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MARITAL PATTERNS OVER SEVERAL
DECADES**

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LABOUR ECONOMICS



Centre for Economic Policy Research

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PARTNER CHOICE AND THE MARITAL COLLEGE PREMIUM: ANALYZING MARITAL PATTERNS OVER SEVERAL DECADES[†]

Abstract

We construct a structural model of household decision-making and matching and estimate the returns to schooling within marriage. We consider agents with idiosyncratic preferences for marriage that may be correlated with education, and we allow the education levels of spouses to interact in producing joint surplus. Using US data on marriages of individuals born between 1943 and 1972, we show that the preference for assortative matching by education has significantly increased for the white population, particularly for highly educated individuals; but not for blacks. Moreover, in line with theoretical predictions, we find that the “marital college-plus premium” has increased for women but not for men.

JEL Classification: D1 and J1

Keywords: college premium and marriage

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1 Introduction

The joint evolution of US male and female demand for college education over the recent decades raises an interesting puzzle. During the first half of the century, college attendance increased for both genders, and slightly faster for men. According to Claudia Goldin and Larry Katz (2008), male and female college attendance rates were about 10% for the generation born in 1900, and reached respectively 55% and 50% for men and women born in 1950. This common trend, however, broke down for the cohorts born in the 50s and later. These individuals faced a market rate of return to schooling (the “college premium”) that was substantially higher than their predecessors; therefore one would have expected their college attendance rate to keep increasing, possibly at a faster pace. This prediction is satisfied for women: 70% of the generation born in 1975 attended college. On the contrary, the male college attendance rate increased at a much slower rate, if at all. The phenomenon is especially striking for post-graduate education (the so-called “college-plus” levels: MAs, MBAs, law and medicine degrees, PhD), for which the labor-market premium has risen even more. The fraction of women aged 30 to 40 with a post college degree exceeds 11% in 2005 (against less than 5% in 1980), while the male proportion has hardly changed over the same period. As a result, in recent cohorts women are more educated than men, by an increasing margin.

To explain these strikingly asymmetric responses to seemingly identical incentives, Pierre-André Chiappori, Murat Iyigun and Yoram Weiss (2009, from now on CIW) stress the role of gender differences in the returns to schooling within marriage¹. They argue that the return to education has two distinct components. One is the standard market college premium, whereby a graduate degree significantly increase wages. This component has evolved in a similar way for men and women²; if anything, men have gained more since they still work more hours on average. Secondly, education has an impact on a person’s situation on the marriage market; it affects the probability of getting married, the characteristics of the future spouse, and the size and distribution of the surplus generated within marriage.

¹Another, largely complementary explanation proposed by Becker, Hubbard and Murphy (2009) relies on the differences between male and female distributions of unobserved ability. Still, these authors also emphasize that educated women must have received some additional, intrahousehold return to their education. It is precisely this additional term that our approach allows to evaluate.

²See for instance Becker, Hubbard and Murphy (2009), Fig 7 and Mary Ann Bronson (2014), Fig 3.

CIW advance the hypothesis that, unlike the market premium, this “marital college-plus premium” may have evolved in a highly asymmetric way between genders. In their paper, agents have heterogeneous attitudes towards marriage and heterogeneous costs of human capital acquisition; their investment in education is based on their (rational) expectations about the total returns, adding labor market and marriage market returns. These in turn are determined in equilibrium by the distribution of education by gender and by preferences. Various technological changes reducing household chores (as in Jeremy Greenwood et al., 2005) as well as progress in birth control technologies, medical techniques and infant feeding methods (stressed by Robert Michael, 2000, Goldin and Katz, 2002, and Stefania Albanesi et al., 2009) can trigger a change in equilibrium, leading to marital returns to education that are higher for women than for men. This type of asymmetry can then generate diverging demands for higher education of women and men³.

While this argument is theoretically consistent, establishing its empirical relevance is a challenging task. In contrast to returns to schooling in the labor market, which can be recovered from observed wages data, returns to schooling within marriage are not directly observed and can only be estimated indirectly from the marriage patterns of individuals with different levels of schooling.

Our paper provides the first such estimates. Specifically, we consider a frictionless matching framework with Transferable Utility (TU). The analysis of the marriage market as a matching process, which dates back to Gary Becker’s seminal contributions (see Becker 1973, 1974, 1991) has recently attracted renewed attention.⁴ Our approach is explicitly

³Our work is related to a recent contribution by Corinne Low (2014), who studies the female demand for graduate education over the last decades. In her model, husbands trade off the education of a potential wife with her age: women who undertake graduate studies typically enter the marriage market later, which reduces their fertility. She argues that the nature of the equilibrium has switched over the last decades. A graduate degree used to be associated with both lower marriage probability and lower spousal income in case of marriage; this trend has totally disappeared over the last decades. Low’s theoretical model is fully compatible with our approach, which can be seen as providing a complementary, structural empirical implementation.

⁴For instance, Jeremy Fox (2010a, 2010b) provides a nonparametric approach that does not explicitly model the stochastic structure of the joint surplus; instead, it relies on a “rank order” property, which postulates that assignments that generate more surplus in a deterministic model are more likely to be observed when stochastic aspects are introduced. The reader is referred to Graham (2011) and Chiappori and Salanié (2014) for a general presentation of different approaches.

structural, in the line of the seminal contribution by Eugene Choo and Aloysius Siow (2006, from now on CS), who study the response of the US marriage market to the legalization of abortion⁵. Their work showed the value of introducing a specific stochastic structure of the surplus generated by any match between two individuals. In their framework, this joint surplus is the sum of a systematic, deterministic component that only depends on observable traits (in our case education levels), and a stochastic part reflecting unobserved heterogeneity. Crucially, CS assumed that the latter is additively separable into a wife- and a husband-specific parts, each of which only depends on the education of the potential spouse. As a consequence, the matching equilibrium conditions translate into a simple, discrete choice structure. Alfred Galichon and Bernard Salanié (2012) generalize the Choo and Siow framework to arbitrary separable stochastic distributions; they also provide a theoretical and econometric analysis of multicriteria matching under the same separability assumption. Mourifié and Siow (2014) extend the Choo and Siow model in another direction, by allowing for peer effects in the joint surplus.

Our paper extends this literature in several directions. First, we clarify the underlying theoretical structure needed for these approaches. We consider a structural model of matching on the marriage market that is close, in spirit, to that adopted by CIW. The model provides an explicit representation of household behavior based on a collective framework, with individual preferences belonging to Theodore Bergstrom and Richard Cornes’s “Generalized Quasi Linear” (GQL) family (2003). Such preferences are known to generate a TU framework; i.e., they admit a cardinal representation in which the Pareto frontier, for all values of wages and incomes, is a straight line with slope -1, and whose intercept is an increasing convex function of the household’s total income. Agents match after choosing their education level, but before their permanent income is revealed; they therefore consider their expected surplus conditional on their educational level and that of potential partners. In addition, still following CIW, we assume that each individual has idiosyncratic preferences for marriage, which are known before investment in human capital is decided. We work out the implications of this framework for the key endogenous variables, namely individual utilities at the stable matching. It is well known that these utilities can be obtained as the dual variables of a linear programming problem: the maximization

⁵See Maristella Botticini and Siow (2008) and Siow (2009) for other applications.

of aggregate surplus over all possible matchings. We show that, under the separability assumption introduced by CS, the stochastic distribution of these dual variables can be fully characterized. In particular, one can identify the distribution of expected utility for any possible choice of a spouse, as well as for the optimal choice—a notion that is directly related to the concept of “marital college premium” introduced by CIW.

Second, we extend the CS approach to a “multi-market” framework⁶; that is, we consider several cohorts of men and women, which introduces variation in the proportions of men and women at all education levels. From a methodological perspective, the multi-market approach allows to relax the very restrictive assumptions imposed by the initial CS contribution. In particular, our model is compatible with class-specific distributions of unobserved heterogeneity (thus allowing for selection into education being correlated with preferences for marriage, a natural consequence of CIW), as well as with class-specific temporal drifts in the systematic component of the surplus. We show that the model is overidentified, and we present specification tests.

From an empirical perspective, the model allows us to study the evolution of matching patterns throughout time, with both gains from marriage and the intra-household allocation of these gains changing over time for each education class. From this information, we can extract the time patterns of the marital education premiums of men and women. We argue that the use of an explicit, structural model helps clarifying some complex issues regarding the level of assortative matching by education and its evolution over the last decades. Several observers have claimed that assortative matching by education is stronger now than four decades ago. Burtless (1999) for instance argues that this evolution complements the increase in the labor market college premium in explaining increased interhousehold income inequality. This view is supported by a recent sociological literature concluding that homogamy has increased in the US and several other countries (see for instance Schwartz and Mare, 2005.) However, these conclusions were drawn from reduced-form, log-linear models with no direct economic interpretation. We argue that, in the absence of a well-defined, structural background, these approaches can be misleading. The mere definition of “stronger assortative matching”—and therefore the metrics used to

⁶See Fox (2010a, 2010b) and Fox and Yang (2012) for different approaches to pooling data from many markets.

quantify the phenomenon—raise complex issues. The percentage of couples in which both spouses have a college or “college-plus” degree has significantly increased over the period. At the same time, however, as women with college degrees became more numerous, the proportion of educated women who marry educated men has *declined*, as more educated women “marry down” (with less educated men.) Part of the increase in the proportion of couples where both partners have similar educations reflects the shifts in the education of women, while some of it may also derive from changes in preferences towards assortative matching. How to formally disentangle these two effects is a crucial aspect of our discussion.

An important advantage of our structural approach is that it provides both a structural definition of the notion of “preferences for assortativeness” (in our context, it is related to the supermodularity of the deterministic part of the surplus function) and an explicit way of distinguishing such changes in preferences from the mechanical effects of the observed changes in male and female education. Empirically, we allow the supermodular part of the surplus to evolve according to a linear educations-dependent trend; we show that it is identified and we estimate it.

Our model enables us to revisit the intriguing differences in marriage patterns of blacks and whites in the US. It is well documented that after World War II, the marriage patterns of whites and blacks started to diverge, as the marriage rates of blacks fell faster for both genders. Starting with Wilson and Neckerman (1986), a growing literature has attributed these trends to the shortage of “marriageable” black men⁷. We bring into this discussion differences across groups in preferences for assortative marriage by education, and the resulting differences in the marriage college premium. Regarding blacks, while marriage rates declined for all education groups (and particularly for the less educated categories), there is no evidence of changes in preferences for assortativeness: the supermodularity of the surplus function hardly changed over time. Moreover, the “marital college premium”, the additional marital surplus received by an agent as a result of acquiring a graduate degree, evolves in pretty much the same way for men and women.

Our conclusions are quite different for whites. First, we find strong evidence of increased

⁷See also Schoen and Klugel (1988), Bennett, Bloom and Craig (1989), Stevenson and Wolfers (2007) and Seitz (2009).

preferences for assortativeness, particularly at the top of the education distribution. Second, the evolution of marital returns to education has been highly gender-specific. For men, it is pretty much constant over the period, whereas it spectacularly increases for women; statistical tests confirm that the difference in trends is very significant. This is fully in line with the argument offered by CIW.

The evolution of the marital college-plus premium can be decomposed into several components. Highly educated white women used to face a handicap in getting married; this has disappeared, as descriptive statistics show. In addition, our estimated model suggests that college-plus educated white women create more surplus than they used to (especially with a husband of similar education), and receive a higher share of that surplus. Only a structural approach such as ours allows to explicitly disentangle these evolutions.

Section 2 presents some stylized facts. Then we introduce our theoretical framework in Section 3, and section 4 describes the basic principles underlying its empirical implementation. In Section 5, we discuss identification issues and present our main theoretical results. Section 6 describes the matching patterns in the data. Our empirical findings are presented in Section 7.

2 The Data

2.1 Constructing the data

We begin by describing our data and some stylized facts about the evolution of matching by education over the last decades in the US. We use the American Community Survey, a representative extract of the Census, which we downloaded from IPUMS (see Ruggles et al (2008).) Unlike earlier waves of the survey, the survey has collected information on current marriage status, number of marriages, and year of current marriage since 2008. Our analysis uses the data from 2008 to 2012. From this population, we extract all white and black adults (aged 18 to 70) who are out of school. For the white population, we therefore start from a sample of 6,518,541 individuals, of which 49.6% are males. We use the “detailed education variable” of the ACS to define five subcategories:

1. High School Dropouts (HSD)

2. High School Graduates (HSG)
3. Some College (SC) — including two-year (associate) degrees
4. Four-year College Graduates (CG)
5. Graduate degrees (“college-plus”, or CG+.)

However, the black population in our sample is smaller and less educated; therefore we merge categories 4 and 5 into a single, “College and college-plus” category.

When empirically studying matching patterns, one has to address several practical issues. One is to decide which matches to consider: the current match of a couple, or earlier unions in which the current partners entered? Also, do we define a single as someone who never married, or as someone who is currently not married? It is notoriously hard to model divorce and remarriage in an empirically credible manner⁸. Since this is not the object of this paper, we chose instead to only keep first matches, and never-married singles. Given this sample selection, in each cohort we miss:

- those individuals who died before the survey;
- those who are single in the survey year but were married before: there are
 - 110,016 white (48,046 black) individuals who are separated from their spouse
 - 842,972 (134,351) who are divorced
 - 196,791 (40,729) who are widowed.

We are therefore left with 5,368,762 (708,299) individuals, out of which 1,777,424 (469,447) are single and 3,591,338 (238,852) are married.

- those who are married in the survey year, but not in a first marriage. This reduces the number of married individuals to 2,327,200 white and 148,606 blacks.

⁸Information on marital dissolution by education of the partners is available in the PSID, SIPP and NSFG but these samples are much smaller than the IPUMS sample that we use to analyze marriages.

Another standard problem is truncation: young men and women who are single in the survey year may still marry. In our figures (and later in our estimates) we circumvent this difficulty by stopping at the male cohort born in 1968 (1963 for blacks); this choice is motivated by the fact that the first union occurs before age 40 (45 for blacks⁹) for most men and women. To examine marriage patterns, we also drop the small number of couples where one partner married before age 16 or after age 40 (recall that these are first unions.) This leaves us with 1,506,384 (75,876) married individuals, 197,080 (31,955) single men, and 153,495 (47,053) single women for whites (blacks).

