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OPTIMAL CAPITAL REQUIREMENTS OVER THE BUSINESS AND FINANCIAL CYCLES

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Optimal capital requirements over the business and financial cycles*

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Abstract

I propose a simple theory of intertwined business and financial cycles, where financial regulation both optimally responds to and influences the cycles. In this model, banks do not internalize the effect of their credit expansion on other banks' expected bankruptcy costs, which leads to excessive aggregate lending. In response, the regulator sets a capital requirement to trade off expected output against financial stability. The capital requirement that ensures investment efficiency depends on the state of the economy and, because of a general equilibrium effect, its stringency increases with aggregate banking capital. A regulation that fails to take this effect into account would exacerbate economic fluctuations and result in excessive aggregate lending during a boom. It would also allow for an excessive build-up of risk in the financial sector, which implies that, at the peak of a boom, even a small adverse shock could trigger a banking sector collapse, followed by an excessively severe credit crunch.

1 Introduction

The recent crisis has exposed how important the interactions between the financial sector and the real side of the economy can be. Yet most of the models used by researchers

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and policy makers to think about these two spheres are separate, and there is no consensus on an integrated approach.[1] I develop here a simple theory of intertwined business and financial cycles, where financial regulation both optimally responds to and influences the cycles.

The questions I seek to address include What are the general equilibrium effects of bank capital requirements? Should bank capital requirements be tightened in "good times" and loosened in "bad times"? (A tightening means that a larger fraction of loans has to be funded by bank equity.) What macroeconomic variables are key for determining the optimal stringency of capital requirements?

To study these questions, I build a model of excessive aggregate lending by the financial sector. In response to the market failure, the regulator sets capital requirements to trade off expected output against financial stability (i.e. lower probability and smaller social cost of a banking sector collapse). The capital requirement that restores investment efficiency depends on the state of the economy and evolves, therefore, over time. Although other tools could equally be used by the regulator to correct the market failure, my focus on capital requirements is motivated by the current policy debate on their effect on the real side of the economy and, in particular, on the cyclical effects of bank regulation.

Cyclically adjusted capital requirements have been used in Spain since 2000 (Jimenez, Ongena, Saurina, and Peydro, 2013) and other countries have started to make discretionary adjustments based on the state of the economy.[2] More generally, the introduction of the so-called *counter-cyclical capital buffers* is an explicit recommendation of "Basel III", the latest version of the Basel Committee on Banking Supervision's international standards for banking regulation.[3] This seems in line with the logic that the benefits, in terms of financial stability, of tight capital requirements have to be balanced with the costs associated with the potential contraction of credit (Kashyap and Stein, 2004).[4] And, if these costs and benefits are dependent on the state of the economy, then the optimal capital requirement may vary over the cycle.

The model includes overlapping generations of risk-neutral savers and bankers in an economy that is subject to random shocks that trigger cyclical fluctuations. Bankers

[1]See for instance Goodhart (2010); Woodford (2010); Galati and Moessner (2013).

[2]For instance, in September 2012, amid credit crunch concerns, the UK Financial Services Authority has decided to soften bank capital requirements for new lending

[3]There is now a consensus that the Basel II regulation has a magnifying effect on real economic fluctuations. This has been dubbed the "pro-cyclical effect of bank regulation" (see Kashyap and Stein (2004); Repullo and Suarez (2013). The buffers recommended in Basel III essentially aim at mitigating this effect. This is why they have been called "counter-cyclical", even though they are supposed to be larger in good times than in bad times. In this paper, to avoid confusion with the business cycle terminology (where a variable is "pro-cyclical" if it is positively correlated with the state of the economy), I refer to the tightness or the stringency of the capital requirement rather than to the size of the associated equity buffer.

[4]See Admati, DeMarzo, Hellwig, and Pfleiderer (2010) and Hellwig (2010) for a critique of the notion that capital requirements are costly.

are protected by limited liability. They collect deposits and competitively lend to firms, which operate a constant returns-to-scale production function. Bank lending is the only source of firm funding. Firms always make zero profits, and bankers are, in effect, the residual claimants of the production. Labor supply is fixed, and decreasing marginal productivity of physical capital translates into decreasing marginal returns to lending. Or, from an opposite standpoint, aggregate bank lending affects the marginal productivity of physical capital. This general equilibrium effect is a key feature of the model. When the proceeds from lending are insufficient to repay depositors in full, the bank is insolvent and must default. Default is costly in the sense that an amount of consumption goods, proportional to the extent of insolvency, is lost in the bankruptcy procedure. This friction interacts with diminishing returns to capital and this results in a typically inefficient competitive outcome. The key mechanism is that bankers do not internalize that credit expansion affects the quality of the marginal loan in the economy, which increases the expected bankruptcy costs of other banks. Bankers' private marginal cost is therefore smaller than social marginal cost, which translates into aggregate over-investment and requires regulatory intervention. Finally, there is a regulator whose mission is to restore investment efficiency.

I find that the capital requirement that ensures investment efficiency is *decreasing in expected productivity*. This is intuitive since an increase in expected productivity makes marginal investments in the economy, and therefore the marginal bank loan, more profitable. All other things equal, regulation should therefore be less stringent when expected productivity is high. This channel suggests that the time-series effects of the Basel II regulation are, to some extent, desirable.[5]

The optimal capital requirement is also *increasing in aggregate bank capital*. This is a key result of the paper and is perhaps slightly less intuitive. On the one hand, more bank capital means that the banking sector can absorb more losses, which decrease the probability and the extent of bank insolvency (hence alleviating deadweight losses). And this suggests that the banking sector could expand. But, on the other hand, there is a general equilibrium effect that dominates the loss absorbing effect. To see the intuition, first consider an atomistic bank that doubles its equity base. It should simply be allowed to double the size of its assets. However, if all banks in the economy double their equity base, and if they are allowed to double the size of their assets, this could double aggregate lending in the economy. Given diminishing returns to capital this cannot be optimal. In fact, the optimal policy is to let the banking sector expand, but

[5]Basel I imposed a capital requirement of 8% on risk weighted bank assets. Risk weights where essentially fixed (that is, each of the five categories of borrowers would have a fixed weight, and a borrower would not change categories). Basel II introduced finer risk weights to better account for loan riskiness *in the cross-section*. Namely, Basel II weights were directly linked to the loan probabilities of default. But probabilities of default tend to come move over time (they go up on average in recession for instance), which affects the effective stringency of such a regulation *in the time-series*. This observation is at the root of the widespread view that Basel II has procyclical effects on the economy.

less than proportionally, which corresponds to an increase in the capital requirement and resonates with the notion of counter-cyclical capital buffers recommended by Basel III.

The dynamics of the model deliver periods of "good times", when productivity, expected productivity, consumption, investment, and physical and bank capital are high, and periods of "bad times", when they are all low. Given the results above, there are therefore two opposite forces. High expected productivity and larger loss absorbing power advocate for lower capital requirements, but the general equilibrium effect of aggregate bank capital goes in the other direction. It turns out that the latter dominates and the optimal capital requirement in the model is therefore tighter in good times than in bad times.

If this general equilibrium effect is overlooked by the regulator, it will magnify economic fluctuations. Aggregate bank lending will be excessive during a boom and the contraction that will follow a bust will be excessive.

To study the potential distortions from government guarantees, I also consider the case where deposits are insured, which makes the interest they are paid insensitive to bank asset quality. Deposit insurance can improve efficiency because lower interest payments reduce the probability and the extent of insolvency, and hence default costs. One way to interpret this is that deposit insurance acts as an implicit subsidy to bankers, which corresponds to an increase in the real value of their equity.

However, under a suboptimal capital requirement, deposit insurance can strongly exacerbate the excessive fluctuations mentioned above. In fact, the implicit subsidy makes banks willing to fund negative net present value investment. In that case, at the peak of a boom, a small adverse shock (or even a shock that is not positive enough) could trigger a banking sector collapse, followed by a severe credit crunch.[6]

Related literature

This paper contributes to the literature on bank capital regulation and on the role of financial frictions in macroeconomics. In terms of approach, the key novelty is to provide a model of bank regulation that reacts to and influences intertwined business and financial cycles.

The most closely related paper is Repullo and Suarez (2013), which studies optimal bank capital regulation over the cycle and compares it to regulations that resemble Basel I, II, and III. They find that counter-cyclical buffers help to mitigate the pro-cyclical effects of regulations such as Basel II. In their set-up, optimal capital requirements are always tighter in bad times than in good times. An important feature

[6]The popular view that risks are being piled up by the banking sector during good times finds some empirical support (see Borio and Drehmann, 2009 and Boissay, Collard, and Smets, 2013 for instance). It has also been suggested that financial booms do not just precede busts but cause them (Borio, 2013) and that the amplitude of the financial cycle is influenced by financial regulation regimes (Borio and Lowe, 2002).

of their model is that the production function is linear in investment, which does not leave much room for interaction between aggregate bank capital and marginal productivity in the real sector. In other words, this assumption restricts interactions between the financial and the real business cycle, and they do not capture the general equilibrium effects I highlight in this paper, which explains the difference in results.

The paper is also related to Martinez-Miera and Suarez (2014) who propose a macroeconomic model of endogenous systemic risk-taking in which correlated risk-shifting by some banks gives an incentive to other banks to play it safe, because the more banks that fail at the same time, the larger the scarcity rent after a crisis. Still, the competitive outcome is inefficient, and the optimal (constant) capital requirement trades off output at steady state with the severity of financial crisis (the time it takes to go back to steady state). They do not consider business cycle dynamics but, in their model, loosening capital requirements after a banking crisis reduces the scarcity rent earned ex post by the "last banks standing" and induces thus more systemic risk-taking ex ante. In contrast, in Dewatripont and Tirole (2012) incentives to gamble for resurrection are stronger after a negative macroeconomic shock. Related to this, Morrison and White (2005) study optimal capital requirements in a model with both moral hazard and adverse selection. They find that the appropriate policy response to a crisis of confidence may be to tighten capital requirements. This happens when the regulator's ability to alleviate adverse selection through banking supervision is relatively low. Repullo (2013) also finds that capital requirements should be loosened after an exogenous negative shock to bank capital. Here again, the relative contribution of my paper is to endogenize aggregate bank capital and to highlight the general equilibrium effects that result from its interaction with the business cycle.[7]

The paper also speaks to studies on the costs and/or benefits of bank capital requirements. These include Van den Heuvel (2008), Admati, DeMarzo, Hellwig, and Pfleiderer (2010), Hellwig (2010), and Morrison and White (2005). It is however important to stress that I only focus on the *cyclical properties* of optimal bank capital requirements. The highly stylized nature of my model makes it less well suited to study the optimal *level* of the requirement.

Many paper in the literature on financial frictions in macroeconomics build on Geanakoplos and Polemarchakis (1986) and study models where the equilibrium level of investment is generally inefficient because investors fail to internalize the social cost of a binding borrowing constraint in the presence of a pecuniary externality. Examples of models that exhibit underinvestment with respect to the first best level, but overinvestment with respect to the second best are Lorenzoni (2008) and Jeanne and Korinek

[7]Other papers provide a rationale for a systemic approach to regulation based on financial institutions' incentives to choose correlated exposures (Farhi and Tirole, 2012, Acharya, 2009), on network externalities (Allen, Babus, and Carletti, 2011), on aggregate demand externalities (Farhi and Werning, 2013).

