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DELIBERATING COLLECTIVE DECISIONS[†]

Abstract

We present a dynamic model of sequential information acquisition by a heterogeneous committee. At each date agents decide whether to vote to adopt one of two alternatives or continue to collect more information. The process stops when a qualified majority vote for an alternative. Three main insights emerge from our analysis and match an array of stylized facts on committee decision making. First, majority rule is more fragile than super-majority rules to impatient committee members. Second, more diverse preferences, more consensual deliberation rules, or more unanimous de- cision voting rules lead to lengthier deliberation and more accurate decisions. Last, balanced committees unanimously prefer to delegate deliberation power to a moderate chairman rather than be governed by a deliberation rule such as unanimity.

JEL Classification: D71, D72 and D83
Keywords: collec- tive learning, optimal stopping, sequential likelihood ratio test and swing voters

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1. Introduction

Juries, boards of directors, standard-setting organizations, congressional and university committees, and government agencies such as the FDA or the EPA serve as examples of committees that spend time deliberating issues and gathering information before reaching a decision or issuing a recommendation. This paper presents a simple model of deliberation that captures some key features of the deliberative process.

Previous literature on deliberation has focused on asymmetric information among members of the deliberating group, on how this information asymmetry can impede effective decision making, and on how different voting rules interact with this information asymmetry.[1] We abstract from private information and concentrate on a simpler aspect of collective action that emerges when studying sequential deliberation; namely, how the dynamics of information collection responds to conflicting preferences. An important aspect of individual decision making involves weighing the cost of obtaining additional information against the value of making more accurate decisions. A classic approach to this question, going back to Wald (1947a, 1947b), is that of sequential analysis. In this approach an individual acquires information sequentially, and at every stage evaluates whether she has sufficient information to make a decision: if she does, she stops and takes a decision; if she does not, she proceeds to acquire additional information at the cost of delaying the decision or paying for an additional sample. Wald showed that the optimal procedure involves a sequential likelihood ratio test, whereby intermediate values of the likelihood ratio require obtaining a new sample, while high (low) values of the likelihood ratio require stopping and taking one (or the other) decision.

Our paper provides an analysis of how collective action affects such information acquisition. We depart from Wald by introducing two important dimensions of heterogeneity: individuals may differ in the urgency with which they wish to reach a decision, i.e., their discount factor, and in the appropriate standard of evidence to apply in order to adopt one of the two decisions, namely, their static preferences over the two alternatives. We show that heterogeneous committees also utilize an appropriately modified sequential likelihood ratio test *in equilibrium*. Our characterization allows us to compare institutions (namely, deliberation protocols and voting rules) and discuss the effects of the committee structure on the length of deliberation, the accuracy of decisions, and the welfare of the committee and of society at large.

Majority and super-majority rules both have their pros and cons. One advantage of majority rule is that it always produces an outcome that is favored by a majority. Indeed, under majority rule, if a majority of agents prefers a different alternative to the one adopted in equilibrium, it can simply adopt the one preferred. Another advantage of majority rule is that all agents of the committee prefer it to stricter majority requirements in

[1] E.g., Austen-Smith and Feddersen (2005, 2006), Coughlan (2000), Gerardi and Yariv (2007, 2008), Meirowitz (2006), and Persico (2004).

symmetric environments where all agents are equally patient and the static preferences for the two alternatives are balanced. In such environments members of a committee would unanimously prefer to delegate deliberation power to a moderate chairman rather than be governed by a deliberation rule such as unanimity.

A potential weakness of majority rule is that it can be excessively influenced by impatient agents who has higher time costs. Since impatient agents prefer quicker decisions, they can be easily persuaded to change their votes. Given that under majority rule it takes the change of only one vote to alter the outcome from adopting one alternative to adopting another, in equilibrium more patient agents who support different alternatives cut short deliberations to capture the impatient member before she changes sides. Because quick decisions are strategic complements—if the supporters of one alternative expect the supporters of the other alternative to decide quickly, they will decide quickly as well—the mere presence of one impatient agent can cause the whole group to rush to a decision, even when all other agents are very patient. This is in stark contrast with one-dimensional static voting environments, where no agent with extreme preferences can influence the outcome under majority rule.

Super-majority rules, by comparison, are more robust against hasty decisions. Under super-majority rules, more than one vote must change to alter the outcome from adopting one alternative to adopting another. Deliberations do not collapse unless the number of impatient agents is sufficiently large. Moreover, because slow decisions are strategic substitutes, one very patient agent would not cause the group to deliberate without end unless unanimity is required. Compared to majority rule, supermajority rules also tend to produce more accurate decisions. Under super-majority rules there are typically two pivotal agents, one for each alternative. Imposing stricter majority requirements has two effects. A direct effect is that individuals with more extreme preferences become pivotal and they prefer to extend deliberations. An indirect, strategic, effect, however, implies that pivotal agents take into account the tendency for prolonged deliberation by their more extreme peers and are therefore willing to halt deliberation earlier than they would have as sole decision makers. We call this strategic response the *moderation effect*. On net, stricter super-majority rules generates longer deliberations and more accurate decisions in symmetric environments. Hence, even when the agents themselves, who bear more of the delay costs, prefer majority rule, the society at large who benefits from a more accurate decision may be better off with a super-majority rule.

Super-majority rules, however, also have drawbacks. In an asymmetric environment, a high majority requirement may allow a small group with extreme preferences to hold out and force other members to accept an alternative that they do not like. This problem is the most severe under unanimity rule. In that case, the deliberation outcome may be preferred by only one member; the rest of group may all prefer the other alternative but vote for the inferior one to avoid costly delay. Thus, while our results support the use of

super-majority rules in situations where agents have diverse time preferences, they also warn against setting a very high majority requirement. In particular, requiring unanimity does not always build consensus.

One way to combine features of majority and supermajority rules is to have a two-stage decision process whereby the agents first vote to end deliberation and then vote again to select an alternative. We model the deliberation process as a threshold rule k_D such that deliberation ends as soon as k_D members of the committee vote to end deliberation. Decision rules are analogously captured by a rule k_d that describes the specific qualified majority required for reaching a decision after deliberation stops. We show that in certain cases, decision rules are irrelevant, while deliberation rules always affect the length of deliberation and accuracy of ultimate decisions. This separation, however, makes it impossible to have outcomes where agents vote against their static preferences in order to avoid costly delays.

Our results are relevant to the design of deliberative mechanisms and for understanding the dynamics of information collection in a variety of collective decision processes such as R&D, hiring decisions, FDA drug approval, and so on. In Section 8 we discuss in more detail two applications: standard-setting organizations and juries. Standard setting processes fit our baseline setting in which deliberation and decision-making are intertwined—decisions are sequential and are often of the form of "continue" or "take a decision" (approve or not). The case of juries fits our two-stage deliberation setting. Juries are an interesting application for our model for three main reasons. First, in juries, the deliberation process is clear-cut and circumscribed: there is a well-defined beginning and end of deliberation, the time it takes the jury to deliberate is measurable, and one single verdict is the typical outcome of such deliberation. Second, the jury setting allows us to contrast our analysis with much of the extant body of literature on deliberation that has focused predominantly on the jury context. Third, the empirical literature has documented some patterns of deliberation in juries that we attempt to explain with our model.

Related Literature

In economics, the past two decades have delivered a rich collection of work on committee decision making; see Li and Suen (2009) for an extended survey. Our paper ties directly to several sets of studies.

In terms of jury decision making, Feddersen and Pesendorfer (1998) study a model in which jurors have private information about the guilt or innocence of the defendant. They show that unanimity leads to less informative outcomes than does majority rule in large juries. Persico (2004) allows for private information collection prior to voting. He characterizes the optimal voting rule and shows that unanimity leads to inferior information collection. Austen-Smith and Feddersen (2006) extend Feddersen and Pesendorfer

(1998) by allowing for a round of cheap-talk communication before voting. They show that unanimity leads to less communication and poorer information aggregation. Gerardi and Yariv (2007, 2008) depart from these papers by studying general communication protocols. They show that the set of equilibrium outcomes is invariant to the voting rules, as long as they are non-unanimous. In fact, unanimous voting rules generate a subset of equilibrium outcomes. In contrast with this literature, we assume that all information in the jury is public: signals are observed by all jurors, and preferences are common knowledge. We view our model as a natural alternative extreme benchmark that is useful for identifying the tensions that arise in collective choice when trading off information collection costs and decision accuracy. Our result that unanimity can lead to more accurate outcomes contrasts with the observations of Feddersen and Pesendorfer (1998), Persico (2004), and Austen-Smith and Feddersen (2005, 2006).

Several studies have looked at the effects of sequential collection of information within groups. Albrecht, Anderson, and Vroman (2010) and Compte and Jehiel (2010) study how group search is affected by voting. Messner and Polborn (2012) study a two-period model where voters receive information over time about the desirability of an irreversible decision. The main message of that paper is that the optimal voting rule requires a supermajority. Bognar, Meyer-ter-Vehn, and Smith (2013) also study a model of dynamic deliberation, but with very different ingredients. In their model jurors have common preferences and private information about a payoff-relevant state. They assume that jurors sequentially exchange coarse messages. In their model there are many equilibria that can be ranked in terms of generated welfare. When there is no discounting, they show that longer conversations are better.[2] Strulovici (2010) analyzes a model of voting over experimentation. He shows that voting by heterogeneous voters, who are learning their preferences, leads to an inefficient level of experimentation. He then describes a voting rule that can restore efficiency. Moldovanu and Shi (2013) study a collective version of a stopping problem in which individual agents can assess the performance of alternatives on one of several dimensions. In their setting, the volume of information collected, as well as the group's welfare, decrease with the heterogeneity in preferences within the group.[3]

From a technical perspective, the starting point of our analysis is Wald (1947a, 1947b), who pioneered the study of sequential testing, and provided a characterization of the optimal test as a sequential likelihood ratio test.[4] Henry and Ottaviani (2014) apply the

[2]A related paper is that of Eso and Fong (2008), who study a dynamic cheap talk model with multiple senders, where the receiver can choose when to make her decision. They show that when the senders are all informed of the state of nature, a perfect Bayesian equilibrium exists with instantaneous, full revelation, regardless of the size and direction of the senders' biases. Wilson (2014) considers exogenous costs for both sending messages and receiving them, and illustrates the dependence of effective communication on agents' quality of information and messaging costs.

[3]Our paper is also related to Gul and Pesendorfer (2012), who study the competition between two political parties to provide public information that may influence voters' chioce. Each party's decision is characterized by a stopping rule, and these stopping decisions are strategically linked.

[4]Dvoretsky, Keifer, and Wolfowitz (1953) gave an early treatment of the hypothesis testing problem in

framework of this paper to a study of the approval process when a firm conducting clinical trials needs approval from a regulator such as the FDA. They also allow for the possibility that the firm may misrepresent the evidence and show that it may not be optimal to forbid such misrepresentation.

2. Model

A group of $2m-1$ agents[5] chooses between two alternatives, α and β. The payoff from each alternative depends on the underlying state $\omega \in \{A, B\}$. In state A, agent i's payoff from α is 1, and his payoff from β is 0. In state B, agent i's payoff from α is 0, and his payoff from β is e^{v_i}. Thus, all agents prefer action α in state A and action β in state B, but agents differ in the intensity of this preference, and $v_i \in \Re$ is a measure of the intensity of preference for β relative to α. Agents' static preferences are heterogenous, with $v_1 < \ldots < v_{2m-1}$. The median static preference is v_m. Agents do not observe the state but share a common prior belief. It is convenient to represent agents' beliefs by the log ratio of the probabilities of the two states. We let $\theta_0 = \log(\Pr[\omega = A]/\Pr[\omega = B])$ represent the initial belief. With this parameterization, the probability of state A is given by $e^\theta/(1+e^\theta)$ when the belief is θ. The immediate expected payoff from choosing α is higher than that from choosing β for agent i if and only if $\theta \geq v_i$.

Time is continuous. At every instant t, each agent independently votes for α, β, or neither. We initially focus on a simple class of decision rules. Under decision rule $k \in \{m, m+1, \ldots, 2m-1\}$, an alternative is adopted at time t if it receives k votes or more. Voting continues until either α or β receive sufficient votes. Voting for neither alternative is effectively a vote in favor of continuing to gather information. Voting for one alternative is also a vote in favor of stopping information acquisition.[6] We call rule $k = m$ majority rule, $k > m$ super-majority rule, and $k = 2m-1$ unanimity rule. The usage is natural in contexts where alternatives are treated symmetrically, and deliberation continues until an alternative has received enough support. In Section 7 we consider an alternative structure by modeling a two-stage deliberation process that separates voting on information gathering from voting on the final decision.

Each agent i discounts the future at a rate r_i. If an alternative is chosen at time t, then agent i's payoff is discounted by the factor $e^{-r_i t}$. In general, agents are heterogeneous

continuous time. See also De Groot (1970) for a modern exposition, and Moscarini and Smith (2001) for an extension that allows for richer sampling strategies.

[5]The assumption of an odd number of agents is used only when we consider majority rule.

[6]Throughout the paper, majority rule or super-majority rule refer to an "absolute" rule. Under an absolute rule, abstention is treated the same as a vote to support neither alternative. In contrast, a "simple" rule discards votes to abstain in the tally. This distinction is based on Riker (1982). Our results would not change if we changed the rule so that each agent can choose to support α, support β, delay, or abstain, and an alternative is adopted if the fraction of votes, excluding abstentions, passes a certain threshold. Because agents do not possess private information in this model, there is no strategic advantage to abstention relative to voting for delay (Feddersen and Pesendorfer 1996).

both in their static preferences v_i and in their time preferences r_i.[7]

Public information arrives continuously as long as deliberation has not stopped and an alternative has been chosen. The arrival of information is represented by a Wiener process $\mathrm{d}S$ that has a positive drift μ and an instantaneous variance ρ^2 if the state is A, or drift $-\mu$ and variance ρ^2 if the state is B. For any time $t > 0$, the accumulated evidence S_t is a sufficient statistic for all the information that has arrived before t. The log-likelihood ratio of observing $S_t = s$ under the two states is

$$\log \frac{h((s-\mu)/\rho)}{h((s+\mu)/\rho)} = \frac{2\mu s}{\rho^2},$$

where $h(\cdot)$ is the standard normal density function. Hence, a higher observed value of the accumulated evidence S_t is stronger evidence in favor of state A. The log posterior probability ratio is given by the sum of the log prior probability ratio and the log-likelihood ratio. So, if we let $S'_t = 2\mu S_t/\rho^2$, the common belief of the group at time t is given by:

$$\theta_t = \theta_0 + S'_t.$$

Denote $\mu' \equiv 2\mu^2/\rho^2$. Then, the agents' common belief is given by a Wiener process, with drift μ' and instantaneous variance $2\mu'$ under state A, and drift $-\mu'$ and variance $2\mu'$ under state B. A higher value of μ' (higher μ or lower ρ) indicates a more informative deliberative process.

When $m = 1$, decisions correspond to an individual, which is the case analyzed by Wald (1947). From Wald, predictions are unique and can be nicely characterized:

Proposition 0 (Wald 1947a, 1947b). *When agent i is the only agent in the group (i.e., $m = 1$), a unique equilibrium exists and is characterized by two thresholds $g_i^* < G_i^*$ such that:*

- *the agent stops information collection and chooses α whenever $\theta_t \geq G_i^*$;*
- *the agent stops information collection and chooses β whenever $\theta_t < g_i^*$;*
- *the agent continues collecting information whenever $\theta_t \in [g_i^*, G_i^*)$.*

When $m > 1$, in principle, an agent's decision at time t could be a function of the entire sample path of S_t, his own decisions, and other agents' decisions before t. However, the difference between the expected payoffs of the two decisions α and β depends solely on

[7] While our setup uses heterogeneous discount rates to model different preferences over how quickly to act, a very similar model can be constructed with explicit information acquisition costs. Suppose that, for agent i, the cost of information collection or deliberation is $\delta \mathrm{d}t$ for a time interval of length $\mathrm{d}t$, and the payoffs from α and β are λ_i and $\lambda_i e^{v_i}$, respectively. Agents with low values of λ_i are low-stake voters. They do not care that much about which alternative is taken; their primary concern is to reduce the explicit information collection cost. In such an alternative setup, low-stake agents would play a similar role to that played by impatient agents in our model.

the current belief θ_t and is strictly increasing in it. We focus on equilibria in which agents adopt Markov cutoff strategies (much like the optimal policies characterized by Wald for an individual decision maker). Formally, agent i's strategy σ_i is represented by a pair of cutoffs (g_i, G_i), with $g_i \leq G_i$. Strategy (g_i, G_i) means voting for α when $\theta_t \geq G_i$, voting for β when $\theta_t < g_i$, and abstaining when $\theta_t \in [g_i, G_i)$.

For any strategy profile $\sigma = (\sigma_1, \ldots, \sigma_{2m-1})$, let $G^{[k]}(\sigma)$ denote the k-th smallest G_i in σ, and let $g^{[k]}(\sigma)$ denote the k-th largest g_i in σ. We call the interval $(g^{[k]}(\sigma), G^{[k]}(\sigma))$ the *waiting region*, because the group does not make a decision as long as the belief stays within this interval. The *width* of the waiting region, $G^{[k]}(\sigma) - g^{[k]}(\sigma)$, is an indicator of the expected time to make a decision. Note, however, that a wider waiting region does not necessarily mean that the group always waits longer, unless two waiting regions are *nested* (i.e., one waiting region contains the other). If two waiting regions are nested, decisions reached with the larger waiting region are more *accurate* because each alternative would be adopted when the probability of its corresponding state is higher.