Finally, we need to define the notion of a “cohort”. From a theoretical perspective, each cohort should represent a “market” (or a matching game), involving specific populations; and our goal is to observe changes in matching patterns resulting from variations in the distribution of education by gender across cohorts. As always, reality is more complex, because the various “cohorts” tend to mix. For instance, if we define a cohort by the year of birth, then the spouse of a man born in 1957 is most likely to be born in 1958: the modal age difference is one year in our data. Yet such a man may well marry a woman born in 1956 or in 1960. Defining broader cohorts (e.g., men born between 1955 and 1960) does not solve the mixing problem, and has the additional drawback of reducing the number of cohorts—which is problematic since, as we shall see, the testability of our approach relies on the restrictions we assume regarding the evolution of economic fundamentals across cohorts. In what follow, we consider two possible solutions. One is to exclusively concentrate on couples in which the difference takes some fixed value (the modal value of one year in our case.) Then we consider men born between 1940 and 1967 and women born between 1941 and 1968, a total of 28 cohorts. Alternatively, one may explicitly model the age difference as a choice parameter, the difference between husband’s and wife’s age as a choice parameter, which can take seven values—from “-1 or less” to “5 or more”; in this case the cohorts become 1940-1967 for men and 1940-1971 for women. Our benchmark model follows the first approach, whereas the second strategy is discussed in the section devoted to extensions. In practice, and reassuringly, the two approaches give similar results.

⁹The data show that blacks continue to marry later than whites.

2.2 Patterns in the data

The trends in education levels of white men and women are shown in figures 1a and 1b. In cohorts born after 1955, women are more likely than men to attend college; for those born after 1965, they are also more likely to achieve a college-plus degree. Not coincidentally, the proportion of marriages in which the husband is more educated than the wife has fallen quite dramatically. Indeed, Figure 2 shows that the percentage of couples in which spouses have the same education is remarkably stable (slightly below 50%) over more than three decades. However, there are now more couples in which the wife is more educated than the opposite.

Fig 1 and 2 about here

Figures 3 and 4 illustrate the decline in marriage by plotting the percentage of individuals who never married by cohort and education. They show that, for both genders, a higher education has tempered the decline in marriage; high-school dropouts, on the other hand, have faced a very steep decline in marriage rates. However, female patterns are very specific. For the older cohorts of our sample, a college-plus degree had a strong, negative effect on the probability of getting married for women, but not for men. This gender difference has largely disappeared in recent cohorts: college-plus women now marry as much as college graduates, and much more than high-school educated women.

Fig 3 and 4 about here

Figures 5a and 5b describe marital patterns by education. It shows that college-educated men are now much less likely to “marry down” (about 25%, against 50% for men born in the early 40s). The pattern for women is opposite; for instance, the chances that a college-educated woman would marry up (to a college-plus husband) has dropped from 40% to 20%, over the period.

Fig 5 about here

Examining the education and marriage patterns of blacks, we find some striking differences from whites. Black women have always been at least as likely as men to get a college or a college-plus degree; and in contrast with the white population, the proportions of a cohort with at least a college degree evolve in a very similar way for men and for women (figures 6a and 6b.) Secondly, marriage rates have declined much faster for blacks than for whites (see figures 7a,7b.) For the cohorts born in the mid 60s, the fraction of never married at age 45 is above 40% for all education classes, and exceeds 80% for high school drop-outs. Thirdly, for the older cohorts, one does not observe the same difference between genders as for whites. While the percentage of couples with equal education are similar for blacks and whites (a stable proportion around 45%), in unequally educated couples the wife is always more likely to be the more educated person among blacks. Finally, the proportion of college educated individuals who never marry was much larger for men than women in the older cohorts; recent percentages are quite similar. In other words, the spectacular differences across genders that characterize the white population cannot be seen in the African-American sample¹⁰.

Fig 6 and 7 about here

Finally, the evolution of age differences between spouses for the white and black populations is given in Tables 1a and 1b, which compare the first and last cohorts.

The age difference has decreased over time, but not by much: the mean difference went from 1.9 years to 1.3 years for whites, and from 2 years to 1.4 years for blacks.

A large literature has evaluated the changes in the value of a higher education on the labor market. While this labor market college premium seems to have evolved in very similar ways across genders and races, the patterns documented here clearly suggest that the effects of a higher education on marital prospects have diverged much more. Yet descriptive statistics alone cannot measure changes in the joint surplus of matches and in its division between the partners; a more precise evaluation of this “marital college

¹⁰We should also note here the increase in interracial marriages that started around 1960 and accelerated recently. Black men are much more likely than black women to marry a white spouse—see Gullickson (2006) and Cherlin (1992, Chapter 4.) Our model does not consider these additional dimensions, which were less prominent in our sample period.

premium” requires an explicit structural model.

3 Theoretical Framework

Our model derives from CIW. We consider an economy with two periods and large numbers of men and women. In period one, agents draw costs of investment in human capital and marital preferences from some random distributions; then they invest in education by choosing from a finite set of possible educational levels. In period 2, agents match on a frictionless marriage market with transferable utility; they each receive an income, the realization of which depends on the agent’s education; and they consume, according to an allocation of resources that was part of the matching agreement.

When investing in human capital, agents must anticipate the outcome of their investment. This outcome has two distinct components. One is a larger future income. In our framework, this effect is taken to be exogenous, and to benefit single and married agents alike. Second, a higher educational level has an impact on marital prospects; it affects the probability of getting married, the expected income of the future spouse, the total utility generated within the household, and the intra-couple allocation of this utility. These “marital gains”, however, depend on the equilibrium reached on the marriage market; this in turn depends on the distribution of education in the two populations, and ultimately of the investment decisions made in the first period. As usual, the model can be solved backwards using a rational expectations assumption; equilibrium is reached when the marital gains resulting from given distributions of education for men and women trigger first period investment decisions that exactly generate these distributions. Note that even if marital preferences and investment cost were independent ex ante, education decisions made during the first period must be correlated with preferences for marriage ex post: since agents with stronger preferences for marriage are more likely to receive the marital gain than agents who prefer to stay single, they have stronger incentives to invest in education.

In the present paper, we aim at estimating and testing the second period behavior described by this model. This choice is mostly dictated by available data: while private costs of human capital investment are not observable, the resulting distribution of education by gender is. In addition, concentrating on the second period allows to introduce a slightly more general framework while addressing the empirical content of the key theoretical con-

cept: the notion of a marital college premium. We therefore consider the situation at the beginning of the second period. Agents are each characterized by their chosen level of education, which belongs to some finite set and is observable by all, and by their preferences for marriage, which are observed by their potential mates but not by the econometrician. Our goal is to identify the underlying structure from observed matching patterns; and we are particularly interested in the marital gains associated with each educational level.

3.1 The model

3.1.1 Preferences

The economy consists of a male population \mathcal{M} , endowed with some continuous, atomless measure $d\mu_{\mathcal{M}}$, and a female population \mathcal{F} , endowed with some continuous, atomless measure $d\mu_{\mathcal{F}}$. Each man i belongs to a class $I = 1, \dots, \mathbf{I}$ which is observed by the econometrician; similarly, each woman j belongs to a class $J = 1, \dots, \mathbf{J}$. These classes may in principle be exogenous, like race. In our case, they refer to education, and are therefore the outcome of individual choices made before the matching stage; this is the gist of CIW's contribution.

Individual utilities consist of an economic and a non economic, predetermined component. Regarding the economic component, there are $(n + N)$ commodities, of which n are privately consumed by each individual and N may be publicly consumed by a couple. The preferences over commodities of each individual k are of the GQL form (Bergstrom and Cornes, 1983):

$$U_k(\mathbf{q}_k, \mathbf{Q}) = w_k(\mathbf{q}_k^{-1}, \mathbf{Q}) + q_k^1 B(\mathbf{Q})$$

where $\mathbf{q}_k = (q_k^1, \dots, q_k^n)$ is the vector of private consumptions by agent k ; the vector $\mathbf{q}_i^{-1} = (q_i^2, \dots, q_i^n)$; $\mathbf{Q} = (Q^1, \dots, Q^N)$ is the vector of household's public consumptions, and B is a positive function.

Since we do not consider price variations, we can normalize all prices to 1. An individual who is single and has an income x_k , would choose $(\mathbf{q}_k, \mathbf{Q})$ to solve

$$\max_{\mathbf{q}_k, \mathbf{Q}} w_i(\mathbf{q}_k^{-1}, \mathbf{Q}) + q_k^1 B(\mathbf{Q}) \text{ such that } \sum_{l=1}^n q_k^l = x_k.$$

Note that, in this program, all commodities (including \mathbf{Q}) are de facto private—that is exactly the economic cost of singlehood. Denote $V_k(x_k)$ the value of this program.

In a couple (i, j) , it is well-known that GQL preferences imply Transferable Utility; as a consequence, any Pareto efficient consumption such that $q_i^1 q_j^1 > 0$ must maximize the *unweighted* sum of utilities. We therefore define:

$$G^{ij}(x_i + x_j) = \max_{\mathbf{q}_i, \mathbf{q}_j, \mathbf{Q}} U_i(\mathbf{q}_i, \mathbf{Q}) + U_j(\mathbf{q}_j, \mathbf{Q}) \text{ s.t. } \sum_k (q_i^k + q_j^k) + \sum_l Q^l = x_i + x_j.$$

It is easily seen by substituting the budget constraint that S_{ij} is the maximum of a family of affine functions of $(x_i + x_j)$ with positive slopes $B(\mathbf{Q})$. As such, it is an increasing and convex function, and it is strictly convex under weak assumptions¹¹ A direct consequence is that the incomes x_i and x_j of the partners are complementary in the production of total, economic utility $G^{ij}(x_i + x_j)$.

By the date of marriage incomes are not yet realized; agents only know the probability distribution of future incomes, conditional on current education level. We therefore define the *expected* economic surplus as:

$$S^{IJ} = E[G_{ij}(x_i + x_j) | I, J] - E[V_i(x_i) | I] - E[V_j(x_j) | J]$$

In addition to (economic) preferences over commodities, each individual has marital preferences which we model by random vectors. Following Choo and Siow (2006), we assume that an individual preferences over potential spouses are individual-specific and only depend on the spouse's (education) class. For instance, a woman j belonging to class J has a vector of marital preferences

$$b_j^J = (b_j^{0J}, b_j^{1J}, \dots, b_j^{IJ})$$

where b_j^{KJ} denotes the utility j derives from marrying a spouse with an education K (and where, by convention, b_j^{0J} denotes the utility j derives from staying single). Similarly, man i 's idiosyncratic marital preferences are described by the vector

$$a_i^I = (a_i^{I0}, a_i^{I1}, \dots, a_i^{IJ})$$

¹¹See for instance Browning, Chiappori and Weiss (2014, ch. 7).

where I denotes i 's education. Note that, in general, the *distribution* of an individual's vector of marital preferences, a_i^I , may depend on the individual's own education I ; that is, more educated men may, on average, value differently an educated wife than less educated men. Not only is this assumption quite plausible empirically, but it is also needed to reflect the endogeneity of education. In CIW, for instance, individual tastes for marriage influence investment in education, because they affect the probability that an individual reaps the benefits of education on the marriage market. Since individuals with different marital tastes invest differently, the conditional distribution of taste given education will typically vary with education, reflecting the selection into educational choices. Consequently, we define

$$A^{IJ} = E(a_i^{IJ}) \text{ and } B^{IJ} = E(b_j^{IJ}).$$

It should be stressed that, in this framework, these idiosyncratic, additively separable shocks are the *only source* of unobserved heterogeneity. This assumption is crucial for a Choo-Siow approach to apply; see Chiappori and Salanié (2015) for a detailed discussion.

Finally, and still following Choo and Siow (2006), we assume that economic and marital preferences are additively separable. To be more precise, the marital surplus s_{ij} generated by the match of man i with education I and woman j with education J is the sum of two components. One is the expected economic surplus S^{IJ} , generated by joint consumption; the other consists of (the sum of) the spouses' idiosyncratic preferences for marriage with each other, relative to singlehood.

$$s_{ij} = S^{IJ} + (a_i^{IJ} - a_i^{I\emptyset}) + (b_j^{IJ} - b_j^{\emptyset J}). \quad (1)$$

or, using the previous definitions:

$$\begin{aligned} s_{ij} = & \left(S^{IJ} + (A^{IJ} - A^{I\emptyset}) + (B^{IJ} - B^{\emptyset J}) \right) \\ & + \left((a_i^{IJ} - A^{IJ}) - (a_i^{I\emptyset} - A^{I\emptyset}) \right) \\ & + \left((b_j^{IJ} - B^{IJ}) - (b_j^{\emptyset J} - B^{\emptyset J}) \right). \end{aligned}$$

The component on the first line

$$Z^{IJ} = S^{IJ} + (A^{IJ} - A^{I\emptyset}) + (B^{IJ} - B^{\emptyset J}) \quad (2)$$

is, by construction, the conditional expectation of the total surplus for matches between classes I and J . Within it, S^{IJ} is the conditional expected economic surplus, while $(A^{IJ} - A^{I\emptyset}) + (B^{IJ} - B^{\emptyset J})$ represents the conditional expected surplus from marital preferences. The components on the last two lines have zero expectation conditional on an (I, J) match. We use

$$\alpha_i^{IJ} = a_i^{IJ} - A^{IJ} \text{ and } \beta_j^{IJ} = b_j^{IJ} - B^{IJ}$$

to denote the within-class variation of marital preferences; note that, by construction,

$$E[\alpha_i^{IJ}] = E[\beta_j^{IJ}] = 0 \text{ for all } i, j, I, J$$

Finally, the total surplus generated by the match between $i \in I$ and $j \in J$ is:

$$s_{ij} = Z^{IJ} + (\alpha_i^{IJ} - \alpha_i^{I\emptyset}) + (b_j^{IJ} - b_j^{\emptyset J}) \quad (3)$$

The matrix $Z = (Z^{IJ})$ will play a crucial role in what follows. As we shall see, the equilibrium matching will depend on preferences through the matrix Z and the distribution of the α 's and β 's. From the definitions above, Z^{IJ} reflects the distribution of income and preferences over commodities of spouses who chose education levels I and J (and each other), as well as the distribution of their marital preferences. It is therefore a complex object; but it is the crucial construct that determines marital patterns in our context. Our goal is to check under which conditions it is identifiable from matching patterns.

