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THE WELFARE STATE AND MIGRATION: A DYNAMIC ANALYSIS OF POLITICAL COALITIONS[†]

Abstract

We develop a dynamic political-economic theory of welfare state and immigration policies, featuring three distinct voting groups: skilled workers, unskilled workers, and old retirees. The essence of inter - and intra-generational redistribution of a typical welfare system is captured with a proportional tax on labor income to finance a transfer in a balanced-budget manner. We provide an analytical characterization of political-economic equilibrium policy rules consisting of the tax rate, the skill composition of migrants, and the total number of migrants. When none of these groups enjoy a majority (50 percent of the voters or more), political coalitions will form. With overlapping generations and policy-determined influx of immigrants, the formation of the political coalitions changes over time. These future changes are taken into account when policies are shaped. Naturally, a lower rate of population growth (that is, an aging population) increases the political clout of the old (the left group). But it also increases the burden on the young (particularly, the skilled).

JEL Classification: E10, F15 and H10
Keywords: and right coalitions, center, dynamics of left and intra- and inter-generational transfers

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Keywords: Generosity of Welfare State; Pay as you go; Political Coalitions; Strategic Voting; and Skill Composition of Migration; Markov-perfect Political Equilibrium

1 Introduction

Milton Friedman reminded us that one cannot have free immigration and a generous welfare state at the same time. Indeed, public opinion in the developed economies with a fairly generous welfare system, favors putting in some way or another restrictions on immigration (see, for example, Hanson, Scheve, and Slaughter (2007, 2009)). A skilled and young immigrant may help the finances of the welfare state; whereas an unskilled and old migrant may inflict a burden on the welfare state. A welfare state with a heterogeneous population, by both age and skills, does not evidently have a commonly accepted attitude towards migration. Hence, this paper develops a framework to study how these inter- and intra-generational conflicts, among different age and skill groups, is resolved in a politico-economic setup. Of a particular interest is how an economy with more than just two groups of voters resolve the conflicts between more demand for welfare state redistribution and and the skill composition of immigration policy. The basic trade-off presented in the paper is as follows: skilled immigrants are helpful in contributing to paying for the welfare state, but their influx could upset the future coalition between unskilled and old individuals that supports redistribution.

To emphasize this trade-offs, our model features two skill levels for labor, skilled and unskilled. People live for two periods in an overlapping-generations manner, which provides another dimension of heterogeneity, between young and old individuals, in addition to the skill levels. The voting on the current migration policies and the generosity of the welfare state takes place in each period. We model a typical welfare state that is characterized by both *inter*-generational redistribution and *intra*-generational redistribution[1] Accordingly, our overlapping generations model is further based on key demographic characteristics: that immigrants are younger and have higher birth rates than the native born population.

Our paper is directly related to two fields of the existing political economy literature: the political economy of the pay-as-you-go (PAYG) social security

[1]Many features of the welfare state, such as national health insurance, involve both inter- and intra-generational redistributions, while some features of the welfare state are also either *inter*- or *intra*-generational, such as old-age social security and income maintenance program.

systems (Cooley and Soares (1999), Boldrin and Rustichini (2000), Galasso (1999), Bohn (2005), Bassetto (2008)) and the political economy of migration (such as Benhabib (1996) and Ortega (2005)). The view that increased migration may come to the rescue of PAYG social security systems reflects the fact that the flow of migrants can alleviate the current demographic imbalance as well, by influencing the age structure of the host economy. A few empirical studies address this point by calibrating the equilibrium impact of a less restrictive policy towards migration according to U.S. data. Storesletten (2000) found in a general equilibrium model that selective migration policies, involving increased inflow of working-age high and medium-skilled migrants, can remove the need for a future fiscal reform. Auerbach and Oreopoulos (1999) performed a similar exercise using *partial* equilibrium generational accounting and arrived at a similar conclusion. By emphasizing the demographic side and abstracting from the migrants' factor prices effects, Lee and Miller (2000) concluded in a similar analysis that a higher number of immigrants admitted into the economy can ease temporarily the projected fiscal burden of retiring babyboomers. There are also a another line of literature which deals with the effect of migrants on the PAYG social security system, such as Razin and Sadka (1999) and Scholten and Thum (1996). However, this literature abstracted from political-economic considerations.

Moreover, there have been previous works on the political economy of immigration and redistribution policies, albeit focussed solely on either inter-generational or intra-generational alone. Razin, Sadka, and Swagel (2002b), and Casarico and Devillanova (2003) focussed on the impact of immigration the political economy of inter-generational redistribution. Sand and Razin (2007) took an additional step to provide a synthesis on the political economy model jointly determining the inter-generational redistribution and immigration. Dolmas and Huffman (2004) analyzed similarly the joint determination of intra-generational redistribution and immigration policy in a dynamic political economy model. None of the works in the literature capture both *inter*- and *intra*-generational redistribution in their studies. Once we incorporate this central feature into the model, we find that the formation of the political coalitions between groups of voters changes over time, which in turns influence how future policies are shaped.

Following a line literature of dynamic political-economic models, we employ the Markov-perfect equilibrium concept, as in Krusell and Rios-Rull (1996), and Hassler et al. (2003) and Bassetto (2008). Similarly to Hassler (2003) and in contrast to Krusell and Rios-Rull (1996) and Bassetto (2008), our model

yields analytical closed-form solutions in political-economic equilibrium, making it highly attractive for further quantitative and qualitative analyses. The forward-looking equilibrium concept means that each young voter takes into account the effect of her vote on the evolution of the economy into the next period; which, in turn, affects the voting outcome in the next period, particularly with respect to the social security benefit that she receives in the next period, when she grows old and retires. Next period voting, in turn, is influenced by the outcome of the voting outcome in the following period, and so on. Since a welfare state will necessarily affect more than two groups of voters, of particular interest is the characterization of possible coalitions which emerge as decisive in the political-economic equilibria, for different demographic and skill-distribution parameters. In this regard, we depart from the typical literature markedly whose focus was mainly on the conflict between only two groups: either young versus old voters (for example, Boldrin and Rustichini (2000), Sand and Razin (2007)), or skilled versus unskilled voters (see for example Hassler et al. (2003), Dolmas and Huffman (2004), and Armenter and Ortega (2011)).

The paper is organized as follows. Section 2 presents the analytical framework. Section 3 characterizes the political process and defines the concept of equilibrium. Section 4 provides a preliminary analysis of the political equilibrium. Section 5 provides a full analysis of the political-economic equilibrium. Lastly, section 6 concludes.

2 The Model

Consider a standard two-period, overlapping-generations model. Each cohort works in the first-period in his life and retires in the second. There are two skill types: skilled and unskilled. The welfare-state[2] is modeled simply by a proportional tax on labor income to finance a lump-sum benefit (demogrant) period-by-period in a balanced-budget manner.[3] With this setting, the simple welfare-state system is quite redistributive.

[2]We draw on Sand and Razin (2007) and Suwankiri (2009).

[3]The lump-sum benefit is perfectly substitutable to private consumption. One can also extend the analysis in a straightforward manner to have public services (e.g. education, health) which are not perfectly substitutable to private consumption.

2.1 Preferences and Technology

The utility of each individual in period t, for young and old, is given, respectively, by

$$U^y(c_t^{y,i}, l_t^i, c_{t+1}^o) = c_t^{y,i} - \frac{\varepsilon (l_t^i)^{\frac{1+\varepsilon}{\varepsilon}}}{1+\varepsilon} + \beta c_{t+1}^o,\ i = s, u \tag{1}$$

$$U^o(c_t^o) = c_t^o. \tag{2}$$

where, s and u denote skilled and unskilled labor. Superscripts y and o stand for "young" and "old", l^i is labor of type $i = s, u$, ε is the elasticity of the labor supply, and $\beta \in (0, 1)$ is the discount factor. Note that c_t^o is the consumption of an old individual at period t (who was born in period $t-1$). Young and old agents in the economy maximize the above utility functions subject to their respective budget constraints as follows:

$$\begin{aligned} c_t^{y,i} + d_t^i &\leq (1-\tau_t) w_t^i l_t^i + b_t \\ c_t^o &\leq (1+r_t) d_{t-1}^i + b_t \end{aligned}$$

where d_t^i, τ_t, w_t^i, and b_t denote, respectively, saving of the young workers of type i, the linear income tax rate, wage for labors of type i, and the lump-sum transfer in the period t.

Given the linearity of U in c_t and c_{t+1}, the only interior equilibrium interest rate r equals to $\frac{1}{\beta} - 1$ and individuals have no incentive to either save or dissave. For simplicity, we set saving at zero.[4] This essentially reduces the two groups of old retirees (skilled and unskilled), potentially distinguishable by their levels of saving, to just one group with identical preference irrespective of their skill type. Individual's labor supplies of the young are given by

$$l_t^i = \left(w_t^i (1-\tau_t)\right)^{\varepsilon},\ i = s, u. \tag{3}$$

There is just one good, which is produced by using the two types of labor as perfect substitute with constant marginal products.[5] The production function is given by

$$Y_t = w^s L_t^s + w^u L_t^u \tag{4}$$

[4] In fact, any saving level is an optimal choice. Assuming no saving is for simplicity. With saving, since old individuals do not work in the last period of their life, they will consume their savings plus any transfer. Through both these channels, the old individuals benefit from migration. See Forni (2005), Sand and Razin (2007), Bassetto (2008) for models with saving and capital accumulation.

[5] This simplification allows us to focus solely on the linkages between the welfare state and migration, leaving aside any effect of migration on wages. For an analysis to the other extreme, in which there are only wage effect, see Ortega (2005).

where L_t^i is the aggregate labor supply of skill $i = s, u$. Labor markets are competitive, ensuring the wages going to the skilled and unskilled workers are indeed equal to their marginal products, w^s and w^u, respectively. We naturally assume that $w^s > w^u$.

Because the old retirees have no labor income and saving, their only source of income comes from the demogrant. Putting all the pieces together, the model yields the following indirect utility functions:

$$V^{y,i} = \frac{\left((1-\tau_t)w^i\right)^{1+\varepsilon}}{1+\varepsilon} + b_t + \beta b_{t+1}$$
$$V^o = b_t,$$

for $i = s, u$. For brevity, we will use V^i to denote $V^{y,i}$ because only the young workers need to be distinguished by their skill level.

In addition to the parameters of the welfare state (τ_t and, consequently, b_t), the political process also determines migration policy which consists of the volume of migration, and the other determines its skill composition. We denote by μ_t the ratio of allowed immigrants to the native-born young population and denote by σ_t the fraction of skilled migrants in the the total number of migrant entering the country in period t. Migrants are assumed to have identical preferences to the native-born. Furthermore, we assume all migrants come young and they are naturalized one period after their entrance. Hence, they gain voting rights when they are old.

2.2 Demographic Equations and Dynamics

Let s_t denote the fraction of native-born skilled workers in the labor force in period t (where $s_0 > 0$). The aggregate labor supply in the economy of each type of labor is given by

$$L_t^s = [s_t + \sigma_t \mu_t] N_t l_t^s \tag{5}$$

and

$$L_t^u = [1 - s_t + (1-\sigma_t)\mu_t] N_t l_t^u, \tag{6}$$

where N_t is the number of native-born young individuals in period t.