Let $u_i(g, G \mid \theta)$ represent the payoff to agent i when the belief is θ and the waiting region is (g, G). If $\theta \geq G$, then α is adopted and the expected payoff is

$$u_i(g, G \mid \theta) = \frac{e^{\theta}}{1 + e^{\theta}}.$$

If $\theta < g$, then β is adopted and the expected payoff is

$$u_i(g, G \mid \theta) = \frac{e^{v_i}}{1 + e^{\theta}}.$$

For $\theta \in [g, G)$, the payoff function satisfies

$$u_i(g, G \mid \theta) = e^{-r_i \mathrm{d}t} \mathbb{E}\left[u_i(g, G \mid \theta + \mathrm{d}S')\right].$$

Because $\mathrm{d}S'$ is a diffusion process with expected drift $\mu'(e^{\theta} - 1)/(1 + e^{\theta})$ and instantaneous variance $2\mu'$, we can use Ito's lemma to derive a differential equation in u_i. Solving this differential equation and imposing the value-matching condition at the two boundaries of the waiting region, we obtain:

$$u_i(g, G \mid \theta) = \frac{e^{\theta}}{1 + e^{\theta}} \Psi_i(g, G \mid \theta) + \frac{e^{v_i}}{1 + e^{\theta}} \psi_i(g, G \mid \theta),$$

where

$$\Psi_i(g, G \mid \theta) = \frac{e^{-R_1(\theta - g)} - e^{-R_2(\theta - g)}}{e^{-R_1(G - g)} - e^{-R_2(G - g)}},$$
$$\psi_i(g, G \mid \theta) = \frac{e^{-R_1(G - \theta)} - e^{-R_2(G - \theta)}}{e^{-R_1(G - g)} - e^{-R_2(G - g)}},$$

with

$$R_1 = \frac{1}{2}\left(1 - \sqrt{1 + \frac{4r_i}{\mu'}}\right), \quad R_2 = \frac{1}{2}\left(1 + \sqrt{1 + \frac{4r_i}{\mu'}}\right).$$

Note that $R_2 > 1$, $R_1 < 0$, and $R_1 + R_2 = 1$. The values of R_1 and R_2 depend on i, but we omit this dependence in the notation to avoid clutter.

The two functions Ψ_i and ψ_i can be given a statistical interpretation (Cox and Miller 1965). When $\theta \in (g, G)$, agent i's payoff depends on which threshold is reached first and on the length of time it takes to reach it. The function $\Psi_i(g, G \mid \theta)$ gives the probability of adopting α in state A when the current belief is θ, times the expected discount factor conditional on the belief reaching the threshold G before reaching g in state A. The function ψ_i can be interpreted analogously.

Definition 1. *Let $(\hat{g}, \hat{G}) = (g^{[k]}(\sigma), G^{[k]}(\sigma))$. A strategy profile σ is an equilibrium under decision rule k if the following conditions are satisfied for any agent i:*

1. *For any belief θ and any strategy σ_i',*

$$u_i(\hat{g}, \hat{G} \mid \theta) \geq u_i(g^{[k]}(\sigma_i', \sigma_{-i}), G^{[k]}(\sigma_i', \sigma_{-i}) \mid \theta).$$

2. *(a) $G_i > (<)\ \hat{G}$ if $\partial u_i/\partial G|_{(\hat{g}, \hat{G};\, \theta = \hat{G})} > (<)\ 0$;*
   (b) $g_i > (<)\ \hat{g}$ if $\partial u_i/\partial g|_{(\hat{g}, \hat{G};\, \theta = \hat{g})} > (<)\ 0$.

Condition 1 of Definition 1 is the standard Nash equilibrium requirement. Condition 2 says that agent i does not vote for an alternative at the threshold if his marginal gain from extending the threshold is positive, and, conversely, that he votes for an alternative before the belief reaches the threshold for that alternative if his marginal gain from contracting the threshold is positive. Note that as long as the decision rule k is not unanimity rule, for any pair of thresholds (g, G), it would be a Nash equilibrium in our model for all agents to adopt the same strategy (g, G), because no agent could unilaterally change the waiting region. Condition 2 rules out trivial equilibria that arise in voting games because of the use of weakly dominated strategies.

3. Best Response in Stopping Decisions

In the classic individual optimal-stopping problem, an agent i chooses (g_i, G_i) to maximize $u_i(g, G \mid \theta)$. The group decision model differs in that the influence of each agent is constrained by the cutoffs of other agents. Under decision rule k, agent i can only decide at what belief between $G^{[k-1]}(\sigma_{-i})$ and $G^{[k]}(\sigma_{-i})$ α is adopted. She can neither stop the adoption of α when the belief exceeds the upper thresholds of k other agents, nor force the adoption of α when fewer than $k-1$ other agents agree to adopt it. We say that agent i's *span of control* over the upper threshold is the interval

$$I^G(\sigma_{-i}) = \left[G^{[k-1]}(\sigma_{-i}), G^{[k]}(\sigma_{-i})\right].$$

Analogously, we define $I^g(\sigma_{-i}) = [g^{[k]}(\sigma_{-i}), g^{[k-1]}(\sigma_{-i})]$ as agent i's span of control over the lower threshold. Then, Condition 1 of the equilibrium definition is equivalent to requiring that $(g^{[k]}(\sigma), G^{[k]}(\sigma))$ be a solution to the following constrained maximization problem for each agent i and for every belief θ:

$$\max_{g,G} u_i(g, G \mid \theta) \quad \text{subject to } (g, G) \in I^g(\sigma_{-i}) \times I^G(\sigma_{-i}). \tag{1}$$

We first study a one-sided stopping problem in which an agent takes one of the stopping boundaries as fixed and chooses the other stopping boundary. We then use the solution to the one-sided problem to characterize the solution to the constrained two-sided stopping problem (1).

Lemma 1. *For $\theta \in (g, G)$, the following properties hold for any agent i:*

1. *Dynamic consistency: The signs of $\partial u_i/\partial G$ and $\partial u_i/\partial g$ do not depend on θ.*
2. *Single crossing: (a) If $\partial u_i/\partial G \geq 0$ at $G' > G$, then $\partial u_i/\partial G > 0$ at G (holding g constant). (b) If $\partial u_i/\partial g \leq 0$ at $g' < g$, then $\partial u_i/\partial g < 0$ at g (holding G constant).*

The partial derivative of u_i with respect to a threshold reflects the trade-off between the cost of delay and the value of additional information. Part 1 of Lemma 1 says that this trade-off is dynamically consistent: if an agent prefers to extend a threshold when his belief reaches it, he prefers to extend this threshold before reaching it. Part 2 of Lemma 1 says that u_i is single-peaked in each threshold, holding the other constant. Intuitively, if an agent prefers waiting to decide α at a later threshold G', he strictly prefers waiting to decide α at an earlier threshold G, where the case for α is weaker.

For any agent i, define $\phi_i(G)$ as the lower best-response function for agent i, which gives, for any fixed G, the optimal lower cutoff g that maximizes $u_i(g, G \mid \theta)$ subject to $g \leq G$. The dynamic consistency and single-crossing properties of Lemma 1 guarantee that $\phi_i(G)$ is well defined and is independent of the current belief θ. Similarly, the upper best-response function $\Phi_i(g)$ is defined as the optimal upper cutoff G that maximizes $u_i(g, G \mid \theta)$ subject to $G \geq g$. Recall that g_i^* and G_i^* are the Wald-optimal cutoffs for agent i when he can choose both thresholds. The point (g_i^*, G_i^*) must be a fixed point of (ϕ_i, Φ_i).

Lemma 2. *The following properties hold for any agent i:*

1. *(a) $\Phi_i(g)$ is continuous everywhere and is twice differentiable except at $g = v_i$, with $\Phi_i(g) > g$ if $g < v_i$ and $\Phi_i(g) = g$ otherwise.*
   (b) $\phi_i(G)$ is continuous everywhere and is twice differentiable except at $G = v_i$, with $\phi_i(G) < G$ if $G > v_i$ and $\phi_i(G) = G$ otherwise.
2. *(a) For $G > v_i$, $\phi_i''(G) > 0$, $\phi_i'(G) < 1$, and $\phi_i(G)$ reaches a minimum when $G = G_i^*$ and goes to ∞ as G goes to ∞.*

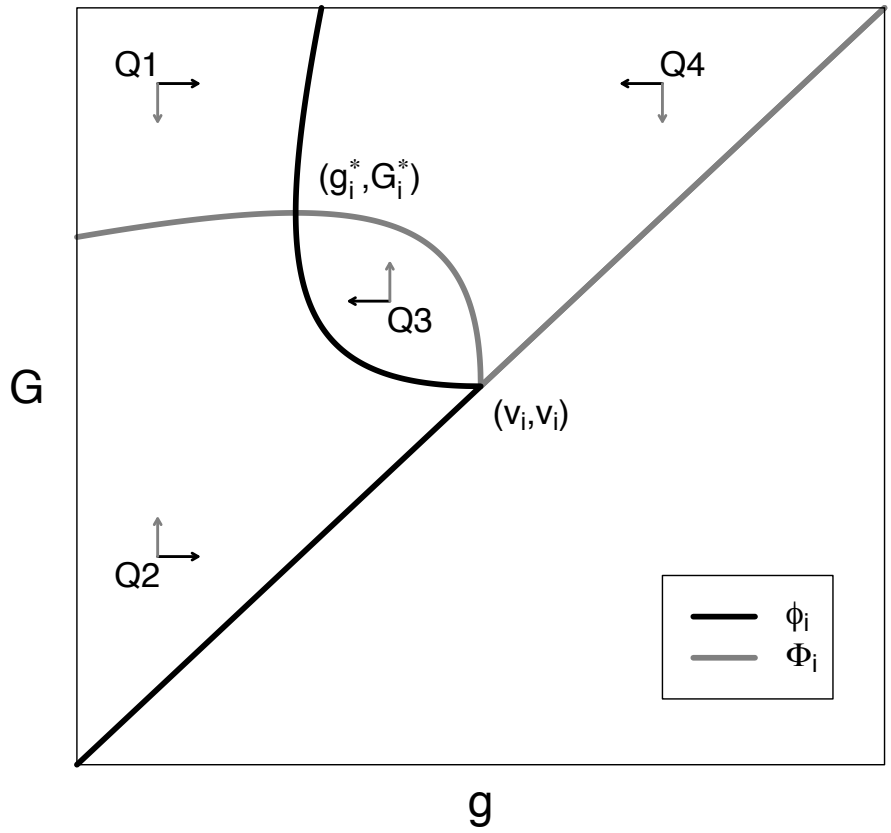

Figure 1. *The upper best-response function $\Phi_i(g)$ is increasing for $g < g_i^*$, decreasing for $g \in (g_i^*, v_i)$, and coincides with the 45-degree line for $g > v_i$. The two best-response functions partition the space into four regions. The arrows associated with each region show that agent i's utility increases if the cutoff moves in the indicated direction. This figure is drawn using the parameters $v_i = 0$, $r_i = 0.039$, and $\mu' = 0.1$.*

(b) For $g < v_i$, $\Phi_i''(g) < 0$, $\Phi_i'(g) < 1$, and $\Phi_i(g)$ reaches a maximum when $g = g_i^*$ and goes to $-\infty$ as g goes to $-\infty$.

3. *(a) For any G, $\phi_i(G)$ increases in v_i and increases in r_i, with $\lim_{v_i \to -\infty} \phi_i(G) = -\infty$ and $\lim_{r_i \to \infty} G - \phi_i(G) = 0$.*
   (b) For any g, $\Phi_i(g)$ increases in v_i and decreases in r_i, with $\lim_{v_i \to \infty} \Phi_i(g) = \infty$ and $\lim_{r_i \to \infty} \Phi_i(g) - g = 0$.

Figure 1 depicts a typical pair of ϕ_i and Φ_i. We focus on the properties of ϕ_i in the following discussion; the properties of Φ_i are analogous.

If the threshold G for adopting α is set below v_i, there is no point in delaying the adoption of β, as agent i would still prefer β to α when his belief reaches the threshold for adopting α. Thus, $\phi_i(G) = G$. By contrast, if $G > v_i$, agent i strictly prefers α to β

when his belief is at G. Hence, when his belief is sufficiently close to G, even an extremely impatient agent i will prefer delaying the decision β, which, at this belief is the "wrong" decision for him, in the hope that the belief will reach G and α will be chosen instead. Thus, $\phi_i(G) < G$. Note that $\phi_i(G)$ is continuous at $G = v_i$. When G is just above v_i, the difference in expected payoff between α and β is very small at G and, hence, agent i gains little from delaying the adoption of β.

Part 2 of Lemma 2 describes agent i's best response when $G > v_i$. Some of these properties are technical, but two features are of economic interest. First is the non-monotonicity of $\phi_i(G)$. Intuitively, α is chosen "too early" when $G < G_i^*$ (the unconstrained optimal cutoff), and is chosen "too late" when $G > G_i^*$. In either case, the agent responds by adopting β earlier. The possibility that a loss of control (over the upper boundary) may reduce the incentive to obtain information is discussed in Albrecht, Anderson, and Vroman (2010) in the context of a collective search model, and by Strulovici (2010) in a two-armed-bandit experiment setting. The non-monotonicity of ϕ_i implies there is a strategic difference between too much and too little waiting. Excessive deliberation is self-limiting, while rushing to a decision is self-reinforcing.

The second feature of economic interest is that $\phi_i(G)$ is unbounded from above. In particular, it is possible that $\phi_i(G) > v_i$ when G is sufficiently large. An agent who prefers α to β may nevertheless vote to adopt β immediately if he anticipates that he would have to wait for a very long time for the group to adopt α.

Part 3 of Lemma 2 describes how ϕ_i varies with v_i and r_i. An increase in v_i shifts both ϕ_i and Φ_i in the north-east direction, causing agent i to adopt β earlier and α later. An increase in r_i "bends" ϕ_i and Φ_i toward the 45-degree line, which prompts agent i to adopt both α and β earlier. As agent i becomes extremely impatient, the best-response lower threshold becomes arbitrarily close to the upper threshold.

Let $I^G(\sigma_{-i}) = [\underline{G}_i, \overline{G}_i]$ and $I^g(\sigma_{-i}) = [\underline{g}_i, \overline{g}_i]$. Given the single-crossing property (Lemma 1), $\hat{g}$ is optimal in $[\underline{g}_i, \overline{g}_i]$ for any belief θ given $\hat{G}$ only if it satisfies:

$$\hat{g} \begin{cases} \geq \phi_i(\hat{G}) & \text{if } \hat{g} = \underline{g}_i, \\ = \phi_i(\hat{G}) & \text{if } \hat{g} \in (\underline{g}_i, \overline{g}_i), \\ \leq \phi_i(\hat{G}) & \text{if } \hat{g} = \overline{g}_i. \end{cases} \tag{2}$$

Similarly, $\hat{G}$ is optimal in $[\underline{G}_i, \overline{G}_i]$ for any belief θ given $\hat{g}$ only if it satisfies:

$$\hat{G} \begin{cases} \geq \Phi_i(\hat{g}) & \text{if } \hat{G} = \underline{G}_i, \\ = \Phi_i(\hat{g}) & \text{if } \hat{G} \in (\underline{G}_i, \overline{G}_i), \\ \leq \Phi_i(\hat{g}) & \text{if } \hat{G} = \overline{G}_i. \end{cases} \tag{3}$$

The following lemma shows that these conditions are also sufficient.

Lemma 3. *A pair of thresholds $(\hat{g}, \hat{G})$, with $\hat{g} < \hat{G}$, solves the constrained optimization problem (1) for all beliefs θ if and only if it satisfies conditions (2) and (3).*

Lemma 3 plays a crucial role in our analysis. Despite the fact that one-sided preferences are dynamically consistent, dynamic consistency does not generally hold when both thresholds change at the same time. An agent may prefer waiting region (g, G) to (g', G') at some belief θ, but have the opposite preference at some other belief θ'. Nevertheless, Lemma 3 shows that a solution to the constrained optimization problem (1) for all θ exists. Furthermore, it reduces our original two-sided collective stopping problem, which is difficult to solve directly, into two readily solvable one-sided problems. In the proof of Lemma 3, we use part 2 of Lemma 2 and the fact that the constraint set $I^g(\sigma_{-i}) \times I^G(\sigma_{-i})$ is a rectangle to establish that only one pair of cutoffs in the constraint set satisfies both (2) and (3). Because (2) and (3) are necessary for constrained maximization, this unique cutoff pair must be optimal in the constraint set.[8]

4. Equilibrium Analysis

Given decision rule k, define the lower *pivotal best-response function* $\phi_{piv}(G; k)$ to be the k-th largest $\phi_i(G)$ for a given G. Thus, in a one-sided stopping problem where α is adopted at G, there are k agents who support β when the belief reaches $\phi_{piv}(G; k)$. Similarly, define the upper pivotal best-response function $\Phi_{piv}(g; k)$ to be the k-th smallest $\Phi_i(g)$ for a given g. In the following, we sometimes suppress the argument k when doing so does not cause confusion. If the static preferences v_i are all the same and agents only differ in their discount factors r_i, then every k is associated with a unique pivotal agent for both thresholds: this is the k-th most impatient agent. If the discount factors r_i are all the same and agents only differ in their static preferences v_i then, as we will see, there are different pivotal agents for the two thresholds: these are the agents with the k-th lowest and the k-th highest static preferences. More generally, however, when agents are heterogeneous in both dimensions, determining the identity of the pivotal agents is not so simple. Furthermore, because individual best-response functions may cross if agents have different time preferences, the identity of the pivotal agent who supports β at $\phi_{piv}(G; k)$ may change as G changes. See Figure 2 for an illustration.