3.2 Matching

A matching consists of

- (i) a measure $d\mu$ on the set $\mathcal{M} \times \mathcal{F}$, such that the marginal of $d\mu$ over \mathcal{M} (resp. \mathcal{F}) is $d\mu_{\mathcal{M}}$ ($d\mu_{\mathcal{F}}$); and
- (ii) a set of payoffs (or imputations) $\{u_i, i \in \mathcal{M}\}$ and $\{v_j, j \in \mathcal{F}\}$ such that

$$u_i + v_j = z_{ij} \text{ for any } (i, j) \in \text{Supp}(d\mu)$$

In words, a matching indicates who marries whom (note that the allocation may be random, hence the measure), and how any married couple shares the surplus z_{ij} generated

by their match. The numbers u_i and v_j are the expected utilities man i and woman j get on the marriage market, on top of their utilities when they remain single; for any pair that marries with positive probability, they must add up to the total surplus generated by the union.

3.2.1 Stable Matchings

A matching is stable if one can find neither a man i who is currently married but would rather be single, nor a woman j who is currently married but would rather be single, nor a woman j and a man i who are not currently married together but would both rather be married together than remain in their current situation. Formally, we must have that:

$$u_i \geq 0, v_j \geq 0 \text{ and} \tag{4}$$

$$u_i + v_j \geq z_{ij} \text{ for any } (i, j) \in \mathcal{M} \times \mathcal{F}. \tag{5}$$

The two conditions in (4) implies that married agents would not prefer remaining single; the third (condition (5)) translates the fact that for any possible match (i, j) , the realized surplus z_{ij} cannot exceed the sum of utilities respectively reached by i and j in their current situation (i.e., a violation of this condition would imply that i and j could both strictly increase their utility by matching together).

As is well known, a stable matching of this type is equivalent to a maximization problem; specifically, a match is stable if and only if it maximizes total surplus, $\int z d\mu$, over the set of measures whose marginal over \mathcal{M} (resp. \mathcal{F}) is $d\mu_{\mathcal{M}}$ (resp. $d\mu_{\mathcal{F}}$). A first consequence is that existence is guaranteed under mild assumptions. Moreover, the dual of this maximization problem generates, for each man i (resp. woman j), a dual variable or “shadow price” u_i (resp. v_j), and the dual constraints these variables must satisfy are exactly (4): the dual variables exactly coincide with payoffs associated to the matching problem.

Finally, is the stable matching unique? With finite populations, the answer is no; in general, the payoffs u_i and v_j can be marginally altered without violating the (finite) set of inequalities (4). However, when the populations become large, the intervals within which u_i and v_j may vary typically shrink; in the limit of continuous and atomless populations, (the distributions of) individual payoffs are exactly determined. On all these issues, the reader is referred to Chiappori, McCann and Nesheim (2009) for precise statements.

3.2.2 A basic lemma

From an economic perspective, our main interest lies in the dual variables u and v . Indeed, v_j is the additional utility provided to woman j by her equilibrium marriage outcome. While this value is individual-specific (it depends on Mrs. j 's preferences for marriage), its expected value conditional of j having reached a given level of education J is directly related to the marital premium associated with education J (more on this below).

In our context, there exists a simple and powerful characterization of these dual variables; it is given by the following Lemma:

Lemma 1 *For any stable matching, there exist numbers U^{IJ} and V^{IJ} , $I = 1, \dots, M$, $J = 1, \dots, N$, with*

$$U^{IJ} + V^{IJ} = Z^{IJ} \quad (6)$$

satisfying the following property: for any matched couple (i, j) such that $i \in I$ and $j \in J$,

$$u_i = U^{IJ} + (\alpha_i^{IJ} - \alpha_i^{0J})$$

and

$$v_j = V^{IJ} + (\beta_j^{IJ} - \beta_j^{0J}) \quad (7)$$

Proof. *Assume that i and i' both belong to I , and their partners j and j' both belong to J . Stability requires that:*

$$u_i + v_j = Z^{IJ} + (\alpha_i^{IJ} - \alpha_i^{0J}) + (\beta_j^{IJ} - \beta_j^{0J}) \quad (8)$$

$$u_i + v_{j'} \geq Z^{IJ} + (\alpha_i^{IJ} - \alpha_i^{0J}) + (\beta_{j'}^{IJ} - \beta_{j'}^{0J}) \quad (9)$$

$$u_{i'} + v_j = Z^{IJ} + (\alpha_{i'}^{IJ} - \alpha_{i'}^{0J}) + (\beta_j^{IJ} - \beta_j^{0J}) \quad (10)$$

$$u_{i'} + v_{j'} \geq Z^{IJ} + (\alpha_{i'}^{IJ} - \alpha_{i'}^{0J}) + (\beta_{j'}^{IJ} - \beta_{j'}^{0J}) \quad (11)$$

Subtracting (1) from (2) and (4) from (3) gives

$$(\beta_{j'}^{IJ} - \beta_{j'}^{0J}) - (\beta_j^{IJ} - \beta_j^{0J}) \leq v_{j'} - v_j \leq (\beta_{j'}^{IJ} - \beta_{j'}^{0J}) - (\beta_j^{IJ} - \beta_j^{0J})$$

hence

$$v_{j'} - v_j = (\beta_{j'}^{IJ} - \beta_{j'}^{0J}) - (\beta_j^{IJ} - \beta_j^{0J})$$

It follows that the difference $v_j - (\beta_j^{IJ} - \beta_j^{0J})$ does not depend on j , i.e.:

$$v_j - (\beta_j^{IJ} - \beta_j^{0J}) = V^{IJ} \quad \text{for all } i \in I, j \in J$$

The proof for u_i is identical. ■

In words, Lemma 1 states that the dual utility v_j of woman j , belonging to class J and married with a husband in education class I , is the sum of two terms. One is woman j 's idiosyncratic preference for a spouse with education I over singlehood, $\beta_j^{IJ} - \beta_j^{0J}$; the second term, V^{IJ} , only depends on the spouses' classes, not on who they are. In terms of surplus division, therefore, the U^{IJ} and V^{IJ} denote how the deterministic component of the surplus, Z^{IJ} , is divided between spouses; then a spouse's utility is the sum of their share of the common component and their own, idiosyncratic contribution.

Finally, for notational consistency, we define $U^{I\emptyset} = V^{\emptyset J} = 0 \forall I, J$.

3.2.3 Stable matching: a characterization

An immediate consequence of Lemma 1 is that the stable matching has a simple characterization in terms of individual choices:

Proposition 2 *A set of necessary and sufficient conditions for stability is that*

1. for any matched couple ($i \in I, j \in J$) one has

$$\alpha_i^{IJ} - \alpha_i^{IK} \geq U^{IK} - U^{IJ} \quad \text{for all } K = \emptyset, 1, \dots, N \quad (12)$$

and

$$\beta_j^{IJ} - \beta_j^{KJ} \geq V^{KJ} - V^{IJ} \quad \text{for all } K = \emptyset, 1, \dots, N \quad (13)$$

2. for any single man $i \in I$ one has

$$\alpha_i^{IJ} - \alpha_i^{I\emptyset} \leq -U^{IJ} \quad \text{for all } J \quad (14)$$

3. for any single woman $j \in J$ one has

$$\beta_j^{IJ} - \beta_j^{0J} \leq -V^{IJ} \quad \text{for all } J \quad (15)$$

Proof. See Appendix A. ■

Stability thus readily translates into a set of inequalities in our framework; and each of these inequalities relates to *one* agent only. This property is crucial, because it implies that the model can be estimated using standard statistical procedures applied at the individual level, without considering conditions on couples. This separation is possible because the endogenous factors U^{IJ} and V^{IJ} adjust to make the separate individual choices consistent with each other.

3.3 Interpretation

Labor economists define the “college premium” as the percentage increase in the wage rate that can be expected from a college education. This wage premium can readily be measured using available data, after controlling for selection into college; existing empirical work suggests that, at the first order, it is similar for singles and married persons and for men and women¹². The point made by CIW is that, in addition to this wage premium, there exists a *marital college premium*: a college education enhances an individual’s marital prospects, via the probability of being married and the expected education (or income) of the spouse, but also the size of the surplus generated and its division within the couple. In other words, it is well-understood that college education benefits individuals in terms of higher wages, better career prospects, etc. What we aim at capturing here are the *additional* benefits that college-educated individuals receive on the marriage market.

The notions previously defined allow a clear definition of the marital college premium. Indeed, the surplus is computed as the difference between the total utility generated within the couple and the sum of individual utilities of the spouses if single, thus capturing exactly the additional gains from education that only benefit married people. Regarding individual well-being, an intuitive interpretation of U^{IJ} (or equivalently of V^{IJ}) would be the following. Assume that a man randomly picked in class I is forced to marry a woman belonging to class J (assuming that the populations are large, so that this small deviation from stability does not affect the equilibrium payoffs). Then his expected utility is exactly

¹²The number of hours worked, and therefore the resulting gain in labor income, of course differ across these populations.

U^{IJ} (the expectation being taken over the random choice of the individual—therefore of his preference vector—within the class).

Note, however, that this value does *not* coincide with the average utility of men in class I who end up being married to women J in a stable matching. The latter value is larger than U^{IJ} , reflecting the fact that agents *choose* their spouses. The expected surplus of an agent with education I is in fact

$$\bar{u}^I = E \max_{J=0,1,\dots,\mathcal{J}} (U^{IJ} + \alpha^{IJ}),$$

where the expectation is taken upon the distribution of the preference shock α . In particular, this expected surplus depends on the distribution of the preference shocks; it will be computed below under a specific assumption on this distribution.

An obvious problem is that the marital college premium is quite difficult to estimate empirically, as intrahousehold allocations cannot be directly observed. We next consider this issue.

4 Empirical implementation

We now describe the econometric model we shall take to data. Galichon and Salanié (2012) proved that separable models are just identified under the strong condition that the econometrician has exact knowledge of the probability distributions of α and β . This is an obvious weakness, since it implies that the model is simply not testable from cross-sectional data. A crucial feature of our approach is that we analyze multiple markets; in practice, we shall consider several cohorts, indexed by $c = 1, \dots, T$, and exploit the time variations in the education profiles of the populations at stake. Such a setting, in principle, should allow us to relax some restrictions while still generating overidentification tests.

Our first task is to describe how the structural components of our statistical framework evolve across cohorts. We start, as a benchmark, with an immediate generalization of CS. We show that this benchmark version generates strong overidentifying restrictions, and we describe a set of overidentification tests. This model fits the data well for the black population, but it is strongly rejected for whites. We then discuss several further extensions of the model, which we go on to test.

4.1 The benchmark version

4.1.1 The structural framework

In a static version of CS, the surplus generated by the match of man i , belonging to class I , with woman j , belonging to class J , takes the form:

$$z_{ij} = Z^{IJ} + \left(\alpha_i^{IJ} - \alpha_i^{I\emptyset} \right) + \left(\beta_j^{IJ} - \beta_j^{\emptyset J} \right)$$

In a multi-cohorts setting, it is natural to assume that the shocks are independent across cohorts, but may not be drawn from the same distribution (more on this later on.) How the deterministic part Z^{IJ} varies with time is less clear. Allowing the entire \mathbf{Z} matrix to vary freely across cohorts would amount to independently repeating static versions of CS, with no gain in terms of testability. Our benchmark model, therefore, introduces category-specific drifts, whereby the Z^{IJ} terms vary according to:

$$Z_c^{IJ} = \zeta_c^I + \xi_c^J + Z^{IJ} \tag{16}$$

so that

$$z_{ij,c} = Z^{IJ} + \zeta_c^I + \xi_c^J + \left(\alpha_{i,c}^{IJ} - \alpha_{i,c}^{I\emptyset} \right) + \left(\beta_{j,c}^{IJ} - \beta_{j,c}^{\emptyset J} \right)$$

In practice, the drifts ζ_c^I and ξ_c^J capture possible changes over time in the surplus generated by marriage. There are several reasons to expect the surplus to vary across periods. One is that technological innovations have drastically altered the technology of domestic production, therefore the respective gender roles within the household (see Greenwood et al 2005.) Other important factors were the evolution of fertility control, as emphasized by Michael (2000) and Goldin and Katz (2002) among others; and improvements in medical techniques and in infant feeding (Albanesi and Olivetti 2009.) Finally, remember that in our framework, the systematic part of the surplus, Z^{IJ} , can be interpreted as a reduced form for more dynamic interactions, including divorce and remarriage; as a consequence, changes in divorce laws or remarriage probabilities may affect the surplus.

It is therefore important to stress what the proposed extension allows and what it rules out. Under (16), the benefits of marriage may evolve over time; and these evolutions may be both gender- and education- specific. We allow, for instance, the gains generated by marriage to decrease less for an educated women than for an unskilled man.

However, all models of the form (16) satisfy an important property, which can be described as follows. For any matrix \mathbf{A} , define the cross-difference operator $D_{IJ,KL}$ by

$$D_{IJ,KL}\mathbf{A} = A^{IJ} - A^{IL} - A^{KJ} + A^{KL}$$

Then (16) is equivalent to assuming that, for all (I, J) and (K, L) , the cross difference

$$D_{IJ,KL}\mathbf{Z}_c = Z_c^{IJ} - Z_c^{IL} - Z_c^{KJ} + Z_c^{KL} \quad (17)$$

is independent of c . This is easily interpreted, given the following definition:

Definition 1 *We say that preferences for assortativeness do not change across cohorts if the deterministic matrices $Z_c = (Z_c^{IJ}, I = 1, \dots, \mathcal{I}, J = 1, \dots, \mathcal{J})$ have a supermodular core*

$$D_{IJ,KL}\mathbf{Z}_c = Z_c^{IJ} - Z_c^{IL} - Z_c^{KJ} + Z_c^{KL}$$

that is independent of c for all I, J, K and L .

