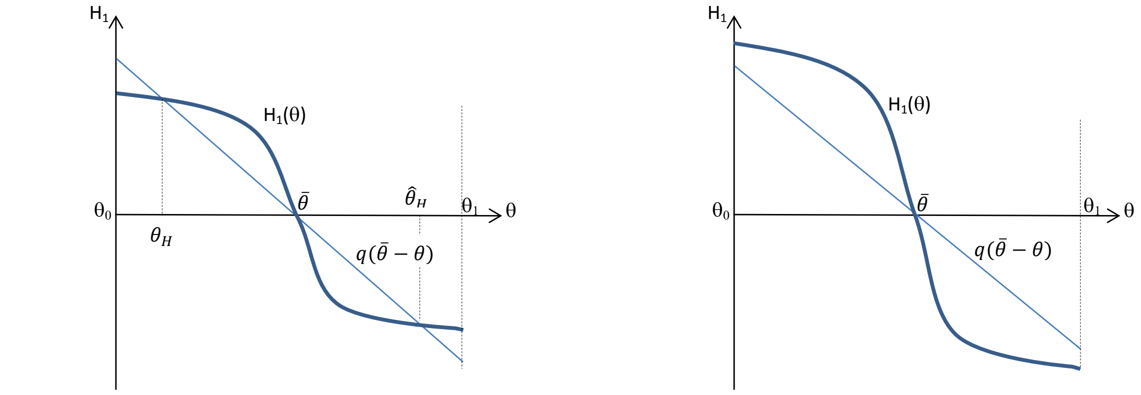
where $M(\theta)=p-(v_0+q\theta)$ and $F(\theta)=(\bar{G}(\theta)-G(\theta))(\gamma_c(v_0+q\theta)+\gamma_p p))$. Because by assumption $\frac{\partial}{\partial\theta}\mathrm{EU}(\theta^*)<0$, we have $F(\theta^*)>M(\theta^*)$. Further, $\mathrm{EU}(\theta^*)\geq 0$ implies that $F(\bar{\theta})=0\geq p-(v_0+q\bar{\theta})=M(\bar{\theta})$. We also have $\frac{d}{d\theta}F(\theta)=-2(\gamma_p p+\gamma_c(v_0+q\theta))g(\theta)+\gamma_c q(\bar{G}(\theta)-G(\theta))$ and $\frac{d^2}{d\theta^2}F(\theta)=-4\gamma_c qg(\theta)-2(\gamma_p p+\gamma_c(v_0+q\theta))g'(\theta)$. Observe that $\frac{d^2}{d\theta^2}<0$ for $\theta<\bar{\theta}$. To sum up, we have $F(\theta^*)>M(\theta^*)$, $F(\cdot)$ is concave over $[\theta^*,\bar{\theta}]$, and $F(\bar{\theta})\geq M(\bar{\theta})$. We conclude that $F(\theta)>M(\theta)$ and $\frac{d}{d\theta}\mathrm{EU}(\theta)=g(\theta)(M(\theta)-F(\theta))<0$ for $\theta\in[\theta^*,\bar{\theta})$. □

Claim 2. *Assume Assumption 1 holds and θ^* is a PE in $[\theta_0,\bar{\theta}]$ such that $\frac{\partial}{\partial\theta}EU(\theta^*)<0$ and $EU(\theta^*)\geq 0$. Then $EU(\theta)<EU(\theta^*)$ for $\theta\in[\bar{\theta},\hat{\theta}^*]$.*