The following proposition establishes some basic properties of equilibrium.

Proposition 1. *For any decision rule k, any $(\hat{g}, \hat{G})$ is an equilibrium outcome of the deliberation game if and only if it is a fixed point of (ϕ_{piv}, Φ_{piv}) and $\hat{g} < \hat{G}$. An equilibrium of the deliberation game exists and the equilibrium waiting region must be non-degenerate. When there are multiple equilibria, the equilibrium waiting regions are nested. Equilibrium is unique if the decision rule*

[8] As the utility function u_i is not quasi-concave in cutoffs, the Kuhn-Tucker conditions in general are not sufficient for optimality in an arbitrary convex constraint set.

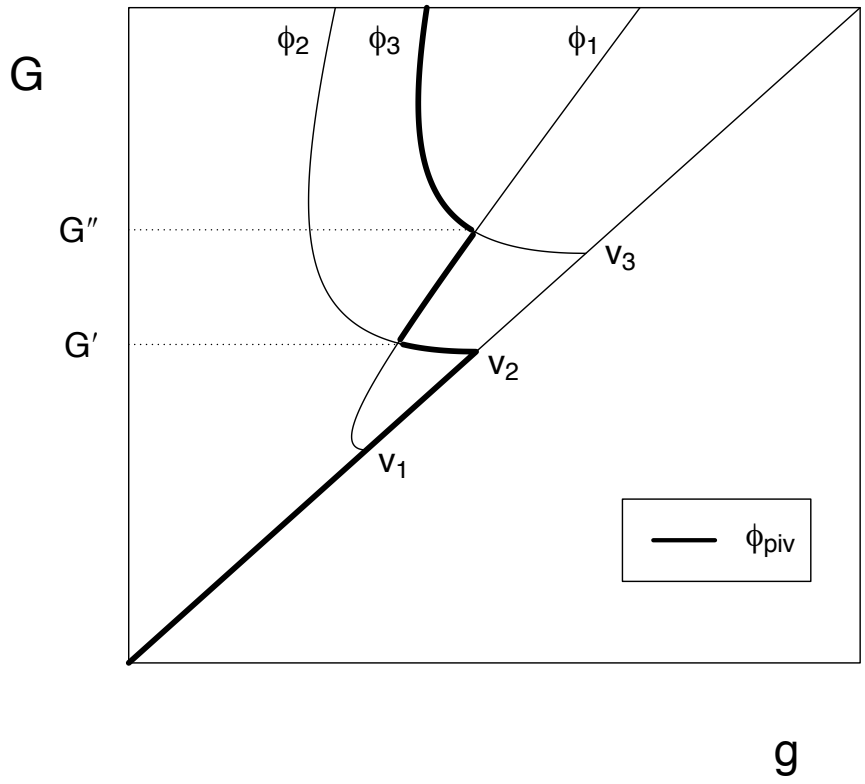

Figure 2. *The bold line shows the lower pivotal best-response function when there are three agents and when the decision rule is majority rule. For $G < v_2$, ϕ_{piv} coincides with the 45-degree line. For fixed G , the median agent (agent 2) is pivotal in the decision to adopt β if $G \in (v_2, G')$. Agent 1 is more impatient than others; he would prefer to adopt β earlier than agent 2 (but later than agent 3) if $G \in (G', G'')$. For $G > G''$, agent 1 prefers to adopt β the earliest and agent 3 becomes pivotal. In this figure, $(v_1, v_2, v_3) = (-0.6, 0, 0.6)$, $(r_1, r_2, r_3) = (0.6, 0.039, 0.039)$, and $\mu' = 0.1$.*

requires unanimity or if there is a single dimension of heterogeneity.

It is obvious that any equilibrium cutoffs must be a fixed point of the pivotal best-response functions. Sufficiency follows from Lemma 3: if $(\hat{g}, \hat{G})$ is an unequal fixed point of (ϕ_{piv}, Φ_{piv}), i.e., $\hat{g} < \hat{G}$, then the strategy profile σ where, for each i,

$$\sigma_i = (\phi_i(\hat{G}), \Phi_i(\hat{g})),$$

satisfies (2) and (3). Under majority rule, (v_m, v_m) is a fixed point of (ϕ_{piv}, Φ_{piv}) but not an equilibrium, because agent m can gain by waiting a little before adopting one of the alternatives.

There may be multiple equilibria for non-unanimous decision rules when there is heterogeneity in both dimensions. Figure 4 provides an example. Thus, expectations can play an important role in deliberations. Agents may decide slowly if they expect others are taking their time, but they may also rush to a decision to preempt others from reaching a different decision in a hurry. When there are multiple equilibria, the equilibrium waiting regions are nested; that is, the *most patient* equilibrium waiting region contains all other equilibrium waiting regions while the *least patient* equilibrium waiting region is contained in all others.

5. Respecting Static Preferences and the Quality of Decisions

In this section, we illustrate two advantages of majority rules. First, they lead to decisions that respect the static preferences of the agents. Second, they are preferred by all members of the committee.

One criterion for evaluating the quality of a collective decision is whether it accords with the static preferences of members of the group. Under majority rule, if a majority of agents prefer a different alternative to the one adopted in equilibrium, they can simply adopt the one they prefer. This simple reasoning immediately implies that majority rule *respects static preferences*, in the sense that at the end of deliberations at least a majority of agents favor the alternative chosen to the one not chosen.

Proposition 2. *Majority rule produces outcomes that always respect static preferences. However, the same is not generally true under super-majority rule $k > m$. Fix the discount rates of all agents and fix the static preferences of $k-1$ agents. If the remaining agents' static preferences for α are sufficiently strong, then under super-majority rule k, all $k-1$ agents in the first group strictly prefer β to α at the cutoff for α.*

Intuitively, under super-majority rules a majority of agents may vote for α when they prefer β because they lack the votes to adopt β. Thus, having a greater majority requirement does not always help build consensus. Although a greater majority requirement may create the appearance of a greater consensus, some of those who vote for an alternative may merely be trying to avoid costly delay.[9]

Two key aspects in the evaluation of different decision rules are the accuracy of the resulting decision and its timeliness. Since more accurate ultimate decisions naturally require more time for discovery, there is an underlying trade-off between the two aspects. We now describe the resolution of this trade-off by the committee itself.

Because unambiguous comparative statics are difficult to obtain when there is two-

[9] In *The Federalist No. 58*, James Madison explained why he did not favor super-majority rules: "It would be no longer the majority that would rule; the power would be transferred to the minority. ...[A]n interested minority might take advantage of it to screen themselves from equitable sacrifices to the general weal, or in particular emergencies to extort unreasonable indulgences" (Hamilton, Madison, and Jay 1982, pp. 298–299).

dimensional heterogeneity, we first discuss the simpler case in which all agents have the same time preferences.

Proposition 3. *When all agents have the same time preference, the equilibrium waiting region $(\hat{g}, \hat{G})$ under decision rule k is the same as the equilibrium waiting region under unanimity rule for a two-agent group consisting of agents $2m-k$ and k. Furthermore, $\hat{g} \geq g^*_{2m-k}$ and $\hat{G} \leq G^*_k$, with equality if and only if $k = m$.*

Because one-sided best-response functions do not cross when agents have the same discount rate, agent $2m-k$ is pivotal for adopting β, while agent k is pivotal for α. The exact intensity of the static preferences of the remaining agents do not matter for the determination of the equilibrium. Proposition 3 further states that, in this environment, the equilibrium waiting region coincides with the optimal waiting region for the median agent under majority rule. Under super-majority rule, the pivotal agent for α (i.e., agent k) adopts α at a belief before reaching the point that he deems optimal. Recall that agent k has a stronger preference for β than agent $2m-k$ does. If agent k were to make the decision alone, he would adopt α only when the belief is sufficiently strong to reach G^*_k. The result that $\hat{G} < G^*_k$ means that group decision-making leads to a moderation effect: the two pivotal agents choose thresholds which are less extreme than those they would choose were they in full control of the deliberation process. This follows from strategic substitution. Because agent $2m-k$ prefers to adopt β later than agent k does, agent k's continuation value from waiting is lowered, and he accommodates agent $2m-k$ by adopting α earlier.[10]

To fix ideas, we also focus on the special case in which the static preferences of agents are symmetric with respect to that of the median agent. We say that a group of agents constitute a *symmetric committee* if $r_1 = \ldots = r_{2m-1}$ and $v_m - v_j = v_{2m-j} - v_m$ for $j = 1, \ldots, m-1$. In a symmetric committee, we say that preferences become *more diverse* when, for any $j = 1, \ldots, m-1$, v_j decreases while v_{2m-j} increases by the same amount. In other words, a more diverse symmetric committee is one in which agents on both sides of the median agent hold stronger opposing preferences for their favored alternative.

Proposition 4. *In a symmetric committee, deliberations are longer but the decisions are more accurate when all agents become more patient, when the size of the majority requirement increases, or when the static preferences of agents become more diverse. In the general environment with two-dimensional heterogeneity, at least one of the alternatives will be adopted later and with a greater accuracy in the most patient and the least patient equilibria when k increases or when r_i for any agent i decreases.*

Since the pivotal agents in a symmetric committee are agents $2m-k$ and k, raising

[10] When agents have different time preferences, we can no longer identify the pivotal agents based on their static preferences alone. Nevertheless, a generalized form of the moderation effect still holds. In any equilibrium under decision rule k, the equilibrium lower threshold is higher than the k-th highest g^*_i and the equilibrium upper threshold is lower than the k-th lowest G^*_i.

the majority requirement from k to k' has the same effect as making preferences in the outcome-equivalent two-person jury more diverse (raising the static preference of one juror from v_{2m-k} to $v_{2m-k'}$, while raising that of another from v_k to $v_{k'}$). In Proposition 4, we show that such a change will expand the equilibrium waiting region (i.e., lower $\hat{g}$ and raise $\hat{G}$), as each of the pivotal agents demands greater evidence before adopting the alternative he is biased against. Deliberations are longer because, starting from any initial belief within the waiting region, the time it takes to reach the boundaries increases. Decisions are more accurate because α is adopted only when the belief reaches a higher upper cutoff $\hat{G}$, which corresponds to a higher probability that the state is A. Likewise, a lower equilibrium $\hat{g}$ means that β is adopted at a point when the probability of state B is higher. This result provides a contrast between our characterization and those pertaining to private information collection, as in Feddersen and Pesendorfer (1998), Persico (2004), and Austen-Smith and Feddersen (2005, 2006).

A symmetric committee provides a particularly simple setting to study the effect of raising the size of the majority requirement k. Perfect symmetry is sufficient but not necessary for Proposition 4 to hold. However, large departures from symmetry may invalidate the result that an increase in k always produces more accurate decisions. Imagine, for example, that agent $2m-k'$ is much more biased for α than agent $2m-k$, while agent k' is only slightly more biased for β than agent k. When the decision rule is raised from k to k', each of the two pivotal agents under decision rule k' wants to extend the threshold before adopting the alternative he controls, but strategic substitution suggests that the less extreme agent k' may want to accommodate the longer expected waiting time caused by the other more extreme pivotal agent by adopting α earlier. If the second effect dominates, then it is possible for the equilibrium upper cutoff $\hat{G}$ to decrease, making the decision for α less accurate when it is adopted. In the general case where preferences are not symmetric or when there is two-dimensional heterogeneity, it is not always the case that increasing k lowers $\hat{g}$ and raises $\hat{G}$. Nevertheless, we can show that the waiting region always become wider when k increases or when r_i decreases for some agent i. This implies that at least one of the alternatives will be adopted later and with a greater accuracy.

We now turn to how members of the committee resolve the accuracy-timeliness tradeoff, maintaining the assumption of symmetry. Because all agents have the same time preference, the median agent is pivotal for both α and β under majority rule. The equilibrium waiting region is simply his Wald-optimal waiting region (g_m^*, G_m^*). Let $(\hat{g}, \hat{G})$ represent the equilibrium waiting region under super-majority rule k. Proposition 4 shows that $\hat{G} - \hat{g} > G_m^* - g_m^*$. Furthermore, by symmetry, these two equilibrium waiting regions are both centered at v_m.

Proposition 5. *In a symmetric committee, there exists an interval of beliefs (containing v_m) such that, if θ_0 belongs to that interval, then all agents prefer majority rule to super-majority rule. For all initial beliefs, a majority of agents prefers majority rule to super-majority rule.*

From the perspective of agent m, it is obvious that the waiting region (g_m^*, G_m^*) is better than $(\hat{g}, \hat{G})$. From the perspective of another agent $j \neq m$, his ideal waiting region is centered at v_j. Thus, both majority rule and super-majority rule give a waiting region which is centered at the "wrong" place for agent j. But since agent j has the same time discount rate as agent m, super-majority rule also produces a waiting region which is "too wide" for him. This does not necessarily mean that agent j prefers majority rule to super-majority rule, because agent j's utility also depends on the initial belief θ_0. However, when the initial belief is near the center of the waiting region, there is unanimity in favor of majority rule: all agents deem the waiting region under super-majority rule too wide and prefer to use majority rule (or, equivalently in in a symmetric committee, to delegate the decision making power to the median voter).

If, on the other hand, the initial belief θ_0 is greater than G_m^* but smaller than $\hat{G}$, majority rule would have adopted α immediately, while there is still some prospect of getting β under super-majority rule. Thus, an agent j with sufficiently strong preference for β may prefer super-majority rule to majority rule despite the longer wait. However, in this example all agents with a preference for α stronger than that of the median agent prefer to adopt α immediately than to wait at this belief θ_0. In other words, even though the preference for majority rule is not unanimous, there is always a majority of group members who prefer majority rule to super-majority rule.

Propositions 4 and 5 suggest that, while committees themselves prefer majority rules, the quality of decisions is actually superior with more restrictive super-majority rules. That increased quality comes at the cost of longer deliberation, which is born by the agents themselves. Whenever there are positive externalities from high quality collective decisions, which may be especially relevant for juries or standard-setting committees as their decisions arguably impact a society of individuals who do not bear the deliberation cost themselves, super-majority rules may be preferable. In the following section, we show that there is another important drawback to majority rules

6. Rushing to Hasty Decisions

In our deliberation game, agents are making two related decisions: which alternative to adopt, and how long to wait. The static preference v_i is related to the first decision, and the time preference r_i is related to the second. When there is two-dimensional heterogeneity, the median voter theorem does not apply; that is, how long the group waits does not depend only on the median discount rate. Because quick decisions are strategic complements, the presence of an impatient agent may cause other agents to stop deliberations early. This, together with the fact that under majority rule it takes only one vote to change the outcome from one alternative to another, implies that the agent with the highest discount rate tends to have a disproportionate influence on the length of collective deliberations.

Figure 3 illustrates how the presence of an impatient agent affects equilibrium outcomes. Agent 1 becomes increasingly impatient across the three panels. Despite having a weaker preference for β, agent 1 is willing to adopt decision β before the other agents do as r_1 increases. He becomes pivotal for β when r_1 increases from panel (a) to panel (b) of Figure 3. Agent 2 responds by stopping earlier to adopt α, because he expects the impatient agent would stop early to adopt β. The waiting region of equilibrium $P2$ is narrower than that of equilibrium $P1$. One may think of the impatient agent 1 as a "swing voter." While he prefers α, he is willing to settle for β as long as the evidence swings a bit in favor of β. His vote, therefore, easily switches between α and β. Hence, both agent 2, who prefers α, and agent 3, who prefers β, have an incentive to stop deliberations early to prevent agent 1 from switching sides. These effects are magnified in equilibrium through strategic complementarity.

In panel (b) of Figure 3, agent 1 is both more impatient and has a stronger preference for α than the other agents. Nevertheless, he continues to influence the deliberation outcome as r_1 increases further. In panel (c), ϕ_1 approaches the 45-degree line, and the equilibrium $P3$ is near (v_m, v_m). In this equilibrium, although the impatient agent 1 has a stronger preference for α, he votes for β as soon as the belief drops slightly below the threshold for α to avoid further delay. In contrast, although agent 2 is patient, he votes for α as soon as the belief goes slightly above his static preference. The value of waiting is low for him because he is almost indifferent between α and β at the belief where β is adopted.

In Figure 3 there are only three agents, but the same logic applies regardless of the size of the group and their preferences.