(2013).[8] The equilibrium in my model exhibit these properties, but the externality and the focus are different.

Finally, my paper also relates to the financial accelerator literature that followed Bernanke and Gertler (1989). In these papers, entrepreneur net worth alleviates an agency problem and aggregate net worth dynamics play a key role in transmission and amplification of shocks. In my model, aggregate bank net worth plays a similar role. However, while the backbone of my model is very close to Bernanke and Gertler (1989), there are key departures that yield a very different mechanism. Furthermore, contrasting the externality in my model with the financial accelerator sheds some new light on that literature. In particular, it makes clear that the way one models default costs can have striking consequences on the form of inefficiency that results.

I present and discus the environment in Section 2. I define the equilibrium and efficient concepts in Section 3. I expose the market failure and analyze the optimal regulatory response in Section 4, and I discuss the results and the policy implications in Section 5. Section 6 concludes.

2 The model

2.1 The basic environment

There are infinite periods indexed by $t = 0, 1, 2, ...$, in which generations of agents born at different dates overlap.

Agents All agents are risk neutral, live two periods, and derive utility from their end-of-life consumption. There is a measure 1 of agents born at the beginning of each period. They are endowed with one unit of labor, which they supply inelastically during the first period of their life for a wage w. Then, these agents incur an "ability" shock. A share $\eta \ll 1$ of these agents is endowed with "banking ability", which enables them to set up a bank and invest in its equity. The remaining share $1 - \eta$ receives no further working ability and retire. I refer to them as savers.

It is convenient to think of each period being divided in two successive phases. During the production phase, firms combine labor with physical capital to produce consumption goods, which they use to pay the factors of production. Then comes the investment and consumption phase.

[8] Other relevant papers that build on pecuniary externality mechanisms include Gromb and Vayanos (2002), Bianchi (2011), Brunnermeier and Sannikov (2014), Korinek (2011), Jeanne and Korinek (2010), Krishnamurthy (2003), Gersbach and Rochet (2012), and Stein (2012); see Hanson, Kashyap, and Stein (2011) for an overview.

Production In each period, there is a continuum of penniless firms that operate a constant-return-to-scale production function. Since labor supply is fixed, there are diminishing returns to capital. The production function takes the form Ak^{α}, where k is physical capital per worker, $0 < \alpha < 1$, and A is a variable that captures aggregate productivity. The physical capital fully depreciates in the production process.

Firms compete for workers and for physical capital (which they borrow from banks). They pay a wage w, and repay R per unit of borrowed capital. Assuming perfect competition on these markets, we have at equilibrium that:

$$\begin{cases} w = (1-\alpha)Ak^{\alpha} \\ R = \alpha Ak^{\alpha-1}, \end{cases} \tag{1}$$

which ensure that firms always make zero profit.

Investment and consumption In the second phase of each period, agents take investment and consumption decisions.

Young savers can choose between depositing their labor income at the bank or using a safe storage technology. The rate of return to storage is normalized to 0. I focus on cases where deposits are in excess supply (i.e. the storage technology is used in equilibrium), so that the return to deposits is the same, in expectation, as the return to storage.[9]

Young bankers can set-up a bank under the protection of limited liability. Hence, they can allocate their wage between bank equity and safe storage.[10] Banks raise deposits (to which they promise a gross return r) and invest in physical capital (their banking ability enables them to transform, one to one, consumption goods into physical capital). Since physical capital will then be competitively lent to firms in the next period, bank investment decisions can therefore be interpreted as lending decisions, where banks take the distribution of marginal return to lending as given. Banks are the only source of funds to firms. Therefore, at equilibrium the realized return to lending is the realized marginal return to capital $R = \alpha Ak^{\alpha-1}$.

Old agents consume their wealth and die. Old savers' wealth consists of their proceeds from storage and deposits, net of government transfers if any (deposit insurance payments and taxes). Old bankers' wealth consists of their proceeds form storage and investment in bank equity. If the bank net worth is negative, bankers can keep their proceeds from storage since they are protected by limited liability.

[9]The economy can be considered as a small open economy with excess savings, facing the world interest rate.

[10]They could also be allowed to deposit at other banks, but given the assumption that deposits are overall in excess supply, this would not change anything to the analysis.

2.2 Frictions and shocks

Costly default Bankruptcies often involve deadweight losses (Townsend (1979)). In the case of financial institutions, losses can be large (James (1991), and banking crises are typically followed by long and painful recessions (Reinhart and Rogoff (2009)) involving permanent output losses (Cerra and Saxena, 2008).[11]

In this model, bank insolvency triggers default. In that case, I assume that the creditors cannot recoup the full value of the assets because an amount $\Psi(z,\gamma) \geq 0$ of consumption goods is destroyed in the bankruptcy procedure. Variable $z \geq 0$ denotes the extent of insolvency, that is, the shortfall in bank asset value with respect to promised repayment to depositors, and $\gamma \geq 0$ is a parameter that captures the intensity of the bankruptcy costs. In particular, I assume that

- $\Psi_z(z,\gamma) \geq 0$ and $\Psi(0,\gamma) = 0$; that is, default costs are increasing in the extent of insolvency. By definition, there are no costs if the bank is solvent.
- $\Psi_\gamma(z,\gamma) \geq 0$ and $\Psi(z,0) = 0$; that is, default costs are increasing in γ and they are nil if $\gamma = 0$;

Deposit insurance I consider two different regimes. Either deposits are insured by the government, or not. In the deposit insurance regime, the government fully compensate depositors for their losses in case of bank insolvency and breaks even by imposing a lump-sum tax on savers. Since I am interested in the distortions created by such insurance, I treat here deposit insurance as an exogenous feature of the environment.

Productivity shocks In line with the business cycle literature, I let aggregate productivity exogenously fluctuate over time: A_t is a random variable distributed over a bounded subset of $\mathbb{R}_0^+$ with some probability distribution function.

Financial shocks The banking sector is also exposed to exogenous shocks.[12] A simple way to capture this is to assume that the proportion of agents that receive banking ability is stochastic. Hence, I let η_t be a random variable distributed over a subset of $(0,1)$, with some probability distribution function.

[11] See Dell'Ariccia, Detragiache, and Rajan (2008) and Kroszner, Laeven, and Klingebiel (2007) for empirical evidence supporting the widespread perception that the relation is causal.

[12] This approach has been adopted by several recent papers. See for instance Jermann and Quadrini (2012).

2.3 Discussion of the economic environment

The backbone of the model is very similar to Bernanke and Gertler (1989). The main differences are that *i)* instead of having an entrepreneur sector, bankers are able to transform consumption goods into capital one for one; *ii)* I impose that banks issue deposit contracts (that may, or may not, be insured), *iii)* in case of bankruptcy, bankers face deadweight costs that increase with the extent of insolvency (in Bernanke and Gertler, 1989, default costs for entrepreneurs are proportional to the value of their production).

The main focus of the paper is the general equilibrium interactions between diminishing returns to capital and a (class of) friction(s) that prevent bankers from fully internalizing the social cost of their lending. The detailed nature of the underlying information asymmetry problems that generates the friction is not central to the analysis. This is why I see my reduced form approach to costly default and deposit insurance (together with the restriction on the contract space) as reasonable and, in fact, desirable because it yields a simple model that delivers transparent insights through closed-form solutions.

It is nevertheless worth providing further motivation for the environment. First of all, there is a vast literature looking at why standard debt contracts (in the model, when they are not insured, deposits are standard debt contracts) may endogenously arise when there are agency problems (Townsend, 1979, Gale and Hellwig, 1985). Building on these, default costs have to be apprehended in this model as reflecting underlying agency problems that make the deposit contract optimal. A concrete benefit of my approach is that it allows me to carefully inspect the market failure and to show, contrasting the key externality to the financial accelerator literature (Bernanke and Gertler, 1989, Carlstrom and Fuerst, 1997, and many other papers), that the particular way one models default (or "verification") costs can have striking consequences on the form of inefficiency that results (see the analysis in Subsection 4.2.2).

Second, I am interested in the distortionary effect of government guarantees, through their impact on financial institutions funding costs. In the real world, such guarantees are explicit in the case of deposit insurance, which is in place in all advanced economies (Demirguc-Kunt, Kane, Karacaovali, and Laeven, 2008).[13] They can also be implicit, for instance due to the too-big-to-fail doctrine (see Acharya, Anginer, and Warburton, 2014, Noss and Sowerbutts, 2012, and Ueda and Weder di Mauro, 2013 for empirical evidence on "systematically important" banks being able to borrow at implicitly subsidized rates). In the model, the distortions created by deposit insurance would also

[13] Note that coverage may be different across countries. Coverage, in terms of maximum amount per person (or account) has generally been extended during the 2008 crisis. In some cases, it has been fully extended ex-post, including to other types of debt. More recently however, in Cyprus, ex-ante uninsured depositors have been excluded ex-post.

arise with implicit guarantees that would, for instance, reflect the inability of the government to fully commit not to bail out bank creditors. Hence, deposit insurance can be interpreted as a reduced form of a more general class of problems.[14]

Finally, I focus on a capital requirement for several reasons. First of all, capital adequacy ratios are one of the main regulatory tools used for prudential purposes in practice. Second, they currently are at the core of several policy debates. Third, in the context of this simple model, they enable the regulator to restore investment efficiency. Last but not least, capital requirements are easy to interpret, they economize on notation, simplify computations, and they make aggregation straightforward.

2.4 Summary of intraperiod time-line

Production

- A_t is realized and publicly observed, firms competitively hire workers and borrow physical capital from banks.
- Production takes place and is allocated: wages are paid, and the share of capital goes to the bankers.
- If solvent, banks repay depositors. If insolvent, banks default and the associated costs are incurred by the depositors. In the deposit insurance regime, the regulator compensates them for their losses.

Investment and consumption

- η_t is realized, young agents learn whether they have banking ability.
- Young bankers make their investment portfolio decision (storage and/or investment in their bank's equity).
- Banks borrow from savers and invest in physical capital. Savers put the remainder of their savings in the storage technology.
- The old generation consumes and leaves the economy.

[14]Other papers that study the distortions caused ex-ante by government guarantees include Merton (1977), Kareken and Wallace (1978), Keeley (1990), Pennacchi (2006), Gete and Tiernan (2014). And Gomes, Michaelides, and Polkovnichenko (2010) attempts to quantify the distortions that arise ex-post, when taxes need to be raised to finance the bailouts.

3 Competitive equilibrium

3.1 The problem of the banker

Because they are protected by limited liability, bankers will never decide to store within the bank. Their relevant decisions are how to allocate their wealth between storage and bank equity and how much the bank lends, given its level of equity. This can be formalized as follows.

Consider a representative bank at date t, and denote e_t its amount of bank capital (or *equity*) and d_t its deposits. Total lending by the bank is then $(d_t + e_t)$. Let v_{t+1} denote the ex-post net worth of the bank, i.e. its value after R_{t+1} is realized. That is,

$$v_{t+1} \equiv (d_t + e_t)R_{t+1} - d_t r_t \ ,$$

where, r_t is the gross interest rate on deposits, which is a promised date $t+1$ payment, made in period t, hence the difference of subscript with R_{t+1}, which is uncertain as of date t (as a convention, the variable time-subscripts reflect the period at which they are realized or determined).

