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BARGAINING UNDER THE ILLUSION OF TRANSPARENCY[†]

Abstract

An uninformed seller offers an object to a privately informed buyer. The buyer projects information and exaggerates the probability that the seller is informed. Letting the buyer bargain and name her own price raises the seller's payoff above the full-commitment payoff. Under seller-offer bargaining, any positive degree of projection implies a full reversal of the Coasian result in stationary strategies. As delay between offers decreases, the seller raises his initial price and, in the limit, extracts the full surplus from trade. Dynamic bargaining without price-commitment is revenue-optimal. Existing experimental evidence is consistent with the comparative static predictions of the model.

JEL Classification: C79 and D03
Keywords: bargaining, Coase conjecture, information projection, pricing

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Bargaining under the Illusion of Transparency

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Abstract

An uninformed seller offers an object to a privately informed buyer. The buyer projects information and exaggerates the probability that the seller is informed. Letting the buyer bargain and name her own price raises the seller's payoff above the full-commitment payoff. Under seller-offer bargaining, *any* positive degree of projection implies a full reversal of the Coasian result in stationary strategies. As delay between offers decreases, the seller raises his initial price and, in the limit, extracts the full surplus from trade. Dynamic bargaining without price-commitment is revenue-optimal. Existing experimental evidence is consistent with the comparative static predictions of the model.

Keywords: Information Projection, Bargaining, Pricing, Coase Conjecture.

1 Introduction

Haggling, as a mode of price formation, has interested economists for a long time. In classic bargaining, underlying monopoly theory, a seller of an object faces an interested but privately informed buyer: a local dealer wants to sell a new car to a client but is uncertain about how much the client is willing to pay; a firm is interested in hiring an expert to complete a task of fixed value, but does not know the expert's reservation wage. What are the benefits for the seller – the uninformed party – from haggling, that is, from establishing a price via bargaining over time instead of committing to a price or a price schedule?

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The standard assumption in bargaining theory is that the buyer correctly understands the extent to which her information is private vis-á-vis the seller. A key result under this assumption is a *no-haggling* result. The optimal way to sell the object requires commitment to prices and typically corresponds to making a single take-it-or-leave-it offer, e.g., Myerson (1981); Riley and Zeckhauser (1983); Skreta (2006).[1]

The fundamental Coase conjecture provides a powerful illustration of some consequences of bargaining in the absence of commitment to prices. The logic is as follows: even if the seller makes all offers, a buyer who fully appreciates the uncertainty the seller faces about her valuation can take advantage of the fact that, since at any given price, a higher buyer type is more likely than a lower one to buy, the uninformed seller becomes pessimistic anytime the buyer rejects the offer. In the absence of commitment, anticipating such future pessimism limits the seller's ability to charge higher prices in the present and hurts his ability to maintain his full-commitment monopoly payoff. In the limit, as friction in the bargaining process goes to zero, it forces the seller to sell immediately at the lowest possible reservation value of the buyer, e.g., Fudenberg, Levine, and Tirole (1985); Gul, Sonnenschein, and Wilson (1986). The Coasian logic and the no-haggling results provide essential insights in microeconomics and establish a strategic rationale for the use of non-negotiable prices. Empirical observation suggests, however, that haggling might be the seller-preferred method of sale even in settings well-approximated by the classic setup, such as the sale of new cars. Furthermore, the resulting price-variation may not be sufficiently explained by observable buyer characteristics – e.g., Goldberg (1996).

In contrast to the standard assumption, evidence from the field and the lab (e.g., Camerer, Loewenstein, and Weber 1989) shows that people may fail to *fully* appreciate informational differences. Instead the typical person *projects* information and too often acts as if others could also act on her private information. When bargaining, a privately informed buyer who projects information thinks that she is more transparent than she truly is. This illusion may play a key role in this context, as bargaining behavior and outcomes may depend critically on just how much information the buyer's actions actually conceal.

This paper incorporates such information projection into sequential bargaining and explores the robustness of the above results to its presence. I show that even a small degree of this mistake provides a powerful channel through which bargaining may greatly benefit the seller. Bargaining over time with a slightly biased buyer not only allows

[1] The optimal auction literature, — e.g., Myerson (1981), Riley and Samuelson (1981) —, studies the optimal selling mechanism and shows, that in the case of a single buyer, the optimal auction is a posted price.

the seller to preserve a larger fraction of his full-commitment (static monopoly) payoff given his true information, but becomes a process that allows the seller to achieve a payoff equivalent to the full surplus from trade. The results also shed novel light on some empirical phenomena, such as the value of commitment and delay, and have implications for a number of applications.

In Section 2, I incorporate information projection into a class of sequential bargaining games.[2] Each player correctly understands the distribution of her own information, but exaggerates the probability that her opponent may act on her private information. Specifically, the privately informed buyer thinks that whenever the seller acts, he might come to condition his strategy on her valuation. The seller is sophisticated: he disagrees with it, but understands the buyer's perception. Given such commonly understood but heterogeneous beliefs, players choose strategies that are sequentially rational given their respective perceptions of Nature's moves. This is true both for histories that could occur in reality and for imaginary histories that could occur with positive probability only in the buyer's imagination.

In Section 3, I apply the model to a simple setting with two periods and two buyer valuations. The Bayesian no-haggling result implies a silence and a Coasian property: the seller's dynamic revenue — either if the parties alternate or if the seller makes all offers — is bounded from above by the revenue from a static take-it-or-leave-it (TIOLI) offer. In contrast, both properties are violated if the buyer projects information, even if she projects only at the initial stage of the game.

To highlight the role of the illusion of transparency versus true initial leakage per se, Section 3 also considers the case in which there is a commonly known chance that before bargaining starts, the salesman may privately infer the buyer's valuation based on some observable characteristics of the buyer. I show that as long as the buyer is well calibrated about the chance of such an initial inference, but does not exaggerate it, the seller's expected revenue is still highest with a take-it-or-leave-it offer following the parties' initial encounter.

To provide some intuition, consider an alternating-offer protocol. Can the seller gain by letting the buyer speak (offer) first? For this to hold, the buyer would need to reveal her private information. If she did, her surplus would be bounded by the seller's cost of delay, that is, the extent to which the seller preferred a lower price today to holding the buyer to her valuation in the next round. The buyer thus reveals herself only if the delay between

[2] I build on a companion paper, Madarász (2014), introducing information projection equilibrium into *static* Bayesian games with partitional information structures.

offers is sufficiently long such that her surplus exceeds the difference between the high and the low type's initial offer times the probability that the seller is uninformed. If the buyer is well-calibrated, this implies, however, that, relative to a TIOLI offer, the surplus transferred to the buyer exceeds the ex-ante expected value of the information gained. In short, it is not beneficial for the seller to let the buyer speak. A biased buyer, in contrast, exaggerates the probability that the seller is informed. Hence she demands a lower surplus conditional on revelation. Positive delay between the bargaining rounds is still needed to induce revelation, but this delay can now be shorter than before. Consequently, the seller's gain can exceed the loss. Although haggling is nominally costly for the seller, engaging in such a 'waiting game' is necessary to capitalize on the buyer's mistaken beliefs. Letting the buyer speak first is a successful sale tactic here. I show a similar result when the seller makes both offers.

Section 4 considers infinite horizon seller-offer bargaining with a continuum of types - the classic domain of the Coase conjecture. Given unbiased expectations, as friction between bargaining rounds decreases, the seller gradually loses his entire ability to extract revenue. In contrast, focusing on stationary strategies, I show that given *any* positive degree of projection - where the buyer wrongly attaches some (arbitrarily small) probability that her valuation might privately leak to the seller from any one round to the next - the opposite happens. Dynamic bargaining without commitment becomes a full rent extracting mechanism (Proposition 5).

Information projection impacts not only the limit results, but the Bayesian comparative static predictions away from the limit as well. Initially, as friction decreases the same result may hold as in the unbiased case. For any positive degree of projection, however, there is some point away from the limit where this comparative static reverses: now as bargaining friction decreases, the seller smoothly increases his initial price and also obtains a larger expected revenue. The results are true despite the fact that, given any positive friction in the bargaining process, the perceived 'mechanical' utility consequence of prospective projection - the discounted total sum of the perceived leakage probabilities expressing the present value of the overall perceived leakage - goes smoothly to zero as projection goes to zero.

The intuition is based on how projection impacts the relative strength of two countervailing effects. As friction (delay) in bargaining decreases, both the buyer's cost of postponing a purchase and the seller's cost of postponing a sale from one round to the next decreases. The sophisticated seller understands that he cannot simply wait for information about the buyer to exogenously arrive. Hence, holding the buyer's strategy constant, the seller's trade-offs remain the same. He still becomes more pessimistic after each rejection and reduces the price over time by an amount that is independent of the buyer's projection. Given unbiased expectations, the buyer correctly understands the extent to which the seller becomes more pessimistic over time. As friction decreases, the differential willingness of different buyer types to wait decreases. This is the classic force. Given information projection, the buyer still understands this pessimism but may underestimate its extent. She believes that with some positive, but potentially arbitrarily small, probability per round the seller might figure her out. Since a higher type has more to lose from such a perceived leakage than a lower type does, this represents a new force that a sophisticated seller can take advantage of.

Key to the intuition is that both the classical force and the force due to projection become stronger as friction decreases. The result is, then, based on the fact that no matter how tiny the projection is, it is sufficient to change the perceived relative strengths of the above two countervailing effects. In particular, if bargaining friction drops below a critical threshold, the cost for the seller to postpone a sale now decreases more quickly than the perceived cost for a given buyer type to postpone a deal. Since, in the limit, the seller cannot make significant drops from one round to the next, the seller's initial price converges to the buyer's highest possible valuation and the buyer's willingness to accept converges to her reservation value. The seller is now able to extract the full surplus.

Section 4.1 further tightens the result and, by allowing projection to vanish as the period length shrinks, it establishes a discontinuity (Proposition 6). It shows that if projection is always smaller than the square-root of the length of the time period, the Coasian result of full revenue loss follows. If it is greater, then full rent extraction follows. Thus even if such mistaken beliefs vanish, smooth haggling without commitment allows the uninformed seller to capture all benefits from trade.