Proposition 6. *Under majority rule, for any $\varepsilon > 0$ there exists $\overline{r}(\varepsilon)$ such that if $r_i \geq \overline{r}(\varepsilon)$ for some agent i, then there is an equilibrium waiting region $(\hat{g}, \hat{G})$ with width less than ε and with $v_m \in (\hat{g}, \hat{G})$.*

We may call the type of equilibrium in Proposition 6 a *hasty equilibrium*, as the equilibrium waiting region can be very small. In a hasty equilibrium, the group decides almost immediately, and which alternative is chosen essentially depends only on whether the initial belief θ_0 is greater or less than the static preference of agent m. The equilibrium outcome is, therefore, almost identical to that of a static problem in which the median voter decides without the benefit of any information collection. Note that the effect of the impatient agent on the equilibrium outcome cannot be counterbalanced by the presence of one or more extremely patient agents in the group. In Proposition 6, $\overline{r}(\varepsilon)$—the discount rate that ensures an equilibrium with an ε-waiting region—does not depend on the preferences of the other agents. Hence, the welfare loss to the other agents could be large. Under majority rule, one agent is enough to trigger a hasty equilibrium outcome, even when every other agent is very patient. In this sense, majority rule is not a robust decision-making mechanism because the length of deliberation can be dominated by one

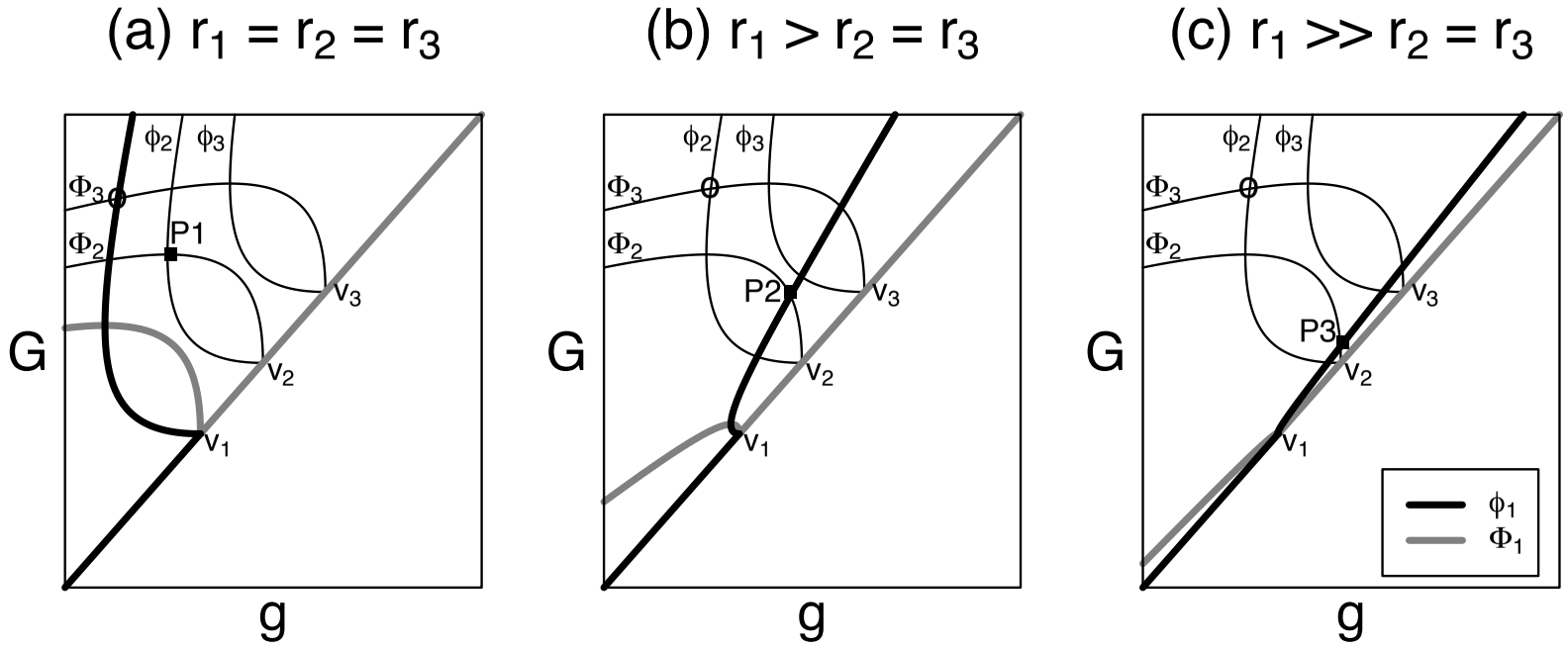

Figure 3. *Deliberation collapses as one of the agents becomes extremely impatient. When $r_1 = r_2 = r_3 = 0.039$ (panel (a)), agent 1 is not pivotal and the equilibrium is $P1$. If r_1 increases to 0.6, his best-response function ϕ_1 bends toward the 45-degree line. Agent 1 becomes pivotal and the equilibrium is $P2$. If r_1 increases further to 9 (panel (c)), ϕ_1 bends further and the equilibrium $P3$ is very close to the 45-degree line. The implied waiting region is very short. The circles in each panel indicate the respective equilibrium points when the decision rule requires unanimity.*

agent with extreme time preference.[11]

As long as there is a very impatient agent, a hasty equilibrium exists. Whether the equilibrium is unique depends on whether the preferences of the other agents are sufficiently different. In Figures 3 and 4, all agents except the impatient agent $i < m$ have the same discount rate. In Figure 3(c), the difference between v_{m+1} and v_m is large, and the hasty equilibrium is the unique equilibrium. In Figure 4, the difference between v_{m+1} and v_m is small, and there exist two other equilibria that are more patient than the hasty equilibrium.

When multiple equilibria exist, these equilibrium outcomes cannot be Pareto ranked. Different agents may have different preferences over different equilibria. More importantly, even the same agent may have different preferences over two equilibria at different beliefs. For example, an agent with strong static preference for α would prefer a hasty equilibrium when the belief is high (so he can obtain α immediately), but would prefer a more patient equilibrium when the belief is low (so he will not get β immediately). Thus, even if a majority of agents initially prefer a more patient equilibrium to a hasty equilibrium initially, some of these agents may change their minds when the belief changes.

The result that one very impatient agent can trigger hasty deliberations depends on two features of our model: first, information arrives slowly and continuously; and second, as information arrives an agent's relative preference between the two alternatives may change. By comparison, in the collective search model of Albrecht, Anderson and Vroman (2010), a new proposal is drawn every period, and in equilibrium the agents never go back and adopt a proposal that they did not accept in the past. As an extremely impatient agent will accept almost any proposal in such a model, having this agent is effectively the same as reducing both the group size and the majority requirement by one, and it does not cause the rest of the group to accept any proposal.

Compared to majority rule, super-majority rules are more resistant to the excessive influence of a few impatient agents. In Figure 3(c), if unanimity rule is used instead of majority rule, the equilibrium, now given by the intersection of ϕ_2 and Φ_3, will no longer be a hasty equilibrium. There is no need under unanimity rule for agents 2 and 3 to rush to a decision because the vote of the impatient voter is no longer sufficient to swing the outcome.

Define the number of *requisite swing votes* of a rule as the minimum number of vote changes to shift the outcome from adopting one alternative immediately to adopting the other alternative immediately. The number of requisite swing votes of majority rule is 1—if α and β each receives m votes, the last vote determines the outcome. More generally,

[11]Since the agents' utility functions are continuous in cutoffs, an equilibrium of the continuous-time model would be an epsilon-Nash equilibrium of the discrete-time model with short time periods. When there is a very impatient agent, we can verify that there is a mixed-strategy discrete-time equilibrium outcome close to the hasty equilibrium outcome in the continuous time model.

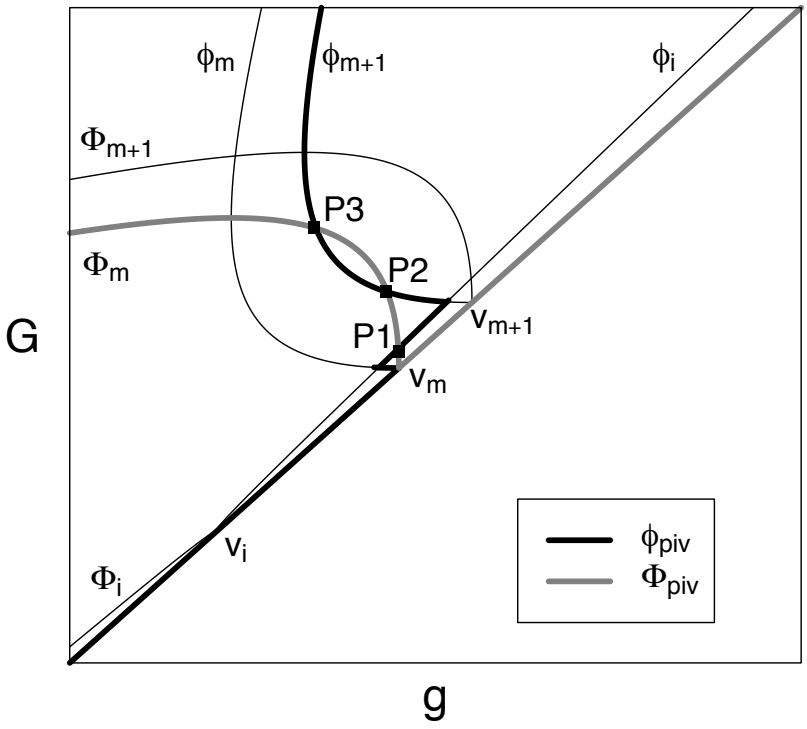

Figure 4. *Multiple equilibria exist when $v_{m+1} - v_m$ is small. The hasty equilibrium is $P1$ in this figure; there are two other more patient equilibria, $P2$ and $P3$. The figure is drawn using the same parameters as in Figure 3(c), but with $v_{m+1} - v_m = 0.4$ instead of 0.6. When $v_{m+1} - v_m$ becomes larger, the best-response functions ϕ_{m+1} and Φ_m are pulled apart, and eventually do not intersect. Then, the hasty equilibrium $P1$ remains the only equilibrium.*

under rule k an alternative must receive at least k votes to be adopted. Because there are altogether $2m-1$ agents, the number of requisite swing votes of rule k is $2k-(2m-1)$.

In what follows, we say that a group of impatient agents can *trigger a collapse of deliberation* if, for any $\varepsilon > 0$, there is an equilibrium with a waiting region no wider than ε when every agent in that group has a discount rate greater than some $\tilde{r}(\varepsilon)$, regardless of the preferences of other agents.

Proposition 7. *A group of impatient agents cannot trigger a collapse of deliberation unless the size of the group is at least as large as the number of requisite swing votes.*

Under rule k, at least $2k-(2m-1)$ agents switch their votes from α at the upper cutoff to β at the lower cutoff. Each of them can unilaterally delay the adoption of either alternative at the equilibrium threshold. It can be shown that, given r_i, agent i always

prefers delaying the adoption of at least one of the alternatives when the waiting region becomes too narrow. Hence, unless all agents who vote for both alternatives at the cutoffs are very impatient, no hasty equilibrium can exist. By contrast, in a hasty equilibrium under majority rule only the very impatient agent is voting for both α and β at the adoption thresholds. Of the remaining $2m-2$ agents, half vote only for α but not β, and the other half vote only for β but not α. While each of these $2m-2$ agents may like to simultaneously delay the adoption of both alternatives, none of them can do so.

Although raising the majority requirement may prevent a few impatient agents from reaching a decision prematurely, it also creates a risk that some very patient agents could drag the whole group into protracted deliberations. However, the effects of very impatient agents and very patient agents are not symmetric. Voting to delay an alternative by a very patient agent would cause other agents to adopt the other alternative more quickly.

Proposition 8. *In any equilibrium under rule k, the waiting region is no wider than the equilibrium waiting region under unanimity rule for any subset of k agents.*

Unless they can block a decision by themselves, a subset of very patient agents cannot prevent the group from reaching a decision. It takes $2m-k$ votes to block a decision under rule k. As the majority requirement is increased by one, the maximum number of very patient agents that can be allowed without causing endless deliberations falls by one. In contrast, the number of requisite swing votes increases by two. Hence, a properly chosen super-majority rule is robust to the presence of a few extreme agents whose discount rates are either very high or very low. For example, suppose the number of very impatient agents and the number of very patient agents each make up $1/4$ of the size of the group. Then a decision rule with less than $3/4$ super-majority requirement would prevent the patient agents from creating protracted deliberations, and the impatient agents cannot trigger a collapse of deliberation as long as the super-majority requirement is greater than $5/8$; see discussion above.

Although we focus on decision rules that treat the two alternatives symmetrically, we note that the issues we describe also exist in asymmetric decision rules. For a rule that requires k_α votes to adopt α and k_β votes to adopt β, the number of requisite swing votes is $k_\alpha + k_\beta - (2m-1)$. As in the symmetric case, one impatient agent may trigger a collapse of deliberation if the number of requisite swing votes is one and, conversely, a group of impatient agents cannot trigger a collapse of deliberation if its size is smaller than the number of requisite swing votes.[12]

7. Two-Stage Process: Deliberation Rules and Decision Rules

In our analysis thus far, a single rule k affects two types of decisions: how long to acquire information, and what action to take. It is natural to consider the consequences of a

[12]We thank a referee for raising this point.

two-stage procedure that distinguishes between a *deliberation rule* and a *decision rule*. A special case of a deliberation rule in a jury setting corresponds to repeated straw polls, with a final vote taken according to the decision rule once the outcome of the straw poll indicates that sufficient consensus has been achieved.[13] In general, deliberation rules may be quite different from decision rules. For instance, many committees feature a committee chairman who has the same power as all the other members over action decisions, but has a special role to play (and more power) in deliberation decisions. The chairman case can be modeled as either a dictator at the deliberation stage, or, if the chairman has been chosen to represent the interest of the majority, as the median voter in the committee.

We model deliberation rules as a threshold rule k_D such that deliberation ends as soon as k_D members of the committee vote to end deliberation. Decision rules are analogously captured by a rule k_d that describes the specific qualified majority required for reaching a decision about which action to take after deliberation stops. In a two-stage process, it might be the case that at the decision stage, there is no k_d-majority of votes for either alternative. In this case, we say that the committee is *indecisive*. We assume that when the committee is indecisive, α or β are determined by the flip of a fair coin.[14] A classic example of an indecisive committee is the case of a hung jury that does not reach the quorum required for conviction or for acquittal.

The analysis of two-stage processes allows us to highlight two distinct issues. First, we saw that in the one stage process it is possible that agents may vote against their static preferences in order to shorten deliberation. This is no longer possible in a two-stage process because the vote on the decision is separated from the vote on ending deliberation. This implies that one way to promote deliberation without giving a minority of agents excessive leverage is to have separate deliberation and decision votes. In order to highlight this effect we only consider the case in which k_d is majority rule since this simplifies the analysis considerably. The second issue that we discuss is the relative importance of deliberation rules and decision rules. This is a complicated issue but we are able to make some headway in the case in which all agents are relatively patient.

We now temporarily assume that the decision rule k_d is majority rule. In the second stage each agent votes for the alternative he likes given the current belief θ. Hence, α is adopted if $\theta \geq v_m$ and β is adopted when $\theta < v_m$. In the first stage each agent knows that voting for ending deliberation when $\theta \geq v_m$ is tantamount to voting for α. Instead of solving the two-stage game, it is more convenient to model the two-stage process as a *constrained deliberation game* where the agents can vote for α only when $\theta \geq v_m$ and β only when $\theta < v_m$. (The agents can still abstain at any point.) Because of the extra con-

[13]The guides distributed to jurors in many U.S. courts indicate that this reflects the deliberation process suggested to juries, see Murphy and Boatright (1999a, 1999b).

[14]The exact assumption we make about the payoff consequences of indecisive committees is for the most part inconsequential as most of our analysis focuses on cases where these do not occur (due to patience of the agents or to simple majority decision making).

straints, we need to modify the second part of our equilibrium condition slightly. Instead of requiring that each agent votes for an alternative before the belief reaches the equilibrium cutoff whenever the marginal gain from an earlier decision is positive, we now only require an agent does so when the constraints are not binding.

Because of the extra requirement that an alternative must be approved by majority rule in the second stage, we may define the constrained best-response functions for agent i:

$$\phi_i^c(G) \equiv \min\{\phi_i(G), v_m\},$$
$$\Phi_i^c(g) \equiv \max\{\Phi_i(g), v_m\}.$$

Define $\phi_{piv}^c(G; k_D)$ to be the k_D-th largest $\phi_i^c(G)$ for a given G, and $\Phi_{piv}^c(g; k_D)$ to be the k_D-th smallest $\Phi_i^c(g)$ for a given g. Using the same reasoning as in the unconstrained case, it is straightforward to see that $(\hat{g}^c, \hat{G}^c)$ is an equilibrium outcome of the two-stage decision process if and only if it is a fixed point of $(\phi_{piv}^c, \Phi_{piv}^c)$ and $\hat{g}^c < \hat{G}^c$.

There are two types of equilibrium outcomes given the two-stage decision process. An *unconstrained outcome* is one where neither cutoff is constrained (i.e., $\hat{g}^c < v_m < \hat{G}^c$), while a *constrained outcome* is one where one of the equilibrium cutoffs is v_m. An unconstrained equilibrium outcome respects static preferences, and hence can be implemented either through deliberation with one stage of voting or through the two-stage decision process. This implies that an unconstrained outcome is an equilibrium outcome under the two-stage decision process if and only if it is also an equilibrium outcome in the unconstrained deliberation game.

When an equilibrium outcome in the unconstrained deliberation game does not respect static preferences, it can no longer be implemented under the two-stage decision process.