Then, consider a representative banker born at date t. After having inelastically supplied his labor and earned a wage w_t, his maximization problem can be written as follows:

$$\max_{e_t, d_t} E_t \left[c_{t+1} \right]$$

subject to the budget constraints and non-negativity conditions:

$$\begin{cases} e_t + s_t = w_t \\ c_{t+1} = v_{t+1}^{+} + s_t \\ e_t,\ d_t,\ s_t,\ c_{t+1} \geq 0 \quad , \end{cases}$$

where v_{t+1}^{+} is the realized (private) value of bank equity, i.e. the positive part of v_{t+1}:

$$v_{t+1}^{+} \equiv \left[(d_t + e_t)R_{t+1} - d_t r_t \right]^{+} ,$$

and s_t denotes the amount stored by the banker from date t to date $t+1$.

Equilibrium definition Given a sequence for the random variables $\{A_t, \eta_t\}_{t=0}^{\infty}$, and initial condition k_0, a competitive equilibrium (without regulator intervention) is a sequence $\{w_t, R_t, e_t, d_t, r_t, \tau_t\}_{t=0}^{\infty}$, such that: vector $\{w_t, R_t\}$ clears the labor and capital markets at date t; vector $\{e_t, d_t\}$ solves the maximization problem of the representative banker born at date t; in the economy without deposit insurance, r_t is such that all savers break even in expectation, and $\tau_t = 0$ at all t; in the economy with deposit insurance, $r_t = 1$ at all t, and τ_t is a lump-sum tax on savers such that the regulator breaks even at all t; and the law of motion for physical capital is given by $k_{t+1} = \eta_t \left(e_t + d_t\right)$.

3.2 Efficiency concepts

3.2.1 First best allocation

Let me consider a benchmark competitive equilibrium in the economy without frictions (that is, without deposit insurance, and with $\gamma = 0$). Since savers must break even in expectation, the cost of deposits for banks must equal the rate of return to storage ($r_t = 1$). The relevant first order condition of the bankers (with respect to d_t) is then:

$$E_t[R_{t+1}] = 1.$$

Substituting the physical capital market clearing condition (1) and solving for k_{t+1} pins down the equilibrium level of aggregate physical capital:

$$k^{FB}_{t+1} = \left(\alpha E_t[A_{t+1}]\right)^{\frac{1}{1-\alpha}} . \tag{2}$$

It is straightforward to show that an equilibrium where this condition is satisfied at all t is Pareto efficient. In short, the Modigliani-Miller theorem holds, bankers are indifferent between storage and investment in bank equity, and they make zero profit in expectation.[15] I will refer to the investment level given by (2) as the *first best* investment level, hence the superscript.

3.2.2 Constrained efficiency

One of the main purposes of this paper is to show how regulatory intervention can improve efficiency, even when the regulator faces similar constraints as those imposed on private agents. In short, the investment level at a given date will be said to be constrained efficient (or *second best*) if it maximizes next period expected output, net of depreciation and bankruptcy costs. Hence, the constrained efficient level can be interpreted as the one that solves a trade-off between expected output and the cost of a banking sector collapse. Formally, it is defined as:

$$k^{SB}_{t+1} \equiv \arg\max_{k_{t+1}} E_t[A_{t+1}]k^{\alpha}_{t+1} - k_{t+1} - E_t\left[\Psi\left(Z(k_{t+1}),\gamma\right)\right],$$

where

[15] Note that there exists a continuum of equilibria because of the indeterminacy implied by the Modigliani-Miller theorem.

$$Z(k_{t+1}) \equiv \left[(k_{t+1} - \eta_t w_t)\, r_t(k_{t+1}) - \alpha A_{t+1} k_{t+1}^{\alpha} \right]^{+}$$

is the aggregate shortfall in bank value with respect to promised repayment to depositors. It is derived from the representative bank extent of insolvency:

$$z_t \equiv \left[d_t r_t - (d_t + e_t) R_{t+1} \right]^{+},$$

together with the law of motion for investment: $k_{t+1} = \eta_t \left(e_t + d_t \right)$ and the promised unit repayment to depositors $r_t(k_{t+1})$, such that they break even in expectation.

Note that the function $Z(k_{t+1})$ implicitly captures the restrictions associated with the environment. The key restriction is that only banker wealth can alleviate bankruptcy costs. In the competitive environment, by imposing a deposit contract between the bank and the savers, I implicitly rule out arrangements that circumvent this restriction. Accordingly, the definition above imposes a repayment consistent with the (insured or not) deposit contract.[16] The other restriction is simply that physical capital be paid its marginal productivity. Finally, note that banker's willingness to participate (and invest their wealth in bank equity) is not an issue here because the other restrictions imply that they make, at worst, zero profit in expectation.[17]

4 Analysis

In this section, I analyze the market failures and I show how the regulator can ensure investment efficiency thanks to a time-varying capital requirement.

The regulator I study the problem of a regulator, whose mission is to restore constrained efficiency, whenever the market outcome is inefficient. The regulatory tool is a capital requirement $x_t \in [0, 1]$, which constraints bank lending to a multiple of its equity.

$$x_t(d_t + e_t) \leq e_t \tag{3}$$

Constrained equilibrium A *constrained equilibrium* is defined as a straightforward extension of the competitive equilibrium. Given the same sequence of random variables and initial condition, it is defined as a sequence of capital requirements $\{x_t\}_{t=0}^{\infty}$

[16]Clearly, a sufficient ex-ante transfer from young savers to young bankers would allow the latter to fully fund investment with equity and implement a first best allocation. Even though I rule out such ex-ante transfers, ex-post transfers may occur in the deposit insurance regime. I analyze their impact on efficiency in Subsection 4.3.

[17]This is because $\Psi_{t+1} \geq 0$ implies $k_{t+1}^{SB} \leq k_{t+1}^{FB}$, which ensures that the expected marginal return to capital is bounded below by one.

and a vector sequence $\{w_t, R_t, e_t, d_t, r_t, \tau_t\}_{t=0}^{\infty}$ satisfying the same conditions, with the only difference that $\{e_t, d_t\}$ must solve the problem of the representative banker born at *t subject to the capital requirement* x_t.

Definition 1. A constrained equilibrium is said to be *efficient at date t* if $k_{t+1} = k_{t+1}^{FB}$ and *constrained efficient at date t* if $k_{t+1} = k_{t+1}^{SB}$.

4.1 Overinvestment and cyclical properties of the regulatory response

In this section, I detail the key mechanism of the paper and derive its implications in terms of regulatory response. Since this mechanism does not hinge on the friction specific form, I focus on the most simple case, which presents the great advantage of being fully solvable in closed form.

First, note that when deposits are insured and bankruptcy is costless ($\gamma = 0$), there is no efficiency trade off between expected output and default costs. The second best corresponds in fact to the first best:

$$k_{t+1}^{SB} = k_{t+1}^{FB}$$

Proposition 1. *Assume deposits are insured and default is costless ($\gamma = 0$).*

The competitive equilibrium at date t cannot be efficient if $x_t = 0$.

Proof. The reason for the inefficiency is that deposits are implicitly subsidized. Therefore, their expected marginal cost for the banker is below the social cost, which generates over-lending. To show this, note that $r_t = 1$ at all t and that the first order condition (with respect to d_t) of the banker can be written:

$$E_t[R_{t+1}] \leq \underbrace{\int_{\hat{R}_{t+1}}^{\infty} f_t(R_{t+1}) dR_{t+1}}_{non-default\ states} + \underbrace{\int_{0}^{\hat{R}_{t+1}} R_{t+1} f_t(R_{t+1}) dR_{t+1}}_{default\ states}, \tag{4}$$

where f_t is the probability distribution function of R_{t+1} conditional to date-t information, and $\hat{R}_{t+1} \equiv \frac{d_t}{e_t + d_t}$ is the solvency threshold (that is, if $R_{t+1} < \hat{R}_{t+1}$ the representative bank is insolvent). The right-hand-side of condition (4) captures the expected marginal cost of lending. It is decreasing in e_t (because the solvency threshold is itself decreasing in e_t and R_{t+1} must be strictly smaller than 1 in default states). Therefore, if $x_t = 0$, bankers optimally choose $e_t = 0$ (it is cheaper to fund lending with deposits), and banks do fail with strictly positive probability in equilibrium. Hence, the right-hand-side must be strictly smaller than 1, and there must be overinvestment at equilibrium.[18] □

[18] See the working paper version for a proof of equilibrium existence (Malherbe, 2014).

4.1.1 Optimal capital requirements

Proposition 2. *Assume deposits are insured and default is costless ($\gamma = 0$).*

The following capital requirement ensures investment efficiency ($k_{t+1} = k_{t+1}^{FB}$) at all t:

$$x_t^* = \min\left\{1, \eta_t w_t \left(\alpha E_t[A_{t+1}]\right)^{\frac{-1}{1-\alpha}}\right\}. \tag{5}$$

Proof. See Appendix A. □

If $x_t^* = 1$, banker wealth is in fact plentiful and the first best level of investment can be financed with bank equity ($\eta_t e_t = k_{t+1}^{FB}$). In the more interesting case where $x_t^* < 1$, the regulator can still implement the first best. First, note that x_t^* ensures that there cannot be overinvestment (by construction, k_{t+1}^{FB} is the investment level that ensues if all bankers invest their whole wealth in bank equity and fully leverage). Second, there cannot be underinvestment either because condition (4) cannot be satisfied for $k_{t+1} < k_{t+1}^{FB}$. Hence bankers invest all their wealth in bank equity and fully leverage, which implies that $k_{t+1} = k_{t+1}^{FB}$.

The case $x_t^* = 1$, where banks are fully funded with equity, is trivial to analyze but is of little empirical relevance. Henceforth, I assume that η_t is small enough to rule it out, and I only focus on the case where $x_t^* < 1$.[19]

Condition 1

$$\eta_t < \frac{\left(\alpha E_t[A_{t+1}]\right)^{\frac{1}{1-\alpha}}}{(1-\alpha)A_t\left(\alpha E_{t-1}[A_t]\right)^{\frac{1}{1-\alpha}}}, \forall t.$$

Condition 1 ensures that the first-best level of investment cannot be fully financed by bank equity.

The following corollary describes the constrained equilibrium in such a case.

Corollary. *Assume Condition 1 holds. Then, $x_t^* < 1$, $e_t = w_t$, the capital requirement is binding, $d_t = \frac{k_{t+1}^{FB}}{\eta_t} - w_t$, and the equilibrium value of w_t and R_t are pinned down by their respective market clearing conditions.*

Interpretation It is useful to write the optimal capital requirement as:

$$x_t^* = \frac{\eta_t e_t}{k_{t+1}^{FB}}. \tag{6}$$

Equation (6) highlights that the dynamic properties of x_t^* are intimately linked to the joint dynamics of $\eta_t e_t$ and k_{t+1}^{FB}. Before exploring these in detail, let me observe that the

[19] See Hanson, Kashyap, and Stein (2011), Stein (2012), and Admati, DeMarzo, Hellwig, and Pfleiderer (2010) for discussions on why bank capital is scarce in reality.

optimal capital requirement x_t^* is *decreasing in expected productivity* $E[A_{t+1}]$ and *increasing in aggregate bank capital* $\eta_t e_t$.