The dynamics of the economy is given by two equations: one governs the *aggregate* population, while the other governs the *skill* composition. We assume here that, for both native-born population and migrants, their offsprings

replicate exactly the skill level of their parents.[6] That is,

$$\begin{aligned} N_{t+1} &= [1 + n + (1 + m)\mu_t] N_t \\ s_{t+1}N_{t+1} &= [(1 + n)s_t + (1 + m)\sigma_t\mu_t] N_t, \end{aligned} \tag{7}$$

where n and m are the population growth rates of the native-born population and of the migrants, respectively. We assume that $n < m \leq 1$. The plausible assumption of differential birth rates provides a room for a sharper analysis on possible future demographic shifts from admitting too many migrants today. We also allow the population growth rates to be negative, to plausibly reflect the current situations faced by many European and Asian countries. Combining the two equations above together, we get the dynamics of the labor supply of skilled native-born as follows:

$$s_{t+1} = \frac{(1 + n)s_t + (1 + m)\sigma_t\mu_t}{1 + n + (1 + m)\mu_t}. \tag{8}$$

Equation (8) implies that the fraction of the native-born skilled in the native-born labor force will be higher in period $t+1$ than in period t if the proportion of skilled migrants in period t is higher than that of the native-born, that is, if $\sigma_t > s_t$. Naturally, when there is no migration the share of skilled workers out of (native-born) young population does not change over time. The evolution of the model thus reduces to a single equation, equation (8), with a single state variable s_t.

2.3 The Welfare State

Being balanced period-by-period, the welfare-state system is a pay-as-you-go system. Therefore, the equation for the demogrant, b_t, is given by

$$b_t = \frac{\tau_t \left((s_t + \sigma_t\mu_t)w^s N_t l_t^s + (1 - s_t + (1 - \sigma_t)\mu_t)\, w^u N_t l_t^u\right)}{(1 + \mu_t)\, N_t + (1 + \mu_{t-1})\, N_{t-1}}, \tag{9}$$

which upon some re-arrangements reduces to

$$b_t = \frac{\tau_t \left((s_t + \sigma_t\mu_t)w^s l_t^s + (1 - s_t + (1 - \sigma_t)\mu_t)\, w^u l_t^u\right)}{1 + \mu_t + \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}}. \tag{10}$$

It is straightforward to see that a larger σ_t increases the demogrant (recall that $w^s l_t^s > w^u l_t^u$). That is, a higher skill composition of migrants brings about

[6] Razin, Sadka, and Swagel (2002b), and Casarico and Devillanova (2003) provide a synthesis with endogenous skill analysis. The first work focussed on the shift in skill distribution of current population, while the latter studied skill-upgrading of future population.

higher tax revenues, and, consequently, enables a more generous welfare state, other things being equal. Similarly, we can conclude that a higher volume of migration enables a more generous welfare system if the share of the skilled among the migrants exceeds the share of the skilled among the native-born workers ($\sigma_t > s_t$).

3 Political Process and Equilibrium

We allow voters to vote strategically. In a simple plurality electoral system, Duverger's Law postulates that the election will tend to be dominated by two main political parties (Duverger, 1954). To Duverger, this is due to "*psychological effect*" of voters who know that only the winning candidate gets to influence policies, so voters are careful to not waste their votes on candidates unlikely to win election. Equipped with this insight, we adopt the following definition of strategic voting from Fisher (2005). Voters are voting strategically if they vote "... for a party they believe is more likely to win than their preferred party, to best influence who wins in the constituency" (Fisher, 2005).[7] To allow for strategic voting in the model, we will borrow heavily from the political process with citizen candidates by Besley and Coate [1997].[8] However, we assume no endogenous candidacy decision by the citizens, both for simplicity in reducing the number of equilibria and to focus more on our question at hands on policy selection.

3.1 The Political Process

The political process in *each period* has three stages. At stage 1, three candidates, one from each respective group, are randomly chosen. We assume that the size of the population is large enough such that the probability of the randomized selection is infinitesimal, hence not taken into the account by each citizen when voting or implementing the policies. At stage 2, the citizens cast their votes for one of the three candidates. In the final stage, the candidate that wins by plurality choose and implement the policy. We will now analyze these stages in reverse order.

[7] The literature does not seem to distinguish between strategic voting, as referred to by political studies in the U.S., and tactical voting, as referred to by political studies in the U.K. Also, we trade for tractability against new advancements in the theory of strategic voting in the literature, which attempt to model strategic voting situation as a global game (Myatt and Fisher, 2002; Myatt, 2007).

[8] For citizen-candidate model with sincere voting, see Osborne and Slivinski (1996).

Policies of Candidates. In the final stage, the winning candidate will always implement his preferred policies. Due to the lack of commitment mechanism such as reputation loss or reelection motive, any promise to deviate from this ideal point will be viewed as cheap talk in equilibrium. For period t, the winning candidate preferred policies are given by a tiplet $(\tau_t^*, \sigma_t^*, \mu_t^*) = \Phi_t$ denoting the three policy choices, namely, the tax rate, the share of skilled immigrant to total immigrants, and the ratio of allowed immigrants to the native-born young population respectively, such that

$$\begin{aligned} \Phi_t^d = \underset{\tau_t, \sigma_t, \mu_t}{\arg\max} \quad & V^d\left(s_t, , \tau_t, \sigma_t, \mu_t, \Phi_{t+1}\right) \\ \text{s.t. } s_{t+1} = & \frac{(1+n)s_t + (1+m)\sigma_t \mu_t}{1+n+\mu_t(1+m)}, \end{aligned} \tag{11}$$

where $d \in \{s, u, o\}$ is the identity of the the winning candidate. Note that the winning candidate must taken into the account how his policy choices will affect the population dynamics and policy choices of the future that will affect the candidate himself in $t+1$ (through Φ_{t+1}). Once policies are implemented, the winning candidate resumes his normal citizenship.

Voting Decisions. In the second stage, three candidates are randomly chosen from each group of voters.[9] Note that there are also only three distinct voting groups: the skilled native-born workers, the unskilled native-born workers, and the old retirees. The set of three political candidates in the election is $\{p^s, p^u, p^o\}$, denoting respectively by their identity. Within each group, the preference of the voters are identical by structure, and hence voters in each group will have identical voting preference. Because identical voters vote identically, we thus focus on the decision of a *representative voter* from each group. We let e_t^i be the vote decision by an individual of type $i \in \{s, u, o\}$ in period t. We allow for a mixed strategy voting decision where the representative voter places probability over the voter for each of the candidate, namely the skilled, unskilled, and old candidate respectively, such that sum of the probabilities placed on the three candidates equals 1. With an abuse of notation, we let $e_t^i = \left(p_t^{s,i}, p_t^{u,i}, p_t^{o,i}\right) \in \triangle^2$ denote the vector of probabilistic votes of voters from group $i \in \{s, u, o\}$ such that $\sum_{k \in \{s,u,o\}} p_t^{k,i} = 1$.[10]

In line with strategic voting, while making the voting decision, the voters will take into the account the probability of winning of the candidate. Given the

[9] Population size is large enough such that no candidate would take into the account the probability of a re-election.

[10] As for an illustration, a pure voting strategy of unskilled voter for its own candidate in period t would be denoted as $e_t^u = (0, 1, 0)$.

assumption that *identical voter vote identically*, the probability of winning of each candidate will be equal to the weighted sum of the probabilistic votes each candidate received, weighted by the share of each group of voting population. We denote by $\mathcal{P}^j(\mathbf{e}_t)$ as the probability of winning of candidate $j \in \{s, u, o\}$, given the vector collecting all voting decisions, $\mathbf{e}_t = \{e_t^s, e_t^u, e_t^o\}$ of the three groups of voters. Then, the voting decision of any individual must be optimal under the *correctly anticipated* probability of winning and policy stance of each candidate.

The voting decisions $\mathbf{e}_t^* = (e_t^{s*}, e_t^{u*}, e_t^{o*})$ form a *voting equilibrium* at time t if

$$e_t^{i*} = \arg\max \left\{ \sum_{j \in \{s,u,o\}} \mathcal{P}^j(e_t^i, \mathbf{e}_{-it}^*) V^i \left(\Phi_t^j, \Phi_{t+1}, \mathbf{e}_{t+1} \right) \mid e_t^i \in \triangle^2 \right\} \tag{12}$$

for $i \in \{s, u, o\}$, where $\mathcal{P}^j(e_t^i, \mathbf{e}_{-it}^*)$ denotes the probability that candidate $j \in \{s, u, o\}$ will win given the voting decisions, and $\mathbf{e}_{-it}^*$ is the optimal voting decision of other groups that is not i, and $\Phi_t^j = \left(\tau_t^j, \sigma_t^j, \mu_t^j \right)$ is the policy vector if candidate j wins, and e_t^{i*} is not a weakly dominated voting strategy.[11] Thus we require that each vote cast by each group is a best-response to the votes by the other groups, as well as be forward looking not only in terms of future policy, but also in terms of future voting decisions. In addition, the representative voter of each group must take into the account the *pivotal* power of their vote, because the entire group will also vote identically. Ruling out weakly dominated strategies ensure a simple majority voting in a two-candidate case, as well as making sure that *the largest group in the demographic always vote for its representative candidate* for consistencies with ideal policies of its respective candidate.[12] In case of a tie, the tie is broken with equal probability.

The voting decision of the old voters is simple special case, because they have no concern for the future,

$$e_t^{o*} = \arg\max \left\{ \sum_{j \in \{s,u,o\}} \mathcal{P}^j(e_t^o, \mathbf{e}_{-ot}^*) V^i \left(\Phi_t^j \right) \mid e_{ot} \in \triangle^2 \right\}.$$

After the election, the votes for each candidate are tallied by adding up the size of votes that each group that has chosen to vote for the candidate. The

[11] The modeling of the voting equilibrium with strategic voting follows Besley and Coate (1997, 1998).

[12] This assumption is in line with the literature on strategic voting which focusses primarily on the votes for the challengers, not for the leading candidate, see Cox (1997), Fisher (2001, 2004), Myatt and Fisher (2002), and Myatt (2007).

candidate with the most votes wins the election and gets to implement his ideal set of policies as described above.