The interpretation of this definition is straightforward, since the supermodularity of the surplus determines the assortativeness of the observed matching. Note that this definition is of independent interest: it provides an explicit characterization of the somewhat hazy notion of “changes in preferences for assortativeness.” Moreover, this characterization is structural: it relies on a model in which assortativeness is related to supermodularity of the surplus function (a natural link indeed), and exploits this relationship to provide a formal definition. In particular, it follows from (17) that our benchmark version assumes that the forces driving the assortativeness of the match are constant for the various cohorts.

In this new setting, Lemma 1 has an immediate generalization:

Corollary 3 *For any cohort c , there exist numbers U_c^{IJ} and V_c^{IJ} , $I = 1, \dots, \mathcal{I}, J = 1, \dots, \mathcal{J}$, with*

$$U_c^{IJ} + V_c^{IJ} = Z_c^{IJ} \quad (18)$$

satisfying the following property: for any matched couple (i, j) in cohort c such that $i \in I$ and $j \in J$,

$$\begin{aligned} u_i &= U_c^{IJ} + (\alpha_i^{IJ} - \alpha_i^{I\emptyset}) \\ &\text{and} \\ v_j &= V_c^{IJ} + (\beta_j^{IJ} - \beta_j^{\emptyset J}) \end{aligned} \tag{19}$$

4.1.2 Distributions

Next, we need to describe the probability distributions of the random terms α and β . Having transformed the problem into a standard discrete choice problem, it is natural to make the following assumption:

Assumption 1 (Gumbel) *The random terms α_i^{IJ} and β_j^{IJ} follow independent Gumbel¹³ distributions $G(-k, 1)$, with $k \simeq 0.5772$ the Euler constant.*

In particular, the α_i^{IJ} and β_j^{IJ} have mean zero and variance $\frac{\pi^2}{6}$. The model can easily be extended to allow for covariates; this extension is fully described in Chiappori, Salanié and Weiss (2011).

A direct consequence of Proposition 2 is that, for any I and any $i \in I$ in cohort c :

$$\begin{aligned} \gamma_c^{IJ} &\equiv \Pr(i \text{ matched with a woman in } J) \\ &= \frac{\exp(U_c^{IJ})}{\sum_K \exp(U_c^{IK}) + 1} \end{aligned}$$

and

$$\gamma_c^{I\emptyset} \equiv \Pr(i \text{ single}) = \frac{1}{\sum_K \exp(U_c^{IK}) + 1}$$

Similarly, for any J and any woman $j \in J$ in cohort c :

$$\delta_c^{IJ} \equiv P(j \text{ matched with a man in } I) \tag{20}$$

$$= \frac{\exp(V_c^{IJ})}{\sum_K \exp(V_c^{KJ}) + 1} \text{ and} \tag{21}$$

$$\delta_c^{\emptyset J} \equiv P(j \text{ single}) = \frac{1}{\sum_K \exp(V_c^{KJ}) + 1}$$

¹³This distribution is also referred to as the “type-I extreme value distribution.” While it has been used in economics since Daniel McFadden (1973), Dagsvik (2000) and Choo and Siow (2006) were the first to apply it to the study of marriage markets.

These formulas can be inverted to give:

$$U_c^{IJ} = \ln \left(\frac{\gamma_c^{IJ}}{1 - \sum_K \gamma_c^{IK}} \right) \quad (22)$$

and

$$V_c^{IJ} = \ln \left(\frac{\delta_c^{IJ}}{1 - \sum \delta_c^{KJ}} \right). \quad (23)$$

In what follows, we assume that there are singles in each class (a claim obviously supported by the data); therefore $\gamma_c^{I\emptyset} > 0$ and $\delta_c^{\emptyset J} > 0$ for each I, J , implying that $\sum_K \gamma_c^{IK} < 1$ and $\sum_K \delta_c^{KJ} < 1$ for all I, J .

We can readily compute the class-specific expected utilities

$$\bar{u}^I = E \left[\max_J (U_c^{IJ} + \alpha_i^{IJ}) \right]$$

Under our assumptions, the difference $\bar{u}^I - \bar{u}^K$ denotes the difference in expected surplus obtained by reaching the education level I instead of K . It therefore represents exactly the marital premium generated by that change in education level—that is, the gain that accrues to married people, on top of the benefits that singles also receive.

From the properties of Gumbel distributions, we have

$$\bar{u}^I = E \left[\max_J (U_c^{IJ} + \alpha_i^{IJ}) \right] = \ln \left(\sum_J \exp(U_c^{IJ}) + 1 \right) = -\ln(\gamma_c^{I\emptyset}) \quad (24)$$

and similarly

$$\bar{v}^J = \ln \left(\sum_I \exp(V_c^{IJ}) + 1 \right) = -\ln(\delta_c^{\emptyset J}). \quad (25)$$

These results illustrate a well-known property of homoskedastic multinomial logit models: the expected utility of any participant is fully summarized by their probability of remaining single. This property, however, no longer holds in an heteroskedastic version of the model (see below). While such a pattern reflects in part the restrictiveness of our benchmark version, it nevertheless authorizes complex dynamics. In particular, it is important to stress that there is no *automatic* relationship between changes in population composition (say, an exogenous increase in the proportion of women with a college degree) and the imputed impact on expected utility. Even in this benchmark version, such an

increase may either boost or deflate expected utility of educated women, depending on its actual consequences on probability of singlehood.

Looking back at the descriptive statistics presented above, one striking fact is the much smaller decline in marriage probability for educated women than uneducated ones. The theoretical interpretation of this fact is that, although the gain from marriage declined for all women, the decline was less pronounced for educated women; this translates into a strong increase in the marital college (and college-plus) premium, which directly reflects the difference between these gains. Moreover, this pattern is gender-specific for the white population, but not for African-Americans; we shall come back later to that aspect.

4.1.3 Empirical tests

Observing several cohorts generates strong overidentifying restrictions on the benchmark model. One can actually give a nice graphic representation of these restrictions. Start with the basic relation (16). Together with (22) and (23), this gives:

$$\ln \left(\frac{\gamma_c^{IJ}}{1 - \sum_H \gamma_c^{IH}} \right) + \ln \left(\frac{\delta_c^{IJ}}{1 - \sum_H \delta_c^{HJ}} \right) = \zeta_c^I + \xi_c^J + Z^{IJ} \quad (26)$$

Applying the cross-difference operator introduced in section 4.1.1 to (26) shows that the expression

$$d_c^{IJ,KL} = D_{IJ,KL} (\ln \gamma_c + \ln \delta_c)$$

should not depend on c .

Two remarks can be made at this point. First, these conditions are necessary and sufficient; if they are satisfied, then one can always find some matrices \mathbf{U} , \mathbf{V} and \mathbf{Z} and some vectors $\boldsymbol{\zeta}$ and $\boldsymbol{\xi}$ such that all previous conditions are satisfied. Second, they are quite restrictive. It is easy to see that with 5 classes, there are only $4 \times 4 = 16$ a priori independent cross-differences (see Fox and Yang (2012).) Take the corresponding 16 T -dimensional vectors $\mathbf{d}^{IJ,KL} = (d_c^{IJ,KL}, c = 1, \dots, T)$; our assumption (16) amounts to requiring that each of them be constant (proportional to $(1, \dots, 1)$.) This generates $16 \times (T - 1)$ restrictions, a large number since we use several decades of yearly data. The results are described in the next section.

4.2 Extensions

We now consider several possible extensions of the model.

4.2.1 Linear and Quadratic Supermodularity Trends

Our benchmark model posits that preferences for assortativeness do not change over the period. As we shall see, this hypothesis is not rejected for the African-American sample. It appears to be excessively restrictive for the white population, however. A natural extension is to allow preferences for assortativeness to change over the period. This should be done parsimoniously, since we want to preserve the strong testability of the model. We first introduce linear time trends in the supermodular core of the deterministic matrix. To do this, we assume that:

$$Z_c^{IJ} = \zeta_c^I + \xi_c^J + a^{IJ} + b^{IJ}c. \quad (27)$$

Then (26) implies that

$$d_c^{IJ,KL} = D_{IJ,KL} (\ln \gamma_c + \ln \delta_c)$$

is a linear function of the cohort index c .

Given the choice of the reference categories, the number of parameters is increased by $4 \times 4 = 16$, and the model remains testable: the 16 a priori independent T -dimensional vectors $\mathbf{d}^{IJ,KL} = (d_c^{IJ,KL}, c = 1, \dots, T)$ must now all belong in the space spanned by the two vectors $(1, \dots, 1)$ and $(1, \dots, T)$. The resulting $16 \times (T - 2)$ conditions are again necessary and sufficient.

Going one step further, one can introduce a quadratic trend in the time variation of the deterministic component of the surplus. Then the vectors $\mathbf{d}^{IJ,KL}$ must lie in the space spanned by the three vectors $(1, \dots, 1)$, $(1, \dots, T)$ and $(1, \dots, T^2)$, for a total of $16 \times (T - 3)$ testable predictions.

4.2.2 Heteroskedasticity

A second generalization introduces class-dependent heteroskedasticity. Specifically, we consider a stochastic formulation of the type:

$$z_{ij,c} = Z^{IJ} + \sigma^I \alpha_{i,c}^{IJ} + \mu^J \beta_{j,c}^{IJ}$$

where the variance of the stochastic component for a man belonging to education class I depends on the parameter σ^I , and the variance of the stochastic component for a woman belonging to education class J is determined by the parameter μ^J . Note that we need to normalize one of these parameters.

The previous relationships can be extended to this more general setting; the reader is referred to Chiappori, Salanié and Weiss (2011). In particular, the expected surplus of a male agent in education class I now is

$$\bar{u}^I = \sigma^I \ln \left(\sum_J \exp(U^{IJ}/\sigma^I) + 1 \right) = -\sigma^I \ln(\gamma_c^{I0})$$

so that the marital education premium between classes I and K (representing the gain in expected surplus from belonging to I instead of K) becomes:

$$\bar{u}^I - \bar{u}^K = \sigma^K \ln(\gamma_c^{K0}) - \sigma^I \ln(\gamma_c^{I0})$$

An important result, proved in Chiappori, Salanié and Weiss (2011), is that this model is again (over) identified.

In practice, we test whether the introduction of heteroskedasticity (resulting in $2 \times 5 - 1 = 9$ additional parameters) significantly improves the fit. For this purpose, we compute a score test from the null of homoskedasticity (see Appendix B for details.)

4.2.3 Age differences

Finally, we may want to consider the age difference within the couple as a choice variable. The natural way to proceed is to assume that the deterministic component of the surplus, Z , also depends on the age difference between husband and wife, which we denote d . Our benchmark model, which allows for category-specific drifts, therefore becomes:

$$Z_{c,d}^{IJ} = \zeta_{c,d}^I + \xi_{c,d}^J + Z^{IJ} \tag{28}$$

so that

$$z_{ij,c,d} = Z^{IJ} + \zeta_{c,d}^I + \xi_{c,d}^J + \left(\alpha_{i,c,d}^{IJ} - \alpha_{i,c,d}^{I\emptyset} \right) + \left(\beta_{j,c,d}^{IJ} - \beta_{j,c,d}^{\emptyset J} \right)$$

Obviously, this model differs a lot from the initial one in both the number of observations and the number of parameters: the set of marriages of a man with education I , born in

cohort c , to a woman with education J is now divided into seven subcategories, each representing a specific age difference (-1 or less, 0, 1, 2, 3, 4, 5 or more.) In order to compare the results of this specification with those of the initial version, we concentrate on the same margins we were trying to fit initially. That is, we take a weighted average of the errors within each cell (I, J, c) over the age differences d , and we use this averaged error to run our tests.

We start, as before, from the benchmark formulation (28), in which the supermodularity of the deterministic component does not change over time. Should this benchmark model be rejected, we can again introduce a linear time trend affecting the supermodularity of the deterministic component—so that (28) becomes

$$Z_{c,d}^{IJ} = \zeta_{c,d}^I + \xi_{c,d}^J + a^{IJ} + b^{IJ}c. \quad (29)$$

5 Results

5.1 Testing the models

5.1.1 Tests of the benchmark model

We start with the benchmark model, tested on the white population. Recall our prediction that each of 16 T -dimensional vectors $\mathbf{d}^{IJ,KL} = (d_c^{IJ,KL}, c = 1, \dots, T)$ be constant. These requirements can readily be checked on the data. Here we take reference categories to be $K = L = 3$ and we plot the $d_c^{IJ,33}$ demeaned over cohorts. Figure 8 shows the graphs corresponding to the “diagonal” elements $d_c^{II,33}$, $I = 1, 2, 4, 5$ of assortatively matched white couples. Under the null, the blue curve (and the dashed smoothed blue curve) should be identically 0; the dotted curves give the 95% confidence band. The property is clearly violated for college and college-plus educated pairs, for which the trend is clearly ascending. This suggests an increase in assortativeness, at least for the more educated fraction of the population.

Insert here Fig 8

Altogether, the graphs suggest that the benchmark model is rejected by the data. A formal test described in Appendix B confirms this claim. The joint test has 432 degrees of freedom, and gives a χ^2 statistic of 1228.2, way above the 5% critical value of 481.5 (the p -value is below $1e - 100$.)

Finally, we also estimate and test the version allowing for age differences. The conclusions are similar; namely, when we average errors over age differences as discussed above, the χ^2 statistic has value 1014.8 with 396 degrees of freedom, while the 5% critical value is 443.4, leading to a p -value that the computer cannot distinguish from 0.

Regarding black couples, however, things are totally different. Given the much smaller sample size, especially for higher education, we only use four education categories, aggregating college and college-plus. The number of degrees of freedom is now 198, leading to a 5% critical value of 231.8. The χ^2 statistic now is 183.5 and corresponds to a p -value of 0.762, indicating very little evidence against the benchmark model. We therefore fail to reject the null that preferences for assortativeness have not changed over the period among the African-American population.