The first observation is intuitive since an increase in expected productivity makes marginal investments in the economy more profitable. Therefore, it makes the marginal loan more profitable and calls for credit expansion. As we will see in the dynamic analysis below, this mechanism captures rather well the logic by which the time-series effects of the Basel II regulation are, to some extent, desirable.

The second observation may appear less intuitive at first, but the underlying logic is very simple. To see it, first consider an atomistic bank that doubles its equity base. It should simply be allowed to double the size of its assets. However, if all banks in the economy double their equity base, and if they are allowed to double the size of their assets, this could double aggregate lending in the economy. Given diminishing returns to capital this would be excessive. And given that banks have incentives to overinvest, this would be problematic indeed. As we will now see, this mechanism can constitute a rationale for the counter-cyclical buffers of Basel III.

4.1.2 Intertwined business and financial cycles

In this subsection, I study the propagation of financial and productivity shocks along the path of the equilibrium derived above. In particular, I show how these shocks affect the dynamics of the optimal capital requirement, and how this can be interpreted as a rationale for key features of the Basel (II and III) regulation.

Shock dynamics Let aggregate productivity A_t follow some random process

$$A_t = A_{t-1}^{\phi} \epsilon_t, \tag{7}$$

defined over a bounded subset of $\mathbb{R}_0^+$, where $\phi \in (0, 1)$ is a parameter that captures the persistence in productivity, and where $\epsilon_t \in \mathbb{R}_0^+$ is a normalized *iid* random variable with a probability distribution function such that $E_{t-1}[\epsilon_t] = 1$ and $E_{t-1}[A_t] = A_{t-1}^{\phi}$.

Let also η_t follow some random process:

$$\eta_t = g(\eta_{t-1}, \theta_t), \tag{8}$$

where θ_t follows an *iid* random process such that $\frac{\partial \eta_t}{\partial \theta_t} > 0$ and $\eta_t \in (0, \bar{\eta})$, where $\bar{\eta} \ll 1$. These restrictions ensures that η_t stays positive and small (a regularity condition) and is increasing in θ_t (so that this shock can be interpreted as a *positive financial shock*).

Optimal capital requirement dynamics

Lemma 1. *Assume Condition 1 holds.*

The optimal capital requirement can be written as an explicit function of the last financial shock and all past productivity shocks:

$$x_t^* = \frac{(1-\alpha)}{\alpha} g\left(\eta_{t-1}, \theta_t\right) \left(\prod_{i=1}^{\infty} \epsilon_{t-i}^{\phi^i} \right)^{\frac{1-\phi}{1-\alpha}} \left(\epsilon_t\right)^{\frac{1-\alpha-\phi}{1-\alpha}} \tag{9}$$

Proof. See Appendix A. □

One can then write this equation for x_{t+1}, x_{t+2}... and take derivatives with respect to ϵ_t and θ_t to explicitly assess the effect of a shock on the stringency of contemporaneous and future capital requirements.

Proposition 3. *Assume deposits are insured, default is costless ($\gamma = 0$), Condition 1 holds, and the processes for the shocks are given by (7) and (8).*

i) A positive productivity shock tightens the contemporaneous optimal capital requirement (x_t^*) if and only if $\alpha + \phi < 1$. However, it tightens all future optimal requirements (x_{t+s}^*, $\forall s > 0$) for any $\alpha, \phi \in (0, 1)$.

ii) A positive financial shock tightens the optimal capital requirement. If the positive effect of the shock on aggregate bank capital is persistent, the tightening is persistent as well.

Proof. Differentiation of (9) gives the results. That is, $\frac{dx_t^*}{d\epsilon_t} > 0 \Leftrightarrow \alpha + \phi < 1$. $\frac{dx_{t+s}^*}{d\epsilon_t} > 0, \forall s > 0$. $\frac{dx_t^*}{d\theta_t} > 0$, $\frac{\partial x_t^*}{\partial g} > 0$. □

Figure 1 illustrates the results for the productivity shocks. The key observation is that the effect is always positive at any strictly positive of lags. Note that the general formulation of the process for η_t allows me to remain agnostic about the long term impact of a financial shock. However, since by construction η_t increases in θ_t, the contemporaneous effect is positive.

A Basel regulation decomposition Expected productivity captures economic prospects and determines the optimal level of investment in the economy. Hence, this investment level depends on past realizations of the productivity shock:

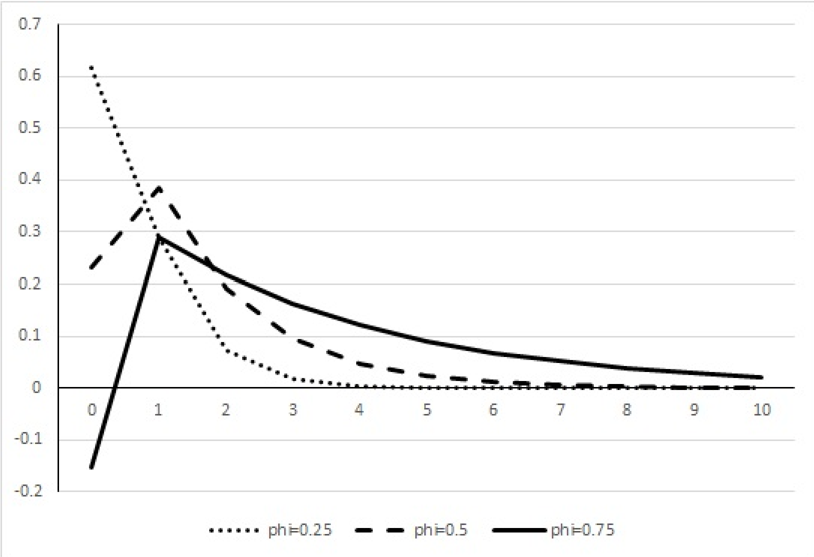
$$k_{t+1}^{FB} = \alpha^{\frac{1}{1-\alpha}} \left(\prod_{i=0}^{\infty} \epsilon_{t-i}^{\phi^i} \right)^{\frac{\phi}{1-\alpha}}.$$

And we have

$$\frac{\partial k_{t+s}^{FB}}{\partial \epsilon_t} > 0; \forall s \geq 0.$$

Therefore, the effect of a productivity shock on x_{t+s}^* through k_{t+s}^{FB} is unambiguously negative (since $x_t^* = \eta_t e_t / k_{t+1}^{FB}$), which captures well the time series properties of Basel

Figure 1: Response of x_t^* to a positive productivity shock



This figure depicts the effect of a shock ϵ_t on x_{t+s} ($s = 0, 1, 2... 10$) for $\alpha = 0.35$ and three different values of ϕ. When ϕ is relatively small (dotted line), the initial effect is the strongest, and monotonically decays over time. At intermediate values of the shock persistence parameter ϕ (dashed line), the effect is always positive and peaks after one period. When ϕ is relatively high (solid line), the initial effect is negative, but it is then positive at all lags.

II risk weights: when productivity goes up, prospects are good, probabilities of default go down, risk-weights decreases, and the effective stringency of capital requirements loosens.

Similarly, one can interpret $\eta_t e_t$ as the *financial muscles* of the banking sector. It also depends on past realizations of the productivity shock:

$$\eta_t e_t = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} \left(\prod_{i=1}^{\infty} \epsilon_{t-i}^{\phi^i} \right)^{\frac{1}{1-\alpha}} \epsilon_t \, g(\eta_t, \theta_{t+1}).$$

And we have

$$\frac{\partial \eta_{t+s} e_{t+s}}{\partial \epsilon_t} > 0; \forall s \geq 0.$$

Therefore, the effect of a productivity shock on x_{t+s}^* through $\eta_{t+s} e_{t+s}$ is unambiguously positive, which captures well the idea that high productivity also makes banking capital "less scarce" (Kashyap and Stein (2004)).

Hence, we have two forces going in opposite directions. From Proposition 3, we know that either can dominate in the very short term (that is for $s = 0$). But we also know that the second always dominates at a longer horizon ($s > 0$), which strongly suggests that in models where persistent productivity shocks generate periods of good and bad times, the optimal capital requirement should be more stringent in good times.

To formalize this and derive intuition on why the financial muscle channel dominates, let me conclude this first exercise with a markov switching example.

4.1.3 Capital requirements in good and bad times

To study the cyclical properties of the optimal capital requirement, let me assume a more stylized law of motion for the productivity shock and temporarily shut down financial shocks (i.e. $\eta_t = \eta, \forall t$).

To capture the idea of booms and busts in the most stylized way, I assume that A_t follows a two-state markov process (without absorbing state) where $A_t \in \{A_L, A_H\}$, with $A_L < A_H$, and with some transition matrix such that $\bar{A}_L \leq 1 \leq \bar{A}_H$, where $\bar{A}_L \equiv E[A_{t+1} \mid A_t = A_L]$ denotes expected productivity in state A_L, and similarly $\bar{A}_H$ denotes expected productivity in state A_H.

Under the optimal capital requirement x_t^*, physical capital k_t can only take two values:

$$\begin{cases} k_H = & (\alpha \bar{A}_H)^{\frac{1}{1-\alpha}} \\ k_L = & (\alpha \bar{A}_L)^{\frac{1}{1-\alpha}} \end{cases},$$

and $e_t = \eta(1-\alpha)A_t k_t^\alpha$ can therefore only take four values. Hence, the economy can only be in four distinct aggregate states, depending on the last two realizations of the productivity shock: HH, HL, LL, and LH.

One can interpret states HH and LL as good times and bad times respectively. Compared to the latter, the former is indeed associated with higher levels of output, wages, consumption, investment, and physical and bank capital.

Proposition 4. *Assume deposits are insured, default is costless ($\gamma = 0$), Condition 1 holds, and A_t follows a two-state markov process. Denote x_{HH} and x_{LL} the optimal capital requirement in good and bad times respectively.*

The optimal capital requirement is tighter in good times than in bad times. That is $x_{HH} > x_{LL}$.

Proof. See Appendix A. □

This result confirms that the *financial muscle channel* dominates the *expected productivity channel* indeed.

The optimal capital requirement is relatively tighter in good times because aggregate bank capital is, in this model, "more procyclical" than the first best level of investment. That is:

$$\frac{e_{HH}}{e_{LL}} > \frac{k_{HH}^{FB}}{k_{LL}^{FB}},$$

with obvious notation.

What happens is that k^{FB} is higher in good times, but e increases relatively more. To gain some intuition, first note that e is directly affected by productivity (one for one), but it also increases with the level of physical capital (which also affects the wage). Hence, any increase in k^{FB} feeds back into e. And it turns out that this prevents the increase in k^{FB} from dominating that in e.[20] To see why, note that at the first best, by definition, the expected marginal return to capital is equal to 1, irrespective of the state:

$$\alpha \bar{A}_{ss} \left(k_{ss}^{FB}\right)^{\alpha-1} = 1.$$

This implies that the ratio of the *expected* wage to physical capital is also constant:

$$\frac{(1-\alpha)\bar{A}_{ss}\left(k_{ss}^{FB}\right)^{\alpha}}{k_{ss}^{FB}} = \frac{(1-\alpha)}{\alpha}$$

But, in good times, realized productivity is above expectations. Therefore, the *realized* wage is also above expectations (and conversely in bad times). Hence, the wage to physical capital ratio is larger in good times than in bad times, which implies that bank capital is more procyclical than physical capital.