Section 5 compares the model's prediction to the evidence. As mentioned, information projection predicts not only a different limit result, but also the reversal of the Bayesian *comparative statics*; this fact allows one to relate the predictions to the existing experimental evidence - Rapoport, Erev, and Zwick (1995). The empirical results display a comparative static that is the reverse of the Bayesian prediction but is consistent with that predicted by the model with projection. Buyers accept offers too soon and, as friction decreases (the discount rate increases), sellers raise rather than lower their initial prices and are able to maintain or increase their revenue.

No-haggling results provide a strong strategic rationale for non-negotiable prices, or price schedules. Haggling, however, despite being time-consuming, is present not only historically in the bazaar, but also in many modern-day settings. It is common in the sale of a variety of large consumer items in the US and smaller items or services also in other countries. It is adopted even for such standardized goods as new cars in local dealerships. As mentioned, here, evidence shows that this practice leads to substantial price variation, dealer margins, across consumers, which need not be sufficiently correlated with readily observable buyer characteristics on the basis of which the seller could discriminate, Goldberg (1996). Information projection provides a structured channel through which an uninformed seller, when interacting with a biased buyer, even if he could just post a price or commit to a no-haggling policy, may greatly benefit from haggling. Relative to a non-negotiable offer the seller may do so more, not simply the larger is the value of the good, but also the *greater* is the private information the buyer truly possesses. I conclude by briefly discussing these and some further implications.

1.1 Evidence on Information Projection

Evidence on information projection comes from a variety of domains. Key manifestations include such findings as the *curse-of-knowledge*, e.g., Camerer, Loewenstein, and Weber (1989), Loewenstein, Moore, and Weber (2006), Birch and Bloom (2007); the *illusion of transparency*, e.g., Gilovich, Medvec, and Savitsky (1998, 2000); the *hindsight bias*, e.g., Fischhoff (1974), Biais and Weber (2010). In all these settings, people's beliefs are biased in that they too often act as if others knew their private information.[3] For a more detailed review, see Madarász (2012).

In the context of strategic bargaining, Samuelson and Bazerman (1985) provide evidence consistent with information projection. In a common-value trade problem, privately informed subjects - sellers in their setting - act as if their uninformed opponents had access to their private information. Keysar, Ginzel, and Bazerman (1995) provide closely related evidence. Less structured but supportive evidence on exaggerated perceptions of transparency in the negotiation process - even in completely constrained situations - comes from e.g., Vorauer and Claude (1998), Vorauer and Ross (1999), Van Boven et al. (2003).[4] Danz, Madarász, and Wang (2014) study a strategic setting with one-sided private information and find not only that privately informed subjects systematically project their

[3] Documentation of such failures of informational perspective-taking goes back to the classic work of Piaget and Inhelder (1948), who studied the *theory of mind* - the ability to attribute different beliefs to others - of children. Recently, Birch and Bloom (2007) have shown that the same structure of mistakes documented in children is present amongst Yale undergraduates in slightly more complex tasks.

[4] Vorauer and Claude (1998) show that even when the behavior of participants was completely constrained by the situations, participants too often felt that observers could accurately detect the true nature of their internal state

information onto less-informed others, but also that less-informed others anticipate such misperceptions of their privately informed opponents, lending support to the assumptions in this paper.

1.2 Related Literature

In a general dynamic environment, Riley and Zeckhauser (1983) characterize the optimal dynamic selling mechanism under commitment and show that it corresponds to making a single take-it-or-leave-it offer. Skreta (2006) considers the case in which the seller can commit only within a period and shows that the optimal sequential mechanism is the seller-offer protocol. The revenue that the seller achieves given such a protocol is bounded from above by the revenue from a single TIOLI offer.[5]

A large and influential literature explores the robustness of the Coasian logic in the absence of commitment. For example, Bond and Samuelson (1984) consider good depreciation in a monopoly market, Sobel (1991) and Fuchs and Skrzypacz (2010) study the arrival of new buyers. Under the no-gap assumption, Ausubel and Deneckere (1989) show, in addition to the Coasian equilibrium, the existence of non-stationary equilibria and that the seller can attain any revenue between the Coasian limit and the static monopoly payoff. Board and Pycia (2014) show that given a privately known outside option for the buyer a no-haggling equilibrium is unique and here the seller charges the static monopoly price. In contrast to these settings, under information, projection bargaining boosts the seller's revenue *above* the full-commitment (static monopoly) payoff.

Feinberg and Skrzypacz (2006) study an infinite horizon seller-offer game with two buyer types who also face initially uncertainty about whether or not the seller is informed. They maintain the assumption that this information structure is common knowledge, and show that such uncertainty produces delay. Bargaining, however, is still a process that leads to a lower ex-ante expected revenue than a take-it-or-leave-it offer following the seller's meeting with the buyer.[6]

Finally, Yildiz (2003) considers biased beliefs in an alternating-offer game with complete information about valuations, but with players who have heterogeneous (and, overall, too optimistic) beliefs about the distribution of their bargaining power – i.e., the probability of being able to make an offer. He provides conditions under which, despite such

[5] On the positive value of commitment see also the settings of e.g., Stokey (1979) or Hart and Tirole (1988).

[6] Admati and Perry (1987) consider a setting with two buyer types and alternating-offers where the buyer has the power to delay a sale, and might do so to signal that she has a low valuation. Under the uniqueness of their separating equilibrium, the seller still achieves a revenue that is bounded from above by the optimal TIOLI offer.

over-optimism, immediate agreement follows. This paper differs in that proposing rights are always common knowledge, but is similar in that players optimize given commonly known heterogeneous beliefs about the underlying environment.

2 Setup

The seller has an object that he values at a normalized amount of 0. The buyer values it at some strictly positive amount θ. Bargaining happens sequentially over time $t = 1, 2, \ldots$. In each round, one of the parties makes a price offer that the other party can accept or reject. Bargaining lasts until an offer is accepted or until some final date T - potentially at infinity - arrives. As standard, friction in bargaining, denoted by Δ, corresponds to some noise or delay in the process. Let $e^{-\Delta}$ denote the probability that bargaining breaks down from one round to the next. Equivalently, Δ will be interpreted as the period length or delay between offers, given time discounting and a normalized interest rate. The terminology that bargaining friction goes to zero refers to the case in which $\Delta \to 0$.

True Information Structure. At time $t = 0$, a game is fixed. The buyer then privately learns θ, which is drawn from a commonly known prior distribution. Nature makes no further moves and bargaining begins.

In Section 3, I allow for a further event. Upon meeting the seller, but before bargaining starts, the seller may privately learn θ with probability α. The case in which $\alpha = 0$ corresponds to the classic setting. The case in which $\alpha > 0$, corresponds to a generalization, with the buyer also facing initial uncertainty as to whether or not the seller is informed. I consider such a generalization to such *initial* uncertainty about the seller's beliefs only to facilitate the interpretation of the results in Section 3. Again, nature makes no further moves and bargaining begins.

Information Projection Information projection by the privately informed buyer corresponds to an exaggerated perception that whenever the seller acts, he can act on the basis of the buyer's information as well. To introduce such beliefs, consider an informational perturbation of the true game. Suppose that at the beginning of each round t, for $t \geq 1$, independent of the history leading to that round, Nature plays a binary leakage lottery whose outcome is observed only by the seller. The realization of this fictional lottery determines whether the buyer's information becomes privately available to the seller. A buyer who exhibits information projection of degree $\rho = \{\rho_t\}_{t=1}^{T} \in [0, 1)^T$ falsely believes that the probability that of leakage in round t is ρ_t. The seller (correctly) believes that this probability, for all $t \geq 1$, is uniformly zero.

Perfect Projection Equilibrium. To solve the model, I introduce perfect ρ-projection equilibrium (equilibrium henceforth) for the games described above. This derives from the perfect equilibrium of the perturbed game with the sole modification that players have the above described commonly known *differing* beliefs about Nature's chance moves. To keep the analysis focused, I present the formal definition in the Appendix, but describe it informally below.[7]

Consider a strategy and belief-system pair $(\sigma^{\rho}, \mu^{\rho})$ in the perturbed game. For this to be a perfect projection equilibrium: (i) each player's strategy has to be sequentially rational given that player's belief system; (ii) each player's belief system must be consistent; beliefs at each information set are derived from Bayes' rule (whenever possible) given the players' strategies and *that* player's belief about the distribution of Nature's moves. Crucially, these apply both to histories that can occur in reality and to histories that only the buyer thinks are possible.[8] A perfect ρ-projection equilibrium of the true game is, then, the natural restriction of such a $(\sigma^{\rho}, \mu^{\rho})$ to the true game.

Discussion. Given the above solution concept, whatever happens in the game is consistent with what players think can happen in the game. Since players know each other's strategies on the real equilibrium path, everything that happens is something that players expect to happen with positive probability. The departure is solely that the biased buyer attaches positive probability to histories occurring that might never occur or occur with a different probability than perceived. Equilibrium is consistent with the players understanding each other's differing perceptions about the chances that the game moves to an imaginary path with positive leakage. If $\rho = 0$, the true game and the perceived game coincide, and the definition is equivalent to that of a perfect Bayesian equilibrium of the true game.[9]

The initial stage is consistent with two interpretations. As is standard in the bargaining literature, one can take a game as *given* and analyze bargaining process there, including comparative statics. Following, Riley and Zeckhauser (1983) and the weaker assumption

[7] Madarász (2014) defines (public) projection equilibrium for two-player *static* games with partitional information structure: there each player exaggerates the probability with which her opponent conditions his strategy on their joint information in the game, i.e., the coarsest common refinement of her partition and her opponent's partition.

[8] While the seller attaches probability zero to achieving information sets with positive leakage histories, his conditional beliefs at such information sets are still well-defined. Note, also, that since these imaginary information sets are singletons, beliefs about the distribution of Nature's future moves there are inconsequential.

[9] The defining feature of this model is that players have false beliefs about the information and, hence, the strategy of their opponent *on average.* This distinguishes the model qualitatively from Jehiel's (2005) model of analogy-based expectations equilibrium, and Eyster and Rabin's (2005) model of cursed equilibrium. In both, it is key that players are correct about their opponents' actions on *average.* In this private value setting, a cursed buyer would behave as rationals do.

considered in Skreta (2006), one can think of the bargaining protocol as the one the seller chooses at the beginning of $t = 0$. Since, at this stage, no uncertainty about θ is resolved, the seller's choice cannot signal the resolution of any uncertainty. Instead, it is consistent with the seller's understanding of the buyer's behavior.

