DISCUSSION PAPER SERIES

No. 10292

INTERGENERATIONAL INEQUALITY AVERSION, GROWTH AND THE ROLE OF DAMAGES: OCCAM'S RULE FOR THE GLOBAL CARBON TAX

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INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

ISSN 0265-8003

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Discussion Paper No. 10292
December 2014
Submitted 26 November 2014

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INTERGENERATIONAL INEQUALITY AVERSION, GROWTH AND THE ROLE OF DAMAGES: OCCAM'S RULE FOR THE GLOBAL CARBON TAX[†]

Abstract

We use the Euler equation to put forward a back-on-the-envelope rule for the global carbon tax based on a two-box carbon cycle with temperature lag, and a constant elasticity of marginal damages with respect to GDP. This tax falls with time impatience and intergenerational inequality aversion and rises with population growth and prudence. It also falls with growth in living standards if inequality aversion is large enough or marginal damages do not react much to GDP. It rises in proportion with GDP if marginal climate damages are proportional to output and has a flat time profile if they are additive. The rule also allows for mean reversion in climate damages. The rule closely approximates the true optimum for our IAM of Ramsey growth, scarce fossil fuel, energy transitions and stranded assets despite it using the more complicated DICE carbon cycle and temperature modules. The simple rule gets close to the social optimum even if damages are much more convex than in DICE.

JEL Classification: H21, Q51 and Q54
Keywords: climate damage specification, intergenerational inequality aversion, optimal energy transitions, Ramsey growth, SCC, simple rule and stranded assets

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[†] We are very grateful to Cees Withagen of VU University Amsterdam for his involvement in the early stages of this project and for his detailed comments on earlier versions. We have benefited from his guidance in those stages of the project and from an earlier paper on additive versus multiplicative damages (Rezai et al., 2014).

1. Introduction

Integrated assessment models of climate change (IAMs) aim to integrate economics and climate science and to assess the impact the economy has on the climate and vice versa. Such IAMs are a crucial step in the design of optimal policies to fight the negative and potentially large effects of climate change on economic well-being. We put forward a micro-founded IAM of Ramsey growth and climate change, which in contrast to many others allows for scarce fossil fuel whose extraction costs rise as reserves are depleted, stranded fossil fuel reserves, optimal transitions between fossil fuel and renewable energy. [4] Furthermore, our IAM allows for substitution possibilities between energy and a capital-labor composite whilst most IAMs set this elasticity of substitution to zero.[5] Fossil fuel demand in most IAMs does not depend on expectations about the price of future renewable energy backstops and thus the energy transition dates simply occur when the price of fossil fuel inclusive of the carbon tax reaches the price of renewable energy. In contrast, our IAM determines the levels and time profiles of the social cost of carbon (SCC) and the market prices of fossil and renewable energy and the effects of expectations. Since global warming stems from the negative externality resulting with the emission of CO2 associated with the use of fossil fuels, it can be corrected by pricing carbon emissions at the SCC either via a carbon tax or a market for carbon emission permits. Without distortions in raising public funds and other second-best issues, the SCC corresponds to the present value of all future global warming damages from burning an extra unit of fossil fuel.[6]

Our first aim is to propose a simple but robust policy rule which approximates the optimal carbon tax. Golosov et al. (2014) offer a tractable Ramsey growth model which generates an optimal carbon tax that is proportional to GDP. Their result depends on bold assumptions: logarithmic utility, Cobb-Douglas production, full

[4] Our IAM is a calibrated and much richer version of the analytical growth and climate model put forward in van der Ploeg and Withagen (2014) and takes into account recent empirical findings by Hassler et al. (2011) on the substitutability of energy in production.

[5] For example, Nordhaus (2014) assumes energy demand is exogenously decreasing.

[6] Determining the SCC is less straightforward with exhaustible fossil fuel, increasing efficiency of carbon-free alternatives, gradual and abrupt transitions from fossil fuel to renewables, fossil extraction cost, and endogenous growth and structural change. One then also needs to consider the interaction between carbon pricing and market prices of fossil and renewable energy. These prices depend on expectations about future prices of fossil energy and the back-stop technology and on learning by doing (Rezai and van der Ploeg, 2014b).

depreciation of capital each period, zero fossil fuel extraction costs, multiplicative production damages captured by a negative exponential function and no temperature lag.[7] Golosov et al. (2014, pp. 76-77) offer an approximate rule for when intergenerational inequality (or what they call risk) aversion and trend growth. Van der Bijgaart et al. (2013) use a similar formula to derive a density distribution of the SCC based on a range of parameter distributions. We generalize the rule to allow for a temperature lag, population growth, climate damages that are not proportional to gross output or GDP, mean reversion in climate damages, and an annual instead of a decadal calibration. Our simple rule is an approximation (rather than an exact characterization) of the first-best optimal carbon tax and performs remarkably well in our general IAM of climate change and Ramsey growth with more realistic and complex carbon and temperature dynamics, especially if damages are not multiplicative exponential, intergenerational inequality aversion differs from unity and there is significant and sustained growth in living standards.

Our second aim is to investigate the implications of different types of modeling damage and production technology for the level and time profile of the optimal SCC and carbon tax. We are particularly interested in the different kinds of modeling of the potential damage influence policy and how well our simple rule can approximate optimal policy in our fully specified IAM. It matters *how* climate damages are specified and there are many ways of specifying them (e.g., Stern, 2013, pp. 846-850). Customary is to assume that marginal damages are proportional to output with the proportion increasing in temperature. This is equivalent to a unit elasticity of substitution between output and marginal damage; Tol (2002), Nordhaus (2008), Stern (2007) and Golosov et al. (2014) are prominent examples of this approach.[8] However, the rate of technical progress rather than the level of output may suffer from global warming. This should capture that the assumption of *exogenous* technical

[7] This formula is used already by others too (e.g., Hassler and Krusell, 2012; Gerlagh and Liski, 2012). Copeland and Taylor (1994) prose a similar framework. The Golosov et al. (2014) framework has already been used to study learning about the climate impact (Gerlagh and Liski, 2014), climate tipping (Engström and Gars, 2014), hyperbolic discounting (Iverson, 2013; Gerlagh and Liski; 2013), and robust stochastic control (Li et al., 2014).
[8] Tol (2002) is noteworthy in explicitly considering effects on ecosystems, vector borne diseases and heat and cold stress rather than subsuming them under production losses.

progress is simply not realistic given the scale of disruption to output that might result from global warming, especially in developing countries (Dell et al., 2012).[9]

Our results demonstrate that, if damages are additive and the economy is along a development path or experiences trend growth, the SCC and optimal carbon tax are smaller than with multiplicative damages as damages do not increase with a growing economy. Furthermore, the time profile of the carbon tax is flat. If, however, climate change has the potential to also affect the growth of productivity, or there is a positive risk of shocks to consumption, the optimal carbon tax is higher, less fossil fuel is used and a higher stock of reserves is abandoned in situ.

Our third aim is to see how much fossil fuel to leave in the crust of the earth and at what date to switch to a carbon-free economy. IPCC (2014) states that cumulative emissions have to be limited to 790 GtC (with an uncertainty range of 700-860 GtC) if global warming is to remain below 2°C. By 2011 520 GtC have been emitted, giving a carbon budget of only 270 GtC. Proven fossil fuel resource exceed this budget by far and we thus try to quantify the optimal amount of stranded assets and policy-induced obsolescence. This enters the price of fossil fuel through its forward-looking scarcity rent (the present discounted value of all future increases in extraction costs resulting from extracting an extra unit of fossil fuel). The consideration of the scarcity rent complicates calculation of transition times, since expectations about future developments such as technological progress in using renewable energy matter. We study the SCC and market prices of all energy sources but also the optimal transition times for abandoning fossil fuel and the amount of untapped fossil fuel.

Section 2 uses the Euler equation to predict the long-run rate of interest that has to be used to discount climate damages and uses this to obtain a simple expression for the SCC that clearly shows how economic growth, intergenerational inequality aversion and temperature lags depress the SCC. Here we also show that the SCC rise in line with GDP if climate damages are multiplicative, but stay flat if damages are additive. The effects of other specifications of climate damages and of uncertainty on interest rate and its term structure and its consequences for the SCC are also reviewed here.

[9] We review different roles of damage in economic models of climate change in section 2 where we will also discuss damages resulting from destruction of the capital stock, infrastructure or land and climate damages that enter directly in the social welfare function or, more implicitly, via a ceiling as is often done in the theoretical literature on climate policy.

Our simple rule reproduces the social optimum exactly under the heroic assumptions of Golosov et al. (2014). Section 3 sets out our general equilibrium IAM of climate change and Ramsey economic growth with different kind of global warming damages, exogenous population growth and potentially endogenous labor productivity growth. Production combines energy with a capital-labor composite. Energy and the composite are imperfect substitutes in production. The two sources of energy, however, carbon-free energy and exhaustible fossil fuel, are perfect substitutes. The cost of carbon-free energy is exogenous and benefits from exogenous technical progress. The extraction cost of fossil fuel increases as fewer reserves are left in the crust of the earth. Our carbon cycle follows Nordhaus (2014) and features positive feedback loops and lags in temperature. Section 4 presents the simulations based on our calibrated IAM paying particular attention to the effect of intergenerational inequality aversion on the level and time profile of the optimal carbon tax as well as to how this tax affects the moments in time that the economy switches from fossil fuel to the carbon-free renewable and the amount of fossil fuel to abandon. We present details for the price dynamics of fossil and renewable energy. Here we also discuss how well our simple rule performs in our full-scale IAM. Sections 5 and 6 list sensitivity studies of how the optimal carbon tax and transition times depend on the different damages specifications. Again, we also present the decentralized market outcomes with the simple rule implemented. In general, the rule is able to approximate the social optimum closely. Section 7 concludes.

2. Simple rules for the social cost of carbon

Let a fraction φ_L of carbon emissions stay up permanently in the atmosphere and of the remainder the fraction ($1-\varphi_0$) be absorbed within the same period and the fraction φ_0 decay at the rate φ, so that of an impulse of 1tC a fraction $z_t \equiv \varphi_L + \varphi_0(1-\varphi_L)(1-\varphi)^{t-1}$ is still in the atmosphere after t periods. We employ an annual time grid and suppose that after 3 decades half of carbon has left the atmosphere ($z_{30} = 0.5$), a fifth of carbon stays up in the atmosphere forever and the remainder has a half-time of 300 years ($(1-\varphi_1)^{300-1} = 0.5$), hence we get $\varphi_L = 0.2$,

$\varphi = 0.00231$ and $\varphi_0 = 0.401$.[10] With a distributed lag between temperature and atmospheric carbon of φ_T = 70 years, $\xi_t = \frac{1}{\varphi_T}\sum_{s=1}^{t}\left(1-\frac{1}{\varphi_T}\right)^{t-s} z_s$ is the remainder relevant for the impact of temperature on marginal climate damages at time *t*, so

$$(1) \qquad \xi_t = \varphi_L\left[1-\left(1-\frac{1}{\varphi_T}\right)^t\right]+\left(\frac{\varphi_0(1-\varphi_L)}{1-\varphi_T\varphi}\right)\left[(1-\varphi)^t-\left(1-\frac{1}{\varphi_T}\right)^t\right].$$

2.1. The social cost of carbon, growth and intergenerational inequality aversion

The SCC is the present value of all future marginal damages from emitting one ton of carbon: $SSC_0 \equiv \sum_{t=0}^{\infty}(1+r)^{-t}\omega_t$ with $\omega_t = \chi GDP_t^{\varepsilon} GDP_0^{1-\varepsilon}\xi_t$ the marginal damage in period *t* and *r* the average discount rate in period *t*. Golosov et al. (2014) have $\chi = 0.02379$ giving marginal climate damages of 2.379% of GDP/TtC. Since initial global GDP is \$70 trillion, initial marginal damages are \$1.67/tC.[11] ε = 1 gives *multiplicative* marginal climate damages as in Golosov et al. (2014), which corresponds closely to the specification used in DICE for the relevant range of stocks of atmospheric carbon and a climate sensitivity of 3 (Nordhaus, 2007). ε = 1 gives *additive* marginal climate damages which do not grow in line with GDP.[12] In general, ε is the elasticity of marginal damages with respect to GDP.

Let the trend exogenous growth rate of consumption be *g*, the rate of population growth be *n*, the rate of time impatience be ρ and the coefficient of relative intergenerational inequality aversion (IIA) be Φ. The Euler equation then gives the long-run interest rate as $r = \rho + \Phi(g-n)$ and we suppose $r > g$ holds (see appendix B). Growth in living standards leads to wealthier future generations that require a

[10] This way of modelling the climate system is based on Golosov et al. (2014) who employ an annual instead of decadal time grid and get $\varphi = 0.0228$ and $\varphi_0 = 0.393$.