3.2 Markov-perfect Property of Political Equilibria

The main problem with ranking the utility streams of the voters is due to the multiplicity of *future* equilibria once we allow for strategic voting. This makes it impossible for the voters to get a precise prediction of what will happen as a result of their action today. Even if we could pin down all the relative sizes of all possible payoffs in the next period, multiple voting equilibria do not allow a prediction of which equilibrium will be selected in the future. To deal with the problem, we restrict the voting equilibrium to satisfy the stationary Markov-perfect property, similarly to the policy choices in previous section. Therefore, we are looking for the a triplet policy function $(\tau_t, \sigma_t, \mu_t) = \Phi(s_t, \mathbf{e}_t^*)$ with the voting decisions $\mathbf{e}_t^*$ that solve the following two problems:

$$\begin{aligned} \Phi^d(s_t, \mathbf{e}_t^*) = \underset{\tau_t, \sigma_t, \mu_t}{\arg\max} \quad & V^d\left(s_t, , \tau_t, \sigma_t, \mu_t, \Phi(s_{t+1}, \mathbf{e}_{t+1}^*)\right) \\ \text{s.t. } s_{t+1} = & \frac{(1+n)s_t + (1+m)\sigma_t\mu_t}{1+n+\mu_t(1+m)}, \end{aligned} \tag{13}$$

where $d \in \{s, u, o\}$ is the identity of the the candidates, and the voting equilibrium $\mathbf{e}_t^*$ that satisfies the Markov perfect property and solves

$$\begin{aligned} e_t^{i*} &= \mathbf{e}^i(s_t) \\ &= \underset{e_t^i \in \triangle^2}{\arg\max} \sum_{j \in \{s,u,o\}} \mathcal{P}^j(e_t^i, \mathbf{e}_{-it}^*) V^i\left(\Phi_t^j, \Phi(s_{t+1}, \mathbf{e}_{t+1}^*), \mathbf{e}^*(s_{t+1})\right) \end{aligned} \tag{14}$$

for $i \in \{s, u, o\}$where $\mathcal{P}^j(e_t^i, \mathbf{e}_{-it}^*)$ denotes the winning probability of the representative candidate $j \in \{s, u, o\}$ given the voting decisions, and $\mathbf{e}_{-it}^*$ is the optimal voting decision of other groups that is not i, and $\Phi_t^j = \left\langle \tau_t^j, \sigma_t^j, \mu_t^j \right\rangle$ is the vector of preferred policy of candidate from group j.

The stationary Markov-perfect equilibrium defined above introduces two functional equation exercise. The first exercise is to find a policy profile that satisfies the Markov-perfect property. This equation states that the decisive (largest) group in period t chooses, given the state of the economy s_t, the most preferred policy variables τ_t, σ_t, and μ_t. In doing so, this group realizes that her utility is affected not only by these (current) variables, but also the policy variables of the next period $(\tau_{t+1}, \sigma_{t+1}, \mu_{t+1})$. This group further realizes that the future policy variables are affected by the current variables according to the policy function $\Phi(s_{t+1}, \tau_t, \sigma_t, \mu_t)$. Furthermore, this inter-temporal functional

relationship between the policy variables in periods $t+1$ and t is the same as the one existed between period t and $t-1$. Put differently, what the decisive group in period t chooses is related to $s_t, \tau_{t-1}, \sigma_{t-1}$, and μ_{t-1} in exactly the same way (through $\Phi(\cdot)$) as what the decisive group in period $t+1$ is expected to be related to $s_{t+1}, \tau_t, \sigma_t$, and μ_t. The second exercise restricts the voting decision to be cast on the belief that individuals in the same situation next period will vote in exactly the same way. With this property, the voters in this period know exactly how future generations will vote in exactly the same manner and can evaluate the stream of payoffs accordingly.

Combining all the analyses of all the three stages, the Markov-perfect political equilibrium is the triplet of policy decisions $\Phi^d(s_t, \mathbf{e}_t^*)$ for all $d \in \{s, u, o\}$ and the vector of voting decisions $\mathbf{e}^i(s_t)$ by for all $i \in \{s, u, o\}$, such that (i) given the stationary Markov-perfect policy decisions, $\mathbf{e}^i(s_t)$ is the stationary Markov-perfect optimal voting decisions for all $i \in \{s, u, o\}$ and (ii) given the stationary Markov-perfect voting decisions, $\Phi^d(s_t, \mathbf{e}_t^*)$ is the stationary Markov-perfect optimal policy decisions for all $d \in \{s, u, o\}$.

4 Political Equilibria: Preliminary Analysis with Full Discounting of the Future

To flash out best the complexity of coalition formation and its impact in shaping the welfare state and the immigration policy, we assume for the moment that the young native-born voters are fully discounting the future ($\beta = 0$). This assumption will break the dynamic political-economy linkages from one period to another, allowing us to focus primarily on just the preferences of candidates, the voting decisions, and the coalition formation without the complication of the dynamic framework. We will complete the analysis in the next section by introducing back the forward looking behavior.

4.1 Sizes of Electorates

We start by looking at which level of the state variable s_t each will group constitute the "largest" group in the population (which may or may not be the majority at ≥ 50 percent).

- The group of the skilled native-born workers is the largest group ("the skilled group") under two conditions. First, its size must dominates the

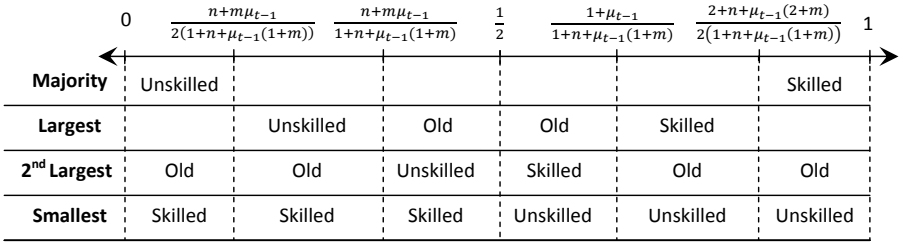

Figure 1: Sizes of each political group for each s_t.

unskilled young, and, second, it must also dominate the old cohort. Algebraically, these are

$$s_t > \frac{1}{2} \tag{15}$$

and

$$s_t > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \tag{16}$$

respectively. It can be shown that, because $n < m \leq 1$, only the second of the two conditions is sufficient.

- The group of unskilled native-born workers is the largest group ("the unskilled group") under two similar conditions; that are reduced to just one:

$$1 - s_t > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}. \tag{17}$$

- The group of old retirees is the largest group ("the old group"), when its size is larger than each one of the former groups, that is,

$$\frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \geq \max\{s_t, 1-s_t\}. \tag{18}$$

One can demonstrate that this partitions the state space $(0,1)$ into three parts: when $s_t < \frac{n+\mu_{t-1}m}{1+n+\mu_{t-1}(1+m)}$ (the unskilled workers are the largest group), when $s_t \in \left[\frac{n+\mu_{t-1}m}{1+n+\mu_{t-1}(1+m)}, \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}\right]$ (the old retirees are the largest group, and when $s_t > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}$ (the skilled workers are the largest group).

Figure 1 summarizes the ranking of the sizes of each group of voters for all s_t. There are considerable shifts in sizes of voters as s_t moves from 0 to 1. Note also that the old retirees will never form the smallest group in the economy. Because our political-economic equilibrium allows for strategic voting, sizes of

these groups at different level of s_t will matter in determining the equilibrium outcomes.

The economy can go through different equilibrium paths depending on n, m, and s_0, as follows:

1. If $n + m \leq 0$, the old group is always the majority. The tax rate is set at the Laffer point and the economy is fully open to skilled migration.

2. If $n+m > 0$, then the dynamics depend on the initial state of the economy, s_0. If $s_0 \geq \frac{1+\frac{n}{2}}{1+n}$, then the skilled workers are the majority (controlling 50 percent of the population), and zero tax rate with limited skilled migration will be observed. If $\frac{n}{2(1+n)} \geq s_0$, the unskilled workers are the majority, and there will be a positive tax rate (less than at the Laffer point) and some skilled migration. If $n < 0$, then *initially* the old cohort is the majority; the tax rate will be set at the Laffer point and the skilled migration will be maximal. When $n \geq 0$, the policies implemented are analyzed in the next subsection.

4.2 Preferred Policies of the Winning Candidate

We now consider the most interesting case of $n \geq 0$ and $m > n > 0$. It is best to break the analysis of political equilibrium down into pieces before reassembling them in the next subsection. In this subsection, we analyze the preferred candidate of each type, which will be taken into account by voters when making the voting decisions. Without future policy consideration, the policy choice of the winning candidate is a straightforward solution to the following:

$$\Phi^d(s_t) = \underset{\tau_t,\sigma_t,\mu_t}{\arg\max} V^d\left\{s_t, \tau_t, \sigma_t, \mu_t\right\}$$

where $d \in \{s, u, o\}$ is the identity of the the winning candidate.

Winning Old Candidate. If the old cohort has the winning candidate, then the policies are straightforward as follows:

$$\Phi^o(s_t) = \left(\tau_t^o = \frac{1}{1+\varepsilon}, \sigma_t^o = 1, \mu_t^o = 1\right). \tag{19}$$

The candidate wants maximal welfare state benefits, which means taxing to the Laffer point ($\frac{1}{1+\varepsilon}$). They also allow the maximal number of skilled migrants in to the economy because of the tax contribution this generates to the welfare system.

Winning Unskilled Candidate. If the winning candidate belongs to the unskilled young-native cohort, the triplet of ideal policies can be described as follows:

$$\Phi^u(s_t) = \left(\tau_t^u = \frac{1 - \frac{1}{J}}{1 + \varepsilon - \frac{1}{J}}, \sigma_t^u = 1, \mu_t^u = 1 \right). \tag{20}$$

where

$$J = \frac{(s_t + \sigma_t \mu_t) \left(\frac{w_t^s}{w_t^u} \right)^{1+\varepsilon} + 1 - s_t + (1 - \sigma_t)\mu_t}{1 + \mu_t + \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}} \tag{21}$$

It is interesting to note that, although the unskilled young are net beneficiaries in this welfare state, they are, nevertheless, still paying taxes. Hence the preferred tax policy of the unskilled voters is smaller than the Laffer point with a wedge $\frac{1}{J}$. Clearly, the unskilled workers also prefer to let in more skilled immigrants due to their contribution to the welfare state.

The preferred tax choice of the unskilled candidate is interpreted as follows. Letting τ_t^u denote the tax rate preferred by the unskilled group, one can verify from equation (21) that $\frac{\partial \tau_t^u}{\partial \sigma_t} > 0$, and there exists $\overline{\sigma}$ such that, for any $\sigma_t < \overline{\sigma}$, we have $\frac{\partial \tau_t^u}{\partial \mu_t} < 0$. Conversely, for any $\sigma_t > \overline{\sigma}$, we would get an expansion of the welfare state, because $\frac{\partial \tau_t^u}{\partial \mu_t} > 0$.[13] Therefore, the higher number of skilled migrants will prompt a higher demand for *intra*-generational redistribution. The fiscal leakage channel shows that unskilled migration creates more fiscal burden, such that the decisive "unskilled" voters would rather have the welfare state shrunken. In addition, an increase in inequality in the economy, reflected in the skill premium $\left(\frac{w_t^s}{w_t^u} \right)$, leads to a larger welfare state demanded by the unskilled.[14]

Winning Skilled Candidate. If the winning candidate represents the skilled young-native cohort, then the ideal policy to be implemented by the candidate will be as follows:

$$\Phi^s(s_t) = \left(\tau_t^s = 0, \sigma_t^s \in [0,1] \ , \mu_t^s \in [0,1] \right). \tag{22}$$

In this policy triplet, the skilled candidate prefer no tax as the skill young workers are the net contributor to the welfare state. Without the tax, there will

[13] Recall that the tax rate preferred by the unskilled young workers is less than the level that is preferred by the old retirees. The tax rate preferred by the old retirees, $\tau_t^o = \frac{1}{1+\varepsilon}$ is the Laffer point that attains the maximum welfare size, given immigration policies. Therefore the size of the welfare state is monotonic in the tax rate when $\tau \in [0, \frac{1}{1+\varepsilon}]$. Thus, our use of "shrink" and "expand" is justified.