Proposition 9. *A constrained outcome $(\hat{g}^c, \hat{G}^c)$ is an equilibrium of the two-stage process if and only if $\hat{g}^c < \hat{G}^c$ and one of the following conditions holds:*

1. $(\hat{g}^c, \hat{G}^c) = (v_m, \Phi_{piv}(v_m))$, *and* $\phi_{piv}(\Phi_{piv}(v_m)) \geq v_m$;
2. $(\hat{g}^c, \hat{G}^c) = (\phi_{piv}(v_m), v_m)$, *and* $\Phi_{piv}(\phi_{piv}(v_m)) \leq v_m$.

A constrained equilibrium exists only if there is an equilibrium outcome $(\hat{g}, \hat{G})$ in the unconstrained deliberation game in which either $\hat{g} \geq v_m$ or $\hat{G} \leq v_m$. The waiting region of a constrained equilibrium in the two-stage decision process is wider than that of the equilibrium it replaces in the unconstrained deliberation game.

Figure 5 depicts a constrained equilibrium with three agents. In the figure the equilibrium outcome in the unconstrained game under unanimity rule is $(\hat{g}, \hat{G})$ (point $P1$). In this equilibrium agents 1 and m prefer α but vote to adopt β at $\hat{g}$. Under the two-stage

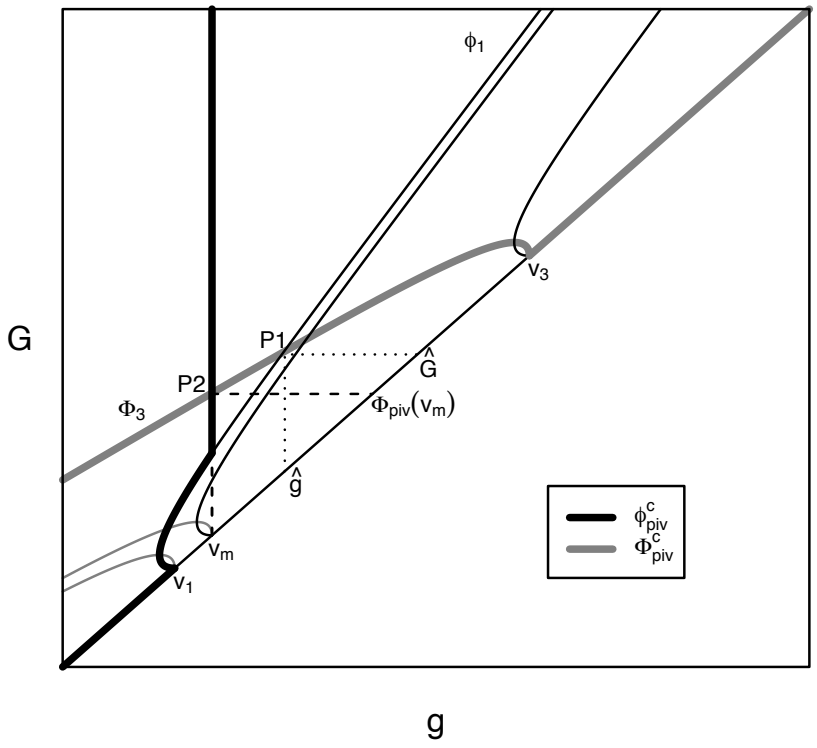

Figure 5. *A two-stage decision process with rule $k_D = 3$, $k_d = 2$ moves the equilibrium outcome from $P1$ to $P2$. Under the one-stage deliberation game, β is adopted at $\hat{g}$, which does not respect majority preferences. Under the two-stage decision process, deliberation stops at v_m and a majority (agents m and 3) vote to adopt β. However, the waiting region is also wider ($P1$ is farther from the 45-degree line than is $P2$). In this figure, $v_1 = -0.2$, $v_m = 0$, $v_3 = 1.7$, and $r_1 = r_m = r_3 = 0.039$.*

decision process the equilibrium outcome is $(v_m, \Phi_{piv}(v_m))$ (point $P2$). Notice that all three agents prefer to adopt β earlier but cannot vote for β until θ reaches v_m. Intuitively, point $P1$ is not an equilibrium outcome in a two-stage decision process because if deliberation ends at $\hat{g}$, agents 1 and m will vote for α in the second stage. Anticipating that, agent 3, who prefers waiting to α at $\hat{g}$, would hold on until either agent m switches and supports β (when $\theta = v_m$) or he switches and prefers α to waiting (when $\theta = \Phi_{piv}(v_m)$).

From Figure 5 it is clear that the waiting region of a constrained equilibrium is wider than that of the equilibrium it replaces in the unconstrained game. Hence, a group of impatient agents cannot trigger a collapse of deliberation in a two-stage process under deliberation rule k_D and decision rule m in the constrained game if it cannot do so under the same rule $k = k_D$ in the one-stage process. If preferences are symmetric, the one-stage

and two-stage processes have the same set of equilibrium outcomes. But when preferences are unbalanced, having a separate decision vote offers extra protection against outcomes dominated by extreme minority preferences. This protection, however, is not without cost: because the length of deliberation is likely to be longer, the agents may actually prefer the unconstrained equilibrium to the constrained one.

We now allow any decision rule and any deliberation rule, but we consider scenarios in which all agents are sufficiently patient. It is easy to see that in any equilibrium of the one-stage deliberation game, the waiting region must contain the static preference v_i for all agents i. This means that when an alternative is adopted at the cutoff, it is unanimously preferred by all agents. We show that such unconstrained outcome of the one-stage game is also an equilibrium of the two-stage decision process regardless of the second-stage decision rule k_d. In particular neither the time to decision nor the relative probability of the two types of errors depend on the decision rule.

Proposition 10. *For any profile of static preferences and any two-stage decision rule (k_D, k_d), there exists $\underline{r}$ such that, whenever $r_i \leq \underline{r}$ for all i, if $(\hat{g}, \hat{G})$ is an equilibrium outcome of the one-stage process under rule k, then it is an equilibrium outcome of the two-stage process for deliberation rule $k_D = k$ and any decision rule k_d. Furthermore, in such an equilibrium, voting is unanimous at the decision stage.*

Proposition 10 implies that, when agents are sufficiently patient, the decision rule has no influence on the equilibrium outcome. This outlines circumstances that are consistent with some of the empirical and experimental evidence on juries that we discuss later.

8. Empirical Relevance

8.1. Application to Juries

In this section we discuss some evidence on juries documented in prior empirical literature. Some of this evidence speaks to the basic structure of the model and some is directly related to our results. This evidence suggests that our model is broadly consistent with a number of different patterns in the data.

In the model, longer deliberation corresponds to additional signals received by the committee. Our interpretation is that this is a reasonable shortcut for thinking about how deliberation helps jurors gain an understanding of the evidence presented at trial. Even though no additional explicit information is received by the jurors during, deliberation has the function of processing information and understanding the relative importance of different, potentially contrasting pieces of evidence.[15] In particular, the model fits a plausible deliberation protocol in which the jury conducts repeated straw polls until a

[15]One instructive way to think of the trial may be the following: the pre-deliberation phase, where all the evidence is presented and discussed, is similar to a lecture given by a professor, and the deliberation phase is similar to a study group that looks through the notes taken during class to gain some further understanding

sufficient consensus emerges.

One strand of literature on juries studies opinion formation by jurors. This is relevant for our model for two reasons. First, it establishes that the deliberation process is important in the formation of jurors' opinions. Second, some features of the opinion formation process seem to mirror the updating process assumed in our model.

Legal scholars emphasize the *fact-finding* role of juries (e.g., Vidmar and Hans 2007). A substantial fraction of deliberation appears to be devoted to a discussion of the facts (Pennington 1983). In fact, starting from Kalven and Zeisel (1966), numerous scholars have argued that juries do a good job at reaching an understanding of the facts. In a study of mock juries, Ellsworth (1989) writes: "In general, over the course of deliberation, jurors appear to focus more on the important facts and issues, come to a clearer understanding of them, and approach consensus on the facts." This literature also suggests the importance of the interaction between decision rules and deliberation protocols. For instance, Hastie, Penrod, and Pennington (1983) point out that the volume of discussion substantially increases with the decision rule.

Importance of deliberation and verdict patterns. Hannaford et al. (2000) studied the timing of jury opinion formation. They used a special case study of a jury reform implemented in Arizona in 1995 that allowed for discussions during civil trial. Their data include survey responses of $1,385$ jurors from 172 trials in four counties. Over 95 percent of jurors reported changing their minds at least once over the course of the trial, and 15 percent reported changing their minds more than once during trial. Importantly, over 40 percent of jurors reported changing their minds during the final deliberations, suggesting that deliberation is a key component of opinion formation.

Hans (2007) studied jury deliberation by using surveys conducted by the National Center for State Courts. The data contain reports from close to $3,500$ jurors who had participated in felony trials in four large, urban courts. Figure 6 summarizes one key finding by Hans (2007). The figure groups each jury into five categories depending on the outcome of an initial straw poll. These go from "strongly favor innocent" where the great majority of jurors initially favored acquittal, to "leaning toward innocent," where a small majority initially favors acquittal, to "closely divided," where the jury is evenly split (5–7, 6–6, or 7–5), to "leaning toward guilty," and finally to "favor guilty." For each initial leaning of the jury, the figure describes the distribution of ultimate outcomes.

There are a number of interesting observations arising from this figure. First, the patterns of opinion change are consistent with collective information acquisition driven by

of the topic. In fact, in the jury setting deliberation is rendered even more important by the fact that, in contrast to a lecture, the jury must sift through the mass of sometimes conflicting evidence presented by two opposing parties during the trial to figure out the relevance of different pieces of information, as well as the appropriate weight to attribute to these in establishing the guilt or innocence of the defendant. It is as if the lectures were held by two professors with opposite views on a policy, and the class had to figure out which arguments by the professors to take more seriously.

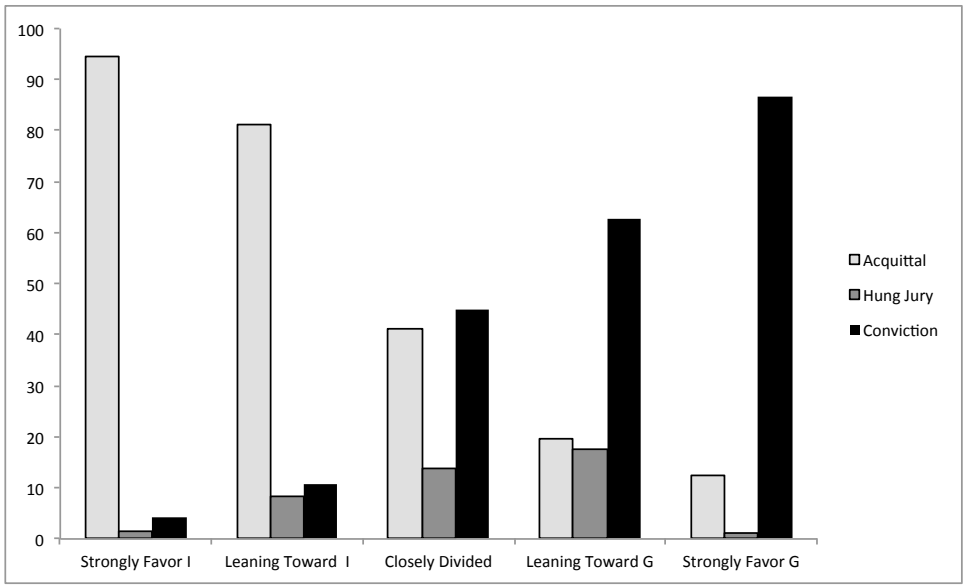

Figure 6. *Distribution of jury outcomes (in percentage) according to initial jury leanings. Source: Hans (2007).*

a Bayesian updating process (as in our model). When the initial vote in the jury strongly supports a particular outcome, that outcome is more likely to ultimately emerge. For instance, 77 of the 89 juries with strong majorities for guilt convicted the defendant. However, the ultimate outcome does change during deliberation: 11 of these 89 juries ended up acquitting the defendant. Recall that the verdict had to be unanimous. Therefore, in all these 11 cases almost all jurors changed their mind during deliberation.[16] Thus, the deliberation process did have a large effect in these juries.

Jury composition. Increased heterogeneity in static preferences has been found to increase quality and length of deliberation; see Goeree and Yariv (2011), who present experimental evidence that increased preference heterogeneity increases deliberation length and accuracy of decisions. In our model, more diverse static preferences increases the length of deliberation since it makes the pivotal members at the deliberation stage more extreme. This translates immediately into longer deliberation and, in symmetric committees, more accurate decisions.

Effect on length of deliberation. Devine et al. (2001) provides a meta-study of empirical jury research and reports that the decision rule affects length and quality of deliberation. On average, using mock juries, deliberation under unanimity tend to take longer than deliberation under majority. The quality, measured by legal experts, exhibits similar patterns—it is higher under unanimity than under majority.

In our model, the length of deliberation can be affected by the decision rule since

[16]These observations should be interpreted with some care, as initial polls within the jury sometimes take place after some amount of deliberations has already taken place. Thus, consensus may be overstated.

pivotal members at the deliberation stage may, if not pivotal at the decision stage, prolong deliberation in order to convince the holdouts at the decision stage.

Effects of the decision rule. Baldwin and McConville (1980) studied a reform in 1974 in England that allowed for majority verdicts in criminal trials, while prior to the reform unanimity was required. A predominance of verdicts (311 out of 326 cases) were unanimous even after the reform, suggesting that the decision rule did not have much of an effect. Kalven and Zeisel (1966) report similar patterns for U.S. states that do not require unanimity for conviction: most verdicts are unanimous anyway.[17] Hastie, Penrod, and Pennington (1983) find, however, that the volume of discussion substantially increases with the decision rule. In laboratory experiments, Goeree and Yariv (2011) find that, when subjects cannot talk before voting, the decision rule has an effect, whereas, when subjects can talk, the decision rule has very little effect.[18] Our model provides a possible explanation for the fact that in some circumstances the decision rule seems to have little effect on decisions. We show that, when agents are sufficiently patient, in equilibrium, deliberation always ends with unanimous decisions: whenever there is disagreement on the appropriate decision to take, members of the committee agree that it is worthwhile to continue deliberating.

8.2. Standard-setting Committees

Standard-setting organizations are voluntary organizations that provide a platform for interested parties to coordinate on technological standards. Some standard-setting organizations consist solely of platform sponsors; others include also user groups, academics, and government agencies (e.g., the American National Standards Institute, or the World Wide Web Consortium). The standard-setting process begins when one or multiple sponsors submit proposed standards. A committee would then be formed to consider the pros and cons of different proposals. Expert opinions play an important role in this process. For example, laboratory and field experimental studies were commissioned to evaluate different bar code designs (Brown 1997).

The evaluation process takes time. The average duration to adopt an internet standard is about 3.5 years (Simcoe 2012). In addition to different preferences over the alternatives, different parties may also disagree over the timing of the decision. In the history of standardization of containers, trucking firms had lower stakes than shipping lines because the latter had to make substantial investments in ships and docks. As a result, trucking firms wanted to reach agreement on container sizes more quickly (Levinson 2006). More

[17] A caveat to this observation is that under majority rule, when a majority of jurors agrees, any other juror's vote cannot affect the final outcome. In particular, those jurors may vote against their private assessment to satisfy social pressures at no consequence to the defendant.

[18] In the context of monetary policy, Blinder and Morgan (2008) look experimentally at two particular decision protocols (ones in which there is a designated leader, and ones in which there is not) and also find little differences in outcomes.

generally, sponsors, who have vested interests in their own technologies tend to be more patient, while users—downstream firms that build applications on the platform—may prefer quicker resolution.

There is a small literature on standard-setting committees. Farrell and Saloner (1988) and Farrell and Simcoe (2012) model the standardization process as a war of attrition between two platform sponsors. The perspective ignores the fact that deliberation, while costly, also produces valuable information. Simcoe (2012) introduces a model of collective search, whereby each sponsor draws a new value for its technology. In reality, the alternatives under consideration are not fixed, but as long as these modifications are marginal rather than radical, our model captures the strategic effects resulting from the evolving nature of the information accumulation process.

There is no consensus on what decision rules a standard-setting organization should use to adopt standards. Chiao, Lerner, and Tirole (2007) report that in their sample of standard-setting organizations, 34 percent use majority rule, 27 percent use super-majority rules, and 13 percent use consensus to adopt standards. Our results suggest that there is no single decision rule that suits all standard-setting organizations. Majority rule may be more suitable to organizations whose members have similar time preferences, such as those that consist solely of technology providers, as it ensures that the adopted standard is preferred by a majority of the members. For standard-setting organizations that have a diverse membership or when the standard affects the welfare of non-members, a super-majority rule may perform better, as it promotes deliberation and prevents low-stake or impatient members from exerting disproportionate influence. Our results, however, also point out the potential danger of a high majority requirement. Instead of promoting consensus, such a rule may also allow a deep-pocketed firm who can afford a drawn-out process to force other interested parties to concede.

9. Conclusions

Deliberation is an integral part of many real-life collective decisions. In addition to the examples mentioned up to now, there has been a more recent surge of interest in the idea of "deliberative democracy" in political science (e.g., Gutmann and Thompson 2004; Fishkin 2009; Steiner 2012), emphasizing consensus building more than preference revelation.