Proof of Claim 2: From Lemma 3, we obtain

$$\frac{d}{d\theta}\mathrm{EU}(\theta)=g(\theta)(H(\theta)+K(\theta)).\tag{17}$$

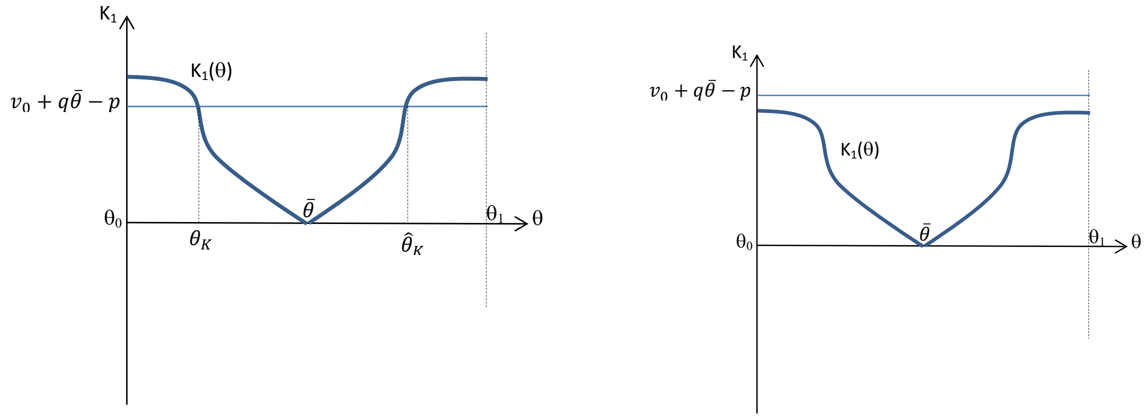
with $H(\theta)=q(\bar{\theta}-\theta)-(\bar{G}(\theta)-G(\theta))(\gamma_c(v_0+q\bar{\theta})+\gamma_p p)$ and $K(\theta)=\gamma_c q(\bar{\theta}-\theta)(\bar{G}(\theta)-G(\theta))-(v_0+q\bar{\theta}-p)$. We further write $H(\theta)=q(\bar{\theta}-\theta)-H_1(\theta)$ and $K(\theta)=K_1(\theta)-(v_0+q\bar{\theta}-p)$. Given these definitions, the following properties hold (see Figures 4 and 5 for schematic representations):



(a) Case: $H(0) < 0$

(b) Case: $H(0) \geq 0$

Figure 4: $H(\theta) = q(\bar{\theta} - \theta) - H_1(\theta)$



(a) Case: $K(0) \geq 0$

(b) Case: $K(0) < 0$

Figure 5: $K(\theta) = K_1(\theta) - (v_0 + q\bar{\theta} - p)$

$(\triangle 1)$: $H_1(\theta)$ is positive over $[\theta_0, \bar{\theta}]$, $\frac{d}{d\theta}H_1(\theta) = -2g(\theta)(\gamma_c(v_0 + q\bar{\theta}) + p\gamma_p) \leq 0$ for $\theta \in [\theta_0, \theta_1]$, $\frac{d^2}{d\theta^2}H_1(\theta) = -2g'(\theta)(\gamma_c(v_0 + q\bar{\theta}) + p\gamma_p) \leq 0$ for $\theta \in [\theta_0, \bar{\theta}]$, and $H_1(\bar{\theta} - x) = -H_1(\bar{\theta} + x)$. Next, we claim that when $H(\theta_0) < 0$ (Panel 4(b)), $H_1(\theta)$ never crosses $q(\theta - \bar{\theta})$ over $[\theta_0, \bar{\theta})$ and when $H(\theta_0) \geq 0$ (Panel 4(a)), $H_1(\theta)$ crosses $q(\theta - \bar{\theta})$ exactly once over $[\theta_0, \bar{\theta})$ at a point that we denote θ_H. Take the latter statement. When $H(\theta_0) \geq 0$, $H_1(\cdot)$ is weakly lower than $q(\theta - \bar{\theta})$ at $\theta = \theta_0$ ($H_1(\theta_0) \leq q\bar{\theta}$), the two are equal at $\theta = \bar{\theta}$ ($H_1(\bar{\theta}) = 0$), and H_1 is decreasing and concave while $q(\theta - \bar{\theta})$ is linear. Thus the two curves cross exactly once.

$(\triangle 2)$: $K_1(\theta) \geq 0$, $\frac{d}{d\theta}K_1(\theta) = -\gamma_c q(\bar{G}(\theta) - G(\theta) + 2g(\theta)(\bar{\theta} - \theta)) \leq 0$ for $\theta \in [\theta_0, \bar{\theta}]$, and $K_1(\bar{\theta} - x) = K_1(\bar{\theta} + x)$. When $K(\theta_0) \geq 0$ (Panel 5(a)), $K_1(\theta)$ intercepts $v_0 + p\bar{\theta} - p$ exactly once in $[\theta_0, \bar{\theta}]$ and we denote that point θ_K. When $K(\theta_0) < 0$ (Panel 5(b)), $K(\theta) < 0$ for $\theta \in [\theta_0, \theta_1]$.