There are two closely-related psychological interpretations of projection. In the first, the privately informed buyer mistakenly thinks that whenever the seller acts, he might condition his choice on the same information she does. Relatedly, the privately informed buyer can have a paranoid belief thinking that valuation can leak to her opponent in any round independent of how she behaves. Crucially, both of these - in accordance with the evidence - imply that such a perception is *independent* of the buyer's behavior. The buyer perceives these as exogenous events by Nature rather than events in her control. Hence she does not think that by changing her observable behavior or by choosing to bargaining through a proxy - as would be the case if leakage was the result of her behavior - she can avoid it.[10]

Notation. Below, by V_F I denote the total *ex-ante* expected full surplus from trade – i.e., the expectation of θ given the prior. I denote by V_M the seller's true ex-ante expected revenue (revenue in short) from the optimal take-it-or-leave-it offer made in $t = 1$. Such V_M is fully separable and increasing in α with $V_M < V_F$. I will also refer to this as the static monopoly payoff.

3 Two-Periods

Consider a two-period setting where $\theta \in \{l, h\}$, and let the prior on the high type be q. I first re-state that under rational expectations and no true leakage, the seller's dynamic revenue is always lower than the expected static revenue (e.g., Skreta 2006). Let $V_H(\Delta)$ denote the seller's maximal ex-ante expected revenue over all perfect Bayesian equilibria that can occur over all fixed sequential bargaining protocols given friction Δ.

Lemma 1 *Suppose $\rho, \alpha = 0$, then $V_H(\Delta) \leq V_M$ for all Δ.*

Two consequences of the above well-known fact are worth highlighting. The first concerns the seller's ex-ante expected dynamic revenue from an alternating-offer protocol.

[10] This paper does not study the case where leakage truly happens in periods after the initial encounter, that is, other than $t = 0$. Crucially, such leakage might not be realistic since beyond some basic observables available at the initial encounter at $t = 0$, the seller may learn about the buyer's valuation during the process of bargaining only from the buyer's endogenous behavior, which the buyer can fully control – e.g., bargaining through a proxy. In contrast, under information projection, the buyer perceives such 'leakage' as an exogenous event controlled by Nature.

The second concerns that from a seller-offer protocol. Specifically, relative to making a single take-it-or-leave-it offer, it follows that

1. **Silence Property:** letting the buyer name her own price does not improve the seller's revenue,

2. **Coasian Property:** the chance of making a second offer by the seller does not improve the seller's revenue.

Suppose, now, that the buyer's valuation might truly leak to the seller with some probability α at $t = 0$. The consequence of such initial leakage is that the buyer now faces uncertainty about the seller's beliefs. The next result shows that as long as the buyer is well-calibrated about such (second-order) uncertainty - which is, thus, common-knowledge - the above properties extend.

Proposition 1 *Suppose $\rho = 0$, then $V_H(\Delta) \leq V_M$ for all Δ.*

To strengthen the results below and to provide a better comparison with the above result, in the remainder of this Section I *restrict* the buyer's projection to round $t = 1$. Formally, I assume that $\rho_2 = 0$. Here, projection effectively implies that the buyer exaggerates the probability of the ex-ante leakage to be $\widehat{\alpha} = \alpha + (1 - \alpha)\rho_1$. All revenue results hold *a fortiori* when $\rho_2 > 0$. For convenience, I denote ρ_1 by ρ below.[11]

3.1 Alternating Offers

When selling a new car, a salesman asks the interested buyer to name her own price first. If the price is not acceptable, the salesman will have to consult his manager. The manager might be away that day, and the salesman may be able to return with his counter-offer only after some delay. Does this practice allow for a higher revenue than a TIOLI offer following the initial encounter?

Consider the class equilibria that can arise in an alternating-offer protocol with friction (delay) Δ. Let $V_A^\rho(\Delta)$ denote the seller's true maximal ex-ante expected revenue over this class given ρ. The next proposition shows that this revenue exceeds V_M whenever there is enough, but not too much, friction in the bargaining process and the buyer is sufficiently biased.

Proposition 2 *Suppose $\rho > \rho_A$, for any $\Delta \in (\widetilde{\Delta}^\rho_{\min}, \widetilde{\Delta}^\rho_{\max})$ it follows that $V_A^\rho(\Delta) > V_M$.*

[11] Under an alternating-offer game, the case with only initial projection and the more general case are isomorphic.

To illustrate, first consider the classic case ($\alpha = 0$). For the seller's dynamic revenue to exceed the static optimum, the seller must obtain information about the buyer's valuation from her offer.[12] In the unbiased case, since the seller can never distinguish between a 'lie' and the 'truth,' no fully-revealing equilibrium exists. Since a semi-revealing equilibrium cannot generate excess revenue either, letting the buyer speak in this fashion can only hurt the seller.

Suppose the buyer projects information. She now wrongly believes that the seller may be able identify her. For a fully-revealing equilibrium to exist, the high type buyer's surplus conditional on revealing herself must exceed her perceived return from pretending to be a low type. The former is bounded by the seller's cost of delay – the extent to which the seller prefers a lower price today to holding the buyer to her valuation in the next round. The latter is the buyer's perception of the probability that the seller is uninformed times the difference between the different types' initial offers. Formally, in a fully-revealing equilibrium, the high type always names a price of $p_h = e^{-\Delta}h$, and the low type some lower price p_l.[13] For this to be (perceived) incentive-compatible, the following must hold:

$$h - p_h \geq (1-\rho)(h - p_l), \tag{3.1}$$

where I rely on the fact that the uninformed seller may attribute any deviation to the high type. Given that $p_h = e^{-\Delta}h$, this constraint implies that delay must exceed:

$$\Delta \geq -\ln(\rho + (1-\rho)\frac{p_l}{h}), \tag{3.2}$$

where this bound is decreasing in ρ going to zero as projection becomes full.

For such an equilibrium to generate excess revenue, there needs to be an upper bound on the delay, as well. Specifically, given that $p_l \geq e^{-\Delta}l$ must hold, whenever delay is shorter than $\Delta \leq \ln(V_F) - \ln(V_M)$, there is excess revenue relative to V_M. Since the bound in Eq.(3.2) is decreasing in ρ, this is always possible if ρ is sufficiently large.

More generally, consider the case where the seller is privately informed initially about θ with probability $\alpha > 0$. Now, even if the buyer is unbiased, there may exist fully-revealing equilibria. Revelation still requires the high type's surplus to exceed the return on pretending to be a low type – $(1-\alpha)(h - p_l)$. It follows that relative to a TIOLI offer in $t = 1$, the buyer's surplus exceeds the ex-ante value of the information revealed — the

[12] It is thus sufficient to consider protocols where the buyer makes the first offer.

[13] Note that p_h is uniquely determined. For this construction if will suffice to consider the lowest possible revealing price for the low type, i.e., $p_l = e^{-\Delta}l$.

gain from price-discrimination by the uninformed seller type. Proposition 1 shows that this also holds for semi-revealing equilibria. In contrast, under information projection the buyer's perceived return on pretending to be a low type is diminished to $(1-\alpha^{\rho})(h-p_l)$. Full revelation can happen given a smaller friction/delay. For the same reason as before, if ρ is sufficiently large and the buyer has enough, but not too much, bargaining power, the seller now obtains excess revenue. The above logic implies the following corollary:

Corollary 1 *For all $\rho > \rho_A$, $V_A^{\rho}(\Delta)$ is maximal given positive but bounded Δ.*

The above claim points to a non-monotone comparative static. If $\rho = 0$, the seller's maximal revenue is achieved under $\Delta = 0$. Under sufficient projection, it may decrease both when delay is shorter and when it is longer than the optimal boundedly positive amount. Positive friction is nominally costly for the seller, but allows him to capitalize on the buyer's mistaken beliefs. By asking the buyer to name her own price first and engaging in such a "waiting game," the seller can boost his revenue. Anecdotal evidence suggests that such a bargaining practice is successful in many settings.[14]

3.2 Seller Offers

Let's turn to the case where only the seller makes offers. When commissioning an expert, the firm may make an initial offer. If it is rejected, although delay on the task is costly to the firm, it may take some time for the firm to assemble a new offer. Does such a sluggish response, which is costly to the firm, combined with uncertainty as to what the second offer would be, help the firm get a better deal?

Consider the class equilibria in the seller-offer protocol given friction Δ. Let $V_{So}^{\rho}(\Delta)$ denote the seller's true maximal ex-ante expected revenue over this class given ρ. The next proposition establishes an analogue of the previous result.

Proposition 3 *Suppose $\rho > \rho_{So}$, for any $\Delta \in (\widehat{\Delta}_{\min}^{\rho}, \widehat{\Delta}_{\max}^{\rho})$ it follows that $V_{So}^{\rho}(\Delta) > V_M$.*

For the result to potentially hold, two facts need to be true. First, there must be price discrimination. Second, the uninformed seller and the (fictional) informed seller - conditional on the buyer being a high type - must pool together in the first round. Otherwise, the high-type buyer always learns whether or not the seller is informed, and the problem becomes separable across seller types. In the biased case, such pooling allows

[14] Note that for the results above, whether, delay is costly for the buyer is irrelevant. Also, I illustrated the result with fully-revealing equilibria where the seller always accepts the buyer's offer. There exists mixed-strategy equilibria, where the seller rejects the low offer with positive probability the seller's ex-ante expected revenue even exceeds that from fully-revealing equilibrium described.

the seller to pretend to be informed more often than he truly is and charge a higher initial price. For such pooling to work, delay between the offers must be positively costly for the seller. This ensures that the high type buyer finds it credible that an informed seller would be willing to pool at a lower price now as opposed to waiting and naming a higher price later. Capitalizing on the buyer's mistaken belief in this fashion, the seller can boost his revenue.

Formally, let $p_{1,h}$ denote such a pooling price in $t = 1$. A high-type buyer is willing to accept this if: (i) her surplus is greater than her perceived continuation value from a rejection:

$$h - p_{1,h} \geq e^{-\Delta}(1 - \alpha^{\rho})(h - l) \tag{3.3}$$

since the uninformed seller asks for l in the next round; and (ii) it is credible that $p_{1,h}$ could be coming from the informed seller, being it is too costly for him to wait to ask for a high price:

$$p_{1,h} \geq e^{-\Delta}h. \tag{3.4}$$

Combining the constraints of Eq.(3.3) and Eq.(3.4) implies a positive *lower* bound on Δ, which is decreasing in ρ. Friction must be positive.