In this paper, we introduce a formal model of open-ended collective deliberations and explain how different decision rules aggregate heterogenous static and time preferences. Our analysis highlights the difference between static voting and dynamic deliberation. Whereas super-majority rules favor a particular alternative in a static environment, they force agents to gather more information in deliberative processes. We show that several features of our model are consistent with juror behavior, and our results explain why many deliberative bodies adopt super-majority rules.

Appendix

Proof of Lemma 1. *Part 1.* A direct calculation gives:

$$\frac{\partial u_i}{\partial G} = \frac{(R_2 - R_1)\, e^{-(G-g)} e^{\theta} \Psi_i}{(1+e^{\theta})\left(e^{-R_1(G-g)} - e^{-R_2(G-g)}\right)} \left(e^{v_i - g} - e^{f(r_i, G-g)}\right),$$

where

$$f(r_i, G-g) = \log \frac{R_2 e^{R_1(G-g)} - R_1 e^{R_2(G-g)}}{R_2 - R_1}.$$

Therefore, the sign of $\partial u_i / \partial G$ depends only on the sign of $v_i - g - f(r_i, G-g)$ and not on the value of θ. Similarly,

$$\frac{\partial u_i}{\partial g} = \frac{(R_2 - R_1)\, e^{-(G-g)} e^{v_i} \psi_i}{(1+e^{\theta})\left(e^{-R_1(G-g)} - e^{-R_2(G-g)}\right)} \left(e^{f(r_i, G-g)} - e^{G - v_i}\right).$$

Therefore, the sign of $\partial u_i / \partial g$ depends only on the sign of $f(r_i, G-g) - G + v_i$ and not on the value of θ.

Part 2. The derivative $\partial u_i / \partial G$ has the same sign as $v_i - g - f(r_i, G-g)$, which is strictly decreasing in G. Similarly, $\partial u_i / \partial g$ has the same sign as $f(r_i, G-g) - G + v_i$, which is strictly decreasing in g. ■

Proof of Lemma 2. We only establish the statements for the upper best-response function; the proof for the corresponding part (b) follows from similar reasoning.

Part 1. From the proof of Lemma 1, the sign of $\partial u_i / \partial G$ depends only on the sign of $v_i - g - f(r_i, G-g)$ when $\theta \in [g, G]$. Suppose $g \geq v_i$. As $f(r_i, G-g) > 0$ when $G > g$, we have $\partial u_i / \partial G < 0$. This shows that $\Phi_i(g) = g$ when $g \geq v_i$.

Suppose $g < v_i$. As $v_i - g - f(r_i, G-g)$ is strictly decreasing in G, and is strictly negative at $G = g$ and strictly positive when G is sufficiently large, there is a unique $G' > g$ such that $\partial u_i / \partial G = 0$ at $G = G'$. By Lemma 1, $u_i(g, G' \mid \theta) \geq u(g, G \mid \theta)$ for any $G \geq g$ and for all $\theta \in [g, \min\{G, G'\}] \cup [\max\{G, G'\}, \infty]$. If $G < \theta < G'$, then $u_i(g, G' \mid \theta) > u_i(g, \theta \mid \theta) = u_i(g, G \mid \theta)$. If $G > \theta > G'$, then $u_i(g, G \mid \theta) < u_i(g, \theta \mid \theta) = u_i(g, G' \mid \theta)$. This shows that $\Phi_i(g) = G' > g$ when $g < v_i$.

The continuity of Φ_i follows from the continuity of $f(r_i, G-g)$ and the fact that $f(r_i, 0) = 0$. Thus, $\lim_{g \uparrow v_i} \Phi_i(g) = v_i$. Twice differentiability follows from the implicit function theorem, since the function defining the first-order condition is smooth.

Part 2. To show that $\lim_{g \to -\infty} \Phi_i(g) = -\infty$, suppose to the contrary that there is a finite lower bound b to $\Phi_i(g)$. We first show that there exists a g' such that $v_i - g - f(r_i, b-g) < 0$

for all $g < g'$. To see this, observe that

$$\lim_{g \to -\infty} \frac{f(r_i, b-g)}{v_i - g} = \lim_{g \to -\infty} f_2(r_i, b-g) = R_2 > 1.$$

Furthermore, for any $g < g'$ and $g < v_i$, $v_i - g - f(r_i, G-g) > 0$ when $G = g$. Thus, there exists a $G' \in (g, b)$ that satisfies $v_i - g - f(r_i, G'-g) = 0$. Because $\Phi_i(g) = G'$ when $g < v_i$, it follows that $\Phi_i(g) < b$ for all $g < g'$, a contradiction.

Let f_2 and f_{22} represent the first and second derivatives of f with respect to $G - g$. For $g < v_i$, differentiate the first-order condition, $v_i - g - f(r_i, \Phi_i(g) - g)) = 0$, with respect to g to get

$$\Phi_i'(g) = 1 - \frac{1}{f_2},$$
$$\Phi_i''(g) = \frac{f_{22}\left(\Phi_i' - 1\right)}{f_2^2}.$$

For $G - g > 0$,

$$f_2 = -R_1 R_2 \frac{e^{R_2(G-g)} - e^{R_1(G-g)}}{R_2 e^{R_1(G-g)} - R_1 e^{R_2(G-g)}} > 0,$$
$$f_{22} = -\frac{R_1 R_2 (R_2 - R_1)^2 e^{G-g}}{\left(R_2 e^{R_1(G-g)} - R_1 e^{R_2(G-g)}\right)^2} > 0.$$

Therefore, $\Phi_i'(g) < 1$ and $\Phi_i''(g) < 0$.

Finally, we show that $\Phi_i(g)$ reaches a maximum at $g = g_i^*$. For any $g < v_i$, the one-sided stopping problem satisfies a smooth pasting condition at the one-sided optimal threshold $\Phi_i(g)$ (see, for example, Dixit 1993). Therefore, at $G = \Phi_i(g)$ and $\theta = \Phi_i(g)$,

$$\frac{\partial u_i}{\partial \theta} = \frac{\mathrm{d}(e^\theta/(1+e^\theta))}{\mathrm{d}\theta}.$$

Differentiate both sides with respect to g to yield, at $G = \Phi_i(g)$ and $\theta = \Phi_i(g)$,

$$\left(\frac{\mathrm{d}^2(e^\theta/(1+e^\theta))}{\mathrm{d}\theta^2} - \frac{\partial^2 u_i}{\partial \theta \partial G} - \frac{\partial^2 u_i}{\partial \theta^2}\right) \Phi_i'(g) = \frac{\partial^2 u_i}{\partial \theta \partial g}.$$

At $g = g_i^*$, we have $G = \Phi_i(g_i^*) = G_i^*$. Both $\partial u_i/\partial G$ and $\partial u_i/\partial g$ are equal to 0 at the optimal thresholds, and dynamic consistency implies that this value does not change with θ. Therefore, at $g = g_i^*$, $G = G_i^*$ and $\theta = G_i^*$, we obtain:

$$\left(\frac{\mathrm{d}^2(e^\theta/(1+e^\theta))}{\mathrm{d}\theta^2} - \frac{\partial^2 u_i}{\partial \theta^2}\right) \Phi_i'(g_i^*) = 0.$$

Because the stochastic process $\mathrm{d}S'$ has an expected drift $\mu'(e^\theta - 1)/(1 + e^\theta)$ and an instantaneous variance $2\mu'$, applying Ito's lemma gives:

$$\frac{r}{\mu'}u_i = \frac{e^\theta - 1}{1 + e^\theta}\frac{\partial u_i}{\partial \theta} + \frac{\partial^2 u_i}{\partial \theta^2}.$$

Using the smooth pasting condition for $\partial u_i/\partial\theta$, we obtain:

$$\frac{\mathrm{d}^2(e^\theta/(1 + e^\theta))}{\mathrm{d}\theta^2} - \frac{\partial^2 u_i}{\partial \theta^2} = -\frac{r}{\mu'}u_i < 0.$$

This establishes that $\Phi_i'(g_i^*) = 0$. When $g < v_i$, as $\Phi_i(g)$ is strictly concave, it reaches a maximum at $g = g_i^*$.

Part 3. For $g < v_i$, $\Phi_i(g)$ satisfies $v_i - g - f(r_i, \Phi_i(g) - g) = 0$. Because the left side of this equation increases without bound in v_i, and because $f_2 > 0$, $\Phi_i(g)$ increases without bound in v_i. Moreover, for $g \leq v_i$, a marginal increase in v_i has no effect on $\Phi_i(g)$, whereas raising v_i to $v_i' > g$ will bring $\Phi_i(g)$ from g to a value strictly greater than g.

Let f_1 be the derivative of f with respect to r_i. From the definitions of R_1 and R_2, we note that $\partial R_2/\partial r_i > 0$ and $\partial R_1/\partial r_i = -\partial R_2/\partial r_i$. Therefore, f_1 has the same sign as:

$$\begin{aligned}
&e^{(R_2 - R_1)(G-g)} - \frac{1 + (R_2 - R_1)R_2(G-g)}{1 - (R_2 - R_1)R_1(G-g)} \\
>& 1 + (R_2 - R_1)(G-g) - \frac{1 + (R_2 - R_1)R_2(G-g)}{1 - (R_2 - R_1)R_1(G-g)} \\
=& \frac{-(R_2 - R_1)^2 R_1 (G-g)^2}{1 - (R_2 - R_1)R_1(G-g)} > 0.
\end{aligned}$$

Because $f(r_i, G - g)$ increases in both r_i and $G - g$, $\Phi_i(g)$ decreases in r_i for $g < v_i$. For $g \geq v_i$, $\Phi_i(g)$ does not change with r_i.

Furthermore, we have $f(0, G - g) = 0$ and

$$\lim_{r_i \to \infty} f(r_i, G - g) = \lim_{r_i \to \infty} \left(1 + \frac{r_i}{\mu'}\right)\left(e^{R_1(G-g)} + e^{R_2(G-g)}\right) = \infty.$$

Thus, for any $g < v_i$ and any $\varepsilon > 0$, we can choose $\overline{r}_i$ such that

$$v_i - g - f(\overline{r}_i, \varepsilon) = 0.$$

Because $f_1 > 0$ and $f_2 > 0$, for any $r_i \geq \overline{r}_i$ we must have $\Phi_i(g) - g \leq \varepsilon$. ■

We use Lemma 2 to establish two additional claims. Claim 1 is concerned with the property of the composite function $\Phi_i(\phi_i(G))$. Claim 2 says that if the waiting region is

narrower than the optimal waiting region for agent i, then he strictly prefers to adopt at least one of the alternatives later. Likewise, if the waiting region is wider than his optimal waiting region, then he prefers to adopt one of the alternatives earlier.

Claim 1. *For any agent i,*

1. $\Phi_i(\phi_i(G)) > G$ *for all* $G \in (v_i, G_i^*)$;
2. $\Phi_i(\phi_i(G)) < G$ *for all* $G > G_i^*$.

Proof. Part 1. Let $h(G) \equiv \Phi_i(\phi_i(G)) - G$. Then for $G \in (v_i, G_i^*)$,

$$h''(G) = \Phi_i''(\phi_i(G)) \left[\phi_i'(G)\right]^2 + \Phi_i'(\phi_i(G))\phi_i''(G) < 0,$$

because by part 2 of Lemma 2, Φ_i is strictly concave, ϕ_i is strictly convex, and $\Phi_i'(\phi_i(G)) < 0$ since $\phi_i(G) > g_i^*$. Thus, h is strictly concave for $G \in (v_i, G_i^*)$. This, together with $h(v_i) = h(G_i^*) = 0$, implies that $h(G) > 0$ for $G \in (v_i, G_i^*)$.

Part 2. Since $\max_g \Phi_i(g) = G_i^*$, we have $h(G) \leq G_i^* - G < 0$ for all $G > G_i^*$. ∎

Claim 2. *For any agent i and any $(g, G) \notin \{(g_i^*, G_i^*), (v_i, v_i)\}$,*

1. *if $G - g \leq G_i^* - g_i^*$, then either $g > \phi_i(G)$ or $G < \Phi_i(g)$;*
2. *if $G - g \geq G_i^* - g_i^*$, then either $g < \phi_i(G)$ or $G > \Phi_i(g)$.*

Proof. Part 1. Suppose $G - g \leq G_i^* - g_i^*$. (i) If $G > G_i^*$, then because $G - \phi_i(G)$ is strictly increasing in G (part 2 of Lemma 2), we have $G - \phi_i(G) > G_i^* - \phi_i(G_i^*) \geq G - g$. Hence, $g > \phi_i(G)$. (ii) If $g < g_i^*$, then because $\Phi_i(g) - g$ is strictly decreasing in g, we have $\Phi_i(g) - g > \Phi_i(g_i^*) - g_i^* \geq G - g$. Hence, $\Phi_i(g) > G$. (iii) Suppose $g \geq g_i^*$ and $G \leq G_i^*$. If $G > g \geq v_i$, then $\phi_i(G) < v_i \leq g$ because ϕ_i is decreasing for $G \in [v_i, G_i^*]$. If $v_i \geq G > g$, then $\Phi_i(g) > v_i \geq G$ because Φ_i is decreasing for $g \in [g_i^*, v_i]$. If $G \geq v_i \geq g$, then by Claim 1, $\Phi_i(\phi_i(G)) > G$ for $G \in (v_i, G_i^*)$. As Φ_i is decreasing for $g \in [g_i^*, v_i]$, if $g \leq \phi_i(G)$ then we must have $\Phi_i(g) \geq \Phi_i(\phi_i(G)) > G_i$.

Part 2. Suppose $G - g \geq G_i^* - g_i^*$. Because $(g, G) \neq (g_i^*, G_i^*)$, either $G > G_i^*$ or $g < g_i^*$. If $G > G_i^*$, then $G > \Phi_i(g)$ for any g. If $g < g_i^*$, then $g < \phi_i(G)$ for any G. ∎

Proof of Lemma 3. A cutoff pair $(\hat{g}, \hat{G})$, with $\hat{g} \neq \hat{G}$, solves the constrained optimization problem for some θ within the boundaries only if it satisfies the Kuhn-Tucker conditions:

$$\frac{\partial u_i}{\partial G} \begin{cases} \leq 0 & \text{if } \hat{G} = \underline{G}_i, \\ = 0 & \text{if } \hat{G} \in (\underline{G}_i, \overline{G}_i), \\ \geq 0 & \text{if } \hat{G} = \overline{G}_i; \end{cases} \qquad \frac{\partial u_i}{\partial g} \begin{cases} \leq 0 & \text{if } \hat{g} = \underline{g}_i, \\ = 0 & \text{if } \hat{g} \in (\underline{g}_i, \overline{g}_i), \\ \geq 0 & \text{if } \hat{g} = \overline{g}_i. \end{cases}$$

From the definition of the the best-response functions, $(\hat{g}, \hat{G})$ satisfies the Kuhn-Tucker conditions if and only if it satisfies (2) and (3).

We now show that there is only one pair of thresholds that satisfies the Kuhn-Tucker conditions. Suppose, by way of contradiction, that (g', G') and $(\hat{g}, \hat{G})$ both satisfy the Kuhn-Tucker conditions. Because the one-sided preferences are single-peaked, $g' \neq \hat{g}$ and $G' \neq \hat{G}$. There are two cases to consider.

Case 1: $g' > \hat{g}$ and $G' > \hat{G}$. Because $g' > \underline{g}_i$ and $G' > \underline{G}_i$, the Kuhn-Tucker conditions require that at the point (g', G'), $\partial u_i/\partial g \geq 0$ and $\partial u_i/\partial G \geq 0$. (This means that (g', G') is in region $Q2$ of Figure 1.) Therefore, $g' \leq \phi_i(G')$ and $G' \leq \Phi_i(g')$. Because $\Phi_i(g') \leq G_i^*$, $\hat{G} < G' \leq \Phi_i(g') \leq G_i^*$. By part 2 of Lemma 2, $\phi_i(\hat{G}) > \phi_i(G') \geq g' > \hat{g}$. Hence, $\partial u_i/\partial g > 0$ at $(\hat{g}, \hat{G})$. (Intuitively, because $(\hat{g}, \hat{G})$ is south-west of (g', G'), it must belong to either region $Q1$ or $Q2$ in Figure 1.) However, because $\hat{g} < \overline{g}_i$, the Kuhn-Tucker conditions require $\partial u_i/\partial g \leq 0$ at that point.

Case 2: $g' > \hat{g}$ and $G' < \hat{G}$. Because $g' > \underline{g}_i$ and $G' < \overline{G}_i$, the Kuhn-Tucker conditions require that at the point (g', G'), $\partial u_i/\partial g \geq 0$ and $\partial u_i/\partial G \leq 0$. (This means that (g', G') is in region $Q1$ in Figure 1.) By part 1 of Claim 2, $G' - g' > G_i^* - g_i^*$. Because $\hat{G} - \hat{g} > G' - g'$, $\hat{G} - \hat{g} > G_i^* - g_i^*$. It then follows from part 2 of Claim 2 that either $\partial u_i/\partial g > 0$ or $\partial u_i/\partial G < 0$ at $(\hat{g}, \hat{G})$. (Intuitively, as Φ_i has a positive slope for $g \leq g_i^*$ and ϕ_i has a positive slope for $G \geq G_i^*$, $(\hat{g}, \hat{G})$, which is north-west of (g', G'), must belong to region $Q1$.) However, as $\hat{g} < \overline{g}_i$ and $\hat{G} > \underline{G}_i$, the Kuhn-Tucker conditions require $\partial u_i/\partial g \leq 0$ and $\partial u_i/\partial G \geq 0$ at that point.