[11] Golosov et al. (2014) derive this damage parameter as the ex-ante value to a high and low damage uncertainty. This value is significantly lower than their value χ= 0.053 which reproduces the damage function of Nordhaus (2008) best.

[12] For example, sea level rise may destroy part of production capacity, but at the cost of building dykes the loss can be mitigated. However, there need be no reason why higher gross production would require higher dykes and, hence, higher losses in net production in absolute terms. Also, in Tol's estimates of the damage cost of climate change some of the costs are expressed in percentages of gross product, but in others, such as for ecosystems, sea level rise, and water resource, the estimates are expressed in (billion) dollars (Tol, 2002a, 2002b).

higher interest rate, especially if IIA is large, since current generations are less prepared to sacrifice current consumption. This gives a simple rule for the SCC.

Proposition 1: A first-order approximation to the SCC along a trend growth path is:

$$(2)\qquad SCC_t \cong \left(\frac{\varphi_L}{\rho - \varepsilon n + (\Phi - \varepsilon)(g-n)} + \frac{\varphi_0(1-\varphi_L)}{\rho - \varepsilon n + (\Phi - \varepsilon)(g-n) + \varphi} \right) \times \left(1 + \varphi_T \left[\rho - \varepsilon n + (\Phi - \varepsilon)(g-n)\right]\right)^{-1} \chi GDP_t^{\varepsilon} GDP_0^{1-\varepsilon},$$

where we assume all denominators in (2) are strictly positive.

Proof: see appendix B.

A special case of (2) prevails with marginal damages are proportional to GDP ($\varepsilon = 1$):

$$(2')\qquad SCC_t \cong \left(\frac{\varphi_L}{\rho - n + (\Phi - 1)(g-n)} + \frac{\varphi_0(1-\varphi_L)}{\rho - n + (\Phi - 1)(g-n) + \varphi} \right) \times \left(1 + \varphi_T \left[\rho - n + (\Phi - 1)(g-n)\right]\right)^{-1} \chi GDP_t.$$

If one also abstracts away from a temperature lag ($\varphi_T = 0$) and population growth (n = 0), and also restricts intergenerational inequality aversion to 1, we get

$$(2'')\quad SCC_t \cong \left(\frac{\varphi_L}{\rho} + \frac{\varphi_0(1-\varphi_L)}{\rho + \varphi} \right) \chi GDP_t \text{ or } SCC_t \cong \beta \left(\frac{\varphi_L}{1-\beta} + \frac{\varphi_0(1-\varphi_L)}{1-(1-\varphi)\beta} \right) \chi GDP_t,$$

where $\beta = 1/(1+\rho)$ is the time impatience factor. Golosov et al. (2014) show that (2″) is indeed the optimal general equilibrium outcome under the additional assumptions that the production function is Cobb-Douglas, there is full depreciation of capital each decade, and energy production does not require any capital inputs. Golosov et al. (2014, p.77) also give an approximate rule which corresponds to general IIA but has no temperature lag, population growth or non-proportional climate damages.[13]

With multiplicative climate damages the SCC (2′) rises in line with GDP. The SCC is high for low time impatience and slow decay of atmospheric carbon. Expression (2″) over-estimates the true SCC since it ignores the temperature lag and the IIA typically

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[13] Their approximate rule is $SCC_t = \left(\frac{\varphi_L}{1-\beta(1+g)^{1-\Phi}} + \frac{\varphi_0(1-\varphi_L)}{1-(1-\varphi)\beta(1+g)^{1-\Phi}} \right) \chi GDP_t$. This can be approximated by $SCC_t \cong \left(\frac{\varphi_L}{\rho + (\Phi-1)g} + \frac{\varphi_0(1-\varphi_L)}{\rho + (\Phi-1)g + \varphi} \right) \chi GDP_t$, a special case of (2′).

exceeds one. If $\beta = 1$, the SCC is infinite, so (2″) cannot speak to what happens if it is unethical to discount welfare of future generations (cf. Stern, 2007; Gollier, 2013).

Equation (2) gives the following additional insights. First, given growth in living standards (G/N), higher population growth (higher N) boosts the SCC only in as far as climate damages increase with GDP ($\varepsilon > 0$). Also, population growth boosts trend growth and thus leads to a higher growth rate of the SCC. Second, with high degrees of IIA or climate damages that do not depend much on GDP ($\Phi > \varepsilon$), growth in living standards (higher G/N) boosts the interest rate and thus depresses the SCC. The lower temperature correction reinforces this effect. As future generations are better off, current generations are less prepared to make sacrifices to combat global warming. However, with low enough IIA, growth in living standards boosts the SCC. Third, higher IIA (higher Φ) means that current generations are less prepared to temper future climate damages and thus the SCC is lower. Fourth, a longer lag between temperature and atmospheric carbon (higher φ_T) depresses the SCC relative to GDP. Finally, if marginal damages react less than proportionally with GDP ($\varepsilon < 1$), the initial SCC will be lower than that with multiplicative climate damages, especially if the rate of economic growth is high. The effective discount rate no longer has to correct for growing climate damages. The growth rate of the SCC is ε times the growth of GDP. Additive marginal damages imply a flat time path for the SCC.

2.2. Mean reversion in climate damages

Pindyck (2014, p.867) claims that temperature does not affect the level but the growth rate of GDP[14] and recent empirical evidence supports this hypothesis, especially for less developed economies (Dell et al., 2012; Bansal and Ochoa, 2011, 2012). Here we introduce mean reversion in climate damages to productivity. To keep it simple, we set $\varepsilon = 1$ and $n = 0$. Gross output is $Y_t = \tilde{D}_t \bar{A}_t Z_t = A_t Z_t$ with $\bar{A}_t = (1+g_A)^t \bar{A}_0$ and $\tilde{D}_t = \tilde{D}_{t-1}{}^{\varphi_\chi} e^{-\chi(E_t - E_0)}$ with g_A trend growth of total factor productivity and φ_χ the degree of mean reversion in climate damages. This formulation implies $A_t = (1+g_A)^{\varphi_\chi} \bar{A}_t^{1-\varphi_\chi} A_{t-1}{}^{\varphi_\chi} e^{-\chi(E_t - E_0)}$. If $\varphi_\chi = 0$, $\omega_t = \chi GDP_t \xi_t$ and a carbon impulse

[14] Adaptation to climate change may require resources that could else be used for R&D.

causes temporary damage. If $\varphi_\chi = 1$, $\tilde{D}_t = \tilde{D}_{t-1} e^{-\chi(E_t - E_0)}$, $A_t = (1+g_A) A_{t-1} e^{-\chi(E_t - E_0)}$ and a carbon impulse causes permanent damage.

Proposition 2: With mean reversion in climate damages, $\varepsilon = 1$ and $n = 0$, we get a first-order approximation to the social cost of carbon along a trend growth path:

$$
\begin{aligned}
&\frac{SSC_t}{\chi GDP_t} = \frac{\varphi_L}{1+\varphi_\chi\left[\rho - 1 + (\Phi-1)g\right]} \frac{1}{\rho + (\Phi-1)g} \\
(3) \qquad &+ \frac{\varphi_0(1-\varphi_L)}{1-\varphi_T\varphi} \frac{1}{1+\varphi_\chi\left[\rho - 1 + (\Phi-1)g + \varphi\right]} \frac{1}{\rho + (\Phi-1)g + \varphi} \\
&- \left\{\varphi_L + \frac{\varphi_0(1-\varphi_L)}{1-\varphi_T\varphi}\right\} \frac{1}{1-\varphi_\chi\left[1-\rho-(\Phi-1)g - \dfrac{1}{\varphi_T}\right]} \frac{1}{\rho + (\Phi-1)g + \dfrac{1}{\varphi_T}}.
\end{aligned}
$$

Proof: See appendix B.

Mean reversion in climate damages give rise to extra terms in the first two lines of (3), which tend to boost the SCC. The negative effects of the temperature lag on the SCC are captured by the third line of (3).

2.3. More general climate damages

In theoretical models climate damages and consumption often enter welfare in separable fashion (e.g., Weitzman, 2009; van der Ploeg and Withagen, 2014). It is important to consider the implications of non-separable welfare functions such as

$$
(4) \qquad U(C_t, Q_t) = \left[\frac{1}{1+\psi} C_t^{\frac{\zeta-1}{\zeta}} + \frac{\psi}{1+\psi} Q_t^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}(1-\Phi)} / (1-\Phi)
$$

with ζ the elasticity of substitution between consumption of final goods C_t and environmental quality Q_t (a negative function of the stock of atmospheric carbon E_t), and Φ the coefficient of IIA as before (e.g., Guesnerie, 2004; Traeger, 2007; Hoel and Sterner, 2007; Sterner and Persson, 2008).[15] This allows marginal utility of consumption to increase or decrease with environmental quality. The parameter ζ is the percentage rate at which the demand for C_t declines if the relative price of

[15] Climate damages have been carefully split up into its multiplicative production and additive utility components (Barrage, 2013). In the model of Golosov et al. (2014) multiplicative production damages are equivalent to additive linear damages in utility.

environmental quality is increased by 1%. If $\zeta \to 1$, utility tends to $C_t^{(1+\psi)^{-1}} Q_t^{\psi(1+\psi)^{-1}}$.[16] Gollier (2010; 2013, Chapter 10, pp. 164-165) gives two results that follow from (4). First, the two goods are substitutes if $\zeta\Phi > 1$ in which case a higher rate of economic growth and anticipated deterioration of environmental quality boost the ecological discount rate and thus depress the SCC. Second, there will be a term structure of interest rates as the Euler equation will have an additional certainty-equivalent term on the right-hand side that decreases in time t if $(\zeta\Phi - 1)(1 - \zeta) > 0$.[17] Climate damages many generations ahead are then not discounted less heavily.

Various IAMs specify damages as shock to the depreciation rate of physical capital (e.g., Fisher and Hanemann, 1993; Hsiang and Jina, 2013; Dietz and Stern, 2014; Fisher and Le, 2014). These damages to the capital stock are one-off and the capital stock typically recovers after some time. The resulting contribution to the SCC would thus not be very substantial (and not rise with GDP). As most IAMs currently used employ a specification where climate change negatively affects net economy-wide production by reducing gross production by a percentage that depends on global mean temperature (see Kopp et al. (2011) for a survey), our focus is on production damages. We do not dwell on climate damages in social welfare or capital destruction shocks any further.

2.4. Uncertainty

Extending the analysis of the SCC based on Euler equation to allow for uncertainty, prudence and risk of climate catastrophe gives insights similar to the ones considered above (Weitzman, 1998, 2001, 2010; Gollier, 2013, Chapter 3 and Part II; Traeger, 2013). First, prudence or downward risk aversion depresses the rate that has to be used to discount production damages by approximately $\frac{1}{2}\Phi(\Phi+1)\sigma_C^2$, where Φ also indicates relative risk aversion, Φ+1 relative prudence, and σ_C^2 the variance of consumption growth.[18] This term is small from a time-series perspective (0.4% per year with $\Phi = 2$), but larger (3.8% per year) from a cross-section perspective as there

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[16] In fact, the model of Golosov et al. (2014) is after taking logs equivalent to a model with separable welfare and a damage term in welfare that is linear in the carbon stock.
[17] Weitzman (2009) uses (4) with $\zeta = 0.5$ and $\Phi = 2$ in which case the term structure is flat.
[18] This term is exact if utility is exponential and the growth rate is normally distributed.

is much more variation in volatility across countries than over time. Second, a small probability of a catastrophic reduction in the growth rate increases the precautionary effect and depresses the monetary discount rate too and thus leads to a bigger social cost of carbon. Third, if growth expectations are diminishing, the stochastic process for the growth rate of the economy displays mean reversion or the future rate of return on capital is governed by a gamma distribution, the precautionary effect is boosted and increases over time due to the persistency of shocks. The rate to discount climate damages thus falls with longer horizons (a decreasing term structure) so that damages that occur many decades ahead are discounted less. All three effects push up the SCC.

Gerlagh and Liski (2014) argue that world GDP has so far grown fast without a big impact of climate damages, but that the growth spell can end at some future time. They thus put forward a hidden-state process and a learning mechanism to assess how robust climate policies are to the delay in hard information on damages. Using the damage specification of Golosov et al. (2014), they find that the SCC should rise *faster* than GDP as long as global warming has not been enough to generate impacts that are informative about the true social cost. This result contrasts with the insight that the SCC taxes should rise *slower* than GDP if damages are additive (or have an elasticity of substitution with output greater than one). We abstract from analysing climate risk in the remainder of this paper.[19]

2.5. Implementation

In optimizing IAMs the social optimum can be replicated in a market economy by levying a carbon tax that is equal to the *optimal* SCC provided all other market externalities and externalities are dealt with too.[20] It would be a big leap of faith to set the global carbon tax to the SCC calculated from (2) using trend rates of growth of the economy and the population. Still, we will see in sections 4 and 5 that these insights with respect to the effects of intergenerational inequality aversion, the rate of economic growth, population growth, and the nature of damages on the SCC hold up fairly well in the calibrated general equilibrium IAM put forward in section 3. One

[19] Dietz and Stern (2014) allow for this numerically (ignoring stranded assets and scarcity rents) using Monte-Carlo simulations but do not solve the Bellman equations (Crost and Traeger, 2013; Jensen and Traeger, 2014; Traeger, 2014).