Note that the same logic applies to the realized return to capital, and therefore to bank profits. Hence, Proposition 4 does not hinge on bankers being active for only one period (and on wages being the only source of equity for banks). One can indeed consider a version of the model where bankers are active for a potentially infinite number of periods and face a constant probability to die δ. In that case, the law of motion for e_t generally takes the form: $\eta e_t = \eta w_t + (1-\delta)v_t^+$ where the last term captures aggregate retained bank profits. This alters short-term dynamics, but does not affect the result that aggregate bank capital is more procyclical than the first best level of physical capital.[21]

It is nevertheless important to stress that such a law of motion for e_t remains simplistic, and that the result in Proposition 4 should be interpreted with caution (see the discussion in Section 5).

4.2 Costly default: the equity buffer channel

The main point of the previous section was to show the basic cyclical properties of the optimal capital requirement in a very simple model of aggregate overinvestment. In that first exercise, the market failure came from the interaction between deposit insurance and diminishing returns to capital. However, the mechanism does not hinge

[20] One can check that the point elasticity of e_{ss} with respect to A_s is equal to 1 plus α times the elasticity of k_{ss}^{FB} with respect to A_s, which is itself equal to $\frac{1}{1-\alpha}$ times the point elasticity of $\bar{A}_s$ with respect to A_s. Since $\alpha < 1$ and A_t is mean reverting (which implies that the point elasticity of $\bar{A}_s$ with respect to A_s is strictly smaller than 1), the point elasticity of e_{ss} must be greater than that of k_{ss}^{FB}.

[21] See the working paper version for more details (Malherbe, 2014).

on deposit insurance. To illustrate that it applies to a larger class of models where banks do not fully internalize the social cost of lending, I now present the version where deposits are not insured but default is costly.

Costly default enriches the analysis in several further dimensions. First of all, it gives an economic role to bank capital, which adds a channel by which the state of the economy affects the optimal capital requirement. Second, while the mechanism by which costly default interacts with diminishing returns is similar to the case above, the externality is different and interesting in itself. Furthermore, the closed form solution analysis sheds a new light on the financial accelerator literature. Finally, costly default also produces interesting interactions with deposit insurance (see Section 4.3 for this last point).

The economic role of bank capital When $\gamma > 0$, bank capital has an economic role because it acts as a buffer that absorbs loan losses, which decreases the probability and the extent of insolvency. Bank capital therefore alleviates deadweight losses, which is a standard result in models where the underlying agency problem is micro-founded.

In the present context, this role suggests that higher levels of aggregate bank capital should be associated with credit expansion (which works in the opposite direction of the financial muscle channel). This channel is not present when default is costless because the first best level of investment is independent of aggregate bank capital (see equation 2).

Savers break even condition To keep things simple, I stick to the two-state markov process introduced above.

Let p_t denote the probability, at date t, that $A_{t+1} = A_H$. This probability takes the value p_H in after a good draw (A_H) and p_L after a bad draw (A_L). First, note that it cannot be efficient for banks to default after a good draw. Otherwise, they would default in all states and make strictly negative profits in expectations. Therefore, one can focus on cases where default may only happen after a bad draw.

The break-even condition of the savers is given by:

$$\begin{cases} r_t = 1 & ; d_t \leq \frac{e_t R^L_{t+1}}{1-R^L_{t+1}} \\ p_t r_t d_t + (1-p_t)\left(d_t + e_t\right) R^L_{t+1} - (1-p_t)\Psi\left(z,\gamma\right) = d_t & ; otherwise, \end{cases} \tag{10}$$

where R^L_{t+1} is the equilibrium return to capital at $t+1$ if $A_{t+1} = A_L$. If leverage is sufficiently low, the bank does not default in the bad state and $r_t = 1$. Otherwise, we have $r_t > 1$ so that savers receive a compensation after a good draw to compensate for the losses they make when the bank defaults.[22]

[22] Note that several values of r_t may satisfy the break-even condition. In such a case, I assume that

4.2.1 Market failure and regulatory response

There are two main cases at each date, depending on whether aggregate bank capital is scarce or not. In short, if it is scarce, the competitive outcome is constrained inefficient (it exhibits overinvestment), if not, it is efficient.

Definition 2. Bank capital is *scarce* at date t if

$$\eta_t < \frac{(\alpha \bar{A}_t)^{\frac{1}{1-\alpha}} - \alpha A_L / \bar{A}_t}{(1-\alpha) A_t \, (\alpha \bar{A}_H)^{\frac{1}{1-\alpha}}}.$$

This condition ensures that the representative bank fails with strictly positive probability if the investment level is the first best.

Proposition 5. *Assume default is costly ($\gamma > 0$).*

i) If bank capital is abundant at date t, the competitive equilibrium investment level is first-best efficient. That is $k_{t+1}^{CE} = k_{t+1}^{SB} = k_{t+1}^{FB}$.

ii) If bank capital is scarce at date t, the competitive equilibrium investment level is inefficiently high. That is: $k_{t+1}^{CE} > k_{t+1}^{SB}$.

iii) The regulator can ensure constrained efficiency, at all t, with the following capital requirement:

$$x_t^* = \min \left\{ 1, \frac{\eta_t w_t}{k_{t+1}^{SB}} \right\}.$$

Proof. See Appendix A. □

This time, the intuition for the market failure goes as follows. When default is costly, bankers do not internalize the fact that credit expansion decreases the quality of the marginal loan in the economy, which increases the expected bankruptcy costs of other banks (this is the general equilibrium effect). Their private marginal cost is therefore smaller than the social marginal cost, which translates in an incentive to over-invest. The capital requirement x_t^* is therefore binding in equilibrium and implements the desired outcome, because of the same logic as in the deposit insurance case.

Before turning to the cyclical properties of x_t^*, let me inspect in greater detail the externality at the source of the market failure, and contrast it with that in Bernanke and Gertler (1989) and the financial accelerator literature that followed.[23]

lending takes place at the lowest of such rates. Otherwise, a bank could convince a saver to lend at the lower rate, leaving the bank strictly better off. Also, for a given R_{t+1}^L, there may be no interest rate that satisfies this condition. In such a case, banks simply could not borrow, but this can not be an equilibrium and R_{t+1}^L will adjust. To avoid dealing with technical complications when the default costs exceed the gross value of the assets, I assume here that savers have unlimited liability.

[23] In the deposit insurance case, the externality is trivial since loan losses exceeding the equity buffer are borne by the tax-payer by construction.

4.2.2 Inspecting the market failure

To illustrate the mechanism behind the market failure, let me follow Townsend (1979), and assume that default costs are simply proportional to the extent of insolvency. That is:

$$\Psi(z, \gamma) = \gamma z.$$

Such an assumption is convenient because one can then solve the break-even condition (10) in closed form for r_t:

$$r_t = \max\left\{1, \frac{1-(1+\gamma)(1-p_t)\frac{d_t+e_t}{d_t}R^L_{t+1}}{p_t-\gamma(1-p_t)}\right\},$$

which provides an intuitive deposit supply function. One can indeed check that r_t is increasing in leverage (that is, increasing in d_t and decreasing in e_t), in default costs γ, and decreasing in p_t and in R^L_{t+1} (which can be interpreted as the gross recovery value).

Using the break-even condition (10), the representative banker objective function corresponds to the social value of the bank:

$$E[c_{t+1}] = E[R_{t+1}](e_t+d_t) - d_t - \gamma(1-p_t)\left[r_t d_t - R^L_{t+1}(e_t+d_t)\right]^+,$$

which reflects the fact that savers make bankers internalize their own expected cost of default.

I am interested here in the case where the period competitive equilibrium exhibits $r_t > 1$. In that case, external finance commands a premium and it is therefore optimal for the representative banker to invest all his wealth in bank equity ($e_t = w_t$). One can then focus on his first order condition with respect to d_t:

$$E_t[R_{t+1}] = 1 + \gamma(1-p_t)\left[\frac{1-(1+\gamma)(1-p_t)R^L_{t+1}}{p_t-\gamma(1-p_t)} - R^L_{t+1}\right]. \tag{11}$$

Using the market clearing condition for physical capital, one can then solve for k^{CE}_{t+1}, the competitive equilibrium level of investment:[24]

$$k^{CE}_{t+1} = \left(\alpha\left[(1-\pi\gamma)\bar{A}_t + \pi\gamma A_L\right]\right)^{\frac{1}{1-\alpha}},$$

where $\pi \equiv \frac{1-p_t}{p_t} > 0$.

In contrast, taking into account the effect of diminishing returns on R^L_{t+1}, the first

[24]Note that in this simple case (discrete state-space and linear cost), individual bank leverage is undetermined over a range. To obtain a uniquely optimal leverage level, one can for instance use a convex cost function and/or a continuous distribution for A_t.

order condition would be:

$$E_t[R_{t+1}] = 1 + \gamma(1-p_t)\left[\frac{1-(1+\gamma)(1-p_t)\alpha R^L_{t+1}}{p_t - \gamma(1-p_t)} - \alpha R^L_{t+1}\right]. \tag{12}$$

To identify the externality, notice that there is a factor α multiplying the terms in R^L_{t+1}. The associated investment level is:

$$k_{t+1} = \left(\alpha\left[(1-\pi\gamma)\bar{A}_t + \alpha\pi\gamma A_L\right]\right)^{\frac{1}{1-\alpha}}, \tag{13}$$

which is strictly smaller than k^{CE}_{t+1}.[25]

Generalizing the externality[26] The difference in first order conditions (11) and (12) comes from the representative banker price taking behavior under my assumption that default costs are proportional to the extent of insolvency ($\Psi = \gamma z$).

Let me look at the total derivative of a general function Ψ that depends on d, $r(d)$, and $R_L(d)$:

$$\frac{\mathrm{d}\Psi\left(d, r(d, R_L(d)), R_L(d)\right)}{\mathrm{d}d} = \frac{\partial\Psi}{\partial d} + \frac{\partial\Psi}{\partial r}\frac{\partial r}{\partial d} + \underbrace{\frac{\partial\Psi}{\partial r}\frac{\partial r}{\partial R_L}\frac{\mathrm{d}R^L}{\mathrm{d}d}}_{ii} + \underbrace{\frac{\partial\Psi}{\partial R_L}\frac{\mathrm{d}R^L}{\mathrm{d}d}}_{i}$$

The last two terms of the total derivative above capture the two channels by which credit expansion endogenously affects default costs through diminishing marginal returns. The term denoted *(i)* is the direct effect through the recovery value, and the term denoted *(ii)* is the indirect effect through interest rate: the decreases in recovery value affects the break-even interest rate, which in turn may affect bankruptcy costs. The banker price taking behavior makes him neglect those two terms. Hence the wedge in the first order condition. This yields overinvestment whenever:

$$\frac{\partial\Psi}{\partial r}\frac{\partial r}{\partial R^L}\frac{\mathrm{d}R^L}{\mathrm{d}d} + \frac{\partial\Psi}{\partial R^L}\frac{\mathrm{d}R^L}{\mathrm{d}d} \geq 0.$$

I am now in a position to identify the key assumptions that yield this result in the case studied above, and to discuss what would happen under alternative assumptions. In particular, I have:

$$\underbrace{\frac{\partial\Psi}{\partial r}}_{\geq 0}\underbrace{\frac{\partial r}{\partial R^L}}_{\leq 0}\underbrace{\frac{\mathrm{d}R^L}{\mathrm{d}d}}_{<0} + \underbrace{\frac{\partial\Psi}{\partial R^L}}_{\leq 0}\underbrace{\frac{\mathrm{d}R^L}{\mathrm{d}d}}_{<0} \geq 0. \tag{14}$$

[25]Note that the investment level in equation (13) is in fact an upper bond for the second best (there are cases where a planner would prefer to restrict the banking sector further and make sure that it does not fail after a bad draw).