[14] This resembles the results of Razin, Sadka, and Swagel (2002a, 2002b).

be no transfer and hence no cost or benefits from having immigrants. Therefore, the skilled candidate preferred immigration policy will be indeterminate.[15]

4.3 Three Types of Political Equilibria

Each individual naturally prefers the ideal policies of their representative candidate. However, strategic voting opens up the possibility of voting for someone else that is not the most preferred candidate in order to avoid the least favorable candidate. For the skilled workers, they prefer the least amount of taxes and some migration. Thus, they will prefer the policy choice of the unskilled over the old candidate because the tax rate will be lower. As for the old retirees, the higher the transfer benefit, the better. Clearly, the unskilled candidate promises some benefits whereas the skilled promises none, so old voters would prefer the policies of the unskilled over the skilled candidates. In sum, since the skilled and the old voters want to defeat the policies of one another, only two coalitions are possible: either unskilled collude with skilled voters to upset the old voters, or the unskilled collude with the old voters to upset the skilled voters.

For the unskilled workers, both rankings are possible: they could either prefer the policy choice of the skilled over the old, or vice versa. The parameters of the model will dictate the direction of their votes. The cut-off tax policy, $\widetilde{\tau}$, is the break-even point for the unskilled between getting taxed but receiving transfer (policies of the old candidate) or pay no tax at all (policies of the skilled candidate). Formally, this tax level, $\widetilde{\tau}$, is defined implicitly by the equation

$$\frac{(w^u)^{1+\varepsilon}}{1+\varepsilon} = \frac{((1-\widetilde{\tau})w^u)^{1+\varepsilon}}{1+\varepsilon} + \frac{\widetilde{\tau}(1-\widetilde{\tau})^{\varepsilon}\left(\left(s_t+\sigma_t\mu_t\right)\left(w^s\right)^{1+\varepsilon}+\left(1-s_t+(1-\sigma_t)\mu_t\right)\left(w^u\right)^{1+\varepsilon}\right)}{1+\mu_t+\frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}}. \tag{23}$$

We know that such a tax policy exists, because, taking next period's policies as given, the payoff in this period to the unskilled is maximized at its preferred policy and zero at $\tau = 1$. Therefore, at some $\widetilde{\tau}$, the equality will hold. This cut-off tax rate will play an important role for the unskilled young' voting decision.

The first equilibrium we look at is dubbed "Intermediate" because it captures the essence that the preferred policies of the unskilled workers are a compromise

[15]Such an indeterminacy is easily resolved by introducing other costs or benefits of immigration, or under a flexible wages scheme in which we provide a full detail in Razin, Sadka, Suwankiri (2010).

from the extremity of the other two groups. We can show that the following strategy profile forms a Markov-perfect equilibrium with strategic voting

$$
\begin{aligned}
e_t^{s*} &= \begin{cases} (1,0,0) & \text{, if } s_t > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \\ (0,1,0) & \text{, otherwise} \end{cases} \\
e_t^{u*} &= (0,1,0) \\
e_t^{o*} &= \begin{cases} (0,0,1), & \text{if } s_t \in \left[\frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}\right] \\ (0,1,0), & \text{otherwise} \end{cases}
\end{aligned} \tag{24}
$$

and the policies implemented when no group is the majority are

$$
\Phi_t = \begin{cases} \left(\tau_t = \frac{1-\frac{1}{J}}{1+\varepsilon-\frac{1}{J}}, \sigma_t = 1, \mu_t = 1\right), & \text{otherwise} \\ \left(\tau_t^s = 0, \sigma_t^s \in [0,1]\ , \mu_t^s \in [0,1]\right), & \text{if } s_t \geq \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \end{cases} \tag{25}
$$

where $J = J(\mu_t, \sigma_t, s_t, \mu_{t-1})$ is as in equation (21).

It is worthnoting here that, the policies preferred by the unskilled young candidate will prevail in equilibrium almost all the time, except in an area where the skilled young voters form the majority in the economy. The equilibrium features the unskilled voters always voting for their representative, whereas the other two groups vote for their respective candidate only if they are the largest group, or for the unskilled candidate otherwise. With these voting strategy, if no group captures 50 percent of the voting populations, the policy choice preferred by the unskilled candidate will prevail no matter who are the largest group in the constituency. Since the old voters will never form a majority in the economy (Figure 1), they will choose to vote for the unskilled young candidate to avoid zero social benefit as preferred by the skilled candidate.

In the preceding equilibrium, we let the preference of the skilled workers and the old retirees decide the fate of the policies. In the following analysis, the unskilled workers consider who they want to vote for. This will depend on how extractive the tax policy preferred by the old is. We call the next equilibrium "Left-winged", because it features a welfare state of the size greater-or-equal to that of the intermediate policy equilibrium. This may arise when the tax rate preferred by the old voters is *not excessively redistributive*.

When $\frac{1}{1+\varepsilon} \leq \widetilde{\tau}$, we have an equilibrium of the following form

$$\begin{aligned} e_t^{s*} &= \begin{cases} (1,0,0) & \text{, otherwise} \\ (0,1,0) & \text{, if } s_t < \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \end{cases} \\ e_t^{u*} &= \begin{cases} (0,1,0) & \text{, if } s_t < \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \\ (0,0,1) & \text{,otherwise} \end{cases} \\ e_t^{o*} &= (0,0,1) \end{aligned} \tag{26}$$

and the policies implemented are

$$\Phi_t = \begin{cases} \left(\tau_t = \frac{1-\frac{1}{J}}{1+\varepsilon-\frac{1}{J}}, \sigma_t = 1, \mu_t = 1\right) & \text{, if } s_t < \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \\ \left(\tau_t^* = \frac{1}{1+\varepsilon}, \sigma_t = 1, \mu_t = 1\right) & \text{, otherwise} \\ \left(\tau_t^s = 0, \sigma_t^s \in [0,1] \text{ , } \mu_t^s \in [0,1]\right), & \text{, if } s_t \geq \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \end{cases} \tag{27}$$

where $J = J(\mu_t, \sigma_t, s_t, \mu_{t-1})$ is as in equation (21) and $\widetilde{\tau}$ is the cut-off tax rate given implicitly in equation (23).

When the tax rate preferred by the old voters is not excessively redistributive in the eyes of the unskilled, we have an equilibrium where the unskilled voters strategically vote for the old candidate to avoid the policies preferred by the skilled voters, resulting in the implementation of the preferred policy of the old candidate even when the skilled voters form the largest group in the constituency. This will be an equilibrium when the size of the skilled is not "too large." Therefore, voting for the old will only be compatible with the interest of the unskilled voters when the tax rate is not excessively high and when the size of the skilled is not too large.

In the last case, when $\frac{1}{1+\varepsilon} > \widetilde{\tau}$, we can show that there is an equilibrium as follows:

$$\begin{aligned} e_t^{s*} &= \begin{cases} (0,1,0) & \text{,if } s_t < \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \\ (1,0,0) & \text{, otherwise} \end{cases} \\ e_t^{u*} &= \begin{cases} (1,0,0) & \text{, if } s_t \in \left[\frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}\right] \\ (0,1,0) & \text{, otherwise.} \end{cases} \\ e_t^{o*} &= \begin{cases} (0,1,0) & \text{,if } s_t > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \\ (0,0,1) & \text{, otherwise} \end{cases} \end{aligned} \tag{28}$$

and the policies implemented are

$$\Phi_t = \begin{cases} \left(\tau_t^s = 0, \sigma_t^s \in [0,1]\ , \mu_t^s \in [0,1]\right), & \text{, if } s_t \in \left[\frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}\right] \\ \left(\tau_t^u = \frac{1-\frac{1}{J}}{1+\varepsilon-\frac{1}{J}}, \sigma_t^u = 1, \mu_t^u = 1\right) & \text{, otherwise} \\ \left(\tau_t^s = 0, \sigma_t^s \in [0,1]\ , \mu_t^s \in [0,1]\right), & \text{, if } s_t \geq \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \end{cases} \tag{29}$$

where $J = J(\mu_t, \sigma_t, s_t, \mu_{t-1})$ is as in equation (21) and $\widetilde{\tau}$ is given in equation (23).

When the Laffer point is higher than $\widetilde{\tau}$, the tax rate is considered as excessive by the unskilled voters.[16] In this case, the unskilled voters will instead choose to vote for the skilled over the old candidate, resulting in the implementation of the preferred policy of the skilled candidate even when the old retirees form the largest group in the constituency. The resulting equilibrium has the size of the welfare state less-than-or-equal to that in the intermediate policy equilibrium, hence we refer to it as "Right-winged." When the tax preferred by the old is excessive from the perspective of the unskilled, the political process could implement the policies preferred by the skilled in order to avoid the worst possible outcome. This happens when the old voters constitute the largest group, and the unskilled voters vote strategically for the skilled candidate. In other cases, however, the policies preferred by the unskilled will be implemented, irrespective of the identity of the largest group in the economy.[17]

[16] This extreme equilibrium in fact the Laffer point is so high beyond the inter-temporal benefit may not in fact be realistic in actual elections due to incomplete information of supports and switching supports, see Myatt and Fisher (2001) and Myatt (2007).

[17] For our results with multidimensional policies, it is important to note here that the ranking of candidates by individual voters allows us to escape the well-known agenda-setting cycle (the "Condorcet paradox"). Such a cycle, which arises when any candidate could be defeated in a pairwise majority voting competition, leads to indeterminacy and non-existence of a political equilibrium. The agenda-setting cycle will have a bite if the rankings of the candidates for all groups are unique: no group occupies the same ranked position more than once. However, this does not arise here, because, in all equilibria, some political groups have a *common* enemy. That is, because they will never vote for the least-preferred candidate (the "common" enemy), the agenda-setting cycle breaks down to determinate policies above, albeit their multiplicity. This occurs when voters agree on who is the least-preferred candidate and act together to block her from winning the election. The literature typically avoids the Condorcet paradox by restricting political preferences with some ad hoc assumptions. For our case, the preferences induced from economic assumption lead to an automatic reduction of dimensions, hence providing an escape from the Condorcet paradox. For further discussions on agenda-setting cycle, see Drazen (2000, page 71-72), and Persson and Tabellini (2000, page 29-31).

5 Political Equilibria under Forward-looking Behavior

In this section, we introduce back the discount factor $\beta \in [0,1]$ so that the young native-born will take into the account how their policy choices today will affect the future political-economic dynamics.

5.1 Preferred Policies of the Winning Candidate

We first summarize what are the variables relevant for each of the three types of voters when casting the vote in period t. First, recall that s_t is the variable which describes the state of the economy. Also, each voter takes into account how his choice of the policy variables in period t will affect the chosen policy variables in period $t+1$ which depends on s_{t+1} (recall that the benefit he will get in period $t+1$, b_{t+1}, depends on τ_{t+1}, σ_{t+1}, and μ_{t+1} as well as the voting strategy $\mathbf{e}^*_{t+1}$). Therefore each voter will cast his vote on the set of policy variables τ_t, σ_t, and μ_t which maximizes his utility given the level of s_t, taking also into account how this will affect s_{t+1} given $\mathbf{e}^*_{t+1}$. Thus, there is a link between the policy chosen in period t to the one chosen in period $t+1$.