Since $\frac{d}{d\theta}\text{EU}(\theta_0) < 0$, we have $H(\theta_0) + K(\theta_0) < 0$. Using this fact we distinguish three cases:

CASE 1: $K(\theta_0) < 0$, $H(\theta_0) \geq 0$, and $\theta^* \leq \theta^H$. (Panels 5(b) and 4(b)). We distinguish three intervals: (1) $\theta \in [\theta^*, \theta_H]$. By Claim 2, $\text{EU}(\theta)$ is decreasing in θ. (2) $\theta \in [\theta_H, 1-\theta_H]$. We have the following properties: $H(\theta_H) = H(1-\theta_H) = 0$ and ($\triangle 2$) implies $\int_{\theta_H}^{\theta} H(\theta)dG(\theta) \leq 0$. Using now the property that $K(\theta) < 0$, we conclude that $\int_{\theta_H}^{\theta} \frac{d}{d\theta}\text{EU}(\theta')d\theta' < 0$. (3) $\theta \in [1-\theta_H, \hat{\theta}^*]$. We now have $H(\theta) \leq 0$ and $K(\theta) < 0$. Thus, $\frac{d}{d\theta}\text{EU}(\theta) < 0$. Putting the conclusions drawn for each interval in 1-3 together, we conclude that $U(\theta) - U(\theta^*) = \int_{\theta^*}^{\theta} \frac{d}{d\theta}\text{EU}(\theta')d\theta' < 0$ for any $\theta \in [\bar{\theta}, \hat{\theta}^*]$.

CASE 2: $H(\theta_0) < 0$, $K(\theta_0) \geq 0$, and $\theta^* \leq \theta^K$. (Panels 4(a) and 5(a)). We distinguish three intervals: (1) $\theta \in [\theta^*, \theta_K]$. By Claim 2, $\text{EU}(\theta)$ is decreasing in θ. (2) $\theta \in [\theta_K, 1-\theta_K]$. We have the following properties: because $\theta_K \leq \bar{\theta}$, ($\triangle 2$) implies that $\int_{\theta_K}^{\theta} H(\theta)dG(\theta) \leq 0$ for any $\theta \in [\theta_K, 1-\theta_K]$. Since $K(\theta) \leq 0$, we conclude that $\int_{\theta_K}^{\theta} \frac{d}{d\theta}\text{EU}(\theta')d\theta' \leq 0$. (3) $\theta \in [1-\hat{\theta}_K, \hat{\theta}^*]$. We now have $H(\theta) \geq 0$ and $K(\theta) \geq 0$ for $\theta \in [1-\theta_K, \theta_1]$. Thus, $\text{EU}(\theta)$ increases over $[1-\theta_K, \theta_1]$ and reaches its maximum $\text{EU}(\theta_1) = 0$ at θ_1. Over that interval, we have $\text{EU}(\theta) \leq \text{EU}(\theta_1) = 0 \leq \text{EU}(\theta^*)$. Putting the conclusions drawn for each interval in 1-3 together, we conclude that $U(\theta) - U(\theta^*) = \int_{\theta^*}^{\theta} \frac{d}{d\theta}\text{EU}(\theta')d\theta' < 0$ for any $\theta \in [\bar{\theta}, \hat{\theta}^*]$.

CASE 3: This includes all remaining cases: (3a) $K(\theta_0) < 0$, $H(\theta_0) \geq 0$, and $\theta^* > \theta^H$ (Panels 5(b) and 4(b)); (3b) $H(\theta_0) < 0$, $K(\theta_0) \geq 0$, and $\theta^* > \theta^K$ (Panels 4(a) and 5(a)); (3c) $H(\theta_0) < 0$ and $K(\theta_0) < 0$ (Panels 4(a) and 5(b)). The argument in all three cases is the same: $K(\theta) < 0$ and $\int_{\theta^*}^{\theta} H(\theta')dG(\theta') \leq 0$ for any $\theta \in [\theta^*, \hat{\theta}^*]$. Again $U(\theta) - U(\theta^*) = \int_{\theta^*}^{\theta} \frac{d}{d\theta}\text{EU}(\theta')d\theta' \leq 0$ for any $\theta \in [\bar{\theta}, \hat{\theta}^*]$.