[20] See Rezai and van der Ploeg (2014b) for an analysis of both a global climate externality and a learning-by-doing externality in using renewable energy.

reason might be that the Ramsey growth dynamics converge in a matter of decades whilst the carbon cycle requires centuries to converge. During the first few decades the interest rate is lower (higher) that to the level it converges to if intergenerational inequality aversion exceeds (is less than) one, hence (2) and (3) give lower (upper) bounds on the SCC. In general, our IAM also allow an exploration of the optimal trade-off between short-term sacrifices in consumption per capita and long-run avoidance of global warming and the costs that are associated with that.

The merit of our simple formula is that is can be easily computed and implemented. Surprisingly, it can be derived from the level of productive capacity minus environmental damage, i.e. GDP, and no knowledge of other state variables, such as the highly uncertain amount of fossil fuel reserves. However, the optimal tax should be based on optimal GDP which, strictly speaking, is not observable when deciding what carbon tax to set. Changing current GDP by last year's GDP plus a correction for trend growth gives almost identical results for the carbon tax and reproduces the first-best policy discussed in later sections well. Since our rule for the optimal carbon tax depends on optimal, not business-as-usual (BAU) level GDP and in period zero GDP is at its BAU level, there may be some cause of concern for the early periods while GDP catches up to its optimal level. However, as optimal GDP in period zero differs by less than 1 percent from BAU GDP, the approximation error is small.

3. An integrated assessment model of Ramsey growth and energy transitions

Our IAM has the same utilitarian social welfare function as used in section 2:

$$(5) \qquad \sum_{t=0}^{\infty} \beta^t L_t U(C_t / L_t) = \sum_{t=0}^{\infty} \beta^t L_t \frac{(C_t / L_t)^{1-\Phi}}{1-\Phi}, \quad \Phi \geq 0,$$

where C_t is aggregate consumption during period t, L_t the population size at the outset of period t, and $\Phi \equiv -\dfrac{U'(C_t / L_t)}{(C_t / L_t)U''(C_t / L_t)} > 0$ the coefficient of relative IIA. The ethics of climate policy depend on how much weight is given to welfare of future generations (i.e., β) and on how small intergenerational inequality aversion (Φ) is or how easy it is to substitute current for future consumption per head. Optimal climate policy maximizes (5) subject to constraints (6)-(9) below.

First, gross output in period t, $Z(K_t, L_t, F_t + R_t)$, is produced using three inputs, manmade capital at the outset of period t, K_t, labor at the outset of period t, L_t, and energy. We model two types of energy: fossil fuels (oil, natural gas and coal), F_t, and renewables, R_t (e.g., wind or solar energy). This aggregate production function allows for imperfect factor substitution between capital, labor and energy, but imposes perfect substitution between fossil fuel and renewables. Renewable energy is infinitely elastically supplied at potentially exogenously decreasing cost, b_t. Fossil fuel extraction cost in period t is $G(S_t)F_t$ with $G'(S_t) < 0$, where S_t denotes remaining fossil fuel reserves at the start of period t, hence extraction becomes more costly as less accessible fields have to be explored. We have sustained technical progress in aggregate and renewable energy production and an exogenous profile for the time path of population growth. We allow for multiplicative and additive damages and allow them to be more convex than the specification used in section 2 and Golosov et al. (2014) to capture catastrophic damages at high temperatures.[21] Production after depreciation at the rate δ, cost of energy use and climate damages is used for consumption C_t and investment:

(6) $$K_{t+1} = Z(K_t, L_t, F_t + R_t) - D(T_t) Z(K_t, L_t, F_t + R_t)^{\varepsilon} Z_0^{1-\varepsilon} - G(S_t)F_t - b_t R_t - C_t + (1-\delta)K_t, \ \forall t \geq 0,$$

where initial capital K_0, energy use $F_0 + R_0$, and gross output $Z_0 = Z(K_0, L_0, F_0 + R_0)$ are given, and climate damages, $D(T_t)$, increase with global mean temperature T_t.

Second, given initial reserves S_0, finite fossil fuel reserves develop as follows:

(7) $$S_{t+1} = S_t - F_t, \quad \forall t \geq 0, \quad \sum_{t=0}^{\infty} F_t \leq S_0.$$

Third, the two-box carbon cycle used in section 2 is based on Golosov et al. (2014):

(8) $$E_{1,t} = E_{1,t-1} + \varphi_L F_t, \qquad E_{2,t} = (1-\varphi)E_{2,t-1} + \varphi_0 (1-\varphi_L) F_t, \quad \forall t \geq 1,$$

where $E_{1,t}$ and $E_{2,t}$ denote the permanent and transient end-of-period stocks of atmospheric carbon and $E_{1,0}$ and $E_{2,0}$ are given (Archer, 2005; Archer et al., 2009). Gerlagh and Liski (2012) point to the importance of the delay between the carbon

[21] We abstract from positive feedback effects and the uncertain climate catastrophes that can occur in climate and growth models once temperature exceeds certain thresholds (e.g. Lemoine and Traeger, 2013; van der Ploeg and de Zeeuw, 2013).

concentration and global warming. We capture this delay by assuming that the effective stock of atmospheric carbon and thus temperature follows from a distributed lag formulation:

(9) $$E_t = \frac{1}{\varphi_T}(E_{1t} + E_{2t}) + \left(1 - \frac{1}{\varphi_T}\right)E_{t-1}, \quad \forall t \geq 1, \quad T_t = \omega \frac{\ln(E_t / 596.4)}{\ln(2)}, \quad \forall t \geq 0,$$

where temperature is the deviation of global mean temperature from pre-industrial temperature in degrees Celcius, 596.4 GtC the IPCC figure for the pre-industrial stock of atmospheric carbon, and ω the climate sensitivity (the rise in temperature after a doubling of the total stock of atmospheric carbon, typically 3).[22]

Proposition 3: *The social optimum chooses* $\{C_t, F_t, R_t, K_t, S_t, E_{1t}, E_{2t}, E_t, \forall t \geq 1\}$ *to maximize (5) and satisfies (6)-(9), the Euler equation for consumption growth*

(10) $$\frac{C_{t+1} / L_{t+1}}{C_t / L_t} = \left[\beta(1 + r_{t+1})\right]^{1/\Phi}, \quad \forall t \geq 0,$$

with $r_{t+1} \equiv \Gamma_{t+1} Z_{K_{t+1}} - \delta$ *and* $\Gamma_t \equiv \left[1 - \varepsilon D(E_t)\left(Z(K_t, L_t, F_t + R_t) / Z_0\right)^{\varepsilon - 1}\right]$, *and the efficiency conditions for energy use*

(11a) $$\Gamma_t Z_{F_t + R_t}(K_t, L_t, F_t + R_t) \leq G(S_t) + s_t + \tau_t, \quad F_t \geq 0, \quad \text{c.s.}, \quad \forall t \geq 1,$$

(11b) $$\Gamma_t Z_{F_t + R_t}(K_t, L_t, F_t + R_t) \leq b_t, \quad R_t \geq 0, \quad \text{c.s.}, \quad \forall t \geq 1.$$

The scarcity rent and SCC are

(12) $$s_t = -\sum_{\zeta=1}^{\infty} (1 + r_{t+1}) \Delta_{t+1+\zeta} G'(S_{t+\zeta}) F_{t+\zeta}, \quad \forall t \geq 1,$$

(13) $$\begin{aligned} &\theta_t = \sum\nolimits_{\varsigma=0}^{\infty} \left[\left\{\varphi_L + \varphi_0(1 - \varphi_L)(1 - \varphi)^{\varsigma}\right\} \Delta_{t+\varsigma} \theta_{Tt+\varsigma}\right] / \varphi_T, \\ &\text{with } \theta_{Tt} = \sum\nolimits_{\varsigma=0}^{\infty} \left[\left(1 - 1/\varphi_T\right)^{\varsigma} \Delta_{t+\varsigma} D'(E_{t+\varsigma}) Z_{t+\varsigma}{}^{\varepsilon} Z_0^{1-\varepsilon}\right] / \varphi_T, \forall t \geq 1, \end{aligned}$$

and the compound discount factors $\Delta_{t+\varsigma} \equiv \prod\nolimits_{\varsigma'=1}^{\varsigma} (1 + r_{t+\varsigma'})^{-1}, \varsigma \geq 1$ *and* $\Delta_t = 1, \forall t \geq 1.$

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[22] Alternatively, we could use $T_t = \frac{\omega}{\varphi_T} \frac{\ln\left((E_{1t} + E_{2t}) / 596.4\right)}{\ln(2)}) + \left(1 - \frac{1}{\varphi_T}\right) T_{t-1},$ which gives a similar expression for the SCC as (13) below (Rezai and van der Ploeg, 2014b).

Proof: see appendix B.

In the numerical simulations of our IAM in sections 4-6 we use the more complex and realistic carbon and temperature cycles and damage specification of Nordhaus (2014). These allow for separate dynamics of the stocks of carbon in the atmosphere and the upper and lower parts of the ocean as well as separate dynamics in the atmospheric and oceanic heat reservoirs and a non-linear forcing relations between carbon and temperature variables (see appendix A for details). One of our numerical sensitivity scenarios also allows for mean reversion in climate damages. For the sake of brevity and analytical clarity, we omitted these details in the derivation of the necessary optimality conditions.

The Euler equation (10) states that growth in consumption per capita increases with the social return on capital and decreases with the rate of time impatience, where the social return on capital is set to the marginal product of capital, net of depreciation. Equation (11a) implies that, if fossil fuel is used, its marginal product should equal its marginal extraction cost, $G(S_t)$, plus its scarcity rent plus the SCC. The scarcity rent and SCC are defined in units of final goods (not utility units). If the marginal product of fossil fuel is below its total marginal cost, it is not used. Equation (11b) states that, if renewable energy is used, its marginal product must equal its marginal cost, b_t. Although our IAM might display simultaneous use of fossil fuel and renewable energy, in practice it rarely occurs and if it does at most during one year.[23]

Equation (12) states that the scarcity rent of keeping an extra unit of fossil fuel unexploited must equal the present discounted value of all future reductions in fossil fuel extraction costs due to this reduction in extraction. It follows from the Hotelling rule which states that the return on extracting an extra unit of fossil, selling it and getting a return on it, the rate of interest $r_{t+1}s_t$ minus the increase in future extraction cost $-G'(S_{t+1})F_{t+1}$, must equal the expected capital gain from keeping an extra unit of fossil fuel in the earth $s_{t+1}-s_t$. Finally, equation (13) states that as in section 2 the SCC equals the present discounted value of all future marginal global warming damages from burning an additional unit of fossil fuel. In contrast to section 2, it allows for the time-varying nature of the gross interest rate as well as the permanent

[23] The period of simultaneous use increases in length with the share of energy in GDP. For historically observed values, this period only lasts for at most one period.

and transient components of atmospheric carbon in the simple two-box carbon cycle and the temperature lag. For the special case of no temperature lag (13) boils down to

(13′) $\theta_t = \sum_{\varsigma=0}^{\infty}\Big[\Big\{\varphi_L + \varphi_0(1-\varphi_L)(1-\varphi)^{\varsigma}\Big\}\Delta_{t+\varsigma}D'(E_{t+\varsigma})Z_{t+\varsigma}{}^{\varepsilon}Z_0{}^{1-\varepsilon}\Big], \quad \forall t \geq 1,.$

In a competitive market economy firms choose inputs to maximize profits, $Z_t - D(E_t)Z_t{}^{\varepsilon}Z_0{}^{1-\varepsilon} - w_tL_t - (r_{t+1}+\delta)K_t - (p_t+\tau_t)F_t - q_tR_t$, taking the wage rate w_t, the market interest rate r_{t+1}, the market price for fossil fuel p_t, the specific tax τ_t on carbon emissions, the market price for renewable energy q_t, and the atmospheric carbon concentration as given. Capital accumulation follows from $K_{t+1} = (1-\delta)K_t + I_t$, where I_t is the investment rate at time t. Fossil fuel owners also operate under perfect competition and maximize the present value of their profits, $\sum_{t=0}^{\infty}\Delta_t\left[p_tF_t - G(S_t)F_t\right]$ with $\Delta_t \equiv \prod_{s=0}^{t}(1+r_{s+1})^{-1}, t \geq 0,$ subject to the depletion equation (7), taking the market price of fossil fuel p_t as given and internalizing the effect of depletion on future extraction costs. Producers of renewable energy also operate under perfect competition and maximize their profits, $\sum_{t=0}^{\infty}\Delta_t\left[q_tR_t - b_tR_t\right]$, taking the market price of renewable energy q_t as given. Households maximize utility (5) subject to their budget constraint $A_{t+1}^H = \left(1+r_{t+1}\right)A_t^H + w_tL_t + \tau_tF_t - C_t$, where A_t^H denotes household assets and τ_tF_t lump-sum carbon tax rebates at time t. Since Ricardian debt neutrality holds, there is no loss of generality in assuming that the government balances its books. Asset market and final good market equilibrium require $A_t^H = K_t$ and $GDP_t = Z_t - D(E_t)Z_t{}^{\varepsilon}Z_0{}^{1-\varepsilon} = C_t + I_t + G(S_t)F_t + b_tR_t$, so the firms produce enough satisfy demand for consumption and investment goods and cover fossil fuel extraction costs and production costs of renewables.