[26]In this section, I omit time subscript for the sake of readability.

Since $\frac{\partial r}{\partial R^L} < 0$ is an obvious feature of a model with risky lending (all other things equal, the equilibrium interest rate decreases with the recovery value), let me focus on the other terms.

First, we have

$$\frac{\mathrm{d}R^L}{\mathrm{d}d} < 0,$$

which comes from diminishing returns to capital, together with the fact that firms need to borrow from banks (I discuss the relevance of these assumptions in Section 5).

Second, since $\Psi = \gamma \left[dr - (d+e)R^L\right]^+$,we have:

$$\begin{cases} \frac{\partial \Psi}{\partial r} \geq 0 \\ \frac{\partial \Psi}{\partial R^L} \leq 0 \end{cases}$$

which reflects that the extent of insolvency increases in promised repayment and decreases in the recovery value.

The assumption that default costs are increasing in the extent of insolvency seems reasonable to me in a bank context. This could for instance reflect that the longer the bankruptcy procedure the larger the forgone profitable investment opportunities by debt-holders, or that the larger the losses incurred by debt-holders the more likely their own borrowing constraints will bind in the future. Still, the decomposition above makes clear that this assumption is not essential and that what ultimately matters is the signs of $\frac{\partial \Psi}{\partial r}$ and $\frac{\partial \Psi}{\partial R^L}$.

Financial accelerator or financial brake? While Townsend used default cost proportional to the extent of insolvency in his seminal costly-state-verification paper (Townsend, 1979), it is interesting to note that Bernanke and Gertler (1989) made a different assumption. In particular, in that paper and in the *financial accelerator* literature that followed (e.g. Carlstrom and Fuerst, 1997; Bernanke, Gertler, and Gilchrist, 1999), verification costs are proportional to the value of production, which makes them *decreasing in the extent of insolvency*. If I were to follow this route in my model, I would have $\frac{\partial \Psi}{\partial r} = 0$ and $\frac{\partial \Psi}{\partial R^L} > 0$, the externality would work in the other direction, and the friction would lead to underinvestment. This suggests that such an externality, which acts as a *brake* on investment, may contribute to the persistence and asymmetry results that are typical in this literature.[27] To the best of my knowledge, this mechanism had not been highlighted yet.

[27]Persistence refers to the protracted effect of shocks (a drop in entrepreneur net worth for instance), and asymmetry refers to the fact that positive shocks have smaller effects.

Fire-sale and other pecuniary externalities Although it shares some of its flavor, the externality in this model is different from a fire-sale externality. In particular, default costs for a bank do not necessarily increase (ex-post) with the extent of insolvency of other banks. This could however easily be the case. Suppose for example that, in the example above, γ is an increasing function of the aggregate shortfall of asset value in the banking sector. This would generate another externality very much in the spirit of fire-sales externalities, and this would magnify the core externality of the model.

Note finally that the competitive equilibrium of my model shares the investment efficiency properties of Lorenzoni (2008) and Jeanne and Korinek (2013). That is, underinvestment with respect to first best, but overinvestment with respect to second best. However, their results are driven by a different kind of pecuniary externalities that do not act through diminishing returns to capital (in their model, the relevant technology is linear in capital).

4.2.3 Cyclical properties of x_t^*

Optimal capital requirements and the business cycle As mentioned above, when bank capital acts as an economic buffer against losses, more bank capital suggests credit expansion. Hence, this channel attenuates the financial muscle channel and affects the relative stringency of the optimal capital requirement in good and bad times. To specifically study the strength of this additional *loss-absorbing channel*, let me again shut down financial shocks. Here, k_{t+1}^{SB} can potentially take more than two values. This is because k_{t+1}^{SB} depends on the size of the aggregate equity buffer ηe_t, which itself depends on past values of k and A. One can however still define a meaningful notion of good and bad times. Good (bad) times is now defined as the state the economy converges to after a sufficiently long series of good (bad) draws for A. With a slight abuse of notation let me still use the subscripts HH and LL to refer to variables in good and bad times respectively.

Proposition 6. *Assume default is costly ($\gamma > 0$), A_t follows a two-state Markov process, and bank capital is scarce in good times.*

The optimal capital requirement is tighter in good times than in bad times. That is $x_{HH} > x_{LL}$ (and bank capital is scarce in bad times too).

Proof. See Appendix A. □

The intuition is the same as for the case with costless default. The feedback effect, by which any increase in k increases e, prevents the increase in k^{FB} in good times from dominating the increase in e, and the optimal capital requirement stays more stringent in good times. Note that, as is the case with the deposit insurance, this logic (and therefore the proposition) easily extends to a version of the model where banks retain profits.

Financial shocks Now, suppose there are financial shocks. First, a positive shock to η_t triggers a tightening of the optimal capital requirement.

Proposition 7. *Assume default is costly ($\gamma > 0$) and bank capital is scarce.*

The optimal capital requirement is increasing in aggregate bank capital, and therefore reacts positively to a financial shock $\frac{dx_t^*}{d\theta_t} > 0$.

Proof. See Appendix A. □

In the costless default case of Section 4.1, the result was obvious since k^{FB} does not depend on aggregate bank capital and the denominator of x_t^* is proportional to η_t. Here however, there is the loss-absorbing channel that goes in the opposite direction since k^{SB} is increasing in $\eta_t e_t$. Still, this channel only produces a second order effect and cannot dominate the financial muscle channel. When bank capital acts as a buffer against losses, if all banks in the economy double their equity base, they can absorb twice as much losses. However, given diminishing returns to capital it can still not be optimal to let banks double aggregate lending in the economy.

Intertwined cycles Finally, assume that financial and productivity shocks are correlated. Then, there are two cases. If the correlation is positive, financial shocks tend to amplify the business cycle fluctuations of the aggregate bank equity buffers. Therefore, we must still have that the optimal capital requirement is more stringent in good times (where the definition of good and bad times is adapted to account for financial shocks). If the correlation is negative, financial shocks attenuate the fluctuations in aggregate bank equity due to productivity shocks, and attenuate the procyclicality of the optimal capital requirement. As an extreme case, suppose that η_t is perfectly and negatively correlated with A_t. Then, one can overturn the cyclicality property of the optimal capital requirements.

Proposition 8. *Assume default is costly ($\gamma > 0$), A_t follows a two-state Markov process, and bank capital is scarce in good and bad times. Denote R_{HH} the realized return to lending in good times, and R_{LL} that in bad times. Suppose financial shocks are perfectly correlated with productivity shocks, so that $\eta \in \{\eta_L, \eta_H\}$, with $\Pr(\eta_s \mid A_s) = 1$. Then,*

$$x_{LL} > x_{HH} \iff \eta_L > \eta_H \left(\frac{R_{HH}}{R_{LL}} \right).$$

Proof. See Appendix A. □

Note that, in equilibrium, R_{HH} is bounded below by one, and R_{LL} is bounded above by one. To have $x_{LL} > x_{HH}$, we need a *negative* correlation between the shocks ($\eta_L > \eta_H$) and a sufficiently large amplitude of the financial shocks. Given that the rhetoric around the current debates points toward a greater scarcity of bank capital in bad times, such a case seems however of little empirical relevance.

4.3 Deposit insurance implicit subsidy and efficiency

In this section, I combine both deposit insurance and costly default.

Deposit insurance improves efficiency under the optimal capital requirement

Deposit insurance can improve efficiency because it reduces default costs. This is simply because the extent of insolvency increase with the interest rate and, while $r_t = 1$ when deposits are insured, we have $r_t \geq 1$ when it is not the case. Since reduced default costs imply that the second best gets closer to the first best, deposit insurance improves efficiency, *under the optimal capital requirement*.

One way to interpret this is that deposit insurance acts as an implicit subsidy to bankers, which corresponds to an increase in the real value of their equity buffer. When bank capital is scarce and ex-ante outright transfers of wealth to bankers are not feasible, deposit insurance can therefore be seen as way to alleviate the scarcity. Note that what matters is not the insurance *per se*, but the subsidy that decreases bank borrowing costs. Arguably, the recent LTRO operations of the ECB have similar consequences: by subsidizing lending, the ECB indirectly contributes to a recapitalization of the European banking system.

Deposit insurance magnifies inefficiencies under suboptimal regulation

Now, suppose that regulation is suboptimal. For instance, suppose that (for reasons outside of the model) $x_t = x \in (x_{HH}, x_{LL})$, $\forall t$, which can be interpreted as a *through-the-cycle* capital requirement. It is straightforward to show that such policy leads to unnecessarily severe credit crunches in bad times (because $x > x_{LL}$) and overinvestment in good times (because $x < x_{HH}$).

Then, whether deposits are insured or not can make a big difference on the extent of inefficiency this generates. When the capital requirement is too tight, say that it binds in bad times under both regimes, then, although the extent of inefficiency is different (see above) allocations will be fairly similar under the two regimes. When the capital requirement is too loose (i.e. in good times) allocations can however be very different. If deposits are not insured, savers make banks internalize the expected costs of bankruptcy. The expected marginal cost for the banks is therefore strictly larger than one, which puts a limit to overinvestment. But, when deposits are insured, the expected marginal cost is below one. Therefore, when x is loose, overinvestment can be severe, and it is possible that the marginal investment has a negative net present value, even before taking bankruptcy cost into account.[28]

[28]From the bank point of view, the net present value is still positive since it includes the benefit from the implicit subsidy.

A number of studies have highlighted the magnifying effect of Basel II requirements (Kashyap and Stein, 2004; Repullo and Suarez, 2013). Other studies have pointed to the fact that risk is built up in the financial sector during good times (Borio and Drehmann, 2009). Also, in a pure real business cycle framework, it would require an implausibly large and persistent negative productivity shock to account for the severity of the downturn that followed the 2007-2009 financial crisis in many countries. The present model offers a simple way to bring those pieces together. The narrative would go as follows:

We start with a regulation that does not take into account the cyclical variations of aggregate bank capital (as was for instance the case of Basel I or II). We start in good times, where the requirement is too loose. As good times continue, aggregate bank capital accumulates, and credit expands. As bank borrowing costs are too low (reflecting the implicit subsidy from government guarantees, either from deposit insurance or due to too-big-to-fail considerations (Acharya, Anginer, and Warburton, 2014; Noss and Sowerbutts, 2012; Ueda and Weder di Mauro, 2013)), this expansion goes too far and ultimately translates into negative net present value real investment (fueling a real estate bubble for instance). In that situation (which corresponds to $k_t > k_t^{FB}$), a small reversal of the business cycle (a small negative productivity shock, or even one that is not positive enough) can trigger a banking sector collapse. Then, aggregate bank capital is severely eroded by the losses, which is not taken into account by the regulator and creates an overly severe credit crunch.