In all three cases, we obtain $U(\theta^*) > U(\theta)$ for $\theta \in [\bar{\theta}, \hat{\theta}^*]$. Thus, the PE θ^* dominates any other candidate PE $\theta \in [\bar{\theta}, \hat{\theta}^*]$. □

Proof of Lemma 5 : Differentiating (11) with respect to θ, we obtain

$$\frac{\partial}{\partial\theta}\text{EU}(\theta) = -g(\theta)(v_0 + q\theta - p + (\bar{G}(\theta) - G(\theta))(\gamma_c(v_0 + q\theta) + p\gamma_p)).$$

Therefore, $\frac{\partial}{\partial\theta}\text{EU}(\theta) < 0$ is equivalent to

$$1 - \gamma_p(\bar{G}(\theta) - G(\theta)) < \frac{v_0 + q\theta}{p}(1 + \gamma_c(\bar{G}(\theta) - G(\theta))).$$

From (6), at an interior PE, we have $\frac{v_0+q\theta^*}{p} = \frac{1+\beta_p+\gamma_p G(\theta^*)}{1+\beta_c+\gamma_c\bar{G}(\theta^*)}$. Evaluating $\frac{\partial}{\partial\theta}\text{EU}(\theta) < 0$ at $\theta = \theta^*$ gives

$$1 - \gamma_p(\bar{G}(\theta^*) - G(\theta^*)) < \frac{1 + \beta_p + \gamma_p G(\theta^*)}{1 + \beta_c + \gamma_c\bar{G}(\theta^*)}(1 + \gamma_c(\bar{G}(\theta^*) - G(\theta^*))).$$

After simplifications, we obtain the inequality in the Lemma. □

Proof of Lemma 6 : We have

$$\frac{R^{\mathrm{LN}}_\theta(\theta)}{R^{\mathrm{LN}}(\theta)} = \frac{q}{v_0+q\theta} - \frac{g(\theta)}{\bar{G}(\theta)}, \qquad \text{and}$$
$$\frac{L_\theta(\theta)}{L(\theta)} = \frac{-(\lambda_c+\lambda_p+\lambda_c\lambda_p-\beta_c\beta_p)g(\theta)}{(1+\beta_p+\gamma_p G(\theta))(1+\beta_c+\gamma_c\bar{G}(\theta))}.$$

Therefore, $\frac{R^{\mathrm{LN}}_\theta(\theta)}{R^{\mathrm{LN}}(\theta)} \leq -\frac{L_\theta(\theta)}{L(\theta)}$ is equivalent to

$$1+\bar{G}(\theta)\frac{\lambda_c+\lambda_p+\lambda_c\lambda_p-\beta_c\beta_p}{(1+\beta_p+\gamma_p G(\theta))(1+\beta_c+\gamma_c\bar{G}(\theta))} \geq \frac{q\bar{G}(\theta)}{(v_0+q\theta)g(\theta)}.$$

A bound for the first fraction in the above inequality is

$$\frac{\lambda_c+\lambda_p+\lambda_c\lambda_p-\beta_c\beta_p}{(1+\beta_p+\gamma_p G(\theta))(1+\beta_c+\gamma_c\bar{G}(\theta))} \geq \frac{\lambda_c+\lambda_p+\lambda_c\lambda_p-\beta_c\beta_p}{(1+\lambda_p)(1+\lambda_c)} = 1 - \frac{1+\beta_c\beta_p}{(1+\lambda_p)(1+\lambda_c)}$$

It follows that $R^{\mathrm{LA}}(\theta)$ is decreasing as long as

$$1+\bar{G}(\theta)\left(1-\frac{1+\beta_c\beta_p}{(1+\lambda_p)(1+\lambda_c)}\right) \geq \frac{q\bar{G}(\theta)}{(v_0+q\theta)g(\theta)}.$$

For $\theta \in [\theta_0, \theta^{\mathrm{LN}})$, we have

$$1+\bar{G}(\theta)\left(1-\frac{1+\beta_c\beta_p}{(1+\lambda_p)(1+\lambda_c)}\right) \geq 1+\bar{G}(\theta^{\mathrm{LN}})\left(1-\frac{1+\beta_c\beta_p}{(1+\lambda_p)(1+\lambda_c)}\right) \geq \epsilon_0^{-1} \geq \frac{q\bar{G}(\theta)}{(v_0+q\theta)g(\theta)}$$

where the middle inequality is by Assumption 4.