5.1.2 White sample: extensions

Since the benchmark model is rejected for the white population, we next consider the linear and quadratic extensions. Figure 9 gives, again for diagonal pairs (I, I) , the standardized residuals of three regressions of the $d_c^{II,33}$, corresponding respectively to the benchmark model, the linear trend, and the quadratic trend extensions. Since the variance is normalized to be 1, theory would require these residuals to remain between -1.96 and 1.96 for 95% of observations. The linear trend models appear to fit data significantly better than the benchmark. Moreover, while the quadratic component further increases the fit, its contribution is much more incremental.

Insert here Fig 9

As before, a formal test is described in Appendix B. While the model is still rejected, the rejection is much less drastic than previously. Specifically, the χ^2 statistic is 575.3, while the 5% critical value (for 416 degrees of freedom) is 464.6. The spectacular decrease in

the χ^2 statistic indicate that preferences for assortativeness did change over the period; an exact description of these changes is provided in the next subsection. Under the quadratic version, the number of degrees of freedom is now 400, giving a 5% critical value of 447.6; the test statistic equals 480.8. In particular, the null that all quadratic terms are zero is rejected. A similar test using age differences gives similar (although slightly better) results. Now the number of degrees of freedom is only 380, corresponding to a 5% critical value of 426.5, while the χ^2 statistic has value 407.4 (corresponding to a p -value equal to 0.1597).

We therefore conclude that in the white population, preferences for assortativeness did change significantly over the period. The direction of these changes is quite interesting. Table 2 gives the estimate of the slope of the linear trend of the deterministic part of the surplus (that is, the coefficient b^{IJ} in (27)) for all pairs of husband and wife educations. One can see in particular that the slopes of diagonal elements are significantly positive, indicating an increase in the marital gain for couples with similar education, while several off diagonal elements have a negative trend. All in all, the supermodularity of the Z matrix tends to increase, which indicates higher preferences for assortativeness, particularly for the more educated part of the sample.

5.1.3 Heteroskedasticity

We next test for heteroskedasticity along the lines suggested in 4.2.2, both in the benchmark model and in its linear trends extension. Remember that the test is a χ^2 statistic with degrees of freedom in both cases under the null of homoskedasticity; its 5% critical value is 16.9. In the benchmark model, the χ^2 statistic takes the value 15.6: the null is not rejected at 5%, although it is at 10% (the p -value is .075). However, this borderline rejection appears to be specific to the benchmark model. Indeed, with linear trends, the χ^2 test statistic takes the value 2.76, way below the critical value (its p -value is .97). We conclude that in our extended framework, there is no evidence of heteroskedasticity; we therefore stick to the homoskedastic formulation with linear trends for the rest of our analysis of the white population.

5.2 The marital college premium: the African-American sample

Since the benchmark model is not rejected for Afro-Americans, we can directly analyze its implications. Figures 10a and 10b shows the evolution of male and female expected surplus by class of education. Two patterns emerge from this diagram. First, the marital surplus declines over the period, more strongly for men than for women. More importantly, however, there is no significant difference between education classes; for each gender, the decline is very similar across education levels. A formal test confirms this impression: the “marital education premia”, defined as the difference in expected marital surplus between education classes, exhibits no significant trend over the period, for men as well as for women. In other words, the additional marital expected utility from a college degree remains constant (and positive) over the period for both men and women.

Insert Here Fig 10

This finding can be related to the specific evolution of the demand for higher education among African-Americans over the period: as discussed earlier, no significant difference appears between men and women, which is fully consistent with our findings.

5.3 The white population

5.3.1 The marital college premium: evolution over cohorts

The estimation of the structural model allows to compute the evolution of the marital college premium by gender over the cohorts under consideration. Figures 11a and 11b represent our estimate of the expected utilities from marriage for men and women. Three facts stand out. First, these gains decrease over the period, for all genders and education levels. Second, the decrease is much stronger for lower levels of education. Thirdly, and more interestingly for our purpose, male and female profiles are dissimilar.

The marital gains of men increase with education for all cohorts. Moreover, while the marital college premium increases over the period, the plots of marital gains for the two highest levels of education—college and college-plus—are almost parallel: the “college-plus premium”, defined as the difference in marital utility between college and college-plus,

remains constant over the period.

For women, however, the picture is radically different. For the older cohorts, the marital gain is inversely related to education (with the exception of high school drop outs.) While marital gains trend downwards, the decrease is much slower for educated individuals; as a result, the more recent cohorts exhibit marital gains that are higher for college-educated women. Even more interesting is the spectacular reduction of the difference between college and college-plus women, which translates into a strong increase in the college-plus premium for women. As a result, the (double) difference between female and male marital college-plus premia increases significantly over the period (see Fig 11c, which only shows one-standard error confidence bands for legibility.)

Insert here Fig 11a, b and c

Formal tests confirm these statements. Using the same notation as above, if \bar{u}_c^I (resp. \bar{v}_c^I) denotes the expected marital gain for men (women) with education I in cohort c , we consider the evolution of the double difference

$$\delta_c^I = (\bar{v}_c^{I+1} - \bar{v}_c^I) - (\bar{u}_c^{I+1} - \bar{u}_c^I)$$

over cohorts. For $I = 4$ for instance, $\bar{u}_c^{I+1} - \bar{u}_c^I$ thus represents the college-plus premium for men, and δ_c^I is the difference in the college-plus premia of women and men. When regressing our estimated δ_c^I on a linear time trend, we find a positive coefficient of .0086, which is highly significant (its t-value is 3.54, and its p -value is .0007—these numbers correct for the prior estimation of the δ 's.)

In summary, we see a significant increase in the college-plus premium for women, whereas no such evolution is apparent for men. It bears repeating that this conclusion obtains from the sole analysis of matching patterns over time, without any other data on intrahousehold allocations. It nevertheless strongly supports CIW's suggestion that the increase in female demand for higher education could be motivated in part by the divergences between the respective evolutions of male and female marital college-plus premia.¹⁴

¹⁴As noted before, a limitation of our analysis is the lack of adequate data on divorce. The study by

5.3.2 Decomposition

The marital college premium can be decomposed into several components. First, education affects the probability of being married. Second, conditional on being married, it also affects the education of the spouse (or more exactly its probability distribution); intuitively, we expect educated women to find a “better” husband, at least in terms of education, and conversely. Third, the impact on the total surplus generated by marriage is twofold. Take women for instance. A wife’s education has a direct impact on the surplus; this impact can be measured, for college-plus education, by the difference $(Z_c^{I5} - Z_c^{I4})$. In addition, since a more educated woman is more likely to marry a more educated husband, the husband’s higher expected education further boosts the surplus, by the average of the $(Z_c^{5J} - Z_c^{4J})$ terms weighted by the difference in probability of marrying a college-plus educated husband instead of a college graduate. Finally, the share of the surplus going to the wife in any given match is also affected by her education. This share is readily measured by the ratio

$$\frac{\bar{v}_c^J}{\bar{u}_c^I + \bar{v}_c^J}$$

which represents the share of the surplus that goes to the wife in an (I, J) marriage.

All these components can readily be computed from our estimates; they are summarized in Tables 3 to 11. The first five tables summarize patterns that we already discussed in the introduction. The marriage probabilities sharply decline (by more than 20 percentage points) for less educated individuals, much less for educated ones, especially women. Regarding spousal education, educated men are more likely to marry an educated wife than they used to be; for college and college-plus educated women, the pattern is opposite, and the probability of a college-plus husband strongly declines.

More interesting are the next four tables, which are specific to our structural approach. The joint surplus declines in general, with the exception of couples formed by college-educated men and college or college-plus educated women. This decline in surplus is always smaller for more educated couples, which is consistent with the increased preferences for assortativeness at the top of the distribution. Lastly, the intra-household distribution of

Schwartz and Han (2014) reports that for recent cohorts, being more educated than the husband is no longer a handicap for women in terms of divorce probability. Although based on very small samples, this finding is consistent with the rising incentive of women to invest in schooling that our model predicts.

the surplus also varies significantly. For all matches between equally educated individuals, the share of the wife increases. Things are more complex off the diagonal. The share received by educated women who marry down decreases. On the other hand, less educated women who marry up benefit from a significantly larger share of a joint surplus which is smaller than it used to be.

Insert here Tables 3 to 11

6 Conclusion

It has long been recognized (at least since Becker’s 1973 seminal contributions) that the division of the surplus generated by marriage should be analyzed as an equilibrium phenomenon. As such, it responds to changes in the economic environment; conversely, investments made before marriage are partly driven by agents’ current expectations about the division of surplus that will prevail after marriage. Theory shows that such considerations may explain the considerable differences in male and female demands for higher education. When deciding whether to go to college, agents take into account not only the labor market college premium (i.e., the wage differential resulting from a college education) but also the “marital college premium”, which represents the impact of education on marital prospects; the latter includes not only marriage probabilities, but also the expected “quality” of the future spouse and the resulting distribution of the marital surplus. We strongly believe that while the first aspect—the labor market college premium—has been abundantly discussed, the second—the marital college premium—has been all but ignored, and that this omission may have severely hampered our understanding of recent evolutions in the demand for higher education.

In this paper, we provide a simple but rich model in which the marital college premium can be econometrically identified. Our framework generalizes the original contribution by Choo and Siow (2006); we show, in particular, that it can be (over)identified using temporal variations in the compositions of the populations. Applying the model to US data, we find gender- and race-specific evolutions. There is no evidence of changes in preference for assortativeness among the black population; among whites, on the contrary, we find a

highly significant increase, particularly among the most educated subsample. In the same way, the evolution of the marital college and college-plus premium is similar across genders for the African-American population; this sharply contrasts with the white sample, where the marital college-plus premium increases much more for women than for men. All in all, our findings support the claim that the marriage market has played an important role in the demand for higher education in the recent decades.

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Appendix A: Proof of Proposition 2

The proof is in several steps. Let $(i \in I, j \in J)$ be a matched couple. Then:

1. First, man i must be better off than being single, which gives:

$$U^{IJ} + \alpha_i^{IJ} - \alpha_i^{I\emptyset} \geq 0$$

hence $\alpha_i^{IJ} - \alpha_i^{I\emptyset} \geq -U^{IJ}$ and the same must hold with woman j .

2. Take some woman j' in J , currently married to some i' in I . Then i must be better off matched with j than j' , which gives:

$$U^{IJ} + (\alpha_i^{IJ} - \alpha_i^{I\emptyset}) \geq z_{ij'} - v_{j'} = Z^{IJ} + (\alpha_i^{IJ} - \alpha_i^{I\emptyset}) + (\beta_j^{IJ} - \beta_j^{\emptyset J}) - (V^{IJ} + (\beta_{j'}^{IJ} - \beta_{j'}^{\emptyset J}))$$

and one can readily check that this inequality is always satisfied as an equality, reflecting the fact that i is indifferent between j and j' , and symmetrically j is indifferent between i and i' .

3. Take some woman k in $K \neq J$, currently married to some i' in I . Then “ i is better off matched with j than k ” gives:

$$U^{IJ} + (\alpha_i^{IJ} - \alpha_i^{I\emptyset}) \geq z_{ik} - v_k = Z^{IK} + (\alpha_i^{IJ} - \alpha_i^{I\emptyset}) + \beta_k^{IK} - (V^{IK} + \beta_k^{IK})$$

which is equivalent to

$$\alpha_i^{IJ} - \alpha_i^{IK} \geq U^{IK} - U^{IJ}$$

and we have proved that the conditions (12) are necessary. The proof is identical for (13).

4. We now show that these conditions are sufficient. Assume, indeed, that they are satisfied. We want to show two properties. First, take some woman j' in J , currently married to some l in $L \neq I$. Then i is better off matched with j than j' . Indeed,

$$U^{IJ} + (\alpha_i^{IJ} - \alpha_i^{I\emptyset}) \geq z_{ij'} - v_{j'} = Z^{IJ} + (\alpha_i^{IJ} - \alpha_i^{I\emptyset}) + (\beta_j^{IJ} - \beta_j^{\emptyset J}) - (V^{IJ} + (\beta_{j'}^{IJ} - \beta_{j'}^{\emptyset J}))$$

is a direct consequence of (13) applied to l . Finally, take some woman k in $K \neq J$, currently married to some l in $L \neq I$. Then i is better off matched with j than j' . Indeed, it is sufficient to show that

$$U^{IJ} + \alpha_i^{IJ} \geq z_{ik} - v_k = Z^{IK} + (\alpha_i^{IJ} - \alpha_i^{I\emptyset}) + \beta_k^{IK} - (V^{IK} + \beta_k^{IK})$$

But from (13) applied to k we have that:

$$\beta_k^{LK} - \beta_k^{IK} \geq V^{IK} - V^{LK}$$

and from (12) applied to i :

$$\alpha_i^{IJ} - \alpha_i^{IK} \geq U^{IK} - U^{IJ}$$

and the required inequality is just the sum of the previous two.

Appendix B: Testing the Models

The Benchmark Model

Start from our benchmark model of section 4.1, with constant supermodular core and homoskedasticity. It is characterized by equation (26), which can be rewritten as

$$\log(\Pr(J|I, c) \Pr(I|J, c)) = Z_c^{IJ} \equiv A_{IJ} + B_c^I + C_c^J. \quad (\text{B.1})$$

This is an identity, in which we introduce the sampling variation that comes from our estimates of the probabilities on the left-hand side: $\widehat{\Pr}(J|I, c)$ and $\widehat{\Pr}(I|J, c)$. These estimators are simple averages; for instance, if there are N_{Ic}^m men of education I in cohort c , the estimator of the probability that one of them marries a woman of education J is

$$\widehat{\Pr}(J|I, c) = \Pr(J|I, c) + \frac{1}{N_{Ic}^m} \sum_{i=1}^{N_{Ic}^m} w_i^{IJc}.$$

For each (I, c) , we first compute the variance-covariance matrix V_{Ic}^m of the estimated probabilities $\widehat{\Pr}(\cdot|I, c)$, taking into account the sampling weights in the American Community Survey. Then we use a second-order Taylor expansion

$$\log \widehat{P} - \log P \simeq \frac{\widehat{P} - P}{P} - \frac{(\widehat{P} - P)^2}{2P^2}$$

to rewrite (B.1) as

$$\begin{aligned} \log \left(\widehat{\Pr}(J|I, c) \widehat{\Pr}(I|J, c) \right) + \frac{V_{Ic}^m(J, J)}{2\widehat{\Pr}(J|I, c)^2} + \frac{V_{Jc}^w(I, I)}{2\widehat{\Pr}(I|J, c)^2} &\simeq A_{IJ} + B_c^I + C_c^J \\ &+ \frac{1}{N_{Ic}^m \widehat{\Pr}(J|I, c)} \sum_{i=1}^{N_{Ic}^m} u_i^{IJc} \\ &+ \frac{1}{N_{Jc}^w \widehat{\Pr}(I|J, c)} \sum_{j=1}^{N_{Jc}^w} v_j^{IJc}. \end{aligned}$$

All of our tests assume away the higher-order terms in $1/\sqrt{N_{Ic}^m}$ in this expansion. With close to 1.7 million observations for white men, each (I, c) cell has on average 12,000 observations and we are not much concerned about higher-order terms. For blacks we only have 50,000 men, and the average cell only has about 350 observations; but we feel that we can still safely truncate the expansion.