The mechanism (policy rule or function) that characterizes the choice of the policy variables (τ_t, σ_t, and μ_t) is invariant over time. This mechanism relates the choice in any period to the choice of the preceding period (τ_{t-1}, σ_{t-1}, and μ_{t-1}). This choice depend also on the current state of the economy, s_t. . With future policy consideration, the policy choice of the winning candidate is a solution to the following: a triplet policy function $(\tau_t, \sigma_t, \mu_t) = \Phi(s_t, \tau_{t-1}, \sigma_{t-1}, \mu_{t-1})$, which is a solution to the following problem:

$$\Phi^d(s_t) = \underset{\tau_t, \sigma_t, \mu_t}{\arg\max} V^d \left\{ s_t, \tau_t, \sigma_t, \mu_t, \Phi(s_{t+1}) \right\} \tag{30}$$
$$\text{subject to } s_{t+1} = \frac{(1+n)s_t + (1+m)\sigma_t \mu_t}{1+n+(1+m)\mu_t},$$

where $d \in \{s, u, o\}$ is the identity of the the winning candidate. Notice that, in contrast with the previous section, the candidate must now take into the account of how his decision influences the evolution of the state variable, s_{t+1} , next period. Note that, the key difference from introducing forward-looking behavior will arise in both the preferred skill composition of immigrants, and the preferred ratio of immigrants to native young as these will directly influence the dynamics of the state variable, but not in the preferred tax rate as the welfare benefit is under the balanced-budget assumption.

Winning Old Candidate. If the old cohort has the winning candidate, then the policies are as before, $\Phi^o(s_t) = \left(\tau_t^o = \frac{1}{1+\varepsilon}, \sigma_t^o = 1, \mu_t^o = 1\right)$, since there are no need for future consideration for the cohort.

Winning Unskilled Candidate. If the winning candidate belongs to the unskilled young-native cohort, the candidate's preferred tax rate is as previously described, since it only has contemporaneous effect on today's welfare benefit. As for the skill composition of immigrants, the unskilled candidate will still preferred maximal, at $\sigma_t^u = 1$, unless if doing so will place the winning policy in period $t+1$ in the hands of skilled candidate whose preferred policies will lead to zero welfare benefit for the retired cohort. Hence, some strategic restriction of immigration quota will occur. Since skilled immigrants still bring in more welfare benefits that unskilled ones, it can be shown that the preferred policy of the unskilled will rather restrict μ_t^u rather than σ_t^u. The preferred unskilled candidate will be careful to choose μ_t^u such that $s_{t+1} \leq \frac{2+n+\mu_t(2+m)}{2(1+n+\mu_t(1+m))}$, a threshold that will place the skilled candidate as a clear winning candidate in period $t+1$'s election, subject to the voting equilibrium that will arise. This leads to $\mu_t^u = \frac{2+n-2(1+n)s_t}{m}$ such that $\mu_t^u \geq 0$. Note that at $s_t = \frac{2+n}{2(1+n)}$, we have $\mu_t^u = 0$ and hence a steady-state, regardless of σ_t. While the preferred tax rate and the preferred skilled-to-unskilled immigrant ratio will remain as before, $\tau_t^u = \frac{1-\frac{1}{j}}{1+\varepsilon-\frac{1}{j}}$ and $\sigma_t^u = 1$, respectively.[18]

Winning Skilled Candidate. If the winning candidate represents the skilled young-native cohort, then the ideal tax rate of the candidate will still be zero, $\tau_t^s = 0$, leading to zero welfare benefit for everyone this period. However, in contrast with the case where $\beta = 0$, the skill composition of immigrants and the ratio of immigrants to native young matter due to their effects on future policies. Since skilled immigrants will give offsprings in period $t+1$ who are also skilled, this expands the welfare state for the next period, a time in which the current skilled cohort would have retired and depended on the welfare benefit for consumption. Therefore, the preferred skill composition of immigrant by the skilled candidate will now be determinate at $\sigma_t^s = 1$, since there is no cost

[18]Since we want to analyze the effect of forward-looking behaviors in the economy, we naturally assume that the benefit to the unskilled young voters of being in control of welfare benefit in two consecutive periods outweighs the benefit of binging on maximum welfare state for only one period. In other words, we assume that, $\Psi = b_t^u + \beta b_{t+1}^u - \widehat{b}_t > 0$ where we denote by $\widehat{b}_t$ the welfare benefit in period t with $\mu_t = 1 = \sigma_t$, and b_t^u the welfare benefit in period t with $\sigma_t = 1$ and $\mu_t = \frac{2+n-2(1+n)s_t}{m}$. Both welfare benefit are associated with the tax rate preferred by the unskilled group. Similarly, $b_{t+1}^o > b_{t+1}^u$are the welfare benefit in period $t+1$ associated with the set of policy variables preferred by the old and the unskilled candidates, respectively. Without this assumption, it trivially leads back to the solutions with $\beta = 0$.

in period t, and will bring about higher benefit in period $t+1$. As such, the skilled candidate will now share a similar dilemma with the unskilled candidate in that, having too many skilled immigrants in period t, may tip the political power in period $t+1$ to the hands of the skilled cohort, leading to zero welfare benefit. Hence, the skilled candidate would also seek to chose $\mu_t^s = \frac{2+n-2(1+n)s_t}{m}$ to ensure an existence of the welfare state next period, ubject to the voting equilibrium that will arise in period $t+1$.

In the case when $s_t > \frac{2+n}{2(1+n)} \geq \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}$, the special set of immigration policies[19] will be preferred by the skilled candidate, similarly to the case of unskilled candidate, with $\widehat{\sigma}^s$ and $\widehat{\mu}^s$ are given implicitly from the maximization problem

$$\begin{aligned} \langle \widehat{\sigma}^s, \widehat{\mu}^s \rangle = \underset{\sigma_t, \mu_t}{\arg\max} V_t^s &= \frac{\left(A_t w_t^s\right)^{1+\varepsilon}}{1+\varepsilon} + \beta b_{t+1}^o \\ \text{subject to} \qquad s_{t+1} &\leq \frac{2+n+\mu_t(2+m)}{2(1+n+\mu_t(1+m))}, \\ \text{and} \qquad \sigma_t &\leq s_t. \end{aligned} \tag{31}$$

where $\sigma_t \leq s_t$ is required to reduce s_{t+1} from s_t. We will attempt to describe the solutions here. Given the constraint $\sigma_t \leq s_t$ which will make s_{t+1} a decreasing function of μ_t, and that $\frac{2+n+\mu_t(2+m)}{2(1+n+\mu_t(1+m))}$ is an increasing function in μ_t, then $\mu_t^s = 1$ will be the solution for any $\widehat{\sigma} \leq s_t$. In other words, when s_t cross the threshold $s_t \geq \frac{2+n}{2(1+n)}$, the skilled candidate will manipulate the skill composition only to ensure the existence of the welfare state. At $\mu_t^s = 1$, the skill composition of the immigrants that ensure the existence of the welfare state next period is given by $\sigma_t^s = \frac{2+\frac{n+m}{2}-(1+n)s_t}{1+m}$. At $\widehat{\sigma}^s = s_t$, when $s_t = \frac{2+\frac{n+m}{2}}{2+n+m}$ and we have a steady-state, regardless of μ_t. But since, we solve this out of the first constraint, given the section, we know that, at the level of $\widehat{\sigma}_t^s = \frac{2+\frac{n+m}{2}-(1+n)s_t}{1+m}$, $s_{t+1} = \frac{2+\frac{n+m}{2}}{2+n+m}$ regardless of s_t and the share of skill workers stops growing. In summary, the preferred policies of the skilled candidate are as follows:

$$\Phi^s(s_t) = \begin{cases} \left(\tau_t^s = 0, \sigma_t^s = 1, \mu_t^s = \frac{2+n-2(1+n)s_t}{m}\right), & \text{if } s_t < \frac{2+n}{2(1+n)} \\ \left(\tau_t^s = 0, \widehat{\sigma}_t^s = \frac{2+\frac{n+m}{2}-(1+n)s_t}{1+m}, \widehat{\mu}_t^s = 1\right) & \text{if } s_t \geq \frac{2+n}{2(1+n)}. \end{cases} \tag{32}$$

[19]Note that $1 > \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} > \frac{2+n}{2(1+n)} \geq \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}$ partitions the area where the skilled voters are the largest group in period t.

5.2 Three Types of Political Equilibria with Forward-looking behavior

We now take a look at the previous three equilibria and highlight the differences that emerged once voters are forward-looking. As with before the cut-off tax policy, $\widetilde{\tau}$ (as described in equation (23)) will determine the strategic support from the unskilled voters.

Intermediate Equilibrium. Recall that in the "Intermediate equilibrium" both extremes, the skilled voters who prefer zero welfare benefit and the old voters who prefer the Laffer point tax rate with maximal welfare benefit, strategically vote to take shelter in the preferred policies of the unskilled candidate against the outcome of either extreme. Hence voting strategy and policy choices the form Markov-perfect equilibrium are as follows:

$$\begin{aligned} e_t^{s*} &= \begin{cases} (1,0,0) & \text{, if } s_t > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \\ (0,1,0) & \text{, otherwise} \end{cases} \\ e_t^{u*} &= (0,1,0) \\ e_t^{o*} &= \begin{cases} (0,0,1), & \text{if } s_t \in \left[\frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}\right] \\ (0,1,0), & \text{otherwise} \end{cases} \end{aligned} \tag{33}$$

and the policies implemented when no group is the majority are

$$\Phi_t = \begin{cases} \left(\tau_t^u = \frac{1-\frac{1}{J}}{1+\varepsilon-\frac{1}{J}}, \sigma_t^u = 1, \mu_t^u = \frac{2+n-(1+n)s_t}{m}\right), & \text{if } s_t < \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \\ \left(\tau_t^s = 0, \sigma_t^s = \frac{2+\frac{n+m}{2}-(1+n)s_t}{1+m}, \mu_t^s = 1\right) & \text{if } s_t \geq \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \end{cases} \tag{34}$$

where $J = J(\mu_t, \sigma_t, s_t, \mu_{t-1})$ is as in equation (21).

It is important to note that, despite the forward-looking behavior of voters, the results will remain similar to the case where $\beta = 0$. However, as previously described, the winning unskilled candidate will restrict the number of entering skilled immigrants to ensure that the welfare state exists in the next period, at the level of preferred by next period's unskilled candidate. Only when the skilled voters form the majority will their candidate gets to implement their preferred policies.