In the uniform case with $\theta_0 = 0$, $\theta_1 = 1$ and $\beta_c = \beta_p = 0$, equation (12) says that the firm revenue as a function of θ is:

$$R^{\mathrm{LA}}(\theta) = \frac{(1-\theta)\,(q\theta+v_0)\,(1+\lambda_c\,(1-\theta))}{\theta\,\lambda_p+1}. \tag{18}$$

Differentiating $R^{\mathrm{LA}}(\theta)$ gives

$$\frac{dR^{\mathrm{LA}}}{d\theta} = \frac{2\lambda_c\lambda_p q\theta^3 - (2\lambda_c\lambda_p q - \lambda_c\lambda_p v_0 - 3\lambda_c q + \lambda_p q)\,\theta^2 - 2\,(2\lambda_c q - \lambda_c v_0 + q)\,\theta - (\lambda_c\lambda_p + 2\lambda_c + \lambda_p + 1)v_0 + (\lambda_c+1)q}{(1+\lambda_p\theta)^2}.$$

We have $\frac{dR^{\mathrm{LA}}}{d\theta}|_{\theta=1} = -\frac{q\lambda_p+\lambda_p v_0+q+v_0}{(\lambda_p+1)^2} < 0$. Further, we have $\frac{dR^{\mathrm{LA}}}{d\theta}|_{\theta=0} = -((1+\lambda_c)(1+\lambda_p)+\lambda_c)v_0 + q(1+\lambda_c)$. The condition in the Lemma, $\frac{q}{v_0} < 1+\lambda_p+\frac{\lambda_c}{1+\lambda_c}$, is necessary for the revenue function to be decreasing at θ_0. With that condition, it suffices to show that the derivative is negative for all $\theta \in (0,1)$. First observe the following properties on the derivative of the numerator of $\frac{dR^{\mathrm{LA}}}{d\theta}$: (1) It is a quadratic and convex function of θ; (2) at $\theta = 0$ it takes the value $2(q(-2\lambda_c-1)+\lambda_c v_0)$ and at $\theta = 1$ the value $2(1+\lambda_p)(q(\lambda_c-1)+\lambda_c v_0)$. (3) $2\,(q(-2\lambda_c-1)+\lambda_c v_0) < 2(1+\lambda_p)(q(\lambda_c-1)+\lambda_c v_0)$.

Now we can distinguish three cases: (1) $2(1+\lambda_p)(q(\lambda_c-1)+\lambda_c v_0) \leq 0$. In this case, the derivative is decreasing in $\theta \in (0,1)$, and, because at $\theta = 0$ it is negative, it follows that it is negative for all $\theta \in [0,1]$. (2) $(q(-2\lambda_c-1)+\lambda_c v_0) \geq 0$. In this case, the derivative is increasing in $\theta \in (0,1)$, and, because at $\theta = 1$ it is negative, it follows that it is negative for all $\theta \in [0,1]$. (3) $(q(-2\lambda_c-1)+\lambda_c v_0) < 0 < 2(1+\lambda_p)(q(\lambda_c-1)+\lambda_c v_0)$. In this case, the derivative first decreases and then increases, and, because at both $\theta = 0$ and $\theta = 1$ it is negative, we conclude that the derivative is negative over $\theta \in [0,1]$. Therefore, in all cases, the R^{LA} is decreasing over $[0,1]$. □

Proof of Lemma 7: The expected utility under full consumption is given by (14). Observe that $\int_{\theta_0}^{\theta_1}(G(\theta')-\bar{G}(\theta'))dG(\theta') = 0$ and $\int_{\theta_0}^{\bar{\theta}}(G(\theta')-\bar{G}(\theta'))(\theta'-\bar{\theta})dG(\theta') = \int_{\bar{\theta}}^{\theta_1}(G(\theta')-\bar{G}(\theta'))(\theta'-\bar{\theta})dG(\theta')$. We can use these identities to rewrite the loss aversion component in (14) as

$$\int_{\theta_0}^{\theta_1}(G(\theta')-\bar{G}(\theta'))\theta' dG(\theta') = \int_{\theta_0}^{\theta_1}(G(\theta')-\bar{G}(\theta'))(\theta'-\bar{\theta})dG(\theta') = 2\int_{\theta_0}^{\bar{\theta}}(G(\theta')-\bar{G}(\theta'))(\theta'-\bar{\theta})dG(\theta').$$