Thus, for any θ, the only (g, G) that satisfies the Kuhn-Tucker conditions with $\theta \in (g, G)$ is the pair $(\hat{g}, \hat{G})$. Because a solution to the constrained maximization problem (1) always exists, $(\hat{g}, \hat{G})$ is a solution to (1) if $u_i(\hat{g}, \hat{G} \mid \theta)$ is greater than $e^{v_i}/(1 + e^{\theta})$ when adopting β immediately is feasible at θ, and greater than $e^{\theta}/(1 + e^{\theta})$ when adopting α immediately is feasible.

Suppose $\theta \in [\hat{g}, \hat{G}]$. Adopting β immediately is feasible if $\theta < \overline{g}_i$. In this case, the Kuhn-Tucker conditions imply that at the point $(\hat{g}, \hat{G}; \theta)$, $\partial u_i/\partial g \leq 0$ (as $\hat{g} \leq \theta < \overline{g}_i$). It follows from Lemma 1 that $u_i(\hat{g}, \hat{G} \mid \theta) \geq u_i(\theta, \hat{G} \mid \theta) = e^{v_i}/(1 + e^{\theta})$. Adopting α immediately is feasible if $\theta \geq \underline{G}_i$. In this case, the Kuhn-Tucker conditions imply that at the point $(\hat{g}, \hat{G}; \theta)$, $\partial u_i/\partial G \geq 0$, and it follows that $u_i(\hat{g}, \hat{G} \mid \theta) \geq u_i(\hat{g}, \theta \mid \theta) = e^{\theta}/(1 + e^{\theta})$.

Suppose $\theta > \hat{G}$. Under $(\hat{g}, \hat{G})$, α is adopted immediately. If adopting β immediately is feasible (i.e., $\theta < \overline{g}_i$), then it must also be feasible at $\hat{G}$. But we have already shown in the last paragraph that agent i prefers α to β at $\hat{G}$ whenever adopting β immediately is feasible. Because the relative payoff of α increases an agent's belief, agent i must also prefer α to β at θ. By similar logic, if α is feasible at some $\theta < \hat{g}$, agent i must prefer β to α. ■

Proof of Proposition 1. *Necessary and sufficient condition for equilibrium.* Sufficiency. Suppose $(\hat{g}, \hat{G})$ is an unequal fixed point of (ϕ_{piv}, Φ_{piv}). Consider the strategy profile σ where, for each i,

$$\sigma_i = (\phi_i(\hat{G}), \Phi_i(\hat{g})).$$

It is straightforward to see that $(g^{[k]}(\sigma), G^{[k]}(\sigma)) = (\hat{g}, \hat{G})$. Furthermore, for each agent i, $\hat{g}$ satisfies (2) given $\hat{G}$, and $\hat{G}$ satisfies (3) given $\hat{G}$. Hence, by Lemma 3, σ is a Nash equilibrium. Furthermore, by the single-crossing property of Lemma 1, σ also satisfies condition 2 of the equilibrium definition. The above argument shows that if $(\hat{g}, \hat{G})$ is an unequal fixed point of (ϕ_{piv}, Φ_{piv}), then it is an equilibrium outcome of the game.

Necessity. Suppose σ is an equilibrium strategy profile. Let $(\hat{g}, \hat{G}) = (g^{[k]}(\sigma), G^{[k]}(\sigma))$. By the single-crossing property of Lemma 1, we can restate condition 2 of Definition 1 as requiring that, for each agent i, $G_i > \hat{G}$ if $\Phi_i(\hat{g}) > \hat{G}$ and $G_i < \hat{G}$ if $\Phi_i(\hat{g}) < \hat{G}$. It follows that a pivotal agent i with $G_i = \hat{G}$ must have $G_i = \Phi_i(\hat{g})$. There are $k-1$ agents with $G_j \leq \hat{G}$; the lower bound of their upper spans of control $I^G(\sigma_{-j})$ is $\hat{G}$ because they can delay but not accelerate the adoption of α. By Lemma 3 we have $\Phi_j(\hat{g}) \leq \hat{G}$ for each of these $k-1$ agents. Similarly, there are $2m-k-1$ agents with $G_j \geq \hat{G}$; the upper bound of their upper spans of control $I^G(\sigma_{-j})$ is $\hat{G}$ because they can accelerate but not delay the adoption of α. By Lemma 3 we have $\Phi_j(\hat{g}) \geq \hat{G}$ for these agents. This argument establishes that agent i's upper best-response function is the k-th smallest upper best-response function when the lower cutoff is at $\hat{g}$. Hence, $\hat{G} = \Phi_i(\hat{g}) = \Phi_{piv}(\hat{g})$. Similar reasoning for the lower best-response shows that $\hat{g} = \phi_{piv}(\hat{G})$. Thus, $(\hat{g}, \hat{G})$ must be a fixed point of (ϕ_{piv}, Φ_{piv}).

Non-degeneracy. We now show that $g^{[k]}(\sigma) < G^{[k]}(\sigma)$. Note that (x, x) is an equal fixed point of (ϕ_{piv}, Φ_{piv}) if $\phi_{piv}(x) = \Phi_{piv}(x) = x$. When $g \geq v_k$, $\Phi_i(g) = g$ for $i \leq k$ and $\Phi_i(g) > g$ for $i > k$. Hence, $\Phi_{piv}(g) = g$ for $g \geq v_k$. In contrast, when $g < v_k$, $\Phi_i(g) > g$ for all $i \geq k$. It follows that $\Phi_{piv}(g) > g$ for $g < v_k$. By similar logic, $\phi_{piv}(G) = G$ when $G \leq v_{2m-k}$ and $\phi_{piv}(G) < G$ when $G > v_{2m-k}$. Because $v_{2m-k} > v_k$ when $k > m$, (ϕ_{piv}, Φ_{piv}) has no equal fixed point when $k > m$.

When $k = m$, the only equal fixed point is (v_m, v_m). Suppose, by way of contradiction, that (v_m, v_m) is an equilibrium outcome. Condition 2 of Definition 1 would require each $i > m$ to choose $G_i > v_m$ and each $j < m$ to choose $g_j < m$. If agent m chooses $G_m > v_m$, then α will not be adopted at v_m. In other words, agent m can delay the adoption of α by choosing $G_m > v_m$. By a similar logic, agent m can delay the adoption of β by choosing $g_m < v_m$. Pick $\varepsilon \in (0, G_m^* - v_m)$ such that agent m can delay the adoption of α by ε and the adoption of β by $v_m - \phi_m(v_m + \varepsilon)$. Because agent m's span of control is non-degenerate, such deviation is feasible if ε is sufficiently small. From Claim 1, $\Phi_m(\phi_m(v_m + \varepsilon)) > v_m + \varepsilon$. Hence, by Lemma 1,

$$u_m\left(\phi_i(v_m+\varepsilon), v_m+\varepsilon \mid v_m\right) > u_m\left(\phi_i(v_m+\varepsilon), v_m \mid v_m\right) = e^{v_m}/(1+e^{v_m}).$$

So (v_m, v_m) cannot be optimal for agent m.

Equilibrium existence. An equilibrium exists if and only if (ϕ_{piv}, Φ_{piv}) has an unequal fixed point. Consider any $k \geq m$. Given that for any $x < v_1$ and all i, $\phi_i(x) = x$ and $\Phi_i(x) > x$, we have $\Phi_{piv}(\phi_{piv}(x)) > x$. Because for all i, $\Phi_i(x) \leq G_i^*$, we have $\Phi_{piv}(\phi_{piv}(x)) < x$ for any $x > \max_i \{G_i^*\}$. By continuity, there exists $x' \in [v_1, \max_i \{G_i^*\}]$, such that $\Phi_{piv}(\phi_{piv}(x')) = x'$. Hence, (ϕ_{piv}, Φ_{piv}) has a fixed point under any rule $k \geq m$.

We have already shown that (ϕ_{piv}, Φ_{piv}) has no equal fixed point under a super-majority rule $k > m$. Hence, any fixed point under rule $k > m$ must be unequal. Finally, consider majority rule $k = m$. Note that

$$\begin{aligned} \Phi_i(g) &= g \quad \text{for } g \geq v_{m-1} \text{ and } i < m; \\ \Phi_i(g) &> g \quad \text{for } g < v_{m+1} \text{ and } i > m. \end{aligned}$$

Because Φ_m is continuous, $\Phi_m(v_m) = v_m$, and $\Phi_m(g) > g$ for $g < v_m$, we have $\Phi_{piv}(g) = \Phi_m(g)$ for g slightly less than v_m. By similar logic, $\phi_{piv}(G) = \phi_m(G)$ for G slightly greater than v_m. Hence, by Claim 1, for G slightly greater than v_m,

$$\Phi_{piv}(\phi_{piv}(G)) = \Phi_m(\phi_m(G)) > G.$$

It follows that under rule $k = m$, $\Phi_{piv}(\phi_{piv}(G')) = G'$ for some $G' \in (G, \max_i \{G_i^*\}]$.

Nested equilibria. For any equilibrium outcome $(\hat{g}, \hat{G})$,

$$\hat{G} - \phi_{piv}(\hat{G}) = \hat{G} - \hat{g} = \Phi_{piv}(\hat{g}) - \hat{g}.$$

Suppose $(\hat{g}', \hat{G}')$ is another equilibrium outcome, and assume without loss of generality that $\hat{G} - \hat{g} \geq \hat{G}' - \hat{g}'$. Because $G - \phi_{piv}(G)$ is strictly increasing in G and $\Phi_{piv}(g) - g$ is strictly decreasing in g (as both Φ_i and ϕ_i have a slope strictly less than 1 for all i), we must have $\hat{G} > \hat{G}'$ and $\hat{g} < \hat{g}'$, which contradicts the initial assumption.

Equilibrium uniqueness. Unanimity. Because any agent can delay a decision under the unanimity rule, $(\hat{g}, \hat{G})$ is an equilibrium only if for all i

$$\hat{g} \leq \phi_i(\hat{G}); \quad \text{and } \hat{G} \geq \Phi_i(\hat{g}).$$

By part 1 of Claim 2, we obtain $\hat{G} - \hat{g} \geq G_i^* - g_i^*$ and, hence, either $\hat{G} \geq G_i^*$ or $\hat{g} \leq g_i^*$. Suppose $\hat{G} \geq G_i^*$. Consider any waiting region (g', G') that strictly contains $(\hat{g}, \hat{G})$. By part 2 of Lemma 2, $G_i^* = \max_g \Phi_i(g)$ and ϕ_i is increasing for $G > G_i^*$. Hence,

$$\begin{aligned} G' > \hat{G} \geq G_i^* \geq \Phi_i(g'); \\ g' < \hat{g} \leq \phi_i(\hat{G}) < \phi_i(G'). \end{aligned}$$

In other words, agent i strictly prefers adopting α before G' and β before g'. By similar logic, the same is true when $\hat{g} \leq g_i^*$. Because given g' every agent i strictly prefers to adopt α before G', (g', G') cannot be an equilibrium. As all equilibria must be nested, $(\hat{g}, \hat{G})$ is unique.

Homogeneous time preference. When all agents have the same discount rate, their one-sided best response functions do not cross. In this case, $\phi_{piv}(\cdot) = \phi_{2m-k}(\cdot)$ and $\Phi_{piv}(\cdot) = \Phi_k(\cdot)$. To prove uniqueness, we need to show that (ϕ_{m-k}, Φ_k) has a unique unequal fixed point. For majority rule $k = m$, it follows from Claim 1 that (ϕ_m, Φ_m) has a unique unequal fixed point at (g_m^*, G_m^*). Consider super-majority rule $k > m$. By Claim 1 and part 3 of Lemma 2, if $G < G_{2m-k}^*$ we have

$$\Phi_k(\phi_{2m-k}(G)) > \Phi_{2m-k}(\phi_{2m-k}(G)) \geq G.$$

Because $\Phi_k(g) \leq G_k^*$ for any g, if $G > G_k^*$ we have

$$\Phi_k(\phi_{2m-k}(G)) < G.$$

Given that $\phi'_{2m-k}(G)$ and $\Phi'_k(\phi_{2m-k}(G))$ are strictly positive and less than 1 for $G \in (G_{2m-k}^*, G_k^*)$, $\Phi_k(\phi_{2m-k}(G)) - G$ is strictly decreasing in that range. Hence, there exists a unique $G' \in [G_{2m-k}^*, G_k^*]$, such that $\Phi_k(\phi_{2m-k}(G')) = G'$.

Homogeneous static preference. When all agents have the same v_i, the pivotal best-response functions are given by the best-response functions of the agent with the k-th most impatient agent. Uniqueness follows Claim 1, which establishes that the best-response functions of any single agent has a unique unequal fixed point at the Wald-optimal thresholds. ∎

Proof of Proposition 2. From Lemma 2, part 1, and from the definition of the pivotal best-response functions,

$$\Phi_{piv}(g;k) \begin{cases} > g & \text{if } g < v_k, \\ = g & \text{if } g \geq v_k; \end{cases} \qquad \phi_{piv}(G;k) \begin{cases} < G & \text{if } G > v_{2m-k}, \\ = G & \text{if } G \leq v_{2m-k}. \end{cases}$$

Because an equilibrium pair of thresholds $(\hat{g}, \hat{G})$ is an unequal fixed point, we must have $\hat{G} = \Phi_{piv}(\hat{g}) > \hat{g}$. This implies that $\hat{g} < v_k$, which means that if β is adopted when the belief hits $\hat{g}$, at least $2m - k$ agents (including agents k to $2m - 1$) must prefer β to α at the current belief. Similarly, the upper threshold must satisfy $\hat{G} > v_{2m-k}$, which implies that at least $2m - k$ agents (including agents 1 to $2m - k$) must prefer α to β when α is adopted. For $k = m$, we must have $v_m \in (\hat{g}, \hat{G})$, which implies that equilibrium outcomes respect the static preferences for a majority of voters.

Fix the discount rates of all agents and fix the static preferences of $k - 1$ agents. We

prove that there exists $\underline{v}$ such that if $v_i \leq \underline{v}$ for $i \leq 2m-k$, then $\hat{G} < v_{2m-k+1}$. By part 2 of Lemma 2, $\Phi_{piv}(g)$ goes to minus infinity as g goes to minus infinity. Therefore, we can define g' such that

$$\Phi_{piv}(g) < v_{2m-k+1}, \quad \text{for } g < g'.$$

By part 3 of Lemma 2, $\phi_i(v_{2m-k+1})$ goes to minus infinity as v_i goes to minus infinity. Therefore, there exists $\underline{v}$ such that, for $i \leq 2m-k$, if $v_i \leq \underline{v}$ then $\phi_i(v_{2m-k+1}) \leq g'$. Under decision rule k, at least one agent from the group $\{1, \ldots, 2m-k\}$ must vote for β in order for β to be adopted by the group. Therefore, $\phi_{piv}(v_{2m-k+1}) \leq g'$. This, together with the displayed inequality above, implies that

$$\Phi_{piv}\left(\phi_{piv}(v_{2m-k+1})\right) < v_{2m-k+1}.$$

Recall from the proof of equilibrium existence in Proposition 1 that for any $x < v_1$, $\Phi_{piv}(\phi_{piv}(x)) - x$ is strictly positive. Therefore, there exists $\hat{G} \in (x, v_{2m-k+1})$ that satisfies $\Phi_{piv}(\phi_{piv}(\hat{G})) - \hat{G} = 0$. Because $\hat{G} < v_{2m-k+1}$, we have $\hat{G} < v_i$ for all $i \geq 2m-k$. All these $k-1$ agents prefer β to α when α is adopted. ■

Proof of Proposition 3. For $k = m$, the equilibrium thresholds is the unequal fixed point of (ϕ_m, Φ_m). Thus, $\hat{g} = g^*_m$ and $\hat{G} = G^*_m$. Part 2 of Lemma 2 shows that $\Phi_k(g) \leq G^*_k$ for any $g < v_k$, with equality if and only if $g = g^*_k$; and $\phi_{2m-k}(G) \geq g^*_{2m-k}$ for any $G > v_{2m-k}$, with equality if and only if $G = G^*_{2m-k}$. For $k > m$, (g^*_{2m-k}, G^*_k) is not a fixed point of (ϕ_{2m-k}, Φ_k), because $\Phi_k(g^*_{2m-k}) < \Phi_k(g^*_k) = G^*_k$. Thus, any unequal fixed point $(\hat{g}, \hat{G})$ of (ϕ_{2m-k}, Φ_k) must satisfy $\hat{g} < g^*_{2m-k}$ and $\hat{G} > G^*_k$. ■

Proof of Proposition 4. *General environment.* We show that the width of the waiting region in the most and least patient equilibria increases whenever Φ_{piv} shifts up or ϕ_{piv} shifts down.