This expansion leads us to regressions of the form

$$\tilde{P}_{IJc} = \tilde{A}_{IJ} + \tilde{B}_c^I + \tilde{C}_c^J + \varepsilon_c^{IJ} \tag{B.2}$$

and we can use the V_{Ic}^m and V_{Jc}^w to compute the variance-covariance matrix of the residuals under the null: it is zero across cohorts, and

$$W_c^{IJ, KL} = \text{cov}(\varepsilon_c^{IJ}, \varepsilon_c^{KL}) = \mathbf{1}(I = K) \frac{V_{Ic}^m(J, L)}{\widehat{\Pr}(J|I, c) \widehat{\Pr}(L|I, c)} + \mathbf{1}(J = L) \frac{V_{Jc}^w(I, K)}{\widehat{\Pr}(I|J, c) \widehat{\Pr}(K|J, c)}.$$

In practice, the $V/(2P^2)$ terms on the left-hand side of (B.2) are very small. The W_c matrices, on the other hand, vary a lot with c , and they are not close to diagonals.

Minimizing

$$\sum_{c=1}^C \hat{\varepsilon}_c' W_c^{-1} \hat{\varepsilon}_c \tag{B.3}$$

yields a consistent estimator of the A , B , and C terms. Moreover, each ε_c^{IJ} is an average of a large number of iid errors; as such, it is approximately normal. This has two consequences: minimizing the quadratic form in (B.3) gives an approximately efficient estimator; and its value at its minimum gives a test statistic which is distributed as a χ^2 under the null, with

$$m^2 C - m^2 - (2m - 1)(C - 1)$$

degrees of freedom (with m the number of education levels and C the number of cohorts.) In our case $m = 5$ and $C = 28$, so that the benchmark model has 432 degrees of freedom.

The models with time-changing complementarities of section 4.2.1 can be dealt with in exactly the same way, and so can the models with preferences for age differences of section 4.2.3. Only the numbers of degrees of freedom of the chi^2 differ, because these extensions have more parameters, and because (for the model with age differences) we fit more margins.

The Heteroskedastic Model

Introducing heteroskedasticity as in section 4.2.2 complicates the test slightly. With time-invariant heteroskedasticity, our basic identity now is

$$Z_{IJ}^c = \sigma^I \log \frac{\Pr(J|I, c)}{\Pr(0|I, c)} + \mu^J \log \frac{\Pr(I|J, c)}{\Pr(0|J, c)}.$$

Take changing complementarities of a given form A_{IJ}^c (constant or linear.) Then we have

$$\sigma^I \log \Pr(J|I, c) + \mu^J \log \Pr(I|J, c) = A_{IJ}^c + B_c^I + C_c^J.$$

We can still do a Taylor expansion, which now gives

$$\begin{aligned} & \sigma^I \log \widehat{\Pr}(J|I, c) + \mu^J \log \widehat{\Pr}(I|J, c) + \\ & \frac{\sigma^I}{2\widehat{\Pr}(J|I, c)^2} V_{Ic}^m(J, J) + \frac{\mu^J}{2\widehat{\Pr}(I|J, c)^2} V_{Jc}^w(I, I) \simeq A_{IJ}^c + B_c^I + C_c^J \\ & + \frac{\sigma^I}{N_{Ic}^m \widehat{\Pr}(J|I, c)} \sum_{i=1}^{N_{Ic}^m} u_i^{IJc} \\ & + \frac{\mu^J}{N_{Jc}^w \widehat{\Pr}(I|J, c)} \sum_{j=1}^{N_{Jc}^w} v_j^{IJc}. \end{aligned}$$

This brings in two new issues. The first one is easy to solve: we only have $(2m - 1)$ additional parameters as we need to keep one of the σ, μ 's equal to one. The more difficult one is that W_c now depends on the unknown σ 's and μ 's:

$$W_c^{IJ, KL} = \text{cov}(\varepsilon_c^{IJ}, \varepsilon_c^{KL}) = \mathbf{1}(I = K) \frac{(\sigma_I)^2 V_{Ic}^m(J, L)}{\widehat{\Pr}(J|I, c) \widehat{\Pr}(L|I, c)} + \mathbf{1}(J = L) \frac{(\mu_J)^2 V_{Jc}^w(I, K)}{\widehat{\Pr}(I|J, c) \widehat{\Pr}(K|J, c)}.$$

Denote θ the vector consisting of the unknown σ and μ 's. Minimizing

$$\sum_{c=1}^C \hat{\varepsilon}'_c(\theta) W_c^{-1}(\theta) \hat{\varepsilon}^c(\theta)$$

will not give a consistent estimator this time. We need to maximize instead the log-likelihood of a normal distribution with unknown variance

$$\frac{1}{C} \sum_{c=1}^C L_c(\theta),$$

where

$$L_c(\theta) = -\frac{1}{2} (\hat{\varepsilon}_c(\theta)' W_c(\theta)^{-1} \hat{\varepsilon}^c(\theta) + \log \det W_c(\theta)).$$

Rather than doing this, we can simply run a score test of the null hypothesis that all σ 's and μ 's equal one, or $\theta \equiv 1$ in abbreviated notation. It is easy to see that the score test statistic is

$$T = C \hat{G}' \hat{S}^{-1} \hat{G},$$

with

$$\hat{G} = \frac{1}{C} \sum_c \frac{\partial L_c}{\partial \theta} \text{ and } \hat{S} = \frac{1}{C} \sum_c \frac{\partial L_c}{\partial \theta} \frac{\partial L_c'}{\partial \theta},$$

taking (numerically) the derivatives in $\theta \equiv 1$. Under homoskedasticity, T should be a $\chi^2(2m - 1)$.

Tables and Figures

Age difference	≤ -1	0	1	2	3	4	≥ 5
First cohort	5.1%	15.9%	21.9%	22.0%	17.3%	11.5%	6.2%
Last cohort	10.7%	23.8%	24.4%	18.8%	12.6%	6.4%	3.3%

Table 1a: Distribution of age differences (husband's - wife's), white sample

Age difference	≤ -1	0	1	2	3	4	≥ 5
First cohort	6.5%	14.5%	19.4%	18.7%	18.6%	13.5%	8.7%
Last cohort	11.1%	21.6%	22.4%	20.8%	12.5%	7.5%	4.1%

Table 1b: Distribution of age differences (husband's - wife's), black sample

		Women				
		HSD	HSG	SC	CG	CG+
	HSD	0.0118*** (0.0015)	0.0067*** (0.0012)	0.0146*** (0.0018)	-0.0023 (0.0017)	-0.0366*** (0.0017)
	HSG	-0.0237*** (0.0011)	0.0024 (0.0008)	0.011*** (0.0008)	-0.0009 (0.0009)	-0.01*** (0.0014)
Men	SC	-0.0198*** (0.0013)	-0.001 (0.0006)	0.0056*** (0.0013)	0.004*** (0.0015)	0.0001 (0.0014)
	CG	0.0187*** (0.0012)	-0.0011 (0.0009)	-0.0093*** (0.0013)	0.0079*** (0.0015)	0.015*** (0.0018)
	CG+	0.0436*** (0.0004)	0.0055*** (0.0006)	-0.0087*** (0.0008)	-0.0059*** (0.001)	0.0149*** (0.0017)