Proposition 4: *The decentralized market outcome satisfies (6)-(9), the Euler equation (10), the energy producers' optimality conditions*

(11a′) $p_t \leq G(S_t) + \theta_t^S, \quad F_t \geq 0, \quad \text{c.s.}, \quad \forall t \geq 1,$

(11b′) $q_t \leq b_t, \quad R_t \geq 0, \quad \text{c.s.}, \quad \forall t \geq 1,$

where the scarcity rent θ_t^S *is (12), and the final good firms' optimality conditions are*

$$\text{(14)} \quad \begin{aligned} &\Gamma_t Z_{L_t} = w_t, \quad \Gamma_t Z_{K_t} = r_t + \delta, \\ &\Gamma_t Z_{F_t+R_t} \le p_t + \tau_t, F_t \ge 0, c.s., \quad \Gamma_t Z_{F_t+R_t} \le q_t, R_t \ge 0, c.s., \quad \forall t \ge 1, \end{aligned}$$

Proof: Follows from the optimality conditions for firms, fossil fuel producers renewable energy producers and households.

Proposition 5: *The social optimum is replicated in the decentralized market economy if* $\tau_t = \theta_t \ \forall t \ge 1,$ *as given by (13).*

Proof: Comparing conditions of proposition 3 with those of proposition 4.

The first best thus prevails if the carbon tax is set to the *SCC* and net revenue is rebated to households.

4. Baseline results: carbon taxes and stranded assets in a growing economy

Here we present the optimal carbon tax and the BAU outcomes from our general equilibrium IAM with stock-dependent extraction costs, optimal energy transitions and time-varying interest and growth rates, and compare them with the outcome under our simple carbon tax rule (2). Our IAM adopts the damage specification and the more realistic carbon cycle with lags and separate temperature dynamics from Nordhaus (2014), but recalibrated to an annual time grid which is crucial to describe the dynamics of the energy transitions accurately.[24] The functional form and calibration of our IAM are discussed in detail in appendix C. In the baseline simulations we assume population to increase from 7 billion to grow to 11 billion over the next century but analyse the effects of stabilization at a lower level in section 6. Total productivity trend growth occurs at a rate of 2% per year. Our baseline parameter values have relatively low damages, low fossil fuel extraction cost and a high cost for renewable energy. This biases our model toward fossil fuel use.

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[24] Nordhaus (2014) now uses a semi-decadal instead of decadal grid in his DICE-2013R model. Cai et al. (2012) show that a decadal grid introduces significant biases in the calculation of the SCC. An annual grid also helps to better to pinpoint the timing of the energy transitions and the level of stranded assets.

We report full optimal policy and BAU simulation results under the assumption of logarithmic utility (Φ = 1) and also under a more widely used intergenerational inequality aversion (Φ = 2). We present three scenarios: (i) the first best optimum where the global carbon tax is set to the optimal SCC derived from our IAM; (ii) BAU with no carbon tax to correct the externality; and (iii) a scenario where the carbon tax is set according to our simple rule (2). Table 1 illustrates the six simulations and the coding that is used to distinguish them in the simulation figures.[25]

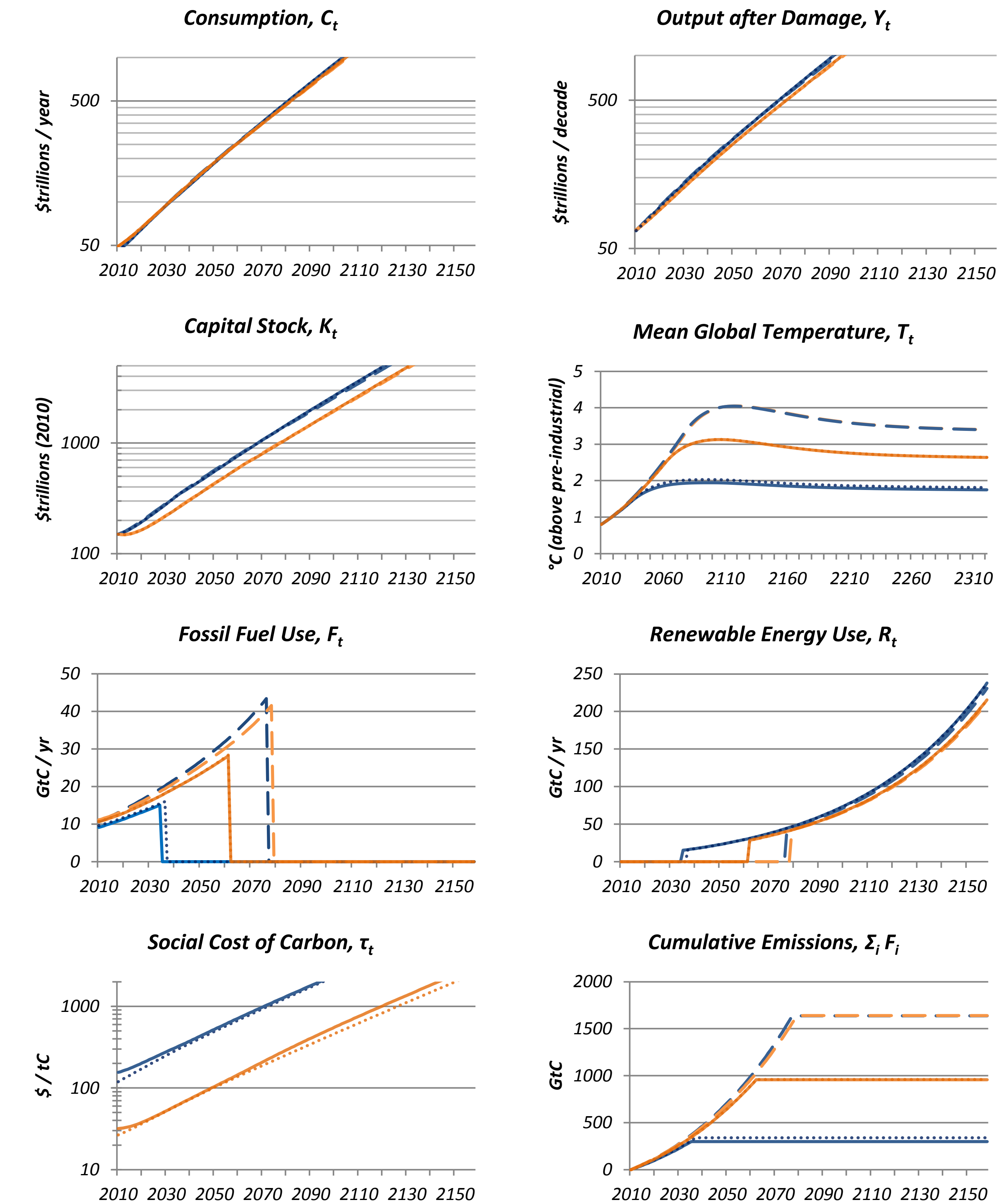
Table 1: Policy Scenarios for the setting of the global carbon tax

	First best	*BAU*	*Simple rule (2)*
Baseline (Φ = 2)	———	— — —	....
Logarithmic utility (Φ = 1)	———	— — —	....

4.1. Role of intergenerational inequality aversion

We start with figure 1 which shows the *first best* and how it depends on the degree of IIA; the solid line corresponds to Φ = 2 and the dark line to Φ = 1. The first, second and third panels show aggregate consumption, total net output and the aggregate capital stock. Consumption and net output rapidly approach their steady growth path of 2 percent per year and are almost the same across scenarios. Given the strong trend in productivity growth, higher IIA lowers concern for future generations and makes climate policy less ambitious. In the baseline (with Φ = 2), table 2 indicates that fossil fuel is phased out in 2060 and temperature increases to slightly above 3°C. Under logarithmic utility, IIA is only half of that in the baseline so current generations can make more sacrifices by producing more costly as fossil fuels are phased out earlier, by 2035. As a result, temperature peaks at a lower level, 2.0°C. These sacrifices also mean that only 325 GtC are burnt which is a third of the 955 GtC of the baseline scenario. In the baseline scenario, higher emission levels imply higher marginal damages along the growth trajectory. The higher concern for intergenerational inequality, however, implies that the interest rate is much higher (by about 2 percentage points for most periods) and that these damages are discounted more heavily, thus leading to a lower SCC. In the baseline scenario the carbon tax starts at

[25] In our numerical simulations and optimizations time runs from 2010 till 2610 and is measured in years, t=1,2,.., 600. The final time period is t=600 or 2610, but we highlight the transitional dynamics in the first 150 years of the simulation.



Key: *Social optima: baseline*, Φ=2 ( ); *logarithmic utility*, Φ=1 ( )
BAU: baseline, Φ=2 ( ); *logarithmic utility*, Φ=1 ( )
Simple rule for the carbon tax: baseline, Φ=2 ( ); *logarithmic utility*, Φ=1 ( )

Figure 1: Baseline simulations for baseline (Φ=2) and logarithmic utility (Φ=1)

$32/tC which is about a fifth of the $155/tC under logarithmic utility. Both carbon taxes then rise at about 2 percent per year in line with the trend growth in productivity and living standards. Without a carbon tax the market price of fossil fuel is significantly lower and firms, not forced to internalize the deleterious effects of fossil fuel, use more of the cheaper input. Cumulative extraction and global temperature are higher under BAU as fossil fuel is used more and longer. Regardless of the degree of IIA, BAU leads to a maximal warming of 4°C and in total as much as 1,640 GtC are burnt which matches closely most recent baseline projections of the IPCC (2014).

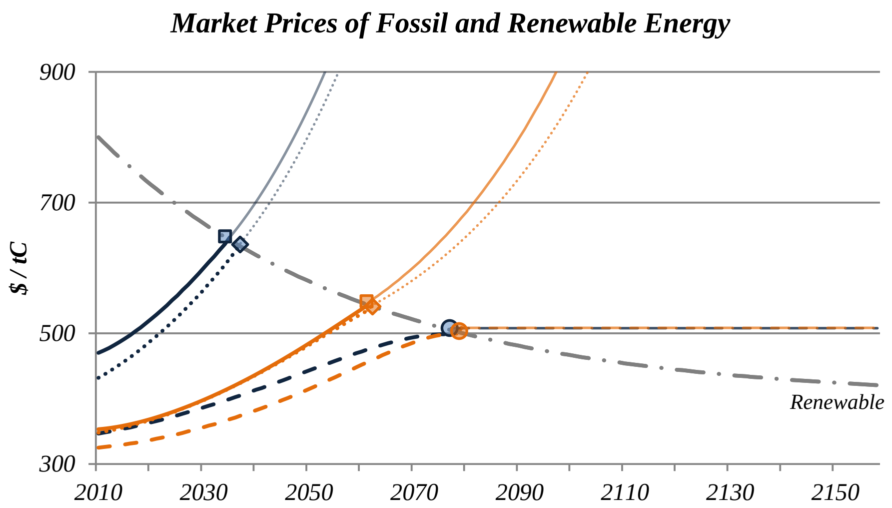
Table 2: Transition times and carbon budget

		Fossil fuel Only	*Renewable Only*	*Carbon used*	*max. T*	*Welfare loss*
Baseline (Φ=2)	*First best*	2010-2060	2061 –	955 GtC	3.1 °C	0%
	BAU	2010-2078	2079 –	1640 GtC	4.0 °C	- 3%
	Simple rule	2010-2061	2062 –	960 GtC	3.1 °C	- 0.001%
Log. Utility	*First best*	2010-2034	2035 –	325 GtC	1.9 °C	0%
	BAU	2010-2076	2077 –	1640 GtC	4.0 °C	- 178%
	Simple rule	2010-2036	2037 –	340 GtC	2.0 °C	- 0.6%

4.2. Dynamics of the energy market

The main reason for imposing a carbon tax is to increase the artificially low market price of fossil energy to take into account the deleterious effects of the emissions resulting from the combustion of fossil fuel. The prices of all energy sources under the different scenarios are depicted in figure 2. The energy price in the first best is the shadow price of fossil fuel: the marginal extraction cost plus the Hotelling rent and the SCC. Under BAU the market price excludes the SCC.