If $\rho = 0$, incentive compatibility implies that the benefit from price discrimination is lower than the cost. The former is given by $(1 - \alpha)\min\{q(h - l), (1 - q)l\}$. The latter is the sum of the information rent demanded by the high type and the cost of selling to the low type (high reservation-wage expert) later.[15] Hence, the Coasian property follows given any positive but bounded friction. The rent demanded by the high type is now decreasing in $\rho > 0$. If ρ is sufficiently high, the information rent transferred becomes smaller than the gain from offering different wages to different types.

For bargaining to generate excess revenue, friction may also need to be bounded from above. This is the case when $l > qh$, where selling to the low type sufficiently early is crucial. If $qh \geq l$, no such upper bound binds and the result holds for all Δ greater than the lower bound. If there is sufficient, but potentially not too much, friction and the buyer is sufficiently biased, the revenue from bargaining now exceeds V_M. The above logic again implies a straightforward corollary.

Corollary 2 *For any $\rho > \rho_{So}$, $V^{\rho}_{So}(\Delta)$ is maximal given positive but bounded friction.*

If $\rho = 0$, no friction or infinite friction is optimal - effectively allowing for commitment. Under information projection, positive but bounded friction is characteristic of the optimal

[15] The same holds when the parties use mixed strategies.

selling protocol. Again, costly delay, in conjunction with the fact that the buyer faces uncertainty about what the next price might be, allows the seller to pretend to be informed more often than he truly is and capitalize on the buyer's biased beliefs. Engaging in such a waiting game, which is nominally costly for the seller, can then improve upon his no-haggling payoff.

4 Infinite Horizon

Above, I described the implications of information projection in a simple two-period bargaining setting. I showed that bargaining over time was beneficial to the seller even if projection was constrained to happen only at the beginning of the initial round. I now turn to the main application and explore the model's predictions to classic infinite-horizon seller-offer bargaining. The seller faces a continuum of buyer types with valuations being distributed uniformly on $(0, 1]$. I maintain the so-called 'gap' assumption, namely that it is common knowledge that there is strict benefit from trade. Bargaining lasts as long as the parties do not agree on an offer. As described in Section 2, I consider prospective projection – that is, the privately informed buyer acts as if in each round t, there was some probability ρ_t with which her valuation leaked to the seller. For convenience, I assume that all $\rho_t = \rho$ so that I can describe projection by a single parameter ρ.[16]

In case projection is almost full, it might be no surprise that such a mistaken perception changes bargaining behavior; and, despite the fact that the seller never exogenously learns the buyer's valuation, and that the buyer understands that the seller thinks so, the absence of commitment might not be as detrimental to the seller's revenue as in the unbiased case. The results below investigate the sensitivity of the key Coasian logic to any such information projection.

Below I describe both the imaginary equilibrium path - which only the buyer thinks happens with positive probability- and the real equilibrium path. I focus on stationary strategies on the real path - the seller's strategy is Markov with respect to the relevant state-variable, which is the highest possible buyer type who has not bought yet (Gul, Sonnenschein, and Wilson (GSW henceforth) 1986).[17] These strategies will be parametrized by ρ and Δ. The buyer employs a cutoff strategy and accepts price p iff her valuation θ is greater than $\lambda(\rho, \Delta)p$. The magnitude $\lambda(\rho, \Delta) \geq 1$ expresses the extent of the buyer's demand withholding. The seller's stationary strategy is based on a state variable that

[16] For simplicity, below I analyze the standard case in which the true initial leakage probability, α, is zero. I suspect that given a continuum of types allowing for $\alpha > 0$ will not change any of the qualitative results.

[17] See, also, Sobel and Takahashi (1983) and Fudenberg, Levine, and Tirole (1985).

corresponds in every period t to the highest possible buyer type that has not bought until that period. Since the skimming property holds, at each period, the seller always faces a left-truncation of the buyer's type distribution. The seller names an initial price of $\gamma(\rho, \Delta)$ and employs a linear pricing rule given the state variable. On the imaginary path, the seller is believed to have a constant strategy holding the buyer to her valuation.[18] When hearing that she is 'identified,' the buyer plans to accept immediately.

The above strategies imply that on the real equilibrium path, there will be a decreasing sequence of prices p_t for rounds $t \in \{1, 2, ...\}$ such that:

$$p_t = \gamma(\rho, \Delta)^t \lambda(\rho, \Delta)^{t-1}$$

Let $V_S^\rho(\Delta)$ be the seller's true expected equilibrium revenue given a ρ-biased buyer and friction (delay) Δ. The following proposition re-states the Coasian result for the unbiased case. As friction decreases, the seller's ability to extract revenue decreases, and, in the limit, he sells immediately at a price equal to the lowest possible buyer valuation.

Proposition 4 (GSW 1986) *Suppose $\rho = 0$. There is a unique perfect equilibrium. Here, $\gamma(0, \Delta)$ increases and $\lambda(0, \Delta)$ decreases in Δ. Furthermore, $V_S^0(\Delta)$ smoothly increases in Δ, and $\lim_{\Delta \to 0} V_S^0(\Delta) = 0$.*

The comparative static properties of the unbiased case imply that as bargaining friction decreases the seller *lowers* his initial price, the buyer's demand withholding increases and the seller's expected revenue decreases. I now turn to the case with information projection.

Proposition 5 *There exists a class of perfect projection equilibria such that,*

1. *Demand-withholding $\lambda(\rho, \Delta)$ smoothly decreases in ρ for any $\Delta \in (0, \infty)$,*

2. *For any $\rho > 0$, there exists $\overline{\Delta}^\rho > 0$ such that $\gamma(\rho, \Delta)$ increases in Δ if $\Delta > \overline{\Delta}^\rho$, and decreases in Δ if $\Delta < \overline{\Delta}^\rho$, where $\overline{\Delta}^\rho$ is increasing in ρ,*

3. *For any $\rho > 0$, and any $\tau > 0$, there exists $\overline{\Delta}^\rho(\tau) > 0$ such that if $\Delta \in [0, \overline{\Delta}^\rho(\tau)]$, then $V_S^\rho(\Delta)$ smoothly decreases in Δ and $\left|V_S^\rho(\Delta) - V_F\right| \leq \tau$.*

In the equilibrium class described, the seller still gradually lowers his price over time until the buyer buys, but this process now has the following properties. First, the extent

[18] Note that since the seller's real path price schedule always contains finitely many prices, the set of types whose valuation is on the price schedule of the real path is measure zero. For such types, the imaginary informed seller names a price that is below their valuation θ but above $e^{-\Delta}\theta$ allowing the imaginary seller type to separate. Such a price always exists for $\Delta > 0$.

to which the buyer withholds demand smoothly decreases in the degree of the buyer's projection. Second, the comparative static on the initial price is non-monotonic. If bargaining friction (delay, noise) is sufficiently high, the seller's initial price decreases as friction decreases in the same qualitative way as in the unbiased case. For any positive degree of projection, however, as friction drops below a certain threshold, the comparative static reverses. Now, the initial price, as well as the entire price sequence, increases as friction decreases, and the seller's expected revenue also increases.[19] Finally, given any positive degree of projection, in the limit, the seller's initial price converges to the buyer's highest possible valuation. The seller extracts all surplus from trade.

Proposition 5 describes both a reversal of the limit result and the comparative static predictions away from the limit. These hold despite the fact that the 'mechanical' utility consequence of projection smoothly goes to zero as the buyer's projection goes to zero. Given projection, the buyer does wrongly believe that if bargaining were to last for infinitely many rounds, the seller would eventually learn her valuation; however, the discounted total sum of the leakage probabilities, $\sum_{t=1}^{\infty} e^{-\Delta(t-1)}\rho(1-\rho)^{t-1}$, given *any* positive delay – $\Delta > 0$ – does smoothly go to zero as projection goes to zero.[20] Hence, anywhere away from the limit, the direct impact of projection is small when ρ is small. Yet the reversal of the comparative static and the limit result holds for any ρ.

The logic is based on how information projection impacts the relative strength of two countervailing effects: as friction decreases, the buyer's willingness to wait for a price reduction increases, and the seller's loss from postponing a sale from one round to the next decreases. Given unbiased expectations, the speed of the former is always greater than that of the latter. Information projection introduces a new force that eventually changes their relative speed and hence the relative strengths of these two effects.

To provide intuition, let me proceed step-by-step. Since the seller correctly understands that exogenous leakage never happens, holding the buyer's strategy constant, his trade-offs are independent of ρ. The seller knows that he cannot just wait for information to exogenously arrive. Instead, since the skimming property holds, implying that in equilibrium higher buyer types accept weakly sooner than lower ones, the seller does become more pessimistic after each rejection by the buyer and reduces the price.

For price-discrimination to be incentive-compatible given stationary strategies, the

[19] If the degree of projection is sufficiently large, this reverse comparative static is global - that is, it holds for any $\Delta > 0$.

[20] Formally, note that for any $\Delta > 0$, the discounted sum of the probability of leakage is such that $\lim_{\rho \to 0} \sum_{t=1}^{\infty} e^{-\Delta(t-1)}\rho(1-\rho)^{t-1} = 0$.

seller's cost from reducing his price in a given round as opposed to in the next one must be less than the associated gain. The gain is the forgone interest loss on the seller's continuation value. The cost is the intertemporal price difference times the probability that the buyer will buy at a higher price now as opposed to at a lower price in the next round. The cost is then proportional to the extent to which the buyer withholds demand. Specifically, price-discrimination requires the ratio of the seller's initial price over his continuation value to exceed the extent of the buyer's demand-withholding. The logic is then based on how information projection affects this constraint.

In the unbiased case, the buyer fully appreciates the pessimism of the seller. Since a decrease in bargaining friction makes it less costly for higher types to imitate lower ones it requires greater information rents to the buyer. This is the classic force. As a result, the buyer's demand withholding increases as delay decreases and the seller's revenue is diminished. In the limit, the buyer's demand withholding becomes unbounded hence the seller's continuation value must converge to zero; the seller sells immediately at the lowest possible valuation of the buyer.

Given limited information projection, the classic force is still present: the buyer correctly believes that with a probability potentially arbitrarily close to one, the seller becomes more pessimistic from one round to the next. At the same time, there is now a new countervailing force due to the buyer's misperception. The fear that the play moves to the imaginary path hurts higher types more than lower ones. This implies the first point above: the buyer's willingness to withhold demand smoothly increases in the degree of projection.