Table 2: Changes in joint surplus—linear extension

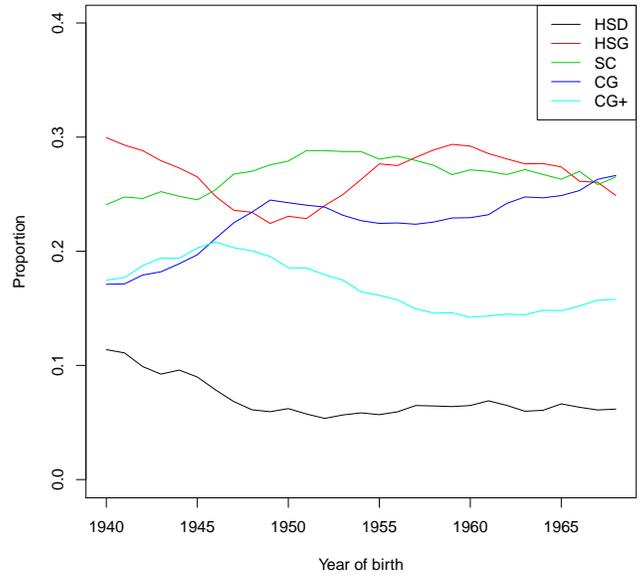


Figure 1a: Education of white men by cohort

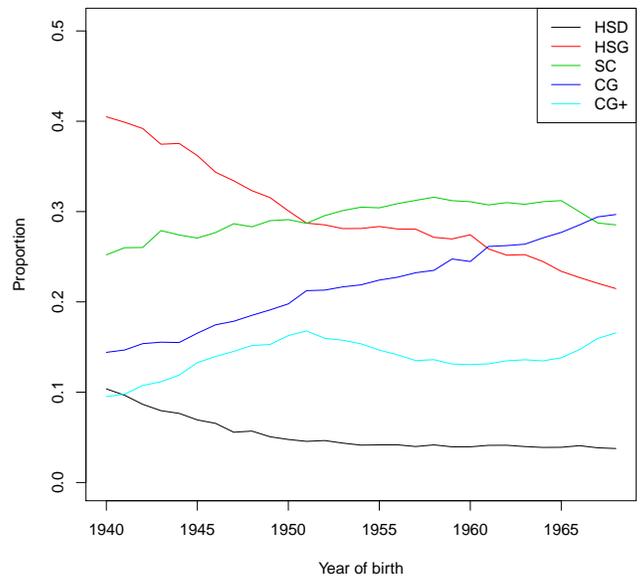


Figure 1b: Education of white women by cohort

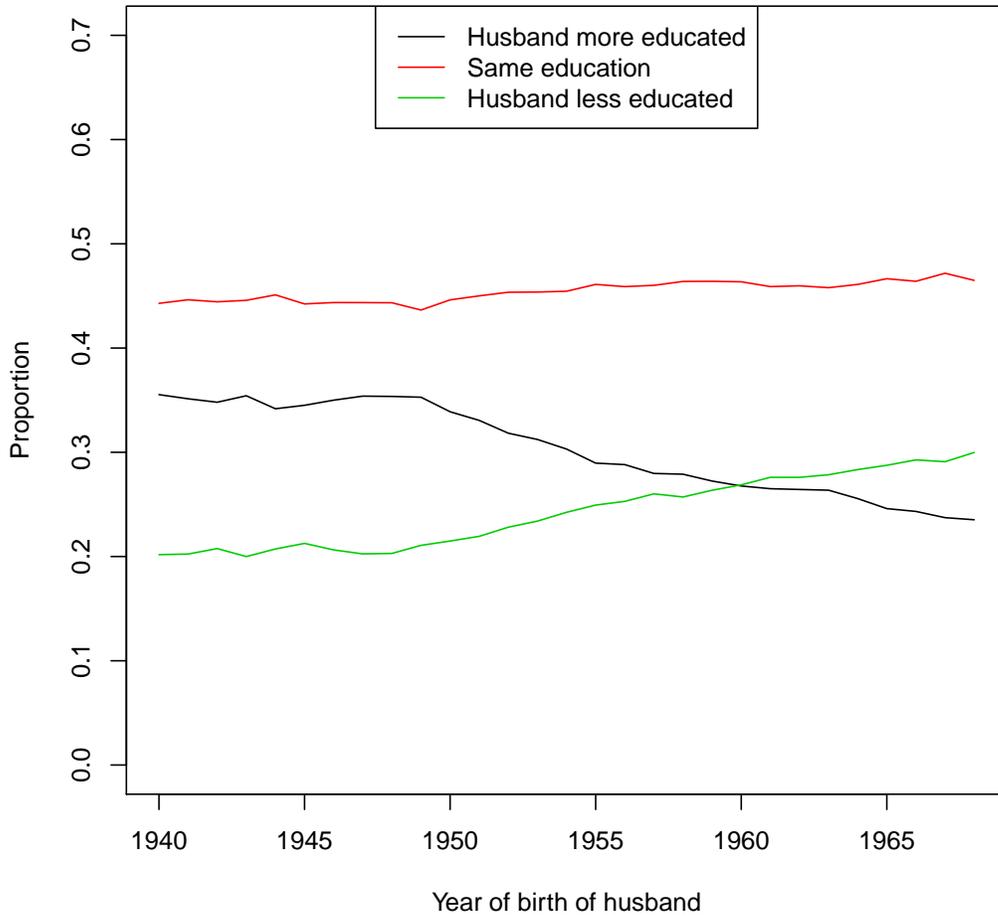


Figure 2: Comparing educations within white couples

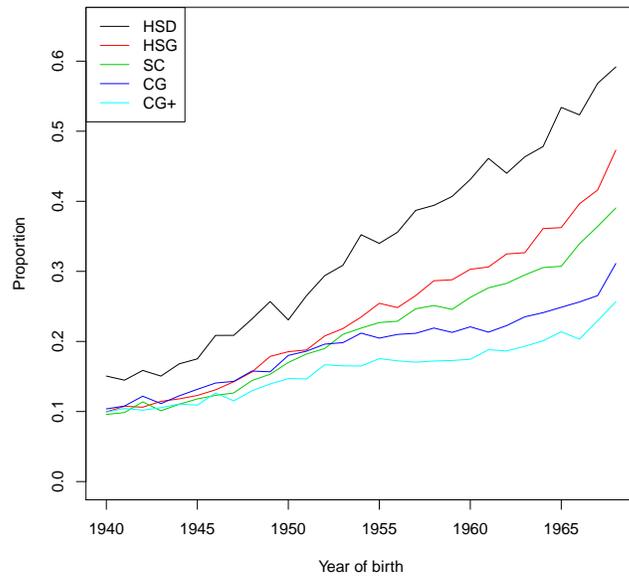


Figure 3a: Proportion of white men who never married by cohort

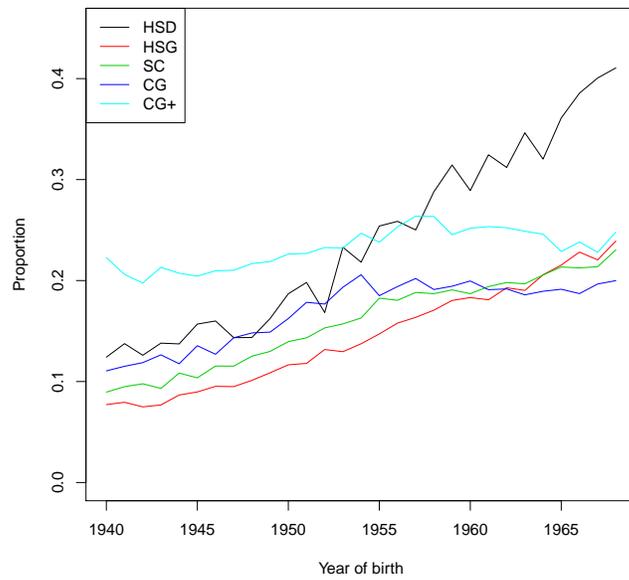


Figure 3b: Proportion of white women who never married by cohort

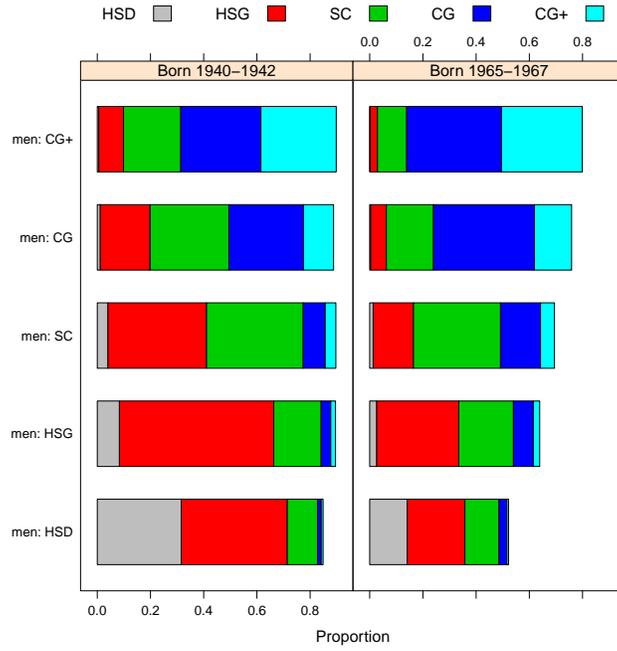


Figure 4a: Marriage patterns of white men

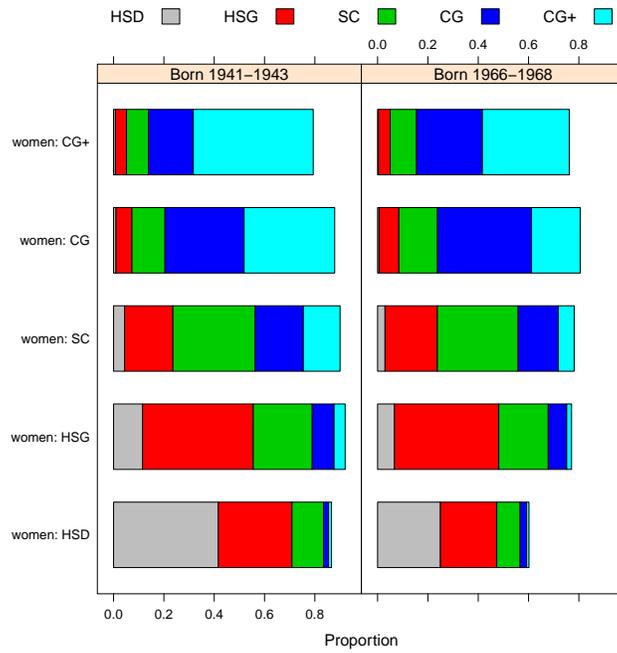


Figure 4b: Marriage patterns of white women

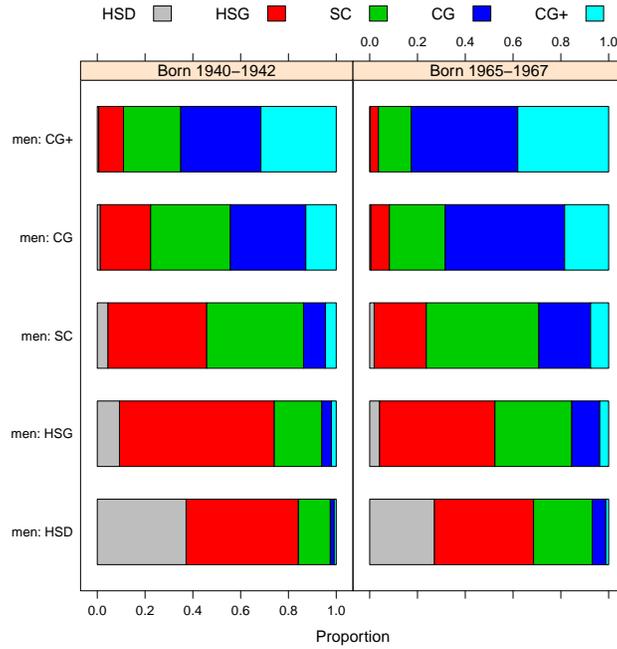


Figure 5a: Marriage patterns of white men who marry

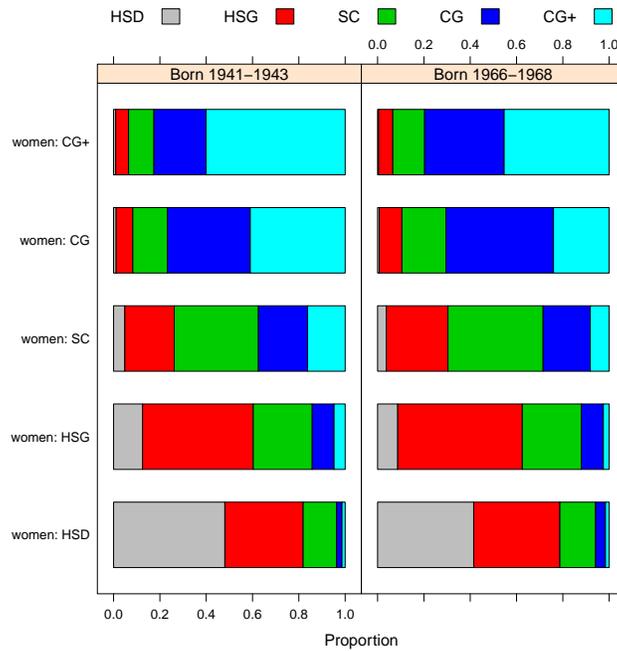


Figure 5b: Marriage patterns of white women who marry

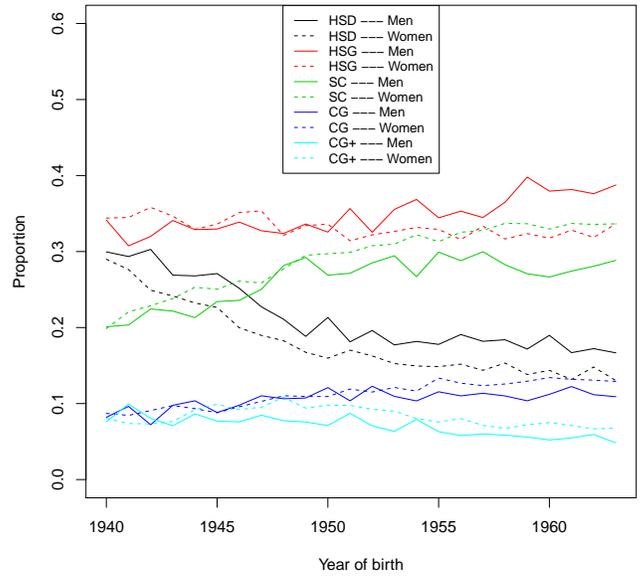


Figure 6a: Education of black men and women by cohort

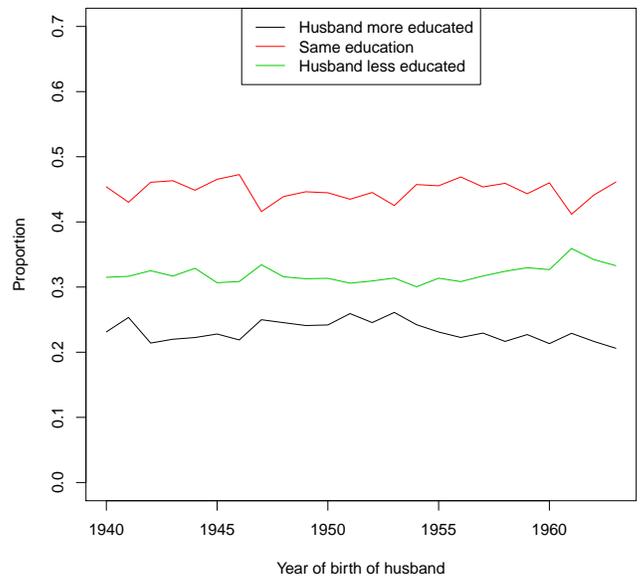


Figure 6b: Comparing educations within black couples

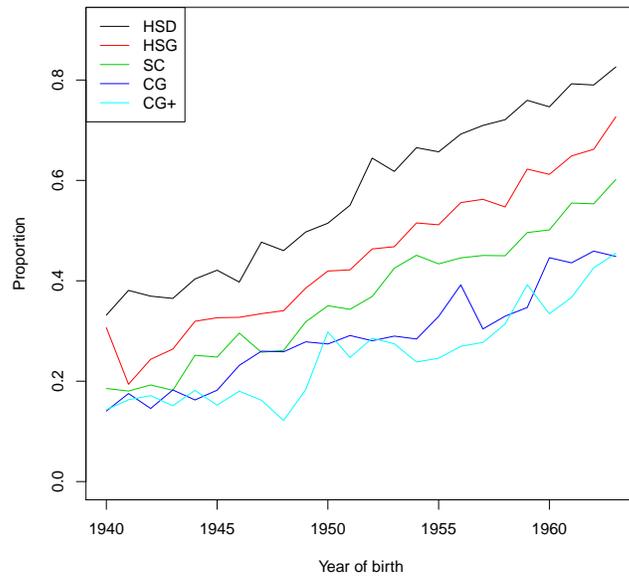


Figure 7a: Proportion of black men who never married by cohort

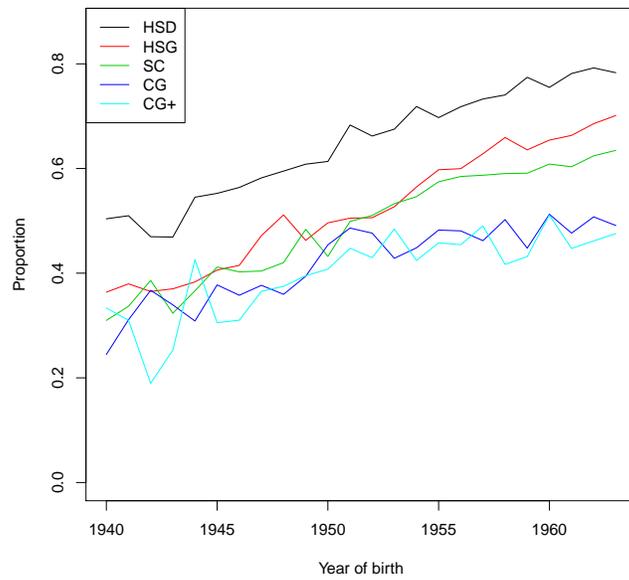


Figure 7b: Proportion of black women who never married by cohort

Testing Benchmark Model for Whites (I=J)