5 Discussion, robustness, and policy insights

The importance of aggregate bank capital

The mechanism behind the general equilibrium effect highlighted in this paper also leads to the main policy insight.

Policy insight 1. *Optimal capital requirements are increasing in aggregate bank capital.*

When aggregate bank capital increases, the banking sector should be allowed to expand but, given diminishing returns to capital, this increase should be less than proportional. This corresponds to an increase in capital requirements. If this is overlooked by the regulator, for instance if capital requirements adjust to expected productivity but not to aggregate bank capital (as was the case of Basel II), regulation will magnify business and financial cycles through this channel.

This result is robust in the sense that it does not hinge on specific parameter values and is in fact due to very few ingredients. First, there is diminishing return to physical capital in the economy. This is perhaps the most standard assumption in macroeconomics, but is often abstracted from the literature on banking and financial regulation (notable exceptions include Martinez-Miera and Suarez, 2014, and Van den Heuvel, 2008).

Second, financial intermediary credit is not irrelevant. In the model, I assume that banks are the only source of funding for firms, but this is not necessary to generate the result. What really matters is that aggregate bank lending affects aggregate investment in physical capital *at the margin*. Even though this seems to be a reasonable starting point, this is often abstracted from the macroeconomic literature. It is however now at the core of current policy debates and there is a fast growing literature on the subject that builds on contributions such as Holmström and Tirole (1997) for instance.[29]

Third, and perhaps most importantly, capital requirements affect aggregate lending, which need not (always) be the case in reality. There is in fact no consensus on the subject (see Repullo and Suarez, 2013). For instance, there is evidence that banks do hold "buffers" above the regulatory level (Gropp and Heider, 2010), but there is also evidence of the relevance of bank capital constraint on the credit supply in general (Bernanke, Lown, and Friedman, 1991; Thakor, 1996; Ivashina and Scharfstein, 2010; Aiyar, Calomiris, and Wieladek, 2012), and in particular that *changes* in capital requirements affect bank lending (Jimenez, Ongena, Saurina, and Peydro, 2013). And indeed, what matters for my analysis is that the requirements are *essentially* constraining lending. That is, what matters is that the capital requirement stance affects their behavior, even if the requirement is not technically binding. The huge resistance of banks (through lobbying for instance) to structural increases in capital adequacy ratios and the strong evidence of "risk-weight optimization" and regulatory arbitrage by the banks operating under the Basel II regulation (buying CDS on ABS from AIG was one typical way to explicitly circumvent the regulation for instance, see Yorulmazer, 2013) all indicate that capital requirements do constrain bank decisions. In the model, capital requirements are binding because banks do not fully internalize the social cost of lending. This is likely to be the case in reality. First, because deposits are insured in most advanced economies, and there is evidence that it does distort their cost of borrowing (Demirguc-Kunt and Detragiache, 2002; Ioannidou and Penas, 2010; Acharya, Anginer, and Warburton, 2014). Second, because large banks benefit from implicit guarantees (Acharya, Anginer, and Warburton, 2014; Kelly, Lustig, and Van Nieuwerburgh, 2011; Laeven, 2000; Noss and Sowerbutts, 2012). And, last but not least, because banks are unlikely to fully internalize the negative spillovers they create when they are distressed. Fire sales externalities are a well understood potential reason for this, but the mechanism I highlight in this paper works in the same direction.

[29]See Brunnermeier, Eisenbach, and Sannikov (2013) for a survey of recent developments, and Suarez (2010) for a great discussion of the various modeling strategies.

Optimal requirements cyclical properties of and link with Basel II and III

Basel I had very coarse risk categories, and Basel II was conceived to better deal with the cross-sectional variation in risks. The idea was to use variables such as an individual loan's probability of default to weight bank assets, and then to apply a flat 8% capital requirement on those weighted assets. However, probabilities of defaults tend to move in the same direction over time (they tend to go up during bad times). Since risk-weights increase on average during bad times, "effective capital requirements" are more stringent in bad times, which tends to contract aggregate credit. In my model, the expected productivity channel suggests that this is, to some extent, desirable.

However, bank capital also tends to be low in bad times. This is not taken into account by Basel II and tighter capital requirements applied to a smaller amount of capital can therefore dramatically contract credit, seriously magnifying economic fluctuations. This is the rationale for Basel III's counter-cyclical buffers, which are supposed to mitigate the effect of increased risk weights. My results strongly suggest that they go in the right direction. In fact, my results suggest that these buffers should more than offset the effect of increased risk weights (so that effective capital requirements be in fact looser in bad times). However, for reasons that I develop below, I would not take this conclusion at face value. I would rather take this as a hint that the general equilibrium effect discussed above is potentially powerful, and I would limit myself to the following statement:

Policy insight 2. *The analysis confirms that the pro-cyclical properties of a capital requirement based on default probabilities (like Basel II) are likely to be excessive. In particular, the general equilibrium effects suggest that the stringency of such capital requirements should be increased in good times and reduced in bad times. In other terms, the "counter-cyclical" buffers of Basel III go in the right direction.*

While the result the capital requirements should *overall* be tighter in good times is robust in the context of the model, one of the reasons for not taking it at face value is that the laws of motion for e_t and k_t are extremely stylized. In fact, they abstract from many ingredients that are potentially relevant. However, that the joint dynamics of e_t and k_t^* are key to the optimal stance of bank capital regulation is a quite general insight and would extend to other models where banks do not internalize the full social cost of borrowing and bank activity has an impact on the quality of the marginal loan in the economy. Therefore:

Policy insight 3. *The joint dynamics of aggregate bank capital and the socially optimal level of aggregate lending are key to the optimal stance of capital regulation.*

6 Conclusion

This paper highlights a simple but potentially powerful general equilibrium effect. When bank capital requirements are binding and there are diminishing returns to physical capital in the economy, an increase in aggregate bank capital will decrease the quality of the marginal loan in the economy. If bankers do not fully internalize the social cost of lending (because bankruptcy procedures entail deadweight losses or because banks enjoy explicit or implicit subsidies from government guarantees, for instance), and if this general equilibrium effect is not accounted for in bank regulation, this will translate in aggregate over-lending. More generally, if the stringency of bank capital requirements does not react to aggregate banking capital, this is likely to unnecessarily magnify business and financial cycle fluctuations.

This general equilibrium effect does not hinge on the specific friction that interacts with diminishing returns to capital to create the wedge in the first order condition of the banker. However, in the presence of government guarantees, the extent of the market failure can be large as banks may then not even internalize *expected* credit losses. In such a case, suboptimal regulation can have potentially high welfare costs.

Quantifying such losses would however require a less stylized approach, and a better understanding of the real-world dynamic behavior of aggregate bank capital.[30] Indeed, our current understanding of bank dynamic capital structure decisions is at best incomplete, especially in general equilibrium (see the discussions in Allen and Carletti, 2013, and Repullo and Suarez, 2013 for instance, and Rampini and Viswanathan, 2014, and He and Krishnamurthy, 2011, for recent advances). Furthermore, there are no widely-accepted historical stylized facts about the dynamics of aggregate bank capital.[31] Even if it was the case, this is not clear that in such a changing environment using past stylized facts as a definite guide to modeling is the most relevant approach. In fact, the recent crisis has challenged previous theories (Acharya, Gujral, Kulkarni, and Shin, 2011; He, Khang, and Krishnamurthy, 2010), perhaps because changes in remuneration practices, and financial innovations have greatly reshaped incentives, and there is therefore a large scope for future research.

[30] More generally, a more sophisticated law of motion for aggregate bank capital would be, in my view, an essential ingredient of a more quantitative study of these issues.

[31] And available data is plagued by measurement issues. For instance, the accuracy of equity book value suffers from the huge lags in loss recognition and many forms of potential window dressing, and the equity market value includes the option value of equity (and therefore, the subsidy from government guarantees (Merton (1977))). See Korinek and Kreamer (2013) for recent US data (market value) based on the Federal Reserve Data base.

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Appendix A: Proofs

Proposition. 2. *Assume deposits are insured and default is costless ($\gamma = 0$). The following capital requirement ensures investment efficiency ($k_{t+1} = k_{t+1}^{FB}$) at all t:*

$$x_t^* = \min\left\{1, \eta_t w_t \left(\alpha E_t[A_{t+1}]\right)^{\frac{-1}{1-\alpha}}\right\}.$$

Proof. If $\eta_t w_t \geq k_{t+1}^{FB}$, bankers have enough wealth to finance the first best level of investment. If $x_t^* = 1$, $d_t = 0$ and the relevant first order condition (with respect to e_t) is $E[R_{t+1}] = 1$, which can only be satisfied with $k_{t+1} = k_{t+1}^{FB}$. Now the case $\eta_t w_t < k_{t+1}^{FB}$. If $e_t < w_t$ or $e_t = w_t$ and $e_t + d_t < k_{t+1}^{FB}$, then under x_t^* we would have $E[R_{t+1}] > 1$, which cannot be an equilibrium. But given x_t^* the maximum possible aggregate lending is k_{t+1}^{FB}, it must therefore be the equilibrium level. □

Lemma. *1. Assume Condition 1 holds. The optimal capital requirement can be written as an explicit function of the last financial shock and all past productivity shocks:*

$$x_t^* = \frac{(1-\alpha)}{\alpha} g\left(\eta_{t-1}, \theta_t\right) \left(\prod_{i=1}^{\infty} \epsilon_{t-i}^{\phi^i}\right)^{\frac{1-\phi}{1-\alpha}} \left(\epsilon_t\right)^{\frac{1-\alpha-\phi}{1-\alpha}}$$

Proof. Under Condition 1, $e_t = \eta_t(1-\alpha)A_t\left(k_t^{FB}\right)^{\alpha}$. Substituting in equation (5), using $k_t^{FB} = (\alpha E_{t-1}[A_t])^{\frac{1}{1-\alpha}}$ and $A_t = \prod_{i=0}^{\infty}\left(\epsilon_{t-i}\right)^{\phi^i}$ (which comes from backward iteration of $A_t = A_{t-1}\epsilon_t$), and rearranging yields the result. □

Comment: (not for publication)

$$x_t^* = (1-\alpha)\left(\alpha^{\frac{\alpha}{1-\alpha}-\frac{1}{1-\alpha}}\right) g\left(\eta_{t-1}, \theta_t\right) \left(\prod_{i=0}^{\infty}\left(\epsilon_{t-i}\right)^{\phi^i}\right) \left(\prod_{i=0}^{\infty}\left(\epsilon_{t-1-i}\right)^{\phi^i}\right)^{\frac{\alpha\phi}{1-\alpha}} \left(\prod_{i=0}^{\infty}\left(\epsilon_{t-i}\right)^{\phi^i}\right)^{-\frac{\phi}{1-\alpha}} \tag{15}$$