Key to the intuition is that both the classic force and the force due to projection become stronger as friction decreases. Initially, as friction decreases, the classic force dominates. As a result, the buyer's demand withholding increases, and the seller lowers his initial price as in the unbiased case. For any $\rho > 0$, however, there is always some critical value $\overline{\Delta}^{\rho}$ where the relative strength of the two forces switches. Now the extent to which different buyer types are willing to accept different prices increases. Since the players understand each other's strategy on the real equilibrium path, the seller now starts to increase his initial price. While both forces get stronger as delay decreases, the force due to projection, independent of its degree, eventually fully overtakes the classic force. In the limit, the buyer ceases to withhold demand, and the incentive-constraint that was binding in the unbiased case is now slack. Since the per-period price gradient on the dynamic price-schedule, independently of the degree of projection, converges to null as delay converges to null, full rent extraction follows.

Proposition 5 relies both on the fact that the seller disagrees with the buyer's perception and that he understands it. The seller knows that he learns about the buyer's valuation only from the buyer's equilibrium behavior and not from waiting for information to exogenously arrive. At the same time, the seller understands the buyer's illusory perception and takes advantage by changing the prices he offers. In the absence of price commitment the seller can capitalize on the buyer's fear induced by information projection.

4.1 Vanishing Projection

The above result is based on the psychologically realistic assumption that the buyer believes that there is some non-zero probability that, in any round, her opponent comes to act on her valuation. Given that each bargaining round is separate, this fits tightly with the interpretation of information projection described. The logic above suggests, however, that an even weaker assumption may suffice.

Suppose that as friction goes to zero, a person's biased perception also converges to zero, $\lim_{\Delta \to 0} \rho(\Delta) = 0$. For example, as offers become more frequent, the buyer attaches a smaller and smaller probability to leakage in a given round, allowing for this perception to converge to zero. Alternatively, since the result below is a limit result, this assumption may also be consistent with a case in which a biased buyer believes that leakage is less and less likely to happen in the future if it did not happen in the past, letting such beliefs converge to zero.

The next result establishes, given the above described class of strategies, an exact threshold on the relative speed at which projection vanishes as bargaining becomes smooth. To highlight the dependence of ρ on Δ notationally, as well, below I make ρ an argument of the seller's expected revenue.

Proposition 6 *Suppose that $\rho = \beta(\Delta)^{\kappa}$. For all $\beta > 0$,*

If $\kappa > 0.5$, $\lim_{\Delta \to 0} V_S(\Delta, \rho(\Delta)) = 0$.

If $\kappa = 0.5$, $\lim_{\Delta \to 0} V_S(\Delta, \rho(\Delta)) = V_F \beta/(1+\beta)$.

If $\kappa < 0.5$, $\lim_{\Delta \to 0} V_S(\Delta, \rho(\Delta)) = V_F$.

For the class of functions parametrized above, Proposition 6 identifies a discontinuity. If projection is smaller than the square-root of the period length, and hence the relative speed at which projection versus delay decreases is less than one-half, then the seller is able to extract all surplus. Here the buyer's perceived cost of waiting decreases more slowly than the seller's. If the relative speed at which projection vanishes is faster than this threshold, then the Coasian result follows. In the knife-edge case, where discontinuity happens,

$\kappa = 0.5$, the distribution of the expected surplus depends on the scaling parameter β. Here, the seller obtains a $\beta/(\beta+1)$ fraction of the expected surplus. For example, when $\beta = 1$, this is exactly one-half of the expected surplus from trade.

Note that, except for the case in which $\kappa = 1$, the limiting *total* undiscounted probability of perceived leakage per unit of real time, as the period length shrinks, is independent of $\kappa < 1$.[21] Hence, the discontinuity does not hinge on this limiting probability, which is the same in both cases. Rather, it depends on how the relative strength of the two opposing forces affect the seller's intertemporal incentive constraint. Finally, note that the above result does not derive from the buyer and the seller implicitly facing different interest rates due to projection. Even when interest rates differ, but the buyer is unbiased, commitment is still strictly valuable and the Coasian limit follows here.[22]

5 Comparative Statics and Evidence

I now turn to the existing evidence on the above game. Proposition 5 implies not only a reversal of the limit result, but also different comparative static predictions away from the limit. This renders the model's predictions readily testable. As highlighted, the comparative static predictions of the case with and without information projection differ. In the unbiased case, as the cost of waiting for a new offer – i.e., Δ – decreases, the seller charges lower initial prices and obtains a lower revenue. Under projection, there is a threshold value $\overline{\Delta}^{\rho}$, increasing in ρ, such that if Δ drops below $\overline{\Delta}^{\rho}$, the process reverses: the seller's initial price and the entire schedule of declining price-sequence increases as Δ decreases.

[21] For $\kappa = 1$, the total undiscounted probability of leakage per unit of real time in the limit is $1 - \lim_{\Delta \to 0}(1-\beta\Delta)^{1/\Delta} = 1 - e^{-\beta}$; for *all* $\kappa < 1$, it converges to 1.

[22] The Appendix contains a proof of the latter statement.

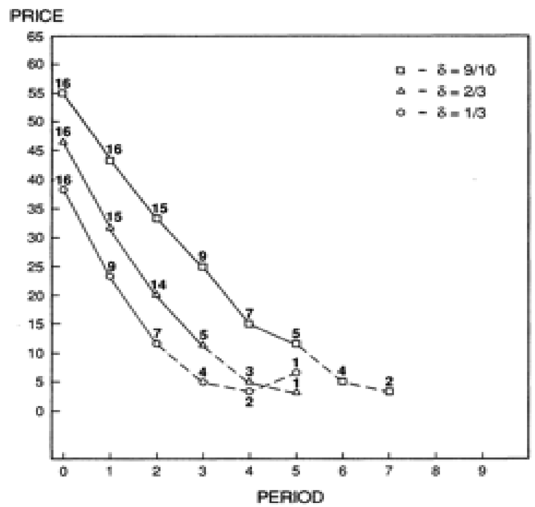

Figure 1: Rapoport, Erev, and Zwick (1995)

Rapoport, Erev, and Zwick (1995) report an experiment with discounting in a game that corresponds to the game studied above. The buyer's private valuation is drawn uniformly from $(0, 100]$ and is never revealed to the seller. The seller makes all offers.[23] The setting is made common knowledge. The treatment variable is the common per-period payoff discount factor $\delta = e^{-\Delta}$. In the three treatments implemented, δ varied over the values of $\{1/3, 2/3, 0.9\}$. The main findings are summarized in Figure 1, which describes the data from sessions with experience. The findings from sessions without experience are qualitatively very similar and, importantly, exhibit even stronger comparative statics in the same direction.

Rapoport et al. (1995) observe that the effect of the treatment variable is the opposite of the unique Bayesian prediction. The authors conclude that even "after considerable experience with the game, all [...] subjects [...] exhibited an ordering of the 3 conditions which is *diametrically opposed* to the equilibrium ordering."[24] In more detail, their results show that: (i) buyers withhold demand but accept offers too soon relative to the actual price path of the sellers; (ii) sellers' price gradient is close to the Bayesian predictions, and (iii) sellers' initial prices increase rather than decrease in δ (Δ^{-1}).

[23]The game terminated either when the buyer accepted the seller's offer or - to deal with finite experimental time - when the highest possible discounted profit became smaller than the smallest unit of the currency used i.e., 1.

[24]See p. 385.

These findings are consistent with the predictions of Proposition 5. First, the result predicts that buyers withhold demand but accept prices too soon. Second, the intertemporal price gradient of the seller's price path is independent of ρ. This is, thus, the same in the biased and the unbiased case. Third, the comparative static predictions are the reverse of the unbiased case but are consistent with positive projection for all $\Delta \geq \overline{\Delta}^{\rho}$. The value of projection above which the reverse comparative static holds for all implemented treatment variables is $\rho \geq 0.32$. The value for which it holds for the higher discount-factor treatments is $\rho \geq 0.17$.

The authors also note that for the highest discount factor, the average initial price is above the static monopoly price, contrary to the point prediction of the unbiased equilibrium, but again consistent with the predictions with information projection. When the discount factor is high, the seller's profit is considerably higher than the unbiased equilibrium prediction. Specifically, for $\delta = 0.9$, the seller's initial price is greater than the static monopoly price and sellers' revenue from bargaining is greater than the value of the full-commitment optimal revenue.

Basic calibrations, given the average price data reported by Rapoport et al., show that initial prices across the higher discount-factor treatments are consistent with the point predictions of the model given significant projection. The fitted degree of projection ρ is higher in the sessions without experience and smaller but still substantial in the sessions with experience, and are sufficiently high for the model to predict a reversal of the Bayesian comparative static. [25]

6 Discussion and Conclusion

This paper explores some consequences of information projection to sequential bargaining with one-sided private information. The model implies - both in a two-period setting with only initial projection and in an infinite-horizon setting with prospective but vanishing projection - that this common psychological phenomenon impacts bargaining behavior. While in the unbiased case, smooth bargaining results in the full loss of the seller's monopoly profit, under any positive projection it can result in full rent-extraction in stationary

[25] Rappoport et al. (1995) report average initial prices from sessions without experience (Iterations 1-3) and from sessions with experience (Iterations 7-9), (see p. 384). For the lowest discount factor used in Iterations 7-9, neither the unbiased nor the biased model may rationalize the data, as this initial price is too low. For the other discount factors, the model can calibrate the data with significant information projection both for (Iterations 1-3) and (Iterations 7-9). Specifically, in sessions without experience (Iterations 1-3), assuming a $\rho \in [0.43, 0.44]$ matches the average initial prices reported for *both* discount factors i.e., $\delta \in \{2/3, 0.9\}$. In sessions with experience (Iterations 7-9), assuming a $\rho \in [0.27, 0.32]$ matches the average initial prices reported for *both* discount factors i.e., $\delta \in \{2/3, 0.9\}$.

strategies. The comparative static predictions of the model match the existing experimental evidence. Given the portable way information projection is introduced, future research can generalize the results and investigate implications of the model to a variety of other bargaining contexts both theoretically and empirically.