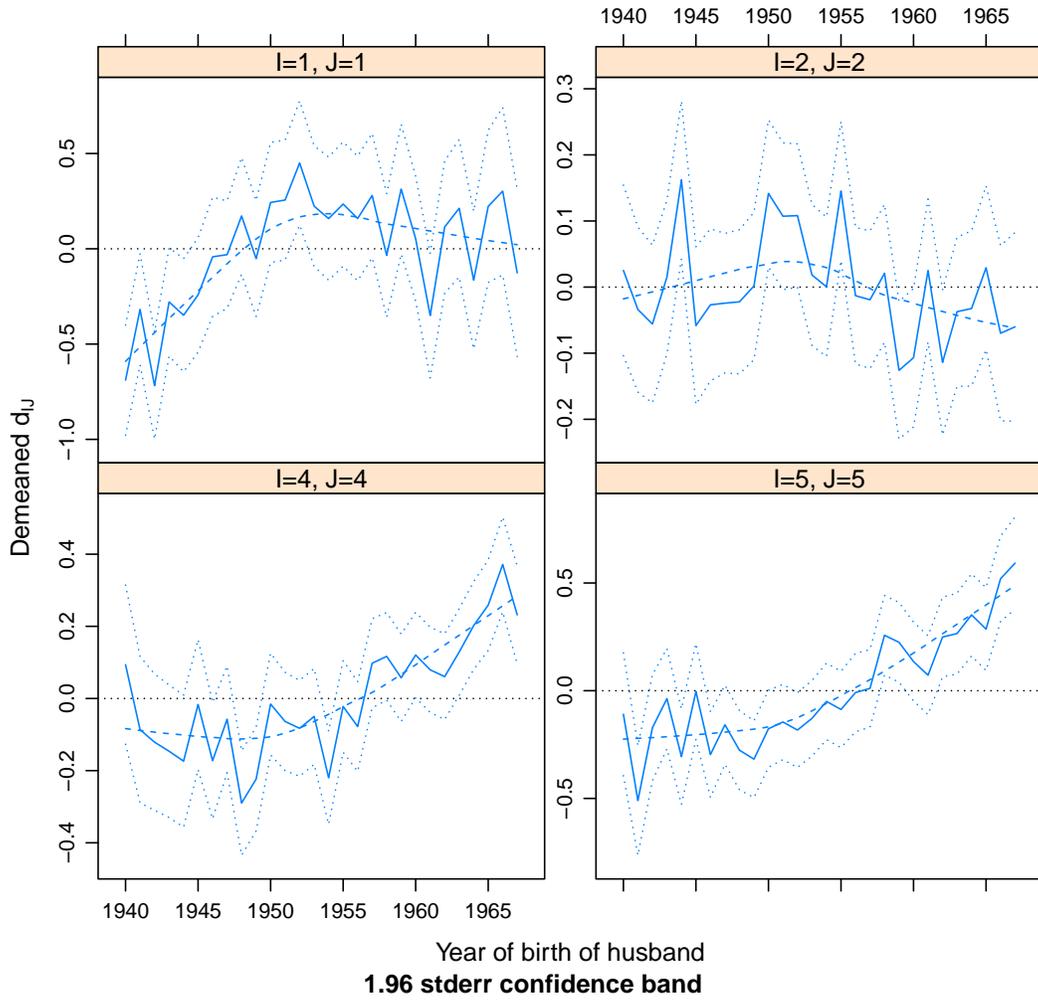


Figure 8: Testing benchmark model for whites ($I=J$)

Testing Various Models for Whites (I=J)

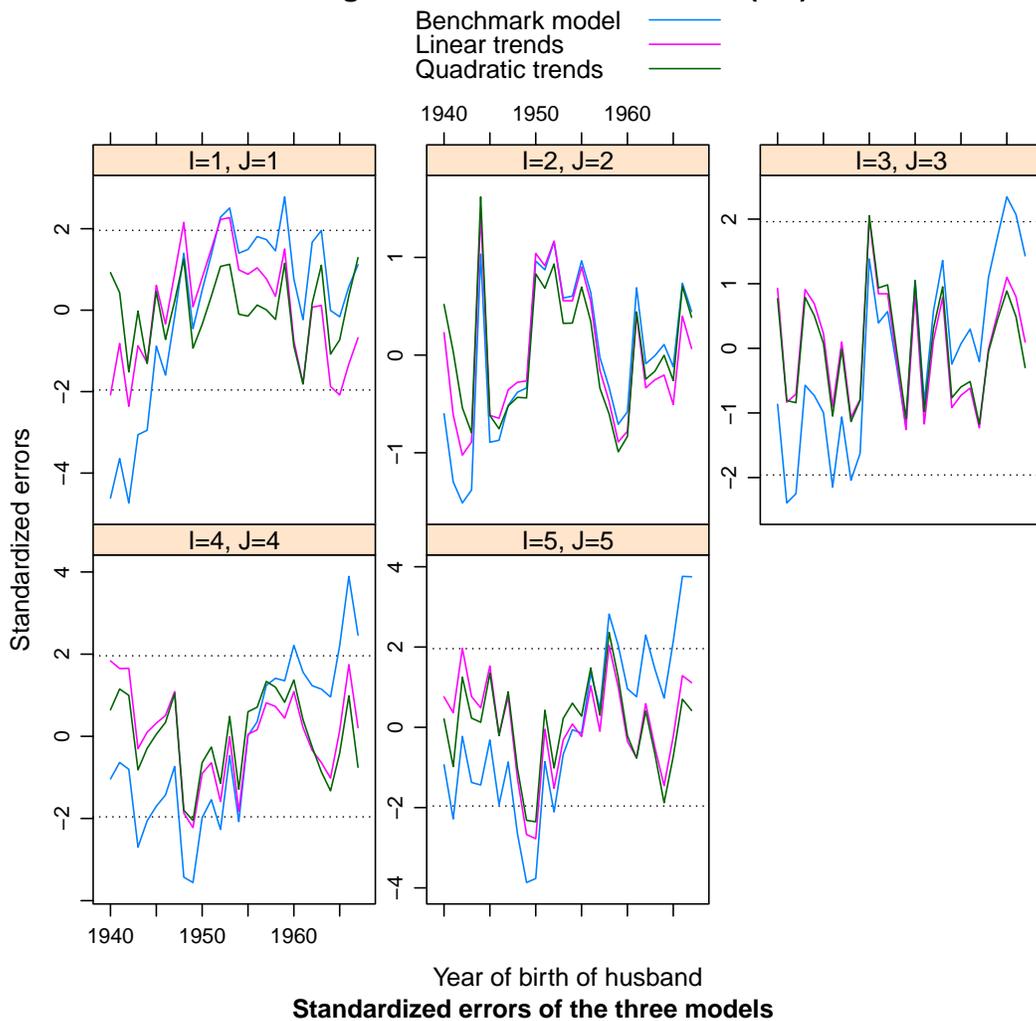


Figure 9: Testing various models for whites ($I=J$)

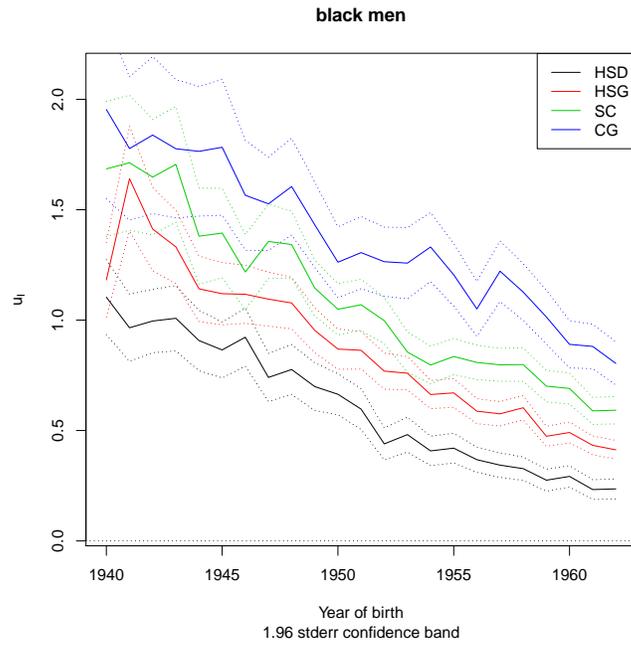


Figure 10a: Expected utilities for black men

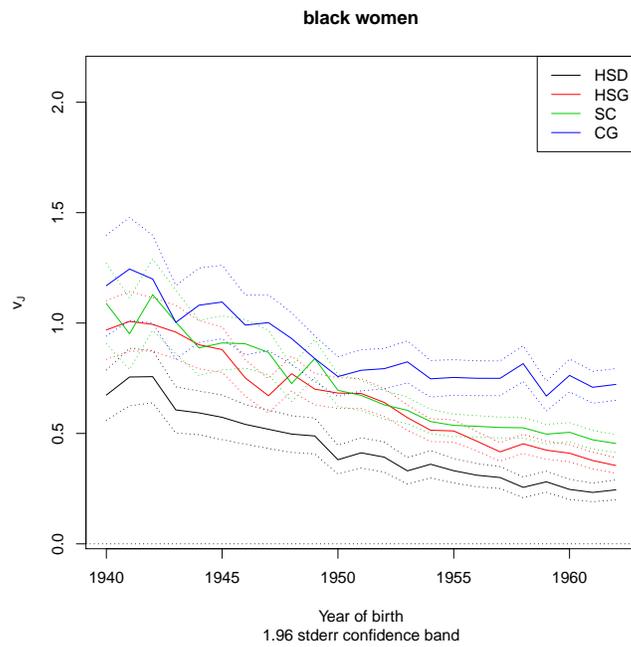


Figure 10b: Expected utilities for black women

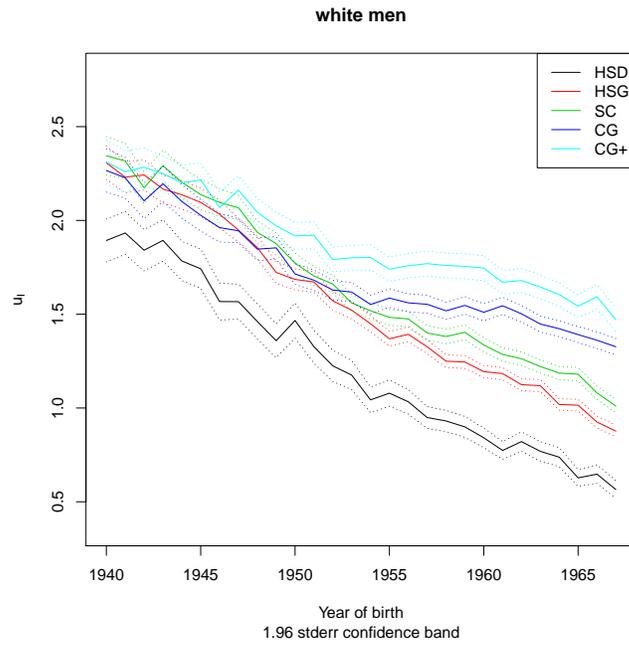


Figure 11a: Expected utilities for white men

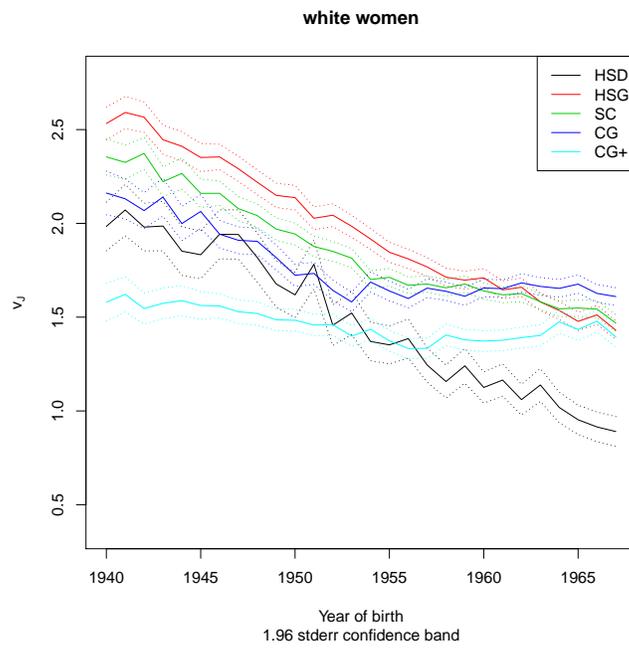


Figure 11b: Expected utilities for white women

whites

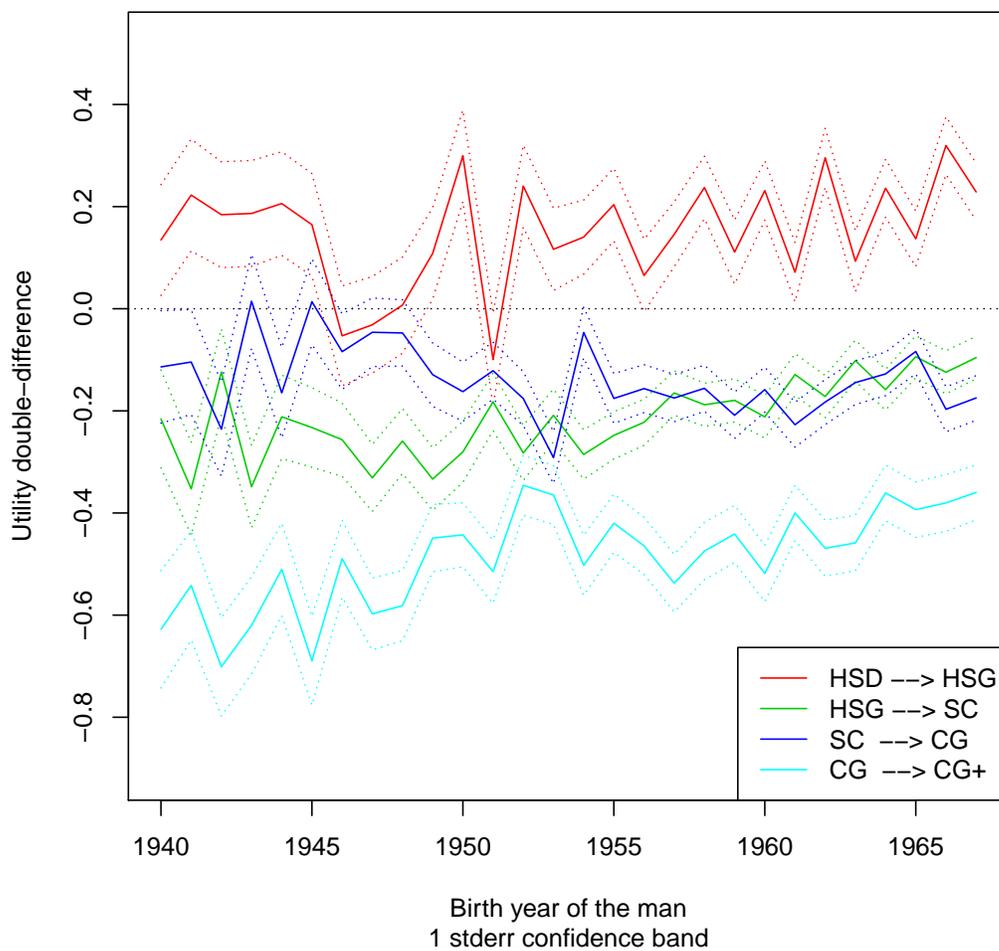


Figure 11c: Excess premia for women