The market prices of fossil fuel increase in all scenarios because all of the three components increase initially. Extraction costs increase as reserves are depleted and the SCC rises due to rising marginal climate damages and sustained consumption growth. The amount of cumulative extraction is also sufficiently high to lead to rising Hotelling rents. As the end of the fossil era draws closer the Hotelling rent falls and from its final period onward the Hotelling rent vanishes because some reserves are left abandoned. This fall in the Hotelling rent is, however, insufficient to induce falling market prices of fossil fuel in our simulations.



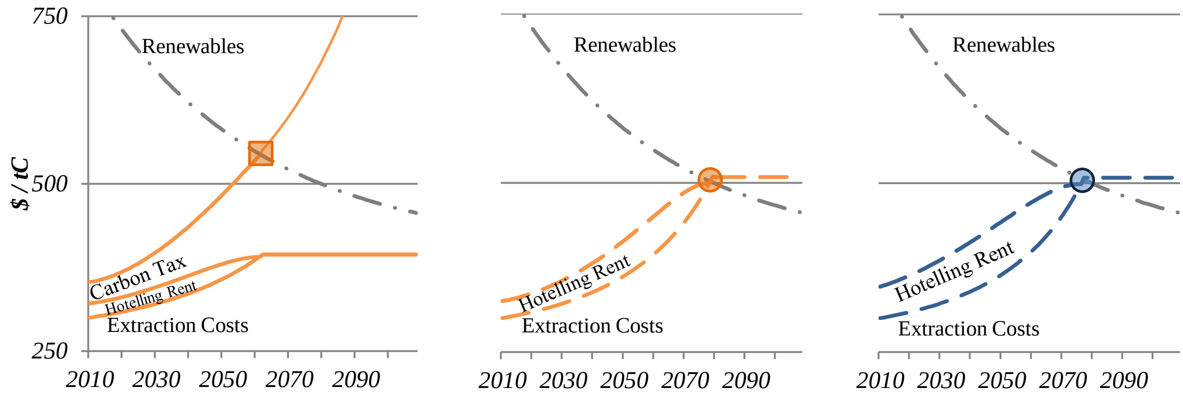
Key: Φ=2: *social optimum* ( ); *simple rule* (2) ( ), *BAU* ( )
Φ=1: *social optimum* ( ); *simple rule* (2) ( ), *BAU* ( )
Shaded areas indicate optimal (squares), simple rule (diamonds), and BAU (circles) switch times. Fainted lines indicate fossil fuel not in use. Market prices for fossil energy rise until they surpass the price of renewable energy. BAU prices do not include the cost of the climate externality. Prices in the social optimum or with the simple policy rule start higher and rise more rapidly, inducing earlier switch times.

Figure 2: Market prices for fossil and renewable energy

Once the market price of fossil energy exceeds the market cost of renewable energy, renewable energy takes over. From then on the marginal extraction cost of fossil fuel is constant. The first-best market price of fossil fuel keeps rising, however, as sustained consumption growth increases the SCC further. Since the SCC is higher with less IIA, the carbon tax is higher initially and rises faster. This leads to a higher market price of fossil energy and induces lower fossil fuel consumption and higher in situ stocks as discussed above. Figure 3 presents a graphical decomposition of the market price of fossil energy for the baseline scenarios.

Output, consumption and capital accumulation take place at very similar levels under the first best and BAU. The reason for this is mainly that we have sustained income growth and low and temporary population growth. Relatively small differences in consumption levels can, however, add up to large welfare losses if differences persist for long periods. The last column of table 3 illustrates these welfare losses relative to the first best. The BAU welfare loss in the baseline is modest with 3 percent of initial

GDP, because low IIA coupled with strong productivity growth and low population growth implies that there is little incentive to place the cost of climate change mitigation on current generations. The material wealth of future generations does not justify stringent climate policy given the discount rate and the magnitude of damages. With less IIA, small reductions in future high consumption levels are valued higher and the transition is initiated much earlier. As a result, the cost of the uncorrected externality under BAU increases to 178 percent of initial GDP.



Key: In the first-best case, the carbon tax increases the market price of energy significantly, capturing a large share of the Hotelling rent. Without climate policy the Hotelling rent increases. Lower IIA increases investment, growth, and fossil fuel use. Hence, the Hotelling rent and market price of fossil fuel increase. Shaded areas indicate switch times.

Figure 3: Decomposition of the market prices of fossil fuel

4.3. Performance of the simple rule for the carbon tax

The simple rule for the optimal carbon tax put forward by Golosov et al. (2014) implies that the SCC rises in line with GDP and offers a useful benchmark for back-of-the-envelope policy appraisal. This rule relies on very restrictive assumptions (logarithmic utility, Cobb-Douglas production, decadal time steps, full depreciation each period, exponential damages, and zero capital intensity of fossil fuel extraction), and especially this rule does not allow for the effects of trend growth and IIA bigger than unity on the SCC (see section 2). Our simple rule (2) is more general as it allows for sustained productivity and population growth and different levels of IIA as well as a temperature lag. The dotted light (for Φ=2) and dark (for Φ=1) lines in figures 2 and

3 represent the BAU outcome under our simple rule (2). We find that our rule predicts the level and growth of the optimal carbon tax accurately and is able to mimic the social optimum very closely. In the baseline the carbon tax according to our simple rule starts out slightly too low at 27$tC, discourages fossil fuel use slightly too little, and induces the transition to renewable energy one year too late.[26] The welfare cost associated with this deviation is negligible, amounting to 0.001 percent of initial GDP (cf. table 2). For logarithmic utility our simple rule is slightly worse. Here our rule (2) is able to reproduce the social optimum, leading to a welfare loss of half a percent of initial GDP. The main reason why our rule (2) performs so well is that the Ramsey dynamics converges much more quickly than the carbon and temperature dynamics.

The rule proposed by GHKT is derived for a simplified decadal two-box carbon cycle with no separate dynamics or lags in global mean temperature. In a calibrated model, it gives an initial SCC of $75/tC which consequently rises at the same rate of growth as GDP. Our IAM is more realistic, since it adopts the five-box carbon and temperature module of Nordhaus (2008) and is recalibrated to an annual time grid. These revisions increase the initial SCC significantly to 155$/tC for IIA = 1 and curb it by half to $32/tC for IIA = 2.

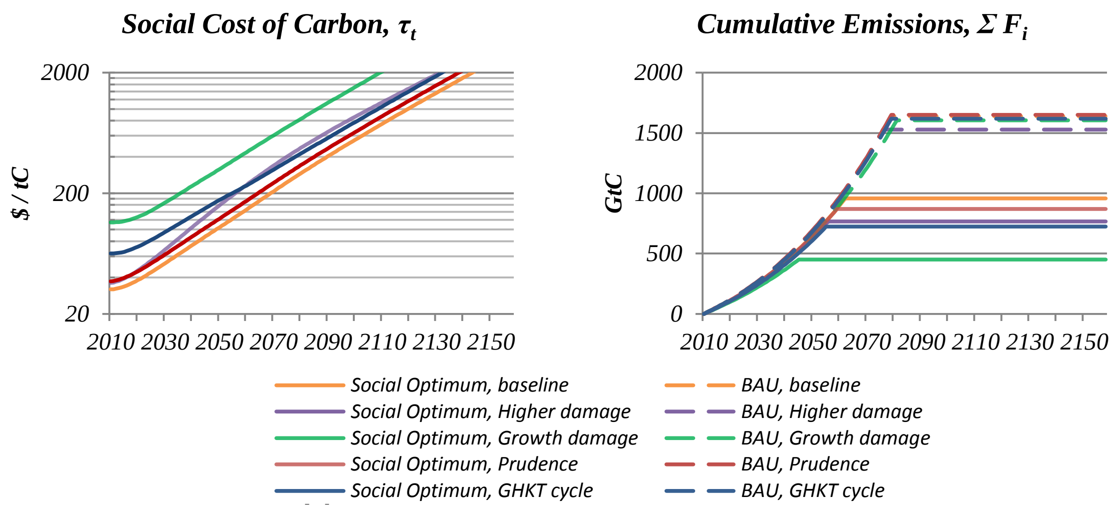
We have established that our simple rule (2) delivers welfare outcomes quite close to the first best. Our simple rule also manages to predict the amount of fossil fuel to be left abandoned quite closely, inducing the transition to renewable energy at most a consequently lowers the initial SCC and global warming increases; in both cases the SCC rises pretty quickly in line with world GDP, as already indicated by (2).

These differences in valuing intergenerational inequality also show up in the use of fossil fuel and the timing of the transition to renewable energy. The relative wealth of future generations implies that current generations are allowed to use more (less) fossil fuel and induce higher (lower) temperature increases, especially if IIA is high decade away from the socially optimal switch point. In section 5 and 6 we also test how our simple rule performs with climate damages hurting productivity growth (cf. Dell et al. (2012)) and additive climate damages (ε=0).

[26] The reason is that with IIA > 1 the initial interest rate is lower than the long-run rate and thus the true optimal SCC is over-estimated. With IIA < 1, we get the opposite result.

5. More ambitious climate policy: convex damages, growth effects and prudence

Section 4 only discussed one representative baseline scenario for which we chose middle-of-the-range parameters. In this and the subsequent sections we explore the robustness of our findings, especially the fitness of the simple policy rule, for a suite of scenarios. Figure 4 shows the sensitivity of the social optimum, the BAU outcome, and the outcome with the simple carbon tax rule (2) to more convex damages, adverse productivity growth effects of climate change, and risk to consumption growth.



Key: Higher instantaneous and growth damages, prudence and more responsive climate system increase the SCC, curb fossil fuel use and lowering the carbon budget.

Figure 4: Sensitivity analysis for the optimal *SCC* and cumulative emissions

The effects of less IIA (e.g., logarithmic utility) are presented in figure 1. These effects make climate policy more ambitious by increasing the SCC and locking more fossil fuel in the ground. Table 3 gives the transition times and carbon budgets for the social optimum and the outcome under the simple policy rule.

The first sensitivity test increases the convexity of damages at high temperatures. Weitzman (2010) and Dietz and Stern (2014) argue that damages rise more rapidly at higher levels of temperature than suggested by Nordhaus (2008), but empirical studies on the costs of global warming at higher temperatures are not available due to lack of evidence (atmospheric carbon concentrations have not exceeded 400ppmv for 800,000 years). Greater convexity increases the SCC and optimal policy responds by limiting emissions, the carbon budget and global warming. Since the damage function does not increase much on the relevant temperature range, the outcome is not very different from the baseline.

Table 3: Robustness of the simple carbon tax rule to more ambitious policy

Scenario	*Carbon used (First best)*	*max. T (First best)*	*Switch Time (First best)*	*Carbon used (simple rule)*	*max. T (simple rule)*	*Welfare loss (simple rule)*	*Welfare loss (BAU)*
Baseline	955 GtC	3.1 °C	2060	960 GtC	3.1 °C	- .001%	- 3%
Higher damage	770 GtC	2.8 °C	2055	960 GtC	3.1 °C	- 1.10%	- 12%
Growth damage	452 GtC	2.3 °C	2044	413 GtC	2.2 °C	- 0.15%	- 29%
Prudence	870 GtC	3.0 °C	2058	870 GtC	3.0 °C	- .002%	- 5%
GHKT cycle	725 GtC	N/A	2054	620 GtC	N/A	- .31%	- 7.8%

Key: Welfare losses are relative to the first best (which has welfare loss of 0%) and are reported as percentage losses of initial GDP.

Hence, even though the simple rule is based on the fairly flat marginal damages of Nordhaus (2014) and Golosov et al. (2014), it still mimics the social optimum quite closely with a welfare loss of 1.1 percent of initial GDP. Climate damages are commonly modelled as flow damages to the level of output. Recent empirical studies, however, found that climate change also has a negative impact on the growth rate of productivity, especially in developing countries (Dell et al., 2012). In the second sensitivity run, we replace the climate damages proposed by Nordhaus (2008) with these persistent damages to productivity.[27] This leads to less fossil fuel use, less short-run growth and an earlier transition to renewable energy. The simple rule (2) adjusts well to the different damage specification with a welfare loss of less than 0.2 percent of initial output. An additional sensitivity run considers a lower productivity growth trend of 1 percent per year which lowers the rate to discount climate damages. This increases the initial SCC but flattens the time profile of the optimal tax due to lower trend growth in output. Here, the welfare loss associated with the approximation (2) is less than 0.1 percent.