A corollary of the results is that individual dynamic price negotiations without are beneficial for an uninformed monopolist when facing privately informed but biased buyers. Haggling allows a salesman to capitalize on the buyer's mistaken beliefs and boost his revenue above the Bayesian full-commitment solution. Since relative to posting a price, such negotiations may well be time-consuming and also associated potential agency costs, the model suggests that the benefits from haggling are more likely to outweigh the costs when the absolute value of the item is greater; and when private information makes buyers *less* transparent implying a greater wedge between full surplus and the monopoly payoff the seller would achieve given his true information.

Evidence from dealerships selling new cars shows that sellers predominantly opt to haggle and allow considerable room for price negotiations. This practice leads to significant variation in dealer discounts - controlling for car characteristics. Using CES data, Goldberg (1996) finds that readily observable socio-economic variables provide an insignificant explanation of the resulting price variation. Scott-Morton et al. (2011) find that an increase in a variable that describes a buyer's fear of being taken advantage of by the seller when bargaining does predict a significant decrease in dealer discounts.[26] This is true even after controlling for the extent to which a buyer was happy about the price negotiated and a variety of other characteristics, including search activities. While further research is needed to more carefully link the model's implications to evidence from the field, information projection does provide a channel whereby such perceptions might arise, and translate endogenously into a form of bargaining ability as a function of the private information possessed.

The model may have implications for other problems as well, such as dynamic contracting e.g., Hart and Tirole (1988). Here, it may affect the trade-off that a monopolist faces when offering the repeat sale of a service and deciding whether to keep consumer anonymity - avoiding Coasian dynamics - or to engage in record keeping i.e., personalized pricing. This is true because consumers who project information will undervalue their own privacy. One might be able to embed the results of this paper into a setting in which the biased buyer could engage in sequential search in a market with haggling sellers. A buyer who projects information might underestimate the return from such an activity by think-

[26] The average dealer-margin in this data set from 2002 is $1565.

ing that another haggling seller might also figure her out. The phenomenon of information projection could also be incorporated into a broader class of bargaining games - relaxing the assumption that the seller's valuation is common knowledge - and to mechanism design more generally.

7 Appendix

7.1 Appendix A

Here I define the notion of a perfect projection equilibrium for the setting introduced in Section 2.[27] Let $\Gamma = \{H, P, I_k, f_N, v_k\}$ be a sequential bargaining game with observable moves by the players: H is the set of histories, Z the set of terminal histories, $P : H \backslash Z \to \{S, B, N\}$ the assignment of histories to the seller, the buyer, and Nature. For each $k \in \{S, B\}$, I_k is the information partition on $\{h \in H : P(h) = k\}$ with generic element $\iota_k \in I_k$. The conditional distribution $f_N(\cdot \mid h)$ is the probability measure describing Nature's chance moves following history h such that $P(h) = N$. In the true games studied, Nature moves only initially assigning the player's information according to $f_N(\mid \emptyset)$. Let $v_k : Z \to \mathbb{R}$ be player k's payoff function over terminal nodes.

Consider an extension of Γ denoted by Γ_+. Suppose at the beginning of each round $t = 1, 2, ..$ Nature makes a binary move $\epsilon_t \in \{0, 1\}$. If $\epsilon_t = 1$, θ is revealed to the seller. If $\epsilon_t = 0$, it is not. The realization of ϵ_t is observed only by the seller. The realizations of ϵ_t and $\epsilon_{t'}$ for any $t \neq t'$ are independent. The seller initially believes that $\Pr(\epsilon_t = 1) = 0$ for all t. A ρ-biased buyer initially believes that $\Pr(\epsilon_t = 1) = \rho_t$. Let μ_k refer to player k's system of beliefs in Γ_+ describing both beliefs at each of k's information sets and also her belief about Nature's moves. To emphasize the dependence of the latter on ρ, μ^{ρ}_B will refer to the belief system of the buyer and μ^0_S to that of the seller.

Let's now partition the seller's information sets in Γ_+ into two disjoint subsets: I^0_S collects those sets where all observed realizations of ϵ_t are zero, I^+_S is the complement collection. This division is always well-defined. Note that at any information set in I^0_S, given μ^0_S, the seller believes that the probability of reaching any information set in I^+_S is zero. The seller's *conditional* beliefs at information sets in I^+_S are still well-defined, however. Note also that all information sets in I^+_S are singletons.[28]

Note that there exists a one-to-one mapping from I^0_S to the set of the seller's information sets in Γ containing equivalent observations of the players' sequence of actions.

[27] This Appendix might be for online publication only.

[28] Hence the seller's beliefs about Nature's future leakage moves once at an information set in I^+_S are inconsequential.

Similarly, there is a one-to-one mapping between the buyer's information sets in Γ_+ and those in Γ containing equivalent observations of the players' sequence of actions. With a slight abuse of notation, I thus also denote the collection of the buyer's information sets in Γ_+ by I_B. Let σ_S denote the seller's strategy, a mapping from histories to behavioral strategies at each of the seller's information sets. Analogously, let σ_B the buyer's strategy. Given the above partition we can also partition the seller's strategy in Γ_+ into $\sigma_S = \{\sigma_S^0, \sigma_S^+\}$ analogously.

Finally, given a strategy profile σ and belief-system μ_k for player k, let $V_k(\sigma, \mu_k \mid \iota_k)$ denote the expected value of the lottery induced over terminal nodes for player k conditional on being at information set ι_k.

Definition 1 *A strategy pair $\sigma = \{\sigma_B, \sigma_S^0\}$ is a perfect ρ projection equilibrium in Γ if there exists σ_S^+ such that for $\{(\sigma_B, \sigma_S), (\mu_B^\rho, \mu_S)\}$ in Γ_+, the following are true:*

1. *$V_B(\sigma_B, \sigma_S, \mu_B^\rho \mid \iota_B) \geq V_B(\sigma'_B, \sigma_S, \mu_B^\rho \mid \iota_B)$ for all σ'_B at any given $\iota_B \in I_B$,*
2. *$V_S(\sigma_B, \sigma_S, \mu_S \mid \iota_S) \geq V_S(\sigma_B, \sigma'_S, \mu_S^0 \mid \iota_S)$ for all σ'_S at any given $\iota_S \in I_S^0 \cup I_S^+$,*
3. *For all $i_B \in I_B$, $\mu_B^\rho(i_B)$ is derived from Bayes' rule (whenever possible) given (σ_B, σ_S) and the assumption that $\Pr(\epsilon_t = 1) = \rho_t$ for any t,*
4. *For all $i_S \in I_S^0 \cup I_S^+$, $\mu_S^0(i_S)$ is derived from Bayes' rule (whenever possible) given (σ_B, σ_S) and the assumption that $\Pr(\epsilon_t = 1) = 0$ for any t.*

7.2 Appendix B

Proof of Proposition 1. The seller's expected revenue given a TIOLI offer in $t = 1$ is $V_M = \alpha V_F + (1 - \alpha) \max\{qh, l\}$. If $\alpha = 0$, the result follows from e.g., Skreta (2006). To show for $\alpha > 0$, consider the four possible sequential bargaining protocols. If the buyer makes all offers, the seller's equilibrium payoff is zero. If they alternate with the seller making the first offer, the informed seller' payoff is bounded by $(1 - e^{-\Delta})V_F$, the uninformed's by $(1 - e^{-\Delta}) \max\{qh, l\}$, because any higher offer will be rejected by the respective buyer type.

Alternating Offer Protocol. Consider the protocol where the buyer makes the first offer. Three types of equilibria can arise: separating, semi-separating and pooling. Consider full separation. Type θ buyer at $t = 1$ names p_θ. Note $p_h = e^{-\Delta}h$ must hold because the seller must accept any price higher and, given revelation, reject any lower price. Furthermore, $p_l \in [e^{-\Delta}l, \min\{l, p_h\}]$, since any lower price will be rejected, and any higher price will violate individual rationality or separation. A tighter upper bound on p_l may hold, and will be considered in Proposition 2, but ignoring it here just strengthens the argument. For buyer separation to be incentive compatible:

$$(1 - e^{-\Delta})h \geq (1 - \alpha)(h - p_l) \tag{7.1}$$

must hold. Re-writing this:

$$\Delta > \Delta_{\min} = \ln h - \ln(\alpha h + (1 - \alpha)p_l)$$

Consider now the seller's revenue. Suppose $l \geq qh$. Simple algebra shows that given a binding Eq.(7.1) for $V_A^0(\Delta) \geq V_M$, it must be that:

$$\Delta \leq \Delta_{\max} = \ln(qh) - \ln(l - p_l(1 - q) + q\alpha(h - l))$$

Since $p_l \leq l$ it follows, however, that $\Delta_{\max} \leq \Delta_{\min}$. Suppose now that $qh > l$. Simple algebra shows that given a binding Eq.(7.1) for $V_A^0(\Delta) \geq V_M$, it must be that:

$$\Delta \leq \Delta_{\max} = \ln(qh) - \ln\left(l\alpha(1 - q) + qh - p_l(1 - q)\right)$$

Since $(qh - l)(1 - \alpha) \geq (1 - q\alpha)(p_l - l)$, it follows that $\Delta_{\max} \leq \Delta_{\min}$.[29]

Consider semi-separation with serious offers. The relevant case is where the high type mixes at $t = 1$ between revelation, p_h , and pooling p_l, doing the latter with probability y. Again $p_h = e^{-\Delta}h$ must hold. If the uninformed seller does not mix, the result follows from the above discussion. Suppose the uninformed seller accepts p_l with probability z. If $p_2 = l$, then $p_l = e^{-\Delta}l$ must hold. Now for the high type buyer to mix the following indifference must hold:

$$(1 - e^{-\Delta})h = (1 - \alpha)(z(h - e^{-\Delta}l) + (1 - z)e^{-\Delta}(h - l))$$

Maximal revenue is affected by z only through its impact on Δ. Specifically, the seller's expected revenue is:

$$\alpha e^{-\Delta}(qh + (1 - q)l) + (1 - \alpha)((q - qy)e^{-\Delta}h + (1 - q + qy)e^{-\Delta}l)$$

[29] Note that if in a fully-revealing equilibrium the seller did not accept the buyer's initial offer, then by revelation the upper-bound on the seller's revenue remained unchanged.