Extract terms in ϵ_t:

$$x_t^* = \frac{(1-\alpha)}{\alpha} g\left(\eta_{t-1}, \theta_t\right) \left(\prod_{i=1}^{\infty}\left(\epsilon_{t-i}\right)^{\phi^i}\right) \left(\prod_{i=0}^{\infty}\left(\epsilon_{t-1-i}\right)^{\phi^i}\right)^{\frac{\alpha\phi}{1-\alpha}} \left(\prod_{i=1}^{\infty}\left(\epsilon_{t-i}\right)^{\phi^i}\right)^{-\frac{\phi}{1-\alpha}} \epsilon_t \left(\epsilon_t\right)^{-\frac{\phi}{1-\alpha}}$$

$$x_t^* = \frac{(1-\alpha)}{\alpha} g\left(\eta_{t-1}, \theta_t\right) \left(\prod_{i=1}^{\infty}\left(\epsilon_{t-i}\right)^{\phi^i}\right) \left(\prod_{i=0}^{\infty}\left(\epsilon_{t-1-i}\right)^{\phi^i}\right)^{\frac{\alpha\phi}{1-\alpha}} \left(\prod_{i=1}^{\infty}\left(\epsilon_{t-i}\right)^{\phi^i}\right)^{-\frac{\phi}{1-\alpha}} \left(\epsilon_t\right)^{\frac{1-\alpha-\phi}{1-\alpha}}$$

Comment: Rewrite the second product from $i = 1$:

$$x_t^* = \frac{(1-\alpha)}{\alpha} g\left(\eta_{t-1}, \theta_t\right) \left(\prod_{i=1}^{\infty} (\epsilon_{t-i})^{\phi^i}\right) \left(\prod_{i=1}^{\infty} (\epsilon_{t-i})^{\phi^{i-1}}\right)^{\frac{\alpha\phi}{1-\alpha}} \left(\prod_{i=1}^{\infty} (\epsilon_{t-i})^{\phi^i}\right)^{-\frac{\phi}{1-\alpha}} (\epsilon_t)^{\frac{1-\alpha-\phi}{1-\alpha}}$$

Combine exponents:

$$x_t^* = \frac{(1-\alpha)}{\alpha} g\left(\eta_{t-1}, \theta_t\right) \left(\prod_{i=1}^{\infty} (\epsilon_{t-i})^{\phi^i\left(1+\frac{\alpha}{1-\alpha}-\frac{\phi}{1-\alpha}\right)}\right) (\epsilon_t)^{\frac{1-\alpha-\phi}{1-\alpha}}$$

$$x_t^* = \frac{(1-\alpha)}{\alpha} g\left(\eta_{t-1}, \theta_t\right) \left(\prod_{i=1}^{\infty} (\epsilon_{t-i})^{\phi^i\left(\frac{1-\phi}{1-\alpha}\right)}\right) (\epsilon_t)^{\frac{1-\alpha-\phi}{1-\alpha}}$$

Proposition. *4. Assume deposits are insured, default is costless ($\gamma = 0$), Condition 1 holds, and A_t follows a two-state markov process. Denote x_{HH} and x_{LL} the optimal capital requirement in good and bad times respectively.*

The optimal capital requirement is tighter in good times than in bad times. That is $x_{HH} > x_{LL}$.

Proof. From Proposition 2, we have: $x_{ss} = \eta \frac{(1-\alpha)}{\alpha} \frac{A_s}{\bar{A}_s}$, with $s = H, L$. Since we have $A_H/\bar{A}_H > 1$ and $A_L/\bar{A}_L < 1$ (because productivity is mean reverting), it must be the case that: $x_H^* > x_L^*$. □

Proposition. *5. Assume default is costly ($\gamma > 0$).*

i) If bank capital is abundant at date t, the competitive equilibrium investment level is first-best efficient. That is $k_{t+1}^{CE} = k_{t+1}^{SB} = k_{t+1}^{FB}$.

ii) If bank capital is scarce at date t, the competitive equilibrium investment level is inefficiently high. That is: $k_{t+1}^{CE} > k_{t+1}^{SB}$.

iii) The regulator can ensure constrained efficiency, at all t, with the following capital requirement:

$$x_t^* = \min\left\{1, \frac{\eta_t w_t}{k_{t+1}^{SB}}\right\}.$$

Proof. i) straightforward; ii) see Subsection 4.2.2; iii) the logic is the same as the one of the proof of Proposition 2. □

Proposition. *6. Assume default is costly ($\gamma > 0$), A_t follows a two-state Markov process, and bank capital is scarce in good times.*

The optimal capital requirement is tighter in good times than in bad times. That is $x_{HH} > x_{LL}$ (and bank capital is scarce in bad times too).

Proof. From Proposition 5, we have: $x_{ss} = \eta e_{ss}/k_{ss}^{SB}$, where e_{ss} and k_{ss} denote the values of e and k in good and bad times (where $ss = HH$ and $ss = HH$, respectively). Substituting the labor market clearing condition gives $x_{ss} = \eta(1-\alpha)A_s \left(k_{ss}^{SB}\right)^{\alpha} / k_{ss}^{SB}$ or $x_{ss} = \eta(1-\alpha)A_s \left(k_{ss}^{SB}\right)^{\alpha-1}$. Hence we have that

$$x_{HH} > x_{LL} \Leftrightarrow \eta(1-\alpha)A_H \left(k_{HH}^{SB}\right)^{\alpha-1} > \eta(1-\alpha)A_L \left(k_{LL}^{SB}\right)^{\alpha-1},$$

but this condition is equivalent to:

$$\alpha A_H \left(k_{HH}^{SB}\right)^{\alpha-1} > \alpha A_L \left(k_{LL}^{SB}\right)^{\alpha-1},$$

which is satisfied since $\alpha A_H \left(k_{HH}^{SB}\right)^{\alpha-1} > \alpha \bar{A}_H \left(k_{HH}^{SB}\right)^{\alpha-1} \geq 1$ (the last inequality follows directly from the definition of the second best) and $\alpha A_L \left(k_{LL}^{SB}\right)^{\alpha-1} < 1$. To understand why the last inequality holds, assume that it does not and notice that $R_{t+1} > 1$ irrespective of the realization of the shock. Therefore $r_t = 1$ ensures that savers break even, and we have $R_{t+1} > r_t$ in all states and banks never defaults. But then a small increase in k would increase output more than one to one in all states, and this cannot be an optimum. □

Proposition. 7. *Assume default is costly ($\gamma > 0$) and bank capital is scarce. The optimal capital requirement is increasing in aggregate bank capital, and therefore reacts positively to a financial shock $\frac{dx_t^*}{d\theta_t} > 0$.*

Proof. Denote $\bar{e}_t \equiv \eta_t e_t$. Then $x_t^*(\bar{e}_t) = \bar{e}_t / k_{t+1}^{SB}(\bar{e}_t)$ and I need to show that $\frac{dk_{t+1}^{SB}}{k_{t+1}^{SB}} / \frac{d\bar{e}_t}{\bar{e}_t} < 1$. To do so, I apply the implicit function theorem to the first order condition that pins down k_{t+1}^{SB}. I provide here a formal proof for the case where deposit are insured (hence $r_t = 1$ and does not depend on leverage) and then argue that the logic applies to the general case.

Denoting $G \equiv \alpha \bar{A} k_{t+1}^{\alpha-1} - 1 - \frac{\partial E_t[\Psi(Z(k_{t+1}^{SB}, \bar{e}_t)]}{\partial k_{t+1}^{SB}}$ (and ignoring henceforth time subscripts and SB superscripts), I need $-\frac{\partial G}{\partial \bar{e}} / \frac{\partial G}{\partial k} < \frac{k}{\bar{e}}$. Since

$$\Psi\left(Z(k, \bar{e})\right) \equiv \gamma \int_0^{(k-\bar{e})/\alpha k^{\alpha}} \left[(k-\bar{e}) - \alpha A k^{\alpha}\right] dA,$$

we have

$$\frac{\partial E_t[\Psi(Z(k_{t+1}^{SB}, \bar{e}_t)]}{\partial k_{t+1}^{SB}} = \gamma \int_0^{\hat{A}} \left(1 - \alpha^2 A k^{\alpha-1}\right) dA,$$

where $\hat{A} \equiv (k - \bar{e}) / \alpha k^{\alpha}$. Hence,

$$\frac{\partial G}{\partial k} = \alpha(\alpha-1)\bar{A}k^{\alpha-2} - \gamma\int_0^{\hat{A}}\left(1-\alpha^2(\alpha-1)Ak^{\alpha-2}\right)dA - \gamma\frac{1-\alpha(1-\bar{e}/k)}{\alpha k^{\alpha}}\left(1-\alpha^2\hat{A}k^{\alpha-1}\right)$$

$$-\frac{\partial G}{\partial k} = \alpha(1-\alpha)\bar{A}k^{\alpha-2} + \gamma\int_0^{\hat{A}}\left(1+\alpha^2(1-\alpha)Ak^{\alpha-2}\right)dA + \gamma\frac{1-\alpha(1-x^*)}{\alpha k^{\alpha}}\left(1-\alpha x^*\right), \tag{16}$$

and

$$\frac{\partial G}{\partial \bar{e}} = -\gamma\frac{-1}{\alpha k^{\alpha}}\left[(1-\alpha x^*)\right].$$

Since the first two terms of the right-hand side of (16) are positive, a sufficient condition for $-\frac{\partial G}{\partial \bar{e}}/\frac{\partial G}{\partial k} < \frac{k}{\bar{e}}$ is

$$x^* < 1-\alpha(1-x^*)$$

which is satisfied when bank capital is scarce because it implies $x^* < 1$.

If deposits are not insured and r increases with leverage, we have:

$$\Psi\left(Z(k,\bar{e})\right) \equiv \gamma\int_0^{(k-\bar{e})r(k,\bar{e})/\alpha k^{\alpha}}\left[(k-\bar{e})\,r(k,\bar{e}) - \alpha A k^{\alpha}\right]dA,$$

which complicates the algebra but does not change the result. This is because we have just established that a small increase in k has a greater impact on the expected default cost than a proportional decrease in $\bar{e}$, while holding r constant. But $r(k,\bar{e})$ depends positively itself on default costs, therefore there is a positive feedback effect between the two, and since $r(k,\bar{e})$ increases in k and decreases in $\bar{e}$, allowing r to adjust will only reinforce the result. □

Proposition. *8. Assume default is costly ($\gamma > 0$), A_t follows a two-state Markov process, and bank capital is scarce in good and bad times. Denote R_{HH} the realized return to lending in good times, and R_{LL} that in bad times. Suppose financial shocks are perfectly correlated with productivity shocks, so that $\eta \in \{\eta_L, \eta_H\}$, with $\Pr(\eta_s \mid A_s) = 1$. Then,*

$$x_{LL} > x_{HH} \Longleftrightarrow \eta_L > \eta_H\left(\frac{R_{HH}}{R_{LL}}\right).$$

Proof. If bank capital is scarce, we have $e_t = w_t$. Then, $x_{ss} = \eta_s(1-\alpha)A_s\left(k_{ss}^{SB}\right)^{\alpha}/k_{ss}^{SB}$. Since $R_{ss} = \alpha A_s\left(k_{ss}^{SB}\right)^{\alpha-1}$, multiplying x_{ss} by $\alpha/(1-\alpha)$ and comparing directly establishes the result. □