The rate of time impatience is a crucial parameter in climate economics. Here we consider the effects of potential shocks to consumption growth (Kocherlokata, 1996; Gollier, 2013). Given that global consumption growth had a standard deviation of 3.6

[27] We keep the same damage function but allow for persistent damages to productivity (see appendix B for details). We take these negative effects to be deviations from trend productivity growth and allow for mean reversion. In the sensitivity runs damages to productivity are assumed persistent with an AR(1) coefficient of 0.75.

percent over much of the past century, we account for prudence and lower the rate of time impatience to 0.6 percent (see section 2.4).[28] The lower discount rate gives greater weight to future marginal climate damages and thus increases the SCC. The carbon budget reduces by nearly 10 percent and global warming is slightly decreased to 3.0°C. The simple rule for the carbon tax aptly adjusts to the lower discount rate and maintains the welfare loss of 0.002 percent.

Table 3 also shows the effects of replacing the DICE carbon cycle with the simpler carbon cycle put forward in Golosov et al. (2014), which was used in the derivations of our simple rules in propositions 1 and 2. Somewhat surprisingly, the welfare loss resulting from the simple rule is somewhat bigger than with the DICE carbon cycle and temperature module. The reason is that the interest rate rises along the transition to the balanced growth path while the long-run rate has been applied in the derivation of propositions 1 and 2. Time lags in the DICE carbon cycle (absent in Golosov et al. (2014)) lessen this approximation error and improve the rule's performance. The immediacy of this carbon cycle variant also explains why the SCC is higher and more carbon is locked up relative to baseline.

6. Less ambitious climate policy: inequality aversion and additive damages

Table 4 and figure 5 present the effects of allowing for additive production damages, population stagnation, and lower productivity growth. These variants give rise to less ambitious climate policy, leading to lower carbon taxes and less fossil fuel abandoned in situ. We have already shown in section 2 that additive marginal climate damages lead to a simple rule with a lower SCC that does not rise with trend GDP growth. Numerical simulations in our IAM confirm this result. Under additive damages the SCC is indeed lower and its time profile is flat. This implies that much more carbon is burnt in a prolonged fossil fuel era. The first-best carbon budget is 1600 GtC which is only 20 GtC less than the 1620 GtC burnt under BAU.[29] The intuition behind this

[28] Effectively, the RTI is reduced by −0.5×IIA×CRP×var, where IIA = 2, CRP = 2 + 1 = 3 and var = 3.6% per year. Stern (2007) discusses further important reasons for lower discounting.

[29] Weitzman (2010) finds that with additive global warming damages (in instantaneous welfare) the willingness to sacrifice current consumption to avoid future global warming is seven times higher than with multiplicative damages. In contrast, we find that additive damages lead to half the SCC at each point of time compared to multiplicative damages.

result is clear, if the economy's endowments and technological change allow the economy to grow, additive marginal climate damages fall as a fraction of GDP over time. In this sense the additive case can be understood as a constant lowering of the multiplicative damage function used in the baseline scenario.

Table 4: Robustness of the proportional tax to less ambitious policy

Scenario	*Carbon used (first best)*	*max. T (first best)*	*Switch Time (first best)*	*Carbon used (simple rule)*	*max. T (simple rule)*	*Welfare loss (simple rule)*	*Welfare loss (BAU)*
Baseline	955 GtC	3.1 °C	2060	960 GtC	3.1 °C	- .001%	- 3%
Additive damage	1600 GtC	4.0 °C	2077	1600 GtC	4.0 °C	- .002%	- .024%
Population stagnation	940 GtC	3.1 °C	2063	970 GtC	3.1 °C	- .006%	- 2.2%
Less prod. growth	822 GtC	2.9 °C	2064	870 GtC	3.0 °C	- 0.06%	- 10%

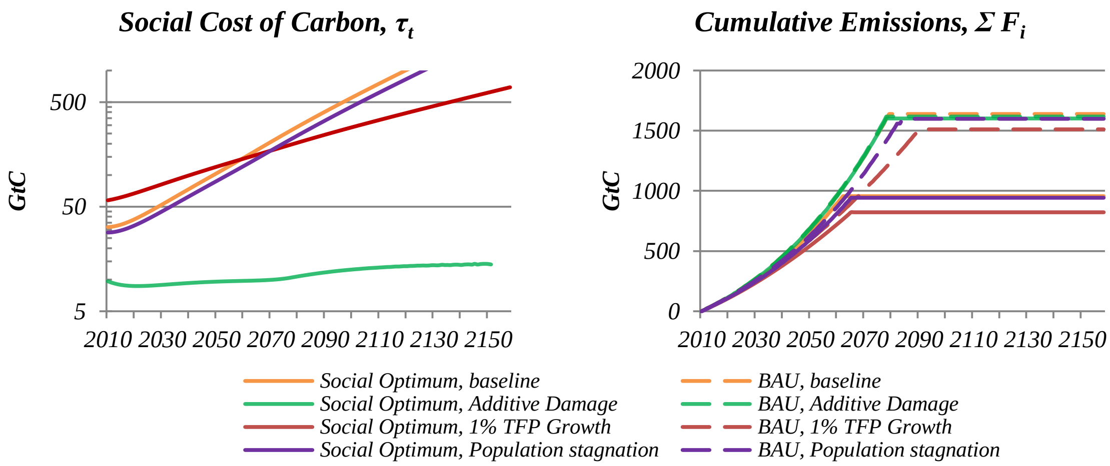
Key: Welfare losses are relative to the first best (which has welfare loss of 0%) and are reported as percentage losses of initial GDP.

Our simple rule (2) for the additive case reproduces the first-best outcome remarkably well with the switch to renewable energy occurring in the socially optimal period and virtually the same amount of carbon used over the complete horizon. The welfare loss of using our simple rule instead of the first-best carbon tax path is thus very low (0.002 percent of initial GDP). Since fossil fuel use is price-elastic, the slight difference in carbon taxes yields differences in the carbon budget due to differences in time profile of fossil fuel use despite the synchronicity in transitions.

The growth of population is an important determinant of climate policy and population projections are prone to large revisions and scientific uncertainty. Here we consider the effect of a stabilization of population at 9 rather than 11 billion people at the end of the century as predicted by Lutz et al. (2014). The existence of fewer people in the future lowers the benefit of climate policy and implies that current generations are allowed to consume fossil fuel for longer. The initial SCC is lower than under baseline. However, as the future population size is lower, the productive capacity is decreased and less fossil fuel used during the longer fossil fuel era. The

However, this stark difference should serve as a reminder that the additive utility damages used in Weitzman and the additive production damages used in our model are not comparable. Weitzman's number applies to the willingness to pay to avoid any change in temperature along an exogenous consumption trajectory, whereas the SCC of our simulations reports the willingness to pay to avoid a marginal increase in atmospheric carbon along the optimal path. Our numerical results are in line with the analytical findings in section 2: the SCC is higher under multiplicative damages provided that the trend rate of economic growth is positive.

overall effect of a significant reduction in future population size is, however, small. In total only 5 GtC more are burnt. The approximate tax rule does not account for population growth as the long run growth rate of population is zero but still manages to capture the extent of the climate externality accurately.



Key: Lower damages and higher population growth lower the SCC and encourage fossil fuel use. Lower growth in living standards flattens the time profile and lowers the carbon budget.

Figure 5: Sensitivity analysis for the optimal *SCC* and cumulative emissions

Table 4 and figure 5 also show the effects of lower trend growth in living standards (1 instead of 2 percent per annum). As future generations are relatively worse off compared to the baseline, climate policy is more ambitious initially. The lower growth rate in productivity, however, flattens the time profile and carbon taxes are lower in the future. In this sense, climate policy is less ambitious. Overall the initially higher carbon tax dominates climate policy and less carbon is burnt and a higher stock of carbon abandoned in situ.[30]

7. Conclusion

The central questions of climate change economics have been what the level and what the time profile of the optimal price of carbon, whether achieved by levying a carbon tax or creating a carbon emission market, should be. We present a simple but robust

[30] We could also have discussed this scenario in section 5.

rule for the optimal SCC which generalizes the exact but less generally valid rules put forward by Golosov et al. (2014) and van der Bijgaart et al. (2013). Our rule accounts not only for geophysical components, such as the dissipation speed of atmospheric carbon and temperature dynamics, but also for climate damages that are non-proportional to gross output and exhibit mean reversion and for socio-economic characteristics, such as the growth rates of population and living standards and society's aversion to intergenerational inequality.

Our rule is easy to understand, calculate and implement. Furthermore, in contrast to the usual discretionary optimal time paths that are calculated from DICE and other IAMs, our feedback rule allows for changing circumstances. It is a restricted feedback rule, since it reacts to GDP (net of climate damage) and does not react to levels of the capital stock, the stock of remaining fossil fuel reserves, atmospheric carbon stocks, or temperature. Due to the carbon cycle dynamics being much more sluggish than the Ramsey growth dynamics, it gets very close to the first-best social optimum even though our specific IAM which is used to capture the climate-economy interactions is much more complicated than the toy model used to calculate our simple rule and in contrast to DICE allows for forward-looking dynamics in scarcity rents and the effect of falling fossil fuel reserves on extraction costs. The rule also performs well in the DICE model as the supplementary material of Golosov et al. (2014) and van der Bijgaard et al. (2014) indicates.[31] The feedback rule is thus robust to model misspecification and may have more to command itself than the true optimal time path of carbon taxes generated by a particular IAM.

How global warming damages are modeled and calibrated matters for the SCC and climate policy. We rely on the common specification of marginal damages that are proportional to production, but also consider additive production damages and damages to productivity growth. We find that the climate policy is less ambitious, energy use higher, the stock of fossil fuel left in situ lower, global mean temperature higher and the optimal carbon tax lower with additive damages provided that the rate

[31] Van der Bijgaard et al. (2013) find that for a mainstream IAM 99 percent of the within-model variation originating from structural uncertainties is explained by a simple rule. Their rule also performs well for other models. Our results generalize their rule to more general damage specifications and for a more sophisticated IAM with forward-looking dynamics of the scarcity rent of fossil fuel, stock-dependent extraction costs and stranded assets.

of economic growth is positive. Climate policy is more ambitious and less fossil fuel is used if damages affect not only the level productivity but also its growth rate. The extended policy rule captures adjusts aptly to these different damage specifications and moves the decentralized economy close to the social optimum with welfare losses amounting to at most 0.15 percent of initial GDP.

Our IAM also indicates that a lower concern of intergenerational inequality and a lower rate of time impatience lead to a higher optimal carbon tax and a quicker phasing in of renewables and more fossil fuel left in the crust of the earth. The lower concern for intergenerational inequality implies that, since society is more concerned with fighting global warming than with avoiding big differences in consumption of different generations, the carbon tax is borne much more by earlier generations than by later generations in the presence of sustained growth. Our simple rule captures this effect correctly for a large range of intergenerational inequality aversion.

Stern (2013) criticizes the current generation of IAMs for focusing on too limited a set of functional and parametric relationships. Our analysis is only a first step in broadening the scope of the type of climate damages that might be considered, admittedly, leaving ample room for further improvement. Our analysis, however, also leads us to conclude that climate change, if addressed through optimal policy, can be avoided at relatively low costs. Depending on the nature of climate damages, the costs of inaction are potentially large even if many additional reasons for concern Stern highlights are not taken into account. Our analysis suggests that further empirical work is needed on whether climate damages are affecting the level or the growth rate of productivity or whether they are multiplicative or additive in production, or, more generally, on how substitutable they are for economic production.

Finally, future developments in the productive capacity of the economy are crucial determinants of the optimal carbon tax. We have assumed a given growth trend of productivity but productivity is clearly the result of innovation based on research and development and of human capital formation. Furthermore, future prices of clean and dirty sources of energy and their necessity in the general production process heavily influence relative prices and the allocation of resources today. We have examined the effects of variations in the substitutability between energy and conventional capital through a CES production function with a fixed rate of technological progress. Recent contributions by Acemoglu et al. (2012) and Mattauch et al. (2012) highlight the importance of learning and lock-in effects by making the rate of technical progress as endogenous. It is possible to use the empirical estimates of the determinants of growth rates in total factor and energy productivities given in Hassler et al. (2011) in our IAM. This will allow much more substitution possibilities between energy and the capital-labor aggregate in the long run than in the short run. The logic of directed technical change suggests that it is more important to have substantial R&D subsidies for green technology to kick-start green innovation and fight global warming.