Furthermore, for $\theta \leq \bar{\theta}$ we have $-1 \leq G(\theta') - \bar{G}(\theta') \leq 0$. Therefore,

$$\int_{\theta_0}^{\bar{\theta}}(G(\theta')-\bar{G}(\theta'))(\theta'-\bar{\theta})dG(\theta') \leq \int_{\theta_0}^{\bar{\theta}}(\bar{\theta}-\theta')dG(\theta').$$

We next establish that $\int_{\theta_0}^{\bar{\theta}}(\bar{\theta}-\theta')dG(\theta') \leq (\bar{\theta}-\theta_0)/4$. To prove, the first step is to show that there must exist a $\theta_k \in [\theta_0, \bar{\theta}]$ such that $g(\theta) \leq \frac{1}{\theta_1-\theta_0}$ for $\theta \leq \theta_k$ and $g(\theta) \geq \frac{1}{\theta_1-\theta_0}$ for $\theta \geq \theta_k$. Assume this is not the case. Then, because $g()$ is positive and increasing on $[\theta_0, \bar{\theta}]$, it must be that either $g(\theta) > \frac{1}{\theta_1-\theta_0}$ for $\theta \in [\theta_0, \bar{\theta}]$, which leads to the contradiction that $G(\bar{\theta}) > 1/2$, or $g(\theta) < \frac{1}{\theta_1-\theta_0}$ for $\theta \in [\theta_0, \bar{\theta}]$, which leads to the contradiction that $G(\bar{\theta}) < 1/2$.

The second step is to show that

$$\int_{\theta_0}^{\bar{\theta}}(\bar{\theta}-\theta')\left(g(\theta')-\frac{1}{\theta_1-\theta_0}\right)d\theta' \leq 0.$$

This is because

$$\int_{\theta_0}^{\bar{\theta}}(\bar{\theta}-\theta')\left(g(\theta')-\frac{1}{\theta_1-\theta_0}\right)d\theta' = \int_{\theta_0}^{\theta_k}(\bar{\theta}-\theta')\left(g(\theta')-\frac{1}{\theta_1-\theta_0}\right)d\theta' + \int_{\theta_k}^{\bar{\theta}}(\bar{\theta}-\theta')\left(g(\theta')-\frac{1}{\theta_1-\theta_0}\right)d\theta',$$

and moreover $\int_{\theta_0}^{\theta_k}(\bar{\theta}-\theta')\left(g(\theta')-\frac{1}{\theta_1-\theta_0}\right)d\theta' \leq (\bar{\theta}-\theta_k)\int_{\theta_0}^{\theta_k}\left(g(\theta')-\frac{1}{\theta_1-\theta_0}\right)d\theta'$ (because $g(\theta') \leq \frac{1}{\theta_1-\theta_0}$ for $\theta \leq \theta_k$) and $\int_{\theta_k}^{\bar{\theta}}(\bar{\theta}-\theta')\left(g(\theta')-\frac{1}{\theta_1-\theta_0}\right)d\theta' \leq (\bar{\theta}-\theta_k)\int_{\theta_k}^{\bar{\theta}}\left(g(\theta')-\frac{1}{\theta_1-\theta_0}\right)d\theta'$ (because $g(\theta') \geq \frac{1}{\theta_1-\theta_0}$ for $\theta \geq \theta_k$). Taking these inequalities into account, we obtain:

$$\int_{\theta_0}^{\bar{\theta}}(\bar{\theta}-\theta')\left(g(\theta')-\frac{1}{\theta_1-\theta_0}\right)d\theta' \leq (\bar{\theta}-\theta_k)\int_{\theta_0}^{\bar{\theta}}\left(g(\theta')-\frac{1}{\theta_1-\theta_0}\right)d\theta' = (\bar{\theta}-\theta_k)(1/2-1/2) = 0.$$

To conclude, note that the inequality $\int_{\theta_0}^{\bar{\theta}}(\bar{\theta}-\theta')\left(g(\theta')-\frac{1}{\theta_1-\theta_0}\right)d\theta' \leq 0$ can be rewritten

$$\int_{\theta_0}^{\bar{\theta}}(\bar{\theta}-\theta')g(\theta')d\theta' \leq \int_{\theta_0}^{\bar{\theta}}\frac{\bar{\theta}-\theta'}{\theta_1-\theta_0}d\theta' = (\theta_1-\theta_0)/8 = (\bar{\theta}-\theta_0)/4.$$

Putting everything together, we obtain an upper bound for the loss aversion component

$$\int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta' dG(\theta') \leq (\bar{\theta} - \theta_0)/2.$$