The dynamics of this politico-economic equilibrium with Markov-perfect property are given by

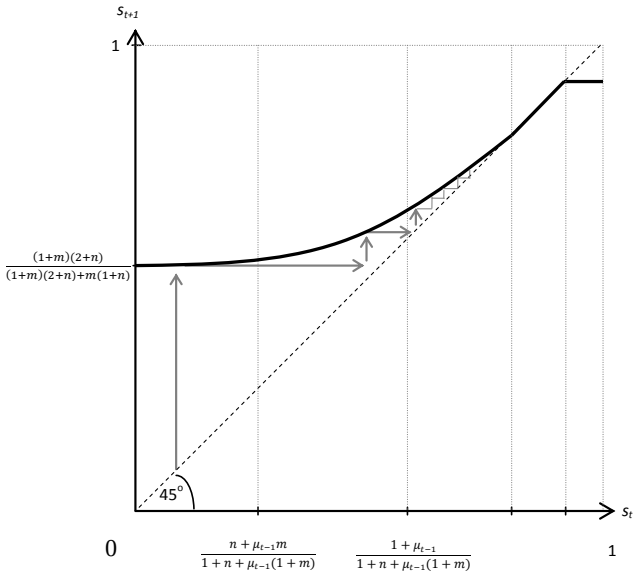

Figure 2: Dynamics under the "Left-winged" Equilibrium

$$s_{t+1} = \begin{cases} \frac{(1+m)(2+n)-(1+n)(2+m)s_t}{(1+m)(2+n)-(1+n)[2(1+m)s_t-m]} & , \text{ if } 0 \leq s_t < \frac{2+n}{2(1+n)} \\ s_t & , \text{ if } \frac{2+n}{2(1+n)} \leq s_t < \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \\ \frac{2+\frac{n+m}{2}}{2+n+m} & , \text{ if } \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \leq s_t \leq 1. \end{cases} \tag{35}$$

Note that the dynamics reaches a steady-state after $s_t \geq \frac{2+n}{2(1+n)}$, which leads to $\mu_t = 0$. Unless, if $s_t \geq \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))}$ with very high s_0, the state of the economy stops at $\frac{2+\frac{n+m}{2}}{2+n+m}$, where the voters will choose to freeze the share of skilled voters in the next period at a threshold that does not make the next period skilled voters the majority.[20] Figure 2 portray the dynamics.

Left-winged Equilibrium. When the Laffer point tax rate is lower than the threshold $\frac{1}{1+\varepsilon} \leq \widetilde{\tau}$, the tax rate preferred by the old candidate is not excessive

[20]To keep the analysis true the strategic intent, we assume that at $s_t = \frac{2+\frac{n+m}{2}}{2+n+m}$, the steady-state, the population dynamics will tilt the tie-breaking towards those generation with strategic intent (that is, the old retirees of period $t+1$). Otherwise, we can formally assume at some algebraic cost that $\sigma_t^s = \frac{2+\frac{n+m}{2}}{2+n+m} - \varepsilon$ for an arbitrarily small $\varepsilon > 0$ to get the same result.

in the eyes of unskilled voters and we have an equilibrium of the following form

$$e_t^{s*} = \begin{cases} (1,0,0) & \text{, otherwise} \\ (0,1,0) & \text{, if } \frac{1-\frac{m-n}{2}}{1+n} \leq s_t \leq \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \end{cases}$$

$$e_t^{u*} = \begin{cases} (0,1,0) & \begin{cases} \text{, if } s_t < \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \\ \text{or } \frac{1-\frac{m-n}{2}}{1+n} \leq s_t \leq \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \end{cases} \\ (0,0,1) & \text{, otherwise} \end{cases} \tag{36}$$

$$e_t^{o*} = \begin{cases} (0,0,1) & \text{, otherwise} \\ (0,1,0) & \text{, if } \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \leq s_t \end{cases}$$

nd the policies implemented are

$$\Phi_t = \begin{cases} \left(\tau_t^u = \frac{1-\frac{1}{J}}{1+\varepsilon-\frac{1}{J}}, \sigma_t^u = 1, \mu_t^u = \frac{2+n-(1+n)s_t}{m}\right) & \text{, if } s_t < \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \\ \left(\tau_t^o = \frac{1}{1+\varepsilon}, \sigma_t^o = 1, \mu_t^o = 1\right) & \text{, if } \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \leq s_t \leq \frac{1-\frac{m-n}{2}}{1+n} \\ \left(\tau_t^u = \frac{1-\frac{1}{J}}{1+\varepsilon-\frac{1}{J}}, \sigma_t^u = 1, \mu_t^u = \frac{2+n-(1+n)s_t}{m}\right) & \text{, if } \frac{1-\frac{m-n}{2}}{1+n} \leq s_t \leq \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \\ \left(\tau_t^s = 0, \sigma_t^s = \frac{2+\frac{n+m}{2}-(1+n)s_t}{1+m}, \mu_t^s = 1\right) & \text{, if } s_t \geq \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \end{cases} \tag{37}$$

where $J = J(\mu_t, \sigma_t, s_t, \mu_{t-1})$ is as in equation (21) and $\widetilde{\tau}$ is the cut-off tax rate given implicitly in equation (23).

One difference with the case with no forward-looking behavior here. Though the tax policy preferred by the old candidate is not extreme in the eyes of the unskilled voters, but the immigration policies are. Letting in too many skilled immigrants could upset the dynamics and lead to zero welfare benefit in period $t+1$. The region to which $\sigma_t^o = 1, \mu_t^o = 1$ will lead to zero welfare benefit in period $t+1$, is when $\frac{1-\frac{m-n}{2}}{1+n} \leq s_t$. Hence, given that the old voters will never form a majority, there is a room of a coalition between the unskilled voters and the skilled voters to vote for the unskilled candidate and implement a more restrictive immigration policy to ensure the existence of the welfare state in the next period.

The dynamics of this politico-economic equilibrium is given as follows:

$$s_{t+1} = \begin{cases} \frac{(1+m)(2+n)-(1+n)(2+m)s_t}{(1+m)(2+n)-(1+n)[2(1+m)s_t-m]} & \text{, if } 0 \leq s_t < \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \\ \frac{(1+n)s_t+(1+m)}{2+n+m} & \text{, if } \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \leq s_t < \frac{1-\frac{m-n}{2}}{1+n} \\ \frac{(1+m)(2+n)-(1+n)(2+m)s_t}{(1+m)(2+n)-(1+n)[2(1+m)s_t-m]} & \text{, if } \frac{1-\frac{m-n}{2}}{1+n} \leq s_t < \frac{2+n}{2(1+n)} \\ s_t & \text{, if } \frac{2+n}{2(1+n)} \leq s_t \leq \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \\ \frac{2+\frac{n+m}{2}}{2+n+m} & \text{, if } \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} < s_t \leq 1. \end{cases} \tag{38}$$

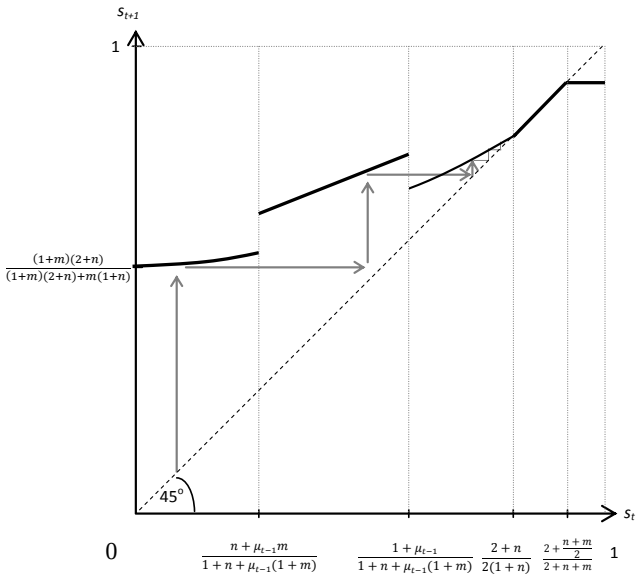

Figure 3: Dynamics under the "Left-winged" Equilibrium

Note that, in the dynamics of this equilibrium, we have a "switchback" in policy supported by majority voters in the case where $\in \left[\frac{1-\frac{m-n}{2}}{1+n}, \frac{2+n}{2(1+n)}\right)$. Since in this range of state variable, the immigration policy of the old retirees will lead directly to zero welfare state benefit in the next period, the young voters in period t colluded to support the policies of the young unskilled candidate instead (alternatively possible is the collusion to support the young skilled candidate). Of the three cases, this case produces the most complex dynamics due to this inter-temporal misalignment of incentives between young and old cohort. Figure 3, we portray what the dynamics of this equilibrium will look like.

Right-winged Equilibrium. When the Laffer point tax rate is higher than the threshold $\frac{1}{1+\varepsilon} > \widetilde{\tau}$, the tax rate preferred by the old candidate is excessive in the eyes of unskilled voters. Hence, the unskilled voters will instead choose to side with the preferred policies of the skilled candidate. When $\frac{1}{1+\varepsilon} > \widetilde{\tau}$, we have

the following equilibrium:

$$
\begin{aligned}
e_t^{s*} &= \begin{cases} (1,0,0) & \text{, otherwise} \\ (0,1,0) & \text{,if } s_t < \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \end{cases} \\
e_t^{u*} &= \begin{cases} (0,1,0) & \text{, otherwise} \\ (1,0,0) & \text{, if } s_t \in \left[\frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}\right]. \end{cases} \\
e_t^{o*} &= \begin{cases} (0,0,1) & \text{, otherwise} \\ (0,1,0) & \text{,if } s_t > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \end{cases}
\end{aligned} \tag{39}
$$

and the policies implemented when no group is the absolute majority are

$$
\Phi_t = \begin{cases} \left(\tau_t^u = \frac{1-\frac{1}{J}}{1+\varepsilon-\frac{1}{J}}, \sigma_t^u = 1, \mu_t^u = \frac{2+n-(1+n)s_t}{m}\right) & \text{,if } 0 \leq s_t < \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \\ \left(\tau_t^s = 0, \sigma_t^s = 1, \mu_t^s = \frac{2+n-(1+n)s_t}{m}\right), & \text{, if } \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \leq s_t < \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \\ \left(\tau_t^u = \frac{1-\frac{1}{J}}{1+\varepsilon-\frac{1}{J}}, \sigma_t^u = 1, \mu_t^u = \frac{2+n-(1+n)s_t}{m}\right) & \text{,if } \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \leq s_t \leq \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \\ \left(\tau_t^s = 0, \sigma_t^s = \frac{2+\frac{n+m}{2}-(1+n)s_t}{1+m}, \mu_t^s = 1\right), & \text{, if } \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} < s_t \leq 1 \end{cases} \tag{40}
$$

where $J = J(\mu_t, \sigma_t, s_t, \mu_{t-1})$ is as in equation (21) and $\widetilde{\tau}$ is the cut-off tax rate given implicitly in equation (23).

The dynamics of the Right-winged equilibrium is given by

$$
s_{t+1} = \begin{cases} \frac{(1+m)(2+n)-(1+n)(2+m)s_t}{(1+m)(2+n)-(1+n)[2(1+m)s_t-m]} & \text{, if } 0 \leq s_t < \frac{2+n}{2(1+n)} \\ s_t & \text{, if } \frac{2+n}{2(1+n)} \leq s_t < \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \\ \frac{2+\frac{n+m}{2}}{2+n+m} & \text{, if } \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))} \leq s_t \leq 1. \end{cases} \tag{41}
$$

Notice that, the dynamics for this equilibrium is identical to the one under the "Intermediate Equilibrium," due to the result that, in both equilibria, the preferred policy of the old candidate never get supported by the voters. Hence, the dynamics is depicted by Figure 2.

It is interesting to note that, once forward looking is introduced back, the skilled and unskilled voters and their candidates now have a common interest in ensuring that the welfare state exist in the next period. Hence, their coalition is formed in a region of the state variables where the old voters are the largest group in the economy to try to block the old candidate's excessive tax rate as well as the extreme immigration policy that could lead to toppling the welfare state system in the next period.