The minimal necessary separating friction $\underline{\Delta}(z)$ is given when $z = 0$. Straightforward algebra shows that $e^{-\underline{\Delta}(0)}V_F \leq V_M$, hence the same is true for all $z > 0$. If $p_2 = h$, it needs to be that $qh > l$. For the high type to mix here the indifference condition is:

$$(1 - e^{-\Delta})h = (1 - \alpha)z(h - p_l)$$

where $p_l = \frac{qy}{qy+(1-q)}e^{-\Delta}h$ must hold for the uninformed seller to mix. It is easy to see that $V_A^0(\Delta) \leq hqe^{-\Delta} + \alpha(1 - q)e^{-\Delta}l \leq V_M$.

Finally, in a pooling equilibrium, $V_A^0(\Delta) \leq \max\{l, \alpha(1 - q)l + e^{-\Delta}qh\} \leq V_M$. Above I used the fact that the informed seller has a dominant strategy. Note also if there is an equilibrium where the buyer makes a non-serious offer, the revenue result holds a fortiori .

Proof Proposition 2. Consider a fully separating equilibrium with serious offers. Note first that $\alpha^\rho = \alpha + (1 - \alpha)\rho$ replaces α in Eq.(7.1). Hence the separating friction here must satisfy:

$$\Delta \geq \ln h - \ln(\alpha^\rho h + (1 - \alpha^\rho)p_l).$$

This constraint implies a positive lower bound for all $\rho < 1$. Note that in this pure strategy equilibrium ρ imposes an upper bound on p_l of the following form:

$$p_l \leq \alpha^\rho e^{-\Delta}l + (1 - \alpha^\rho)l$$

because an informed seller type cannot reject any initial offer greater than $e^{-\Delta}l$ from the low type in a perfect equilibrium. As before, $p_l = e^{-\Delta}l$ is always feasible for any α^ρ. Note that this upper-bound is decreasing in ρ. Simple substitution shows, however, that, given binding incentive constraints, the lower bound on the friction in this equilibrium class, $\Delta^\rho_{\min}$, is decreasing in ρ with $\Delta^1_{\min} = 0$. The revenue conditions are the same before, except for the constraint on p_l, which implies that $\Delta^\rho_{\max}$ is decreasing in ρ. Nevertheless, $\Delta^\rho_{\max}$ remains greater than the positive friction described by the revenue constraint with $p_l = e^{-\Delta}l$. This already implies that there exists ρ_A such that for if $\rho > \rho_A$, then $\widetilde{\Delta}^\rho_{\min} < \widetilde{\Delta}^\rho_{\max}$. The corollary then follows form that fact that under $\Delta = 0$ or $\Delta = \infty$ no pooling or semi-separating equilibrium can generate a revenue higher than V_M for any $\rho < 1$.

Proof of Proposition 1 (Continued). Seller-Offer Protocol. For $V_{So}(\Delta) > V_M$, two facts must hold. First, the uninformed seller needs to sell to both types at different prices with positive probability. Second, there has to be some pooling between the uninformed and the informed seller conditional on $\theta = h$ since otherwise the high type buyer always learns the seller's type. Absent such pooling, the seller's revenue becomes separable in α, and hence the result follows immediately from Lemma 1. Let such a pooling price at $t = 1$ be $p_{1,h}$.

Consider first such equilibria without mixing. Let the price named by the informed seller conditional on $\theta = l$ in $t = 1$ be $p_{1,l}$. At $t = 2$, the uninformed seller's price, given price discrimination, must be $p_2 = l$. The informed seller has a dominant strategy. For the informed seller to pool on $p_{1,h}$ in $t = 1$, the inequality $p_{1,h} \geq \max\{e^{-\Delta}h, p_{1,l}\}$ must hold. For the high type buyer to accept $p_{1,h}$ the following IC constraint must hold:

$$h - p_{1,h} \geq e^{-\Delta}(1-\alpha)(h-l). \tag{7.2}$$

Given a binding Eq.(7.2), and setting $p_{1,l} = l$, we obtain an upper-bound on revenue in this equilibrium class.[30] Let this be $\widehat{V}^0_{So}(\Delta)$.Simple algebra shows that:

$$\text{if } l \geq qh, \text{ then } \widehat{V}^0_{So}(\Delta) - V_M = \left(e^{-\Delta} - 1\right)(1-\alpha)(l - qh) \leq 0,$$
$$\text{if } qh > l, \text{ then } \widehat{V}^0_{So}(\Delta) - V_M = e^{-\Delta}(1-\alpha)(l - qh) \leq 0.$$

Consider mixed equilibria. If the uninformed seller mixes between h and l in $t = 2$, the IC constraint on $p_{1,h}$, Eq.(7.2), can be relaxed. For this to be possible, $qh > l$ must hold. The high type buyer is indifferent between accepting or rejecting $p_{1,h}$ if:

$$h - p_{1,h} = (1-\alpha)ke^{-\Delta}(h-l),$$

where k is the probability that the uninformed seller names l in $t = 2$. For the uninformed seller to mix in $t = 2$, it has to be true that $\frac{q(1-j)}{q(1-j)+(1-q)} = l/h$ where j is the probability that the high type buyer accepts $p_{1,h}$. This is independent of α. If $k = 0$, then $p_{1,h} = h$. Here it is straightforward that the seller's revenue is below V_M. Since $p_{1,h}$ is maximal for $k = 0$, and all other prices and j are independent of k, the same holds for all k provided $p_{1,h} \geq e^{-\Delta}h$ so that the informed seller is still willing to pool. Hence again revenue is bounded from above by V_M for all α. Considering mixing in $t = 1$ alone will not relax the IC constraint on the maximal price at which the high type is willing to buy in $t = 1$. Hence it cannot boost the revenue above $\widehat{V}^0_{So}(\Delta)$.

[30] Note that $p_{1,h} \geq \alpha h + (1-\alpha)l \geq l$ and hence separation aways holds.

Proof of Proposition 3. Consider equilibria without mixing and with price discrimination and pooling in $t = 1$ conditional on $\theta = h$ as before. Such pooling again requires that

$$p_{1,h} \geq e^{-\Delta} h \tag{7.3}$$

For the high type buyer to accept $p_{1,h}$ the IC constraint is:

$$h - p_{1,h} \geq e^{-\Delta}(1 - \alpha^{\rho})(h - l) \tag{7.4}$$

Combining Eq.(7.3) and Eq.(7.4) one gets the following constraint

$$\Delta \geq \Delta^{\rho}_{\min} = \ln(h + (1 - \alpha^{\rho})(h - l)) - \ln h$$

where this $\Delta^{\rho}_{\min}$ is decreasing in ρ and becomes 0 as ρ goes to 1.[31]

Given a binding Eq.(7.4), setting $p_{1,l} = l$, we get an equilibrium revenue. Let's denote this by $\widehat{V}^{\rho}_{So}(\Delta)$. Suppose $qh \geq l$. Simple algebra shows that $\widehat{V}^{\rho}_{So}(\Delta) > V_M$ for all $\rho > (qh - l)/(qh - ql)$ as long as $\Delta \in (\Delta^{\rho}_{\min}, \infty)$. Suppose $l > qh$, here $\widehat{V}^{\rho}_{So}(\Delta) > V_M$ requires that $\Delta < \Delta^{\rho}_{\max} = \ln(l - qh + q\rho(h - l)) - \ln(l - qh)$. The condition that $\Delta^{\rho}_{\max} > \Delta^{\rho}_{\min}$ is equivalent to $\rho > (l - qh)/(l + qh\alpha/(1 - \alpha)) < 1$. It follows that there exists ρ_{S_0} such that if $\rho > \rho_{S_0}$, then there exists $\widehat{\Delta}^{\rho}_{\min} < \widehat{\Delta}^{\rho}_{\max}$ such that $V^{\rho}_{So}(\Delta) > V_M$ for all $\Delta \in (\widehat{\Delta}^{\rho}_{\min}, \widehat{\Delta}^{\rho}_{\max})$. Note $\widehat{V}^{\rho}_{So}(\Delta)$ is maximal at $\Delta^{\rho}_{\min}$. This is true because $\widetilde{V}^{\rho}_{So}(\Delta)/\partial\Delta < 0$ whenever $l \geq hq$ and provided that $\rho > \rho_{So}$ also when $qh > l$. Finally, for the corollary note that if $\Delta = \infty$, the uninformed seller's maximal revenue is bounded by $\max\{qh, l\}$. If $\Delta = 0$, pooling requires $p_{1,h} = h$ which violates Eq.(7.3) for any $\rho < 1$.

Proof of Proposition 5. Given the stationary strategies described, the skimming property holds on the real path. This implies that in the beginning of period t, on the real path the seller will face a uniform truncation of the type distribution, $[0, \theta_t]$ where θ_t is then the state-variable. In round t the seller offers $p_t = \gamma(\rho, \Delta)\theta_t$ and the buyer of type θ accepts a price of p iff $p \leq \frac{1}{\lambda(\rho,\Delta)}\theta$ where $\lambda(\rho, \Delta) \geq 1$. Given dynamic optimization by the real seller, who knows that Nature never leaks θ, under stationary strategies the following maximization needs to be true. Given any positive value of the state variable θ_t at any round t:

$$V_s(\theta_t) = \max_{p_t}\{(\theta_t - \lambda p_t)p_t + e^{-\Delta}V_s(\lambda p_t)\}$$

[31] Note again that $p_{1,h} \geq l + (h - l)(\alpha + \rho - \alpha\rho) \geq p_{1,l}$.

since $\theta_{t+1} = \lambda p_t$. Taking the first-order condition with respect to p_t, we get that:

$$\theta_t - 2\lambda p_t + e^{-\Delta}\lambda V_s'(\lambda p_t) = 0$$

where from the envelop theorem, given the buyer's strategy and optimality, it follows that:

$$V_s'(\theta_{t+1}) = \gamma\theta_{t+1}$$

hence:

$$\theta_t - 2\lambda p_t + e^{-\Delta}\lambda^2\gamma p_t = 0$$

Consider now the ρ-biased buyer's strategy on the real equilibrium path. Given the strategies on the imaginary path, in any round t - where the price does not equal her reservation value, except for a measure-zero set of types - the buyer will be indifferent between accepting the current price and rejecting the offer if her value is, $\theta_{t+1} = \lambda p_t$, and thus:

$$\lambda p_t - p_t = e^{-\Delta}(1-\rho)(\lambda p_t - \gamma\lambda p_t)$$

must hold, since she believes that with probability ρ she might be detected in the next round. We thus have two equations for the parameters λ and γ, from which we then obtain:

$$\lambda(\rho,\Delta) = \frac{1}{e^{-\Delta}(\rho-1)+1}\left(\rho(1-\sqrt{1-e^{-\Delta}}) + \sqrt{1-e^{-\Delta}}\right)$$
$$\gamma(\rho,\Delta) = \left(1-e^{-\Delta}(1-\rho)\right)\frac{1-e^{-\Delta}(1-\rho)-\sqrt{1-e^{-\Delta}}}{e^{-2\Delta}(1-\rho)^2+e^{-\Delta}(2\rho-1)}$$

Let's return to the imaginary equilibrium path ensuing positive leakage. By naming θ the imaginary informed seller and the uninformed seller always separate - except for an ex-ante measure zero set of types. Hence it is a best-response by the buyer to accept such a price. For the set of measure zero types whose valuation is part of the support of the real path price schedule, the fictional informed seller is believed to name first a specific price that is arbitrarily smaller than θ, but greater than $e^{-\Delta}\theta$ and is not on the real path, and θ forever thereafter. Given that the seller's real price schedule has finitely many points, for all $\Delta > 0$, and that here $\lambda(\rho,\Delta) > 1$ if $\rho < 1$, such a price exists and is thus incentive compatible for the fictional informed seller to offer. This ensures separation. Thus in each round, the buyer knows the type of the seller in that period, hence whenever she hears a price in equilibrium that comes from an informed seller it is a best-response for her to accept.