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Appendix A: Carbon and temperature cycles used in DICE

The DICE-07 and DICE-13R model approximate the geophysical dynamics using a three-box carbon cycle, radiative forcing, and a two-box temperature cycle. In the carbon cycle, carbon diffuses between the atmosphere (M_{AT}), and the upper (M_{UP}) and lower (M_{LO}) oceans following a Markov process. Emissions E stem from fossil fuel (E_{ind}) and land use (E_{land}) and are emitted initially into the atmosphere. We ignore the dynamics of land use emissions, $E_{land} = 0$. With the 3×3 transition matrix $\boldsymbol{\Upsilon}$, the transition equations are

(A1) $$\begin{pmatrix} M_{AT}(t) \\ M_{UP}(t) \\ M_{LO}(t) \end{pmatrix} = E(t) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \boldsymbol{\Upsilon} \begin{pmatrix} M_{AT}(t-1) \\ M_{UP}(t-1) \\ M_{LO}(t-1) \end{pmatrix}$$

with $\boldsymbol{\Upsilon} = \begin{pmatrix} 0.91 & 0.038 & 0 \\ 0.09 & 0.959 & 0.0003 \\ 0 & 0.003 & 0.9997 \end{pmatrix}$. The carbon cycle is structured such that interaction between the atmosphere and the lower oceans only occurs through the upper oceans. The carbon cycle is closed and, in equilibrium, only 4.9% of emissions remain in the atmosphere, 11.3% transition to the upper and 83.8% to the lower oceans. Initial conditions for the carbon stocks in 2010 are: $M_{\mathrm{AT}}(0) = 830.4$ GtC, $M_{\mathrm{UP}}(0) = 1527$ GtC, and $M_{\mathrm{LO}}(0) = 10010$ GtC.

Nordhaus (2014) assumes a simple first-order approximation of radiative forcing increases, $F(t)$, due to deviations from the pre-industrial stock of atmospheric carbon and captures the evolution of other climate-active gases, such as CH4, N20, SO2 and SF6, and other forcing components, such as ozone and albedo, through the exogenous forcing component, $\mathrm{F_{EX}(t)}$:

(A2) $$F(t) = \eta_2 Ln\left[\frac{M_{AT}(t)}{M_{AT1750}}\right] / Ln[2] + F_{EX}(t)$$

with $\eta_2 = 3.8$ W/m² and $M_{\mathrm{AT1750}} = 588$ GtC.[32] Exogenous forcing, $F_{\mathrm{EX}}(t)$, linearly increases from 0.25 W/m² in 2010 to 0.7 W/m² in 2200 after which it is assumed constant. The exogenous forcing trajectory is based on the RCP 6.0 W/m²

[32] Radiative forcing equations are gas-specific. IPCC (2001, ch. 6) provides a general overview and lists the most important relationships in Table 6.2: http://www.grida.no/publications/other/ipcc_tar/?src=/climate/ipcc_tar/wg1/222.htm

representative scenario and amounts to at most 15% of total forcing. Nordhaus (2014) acknowledges the difficulty of accounting for the forcing components that are not directly related to CO2.

The oceanic diffusion process is approximated by introducing a second temperature box representing the deep ocean. As increasing radiative forcing heats the global atmospheric temperature, $T_{\mathrm{AT}}(t)$, some of this additional energy is taken up by the ocean, $T_{\mathrm{LO}}(t)$. This extra box brings the model closer to the geophysics and introduces additional inertia in the climate system. Energy moves between the two temperature boxes as to equilibrate the two. Radiative forcing increases atmospheric temperature linearly:

(A3) $$\begin{pmatrix} T_{AT}(t) \\ T_{LO}(t) \end{pmatrix} = \begin{pmatrix} T_{AT}(t-1) \\ T_{LO}(t-1) \end{pmatrix} - \xi_1 \left[\Delta(t-1) \begin{pmatrix} \xi_3 \\ \frac{-\xi_4}{\xi_1} \end{pmatrix} + \begin{pmatrix} F(t) - \xi_2 T_{AT}(t-1) \\ 0 \end{pmatrix} \right]$$

with $\Delta(t) = T_{\mathrm{AT}}(t) - T_{\mathrm{LO}}(t)$ the difference in atmospheric and oceanic temperature. The Equilibrium Climate Sensitivity in DICE-2013R is set to 2.9°C per equilibrium CO2 doubling, which gives $\xi_2 = \eta_2/2.9 = 1.31$. ξ_1 captures the transient climate response (TCR) and is set to 0.98 to produce a TCR of 1.7°C which is mid of the range of estimates given in IPCC (2013). Nordhaus (2014) cites regression analysis for the determination of temperature parameters, $\xi_3 = 0.088$ and $\xi_3 = 0.025$. The temperature cycle is not closed and in equilibrium $T_{\mathrm{AT}} = T_{\mathrm{LO}} = \mathrm{F}/\xi_2$.

Appendix B: Proofs

Proof of proposition 1:

From (1) we get the following expression for the social cost of carbon:

$$\frac{SCC_t}{\chi Y_t^{\varepsilon} Y_0^{1-\varepsilon}} = \left(\frac{\varphi_L}{1 - \dfrac{(1+g)^{\varepsilon}}{1+r}} + \frac{\varphi_0 (1-\varphi_L)}{1 - \dfrac{(1+g)^{\varepsilon}}{1+r}(1-\varphi)} \right) \left(\frac{(1+g)^{\varepsilon}}{(1+g)^{\varepsilon} + \varphi_T \left[1 + r - (1+g)^{\varepsilon} \right]} \right).$$

Per-capita consumption growth $(1+g)/(1+n) = \left[(1+r)/\beta\right]^{1/\Phi}$ follows from the Euler equation, so the interest rate is $1 + r = \left[(1+g)/(1+n)\right]^{\Phi} / \beta$ and we get

$$SCC_t = \left(\frac{\varphi_L}{1 - \beta(1+n)^{\varepsilon} \left(\dfrac{1+g}{1+n} \right)^{\varepsilon - \Phi}} + \frac{\varphi_0 (1-\varphi_L)}{1 - \beta(1+n)^{\varepsilon} \left(\dfrac{1+g}{1+n} \right)^{\varepsilon - \Phi} (1-\varphi)} \right) \times$$
$$\left(1 + \varphi_T \left[\left(\frac{1+g}{1+n} \right)^{\Phi - \varepsilon} \frac{(1+n)^{-\varepsilon}}{\beta} - 1 \right] \right)^{-1} \chi Y_t^{\varepsilon} Y_0^{1-\varepsilon},$$

For small φ, r, g and $\rho \equiv \beta - 1$ (ignoring second-order terms), the Euler equation becomes $g - n = (r - \rho)/\Phi$ and this expression for the SCC boils down to (2). □

Proof of proposition 2:

E_t affects all future damages at time $s \geq t$, so we need $\omega_t = \sum_{s=t}^{\infty} (1+r)^{-(s-t)} \dfrac{\partial D_s}{\partial E_t} =$ $-Y_t \sum_{s=t}^{\infty} \left(\dfrac{1+g}{1+r} \right)^{s-t} \dfrac{\partial A_s}{\partial E_t} \xi_s$. If $\bar{A}_t = (1+g)^t \bar{A}_0$, we get

$$A_s = \left(\prod_{t=0}^{s} e^{-\chi\varphi_\chi^{s-t}(E_t - E_0)} \left[(1+g)^t \bar{A}_0 \right]^{(1-\varphi_\chi)\varphi_\chi^{s-t}} \right) \bar{A}_0^{\varphi_\chi s}$$ so that

$-\dfrac{\partial A_s}{\partial E_t} = \chi \varphi_\chi^{s-t} A_s,\ s \ge t.$ Marginal damages of emitting one 1 tC at time *t* is

$$\omega_t = \chi Y_t \sum_{s=t}^{\infty} \left(\frac{1+g}{1+r} \varphi_\chi \right)^{s-t} \xi_s, \text{ so}$$

$$\frac{\omega_t}{\chi Y_t} = \varphi_L \left[\frac{1}{1 - \left(\frac{1+g}{1+r}\right)\varphi_\chi} - \frac{1}{1 - \left(\frac{1+g}{1+r}\right)\varphi_\chi \left(1 - \frac{1}{\varphi_T}\right)} \left(1 - \frac{1}{\varphi_T}\right)^t \right] +$$

$$\left(\frac{\varphi_0 (1-\varphi_L)}{1-\varphi_T \varphi} \right) \left[\frac{1}{1 - \left(\frac{1+g}{1+r}\right)\varphi_\chi (1-\varphi)} (1-\varphi)^t - \frac{1}{1 - \left(\frac{1+g}{1+r}\right)\varphi_\chi \left(1 - \frac{1}{\varphi_T}\right)} \left(1 - \frac{1}{\varphi_T}\right)^t \right].$$

Substituting this into $SSC_t \equiv \sum_{s=0}^{\infty} (1+r)^{-(s-t)} \omega_s$, we get

$$\frac{SSC_t}{\chi Y_t} = \frac{\varphi_L}{1 - \left(\frac{1+g}{1+r}\right)\varphi_\chi} \frac{1}{1 - \left(\frac{1+g}{1+r}\right)} - \frac{\varphi_L}{1 - \left(\frac{1+g}{1+r}\right)\varphi_\chi \left(1 - \frac{1}{\varphi_T}\right)} \frac{1}{1 - \left(\frac{1+g}{1+r}\right)\left(1 - \frac{1}{\varphi_T}\right)}$$

$$\frac{\varphi_0 (1-\varphi_L)}{1-\varphi_T \varphi} \left[\frac{1}{1 - \left(\frac{1+g}{1+r}\right)\varphi_\chi (1-\varphi)} \frac{1}{1 - \left(\frac{1+g}{1+r}\right)(1-\varphi)} - \frac{1}{1 - \left(\frac{1+g}{1+r}\right)\varphi_\chi \left(1 - \frac{1}{\varphi_T}\right)} \frac{1}{1 - \left(\frac{1+g}{1+r}\right)\left(1 - \frac{1}{\varphi_T}\right)} \right].$$

Upon substitution of the Euler equation $\dfrac{1+g}{1+r} = (1+g)^{1-\Phi} / (1+\rho) \cong 1 - \rho - (\Phi - 1)g,$ and ignoring other second-order terms, we get (3). □

Proof of proposition 3:

The Lagrangian for the problem of choosing $\{C_t, F_t, R_t, \forall t \ge 1\}$ to maximize social welfare (5) subject to equations (6)-(9) is:

$$
\begin{aligned}
L \equiv & \sum_{t=0}^{\infty} \beta^t \left[L_t \frac{(C_t / L_t)^{1-\Phi}}{1-\Phi} - \mu_t (S_{t+1} - S_t + F_t) \right] \\
& + \sum_{t=0}^{\infty} \beta^t \left[\nu_{1,t} \left(E_{1,t} - E_{1,t-1} - \varphi_L F_t \right) + \nu_{2,t} \left\{ E_{2,t} - (1-\varphi) E_{2,t-1} - \varphi_0 (1-\varphi_L) F_t \right\} \right] \\
& + \sum_{t=0}^{\infty} \beta^t \left[\nu_t \left(E_t - \frac{1}{\varphi_T} (E_{1,t} + E_{2,t}) - \left(1 - \frac{1}{\varphi_T} \right) E_{t-1} \right) \right] \\
& - \sum_{t=0}^{\infty} \beta^t \lambda_t \left[K_{t+1} - (1-\delta) K_t - Z(K_t, L_t, F_t + R_t) + D(E_t) Z(K_t, L_t, F_t + R_t)^{\varepsilon} Z_0^{1-\varepsilon} + G(S_t) F_t + b_t R_t + C_t \right],
\end{aligned}
$$

where μ_t denotes the shadow value of in-situ fossil fuel, ν_{1t}, ν_{2t} and ν_t the shadow disvalues of the permanent, transient and total delayed stocks of atmospheric carbon, and λ_t the shadow value of manmade capital. Consumption, fossil fuel use and renewable use in period zero, C_0, F_0 and R_0, the capital stock at the start of period zero, K_0, the stock of fossil fuel reserves at the start of period zero, S_0, and the carbon stocks at the end of period zero, E_{10}, E_{20}, and E_0, are given, and thus not part of the optimization program. It follows that gross output, Z_0, fossil fuel extraction costs, $G(S_0)$, and GDP in period zero are given too. We use the same timing conventions as Golosov et al. (2014).

Necessary optimality conditions for $\{C_t, F_t, R_t, \forall t \geq 1\}$ are thus given by:

(B.1) $\quad U'(C_t / L_t) = (C_t / L_t)^{-\Phi} = \lambda_t, \quad \forall t \geq 1,$

(B.2a) $\quad \Gamma_t Z_{F_t + R_t}(K_t, L_t, F_t + R_t) \leq G(S_t) + \left[\mu_t + \varphi_L \nu_{1t} + \varphi_0 (1-\varphi_L) \nu_{2t} \right] / \lambda_t,$
$\quad F_t \geq 0, \quad \text{c.s.}, \quad \forall t \geq 1,$

(B.2b) $\quad \Gamma_t Z_{F_t + R_t}(K_t, L_t, F_t + R_t) \leq b_t, \quad R_t \geq 0, \quad \text{c.s.}, \quad \forall t \geq 1.$

The co-state equations for this optimization problem are:

(B.3) $\quad \beta \left\{ 1 - \delta + \Gamma_{t+1} Z_{K_{t+1}}(K_{t+1}, L_{t+1}, F_{t+1} + R_{t+1}) \right\} \lambda_{t+1} = \lambda_t, \quad \forall t \geq 1,$

(B.4) $\quad \mu_{t+1} = \beta^{-1} \mu_t + G'(S_{t+1}) F_{t+1} \lambda_{t+1}, \quad \forall t \geq 1,$

(B.5) $\quad \nu_{1t} = \beta \nu_{1t+1} + \dfrac{1}{\varphi_T} \nu_t, \quad \forall t \geq 1,$

(B.6) $\quad \nu_{2t} = \beta(1-\varphi)\nu_{2t+1} + \frac{1}{\varphi_T}\nu_t, \quad \forall t \geq 1,$

(B.7) $\quad \nu_t = \beta\left(1-\frac{1}{\varphi_T}\right)\nu_{t+1} + D'(E_t)Z_t(K_t, L_t, F_t + R_t)^{\varepsilon} Z_0^{1-\varepsilon}\lambda_t \quad \forall t \geq 1.$

Equations (B.1) and (B.3) give $U'(C_t / L_t) = \beta(1+r_{t+1})U'(C_{t+1} / L_{t+1})$ with $r_{t+1} \equiv \Gamma_{t+1}Z_{K_{t+1}} - \delta$ or $\frac{C_{t+1} / L_{t+1}}{C_t / L_t} = \left[\beta(1+r_{t+1})\right]^{1/\Phi}$ which is (10).