5.3 Interpreting the Results: Migration and Tax Policies

The intuition for the aforementioned results is as follows. The skilled is the net contributor to the welfare state, while the other two groups are net beneficiaries. We refer to Figure 1 for illustration. Preferences of the old retirees are simple. If the old cohort is the largest, it wants maximal welfare state benefits, which means taxing to the Laffer point ($\frac{1}{1+\varepsilon}$). They also allow the maximal number of skilled migrants in to the economy because of the tax contribution this generates to the welfare system.

It is interesting to note that, although the unskilled young are net beneficiaries in this welfare state, they are, nevertheless, still paying taxes. Hence the preferred tax policy of the unskilled voters is smaller than the Laffer point with a wedge $\frac{1}{J}$, as previously described. Clearly, the unskilled workers also prefer to let in more skilled immigrants due to their contribution to the welfare state. How many will they let in depends on the function Ψ, which weighs the future benefits against the cost at the present. Basically, if the unskilled workers are not forward-looking, it is in their best interest to let in as many skilled migrants as possible. However, this will lead to no redistribution in the next period because the skilled workers will be the largest. Hence, the function Ψ is the difference between the benefits they get by being, as they are, forward-looking and being myopic.

The skilled native-born young prefer more skilled migrants for a different reason than the earlier two groups. They prefer to let in skilled migrants in this case because this will provide a higher number of skilled native workers in the *next* period. Thus, because the skilled are forward-looking, they too will prefer to have more skilled workers in their retirement period. However, they cannot let in too many of them because their high birth rate may render the skilled young in the next period as the largest group who will vote to abolish the welfare state altogether.

A common feature among models with Markov-perfect equilibrium is the idea that today's voters have the power to influence the identity of future policy makers. Such feature is also prominent in our analysis here. As previously pointed out in Dolmas and Huffman (2004), Ortega (2005), and Sand and Razin (2007), future political influence of migrants will matter for the decision on immigration policy today. The migration policy of either young group reflects this fact that they may want to put themselves as the largest group in the next period. Thus, instead of letting in too many migrants, who will give birth to a large new skilled generation, they will want to let in as much as

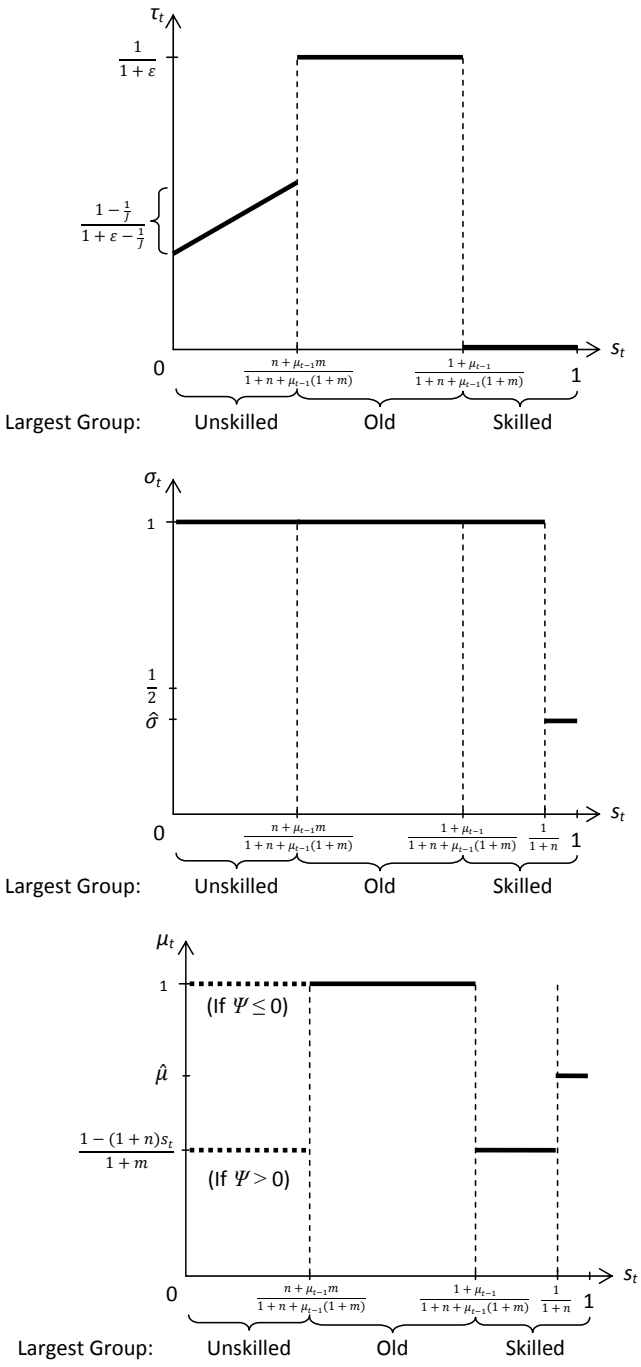

Figure 4: Policy Functions for each s_t

possible before the threshold is crossed. This threshold is $\frac{2+n-(1+n)s_t}{m}$.[21] Letting $s_t = 1$ gets the same result as Sand and Razin (2007). There are two differences nonetheless. First, the equilibrium here has a bite even if the population growth rate is *positive*, which cannot be done when there are only young and old cohort, unless there is a negative population growth rate as in their work. Another fundamental difference is that, in order to have some transfer in the economy, the young decisive largest group has a choice of placing the next period's decisive power either in the hand of next period's unskilled or the old. So we need to verify an additional condition that it is better for this period's decisive young to choose the old generation next period, which is the case.

When $s_t \geq \frac{1}{1+n}$, we have a unique situation (which is only possible when $n > 0$). In this range of values, the number of skilled is growing too fast to be curbed by reducing migration volume alone. To ensure that the decisive power of the next period lands in the hands of , that is, the old, the skilled voters (who are the largest in this period) must make the unskilled cohort grow to weigh down the growth rate of the skilled workers. This is done by restricting both the skill composition as well as the size of total migration. Only when $\sigma_t < s_t$ can the growth in skilled population can be reversed. [22]

In the appendix, we analyze how the political equilibrium moves from the most preferred policies to the policies implemented by the various coalitions. That is, how the political-economy equilibrium depends on the value of state variables, s_t.

6 Concluding Remarks

We develop and analyze a Pay-as-you-go (PAYG) welfare system and migration policies in a political economy model that provides a resolution to tensions across generations and income groups. We built a dynamic political-economic model featuring three distinct voting groups: skilled workers, unskilled workers, and retirees, with both *inter*- and *intra*-generational redistribution, resembling a welfare state. The skilled workers are net contributors to the welfare state

[21] One can easily verify that, given $\sigma_t = 1$, the immigration quota that will put the old generation as the largest group in the next period must satisfy $\mu_t \in \left[n - (1+n)s_t, \frac{1-(1+n)s_t}{m}\right]$.

[22] Empirically, with the population growth rate of the major host countries for migration like the U.S. and Europe going below 1%, it is unlikely that this case should ever be of much concern. Barro and Lee (2000) provides an approximation of the size of the skilled. While Barro and Lee statistics capture those 25 years and above, they also cite OECD statistics which capture age group between 25 and 64. The percentage of this group who received tertiary education or higher in developed countries falls in the range of 15 to 47 percent.

whereas the unskilled workers and old retirees are net beneficiaries. We provide the analytical characterization of the political-economic equilibria of the tax rate, skill composition, and the total number of immigrants.

We adopt the electoral system as studied by Besley and Coate (1997), known as the citizen-candidates model with strategic voting behaviors (also similarly by Osborne and Slivinski (1996) albeit under sincere-voting behaviors). Each of the three distinct voting groups presents a candidate who will implement the most preferred policy of his group, if elected. When one of these groups enjoy a majority (that is, it constitutes more than 50 percent of the voters), then its candidate automatically wins the election and implements his most preferred policy. We note that the current preferred policy (especially with respect to immigration) takes in to account how immigration may change the composition of the voters in each group and, consequently, the policy that will be implemented in the future.[23]

When no group enjoys the majority, there is effectively a ("second round") run-off between two candidates representing the two largest group of voters. Despite having its representative candidate, the third group, the smallest, will vote for the candidate of that one of the two largest groups whose most preferred policy is better for the third group, even though this policy is not the most preferred policy by the third group.

The model is designed to make a three dimensional policy choice in such a way that there are a clear "left" group, a "center" group, and a "right" group. The left group consists of the old native-born and the old first-generation immigrants (both skilled and unskilled) who earn no income and wish to extend as much as possible the generosity of the welfare state. They prefer to admit as much as possible skilled immigrants to help finance the generosity of the welfare state. The right group consists of the native-born skilled workers who bear the lion share of financing the welfare state and wish therefore to downscale its generosity as much as possible. The attitude of this group toward skilled immigrants is subject to two conflicting considerations. On the one hand, they benefit from the contribution of the skilled immigrants to the financing of the welfare state which alleviate the burden on them. On the other hand, they are aware that the offspring of the skilled immigrants will vote to downscale the generosity of the welfare state in the next period, when the members of this right group turn older and benefit from the generosity of the welfare state. This

[23]For instance, the current political debate in the U.S. about the path to citizenship of the existing illegal immigrants is affected by current expectations about how these new citizens may affect the composition of the future voters.

consideration is amplified by the fact that the fertility rate of immigrants is higher than that of the native-born.

The center group consists of the native-born unskilled young. They do like the generous welfare state but not as much as the old because they also pay for it. They like it more than the native-born skilled young because they pay less for it making them still a net beneficiary. With respect to immigration, they (like the native-born skilled young) face two conflicting effects. On the one hand, they would like to admit skilled immigrants who contribute positively towards the finances of the welfare state at the current period. But, on the other hand, they are concerned that the skilled offsprings of these skilled migrants will tilt the political balance of power in favor of the skilled in the next period; and, consequently, against the generosity of the welfare state. The center group is more pro-skilled immigrant than the left group, but similar in attitude to the right group.

The evolution of the fiscal and immigration policy of the economy over time depends naturally on the state at which it starts. The state of this stylized economy depends exclusively on the share s_t of the native-born skilled young in the total native-born young population.

We find that there are several decisive ranges for this share which determine which of the three groups most preferred policies will be implemented. These ranges are arranged from the lowest values of s_t (starting from 0) to the largest values of s_t (ending at 1). We note also that these ranges depend on the fertility rates of the native-born and the first-generation immigrants.

When s_t falls in the lowest range, the most preferred policy that will be implemented is that of the center group (the native-born unskilled young). In this case, this group forms a majority and its candidate is able to implement his most preferred policy: moderate welfare-state generosity with large, but no extreme, influx of skilled immigrants only. Therefore, the share of the native-born skilled grows over time. Eventually s_t enters the next range.

When s_t falls in the next range, the center group is still the largest group, but does not constitute majority. The native-born skilled is the smallest group in this case. This latter group, being on the right, always prefers the most preferred policy of the center group than the most preferred policy of the left group (the old). Therefore, the most preferred policy of the center will still be a winning one, though by a coalition (with the right) in this case, rather than by a sheer majority of the center group. Note that this policy increases over time the share s_t of the native-born skilled, and eventually s_t enters the next range.