Off-equilibrium path beliefs of the buyer on, following any deviation before separation, assign full weight to these originating from the uninformed seller. The buyer here will follow her cut-off strategy given by $\lambda(\rho, \Delta)$. This is optimal since the continuation value from rejecting remains unchanged for the buyer, since she believes that leakage still happens with probability ρ next round. Once a seller has revealed himself to be informed on the imaginary path, the buyer maintains this belief about the seller's type given any deviation. Finally, the real seller would never want to deviate to a price of the imaginary seller given that the real seller's belief never has atoms and thus given the seller's equilibrium beliefs the probability of positive profit given deviation is zero.

It is easy to check that given the above conditions, and since

$$\frac{p_{t+1}}{p_t} = \lambda(\rho, \Delta) * \gamma(\rho, \Delta) = \frac{1}{e^{-\Delta}} \left(1 - \sqrt{1 - e^{-\Delta}}\right)$$

is independent of ρ, the second-order condition on the value function is satisfied. Furthermore, this gradient is increasing in Δ for all $\rho \geq 0$. To show that $\lambda(\rho, \Delta)$ is decreasing in ρ, note that:

$$\lambda_\rho(\rho, \Delta) = -\frac{1}{\left(e^{-\Delta}(\rho - 1) + 1\right)^2} \left(\sqrt{1 - e^{-\Delta}} - (1 - e^{-\Delta})\right) < 0$$

with $\lim_{\rho \to 1} \lambda(\rho, \Delta) = 1$.

Consider now the impact of a change in Δ on $\gamma(\rho, \Delta)$. To make calculations more concise, I alternate in notation and whenever more convenient, I denote $e^{-\Delta}$ equivalently by δ. Note first that, after some algebra, we obtain that:

$$\gamma_\delta(\rho, \delta) = \overbrace{\frac{1}{\delta^2 \left(\delta(1-\rho)^2 + 2\rho - 1\right)^2}}^{I} * \overbrace{\frac{\delta(1-\rho) + 2\rho - 1}{2\sqrt{1-\delta}}}^{II} \overbrace{* \delta^2(1-\rho)^2 + 2(1 - \delta + \delta\rho)(1 - \sqrt{1-\delta}) - \delta}^{III} \tag{7.5}$$

The first term in this expression, Term I, is always positive. Furthermore, some algebra shows that term III can be re-written as[32]:

$$(\sqrt{1-\delta} - 1)^2 (\sqrt{1-\delta}(\rho - 1) + \rho)^2 \geq 0.$$

which is thus always positive - except at the two roots where it is zero. Hence the sign

[32] Note that by substituting $x = \sqrt{1-\delta}$ and $y = \rho - 1$, we can re-write term III as $(-1+x)^2(1+y+yx)^2 = (-1 + \sqrt{1-\delta})^2(\sqrt{1-\delta}(\rho - 1) + \rho)^2 \geq 0$.

of this comparative static is determined by the sign of term II. This term is positive iff $\delta \geq \max\{\frac{1-2\rho}{1-\rho}, 0\}$. Hence, it follows that if $\delta \geq \overline{\delta}^{\rho}$ with $\overline{\delta}^{\rho} = \max\{\frac{1-2\rho}{1-\rho}, 0\}$, then $\gamma_{\delta}(\rho, \delta) > 0$, and if $\delta < \overline{\delta}^{\rho}$, then $\gamma_{\delta}(\rho, \delta) \leq 0$. It is easy to see then that $\gamma_{\delta}(0, \delta) < 0$ and that $\lim_{\delta \to 1} \gamma_{\delta}(\rho, \delta) > 0$ iff $\rho > 0$.

Finally, note that $\lim_{\Delta \to 0} \gamma(\rho, \Delta) = 1$ for any $\rho > 0$. Given that $\lim_{\Delta \to 0} \frac{p_{t+1}}{p_t} \to 1$, it follows by the continuity of $V_S^{\rho}(\Delta)$ in Δ that for any $\rho > 0$ and any $\tau > 0$, there exists $\widehat{\Delta}^{\rho}(\tau) > 0$ such that if $\Delta \leq \widehat{\Delta}^{\rho}(\tau)$, then $\left|V_S^{\rho}(\Delta) - V_F\right| \leq \tau$. To show this formally, note that the seller's ex-ante expected revenue can be approximated as:

$$V_S^{\rho}(\Delta) = \sum_{t=1}^{\infty} e^{-\Delta(t-1)}(\lambda(\rho, \Delta)^{t-1}\gamma(\rho, \Delta)^t)(\lambda(\rho, \Delta)^{t-1}\gamma(\rho, \Delta)^{t-1} - \lambda(\rho, \Delta)^t\gamma(\rho, \Delta)^t) =$$

$$= \gamma(\rho, \Delta)(1 - \frac{1}{e^{-\Delta}}(1 - \sqrt{1 - e^{-\Delta}}) \sum_{t=1}^{\infty}(e^{-\Delta}\lambda(\rho, \Delta)^2\gamma(\rho, \Delta)^2)^{t-1}$$

Note that whether the sum converges to V_F or 0 depends solely on $\lim_{\Delta \to 0} \gamma(\rho, \Delta)$. As long as $\lim_{\Delta \to 0} \gamma(\rho, \Delta) = 1$ the revenue converges to V_F. Hence for any $\rho > 0$, $\lim_{\Delta \to 0} V_S^{\rho}(\Delta) = V_F$. Furthermore, simple algebra shows that in the case where $\rho = 0$, since $\gamma(0, \Delta)$ is increasing in Δ, $V_S^0(\Delta)$ is increasing in Δ as well[33] .

Proof of Proposition 6. First note that $\gamma(\rho, \Delta)$ can be re-written as:

$$\gamma(\rho, \delta) = \overbrace{\frac{(1 - (1 - \rho)\delta)^2}{\delta^2(1 - \rho)^2 - \delta(1 - \rho) + \rho\delta}}^{IV} - \overbrace{\frac{(\sqrt{1 - \delta})\,(\delta\rho - \delta + 1)}{\delta^2(1 - \rho)^2 - \delta(1 - \rho) + \rho\delta}}^{V} \tag{7.6}$$

When considering the limit as $\delta \to 1$, we can ignore Term V in Eq.(7.6) as it always converges to zero independent of ρ. By substituting $\rho = \beta(-\ln\delta)^{\kappa}$, given that $-\Delta = \ln\delta$, and setting $\beta = 1$, and re-arranging terms in Term IV we get that in the limit Eq.(7.6) becomes:

$$\lim_{\delta \to 1} \gamma(\rho, \Delta) = \lim_{\delta \to 1}(1 - \overbrace{\frac{\delta - 1}{\delta \ln^{\kappa}\frac{1}{\delta} + \delta^2\left(\ln^{\kappa}\frac{1}{\delta} - 1\right)^2 + \delta\left(\ln^{\kappa}\frac{1}{\delta} - 1\right)}}^{VI})$$

[33] To see that in the case $\rho = 0$ the results extend to different positive interests rate, let's normalize the interest rate faced by the seller to be 1 and let's denote the interest rate for the buyer to be $b > 0$. The initial price of the seller n the stationary equilibrium is given by $\gamma^b(\Delta, 0) = -\left(e^{b\Delta} - 1\right) \frac{e^{\Delta} + e^{b\Delta}\sqrt{e^{\Delta}\left(e^{\Delta} - 1\right)} - e^{b\Delta}e^{\Delta}}{e^{\Delta} + e^{2(b\Delta)} - 2e^{b\Delta}e^{\Delta}}$ and it is easy to see that $\lim_{\Delta \to 0} \gamma^b(\Delta, 0) = 0$ for all $b > 0$.

Applying l'Hôpital's rule on the Term VI above, after dividing both the numerator and the denominator by δ, we get that:

$$\lim_{\delta\to 1} \frac{1}{\delta\left(\delta - 2\delta\ln^{\kappa}\frac{1}{\delta} + \delta\ln^{2\kappa}\frac{1}{\delta} + 2\kappa\left(\ln^{\kappa-1}\frac{1}{\delta}\right)(\delta-1) - 2\kappa\delta\ln^{2\kappa-1}\frac{1}{\delta}\right)}$$

The terms inside the bracket of the denominator in this expression, except for the first and the last ones, converge to zero. The last term goes to 0 if $\kappa > 0.5$, and becomes unbounded if $\kappa < 0.5$. In the former case, we then have that $\lim_{\Delta\to 0}\gamma(\rho(\Delta),\Delta) = 0$. In the latter case, we then have that $\lim_{\Delta\to 0}\gamma(\rho(\Delta),\Delta) = 1$. In these cases the results hold for any $\beta > 0$, as it does not impact the limit. In the case where $\kappa = 0.5$, it is easy to show that $\lim_{\Delta\to 0} V_S^{\rho}(\Delta)$ converges to $V_F\beta(1+\beta)^{-1}$.

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