The Kuhn-Tucker conditions (B.2a) and (B.2b) give (11a) and (11b) after defining $s_t \equiv \mu_t / \lambda_t$ and $\theta_t \equiv \left[\varphi_L \nu_{1t} + \varphi_0(1-\varphi_L)\nu_{2t}\right] / \lambda_t$ in final good units.

Equation (B.4) gives $\left[s_{t+1} - G'(S_{t+1})F_{t+1}\right]\frac{\lambda_{t+1}}{\lambda_t} = \beta^{-1}s_t, \forall t \geq 1.$ Upon substitution of (B.3), we get $(1+r_{t+1})s_t = s_{t+1} - G'(S_{t+1})F_{t+1}, \forall t \geq 1.$ Solving this difference equation forwards, we get $s_t = -\sum_{\zeta=1}^{\infty}(1+r_{t+1})\Delta_{t+1+\zeta}G'(S_{t+\zeta})F_{t+\zeta}$ or (12).

Finally, dividing (B.5)-(B.7) by λ_t and using (B.3) yields $\theta_{1t+1} = (1+r_{t+1})\left(\theta_{1t} - \frac{\varphi_L}{\varphi_T}\theta_{Tt}\right), \qquad (1-\varphi)\theta_{2t+1} = (1+r_{t+1})\left[\theta_{2t} - \frac{\varphi_0(1-\varphi_L)}{\varphi_T}\theta_{Tt}\right]$ and $\left(1-\frac{1}{\varphi_T}\right)\theta_{Tt+1} = (1+r_{t+1})\left[\theta_{Tt} - D'(E_t)Z_t^{\varepsilon}Z_0^{1-\varepsilon}\right].$ Solving the first two difference equations forwards and using $\Delta_{t+\varsigma} \equiv \prod_{\varsigma'=1}^{\varsigma}(1+r_{t+\varsigma'})^{-1}, \varsigma \geq 1$ and $\Delta_t = 1,$ we get

$\theta_t \equiv \theta_{1t} + \theta_{2t} = \sum_{\varsigma=0}^{\infty}\left[\left\{\varphi_L + \varphi_0(1-\varphi_L)(1-\varphi)^{\varsigma}\right\}\Delta_{t+\varsigma}\theta_{Tt+\varsigma}\right] / \varphi_T.$ The third difference equation gives $\theta_{Tt} = \sum_{\varsigma=0}^{\infty}\left[\left(1-\frac{1}{\varphi_T}\right)^{\varsigma}\Delta_{t+\varsigma}D'(E_{t+\varsigma})Z_{t+\varsigma}{}^{\varepsilon}Z_0^{1-\varepsilon}\right] / \varphi_T.$ □

Appendix C: Functional forms and calibration

We have a CES utility function and set IIA = Φ = 2 and β = 0.99. The initial capital stock is set to 150 (US$ trillion) and set δ = 0.1 per year. Output before damages is

$$Z_t = A_t \left[(1-\beta)\left(A_0 K_t^{\alpha} (L_t)^{1-\alpha} \right)^{1-1/\vartheta} + \beta\left(\frac{F_t + R_t}{\sigma}\right)^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}}, \qquad \vartheta \geq 0,\ 0 < \alpha < 1 \qquad \text{and}$$

$0 < \beta < 1.$ This is a constant-returns-to scale CES production function in energy and a capital-labour composite with θ the elasticity of substitution and β the share the parameter for energy. The capital-labor composite is defined by a constant-returns-to-scale Cobb-Douglas function with α the share of capital and A_t total factor productivity. The two types of energy are perfect substitutes in production. Damages are calibrated so that they give the same level of global warming damages for the initial levels of output and mean temperature. It is convenient to rewrite production before damages as $Z_t = Z_0 A_t \left[(1-\beta)\left(\frac{A_0 K_t^{\alpha} L_t^{1-\alpha}}{Z_0} \right)^{1-1/\vartheta} + \beta\left(\frac{F_t + R_t}{\sigma Z_0} \right)^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}}$. We set the share of capital to α = 0.35 and the energy share parameter to β = 0.06. Golosov et al. (2014) use a Cobb-Douglas aggregate production function and thus have ϑ = 1. Nordhaus (2014) assume a Leontief production function and thus ϑ = 0. Hassler et al. (2012) give empirical evidence that suggests that ϑ > 0 but small. Hence, we have decided to simply use ϑ = 0.5. Initial world GDP in 2010 is $63 trillion. Given $A_1 = 1$, we calibrate A = 3.78 to yield initial output under BAU. The energy intensity of output σ is calibrated to an initial energy use of 9 GtC under BAU, $\sigma = 0.15$ GtC/$T.

Population growth and technical progress

Population in 2010 (L_1) is 7 billion people. In the baseline scenario we follow Nordhaus (2014) and UN projections where population growth is given by $L_t = 11 - 4e^{-7\,0.01/2.1t}$. Population growth starts at around 1% per year and falls below 0.1% percent within six decades and flattens out at 11 billion people. In a sensitivity scenario we lower the upper limit to 9 billion. The exogenous trend in total factor productivity growth is 2% per year. In a sensitivity scenario we lower this trend growth rate to 1% per year.

Cost of energy

We employ an extraction technology of the form $G(S) = \gamma_1 (S_0 / S)^{\gamma_2}$, where γ_1 and γ_2 are positive constants. This specification implies that reserves will not be fully be extracted; some fossil fuel remains untapped in the crust of the earth. Extraction costs are calibrated to give an initial share of energy in GDP between 5%-6% depending on the policy scenario. This translates to fossil production costs of $300/tC ($30/barrel of oil), where we take one barrel of oil to be equivalent to 1/10 ton of carbon, giving $G(S_0) = \gamma_1 = 0.3$. The IEA (2008) long-term cost curve for oil extraction gives a doubling to quadrupling of the extraction cost of oil if another 1000 GtC are extracted. Since we are considering all carbon-based energy sources (not only oil) which are more abundant and cheaper to extract, we assume only a doubling of production costs if a total 2000 GtC is extracted. With $S_0 = 4000$ GtC,[33] this gives $\gamma_2 = 1$.[34] This implies that we assume very low extraction costs and a high initial stock of reserves which biases our findings toward using more fossil fuel longer.

The unit cost of renewable energy is calibrated to the percentage of GDP necessary to generate all energy demand from renewables. It customary to calibrate the cost of mitigation in IAMs as a share of output, e.g. Nordhaus (2008) assumes that it costs 5.6% of GDP to achieve full decarbonisation today with this share falling to 0.9% in 300 years and below 0.4% in 600 years. We want to study the energy market in more detail and explicitly model the production costs of fossil and renewable energy. To make our IAM comparable to the wider literature, we calibrate to the extreme but customary case of Leontief technology ($\vartheta \to 0$) in which energy demand equals σZ_t with $Z_t = AK_t^{\alpha}(A_t^L L_t)^{1-\alpha}$ pre-damage capital-labour aggregate and σ the carbon intensity of output. Under Leontief technology, the cost of generating all energy carbon-free as a percentage of GDP reduces to $\sigma Z_t b_t / Z_t = \sigma b_t$. We are more

[33] Stocks of carbon-based energy sources are notoriously hard to estimate. IPCC (2007) assumes in its A2- scenario that 7000 $GtCO_2$ (with 3.66 tCO2 per tC this equals 1912 GtC) will be burnt with a rising trend this century alone. We roughly double this number to get our estimate of 4000 GtC for initial fossil fuel reserves. Nordhaus (2008) assumes an upper limit for carbon-based fuel of 6000 GtC in the DICE-07.

[34] Since $G(2000)/G(4000) = (4000/2000)^{\gamma_2} = 2^{\gamma_2}$ and $2^{\gamma_2} = 2$.

conservative than Nordhaus and assume that it costs 6% to achieve this. Adding the existing cost of energy, we take $\sigma b_1 = 0.12$ (i.e. we assume 12% of GDP would go to the energy sector if all carbon emissions where mitigated today) or, with $\sigma = 0.15$, $b_1 = 0.8$. In the future this cost falls to current prices of fossil energy (with energy amounting to about 6% of GDP), that is b_t approaches 0.4). We assume that exogenous technical progress lowers the unit cost at a falling rate starting at a reduction of 1% per year. Specifically, $b_t = 0.4 + 0.4e^{-0.8\ 0.01/0.4\ t}$. This calibration assumes that renewable energy is initially very expensive and falls to current levels only in the very long run. Together with the assumption about fossil energy, this biases the model against rapid de-carbonization. The calibration is done for a Leontief technology to make our calibration comparable to that of other IAMs. We assume that for a more general technology the same parameter values can be applied.

Damages

Nordhaus (2008) has combined detailed micro estimates of the costs of global warming to get aggregate macro costs of global warming of 1.7% of world GDP when global warming is 2.5^o C. This figure is used to calibrate the fraction of production output that is left after damages from global warming: $\tilde{Z}(T_t) = \dfrac{1}{1 + \zeta_1 T_t^{\zeta_2} + \zeta_3 T_t^{\zeta_4}}$ with ζ_1 = 0.00284, ζ_2 = 2, and ζ_3= ζ_4 = 0. This is our baseline damage calibration. Weitzman (2010) and Dietz and Stern (2014) argue that damages rise more rapidly at higher levels of temperature than suggested by (5), but empirical studies on the costs of global warming at higher temperatures are not available. With the heroic assumptions that damages are 50% of world GDP at 6^o C and 99% at 12.5^o C, Ackerman and Stanton (2012) recalibrate (5) with ζ_1 = 0.00245, ζ_2 = 2, ζ_3 = 5.021 x 10^{-6}, and ζ_4 = 6.76. The extra term in the denominator captures potentially catastrophic losses at high temperatures. This revised calibration, which we utilize in our of sensitivity runs in section 7, is arguably more appropriate for higher temperatures and but matches baseline damages very closely with deviations of less than 0.5%-point up to 3°C of warming.

Appendix D: Computational implementation (for online publication only)

In our simulations we solve the model for finite time and rely on the turnpike property to approximate the infinite-horizon problem. All equilibrium paths approach the steady growth path quickly such that the turnpike property renders terminal conditions essentially unimportant. We allow for continuation stocks to reduce the impact of the terminal condition on the transitions paths in the early periods of the program. We use the computer program GAMS and its optimization solver CONOPT3 to solve the model numerically. The social planner solution in which the externality is taken into account fit the program structure readily. To solve the "laissez-faire" equilibrium paths, we introduce an additional flow variable, F_t^{Exog}. We set F_t^{Exog} exogenously and only F_t^{Exog} but not F_t enters the atmosphere. F_t has no deleterious effects on the climate, only positive ones on production in this scenario. That is, we render the carbon cycle equation to read

$$\text{(A.1}'\text{)} \quad \begin{pmatrix} M_{AT}(t) \\ M_{UP}(t) \\ M_{LO}(t) \end{pmatrix} = F(t)^{Exog} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \Phi \begin{pmatrix} M_{AT}(t-1) \\ M_{UP}(t-1) \\ M_{LO}(t-1) \end{pmatrix}$$

Using an iterative approach similar to the one discussed in detail in Rezai (2011). Starting with an initial guess ${}^{0}F_t^{Exog}$, where the left superscript denotes the index of iteration, the optimal program is solved and the optimal fossil fuel use, ${}^{0}F_t$, is used as next iterations exogenous emissions trajectory, ${}^{1}F_t^{Exog}$. This iterative routine continues until the exogenous emissions and fossil fuel use trajectories converge, i.e. the equilibrium condition ${}^{i}F_t^{Exog} = {}^{i}F_t$ holds. In our simulations, the iteration is repeated 10 times after which the condition is always satisfied at the precision the solver applies to all equilibrium conditions.