After replacing the loss aversion component in equation (14), we obtain

$$\mathrm{EU}\left(\theta_0, p_0^{\mathrm{LA}}\right) \geq v_0 + q\bar{\theta} - p_0^{\mathrm{LA}} - \gamma_c q(\bar{\theta} - \theta_0)/2.$$

Thus $\mathrm{EU}(\theta_0, p_0^{\mathrm{LA}}) \geq 0$ for any $G(\cdot)$ if

$$v_0 + q\bar{\theta} - p_0^{\mathrm{LA}} - \gamma_c q(\bar{\theta} - \theta_0)/2 \geq 0.$$

This expression simplifies to Assumption 5 after plugging the value of p_0^{LA}. $\square$

Proof of Proposition 3: (a) $p_0^{\mathrm{LA}} = \frac{1+\lambda_c}{1+\beta_p} v_0$ and $p_q = 0$. (b) We have $p_q^{\mathrm{LN}}(q) = \theta^{\mathrm{LN}}(q) + q\theta_q^{\mathrm{LN}}(q)$, $\theta_q^{\mathrm{LN}}(q) = -\frac{g(\theta_0)v_0}{qR_{\theta\theta}^{\mathrm{LN}}} > 0$. If $\theta_0 = 0$, we have

$$p_q^{\mathrm{LN}}(q) > 0 = p_q^{\mathrm{LA}}(q).$$

If $\theta_0 > 0$, we have $\theta^{\mathrm{LN}}(q) \geq \frac{1+\lambda_c}{1+\beta_p}\theta_0$. Puting these together,

$$p_q^{\mathrm{LN}}(q) = \theta^{\mathrm{LN}}(q) + q\theta_q^{\mathrm{LN}}(q) \geq \theta^{\mathrm{LN}}(q) \geq \frac{1+\lambda_c}{1+\beta_p}\theta_0 = p_q^{\mathrm{LA}}(q). \quad \square$$

Proof of Corollary 1: Lemma 6 says that the $R^{LA}(\theta)$ is decreasing in θ when $q/v_0 < 1 + \lambda_p + \frac{\lambda_c}{1+\lambda_c}$. We need to check that PC holds at PE θ_0 for $p = (1+\lambda_c)v_0$. If that's the case, then Proposition 1 says that θ_0 is a PPE for $p = (1+\lambda_c)v_0$. We have $\mathrm{EU}(\theta = 0) = \frac{q}{2} - p + v_0 - \frac{\lambda_c q}{6}$. Plugging price $p = (1+\lambda_c)v_0$ gives $\mathrm{EU}(\theta = 0, p = (1+\lambda_c)v_0) = \frac{q}{2} - (1+\lambda_c)v_0 + v_0 - \frac{\lambda_c q}{6}$ and PC is equivalent to $\frac{6\lambda_c}{3-\lambda_c} \leq \frac{q}{v_0}$. Since $R^{LA}(\theta)$ is decreasing in θ, there is no other PE or PPE that gives a higher revenue than $(1+\lambda_c)v_0$. $\square$

Proof of Lemma 8:

$$\frac{\partial}{\partial q}\tilde{\mathrm{EU}}(q) = \bar{\theta} - \frac{1+\lambda_c}{1+\beta_p}\theta_0 - \gamma_c \int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta' dG(\theta')$$

Assumption 5 implies that PC holds. That is,

$$\mathrm{EU}\left(\theta_0, p_0^{\mathrm{LA}}\right) = v_0 + q\bar{\theta} - \frac{1+\lambda_c}{1+\beta_p}(v_0 + q\theta_0) - \gamma_c q \int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta' dG(\theta') \geq u_0$$

$$q\bar{\theta} \geq v_0 \left(\frac{1+\lambda_c}{1+\beta_p} - 1\right) + \frac{1+\lambda_c}{1+\beta_p} q\theta_0 - \gamma_c q \int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta' dG(\theta') + u_0$$

and if $\lambda_c \geq \beta_p$

$$\bar{\theta} \geq \frac{1+\lambda_c}{1+\beta_p}\theta_0 - \gamma_c \int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta' dG(\theta')$$

Using the derivations from Lemma 7

$$\bar{\theta} - \frac{1+\lambda_c}{1+\beta_p}\theta_0 - \gamma_c \int_{\theta_0}^{\theta_1} (G(\theta') - \bar{G}(\theta'))\theta' dG(\theta') \geq \bar{\theta} - \frac{1+\lambda_c}{1+\beta_p}\theta_0 - \gamma_c(\bar{\theta} - \theta_0)^2 \geq 0$$