Let $(\hat{g}, \hat{G})$ be the fixed point of (ϕ_{piv}, Φ_{piv}) in the most patient equilibrium. Let (g', G') be the fixed point of $(\phi_{piv}, \tilde{\Phi}_{piv})$ in the most patient equilibrium, where $\tilde{\Phi}_{piv}(g) \geq \Phi_{piv}(g)$ for all g. We have

$$\tilde{\Phi}_{piv}(\phi_{piv}(\hat{G})) - \hat{G} \geq \Phi_{piv}(\phi_{piv}(\hat{G})) - \hat{G} = 0.$$

In the proof of equilibrium existence in Proposition 1, we have shown that $\tilde{\Phi}_{piv}(\phi_{piv}(G)) - G < 0$ for G that is sufficiently large. Because multiple equilibrium thresholds are nested, G' is the largest solution to $\tilde{\Phi}_{piv}(\phi_{piv}(G)) - G = 0$. It follows that $G' \geq \hat{G}$. Furthermore, $G - \phi_{piv}(G)$ is increasing in G because the slope of $\phi_i(G)$ is less than 1 for every i. Therefore,

$$G' - g' = G' - \phi_{piv}(G') \geq \hat{G} - \phi_{piv}(\hat{G}) = \hat{G} - \hat{g}.$$

Similar reasoning applies to the least patient equilibrium and to the case when ϕ_{piv} shifts down.

Because $\Phi_{piv}(g;k') \geq \Phi_{piv}(g;k)$ and $\phi_{piv}(G;k') \leq \phi_{piv}(G;k)$ for $k' > k$, the width of the equilibrium waiting region is increasing in k both in the most patient equilibrium and in the least patient equilibrium. Similarly, a decrease in r_i shifts up Φ_i and shifts down ϕ_i. As a result, Φ_{piv} also shifts up and ϕ_{piv} shifts down (weakly). The waiting region in the most patient and the least patient equilibria becomes wider.

Symmetric committee. An increase in k, a decrease in the common discount rate r, or more diverse preferences in a symmetric jury shifts up Φ_{piv} and shifts down ϕ_{piv}. Thus, $\hat{G} - \hat{g}$ increases. From the proof of Lemma 1, the equilibrium cutoffs must satisfy these first-order conditions:

$$\begin{aligned} v_k - \hat{g} - f(r, \hat{G} - \hat{g}) &= 0, \\ f(r, \hat{G} - \hat{g}) - \hat{G} + v_{2m-k} &= 0. \end{aligned}$$

Adding these two equations implies that $\hat{g} + \hat{G} = v_k + v_{2m-k} = 2v_m$, which is fixed. Thus, $\hat{G}$ must increase and $\hat{g}$ must fall. ■

Proof of Proposition 5. For $\theta_0 \geq \hat{G}$ or $\theta_0 < \hat{g}$, majority rule and super-majority rule yield the same outcome. There are four other cases to consider.

Case 1: $\theta_0 \in [G^*_m, \hat{G})$. For belief in this region, super-majority rule will lead to some waiting while majority rule leads to immediate adoption of α. By Lemma 2, for $j \leq m$.

$$\Phi_j(\hat{g}) \leq \Phi_m(\hat{g}) < G^*_m.$$

By the single-crossing property, $u_j(\hat{g}, G \mid \theta_0)$ is decreasing in G if $G > \Phi_j(\hat{g})$. Therefore, for $\theta_0 \in [G^*_m, \hat{G})$,

$$u_j(\hat{g}, \hat{G} \mid \theta_0) < u_j(\hat{g}, \theta_0 \mid \theta_0) = e^{\theta_0}/(1 + e^{\theta_0}).$$

Therefore agents 1 to m strictly prefer majority rule to super-majority rule.

Case 2: $\theta_0 \in [v_m, G^*_m]$. Instead of writing the utility of agent j as a function of the cutoffs g and G, we can define $c = (g + G)/2$ and $y = G - g$ to express utility as a function of the center and the width of the waiting region. Using the formula in the text, and after some re-arrangement, we obtain:

$$u_j(g, G \mid \theta_0) = \frac{1}{1 + e^{\theta_0}} \frac{\tilde{q}_j(c, y; \theta_0)}{q(y)};$$

where

$$\begin{aligned} \tilde{q}_j(c, y; \theta_0) &= \frac{Q_1 e^{-R_1 y/2} - Q_2 e^{-R_2 y/2}}{e^{-R_1 y/2} - e^{-R_2 y/2}}, \\ q(y) &= e^{-R_1 y/2} + e^{-R_2 y/2}, \end{aligned}$$

with $Q_1 = e^{v_j}e^{R_1(\theta_0-c)} + e^c e^{R_2(\theta_0-c)}$ and $Q_2 = e^c e^{R_1(\theta_0-c)} + e^{v_j}e^{R_2(\theta_0-c)}$. In the proof of Proposition 4 for a symmetric committee, we show that $c = v_m$ under both majority rule and super-majority rule. Moreover, the width of the equilibrium waiting region under majority rule, denoted y^*, is smaller than that under super-majority rule, denoted $\hat{y}$. At $\theta_0 = v_m$, the utility of agent j is

$$u_j(g, G \mid v_m) = \frac{1}{1+e^{v_m}} \frac{e^{v_j} + e^{v_m}}{q(y)}.$$

The function $q(y)$ is decreasing for $y < y^*$ and increasing for $y > y^*$. Since $\hat{y} > y^*$, we have $u_j(g^*, G^* \mid v_m) > u_j(\hat{g}, \hat{G} \mid v_m)$ for all j. Moreover, $\tilde{q}_j$ decreases in y for $\theta_0 > c$ and $v_j < c$. Therefore, $\hat{y} > y^*$ implies $u_j(g^*, G^* \mid \theta_0) > u_j(\hat{g}, \hat{G} \mid \theta_0)$ for $j = 1, \ldots, m$.

Case 3: $\theta_0 \in [g_m^*, v_m)$. Recall that $u_j(g^*, G^* \mid v_m) > u_j(\hat{g}, \hat{G} \mid v_m)$ for all j. Since $\tilde{q}_j$ decreases in y for $\theta_0 < c = v_m$ and $v_j > c = v_m$, and since $q(y)$ is decreasing for $y > y^*$, $\hat{y} > y^*$ implies $u_j(g^*, G^* \mid \theta_0) > u_j(\hat{g}, \hat{G} \mid \theta_0)$ for $j = m, \ldots, 2m-1$.

Case 4: $\theta_0 \in [\hat{g}, g_m^*)$. Super-majority rule will lead to some waiting while majority rule leads to immediate adoption of β. By Lemma 2, for $j \geq m$, $\phi_j(\hat{G}) \geq \phi_m(\hat{G}) > g_m^*$. Since $u_j(g, \hat{G} \mid \theta_0)$ is increasing in g if $g < \phi_j(\hat{G})$, for $\theta_0 \in [\hat{g}, g_m^*]$ we have $u_j(\hat{g}, \hat{G} \mid \theta_0) < e^{v_j}/(1+e^{\theta_0})$. Therefore agents m to $2m-1$ strictly support majority rule.

Note that case 2 shows that all agents strictly prefer majority rule to super-majority rule when $\theta_0 = v_m$. Since preferences are continuous in θ_0, the second part of the proposition follows. ∎

Proof of Proposition 6. Let i be an impatient agent. Assume that $i \leq m$. The case in which $i \geq m$ can be treated symmetrically. Pick any $\varepsilon \leq (0, v_{m+1} - v_m)$. By part 3 of Lemma 2 there exists $\overline{r}(\varepsilon)$ such that for any $r_i \geq \overline{r}(\varepsilon)$ and $G \leq v_{m+1}$, we have $\phi_i(G) > G - \varepsilon$.

For all agents $j \geq m+1$, $\phi_j(G) = G$ if $G < v_{m+1}$. Thus, $\phi_{piv}(G) \geq \phi_i(G)$ if $G < v_{m+1}$. Given that $v_m + \varepsilon < v_{m+1}$, we have

$$\phi_{piv}(v_m + \varepsilon) \geq \phi_i(v_m + \varepsilon) > v_m + \varepsilon - \varepsilon = v_m.$$

For all agents $j \leq m$, $\Phi_j(g) = g$ if $g > v_m$. Thus, $\Phi_{piv}(g) = g$ if $g > v_m$. This, together with the displayed inequality above, implies that

$$\Phi_{piv}(\phi_{piv}(v_m + \varepsilon)) = \phi_{piv}(v_m + \varepsilon) < v_m + \varepsilon,$$

where the last inequality follows because $\phi_{piv}(G) < G$ for any $G > v_m$.

We have already shown in the proof of equilibrium existence in Proposition 1 that $\Phi_{piv}(\phi_{piv}(G')) > G'$ for G' slightly above v_m. Therefore, there is an upper threshold $\hat{G} \in (G', v_m + \varepsilon)$ that satisfies the equilibrium condition. Furthermore, because $\hat{G} < v_{m+1}$,

$\phi_{piv}(\hat{G}) \geq \phi_i(\hat{G})$. Therefore, $\hat{G} - \phi_{piv}(\hat{G}) \leq \hat{G} - \phi_i(\hat{G}) < \varepsilon$. The equilibrium waiting region is no wider than ε. Finally, the fact that $v_m \in (\hat{g}, \hat{G})$ is established in Proposition 2. ■

Proof of Proposition 7. Because (g_i^*, G_i^*) is the unique unequal fixed point of (ϕ_i, Φ_i), by Proposition 4, $G_i^* - g_i^*$ decreases in r_i. Therefore, there exists $\tilde{r}(\varepsilon)$ such that $G_i^* - g_i^* \geq \varepsilon$ for any agent i with $r_i \leq \tilde{r}(\varepsilon)$.

Suppose the equilibrium waiting region is $(\hat{g}, \hat{G})$ and the number of agents with $r_i \geq \tilde{r}(\varepsilon)$ is strictly less than the number of requisite swing votes under rule k. Then, there must be at least one agent with $r_i < \tilde{r}(\varepsilon)$ who both votes for α at $\hat{G}$ and votes for β at $\hat{g}$. Suppose by way of contradiction that $\hat{G} - \hat{g} < \varepsilon$. Then, for an agent with $r_i < \tilde{r}(\varepsilon)$, we have $\hat{G} - \hat{g} < \varepsilon \leq G_i^* - g_i^*$. But for such $(\hat{g}, \hat{G})$, by part 1 of Claim 2, either $\partial u_i / \partial g < 0$ or $\partial u_i / \partial G > 0$. This agent cannot both vote for α at $\hat{G}$ and vote for β at $\hat{g}$, a contradiction. ■

Proof of Proposition 8. Let $\mathcal{C}$ denote a subset of k agents. The upper and lower pivotal functions for the group $\mathcal{C}$ under the unanimity rule are $\Phi_*(g) \equiv \max_{i \in \mathcal{C}} \Phi_i(g)$ and $\phi_*(G) \equiv \min_{i \in \mathcal{C}} \phi_i(G)$, respectively. Let (g_*, G_*) denote the unique unequal fixed point of (ϕ_*, Φ_*).

In the original deliberation game with $2m - 1$ agents, $\Phi_{piv}(g) \leq \Phi_*(g)$ for all g and $\phi_{piv}(G) \geq \phi_*(G)$ for all G under rule k, regardless of the preferences of the agents outside $\mathcal{C}$. From the proof of Proposition 4, the width of the waiting region in the most patient equilibrium increases whenever Φ_{piv} shifts up or ϕ_{piv} shifts down. Because (g_*, G_*) is the unique, and hence most patient, fixed point of (ϕ_*, Φ_*), we have

$$\hat{G} - \hat{g} \leq G_* - g_*,$$

for any fixed point $(\hat{g}, \hat{G})$ of (ϕ_{piv}, Φ_{piv}). ■

Proof of Proposition 9. We prove the proposition for the case where the lower cutoff is constrained. The proof of the other case is similar.

If $\phi_{piv}(G) \geq v_m$, then $\phi_{piv}^c(G) = \min\{\phi_{piv}(G), v_m\} = v_m$. Since $\Phi_{piv}(v_m) > v_m$, we have $\Phi_{piv}^c(v_m) = \Phi_{piv}(v_m)$. This proves that $(v_m, \Phi_{piv}(v_m))$ is an unequal fixed point of $(\phi_{piv}^c, \Phi_{piv}^c)$ if $\phi_{piv}(\Phi_{piv}(v_m)) \geq v_m$ and $\Phi_{piv}(v_m) > v_m$.

Conversely, $(v_m, \Phi_{piv}(v_m))$ cannot be an unequal fixed point of $(\phi_{piv}^c, \Phi_{piv}^c)$ if either $\phi_{piv}(\Phi_{piv}(v_m)) < v_m$ or $\Phi_{piv}(v_m) \leq v_m$. If $\phi_{piv}(\Phi_{piv}(v_m)) < v_m$, then $\phi_{piv}^c(\Phi_{piv}(v_m)) = \phi_{piv}(\Phi_{piv}(v_m)) < v_m$. Recall that $\Phi_{piv}(v_m) \geq v_m$ for all $k_D \geq m$. Hence, $\Phi_{piv}(v_m) = v_m$ if $\Phi_{piv}(v_m) \leq v_m$. But then (v_m, v_m) is not unequal.

Finally, $\Phi_{piv}(v_m) > v_m$ holds only when $k_D > m$. For any rule $k_D > m$,

$$\phi_{piv}(\Phi_{piv}(v_{k_D})) = \phi_{piv}(v_{k_D}) < v_{k_D}.$$

It follows that if $(v_m, \Phi_{piv}(v_m))$ is an unequal fixed point of $(\phi^c_{piv}, \Phi^c_{piv})$, then

$$\phi_{piv}(\Phi_{piv}(v_m)) \geq \phi^c_{piv}(\Phi_{piv}(v_m)) = v_m.$$

Hence, (ϕ_{piv}, Φ_{piv}) must have an unequal fixed point $(\hat{g}, \hat{G})$ with $\hat{g} \in [v_m, v_{k_D})$. At this equilibrium outcome, at most m agents prefers β at $\hat{g}$. Furthermore, since $\Phi^c_{piv}(g) \geq \Phi_{piv}(g)$ and $\phi^c_{piv}(G) \leq \phi_{piv}(G)$, the same reasoning as in the proof of Proposition 4 establishes that the waiting region is wider under the constrained two-stage decision process. ■

Proof of Proposition 10. Suppose $(\hat{g}, \hat{G})$ is an equilibrium in the one-stage deliberation game with rule $k = k_D$. Suppose agent j is pivotal for the upper cutoff in this equilibrium. For any fixed g, the best response of agent j diverges to infinity as r_j goes to zero. Because $\lim_{r_j \to 0} R_1 = 0$ and $\lim_{r_j \to 0} R_2 = 1$, we have

$$\lim_{r_j \to 0} f(r_j, G - g) = \lim_{r_j \to 0} \log \frac{R_2 e^{R_1(G-g)} - R_1 e^{R_2(G-g)}}{R_2 - R_1} = 0.$$

The sign of $\partial u_j / \partial G$ depends only on the sign of $v_j - g - f(r_j, G - g)$. This implies that, for any $g < v_j$, $\lim_{r_j \to 0} \partial u_j / \partial G > 0$ for any G. Thus G must increase without bound as r_j goes to 0. A similar argument establishes that g must decrease without bound as the pivotal agent for the lower cutoff has a discount rate that goes to 0. Finally, since $f(r, G - g)$ is monotone increasing in r (proof of Lemma 2, part 3), there exists $\underline{r}$ such that $\hat{G} > v_{2m-1}$ and $\hat{g} < v_1$ whenever $r_i \leq \underline{r}$ for all agents i.

Next, we show that such $(\hat{g}, \hat{G})$ is an equilibrium outcome of the two-stage decision process with any decision rule k_d. We first note that it is an equilibrium for each agent i to vote for α if $\theta \geq v_i$ and to vote for β otherwise, because α is preferred to flipping a coin (which is in turn preferred to β) whenever $\theta \geq v_i$, while β is preferred to flipping a coin (which is in turn preferred to α) whenever $\theta < v_i$. Given such equilibrium second-stage strategy, and given the fact that $\hat{G} > v_{2m-1}$, α will be adopted with unanimous support for any k_d if the belief reaches the upper cutoff. Likewise, β will be adopted with unanimous support for any k_d if the belief reaches the lower cutoff.

In the first stage, suppose each agent i adopts the strategy $(g_i, G_i) = (\phi_i(\hat{G}), \Phi_i(\hat{g}))$. As r_i goes to 0, we have $G_i > v_{2m-1}$ for every i. Given this first-stage strategy, the span of control $I^G(\sigma_{-i})$ for any agent i over the upper cutoff is above v_{2m-1}. Similarly, the span of control for any agent over the lower cutoff is below v_1. For any decision rule k_d which is non-unanimous, it is not feasible for any agent to unilaterally deviate to obtain an indecisive outcome, given the strategy profile of other agents. Furthermore, it is not profitable to unilaterally deviate to delay or hasten the adoption of either alternative, because $(\hat{g}, \hat{G})$ solves the constrained maximization problem 1 for the one-stage problem. If the decision rule k_d is unanimous, it is feasible for an agent to deviate by changing his thresholds in the first stage and withholding his support for an alternative in the second stage to ob-

tain an indecisive outcome. But doing so is worse than just changing the thresholds in the first stage and voting sincerely in the second stage, because an indecisive outcome is worse than α for any $\theta \in I^G(\sigma_{-i})$, and is worse than β for any $\theta \in I^g(\sigma_{-i})$. Since the latter deviation is unprofitable, deviation to force flipping a coin cannot be profitable for any belief within an agent's span of control. ■

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