When s_t moves into the next range, then the left group (the old) is the largest

group, but does not constitute a majority. The right group (the native-born skilled) is the smallest group. In this case, the center group (the native-born unskilled) will join a coalition led by the left, provided that the preferred tax rate of the left group is not excessive. The candidate representing the left group wins and the most preferred policy of the left will be implemented: an extreme generosity of the welfare state and an extreme influx of skilled immigrants. Consequently, s_t continues to rise, and so on. Eventually, when s_t becomes sufficiently large, the right group (the native-born skilled) becomes the largest group and its candidate will get to implement the group's most preferred policy: the generosity of the welfare state will be severely downscaled. All will be concerned that admitting more skilled immigrants; it will render the skilled in the next period an unbeatable majority, who will cut severely their benefit in the next period, when they grow old. This future threat on the welfare state balances the dynamics forces to further changes in s_t and it stops rising. Only limited skilled immigrants will be allowed, and s_t will enter the steady-state. [24]

Naturally, a lower rate of population growth (that is, an aging population) increases the political clout of the old (the left group). But it also increases the burden on the young (particularly, the skilled). The implications of aging for the political-economy evolution of coalitions are left for further research.

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Appendices

A Skill Dynamics

In this appendix, we turn our attention to the dynamics of the political-economic equilibrium. With multiple equilibria and no equilibrium selection mechanism, it is almost impossible to precisely trace out the dynamics of economy. Fortunately, we can impose one additional assumption about strategic voting to narrow immensely down the dimensions spanned by all equilibrium paths. Theoretical studies into Duvergerian outcome such as Cox (1997) led to a conclusion that supporters of lower-ranked challengers may choose to abandon their preferred candidate to vote instead for one of the two most popular leading candidates. As Myatt and Fisher (2001) put it, "[s]ometimes voters would rather abandon their preferred candidate to vote for another with a better chance of winning so as best to influence the outcome of the election." Such a switch may further trigger a bandwagon effect of more switching from lower-ranked to the top-two runners. In the extreme case, the process continues until we reach a Duvergerian equilibrium, only two candidates receive votes, while the rest receive none. In light of this literature, we further assume that *in most of the time only the smallest group of the time t can vote strategically at that time t unless not supported by the voting equilibrium, in which case, we will apply the voting equilibrium which appears to most resemble a Schelling's focal point* (Schelling, 1980). Equipped with this new assumption, we can partition the state space into different segments according voting strategy as follows.

First, when $s_t \in \left[0, \frac{n+m\mu_{t-1}}{2(1+n+\mu_{t-1}(1+m))}\right)$, the unskilled workers from the decisive majority, and elect their candidate to implement their most preferred policies: intermediate welfare state with some skilled immigrants, but not the maximal admission (recall that they must be careful with skilled immigration policy not to place the skilled offsprings as the decisive majority in the next period when they are old). In this region, strategic voting does not matter. When $s_t \in \left[\frac{n+m\mu_{t-1}}{2(1+n+\mu_{t-1}(1+m))}, \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}\right)$, the unskilled group is the largest, but not majority, the skilled group is the smallest. However, there is no strategic voting from the skilled voters to the old candidate as they stand on the opposite end of political spectrum. To the skilled, policies of the unskilled candidate are already more attractive than the policies of the old candidate. The next region is when $s_t \in \left[\frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \frac{1}{2}\right)$, in which the old forms the largest group in the economy with the skilled being the smallest group. In this case, the skilled voters will strategically vote for the unskilled candidate to avoid Laffer point

tax rate and maximal skilled migration. The collusion will be enough to overcome the size of old voters and push for implementation of the policies preferred by the unskilled candidate (given that $n+m>0$). As $s_t \in \left[\frac{1}{2}, \frac{1-\frac{m-n}{2}}{1+n}\right)$, the largest group is still the old voters, while the smallest group switches to be the unskilled voters. If the Laffer tax rate is *not* too extractive ($\frac{1}{1+\varepsilon} \leq \widetilde{\tau}$), the unskilled will not vote strategically for the skilled candidate. But if the tax rate is too extractive ($\frac{1}{1+\varepsilon} > \widetilde{\tau}$), the unskilled will strategically vote for the skilled candidate to defeat the old candidate, hence implementing the preferred policies of the skilled candidate. A special case arises when $s_t \in \left[\frac{1-\frac{m-n}{2}}{1+n}, \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}\right]$, the region in which the size of skilled is currently too large for the unskilled voters to risk putting the skilled voters as the decisive majority in the next period, killing the chance of having any welfare state in retirement. Hence both voting strategically or sincerely by the unskilled voters are no longer supported as an equilibrium unless the skilled voters choose to vote strategically for the unskilled candidate (the candidate with the smallest support) in order to defeat the old candidate.[25] When $s_t \in \left(\frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))}\right)$, the skilled is now the largest group while the unskilled is the smallest group. However, since the size of skilled group is already so large, the unskilled voters will not vote strategically for the old candidate who will maximal open the economy to skilled migrants. Here, we again appeal the Schelling point's argument. While after observing the state variable, the old voters realize that their old candidate will not receive the strategic support from the unskilled voters and has no chance of defeating the skilled candidate. However, since the old voters prefer the policies of the unskilled over the skilled candidate, the unskilled voters know that the old voters are willing to support the unskilled candidate and its policy. Hence, the coalition with the old voters voting strategically for the unskilled candidate will be enough to defeat the status quo, the preferred policies of the skilled candidate. Lastly, when $s_t \in \left[\frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))}, 1\right]$, the skilled voters form the decisive majority and implement their preferred policies irrespective of strategic voters.[26]

[25] Alternatively, the unskilled may choose to take a bitter pill by voting strategically for the skilled candidate in order to defeat the old candidate under the condition that the forgone welfare benefit today must be less than the discounted benefit in retirement. Otherwise, we revert back to what we think as more sensible collusion of skilled and unskilled for the intermediate policies preferred by unskilled candidate (Schelling point). This after all is an improvement for both the skilled and unskilled voters over the status quo (policies of the old).

[26] You may notice that the old voters are never the smallest group. This can be proven formally as follows. Suppose to the contrary that the old voters are the smallest group, that is $s_t > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}$ and $1-s_t > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}$. Adding the two inequalities yield

Below we explicitly write out the voting equilibrium associated with each partition of the state space. Voting vectors are in rows, so the first, second, and last row correspond to votes cast by the skilled, unskilled, and old voters respectively. The columns denote to whom the votes were casted for with the first, second, and last column correspond to voting for skilled, unskilled, and old candidate, respectively. Therefore, any off-diagonal vote imply strategic voting. The voting equilibria for the case of $\frac{1}{1+\varepsilon} \leq \widetilde{\tau}$ are as follows:

$$\mathbf{e}^*(s_t) = \begin{cases} \begin{pmatrix} \eta^s & \xi^s & 1-\eta^s-\xi^s \\ 0 & 1 & 0 \\ \eta^o & \xi^o & 1-\eta^o-\xi^o \end{pmatrix} & , \text{ if } s_t \in \left[0, \frac{n+m\mu_{t-1}}{2(1+n+\mu_{t-1}(1+m))}\right) \\ \begin{pmatrix} \eta^s & 1-\eta^s & 0 \\ 0 & 1 & 0 \\ 0 & \xi^o & 1-\xi^o \end{pmatrix} & , \text{ if } s_t \in \left[\frac{n+m\mu_{t-1}}{2(1+n+\mu_{t-1}(1+m))}, \frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}\right) \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} & , \text{ if } s_t \in \left[\frac{n+m\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \frac{1}{2}\right) \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} & , \text{ if } s_t \in \left[\frac{1}{2}, \frac{1-\frac{m-n}{2}}{1+n}\right) \\ \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} & , \text{ if } s_t \in \left[\frac{1-\frac{m-n}{2}}{1+n}, \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}\right] \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} & , \text{ if } s_t \in \left(\frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}, \frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))}\right) \\ \begin{pmatrix} 1 & 0 & 0 \\ \eta^u & \xi^u & 1-\eta^u-\xi^u \\ \eta^o & \xi^o & 1-\eta_o-\xi^o \end{pmatrix} & , \text{ if } s_t \in \left[\frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))}, 1\right] \end{cases} \tag{42}$$

where $0 \leq \eta^i, \xi^i \leq 1$ are probabilistic vote of type $i \in \{s, u, o\}$ voters for the skilled and unskilled candidate, respectively.

The dynamics of the economy are given as follows for the case of $\frac{1}{1+\varepsilon} \leq \widetilde{\tau}$:

$\frac{1}{2} > \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}$. However, some algebra will reveal that $\frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)} \in [\frac{1}{2}, 1]$ which produces a contradiction. Intuitively, for the old voters to be the smallest group, population growth rates must be 100% or higher! Otherwise, which ever way we slice the young generation into skilled and unskilled, one group will always be smaller than the old group.

$$s_{t+1} = \begin{cases} \frac{(1+m)(2+n)-(1+n)(2+m)s_t}{(1+m)(2+n)-(1+n)(2+m(2-s_t))} & , \text{ if } s_t \in \left[0, \frac{1}{2}\right) \\ \frac{(1+n)s_t+(1+m)}{2+n+m} & , \text{ if } s_t \in \left[\frac{1}{2}, \frac{1-\frac{m-n}{2}}{1+n}\right) \\ \frac{(1+m)(2+n)-(1+n)(2+m)s_t}{(1+m)(2+n)-(1+n)(2+m(2-s_t))} & , \text{ if } s_t \in \left[\frac{1-\frac{m-n}{2}}{1+n}, \frac{1}{1+n}\right] \\ \frac{(1+n)s_t+(1+m)\widehat{\sigma}\widehat{\mu}}{1+n+(1+m)\widehat{\mu}} & , \text{ if } s_t \in \left(\frac{1}{1+n}, 1\right]. \end{cases} \tag{43}$$

It is worthwhile to note that, although the dynamics appear similar to the sincere voting case, the voting game is much more complex than the above equations appear.

For the case $\frac{1}{1+\varepsilon} > \widetilde{\tau}$, the Laffer tax rate to too extractive for the unskilled workers, so the skilled and unskilled have incentives to try to block the old candidate from coming into power. In this case, there is no strategic voting from the unskilled voters to the old candidate, only for the skilled candidate. In this case, when $s_t \in \left[\frac{1}{2}, \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}\right)$, the strategic coalition would successfully push for to implement of policies of the skilled candidate (unskilled voters vote strategically under the assumption that the smallest group will do so), hence $s_{t+1} = \frac{(1+m)(2+n)-(1+n)(2+m)s_t}{(1+m)(2+n)-(1+n)(2+m(2-s_t))}$. As the skilled voters form the largest group, or $s_t \geq \frac{1+\mu_{t-1}}{1+n+\mu_{t-1}(1+m)}$, two possible coalitions could form in equilibrium. First, as in the "Right-winged" voting equilibrium above, the old voters could strategically vote for the unskilled candidate on top of votes by the unskilled voters to ensure the survival of the welfare state. However, if $s_t \geq \frac{1-\frac{m-n}{2}}{1+n}$, the size of skilled voters is already too large for fully opening the economy to skilled migration, so a coalition between skilled and unskilled voters comes into effect (which fits the above assumption that only the smallest group vote strategically). The second type of coalition continues until the skilled voters form the majority or $s_t \in \left[\frac{2+n+\mu_{t-1}(2+m)}{2(1+n+\mu_{t-1}(1+m))}, 1\right]$.