We conclude that $\frac{\partial}{\partial q}\tilde{\mathrm{EU}}(q) \geq (\bar{\theta} - \theta_0)(1 - \gamma_c(\bar{\theta} - \theta_0)) - \frac{\lambda_c - \beta_p}{1+\beta_p}\theta_0 \geq 0.$ □

C PC Binds or Interior Equilibrium

As mentioned in the text, it is not possible to fully characterize the firms's optimal consumption threshold θ^{LA} in general. We can, however, make specific statements. Recall that the firm maximizes $R^{\mathrm{LA}}(\theta)$ subject to $\theta \in \mathbf{\Theta}^U \cap \mathbf{\Theta}^{\mathrm{PPE}}$. The optimal threshold may be an interior value or a corner at the boundary of the set $\mathbf{\Theta}^U \cap \mathbf{\Theta}^{\mathrm{PPE}}$. Consider the case where PC binds: θ^{LA} is a corner located on the boundary of $\mathbf{\Theta}^U$. In Figure 3(a), this corresponds to the section of the curves to the left of the flat segments. An increase in consumption loss aversion reduces the slope of the price schedule.

Proposition 4. $p^{LA}_{q,\lambda_c}(q) < 0$ *when PC binds.*

Proof of Proposition 4: The participation constraint, equation 3, binds.

$$\int_{\theta}^{\theta_1} (v_0 + q\theta' - p^{\mathrm{LA}}(q,\lambda_c))dG(\theta') - \gamma_c \int_{\theta}^{\theta_1} (G(\theta') - \bar{G}(\theta'))(v_0 + q\theta')dG(\theta') - \gamma_p p^{\mathrm{LA}}(q,\lambda_c)G(\theta)\bar{G}(\theta) = 0$$

We have $p^{\mathrm{LA}}_{q,\lambda_c}(q) < 0$ and $p^{\mathrm{LA}}_{q,\lambda_p}(q) = 0$. □

In Figure 3(a), the curves get flatter to the left of the first kink. This will be the case for low quality products or when the consumption loss aversion coefficient is large so that EU from equation (14) is negative. Monetary loss aversion has no impact on the slope of the price schedule $p^{\mathrm{LA}}_{q,\lambda_p}(q) = 0$.

Next, consider the case of interior consumption thresholds. In Figure 3(a), this corresponds to the section of the curves to the right of the flat segments. Interior PPEs are also always chosen for small enough values of loss aversion. Consumption and price loss aversion increase consumption.

Proposition 5. $\frac{\partial}{\partial \lambda_c}\theta^{LA} < 0$, $\frac{\partial}{\partial \lambda_p}\theta^{LA} < 0$.

Proof of Proposition 5: Assume the first order approach holds. The derivative of the firm's revenue (equation 12) with respect to θ is

$$R^{\mathrm{LA}}_{\theta} = R^{\mathrm{LN}}_{\theta} L + R^{\mathrm{LN}} L_{\theta}.$$

An interior solution is characterized by $R^{\mathrm{LA}}_{\theta} = 0$ or

$$\frac{R^{\mathrm{LN}}_{\theta}}{R^{\mathrm{LN}}} = -\frac{L_{\theta}}{L}.$$

Consider an increase in loss aversion, λ_c or λ_p. This does not change the LHS while we have $-\frac{d}{d\lambda}\frac{L_\theta}{L} > 0$ for λ equals λ_c or λ_p. An increase in loss aversion increases the RHS. Since R^{LN}_{θ} is decreasing in θ, it must be the case that θ decreases. □

The consumption threshold decreases with consumption and price loss aversion. The effect on the slope of the price schedule, however, is not possible to sign. For interior equilibria, we have

$$p_q^{\text{LA}}(q) = \theta^{\text{LA}}(q)L(\theta^{\text{LA}}(q)) + \left(qL(\theta^{\text{LA}}(q)) + (v_0 + q\theta^{\text{LA}}(q))L_\theta(\theta^{\text{LA}}(q))\right)\theta_q^{\text{LA}}(q).$$

An increase in loss aversion, λ_c or λ_p, decreases $\theta^{\text{LA}}(q)$, but its impact on the other terms cannot be signed.